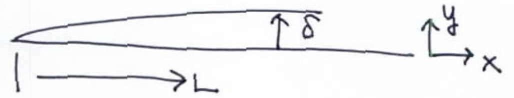
Chap. 8. Velocity Gradients and Boundary Layers

* Boundary - Layer Theory.



$\frac{\delta}{L} \ll 1$

$\frac{\partial p}{\partial y} \ll \frac{\partial p}{\partial x}$ $\qquad p = p(x)$

momentum eq : $\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = - \frac{dp}{dx} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$

continuity : $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0.$

$$\frac{O\left(\frac{\partial^2 u}{\partial x^2}\right)}{O\left(\frac{\partial^2 u}{\partial y^2}\right)} \sim \frac{\frac{U}{L^2}}{\frac{U}{\delta^2}} \sim \left(\frac{\delta}{L}\right)^2 \ll 1.$$

momentum eq. : $\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = - \frac{dp}{dx} + \mu \frac{\partial^2 u}{\partial y^2}$

on flat plate : $\frac{dp}{dx} = 0$

scaling analysis.

$$\rho U \frac{U}{L} \sim \mu \frac{U}{\delta^2}$$

$$\delta^2 \sim \frac{\mu L}{\rho U}$$

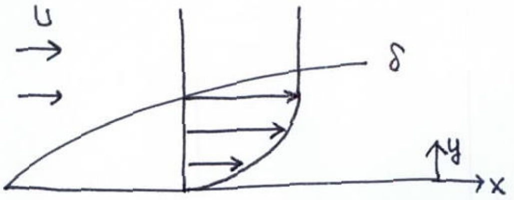
$$\delta_x \sim \left(\frac{x \mu}{\rho U} \right)^{1/2}$$

$$u(\delta_{99}) = 0.99\, U_\infty$$

$$\delta_{99\%} = 5.0 \left(\frac{x\mu}{\rho U} \right)^{1/2}.$$

$$\frac{\delta}{x} = 5.0\, Re_x^{-1/2}$$ Blasius (1908)

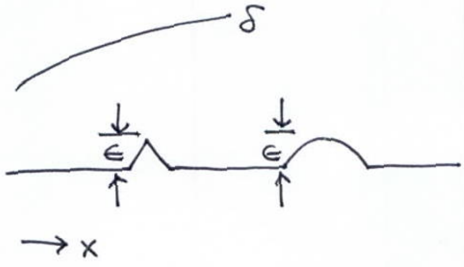
: laminar B.L. ($Re_x < \sim 5 \times 10^5$)



· velocity approximation.

$$y < \frac{\delta}{2} : \qquad \frac{u}{U} = 0.32\, y \left(\frac{\rho U}{x \mu} \right)^{1/2}$$

· How rough can a surface be without materially affecting the flow over it ?



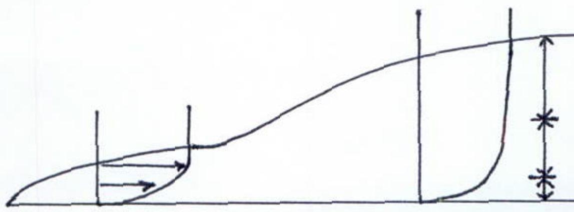
$$\frac{\epsilon}{x} \le 9.5\, Re_x^{-3/4} \qquad (\text{pointed})$$

$$\frac{\epsilon}{x} \le 12.2\, Re_x^{-3/4} \qquad (\text{rounded})$$

$$Re_x \uparrow \; - \; \frac{\epsilon}{x} \downarrow \qquad (\text{for fixed } \epsilon - x \uparrow)$$

* Turbulent boundary layer

$$\frac{\delta}{x} = 0.376\, Re_x^{-1/5}$$
$$= 0.16\, Re_x^{-1/7}$$



outer layer

logarithmic overlap layer

viscous wall layer

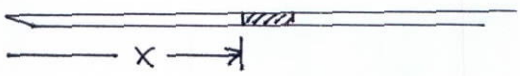
· real biological flat surfaces (leaves)

: turbulence occurs $Re < Re_{tr}$ (flat plate)

heat/mass transfer > theory.

§ Forces at and near surfaces

* drag on a spot on a flat plate



$$\tau_w = \mu \frac{\partial u}{\partial y}\Big|_{y=0}$$

$$= 0.32\ U \left(\frac{\rho U}{x\mu}\right)^{1/2} \cdot \mu \qquad \text{(linear vel. approximation)}$$

local drag coefficient C_{Dl}

$$C_{Dl} = \frac{F_D}{\frac{1}{2}\rho U^2 S} = \frac{2\tau_w}{\rho U^2}$$

$$= 0.64 \left(\frac{\mu}{\rho U x}\right)^{1/2} = 0.64 \cdot Re_x^{-1/2}$$

total drag $(0 \rightarrow x)$

$$F_D = \int_0^x \tau_w \, dx$$

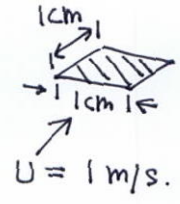
$$= 0.32 \int_0^x \frac{\rho^{1/2} U^{3/2} \mu^{1/2}}{x^{1/2}} dx$$

$$= 0.64\ \rho^{1/2} U^{3/2} \mu^{1/2} x^{1/2}$$

$$C_D = \frac{F_D}{\frac{1}{2}\rho U^2 x} = 1.28\ Re^{-1/2} \qquad \sim (7.1)$$

$(1.33\ Re^{-1/2})$ p. 135

e.g.



(1) on flat plate. $x = 0.5$ m

$Re_x = 5 \times 10^5$

$C_{Dl} = 9 \times 10^{-4} = 0.64\ Re_x^{-1/2}$

$F_D = 4.5 \times 10^{-5}$ N.

(2) In the free stream,

$$Re = \frac{\rho U (1cm)}{\mu} = \frac{10^3 \times 10^{-2}}{10^{-3}} = 10^4.$$

$$C_D = 1.33\ Re^{-1/2} = 0.0133$$

$$F_D = \frac{1}{2}\rho U^2 (2cm^2)\ C_D = \frac{1}{2}(10^3)(10^{-4})(0.0133)\ (2)$$

↑ wetted area

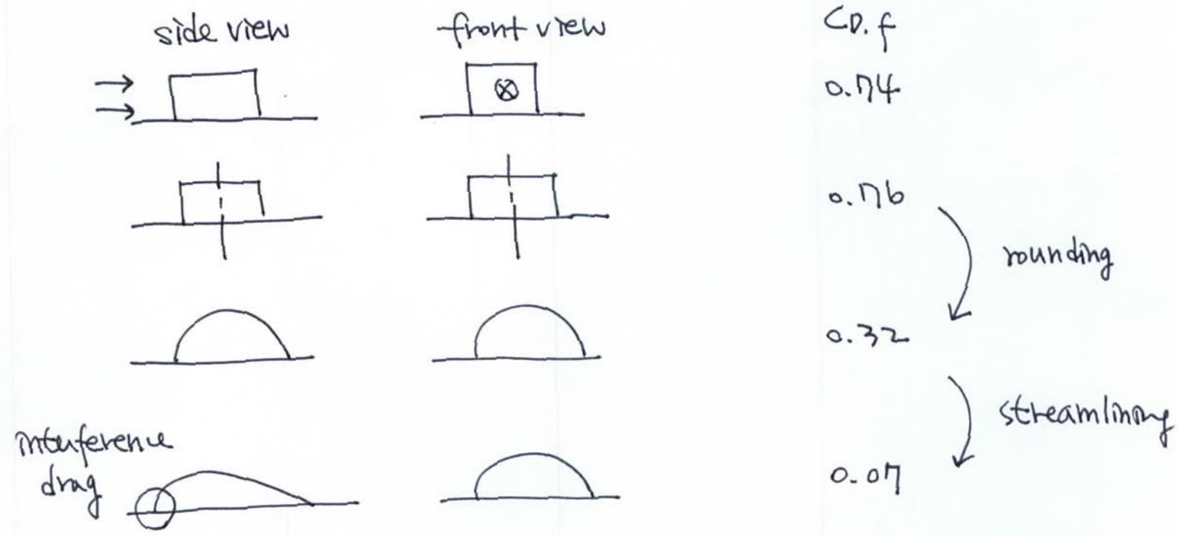
$$= 0.00133\ N$$

$$= 1.33 \times 10^{-3}\ N.$$

$$F_{D,(2)} \approx 30\ F_{D,(1)}$$

* Drag coefficients of protuberances on flat plates

Fig. 8.4



$$Re_{dia} = 2\times10^4 \sim 5\times10^4.$$

$$\delta_{BL} \ll \text{height}$$

↑
lengthen and flatten the leading edge $\sim C_{D_f} = 0.03$

§ Unbounded boundary layers

- flat plate with no leading edge

  e.g. surface of the earth (ex : Table 8.1)

$$u_x = \frac{u^*}{\kappa} \ln\left(\frac{z-d}{z_0}\right)$$: turbulent B.L. logarithmic overlap layer

$\kappa \simeq 0.41$

u^* (friction velocity / shear velocity)

$= \sqrt{\frac{\tau}{\rho}}$

§ Flow right on bumpy bottoms

e.g. canopy of forest, top leaves of field of crop, top of forest of tall sea anemones on a subtidal rock face

Fig. 8.7