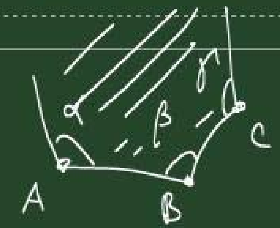
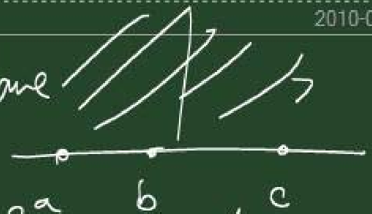
Schwarz - Christoffel transf.

노트 제목 2010-04-20

z plane

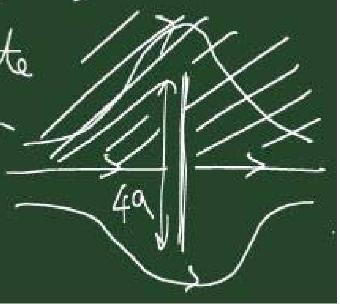


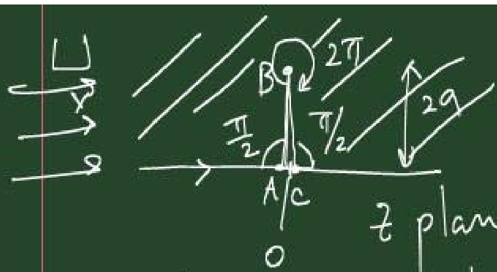
ζ plane



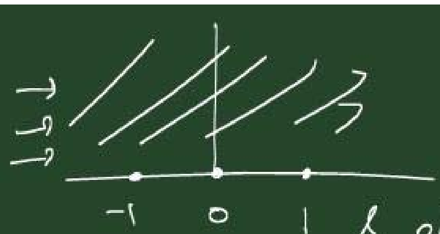
$$\frac{dz}{d\zeta} = K(\zeta-a)^{\frac{\alpha}{\pi}-1}(\zeta-b)^{\frac{\beta}{\pi}-1}(\zeta-c)^{\frac{\gamma}{\pi}-1} \cdots\cdots$$

- Flow around a vertical flat plate assuming non-separated flow





z plane



ζ plane

$$\frac{dz}{d\zeta} = K(\zeta+1)^{-\frac{1}{2}}(\zeta-0)^{1}(\zeta-1)^{-\frac{1}{2}} = K\frac{\zeta}{\sqrt{\zeta^2-1}}$$

$$\rightarrow z = K\sqrt{\zeta^2-1} + D$$

Condition; $z=0$ @ $\zeta=\pm 1 \rightarrow D=0$

$z = i2a$ @ $\zeta = 0 \rightarrow K = 2a$

$$\rightarrow z = 2a\sqrt{\zeta^2-1} \quad \left(\text{or } \zeta = \pm\frac{1}{2a}\sqrt{z^2+4a^2}\right)$$

Uniform flow in the ζ plane

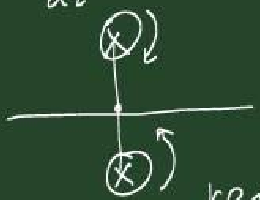
$$W(z) = \frac{d\zeta}{dz} W(\zeta) \to W(\zeta) = \frac{dz}{d\zeta} W(z)$$

as $\zeta \to \infty$, $W(\zeta) = 2a \underset{Ui}{\underline{\underline{W(z)}}}$

$$\therefore F(\zeta) = 2aU\zeta$$

$$F(z) = 2aU\left(\pm \frac{1}{2a}\sqrt{z^2+4a^2}\right) = \pm U\sqrt{z^2+4a^2}$$

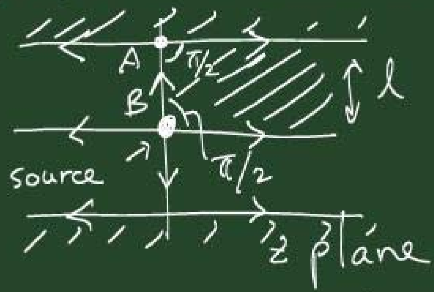
$\frac{dF}{dz} \Rightarrow$ singularities at $y = \pm 2a$ ($z = \pm 2ai$)

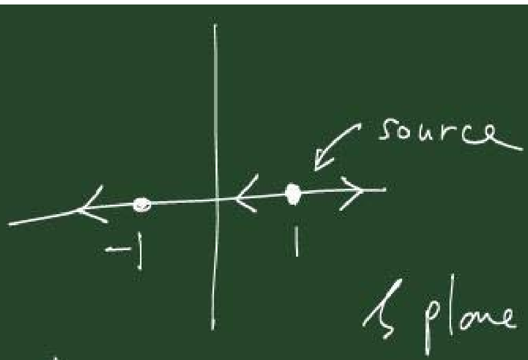


Kutta condition cannot handle this problem.

real flow separates at these points.

• Source in a channel





$$\frac{dz}{d\zeta} = K(\zeta+1)^{-\frac{1}{2}}(\zeta-1)^{-\frac{1}{2}} = \frac{K}{\sqrt{\zeta^2-1}}$$

$\longrightarrow z = K\cosh^{-1}\zeta + D$

cond. $z=0$ @ $\zeta=1 \to D=0$

$z=il$ @ $\zeta=-1 \to K = l/\pi$

$$\therefore\ z = \frac{\ell}{\pi}\cosh^{-1}\zeta \quad \left(\text{or } \zeta = \cosh\frac{\pi z}{\ell}\right)$$

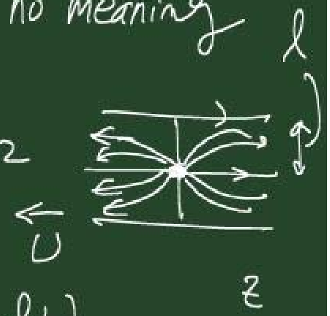
$$F(\zeta) = \frac{m}{2\pi}\ln(\zeta - 1)$$

$$\rightarrow F(z) = \frac{m}{2\pi}\ln\left(\cosh\frac{\pi z}{\ell} - 1\right)$$

$$\vdots$$

$$= \frac{m}{\pi}\ln\left(\sinh\frac{\pi z}{2\ell}\right) + \frac{m}{2\pi}\ln 2$$

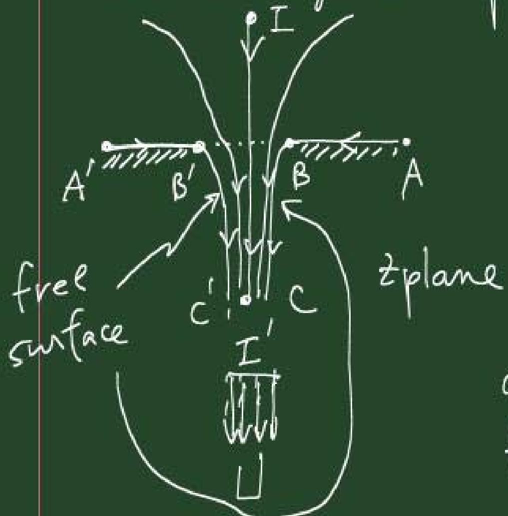
no meaning



$$\rightarrow F(z) = \frac{m}{\pi}\ln\left(\sinh\frac{\pi z}{2\ell}\right)$$

$$= \frac{4\ell U}{\pi}\ln\left(\sinh\frac{\pi z}{2\ell}\right) \qquad m = 4\ell U$$

- Flow through an aperture



uncertainty in the location of free surface

→ cure : hodograph plane

uses the fact that along such free streamlines, the press. is constant.

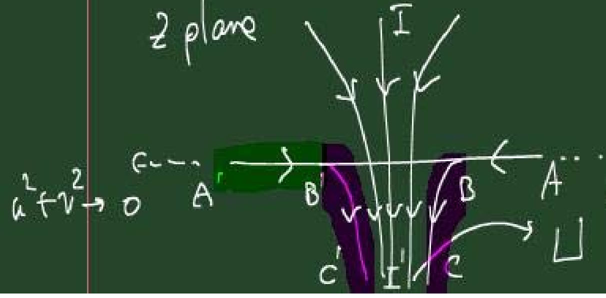
$$\zeta \doteq U\frac{dz}{dF} = \frac{U}{W} = \frac{U}{u-iv} = \frac{U}{\sqrt{u^2+v^2}\,e^{-i\theta}} = \frac{U}{\sqrt{u^2+v^2}}e^{i\theta}$$

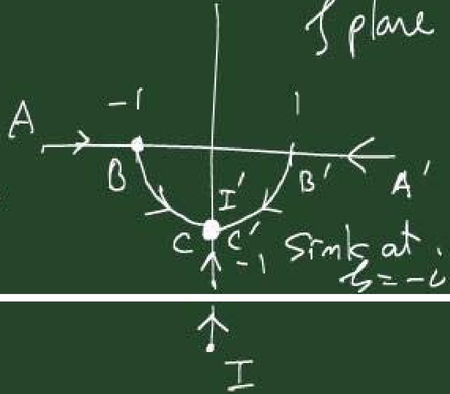
$$= re^{i\theta}$$; θ comes from the velocity.

<u>on free surface</u>, press. is const.

$\rightarrow u^2+v^2$ is const $(=U^2)$

$\therefore\ \zeta = e^{i\theta}$ on free surface





on AB : $\theta = \pi$

on A'B' : $\theta = 0$ (or 2π)

on B'C' : $-\frac{\pi}{2} < \theta < 0$, $r=1$

on BC : $\pi < \theta < \frac{3}{2}\pi$, $r=1$

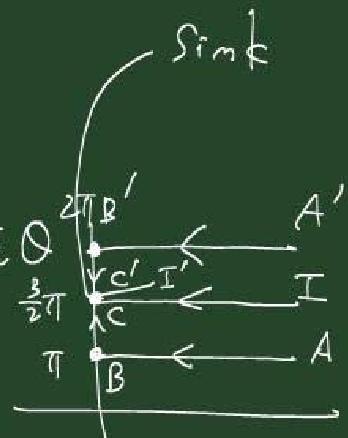
on II' : $\theta = \frac{3}{2}\pi$, $r \to \infty$ (I)

$r \to 1$ (I')

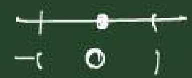
$$\zeta' = \ln\zeta = \ln(re^{i\theta}) = \ln r + i\theta$$

sink in a channel

ζ'



$$\zeta'' = \cosh(\zeta' - i\pi) = -\cosh\zeta'$$



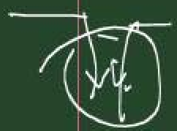
$$F(\zeta'') = -\frac{m}{2\pi}\ln\zeta'' + K$$

$\vdots$

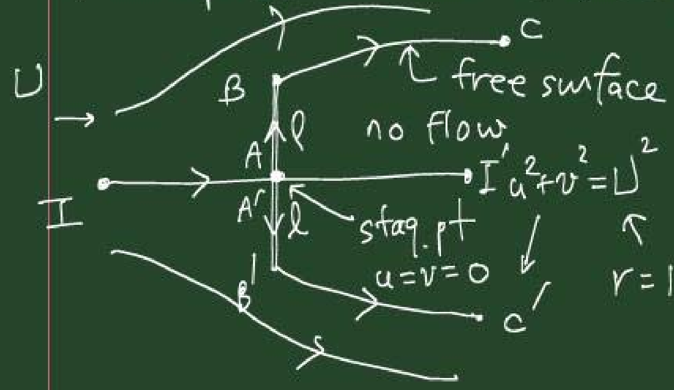
$$F(z) = -\frac{2C_c l U}{\pi}\ln\left\{\cosh\left[\ln\left(U\frac{dz}{dF}\right) - i\pi\right]\right\} + iC_c l U$$

C_c (contraction coeff) $= \dfrac{\pi}{\pi+2}$

agrees well with exp.

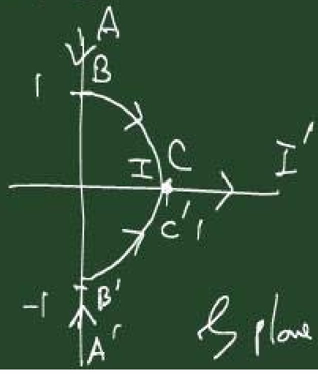


- Flow past a vertical flat plate



→ hodograph plane

$$\zeta = U\frac{dz}{dF} = \frac{U}{\sqrt{u^2+v^2}}e^{i\theta}$$
$$= re^{i\theta}$$



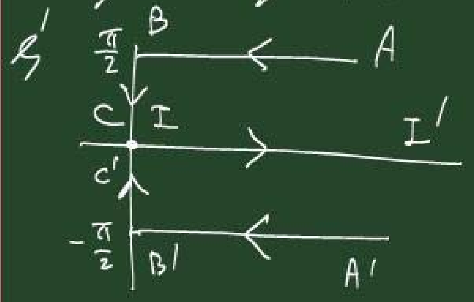
on AB : $\theta = \frac{\pi}{2}$ at A, $r \to \infty$

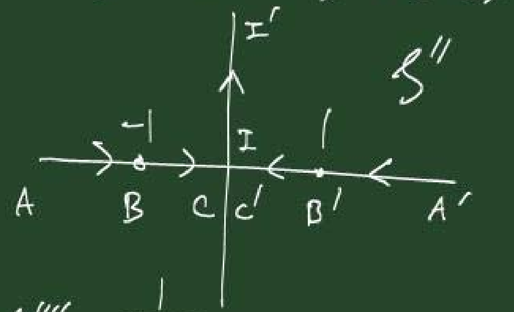
on BC : $0 < \theta < \frac{\pi}{2}$, $r = 1$

at I : $u = U$, $v = 0 \to r = 1$, $\theta = 0$

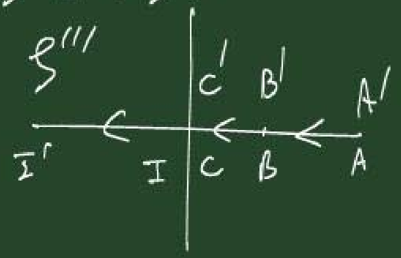
at I' : $u = v = 0 \to r \to \infty$, $\theta = 0$

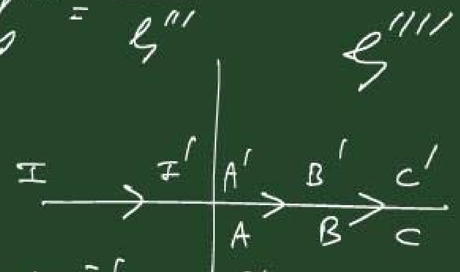
$$\zeta' = \ln\zeta = \ln r + i\theta \rightarrow \zeta'' = \cosh\left(\zeta' + i\frac{\pi}{2}\right)$$





$$\zeta''' = (\zeta'')^2 \qquad \zeta'''' = \frac{1}{\zeta'''}$$



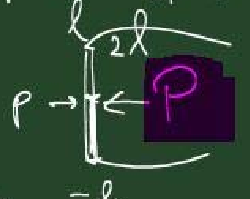


uniform flow

$$F(\zeta'''') = \underline{\underline{k}}\,\zeta'''' \cdots \rightarrow k = \frac{2Ul}{\pi+4}$$

$$F(z) = -\frac{2Ul}{\pi+4}\,\frac{1}{\sinh^2\left\{\log\left[U\frac{dz}{dF}\right]\right\}}$$

drag force



$$\frac{u^2+v^2}{2} + \frac{p}{\rho} = \frac{U^2}{2} + \frac{P}{\rho}$$

$$X = 2\int_{-l}^{0} (p-P)\,dy$$

$\therefore p < P$

$$= \cdots = \frac{2\pi}{\pi+4}\rho U^2 l$$

cf. $C_D \simeq 2.0$ in real flow

drag coeff. $C_X = \dfrac{X}{\frac{1}{2}\rho U^2 \cdot 2l} = \dfrac{2\pi}{\pi+4} \simeq 0.88$