inner layer ($\frac{y}{\delta} < 0.1$)

노트 제목 2010-05-06

$$\frac{d\langle U\rangle}{dy} \Big/ \frac{u_\tau}{y} = \Phi\left(\frac{y}{\delta_\nu}, \frac{y}{\delta}\right)$$

δ ↕ $y^+ = \frac{y}{\delta_\nu} \gg 1$ indep of ν

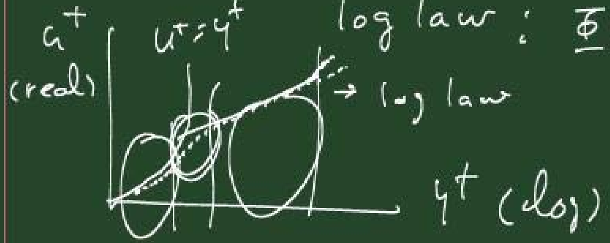
Law of the wall ⇒ $\Phi\left(\frac{y}{\delta_\nu}\right)$

$\rightarrow u^+ = f_w(y^+)$

$\frac{\langle U\rangle}{u_\tau}$ //

viscous sublayer : $u^+ = y^+ + O(y^{+2})$

log law : $\Phi\left(\frac{y}{\delta_\nu}\right) = \frac{1}{\kappa} \Rightarrow u^+ = \frac{1}{\kappa}\ln y^+ + B$



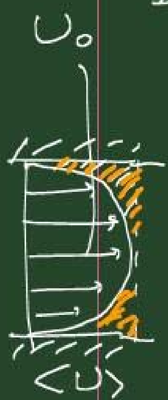
- Velocity-defect law

In outer layer ($y^+ > 50$), $\Phi\left(\cancel{\frac{y}{\delta_\nu}}, \frac{y}{\delta}\right)$ is indep. of ν.

$$\rightarrow \frac{d\langle U\rangle}{dy} \Big/ \frac{u_\tau}{y} = \Phi_0\left(\frac{y}{\delta}\right)$$

integration from y to δ

$$\rightarrow \boxed{\frac{U_0 - \langle U\rangle}{u_\tau} = F_0\left(\frac{y}{\delta}\right)}$$ velocity-defect law



$y = \delta$

$y = 0$

At sufficiently high Re, there is an overlap region bet. inner layer ($y/\delta < 0.1$) and outer layer ($y/\delta_\nu > 50$).

y^{+} "

In this region, $\dfrac{d\langle U\rangle}{dy} \Big/ \dfrac{u_\tau}{y} = \Phi_1\left(\dfrac{y}{\delta_v}\right)$ for $\delta_v \ll y \ll \delta$

$$\quad \parallel \quad = \Phi_o\left(\frac{y}{\delta}\right)$$

$$\parallel$$

$$\text{const}$$

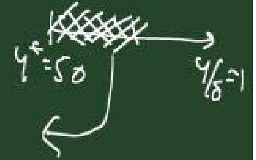
$\rightarrow \dfrac{d\langle U\rangle}{dy} \Big/ \dfrac{u_\tau}{y} = \dfrac{1}{\kappa}$

for $\delta_v \ll y \ll \delta$

Millikan (1938)

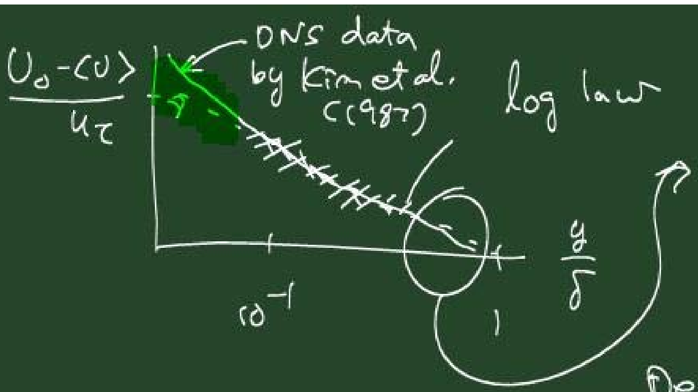
$\downarrow$

again log law!



or, $\dfrac{U_0-\langle U\rangle}{u_\tau} = F_D\left(\dfrac{y}{\delta}\right) = -\dfrac{1}{\kappa}\ln\dfrac{y}{\delta} + B$

for $y/\delta \ll 1$.



there exists a deviation from log law (but small)

Deviation is larger in the outer layer of turb. boundary layer.

● Friction law & Reynolds number

Relations among U_0, $\bar{U}$ & u_τ.

- bulk velocity $\bar{U}$ from log law

$$\frac{U_0 - \langle U \rangle}{u_\tau} = -\frac{1}{\kappa}\ln\frac{y}{\delta} + B_1$$

✓ @ $y=\delta$, $\langle U \rangle \doteq U_0 \Rightarrow B_1 = 0$

$$\frac{U_0 - \bar{U}}{u_\tau} = \frac{U_0 - \frac{1}{\delta}\int_0^\delta \langle U \rangle dy}{u_\tau} = \frac{1}{\delta}\int_0^\delta \frac{U_0 - \langle U \rangle}{u_\tau}\,dy$$

$$\simeq \frac{1}{\delta}\int_0^\delta \left(-\frac{1}{\kappa}\ln\frac{y}{\delta}\right)dy = \frac{1}{\kappa} \simeq 2.4$$

error near the wall is small

agrees well with exp. data.

Inner layer $\quad \dfrac{\langle U \rangle}{u_\tau} = \dfrac{1}{\kappa}\ln\dfrac{y}{\delta_\nu} + B$

Outer layer $\quad + \; \dfrac{U_0 - \langle U \rangle}{u_\tau} = -\dfrac{1}{\kappa}\ln\dfrac{y}{\delta} + B_1$

$$\frac{U_0}{u_\tau} = \frac{1}{\kappa}\ln\frac{\delta}{\delta_\nu} + B + B_1$$

$$\left(\frac{\delta}{\delta_\nu} = \delta \cdot \frac{1}{\nu/u_\tau} = \frac{\delta u_\tau}{\nu} = \frac{\delta U_0}{\nu}\cdot\frac{u_\tau}{U_0} \right)$$

$$\rightarrow \boxed{\frac{U_0}{u_\tau} = \frac{1}{\kappa}\ln\left[Re_0\left(\frac{U_0}{u_\tau}\right)^{-1}\right] + B + B_1}$$

$$\rightarrow \frac{U_0}{u_\tau} = f(Re_0)$$

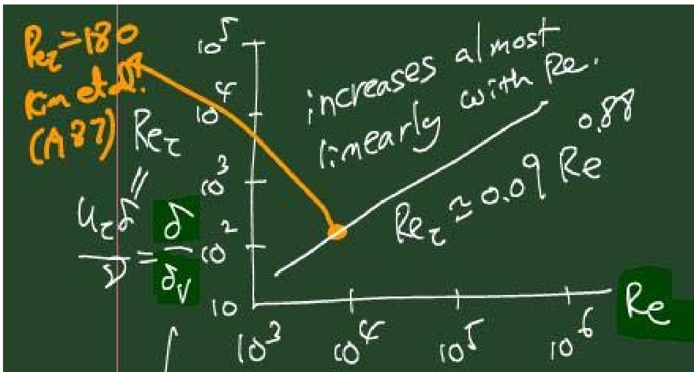
$$c_f \equiv \frac{\tau_w}{\frac{1}{2}\rho U_0^2} = 2\left(\frac{u_\tau}{U_0}\right)^2 \Rightarrow \frac{U_0}{u_\tau} = \sqrt{\frac{2}{c_f}}$$

friction coeff.

$$\boxed{\sqrt{\frac{2}{C_f}} = \frac{1}{\kappa}\ln\left[Re_\theta\sqrt{\frac{C_f}{2}}\right] + B + B_1}$$

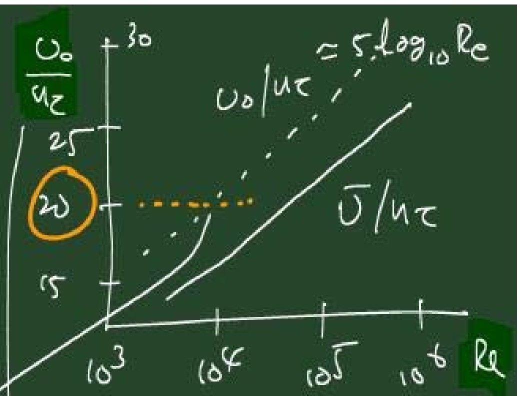
↳ $C_f = f(Re_\theta)$

good agreement at Re > 3000



$\frac{u_\tau \delta}{\nu} = \frac{\delta}{\delta_\nu}$

↳ ratio of outer to inner lengthscales



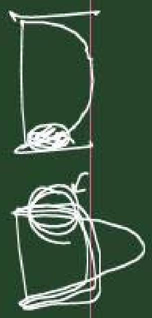
ratio of outer- to inner velocity scales

velocity ratios increase very slowly with Re.

At high Re, viscous lengthscale is very small.

e.g. $\delta = 2\,\text{cm}$, $Re = 10^5 \rightarrow \delta_\nu = 10^{-5}\,\text{m}$

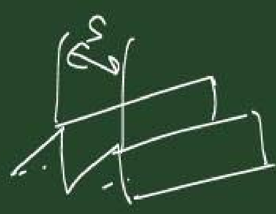
$y^+ = 100 \rightarrow 1\,\text{mm}$



Significant fraction of the increase in the mean velocity bet. wall and centerline occurs in the viscous wall region.

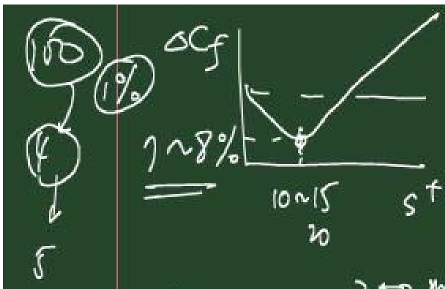
e.g. $\delta = 2\,\text{cm}$, $Re = 10^5 \rightarrow y^+ = 10 \rightarrow y \simeq 0.1\,\text{mm}$

$\langle U \rangle_{y=0.1\,\text{mm}} \simeq \underline{0.3\, U_0}$!



shark skin

riblet

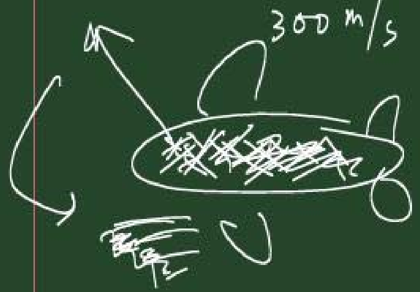


airplane

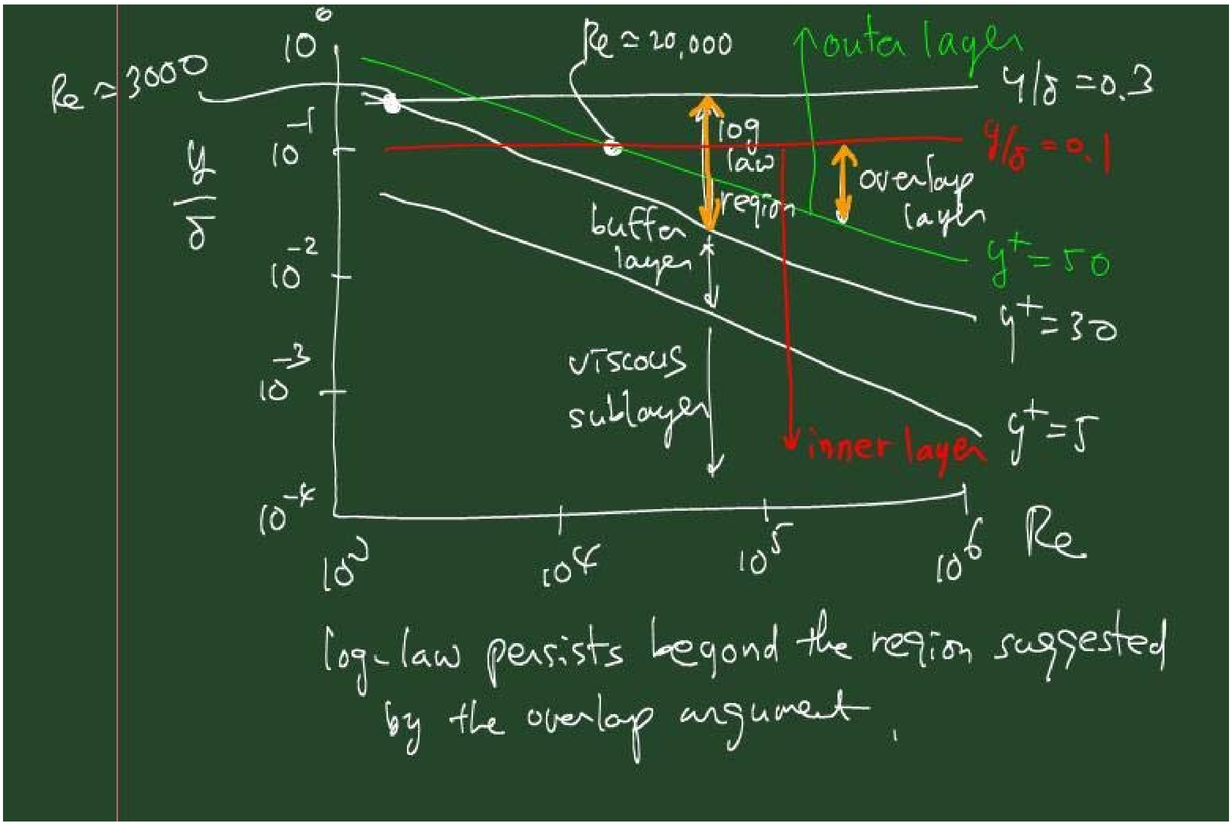
$s^+ = \frac{s u_\tau}{\nu} \Rightarrow s = \frac{s^+ \nu}{u_\tau} = \frac{15 \times 1.5 \times 10^{-5}}{u_\tau}$

$\boxed{\frac{U_0}{u_\tau} \doteq 20}$ $\quad u_\tau = \frac{300}{20} = 15\,\text{m/s}$

$s = \frac{15 \times 1.5 \times 10^{-5}}{15} = 1.5 \times 10^{-5} = 15\,\mu\text{m}$



Gave up


Re ≃ 3000
Re ≃ 20,000
outer layer
y/δ = 0.3
y/δ = 0.1
log law region
overlap layer
buffer layer
viscous sublayer
inner layer
y+ = 50
y+ = 30
y+ = 5
y/δ
10^0
10^-1
10^-2
10^-3
10^-4
10^3
10^4
10^5
10^6
Re
log-law persists beyond the region suggested
by the overlap argument.