§ Line patterns to visualize flows

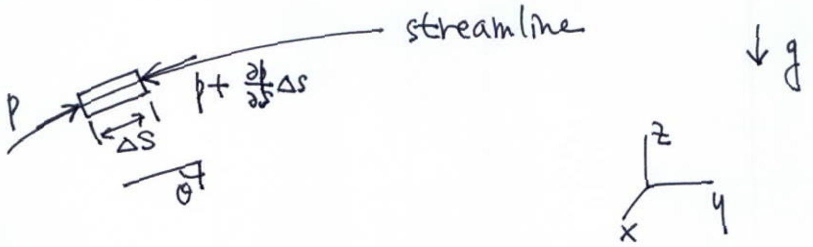
- streamline
- pathline
- streakline
- timeline

Chap. 4. Pressure and Momentum

§ Bernoulli's principle

· Euler's equation in curved streamline (steady)

(1) pressure gradient along streamline



$$\nearrow \Sigma F = pA - \left(p + \frac{\partial p}{\partial s}\Delta s\right)A - \rho A\Delta s\, g\cos\theta = \rho A \Delta s \frac{dv}{dt}$$

$$-\frac{dp}{ds} - \rho g\cos\theta = \rho\frac{dv}{dt}$$

$$\frac{dp}{ds} + \rho\frac{dv}{dt} + \rho g\cos\theta = 0$$

$$dp + \rho\left(\frac{ds}{dt}\right)dv + \rho g\, ds\cos\theta = 0$$

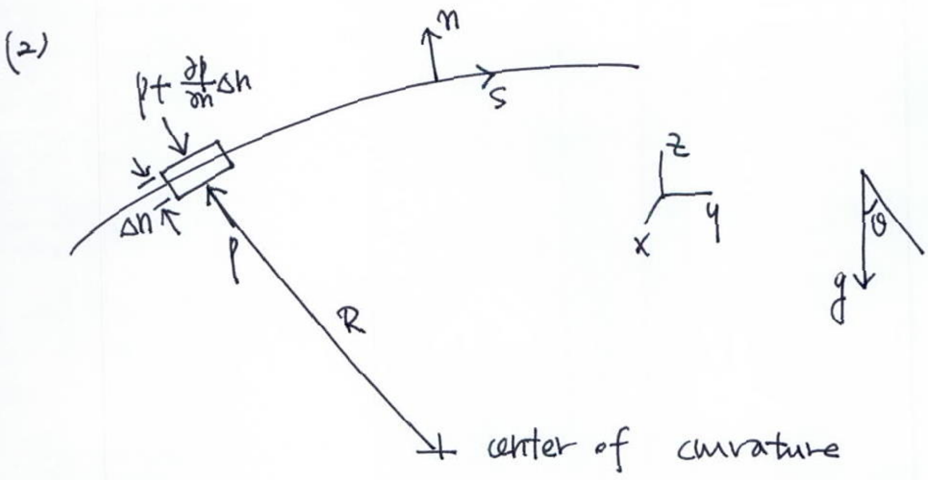
$$dp + \rho v\, dv + \rho g\, dh = 0$$

$$d\left(p + \frac{1}{2}\rho v^2 + \rho g h\right) = 0$$

OR $$\boxed{\frac{d}{ds}\left(p + \frac{1}{2}\rho v^2 + \rho g z\right) = 0}$$

Bernoulli's eq.

(2)

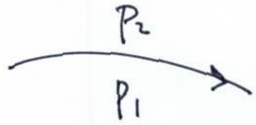


$$\searrow \Sigma F = -pA + \left(p+\frac{\partial p}{\partial n}\Delta n\right)A + \rho g \Delta n A \cos\theta$$

$$= \left(\frac{\partial p}{\partial n} + \rho g \cos\theta\right) A \Delta n$$

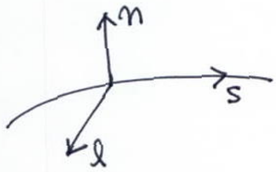
$$= \rho A \Delta n \frac{v^2}{R}$$

$$\frac{\partial}{\partial n}(p+\rho g z) = \frac{\rho v^2}{R}$$



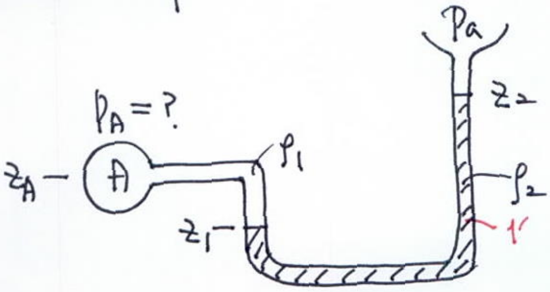
$p_2 > p_1$

(3)



$$\frac{\partial}{\partial l}(p+\rho g z) = 0.$$

§ Manometry



$p_1 = p_1'$ (Pascal's law)

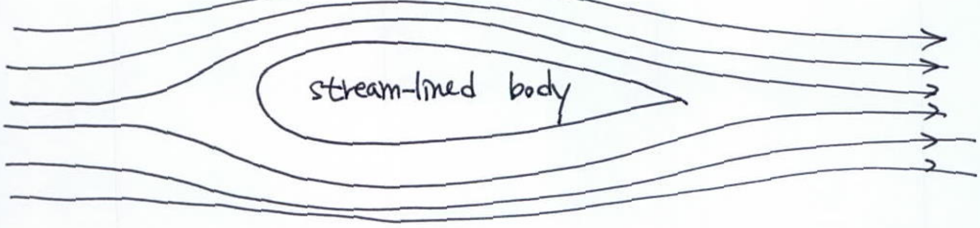
$p_1' = p_a + \rho_2 g (z_2 - z_1)$

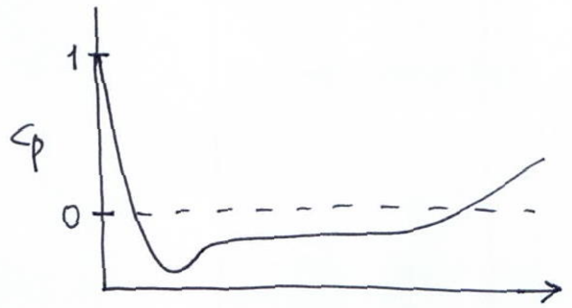
$p_1 = p_A + \rho_1 g (z_A - z_1)$

$\therefore p_A = p_a + \rho_2 g (z_2 - z_1) - \rho_1 g (z_A - z_1)$

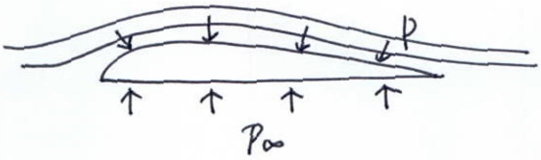
§ Pressure coefficients

- $$C_p = \frac{p - p_\infty}{\frac{1}{2}\rho U^2}$$



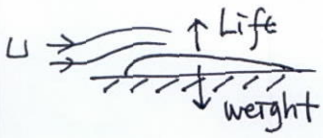


· Lift



$$L = \int_A (p_\infty - p)\, dA_\perp = \int_A \left(\tfrac{1}{2} C_p \rho U^2\right) dA$$
$$= \tfrac{1}{2}\rho U^2 \int_A C_p(x)\, dA$$

e.g. plaice (가자미), ray (가오리)

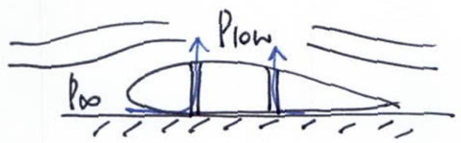


as U ↑ – Lift ↑

U : 0 — (slip speed) 0.2 m/s — (lift-off speed) 0.5 m/s —

- 0 – 0.2 m/s: quiescent
- 0.2 – 0.5 m/s: fin beating to stay on bottom
- above 0.5 m/s: actively dig in or take-off

e.g. slotted sand dollar (성게)

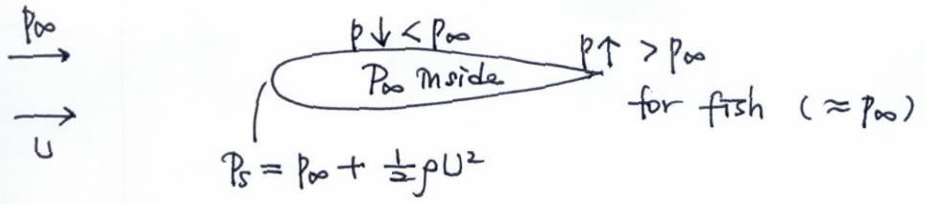


presence of slots – reduces lift

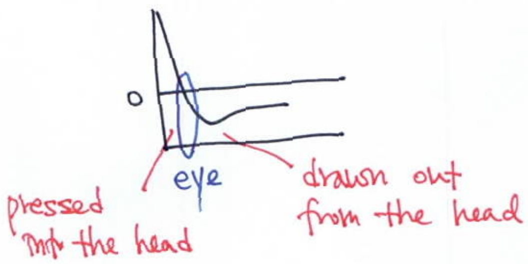
– raise the dislodgement speed by ~ 20%.

§ Pressure distributions around flexible organisms

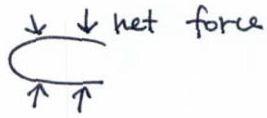
e.g. the form of fishes



· fish eye location : $C_p = 0$ or $p = P_\infty$



· where $p > P_\infty$

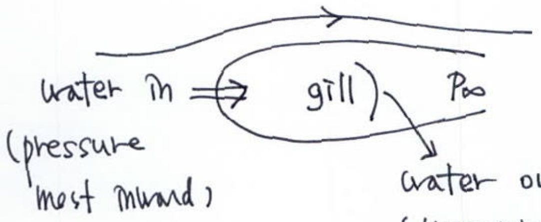


~ compression – resistant skull

· where $p < P_\infty$

~ flexible but minimally extensible skin to maintain normal shape

· ram ventilation (augmentation of ventilation by the motion of fish)



e.g. engulfing whales . (Fig. 4.11)

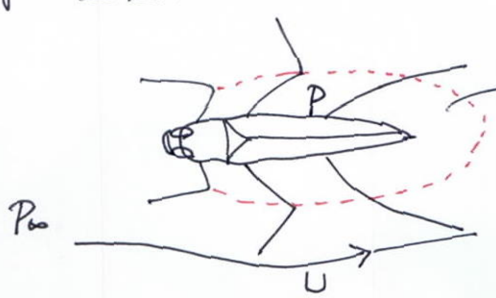
$$\text{stretching force} = \int (p_\infty - p)\, dA$$

$$= \frac{1}{2}\rho U^2 \int_A C_p(x)\, dA$$

sufficient force to expand cavity if $U = 3\,m/s$

≈ speed of feeding fin whales

e.g. beetle's bubble



— to maintain the bubble, dissolved gas must diffuse inward

$P < P_\infty$

$P < P_{atm}$

bubble = physical gill

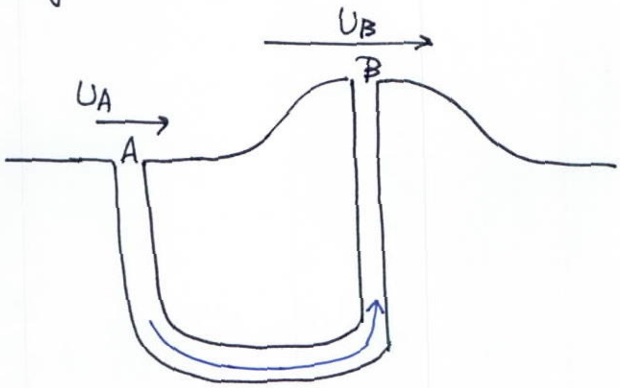
(permanent only in moving water saturated with air at P_{atm})

* to be subatmospheric, live in very shallow ($h \downarrow$) and very rapid water

h

$P_\infty = P_{atm} + \rho g h$

§ Forcing further flows



$U_A < U_B$

$P_A > P_B$

e.g. prairie dog's burrows : ventilation, olfactory assistance

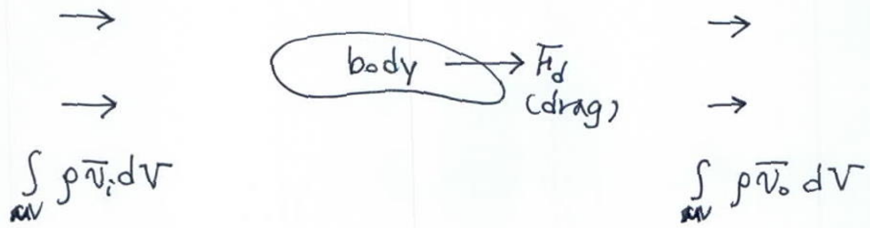
e.g. African termites' mounds : ventilation

e.g. seawater moving across ripples of sand

e.g. breathing volcano

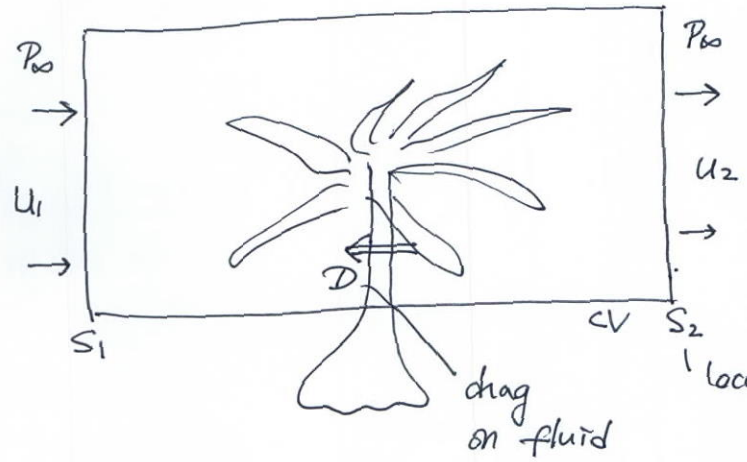
§ Momentum

§ Indirect force measurements



$$\frac{D}{Dt}\int_{CV} \rho \bar{v}\, dV = F_d$$

If the moving fluid exerts a drag on a body, then the body must remove momentum from the fluid at a rate that just balances its drag.



location should be wisely chosen

$$\rightarrow \quad -D = \int_{CS} \rho v\, v_{rn}\, dA$$

$$-D = \int_{S_1} \rho U_1 (-U_1)\, dA + \int_{S_2} \rho U_2 \cdot U_2\, dA$$

if $S_1 = S_2 = S$

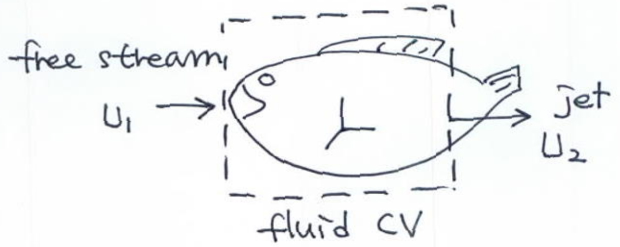
$$D = \int_S \rho (U_1^2 - U_2^2)\, dA$$

continuity : $\int_{S_1} \rho U_1 dA = \int_{S_2} \rho U_2 dA$

unknowns : U_2, D , # eqs : 2

§ Jet propulsion

Fig. 4.14 examples



$\rightarrow$ $T = \int_{CS_1} \rho U_1 (-U_1)\, dA + \int_{CS_2} \rho U_2 (U_2)\, dA$

U_1, U_2 uniform across CS_1 and CS_2, respectively.

$T = \dot{m}(U_2 - U_1) > 0$: force on fluid →

where $\dot{m} = \int_{CS_1} \rho U_1\, dA = \int_{CS_2} \rho U_2\, dA$

by continuity

∴ force on the swimmer ← : thrust

- power output : $P_{out} = T \cdot U_1 = \dot{m} U_1 (U_2 - U_1)$
- power input : $P_{in} = \frac{1}{2} \dot{m} (U_2^2 - U_1^2)$
- Froude propulsion efficiency

$$\eta_f = \frac{P_{out}}{P_{in}} = \frac{2U_1}{U_2 + U_1}$$

$= 1$ if $U_1 = U_2$

↓ as $U_2 \uparrow$ ($> U_1$)

: propellers or fins achieve high η_f with only a small incremental velocity.

- real η_f of jet propulsion $> \frac{2U_1}{U_2 + U_1}$

∵ water entrainment as rolling up into toroidal vortices