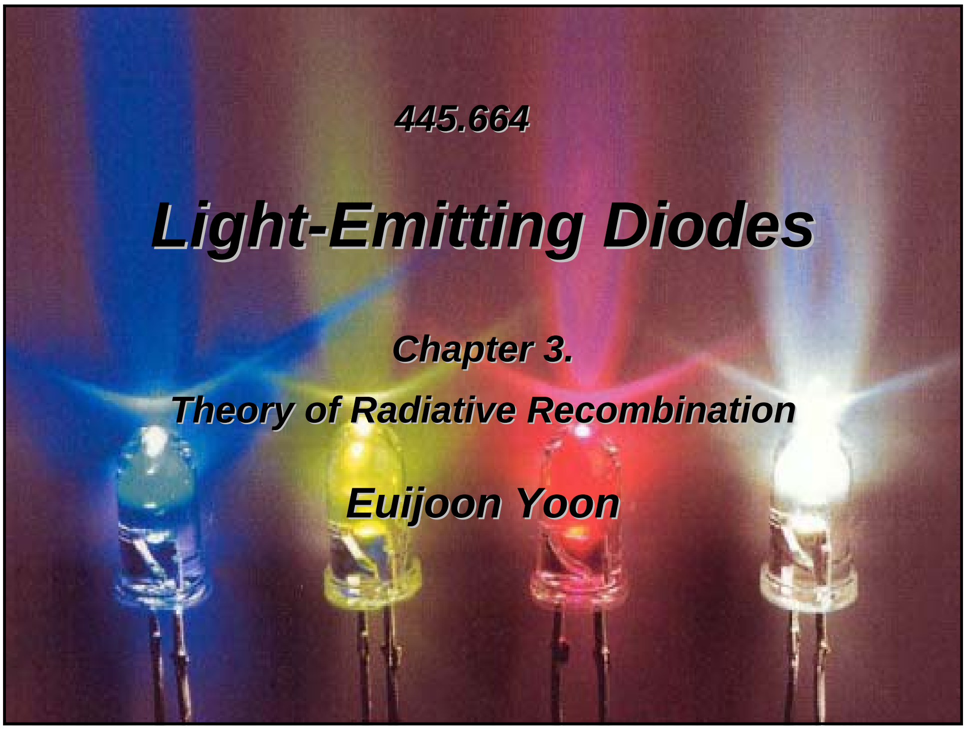

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Light-Emitting Diodes

Chapter 3.
Theory of Radiative Recombination

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Theory of Radiative Recombination

Semiclassical model of radiative recombination based on equilibrium generation and recombination

- **van Roosbroeck-Shockley model**

  ➔ Calculation of the spontaneous radiative recombination rate under equilibrium and non-equilibrium conditions

- **Einstein model**

  ➔ Calculation of the spontaneous and stimulated transitions in a two-level atom

The van Roosbroeck-Shockley Model

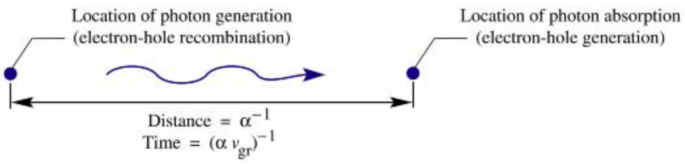


Fig. 3.1. Illustration of distance and elapsed time between a photon generation and absorption event.

$\alpha(\nu)$	**absorption coefficient [cm^{-1}]**
$\alpha(\nu)^{-1}$	**mean distance that a photon travels before being absorbed**
υ_{gr}	**group velocity of photons propagating in the semiconductor**
$\tau(\nu)$	**time that it takes for a photon to be absorbed = $(\alpha\, \upsilon_{gr})^{-1}$**
ν	**frequency of a generated photon**

Photon Absorption Probability

group velocity of photons

$$\upsilon_{gr} = \frac{\mathrm{d}\omega}{\mathrm{d}k} = \frac{\mathrm{d}\nu}{\mathrm{d}(1/\lambda)} = c\,\frac{\mathrm{d}\nu}{\mathrm{d}(\bar{n}\,\nu)}$$

$\omega \ (= 2\pi\nu)$	**angular frequency**
$k \left(= \dfrac{2\pi}{\lambda}\right)$	**wave vector**
$\bar{n} \left(= \dfrac{c}{\lambda\nu}\right)$	**refractive index**

inverse photon lifetime

$$\frac{1}{\tau(\nu)} = \alpha(\nu)\,\upsilon_{gr} = \alpha(\nu)\,c\,\frac{\mathrm{d}\nu}{\mathrm{d}(\bar{n}\,\nu)}$$

➔ ***photon absorption probability per unit time***

Density of Photons

$$\bar{n} = \frac{c}{\lambda \nu} \quad \Rightarrow \quad \lambda = \frac{c}{\bar{n}\,\nu}$$

$$\mathrm{d}\lambda = -\frac{c}{(\bar{n}\,\nu)^2}\,\frac{\mathrm{d}(\bar{n}\,\nu)}{\mathrm{d}\nu}\,\mathrm{d}\nu$$

Density of photons per unit volume (equilibrium conditions)
by Planck's black body radiation formula

$$N(\lambda)\,\mathrm{d}\lambda = \frac{8\pi}{\lambda^4}\,\frac{1}{e^{h\nu/kT}-1}\,\mathrm{d}\lambda$$

$$N(\nu)\,\mathrm{d}\nu = \frac{8\pi\,\nu^2\,\bar{n}^2}{c^3}\,\frac{\mathrm{d}(\bar{n}\,\nu)}{\mathrm{d}\nu}\,\frac{1}{e^{h\nu/kT}-1}\,\mathrm{d}\nu$$

The van Roosbroeck-Shockley Equation

Phonon absorption rate per unit volume
= Absorption probability × Photon density

Absorption rate per unit volume in the frequency interval ν and $\nu + d\nu$

$$\begin{aligned}
R_0(\nu)\,\mathrm{d}\nu &= \frac{N(\nu)\,\mathrm{d}\nu}{\tau(\nu)} \\
&= \left(\frac{8\pi\,\nu^2\,\bar{n}^2}{c^3}\,\frac{\mathrm{d}(\bar{n}\,\nu)}{\mathrm{d}\nu}\,\frac{1}{e^{h\nu/kT}-1}\,\mathrm{d}\nu\right)\cdot\left(\alpha(\nu)\,c\,\frac{\mathrm{d}\nu}{\mathrm{d}(\bar{n}\,\nu)}\right) \\
&= \frac{8\pi\,\nu^2\,\bar{n}^2}{c^2}\,\frac{\alpha(\nu)}{e^{h\nu/kT}-1}\,\mathrm{d}\nu
\end{aligned}$$

Absorption rate per unit volume

$$R_0 = \int_0^\infty R_0(\nu)\,\mathrm{d}\nu = \int_0^\infty \frac{8\pi\,\nu^2\,\bar{n}^2}{c^2}\,\frac{\alpha(\nu)}{e^{h\nu/kT}-1}\,\mathrm{d}\nu$$

van Roosbroeck-Shockley Equation

The Simplified van Roosbroeck-Shockley Model

Absorption coefficient

$$\alpha = \alpha_0 \sqrt{\frac{E - E_g}{E_g}}$$

E_g band gap energy of the semiconductor

α_0 absorption coefficient at $h\nu = 2E_g$

Neglecting the frequency dependence of the refractive index

Using the absorption coefficient at the band edge ($\alpha = \alpha_0$)

$$x = h\nu / kT = E / kT$$

$$x_g = E_g / kT$$

$$\mathrm{d}x = (h/kT)\,\mathrm{d}\nu$$

$$R_0 = 8\pi c\, \bar{n}^2 \alpha \sqrt{\frac{kT}{E_g}} \left(\frac{kT}{c\,h}\right)^3 \int_{x_g}^{\infty} \frac{x^2\sqrt{x - x_g}}{e^x - 1}\,\mathrm{d}x$$

The simplified van Roosbroeck-Shockley Equation

Bimolecular Recombination Rate

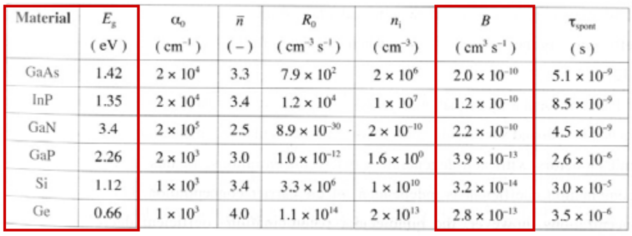
Under equilibrium conditions,

$$\begin{array}{ccc} \text{photon absorption rate } (R_0) & = & \text{photon emission rate} \\ \| & & \| \\ \text{carrier generation rate} & = & \text{carrier recombination rate } (R) \end{array}$$

Equilibrium carrier recombination rate

$$R = B\,n\,p = B\,n_i^2 = R_0$$

B bimolecular coefficient



Material	E_g (eV)	α_0 (cm^{-1})	$\bar{n}$ (–)	R_0 (cm^{-3} s^{-1})	n_i (cm^{-3})	B (cm^3 s^{-1})	τ_{spont} (s)
GaAs	1.42	2×10^4	3.3	7.9×10^2	2×10^6	2.0×10^{-10}	5.1×10^{-9}
InP	1.35	2×10^4	3.4	1.2×10^4	1×10^7	1.2×10^{-10}	8.5×10^{-9}
GaN	3.4	2×10^5	2.5	8.9×10^{-30}	2×10^{-10}	2.2×10^{-10}	4.5×10^{-9}
GaP	2.26	2×10^3	3.0	1.0×10^{-12}	1.6×10^0	3.9×10^{-13}	2.6×10^{-6}
Si	1.12	1×10^3	3.4	3.3×10^6	1×10^{10}	3.2×10^{-14}	3.0×10^{-5}
Ge	0.66	1×10^3	4.0	1.1×10^{14}	2×10^{13}	2.8×10^{-13}	3.5×10^{-6}

Table 3.1. Bimolecular recombination coefficient at 300 K for different semiconductors as calculated from the energy gap, absorption coefficient, and refractive index at the bandgap energy. The spontaneous lifetime is given by $B^{-1} N_{D,A}^{-1}$ and it is calculated for a majority carrier concentration of 10^{18} cm^{-3}.

Direct band gap
$B = 10^{-9} \sim 10^{-11}$ cm^3/s

Indirect band gap
$B = 10^{-13} \sim 10^{-15}$ cm^3/s

Einstein Model

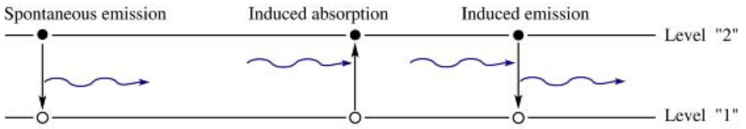


Fig. 3.2. Spontaneous emission, induced absorption, and induced emission events in the two-level atom model.

A **Spontaneous transition rate**
B **Induced (stimulated) transition rate ∝ photon density, $\rho(\nu)$**

Probability per unit time

downward transition (2 → 1) $$W_{2\to1} = \overset{\text{Induced emission}}{B_{2\to1}}\,\rho(\nu) + \overset{\text{spontaneous emission}}{A}$$

upward transition (1 → 2) $$W_{1\to2} = \underset{\text{Induced absorption}}{B_{1\to2}}\,\rho(\nu)$$

Einstein showed that

1. $B = B_{2\to1} = B_{1\to2}$ ➔ **Stimulated absorption and stimulated emission are complementary processes.**
2. $A/B = 8\pi\, \overline{n}^3\, h\, \nu^3 / c^3$ = **constant in an isotropic medium**