13. Prestressed Concrete

INTRODUCTION
EFFECTS OF PRESTRESSING
PRESTRESSING METHODS
PRESTRESSING STEEL
CONCRETE
ELASTIC FLEXURAL ANALYSIS
FLEXURAL STRENGTH
PARTIAL PRESTRESSING
FLEXURAL DESIGN BASED ON ALLOWABLE STRESS
LOSS OF PRESTRESS

M1586.002400
Planning of Structure System

6. Serviceability

Effective moment of inertia I_e

$$I_{cr} < I_e < I_{ut} \tag{15}$$

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_{ut} + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \leq I_{ut} \tag{16}$$

In RC, whole cross section cannot be utilized for reducing the deflection due to cracking

- Significant deflection in long-span bridge
- Increase of dead load due to using large cross section

⇨ How to maintain long-span RC section not cracked ??

13. Prestressed Concrete

INTRODUCTION

Prestressed concrete members can be defined as one that has internal stresses induced to balance out stresses due to externally loads to a desired degree.

Prestressing applies a precompression to the member that reduces or eliminates undesirable tensile stresses that would otherwise be present.

⇨ less cracks, less diagonal tension stresses, less deflection, smaller section, less dead weight, longer span

⇦ High-strength material and improved design tech.

INTRODUCTION

Eugene Freyssinet (1879～1962)

French Structural Engineer

Inventor of Prestressed Concrete

Pont Saint-Michel (Toulouse)

Pont de la Libération (France,1919)

13. Prestressed Concrete

INTRODUCTION

Pont de Veurdre (72.5m)

PSC arch bridge built in 1911 by Freyssinet

Pont de Grafton (97.6m)

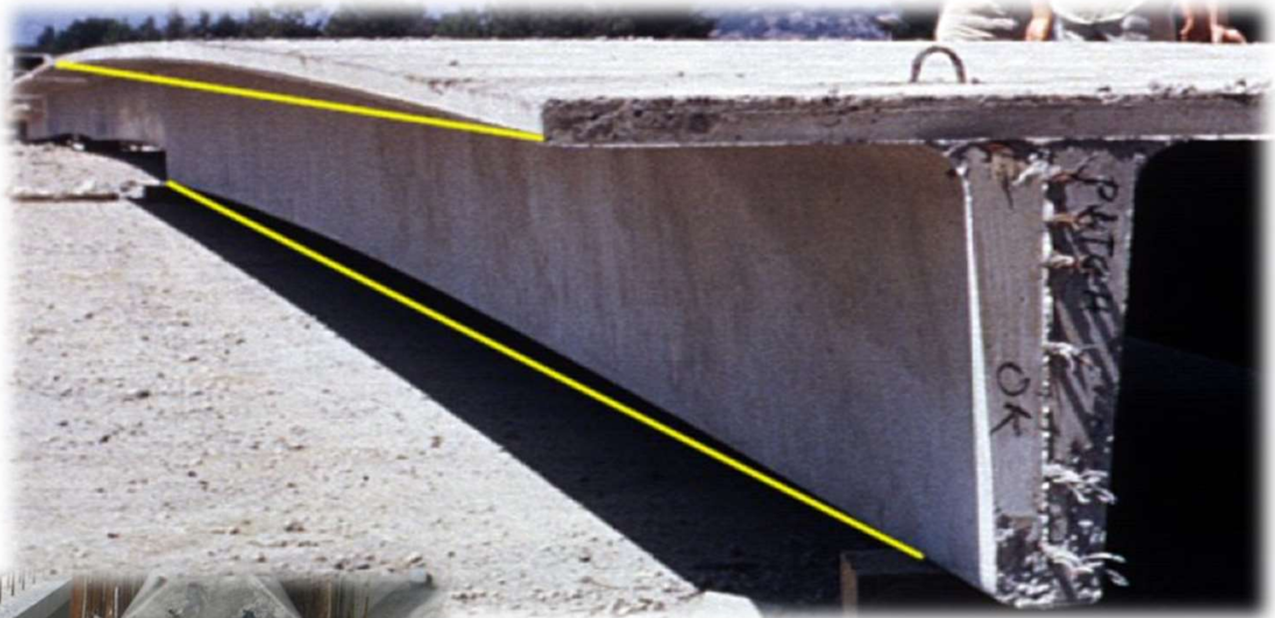
RC arch bridge built in 1910

Thickness and height/span ratio of arch rib are dramatically decreased

⇨ Due to an optimized usage of whole RC cross-sections

13. Prestressed Concrete

INTRODUCTION



INTRODUCTION

BUT the followings should be considered.

1) The higher unit cost of stronger materials.
2) The needs for expensive accessories.
3) The necessity for close inspection and quality control
4) In the case of precasting, a higher initial investment in plant.

13. Prestressed Concrete

PRESTRESSING STEEL

Essential parts for inducing PRESTRESS

- Tensile stress is induced before concrete placing
- Tensile strength is significantly larger than ordinary steel reinforcement

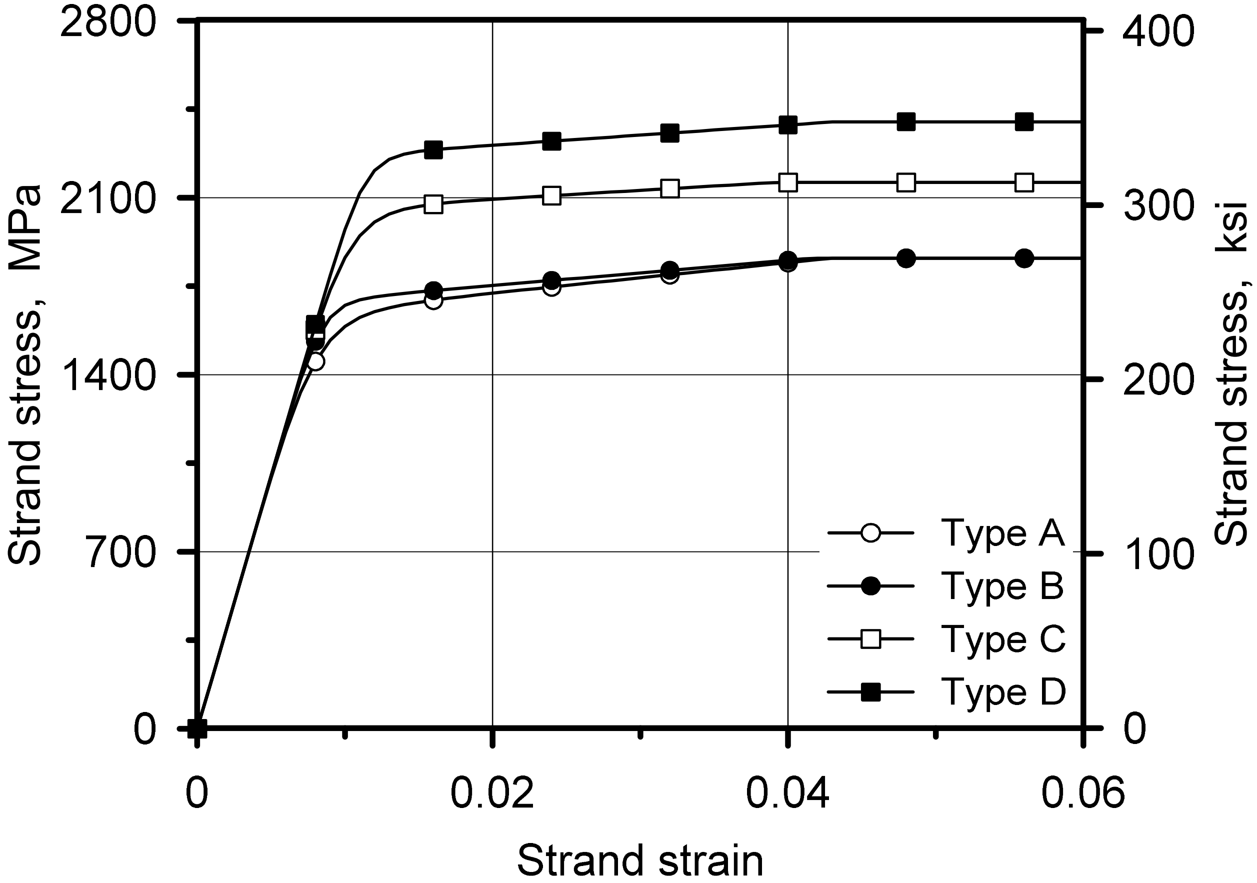
  ⇨ Recently, 1860 & 2140 MPa strands are developed

7 Wire strands

DYWIDAG threaded bar

13. Prestressed Concrete

PRESTRESSING STEEL



Where is yielding point??

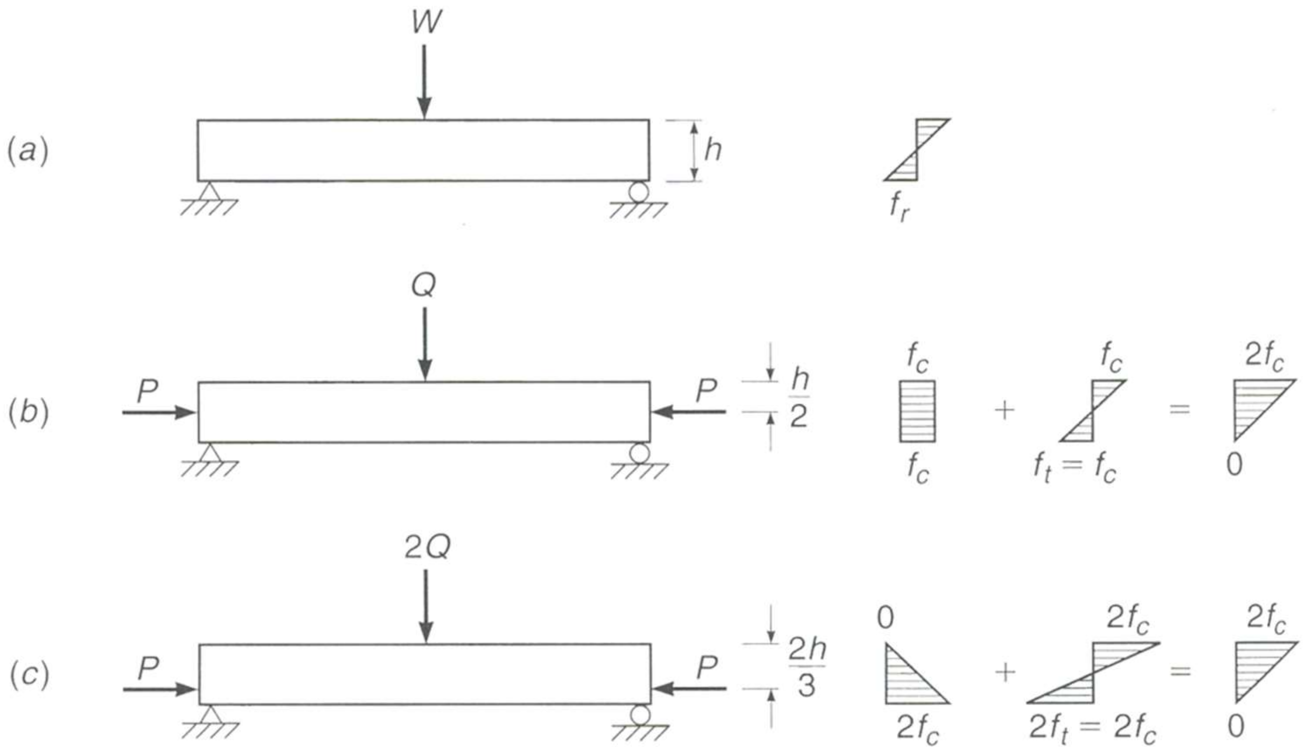
EFFECTS OF PRESTRESSING

Alternative schemes for prestressing a rectangular beam

Plane concrete beam

Axially prestressed beam

Eccentrically prestressed beam

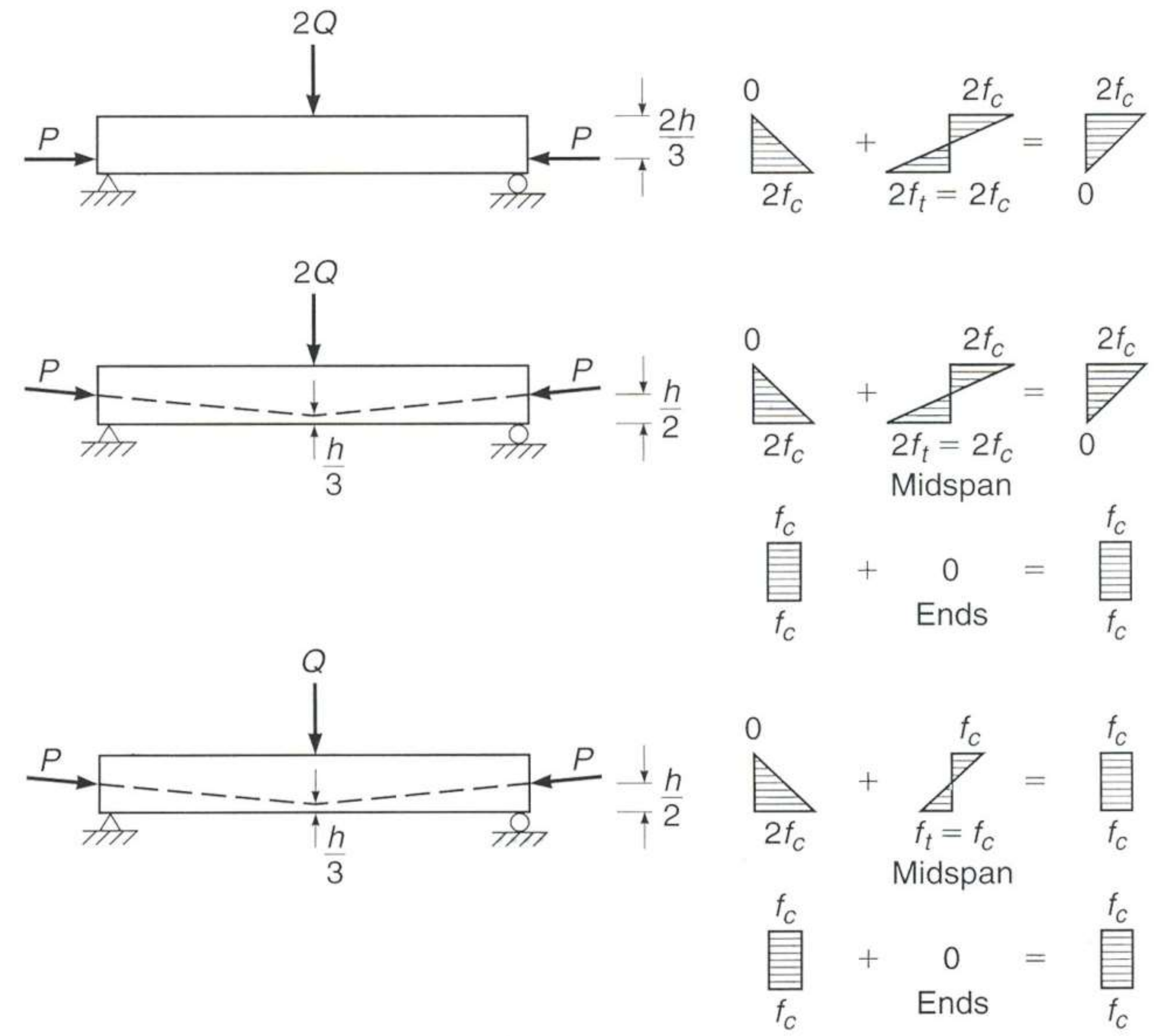


EFFECTS OF PRESTRESSING

Eccentrically prestressed beam (c)

Prestressed beam with variable eccentricity (d)

Balanced load stage for beam with variable eccentricity (e)



EFFECTS OF PRESTRESSING

The best tendon profile

; produces a prestress moment diagram that corresponds to that of the applied load.

; If the prestress counter-moment is made exactly equal and opposite to the load-induced moment, axial compressive stress is uniform all along the span.

(See figure (e) again)

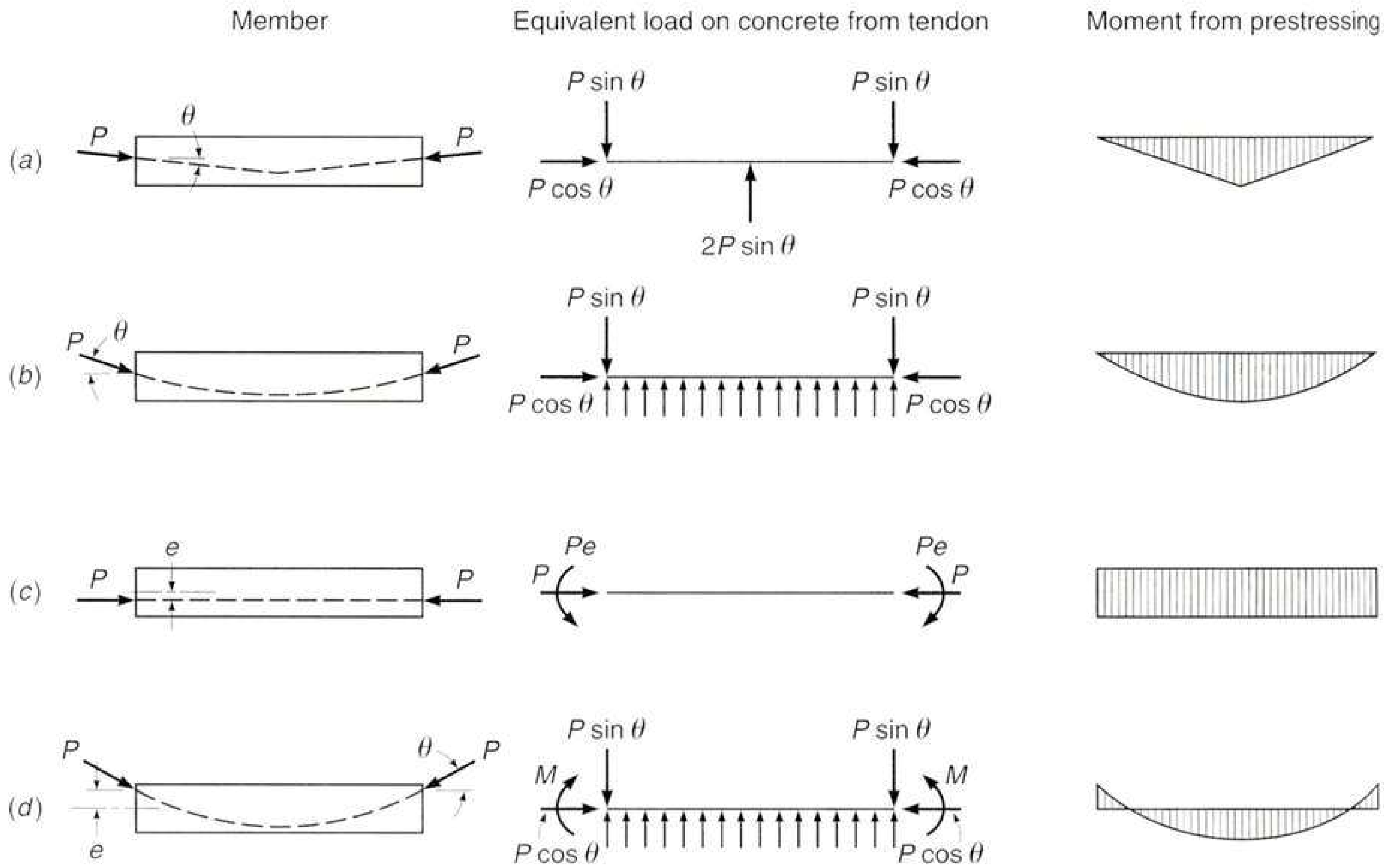
EFFECTS OF PRESTRESSING

Note

1. Prestressing can control or even eliminate concrete tensile stress for specified loads.
2. Eccentric prestress is usually much more efficient than concentric prestress.
3. Variable eccentricity is usually preferable to constant eccentricity, from the view point of both stress control and deflection control.

EFFECTS OF PRESTRESSING

Equivalent loads and moments produced by prestressing



EFFECTS OF PRESTRESSING

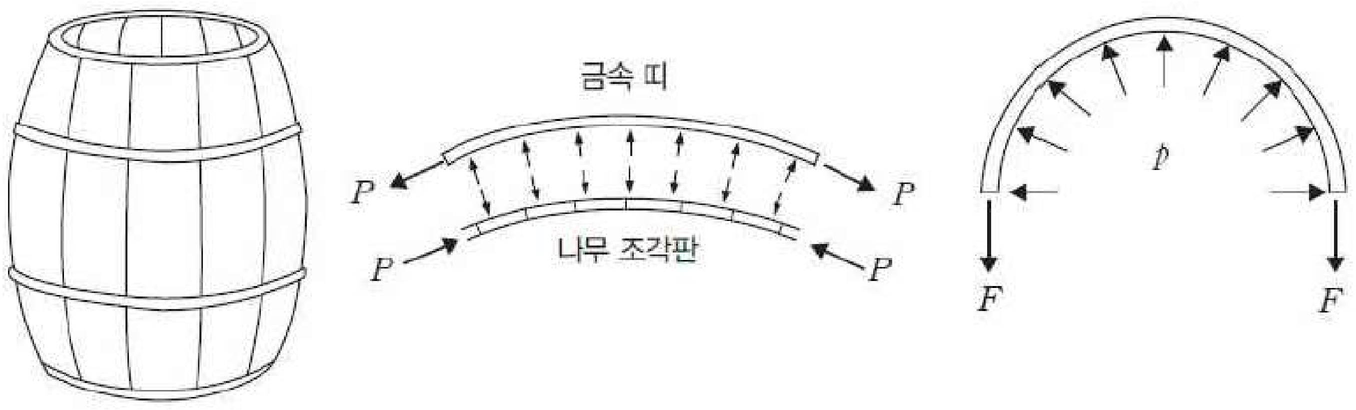
It may be evident that for any arrangement of applied loads, a tendon profile can be selected so that the equivalent loads acting on the beam from the tendon are just equal and opposite to the applied loads

⇨ pure compressive stress in concrete

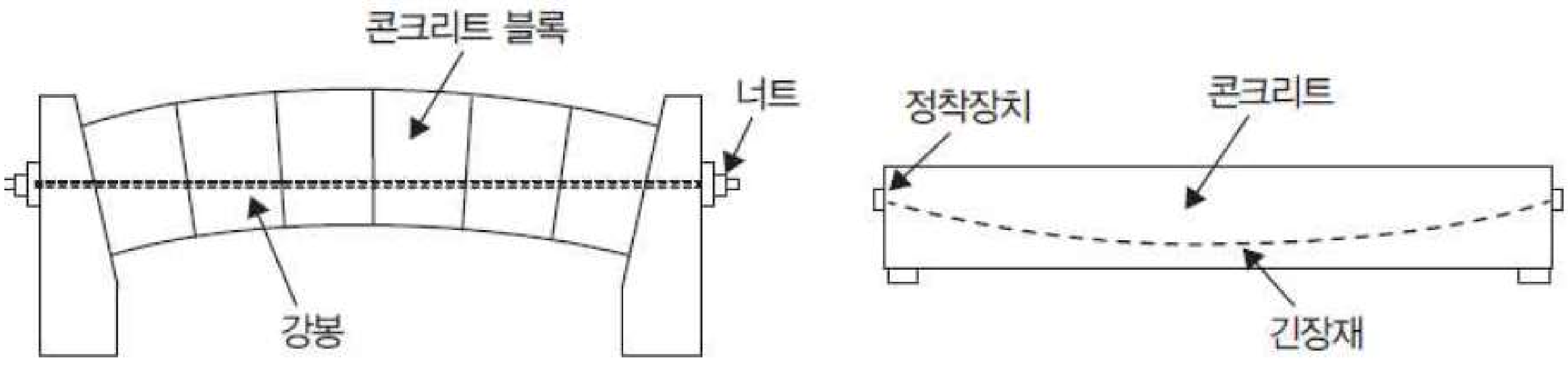
An advantage of the equivalent load concept is that it leads the designer to select what is the best tendon profile for a particular loading.

⇨ Tendon profile needs not to be straight and linear

TYPE OF PRESTRESSING



(a) 통의 형상 (b) 힘의 평형 (c) 후프 응력과 힘



(a) Jackson의 특허[1.8] (b) 현대의 포스트텐션 부재

13. Prestressed Concrete

MODERN APPLICATIONS of PSC


(a) 플랫 플레이트 건물(Freyssinet)


(b) 주차장 건물


(e) 인천대교


(f) 해양 구조물(노르웨이, VSL)

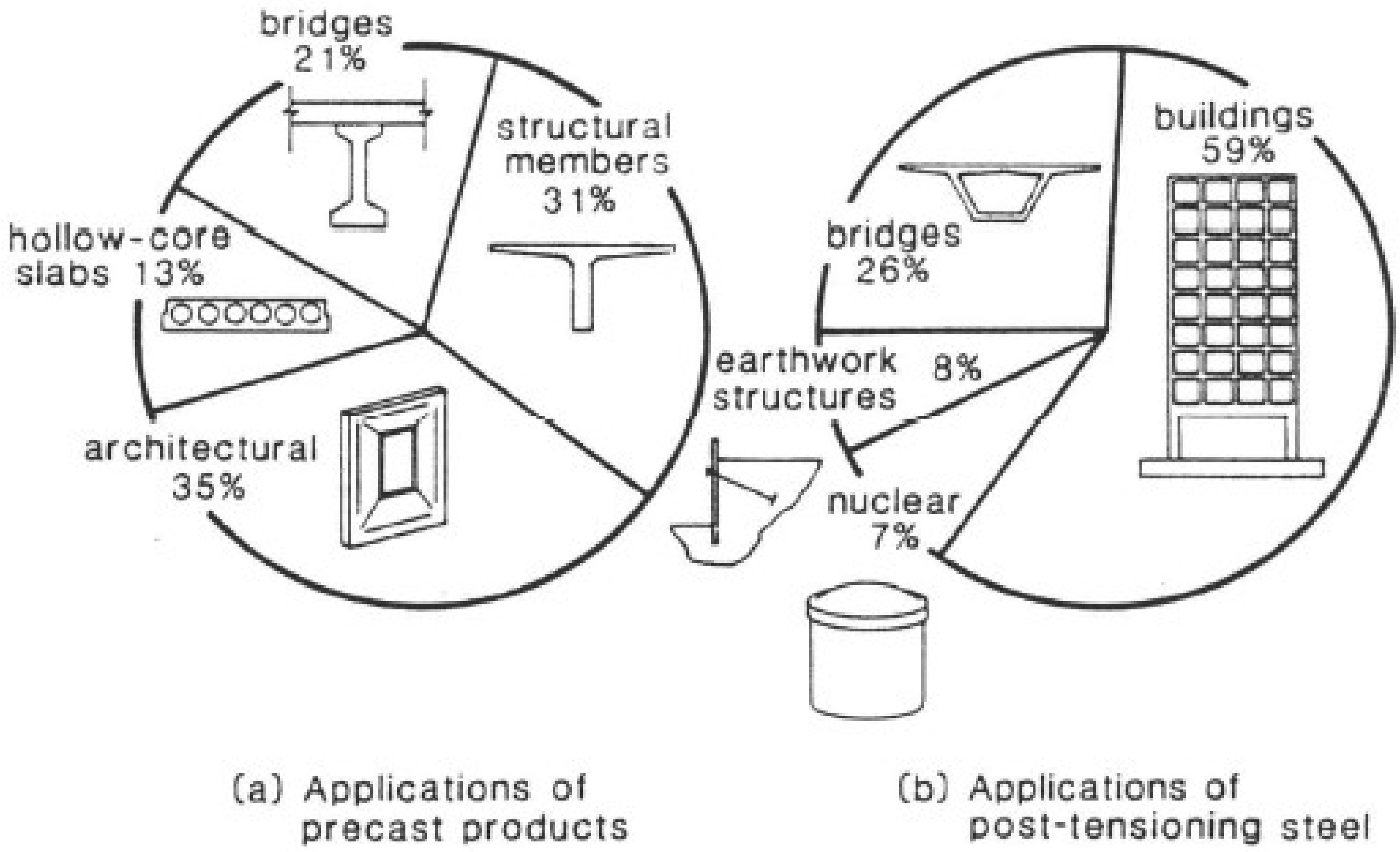

(c) 원효대교


(d) 서해대교


(g) 신고리 원자력발전소

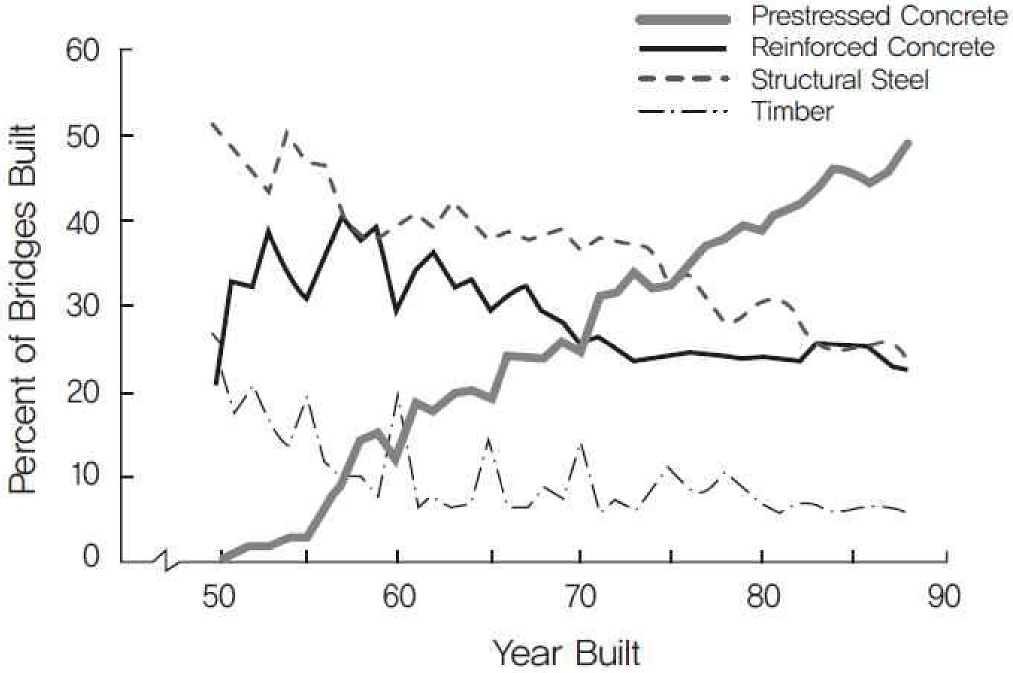

(h) 하수처리 탱크(독일, VSL)

MODERN APPLICATIONS of PSC

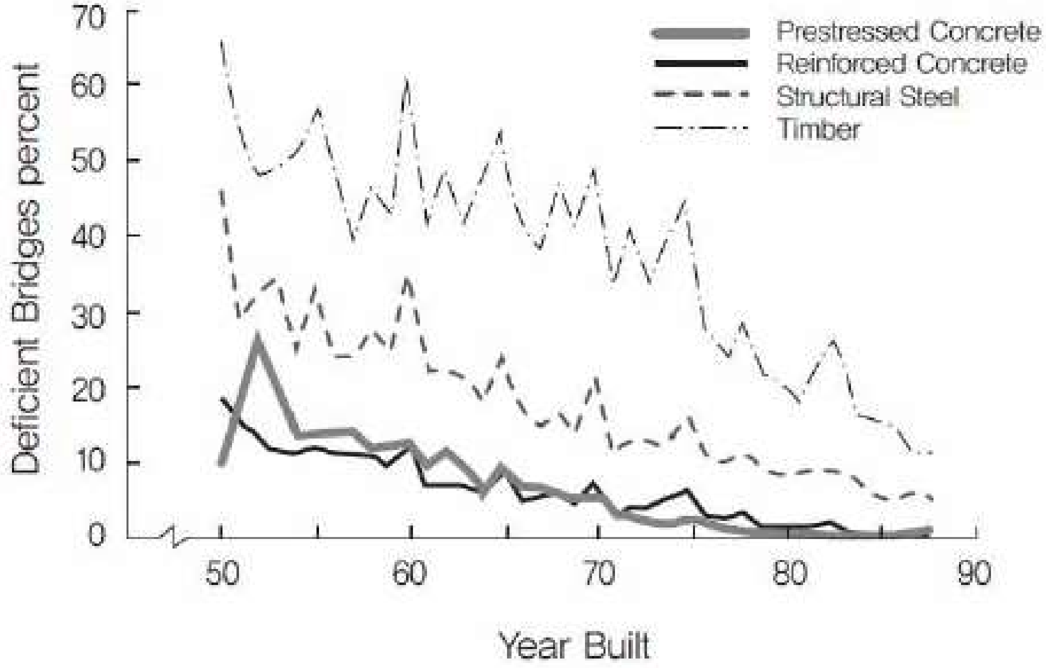


Applications of prestressed concrete in North America

ECONOMIC ADVANTAGES of PSC



(a) 연도별 형식별 건설 현황



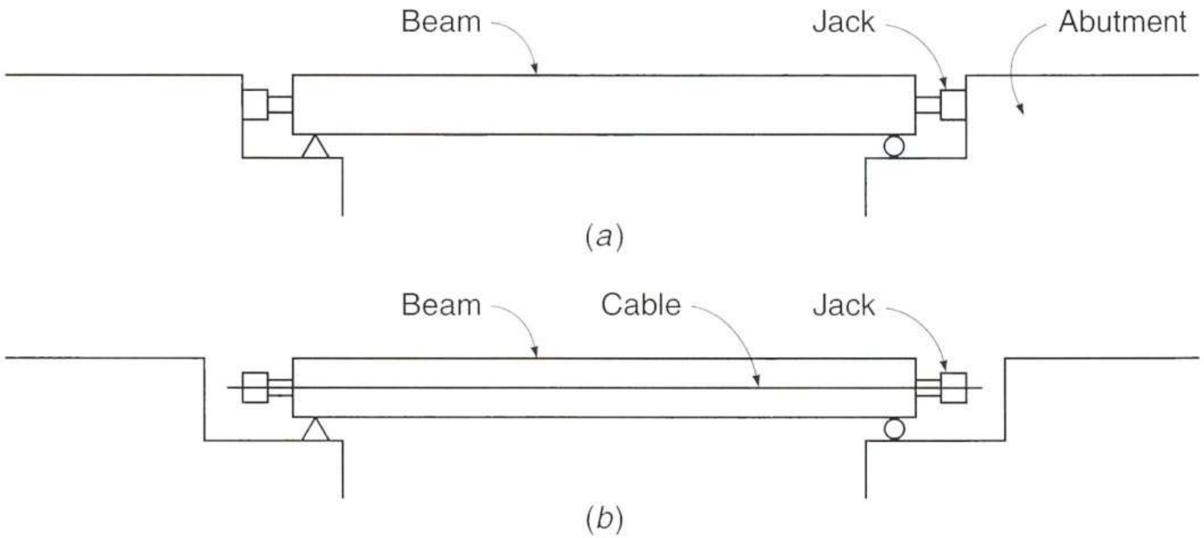
(b) 준공 연도별 노후화 현황

PRESTRESSING METHODS

1) Post-tensioning

The tendons are tensioned after the concrete is placed and has gained their strength.

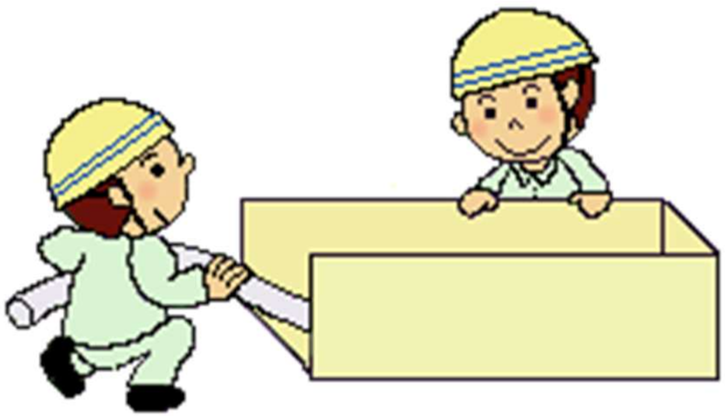
A significant advantage of all post-tensioning schemes is the ease with which the tendon eccentricity can be carried along the span.



PRESTRESSING METHODS

1) Post-tensioning

① Placing of Sheath



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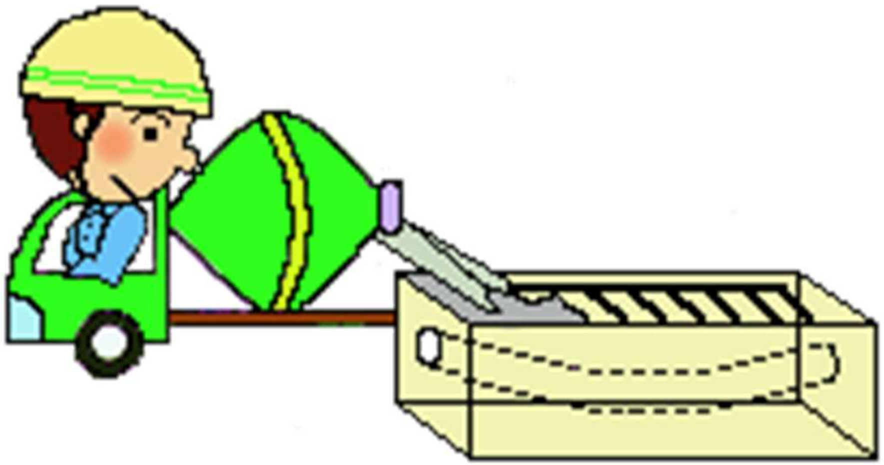
PRESTRESSING METHODS

1) Post-tensioning

② **Concrete Pouring**

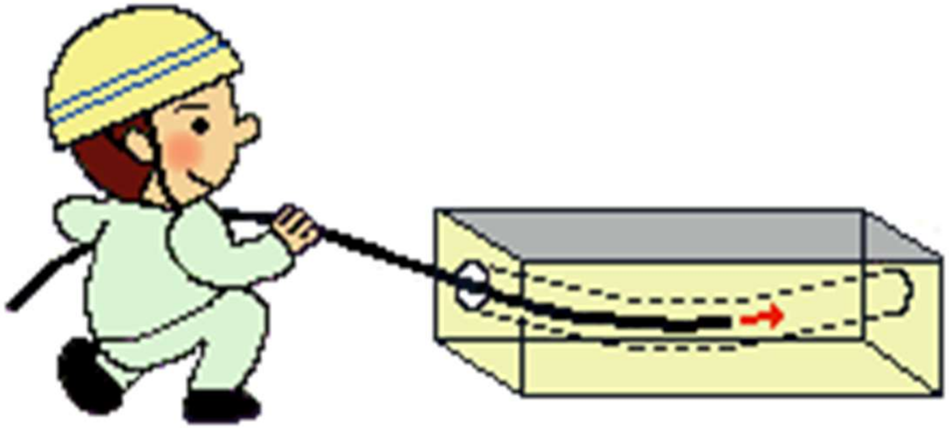


13. Prestressed Concrete

PRESTRESSING METHODS

1) Post-tensioning

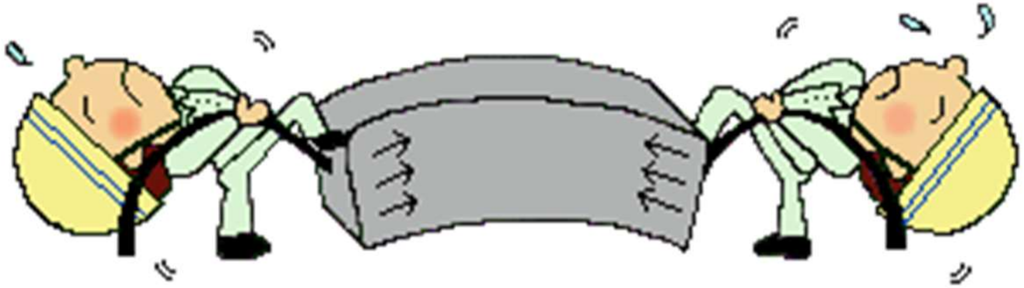
③ Installation of Tendons

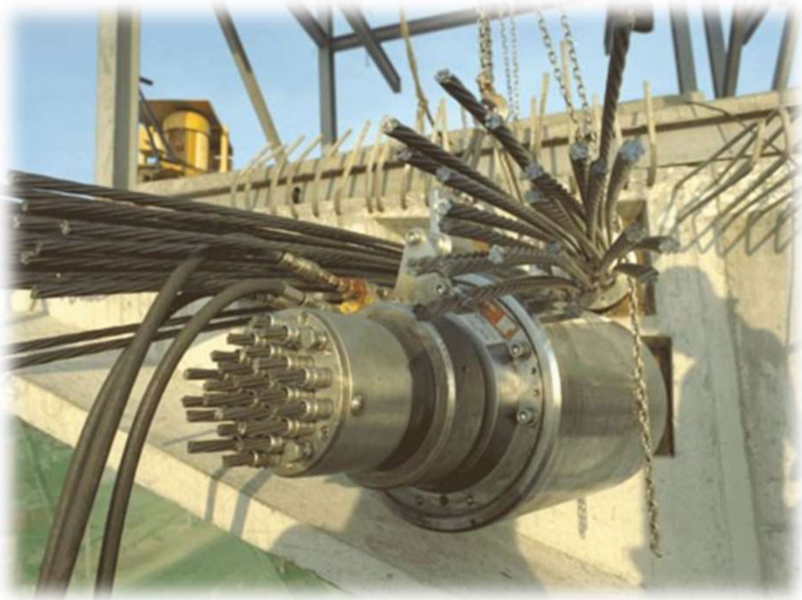


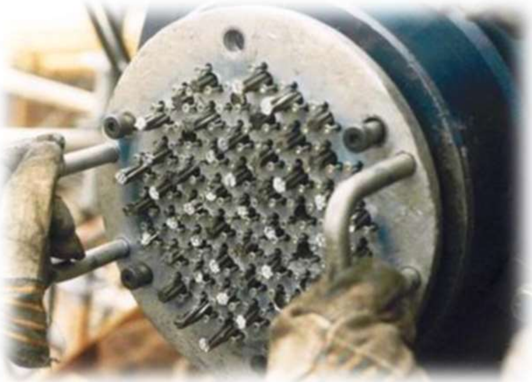
PRESTRESSING METHODS

1) Post-tensioning

④ Prestressing







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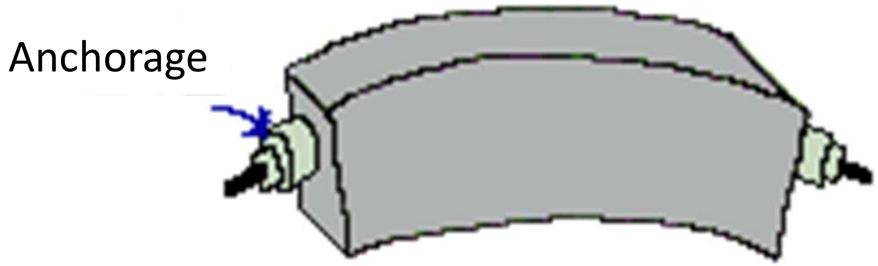
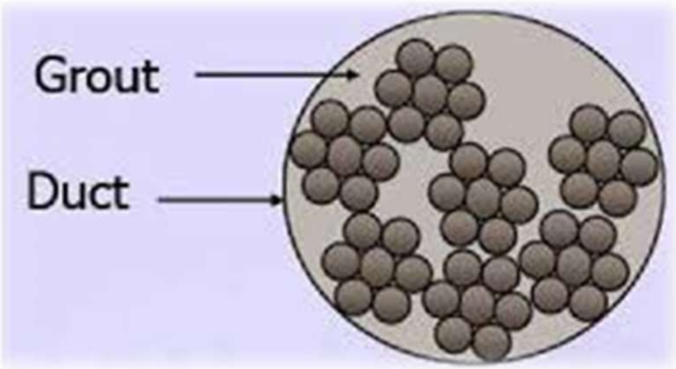
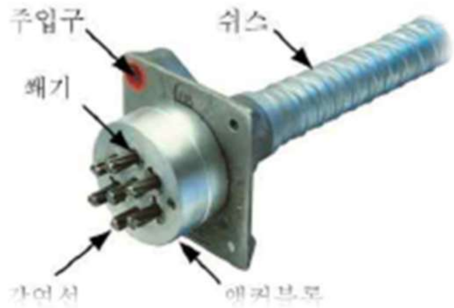
PRESTRESSING METHODS

1) Post-tensioning

⑤ Installation of Anchorage



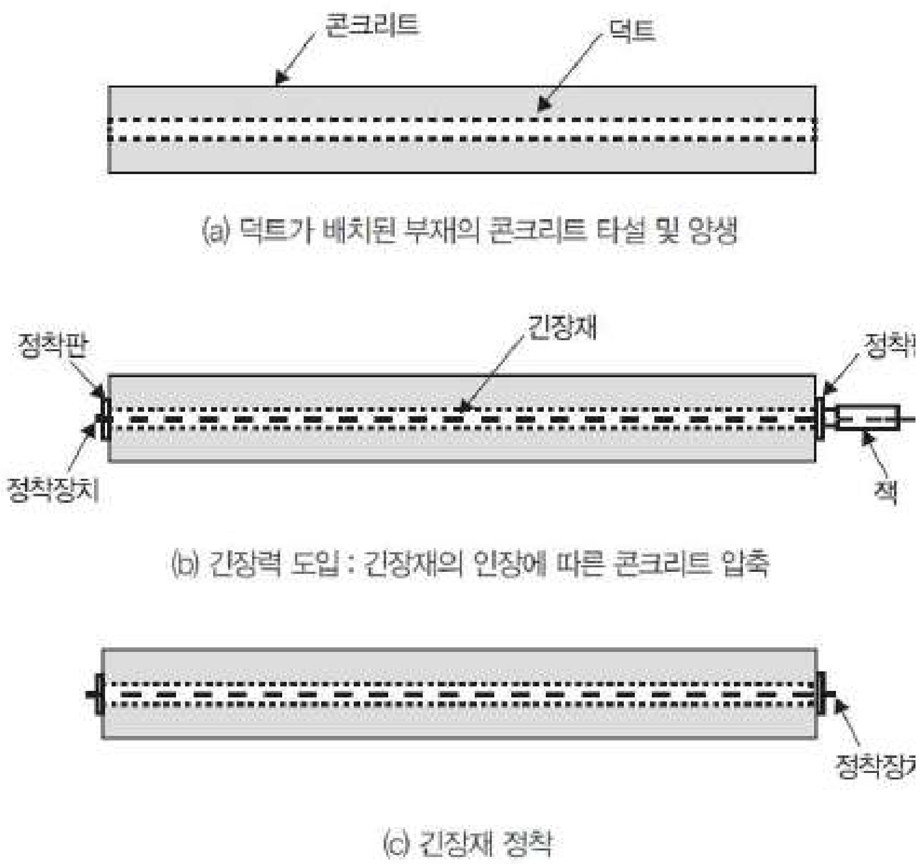


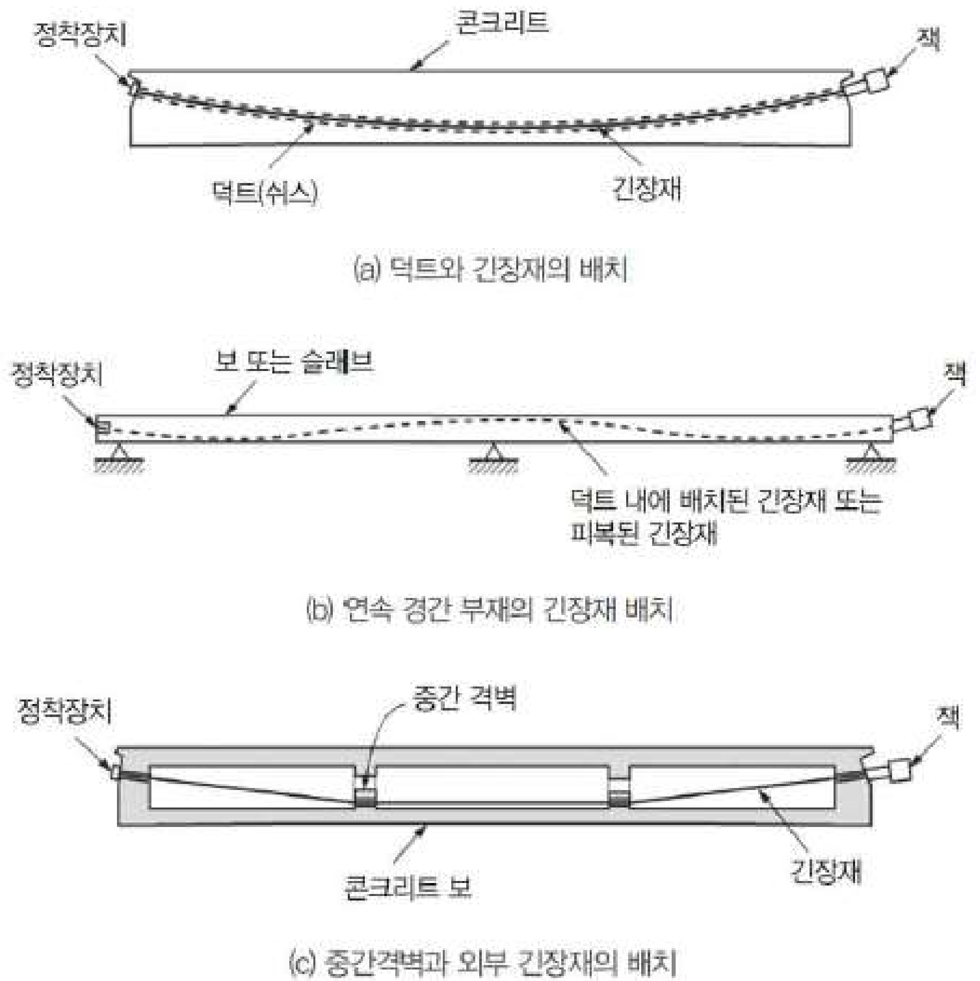


13. Prestressed Concrete

PRESTRESSING METHODS

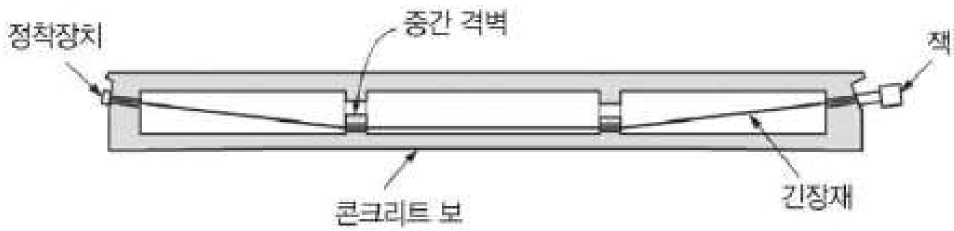
1) Post-tensioning



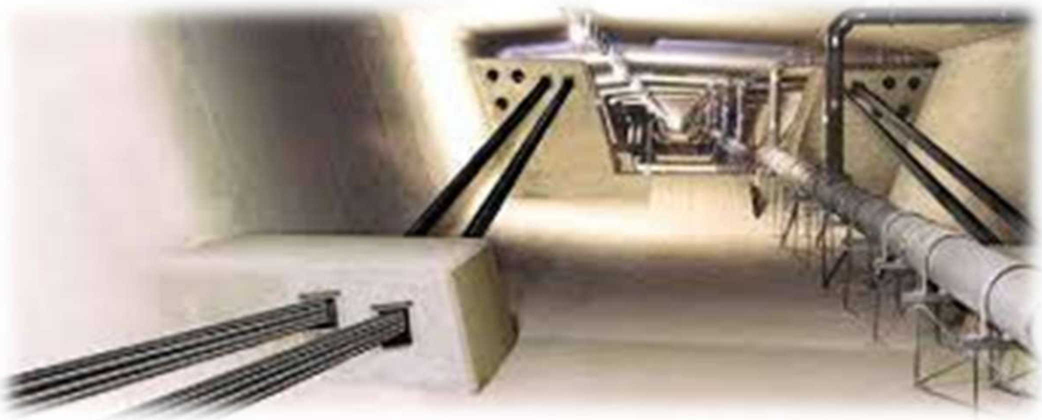


PRESTRESSING METHODS

1) Post-tensioning



(c) 중간격벽과 외부 긴장재의 배치

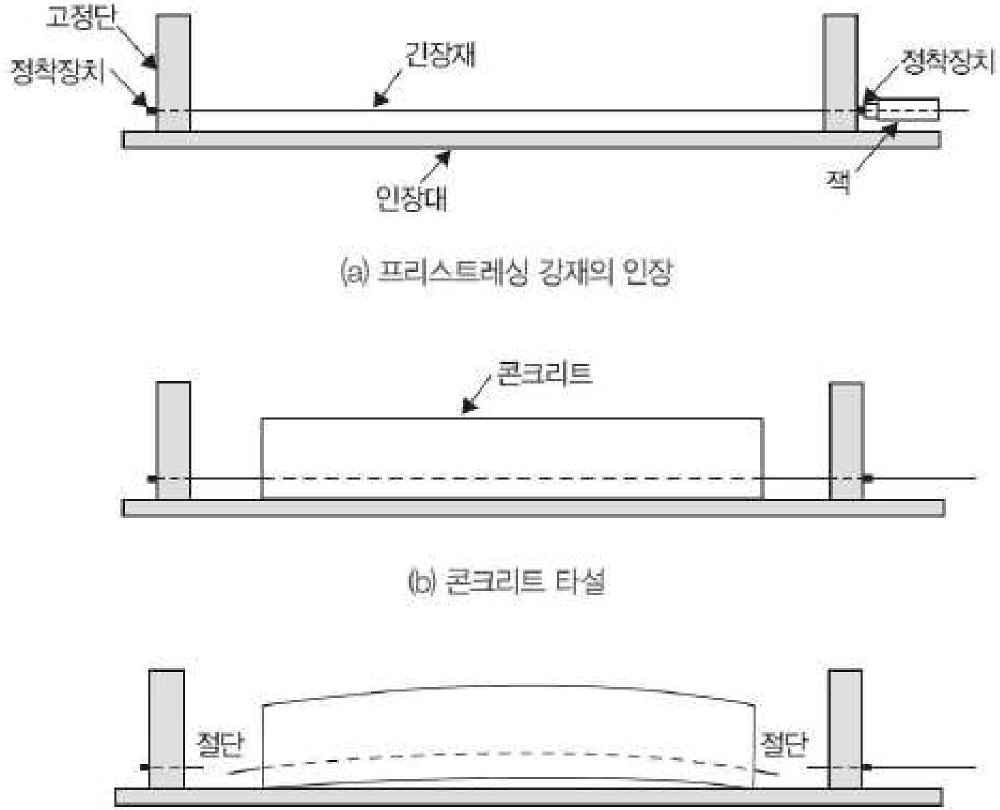


PRESTRESSING METHODS

2) Pre-tensioning

An economical method of prestressing

- Permits reusable steel or fiberglass forms
- Permits the simultaneous prestressing of many members at once
- Expensive and anchorage hardwares not required



(a) 프리스트레싱 강재의 인장

(b) 콘크리트 타설

13. Prestressed Concrete

PRESTRESSING METHODS

2) Pre-tensioning

PRESTRESSING STEELS

Importance of High Strength Steel

Low prestress using ordinary structural steel may be quickly lost due to shrinkage and creep in the concrete

Type of Prestressing Steels

Individual wires	- KS D 7002
Strands made up of seven wires	- KS D 7002
Alloy-steel bars	- KS D 3505

PRESTRESSING STEELS

Serviceability Requirements (KCI 9.3.1)

Maximum Permissible Stresses in Prestressing Steel (KCI 9.3.2)

shall not exceed the followings

1) Due to the prestressing steel jacking force $0.94f_{py}$
   but not greater than the lesser of $0.80f_{pu}$ and
   the max. value recommended by the manufactures

2) Immediately after prestress transfer $0.82f_{py}$
   but not greater than $0.74f_{pu}$

3) Post-tensioning tendons, at anchorage devices
   and couplers, immediately after transfer $0.70f_{pu}$

13. Prestressed Concrete

CONCRETE

Most prestressed construction is designed for a compressive strength above 35 MPa. Why?

1) The higher strength, the higher modulus of elasticity.
   ⇨ A reduction in creep strain which is proportional to elastic strain
   ⇨ A reduction in loss of prestress
2) In post-tensioned construction, the bearing capacity of he concrete can be increased by increasing its compressive strength.
3) In pre-tensioned construction, high-strength concrete will permit the development of higher bond stress.
4) A substantial part of the prestressed construction is precast.

13. Prestressed Concrete

CONCRETE

Reinforcement for increasing bearing capacity near post-tension anchorage

CONCRETE

Classification of Prestressed Flexural Members (KCI 9.2.2)

Based on f_t , the computed extreme fiber stress in tension in the precompressed tensile zone calculated at service loads.

Class U (uncracked, 비균열등급) $f_t \le 0.63\sqrt{f_{ck}}$

Class T (transition b/w U and C, 부분균열등급) $0.63\sqrt{f_{ck}} < f_t \le 1.0\sqrt{f_{ck}}$

Class C (cracked, 완전균열등급) $f_t > 1.0\sqrt{f_{ck}}$

Note

- Class C members are principally designed based on strength.
- Class U & T are designed so that stresses in concrete and steel at actual service loads are within permissible limit.

⇦ An important objective of prestressing is to improve the performance of members at service loads.

CONCRETE

Classification of Prestressed Flexural Members (KCI 9.2.2)

	Prestressed			Nonprestressed
	Class U	Class T	Class C	
Assumed behavior	Uncracked	Transition between uncracked and cracked	Cracked	Cracked
Section properties for stress calculation at service loads	Gross section 24.5.2.2	Gross section 24.5.2.2	Cracked section 24.5.2.3	No requirement
Allowable stress at transfer	24.5.3	24.5.3	24.5.3	No requirement
Allowable compressive stress based on uncracked section properties	24.5.4	24.5.4	No requirement	No requirement
Tensile stress at service loads 24.5.2.1	$\leq 0.62\sqrt{f_c'}$	$0.62\sqrt{f_c'} < f_t \leq 1.0\sqrt{f_c'}$	No requirement	No requirement
Deflection calculation basis	24.2.3.8, 24.2.4.2 Gross section	24.2.3.9, 24.2.4.2 Cracked section, bilinear	24.2.3.9, 24.2.4.2 Cracked section, bilinear	24.2.3, 24.2.4.1 Effective moment of inertia
Crack control	No requirement	No requirement	24.3	24.3
Computation of Δf_{ps} or f_s for crack control	—	—	Cracked section analysis	$M/(A_s \times$ lever arm), or $2/3 f_y$
Side skin reinforcement	No requirement	No requirement	9.7.2.3	9.7.2.3

13. Prestressed Concrete

CONCRETE

Serviceability Requirements (KCI 9.3.1)

Permissible concrete stress at transfer of prestress

Calculated extreme concrete fiber stress immediately after transfer of prestress but before time dependent prestress losses shall not exceed the followings

	Tensile	Compressive
1) End of simply supported member	$0.50\sqrt{f_{ci}}$	$0.70 f_{ci}$
2) all other locations	$0.25\sqrt{f_{ci}}$	$0.60 f_{ci}$

f_{ci}: Compressive strength of concrete at time of initial prestress

13. Prestressed Concrete

CONCRETE

<u>Serviceability Requirements</u> (KCI 9.3.1)

Permissible concrete compressive stress at service loads

For Class U and T members, the calculated extreme concrete fiber stress in compression at service loads after allowance for all prestress losses shall not exceed the followings

1) Prestress plus sustained load $0.45 f_{ck}$

2) prestress plus toal load $0.60 f_{ck}$

f_{ck}: Compressive strength of concrete

CONCRETE

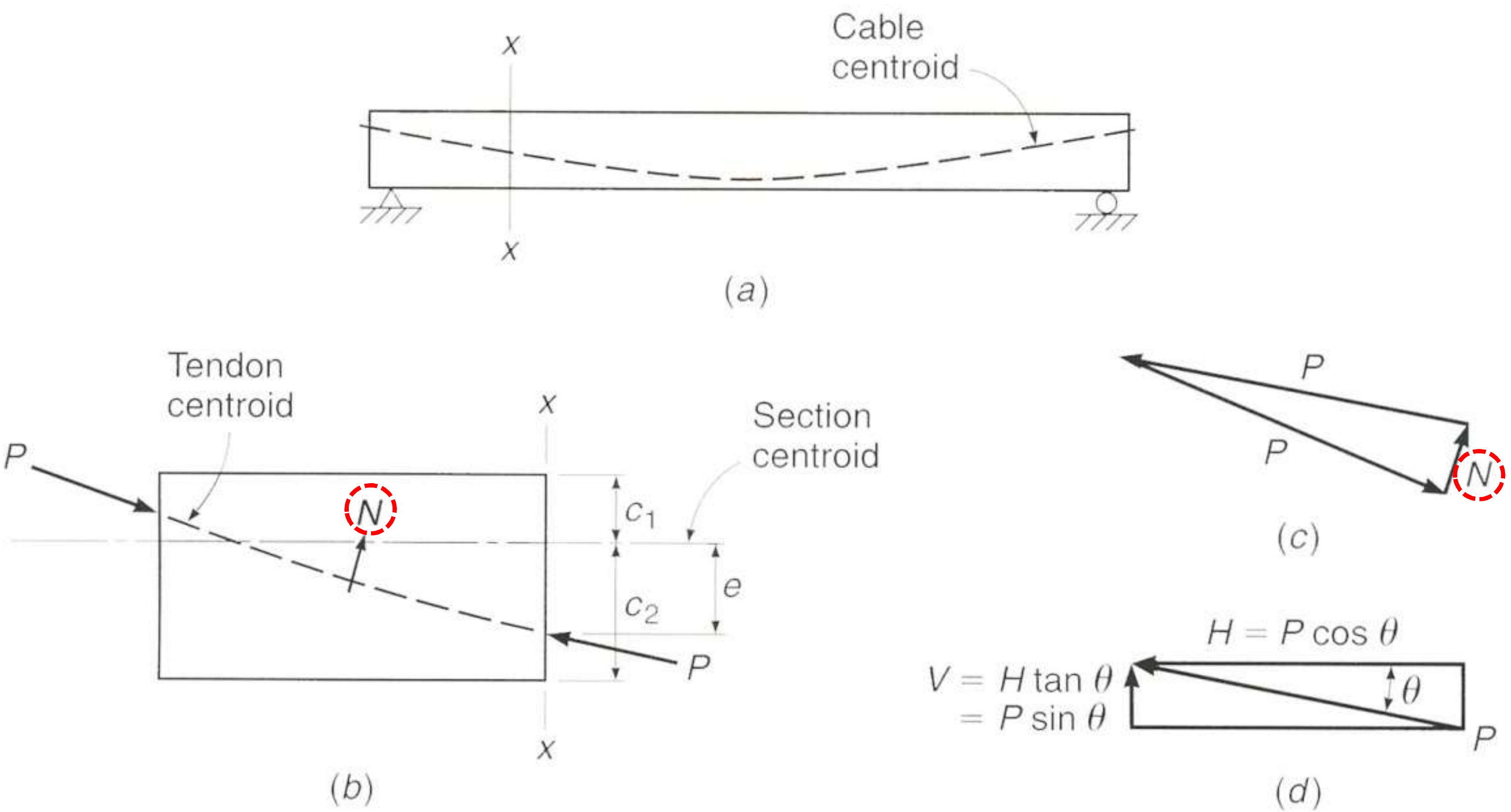
Serviceability Requirements (Summary)

Permissible stressed in prestressed flexural member

Conditions	Class		
	U	T	C
Extreme fiber stress in compression immediately after transfer	$0.60 f_{ci}$	$0.60 f_{ci}$	$0.60 f_{ci}$
Extreme fiber stress in compression immediately after transfer at the end of a simple supported member	$0.70 f_{ci}$	$0.70 f_{ci}$	$0.70 f_{ci}$
Extreme fiber stress in tension immediately after transfer	$0.25\sqrt{f_{ci}}$	$0.25\sqrt{f_{ci}}$	$0.25\sqrt{f_{ci}}$
Extreme fiber stress in tension immediately after transfer at the end of a simply supported member	$0.50\sqrt{f_{ci}}$	$0.50\sqrt{f_{ci}}$	$0.50\sqrt{f_{ci}}$
Extreme fiber stress in compression due to prestress plus sustained load	$0.45 f_{ck}$	$0.45 f_{ck}$	-
Extreme fiber stress in compression due to prestress plus total load	$0.60 f_{ck}$	$0.60 f_{ck}$	-

13. Prestressed Concrete

ELASTIC FLEXURAL ANALYSIS



Prestressing forces acting on concrete

ELASTIC FLEXURAL ANALYSIS

Partial loss of Prestressing Forces

The magnitude of the prestress force is not constant.

① P_j : jacking force

② P_i : initial prestress

③ P_e : effective prestress

$P_j \Rightarrow P_i$ ⇦ elastic shortening of the concrete, slip of the tendon as the force is transferred from jacks to beam end

$P_i \Rightarrow P_e$ ⇦ concrete creep and shrinkage, relaxation of stress in steel

ELASTIC FLEXURAL ANALYSIS

Several Stages to be Considered

1) Initial prestess, immediately after transfer, when P_i alone may act on the concrete. P_i

2) Initial prestress plus self-weight of the member. $P_i + M_o$

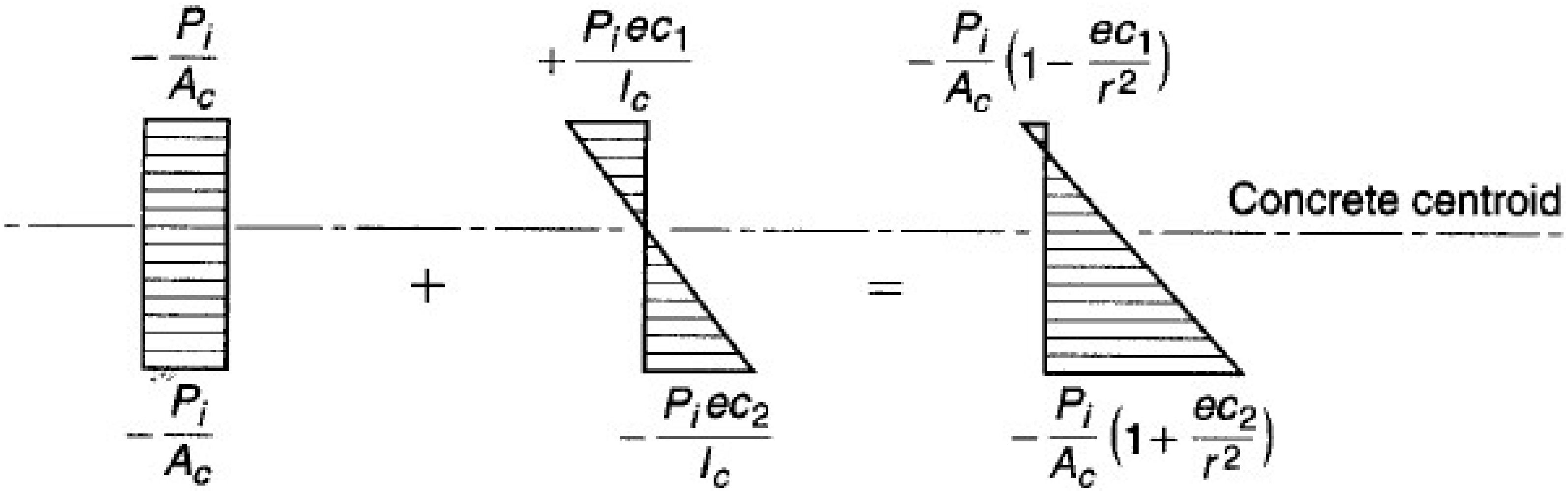
3) Initial prestress plus full dead load. $P_i + M_o + M_d$

4) Effective prestress P_e, after loses plus service loads consisting of full dead and expected live load. $P_e + M_o + M_d + M_i$

5) Ultimate load, when the expected service loads are increased by load factors, and the member is at initial failure. P_u

ELASTIC FLEXURAL ANALYSIS

1st Stage 1) initial prestress force P_i



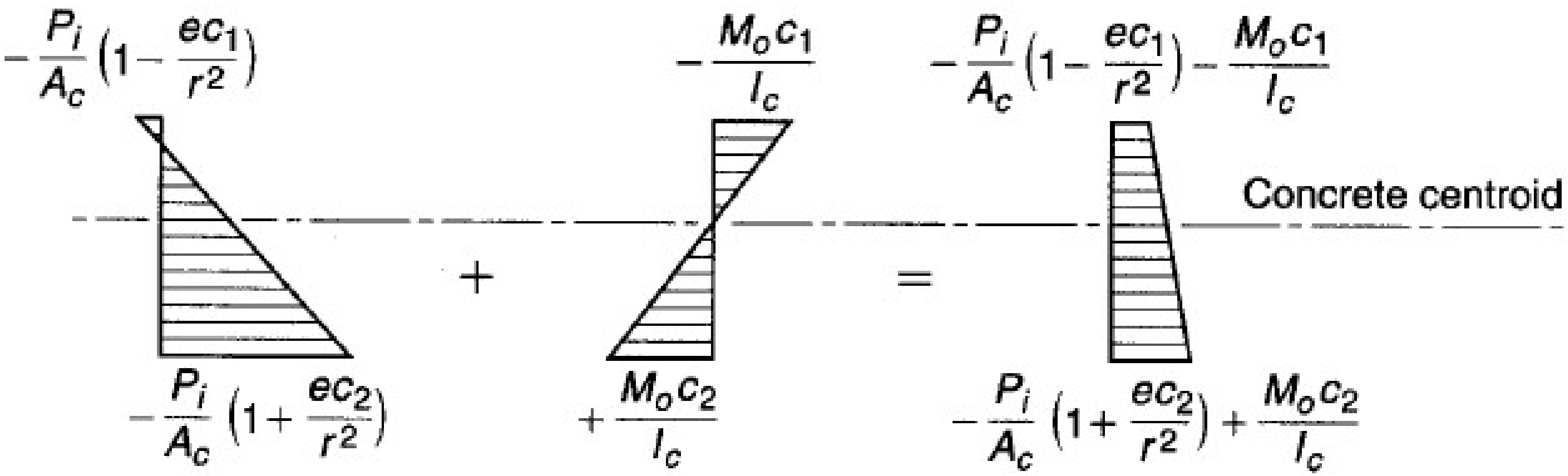
Top fiber stress $$f_1 = -\frac{P_i}{A_c} + \frac{P_i e c_1}{I_c} = -\frac{P_i}{A_c}\left(1 - \frac{ec_1}{r^2}\right) \qquad (1)$$

Bottom fiber stress $$f_2 = -\frac{P_i}{A_c} - \frac{P_i e c_2}{I_c} = -\frac{P_i}{A_c}\left(1 + \frac{ec_2}{r^2}\right) \qquad (2)$$

where $r^2 = \frac{I_c}{A_c}$

ELASTIC FLEXURAL ANALYSIS

2nd Stage 2) & 3)



Top fiber stress $$f_1 = -\frac{P_i}{A_c}\left(1-\frac{ec_1}{r^2}\right)-\frac{M_o c_1}{I_c}-\frac{M_d c_1}{I_c} \quad (3)$$

Bottom fiber stress $$f_2 = -\frac{P_i}{A_c}\left(1+\frac{ec_2}{r^2}\right)+\frac{M_o c_2}{I_c}+\frac{M_d c_2}{I_c} \quad (4)$$

At this stage time-dependent losses due to shrinkage, creep, and relaxation commence

ELASTIC FLEXURAL ANALYSIS

2nd Stage 2) & 3)

It is usually acceptable to assume that all time-dependent losses occur prior to the application of service loads, since the concrete stress at service loads will be CRITICAL after losses, not before.

Top fiber stress

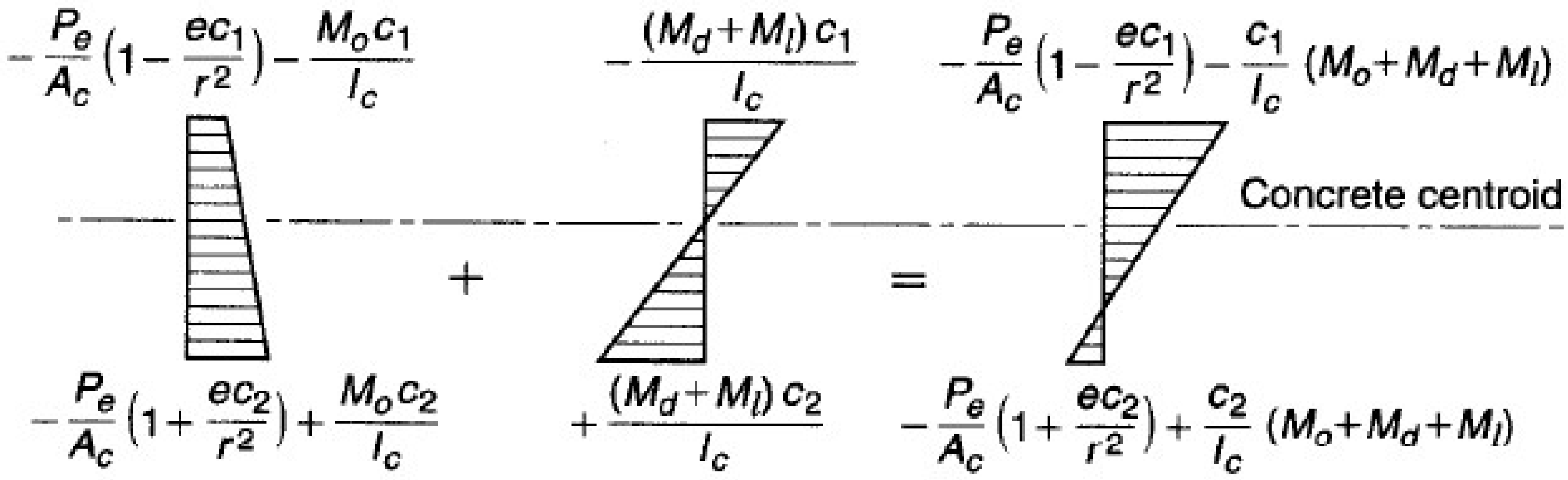
$$f_1 = -\frac{P_e}{A_c}\left(1 - \frac{ec_1}{r^2}\right) - \frac{M_o c_1}{I_c} \qquad (5)$$

Bottom fiber stress

$$f_2 = -\frac{P_e}{A_c}\left(1 + \frac{ec_2}{r^2}\right) + \frac{M_o c_2}{I_c} \qquad (6)$$

ELASTIC FLEXURAL ANALYSIS

3rd Stage 4) Service load stage



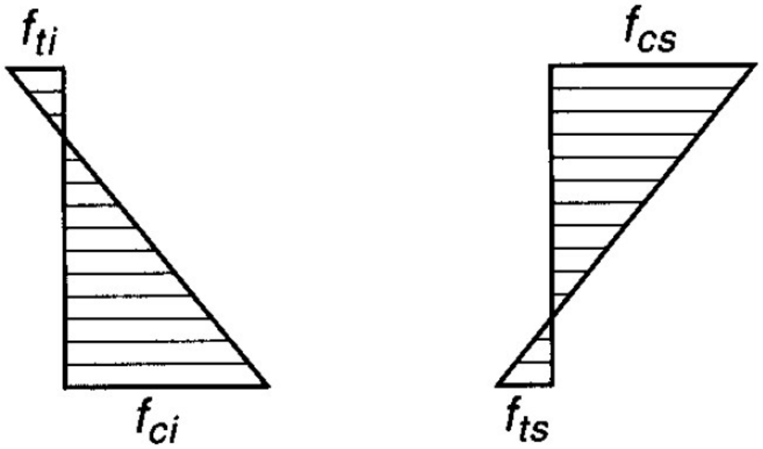
Top fiber stress
$$f_1 = -\frac{P_e}{A_c}\left(1-\frac{ec_1}{r^2}\right)-\frac{(M_o+M_d+M_l)c_1}{I_c} \quad (7)$$

Bottom fiber stress
$$f_2 = -\frac{P_e}{A_c}\left(1+\frac{ec_2}{r^2}\right)+\frac{(M_o+M_d+M_l)c_2}{I_c} \quad (8)$$

ELASTIC FLEXURAL ANALYSIS

Check serviceability (KCI 9.3.1)

Unloaded



Loaded

f_{ci} : Permissible compressive stress in the concrete immediately after transfer

f_{ti} : Permissible compressive stress in the concrete immediately after transfer

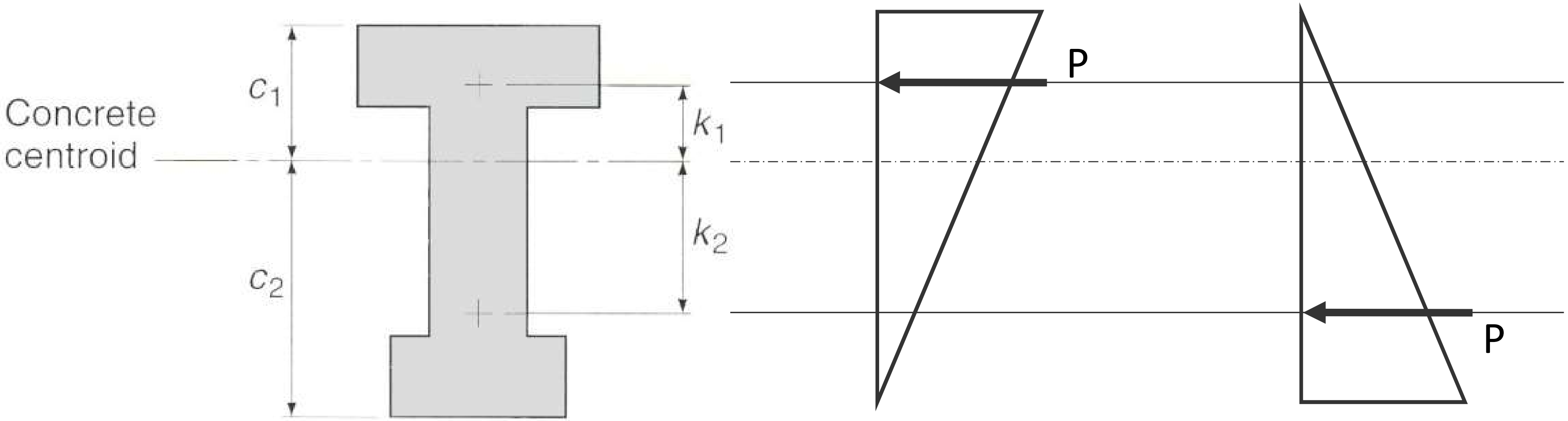
f_{cs} : Permissible compressive stress at service loads

f_{ts} : Permissible tensile stress at service loads

ELASTIC FLEXURAL ANALYSIS

Cross Section kern

When the prestressing force, acting alone, causes no tension in the cross section, it is said to be acting within the “kern” of the cross section.



Limiting points inside which the prestress force resultant may be applied without causing tension anywhere in the cross section.

ELASTIC FLEXURAL ANALYSIS

Cross Section kern

To find upper kern-point distance k_1 , let the prestress force resultant act at that point. Then the bottom fiber stress is zero.

$$f_2 = -\frac{P}{A_c}\left(1 + \frac{ec_1}{r^2}\right) = 0 \tag{9}$$

$$\rightarrow \left(1 + \frac{ec_1}{r^2}\right) = 0 \tag{10}$$

$$\rightarrow e = k_1 = -\frac{r^2}{c_2} \tag{11}$$

Similarly, the low kern-point k_2 is

$$e = k_2 = \frac{r^2}{c_1} \tag{12}$$

ELASTIC FLEXURAL ANALYSIS

Cross Section kern

Note

It should not be implied that the steel centroid must remain within the kern. However, the kern limits after serve as convenient reference points in the design of beams.

Example 5.1> Pretensioned I beam with constant eccentricity.

A simple supported symmetrical I beam (span =13 m) is to carry a superimposed dead plus live load of 10 kN/m in addition to the self-weight.

Multiple seven wire strands with a constant e = 200 mm

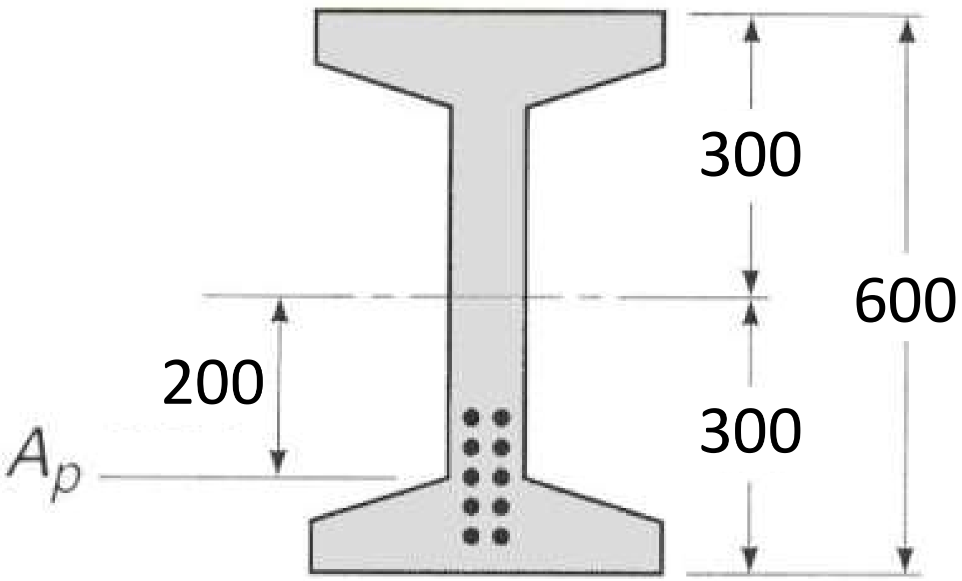
P_i = 700 kN , P_e = 600 kN

The specified strength of concrete f_{ck} = 35 MPa
At the time of prestressing f_{ci} = 27 MPa

I_c = 5,000,000,000 mm^4
A_c = 115,000 mm^2 w_c = 2400 N/m
r^2 = 44,000 mm^2 S = 16,400,000 mm^3

Example 5.1>

Calculate the concrete flexural stresses at the midspan section at the time of transfer, and after all losses with full service load in place.



Solution

(1) From initial prestress force P_i using Eq. (1) & (2)

$$f_1 = -\frac{700\times10^3}{115,000}\left(1-\frac{(200)(300)}{44,000}\right) = \underline{2.21MPa}$$

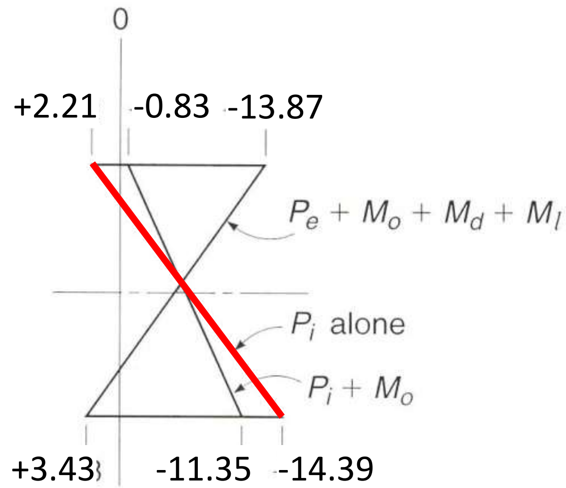
$$f_2 = -\frac{700\times10^3}{115,000}\left(1+\frac{(200)(300)}{44,000}\right) = \underline{-14.39MPa}$$

Immediate moment due to self-weight M_o

$$M_o = \frac{1}{8}(2,400)(13)^2 = 50.7kN\cdot m$$

Corresponding stress is

$$(50.7\times10^6)\frac{300}{5,000,000,000} = 3.04MPa$$



Midspan

Solution

(2) From initial prestress and self-weight ← $P_i + M_o$

$$f_1 = +2.21 - 3.04 = \underline{-0.83MPa}$$

$$f_2 = -14.39 + 3.04 = \underline{-11.35MPa}$$

(3) After losses, P_i is reduced to P_e

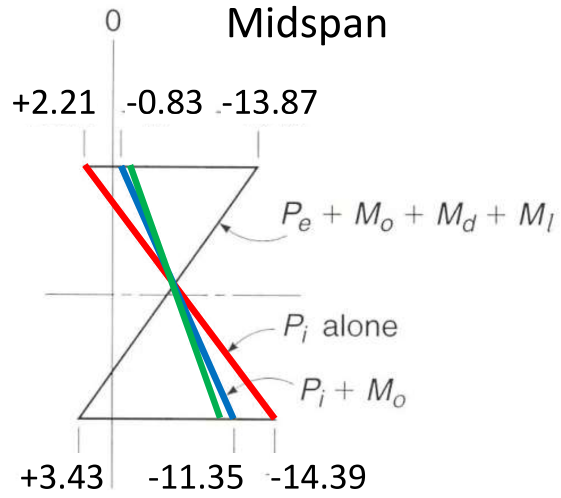
$$f_1 = +2.21 \times \frac{600}{700} - 3.04 = \underline{-1.15MPa}$$

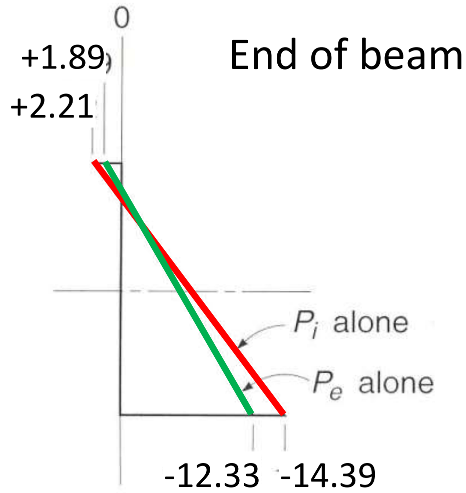
$$f_2 = -14.39 \times \frac{600}{700} + 3.04 = \underline{-9.29MPa}$$

Stresses at the end of the beam

$$f_1 = 2.21 \times \frac{600}{700} = \underline{1.89MPa}$$

$$f_2 = -14.39 \times \frac{600}{700} = \underline{-12.33MPa}$$





Solution

(4) Superimposed load → $M_d + M_l$

$$M_d + M_l = \frac{1}{8}(10,000)(13)^2 = 212\,kN \cdot m$$

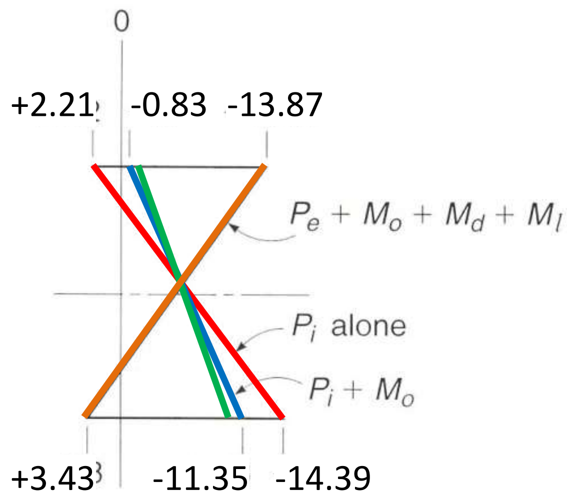
Corresponding stress is

$$\left(212 \times 10^6\right) \times \frac{300}{5,000,000,000} = 12.72 MPa$$

From the superimposed load

$$f_1 = -1.15 - 12.72 = \underline{-13.87 MPa}$$

$$f_2 = -9.29 + 12.72 = \underline{3.43 MPa}$$



Midspan

Serviceability Requirements (Summary)

Permissible stressed in prestressed flexural member

Conditions	Class		
	U	T	C
Extreme fiber stress in compression immediately after transfer	$0.60 f_{ci}$	$0.60 f_{ci}$	$0.60 f_{ci}$
Extreme fiber stress in compression immediately after transfer at the end of a simple supported member	$0.70 f_{ci}$	$0.70 f_{ci}$	$0.70 f_{ci}$
Extreme fiber stress in tension immediately after transfer	$0.25\sqrt{f_{ci}}$	$0.25\sqrt{f_{ci}}$	$0.25\sqrt{f_{ci}}$
Extreme fiber stress in tension immediately after transfer at the end of a simply supported member	$0.50\sqrt{f_{ci}}$	$0.50\sqrt{f_{ci}}$	$0.50\sqrt{f_{ci}}$
Extreme fiber stress in compression due to prestress plus sustained load	$0.45 f_{ck}$	$0.45 f_{ck}$	-
Extreme fiber stress in compression due to prestress plus total load	$0.60 f_{ck}$	$0.60 f_{ck}$	-

Solution

(5) Checking the permissible stress

Tension at transfer

$$f_{ti} = 0.25\sqrt{f_{ci}} = 0.25\sqrt{27} = 1.27 > -0.83MPa$$

Compression at transfer

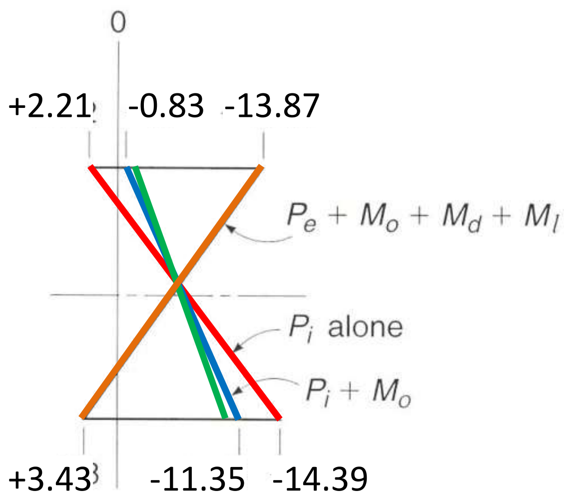
$$f_{ci} = 0.6 \times f_{ci} = 21 > 11.35MPa$$

Tension at service load

$$f_{ts} = 0.63\sqrt{f_{ck}} = 0.63\sqrt{35} = 3.72 > 3.43MPa$$

Compression at service load

$$f_{cs} = 0.45 f_{ck} = 15.75 > 13.87MPa$$



Midspan

Solution

Note

$$f_{ti} = 1.27MPa > -0.83MPa$$

While more prestress or more eccentricity might be suggested to more fully utilize the section, to attempt to do so in this beam, with constant eccentricity would violate limits at the SUPPORT.

Extreme fiber allowable stress in tension at ends of simply supported members

$$0.5\sqrt{f_{ci}} = 0.5\sqrt{26} = 2.55MPa > 2.21MPa$$ Slightly/Barely larger !!

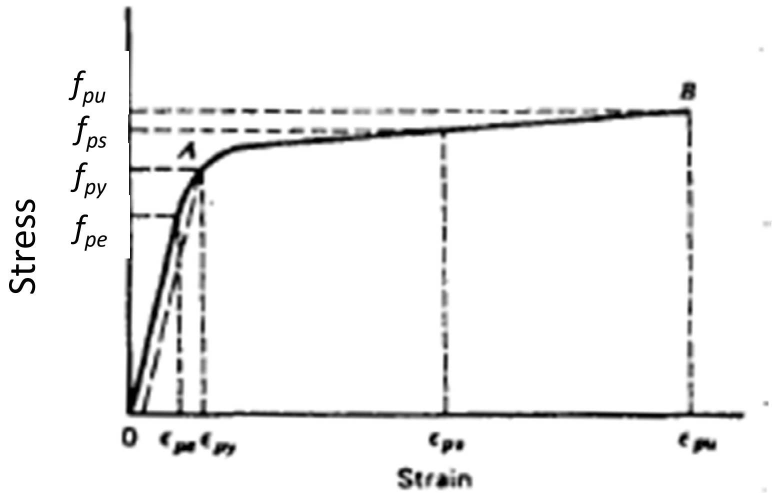
FLEXURAL STRENGTH

Difference from the Ordinary RC Beam

1) Prestressing steel has different shape of stress-strain curve compared with typical reinforcing steel

2) Tensile strain is already present in the prestressing steel even before the beam is loaded

<Typical S-S curve>



f_{pe} : due to P_e after all losses

f_{py} : yield strength

f_{pu} : ultimate tensile strength

f_{ps} : stress when the beam fails

FLEXURAL STRENGTH

Difference from the Ordinary RC Beam

Highly accurate prediction of the flexural strength of prestressed beams can be made on a STRAIN COMPA-TIBILITY ANALYSIS. (Recommended)

Ref.) Arthur H. Nilson, Design of Prestressed Concrete, 1987, chapter 3.7.

FLEXURAL STRENGTH

Stress in the Prestressed Steel at Flexure Failure

KCI code 9.5 indicates that within certain limitations, an approximate determination may be made instead of using STRAIN COMPA-TIBILITY ANALYSIS.

Provided the effective prestress , $f_{pe} \geq 0.5 f_{pu}$ the steel stress at failure, f_{ps} (Note : not f_{py}) can be taken equal to the followings (KCI 9.5.1)

FLEXURAL STRENGTH

Stress in the Prestressed Steel at Flexure Failure

1) For bonded tendon

$$f_{ps} = f_{pu}\left[1-\frac{\gamma_p}{\beta_1}\left\{\rho_p\frac{f_{pu}}{f_{ck}}+\frac{d}{d_p}(\omega-\omega')\right\}\right] \tag{13}$$

γ_p: factor for type of prestressing steel

ρ_p : A_{ps}/bd_p

d : distance from extreme compressive fiber to centroid of NON prestressed tension bars

d_p: distance from extreme compressive fiber to centroid of prestressed tension bars

ω : $\rho f_y / f_{ck}$ for tensile rebar, ω' : $\rho' f_y / f_{ck}$ for compressive reinforcement

13. Prestressed Concrete

FLEXURAL STRENGTH

Stress in the Prestressed Steel at Flexure Failure

1) For bonded tendon

$$f_{ps} = f_{pu}\left[1 - \frac{\gamma_p}{\beta_1}\left\{\rho_p \frac{f_{pu}}{f_{ck}} + \frac{d}{d_p}(\omega - \omega')\right\}\right] \qquad (13)$$

γ_p : factor for type of prestressing steel

= 0.55 if $f_{py} / f_{pu} \geq 0.80$

= 0.40 if $f_{py} / f_{pu} \geq 0.85$

= 0.28 if $f_{py} / f_{pu} \geq 0.90$

if ω' is not zero, $\left\{\rho_p \frac{f_{pu}}{f_{ck}} + \frac{d}{d_p}(\omega - \omega')\right\} \geq 0.17$ and $d' \leq d_p$

FLEXURAL STRENGTH

<u>Stress in the Prestressed Steel at Flexure Failure</u>

2) For unbounded tendon

(a) For span-to-depth ratio ≤ 35

$$f_{ps} = f_{pe} + 70 + \frac{f_{ck}}{100\rho_p} \qquad (14)$$

$$f_{ps} \le f_{py} \quad \text{or} \quad (f_{pe} + 420)\,MPa$$

f_{pe} : effective stress in prestressed reinforcement

(b) For span-to-depth ratio 35

$$f_{ps} = f_{pe} + 70 + \frac{f_{ck}}{300\rho_p} \qquad (15)$$

$$f_{ps} \le f_{py} \quad \text{or} \quad (f_{pe} + 210)\,MPa$$

FLEXURAL STRENGTH

Nominal Flexural Strength

I) a (stress block) < h_f (compression flange)

The nominal flexural strength is

$$M_n = A_{ps} f_{ps} \left(d_p - \frac{a}{2} \right) \tag{16}$$

$$a = \frac{A_{ps} f_{ps}}{0.85 f_{ck} b} \tag{17}$$

II) $a > h_f$

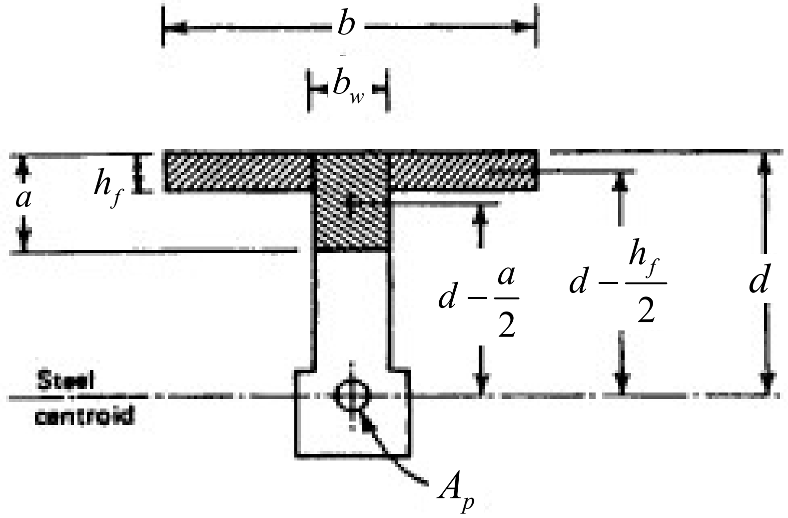
The total prestressed tensile steel area is divided into two parts.

FLEXURAL STRENGTH

Nominal Flexural Strength

(a) Overhanging part

$$A_{pf} = 0.85\frac{f_{ck}}{f_{ps}}(b-b_w)h_f \qquad (18)$$



(b) Web part

$$A_{pw} = A_{ps} - A_{pf} \qquad (19)$$

$$M_n = A_{pw}f_{ps}\left(d_p-\frac{a}{2}\right) + A_{pf}f_{ps}\left(d_p-\frac{h_f}{2}\right) \qquad (20)$$

$$= A_{pw}f_{ps}\left(d_p-\frac{a}{2}\right) + 0.85f_{ck}(b-b_w)h_f\left(d_p-\frac{h_f}{2}\right) \qquad (21)$$

where $a = \frac{A_{pw}f_{ps}}{0.85f_{ck}b_w}$

FLEXURAL STRENGTH

Nominal Flexural Strength

Note

If, after a prestressed beam is designed by elastic methods at service loads, it has inadequate strength to provide the required safety margin under factored load, NON prestressed reinforcement can be added on the tension side and will work in combination with the prestressing steel to provide the needed strength.

FLEXURAL STRENGTH

Limits for reinforcements

Ductile failure: Due to the complexity of computing *net tensile strain* in prestressed members, it is easier to perform the check using the c/d_t ratio.

for ductile failure
$$\frac{c}{d_t} \le \frac{0.003}{0.003 + \boxed{0.005}} = 0.375 \qquad (22)$$

Note

- In many cases, d_t is the same as d_p
- If nonprestressed steel is used, d_t will be greater than d_p
- If Eq.(22) is not satisfied, Φ should be calculated

FLEXURAL STRENGTH

Limits for reinforcements

Minimum tensile reinforcement ratio is required for the safety from sudden failure upon the formation of flexural cracks.

KCI Code 9.5.2 required that the total tensile reinforcement must be adequate to support a factored load of at least 1.2 times the cracking load of the beam calculated on the basis of a modulus of rupture f_r .

FLEXURAL STRENGTH

Minimum Bonded Reinforcement

To control cracking in beams and one-way PSC slabs with UNBONDED tendons, some *bonded non-prestressed* reinforcement must be added.

The minimum amount of such reinforcement is

$$A_s = 0.004 A_{ct} \tag{23}$$

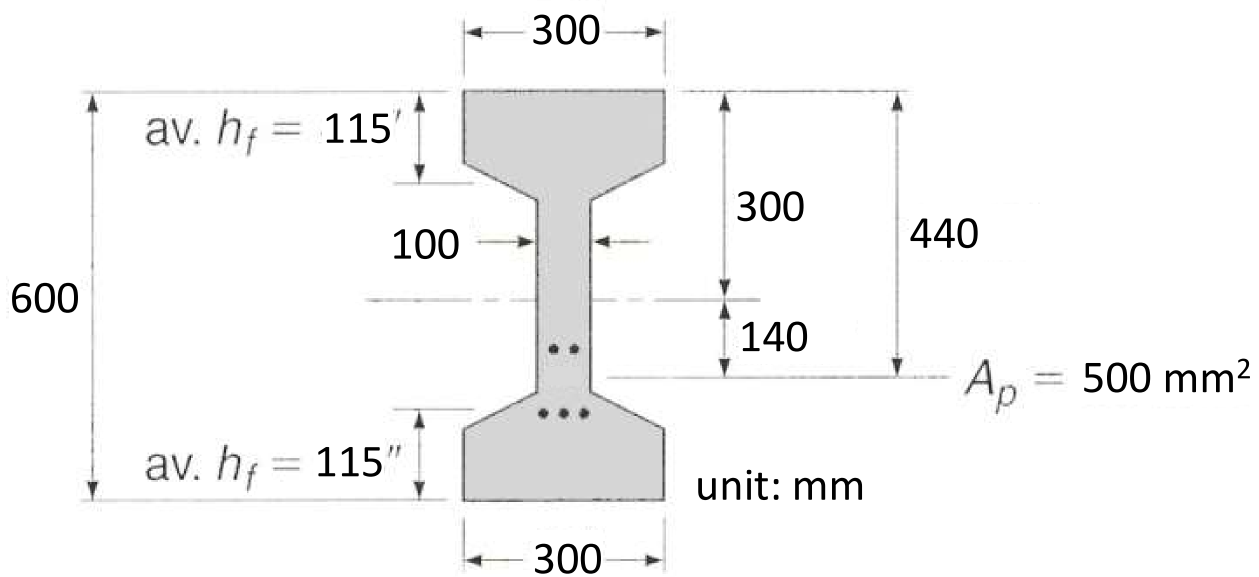
where A_{ct} is the area of that part of the cross section between the tension section face and the centroid of the gross concrete cross section.

Example 5.2> Flexural strength of pretensioned I beam

The prestressed I beam is pretensioned using five low relaxation Grade 270 13 mm diameter strands.

f_{pu}= 1,100 MPa, f_{pe}= 1,100 MPa, f_{ck} = 27 MPa $\gamma_p = 0.28$

Calculate the design strength of the beam.



13. Prestressed Concrete

Solution

The ratio of effective prestress to ultimate strength of the steel

$$\frac{f_{pe}}{f_{pu}} = \frac{1,100}{1,860} = 0.59 > 0.5$$

⇨ The approximate KCI equations are applicable.

For the basic case, in which the prestressed steel provides ALL of the flexural reinforcement, Eq.(13) can be simplified as

$$f_{ps} = f_{pu}\left[1 - \frac{\gamma_p}{\beta_1}\rho_p \frac{f_{pu}}{f_{ck}}\right]$$

$$f_{ps} = f_{pu}\left[1 - \frac{\gamma_p}{\beta_1}\left\{\rho_p \frac{f_{pu}}{f_{ck}} + \frac{d}{d_p}(\omega - \omega')\right\}\right] \quad (13)$$

Solution

$$f_{ps} = f_{pu}\left[1-\frac{\gamma_p}{\beta_1}\rho_p\frac{f_{pu}}{f_{ck}}\right] \leftarrow \rho_p = \frac{500}{300\times440} = 0.0038$$

$$= 1860\left(1-\frac{0.28}{0.85}\rho_p\frac{1860}{27}\right) = 1,700\,MPa$$

Check if $a > h_f$

First on the assumption that $a < h_f$

$$a = \frac{A_{ps}f_{ps}}{0.85f_{ck}b} = \frac{(500)(1,700)}{(0.85)(27)(300)} = 123\,mm > 115\,mm = h_f$$

⇨ Equations for flanged members must be used.

Solution

The steel that acts with the overhanging flange

$$A_{pf} = 0.85\frac{f_{ck}}{f_{ps}}(b-b_w)h_f$$

$$= 0.85\frac{27}{1,700}(300-100)(115) = 311\ mm^2$$

$$A_{pw} = A_p - A_{pf}$$

$$= 500 - 311 = 189\ mm^2$$

Solution

The actual stress block depth is

$$a = \frac{A_{pw} f_{ps}}{0.85 f_{ck} b_w} = \frac{(189)(1700)}{(0.85)(27)(100)} = 140\,mm$$

$$\Rightarrow c = \frac{a}{\beta_1} = \frac{140}{0.85} = 164.7\,mm$$

Now a check should be made to determine if the beam can be considered underreinforced.

$$\frac{c}{d_t} = \frac{164.7}{\underline{440}} = 0.374 < 0.375 \qquad \text{for} \qquad \varepsilon_t \geq 0.005$$

actually this is d_p

⇨ OK. $\phi = 0.85$, if c/dt is greater than 0.375??

Solution

$$M_n = A_{pw} f_{ps}\left(d_p - \frac{a}{2}\right) + 0.85 f_{ck}(b - b_w)h_f\left(d - \frac{h_f}{2}\right)$$

$$= (189)(1,700)\left(440 - \frac{140}{2}\right) + 0.85(100)(300 - 100)(115)\left(440 - \frac{115}{2}\right)$$

$$= 118\,kN \cdot m \; + \; 202\,kN \cdot m = 320\,kN \cdot m$$

$$M_u = \phi M_n = 0.85 \times 320 = 286.4\,kN \cdot m$$

PARTIAL PRESTRESSING

Early concept of prestressing is full prestressing.
: NO tension stress at service load.

- In case that full live load is seldom in place, excessive large upward deflection occurs due to concrete creep.
- In addition, longitudinal shortening occurs which may causes prestress losses due to elastic and creep deformation.
- Such a heavily prestressed beams may fail in brittle mode.

PARTIAL PRESTRESSING

Today partial prestressing

: Flexural tensile stress and some limited cracking is permitted under full service load.

- With partial prestressing, excessive camber and troublesome axial shortening are avoided.

Note

While tensile stress and possible crack may be allowed at full service load, it is also recognized that such full service load may be INFREQUENTLY applied.

PARTIAL PRESTRESSING

Note

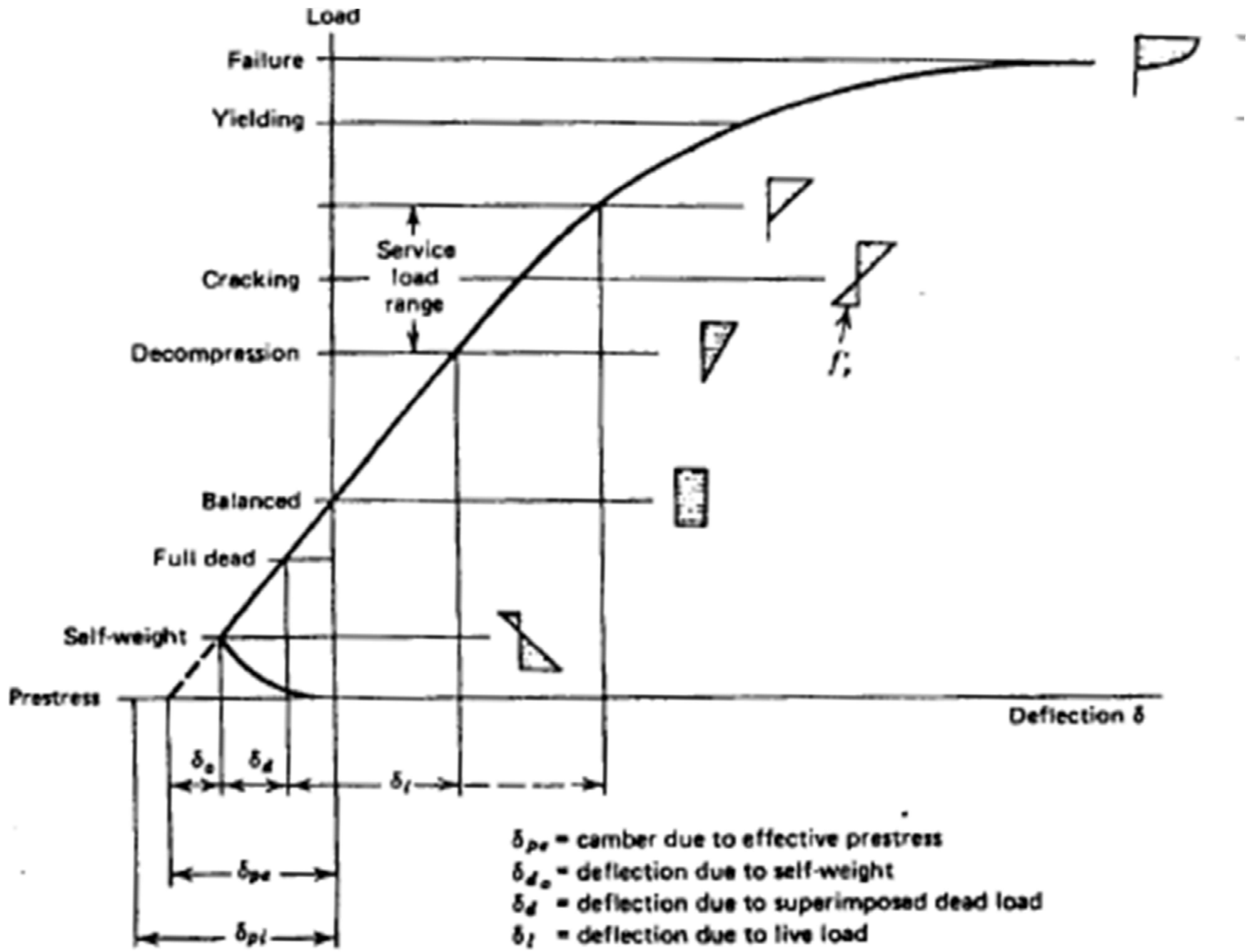
Regardless of the amount of prestress force used, the amount of steel must be such as to provide adequate flexural strength when the beam is overloaded so that the desired factor of safety is obtained.

This requirement may determine the total steel area to be used. Then, the amount of prestressing force may be controlled.

a) By stressing all tendons to less than the full permitted value.
b) By stressing some tendons fully, leaving others free of stress.
c) By providing the desired steel area partially by fully stressed tendons and partially by ordinary unstressed reinforcing bars.

FLEXURAL DESIGN BASED ON ALLOWABLE STRESS

Basis of Design



FLEXURAL DESIGN BASED ON ALLOWABLE STRESS

Three Practical Approaches to the Flexural Design

1) Assume the concrete section, calculate the required prestress force and eccentricity, then check the stresses, and finally check the flexural strength. The trial section then revised if necessary.

   ⇨ This method will be the best for shorter span and ordinary loads

2) For longer or when customized shapes are used, design the cross section so that the specified concrete stress limits (allowable stresses) are closely matched.

   Then modified to meet functional requirements
   (e.g. providing a broad top flange for a bridge deck) or
   to meet strength requirement, if necessary.

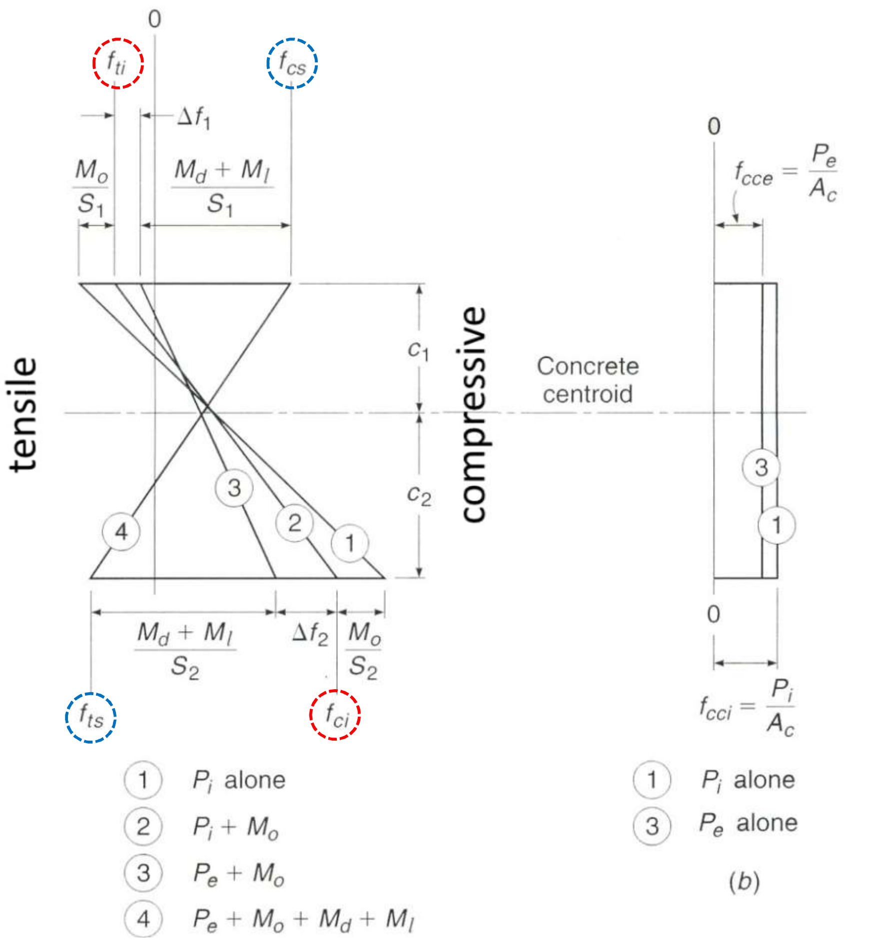
FLEXURAL DESIGN BASED ON ALLOWABLE STRESS

Three Practical Approaches to the Flexural Design

3) Load balancing method using the equivalent loads. Trial section is chosen, after which the prestress force and tendon profiles are selected to provide uplift forces as to just balance a specified load. Modification may then be made if needed to stress limits a strength requirement.

FLEXURAL DESIGN BASED ON ALLOWABLE STRESS

Design of Beams with Variable Eccentricity



Max. moment sec. Support sec.

- Distr. ① initial value P_i
- Distr. ② upward camber
  → the self-weight M_o
  : the actual first stage
- Distr. ③ all losses occur
- Distr. ④ superimposed dead load
  + service live load

FLEXURAL DESIGN BASED ON ALLOWABLE STRESS

Design of Beams with Variable Eccentricity

Note

· Stage ② : should not exceed f_{ti} and f_{ci}
· Stage ④ : should not exceed f_{ts} and f_{cs}

The requirements for the section moduli S_1 and S_2 are

$$S_1 \geq \frac{M_d + M_l}{f_{1r}} \tag{24}$$

$$S_2 \geq \frac{M_d + M_l}{f_{2r}} \tag{25}$$

Where the available stress ranges f_{1r} and f_{2r} can be calculated from the specified stress limits f_{ti} , f_{cs} , f_{ts} and f_{ci} , once the stress changes Δf_1 and Δf_2 are known.

associated with prestress loss

FLEXURAL DESIGN BASED ON ALLOWABLE STRESS

Design of Beams with Variable Eccentricity

-

The effectiveness ratio

$$R = \frac{P_e}{P_i} \quad (26)$$

Thus the loss in prestressing force is

$$P_i - P_e = (1-R)P_i \quad (27)$$

Δf_1 and Δf_2 are equal to (1-R) times the corresponding stresses due to the initial prestress force P_i acting alone

$$\Delta f_1 = (1-R)\left(f_{ti} + \frac{M_o}{S_1} \right)$$: reduction of tension (28)

$$\Delta f_2 = (1-R)\left(-f_{ci} + \frac{M_o}{S_2} \right)$$: reduction of compression (29)

FLEXURAL DESIGN BASED ON ALLOWABLE STRESS

Design of Beams with Variable Eccentricity

The stress ranges available as the superimposed load moments $M_d + M_l$ are applied are

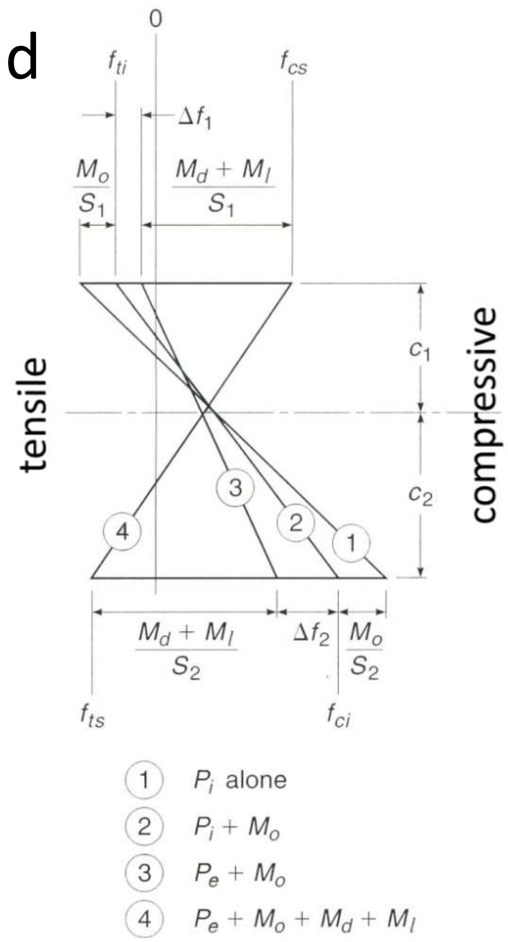
$$f_{1r} = f_{ti} - \Delta f_1 - f_{cs} \tag{30}$$

$$= Rf_{ti} - (1-R)\left(\frac{M_o}{S_1}\right) - f_{cs} \tag{31}$$

Similarly,

$$f_{2r} = f_{ts} - \Delta f_2 - f_{ci} \tag{32}$$

$$= f_{ts} - (1-R)\left(\frac{M_o}{S_2}\right) - Rf_{ci} \tag{33}$$



Max. moment sec.

FLEXURAL DESIGN BASED ON ALLOWABLE STRESS

Design of Beams with Variable Eccentricity

-

The minimum acceptable value of S_1 is

$$S_1 \geq \frac{M_d + M_l}{Rf_{ti} - (1-R)\dfrac{M_o}{S_1} - f_{cs}} \quad \text{or} \quad S_1 \geq \frac{(1-R)M_o + M_d + M_l}{Rf_{ti} - f_{cs}} \qquad (34)$$

Similarly

$$S_2 \geq \frac{(1-R)M_o + M_d + M_l}{f_{ts} - Rf_{ci}} \qquad (35)$$

From $I_c = S_1 c_1 = S_2 c_2$, the centroidal axis must be located

$$\frac{c_1}{c_2} = \frac{S_2}{S_1} \qquad (36)$$

FLEXURAL DESIGN BASED ON ALLOWABLE STRESS

Design of Beams with Variable Eccentricity

-

in terms of the total section depth $h = c_1 + c_2$

$$\frac{c_1}{h} = \frac{S_2}{S_1 + S_2} \qquad (37)$$

The concrete centroidal stress under initial condition

$$f_{cci} = f_{ti} - \frac{c_1}{h}\left(f_{ti} - f_{ci}\right) \qquad (38)$$

Then initial prestress force P_i is obtained by

$$P_i = A_c f_{cci} \qquad (39)$$

FLEXURAL DESIGN BASED ON ALLOWABLE STRESS

Design of Beams with Variable Eccentricity

-

The eccentricity of prestress force may be found by considering the flexural stresses that must be imparted by the ending moment $P_i e$.

The flexural stress at the top surface of the beam resulting from the eccentric prestress force alone is

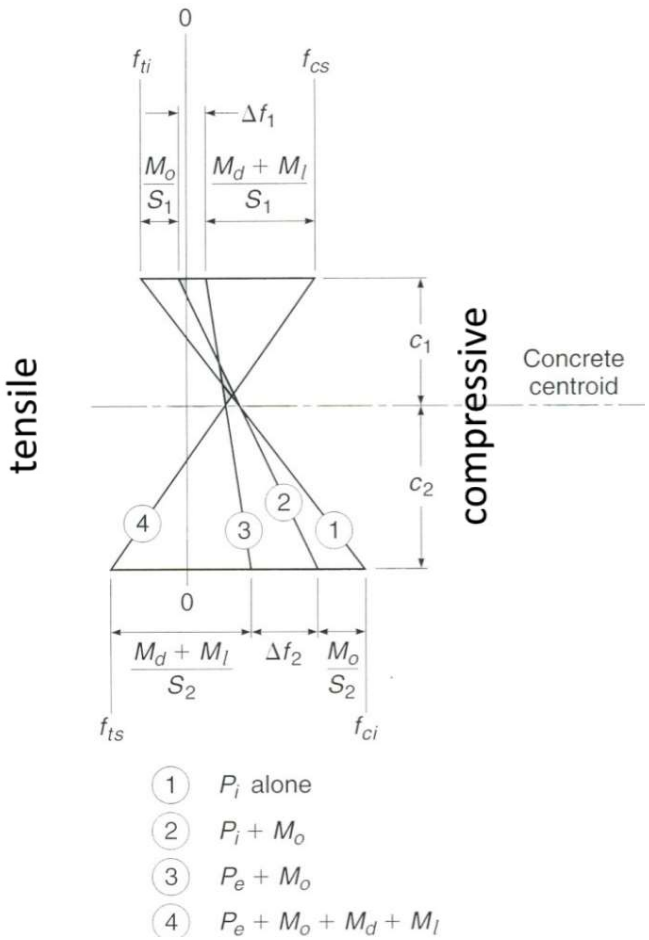
$$\frac{P_i e}{S_1} = \left(f_{ti} - f_{cci}\right) + \frac{M_o}{S_1} \qquad (40)$$

From which the required eccentricity is

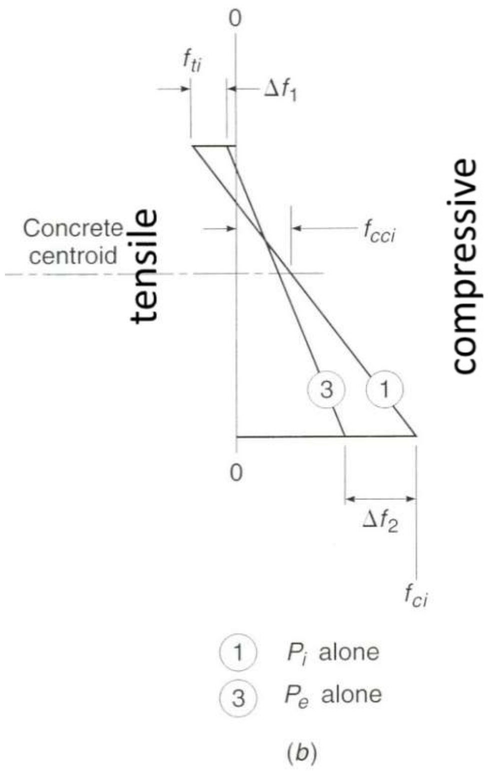
$$e = \left(f_{ti} - f_{cci}\right)\frac{S_1}{P_i} + \frac{M_o}{P_i} \qquad (41)$$

FLEXURAL DESIGN BASED ON ALLOWABLE STRESS

<u>Design of Beams with Constant Eccentricity</u>



Max. moment sec.



Support sec.

FLEXURAL DESIGN BASED ON ALLOWABLE STRESS

Design of Beams with Constant Eccentricity

If P_i and e were to be held constant along the span, the stress limits f_{ti} and f_{ci} would be EXCEEDED elsewhere along the span, where M_o is less than its maximum value.

Design concept is to avoid such a condition Eq.(41)

$$e < \left(f_{ti} - f_{cci}\right)\frac{S_1}{P_i} + \frac{M_o}{P_i} \tag{42}$$

The stress changes Δf_1 and Δf_2

$$\Delta f_1 = (1-R)\left(f_{ti} + \frac{M_o}{S_1}\right) \tag{43}$$

$$\Delta f_2 = (1-R)\left(-f_{ci} + \frac{M_o}{S_2}\right) \tag{44}$$

FLEXURAL DESIGN BASED ON ALLOWABLE STRESS

Design of Beams with Constant Eccentricity

In this case, the available stress ranges between limit stresses must provide for the effect of "M_o" as well as M_d and M_l

$$\begin{aligned} f_{1r} &= f_{ti} - \Delta f_1 - f_{cs} \\ &= Rf_{ti} - f_{cs} \end{aligned} \tag{45}$$

$$\begin{aligned} f_{2r} &= f_{ts} - \Delta f_2 - f_{ci} \\ &= f_{ts} - Rf_{ci} \end{aligned} \tag{46}$$

And the requirement on the section moduli are

$$S_1 \geq \frac{M_o + M_d + M_l}{Rf_{ti} - f_{cs}} \tag{47}$$

$$S_2 \geq \frac{M_o + M_d + M_l}{f_{ts} - Rf_{ci}} \tag{48}$$

FLEXURAL DESIGN BASED ON ALLOWABLE STRESS

Design of Beams with Constant Eccentricity

The concrete centroidal stress is the same as before

$$f_{cci} = f_{ti} - \frac{c_1}{h}\left(f_{ti} - f_{ci}\right) \quad (49)$$

And initial prestress force P_i

$$P_i = A_c f_{cci} \quad (50)$$

BUT the required eccentricity is

$$\frac{P_i \cdot e}{S_1} = \left(f_{ti} - f_{cci}\right) + \frac{M_o}{S_1} \quad (51)$$

$$\Rightarrow \quad e = \left(f_{ti} - f_{cci}\right)\frac{S_1}{P_i} \quad (52)$$

Example 5.3>

Design of beam with variable eccentricity tendons.

Consider a post-tensioned PSC beam with 12 m simple span.

Intermittent live load 14.5 kN/m

Superimposed dead load 7.3 kN/m (not including self-weight)

f_{ck} = 42 MPa and at the time of transfer 29 MPa

Time dependent loss of 15 percent of initial prestress

→ R = 0.85

Determine

- The required concrete dimensions
- Magnitude of prestress force
- Eccentricity of the steel centroid.

Solution

· Stress limits

$$f_{ci} = -0.6 \times 29 = -17.4\,MPa$$

$$f_{ti} = 0.25\sqrt{29} = 1.35\,MPa$$

$$f_{cs} = -0.6 \times 42 = -25.2\,MPa$$

$$f_{ts} = 0.63\sqrt{42} = 3.88\,MPa$$

· The self weight is estimated at 3.65 kN/m
The service moment due to transverse loading are

$$M_o = \frac{1}{8} \times 3.65 \times 12^2 = 65.7\,kN \cdot m$$

$$M_d + M_l = \frac{1}{8} \times 21.8 \times 12^2 = 392.3\,kN \cdot m$$

Solution

The required section moduli are

$$S_1 \geq \frac{(1-R)M_o + M_d + M_l}{Rf_{ti} - f_{cs}} = \frac{\left((0.15)(65.7)+392.3\right)\times 10^6}{(0.85)(1.35)+25.2}$$

$$= 1.53\times 10^7 mm^3$$

$$S_2 \geq \frac{(1-R)M_o + M_d + M_l}{f_{ts} - Rf_{ci}} = \frac{\left((0.15)(65.7)+392.3\right)\times 10^6}{3.88+(0.85)(17.4)}$$

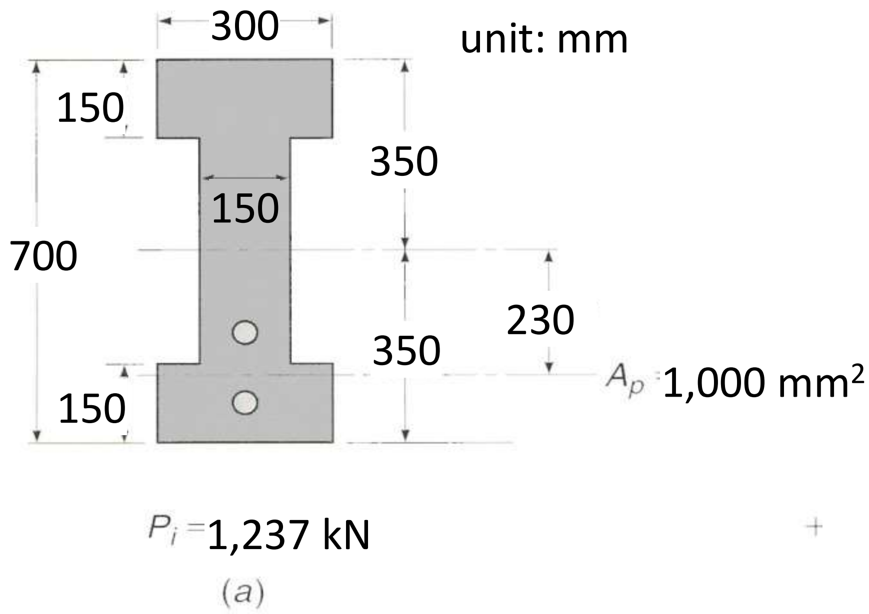
$$= 2.15\times 10^7 mm^3$$

⇨ Asymmetric section is appropriate.

Solution

But, a symmetric section is selected for simplicity.

$$I_c = 8.28\times10^9\ mm^4$$
$$S = 2.33\times10^7\ mm^3$$
$$A_c = 1.54\times10^5\ mm^2$$
$$r^2 = 5.35\times10^4\ mm^2$$
$$\omega_o = 3.6\,kN/m$$ (as assumed)



Solution

· The concrete centroidal stress is

$$f_{cci} = f_{ti} - \frac{c_1}{h}\left(f_{ti} - f_{ci}\right)$$

$$= 1.35 - \frac{350}{700}(1.35 + 17.4) = -8.03\,MPa$$

⇨ initial prestress $P_i = A_c \; f_{cci} = (1.54 \times 10^5)(8.03) = 1,237 kN$

· Required tendon eccentricity at the max. moment section is

$$e = (f_{ti} - f_{cci})\frac{S_1}{P_i} + \frac{M_o}{P_i}$$

$$= (1.35 + 8.03)\frac{2.33 \times 10^7}{1,237 \times 10^3} + \frac{65.7 \times 10^6}{1,237 \times 10^3} = 230\,mm$$

Elsewhere along the span, the eccentricity will be reduced so that the concrete stress limits will not violated.

Solution

· Determination of prestressing steel

$$A_p = 100 mm^2$$

$P_i = 1,237 kN$ → 12 mm. Grade 270 low-relaxation strand

$$f_{pu} = 1,860\, MPa$$

$$f_{py} = 0.9 f_{pu} = 0.9 \times 1,860 = 1,674\, MPa$$

· Codes provides that the permissible stress in the strand immediately after transfer must not exceed.

$0.74 f_{pu} = 1,376\, MPa$ or $0.82 f_{py} = \underline{1,327\, MPa}$

$$\therefore\ A_{ps} = \frac{1,237 \times 10^3}{1,327} = 932\, mm^2$$

Solution

· The number of strands required is

$$\frac{A_{ps}}{A_p} = \frac{932}{100} = 9.32$$

⇨ Two FIVE–strand tendons will be used.

Each will be stressed to 619 kN (= 0.5P_i)

· Check the stress limits at CRITICAL stages.

Calculate f_1 & f_2 at P_i, P_e, M_o, M_d+ M_l

⇨ $P_i + M_o$ & $P_e + M_o + (M_d + M_l)$ at midspan

13. Prestressed Concrete

LOSS OF PRESTRESS

How to Estimate prestress losses ?

$P_j \longrightarrow P_i \longrightarrow P_e$

Jacking　　Initial prestress　　Effective prestress

Loss occurred due to:

1) Anchorage slip, Elastic shortening, Friction b/w duct and tendon (Instant)
2) Creep, Shrinkage, Relaxation (Time dependent)

LOSS OF PRESTRESS

Estimates of Separate Losses

INTERDEPENDENCE b/w time dependent losses

The rate of loss due to one effect is CONTINUOUSLY being altered by changes in stress due to other causes.

e.g. the relaxation of stress in the tendons is affected by length changes due to creep of concrete. Rate of creep, in turn, is altered by change in tendon stress.

LOSS OF PRESTRESS

Estimates of Separate Losses

(1) Anchorage slip

Mainly corresponds to post-tensioned members and dependent on hardwares selected

The amount of movement ΔL is determined by test and provided by manufactures

$$\Delta f_{s,slip} = \frac{\Delta L}{L} E_p \tag{74}$$

where L is tendon length and

E_p is elastic modulus of the tendon

LOSS OF PRESTRESS

Estimates of Separate Losses

(1) Anchorage slip

Note

1) ΔL is independent of L. Therefore, the stress loss is large for short tendons.
2) If frictional losses are high, the anchorage slip loss may be concentrated mostly near the end of the tendon, requiring special consideration.

LOSS OF PRESTRESS

Estimates of Separate Losses

(2) Elastic shortening of the concrete

should be considered for pre-tensioned beams and post-tensioned members in which ALL tendons are prestressed AT ONCE.

First considering pretensioned beams, the compressive stress at the level of steel centroid,

$$f_2 = -\frac{P_i}{A_c}\left(1+\frac{e\,c_2}{r^2}\right)+\frac{M_o c_2}{I_c}$$

⇨ $$f_c = -\frac{P_i}{A_c}\left(1+\frac{e^2}{r^2}\right)+\frac{M_o e}{I_c} \qquad (75)$$

LOSS OF PRESTRESS

Estimates of Separate Losses

(2) Elastic shortening of the concrete

Introducing $n = E_p / E_c$, the loss of stress in the tendon due to elastic shortening of the concrete is

$$\Delta f_{s,ela} = E_p \varepsilon_P = E_p \frac{f_c}{E_c} = nf_c \tag{76}$$

It should be noted that E_c must be that of concrete at the time of (DE)TENSIONING.

For post–tensioned beam with the tendons tensioned IN SEQUANCE, there will be the losses.
In the most case, it is sufficiently accurate to calculate the loss in the first strand and to apply one half that value to all strands.

LOSS OF PRESTRESS

Estimates of Separate Losses

(3) Friction losses

The total friction loss is the sum of the wobble friction due to *unintentional misalignment* and the curvature friction due to the *intentional curvature of tendons*.

Even a straight tendon duct will have some unintentional misalignment.

⇨ Wobble friction must always be considered in post-tensioned work.

LOSS OF PRESTRESS

Estimates of Separate Losses

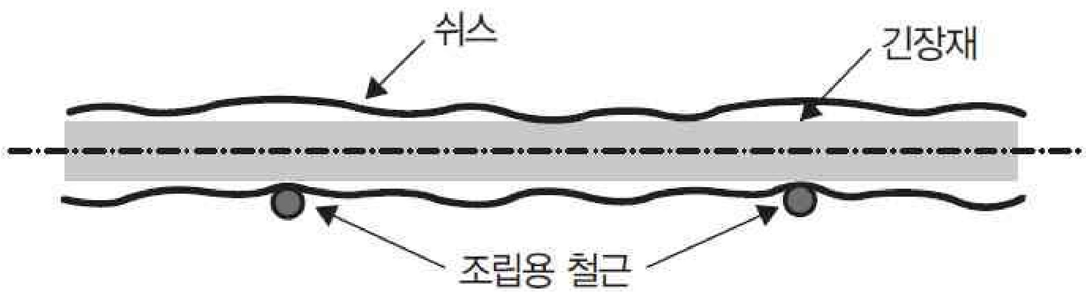
(3) Friction losses

i) Wobble friction coefficient *K*

It depends on the type of tendon and duct used and on the care taken during construction.

The incremental stress loss *dP* due to wobble friction in a short length *dx* of tendon is

$$dP = KPdx \tag{77}$$



LOSS OF PRESTRESS

Estimates of Separate Losses

(3) Friction losses

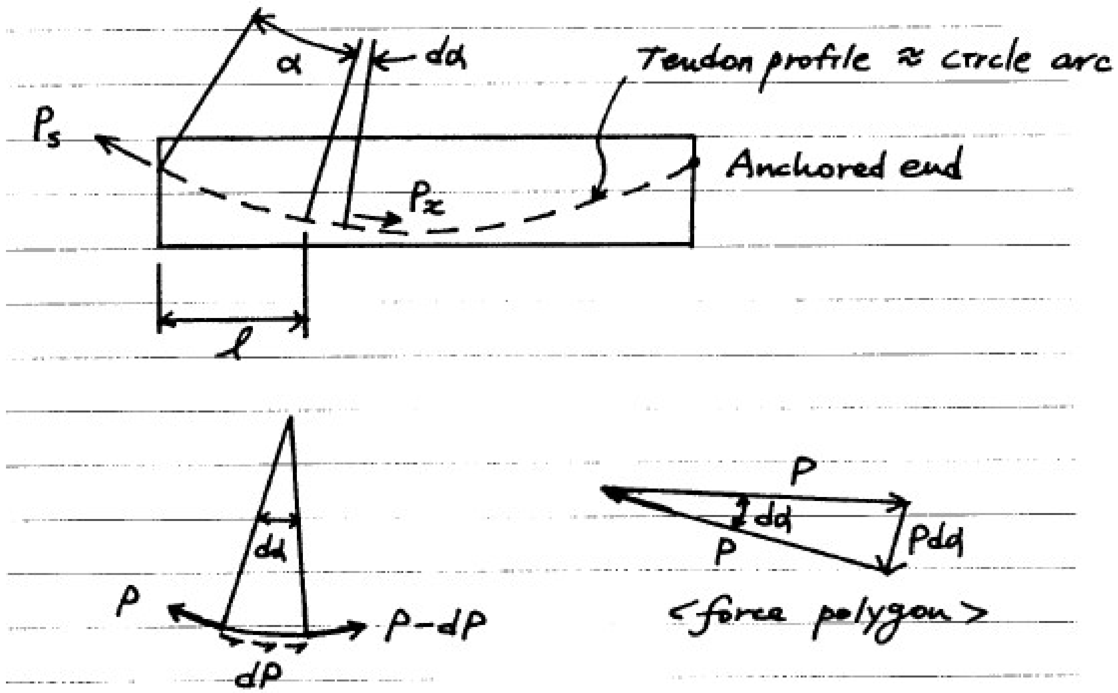
i) Wobble friction coefficient *K* (KCI 9.4.2.1)

		PS 강재의 종류	파상마찰계수 K	곡률마찰계수 μ
부착긴장재		강선 강봉 강연선	0.0033~0.00500. 0003~0.0020 0.0015~0.0066	0.15~0.25 0.15~0.25 0.15~0.25
비부착 긴장재	수지, 방수, 피복	강선 강연선	0.0033~0.0066 0.0033~0.0066	0.05~0.15 0.05~0.15
	그리스로 미리 도포된 경우	강선 강연선	0.0010~0.0066 0.0010~0.0066	0.05~0.15 0.05~0.15

LOSS OF PRESTRESS

Estimates of Separate Losses

ii) Curvature friction



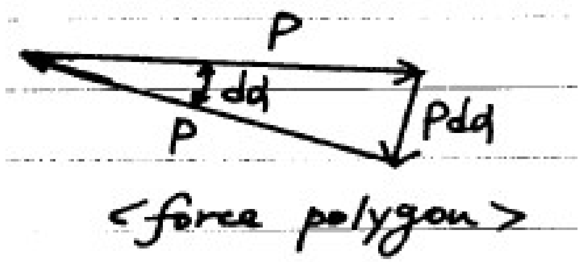
The loss of force in the short length $d\alpha$ (defined by the angle change) is dP. Here P is the value of prestress force at the location considered.

LOSS OF PRESTRESS

Estimates of Separate Losses

ii) Curvature friction

The equilibrium polygon of force acting on the short segment indicates that the component of force normal to the tendon is equal to *P*d*α*



If the frictional coeff. Between tendon and duct is μ, the incremental stress loss dP due to curvature is

$$dP = \mu P d\alpha \quad (78)$$

LOSS OF PRESTRESS

Estimates of Separate Losses

(3) Friction losses

Combining the effects by wobble and curvature friction

$$dP = KPdx + \mu Pd\alpha \tag{79}$$

the friction loss is conveniently expressed *dP/P* at the location considered. Then integrate between proper limits.

$$\int_{P_x}^{P_s} \frac{dP}{P} = \int_0^l Kdx + \int_0^\alpha \mu d\alpha \tag{80}$$

$$\ln \frac{P_s}{P_x} = Kl + \mu\alpha \tag{81}$$

⇨ $P_s = P_x e^{(Kl + \mu\alpha)}$ relation between prestress force P_s at the jack and reduced P_x at a distance

LOSS OF PRESTRESS

Estimates of Separate Losses

(3) Friction losses

If frictional losses are sufficiently low, it is satisfactory to calculate the losses based on the tension P_x at the distance from the jack.

$$P_s - P_x = KP_x l + \mu P_x \alpha \qquad (83)$$

⇨ $$P_s = P_x(1 + Kl + \mu\alpha) \quad \text{(Approximation)} \qquad (84)$$

The KCI Code 9.4.2 permit the use of this simplified form, if the value of $Kl + \mu\alpha \leq 0.3$

LOSS OF PRESTRESS

Estimates of Separate Losses

(3) Friction losses

The above eq. can be expressed in terms of loss of stress rather than loss in force. Thus loss in force due to friction is

$$\Delta P_{fr} = P_s - P_x = P_s\left(1-e^{-(Kl+\mu\alpha)}\right) \quad (85)$$

$$\Rightarrow \quad \Delta f_{s,fr} = \frac{\Delta P_{fr}}{A_p} = f_s\left(1-e^{-(Kl+\mu\alpha)}\right) \quad (86)$$

where f_s is the tendon stress at the jack.

LOSS OF PRESTRESS

Estimates of Separate Losses

(3) Friction losses

For approximated eq.

$$\Delta P_{fr} = P_s - P_x = P_x\left(Kl + \mu\alpha\right)$$

$$\approx P_s\left(Kl + \mu\alpha\right) \qquad (85)$$

$$\Rightarrow \quad \Delta f_{s,fr} = f_s\left(Kl + \mu\alpha\right) \qquad (86)$$

LOSS OF PRESTRESS

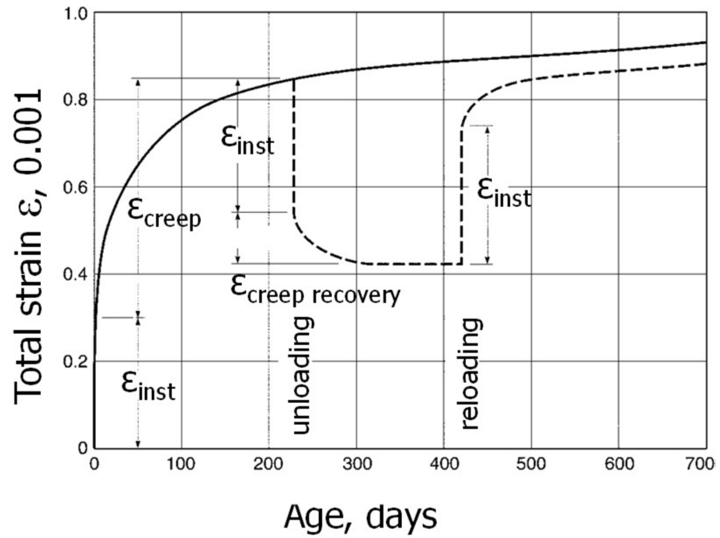
Estimates of Separate Losses

(4) Losses due to creep

The ultimate creep coeff.

$$C_u = \frac{\varepsilon_{cu}}{\varepsilon_{ci}} = \frac{\varepsilon_{creep}}{\varepsilon_{inst}} \qquad (87)$$

Typical value of C_u ranges from 2 to 4. Average 2.35



The interdependence of time–dependent losses
: Compressive force causing creep is NOT constant, but diminishes with the passage of time, because of RELAXATION of steel and SHRINKAGE of concrete.

⇨ To account for this, the prestress force causing creep should be assumed to be $0.9P_i$

LOSS OF PRESTRESS

Estimates of Separate Losses

Note

Therefore, step–by–step approach is adopted, e.g. PCI method

Step 1) For pretensioned members
; from the time of anchorage of the prestess steel until the age of prestressing the concrete
For post-tensioned members
; from the time when curing ends until the age of prestressing the concrete

Step 2) From the end of Step 1) until age 30 days, or the time when a member is subjected to load in addition to its own weight

LOSS OF PRESTRESS

Estimates of Separate Losses

Note

Step 3) From the end of Step 2) until age 1 year

Step 4) From the end of Step 3) until the end of service life

After f_c (concrete stress at the level of steel centroid) is found, the loss of steel stress associated with concrete creep can be determined.

$$\Delta f_{s,creep} = C_u n f_c \quad (88)$$

LOSS OF PRESTRESS

Estimates of Separate Losses

(4) Losses due to creep

After f_c (concrete stress at the level of steel centroid) is found, the loss of steel stress associated with concrete creep can be determined.

$$\Delta f_{s,creep} = C_u n f_c \qquad (88)$$

Note

- For post–tensioned (bonded) and pre-tensioned member, the loss due to creep is dependent on the concrete stress at the section of maximum moment.

- For unbounded post–tensioned member, the stress reduction at steel is more or less uniform. So average value of f_c between anchorages may be used.

LOSS OF PRESTRESS

Estimates of Separate Losses

(5) Losses due to shrinkage

Only the part of the drying shrinkage that occurs after transfer of prestressing force to the member need to be considered.

For pre-tensioned members, transfer commonly take place just 18 hours after pouring the concrete and nearly ALL the shrinkage takes place after that time.

On the other hand, post–members are SELDOM stressed at an early age than 7 days and often much later than that.
: Typically 15% of ultimate shrinkage by 7 days
40% of ultimate shrinkage by 28 days

LOSS OF PRESTRESS

Estimates of Separate Losses

(5) Losses due to shrinkage

Once the amount of concrete shrinkage has been determined

$$\Delta f_{s,sh} = E_p \varepsilon_{sh} \tag{89}$$

The shrinkage strain ε_{sh}=0.0004~0.0008 (0.0006 average)

LOSS OF PRESTRESS

Estimates of Separate Losses

(6) Losses due to relaxation

The relaxation calculation can be based on a $0.9P_i$ considering the interdependent effects.

The ratio of reduced stress f_p to initial stress f_{pi} can be estimated

$$\frac{f_p}{f_{pi}} = 1 - \frac{\log^t}{10}\left(\frac{f_{pi}}{f_{py}} - 0.55\right) \quad (90)$$

In terms of the loss in stress

$$\Delta f_{s,rel} = f_{pi}\frac{\log^t}{10}\left(\frac{f_{pi}}{f_{py}} - 0.55\right) \quad (91)$$

LOSS OF PRESTRESS

Estimates of Separate Losses

(6) Losses due to relaxation

Note

The largest part of relaxation loss occurs SHORTLY AFTER the steel is stretched.

For stresses of $0.8f_{pu}$ and higher, even a very short period loading will substantial relaxation.