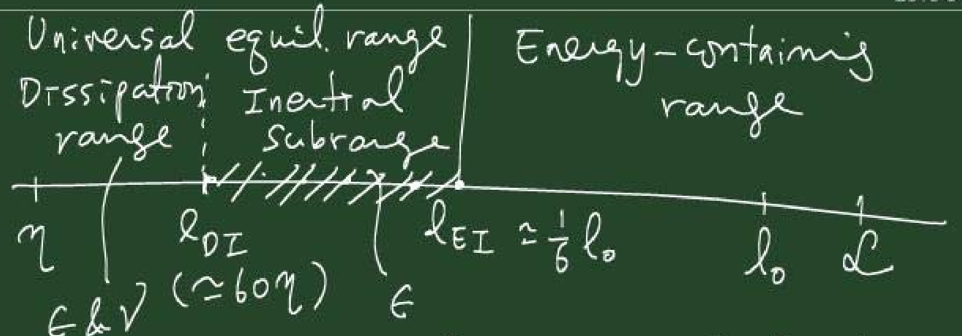
노트 제목 2010-04-20



$2\nu s_{ij} s_{ij}$ $\parallel$

- Given an eddy size l in the inertial subrange,

$\varepsilon = [L^2 T^{-3}]$ $\quad u(l) = (\varepsilon l)^{\frac{1}{3}} = u_\eta (l/\eta)^{\frac{1}{3}} \sim u_0 (l/l_0)^{\frac{1}{3}}$

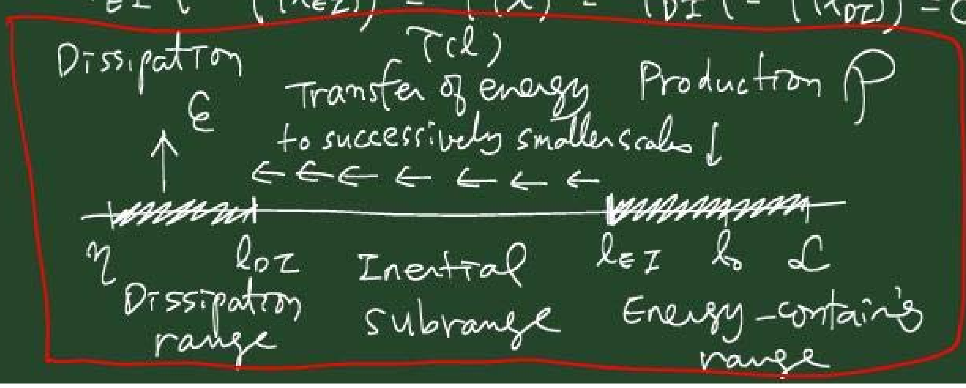
$$\tau(l) = (l^2/\varepsilon)^{\frac{1}{3}} = \tau_\eta (l/\eta)^{\frac{2}{3}} \sim \tau_0 (l/l_0)^{\frac{2}{3}}$$

$\therefore$ As l decreases, u and τ decrease.

$\mathcal{T}(l)$: energy transferred from eddies larger than l to those smaller than l.

$$\mathcal{T}(l) \sim u(l)^2/\tau(l) = \varepsilon, \text{ indep. of } l.$$

$$\mathcal{T}_{EI} (= \mathcal{T}(l_{EI})) = \mathcal{T}(l) = \mathcal{T}_{DI} (= \mathcal{T}(l_{DI})) = \varepsilon$$



- Energy spectrum $E(k)$

  length scale $\ell \longrightarrow$ wavenumber $\underline{k = 2\pi/\ell}$

  e^{ikx}, 2π

  energy in (k_a, k_b): $k_{(k_a,k_b)} = \int_{k_a}^{k_b} E(k)\, dk$

  $u(x)$ $\downarrow$ FT $\hat{u}(k)$

  dissipation rate ε in (k_a, k_b):

  $$\varepsilon_{(k_a,k_b)} = \int_{k_a}^{k_b} \boxed{2\nu k^2 E(k)}\, dk .$$

  $\hat{u}(k)\,\hat{u}^{T}(k)$

  $\varepsilon = 2\nu \underbrace{s_{ij}}_{ik\hat{u}} \underbrace{s_{ij}}_{ik\hat{u}}$, $k^2 \hat{u}\hat{u}^T$

  In the universal equil range,

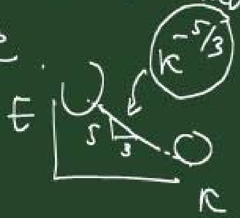
  $(k > k_{EI} = 2\pi/\ell_{EI})$

  spectrum is a universal ft. of ε & ν.

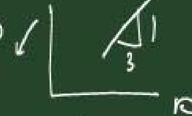
- In the inertial subrange $(k_{EI} < k < k_{DI} = 2\pi/\ell_{DI})$

  spectrum is a universal ft. of ε.

  $\rightarrow \underline{E(k) = C\, \varepsilon^{\frac{2}{3}} k^{-\frac{5}{3}}}$ (§ 6.5)



  $2\nu k^2 E(k) \sim k^{\frac{1}{3}}$ $\rightarrow$ D



  $\rightarrow$ bulk of energy in the large scales

  " " dissipation in the small scales

- $\underline{\text{structure fts}}$ — skip

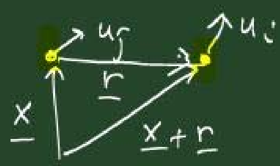
⊙ Two-point correlation

· Auto-correlation fts.

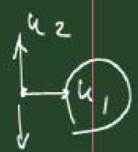
Consider homogeneous, isotropic turbulence with zero mean, rms vel. $u'(t)$ and $\varepsilon(t)$.

$$R_{ij}(\underline{r},t) \equiv \langle u_i(\underline{x}+\underline{r},t)\, u_j(\underline{x},t)\rangle$$

$\cancel{\underline{x}} \longrightarrow$ homogeneity



$R_{12}(0,t)=0$ $\quad R_{ij}(0,t) = \langle u_i u_j\rangle = u'^2\delta_{ij}$ $\quad (\because$ isotropic$)$



no production $\rightarrow$ $\dfrac{dk}{dt} = -\varepsilon$ $\quad \left(k = \frac{3}{2}u'^2(t)\right)$

$\rightarrow R_{ij}(\underline{r},t) = u'^2\left(\underset{a(r)}{a}\,\delta_{ij} + \underset{b(r)}{b}\,\dfrac{r_i r_j}{r^2}\right)$ $\quad r_i r_j = \begin{pmatrix} r_1r_1 & r_1r_2 & r_1r_3 \\ r_2r_1 & r_2r_2 & r_2r_3 \\ r_3r_1 & r_3r_2 & r_3r_3 \end{pmatrix}$

(second-order tensors can be formed from δ_{ij} and $r_i r_j$)

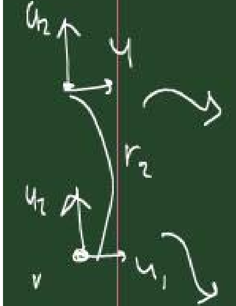
$$R_{11} = u'^2\left(a + b\frac{r_1^2}{r^2}\right)$$

$$R_{22} = u'^2\left(a + b\frac{r_2^2}{r^2}\right)$$

for $\underline{r} = r_1\underline{e}_1$, $\Downarrow$ $(r_2 = r_3 = 0)$

$$\left.\begin{array}{l} R_{11} = u'^2(a+b) \\ R_{22} = u'^2 a \end{array}\right\} \quad \begin{array}{l} a \equiv g = \dfrac{R_{22}}{u'^2} \\ b = f - g \\ (f \equiv R_{11}/u'^2) \end{array}$$

$$\rightarrow \underline{R_{ij}(\underline{r},t) = u'^2\left[g(r,t)\delta_{ij} + (f(r,t)-g(r,t))\frac{r_i r_j}{r^2}\right]}$$



$$f \equiv R_{11}/u'^2 = \langle u_1(\underline{x}+\underline{e}_1 r_1, t)\, u_1(\underline{x},t)\rangle / u_1'^2$$

$(r=r_1)$

longitudinal auto-correlation ft.

$$g \equiv R_{22}/u'^2 = \langle u_2(\underline{x}+\underline{e}_1 r_1, t)\, u_2(\underline{x},t)\rangle / u_2'^2$$

transverse auto-corr. ft.

$r=r_1$

$R_{22}=R_{33}$ $\qquad f(0,t)=1$

$R_{ij}=0$ for $i\neq j$ $\qquad g(0,t)=1$

$$R_{ij} = \langle u_i(\underline{x}+\underline{r},t)\, u_j(\underline{x},t)\rangle = \langle u_i(\underline{x})\, u_j(\underline{x}-\underline{r})\rangle$$ Homo.

$$\frac{\partial R_{ij}}{\partial r_j} = \frac{\partial}{\partial r_j}\langle u_i(\underline{x})u_j(\underline{x}-\underline{r})\rangle = \langle u_i(\underline{x})\, \cancel{\frac{\partial u_j}{\partial r_j}}^{\,0}(\underline{x}-\underline{r})\rangle = 0$$

from cont.

$$R_{ij}(\underline{r},t) = u'^2\left[g(r,t)\delta_{ij} + (f(r,t)-g(r,t))\frac{r_i r_j}{r^2}\right]$$

$$\frac{\partial R_{ij}}{\partial r_j} = \frac{\partial R_{ij}}{\partial r}\frac{\partial r}{\partial r_j} = u'^2\left[g'\delta_{ij} + (f'-g')\frac{r_i r_j}{r^2}\right]\frac{r_j}{r} + u'^2(f-g)\underline{\frac{\partial}{\partial r_j}\left(\frac{r_i r_j}{r^2}\right)}$$

$$\left(\frac{\partial r}{\partial r_j} = \frac{\partial}{\partial r_j}(r_j r_j)^{\frac{1}{2}} = \frac{r_j}{r}\right)$$

$$\frac{\partial}{\partial r_j}\left(\frac{r_i r_j}{r^2}\right) = \frac{\delta_{ij} r_j}{r^2} + \frac{r_i}{r^2} = \frac{2r_i}{r^2}$$

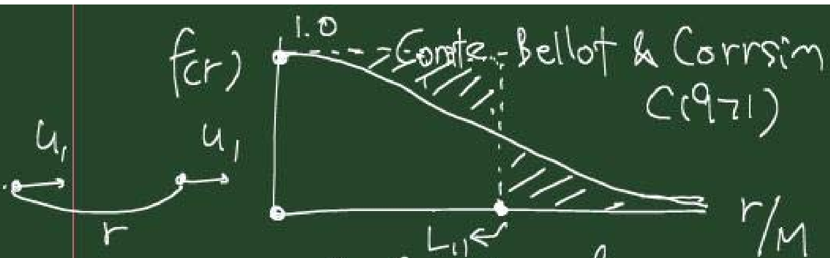
$$\frac{\partial R_{ij}}{\partial r_j} = u'^2 \left[\cancel{g' \frac{r_i}{r}} + \cancel{(f'-g') \frac{r_i}{r}} + (f-g) \frac{2 r_i}{r^2} \right] = 0$$

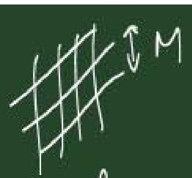
$$\rightarrow f' + (f-g)\frac{2}{r} = 0$$

$$\rightarrow g = f + \frac{r}{2} f'$$

$$\boxed{g(r,t) = f(r,t) + \frac{1}{2} r \frac{\partial}{\partial r} f(r,t)}$$

$\therefore$ In isotropic turb., $R_{ij}(\underline{r},t)$ is completely determined by $f(r,t)$.





char. of large eddies

- Integral lengthscale

longitudinal int. lengthscale $L_{11}(t) = \int_0^\infty f(r,t)\,dr$

transverse " " $L_{22}(t) = \int_0^\infty g(r,t)\,dr$

$= \frac{1}{2} L_{11}(t)$

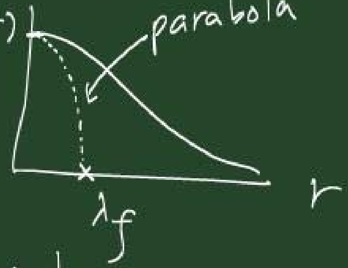
for isotropic turb.

(Ex 6.4)

- Taylor ~~micro~~scale

longitudinal Taylor microscale $\lambda_f(t)$

$$\lambda_f(t) \equiv \left[-\frac{1}{2} f''(0,t)\right]^{-\frac{1}{2}}$$



$$-u'^2 f''(0,t) = -u'^2 \frac{\partial^2}{\partial r^2} f(r,t)\Big|_{r=0}$$

$$= -\frac{\partial^2}{\partial r^2} \langle u_1(\underline{x}+\underline{e}_1 r, t)\, u_1(\underline{x},t)\rangle \Big|_{r=0}$$

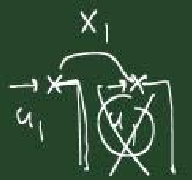
$$= -\left\langle \frac{\partial^2 u_1}{\partial x_1^2} u_1 \right\rangle = -\left\langle \cancel{\frac{\partial}{\partial x_1}\left(u_1 \frac{\partial u_1}{\partial x_1}\right)} - \left(\frac{\partial u_1}{\partial x_1}\right)^2 \right\rangle$$

$\triangleright$ o Homo.

$$= \left\langle \left(\frac{\partial u_1}{\partial x_1}\right)^2 \right\rangle$$

$$\therefore \left\langle \left(\frac{\partial u_1}{\partial x_1}\right)^2 \right\rangle = \frac{2u'^2}{\lambda_f^2}.$$

$u_1(t) \xrightarrow{\text{Taylor hypo.}} u_1(x_1)$

$\Longrightarrow$ 

$\cancel{\frac{\partial u_1}{\partial x_1}}$ ~~Hot-wire probe~~

LDV o

$\lambda_f \longleftarrow \left\langle \left(\frac{\partial u_1}{\partial x_1}\right)^2 \right\rangle$