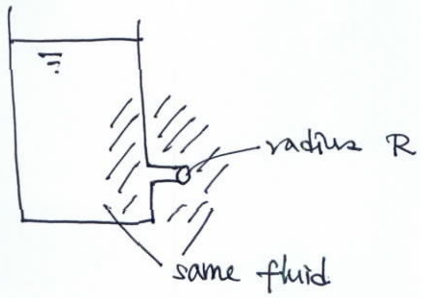
§ Flow through circular apertures (submerged)



Re < 3 : $Q = \dfrac{R^3 \Delta p}{3\mu}$

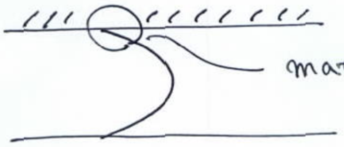
higher Re : $Q = C_0 \pi R^2 \left(\dfrac{2\Delta p}{\rho}\right)^{1/2}$

C_0 : orifice coefficient $= fn(Re_d)$

Chap. 14. Internal flows in organisms

- most circulatory systems of animals
  ~ pulsatile flow of non-Newtonian fluids in pipes of time varying cross-sectional areas and shapes

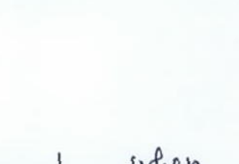
§ Circumventing the parabolic profile



material/heat exchange ~ $\dfrac{\partial u}{\partial y}$ ↓ when parabolic

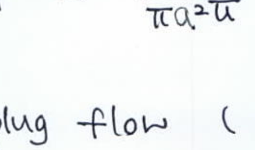
(1) How far is flow from a wall ?

- distance index



$$Di = \frac{1}{\pi a^2 \bar{u}} \int_0^a u\,(a-r)\cdot 2\pi r\,dr \;/ a$$

① plug flow (slug flow)



$\bar{u} = u = \text{const.}$

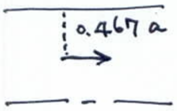
$$Di = \frac{2\pi}{\pi a^2 u}\, u \int_0^a (ar - r^2)\,dr$$

$$= \frac{2}{a^3}\cdot\left(\frac{a^3}{2} - \frac{a^3}{3}\right) = \frac{1}{3}$$

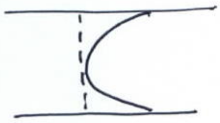
② fully developed laminar flow (parabolic)

$$D_i = \frac{2}{a^3 \bar{u}} \int_0^a \frac{\Delta P}{4\mu L}(a^2 - r^2)(ar - r^2)\,dr. \qquad \bar{u} = \frac{\Delta p \cdot a^2}{L \cdot 8\mu}$$

$$= \frac{7}{15} = 0.467$$



③ reversed parabolic profile : wall forcing the flow (cilia)



$D_i = 0.2$

· $D_i \downarrow$ – exchange of material/ heat $\uparrow$

(2) Using noncircular cross sections

· circular cross section :

good – mechanical robustness
cost of construction

worst – exchange

· large pipes through which exchange takes place

~ closer to parallel plates than to circles

e.g. internal gills of fish
nasal passages
intestine of earthworm Fig. 14.2

(3) Making the pipes very small

a $\downarrow$ – D_i the same

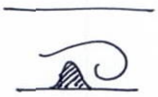
–  total cross-sectional area $\uparrow$ (good for exchange)

~ vel. $\downarrow$

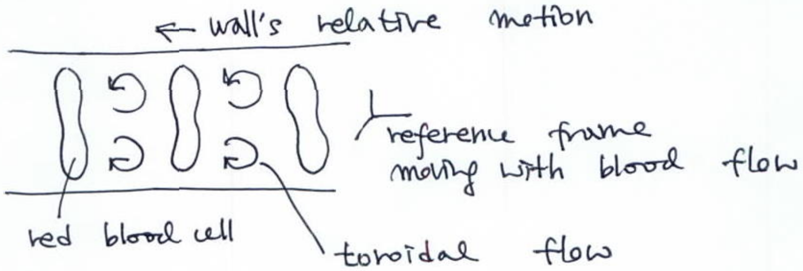
~ time for exchange $\uparrow$

(4) Eddies and turbulunce

- turbulunce : ↗ vel. : greatly enhance exchange
  but Re of biological pipes too low

- eddies  Re > 30.

(5) Periodic boluses



* Peclet number

$$Pe = \frac{UL}{D} = \frac{\text{convection}}{\text{diffusion}}$$

D : diffusion coefficient of the dissolved substance

(6) Pumping at the wall

- cilia / flagella coatings on the walls
  ∴ steep velocity gradient → efficiency of exchange ↑
  but cost of operation high

  e.g. oviducts
  gastroderm of coelenterates
  gills of mollusks

§ Efficient branching arrays of pipes

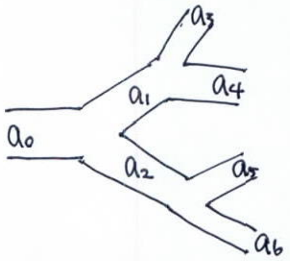
(1) Murray's law

- optimal design of a circulatory system based on minimization of cost factors

- cost factors
  1. keeping the blood going against the pressure losses consequent to Hagen-Poiseuille eq
  2. construction and maintenance cost proportional to the volume of the system.

$$\boxed{Q = k a^3}$$

e.g. branching tubes



by continuity, $a_0^3 = a_1^3 + a_2^3 = a_3^3 + a_4^3 + a_5^3 + a_6^3$

- consequences

1. if $a_1 = a_2$, $2a_1^3 = a_0^3$, $a_1^3 = \frac{1}{2} a_0^3$

   cross-sectional area : $a_1^2 = \left(\frac{1}{2}\right)^{2/3} a_0^2 = 0.63\, a_0^2$

2. $U_0 a_0^2 = 2 U_1 a_1^2$

   $= 1.26\, U_1 a_0^2$

   $U_1 = \frac{1}{1.26} U_0 = 0.79\, U_0$

3. $Q = a^2 U = k a^3$

   $U \sim a$

4. same specific parabolic profile $\left(\frac{u}{a}\right)$ for every vessel

5. same velocity gradient at the wall

   $\frac{du}{dr}\Big|_{r=a}$

6. same shear stress $\sim \mu \frac{du}{dr}\Big|_{wall}$

(2) Do real systems really follow Murray's law?

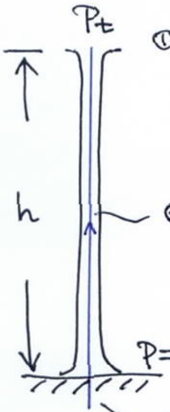
almost yes. Table 14.1

except arterioles and capillaries
(flow profile not parabolic)

§ The ascent of sap in trees

· sap : liquid that rises in the xylem from roots to leaves

↔ fluid in phloem (체관부)



① hydrostatic pressure : $\rho g h$

② pressure drop due to viscosity , ΔP_v

③ pressure needed to draw water from the interstices of soil . ΔP_r

$$P_t = P_a - \rho g h - \Delta P_v - \Delta P_r$$

for tall trees ($h = 100\,m$)

$P_a = 1\,atm$

$\rho g h = 10\,atm$

$P_t \approx -80\,atm$ (absolute)

· driving pressure : $P_a - P_t \approx 80\,atm$

← evaporation of water in the interstices of leaves if $U_{sap} = 0.1$ m/s

$d_{xylem} = 0.25$ mm

$$\frac{\Delta p_{ideal}}{L} = \frac{8\mu U}{a^2} = \frac{8(10^{-3})(0.1)}{(0.125\times10^{-3})^2} = 51.2 \text{ kPa/m}$$

$$\frac{\Delta p_{real}}{L} = (0.7)^{-1}\frac{\Delta p_{ideal}}{L} = 73 \text{ kPa/m} \neq 7.3 \text{ kPa/m}$$

(in textbook)

Chap. 15. Flow at very low Reynolds numbers

- creeping flow

  N.-S. eq

  $$0 = -\nabla p + \mu \nabla^2 \bar{u} \quad : \text{ Stokes eq}$$

  e.g. microscopic organisms
  proteins. cells. DNA, ...

- characteristics

  ① reversible

  ② mixing extremely difficult

  ③ significant wall effect

- propulsion slow

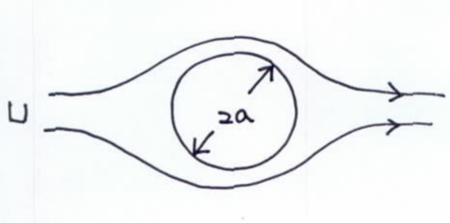
  Drag $\sim L^2$

  Thrust $\sim$ volume $\sim L^3$ (engine size)

  $\frac{T}{D} \sim L$ $\downarrow$ as $L \downarrow$

§ Drag

(1) Spheres



$$\boxed{D = 6\pi\mu U a}$$ $(Re < 1)$

Stokes' law