if $U_{sap} = 0.1$ m/s

$d_{xylem} = 0.25$ mm

$$\frac{\Delta p_{ideal}}{L} = \frac{8\mu U}{a^2} = \frac{8(10^{-3})(0.1)}{(0.125\times10^{-3})^2} = 51.2 \text{ kPa/m}$$

$$\frac{\Delta p_{real}}{L} = (0.7)^{-1}\frac{\Delta p_{ideal}}{L} = 73 \text{ kPa/m} \neq 7.3 \text{ kPa/m}$$

(in textbook)

Chap. 15. Flow at very low Reynolds numbers

- creeping flow

  N.-S. eq

  $$0 = -\nabla p + \mu \nabla^2 \bar{u} \quad : \text{Stokes eq}$$

  e.g. microscopic organisms

  proteins, cells, DNA, …

- characteristics

  ① reversible

  ② mixing extremely difficult

  ③ significant wall effect

- propulsion slow

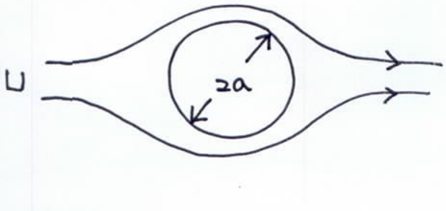
  Drag $\sim L^2$

  Thrust $\sim$ volume $\sim L^3$ (engine size)

  $\frac{T}{D} \sim L$ ↓ as L ↓

§ Drag

(1) Spheres



$$\boxed{D = 6\pi\mu U a} \quad (Re < 1)$$

Stokes' law

$$C_{D_f} = \frac{24}{Re} + \frac{6}{1+Re^{1/2}} + 0.4 \qquad (1 < Re < 2\times10^5)$$

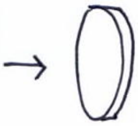
$$Re = \frac{U(2a)}{\nu}$$

$$= \frac{D}{\frac{1}{2}\rho U^2 S}.$$

Stokes' drag = skin friction + fore-and-aft pressure drop

$(\frac{2}{3})$ $(\frac{1}{3})$

(2) circular disks



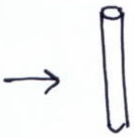
$D = 16\mu U a$

} difference not large

$D = 10.67\,\mu U a$

(3) Cylinders

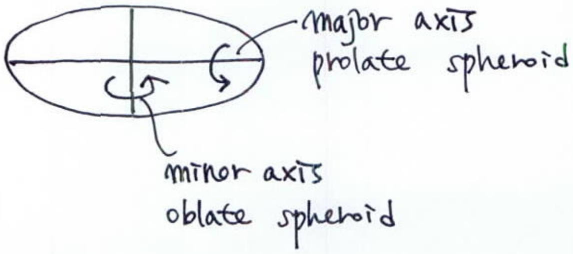
$$D = \frac{2\pi\mu U l}{\ln(\frac{l}{a}) - 0.807}$$



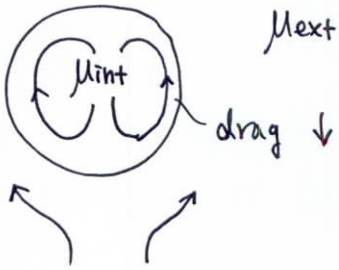
$$D = \frac{4\pi\mu U l}{\ln(\frac{l}{a}) + 0.193}$$

(4) Spheroids

ellipse



(5) Fluid spheres



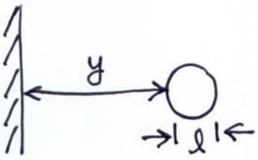
$$D = 6\pi\mu_e U a \, \frac{1+\frac{2}{3}(\mu_e/\mu_i)}{1+\mu_e/\mu_i}$$

for gas bubbles

$$\frac{\mu_e}{\mu_i} \to \infty$$

$$D = 4\pi\mu_e U a$$

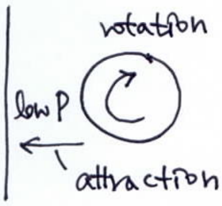
(6) Wall effects

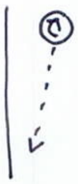


$\text{Drag}_{\text{near wall}} > \text{Drag}_{\text{isolated}}$

to neglect wall effect, $\frac{y}{l} > \frac{20}{Re}$

$(Re < 1)$

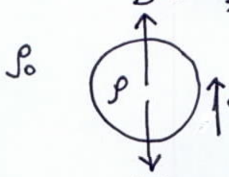




§ Terminal velocity

force balance for a sphere falling at a constant vel.

$B = \frac{4}{3}\pi a^3 \rho_0 g$



$D = 6\pi\mu U a$

$W = \frac{4}{3}\pi a^3 \rho g$

$$W = B + D$$

$$\frac{4}{3}\pi a^3 (\rho - \rho_0) g = 6\pi\mu U a$$

terminal vel.

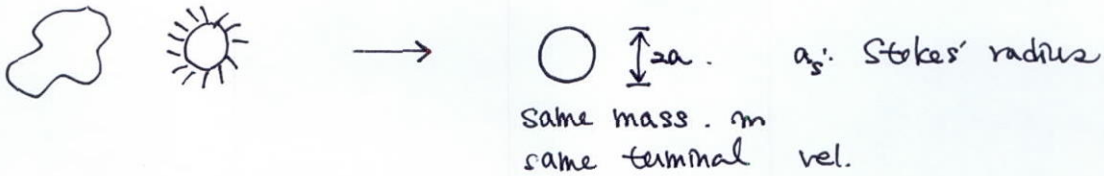
$$U = \frac{2a^2 g(\rho - \rho_0)}{9\mu} \quad (Re < 0.5)$$

Note: $U_t \sim a^2$

$\because$ $W - B \sim a^3$

$D \sim a$

(1) Stokes' radius

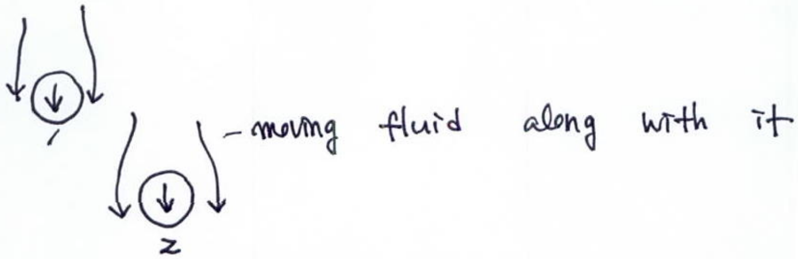


$$mg - \frac{4}{3}\rho_0 g \pi a_s^3 = 6\pi\mu U a_s$$

for air, ρ_0 term negligible

$$a_s = \frac{mg}{6\pi\mu U}$$

(2) Interactions among falling objects

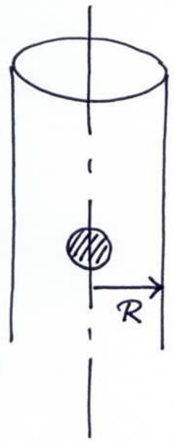


$U_{1, pair} > U_{1, isolated}$

$U_{2, pair} < U_{2, isolated}$

(3) Measuring terminal velocity

- wall effect !



$$\mu_{true} = \frac{\mu_{apparent}}{1 + 2.4\frac{a}{R}}$$

(4) When terminal velocity matters

- small sinkers – reduce sinking rate to travel longer distance

ways

① swim upward (negative geotaxis)

② mucus thread

③ enveloped in mucilage

④ irregular shapes

§ Propulsion at low Re

(circulation-based lift not efficient (not working)
drag-based scheme

- velocity gradient : wide, gentle
~ to push against free stream, propulsive appendage should protrude through / beyond the gradient region

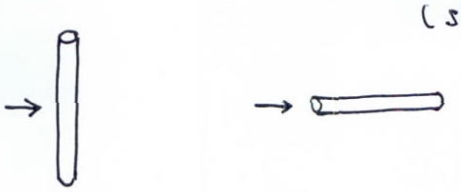
(1) Changing the area of appendages : paddles and bristles

bristles – power stroke : spread
bristles – recovery stroke : fold back

Fig. 15.3

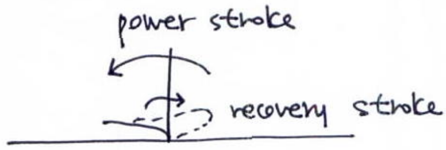
(2) Changing the orientation of appendages : cilia and flagella
: cylinders !

* cilium : reciprocating beat
(short : 5~10 μm in length)



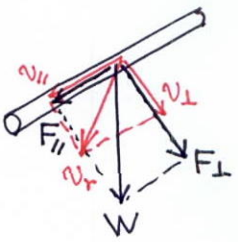
$D_\perp > D_\parallel$

Fig. 15.4

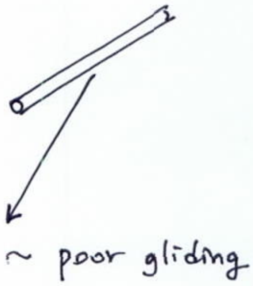


* flagellum
- planar, undulating waves
- helical waves from base to tip
- spiral waves

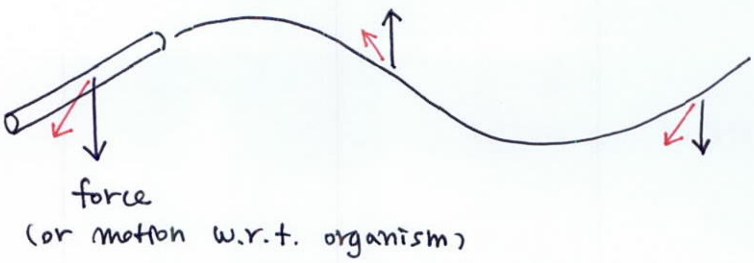
· obliquely falling cylinder



∴



· moving oblique portion of appendage sideways



for cylinders (Re < 1)

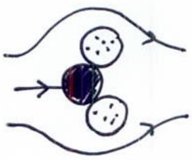
$$\frac{D_\perp}{D_\parallel} = 1.75 \qquad l/a = 100$$

$$1.80 \qquad l/a = 300$$

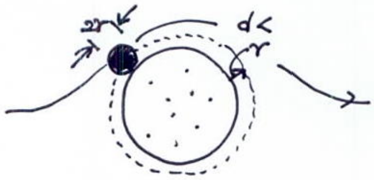
§ Filtration (suspension feeding)

- 5 mechanisms

① sieving



② direct interception



③ inertial impaction

④ gravitational deposition

⑤ diffusional deposition

Brownian motion