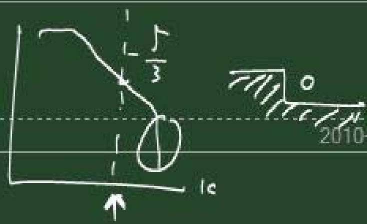
노트 제목 2010-06-03



⊙ issues related to LES

· Most flows contain non-isotropic flow regions

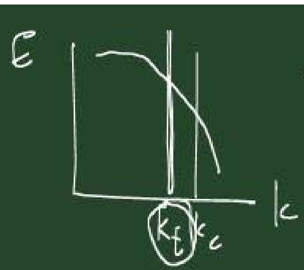
– invalid for the usage of Smagorinsky model

· Modified wavenumber of 2nd-order central difference method is bad at high wavenumbers

CD2 + LES
↓
OK.

spectral method → good for homo. flow,
cannot be applied to complex geometry

2nd-order CD (CD2)



– invalid for the concept of dynamic model
(good for spectral method)

• Upwind-type scheme may produce~~s~~ numerical dissipation larger than the subgrid-scale eddy dissipation
(Park et al JCP 2004)

– invalid for the concept of subgrid-scale eddy dissipation.

– should not use upwind-type schemes

– but sol. may be difficult to obtain

without using upwind-type schemes.

- How to evaluate good SGS model?
  (Park et al. POF 2005)

- Number of grid pts requirement (Chapman 1979) AIAA J

  $N \sim Re^{1.8}$

  $N \sim Re^{0.4}$ with wall layer modeling

  (cf. $N \sim Re^{\frac{9}{4}}$ for DNS)

- Wall modeling + LES

  DES (detached eddy simulation)

VLES (very LES)

Hybrid LES → LES w/ wall modeling

⋮

A lot to do for high Re # + complex geometry LES.

⓪ Turbulence models & Their Application in Hydraulics by Wolfgang Rodi (1984)

ch.2 Turbulence modelling

Mean-flow eqs and the problem of closure

$$\hat{u}_i = U_i + u_i \qquad \hat{p} = P + p' \qquad \hat{\phi} = \Phi + \phi'$$

inst. mean fluc.

buoyancy term

$$\begin{cases} \dfrac{\partial \hat{u}_i}{\partial x_i} = 0 \\ \dfrac{\partial \hat{u}_i}{\partial t} + \hat{u}_j \dfrac{\partial \hat{u}_i}{\partial x_j} = -\dfrac{1}{\rho_r}\dfrac{\partial \hat{p}}{\partial x_i} + \nu \nabla^2 \hat{u}_i + g_i \dfrac{\hat{\rho} - \hat{\rho}_r}{\hat{\rho}_r} \\ \dfrac{\partial \hat{\phi}}{\partial t} + \hat{u}_j \dfrac{\partial \hat{\phi}}{\partial x_j} = \lambda \nabla^2 \hat{\phi} + S_\phi \end{cases}$$

$\hat{\phi}$: temp or concentration

source

$$\rightarrow \begin{cases} \dfrac{\partial U_i}{\partial x_i} = 0 \\ \dfrac{\partial U_i}{\partial t} + U_j \dfrac{\partial U_i}{\partial x_j} = -\dfrac{1}{\rho_r}\dfrac{\partial P}{\partial x_i} + \dfrac{\partial}{\partial x_j}\left(\nu \dfrac{\partial U_i}{\partial x_j} - \overline{u_i u_j}\right) + g_i \dfrac{\rho - \rho_r}{\rho_r} \\ \dfrac{\partial \Phi}{\partial t} + U_j \dfrac{\partial \Phi}{\partial x_j} = \dfrac{\partial}{\partial x_j}\left(\lambda \dfrac{\partial \Phi}{\partial x_j} - \overline{u_j \phi}\right) + S_\phi \end{cases}$$

- Eddy-viscosity concept : Boussinesq (1877)

$$-\overline{u_i u_j} = \nu_t \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) - \frac{2}{3} k \delta_{ij}$$

ν_t : turb. viscosity (eddy ")

$$\frac{\partial}{\partial x_j}\left(-\overline{u_i u_j}\right) = \frac{\partial}{\partial x_j}\left[\nu_t\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) - \frac{2}{3}k\delta_{ij}\right]$$

$$-\frac{\partial}{\partial x_i}\left(\frac{P}{\rho_r} + \frac{2}{3}k\right) \leftarrow -\frac{\partial}{\partial x_i}\left(\frac{2}{3}k\right)$$

∴ the appearance of k does not need the determination of k.

⇒ determine ν_t. $\nu_t \propto \tilde{V} L$

not fluid ← flow property

vel. scale

length scale

$$\underbrace{-\overline{u_i u_j}}_{\text{tensor}} = \underbrace{\nu_t}_{\text{scalar}} \underbrace{\left(\frac{\partial U_i}{\partial x_j} + \cdots\right)}_{\text{tensor}} - \frac{2}{3}k\delta_{ij}$$

↳ isotropic → not true in reality

→ limitation of this approach.

→ Anyway, this approach is the basis of most turb. models in use today.

- Eddy-diffusivity

$$-\overline{u_j \phi} = \Gamma \frac{\partial \phi}{\partial x_j} \qquad \Gamma\text{: turb. diffusivity}$$

$$\Gamma \equiv \frac{\nu_t}{\sigma_t} \qquad \sigma_t = \frac{\nu_t}{\Gamma}\text{ : turb. Prandtl number (or Schmidt)}$$

exps. → σ_t varies only little across flow and also little from flow to flow.

⇒ many models assume σ_t as a constant.
Buoyancy & streamline curvature affect the value of σ_t.

$\nu_t \sim \hat{V} \cdot L$

- 0-eq model : no diff'l eq.
- 1-eq " : diff'l eq for $\hat{V}$
- 2-eq " : two " "s for $\hat{V}$ & L.

other models

- Zero-equation models

mixing-length models (Prandtl 1925)

$\nu_t \propto \hat{V} \; L$ ← mixing length l_m

($\hat{V}$: mean fluct. vel. scale)

v shear layer 

$-\overline{uv}$ $\quad \dfrac{\partial U}{\partial y}$

↳ $\hat{V} = l_m \left| \dfrac{\partial U}{\partial y} \right|$

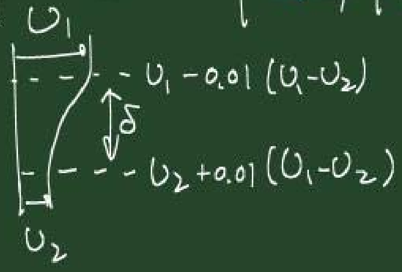
→ $\nu_t = l_m^2 \left| \dfrac{\partial U}{\partial y} \right|$: Prandtl mixing-length hypothesis.

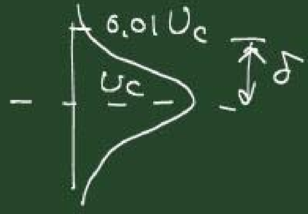
↳ l_m unknown

↳ specified by simple empirical formulae

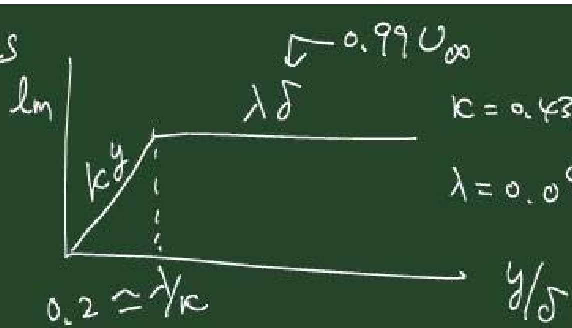
In free shear layers, $l_m \sim$ const across the layer and $l_m \propto \delta(x)$

Flow	Plane mixing layer	plane jet	Round jet	Radial jet	Plane wake
l_m/δ	0.07	0.09	0.075	0.125	0.16





In wall bdry layers



$\kappa = 0.435$

$\lambda = 0.09$

very close to the wall,

$$l_m = \kappa y \left[1 - \exp\left(-y^+/A^+\right)\right], \quad A^+ = 26$$

van Driest's damping ft.

General flows

$$\nu_t = l_m^2 \left[\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) \frac{\partial U_i}{\partial x_j}\right]^{\frac{1}{2}} \Leftarrow \text{difficult to define } l_m.$$

Heat & mass transfer

$\sigma_t \simeq 0.9$ near-wall flows

0.5 plane jets & mixing layers

0.7 round jets.

$\boxed{\text{June 14}}$ – <u>6 pm</u> : final exam.