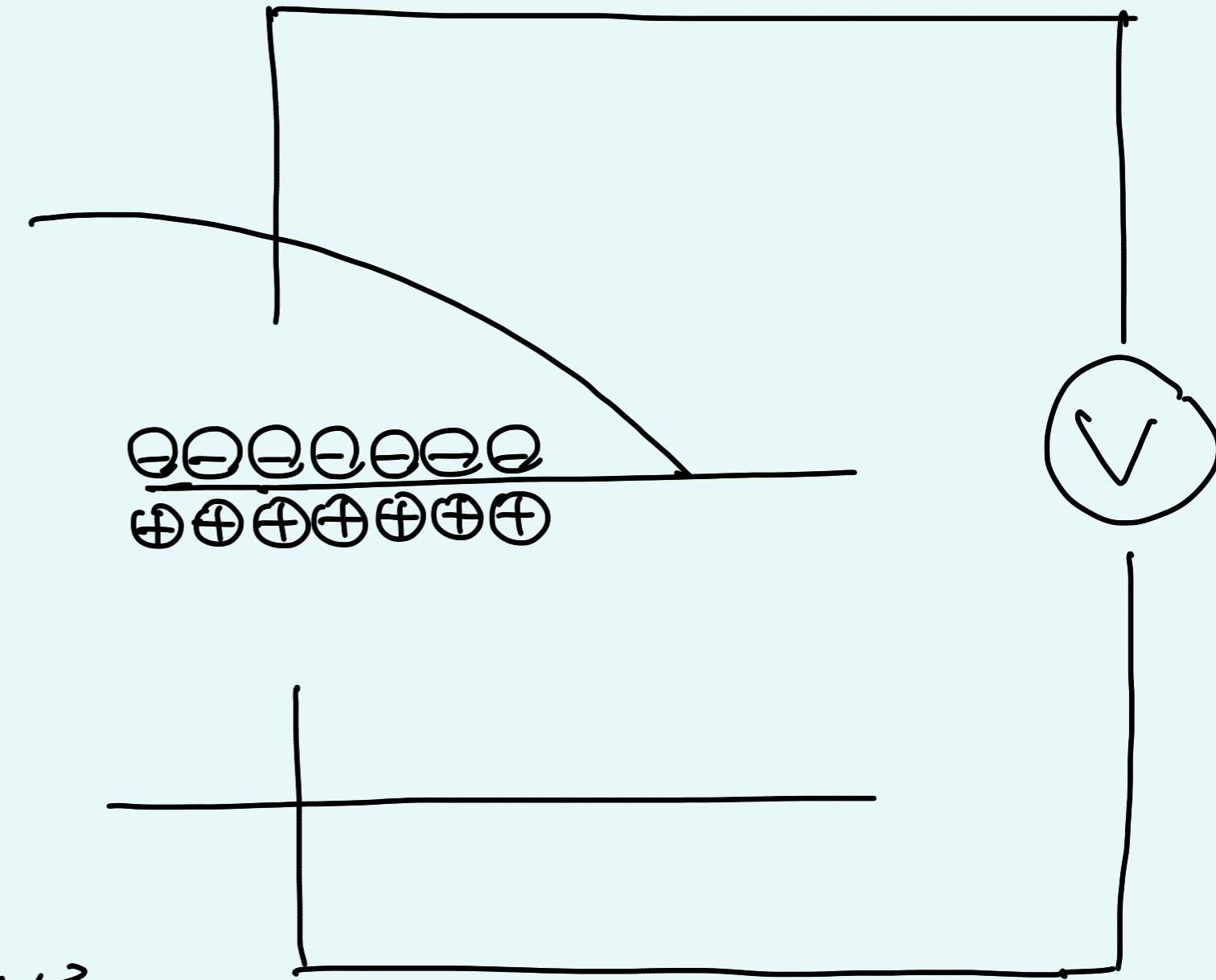
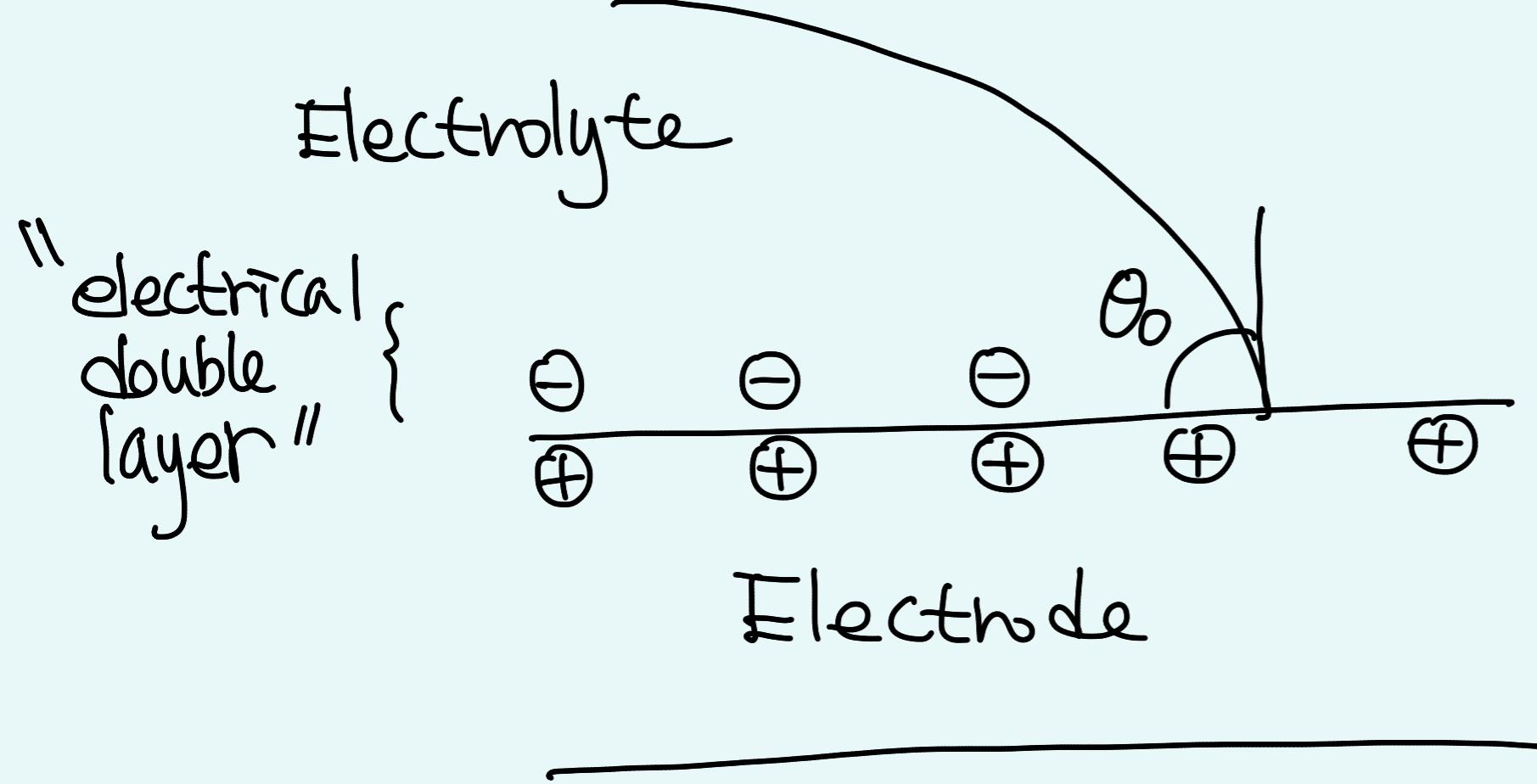


Electrowetting



$$\gamma_{SL}(V) = \gamma_{SL}(V=0) - \frac{C}{2}V^2$$

• Young's eq.

$$\gamma_{LG} \cos \theta = \gamma_{SG} - \gamma_{SL}$$

$$= \underbrace{\gamma_{SG} - \gamma_{SL,0}}_{\gamma_{SL,0}} + \frac{C}{2} V^2$$

$$= \gamma_{LG} \cos \theta_0 + \frac{C}{2} V^2$$

$$\gamma_{LG} (\cos\theta - \cos\theta_0) = \frac{c}{2} V^2$$

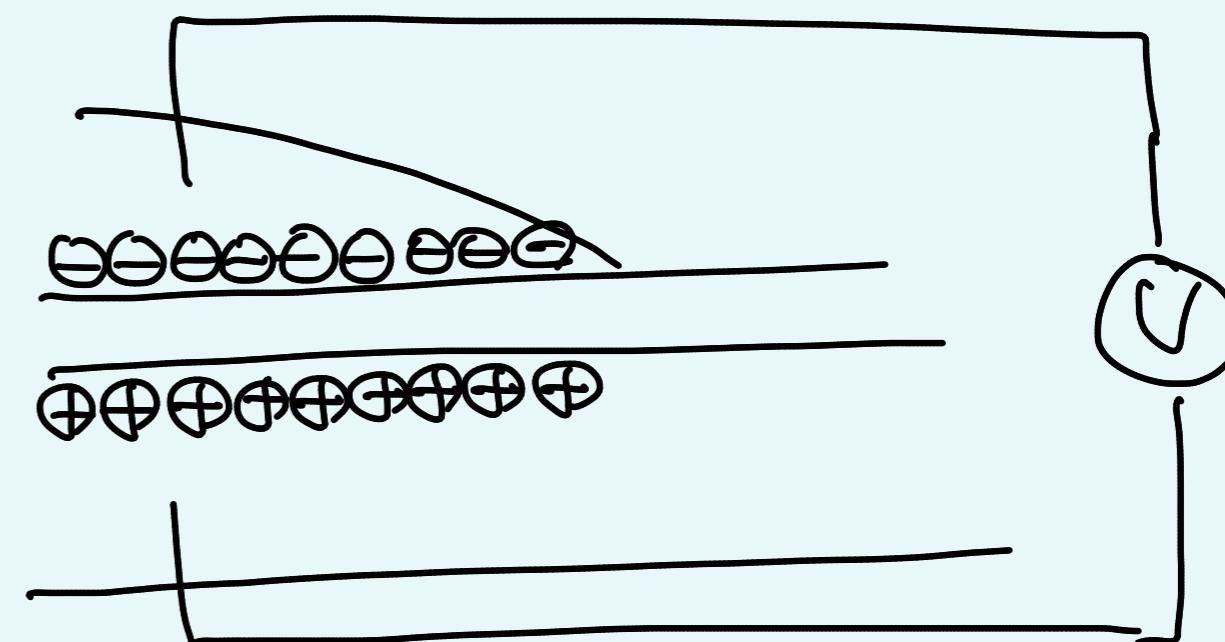
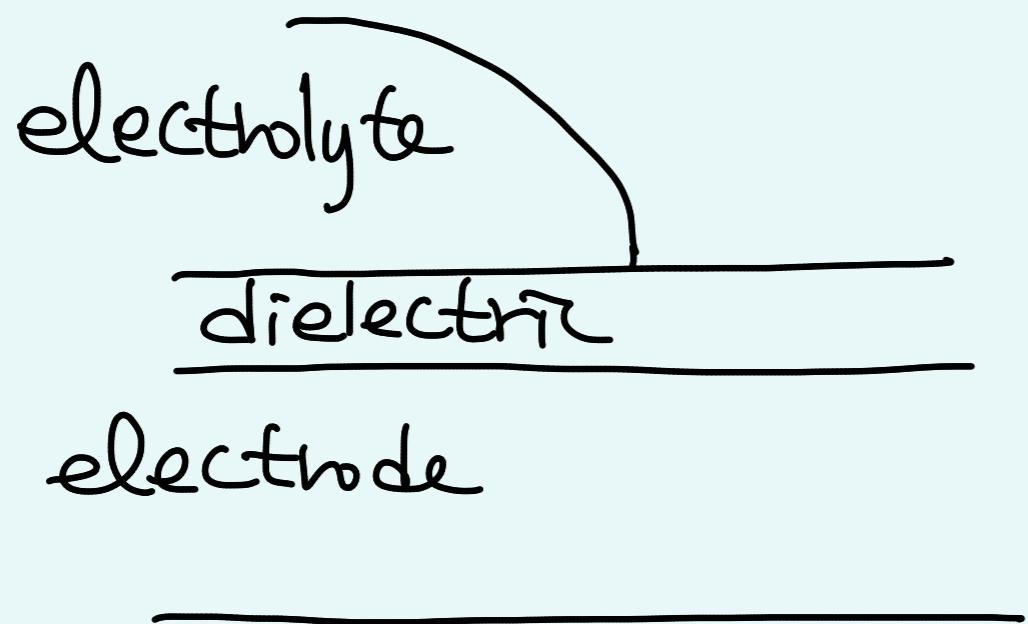
$$\cos\theta = \cos\theta_0 + \frac{c}{2\gamma_{LG}} V^2$$

$$\theta < \theta_0$$

"c" = specific capacitance of the layer (EDL)

"V" across EDL : too low , $\Delta\theta$ small

* Electrowetting on dielectric (EWOD)



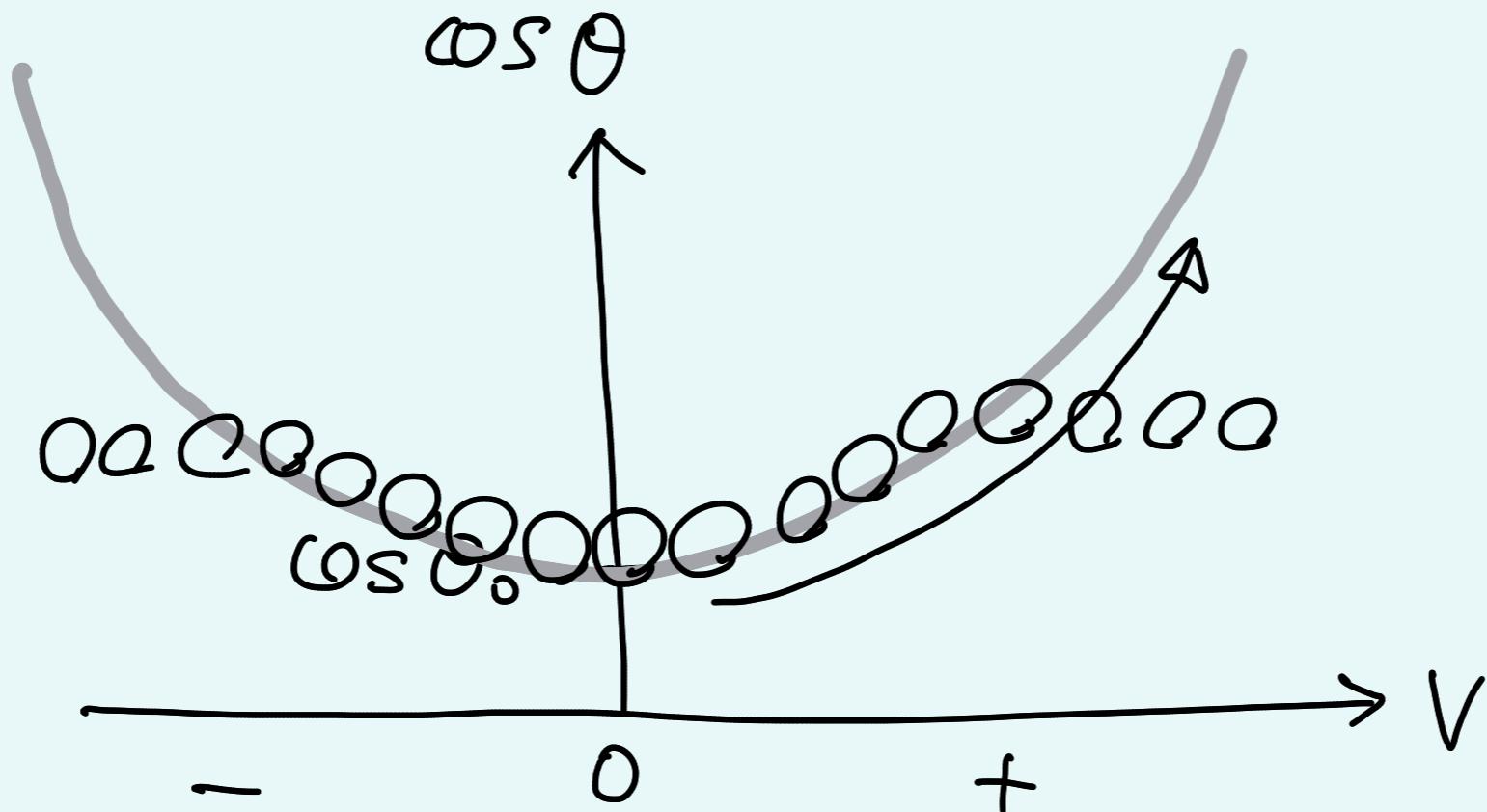
$$C = \frac{\epsilon_0 \epsilon}{t}$$

[ϵ_0 : vacuum permittivity]

[ϵ : dielectric constant]

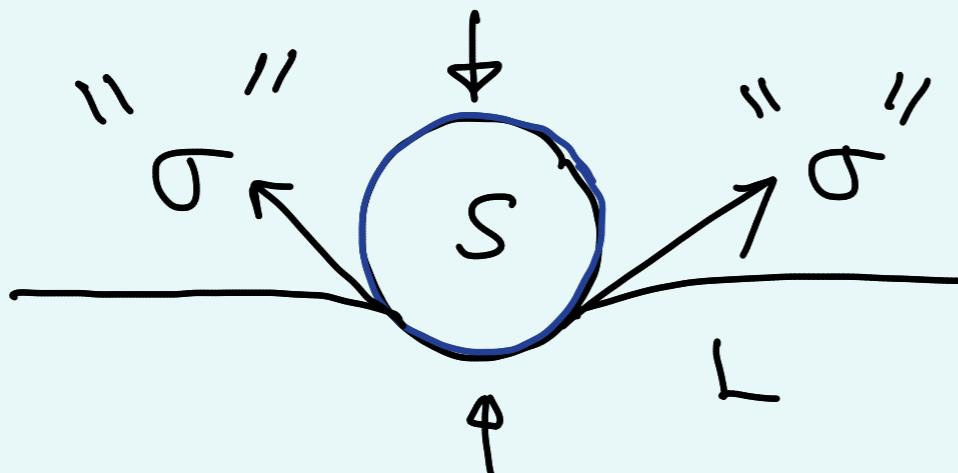
t : thickness

$$\cos \theta - \cos \theta_0 = \frac{\epsilon_0 \epsilon}{2 \gamma_{LG} t} V^2$$

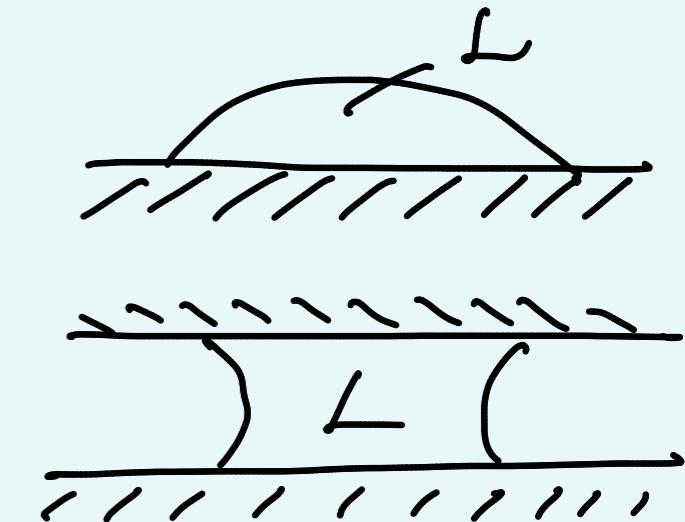


Midterm Exam Apr. 23rd. in class.

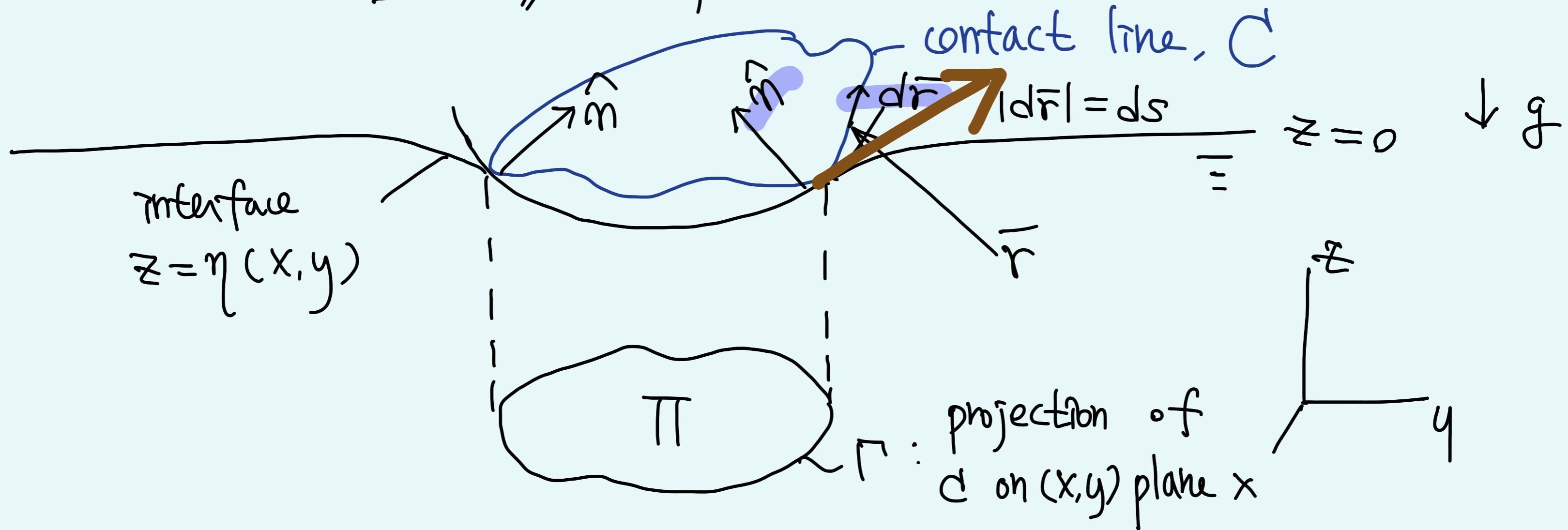
Force on a partly submerged body

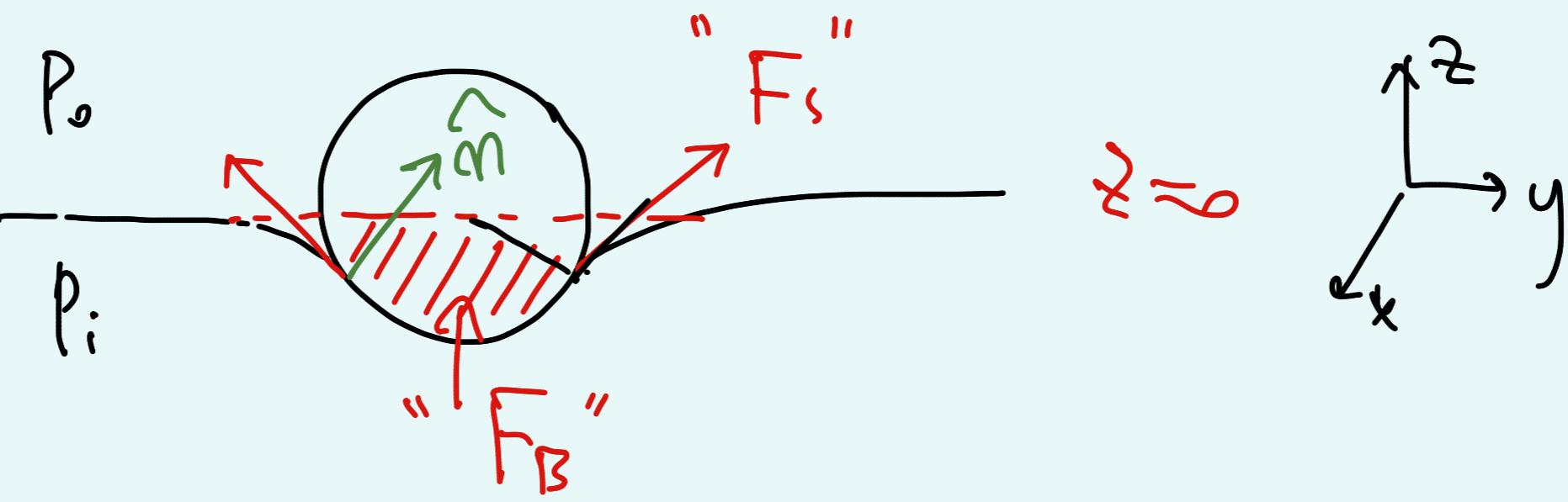


vs.



J.B. Keller, Physics of Fluids, 10, 3009 (1998)





* Force due to surface tension

Young-Laplace eq

$$P_i - P_o = -\sigma \kappa$$

$$\rho g \eta = P_o - P_i$$

$$\rho g \eta(x, y) = -\sigma \nabla \cdot \hat{n}$$

$$\kappa = -\nabla \cdot \hat{n} \quad \dots (1)$$

\hat{n} : unit normal vector to the interface pointing out of the liquid

$$\hat{n}(x,y) = n_1 \hat{i} + n_2 \hat{j} + n_3 \hat{k}$$

$$= \frac{1}{\sqrt{1+n_x^2+n_y^2}} (-n_x \hat{i} - n_y \hat{j} + \hat{k})$$

$$\nabla \cdot \hat{n} = \left(\frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \right) \cdot \hat{n}$$

$$= \frac{\partial n_1}{\partial x} + \frac{\partial n_2}{\partial y}$$

integrating (i) over (x,y) plane outside Γ

$$-\int_{R^2/\pi} \sigma \nabla \cdot \hat{n} dA = \int_{R^2/\pi} \rho g n dA$$

$$\text{LHS} = \sigma \int_{\Gamma} \hat{i} \cdot \hat{n} dt$$

by divergence theorem

RHS = $\rho g \cdot$ (volume between the interface $z=\eta$ and the surface $z=0$)

$$|\vec{a} \times \vec{b}| \\ = |\vec{a}| |\vec{b}| \sin \theta$$

$= W_M$: weight of the liquid displaced by the meniscus

We will show that $\Delta H_S = \hat{k} \cdot \vec{F}_S$: vertical component of surface tension force

• surf. ten. force on $ds \perp d\vec{r}, \perp \hat{n}$

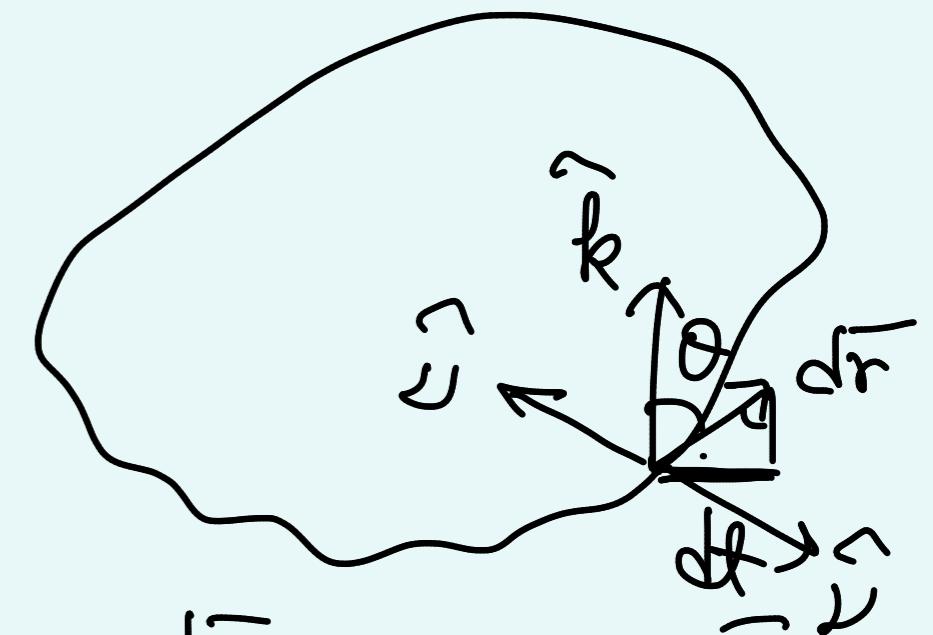
$$d\vec{F}_S = \sigma d\vec{r} \times \hat{n}$$

$$\vec{F}_S = \sigma \int_C \frac{d\vec{r}}{ds} \times \hat{n} ds$$

$$= \sigma \int_C \dot{\vec{r}}(s) \times \hat{n} ds. \quad \dot{\vec{r}} = \frac{d\vec{r}}{ds}$$

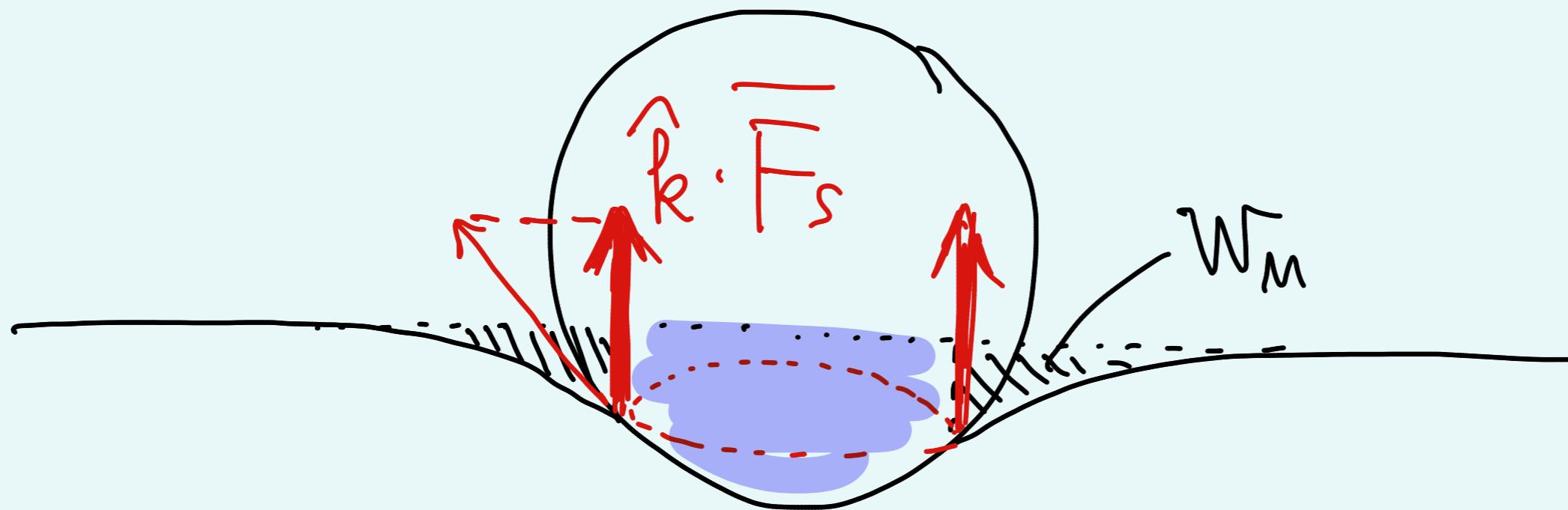
vector identity

$$\hat{k} \cdot \vec{F}_S = \sigma \int_C \hat{k} \cdot \dot{\vec{r}} \times \hat{n} ds \stackrel{\leftarrow}{=} \sigma \int_C \hat{n} \cdot \underbrace{\hat{k} \times \dot{\vec{r}}}_{= \hat{k} \times d\vec{r}} ds = \hat{k} \times d\vec{r} = \hat{i} dt$$

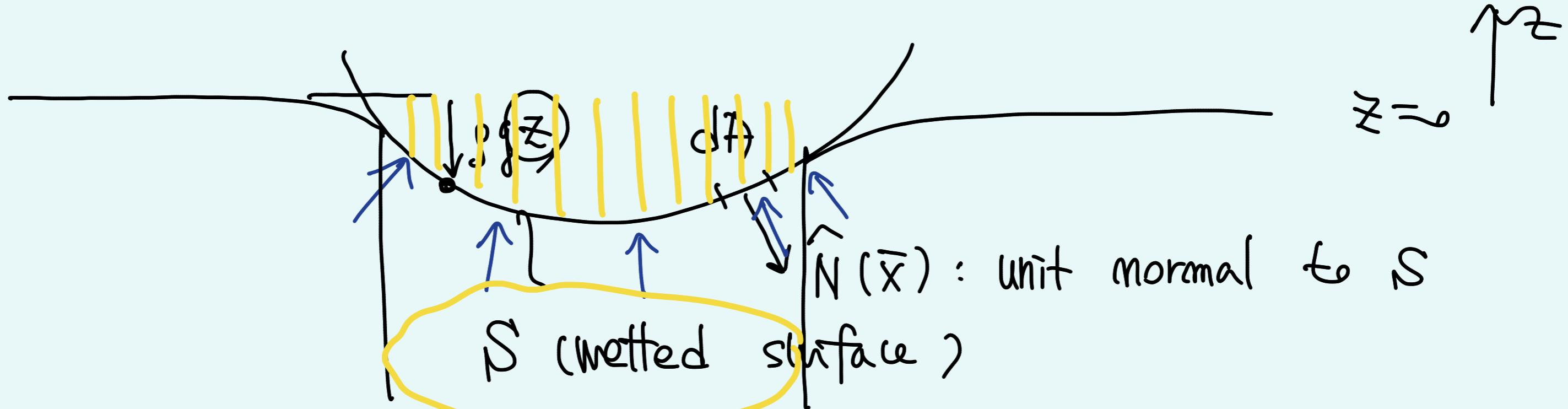


$\hat{\nu}$: inward normal vector of $\hat{\Sigma}$

$$\therefore \underbrace{\hat{k} \cdot \bar{F}_s}_{\Gamma} = \sigma \int_{\Gamma} \hat{n} \cdot \hat{\nu} dt$$
$$= W_M$$



* Force due to hydrostatic pressure



$$\bar{F}_p = \int_S \rho g z \hat{N}(\bar{x}) dA$$

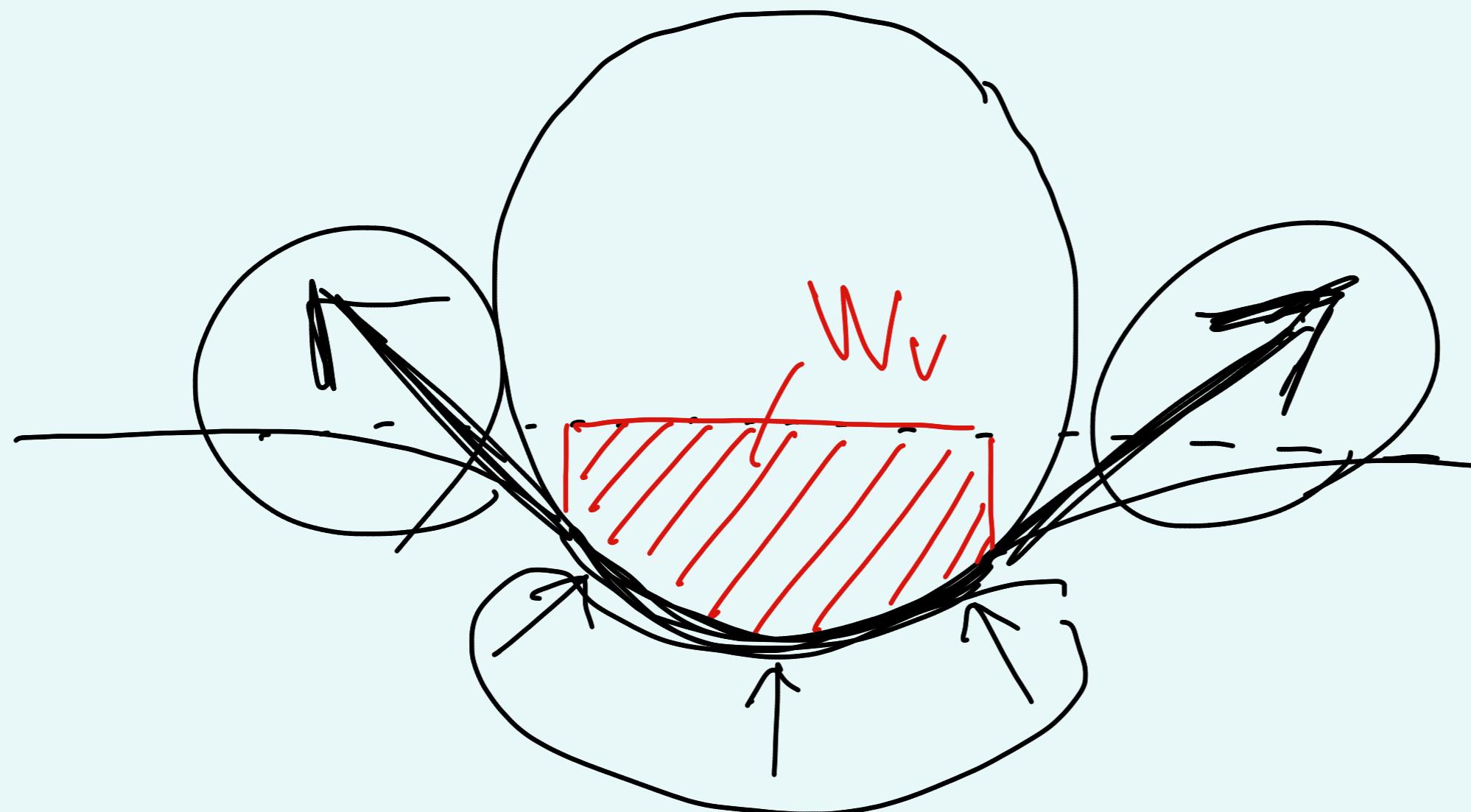
vertical component of \bar{F}_p

$$\hat{k} \cdot \bar{F}_p = \rho g \int_S \hat{k} \cdot \hat{N}(\bar{x}) dA$$

$$= -\rho g \int_{\Pi} z(x,y) dx dy$$

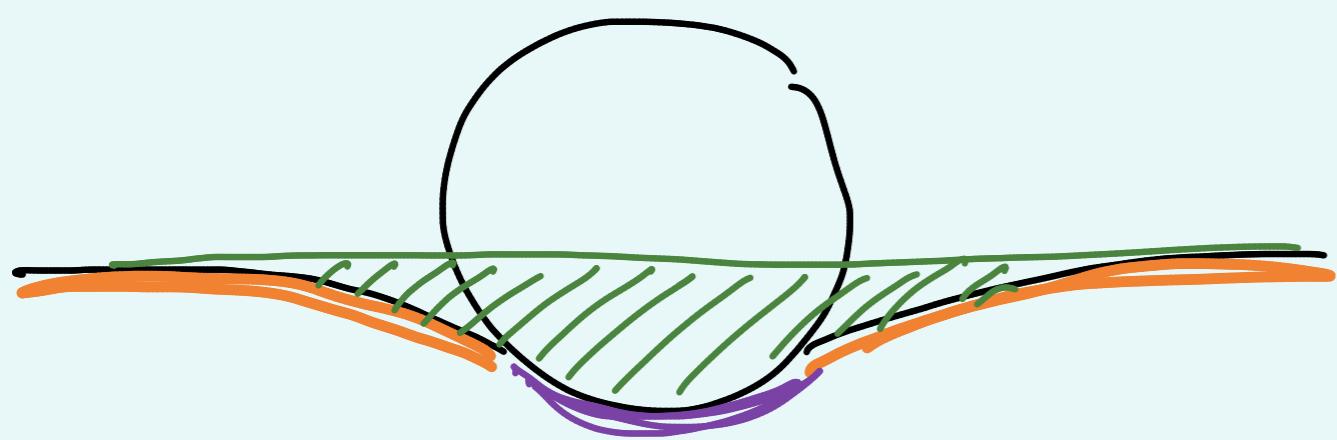
\hat{k}
 θ
 ϕ
 $\hat{N} dA$
 $\hat{k} \cdot \hat{N} dA = -\cos\phi dA$
 $= -dx dy$

$= W_v$: weight of liquid in the vertical cylinder
through C , which intersects the
horizontal plane in the curve Γ

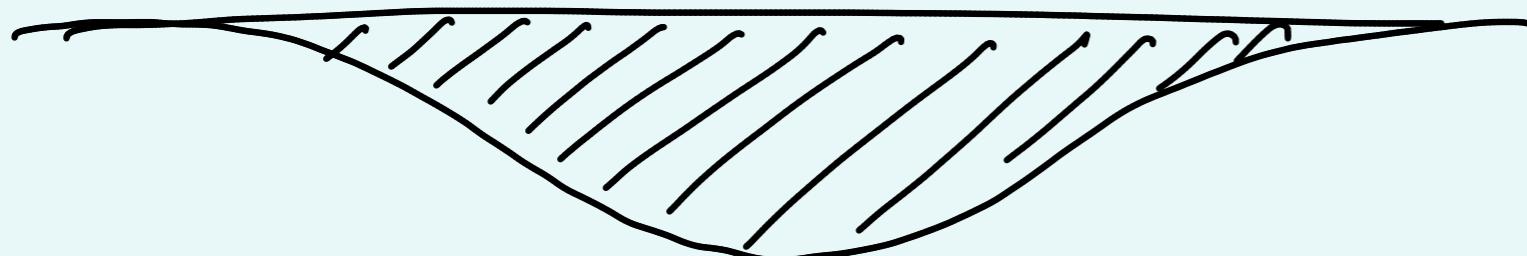


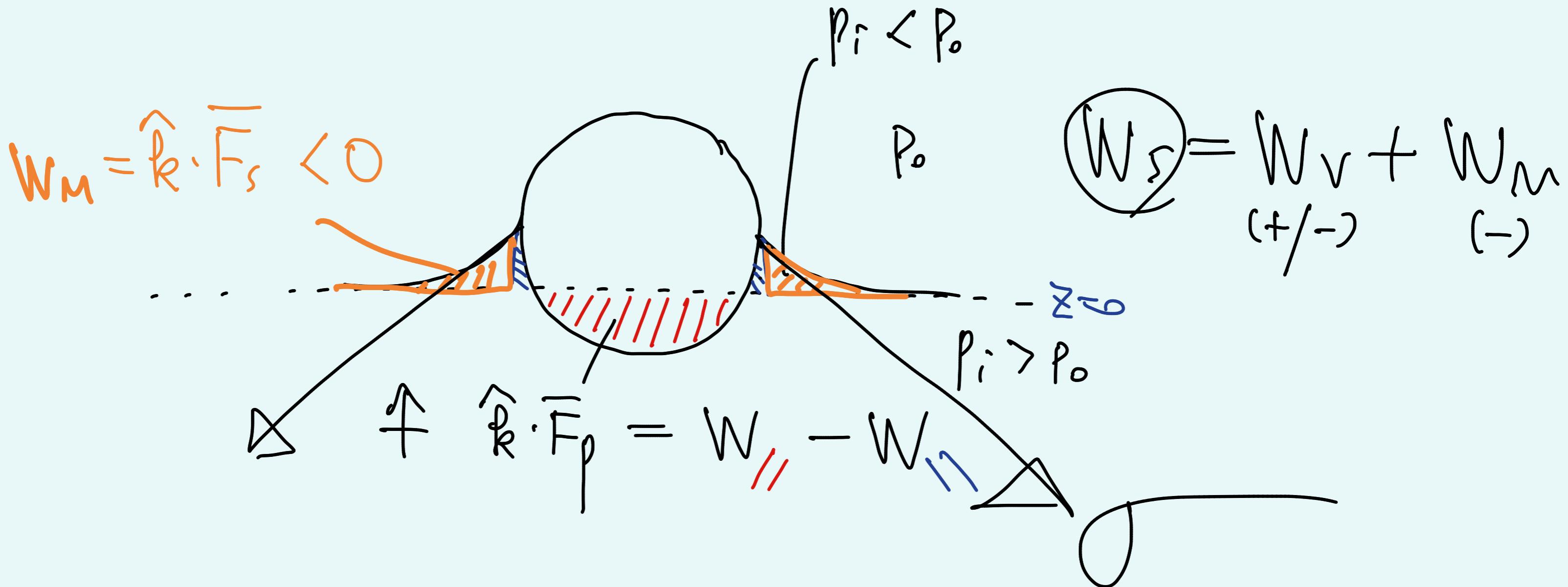
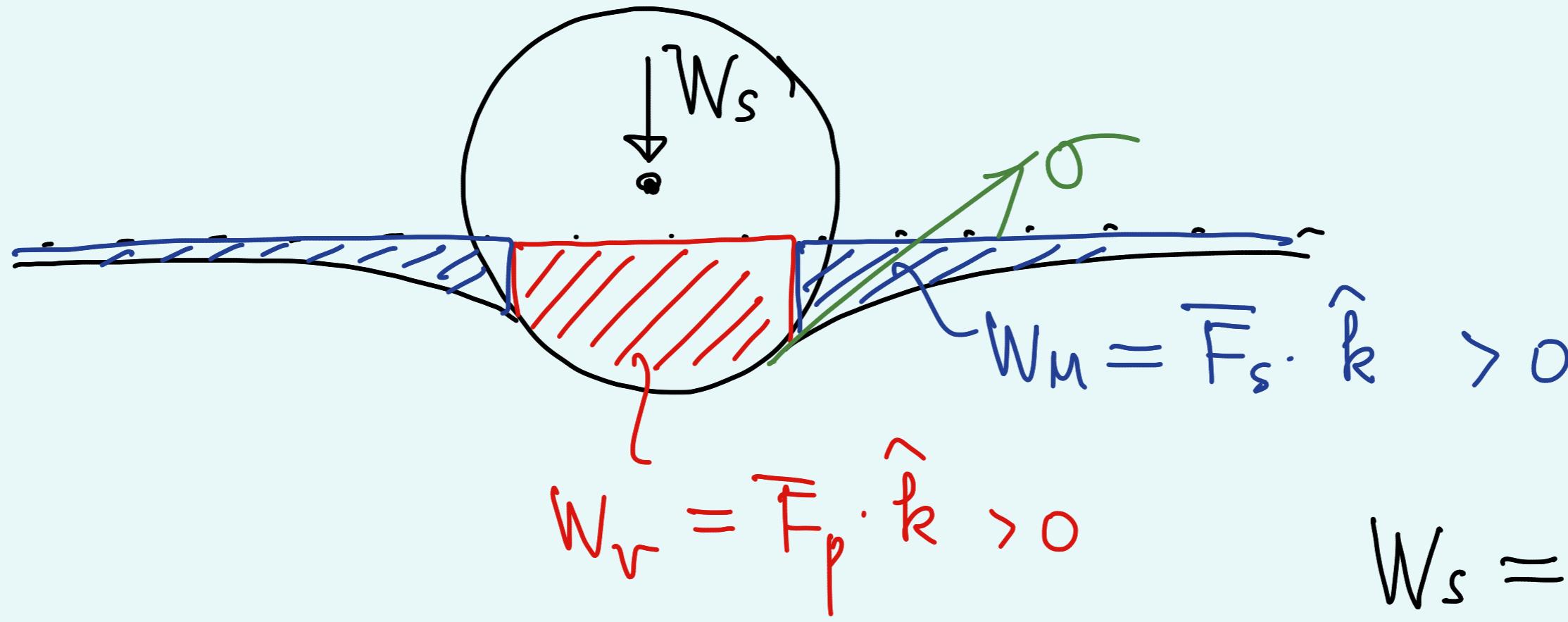
* Total vertical force on the body

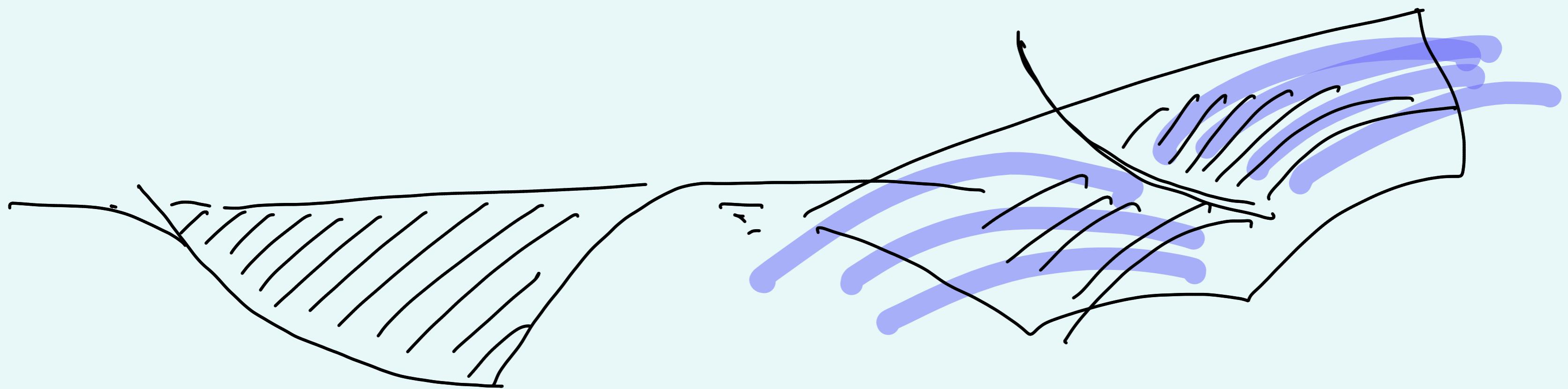
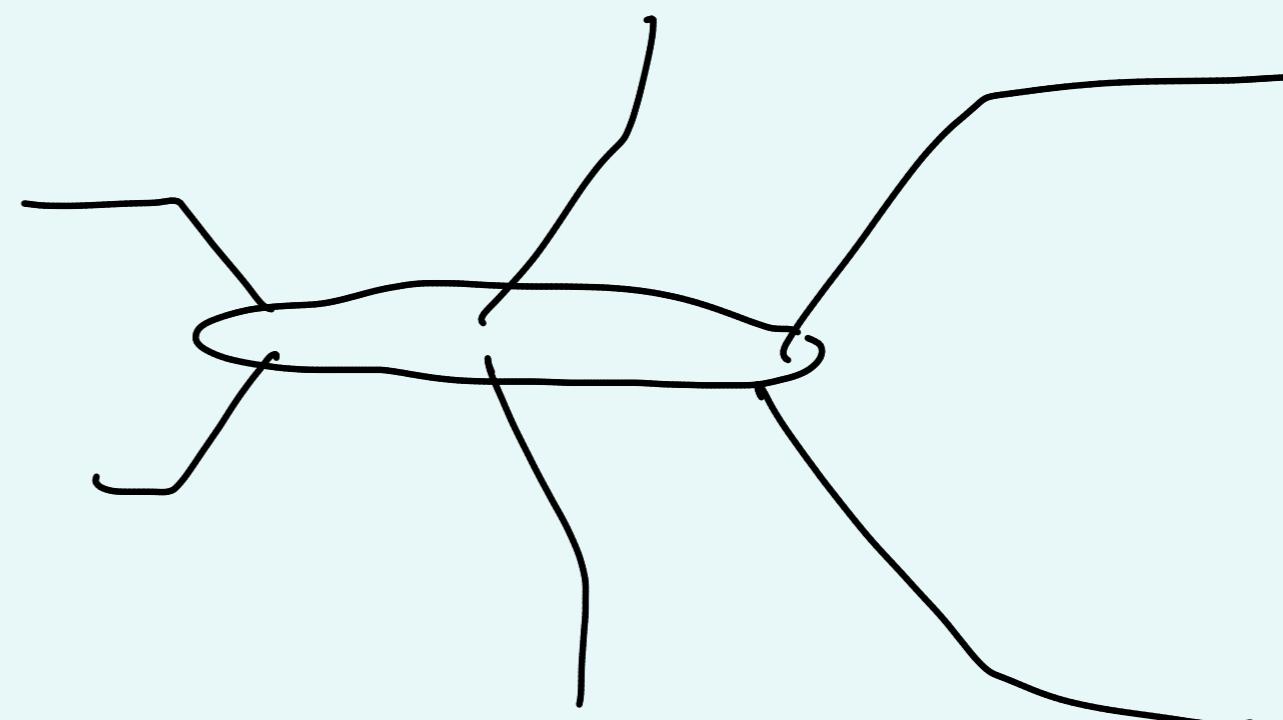
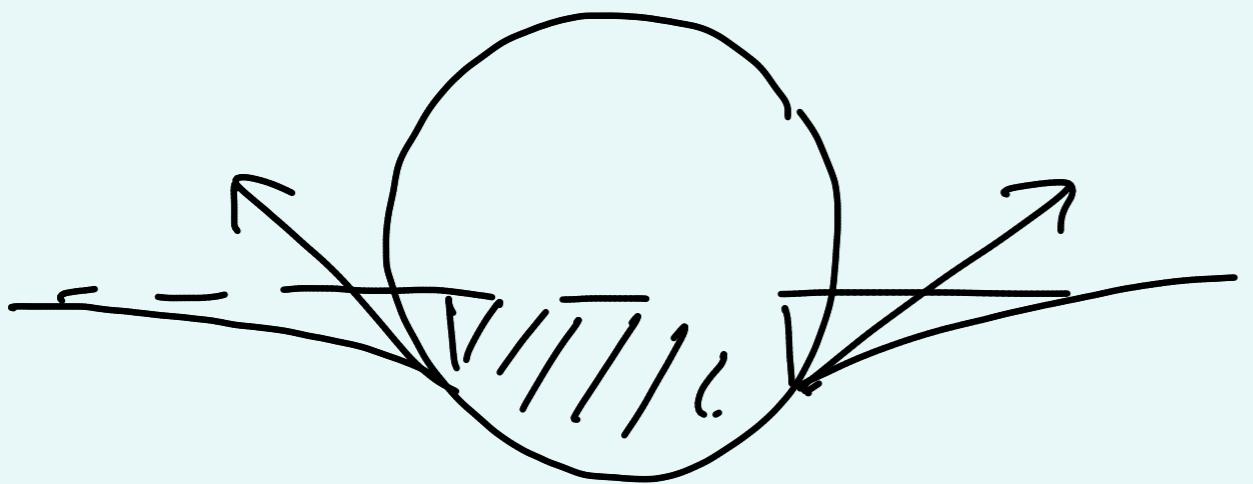
$$\hat{\rho} \cdot (\underline{F_s} + \underline{\overline{F_p}}) = \underline{W_M} + \underline{W_v}$$



: weight of liquid that would fill the
region bounded below by the
interface and the wetted surface
of the body , and above
by the undisturbed interface
 $\nabla = 0$.







Mid-term exam