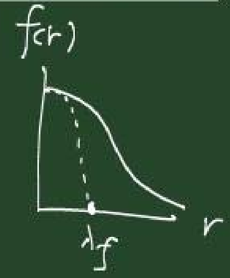
노트 제목 2010-04-22

- Taylor microscale



① longitudinal Taylor microscale λ_f

$$\lambda_f(t) = \left(-\frac{1}{2} f''(0,t)\right)^{-\frac{1}{2}}$$

$$\left\langle \left(\frac{\partial u_1}{\partial x_1}\right)^2 \right\rangle = \frac{2u'^2}{\lambda_f^2} \quad \text{— (i)}$$

② transverse Taylor microscale $\lambda_g(t)$ — (ii)

$$\lambda_g(t) = \left(-\frac{1}{2} g''(0,t)\right)^{-\frac{1}{2}} = \frac{1}{\sqrt{2}} \lambda_f(t) \text{ for isotropic turbulence.}$$

(i) & (ii)

$$\left.\begin{array}{l} \\ \varepsilon = 15\nu \left\langle \left(\frac{\partial u_1}{\partial x_1}\right)^2 \right\rangle \end{array}\right\} \rightarrow \varepsilon = 15\nu u'^2 / \lambda_g^2$$

Eq. (5.171)

Taylor (1935) first defined λ_g and obtained this eq., and stated that λ_g may be regarded as a measure of the diameter of the smallest eddies which are responsible for the dissipation of energy.

$\longrightarrow$ wrong! because u' is not the char. vel. of dissipative eddies.

• $L \equiv k^{\frac{3}{2}}/\varepsilon$: length scale characterizing large eddies

$$Re_L = \frac{k^{\frac{1}{2}}L}{\nu} = \frac{k^2}{\varepsilon\nu}$$

$$\rightarrow \frac{\lambda_g}{L} = \sqrt{10}\, Re_L^{-\frac{1}{2}}$$

$$\frac{\eta}{L} = Re_L^{-\frac{3}{4}}$$

$$\rightarrow \lambda_g = \sqrt{10}\,\eta^{\frac{2}{3}} L^{\frac{1}{3}}$$

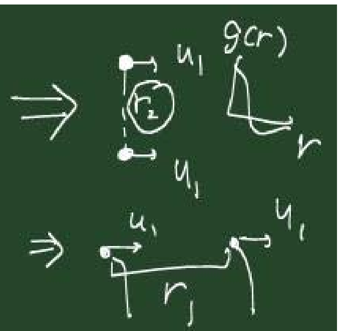
$$\frac{\lambda_g}{\eta} = \sqrt{10}\, Re_L^{\frac{1}{4}}$$

$$\therefore \underline{\eta < \lambda_g < L}$$

→ Taylor scale has no clear physical interpretation, but is often used (e.g. grid turb.)

Taylor-scale Reynolds number

$$R_\lambda = \underline{u'}\lambda_g/\nu \;,,$$



6.4 Fourier modes

Discrete Fourier transformation

$$u_j(\underline{x},t) = \sum_{\underline{k}} e^{i\underline{k}\cdot\underline{x}}\, \hat{u}_j(\underline{k},t) \qquad \underline{k}\text{ : wavenumber}$$

$$\hat{u}_j(\underline{k},t) = \frac{1}{L^3}\int_0^L\int_0^L\int_0^L u_j(\underline{x},t)\, e^{-i\underline{k}\cdot\underline{x}}\, dx_1 dx_2 dx_3$$

$$\hat{u}_j^*(\underline{k},t) = \hat{u}_j(-\underline{k},t) \quad \text{from real } u_j$$

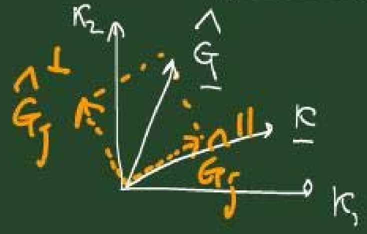
$\hat{E}(\underline{k},t) = \hat{u}_j^{*}(\underline{k},t)\,\hat{u}_j(\underline{k},t)$

$\underline{k}\cdot\hat{\underline{u}} = 0$

- Continuity $\frac{\partial u_j}{\partial x_j} = 0 \xrightarrow{FT} i k_j \hat{u}_j = 0 \rightarrow \boxed{k_j \hat{u}_j = 0}$

- Any vector $\hat{G}_j = \hat{G}_j^{\parallel} + \hat{G}_j^{\perp}$

parallel to $\underline{k}$ normal to $\underline{k}$



$\hat{G}_j^{\parallel} = \underline{e}\,(\underline{e}\cdot\hat{\underline{G}})$ $\underline{e} = \underline{k}/k$ $(k = |\underline{k}|)$

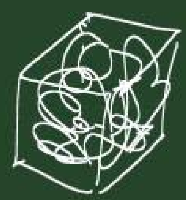
$= e_j (e_k \hat{G}_k) = \frac{k_j k_k}{k^2}\hat{G}_k$ $e_j = k_j/k$

$\hat{G}_j^{\perp} = \hat{G}_j - \hat{G}_j^{\parallel} = \left(\delta_{jk} - \frac{k_j k_k}{k^2}\right)\hat{G}_k \equiv P_{jk}\hat{G}_k$

projection tensor

- Navier-Stokes eq.

$$\frac{\partial u_j}{\partial t} + \frac{\partial}{\partial x_k}(u_j u_k) = \nu \frac{\partial^2 u_j}{\partial x_k \partial x_k} - \frac{1}{\rho}\frac{\partial p}{\partial x_j}$$



FT ↓ $(ik_k)(ik_k) = -k^2$ ik_j

$$\frac{d\hat{u}_j}{dt} + \hat{G}_j = -\nu k^2 \hat{u}_j - \frac{1}{\rho} i k_j \hat{p}$$

$$k_j\left(\frac{d\hat{u}_j}{dt} + \nu k^2 \hat{u}_j = -\frac{1}{\rho} i k_j \hat{p} - \hat{G}_j\right)$$

$$\rightarrow \quad 0 = -\frac{1}{\rho} i k^2 \hat{p} - k_j \hat{G}_j \longrightarrow \hat{p} = i\rho \frac{k_j}{k^2} \hat{G}_j$$

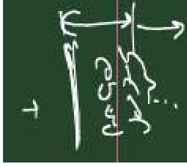
$$\rightarrow \quad -\frac{1}{\rho} i k_j \hat{p} = \frac{k_j k_k}{k^2} \hat{G}_k = \hat{G}_j^{\parallel}$$

∴ press term exactly balances $+\hat{G}^{\parallel}$.

$$\frac{d\hat{u}_j}{dt} + \nu k^2 \hat{u}_j = -\left(\delta_{jk} - \frac{k_j k_k}{k^2}\right) \hat{G}_k = -P_{jk} \hat{G}_k = -\hat{G}_j^{\perp}$$

N-S eq. in wavespace.

· final period of decay of isotropic turbulence



in which Re is very low → conv. is negligible.

$$\rightarrow \quad d\hat{u}_j/dt = -\nu k^2 \hat{u}_j$$

$$\rightarrow \quad \hat{u}_j(\mathbf{k}, t) = \hat{u}_j(\mathbf{k}, t_0)\, e^{-\nu k^2 (t - t_0)}$$

→ in the final period of decay, each Fourier coeff. evolves independently of all other modes, decaying exponentially in time.

high-wavenumber modes (small scales) decay more rapidly than low-wavenumber modes (large scales).

· $\hat{G}_j(\underline{k},t) = FT\left(\frac{\partial}{\partial x_k} u_j u_k\right)$

$= i k_k \, FT(u_j u_k)$

$= i k_k \, FT\left(\underbrace{\sum_{\underline{k}'} \hat{u}_j(\underline{k}') e^{i\underline{k}'\cdot\underline{x}} \cdot \sum_{\underline{k}''} \hat{u}_k(\underline{k}'') e^{i\underline{k}''\cdot\underline{x}}}_{\text{use orthogonality}}\right)$

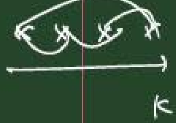
$= \cdots$

$= i k_k \sum_{\underline{k}'} \hat{u}_j(\underline{k}')\, \hat{u}_k(\underline{k}-\underline{k}')$

convolution sum.

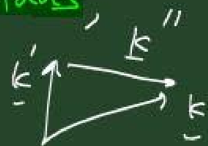
Back to N-S eq. in wave space.

$$\left(\frac{d}{dt} + \nu k^2\right)\hat{u}_j(\underline{k},t) = -i k_\ell \, P_{ij}(\underline{k}) \sum_{\underline{k}'} \hat{u}_k(\underline{k}',t)\, \hat{u}_\ell(\underline{k}-\underline{k}',t)$$



Convection term is nonlinear and non-local in wavenumber space, involving the interaction of wavenumber triads,

$\underline{k}$, $\underline{k}'$ and k'' $(= \underline{k}-\underline{k}')$.



6.5 Velocity spectra

• Two-pt. corr. $R_{ij}(\underline{r},t) \equiv \langle u_i(\underline{x},t)\, u_j(\underline{x}+\underline{r},t)\rangle$

$$R_{ij}(\underline{r}) = \iiint_{-\infty}^{\infty} \underbrace{\phi_{ij}(\underline{k})}\, e^{i\underline{k}\cdot\underline{r}}\, d\underline{k}$$

$$\hat{u}_i^{*}(\underline{k},t)\,\hat{u}_j(\underline{k},t)$$

$$\| $$

$$= \hat{u}_i(-\underline{k},t)\,\hat{u}_j(\underline{k},t)$$; velocity spectrum tensor

$$\phi_{ij}(\underline{k}) = \frac{1}{(2\pi)^3} \iiint_{-\infty}^{\infty} R_{ij}(\underline{r})\, e^{-i\underline{k}\cdot\underline{r}}\, d\underline{r}$$

✓ $$\left\langle \frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_l} \right\rangle = \iiint_{-\infty}^{\infty} k_k k_l\, \phi_{ij}(\underline{k})\, d\underline{k}$$

$$\rightarrow \varepsilon = \iiint_{-\infty}^{\infty} 2\nu k^2 \cdot \frac{1}{2}\phi_{ii}(\underline{k})\, d\underline{k}$$

model

$$\phi_{ij}(\underline{k}) = ?$$