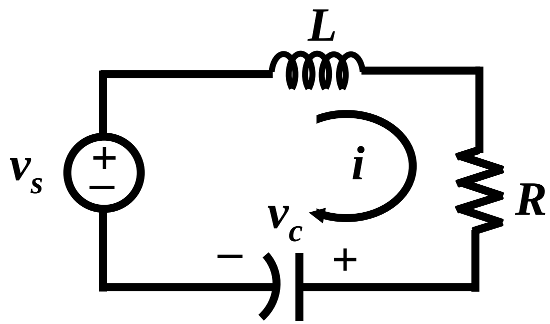
Communications and Power System

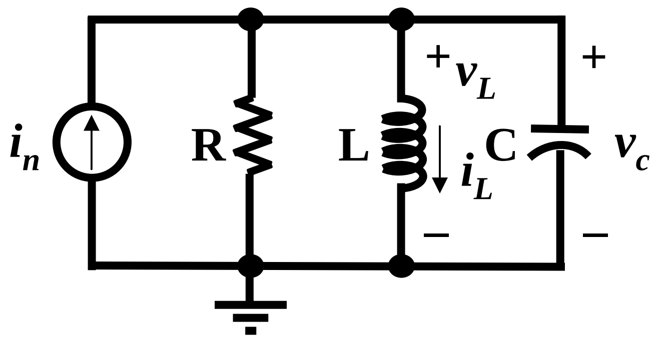
- 전기 회로는 **electrical signal** 또는 **electrical power**를 전송하는 데 사용한다.

Electrical system

- 통신 시스템 **(예: Morse code, radio)**에서는 전압 신호 등과 같은 입력 신호가 전원이 된다.
- 변환기는 전파 매체에 적당하도록 신호를 변환한다.
- 변환기의 출력은 수신기에 도착할 때까지 매체를 진행한다.
- 수신기는 사용자가 쓰기에 적절한 형태로 변환시킨다.
- 전력 시스템 에서는 발전기가 **30 - 70 MW**의 전력을 발생시킨다.
- 전선을 통해서 전력을 효율적으로 수용가에 수송한다.
- 통신시스템**: undistorted transmission,** 전력시스템**: efficient power transmission.**

Natural Response of Second-Order Circuits





KVL

$$L\frac{di}{dt} + Ri + v_c - v_s = 0$$

$$i = C\frac{dv_c}{dt}$$

$$LC\frac{d^2 v_c}{dt^2} + RC\frac{dv_c}{dt} + v_c = v_s$$

$$\frac{d^2 v_c}{dt^2} + \frac{R}{L}\frac{dv_c}{dt} + \frac{v_c}{LC} = \frac{v_s}{LC}$$

KCL

$$(-i_n) + \frac{v_c}{R} + i_L + C\frac{dv_c}{dt} = 0$$

$$v_c = v_L, \quad v_L = L\frac{di_L}{dt}$$

$$\frac{v_L}{R} + i_L + C\frac{dv_L}{dt} = i_n$$

$$\frac{d^2 i_L}{dt^2} + \frac{1}{RC}\frac{di_L}{dt} + \frac{i_L}{LC} = \frac{i_n}{LC}$$

$$\frac{d^2 x}{dt^2} + 2\alpha\frac{dx}{dt} + \omega_0^2 x = f(t)$$

$$\alpha = \frac{R}{2L}, \quad \omega_0^2 = \frac{1}{LC}, \quad f = \frac{v_s}{LC}$$

$$\alpha = \frac{1}{2RC}, \quad \omega_0^2 = \frac{1}{LC}, \quad f = \frac{i_n}{LC}$$

Second-Order Differential Equation

$$\frac{d^2x}{dt^2}+2\alpha\frac{dx}{dt}+\omega_0^2x=f, \quad x=x_h+x_p$$

$$\frac{d^2x_h}{dt^2}+2\alpha\frac{dx_h}{dt}+\omega_0^2x_h=0 \quad \Rightarrow \quad x_h=Ke^{st}$$ 로 가정.

$$Ke^{st}(s^2+2\alpha s+\omega_0^2)=0 \quad \Rightarrow$$ 특성방정식 $$s^2+2\alpha s+\omega_0^2=0$$

$$s_1=-\alpha+\sqrt{\alpha^2-\omega_0^2}, \quad s_2=-\alpha-\sqrt{\alpha^2-\omega_0^2}$$

damping ratio $\zeta=\alpha/\omega_0$

1. Over damped $(\zeta>1, \alpha>\omega_0)$ s_1, s_2 **: negative real**

2. Critically damped $(\zeta=1, \alpha=\omega_0)$ s_1 **: negative real**

$$x_h=K_1e^{s_1t}+K_2t\,e^{s_1t}$$

3. Under damped $(\zeta<1, \alpha<\omega_0)$ s_1, s_2 **: complex with negative real**

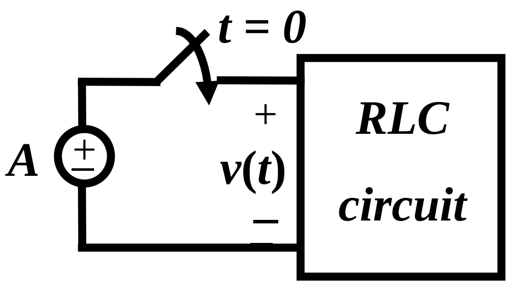
$$x_h=K_1e^{(-\alpha+j\omega_d)t}+K_2e^{(-\alpha-j\omega_d)t}$$

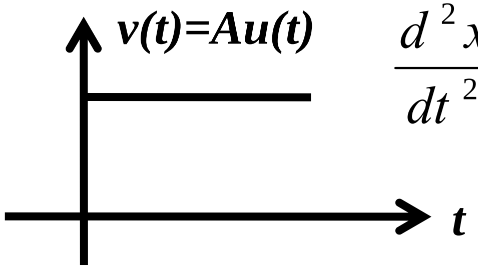
$(\omega_d=\sqrt{\omega_0^2-\alpha^2})$ **damped resonant frequency**

$$=K_3e^{-\alpha t}\cos\omega_dt+K_4e^{-\alpha t}\sin\omega_dt$$

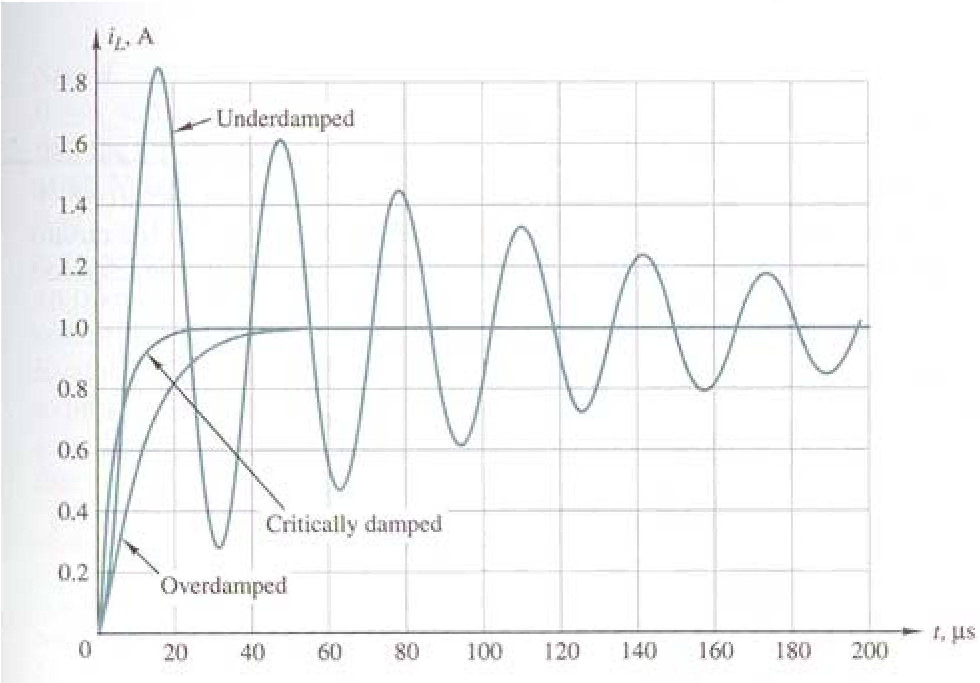
Normalized Step Response of Second-Order Systems

- **RLC** 회로에 **step function**의 전압을 가했다.





$$\frac{d^2x}{dt^2} + 2\alpha\frac{dx}{dt} + \omega_0^2 x = Au(t)$$



– $\zeta = \alpha / \omega_0$의 값에 따라 출력 값에 변화가 있음.

- 특히 $\zeta = 1$ 을 경계로 **overshoot** 보인다.

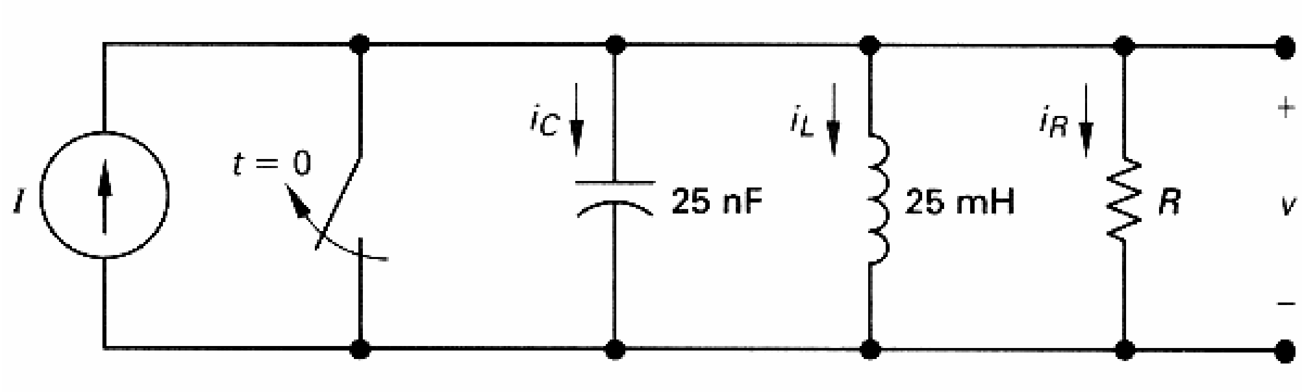
- 응용의 특성에 따라 **damping ratio**을 변화시킨다.

Underdamped, critically damped, and overdamped response curves for parallel RCL circuit of example 9.7.

DeCarlo 책 351쪽 Fig. 9.9

Parallel RLC Circuit (I)

$t = 0$ 인 순간 switch가 열리고 전류 **24** mA가 회로에 가해진다.
저항 값은 **400 Ω** 이다.



$$a)\ i_L(0^+) = ?\quad i_L(0^-) = 0$$ 이므로 $$i_L(0^+) = i_L(0^-) = 0.$$

$$b)\ \left.\frac{di_L}{dt}\right|_{t=0^+} = ?\quad v(0^-) = 0$$ 이므로 $$v(0^+) = 0 \therefore \left.\frac{di_L}{dt}\right|_{t=0^+} = 0.$$

$$c)\ C\frac{dv_c}{dt} + i_L + \frac{v_c}{R} = I,\quad L\frac{di_L}{dt} = v_c,\quad LC\frac{d^2 i_L}{dt^2} + i_L + \frac{L}{R}\frac{di_L}{dt} = I.$$

$$\frac{d^2 i_L}{dt^2} + \frac{1}{RC}\frac{di_L}{dt} + \frac{1}{LC}i_L = \frac{I}{LC}$$

Parallel RLC Circuit (II)

특성방정식

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0, \quad s^2 + 100000s + 16\times10^8 = 0$$

$$\alpha = \frac{1}{2RC} = 5\times10^4, \omega_0 = \frac{1}{\sqrt{LC}} = 4\times10^4$$

$$s = -5\times10^4 \pm 3\times10^4, s_1 = -20000, s_2 = -80000$$

$$i_L = 24\times10^{-3} + A_1 e^{-20000t} + A_2 e^{-80000t}$$

$$i_L(0^+) = 0, \quad \frac{di_L}{dt}(0^+) = 0$$

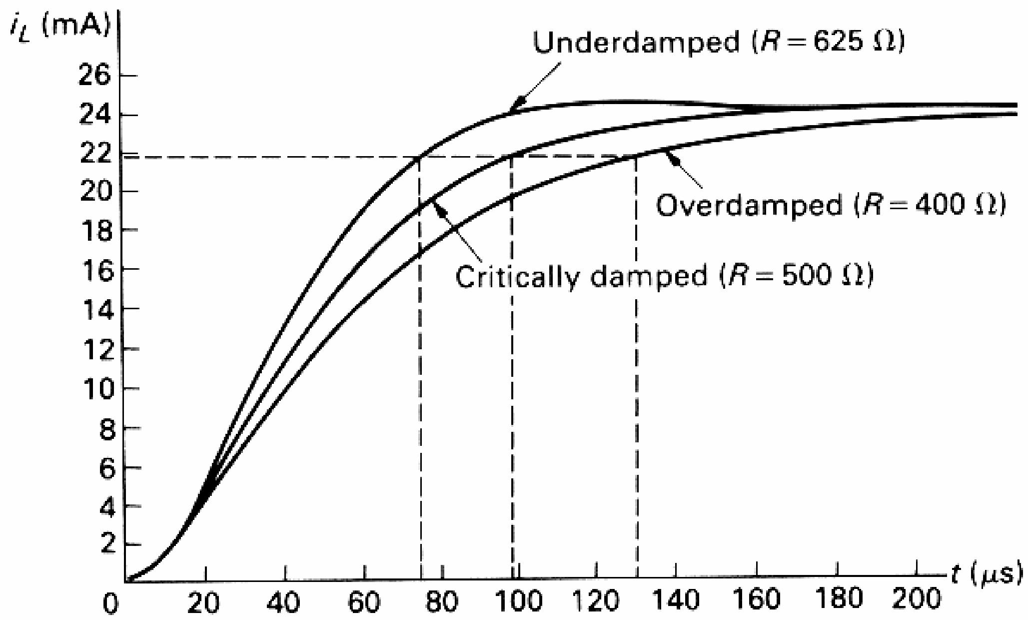
$$\Rightarrow A_1 = 8\times10^{-3}, A_2 = -32\times10^{-3}$$

$$If\ R = 625\,\Omega,$$

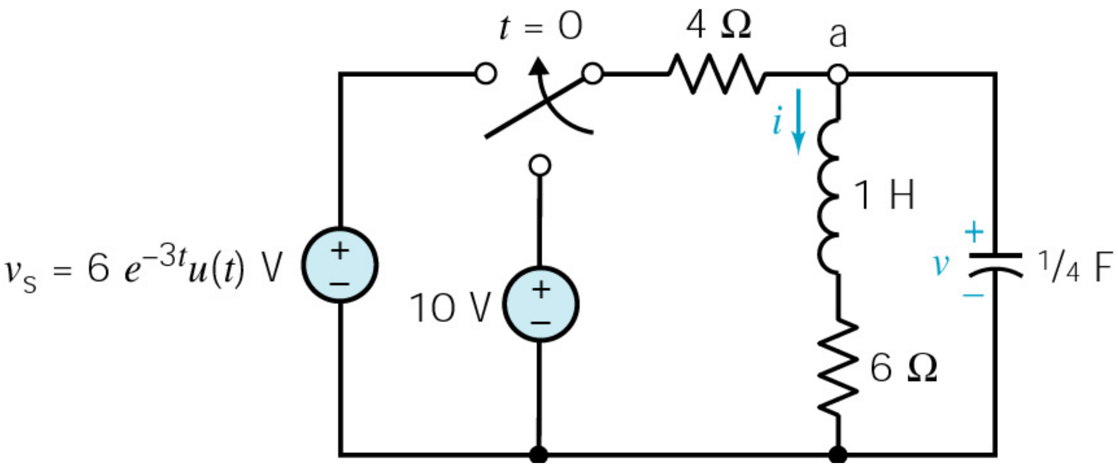
$$\alpha = 3.2\times10^4 < \omega_0$$

$$If\ R = 500\,\Omega,$$

$$\alpha = 4\times10^4 = \omega_0$$



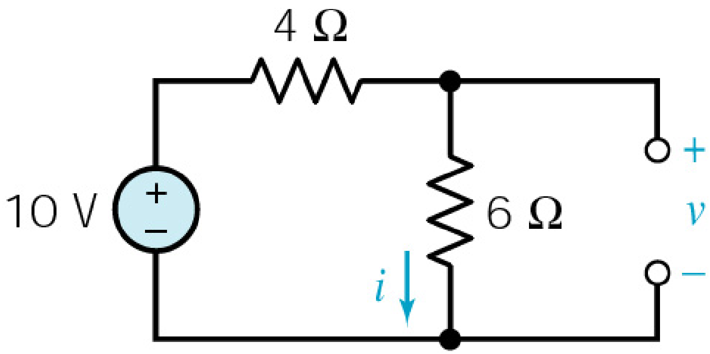
Complete Response of RLC Circuits (I)



전압 ***v***를 구하라.

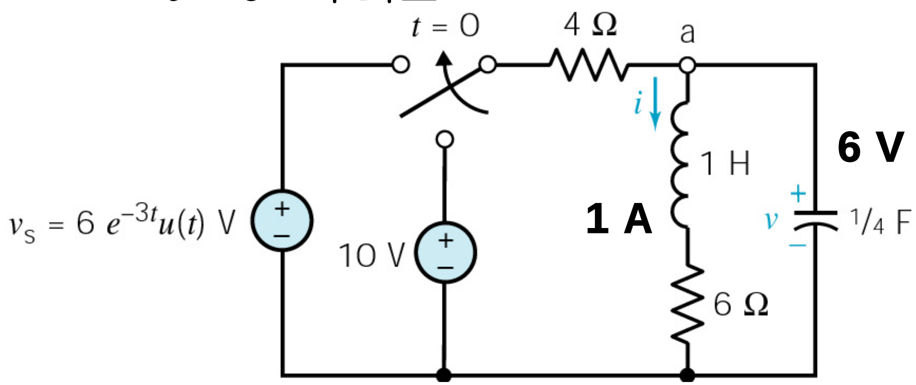
(a) 초기조건을 구하라.

- *t* = 0⁻ 의 회로



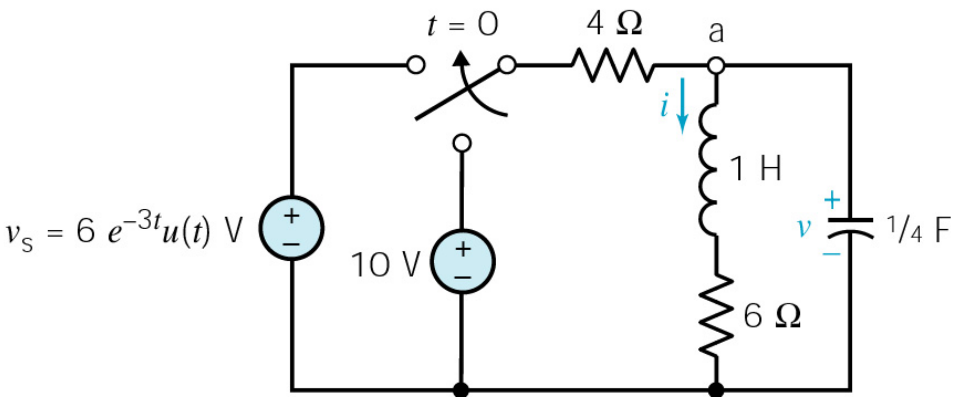
$$v_C(0^-) = 6\,\mathrm{V},\ \ i_L(0^-) = 1\,\mathrm{A}$$

- *t* = 0⁺ 의 회로



$$v_L(0^+) = L\left.\frac{di_L}{dt}\right|_{0^+} = 0 \quad \therefore \left.\frac{di_L}{dt}\right|_{0^+} = 0$$

Complete Response of RLC Circuits (II)



(b) KCL 로 식을 세워라.

$$\frac{v_C - v_s}{4} + i_L + 0.25\frac{dv_C}{dt} = 0$$

$$v_C = 1\frac{di_L}{dt} + 6i_L$$

$$(\frac{di_L}{dt} + 6i_L) - v_s + 4i_L + \frac{d}{dt}(\frac{di_L}{dt} + 6i_L) = 0$$

$$\frac{d^2 i_L}{dt^2} + 7\frac{di_L}{dt} + 10i_L = v_s = 6e^{-3t}u(t)$$

$$i_L = i_{Lh} + i_{Lp}$$

set $i_{Lp} = Ae^{-3t}$

$$9A + (-21A) + 10A = 6$$

$$A = -3$$

set $i_{Lh} = Ke^{st}$

$$s^2 + 7s + 10 = 0$$

$$s = -2, -5$$

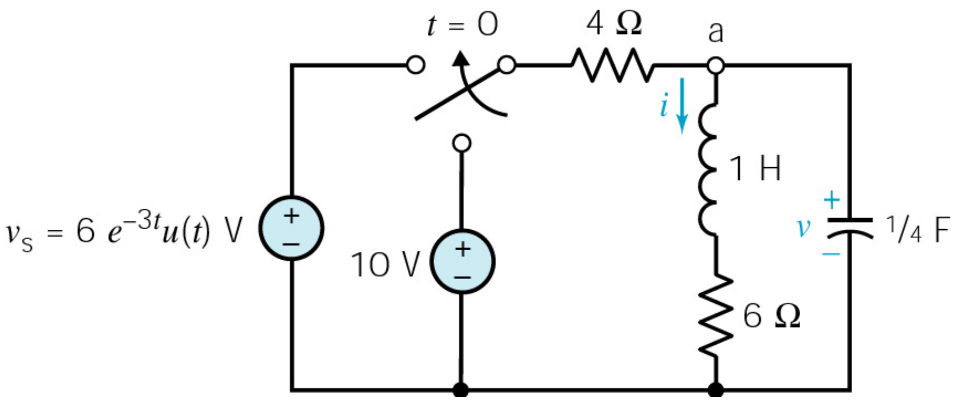
$$i_{Lh} = K_1 e^{-2t} + K_2 e^{-5t}$$

$$i_L = K_1 e^{-2t} + K_2 e^{-5t} - 3e^{-3t}$$

$$i_L(0^+) = 1 = K_1 + K_2 - 3$$

$$i_L'(0^+) = 0 = -2K_1 - 5K_2 + 9$$

Complete Response of RLC Circuits (III)



$$K_1 + K_2 = 4$$

$$2K_1 + 5K_2 = 9$$

$$K_1 = 11/3, \quad K_2 = 1/3$$

$$i_L = \frac{11}{3}e^{-2t} + \frac{1}{3}e^{-5t} - 3e^{-3t}$$

$$v_C = 1\frac{di_L}{dt} + 6i_L$$ 이므로

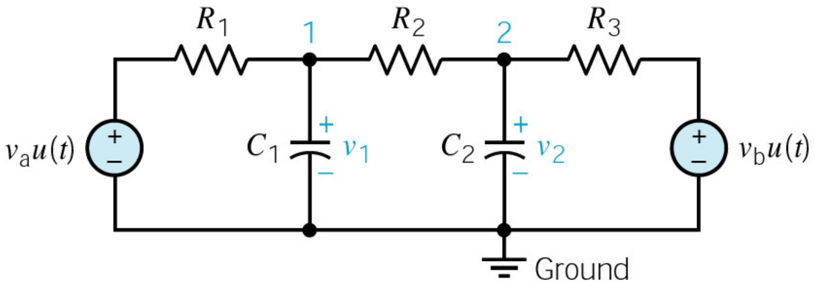
$$v_C = \frac{d}{dt}\left(\frac{11}{3}e^{-2t} + \frac{1}{3}e^{-5t} - 3e^{-3t}\right) + 6\left(\frac{11}{3}e^{-2t} + \frac{1}{3}e^{-5t} - 3e^{-3t}\right)$$

$$= -\frac{22}{3}e^{-2t} - \frac{5}{3}e^{-5t} + 9e^{-3t} + \frac{66}{3}e^{-2t} + \frac{6}{3}e^{-5t} - 18e^{-3t}$$

$$= \frac{44}{3}e^{-2t} + \frac{1}{3}e^{-5t} - 9e^{-3t}$$

State Variable Approach to Circuit Analysis

- **State variable method :** 회로의 전체 응답을 구하기 위해서 **state variable**의 **1**계 미분방정식을 이용한다.
- **Inductor**의 전류나 **capacitor**의 전압을 **state variable**로 이용한다.



KCL

$$node\,1: \quad C_1\frac{dv_1}{dt}+\frac{v_1-v_a}{R_1}+\frac{v_1-v_2}{R_2}=0$$

$$node\,2: \quad C_2\frac{dv_2}{dt}+\frac{v_2-v_1}{R_2}+\frac{v_2-v_b}{R_3}=0$$

$\frac{d}{dt}$ 를 **operator s**로 써서 정리하면,

$$\begin{bmatrix} C_1 s+1/R_1+1/R_2, & -1/R_2 \\ -1/R_2, & C_2 s+1/R_2+1/R_3 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}=\begin{bmatrix} 1/R_1, & 0 \\ 0, & 1/R_3 \end{bmatrix}\begin{bmatrix} v_a \\ v_b \end{bmatrix}$$

v_1과 v_2는 아래의 꼴로 쓰여진다.

$$v_1, v_2=\frac{(Cs+D)v_a+(Es+F)v_b}{s^2+As+B}$$

이것의 해를 구하는 방법은 **14**장 **Laplace transform**에서 다루겠다.

Roots in the Complex Plane (I)

- **2계 미분 방정식 시스템**

(parallel RLC 회로)

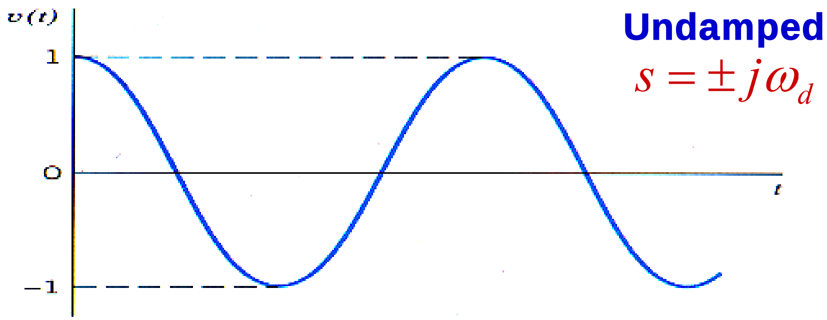
$$s^2 + \frac{s}{RC} + \frac{1}{LC} = 0$$

$$s = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

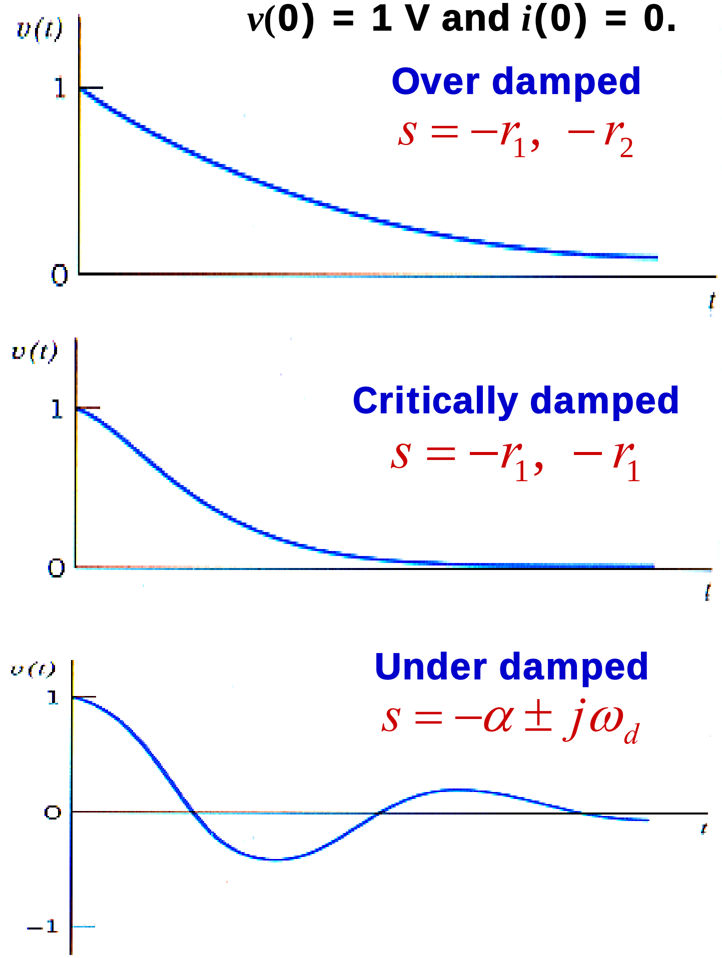
$$if \quad \omega_0 \rangle \alpha, \quad s = -\alpha \pm j\sqrt{\omega_0^2 - \alpha^2} = -\alpha \pm j\omega_d$$

– 해를 복소 평면에 그릴 수 있다.

– 실수 축과 허수 축, σ and $j\omega$

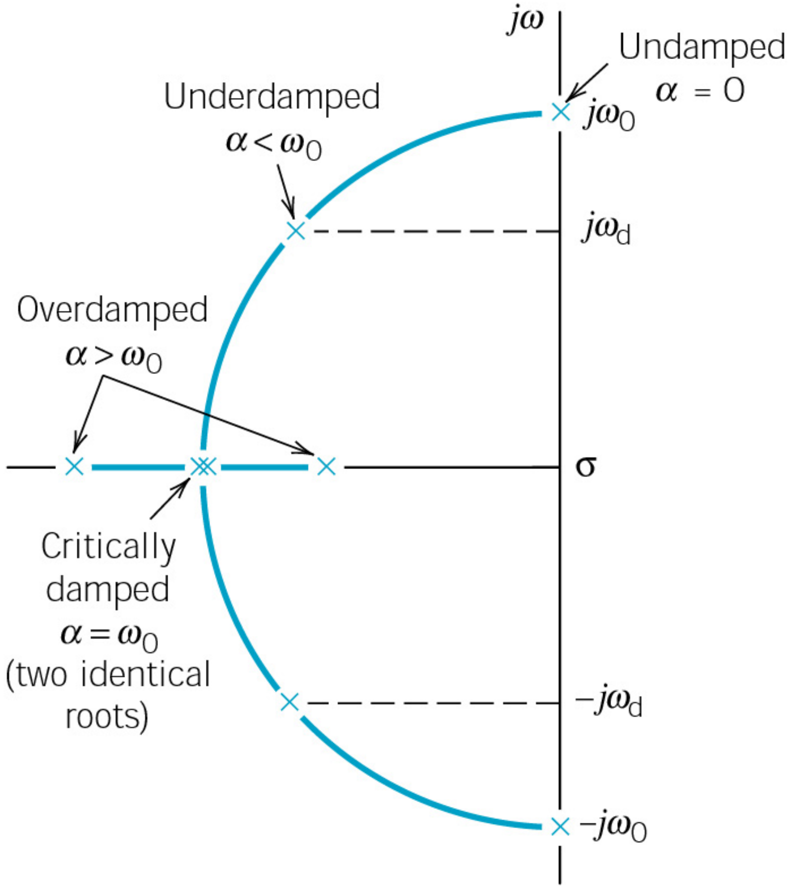


Form of response for $v(0)$ = 1 V and $i(0)$ = 0.



Natural response of a parallel RLC circuit

Roots in the Complex Plane (II)



The complete s-plate showing the location of the two roots, s_1 and s_2, of the characteristic equation in the left-hand portion of the s-plane. The roots are designated by the × symbol.

해의 각 경우를 복소 평면에 그려 보면

i) Over damped

$s = -r_1 , -r_2$ **(음의 실수 축 위의 두 점)**

ii) Critically damped

$s = -r_1 , -r_1$ **(음의 실수 축 위의 한 점)**

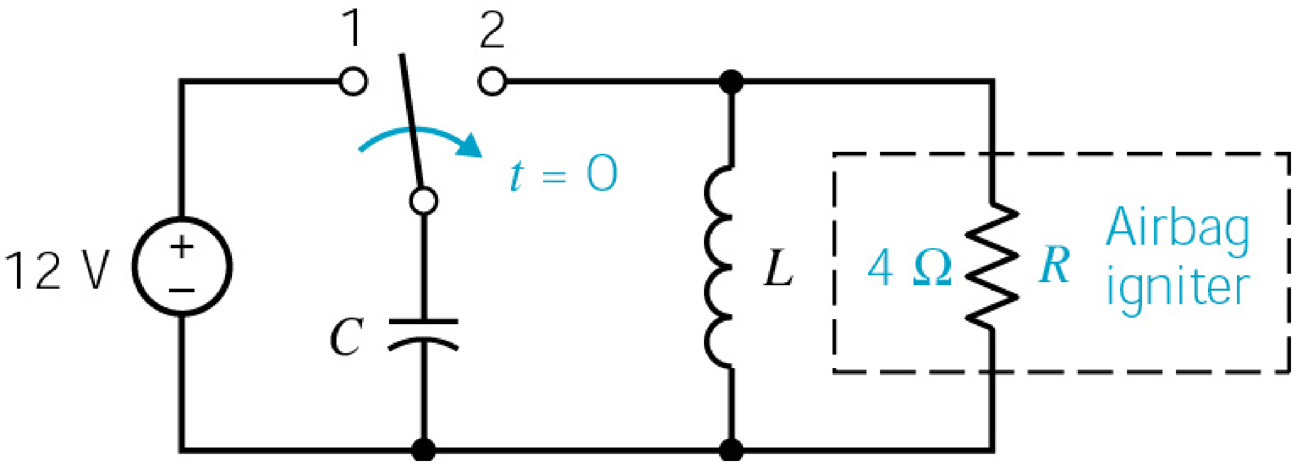
iii) Under damped

$s = -\alpha \pm j\, \omega_d$ **(음의 실수 평면 위의 두 점)**

iv) Undamped

$s = \pm j\, \omega_d$ **(허수 축 위의 두 점)**

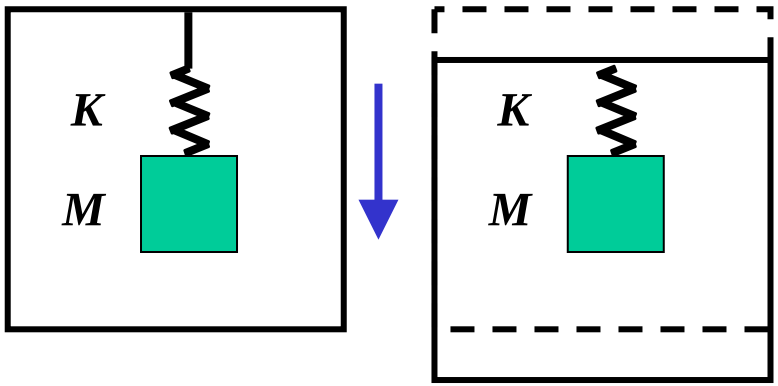
Auto Airbag Igniter (I)



An automobile airbag ignition device

- **Airbag**은 운전자의 안전을 위해서 이용된다.
- **Pendulum**이 **capacitor energy**를 **igniter**로 보내도록 스위치한다.
- 저항 ***R***에서 받은 에너지로 화약 등을 폭발시켜 **airbag**을 팽창시킨다.
- 저항 ***R***에서 **1 J**의 에너지를 소모해야 하고, **0.1** 초 이내에 트리거해야 한다.
- ***L*, *C***의 값을 정하라.

Auto Airbag Igniter (II)



- **Box** 안에 **pendulum** 이 매달려 있다.
- 가속도가 가해지면 관성력에 의하여 **pendulum** 과 **box**벽의 거리가 변화한다.

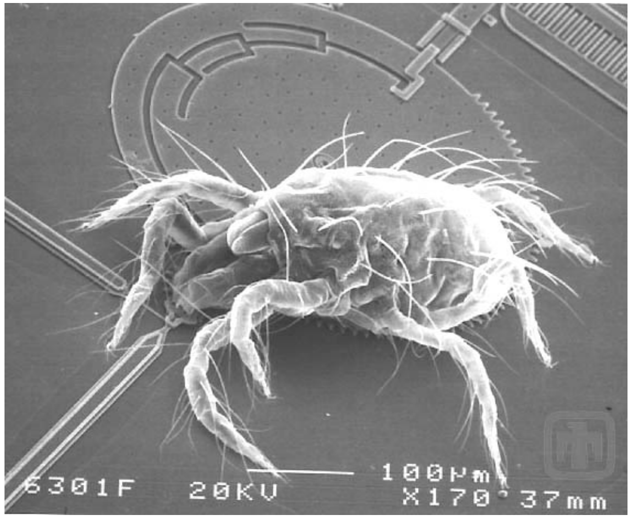
$$ma = k\Delta x \;\Rightarrow\; a = \frac{k}{m}\Delta x$$

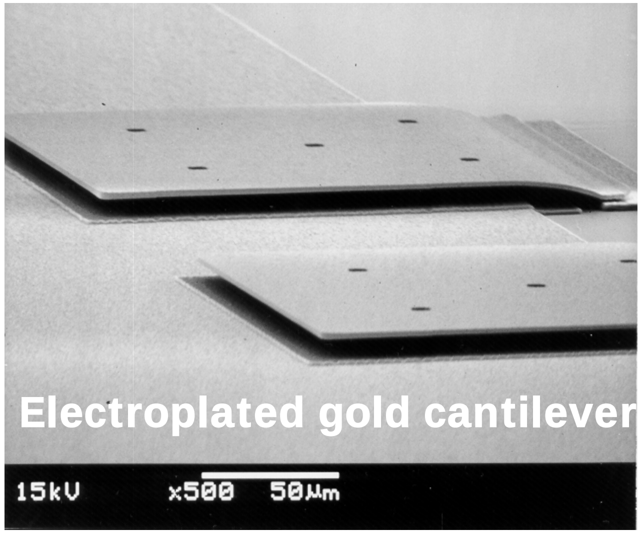
- **Box**의 위, 아래 벽에 전극을 설치하고, **pendulum**을 전극으로 이용한다.
- **Box**의 위, 아래 벽 전극과 **pendulum** 전극에 전압을 인가한다.
- **pendulum**의 위치 변화에 따라 위 아래 전극과의 정전용량이 변화한다.
- 이들 값의 차분을 구하면 위치변화분을 알 수 있다.

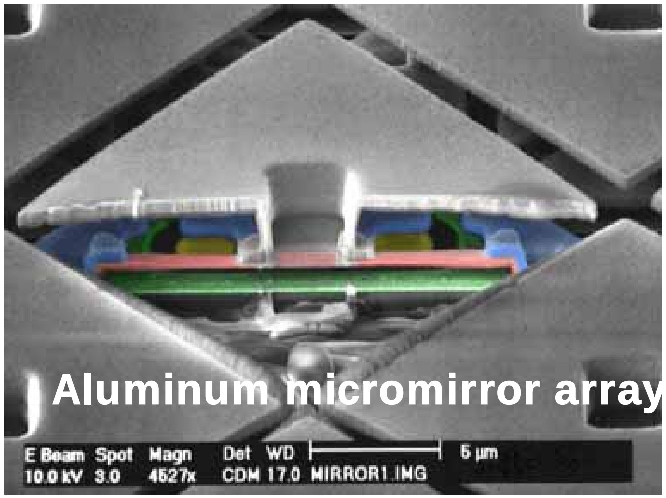
$$C_1 = \frac{\varepsilon_0 A}{d - \Delta x}, \quad C_2 = \frac{\varepsilon_0 A}{d + \Delta x}$$

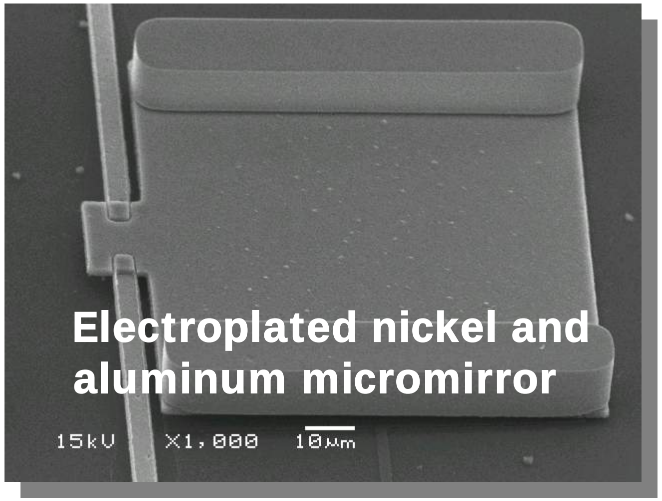
$$\begin{aligned}
\Delta C &= C_1 - C_2 \\
&= \frac{\varepsilon_0 A}{d - \Delta x} - \frac{\varepsilon_0 A}{d + \Delta x} \\
&= \frac{\varepsilon_0 A}{d}\left(\frac{1}{1 - \Delta x/d} - \frac{1}{1 + \Delta x/d}\right) \\
&\approx \frac{\varepsilon_0 A}{d}\left[(1 + \Delta x/d) - (1 - \Delta x/d)\right] \\
&= \frac{\varepsilon_0 A}{d}\frac{2}{d}\Delta x
\end{aligned}$$

Auto Airbag Igniter (III)

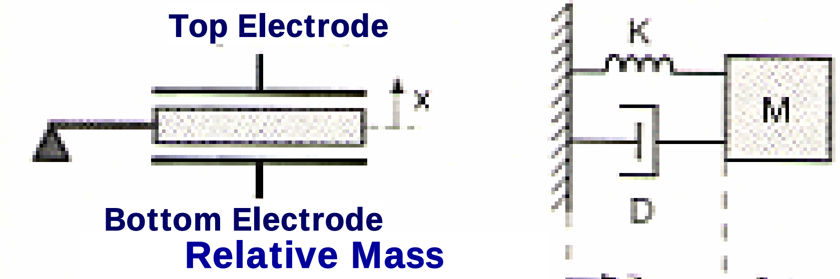






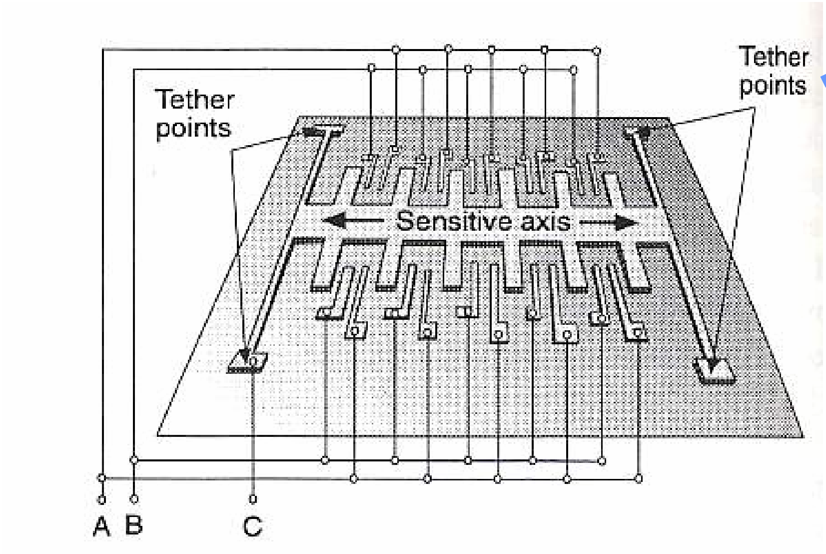


MEMS Accelerometer

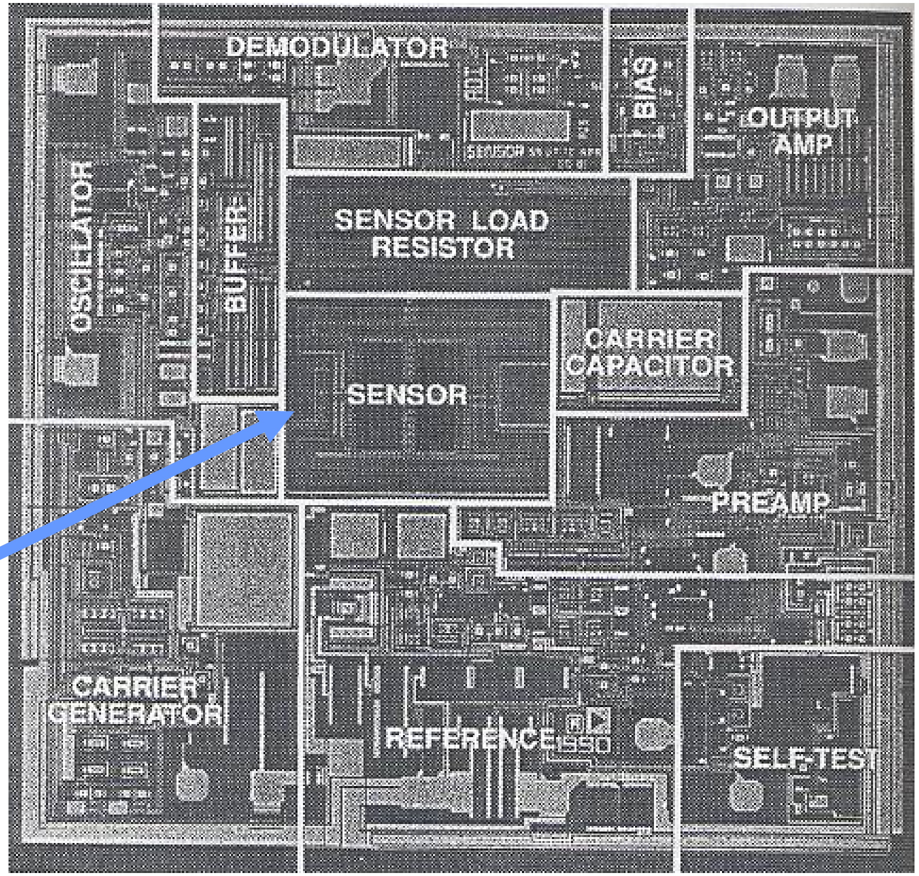


Relative Mass Displacement=x=y=z

General accelerometer structure and is mechanical lumped model.



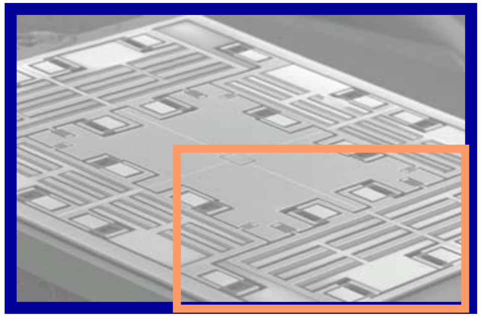
ADXL 50 accelerometer

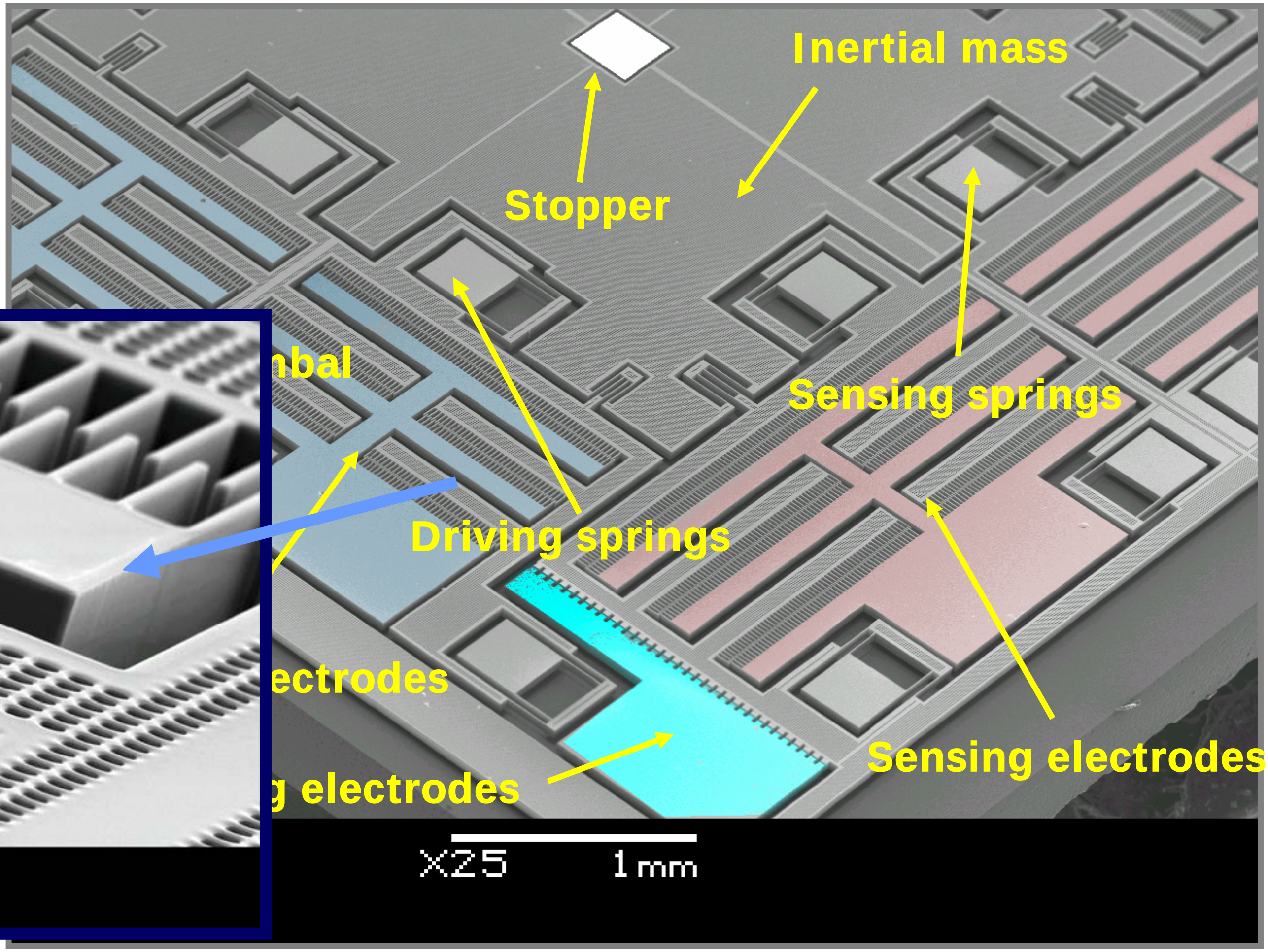


ADXL 50 accelerometer. The sensing element in the center is surrounded by active electronics. Chip size is 3 mm x 3 mm.

From Analog Devices, Inc.

Non-Vacuum Sealed MEMS Gyroscope

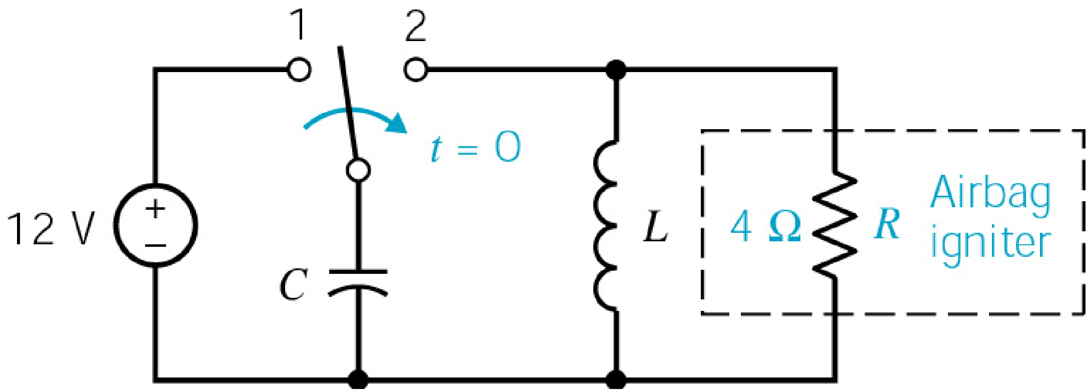




Comb electrodes after deep RIE(DRIE)

From Lab. for MiSA, SNU

Auto Airbag Igniter (IV)



$$v_C(0^-) = 12\text{ V} = v_C(0^+)$$

$$i_L(0^-) = 0\text{ A} = i_L(0^+)$$

- **0.1** 초 이내에서 트리거되려면 **under damped response**를 보여야 한다**.**
- **0.1** 초가 주기의 ¼ 정도가 되어야 한다**.**

$$C\frac{dv}{dt} + i_L + \frac{v}{R} = 0, v = L\frac{di_L}{dt}$$

$$\therefore LC\frac{d^2i_L}{dt^2} + \frac{L}{R}\frac{di_L}{dt} + i_L = 0$$

$$\frac{d^2i_L}{dt^2} + \frac{1}{RC}\frac{di_L}{dt} + \frac{i_L}{LC} = 0$$

$$\alpha = 1/2RC, \omega_0 = 1/\sqrt{LC}$$

underdapmed condition : $\alpha < \omega_0$

$$i_L = K_1 e^{-\alpha t}\cos\omega_d t + K_2 e^{-\alpha t}\sin\omega_d t$$

$$(\omega_d = \sqrt{\omega_0^2 - \alpha^2})$$

Auto Airbag Igniter (V)

$$i_L = K_1 e^{-\alpha t} \cos \omega_d t + K_2 e^{-\alpha t} \sin \omega_d t$$

- 빠른 응답을 위하여 α = 2 로 선택한다.

$\alpha = 1/2RC, \omega_0 = 1/\sqrt{LC}$ 이므로

$C = 1/2R\alpha = 1/16$ F 이고

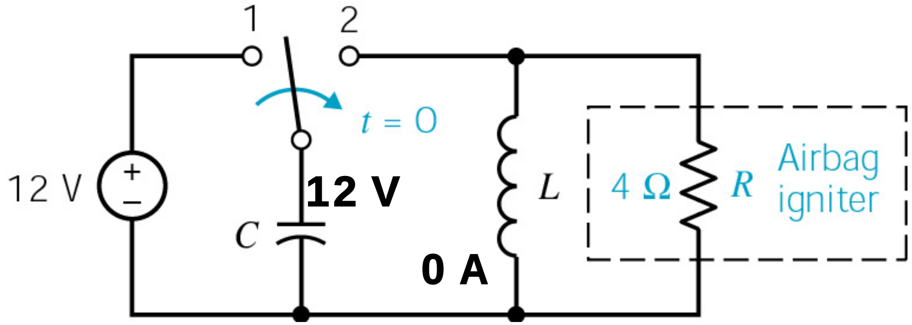
$T = \dfrac{1}{f} = \dfrac{2\pi}{\omega_d} \approx \dfrac{2\pi}{\omega_0} = 0.4$ 이므로

$L = (0.4/2\pi)^2 / C = 16(0.4/2\pi)^2 = 65$ mH

$$i_L = K_1 e^{-2t} \cos 15.57t + K_2 e^{-2t} \sin 15.57t$$

$$(\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 15.57)$$

***t* = 0⁺**



$i_L(0^-) = 0$ A $= i_L(0^+)$

$v_L(0^+) = 12$ V $= L\left.\dfrac{di_L}{dt}\right|_{0^+}$

$K_1 = 0$

$$\left.\frac{di_L}{dt}\right|_{0^+} = \frac{12}{L} = K_2 15.57$$

$$K_2 = \frac{12}{15.57L} = 11.86$$

Auto Airbag Igniter (VI)

$$i_L = 11.86e^{-2t}\sin 15.57t$$

$$v = L\frac{di_L}{dt}$$

$$= 0.065 \times 11.86(-2e^{-2t}\sin 15.57t + 15.57e^{-2t}\cos 15.57t)$$

$$\approx 12e^{-2t}\cos 15.57t$$

$$p = v^2/R = 36e^{-4t}\cos^2 15.57t$$

- 저항의 전압과 전류를 보여준다.
- **0** 초에서 **0.1** 초 까지 거의 직선적으로 변하기 때문에 전력을 삼각형으로 여기고 에너지를 구한다.

$$W = \int_0^{0.1} pdt = \int_0^{0.1} vidt$$

$$= \frac{1}{2}36 \times 0.095 = 1.71\ \text{J} > 1\ \text{J}$$