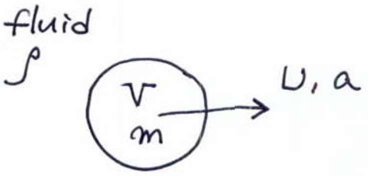
Chap. 16. Unsteady Flows

§ Added mass and acceleration reaction



- the force needed to accelerate a body in a fluid

$$F = \underbrace{\tfrac{1}{2} C_D \rho S U^2}_{\text{drag}} + m a + \underbrace{C_a \rho V a}_{\text{force needed to accelerate the added mass of fluid backward.}}$$

∴ acceleration reaction
(added inertia)

- Ca : added mass coefficient

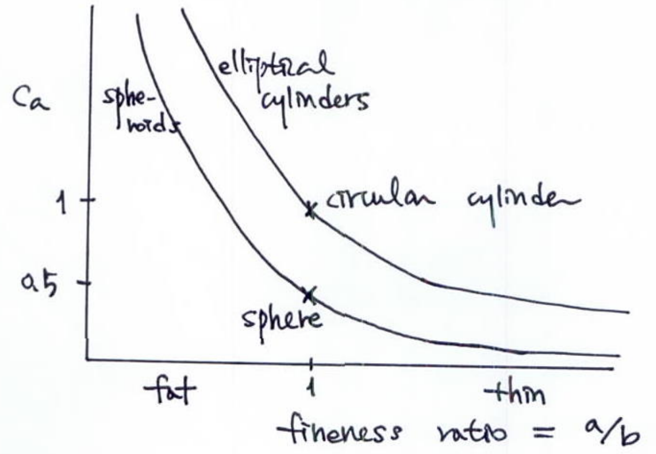


Fig. 16.1.

- Direction of forces
  - drag : always slows a body down
  - acceleration reaction : always opposes any change in speed

(1) The beginning and end of a swim

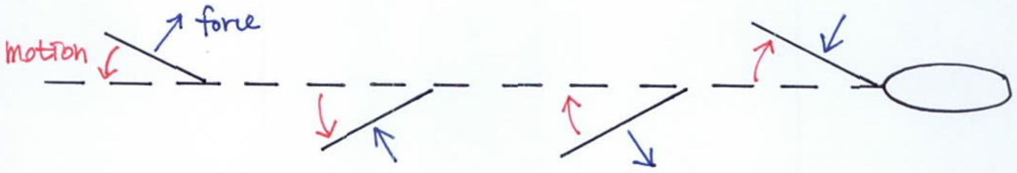
- added inertia important for (escape responses
  lunging predatory strikes

(2) Making paddles and tails work

- How a toyfish swims ?

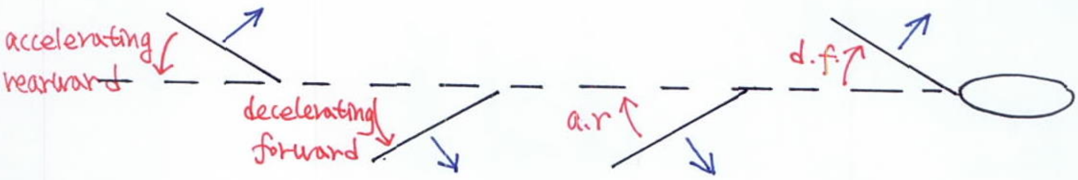
Fig. 16.2

a. quasi-steady view

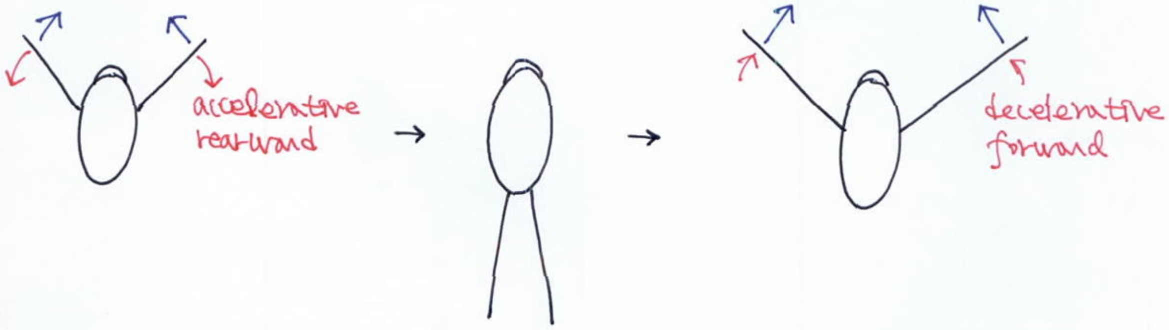


: all forces cancel , no net thrust

b. unsteady view



- dytiscid beetle



(3) Accelerative forces on stationary objects

an attached organism suddenly subjected to a surge of water

$$F = \frac{1}{2}\rho S U^2 C_D + \rho V a + C_a \rho V a$$

if the object weren't there, a body of water of its volume would be accelerated along with the remaining water.

~ stationary body ≡ accelerating the same volume of water in the opposite direction

$$= \frac{1}{2} C_D \rho S U^2 + (1 + C_a)\rho V a$$

- $\frac{\text{attachment tenacity}}{\text{drag}} = \frac{L^2}{L^2}$ ~ indep. of L

- $\frac{\text{att. tenacity}}{\text{acceleration reaction}} = \frac{L^2}{L^3} \sim \frac{1}{L}$ ↓ as L ↑

(4) Added mass in air

negligible in many cases

i) speeds of flow much higher in air : U^2

ii) accelerations not large

iii) organism mass ≫ air mass displaced

§ Consequences in the wake of vortices

(1) Vortex shedding

von Kármán vortex street

- dimensionless frequency, Strouhal number

$$St = \frac{n l}{U} = fn\ (\text{shape}, Re)$$

n : frequency

l : characteristic length

U : free-stream velocity.

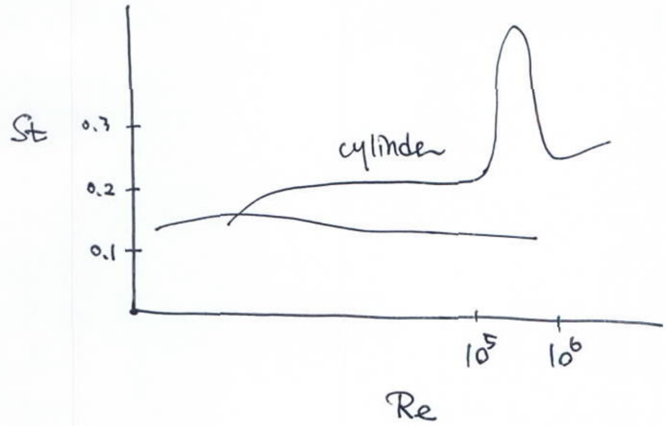
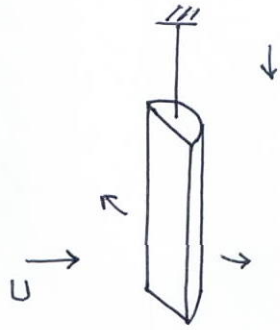
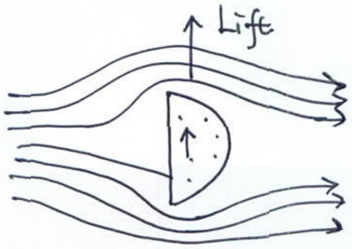


Fig. 16.5.

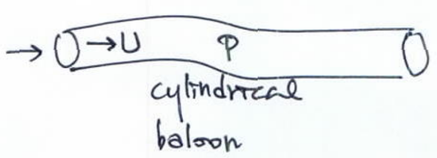
(2) Self-excited oscillators and aeroelasticity.

Ex1.





Ex2.



U ↑ − p ↓ − collapse

− resistance ↑ − U ↓

− p ↑ − expansion

e.g. pathological stenoses in human circulation
respiratory wheezes

§ When is flow unsteady enough to matter?

(1) Lighthill : aerodynamic frequency parameter
eg beating wing

$$f_a = \frac{2\pi n c}{U} > 0.5$$: unsteady effects likely to be significant

n : wingbeat frequency

c : wing chord

(2) Womersley : Wo. - pulsating flow in circulatory systems

$$Wo = a\sqrt{\frac{2\pi n \rho}{\mu}}$$

a : pipe radius

n : frequency of sinusoidally applied pressure gradient.

$Wo < 1$: quasi-steady. parabolic profile

> 1 : inertia distorts the profile

$$Wo^2 = \frac{2\pi n \rho}{\mu} \frac{a^2 U}{U} = \left(\frac{\rho U a}{\mu}\right) \frac{2\pi n a}{U}$$

$$= \left(\frac{\rho U \cdot 2a}{\mu}\right)\left(\frac{n \cdot 2a}{U}\right) \cdot \frac{\pi}{2} = \frac{\pi}{2} Re_d \, St_d$$

$d = 2a$