# DIELECTROPHORESIS

- Background

(1) Complex permittivity

Maxwell's Equations

$$\begin{cases} \nabla \times \overline{E} = -\dfrac{\partial \overline{B}}{\partial t} & : \text{Faraday's law} \\ \nabla \times \overline{H} = \dfrac{\partial \overline{D}}{\partial t} + \overline{J} & : \text{Ampere's law} \\ \nabla \cdot \overline{D} = \rho_E & \\ \nabla \cdot \overline{B} = 0. & \end{cases} \Big\} \text{ Gauss' laws}$$

$\overline{E}$ : electric field (V/m)

$\overline{B} = \mu \overline{H}$ : magnetic flux density (Webers/m$^2$)

$\overline{H}$ : magnetic field (A/m)

$\overline{D} = \epsilon \overline{E}$ : electric displacement (C/m$^2$)

$\overline{J}$ : electric current density (A/m$^2$)

$\rho_E$ : electric charge density (C/m$^3$)

In free space

$\mu_0$ (permeability) $= 4\pi \times 10^{-7}$ henry/m

$\epsilon_0$ (permittivity) $\fallingdotseq 8.85 \times 10^{-12}$ Farad/m

For a time-harmonic field with angular frequency $\omega$,

$\overline{E}(\overline{r}, t) = \mathrm{Re}\{\overline{E}(\overline{r})\, e^{-i\omega t}\}$

$\overline{H}$

$\overline{B}$ 〃

$\overline{D}$

$\overline{J}$

$$\frac{\partial}{\partial t} = -i\omega$$

Ampere's law

$$\nabla \times \overline{H} = -i\omega \overline{D} + \overline{J}$$

In a conducting medium governed by Ohm's law

$$\overline{J} = \sigma \overline{E}$$

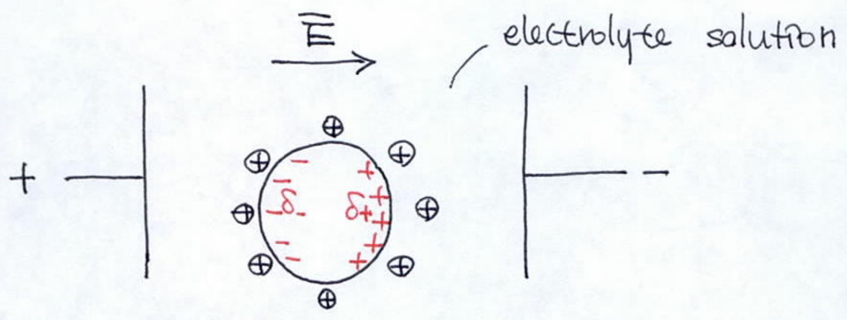
$$\nabla \times \overline{H} = -i\omega \epsilon \overline{E} + \sigma \overline{E}$$

$$= -i\omega \left(\epsilon + \frac{i}{\omega}\sigma\right) \overline{E}$$

$$= -i\omega\, \epsilon^*(\omega)\, \overline{E}$$

complex permittivity $\quad \epsilon^*(\omega) = \epsilon + i\frac{\sigma}{\omega}$

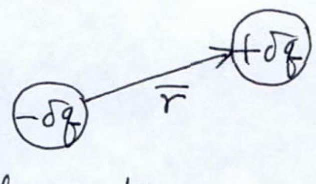
(2) Electrical dipole moment on a particle




① distortion of EDL
② Interfacial charges induced at the particle boundary
↳ Maxwell-Wagner interfacial polarization

→ electric dipole moment

$\overline{m} = \delta q \overline{r}$




$r$: particle radius

- Effects of A.C

  - electrophoresis vanishingly small $f > 1\,\mathrm{kHz}$
    ∵ particle's inertia
  - distortion of EDL negligible $f > 50\,\mathrm{kHz}$
  - M-W. polarization persists up to $> 50\,\mathrm{MHz}$

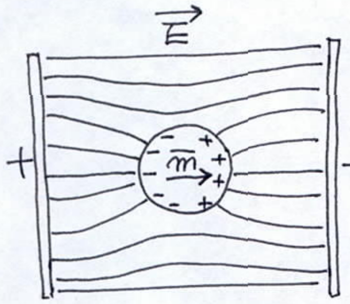
- Dipole moment arising from M-W. polarization

medium $\varepsilon_m$

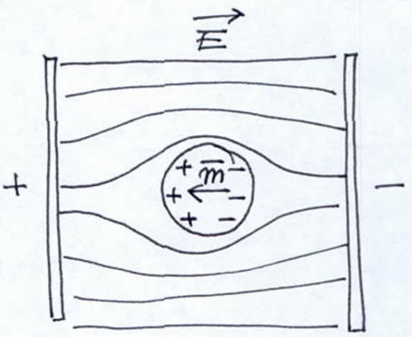
$$\overline{m} = 4\pi \varepsilon_m \left( \frac{\varepsilon_p^* - \varepsilon_m^*}{\varepsilon_p^* + 2\varepsilon_m^*} \right) r^3 \overline{E}$$

if $|\varepsilon_p^*| > |\varepsilon_m^*|$ : $\overline{m} \parallel \overline{E}$ (a)

$|\varepsilon_p^*| < |\varepsilon_m^*|$ : $\overline{m} \parallel -\overline{E}$ (b)



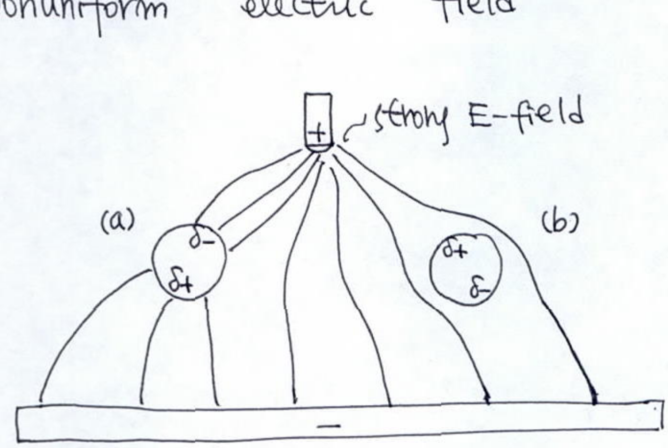
(a)



(b)

uniform E-field → symmetric system
→ no net electrical force on the particle

· Nonuniform electric field




(a) particle is more polarizable than medium
→ attracted toward the strong field
→ positive DEP

(b) particle is less polarizable than medium
→ directed away from the strong field
→ negative DEP.

Total electric force on a particle

$$\overline{F} = Q\overline{E} + \delta q\,\overline{E}(\overline{r_+}) - \delta q\,\overline{E}(\overline{r_-})$$

$$= Q\overline{E} + (\overline{m}\cdot\nabla)\,\overline{E}$$

$\underbrace{Q\overline{E}}_{\text{electrophoretic effect}}$ electrophoretic effect → negligible ($f > 1\,kHz$)

Time-averaged force

$$\overline{F}(\omega) = Re\{m(\omega)\}\,\frac{\nabla E^2}{2E}$$

$$m(\omega) = 4\pi\epsilon_m\left(\frac{\epsilon_p^* - \epsilon_m^*}{\epsilon_p^* + 2\epsilon_m^*}\right) r^3 E$$

$$\overline{F}(\omega) = 2\pi\epsilon_m r^3\, Re\{f_{CM}\}\,\nabla|E_{rms}|^2$$

$$f_{CM} = \frac{\epsilon_p^* - \epsilon_m^*}{\epsilon_p^* + 2\epsilon_m^*}$$ : Clasius-Mossotti factor