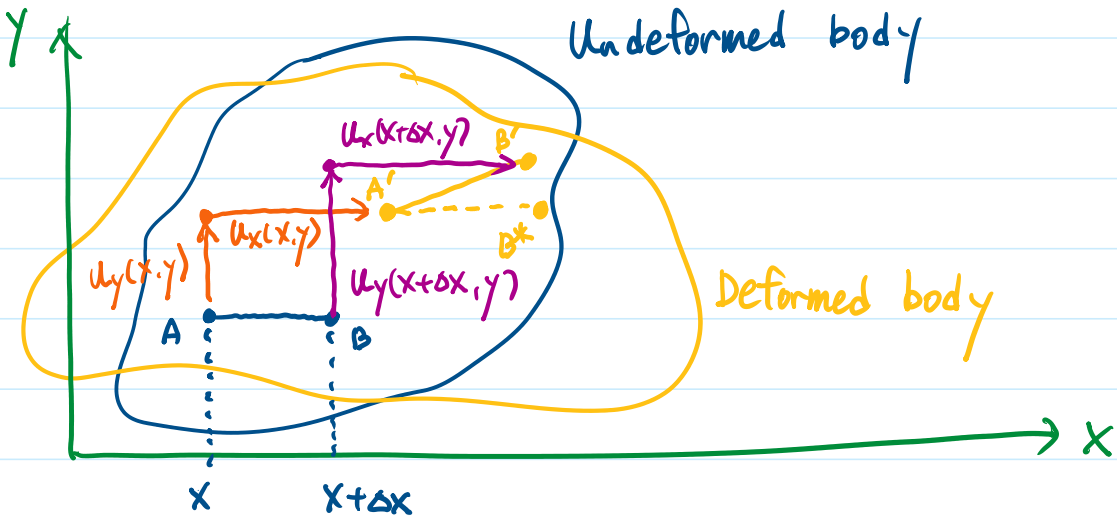
Relation between Strain and Displacement

2019년 7월 15일 월요일 오후 2:24

* What is Normal strain ?
* What is Shear strain ?

Normal strain : 원래 길이에 비교하였을 때 길이가 얼마나 변형됐는지

shear strain : 원래 각도에 비교하였을 때 각도가 얼마나 변형 됐는지



* Normal strain - Displacement

After deformation, the line segment AB is deformed to A'B'

$A \to A'$: $u_x(x, y)$

$B \to B'$: $u_x(x+\Delta x, y)$

Normal strain $\varepsilon_{xx} = \dfrac{A'B' - AB}{AB} = \dfrac{u_x(x+\Delta x, y) - u_x(x, y)}{\Delta x}$

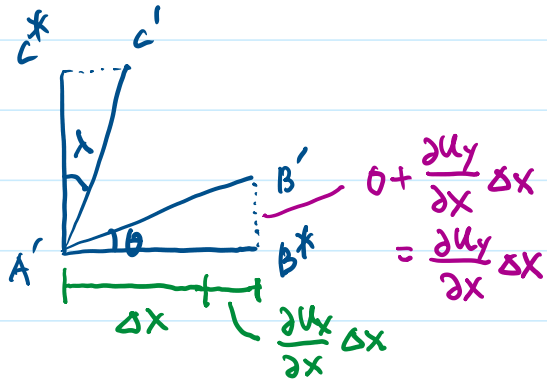
$\Delta x \to 0$, $\varepsilon_{xx} = \dfrac{\partial u_x}{\partial x}$

$\varepsilon_{yy} = \dfrac{\partial u_y}{\partial y}$

$\varepsilon_{zz} = \dfrac{\partial u_z}{\partial z}$

$$\varepsilon_{zz} = \frac{\partial u_z}{\partial z}$$

* Shear strain - Displacement



$$\theta + \frac{\partial u_y}{\partial x}\Delta x = \frac{\partial u_y}{\partial x}\Delta x$$

$$\theta \approx \tan\theta = \frac{\frac{\partial u_y}{\partial x}\Delta x}{\Delta x + \frac{\partial u_x}{\partial x}\Delta x}$$

$$= \frac{\partial u_y/\partial x}{1 + \underbrace{\partial u_x/\partial x}_{\text{extremely small compare to 1}}} = \frac{\partial u_y}{\partial x} = \omega_{xy}$$

Likewise, $\lambda = \frac{\partial u_x}{\partial y}$

How can the angular deformation be expressed in terms of orthogonal displacements?

$$\gamma_{xy} = 2\varepsilon_{xy} = \frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y}$$

For small angles, $\theta \approx \tan(\theta)$

$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y}\right)$$

$\varepsilon_{xy} = \omega_{yx}$ when $\Sigma M = 0$

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$$\varepsilon = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix}$$

$\varepsilon_{xy} = \varepsilon_{yx}$

$$= \begin{bmatrix} \frac{\partial u_x}{\partial x} & \frac{1}{2}\left(\frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y}\right) & \frac{1}{2}\left(\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z}\right) \\ \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right) & \frac{\partial u_y}{\partial y} & \frac{1}{2}\left(\frac{\partial u_z}{\partial y} + \frac{\partial u_y}{\partial z}\right) \\ \frac{1}{2}\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right) & \frac{1}{2}\left(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}\right) & \frac{\partial u_z}{\partial z} \end{bmatrix}$$

$$\nabla u = \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{bmatrix} \begin{bmatrix} u_x & u_y & u_z \end{bmatrix} = \begin{bmatrix} \partial u_x/\partial x & \partial u_y/\partial x & \partial u_z/\partial x \\ \partial u_x/\partial y & \partial u_y/\partial y & \partial u_z/\partial y \\ \partial u_x/\partial z & \partial u_y/\partial z & \partial u_z/\partial z \end{bmatrix}$$

$$\varepsilon = \frac{1}{2}\left[\nabla u + \nabla u^T\right]$$

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Strains can be expressed using displacements