Ch 3. The Quantum Theory of Light

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Light

하나님이 이르시되 빛이 있으라 하니 빛이 있었고

빛이 하나님이 보시기에 좋았더라 하나님이 빛과 어둠을 나누사

(창세기 1:3-4)

And God said, “Let there be light,” and there was light.

God saw that the light was good, and he separated the light from the darkness.

(Genesis 1:3-4)

Light

•We all *know* what light is; but it is not easy to *tell* what it is.

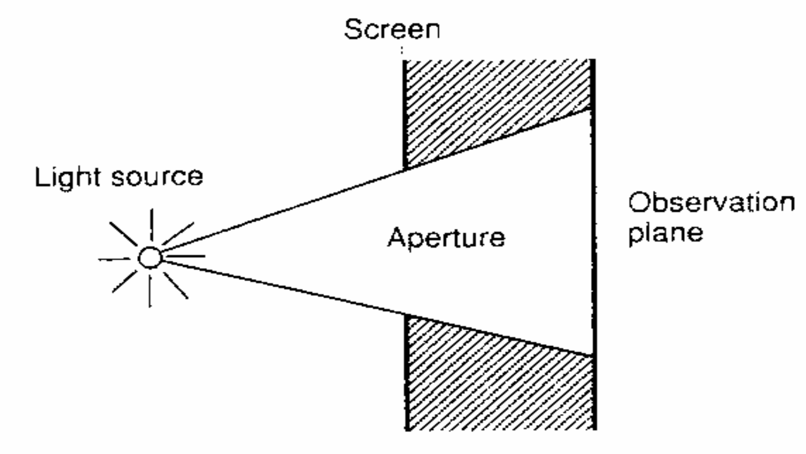
- Samuel Johnson

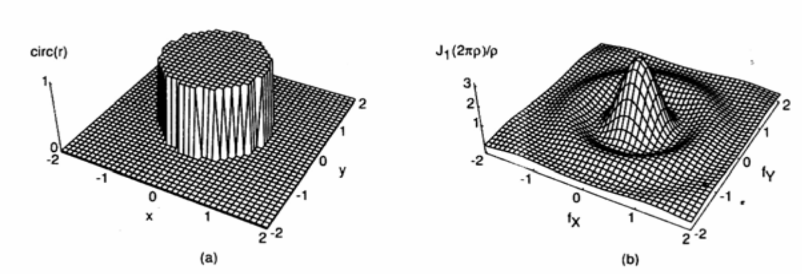
• 파동성(Wave : 전자파(Electromagnetic Wave)의 일종)

• 입자성(Particle : Photon; 光子)

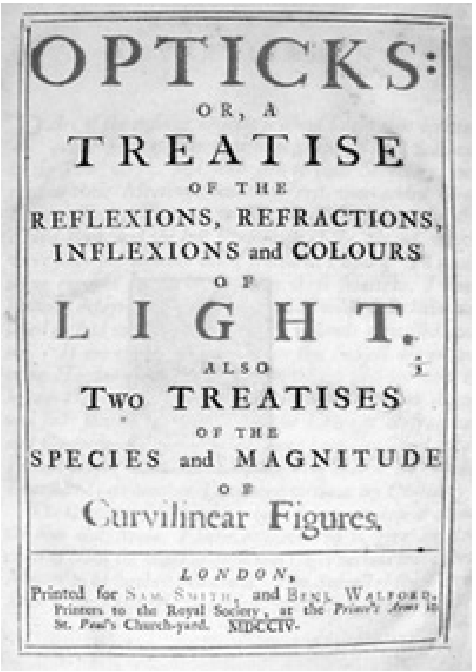
History of the Study on Light

- 1665 Grimaldi
- 1678 Huygens
- 1704 Newton
- 1804 Young
- 1818 Fresnel
- 1860 Maxwell
- 1905 Einstein





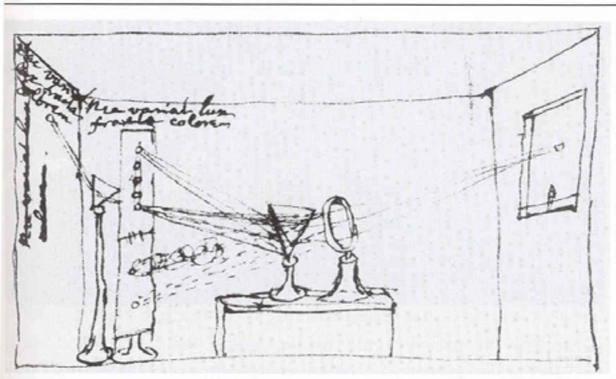
Newton


OPTICKS:
OR, A
TREATISE
OF THE
REFLEXIONS, REFRACTIONS,
INFLEXIONS and COLOURS
OF
LIGHT.
ALSO
Two TREATISES
OF THE
SPECIES and MAGNITUDE
OF
Curvilinear Figures.

LONDON,
Printed for Sam. Smith, and Benj. Walford,
Printers to the Royal Society, at the Prince's Arms in
St. Paul's Church-yard. MDCCIV.

Isaac Newton

(1642-1727)

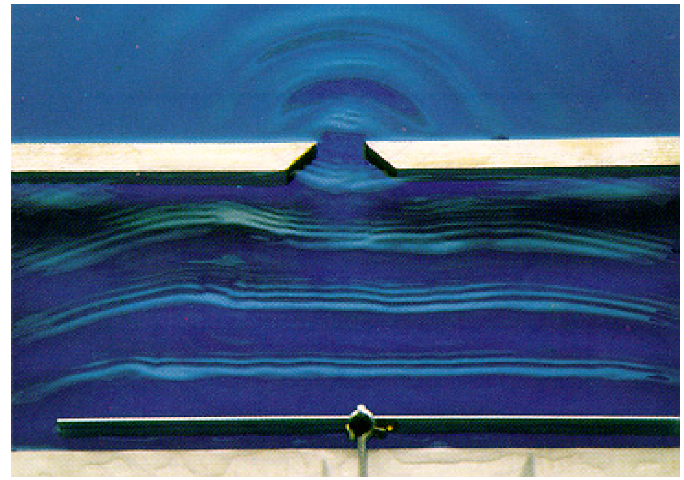


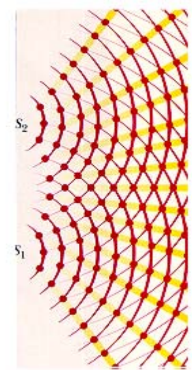
A sketch (left) from Newton's 1672 notebook shows sunlight entering through the window at right, passing through a triangular prism, and splitting into a spectrum of colors. One of the earliest known studies of optics (the science of light and vision) was done by Islamic mathematician Ibn al-Haytham (965–1040), also known as Alhazen. His sketch of lenses is below.

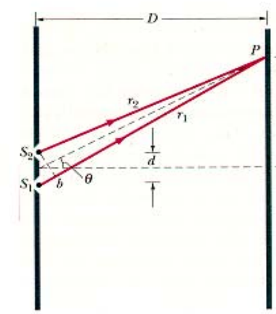
J. Hakim, The Story of Science – Newton at the Center, Smithsonian Books, Washington DC, USA, 2005

Young

Thomas Young (1773-1829)







Electromagnetic Wave

James Clerk Maxwell (1831–1879)

Heinrich Hertz (1857–1894)

Tragically, Hertz was as unfortunate with his health as he was fortunate with his talent. The first sign of trouble was a series of toothaches in 1888, which led to removal of all his teeth in 1889. By 1892, he was suffering from pains in his nose and throat, and was often depressed. His doctors could give him no satisfactory diagnosis. Several operations failed to provide permanent relief. By December 1893, he knew he would not recover, and in a letter he asked his parents *"not to mourn... rather you must be a little proud and consider that I am among the especially elect destined to live for only a short while and yet to live enough. I did not choose this fate, but since it has overtaken me, I must be content; and if the choice had been left to me, perhaps I should have chosen it myself."*

Hertz died of blood poisoning on New Year's Day, 1894; he was thirty-six years old.

--- From 'Great Physicists' by W. H. Cropper

Electromagnetic Wave

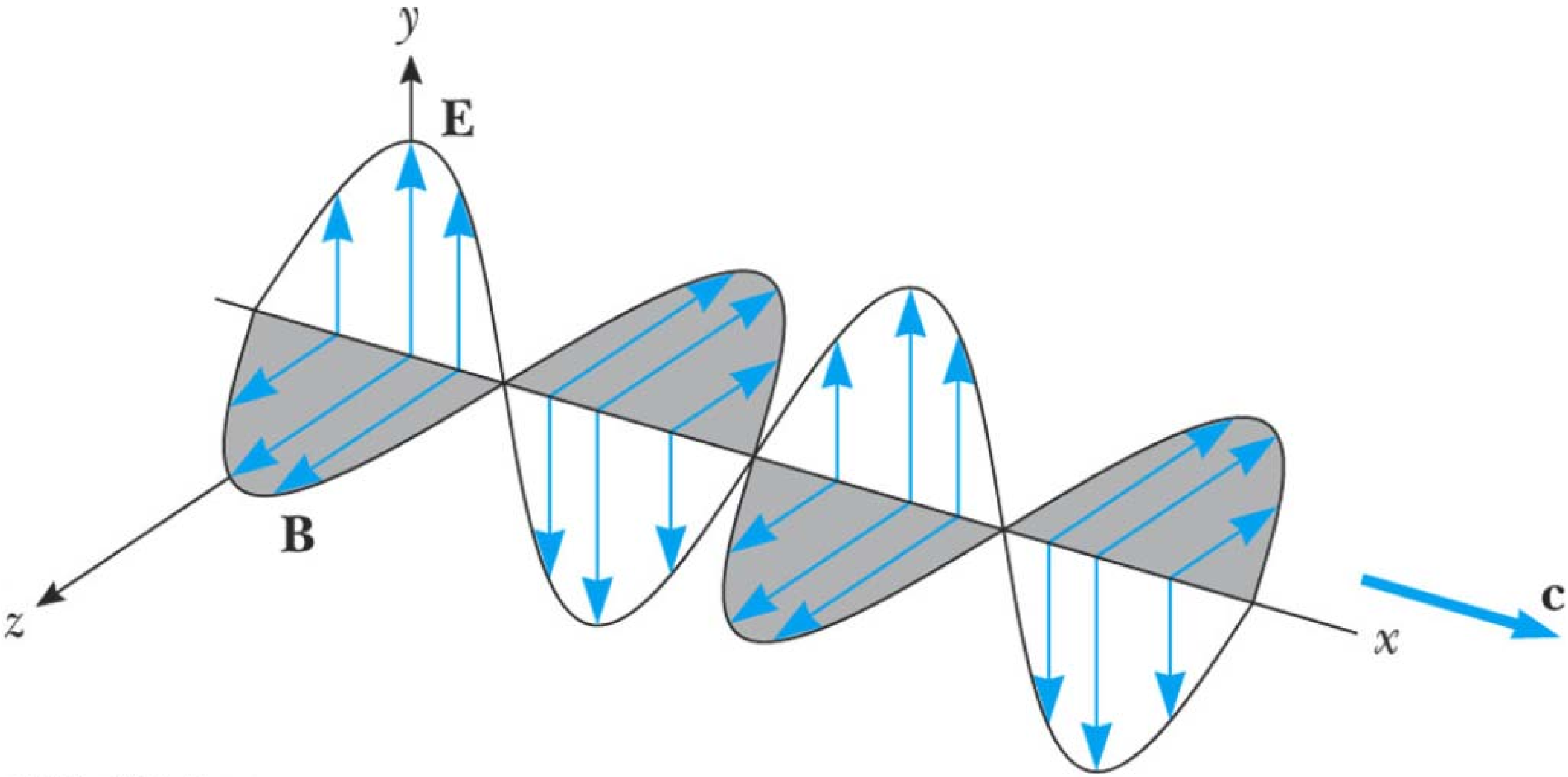


Fig. 3-2, p. 67

파장과 주파수

주기 T

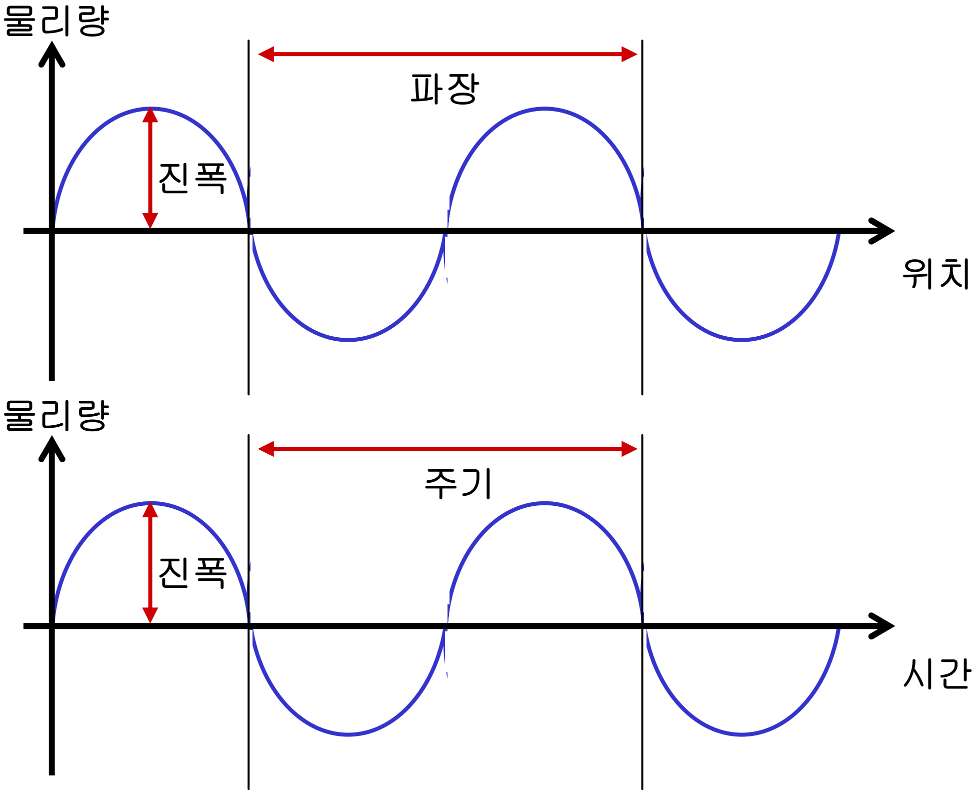
주파수 f

파장 λ

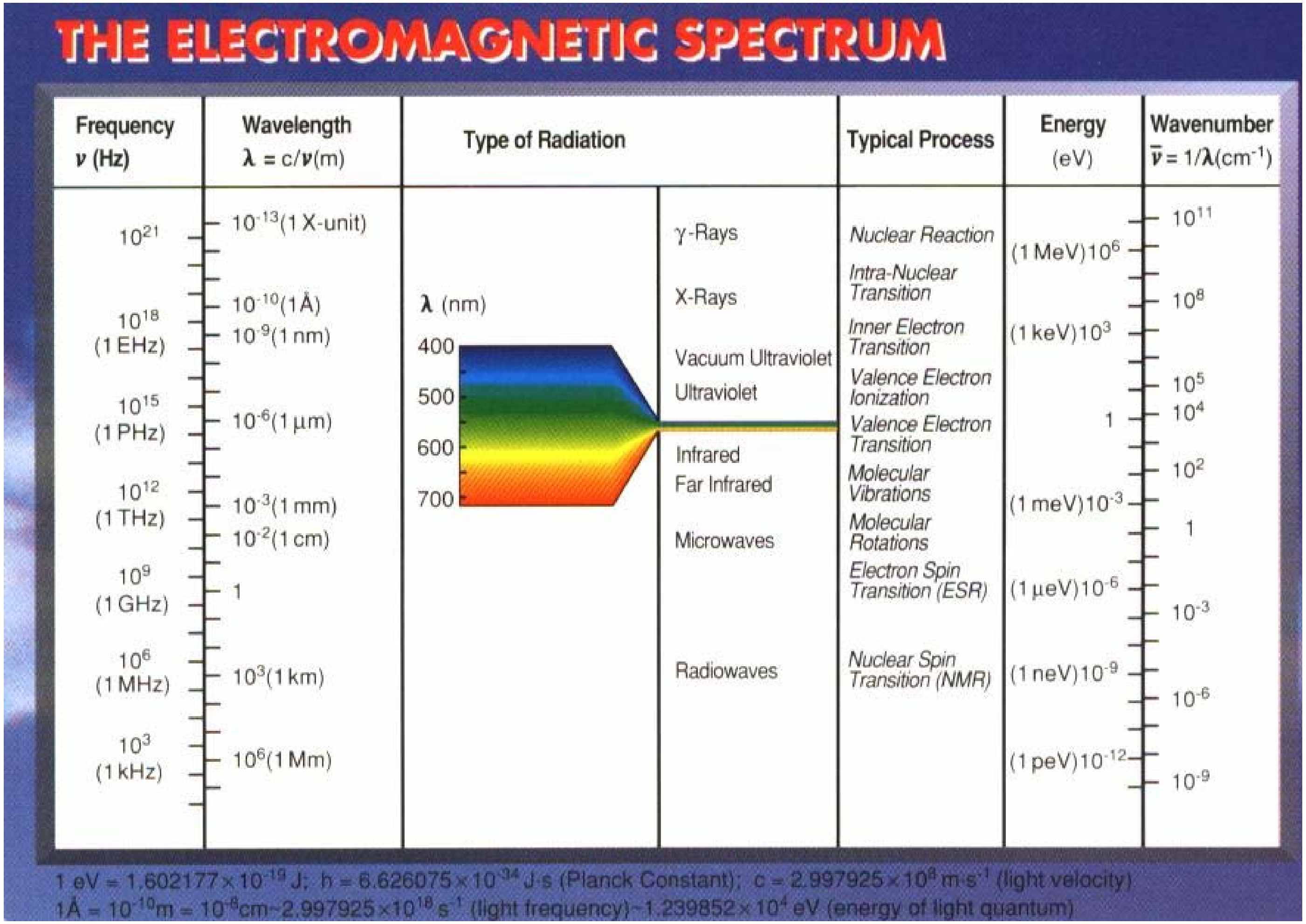
진공에서의 빛의 속력 c

$$f = \frac{1}{T}$$

$$c = \frac{\lambda}{T} = \lambda f$$

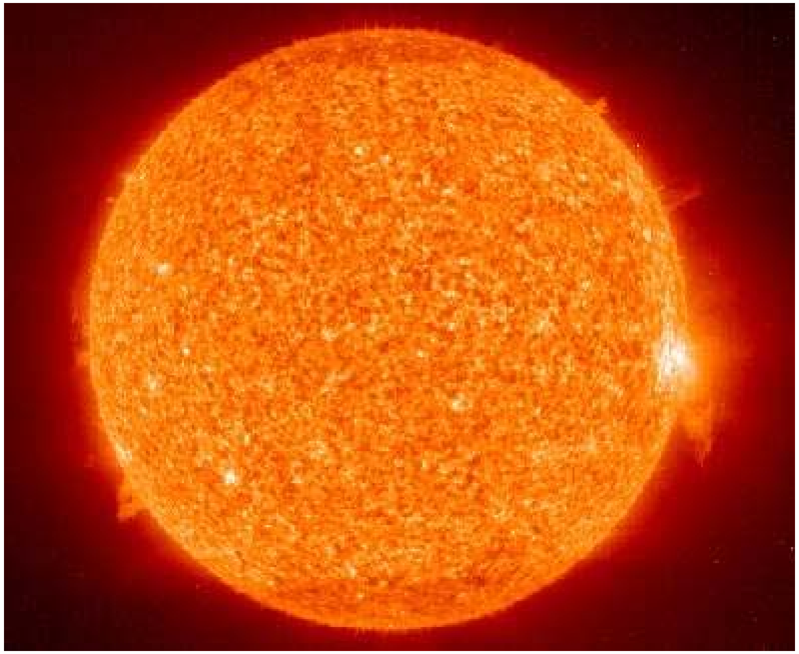


전자파의 스펙트럼



THE ELECTROMAGNETIC SPECTRUM
Frequency ν (Hz)
Wavelength λ = c/ν(m)
Type of Radiation
Typical Process
Energy (eV)
Wavenumber ν̄ = 1/λ(cm⁻¹)
10^21
10^18 (1 EHz)
10^15 (1 PHz)
10^12 (1 THz)
10^9 (1 GHz)
10^6 (1 MHz)
10^3 (1 kHz)
10^-13 (1 X-unit)
10^-10 (1 Å)
10^-9 (1 nm)
10^-6 (1 μm)
10^-3 (1 mm)
10^-2 (1 cm)
1
10^3 (1 km)
10^6 (1 Mm)
λ (nm)
400
500
600
700
γ-Rays
X-Rays
Vacuum Ultraviolet
Ultraviolet
Infrared
Far Infrared
Microwaves
Radiowaves
Nuclear Reaction
Intra-Nuclear Transition
Inner Electron Transition
Valence Electron Ionization
Valence Electron Transition
Molecular Vibrations
Molecular Rotations
Electron Spin Transition (ESR)
Nuclear Spin Transition (NMR)
(1 MeV) 10^6
(1 keV) 10^3
1
(1 meV) 10^-3
(1 μeV) 10^-6
(1 neV) 10^-9
(1 peV) 10^-12
10^11
10^8
10^5
10^4
10^2
1
10^-3
10^-6
10^-9
1 eV = 1.602177 × 10^-19 J; h = 6.626075 × 10^-34 J·s (Planck Constant); c = 2.997925 × 10^8 m·s^-1 (light velocity)
1Å = 10^-10 m = 10^-8 cm ~ 2.997925 × 10^18 s^-1 (light frequency) ~ 1.239852 × 10^4 eV (energy of light quantum)

Thermal Radiation



Blackbody

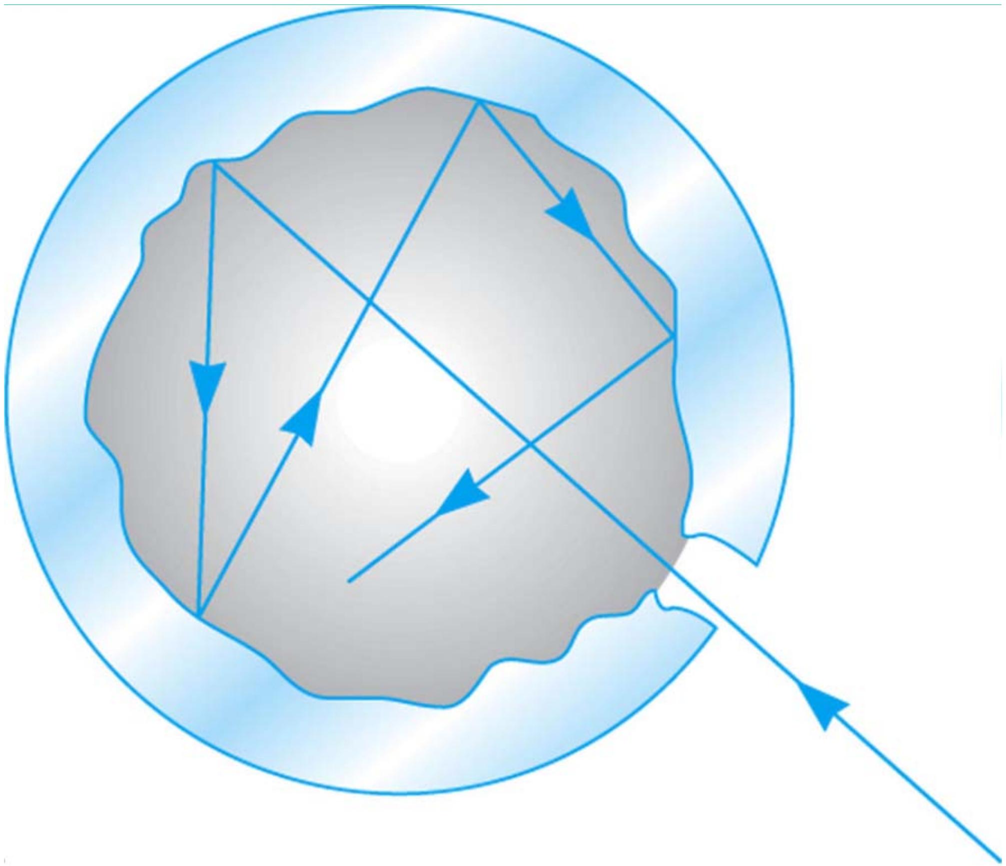


Fig. 3-4, p. 69

Blackbody Radiation

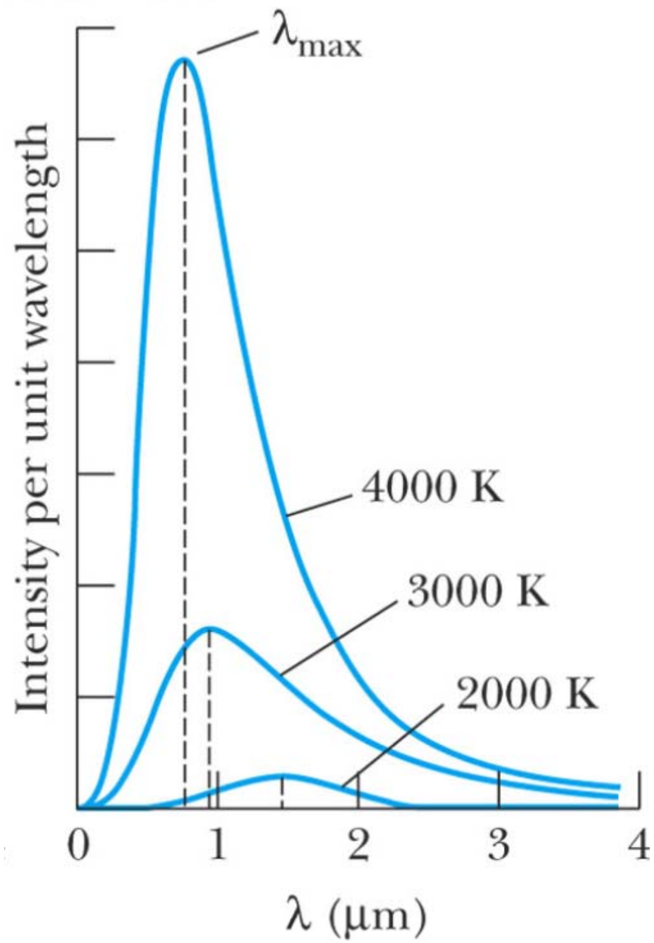
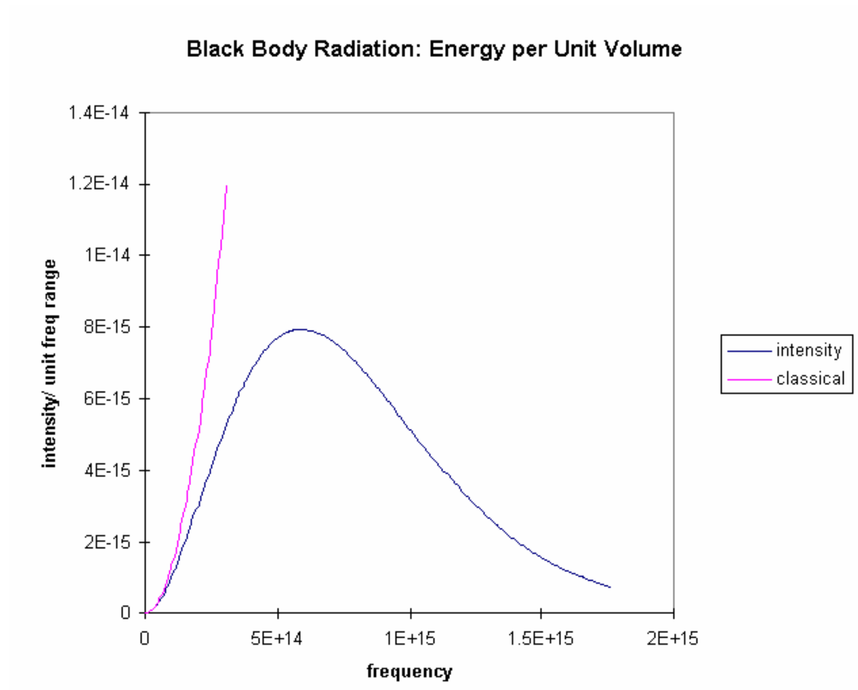


Fig. 3-3, p. 68



Harmonic Oscillator

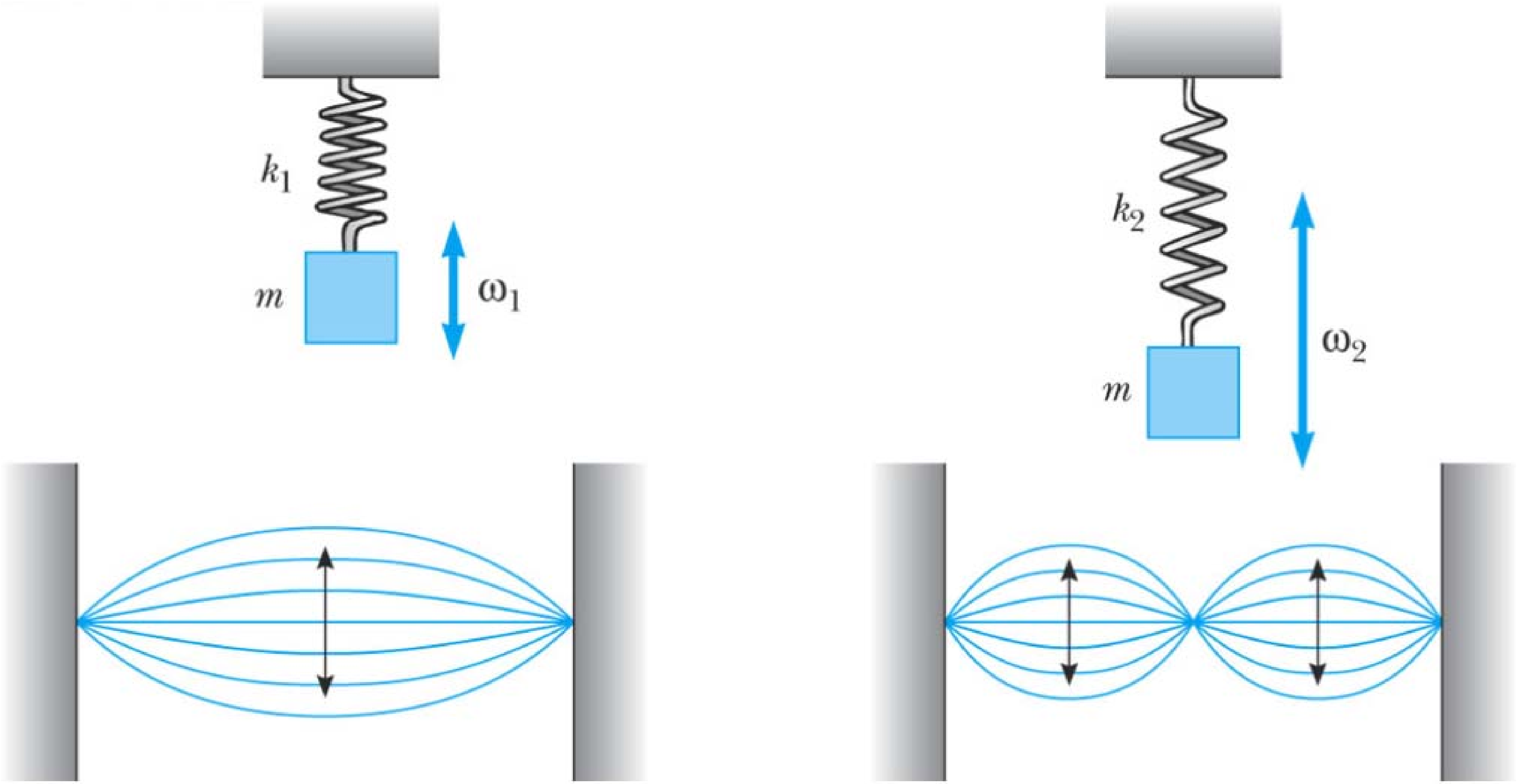


Fig. 3-11, p. 78

Cavity Modes I

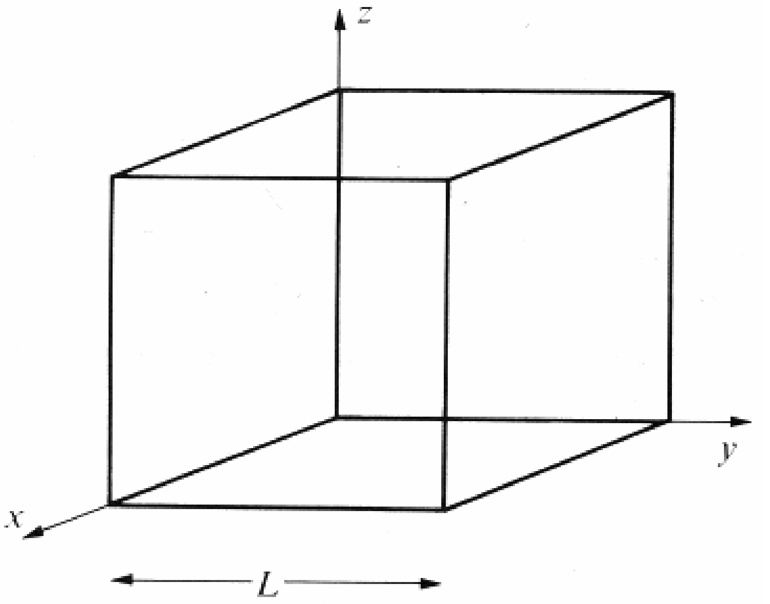
$$\nabla^2 \mathbf{E}(\mathbf{r},t) = \frac{1}{c^2}\frac{\partial^2 \mathbf{E}(\mathbf{r},t)}{\partial t^2}$$

$$\nabla \cdot \mathbf{E}(\mathbf{r},t) = 0 \qquad k = \frac{2\pi}{\lambda} = \frac{2\pi f}{c} = \frac{\omega}{c}$$

$$E_x(\mathbf{r},t) = E_x(t)\cos(k_x x)\sin(k_y y)\sin(k_z z)$$

$$E_y(\mathbf{r},t) = E_y(t)\sin(k_x x)\cos(k_y y)\sin(k_z z)$$

$$E_z(\mathbf{r},t) = E_z(t)\sin(k_x x)\sin(k_y y)\cos(k_z z)$$

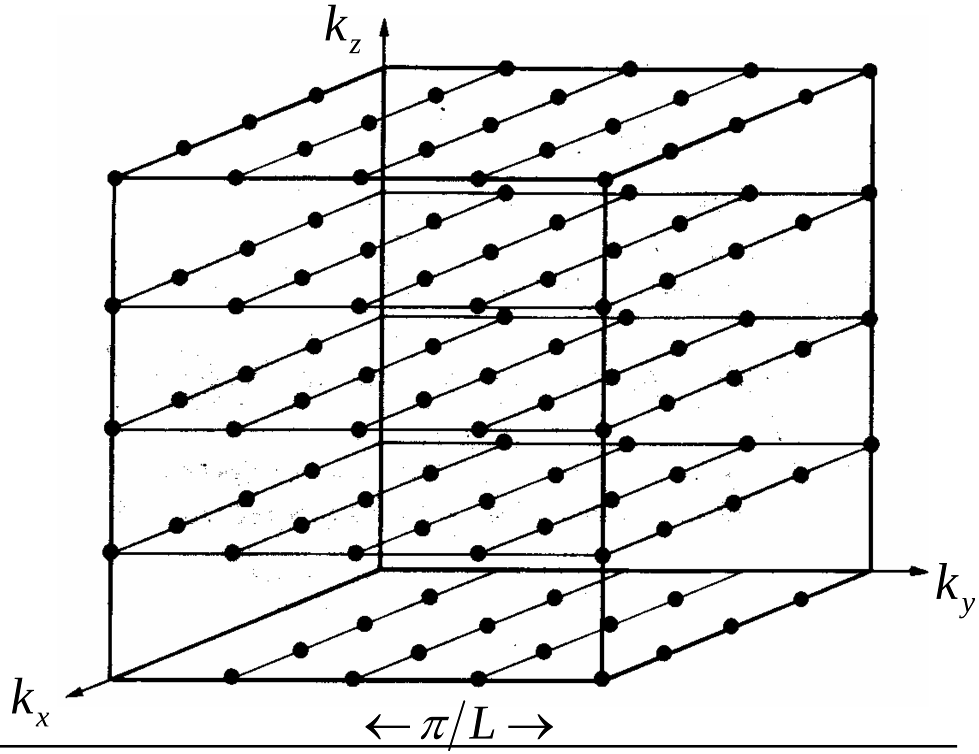


Cavity Modes II

$$k_x = \pi n_x / L, \quad k_y = \pi n_y / L, \quad k_z = \pi n_z / L$$

$$n_x, n_y, n_z = 0, 1, 2, 3, \ldots$$

$$\mathbf{k} \cdot \mathbf{E}(t) = 0$$



Mode Density I

$$\frac{\frac{1}{8}\left(4\pi k^2 dk\right)}{\left(\pi / L\right)^3} \times 2$$

Mode density $N(k)$

Number of modes between k and $k+dk$ (per unit volune) $= N(k)dk$

$$N\left(k\right)dk = \frac{k^2}{\pi^2}dk \qquad N(k) = \frac{k^2}{\pi^2}$$

Many books use the notation of $\rho(k)$ rather than $N(k)$.

Mode Density II

Mode density $N(f)$

Number of modes between f and $f + df$ (per unit volune) $= N(f)df$

$$f = \frac{ck}{2\pi}$$

$$N(f)df = N(k)dk$$

$$N(f) = N(k)\frac{dk}{df} = \frac{k^2}{\pi^2}\frac{2\pi}{c} = \frac{8\pi f^2}{c^3}$$

Spectral Energy Density

Spectral energy density $u(f,T)$

Energy of radiation fields between f and $f+df$ (per unit volune) $=u(f,T)df$

$$u(f,T)df = \bar{E}N(f)df$$

Rayleigh-Jeans Law

$$\bar{E} = \frac{\int_0^\infty E e^{-E/(k_B T)} dE}{\int_0^\infty e^{-E/(k_B T)} dE} = k_B T$$

(By Boltzmann Distribution
or by the equipartition principle)

$$u(f,T)df = \bar{E}N(f)df = \frac{8\pi^2 f^2}{c^3} k_B T df$$

$$u(\lambda,T)d\lambda = \bar{E}N(f)df = \frac{8\pi}{\lambda^4} k_B T d\lambda$$

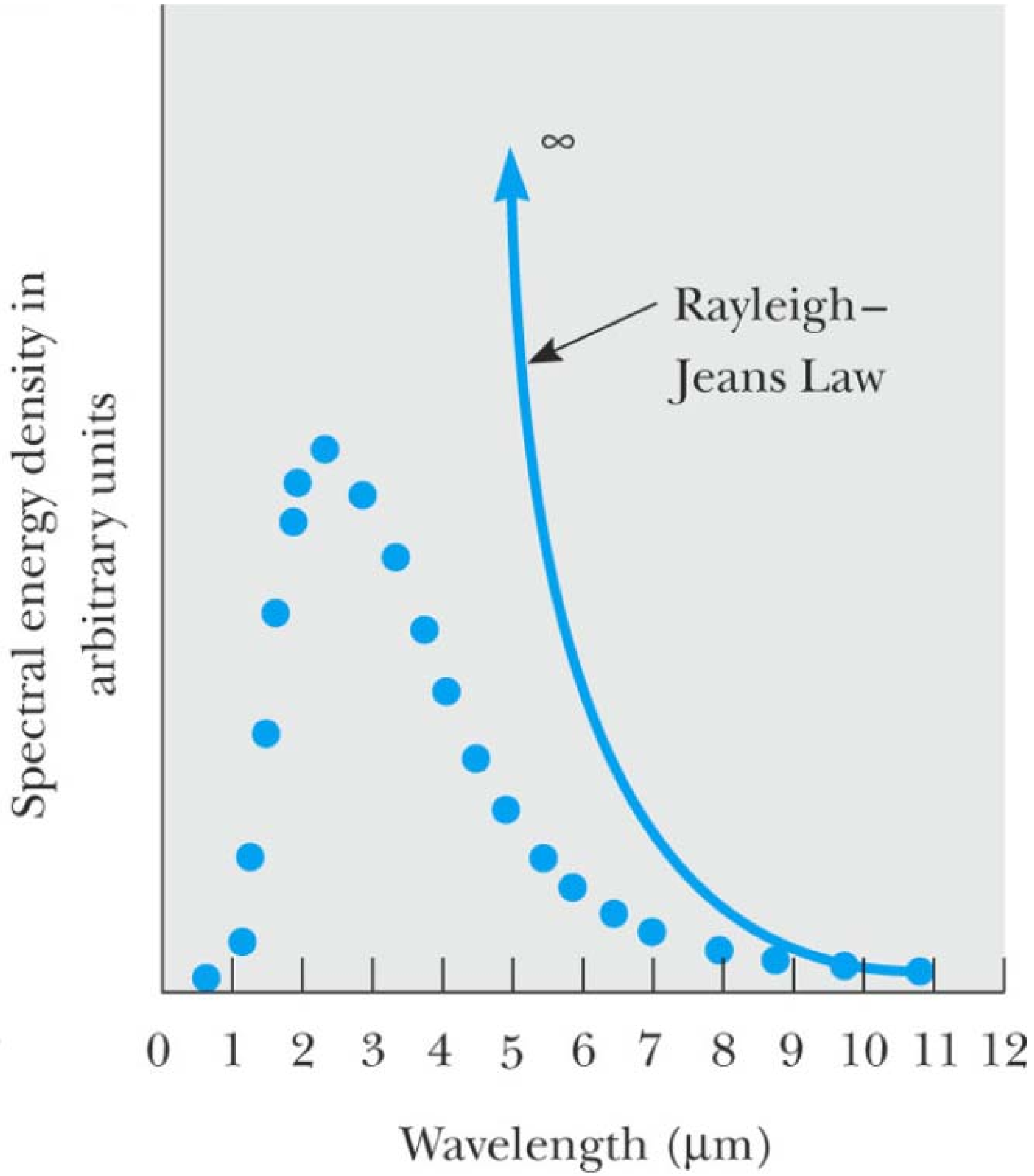

∞
Rayleigh–
Jeans Law
Spectral energy density in
arbitrary units
0 1 2 3 4 5 6 7 8 9 10 11 12
Wavelength (μm)

Fig. 3-12, p. 79