Engineering Economic Analysis

2019 SPRING

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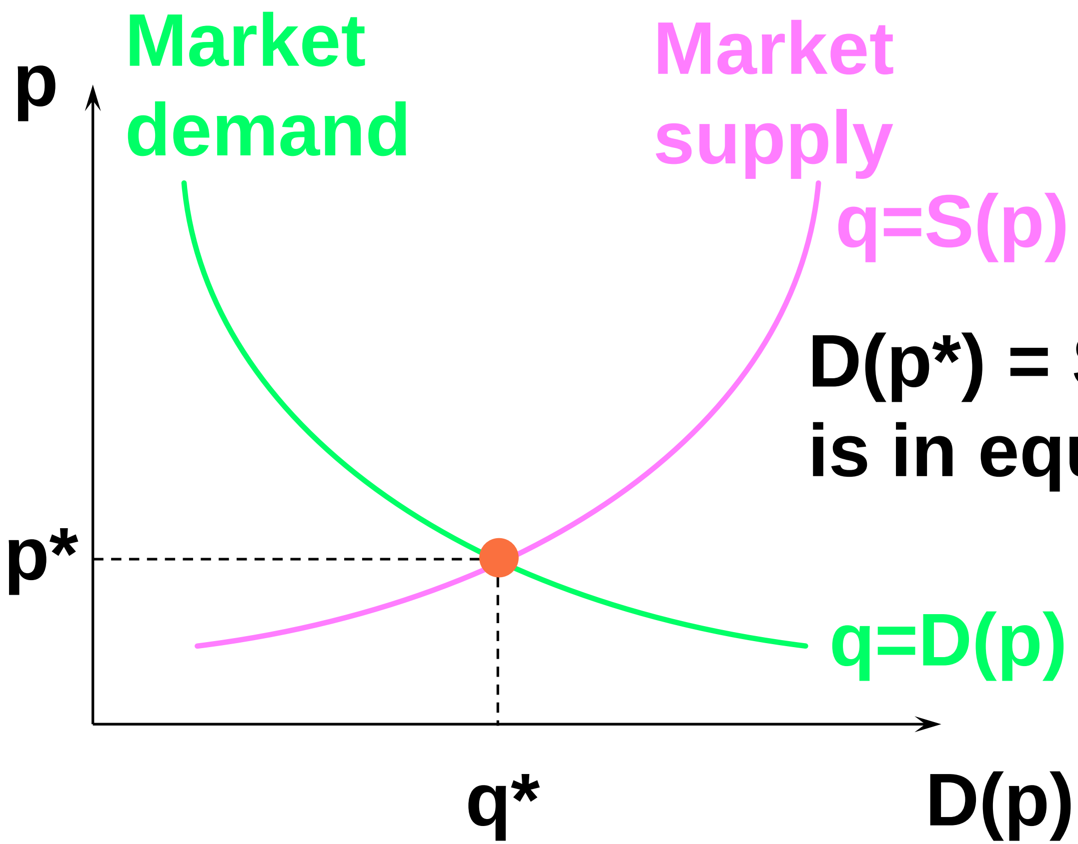
Chap. 16

EQUILIBRIUM

Market Equilibrium

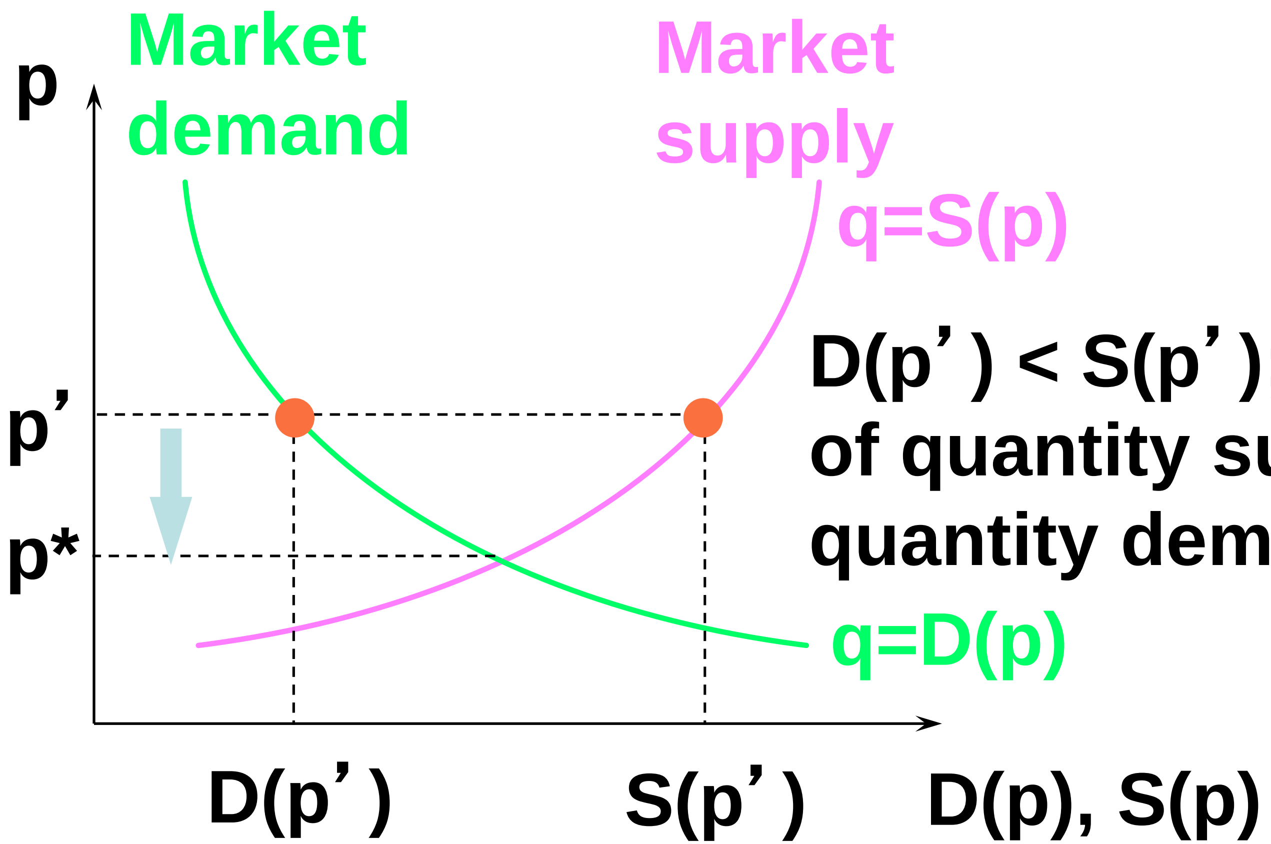
- Competitive Market: where each economic agent takes the market price as outside of his (her) control
  - Each consumer or provider is so small that has a negligible effect on the market price
  - In competitive market, each economic agent simply determines their best response given those market prices
- Market demand curve: D(p)
- Market supply curve: S(p) measures how much the provider (or firms) is willing to supply of a good at each possible market price

Market Equilibrium



D(p*) = S(p*); the market is in equilibrium.

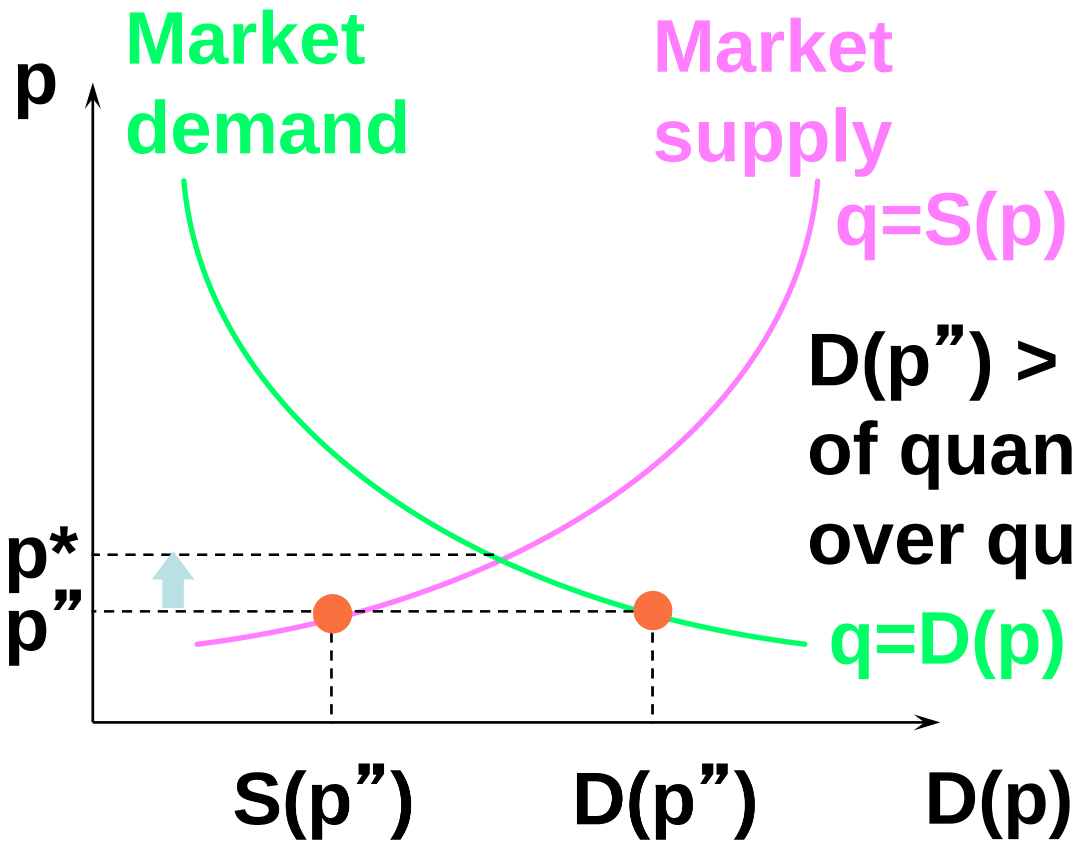
Market Equilibrium



D(p’) < S(p’); an excess of quantity supplied over quantity demanded.

Market price must fall towards p*.

Market Equilibrium



D(p”) > S(p”); an excess of quantity demanded over quantity supplied.

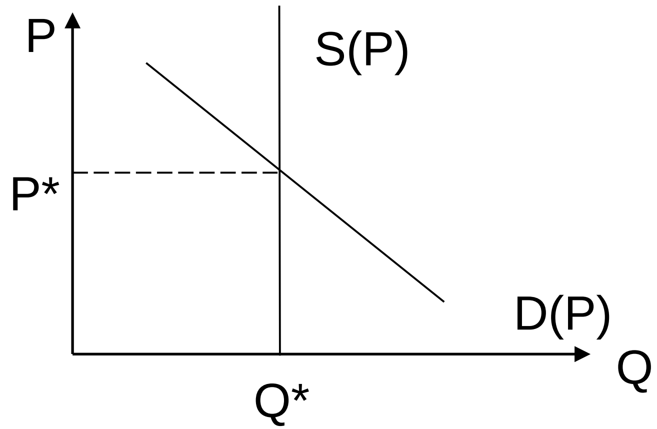
Market price must rise towards p*.

Market Equilibrium

- "An economic equilibrium is a situation where all agents are choosing the best possible action for themselves and each person's behavior is consistent with that of others"
  - At any price other than an equilibrium price, some agent's behaviors would be infeasible, and there would therefore be a reason for their behavior change.
  - Thus, a price that is not an equilibrium price cannot be expected to persist since at least some agents would have an incentive to change their behavior

Two Special Cases

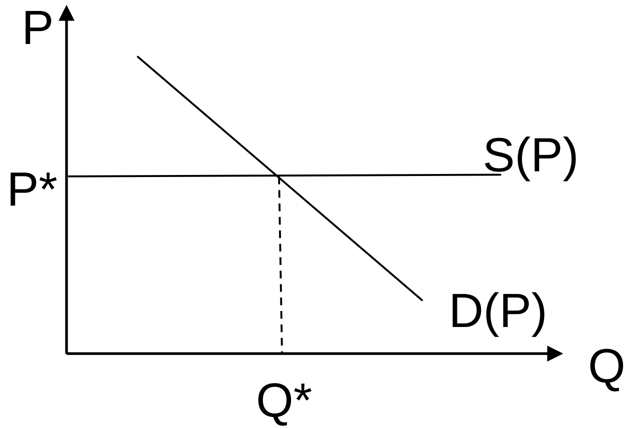
- Fixed supply curve



  - Q* is determined entirely by supply
  - P* is determined entirely by demand

Two Special Cases

- Horizontal supply curve



  - P* is determined entirely by supply
  - Q* is determined entirely by demand

- In general, P* and Q* are jointly determined by the demand and supply.

Demand & Supply Curve

- Example: Linear curves

$$D(P) = a - bP$$
$$S(P) = c + dP$$

  - Equilibrium condition

$$D(P) = S(P) \quad \Rightarrow \; a - bP = c + dP$$

  - Equilibrium price

$$P^* = \frac{a-c}{d+b}$$

  - Equilibrium quantity

$$D(P^*) = a - b\frac{a-c}{d+b} = \frac{ad+bc}{d+b}$$

Inverse Demand & Supply Curve

- Inverse curves

$$D(p) \xrightarrow{inverse} P_D(q)$$

$$S(p) \xrightarrow{inverse} P_S(q)$$

  - Equilibrium

$$P_D(q^*) = P_S(q^*)$$

- Example: Linear curves

$$D(P) = a - bP \qquad \xrightarrow{\text{inverse}} \qquad P_D(q) = \frac{a-q}{b}$$

$$S(P) = c + dP \qquad\qquad\qquad P_S(q) = \frac{q-c}{d}$$

Demand & Supply Curve

- Equilibrium condition

$$P_D(q) = P_S(q) \quad \Rightarrow \frac{a-q}{b} = \frac{q-c}{d}$$

- Equilibrium quantity

$$q^* = \frac{ad+bc}{d+b}$$

- Equilibrium price

$$P_D(q^*) = \frac{a}{b} - \frac{1}{b}\frac{ad+bc}{d+b} = \frac{a-c}{d+b}$$

Tax

- Quantity taxes: tax levied per unit of quantity bought or sold
  - t: amount of quantity tax per unit sold

$$P_D = P_S + t$$

  - If the tax is levied on sellers then it is an *excise tax*.
  - If the tax is levied on buyers then it is a *sales tax*.

- Value taxes: tax expressed in percentage units
  - τ: tax rate

$$P_D = (1+\tau)P_S$$

Tax

- What happens in a market when a quantity tax is imposed.
- Suppose the supplier is required to pay the tax.
  - Supply depends on the supply price (the amount of the supplier actually gets after paying tax), $S(P_S)$
  - Demand depends on the demand price (the amount that the demander actually pays), $D(P_D)$

$$D(P_D) = S(P_S)$$

$$P_S = P_D - t$$

  - Thus,

$$D(P_D) = S(P_D - t) \text{ or}$$

$$D(P_S + t) = S(P_S)$$

Tax

- When the demander has to pay the tax.
  - Demander should pay P_D which includes the tax to buy a good.
  - Tax goes to the government and the price supplier can get will be only the remaining amount

$$P_D - t = P_S$$

  - Thus, $D\left(P_D\right) = S\left(P_D - t\right)$
- It doesn't matter who is responsible for paying the tax

Tax

- When we use the inverse functions,

$$P_D(q), P_S(q)$$

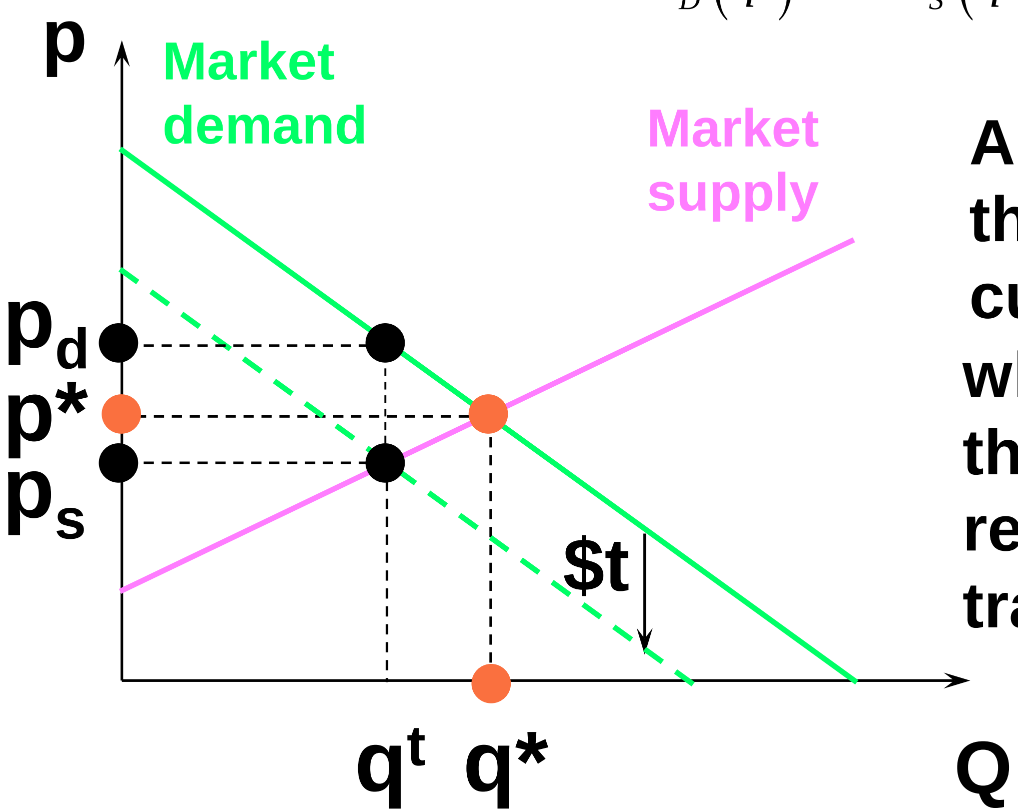
  - Note that the quantity supplied must be the same with the demanded amount, q^t
  - the equilibrium quantity traded is that q^t such that the demand price at q^t minus the tax being paid is just equal to the supply price at q^t

$$\begin{cases} P_D(q^t) - t = P_S(q^t), & \text{when sales tax} \\ P_S(q^t) + t = P_D(q^t), & \text{when excise tax} \end{cases}$$

- Also same equations!!

Tax(sales tax)

$$\text{Sales tax}: P_D\left(q^t\right) - t = P_S\left(q^t\right)$$

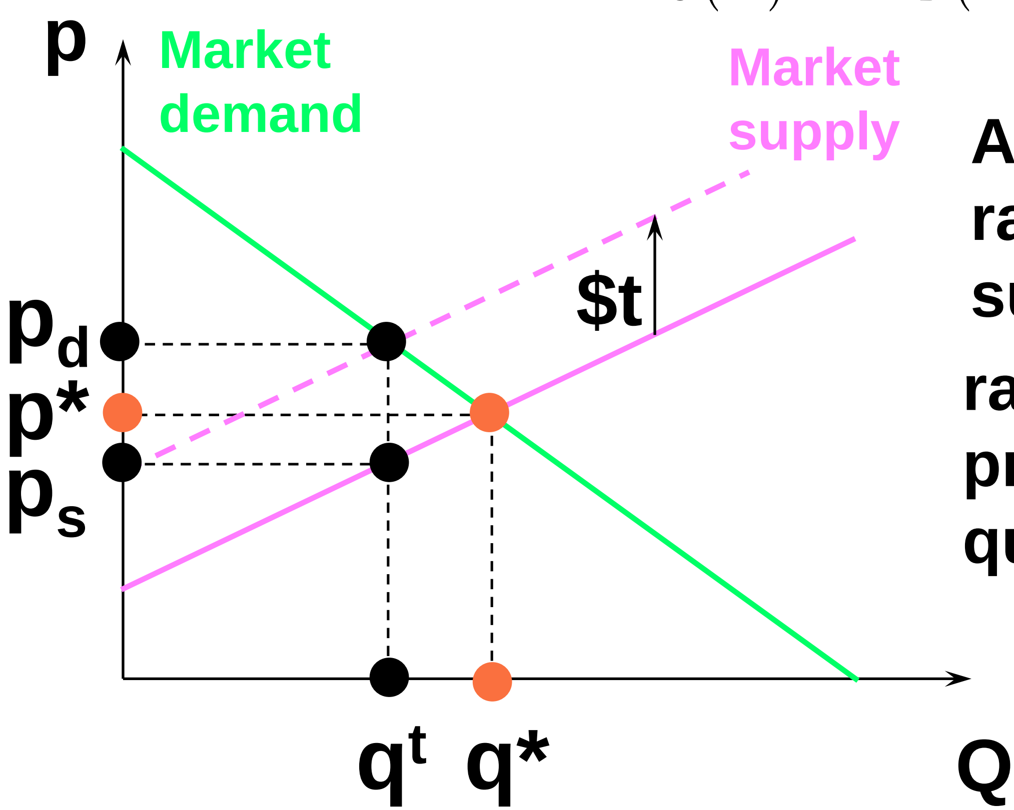


A sales tax lowers the market demand curve by $t, which lowers the sellers' price and reduces the quantity traded.

And demanders pay $p_d = p_s + t$.

Tax(excise tax)

$$\text{Excise tax}: P_S\left(q^t\right)+t=P_D\left(q^t\right)$$

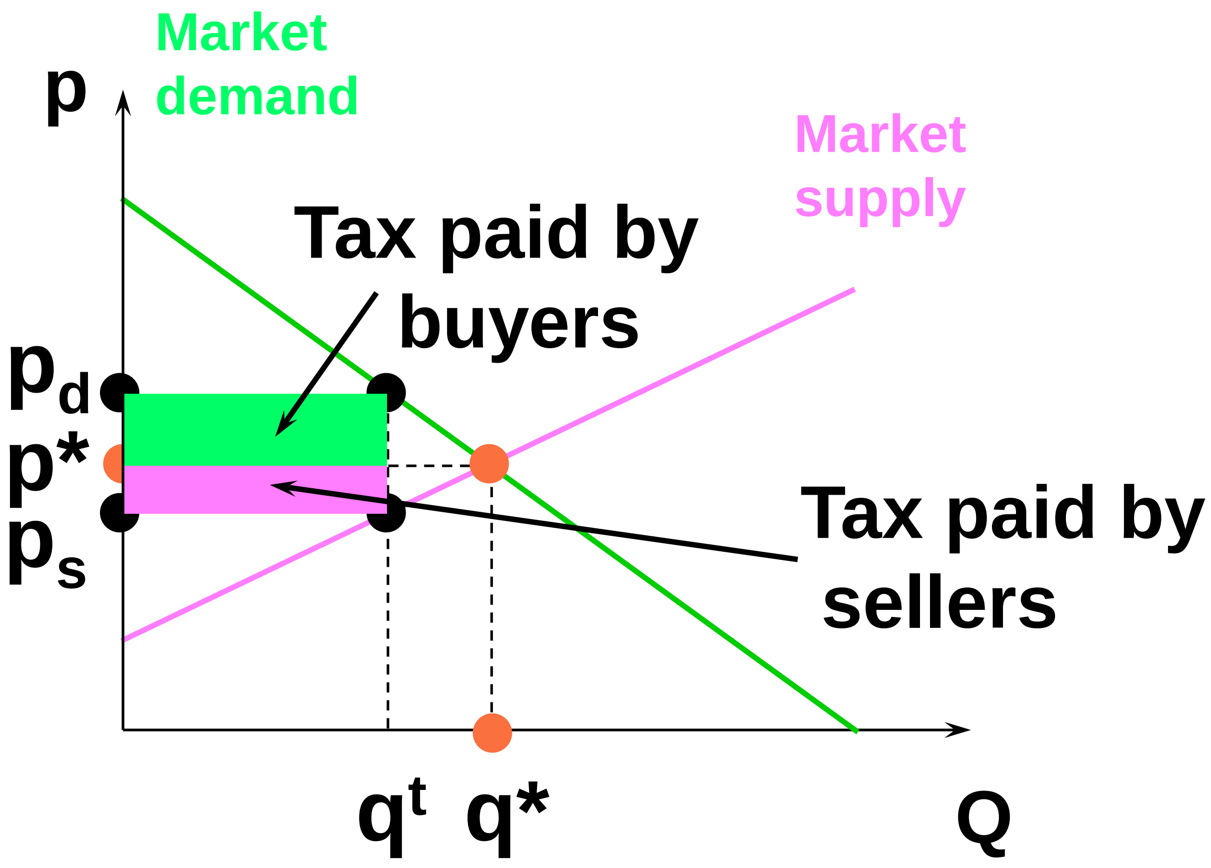


An excise tax raises the market supply curve by $t,

raises the demanders' price and lowers the quantity traded.

And sellers receive only $p_s = p_d - t$.

Tax



Market demand
p
Market supply
Tax paid by buyers
p_d
p*
p_s
Tax paid by sellers
q^t
q*
Q

Tax(example)

- Linear functions

$$D(P) = a - bP_D$$
$$S(P) = c + dP_S$$

- Equilibrium condition after quantity tax

$$\begin{cases} a - bP_D = c + dP_S \\ P_D = P_S + t \end{cases}$$

- Solving

$$a - b(P_S + t) = c + dP_S$$

- Equilibrium prices

$$P_S^* = \frac{a - c - bt}{d + b}$$

$$P_D^* = P_S^* + t = \frac{a - c - bt}{d + b} + t = \frac{a - c + dt}{d + b}$$

Tax(example)

- Equilibrium quantity

$$q^t = D(p_D) = S(p_s)$$
$$= \frac{ad + bc - bdt}{b + d}$$

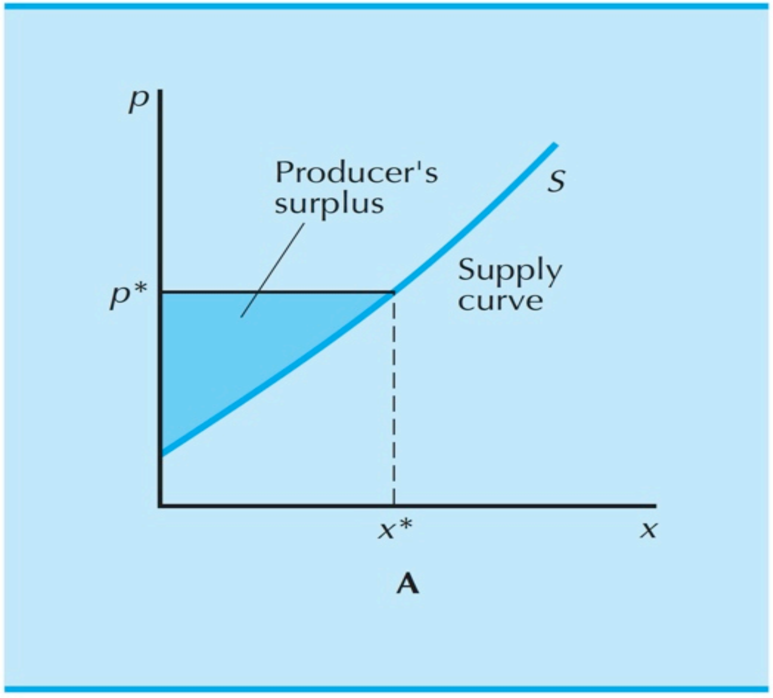
- As t increases, p_s falls, p_D rises, and q^t falls

Deadweight Loss of a Tax

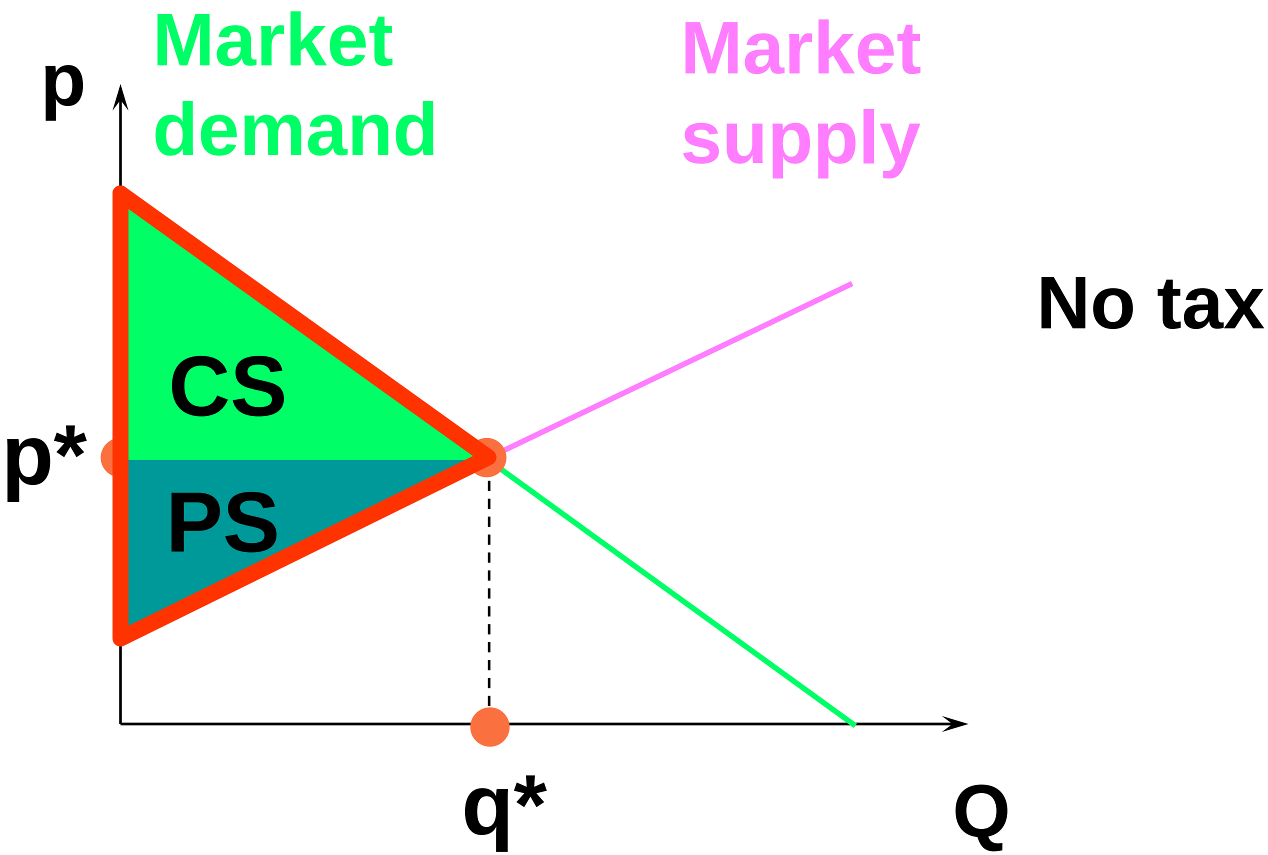
- Social cost of a tax using the consumer's and producers' surplus
- A quantity tax imposed on a competitive market reduces the quantity traded and so reduces gains-to-trade (*i.e.* the sum of Consumers' and Producers' Surplus).
- The lost total surplus is the tax's deadweight loss, or excess burden.

Deadweight Loss of a Tax

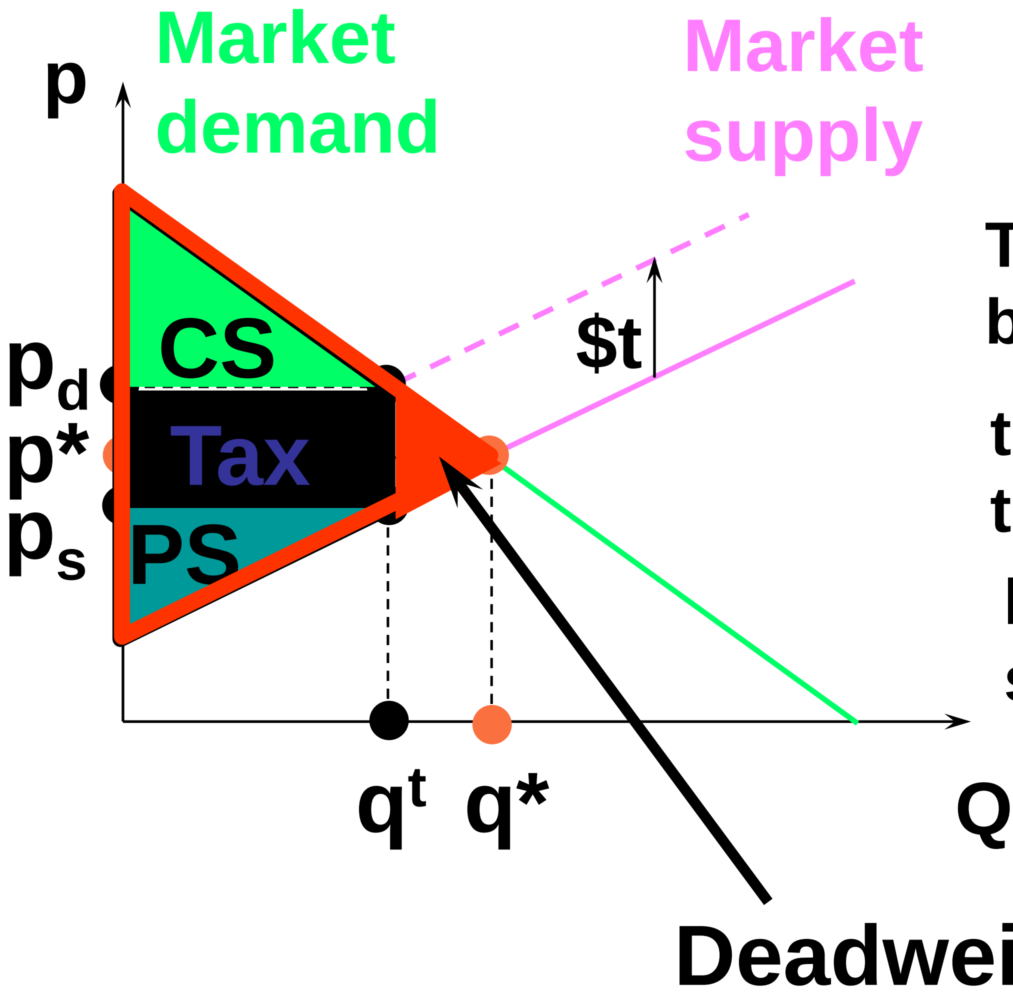
- Producers’ surplus: changes in a firm’s welfare can be measured in dollars much as for a consumer.
  - Profits



Deadweight Loss of a Tax



Deadweight Loss of a Tax

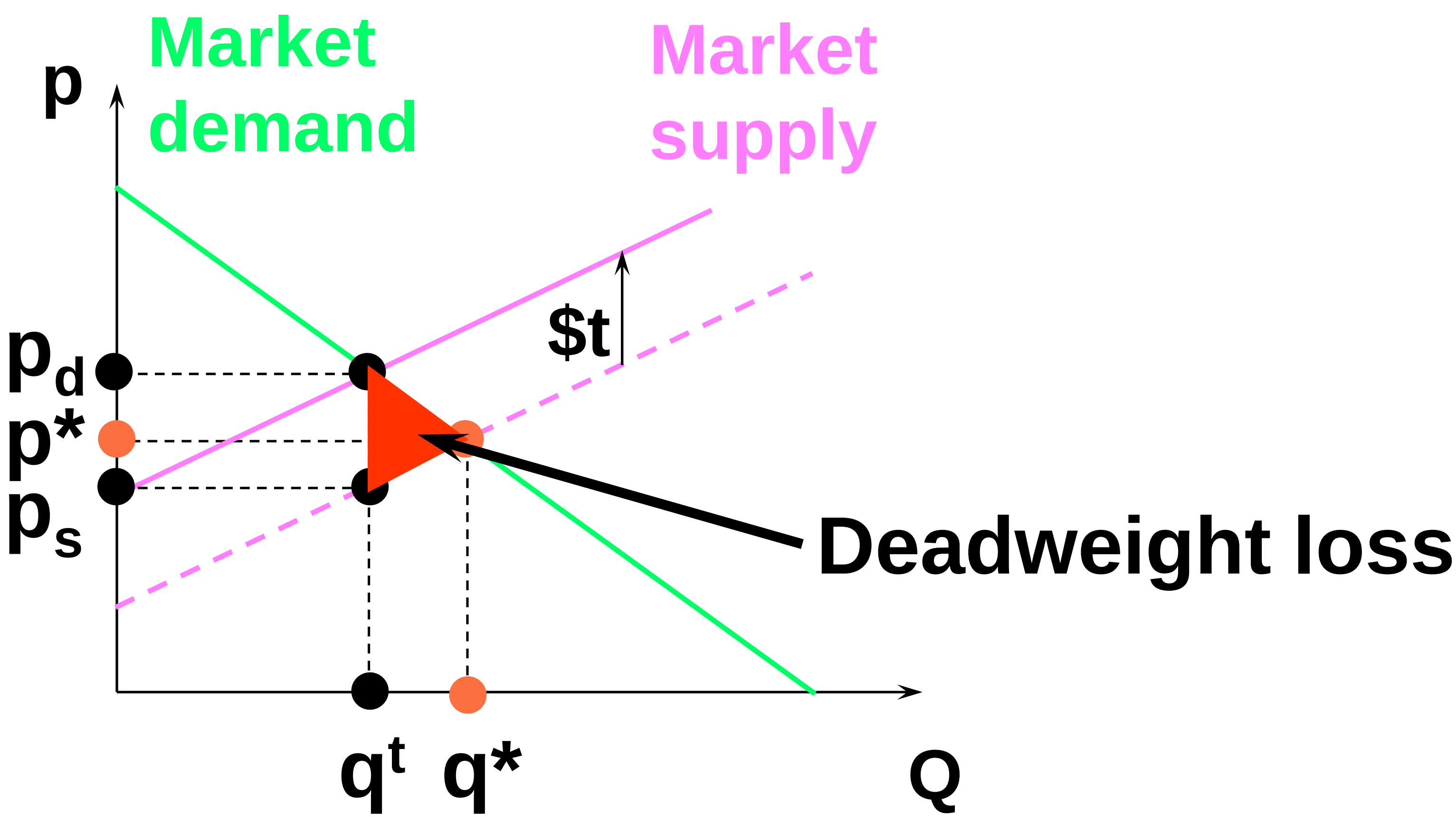


The tax reduces both CS and PS,

transfers surplus to government

lowers total surplus

Deadweight Loss of a Tax



Example: Market for loans

- The interest rate serves as a price in the market for loans
  - $D(r)$= demand for loans by borrowers
  - $S(r)$=supply of loans by lenders
  - Equilibrium interest rate $r*$

$$D(r^*) = S(r^*)$$

- Tax system
  - There is a income tax on the interest earned for lenders
    - Income from interest will decrease from r to (1-t)r
  - There is a tax deduction on the interest charges for borrowers
    - Expense by interest will decrease from r to (1-t)r

Example: Market for loans

- Equation for interest rate determination

$$D((1-t)r') = S((1-t)r')$$

  - Note that $D(r^*) = S(r^*)$
  - Therefore, if $r^* = (1-t)r'$

$$D((1-t)r') = S((1-t)r')$$

  - Thus the interest rate in the presence of tax will be higher

$$r' = \frac{r^*}{(1-t)}$$

Example: Market for loans

- Inverse functions approach
  - *Inverse demand function ($r_b(q)$): what the after-tax interest rate would have to be to induce borrowers to borrow q*
  - *Inverse supply function ($r_l(q)$)*
- Equilibrium amount lent, *q**

$$r_b(q^*) = r_l(q^*)$$

- Tax system when market interest rate is *r*
  - After-tax rate facing borrowers will be $(1-t_b)r$
  - The quantity of borrowing will be determined by

$$(1 - t_b)r = r_b(q) \quad \text{or} \quad r = \frac{r_b(q)}{1 - t_b}$$

Example: Market for loans

- Tax system when market interest rate is r
  - After-tax rate facing lenders will be $(1-t_l)r$
  - The quantity of lending will be determined by

$$(1-t_l)r = r_l(q) \quad \text{or} \quad r = \frac{r_l(q)}{1-t_l}$$

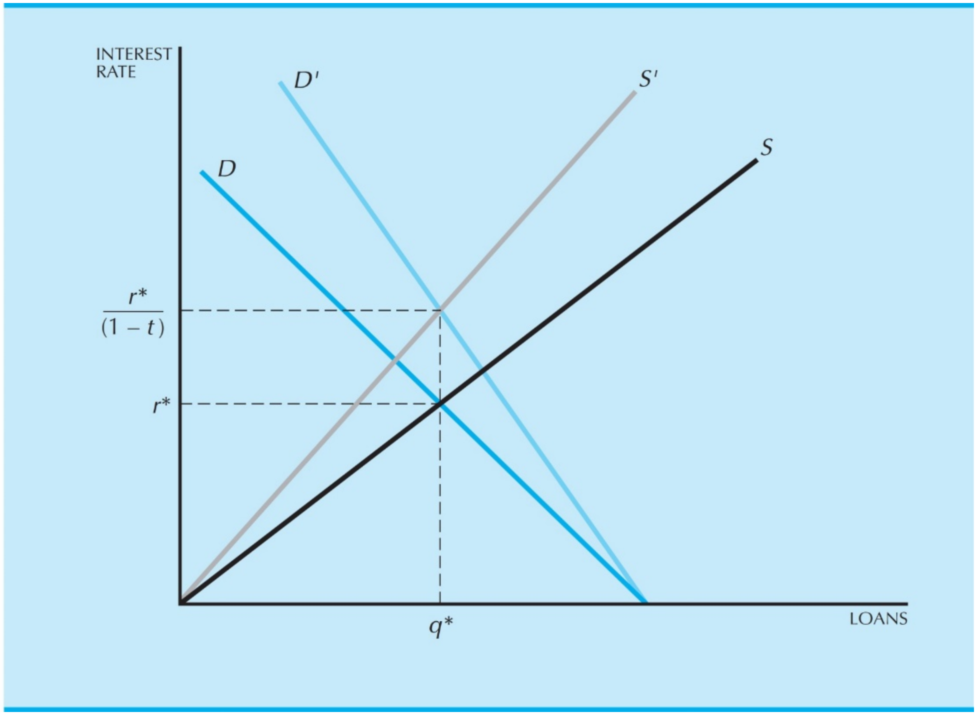
  - Combining two equations gives

$$r = \frac{r_b(\hat{q})}{1-t_b} = \frac{r_l(\hat{q})}{1-t_l}$$

  - If $t_b = t_l$, *then* $\hat{q} = q^*$

Example: Market for loans

- Equilibrium in the loan market when $t_b = t_l = t$



  - After-tax interest rate and the amount borrowed are unchanged

Example: Market for loans

- Tax in the market for loans has impacts on
  - Subsidizing borrowers
  - Taxing lenders
- How about net effects?
  - When $r_b(q)>r_l(q)$, then net taxing (and vice versa). *Why?*
  - We know that

$$r = \frac{r_b(\hat{q})}{1-t_b} = \frac{r_l(\hat{q})}{1-t_l} \quad \Rightarrow \quad r_b(\hat{q}) = \frac{1-t_b}{1-t_l} r_l(\hat{q})$$

  - Therefore, $r_b(q)>r_l(q)$ implies

$$\frac{1-t_b}{1-t_l} > 1 \quad \Rightarrow \quad t_l > t_b$$