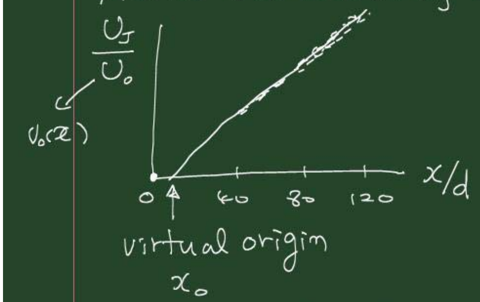
· Axial variation of scales



$$\rightarrow \frac{U_0(x)}{U_J} = \frac{B}{(x-x_0)/d}$$ B ← empirical const.

$\exists\ U_J - U_0(x)$ $\quad\boxed{U_0 \sim x^{-1}}$

It is also found that the spreading rate

$$S \equiv \frac{d r_{\frac{1}{2}}(x)}{dx} = \text{const.} \quad \text{or} \quad \underline{r_{\frac{1}{2}} = S(x-x_0)}$$

@ $r = r_{\frac{1}{2}}$, $\langle U \rangle = \frac{1}{2} U_0$

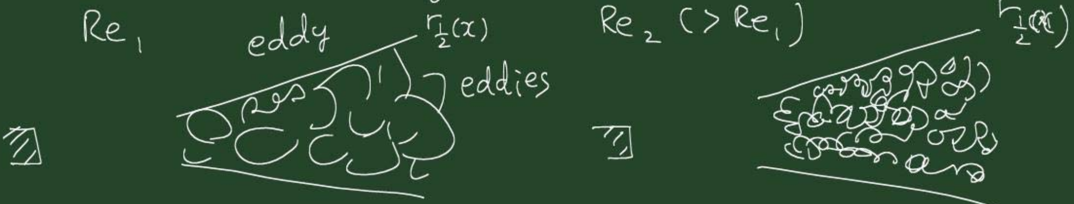
$\Rightarrow r_{\frac{1}{2}} \sim x$, $U_0 \sim x^{-1}$ $\Rightarrow r_{\frac{1}{2}} U_0$ is indep. of x.

local Reynolds number $Re_0(x) \equiv \dfrac{U_0(x)\, r_{\frac{1}{2}}(x)}{\nu}$ is indep. of x!

· S and B have no dependence on Re.

(See Table 5.1) $B = 5.8$, $S = \underline{0.094}$

That is, the mean vel. profile and the spreading rate are indep. of Re, although small-scale structures are smaller at larger Re number.

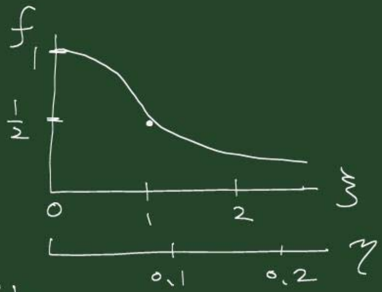


- $\xi = r/r_{\frac{1}{2}}$ or $\eta = r/(x-x_0)$

  $\rightarrow \eta = S\xi$

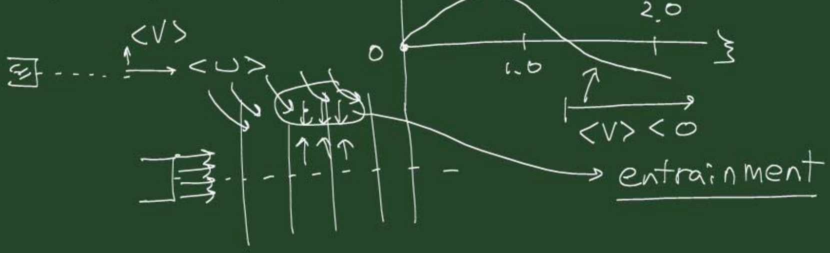
  $f(\eta) = \bar{f}(\xi) \equiv \dfrac{\langle U(x,r)\rangle}{U_0(x)}$

  Self-similar mean vel. profile

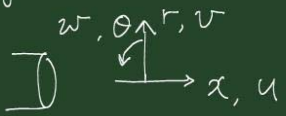


mean lateral vel.

$\langle U\rangle \xrightarrow[\text{cont. eq}]{} \langle V\rangle \ll \langle U\rangle$



⊙ Reynolds stresses



$$\begin{bmatrix} \langle u^2\rangle & \langle uv\rangle & \cancel{\langle uw\rangle} \\ \langle uv\rangle & \langle v^2\rangle & \cancel{\langle vw\rangle} \\ 0 & 0 & \langle w^2\rangle \end{bmatrix}$$

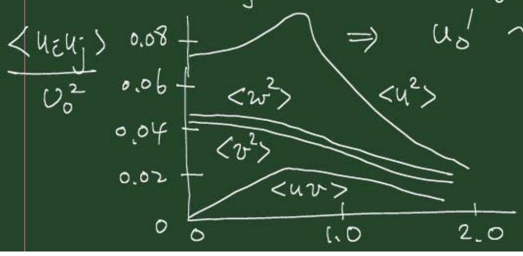
due to circumferential symmetry

$\langle uw\rangle = -\langle uw\rangle$

$U_0(x)$

$u_0'(x) \equiv \langle u^2\rangle^{\frac{1}{2}}_{r=0}$

in self-similar region, $u_0'(x)/U_0(x) \sim 0.25$



$\Rightarrow u_0' \sim x^{-1}$ $\quad (\because U_0 \sim x^{-1})$

$\langle w^2\rangle > \langle v^2\rangle$

$\langle u^2\rangle > \langle w^2\rangle > \langle v^2\rangle$

$\rightarrow$ anisotropic

$$\left(\langle u^2 \rangle \simeq \langle v^2 \rangle \simeq \langle w^2 \rangle \rightarrow \text{isotropic} \right)$$

$$\langle uv \rangle = -\nu_T \frac{\partial \langle U \rangle}{\partial r} > 0$$

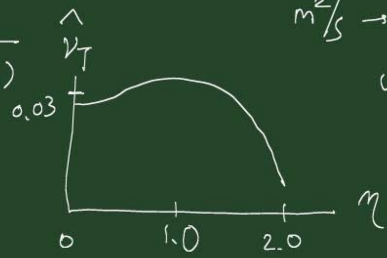
$$\frac{\partial \langle U \rangle}{\partial r} < 0$$

$$\hat{\nu}_T(\eta) = \frac{\nu_T(x,r)}{U_0(x)\, r_{\frac{1}{2}}(x)}$$

self-similar

$m^2/s \rightarrow m/s \cdot m$

vel length

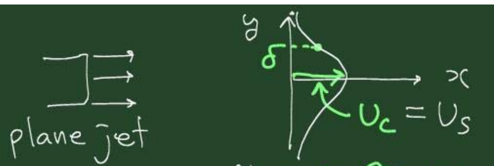


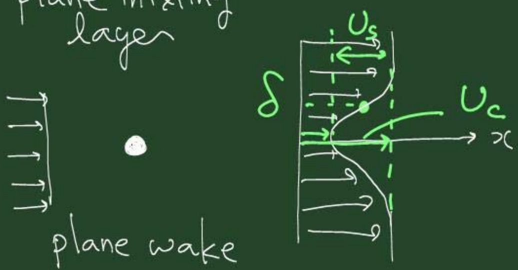
5.2 Round jet: mean momentum

① Boundary layer eqs.

in turbulent round jet, $|\langle V \rangle| \simeq 0.03\, |\langle U \rangle|$

$\frac{dr_{\frac{1}{2}}}{dx} \sim 0.1$ axial gradient is very small.

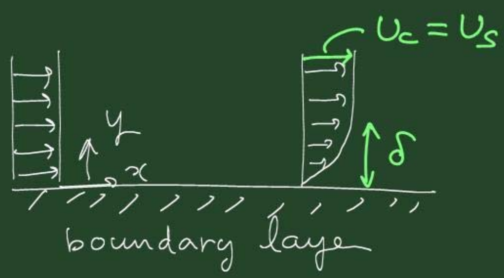




δ : characteristic flow width

U_c : " convection velocity

U_s : " velocity difference.



boundary-layer approx. $\underline{\frac{\partial}{\partial x} \ll \frac{\partial}{\partial y}}$

$|\langle V\rangle| \ll |\langle U\rangle|$ $\quad \langle v^2\rangle \not\ll \langle u^2\rangle$

- $\frac{\partial\langle U\rangle}{\partial x} + \frac{\partial\langle V\rangle}{\partial y} = 0$
- $\langle U\rangle\frac{\partial\langle U\rangle}{\partial x} + \langle V\rangle\frac{\partial\langle U\rangle}{\partial y} = -\frac{1}{\rho}\frac{\partial\langle p\rangle}{\partial x} + \underbrace{\cancel{\nu\frac{\partial^2\langle U\rangle}{\partial x^2}}}_{0} + \nu\frac{\partial^2\langle U\rangle}{\partial y^2} - \frac{\partial\langle u^2\rangle}{\partial x} - \frac{\partial\langle uv\rangle}{\partial y}$
- $\langle U\rangle\underbrace{\cancel{\frac{\partial\langle V\rangle}{\partial x}}}_{0} + \langle V\rangle\underbrace{\cancel{\frac{\partial\langle V\rangle}{\partial y}}}_{0} = -\frac{1}{\rho}\frac{\partial\langle p\rangle}{\partial y} + \nu\underbrace{\cancel{\frac{\partial^2\langle V\rangle}{\partial x^2}}}_{0} + \nu\underbrace{\cancel{\frac{\partial^2\langle V\rangle}{\partial y^2}}}_{0} - \underbrace{\cancel{\frac{\partial\langle uv\rangle}{\partial x}}}_{0} - \frac{\partial\langle v^2\rangle}{\partial y}$

neglect

y-mtm eq : $+\frac{1}{\rho}\frac{\partial\langle p\rangle}{\partial y} + \frac{\partial\langle v^2\rangle}{\partial y} = 0$

integrate : $\int_y^\infty \frac{1}{\rho}\frac{\partial\langle p\rangle}{\partial y}dy + \int_y^\infty \frac{\partial\langle v^2\rangle}{\partial y}dy = 0$

$$\frac{1}{\rho}\underset{\uparrow\ \langle p\rangle_{y\to\infty}}{p_0} - \frac{1}{\rho}\langle p\rangle + \underbrace{\cancel{\langle v^2\rangle_\infty}}_{0} - \langle v^2\rangle = 0$$

$\rightarrow \frac{1}{\rho}\langle p\rangle = \frac{1}{\rho}p_0 - \langle v^2\rangle$

$\rightarrow \frac{1}{\rho}\frac{\partial\langle p\rangle}{\partial x} = \frac{1}{\rho}\frac{dp_0}{dx} - \frac{\partial\langle v^2\rangle}{\partial x}$

x-mtm :

$$\langle U\rangle\frac{\partial\langle U\rangle}{\partial x} + \langle V\rangle\frac{\partial\langle U\rangle}{\partial y} = -\frac{1}{\rho}\frac{dp_0}{dx} + \nu\frac{\partial^2\langle U\rangle}{\partial y^2} - \frac{\partial\langle uv\rangle}{\partial y} - \underbrace{\cancel{\frac{\partial}{\partial x}\left(\langle u^2\rangle - \langle v^2\rangle\right)}}_{0}$$

neglect

$\sim \nu\frac{U_s}{\delta^2} \sim O(Re^{-1})$ negligible as compared to other terms

but not negligible near the wall

in case of unif. stream ($dP_0/dx = 0$)

$$\langle U \rangle \frac{\partial \langle U \rangle}{\partial x} + \langle V \rangle \frac{\partial \langle U \rangle}{\partial y} = \nu \frac{\partial^2 \langle U \rangle}{\partial y^2} - \frac{\partial}{\partial y} \langle uv \rangle$$

For (x, r, θ) coord.