Nonlinear Optical Engineering

Second-Harmonic Generation (1)

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Orthogonality of Modes

Maxwell's equations:

$$\nabla\times\mathbf{E}+\frac{\partial\mathbf{B}}{\partial t}=0$$

$$\nabla\times\mathbf{H}-\frac{\partial\mathbf{D}}{\partial t}=\mathbf{J}$$

$$\nabla\cdot\mathbf{D}=\rho$$

$$\nabla\cdot\mathbf{B}=0$$

Constitutive relations:

$$\mathbf{D}=\varepsilon\mathbf{E}=\varepsilon_o\mathbf{E}+\mathbf{P}$$

$$\mathbf{B}=\mu\mathbf{H}=\mu_o(\mathbf{H}+\mathbf{M})$$

Lorentz reciprocity theorem:

$$\nabla\cdot(\mathbf{E}_q\times\mathbf{H}_p^*+\mathbf{E}_p^*\times\mathbf{H}_q)\quad(p,q=1,2,3,\ldots)$$ $\leftarrow$ Eigenmodes

$$=\mathbf{H}_p^*\cdot(\nabla\times\mathbf{E}_q)-\mathbf{E}_q\cdot(\nabla\times\mathbf{H}_p^*)+\mathbf{H}_q\cdot(\nabla\times\mathbf{E}_p^*)-\mathbf{E}_p^*\cdot(\nabla\times\mathbf{H}_q)$$

$$=-\mathbf{H}_p^*\cdot\frac{\partial\mathbf{B}_q}{\partial t}-\mathbf{E}_q\cdot\frac{\partial\mathbf{D}_p^*}{\partial t}-\mathbf{H}_q\cdot\frac{\partial\mathbf{B}_p^*}{\partial t}-\mathbf{E}_p^*\cdot\frac{\partial\mathbf{D}_q}{\partial t}$$ $\leftarrow$ Monochromatic radiation

$$=-i\omega(H_{p,i}^*\mu_{ij}H_{q,j}-E_{q,i}\varepsilon_{ij}^*E_{p,j}^*-H_{q,i}\mu_{ij}^*H_{p,j}^*+E_{p,i}^*\varepsilon_{ij}E_{q,j})$$

$$=-i\omega(H_{p,i}^*\mu_{ij}H_{q,j}-E_{q,j}\varepsilon_{ji}^*E_{p,i}^*-H_{q,j}\mu_{ji}^*H_{p,i}^*+E_{p,i}^*\varepsilon_{ij}E_{q,j})$$

$$=0$$

$\rightarrow$ Note: Source free and Hermitian tensors of ε and μ

Coupled-Mode Equations

Perturbation in permittivity:

$$\varepsilon' = \varepsilon + \Delta\varepsilon, \quad \mu' = \mu \quad \leftarrow \text{Perturbed medium}$$

$$\mathbf{E}', \mathbf{H}' \quad \leftarrow \text{Perturbed fields}$$

Coupled-mode equations:

$$\begin{aligned}
&\nabla \cdot (\mathbf{E}' \times \mathbf{H}_p^* + \mathbf{E}_p^* \times \mathbf{H}') \quad (p = 1, 2, 3, \ldots) \\
&= \mathbf{H}_p^* \cdot (\nabla \times \mathbf{E}') - \mathbf{E}' \cdot (\nabla \times \mathbf{H}_p^*) + \mathbf{H}' \cdot (\nabla \times \mathbf{E}_p^*) - \mathbf{E}_p^* \cdot (\nabla \times \mathbf{H}') \\
&= -\mathbf{H}_p^* \cdot \frac{\partial \mathbf{B}'}{\partial t} - \mathbf{E}' \cdot \frac{\partial \mathbf{D}_p^*}{\partial t} - \mathbf{H}' \cdot \frac{\partial \mathbf{B}_p^*}{\partial t} - \mathbf{E}_p^* \cdot \frac{\partial \mathbf{D}'}{\partial t} \\
&= -i\omega(\mathbf{H}_p^* \cdot \mathbf{B}' - \mathbf{E}' \cdot \mathbf{D}_p^* - \mathbf{H}' \cdot \mathbf{B}_p^* + \mathbf{E}_p^* \cdot \mathbf{D}') \\
&= -i\omega \mathbf{E}_p^* \Delta\varepsilon \mathbf{E}' = -i\omega \mathbf{E}_p^* \Delta \mathbf{P}'
\end{aligned}$$

$$\leftarrow \mathbf{D}' = \varepsilon' \mathbf{E}' = (\varepsilon + \Delta\varepsilon)\mathbf{E}' = \varepsilon \mathbf{E}' + \Delta \mathbf{P}', \quad \mathbf{B}' = \mu \mathbf{H}'$$

In result:

$$\nabla \cdot (\mathbf{E}' \times \mathbf{H}_p^* + \mathbf{E}_p^* \times \mathbf{H}') = -i\omega \mathbf{E}_p^* \Delta\varepsilon \mathbf{E}' \quad (p = 1, 2, 3, \ldots)$$

Coupled-Mode Equations

Perturbed fields:

$$\begin{pmatrix} \mathbf{E}' \\ \mathbf{H}' \end{pmatrix} \equiv \sum_q a_q(z) \begin{pmatrix} \mathbf{E}_q \\ \mathbf{H}_q \end{pmatrix} = \sum_q a_q(z) \begin{pmatrix} \mathbf{e}_q(x,y)\exp(-i\beta_q z) \\ \mathbf{h}_q(x,y)\exp(-i\beta_q z) \end{pmatrix}$$

Coupled-mode equations:

$$\sum_q a_q(z)\nabla\cdot(\mathbf{E}_q \times \mathbf{H}_p^* + \mathbf{E}_p^* \times \mathbf{H}_q) + \sum_q \frac{da_q(z)}{dz}(\mathbf{E}_q \times \mathbf{H}_p^* + \mathbf{E}_p^* \times \mathbf{H}_q)\cdot\hat{z}$$

$$= -i\omega \sum_q a_q(z)\mathbf{E}_p^* \cdot \Delta\varepsilon(x,y,z)\mathbf{E}_q$$

$$\leftarrow \ \frac{1}{4}\int (\mathbf{E}_q \times \mathbf{H}_p^* + \mathbf{E}_p^* \times \mathbf{H}_q)\cdot\hat{z}\ dxdy = \begin{cases} 1, & p = q \\ 0, & p \neq q \end{cases}$$

In result:

$$\frac{da_p(z)}{dz}\exp(-i\beta_p z) = -i\frac{\omega}{4}\sum_q a_q(z)\exp(-i\beta_q z)\int \mathbf{e}_p^*(x,y)\cdot\Delta\varepsilon(x,y,z)\mathbf{e}_q(x,y)dxdy$$

Two-Mode Coupling Approximation (TMCA)

Coupled equations:

$$\frac{da_p(z)}{dz} = -i\frac{|\beta_p|}{\beta_p}\sum_q \kappa_{pq}(z)a_q(z)\exp[-i(\beta_q-\beta_p)z], \quad p=1,2,3,\ldots$$

$$\leftarrow \ \kappa_{pq}(z) = \frac{\pi c}{2\lambda}\iint \mathbf{e}_p^*(x,y)\cdot\Delta\varepsilon(x,y,z)\mathbf{e}_q(x,y)\,dxdy$$

Two-mode coupling approximation:

Consider only for the dominant two modes and their coupling constants which can satisfy the longitudinal phase matching condition

Nonlinear Perturbation

Coupled-mode theory with nonlinear perturbation terms:

$$\nabla\cdot(\mathbf{E}'_{\omega}\times\mathbf{H}^{*}_{\omega,p}+\mathbf{E}^{*}_{\omega,p}\times\mathbf{H}'_{\omega})=-i\omega\mathbf{E}^{*}_{\omega,p}\cdot\Delta\mathbf{P}'_{\omega},\quad (p=1,2,\ldots),$$

$$\rightarrow\ \Delta\mathbf{P}'_{\omega_1}=\Delta\varepsilon_{ij}(\omega_1)E'_{\omega_1,j}+2d_{ijk}(-\omega_1,\omega_2,-\omega_1)E'_{\omega_2,j}E'^{*}_{\omega_1,k}$$
$$+\left\{3\chi_{ijkl}(-\omega_1,\omega_1,-\omega_1,\omega_1)E'_{\omega_1,j}E'^{*}_{\omega_1,k}+6\chi_{ijkl}(-\omega_1,\omega_2,-\omega_2,\omega_1)E'_{\omega_2,j}E'^{*}_{\omega_2,k}\right\}E'_{\omega_1,l},$$

$$\rightarrow\ \Delta\mathbf{P}'_{\omega_2}=\Delta\varepsilon_{ij}(\omega_2)E'_{\omega_2,j}+d_{ijk}(-\omega_2,\omega_1,\omega_1)E'_{\omega_1,k}E'_{\omega_1,k}$$
$$+\left\{3\chi_{ijkl}(-\omega_2,\omega_2,-\omega_2,\omega_2)E'_{\omega_2,j}E'^{*}_{\omega_2,k}+6\chi_{ijkl}(-\omega_2,\omega_1,-\omega_1,\omega_2)E'_{\omega_1,j}E'^{*}_{\omega_1,k}\right\}E'_{\omega_2,l}.'$$

$\leftarrow$ With second- and third-order nonlinear susceptibilities

Second-Harmonic Generation

Plane waves:

$$E^{\omega_j}(t,z) = E_j(z)\exp[i(\omega_j t - k_j z)] \quad (j = 1, 2,\ \omega_2 = 2\omega_1)$$

Second-order nonlinear coupled-wave equations:

$$\frac{dE_1}{dz} = -i\frac{\omega_1}{\varepsilon_o n_1 c} d_1 E_2 E_1^* \exp(-i\Delta k z)$$

$$\frac{dE_2}{dz} = -i\frac{\omega_2}{2\varepsilon_o n_2 c} d_2 E_1^2 \exp(i\Delta k z) \qquad \leftarrow \ \Delta k = k_2 - 2k_1$$

Phase-matching condition: $\Delta k = k_2 - 2k_1 = 0$

Birefringence: Type I or Type II

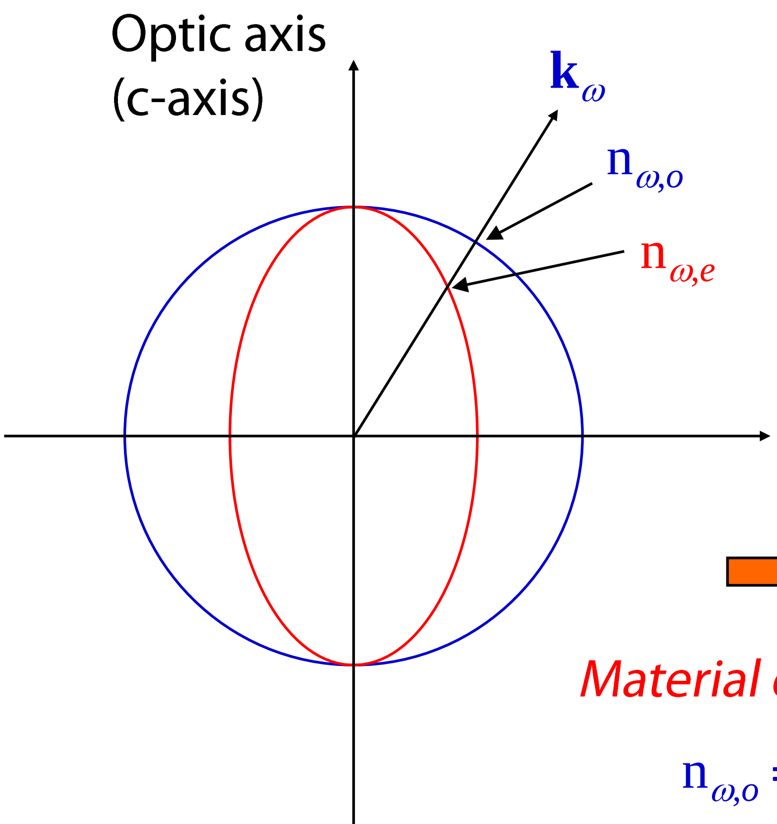
Modal dispersion: Waveguide modes

Quasi-phase matching: Modulation of nonlinearity

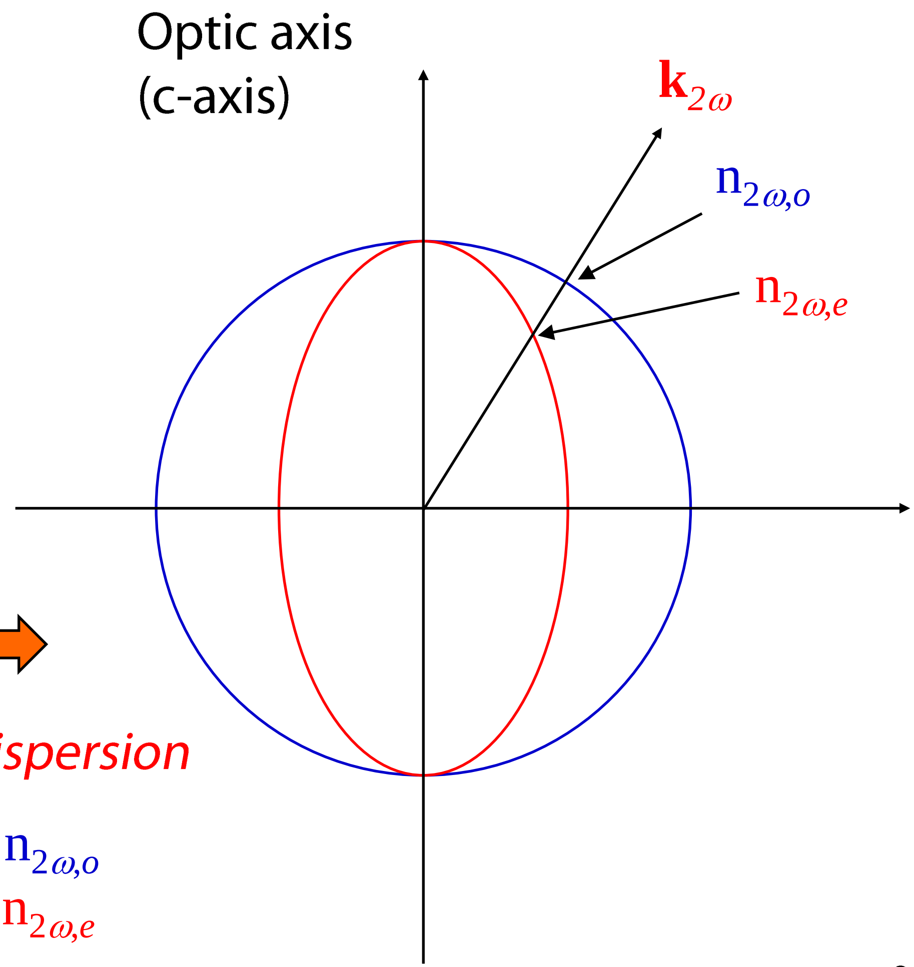
Phase Matching in Birefringent Media (1)

Example: Negative uniaxial crystal

For fundamental harmonic:



For second harmonic:



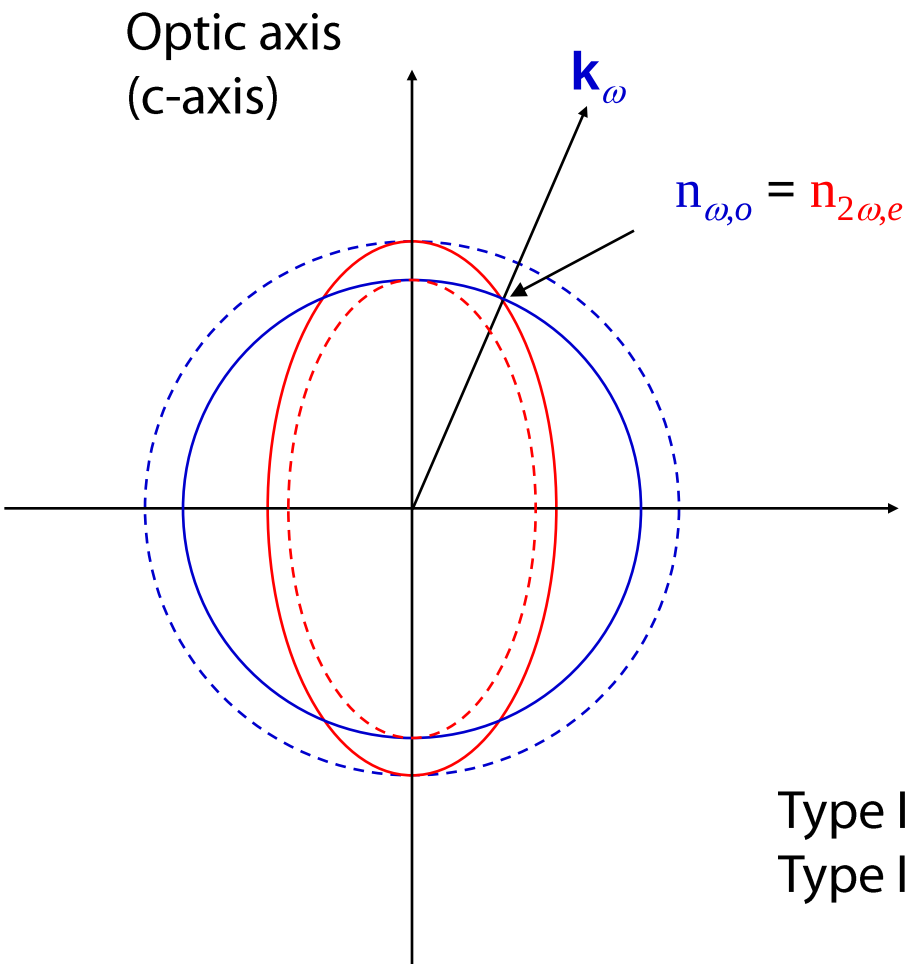
Material dispersion

$n_{\omega,o} \neq n_{2\omega,o}$

$n_{\omega,e} \neq n_{2\omega,e}$

Phase Matching in Birefringent Media (2)

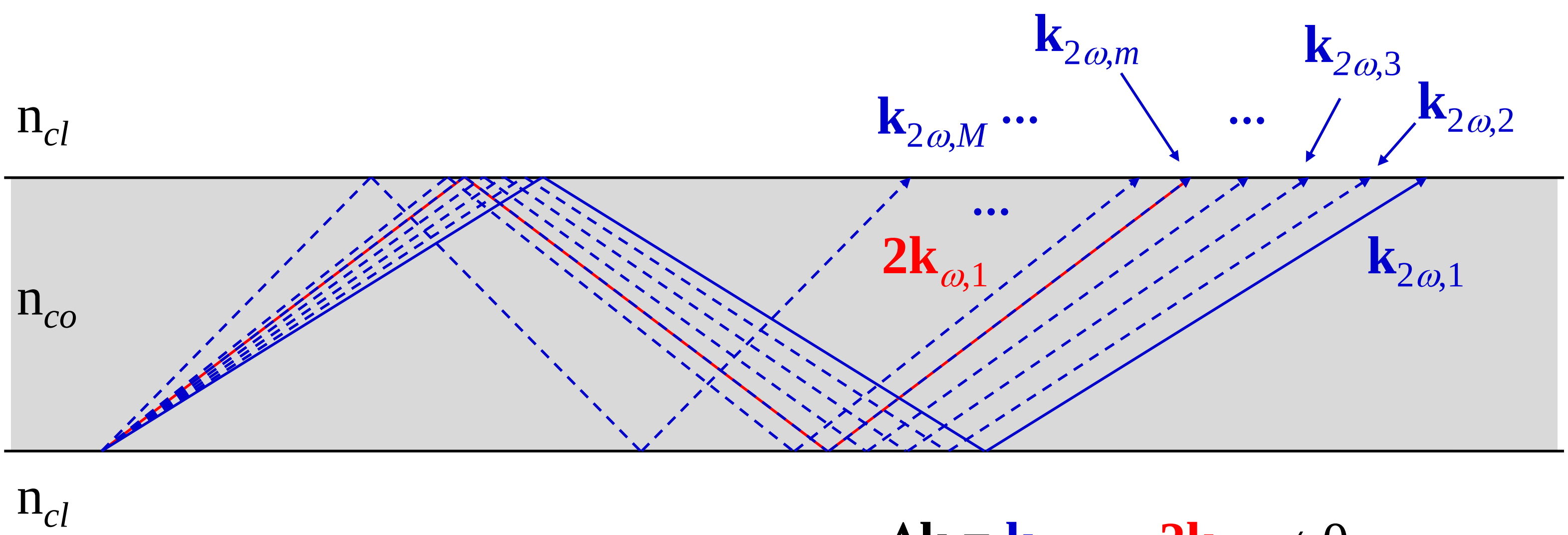
Example: Negative uniaxial crystal



Type I: ooe, eeo
Type II: oeo, oee

Phase Matching in Waveguides

Example: Dispersive waveguide



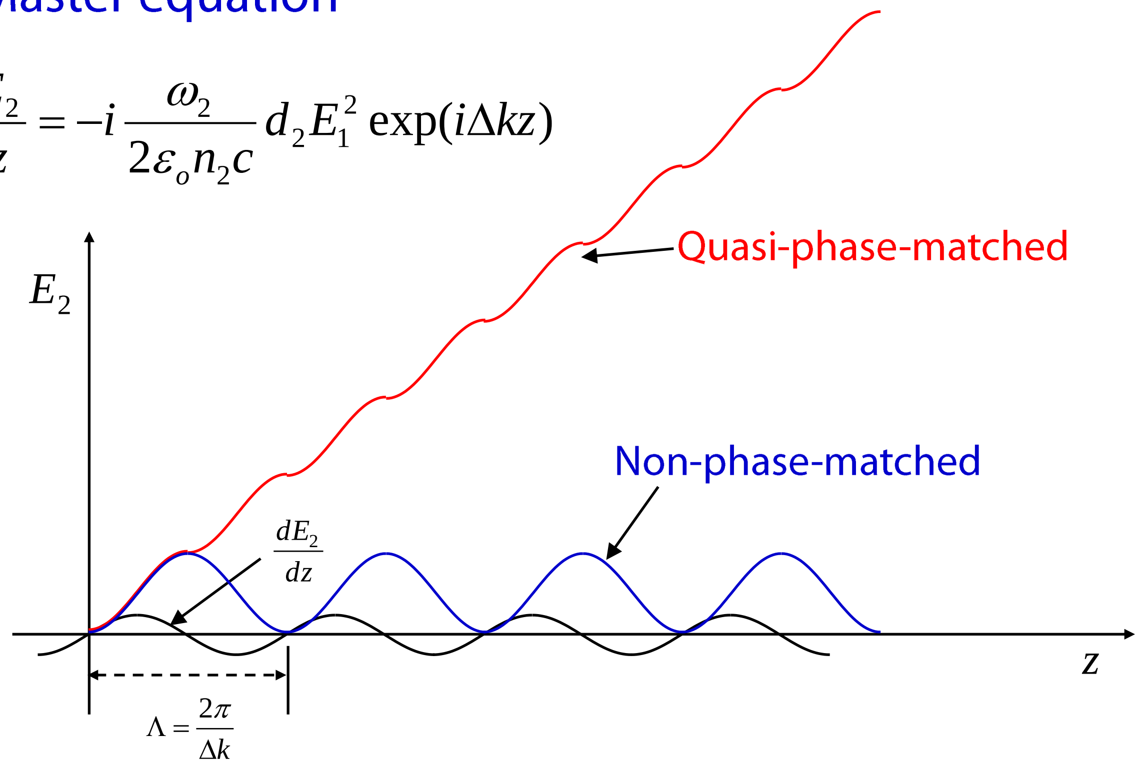
$\Delta\mathbf{k} = \mathbf{k}_{2\omega,1} - \mathbf{2k}_{\omega,1} \neq 0$

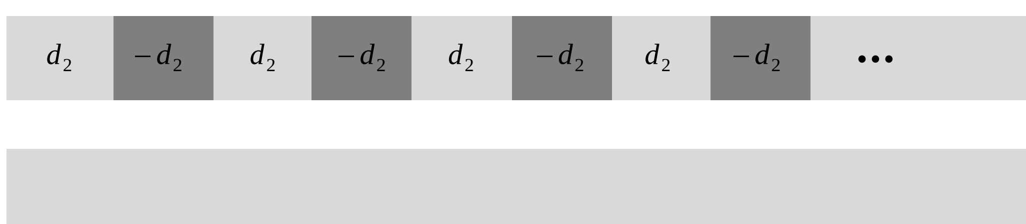
$\Delta\mathbf{k} = \mathbf{k}_{2\omega,m} - \mathbf{2k}_{\omega,n} = 0$, i.e. the phase-matching condition can be satisfied for some set of modes of *m* and *n, exploiting the modal dispersion in waveguides.*

Quasi-Phase Matching

Recall: Master equation

$$\frac{dE_2}{dz} = -i\,\frac{\omega_2}{2\varepsilon_o n_2 c}\,d_2 E_1^2 \exp(i\Delta k z)$$





Quasi-phase-matched

Non-phase-matched

→ Periodic spatial modulation of the nonlinear coefficient: e.g. PPLN (periodically-poled lithium niobate)