MIXING IN MICROFLUIDICS

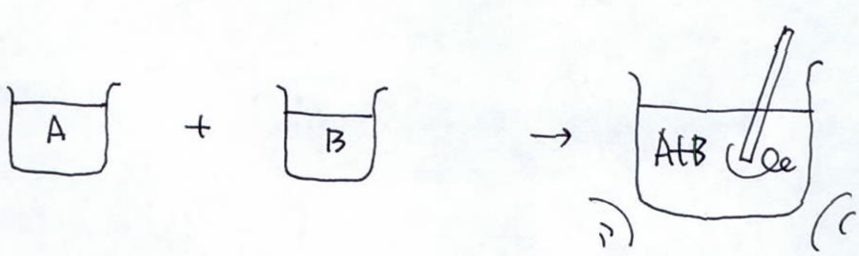
- Lab-on-a-chip
- μTAS

} biochemical reaction

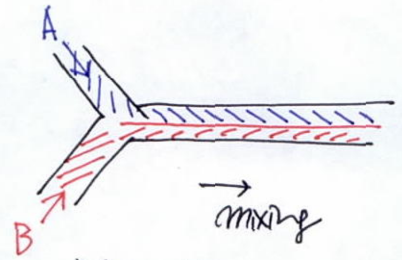
- biochemical analysis
- drug delivery
- sequencing / synthesis of nucleic acids
- cell activation
- enzyme reaction
- protein folding
- hazardous chemical detection

"A + B → AB"

macroscale:



microscale:



Y-junction

T-junction

surface-to-volume ratio
(area)

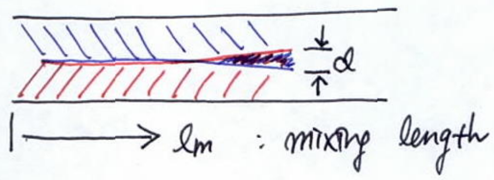
$$\frac{A}{V} \sim \frac{l^2}{l^3} \sim \frac{1}{l} \quad \uparrow \text{ as } l \downarrow$$

Mass transfer mechanism of mixing

1. molecular diffusion — low Re
2. eddy diffusion
3. bulk diffusion

(2, 3) ~ turbulence

Simplest case



molecular diffusion:

$$\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial y^2}$$ D: molecular diffusivity

scaling analysis

$$\frac{\Delta c}{\tau} \sim D \frac{\Delta c}{d^2}$$

$$\boxed{d \sim \sqrt{D\tau}}$$ $\tau \sim \frac{l}{U}$

$$d \sim \sqrt{D \cdot \frac{l}{U}}$$

$$\boxed{l_m \sim \frac{U}{D} d^2}$$

e.g. $U = 1\ mm/s$, d (channel width) $= 100 \mu m$,

$D \approx 10^{5}\ cm^2/s = 10^{-9}\ m^2/s$ (small molecules)

$10^{-7}\ cm^2/s$ (large molecules, e.g. haemoglobin in water)

channel length for complete mixing

$$l_m \sim \frac{10^{-3}}{10^{-9}} (10^{-4})^2 \sim 10^{-2}\ m \sim 1\ cm.$$ "too long for multi-step reactions"

* Peclet number. $Pe = \frac{Ud}{D} = \frac{\text{convective transport}}{\text{diffusive transport}}$.

$$l_m \sim Pe \cdot d$$

$$l_m \propto Pe.$$

* Chaotic advection / Lagrangian chaos

- laminar flow at low Re → chaotic particle trajectories
- velocity field [2-D , time-dependent / 3-D.
- Analysis : hydrodynamics + nonlinear dynamics

* Mixing devices

[active mixers (moving parts, varying pressure gradient)
[passive mixers

* Evaluation of mixing

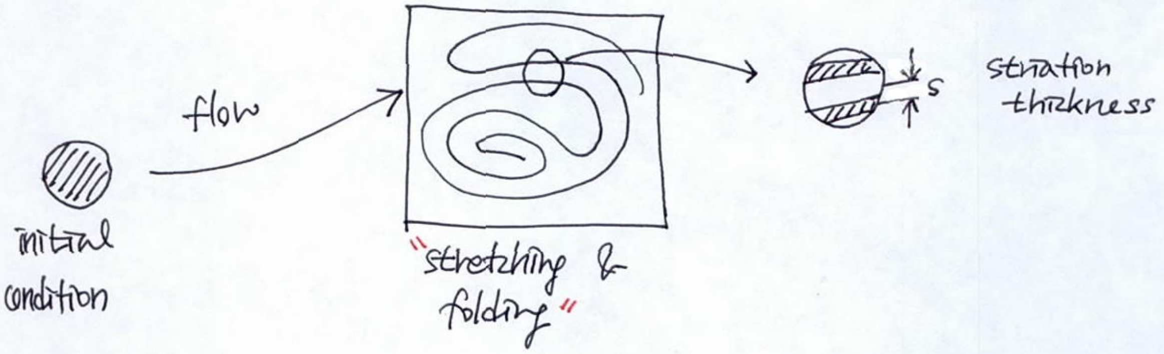
- observing color or intensity variations of a dye or pH indicator
- indicator
  - bromothymol blue (pH)
  - fluorescein (angiography, fluorescent tag) 혈관조영법
  - potassium permanganate ($KMnO$, oxidizing agent)
  - rhodamine / marine dyes
  - phenolphthalein (pH) : acid (colorless) → base (pinkish) ~pH9

* Necessary condition for chaos :

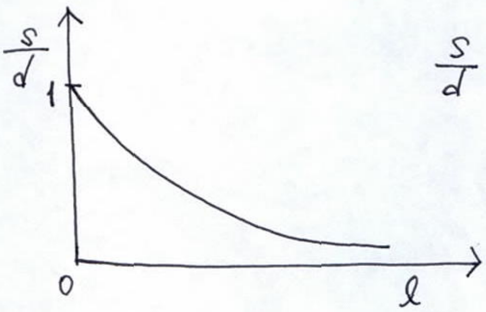
'crossing' of streamlines at different times.

~ two successive streamline portraits, say at t and $t+\Delta t$ for time-periodic 2-D flows, or at z and $z+\Delta z$ for spatially periodic flows, when superimposed, should show intersecting streamlines when projected onto the (x, y)-plane.

· Basic processes during mixing (except molecular diffusion) by flow



In a static chaotic flow,



$$\frac{s}{d} = \exp\left(-\frac{l}{\lambda}\right)$$

λ: characteristic length determined by the geometry of trajectories in the chaotic flow

$l_m(d)$?

· residence time $\tau_r = \frac{l}{U}$

· diffusion time $\tau_d = \frac{s^2}{D} = \frac{d^2}{D}\exp\left(-\frac{2l}{\lambda}\right)$

$$\tau_r \sim \tau_d$$

$$\frac{l}{U} \sim \frac{d^2}{D}\exp\left(-\frac{2l}{\lambda}\right)$$

$$\frac{Ud}{D} \sim \frac{l}{d}\exp\left(\frac{2l}{\lambda}\right)$$

$$\ln(Pe) \sim \ln\left(\frac{l}{d}\right) + 2\frac{l}{\lambda}$$

if $\ln\left(\frac{l}{d}\right) \ll 2\frac{l}{\lambda}$

$$l_m \sim \lambda \ln(Pe)$$