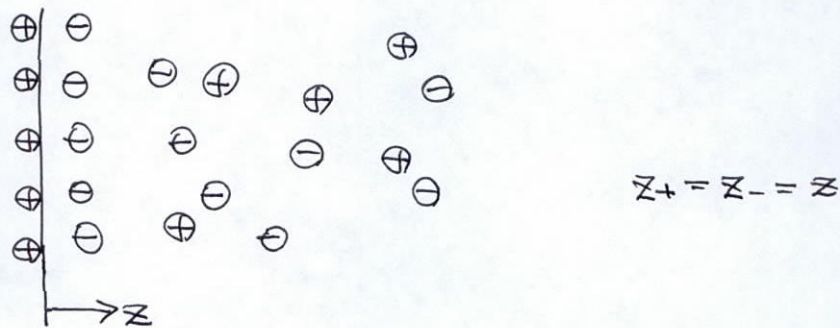


Electrokinetic Phenomena

• Electrokinetics: coupling between electric currents and fluid flow in liquids containing electrolytes

• Electric double layer



- Rough estimation

Poisson's eq

$$\frac{d^2\phi}{dx^2} = \frac{FzC}{\epsilon} \quad c = \text{const.}$$

- Detailed calculation

• Concentration of ions: Boltzmann distribution
(probability of finding an ion at a particular point depends on the local potential)

$$\begin{cases} c_+ = G \exp\left(-\frac{z_+ F \phi}{RT}\right) \\ c_- = G \exp\left(+\frac{z_- F \phi}{RT}\right) \end{cases} \quad c \rightarrow G \text{ as } \phi \rightarrow 0$$

• charge density $\rho_E = F \sum z_i c_i$

$$\begin{aligned} \rho_E &= F(z_+ c_+ - z_- c_-) \\ &= z_+ F G \left[\exp\left(-\frac{z_+ F \phi}{RT}\right) - \exp\left(+\frac{z_- F \phi}{RT}\right) \right] \end{aligned}$$

$$\rho_E = -z F z C \sinh\left(\frac{z F \phi}{RT}\right)$$

Poisson's eq

$$\frac{d^2\phi}{dx^2} = -\frac{\rho_E}{\epsilon}$$

$$= \frac{z z F C}{\epsilon} \sinh\left(\frac{z F \phi}{RT}\right) \quad : \text{Poisson-Boltzmann Eq}$$

For small potentials, $z F \phi \ll RT$

$$\sinh x = \sum_{n=0}^{\infty} \frac{1}{(2n+1)!} x^{2n+1}$$

$$= x + \frac{1}{3!} x^3 + \frac{1}{5!} x^5 + \dots$$

$$\frac{d^2\phi}{dx^2} = \frac{z z F C}{\epsilon} \cdot \frac{z F \phi}{RT} = \frac{z z^2 F^2 C}{\epsilon RT} \cdot \phi$$

Recalling $\lambda_D \equiv \left(\frac{\epsilon RT}{2 F^2 z^2 C}\right)^{1/2}$

$$\frac{d^2\phi}{dx^2} = \frac{\phi}{\lambda_D^2} \quad : \text{Debye-Hückel approximation}$$

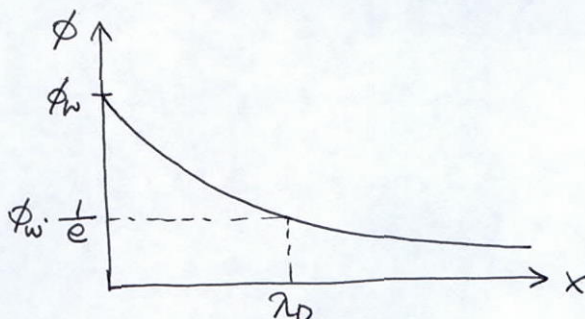
Integrating,

$$\phi = a \exp\left(\frac{x}{\lambda_D}\right) + b \exp\left(-\frac{x}{\lambda_D}\right)$$

B.C. $x=0$: $\phi = \phi_w$

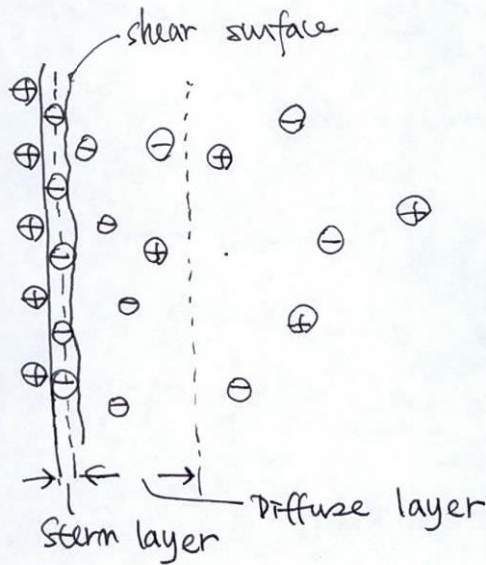
$x \rightarrow \infty$: $\phi = 0$, $\frac{d\phi}{dx} = 0$

$$\therefore \phi = \phi_w \exp\left(-\frac{x}{\lambda_D}\right)$$

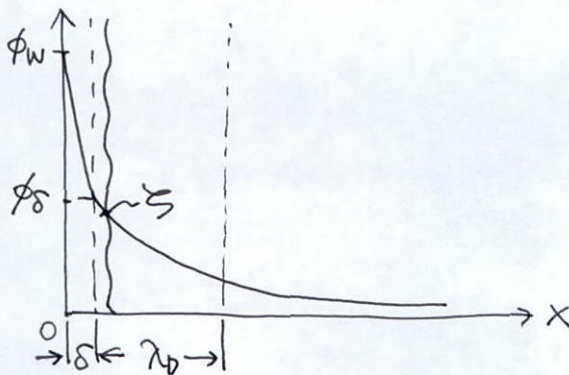


: diffuse double layer
(ions: point charges)

- Ions : finite size



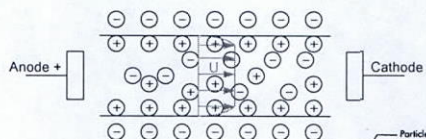
- Stern layer: Inner part of the double layer next to the surface, the outer boundary of which is approximately a hydrated ion radius from the surface
- Stern plane: the plane separating the inner layer and outer diffuse layer
- zeta (ζ) potential = electrokinetic potential :
the potential at the shear surface between the charge surface and the electrolyte solution.
- shear surface : the plane at which the mobile portion of the diffuse layer can "slip" or flow past the charged surface



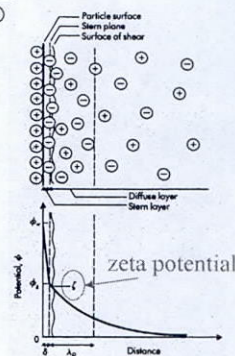
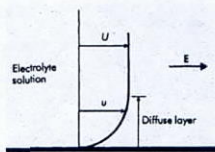
Lecture 9

Electrokinetically driven flows

- Basic concept

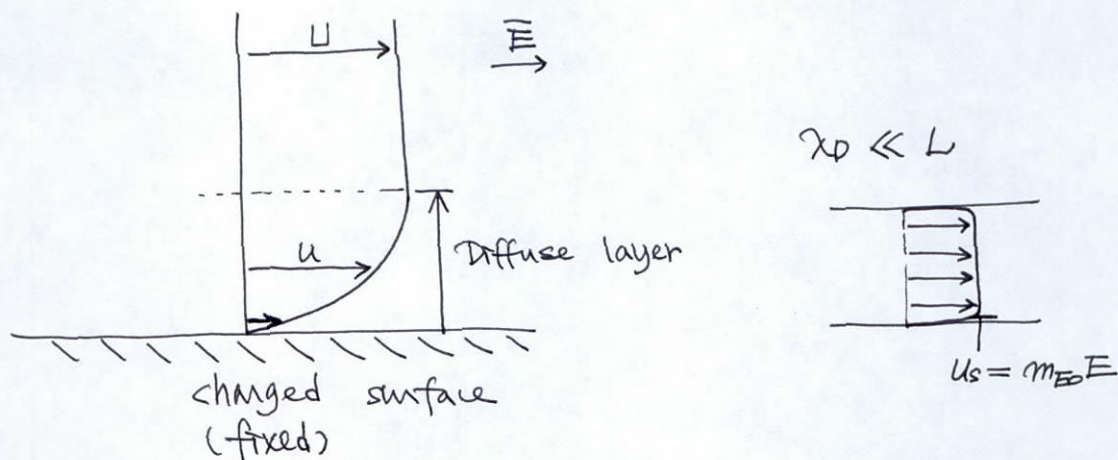


- Structure of electric double layer
- Flow produced by E-field



Four categories for electrokinetic phenomena

- Electroosmosis
 - motion of ionized *liquid* relative to the stationary *charged surface* by an applied *electric field*
- Electrophoresis
 - motion of the *charged surface* relative to the stationary *liquid* by an applied *electric field*
- Streaming potential
 - *electric field* created by the motion of ionized *liquid* along stationary *charged surfaces* ↔ electroosmosis
- Sedimentation potential
 - *electric field* created by the motion of *charged particles* relative to a stationary *liquid* ↔ electrophoresis



- Four categories of electrokinetic phenomena

	Nature of solid surface	
	stationary ^a	Moving ^b
Potential		
Applied	Electroosmosis	Electrophoresis
Induced	streaming potential	Sedimentation potential

(a) e.g. microchannel surface

(b) e.g. colloidal particles, protein molecules