"No Class on Thursday (March 18)."

$R(s) = \langle u(t)\, u(t+s) \rangle$

노트 제목 2010-03-16

- Frequency spectrum $E(\omega)$

$$\left(\text{Fourier transform} \quad \begin{aligned} f(t) &= \int_{-\infty}^{\infty} g(\omega) e^{i\omega t}\, d\omega \\ g(\omega) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t) e^{-i\omega t}\, dt \end{aligned} \right)$$

$$\rightarrow E(\omega) \equiv \frac{1}{\pi}\int_{-\infty}^{\infty} R(s) e^{-i\omega s}\, ds = \frac{2}{\pi}\int_{0}^{\infty} R(s)\cos(\omega s)\, ds$$

$$\rightarrow R(s) = \frac{1}{2}\int_{-\infty}^{\infty} E(\omega) e^{i\omega s}\, d\omega = \int_{0}^{\infty} E(\omega)\cos(\omega s)\, d\omega$$

$$\left(\begin{aligned} & u(t) \xrightarrow{FT} \hat{u}(\omega) \qquad \hat{u}(\omega)\hat{u}^{*}(\omega) = E(\omega) \\ & R(s) = \langle u(t) u(t+s) \rangle \end{aligned} \right)$$

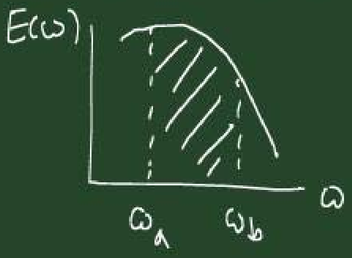
$$\left.\begin{aligned} u(t) &= \int_{-\infty}^{\infty} \hat{u}(\omega) e^{i\omega t}\, d\omega \\ u(t+s) &= \int \quad '' \quad e^{i\omega(t+s)}\, d\omega \end{aligned}\right\} \; \langle u(t)\, u(t+s) \rangle^{t} = \cdots\cdots$$

orthogonality

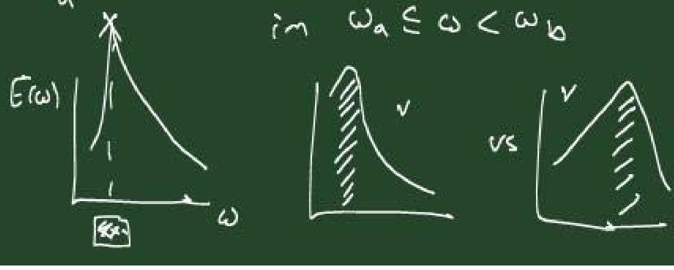
$$R(s) = \frac{1}{2}\int_{-\infty}^{\infty} E(\omega) e^{i\omega s}\, d\omega$$

$$R(s=0) = R(0) = \int_{0}^{\infty} E(\omega)\, d\omega \quad \Rightarrow \quad \langle u^2 \rangle = \int_{0}^{\infty} E(\omega)\, d\omega$$

$$R(0) = \langle u(t) u(t) \rangle = \langle u^2 \rangle$$



$\int_{\omega_a}^{\omega_b} E(\omega)\, d\omega$: contribution to $\langle u^2 \rangle$ in $\omega_a \le \omega < \omega_b$



$$E(0) = \frac{2}{\pi}\int_0^\infty R(s)\,ds$$

$$\bar{\tau} = \int_0^\infty \rho(s)\,ds = \int_0^\infty \frac{R(s)}{\langle u^2\rangle}\,ds = \frac{\pi E(0)}{2\langle u^2\rangle}$$

③ Random fields

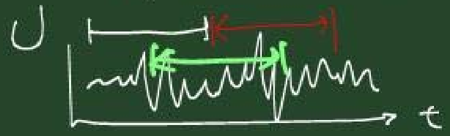
- One-point statistics

ex. velocity covariance $\langle u_i(\underline{x},t)\,u_j(\underline{x},t)\rangle$

$= \langle u_i u_j\rangle$: Reynolds stresses

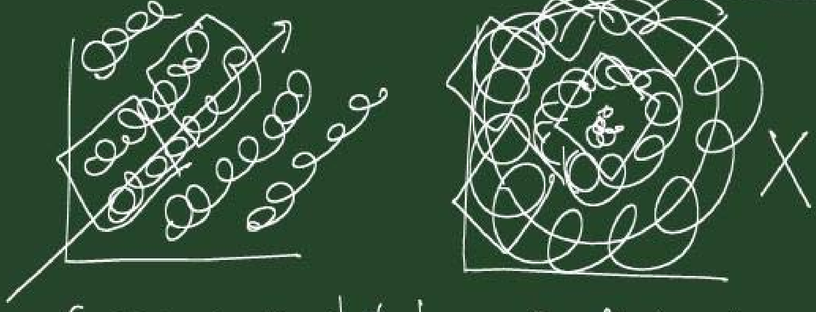
- $U(\underline{x},t)$ is <u>statistically stationary</u>

→ all statistics are invariant under a shift in time.



- $U(\underline{x},t)$ is <u>statistically homogeneous</u>.

→ all statistics are invariant <u>under a shift</u> in position.

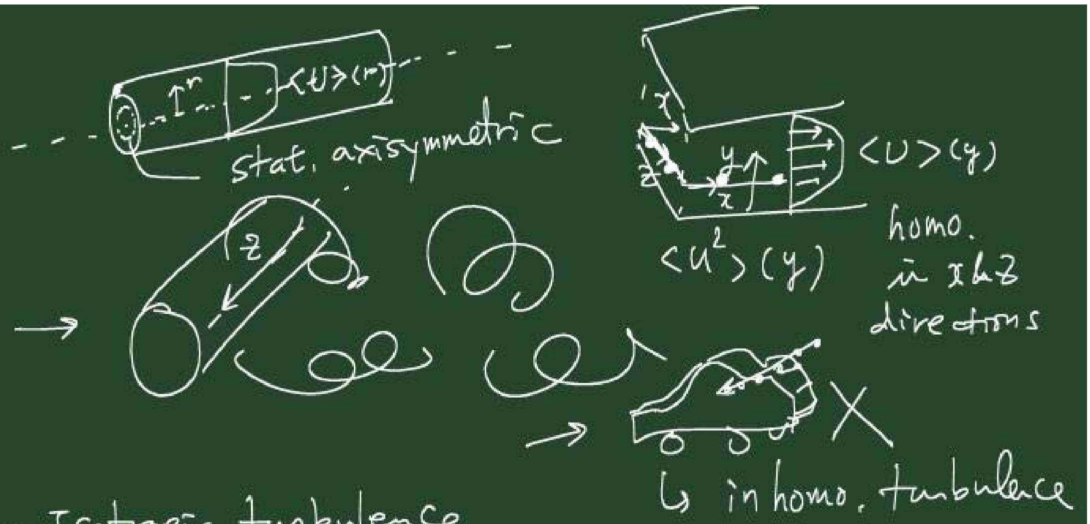


If $U(\underline{x},t)$ is stat. homo. in all directions, $\langle U\rangle$ is uniform.

* <u>Homogeneous turbulence</u> (definition is less restrictive)

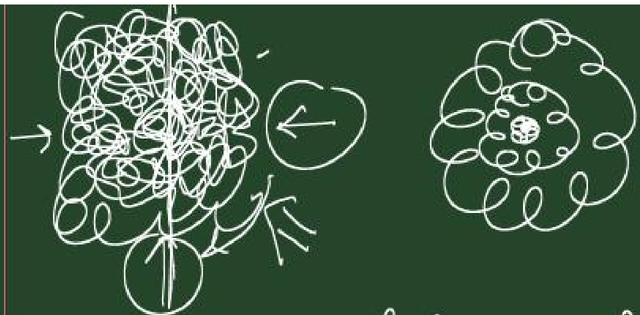
Fluctuating velocity $u(\underline{x},t)$ is stat. homo.

(but $\langle U\rangle$ does not have to be uniform)



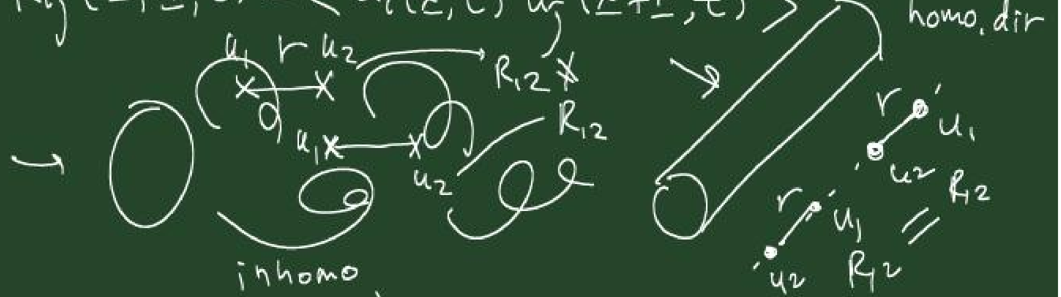
· <u>Isotropic turbulence</u>

turbulent flow is statistically invariant under <u>rotations</u> and <u>reflections</u> of the coordinate system.

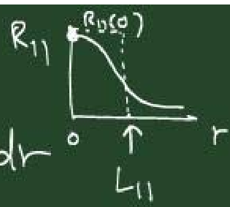


· <u>Two-point correlation</u> : <u>information on spatial structure of field</u>.

$$R_{ij}(\underline{r}, \underline{x}, t) \equiv \langle u_i(\underline{x}, t)\, u_j(\underline{x}+\underline{r}, t) \rangle$$



Integral length scale



$$L_{11}(\underline{x},t) = \frac{1}{R_{11}(0,\underline{x},t)} \int_0^\infty R_{11}(\underline{e}_1 r, \underline{x}, t)\, dr$$

- Wavenumber spectra

For homo. turbulence, $R_{ij}(\underline{r},t)$ is indep. of $\underline{x}$.

→ wavenumber spectrum

$$\Phi_{ij}(\underline{k},t) = \frac{1}{(2\pi)^3} \iiint_{-\infty}^{\infty} R_{ij}(\underline{r},t)\, e^{-i\underline{k}\cdot\underline{r}}\, d\underline{r}$$

$$R_{ij}(\underline{r},t) = \iiint_{-\infty}^{\infty} \Phi_{ij}(\underline{k},t)\, e^{i\underline{k}\cdot\underline{r}}\, d\underline{k}$$

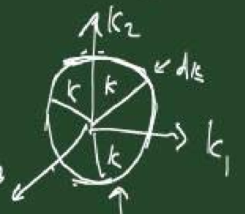
$\underline{k}$: wavenumber vector

$$\langle u_i u_j \rangle = R_{ij}(0,t) = \iiint_{-\infty}^{\infty} \Phi_{ij}(\underline{k},t)\, d\underline{k}$$

Φ_{ij} : contribution to $\langle u_i u_j \rangle$ with wavenumber $\underline{k}$

- Energy spectrum function

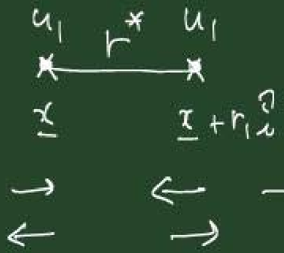
$$E(k,t) \equiv \iiint_{-\infty}^{\infty} \frac{1}{2}\Phi_{ii}(\underline{k},t)\, \delta(|\underline{k}|-k)\, d\underline{k}$$

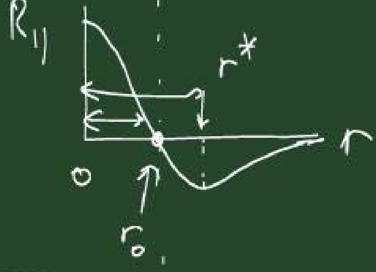


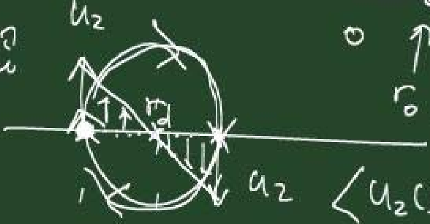
$$\int_0^\infty E(k,t)\, dk = \frac{1}{2} R_{ii}(0,t) = \frac{1}{2}\langle u_i u_i \rangle \;:\; \text{turbulent kinetic energy}$$

→ $E(k,t)$: contribution to turb. kinetic energy $\frac{1}{2}\langle u_i u_i \rangle$ from all modes with $|\underline{k}|$ in the range of $k \le |\underline{k}| < k+dk$.

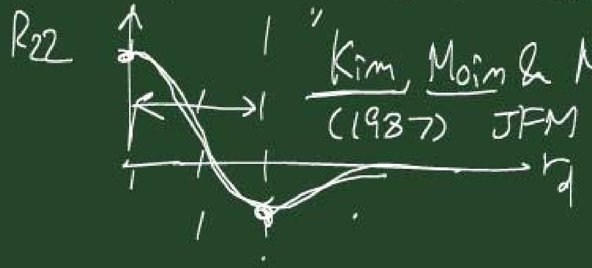
$R_{11}(r_1) = \langle u_1(\underline{x})\, u_1(\underline{x} + r_1 \hat{i}) \rangle$







$\langle u_2(\underline{x})\, u_2(\underline{x} + \hat{r}_d) \rangle$



Kim, Moin & Moser (1987) JFM

⊙ Probability and averaging

- time average for statistically stationary flows

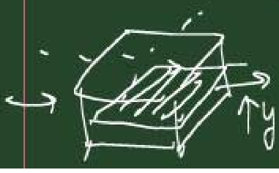
$$\langle U(t) \rangle_T \equiv \frac{1}{T} \int_t^{t+T} U(t')\, dt'$$

- ensemble average for repeatable flows

$$\langle U(t) \rangle_N = \frac{1}{N} \sum_{n=1}^{N} U^{(n)}(t)$$

- Spatial average for homo. turbulence ∈ numerical simulation

$$\langle U(t) \rangle_L = \frac{1}{L^3} \iiint_0^L U(\underline{x}, t)\, dx_1\, dx_2\, dx_3$$



$$\langle U \rangle (y) = \frac{1}{L_x L_z} \int_0^{L_z} \int_0^{L_x} U(\underline{x}, t)\, dx\, dz$$