2012년 1학기 창의적 선박설계 강의자료
(Innovative Ship Design Lecture Note)

서울대학교 조선해양공학과

선박설계자동화연구실

이규열

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Naval Architecture & Ocean Engineering

Ship Design

Spring 2012

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본 강의의 평가 지표

☑ 이 교과목의 수강이 **필요한가?**

☑ 이 교과목으로부터 어떤 **성과(이익)을 얻을 것인가?**

☑ 이 교과목의 내용을 **어떻게 활용할 것인가?**

➡ "무엇을 알고 있느냐"가 아니고
"알고 있는 것으로 무엇을 할 수 있느냐"가 중요

2

본 강의의 담당 강사(Instructor)의 임무

☑ 강사가 가르친 내용
- ➡ 수강생 이해
- ➡ 수강생이 강사가 되어 가르칠 때
- ➡ 다른 사람이 이해되어야!!!

본 강의의 진행 방향

☑ 확실한 이해 중심의 강의

"들은 것은 잊어버리고,
본 것은 기억만 되나
직접 해 본 것은 이해된다"
- 공자 -

본 강의의 진행 방향

☑ Capstone Design 중심의 강의

- **Capstone Design팀 구성**
- **Capstone Design 목표로 강의 내용 구성**
- **A capstone design that integrates each functional area of the naval architecture and emphasizes the role of strategic planning**
- **The capstone design in this course is the Design Project in which each team performs a ship design, complete in all major respects.**

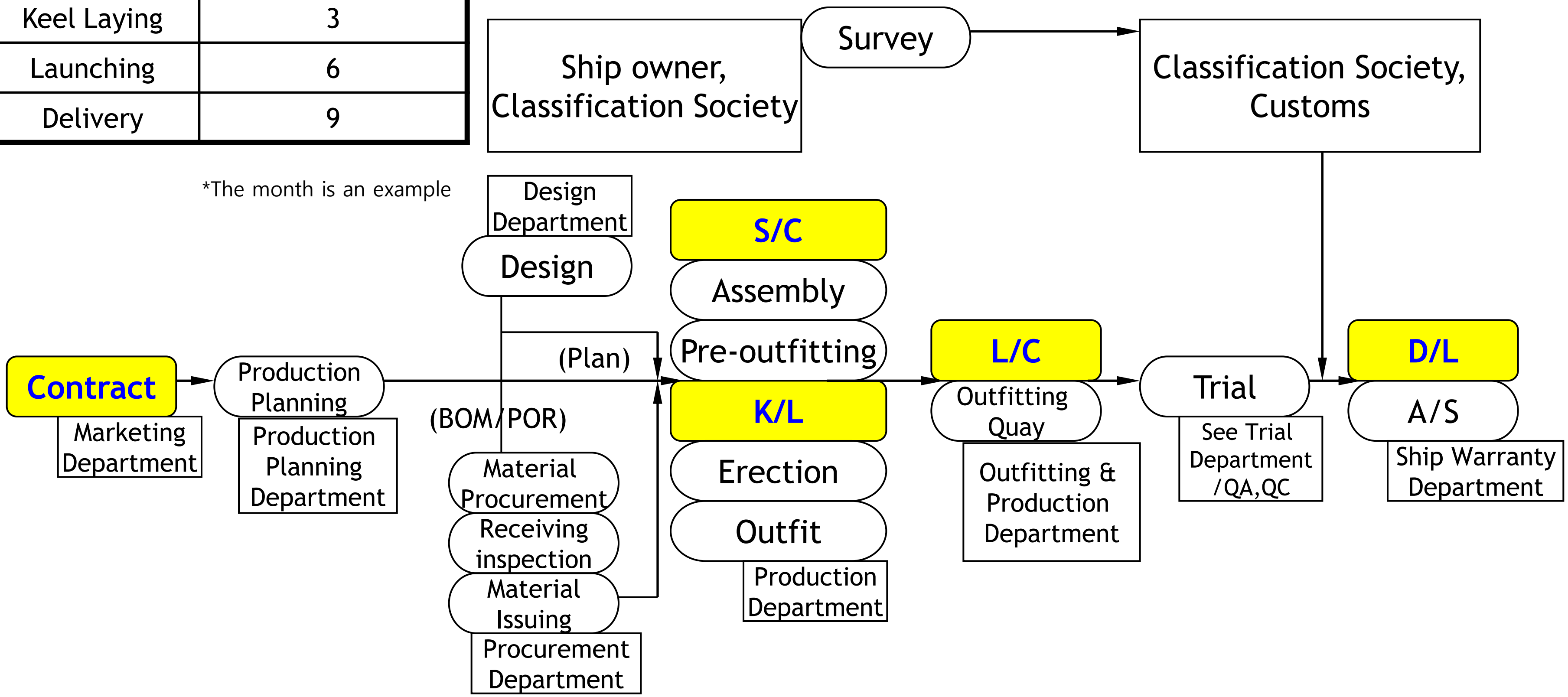
Chapter 1. Shipbuilding Schedule

Main Events of Shipbuilding

Event	Month (From Steel Cutting)
Contract	-12
Steel Cutting	0
Keel Laying	3
Launching	6
Delivery	9

*The month is an example

Shipyards offer financing schemes in which the ship owner pays at the main events.



•S/C: Steel Cutting, K/L: Keel Laying, L/C: Launching, D/L: Delivery
* BOM: Bill Of Material, POR: Purchase Order Request, QA: Quality Assurance, QC: Quality Control
•A/S: After Sales Service

Main Events of Shipbuilding

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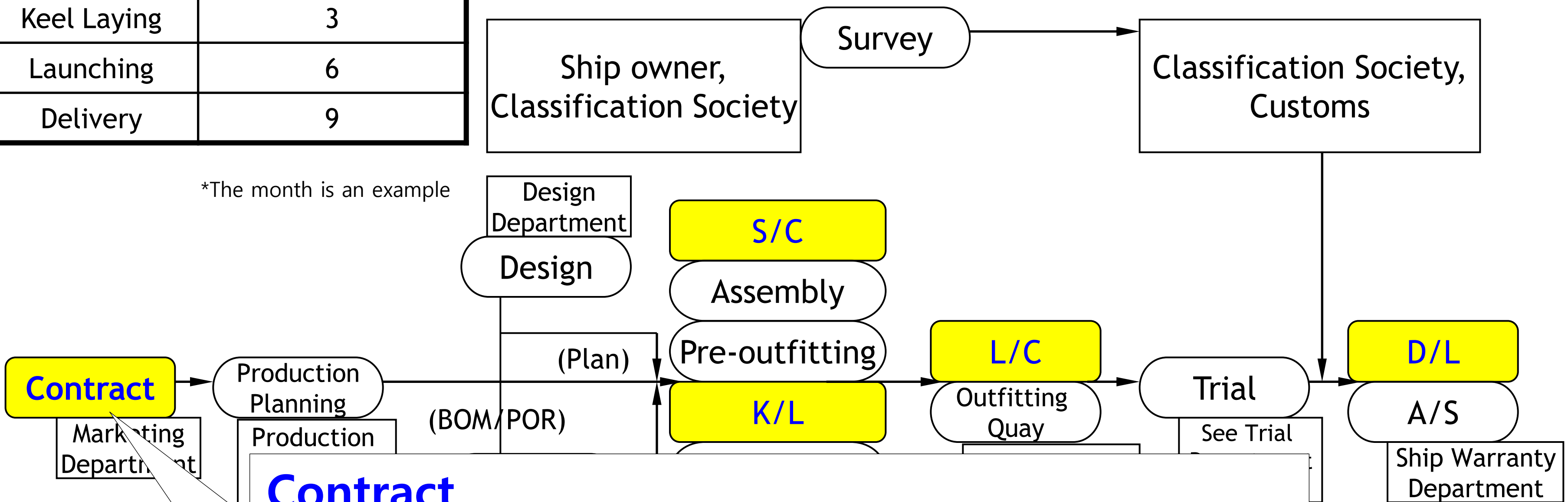

Contract

: Contract is the signing the agreement between shipyard and ship owner for the actual construction of the ship.

Prior to the signing the contract, the shipping company, financer and future owners have completed a long road of negotiations and considerations.

•S/C: Steel Cutting,
* BOM: Bill Of Materi
•A/S: After Sales Ser
ntrol

Main Events of Shipbuilding

Event	Month (From Steel Cutting)
Contract	-12
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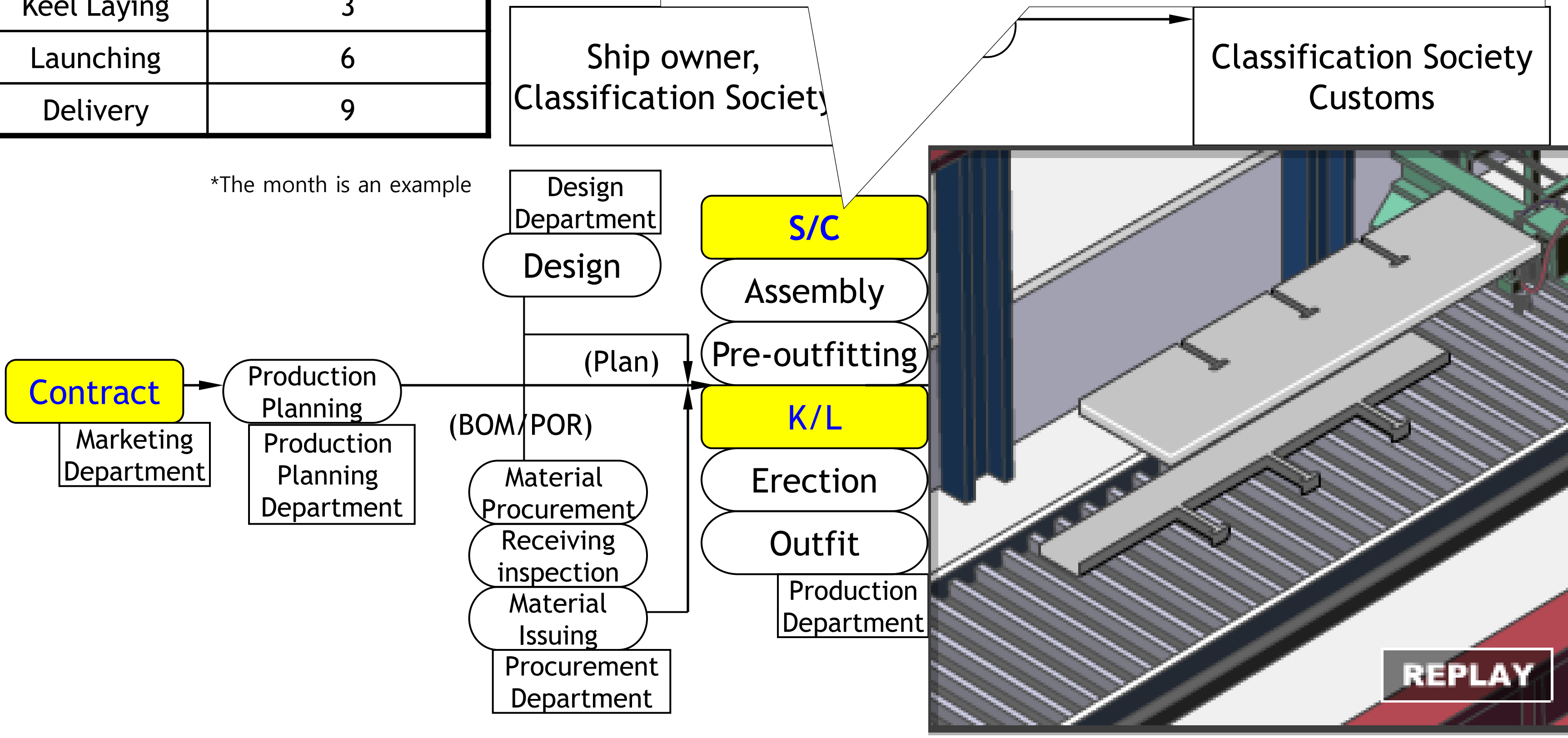
*The month is an example

Steel Cutting

: Steel Cutting is the process that steel plates and stiffeners are cut according to the manufacturing plans.



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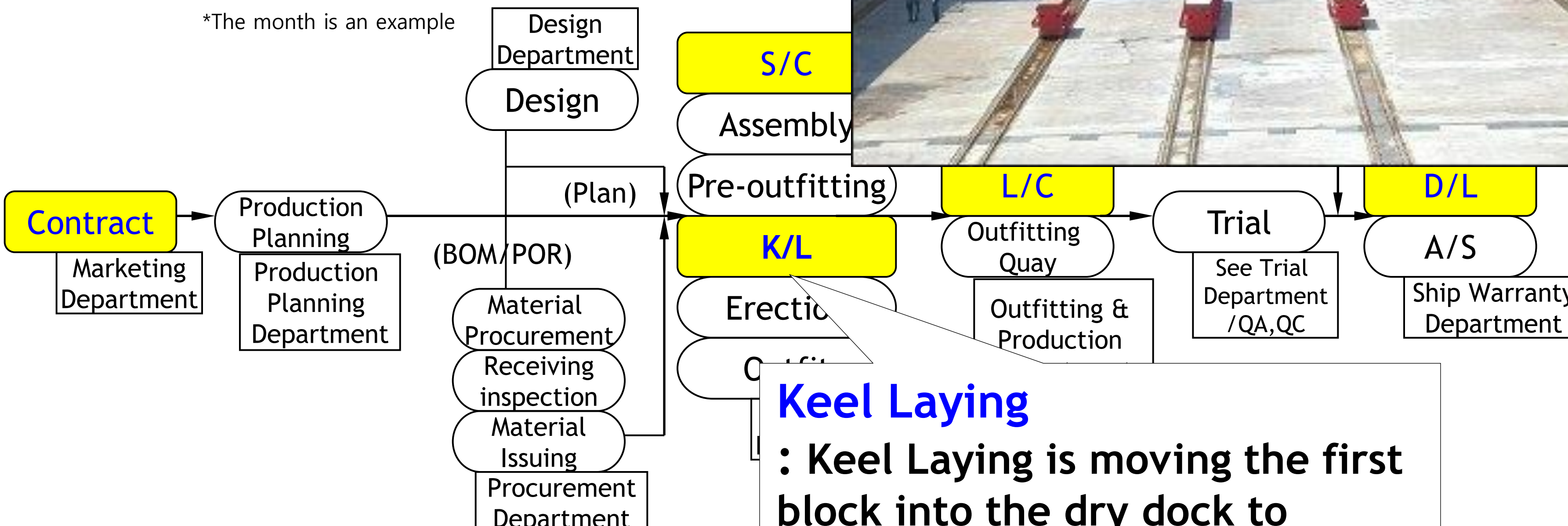

Keel Laying

: Keel Laying is moving the first block into the dry dock to assemble the whole ship.

•S/C: Steel Cutting, K/L: Keel Laying, L/C: Launching,

* BOM: Bill Of Material, POR: Purchase Order Request, QA: Quality Assurance, QC: Quality Control

•A/S: After Sales Service

Main Events of Shipbuilding

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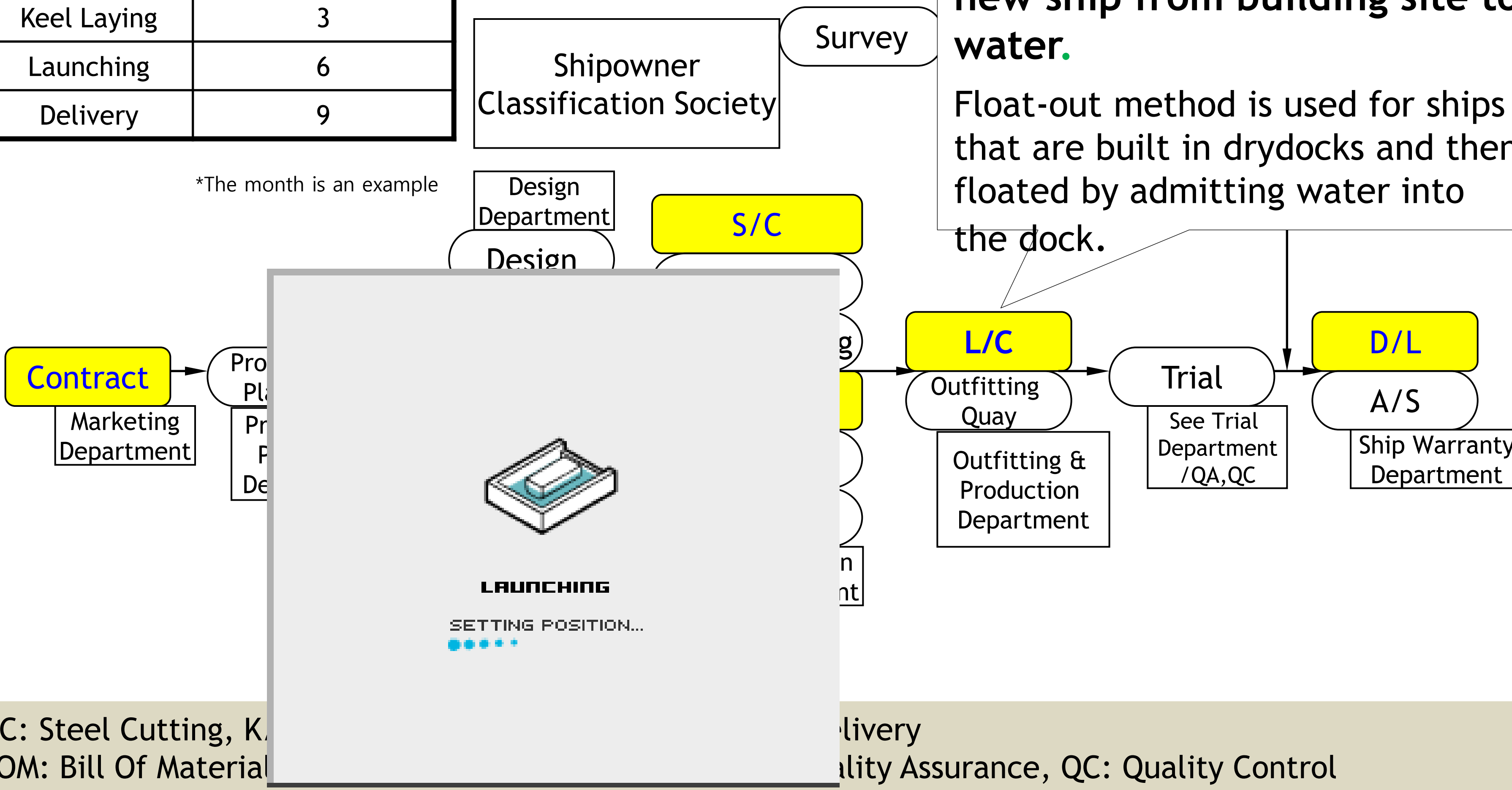
Delivery	9

*The month is an example

Launching(L/C)
: Launching is conveying(transporting) a new ship from building site to water.

Float-out method is used for ships that are built in drydocks and then floated by admitting water into the dock.



•S/C: Steel Cutting, K... ...livery
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Main Events of Shipbuilding

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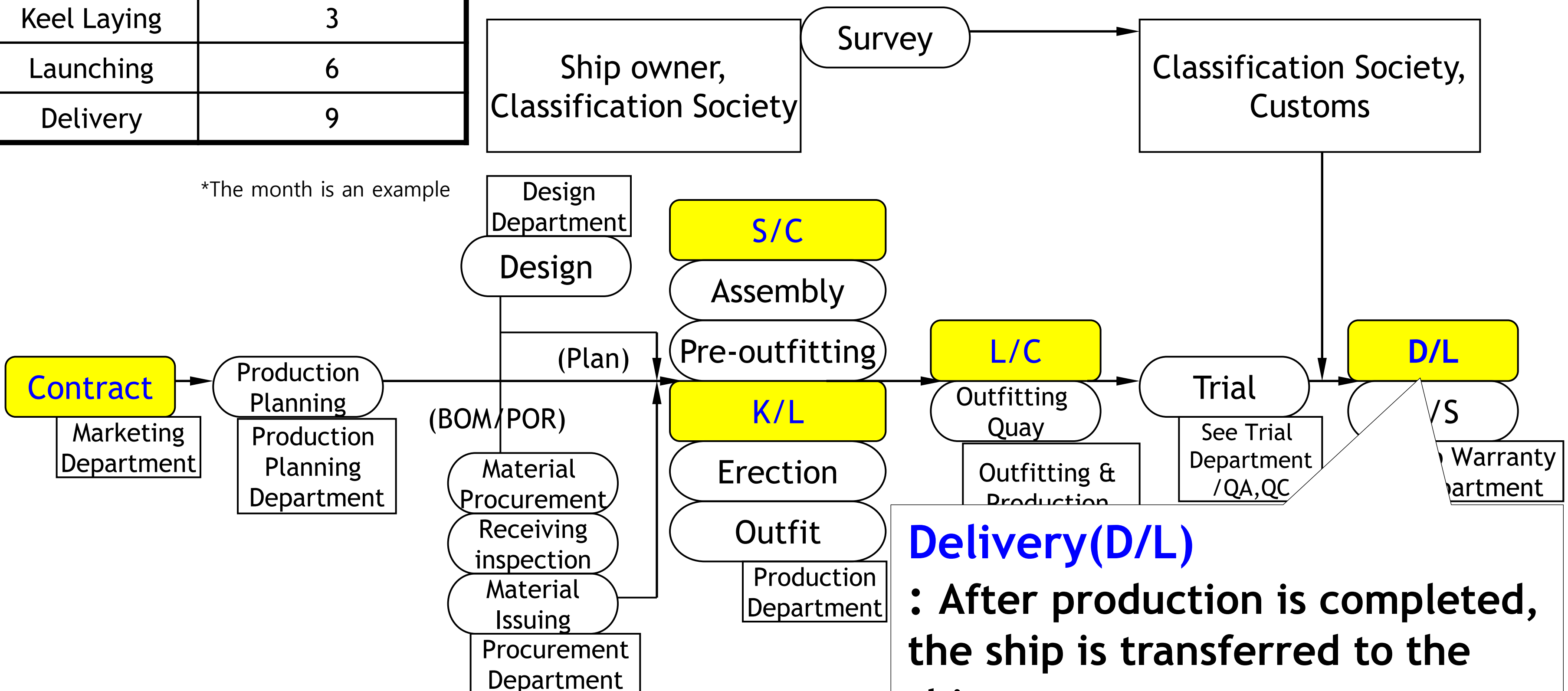


Delivery(D/L)

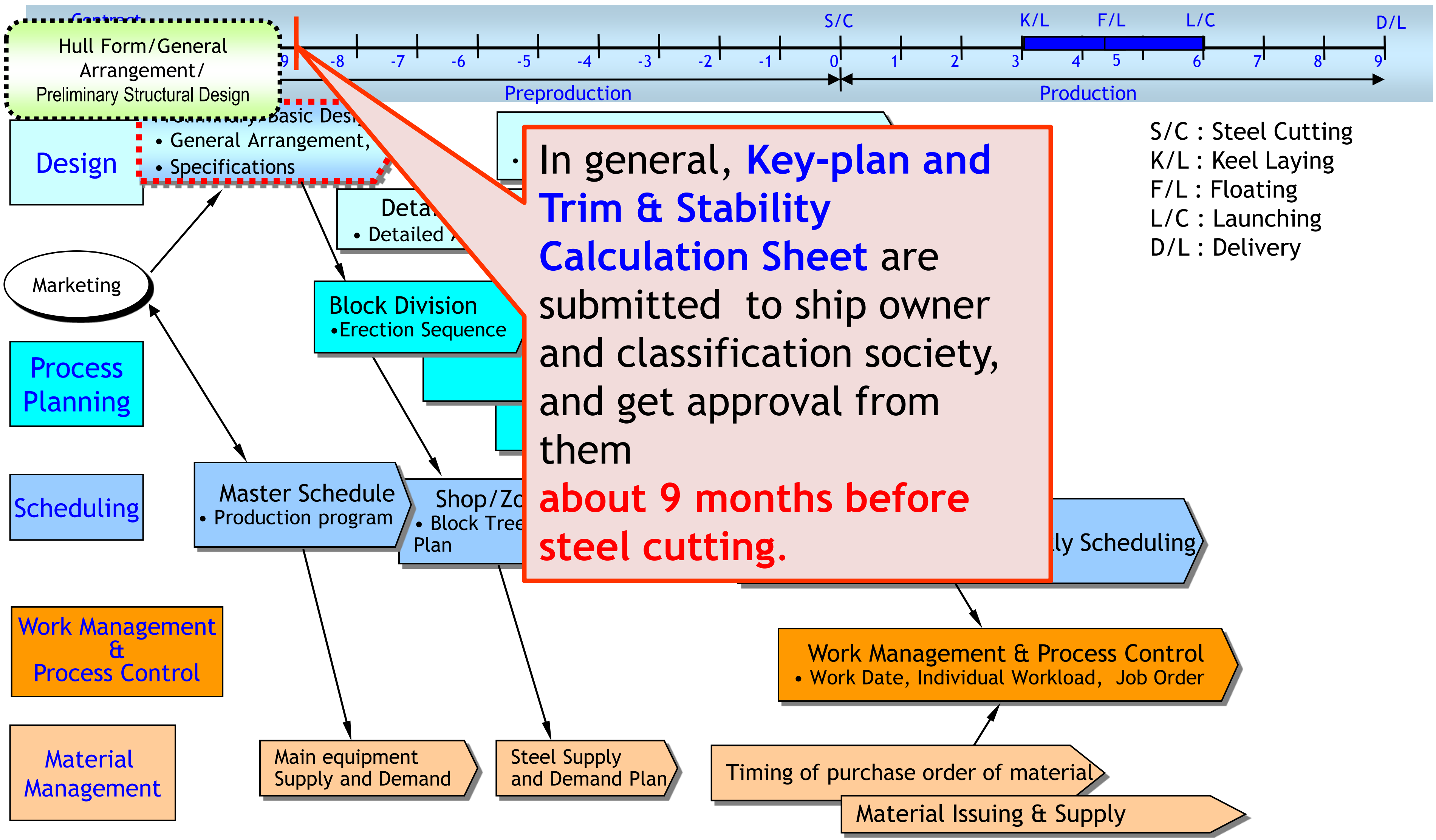
: After production is completed, the ship is transferred to the ship owner.

•S/C: Steel Cutting, K/L: Keel Laying, L/C: Launching, D/L: Delivery

* BOM: Bill Of Material, POR: Purchase Order Request, QA: Quality Assurance, QC: Quality Control

•A/S: After Sales Service

Shipbuilding Workflow



Chapter 2. Design Equations (Design Constraints)

2-1 Owner's Requirements

Owner's Requirements

☑ **Owner's Requirements**

- **Ship's Type**
- **Deadweight(DWT)**

- **Cargo Hold Capacity(V_{CH})**
  - Cargo Capacity: Cargo Hold Volume / Containers in Hold & on Deck / Car Deck Area.
  - Water Ballast Capacity.

- **Service Speed (V_s)**
  - Service Speed at Design Draft with Sea Margin, MCR/NCR Engine Power & RPM.

- **Dimensional Limitations : Panama canal, Suez canal, Strait of Malacca, St. Lawrence Seaway, Port limitations.**

- **Maximum Draft(T_{max})**

- **Daily Fuel Oil Consumption(DFOC) : Related with ship's economy.**

- **Special Requirements**
  - Ice Class, Air Draft, Bow/Stern Thruster, Special Rudder, Twin Skeg.

- **Delivery Day**
  - Delivery day, with (   )$ penalty per delayed day.
  - Abt. 21 months from contract.

- **The Price of a ship**
  - Material & Equipment Cost + Construction Cost + Additional Cost + Margin.

Principal Particulars of a Basis Ship

At early design stage, there are few data available to determine the principal particulars of the design ship. Therefore, initial values of the principal particulars can be estimated from the basis ship (called also as 'parent ship' or 'mother ship'), whose main dimensional ratios and hull form coefficients are similar with the ship being designed.

The principal particulars include main dimensions, hull form coefficients, speed and engine power, DFOC, capacity, cruising range, crew, class, etc.

Example) VLCC(Very Large Crude Carrier)

308.000DWT

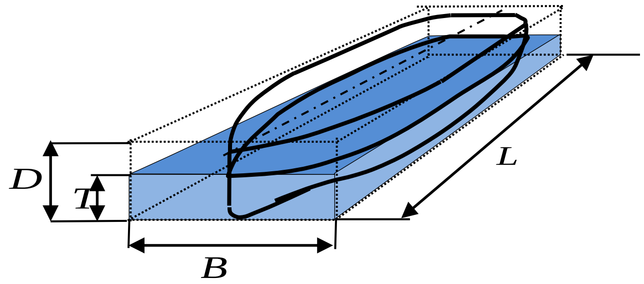
318.000DWT

Principal Dimensions & Hull Form Coefficients

The principal dimensions and hull form coefficients decide many characteristics of a ship, e.g. stability, cargo hold capacity, resistance, propulsion, power requirements, and economic efficiency.

Therefore, the determination of the principal dimensions and hull form coefficients is most important in the ship design.

The length L, breadth B, depth D, immersed depth(draft) T, and block coefficient C_B should be determined first.



2-2 Design Constraints

Design Constraints

In the ship design, the principal dimensions cannot be determined arbitrarily; rather, they have to satisfy following design constraints:

1) Physical constraint

- **Floatability : Hydrostatic equilibrium -> "Weight Equation"**

2) Economical constraints

- **Owner's requirements**

Ship's type, Deadweight(DWT)[ton],

Cargo hold capacity (V_{CH})[m^3] -> **"Volume Equation"**

Service speed (V_S)[$knots$] -> **Daily fuel oil consumption(DFOC)**[ton/day])

Maximum draft(T_{max})[m],

Limitations of main dimensions(Canals, Sea way, Strait, Port limitations :e.g. Panama canal, Suez canal, St. Lawrence Seaway, Strait of Malacca,

Endurance[$n.m$[1)]],

3) Regulatory constraints

International **M**aritime **O**rganization [**IMO**] regulations,
International Convention for the **S**afety **O**f **L**ife **A**t **Sea**[**SOLAS**],
International Convention for the Prevention of **Mar**in **Pol**lution from Ships[**MARPOL**],
International **C**onvention on **L**oad **L**ines[**ICLL**]
Rules and Regulations of **Classification** Societies

1) n.m = nautical mile
1 n.m = 1.852 km

2-3 Physical Constraints

Physical constraint

- **Physical constraint**

  - **Floatability**

    For a ship to float in sea water, the total weight of the ship(W) must be equal to the buoyant force(F_B) on the immersed body →Hydrostatic equilibrium :

$$F_B \overset{!}{=} W \quad \dots(1)$$

$$\boxed{W = LWT + DWT}$$

***Lightweight**(LWT) reflects the weight of vessel being ready to go to sea without cargo and loads. And lightweight can be composed of:
$LWT = \text{Structural weight} + \text{Outfit weight} + \text{Machinery weight}$

***Deadweight**(DWT) is the weight that a ship can load till the maximum allowable immersion(at the scantling draft(T_s)).
And deadweight can be composed of:
$DWT = \text{Payload} + \text{Fuel oil} + \text{Diesel oil} + \text{Fresh water} + \text{Ballast water} + \text{etc.}$

- **Physical constraint : hydrostatic equilibrium**

$$F_B = W \quad ...(1)$$

$$W = LWT + DWT$$

∇ **: the immersed volume of the ship.**

ρ : density of sea water = 1.025 Mg/m^3

(L.H.S) **What is the buoyant force(F_B)?**

According to the Archimedes' principle,

the buoyant force on an immersed body has the same magnitude as the weight of the fluid displaced by the body.

$$\boxed{F_B = g \cdot \rho \cdot V}$$

Volume

Mass

Buoyant Force is the weight of the displaced fluid.

In shipbuilding and shipping society, those are called as follows :

- **Displacement volume** ∇
- **Displacement mass** Δ_m
- **Displacement** Δ

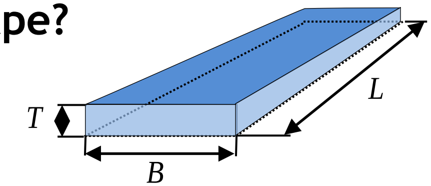
In shipbuilding and shipping society, **buoyant force** is called in another word, **displacement(△)**.

2-4 Weight Equation

Block coefficient(C_B)

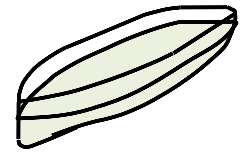
V : immersed volume
V_{box} : volume of box
L : length, B : breadth
T : draft

Does a ship or an airplane usually have box shape?



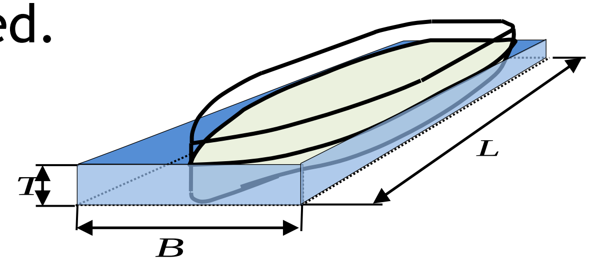
No.

They have a streamlined shape.



Why does a ship or an airplane has a streamlined shape?

They have a streamlined shape **to minimize the drag force** experienced when they travel, so that the propulsion engine needs a smaller power output to achieve the same speed.



Block coefficient(C_B) is **the ratio of the immersed volume to the box bounded by L, B, T.**

$$C_B \equiv \frac{V}{V_{box}} = \frac{V}{L \cdot B \cdot T}$$

Shell Appendage Allowance

$$C_B = \frac{V}{L \cdot B \cdot T}$$

V : immersed volume
V_{box} : volume of box
L : length, B : breadth
T : draft
C_B block coefficient

The immersed volume of the ship can be expressed by block coefficient.

$$V_{molded} = L \cdot B \cdot T \cdot C_B$$

In general, we have to consider the **displacement of shell plating and appendages such as propeller, rudder, shaft, etc.** additionally.
Thus, The total immersed volume of the ship can be expressed as following:

$$V_{total} = L \cdot B \cdot T \cdot C_B \cdot (1+\alpha)$$

Where the hull dimensions length L, beam B, and draft T are the **molded** dimensions of the immerged hull to the inside of the shell plating,

thus α is a fraction of the shell appendage allowance which adapts the **molded volume to the actual volume** by accounting for the volume of the shell plating and appendages (typically about 0.002~0.0025 for large vessels).

$$\Rightarrow \boxed{F_B = g \cdot \rho \cdot V_{total} = \rho \cdot g \cdot L \cdot B \cdot T \cdot C_B \cdot (1+\alpha)}$$

Design Equations
- Weight Equation

- **Physical constraint : hydrostatic equilibrium**

$$F_B = W \quad ...(1)$$

(R.H.S) $W = LWT + DWT$

(L.H.S) $F_B = \rho \cdot g \cdot L \cdot B \cdot T \cdot C_B \cdot (1+\alpha)$

ρ : density of sea water = 1.025 Mg/m^3
α : displacement of shell, stern and appendages
C_B : block coefficient
g : gravitational acceleration

$$\boxed{\rho \cdot g \cdot L \cdot B \cdot T \cdot C_B \cdot (1+\alpha) = LWT + DWT \; ...(2)}$$

The equation (2) describes the physical constraint to be satisfied in ship design.

Unit of the Lightweight and Deadweight

- **Physical constraint** : hydrostatic equilibrium

$$F_B = W \quad ...(1)$$

$$\boxed{\rho \cdot g \cdot L \cdot B \cdot T \cdot C_B \cdot (1+\alpha) = LWT + DWT \quad ...(2)}$$

What is the unit of the lightweight and deadweight ?

Weight vs Mass

Question: Are the “weight” and “mass” the same? 

Answer : No!

Mass is a measure of the amount of matter in an object.

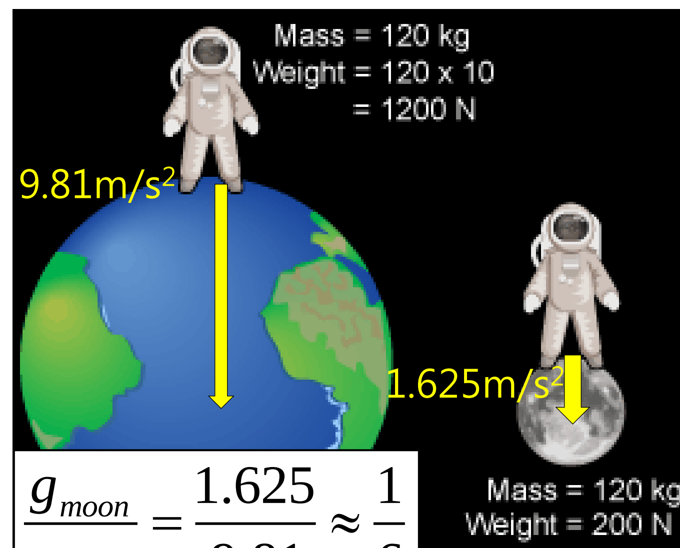
Weight is a measure of the force on the object caused by the universal gravitational force.

Gravity causes weight

Mass of an object does not change , but its weight can change.

For example, an astronaut’s weight on the moon is one-sixth of that on the Earth.

But the astronaut’s mass does not change.



$$\frac{g_{moon}}{g_{earth}} = \frac{1.625}{9.81} \approx \frac{1}{6}$$

g_{moon} : gravitational acceleration on the moon

g_{earth} : gravitational acceleration on the earth

Design Equations
- Mass Equation

- **Physical constraint** : hydrostatic equilibrium

$$F_B = W \quad \ldots(1)$$

In shipping and shipbuilding world, "ton" is used instead of "Mg (mega gram)" for the unit of the lightweight and deadweight in practice.

Actually, however, the weight equation is "mass equation".

$$\boxed{\rho \cdot L \cdot B \cdot T \cdot C_B \cdot (1+\alpha) = LWT + DWT \quad \ldots(3)}$$

"Mass equation"

where ρ := 1.025 Mg/m^3

2-5 Volume Equation

Economical constraints :Required Cargo Hold Capacity → Volume Equation

•Economical constraints

- Owner's requirements (Cargo hold capacity[m^3]**)**
- The main dimensions have to satisfy the required cargo hold capacity(V_{CH}**).**

$$\boxed{V_{CH} = f(L, B, D)}$$

: Volume equation of a ship

- It is checked whether the depth will allow the <u>required cargo hold capacity</u>.

2-6 Service Speed & DFOC
(Daily Fuel Oil Consumption)

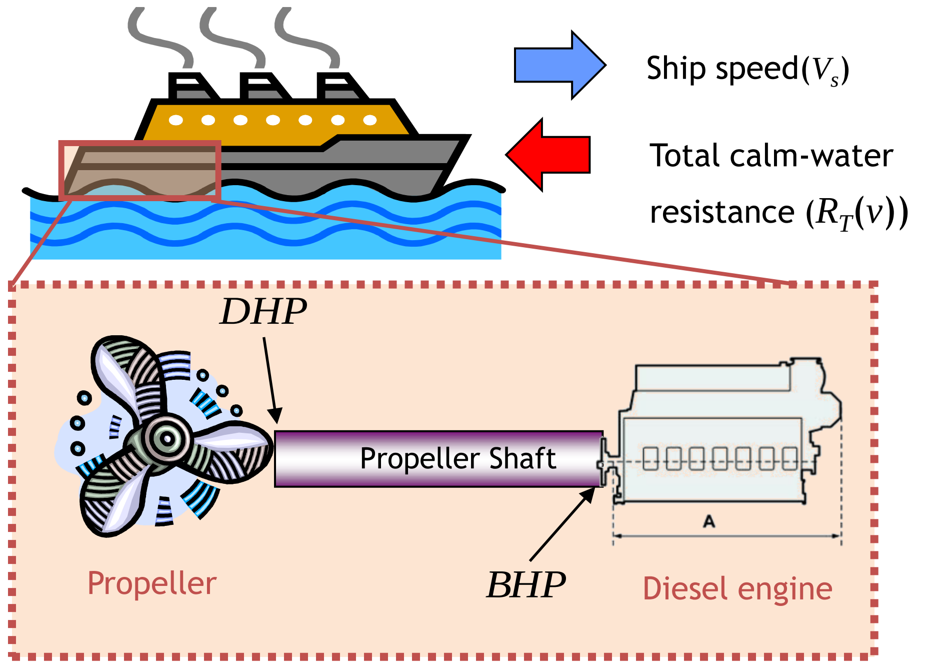
Economical constraints :Required DFOC (Daily Fuel Oil Consumption) → Hull Form Design and Hydrodynamic Performance Equation

☑Goal : Meet the Required DFOC

At first, we have to estimate total calm-water resistance of a ship

$$EHP = \boxed{R_T(v)} \cdot V_s$$

Then, the required brake horse power (BHP) can be predicted by estimating propeller efficiency, hull efficiency, relative rotative efficiency, shaft transmission efficiency, sea margin, and engine margin.

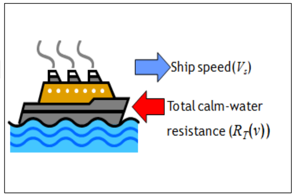


Economical constraints : Required DFOC (Daily Fuel Oil Consumption) → Propeller and Engine Selection

① EHP (Effective Horse Power)

$$EHP = \boxed{R_T(v) \cdot} V_s$$ (In calm water)

Resistance estimation



② DHP (Delivered Horse Power)

$$DHP = \frac{EHP}{\boxed{\eta_D}}$$

(η_D: Propulsive efficiency)

$$\eta_D = \eta_O \cdot \eta_H \cdot \eta_R$$

η_O : Open water efficiency

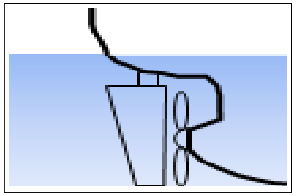
η_H : Hull efficiency

η_R : Relative rotative efficiency

Propeller efficiency

Thrust deduction and wake (due to additional resistance by propeller)

Hull-propeller interaction



③ BHP (Brake Horse Power)

$$BHP = \frac{DHP}{\eta_T}$$

(η_T : Transmission efficiency)

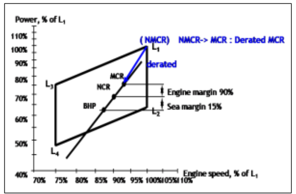
④ NCR (Normal Continuous Rating)

$$NCR = BHP\left(1 + \frac{\text{Sea Margine}}{100}\right)$$

⑤ DMCR (Derated Maximum Continuous Rating)

$$\boxed{DMCR = \frac{NCR}{\text{Engine Margin}}}$$

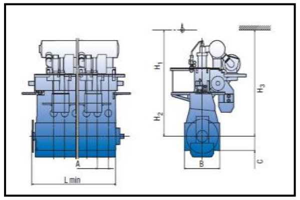
Engine Selection



⑥ NMCR (Nominal Maximum Continuous Rating)

$$NMCR = \frac{DMCR}{\text{Derating rate}}$$

Engine Data



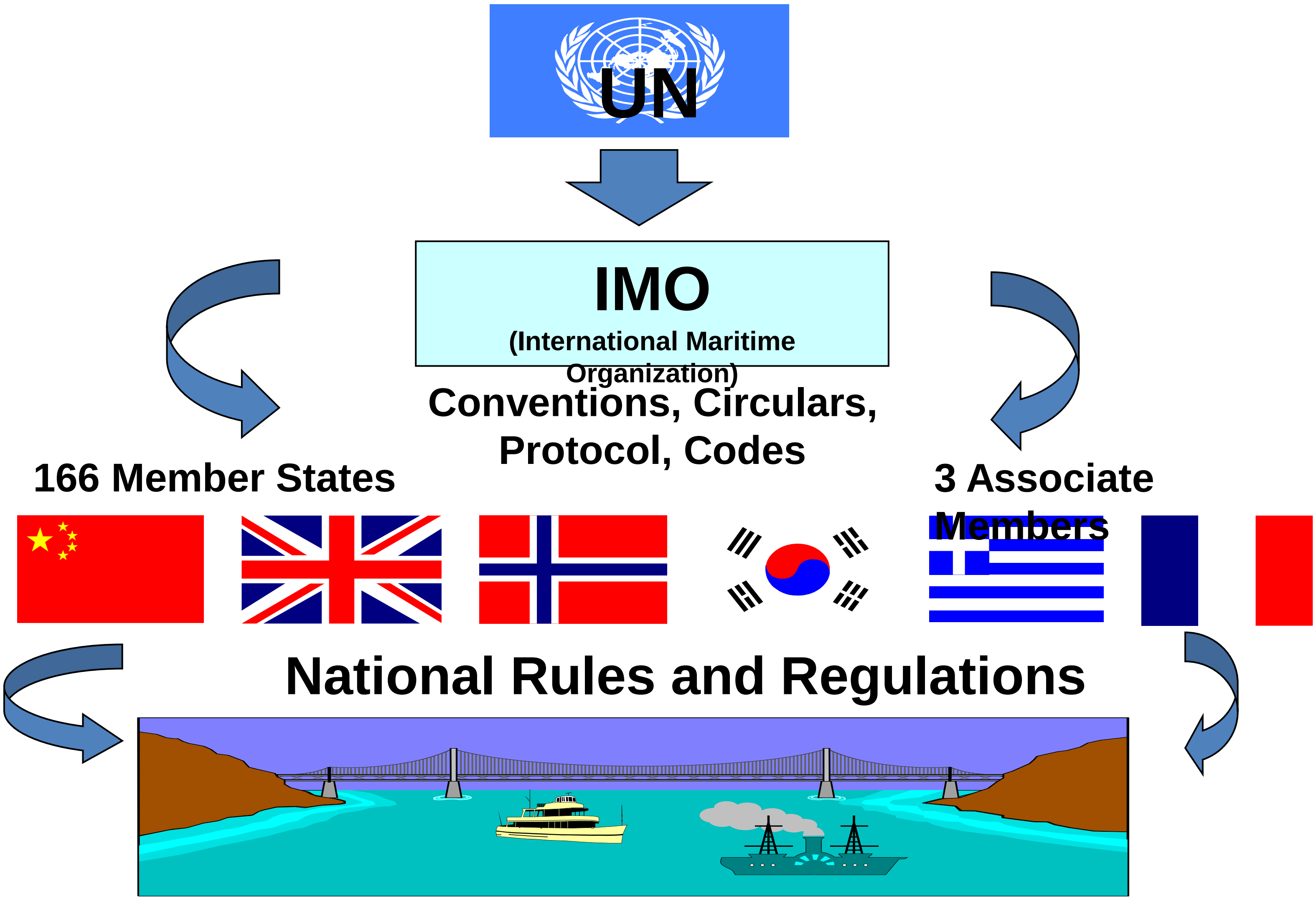
2-7 Regulatory constraints

1) Regulatory constraints : Organizations

- International Maritime Organizations(IMO)
- International Labour Organizations (ILO)
- Regional Organizations (EU,…)
- Administrations (Flag, Port)
- Classification Societies
- International Standard Organizations (ISO)

Rules and Regulations : IMO

Rules and Regulations - IMO



Rules and Regulations – IMO Instruments

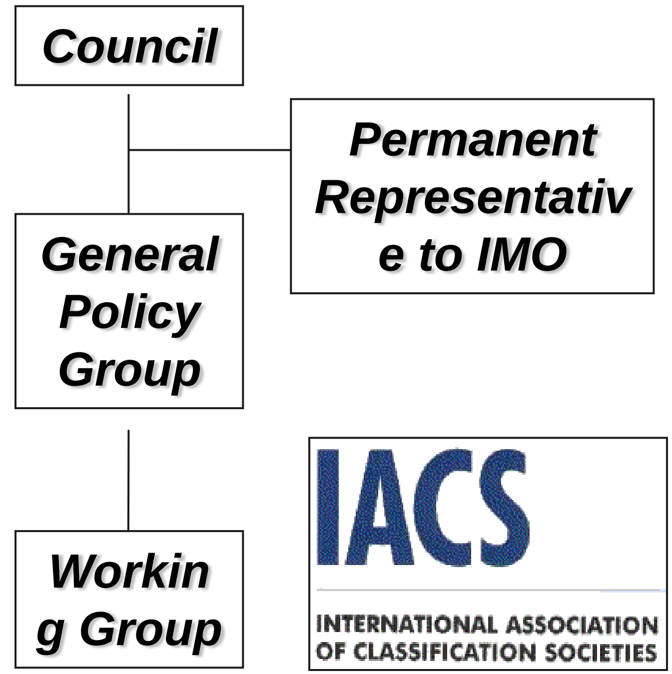
- ☑ **Conventions**
  - **SOLAS / MARPOL / ICLL / COLREG / ITC / AFS / BWM ……**
- ☑ **Protocols**
  - **MARPOL Protocol 1997 / ICLL Protocol 1988**
- ☑ **Codes**
  - **ISM / LSA / IBC / IMDG / IGC / BCH / BC / GC ……**
- ☑ **Resolutions**
  - **Assembly / MSC / MEPC**
- ☑ **Circulars**
  - **MSC / MEPC / Sub-committees ……**

2) Classification Society

Rules & Regulations – IACS

☑ 10 Members

- ABS (American Bureau of Shipping)
- DNV (Det Norske Veritas)
- LR (Lloyd's Register)
- BV (Bureau Veritas)
- GL (Germanischer Lloyd)
- KR (Korean Register of Shipping)
- RINA (Registro Italiano Navale)
- NK (Nippon Kaiji Kyokai)
- RRS (Russian Maritime Register of Shipping)
- CCS (China Classification Society)



☑ 2 Associate Members

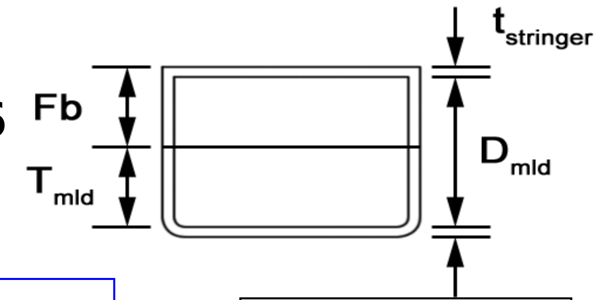
- CRS (Croatian Register of Shipping)
- IRS (Indian Register of Shipping)

2-8 Required Freeboard

2.8 Required Freeboard of ICLL 1966

- **Regulatory constraints**

  - International Convention on Load Lines(ICLL)1966



$$D_{Fb} - T \geq Fb_{ICLL}(L, B, D_{mld}, C_B)$$

$$D_{Fb} = D_{mld} + t_{stringer}$$

: Freeboard Equation

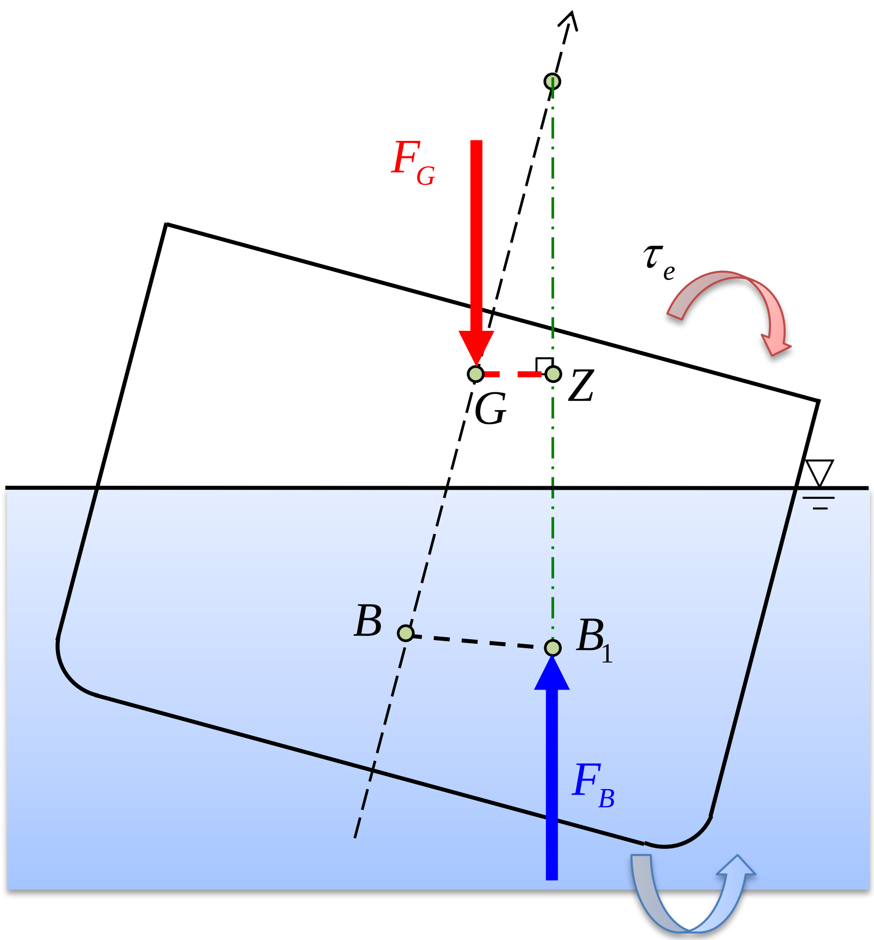
✓ **Check: Actual freeboard($D_{Fb} - T$) of a ship should not be less than the freeboard required by the ICLL 1966 regulation(Fb_{ICLL}).**

Freeboard(Fb) means the distance between the water surface and the top of the deck at the side(at the deck line). It includes the thickness of freeboard deck plating.

- The freeboard is closely related to the draught.
  A 'freeboard calculation' in accordance with the regulation determines whether the desired depth is permissible.

2-9 Required Stability

Definition of GZ(Righting Arm)



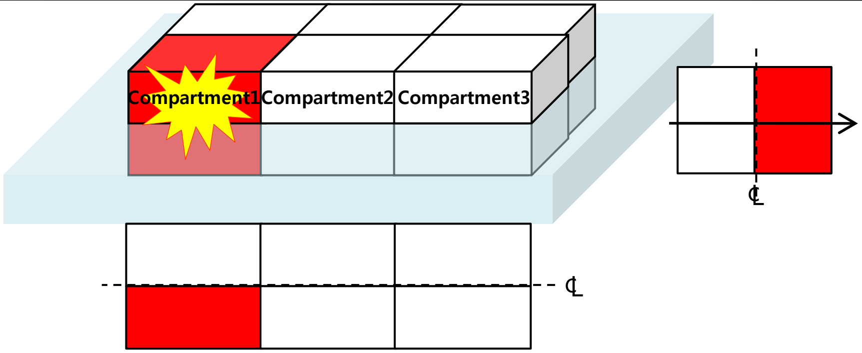
- Overview of Ship Stability

$$\tau_r = GZ \cdot F_B$$

τ_r : **Righting Moment**

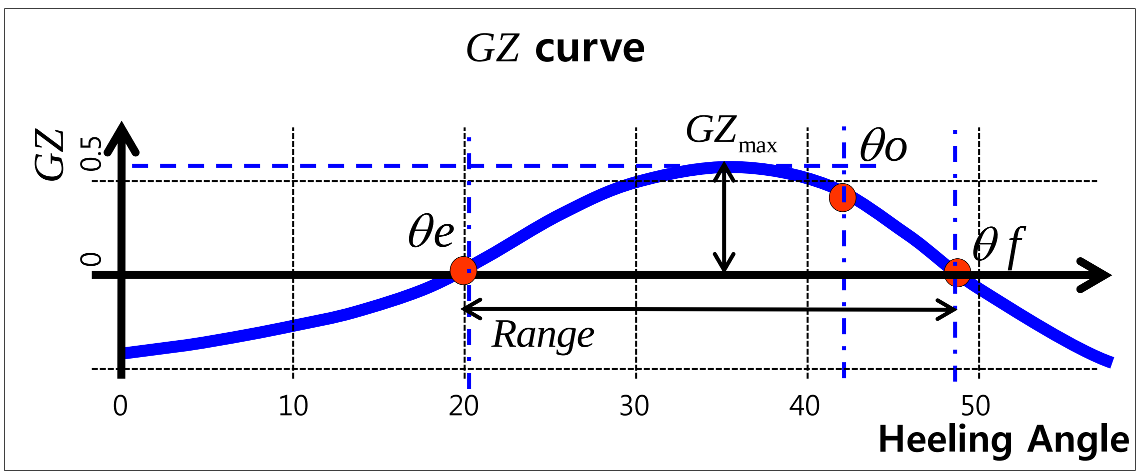
GZ **: Righting Arm**

Required GZ in Damaged condition



Immersion + Trim + Heel

✓ To measure the damage stability, **calculate the GZ curves** of this damage case by calculating the new center of buoyancy and center of mass.



θe : Equilibrium heel angle.
θv : $\theta v = \text{minimum}(\theta f, \theta o)$
(in this case, θv equals to θo)
GZ_{max} : Maximum value of GZ.
Range : Range of positive righting arm.
Flooding stage : Discrete step during the flooding process.

θf : angle of flooding (righting arm becomes negative)
θo : angle at which an **"opening"** incapable of being closed weathertight becomes submerged.

2-10 Structural Design in accordance with the Rule of the Classification Society

Regulatory Constraint :
Ship Structural Design in accordance with Rule of the Classification Society

● Ship Structural Design

what is designer's **major** interest?

● **Safety :**

Won't 'IT' fail under the load?

a ship
a stiffener
a plate

a ship } global

a stiffener, a plate } local

a ship

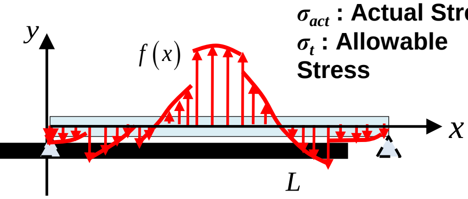
$Load: f(x)$

cause

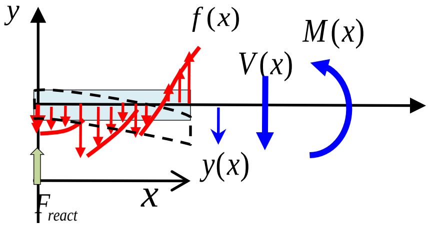
Shear Force: $V(x)$
Bending Moment: $M(x)$
Deflection : $y(x)$

'relations' of load, S.F., B.M., and deflection

$$\frac{dV(x)}{dx} = -f(x), \quad \frac{dM(x)}{dx} = V(x), \quad EI\frac{d^2y(x)}{dx^2} = M(x)$$



σ_{act} : Actual Stress
σ_l : Allowable Stress



what is our interest?

● **Safety :**
Won't it fail under the load?

Stress should meet :

$$\boxed{\sigma_{act} \le \sigma_l}$$

, where $\sigma_{act} = \frac{M}{I_{\bar{y}}/\bar{y}_i} = \frac{M}{Z}$

● **Geometry :**
How much it would be bent under the load?

Differential equations of the defection curve

$$\boxed{EI\frac{d^4y(x)}{dx^4} = -f(x)}$$

what kinds of load f cause hull girder moment?

$$\boxed{\sigma_{act.} \le \sigma_l}, \quad \sigma_{act.} = \frac{M_S + M_W}{Z_{mid}}$$

, M_S = Still water bending moment
M_W = Vertical wave bending moment

$$f(x) = f_S(x) + f_W(x)$$
$$V(x) = V_S(x) + V_W(x)$$
$$M(x) = M_S(x) + M_W(x)$$

Hydrostatics

$f_S(x)$: load in still water
= weight + buoyancy

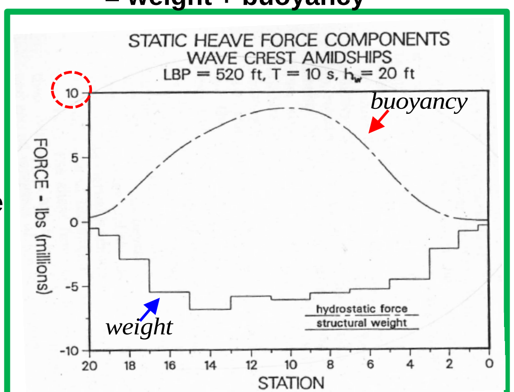
$f_S(x)$: load in still water

$$V_S(x) = \int_0^x f_S(x)dx$$

$V_S(x)$: still water shear force

$$M_S(x) = \int_0^x V_S(x)dx$$

$M_S(x)$: still water bending moment



Hydrodynamics

$f_W(x)$: load in wave
= added mass + diffraction
+ damping + Froude-Krylov + mass iner

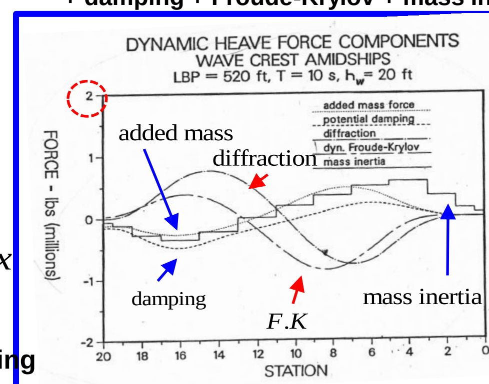
$f_W(x)$: load in wave

$$V_W(x) = \int_0^x f_W(x)dx$$

$V_W(x)$: wave shear force

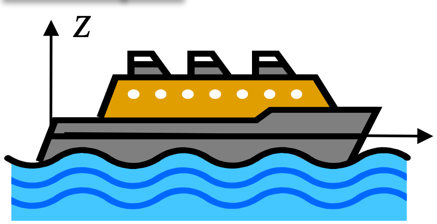
$$M_W(x) = \int_0^x V_W(x)dx$$

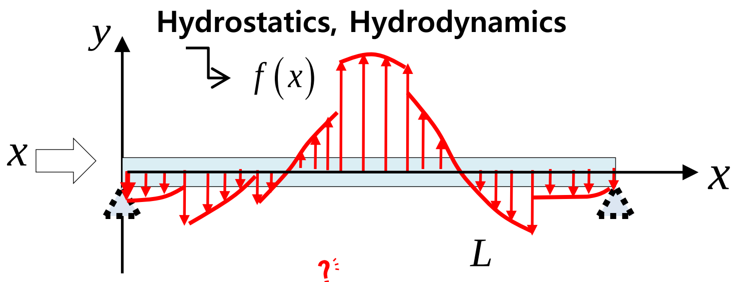
$M_W(x)$: vertical wave bending moment

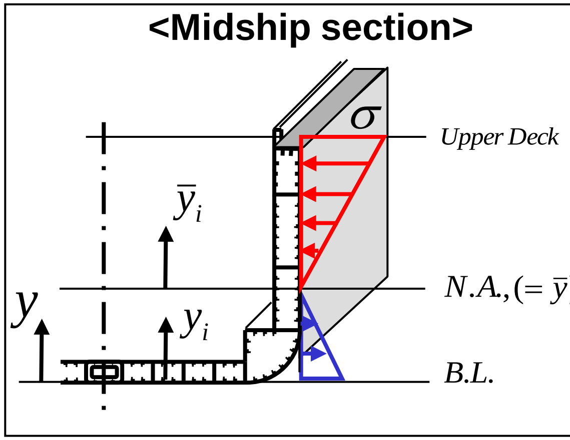


a ship	global
a stiffener a plate	local

a ship







Actual stress on midship section should be less than allowable stress

$$\boxed{\sigma_{act.} \leq \sigma_{allow}}$$

$$, \sigma_{act.} = \frac{M_{mid}}{Z_{mid}} = \frac{M_s + M_w}{I_{ship,N.A} / \bar{y}_i}$$

Allowable stress by Rule : (for example)

$$, \sigma_{allow} = 175 f_1 \ \ [N/mm^2]$$

How we can meet the rule?

'Midship design' is to <u>arrange</u> the structural members and <u>fix the thickness</u> of them to secure enough section modulus to the rule.

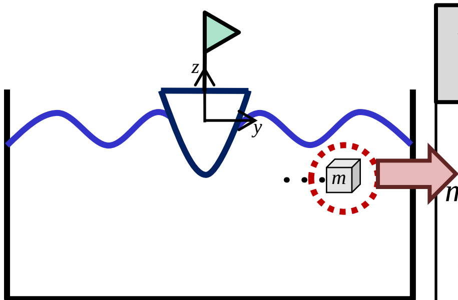
$, M_w$: vertical wave bending moment

$, M_s$: still water bending moment

$, I_{ship,N.A.}$: moment of inertia from N.A. of Midship section

2-11 Hydrostatic and Hydrodynamic Forces acting on a Ship in Waves

Equations of Motion of a Fluid Element(Cauchy eq. ~ Bernoulli eq.)



✓Equations of motion of a Fluid Element

Lagrangian & Eulerian Description

Shear force

Curl & Rotation

Newton's 2nd Law

$m\ddot{\mathbf{r}} = \sum \mathbf{F} = (\text{Body force} + \text{Surface force})$

→ Cauchy equation → ①② Navier-Stokes equation → ③ Euler equation → ④⑤ Bernoulli equation

$$\rho \frac{\partial \Phi}{\partial t} + P + \frac{1}{2}\rho \left|\nabla \Phi\right|^2 + \rho g z = 0$$

Microscopic/ Macroscopic Derivation(RTT[1])

Mass Conservation Law → ④⑤ Laplace Equation

$$\nabla^2 \Phi = 0 \quad (\mathbf{V} = \nabla \Phi)$$

① Newtonian fluid : fluid whose stress versus strain rate curve is linear.

② Stokes Assumption: Definition of viscosity coefficient(μ,λ) due to linear deformation and isometric expansion

③ inviscid fluid

④ irrotational flow

⑤ incompressible flow

$\mathbf{r}$: displacement of a fluid particle with respect to the time

$$\mathbf{V} = \frac{d\mathbf{r}}{dt}, \mathbf{a} = \frac{d^2\mathbf{r}}{dt^2}$$

1) RTT : Reynold Transport Theorem

1) Kundu,P.K., Cohen,I.M., Fluid Mechanics 5th, Academic Press,2012

* A **Newtonian fluid** : fluid whose stress versus strain rate curve is linear.

**Definition of viscosity coefficient(μ,λ) due to linear deformation and isometric expansion

Equations of Motion of a Fluid Element and Continuity Equation

Cauchy Equation : $\rho\frac{d\mathbf{V}}{dt}=\rho\mathbf{g}+\nabla\bullet\sigma \quad ,\left(\mathbf{V}=[u,v,w]^T\right)$

① Newtonian fluid*
② Stokes assumption**

Navier-Stokes Equation : (in general form) $\rho\frac{d\mathbf{V}}{dt}=\rho\mathbf{g}-\nabla P+\mu\left(\frac{1}{3}\nabla(\nabla\cdot\mathbf{V})+\nabla^2\mathbf{V}\right)$

$(\mu=0)$ ③ Inviscid fluid

Euler Equation : $\rho\frac{d\mathbf{V}}{dt}=\rho\mathbf{g}-\nabla P$

$\rho=\rho(P)$ ④ barotropic flow

Euler Equation : (another form) $\frac{\partial\mathbf{V}}{\partial t}+\nabla B=\mathbf{V}\times\boldsymbol{\omega} \quad ,\left(B=\frac{1}{2}q^2+gz+\int\frac{dP}{\rho} \;,\; q^2=u^2+v^2+w^2\right)$

$\boldsymbol{\omega}=\nabla\times\mathbf{V}$
If $\boldsymbol{\omega}=0$, (irrotational flow)
then $\nabla\times\mathbf{V}=0$
If $\nabla\times\mathbf{V}=0$, then $\mathbf{V}=\nabla\Phi$.

⑤ Steady flow $\left(\frac{\partial\mathbf{V}}{\partial t}=0\right)$

along the streamlines and vortex lines

Bernoulli equation (case1) $B=\text{Constant}$

$\left(\frac{1}{2}q^2+gz+\int\frac{dP}{\rho}=C\right)$

$\left(q^2=|\nabla\Phi|^2\right)$ ⑥ unsteady, irrotational flow

Bernoulli equation (case2) $\frac{\partial\Phi}{\partial t}+\frac{1}{2}|\nabla\Phi|^2+gz+\int\frac{dP}{\rho}=F(t)$

$(\rho=constant)$ ⑦ incompressible flow

Bernoulli equation (case3) $\frac{\partial\Phi}{\partial t}+\frac{1}{2}|\nabla\Phi|^2+gz+\frac{P}{\rho}=F(t)$

Continuity Equation $\frac{\partial\rho}{\partial t}+\nabla\bullet\rho\mathbf{V}=0$

⑦ incompressible flow
$\rho=constant, \left(\frac{\partial\rho}{\partial t}=0\right)$

$\nabla\bullet\mathbf{V}=0$

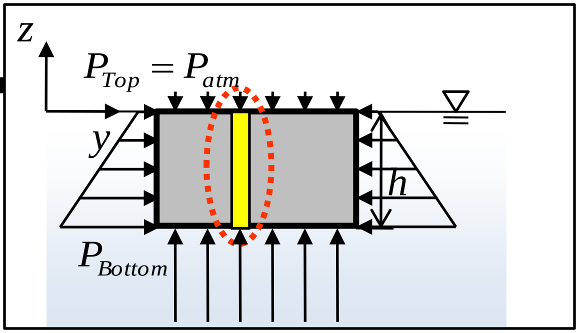
⑥ irrotational flow $(\mathbf{V}=\nabla\Phi)$

Laplace Equation $\nabla^2\Phi=0$

Newtonian fluid
Stokes assumption
inviscid fluid
unsteady flow
irrotational flow
incompressible flow

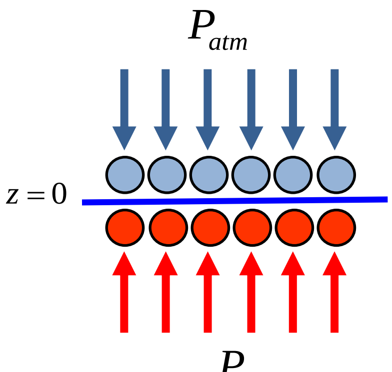
Meaning of F(t) in Bernoulli Equation and Gauge Pressure

1) Gauge pressure : The net pressure of the difference of the total pressure and atmospheric pressure



Bernoulli Equation

$$\frac{\partial \Phi}{\partial t} + \frac{P}{\rho} + \frac{1}{2}|\nabla \Phi|^2 + g\,z = F(t)$$



If a fluid element is in staitc equilibrium state on the free surface (z=0), then

$$\frac{\partial \Phi}{\partial t} = 0,\ \nabla\Phi = 0,\ \ P = P_{atm}$$

$$\cancel{\frac{\partial \Phi}{\partial t}} + \frac{P}{\rho} + \frac{1}{2}|\cancel{\nabla \Phi}|^2 + g\cancel{z} = F(t) \longrightarrow \frac{P_{atm}}{\rho} = F(t)$$

(Atmospheric pressure(P_{atm})) = (Pressure at $z=0$)

$$\therefore \frac{\partial \Phi}{\partial t} + \frac{P}{\rho} + \frac{1}{2}|\nabla \Phi|^2 + g\,z = \frac{P_{atm}}{\rho}$$

✓What is the pressure on the bottom of an object ?

$$\frac{\partial \Phi}{\partial t} + \frac{P_{Bottom}}{\rho} + \frac{1}{2}|\nabla \Phi|^2 + gz = \frac{P_{atm}}{\rho}$$

$$\frac{\partial \Phi}{\partial t} + \frac{\cancel{P_{atm}} + P_{Fluid}}{\rho} + \frac{1}{2}|\nabla \Phi|^2 + gz = \frac{\cancel{P_{atm}}}{\rho}$$

$$\therefore \frac{\partial \Phi}{\partial t} + \boxed{\frac{P_{Fluid}}{\rho}} + \frac{1}{2}|\nabla \Phi|^2 + gz = 0$$

'gauge pressure'

※ In case that R.H.S of Bernoulli equation is expressed by zero, pressure P means the pressure due to the fluid which excludes the atmospheric pressure.

If the motion of fluid is small, square term could be neglected.

$$\frac{\partial \Phi}{\partial t} + \frac{P_{Fluid}}{\rho} + \cancel{\frac{1}{2}|\nabla \Phi|^2} + gz = 0$$

$$P_{Fluid} = \underbrace{-\rho \frac{\partial \Phi}{\partial t}}_{P_{dynamic}} \underbrace{- \rho g z}_{P_{static}} = 0$$

'Linearized Bernoulli Equation'

Pressure and Force acting on a Fluid Element

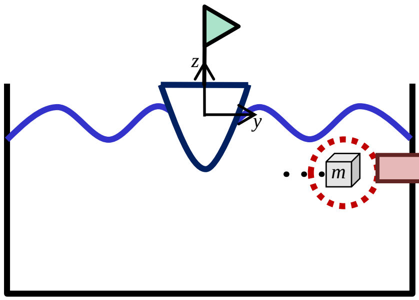
$\mathbf{r}$: displacement of particle with respect to time

$\mathbf{V} = \frac{d\mathbf{r}}{dt}, \mathbf{a} = \frac{d^2\mathbf{r}}{dt^2}$

$\mathbf{F}_{F.K}$: Froude- krylov force
$\mathbf{F}_D$: Diffraction force
$\mathbf{F}_R$: Radiation force

1) RTT : Reynold Transport Theorem
2) SWBM : Still Water Bending Moment
3) VWBM : Vertical Wave Bendidng Moment

✓ Assumption
① Newtonian fluid*
② Stokes Assumption**
③ invicid fluid
④ Irrotational flow
⑤ Incompressible flow



$\mathbf{r} = [\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6]^T$

ξ_1 : surge , ξ_4 : roll
ξ_2 : sway , ξ_5 : pitch
ξ_3 : heave , ξ_6 : yaw

Microscopic/ Macroscopic Derivation(RTT[1])

✓Equations of motions of Fluid Particles

Lagrangian & Eulerian Description

Shear force

Curl & Rotation

Newton's 2nd Law
$m\ddot{\mathbf{r}} = \sum \mathbf{F} = (\text{Body force} + \text{Surface force})$

Cauchy equation ①②

Navier-Stokes equation ③

Euler equation ④⑤

Bernoulli equation

Mass Conservation Law ④⑤

Laplace Equation

$$\nabla^2\Phi = 0 \quad (\mathbf{V} = \nabla\Phi)$$

$$\rho\frac{\partial\Phi}{\partial t} + P + \frac{1}{2}\rho|\nabla\Phi|^2 + \rho g z = 0$$

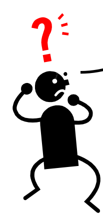
Velocity potential Φ

✓Calculation of Fluid Force

$\Phi = \Phi_I$ (Incident wave potential)
$+\Phi_D$ (Diffraction potential)
$+\Phi_R$ (Radiation potential)

Linearization
$\left(\frac{1}{2}\rho|\nabla\Phi|^2 = 0\right)$

$$P = -\rho g z - \rho\frac{\partial\Phi}{\partial t}$$



$$\mathbf{F}_{Fluid}(\mathbf{r}, \dot{\mathbf{r}}, \ddot{\mathbf{r}}) = \iint_{S_B} P\mathbf{n}dS = \mathbf{F}_{Buoyancy}(\mathbf{r}) + \mathbf{F}_{F.K}(\mathbf{r}) + \mathbf{F}_D(\mathbf{r}) + \mathbf{F}_R(\mathbf{r}, \dot{\mathbf{r}}, \ddot{\mathbf{r}})$$

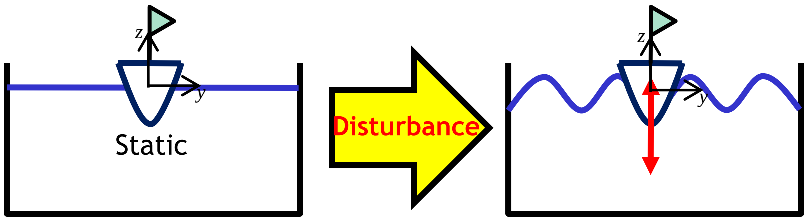
(S_B : wetted surface Area)

* A **Newtonian fluid** : fluid whose stress versus strain rate curve is linear.
**Definition of viscosity coefficient(μ,λ) due to linear deformation and isometric expansion

- Hydrostatic Pressure, Force and Moment on a Floating Body

Forces acting on a ship in Waves

$$\rho\frac{\partial \Phi}{\partial t}+P+\frac{1}{2}\rho\left|\nabla\Phi\right|^2+\rho g z=0$$

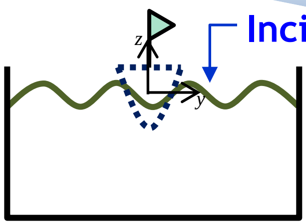


✓ **Pressure due to the fluid Elements around the ship in wave**

: Velocity, acceleration, pressure of the fluid elements are changed due to the motion of fluid, then the pressure of fluid elements acting on the ship is changed.

Linearization

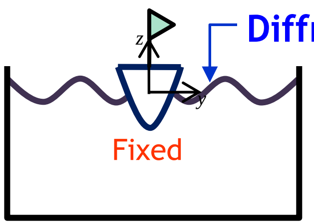
Incident wave velocity potential (Φ_I)



✓ **Velocity potential of incoming waves that are independent of the body motion**

$+$

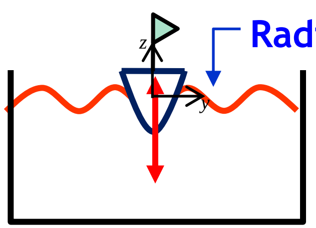
Diffraction wave velocity potential (Φ_D)



✓ **Velocity potential of the disturbance of the incident waves by the body that is fixed in position**[1]

$+$

Radiation wave velocity potential (Φ_R)



✓ **Velocity potential of the waves that are induced due to the body motions, in the absence of the incident waves.**[1]

✓ **Total Velocity Potential**

$$\Phi_T=\Phi_I+\Phi_D+\Phi_R$$

Superposition theorem

For homogeneous linear PDE, the superposed solution is also a solution of the linear PDE[2]

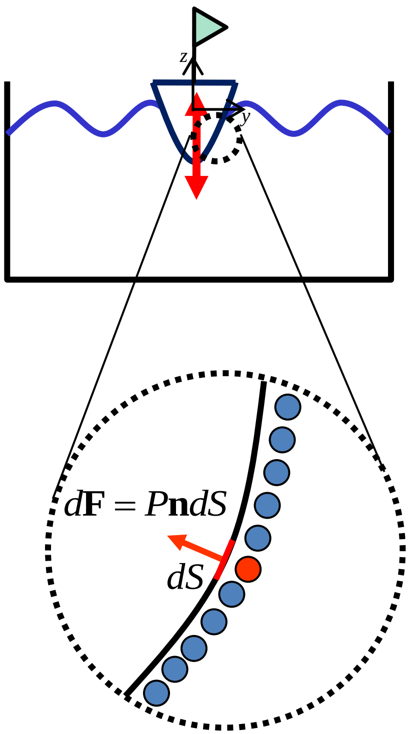
$$P=-\rho g z-\rho\frac{\partial \Phi_T}{\partial t}$$

$$\mathbf{F}_{Fluid}=\iint_{S_B}P\mathbf{n}dS$$

$$=\mathbf{F}_{static}+\mathbf{F}_{F.K}+\mathbf{F}_D+\mathbf{F}_R$$

1) Newman, J.N. , Marine Hydrodynamics, The MIT Press, Cambridge, 1997, pp 287
2) Erwin Kreyszig, Advanced Engineering Mathematics, Wiley, 2005, Ch 12.1 (pp 535)

Φ_I : Incident wave velocity potential
Φ_D : Diffraction potential
Φ_R : Radiation potential



$d\mathbf{F}$: Infinitesimal force of the fluid elements acting on the ship

dS : Infinitesimal Area

$\mathbf{n}$: Normal vector of the infinitesimal Area

$\mathbf{r} = [\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6]^T$

ξ_1 : surge , ξ_4 : roll
ξ_2 : sway , ξ_5 : pitch
ξ_3 : heave , ξ_6 : yaw

✓ Bernoulli Equation

$$\rho\frac{\partial\Phi}{\partial t} + P + \cancel{\frac{1}{2}\rho|\nabla\Phi|^2} + \rho g z = 0$$

Linearization

✓ Laplace Equation

$$\nabla^2\Phi = 0$$

$$\Phi = \Phi_I + \Phi_D + \Phi_R$$

Linear combination of the Basis solutions

Basis solutions

$$P_{Fluid}(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}}) = -\rho g z - \rho\frac{\partial\Phi}{\partial t} = \boxed{-\rho g z} - \rho\left(\frac{\partial\Phi_I}{\partial t} + \frac{\partial\Phi_D}{\partial t} + \frac{\partial\Phi_R}{\partial t}\right)$$

Pressure of the fluid elements acting on the ship

$$= \boxed{P_{Buoyancy}(\mathbf{r})} + \underbrace{P_{F.K}(\mathbf{r}) + P_D(\mathbf{r}) + P_R(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}})}_{P_{dynamic}}$$

$$P_{Fluid} \quad = \mathbf{F}_{Buoyancy}(\mathbf{r}) + \mathbf{F}_{F.K}(\mathbf{r}) + \mathbf{F}_D(\mathbf{r}) + \mathbf{F}_R(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}})$$

(S_B: wetted surface)

Integration over the wetted surface area of the ship
(**Forces and moments acting on the ship due to the fluid elements**)

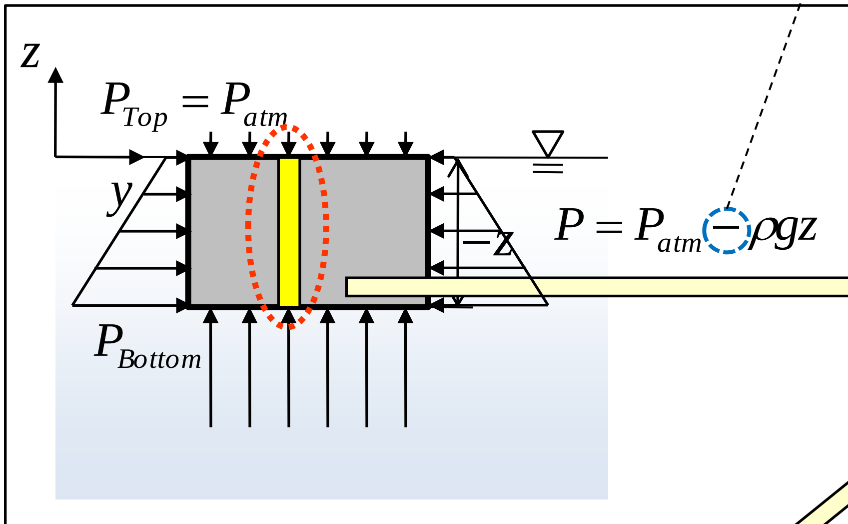
$$\mathbf{F}_{Fluid}(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}}) = \iint_{S_B} P \, \mathbf{n} dS$$

Hydrostatic Pressure and Buoyant Force acting on a Ship

※ Pressure : force per unit area applied in a direction perpendicular to the surface of an object.
To calculate force, we should multiply pressure by area and na ormal vector of the area.

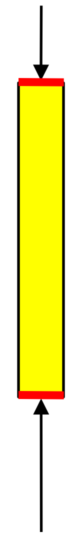
According to the reference frame, (-) sign is added because the value of z is (-).

✓ What is the force acting on the bottom of an object?



: Force acting on the upper differential area

$$d\mathbf{F}_{Top} = P_{Top} \cdot \mathbf{n}_1 dS \quad \begin{pmatrix} P_{Top} = P_{atm} - \rho g \cdot 0 \\ \mathbf{n}_1 = -\mathbf{k} \end{pmatrix}$$



$\mathbf{n}_1$: Normal vector

dS : Surface area

$$d\mathbf{F}_{Bottom} = P_{Bottom} \cdot \mathbf{n}_2 dS \quad \begin{pmatrix} P_{Bottom} = P_{atm} - \rho g z \\ \mathbf{n}_2 = \mathbf{k} \end{pmatrix}$$

: Force acting on the lower differential area

$$\begin{aligned} d\mathbf{F} &= d\mathbf{F}_{Top} + d\mathbf{F}_{Bottom} \\ &= P_{Top} \cdot \mathbf{n}_1 dS + P_{Bottom} \cdot \mathbf{n}_2 dS \\ &= \cancel{P_{atm}(-\mathbf{k})dS} + (\cancel{P_{atm}} - \rho g z)\mathbf{k}dS \\ &= -\rho g z \mathbf{k} dS = \mathbf{k}(-\rho g z \cdot dS) \end{aligned}$$

: Force due to the atmospheric pressure is vanished.

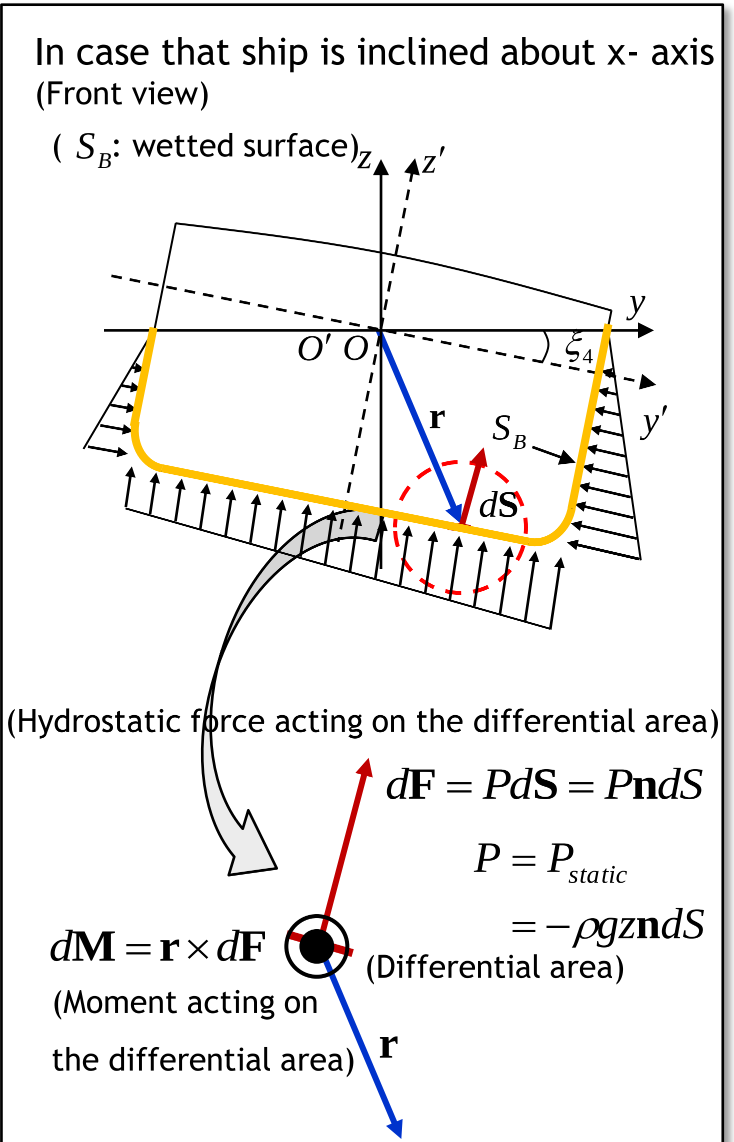
$$\mathbf{F} = \int d\mathbf{F} = \iint_{S_B} P\mathbf{n}dS \quad ,(P = P_{static} = -\rho g z)$$

Static fluid pressure excluding the atmospheric pressure.

$$= -\rho g \iint_{S_B} \mathbf{n} z dS$$

Cf) Linearized Bernoulli eq

$$P = \underbrace{-\rho g z}_{P_{static}} \underbrace{- \rho \frac{\partial \Phi}{\partial t}}_{P_{dynamic}}$$



- Hydrostatic force (Surface force) is calculated by integrating
- the differential force over the wetted surface area.

✓ Hydrostatic force acting on the differential area :

$$d\mathbf{F} = P \cdot d\mathbf{S} = P \cdot \mathbf{n}dS$$

P is hydrostatic pressure, P_{static}.

$$P = P_{static} = -\rho gz$$

$$d\mathbf{F} = P_{static} \cdot \mathbf{n}dS = -\rho gz \cdot \mathbf{n}dS$$

✓ Total force : (S_B : wetted surface area)

$$\mathbf{F} = \iint_{S_B} P\mathbf{n}dS \Rightarrow \boxed{\mathbf{F} = -\rho g \iint_{S_B} z\mathbf{n}dS}$$

- Hydrostatic Moment : (Moment)=(Position vector) X (Force)

✓ Moment acting on the differential area :

$$d\mathbf{M} = \mathbf{r} \times d\mathbf{F} = \mathbf{r} \times P\mathbf{n}dS = P(\mathbf{r} \times \mathbf{n})dS$$

✓ Total moment :

$$\mathbf{M} = \iint_{S_B} P(\mathbf{r} \times \mathbf{n})dS \Rightarrow \boxed{\mathbf{M} = -\rho g \iint_{S_B} z(\mathbf{r} \times \mathbf{n})dS}$$

Buoyant Force

1) Erwin Kreyszig, Advanced Engineering Mathematics 9th ,Wiley,Ch10.7(p458~463)
2) Erwin Kreyszig, Advanced Engineering Mathematics 9th ,Wiley,Ch9.9(p414~417)

✓ Hydrostatic force

$$\mathbf{F} = -\rho g \iint_{S_B} z\mathbf{n}dS$$

(S: wetted surface area)

By divergence theorem[1] ,

$$\left(\iint_S f \cdot \mathbf{n}dA = \iiint_V \nabla f dV \right)$$

$$\mathbf{F} = \rho g \iiint_V \nabla z dV \qquad \left(\nabla z \overset{2)}{=} \frac{\partial z}{\partial x}\mathbf{i} + \frac{\partial z}{\partial y}\mathbf{j} + \frac{\partial z}{\partial z}\mathbf{k} = \mathbf{k} \right)$$

$$= \mathbf{k}\rho g \iiint_V dV$$

$$= \mathbf{k}\rho g V(t)$$

When ship moves, the displacement volume(V) of the ship is changed with time.
That means V is the function of time, $V(t)$.

: The buoyant force on an immersed body has the same magnitude as the weight of the fluid displaced by the body1). And the direction of the buoyant force is opposite to the gravity (≒Archimedes' Principle)

※ The reason why (-) sign is disappeared

: Divergence theorem is based on the outer unit vector of the surface.
Normal vector for the calculation of the buoyant force is based on the inner unit vector of the surface, so (-)sign is added, and then divergence theorem is applied.

Hydrostatic Moment

1) Erwin Kreyszig, Advanced Engineering Mathematics 9th ,Wiley,Ch10.7(pp.458~463)
2) Erwin Kreyszig, Advanced Engineering Mathematics 9th ,Wiley,Ch9.9(pp.414~417)

✓ Hydrostatic moment

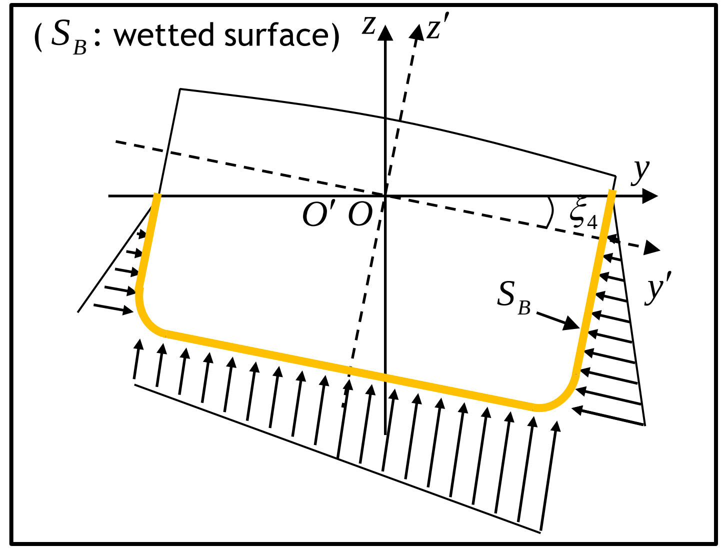
$$\mathbf{M} = -\rho g \iint_{S_B} (\mathbf{r} \times \mathbf{n}) z dS = \rho g \iint_{S_B} (\mathbf{n} \times \mathbf{r}) z dS$$

By divergence theorem[1] ,

$$\left(\iiint_V \nabla \times \mathbf{F} dV = \iint_S \mathbf{n} \times \mathbf{F} dA \right)$$

$$\mathbf{M} = -\rho g \iiint_V (\nabla \times \mathbf{r}) z dV$$

Because direction of normal vector is opposite, (-) sign is added



$$\left(\nabla \times \mathbf{r} z \overset{2)}{=} \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xz & yz & z^2 \end{vmatrix} = \mathbf{i}\left(\frac{\partial}{\partial y} z^2 - \frac{\partial}{\partial z} yz \right) + \mathbf{j}\left(\frac{\partial}{\partial z} xz - \frac{\partial}{\partial x} z^2 \right) + \mathbf{k}\left(\frac{\partial}{\partial x} yz - \frac{\partial}{\partial y} xz \right) = -\mathbf{i}y + \mathbf{j}x \right)$$

$$\therefore \mathbf{M} = -\rho g \iiint_V [-\mathbf{i}y + \mathbf{j}x] dV$$

Hydrodynamic Forces

calculated from 6 DOF(Degree of Freedom)

Equations of ship motions

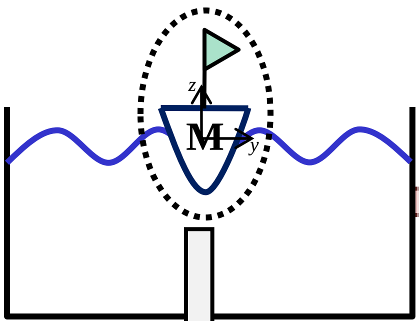
r : displacement of particle with respect to time

$\mathbf{V} = \frac{d\mathbf{r}}{dt}, \mathbf{a} = \frac{d^2\mathbf{r}}{dt^2}$

$\mathbf{F}_{F.K}$: Froude- krylov force
$\mathbf{F}_D$: Diffraction force
$\mathbf{F}_R$: Radiation force

1) RTT : Reynold Transport Theorem
2) SWBM : Still Water Bending Moment
3) VWBM : Vertical Wave Bending Moment

✓ Assumption
① Newtonian fluid*
② Stokes Assumption**
③ inviscid fluid
④ Irrotational flow
⑤ Incompressible flow



✓Equations of motions of Fluid Particles

Lagrangian & Eulerian Description

Shear force

Curl & Rotation

Newton's 2nd Law
$m\ddot{r} = \sum F = (\text{Body force} + \text{Surface force})$

Cauchy equation → ①② Navier-Stokes equation → ③ Euler equation → ④⑤ Bernoulli equation

Microscopic/ Macroscopic Derivation(RTT[1])

Mass Conservation Law → ④⑤ Laplace Equation

$$\nabla^2\Phi = 0 \quad (\mathbf{V} = \nabla\Phi)$$

$$\rho\frac{\partial\Phi}{\partial t} + P + \frac{1}{2}\rho|\nabla\Phi|^2 + \rho g z = 0$$

✓6 D.O.F equations of motions

① Coordinate system(Reference frame)
(Water surface-fixed & Body-fixed frame)

② Newton's 2nd Law

$$\mathbf{M}\ddot{\mathbf{r}} = \sum\mathbf{F} = (Body\ \ Force) + (Surface\ \ Force)$$

$$= \mathbf{F}_{gravity}(\mathbf{r}) + \boxed{\mathbf{F}_{Fluid}(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}})}$$

$$= \mathbf{F}_{gravity} + \mathbf{F}_{Buoyancy}(\mathbf{r}) + \mathbf{F}_{F.K}(\mathbf{r}) + \mathbf{F}_D(\mathbf{r})$$
$$+ \mathbf{F}_{R,Damping}(\mathbf{r},\dot{\mathbf{r}}) + \mathbf{F}_{R,Mass}(\mathbf{r},\ddot{\mathbf{r}})$$

Nonlinear terms → Nonlinear equation
→ Difficulty of getting analytic solution

Numerical Method

Velocity potential Φ

✓Calculation of Fluid Force

$\Phi = \Phi_I$ (Incident wave potential)
$+\Phi_D$ (Diffraction potential)
$+\Phi_R$ (Radiation potential)

Linearization

$\left(\frac{1}{2}\rho|\nabla\Phi|^2 = 0\right)$

$$P = -\rho g z - \rho\frac{\partial\Phi}{\partial t}$$

$$\mathbf{F}_{Fluid}(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}}) = \iint_{S_B} P\mathbf{n}dS = \mathbf{F}_{Buoyancy}(\mathbf{r}) + \mathbf{F}_{F.K}(\mathbf{r}) + \mathbf{F}_D(\mathbf{r}) + \mathbf{F}_R(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}})$$

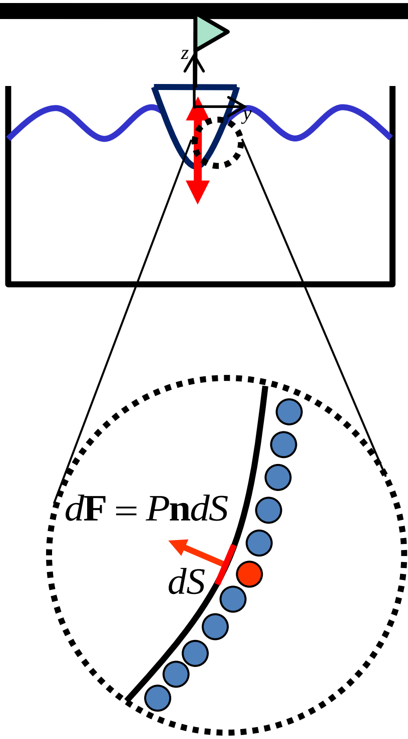
(S_B: wetted surface)

(displacement : $\mathbf{r} = [\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6]^T$)

$\xi_1 : surge$
$\xi_2 : sway$
$\xi_3 : heave$

$F_{F.K}$: Froude- krylov force
F_D: Diffraction force
F_R: Radiation force

Φ_I : Incident wave velocity potential
Φ_D : Diffraction potential
Φ_R : Radiation potential



✓ Surface forces: Fluid forces acting on a ship

$$\mathbf{F}_{Fluid}(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}}) = \iint_{S_B} P\mathbf{n}dS = \mathbf{F}_{Buoyancy}(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}}) + \mathbf{F}_{F.K}(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}}) + \mathbf{F}_D(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}}) + \mathbf{F}_R(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}})$$

: Fluid forces are obtained by integrating the fluid hydrostatic and hydrodynamic pressure over the wetted surface of a ship.

✓ 6 D.O.F equations of motion

Newton's 2[nd] Law

$$\mathbf{M}\ddot{\mathbf{r}} = \sum \mathbf{F} = \text{(Body Force)} + \boxed{\text{(Surface Force)}}$$

$$= \mathbf{F}_{gravity} + \mathbf{F}_{Fluid} + \mathbf{F}_{external}$$

Body force **Surface force**

$$\mathbf{M}\ddot{\mathbf{r}} = \boxed{\mathbf{F}_{Gravity}(\mathbf{r})} + \boxed{\mathbf{F}_{Buoyancy}(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}}) + \mathbf{F}_{Hydrodynamic}(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}})} + \boxed{\mathbf{F}_{external,\,dynamic} + \mathbf{F}_{external,\,static}}$$

Assume that forces are constant or proportional to the displacement, velocity and acceleration of the ship.

$$\mathbf{M}\ddot{\mathbf{r}} = \boxed{\mathbf{F}_{gravity}} + \boxed{\mathbf{F}_{Buoyancy}(\mathbf{r}) + \mathbf{F}_{F.K}(\mathbf{r}) + \mathbf{F}_D(\mathbf{r}) + \mathbf{F}_{R,Damping}(\mathbf{r},\dot{\mathbf{r}}) + \mathbf{F}_{R,Mass}(\mathbf{r},\ddot{\mathbf{r}})} + \boxed{\mathbf{F}_{ext,\,dynamic} + \mathbf{F}_{ext,\,static}}$$

$d\mathbf{F}$: Force of fluid elements acting on the infinitesimal surface of a ship

dS : Infinitesimal surface area

$\mathbf{n}$: Normal vector of the infinitesimal surface area

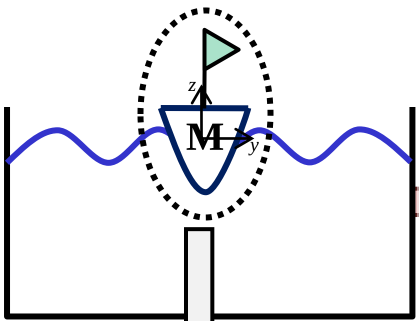
$$\mathbf{r} = [\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6]^T$$

ξ_1 : surge ξ_4 : roll
ξ_2 : sway ξ_5 : pitch
ξ_3 : heave ξ_6 : yaw

$\mathbf{M}_A$: 6×6 added mass matrix
$\mathbf{B}$: 6×6 damping coeff. matrix
$\mathbf{C}$: 6×6 restoring coeff. matrix

: Gravity Constant

Shear Force and Bending Moment in Waves



✓Equations of motions of Fluid Particles

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Shear force

Curl & Rotation

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Microscopic/ Macroscopic Derivation (RTT[1])

Mass Conservation Law ④⑤ → Laplace Equation

$$\nabla^2\Phi = 0 \quad (\mathbf{V} = \nabla\Phi)$$

$$\rho\frac{\partial\Phi}{\partial t} + P + \frac{1}{2}\rho|\nabla\Phi|^2 + \rho g z = 0$$

✓6 D.O.F equations of motions

① Coordinate system
(Waterplane Fixed & Body-fixed frame)

② Newton's 2nd Law

$$\mathbf{M}\ddot{\mathbf{r}} = \sum \mathbf{F} = (Body\ \ Force) + (Surface\ \ Force)$$

$$= \mathbf{F}_{gravity}(\mathbf{r}) \qquad + \qquad \boxed{\mathbf{F}_{Fluid}(\mathbf{r},\dot{\mathbf{r}},\ddot{\mathbf{r}})}$$

$$= \mathbf{F}_{gravity} + \mathbf{F}_{Buoyancy}(\mathbf{r}) + \mathbf{F}_{F.K}(\mathbf{r}) + \mathbf{F}_{D}(\mathbf{r})$$

$$+ \mathbf{F}_{R,Damping}(\mathbf{r},\dot{\mathbf{r}}) + \mathbf{F}_{R,Mass}(\mathbf{r},\ddot{\mathbf{r}})$$

Non-linear terms → Non-linear equation
→ Difficulty of getting analytic solution

Numerical Method

Velocity potential Φ

✓Calculation of Fluid Force

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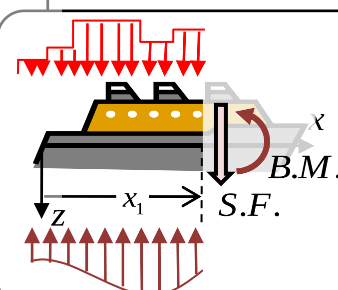
(S_B: wetted surface)



$-m(x)a_z$, $f_{Gravity}$

(a_z: Acceleration induced due to the heave & pitch motion)

$f_{F.K.}$, f_D , $-a_{33}a_z$,

$-b_{33}v_z$, f_{Static}

✓Shear force(S.F.) & Bending moment(B.M.)

Shear force(*S.F.*)

↓ Integration

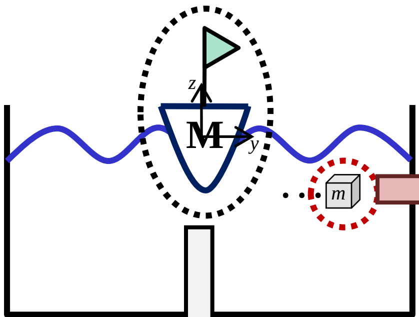
Bending moment(*B.M.*)

Shear Forces and Bending Moment in Waves
: Relationship with NAOE Undergraduate Courses

$F_{F.K}$: Froude- krylov force
F_D: Diffraction force
F_R: Radiation force

1) RTT : Reynold Transport Theorem
2) SWBM : Still Water Bending Moment
3) VWBM : Vertical Wave Bendidng Moment

✓ Assumption
① Newtonian fluid*
② Stokes Assumption**
③ invicid fluid
⑤ Incompressible flow



$\mathbf{r} = [\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6]^T$
ξ_1 : surge , ξ_4 : roll
ξ_2 : sway , ξ_5 : pitch
ξ_3 : heave , ξ_6 : yaw

Ship Hydrodynamics, Dynamics
(2nd-year undergraduate)

✓Equations of motions of Fluid Particles

Lagrangian & Eulerian Description

Shear stress

Curl & Rotation

Newton's 2nd Law
$m\ddot{\mathbf{r}} = \sum \mathbf{F} = (\text{Body force} + \text{Surface force})$

Cauchy equation → ①② Navier-Stokes equation → ③ Euler equation → ④⑤ Bernoulli equation

Mass Conservation Law ④⑤ Laplace

$(\mathbf{V} = \nabla\Phi)$

$\nabla^2\Phi = 0$

$$\rho\frac{\partial\Phi}{\partial t} + P + \frac{1}{2}\rho|\nabla\Phi|^2 + \rho g z = 0$$

Engineering Math.
(2nd-year undergraduate)

Microscopic/
Macroscopic Derivation(RTT[1])

✓6 D.O.F equations of motion

Behavior of ship and its control
(3rd -year undergraduate)
Dynamics (2nd -year undergraduate)

① Coordinate system
(Waterplane Fixed & Body-fixed frame)

② Newton's 2nd Law

$$\mathbf{M}\ddot{\mathbf{r}} = \sum \mathbf{F} = (Body\ \ Force) + (Surface\ \ Force)$$

$$= \mathbf{F}_{gravity}(\mathbf{r}) + \mathbf{F}_{Fluid}(\mathbf{r}, \dot{\mathbf{r}}, \ddot{\mathbf{r}})$$

$$= \mathbf{F}_{gravity} + \mathbf{F}_{Buoyancy}(\mathbf{r}) + \mathbf{F}_{F.K}(\mathbf{r}) + \mathbf{F}_D(\mathbf{r})$$

$$\ldots(\mathbf{r}, \dot{\mathbf{r}}) + \mathbf{F}_{R,Mass}(\mathbf{r}, \ddot{\mathbf{r}})$$

Planning of NAOE(Ship Stability)
(2nd-year undergraduate)

Non-linear terms → Non-linear equation
→ Difficulty of getting analytic solution

Numerical Method

Computer aided ship design
(3rd-year undergraduate)

Velocity potential Φ

$\Phi = \Phi_I$ (Incident wave potential)
$+\Phi_D$ (Diffraction potential)
$+\Phi_R$ (Radiation potential)

✓Calculation of Fluid Force

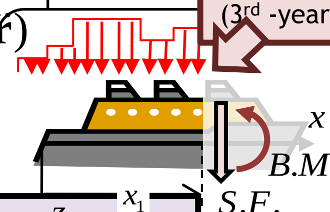
Ocean environment Information system
(3rd -year undergraduate)

$$P = -\rho g z - \rho\frac{\partial\Phi}{\partial t}$$

$$\mathbf{F}_{Fluid}(\mathbf{r}, \dot{\mathbf{r}}, \ddot{\mathbf{r}}) = \iint_{S_B} P\mathbf{n}dS = \mathbf{F}_{Buoyancy}(\mathbf{r}) + \mathbf{F}_{F.K}(\mathbf{r}) + \mathbf{F}_D(\mathbf{r}) + \mathbf{F}_R(\mathbf{r}, \dot{\mathbf{r}}, \ddot{\mathbf{r}})$$

Ship Structural Design system
(3rd -year undergraduate)

Fundamental of maritime Structural statics
(2nd -year undergraduate)



$f_{Gravity}$
(a_z : Acceleration of z direction by heave& pitch motion)

$f_{F.K.}$, f_D , $-a_{33}a_z$,
$-b_{33}v_z$, f_{Static}

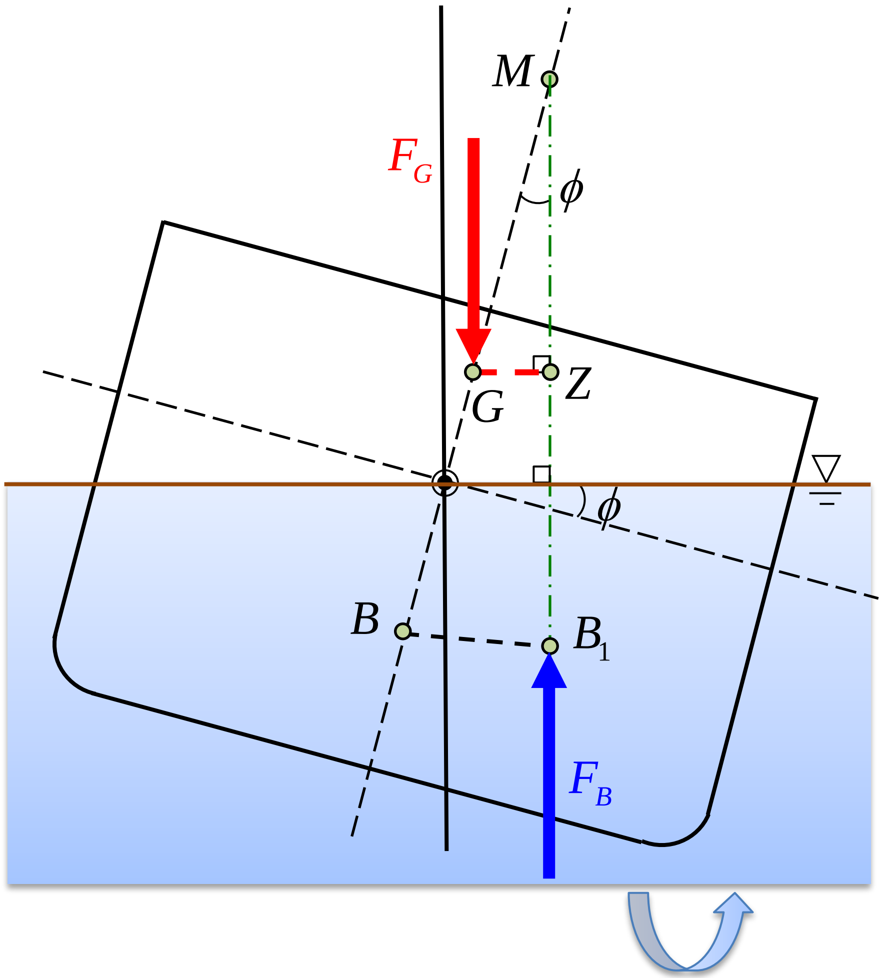
bending moment(B.M.)

Shear force(S.F.)
↓ Integral
Bending moment(B.M.)

2-12 Roll Period

Required Min. Roll Period , for example, min. 12 sec:

I_a : added moment of inertia
b : damping moment coefficient



$$\tau_r = GZ \cdot F_B$$

G: Center of mass of a ship
B: Center of buoyancy at initial position
$\mathbf{F}_G$: Gravitational force of a ship
$\mathbf{F}_B$: Buoyant force acting on a ship
M : Metacenter

Derivation of the equation of roll motion of a ship :

$I\ddot{\phi} = \sum \tau$ **(Euler equation)**

$$= \tau_{body} + \tau_{surface}$$

$$= \tau_{gravity} + \tau_{fluid}$$

$$= \tau_{gravity} + \tau_{hydrostatic} + \tau_{F.K} + \tau_{diffraction} + \tau_{radiation}$$

$$= \tau_r + \tau_{exciting} - I_a\ddot{\phi} - b\dot{\phi}$$

$$\tau_r = GZ \cdot F_B$$
$$\approx GM \cdot \sin\phi \cdot F_B$$
$$= GM \cdot \sin\phi \cdot \rho g V$$
$$\approx GM \cdot \phi \cdot \rho g V$$

For small ϕ $\sin\phi \cong \phi$

$$= -\rho g V \cdot GM \cdot \phi + \tau_{exciting} - I_{add}\ddot{\phi} - b\dot{\phi}$$

Equation of roll motion of a ship

$$\therefore (I + I_a) \cdot \ddot{\phi} + b\dot{\phi} + (\rho g V \cdot GM) \cdot \phi = \tau_{exciting}$$

Calculation of Natural Roll Period

$$(I+I_a)\cdot\ddot{\phi}+b\dot{\phi}+\left(\rho gV\cdot GM\right)\cdot\phi=\tau_{exciting}$$ ← **Second order Linear Ordinary Differential Equation**

- **Objectives : Find the natural frequency of roll motion**
  : No exciting moment $\left(\tau_{exciting}=0\right)$
- **Assumption : No damping moment** $\left(b\dot{\phi}=0\right)$

$$(I+I_a)\cdot\ddot{\phi}+\left(\rho gV\cdot GM\right)\cdot\phi=0$$

Try: $\phi=e^{\lambda t}$

$$(I+I_a)\cdot\lambda^2\cdot e^{\lambda t}+\left(\rho gV\cdot GM\right)\cdot e^{\lambda t}=0$$

$$\left\{(I+I_a)\cdot\lambda^2+\left(\rho gV\cdot GM\right)\right\}\cdot e^{\lambda t}=0$$

$$(I+I_a)\cdot\lambda^2+\left(\rho gV\cdot GM\right)=0\ ,(\because e^{\lambda t}\neq 0)$$

$$\lambda_{1,2}=\pm\sqrt{\frac{\rho gV\cdot GM}{I+I_a}}\cdot i$$

$$\therefore\phi=C_1e^{\lambda_1\cdot t}+C_2e^{\lambda_2\cdot t}=C_1e^{\sqrt{\frac{\rho gV\cdot GM}{I+I_a}}\cdot i\cdot t}+C_2e^{-\sqrt{\frac{\rho gV\cdot GM}{I+I_a}}\cdot i\cdot t}$$

$$\phi = C_1 e^{\sqrt{\frac{\rho g V \cdot GM}{I + I_a}} \cdot i \cdot t} + C_2 e^{-\sqrt{\frac{\rho g V \cdot GM}{I + I_a}} \cdot i \cdot t}$$

Euler's formula $\left(e^{i\phi} = \cos\phi + i\sin\phi\right)$

$$\boxed{\phi = C_1 \cos\left(\boxed{\sqrt{\frac{\rho g V \cdot GM}{I + I_a}}} \cdot t\right) + C_2 \sin\left(\boxed{\sqrt{\frac{\rho g V \cdot GM}{I + I_a}}} \cdot t\right)}$$

Angular frequency (ω)

Because $\omega = \dfrac{2\pi}{T_\phi}$ **, the natural roll period is as follows:**

$$T_\phi = \frac{2\pi}{\omega}$$

$$= 2\pi \sqrt{\frac{I + I_a}{\rho g V \cdot GM}}$$

Effect of GM on the Natural Roll Period

Roll period

$$T_\phi = 2\pi\sqrt{\frac{I+I_a}{\rho g V \cdot GM}}$$

(Assumption) $I + I_a \cong (k \cdot B)^2 \cdot \rho \cdot V$

✓ 2k : 0.32 ~ 0.39 for full load condition
✓ 2k : 0.37 ~ 0.40 for ballast condition

$$T_\phi = 2\pi\sqrt{\frac{(k \cdot B)^2 \cdot \rho \cdot V}{\rho \cdot g \cdot V \cdot GM}}$$

$$\approx \frac{2\pi \cdot k \cdot B}{\sqrt{g \cdot GM}}$$

$$\boxed{= \frac{2 \cdot k \cdot B}{\sqrt{GM}}} \quad ,(\because \sqrt{g} \approx \pi)$$

Does a ship in a light condition roll quickly or slowly? What does this indicate?

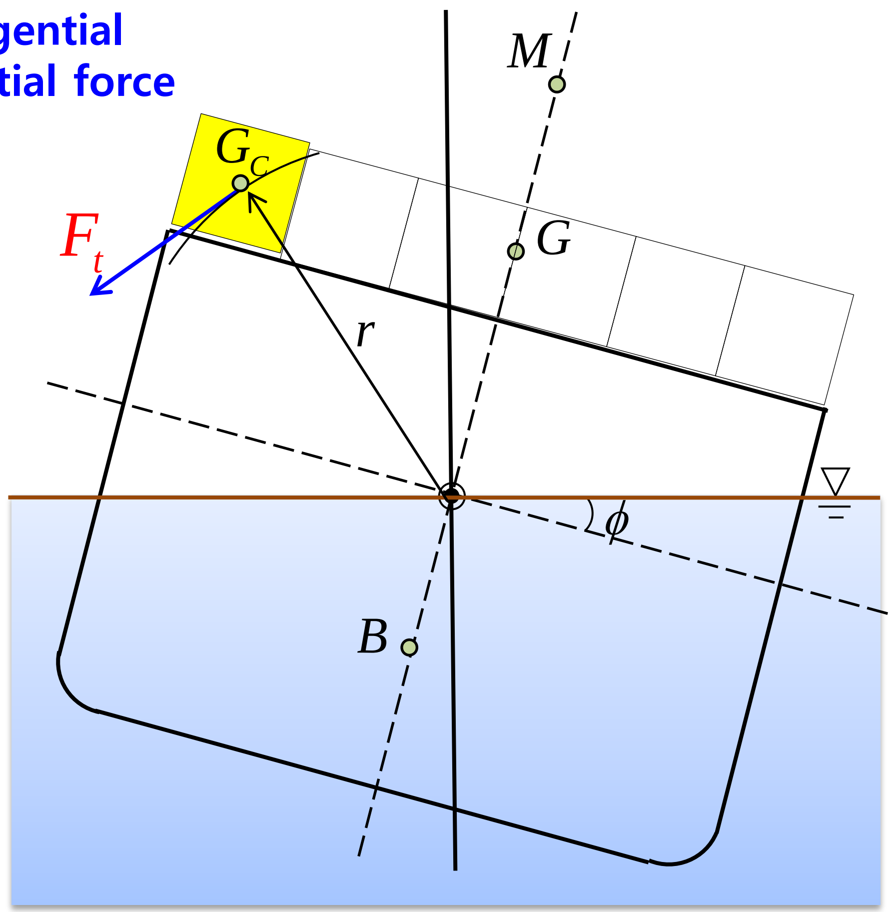
Approximate Roll period of ship

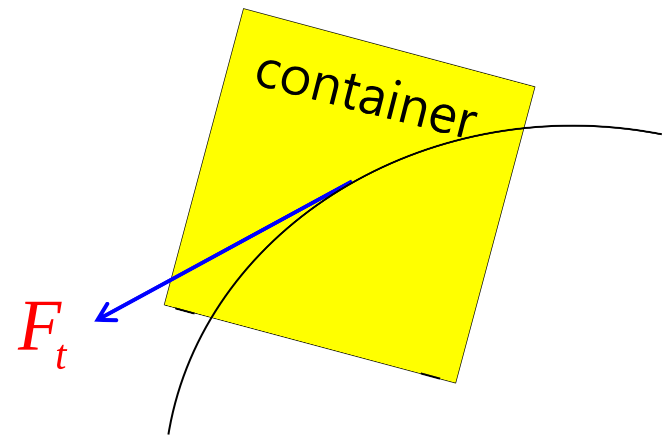
That is, a stiff ship or crank ship, one with a large metacentric height will roll quickly whereas a tender ship, one with a small metacentric height, will roll slowly.

Effect of GM on the Tangential Inertial Force due to the Roll Motion

< Container Carrier >

Tangential inertial force





Effect of GM on the Angular acceleration of a ship

$$\omega = \sqrt{\frac{\rho g V \cdot GM}{I + I_A}}$$

Roll motion of a ship

$$\phi = C_1 \cos(\omega \cdot t) + C_2 \sin(\omega \cdot t)$$

$$\Downarrow$$

$$\phi = \sqrt{C_1^{\ 2} + C_2^{\ 2}} \cos(\omega \cdot t + \beta) \qquad , \beta : \text{phase}$$

Angular acceleration of a ship :

$$\ddot{\phi} = \sqrt{C_1^{\ 2} + C_2^{\ 2}}\, \omega^2 \cos(\omega \cdot t + \beta)$$

$$= A\omega^2 \cos(\omega \cdot t + \beta) \quad , (A = \sqrt{C_1^{\ 2} + C_2^{\ 2}})$$

Chapter 3. Design Model

Problem Statement for Ship Design

✓ **Given:**

✓ **Deadweight(DWT),**

✓ **Cargo hold capacity(V_{CH}),**

✓ **Service speed(V_s),**

✓ **Daily Fuel Oil Consumption(DFOC), Endurance, etc.**

✓ **Determine:** L, B, D, T, C_B

3-1 Determination of the Principal Dimensions by the Weight Equation

Determination of the Principal Dimensions by the Weight Equation

- **Weight equation**

Dimensions of a deadweight carrier whose design is weight critical are determined by the following equation.

$$\rho \cdot L \cdot B \cdot T \cdot C_B \cdot (1+\alpha) = \boxed{DWT + LWT} \quad \ldots(3)$$

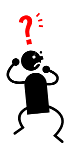
✓ **Given:** $\boldsymbol{DWT}$ **(owner's requirement)**

✓ **Find:** $\boldsymbol{L, B, T, C_B}$

ρ :density of sea water = 1.025 Mg/m^3 = 1.025 ton/m^3
α : **a fraction of the shell appendage allowance** , displacement of shell plating and appendages as a fraction of the moulded displacement

$$\boxed{DWT + LWT} = W_{Total}$$

Deadweight is given by owner's requirement, whereas total weight is not a given value.
Thus, lightweight should be estimated by appropriate assumption.



How can you estimate the LWT **?**

Assume that the lightweight is the same as that of the basis ship

At the early design stage, there are few data available for the estimation of the lightweight.
The simplest possible way of estimating the lightweight is to assume that the lightweight does not change in the variation of the principal dimensions.

<u>Method</u> 1 : Assume that the lightweight is the same as that of the basis ship.

$$LWT = LWT_{Basis}$$

$$L \cdot B \cdot T \cdot C_B \cdot \rho \cdot (1+\alpha) = DWT + LWT_{Basis} \qquad ...(4.1)$$

It will be noted that finding a solution for this equation is a complex matter, because there are 4 unknown variables (L, B, T, C_B) with one equation, that means this equation is a kind of indeterminate equation.
Moreover, the unknown variables are multiplied by each other, that means this equation is a kind of nonlinear equation.

$$L \cdot B \cdot T \cdot C_B \cdot \rho \cdot (1+\alpha) = W$$
$$= DWT + LWT_{Basis} \ldots(4.1)$$

The equation (4.1) is called nonlinear indeterminate equation which has infinitely many solutions.

- **Therefore, we have to assume three unknown variables to solve this indeterminate equation.**

- **The principal dimensions must be obtained by successive iteration until the displacement becomes equal to the total weight of ship.** (∵ nonlinear equation)

- **We can have many sets of solution by assuming different initial values.**
  (∵ indeterminate equation)
  Thus, we need a certain criteria to select proper solution.

For example, this is the first set of solution.

The ratios of the principal dimensions L/B, B/T, B/D and C_B can be obtained from the basis ship.

Substituting the ratios obtained from the basis ship into the equation (4.1), the equation can be converted to a cubic equation in L.

$$L \cdot B \cdot T \cdot C_B \cdot \rho \cdot (1+\alpha) = W$$

$$L \cdot \left(L \cdot \frac{B}{L} \right) \cdot \left(\boxed{B} \cdot \frac{T}{B} \right) \cdot C_B \cdot \rho \cdot (1+\alpha) = W$$

$$L^2 \cdot \left(\frac{B}{L} \right) \cdot \left(\boxed{L. \frac{B}{L}} \right) \cdot \left(\frac{T}{B} \right) \cdot C_B \cdot \rho \cdot (1+\alpha) = W$$

$$L^3 \cdot \left(\frac{B}{L} \right)^2 \cdot \left(\frac{T}{B} \right) \cdot C_B \cdot \rho \cdot (1+\alpha) = W$$

$$\Rightarrow L = \left(\frac{W \cdot (L/B)_{Basis}{}^2 \cdot (B/T)_{Basis}}{\rho \cdot C_{B\_Basis} \cdot (1+\alpha)} \right)^{1/3}$$

Assume that the total weight(W) is proportional to the deadweight.

- **Weight equation of a ship**

$$\rho \cdot L \cdot B \cdot T \cdot C_B \cdot (1+\alpha) = W$$
$$= DWT + LWT \quad ...(3)$$

Given: DWT **,Find:** L, B, T, C_B

Method ②: $W = \dfrac{W_{Basis}}{DWT_{Basis}} \cdot DWT$

Since the lightweight is assumed to be invariant in the 'Method 1', even though the principal dimensions are changed, the method might give too rough estimation.

How can you estimate the lightweight more accurately than the 'Method 1'?

Method 2: Design ship and basis ship are assumed to have the same ratio of deadweight to total weight.

$$\boxed{\frac{DWT_{Basis}}{W_{Basis}} = \frac{DWT}{W}}$$

Therefore, the total weight of design ship can be estimated by the ratio of deadweight to total weight of the basis ship.

$$L \cdot B \cdot T \cdot C_B \cdot \rho \cdot (1+\alpha) = W \quad ...(4.2)$$

Assume that the lightweight could vary as the volume of the ship

- **Weight equation of a ship**

$$\rho \cdot L \cdot B \cdot T \cdot C_B \cdot (1+\alpha) = W = DWT + LWT \quad ...(3)$$

Given: DWT **,Find:** L, B, T, C_B

Method ③: $LWT = C_{LWT} L \cdot B \cdot D$

The lightweight estimated in the 'Method 2' still has nothing to do with the variation of the principal dimensions.

How can you estimate the lightweight more accurately than the 'Method 2'?

Assume that the lightweight is dependent on the principal dimensions such as L, B and D.

$$LWT = f(L, B, D)$$

To estimate the lightweight, we will introduce the volume variable L·B·D and assume that LWT is proportional to L·B·D

$$\boxed{LWT = C_{LWT} \cdot L \cdot B \cdot D}$$

where coefficient C_{LWT} can be obtained from the basis ship.

$$L \cdot B \cdot T \cdot C_B \cdot \rho \cdot (1+\alpha) = DWT + C_{LWT} \cdot L \cdot B \cdot D \quad ...(4.3)$$

Estimate the structural weight(W_s), outfit weight(W_o) ,machinery weight(W_m) in components.

- **Weight equation of a ship**

$$\rho \cdot L \cdot B \cdot T \cdot C_B \cdot (1+\alpha) = W$$
$$= DWT + \boxed{LWT} \quad \ldots(3)$$

Given: DWT **,Find:** L, B, T, C_B

Method ④: $LWT = \boxed{W_s} + W_o + W_m$

How can you estimate lightweight more accurately?

We assume that a ship is composed of hull structure, outfit ,and machinery. Based on this assumption, the lightweight estimation would be more accurate, if we could estimate the weight of each components.

Method 4 : Estimate the structural weight(W_s), outfit weight(W_o), and machinery weight(W_m) in components.

$$\boxed{LWT = W_s + W_o + W_m}$$

How can you estimate W_s, W_o, W_m ?

Assume that W_s, W_o, W_m are dependent on the principal dimensions

Steel Weight Estimation : Method 4-(1)

$$LWT = \boxed{W_s} + W_o + W_m$$

Assume that the structural weight(W_s) is a function of L, B, D as follows:

$$W_s = f(L, B, D)$$

Since the structural weight of a ship is actually composed of stiffened plate surfaces, some type of 'area variables' would be expected to provide a better correlation.

To estimate the structural weight, we will introduce an '<u>area variables' such as L·B or B·D.</u>

$$W_s = f(L \cdot B, B \cdot D)$$

For example, assume that structural weight <u>is proportional to L^α, $(B+D)^\beta$</u>

<u>Method 4-(1)</u>

$$\boxed{W_s = C_s \cdot L^\alpha (B+D)^\beta}$$

Unknown parameters (C_s, α, β) can be obtained from as-built ship data <u>by regression analysis*.</u>

*Regression analysis is a numerical method which can be used to develop equations or models from data when there is no or limited physical or theoretical basis for a specific model.
It is very useful in developing parametric models for use at the early design stage.

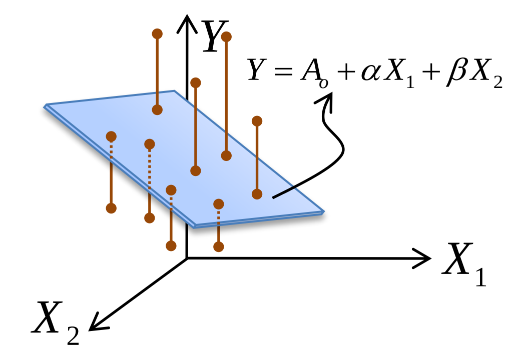
Regression Analysis to obtain a Formula for the Steel weight Estimation

$$\boxed{W_s = C_s L^{\alpha} (B+D)^{\beta}}$$

a) In order to perform the regression analysis, we transform the above nonlinear equation into the linear equation by applying logarithmic operation on both sides, then we have a logarithmic form

$$\underbrace{\ln W_s}_{Y} = \underbrace{\ln C_S}_{A_0} + \alpha \underbrace{\ln L}_{X_1} + \beta \underbrace{\ln(B+D)}_{X_2} \quad \text{: Logarithmic Form}$$

$$\rightarrow Y = A_o + \alpha X_1 + \beta X_2 \quad \text{: Linear Equation}$$



Regression analysis plane for data on the variables Y, X_1 and X_2

b) If sets of as-built ship data ($X_{1i}, X_{2i}; Y_i$) are available,

then, the parameters can be obtained by finding a function that minimize the sum of the squared errors, "least square method", which is the difference between the sets of the data and the estimated function values.

$$\rightarrow C_S, \alpha = 1.6, \ \beta = 1$$

$$\boxed{W_s = C_s \cdot L^{1.6} \cdot (B+D)}$$

e.g. 302K VLCC : $C_s = 0.0414$

Above equation reflects that length(L) will exponentially affect on the steel weight much more than other variables, B and D.

Outfit Weight Estimation : Method 4-(2)

$$LWT = W_s + \boxed{W_o} + W_m$$

Assume that the outfit weight(W_o) is a function of L, B **:** $W_o = f(L, B)$

To estimate the outfit weight, we will use the area variable $L \cdot B$**.**

$$W_o = f(L \cdot B)$$

For example, assume that outfit weight (W_o) is proportional to $L \cdot B$

$$\boxed{W_o = C_o \cdot L \cdot B}$$

where coefficient C_o **can be obtained from the basis ship.**

W_s : structural weight
W_o : outfit weight
W_m : machinery weight

Machinery Weight Estimation : Method 4-(3)

$$LWT = W_s + W_o + \boxed{W_m}$$

To estimate the machinery weight, assume that the machinery weight(W_s) is a function of $NMCR$:

$$W_m = f(NMCR)$$

For example, assume that machinery weight is proportional to $NMCR$:

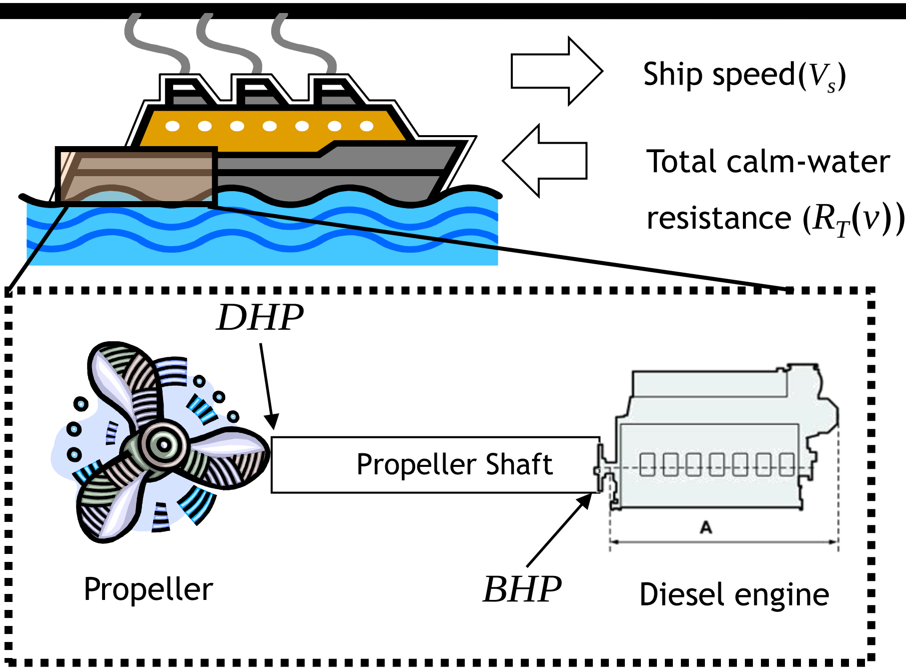
$$\boxed{W_m = C_m \cdot NMCR}$$

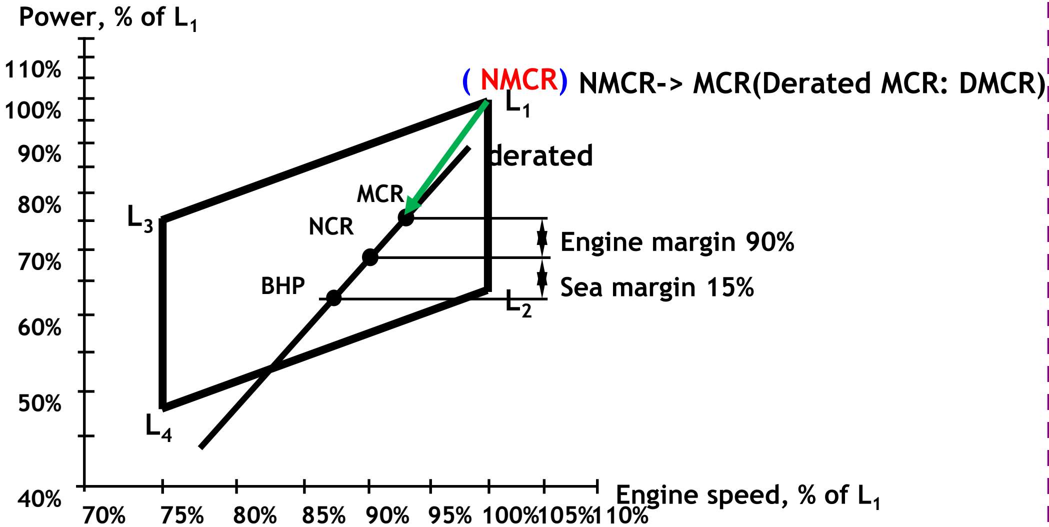
where coefficient Cm **can be obtained from the basis ship.**

*** NMCR (Nominal maximum continuous rating) is the maximum power/speed combination available for the engine and is a criteria for the dimensions, weight, capacity, and cost of the engine.**

Then, how can you estimate the $NMCR$**?**

Estimation of the NMCR (Nominal Maximum Continuous Rating)





① EHP (Effective Horse Power)

$$EHP = R_T(v) \cdot V_s \quad \text{(In Calm water)}$$

② DHP (Delivered Horse Power)

$$DHP = \frac{EHP}{\eta_D} \quad (\eta_D : \text{Propulsive efficiency})$$

③ BHP (Brake Horse Power in calm water)

$$BHP = \frac{DHP}{\eta_T} \quad (\eta_T : \text{Transmission efficiency})$$

④ NCR (Normal Continuous Rating)

$$NCR = BHP\left(1 + \frac{\text{Sea Margin}}{100}\right)$$

⑤ DMCR (Derated Maximum Continuous Rating)

$$DMCR = \frac{NCR}{\text{Engine Margin}}$$

⑥ NMCR (Nominal Maximum Continuous Rating)

$$\boxed{NMCR = \frac{DMCR}{\text{Derating rate}}}$$

Estimation of the NMCR by Admiralty formula

$$\boxed{W_m = C_m \cdot NMCR}$$

NMCR **can be estimated based on the prediction of resistance and propulsion power. However, there are few data available for the estimation of the *NMCR* at the early design stage, *NMCR* <u>can be approximatelty estimated by empirical formula such as 'Admiralty formula'</u>**

Admiralty formula :

$$DHP_{Calm\,water} = f(\Delta, V_s)$$

C_{ad} : Admiralty coefficient
V_s : speed of ship [knots]
Δ : displacement [ton]

$$\Downarrow$$

$$DHP_{Calm\,water} = C_{DHP} \cdot \Delta^{2/3} \cdot V_s^{\,3}$$

$$\Downarrow$$

Define $C_{ad} \equiv \dfrac{1}{C_{DHP}}$

C_{ad} **is called "Admiralty coefficient".**

$$\boxed{DHP_{Calm\,water} = \frac{\Delta^{2/3} \cdot V_s^{\,3}}{C_{ad}}}$$

Admiralty coefficient : A Kind of Propulsive Efficiency(η_D)

Admiralty formula :

$$\boxed{DHP_{Calm\,water} = \frac{\Delta^{2/3} \cdot V^3}{C_{ad}}}$$

C_{ad} **: Admiralty coefficient**

$$C_{ad} = \frac{\Delta^{2/3} \cdot V^3}{DHP_{Calm\,water}}$$

-Since $\Delta^{2/3} \cdot V_s^3$ is proportional to $\boldsymbol{EHP}$, the Admiralty coefficient can be regarded as a kind of the propulsive efficiency(η_D).

$$\eta_D = \frac{EHP}{DHP}$$

- However, this should be used only for a rough estimation. After the principal dimensions are determined, $\boldsymbol{DHP}$ needs to be estimated more accurately based on the resistance and power prediction.

(ref. : Resistance estimation, Speed-Power Prediction)

Machinery Weight in terms of Principal Dimensions

$$W_m = C_m \cdot NMCR$$

$$NMCR = \frac{1}{\eta_T} \cdot (1 + \frac{\text{Sea Margine}}{100}) \cdot \frac{1}{\text{Engine Margin}} \cdot \frac{1}{\text{Derating ratio}} \cdot DHP_{Calm\,water}$$

$$= C_1 \cdot DHP_{Calm\,water}$$

$$DHP_{Calm\,water} = \frac{\Delta^{2/3} \cdot V_s^{\,3}}{C_{ad}}$$,(Admiralty formula)

$$\Delta = \rho \cdot L \cdot B \cdot T \cdot C_B \cdot (1 + \alpha)$$

$$W_m = C_m \cdot \frac{C_1}{C_{ad}} \cdot (\rho \cdot L \cdot B \cdot T \cdot C_B \cdot (1+\alpha))^{2/3} \cdot V_s^{\,3}$$

Define

$$C_{power} \equiv C_m \cdot \frac{C_1}{C_{ad}}$$

$$\boxed{W_m = C_{power} \cdot (\rho \cdot L \cdot B \cdot T \cdot C_B \cdot (1+\alpha))^{2/3} \cdot V_s^{\,3}}$$

- If the machinery weight is changed due to the changed $NMCR$, the principal dimension must be adjusted to the changed machinery weight.

Determination of the principal dimensions by the weight equation

- **Weight equation of a ship**

$$\rho \cdot L \cdot B \cdot T \cdot C_B \cdot (1+\alpha) = W$$
$$= DWT + LWT \quad \ldots(3)$$

Given: DWT **,Find:** L, B, T, C_B

Method ④: $LWT = W_s + W_o + W_m$

$W_m = C_m \cdot NMCR$

$$L \cdot B \cdot T \cdot C_B \cdot \rho \cdot (1+\alpha) = DWT + LWT \cdots (3)$$

$$LWT = W_s + W_o + W_m$$

$$\begin{cases} W_s = C_s \cdot L^{1.6} \cdot (B+D) \\ W_o = C_o \cdot L \cdot B \\ W_m = C_m \cdot NMCR \\ \quad = C_{power} \cdot (L \cdot B \cdot T \cdot C_B \cdot (1+\alpha))^{2/3} \cdot V_s^{\,3} \end{cases}$$

W_s : structural weight
W_o : outfit weight
W_m : machinery weight
V_s : speed of ship
Δ : displacement
ρ : density of sea water [ton/m^3]

$$L \cdot B \cdot T \cdot C_B \cdot \rho \cdot (1+\alpha) = DWT + C_s \cdot L^{1.6} \cdot (B+D) + C_o \cdot L \cdot B + C_{power} \cdot (\rho \cdot L \cdot B \cdot T \cdot C_B \cdot (1+\alpha))^{2/3} \cdot V_s^{\,3} \quad \ldots(4.4)$$

It will be noted that finding a solution for this equation is a complex matter, because there are 5 unknown variables (L, B, D, T, C_B) with one equation, that means this equation is a kind of indeterminate equation. Moreover, the unknown variables are multiplied by each other, that means this equation is a kind of nonlinear equation. Therefore, we have to assume four unknown variables to solve this indeterminate equation. The principal dimensions must be obtained by successive iteration until the displacement becomes equal to the total weight of ship(∵ nonlinear equation). We can have many sets of solution by assuming different initial values. (∵ indeterminate equation). Thus, we need a certain criteria to select proper solution.

Criteria to select proper solution: Objective Function

What kind of Criteria is available to select proper solution?

Possible Criteria (Objective Function)

- For Shipbuilding Company : Shipbuilding Cost. (See Ref.)
- For Shipping Company :
  - Less Power→ Less Energy Consumption→ Minimum Operational Expenditure (OPEX) (See Ref.)
  - Operability→ Required Freight Rate(RFR).
  - Minimum Capital Expenditure(CAPEX)
  - Minimum Main Engine Power/ DWT

For example, shipping company will adopt objective function as RFR, then the design ship should have the least RFR expressed as:

$$\boxed{RFR = \frac{\text{Capital cost+Annual operating cost}}{\text{Annual transported cargo quantity}}}$$

*Capital cost=Building cost $\times$ Capital recovery factor.

*CRF(Capital Recovery Factor) $= \dfrac{i(1+i)^n}{(1+i)^n - 1}$

3-2 Block Coefficient

Recommended Value for Block Coefficient

- **Recommended value for obesity coefficient considering maneuverability:**

$$C_B/(L/B) \le 0.15$$

- **Recommended value for C_B proposed by Watson & Gilfillan:**

This formula seems to confirm its continuing validity and many naval architects are using this equation up to now.

$$C_B \le 0.70 + 0.125\tan^{-1}((23 - 100Fn)/4)$$

3-3 Determination of the Principal Dimensions by the Volume Equation

Determination of the Principal Dimensions by the Volume Equation

- **Economical constraint : Required cargo hold capacity[m^3]**

  - **P**rincipal dimensions have to satisfy the required cargo hold capacity.

  The dimensions of a **volume carrier** whose design is **volume critical** can be determined by the following equation.

$$\boxed{V_{CH} = f(L, B, D)}$$ ⇨ **Volume equation of a ship**

✓ **Given: Cargo hold capacity(Vch)[m^3]**

✓ **Find:** $\boldsymbol{L, B, D}$

How can you represent the cargo hold capacity in terms of the principal dimensions?

Determination of the Principal Dimensions by the Volume Equation – Method 1

- Volume equation of a ship

$$V_{CH} = f(L, B, D)$$

Given: *Cargo hold capacity*, Find: L, B, D

Method ①: $f(L,B,D) = C_{CH} \cdot L \cdot B \cdot D$

How can you estimate the cargo hold capacity?

Method 1 : Assume that the cargo hold capacity is proportional to $(L \cdot B \cdot D)$.
$V_{CH} = C_{CH} \cdot L \cdot B \cdot D$

where coefficient C_{CH} can be obtained from the basis ship.

It will be noted that finding a solution to this equation is a complex matter, because there are 3 unknown variables L, B, D with one equation, that means this equation is also a kind of indeterminate equation.
Moreover, the unknown variables are multiplied by each other, that means this equation is a kind of nonlinear equation.

This kind of equation is called a nonlinear indeterminate equation, which has infinitely many solutions.

Determination of the Principal Dimensions by the Volume Equation – Method 2

- **Volume equation of a ship**

$$V_{CH} = f(L, B, D)$$

Given: *Cargo hold capacity* **,Find:** L, B, D

Method ②: $f(L, B, D) = C_{CH} \cdot L_H \cdot B \cdot D \cdot C_{MD}$

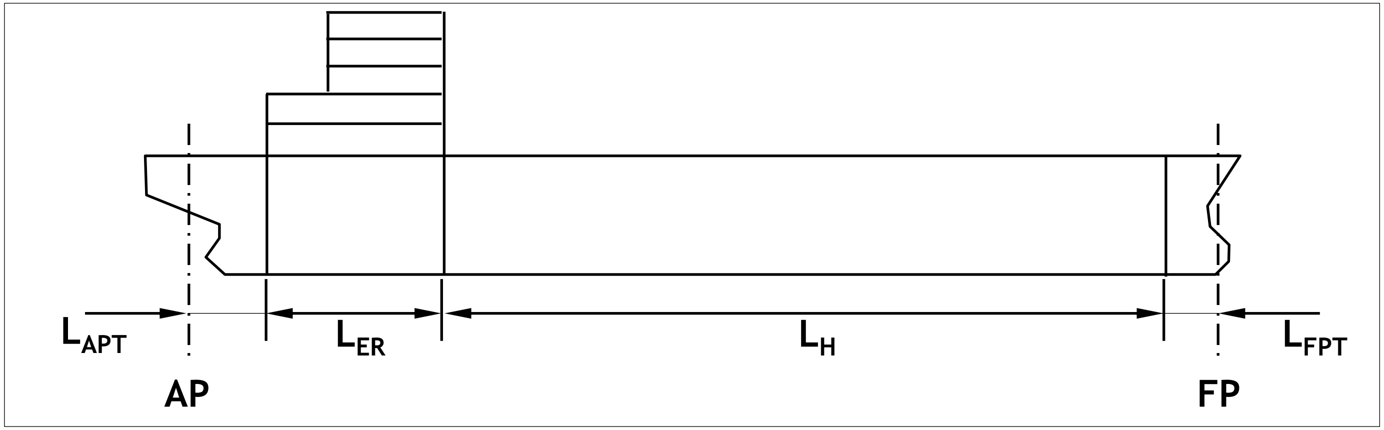
Hold capacity can be **estimated more accurately** by using the length of cargo hold(L_{H}) instead of the ship's length(L)

$$V_{CH} = C_{CH} \cdot L_H \cdot B \cdot D$$

L_H : Length of the cargo hold

The Length of cargo hold(L_H) is defined as being L_{BP} subtracted by L_{APT}, L_{ER} and L_{FPT} :

$$L_H = L_{BP} - L_{APT} - L_{ER} - L_{FPT}$$



L_{BP} : Length between perpendicular
L_{APT} : Length between aft perpendicular to aft bulkhead
L_{FPT} : Length between forward perpendicular to collision bulkhead
L_{ER} : Length of engine room

The coefficients(C_{CH}) and partial lengths, L_{APT}, L_{ER} and L_{FPT} can be obtained from the basis ship.

Summary:
Determination of the Principal Dimensions by the Volume Equation

Method 1 : Assume that the cargo hold capacity is proportional to L·B·D.

$$V_{CH} = C_{CH} \cdot L \cdot B \cdot D$$

Method 2 : Assume that the cargo hold capacity is proportional to LH·B·D.

$$V_{CH} = C_{CH} \cdot L_{CH} \cdot B \cdot D$$

Since the method 1 and 2 are used for a rough estimation, cargo hold capacity should be estimated more accurately after the arrangement of compartment has been made.

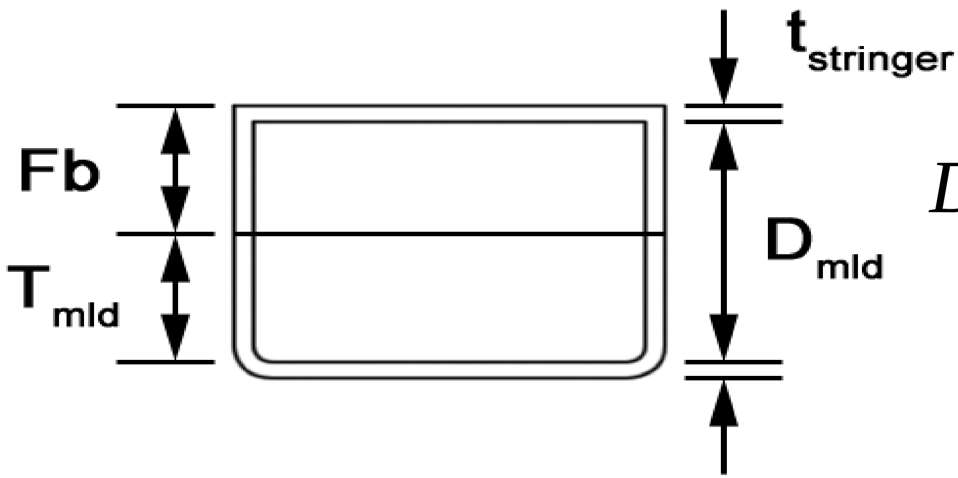
3-4 Freeboard

What is Freeboard* ?

•ICLL(International Convention on Load Lines) 1966 Regulatory constraint

- Ships need safety margin to maintain buoyancy and stability while operating at sea.

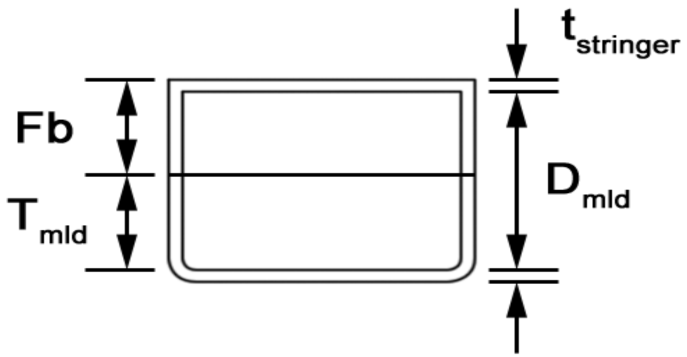
- This safety margin is provided by the reserve of buoyancy of the hull located above the water surface.



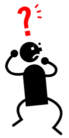
$$D_{Fb} = D_{mld} + t_{stringer}$$

*Freeboard(Fb) means the distance between the water surface and the top of the deck at the side(at the deck line). It includes the thickness of freeboard deck plating.

✓ **Actual freeboard ($D_{Fb} - T$) of a ship should not be less than the required freeboard(Fb) determined in accordance with the freeboard regulation.**



$$\boxed{D_{Fb} - T \;\geq\; Fb\,(L, B, D_{mld}, C_B)}$$



How can you determine the required freeboard(Fb) ?

- **Volume equation of a ship**

$$D_{Fb} \geq T + Fb(L, B, D_{mld}, C_B)$$

Given: $L, B, D(=D_{mld}), T, C_B$,**Check:** Satisfaction of the freeboard regulation

$$Fb(L, B, D, C_B) = C_{FB} \cdot D$$

How can you determine the required freeboard(Fb)?

At the early design stage, there are few data available to calculate required freeboard. Thus, the required freeboard can be roughly estimated from the basis ship.

Assume that the freeboard is proportional to the depth.

$$Fb(L, B, D_{mld}, C_B) = C_{Fb} \cdot D_{mld}$$

$$D_{Fb} \geq T + C_{Fb} \cdot D_{mld}$$

where coefficient C_{Fb} can be obtained from the basis ship.

In progress of the design, however, the required freeboard has to be calculated in accordance with ICLL 1966.

$$Fb(L, B, D_{mld}, C_B) = f(L_f, D_{mld}, C_B, \text{Superstructure}_{\text{Length}}, \text{Superstructure}_{\text{Height}}, \text{Sheer})$$

If ICLL 1966 regulation is not satisfied, the depth should be changed.

3-5 Estimation of Shipbuilding Cost

Objective Function(criteria to select the proper main dimensions)

Assume that the shipbuilding cost is proportional to the weight of the ship.

$$Building\ \ Cost = C_{PS} \cdot W_S + C_{PO} \cdot W_O + C_{PM} \cdot W_M$$

If the weight of the ship is represented by the main dimensions of the ship, the shipbuilding cost can be represented by them as follows:

$$\begin{aligned} Building\ \ Cost &= C_{PS} \cdot C_s \cdot L^{1.6}(B+D) + C_{PO} \cdot C_o \cdot L \cdot B + C_{PM} \cdot C_{ma} \cdot NMCR \\ &= C_{PS} \cdot C_s \cdot L^{1.6}(B+D) + C_{PO} \cdot C_o \cdot L \cdot B \\ &\quad + C_{PM} \cdot C_{power} \cdot (L \cdot B \cdot T \cdot C_B)^{2/3} \cdot V^3 \end{aligned}$$

C_{PS} : Coefficient related with the cost of the steel(structural)

C_{PO} : Coefficient related with the cost of the outfit

C_{PM} : Coefficient related with the cost of the machinery

Coefficients can be obtained from the as-built ship data

Ex) The value of the coefficients obtained from the 302K VLCC

$$C_{PS} = 2,223,\ C_{PO} = 4,834,\ C_{PM} = 17,177$$

Method to obtain the coefficient related with the cost

The shipbuilding cost is composed as follows:

$$Shipbuilding\ \ Cost = \text{(Man-hour for the steel structure + Material cost for the steel structure)}$$
$$+\text{(Man-hour for the outfit +Material cost for the outfit)}$$
$$+\text{(Man-hour for the machinery +Material cost for the machinery)}$$
$$+\text{Additional cost}$$

※ The shipbuilding cost of the VLCC is about $130,000,000.

If we assume that the shipbuilding cost is proportional to the weight of the ship and the weight of the ship is composed of the steel structure weight, outfit weight and machinery weight, the shipbuilding cost can be represented as follows.

$$Building\ \ Cost = C_{PS} \cdot W_S + C_{PO} \cdot W_O + C_{PM} \cdot W_M$$

C_{PS} : Coefficient related with the cost of the steel structure

C_{PO} : Coefficient related with the cost of the outfit

C_{PM} : Coefficient related with the cost of the machinery

$$C_{PS} = \frac{\text{(Man-hour for the steel structure + Material cost for the steel structure)}}{W_S}$$

$$C_{PO} = \frac{\text{(Man-hour for the outfit +Material cost for the outfit)}}{W_O}$$

$$C_{PM} = \frac{\text{(Man-hour for the machinery +Material cost for the machinery)}}{W_M}$$

☑ Comparison of the building cost [Unit: %]

		Korea	Japan	China
Material Cost	Steel	17	17	18
	Equipment	42	43	47
	Sub sum	59	60	65
Labor Cost		27	29	19
General Cost		14	13	16
Total sum		100	100	100

3-6 Design Model for the Determination of the Optimum Main Dimensions(L,B,D,T,C_B)

Design Model for the Determination of the Optimum Main Dimensions(L,B,D,T,C_B)

Find(Design variables) L, B, D, C_B T_d
length breadth depth block coefficient design draft

Given(Owner's requirement) DWT, V_{H_req}, $T_s (= T_{max})$, V
deadweight, Required cargo hold capacity, Scantling Draft (maximum), ship speed

Physical constraint

→ Hydrostatic equilibrium(Weight equation) – Equality constraint

$$L \cdot B \cdot T_d \cdot C_B \cdot \rho_{sw} \cdot C_\alpha = DWT_{given} + LWT(L, B, D, C_B)$$
$$= DWT_{given} + C_s \cdot L^{1.6}(B + D) + C_o \cdot L \cdot B$$
$$+ C_{power} \cdot (L \cdot B \cdot T_d \cdot C_B)^{2/3} \cdot V^3 \quad \cdots (2.3)$$

Economical constraints(Owner's requirements)

→ Required cargo hold capacity(Volume equation) - Equality constraint

$$V_{H\_req} = C_H \cdot L \cdot B \cdot D \cdots (3.1)$$

- DFOC(Daily Fuel Oil Consumption)
  : It is related with the resistance and propulsion.
- Delivery date
  : It is related with the shipbuilding process.

Regulatory constraint

→ Freeboard regulation(1966 ICLL) - Inequality constraint

$$D \geq T_s + C_{FB} \cdot D \cdots (4)$$

Objective Function(Criteria to determine the proper main dimensions)

$$Building\ Cost = C_{PS} \cdot C_s \cdot L^{1.6}(B + D) + C_{PO} \cdot C_o \cdot L \cdot B + C_{PM} \cdot C_{power} \cdot (L \cdot B \cdot T_d \cdot C_B)^{2/3} \cdot V^3$$

Chapter 4. Deadweight carrier & Volume Carrier

4-1 Charcteristcs of Deadweight Carrier & Volume Carrier

Deadweight Carrier vs Volume Carrier

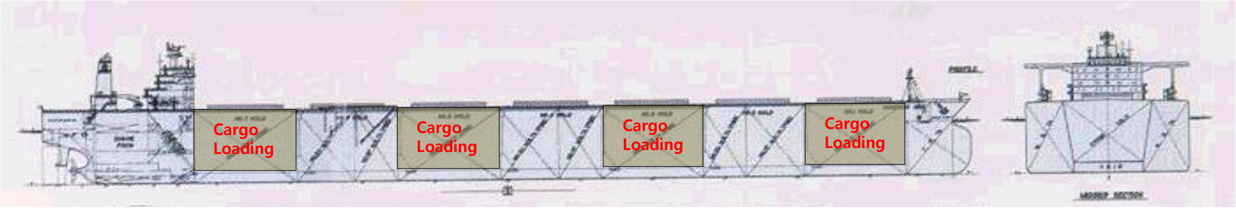
Deadweight Carrier

is a ship whose weight is a critical factor when the cargo to be carried is "**heavy**" in relation to the space provided for it.

The ship will be **weight critical** when the ship carries a cargo which has a density greater than **0.77ton/m³** or inversely lesser than **1.29 m³/ton**.

Ore Carrier

For an example, ore carrier loads the iron ore (density ≈ 2.5 ton/m³) in alternate holds, **"alternated loading"**, therefore this kind of ship needs less than a half of the hold volume.



<Alternated loading in ore carrier>

※ **Approximate formula of roll periods (T_r)**

$$T_r = \frac{2k \cdot B}{\sqrt{GM}}$$

GM : Metacentric height
B : Breadth,
k : 0.32~0.39 for full loading
0.37~.040 for ballast condition

Volume Carrier

is a ship whose volume is a critical factor when the cargo to be carried is "**light**" in relation to the space provided for it.

Membrane-type LNG Carrier

Examples of Volume Carriers

➢ Container Carrier

Containers are arranged in bays in lengthwise, rows in beam wise, tiers in depth wise.
Therefore, length, breadth and depth of a container carrier vary **stepwise** according to the number and size of containers.

Container Carrier

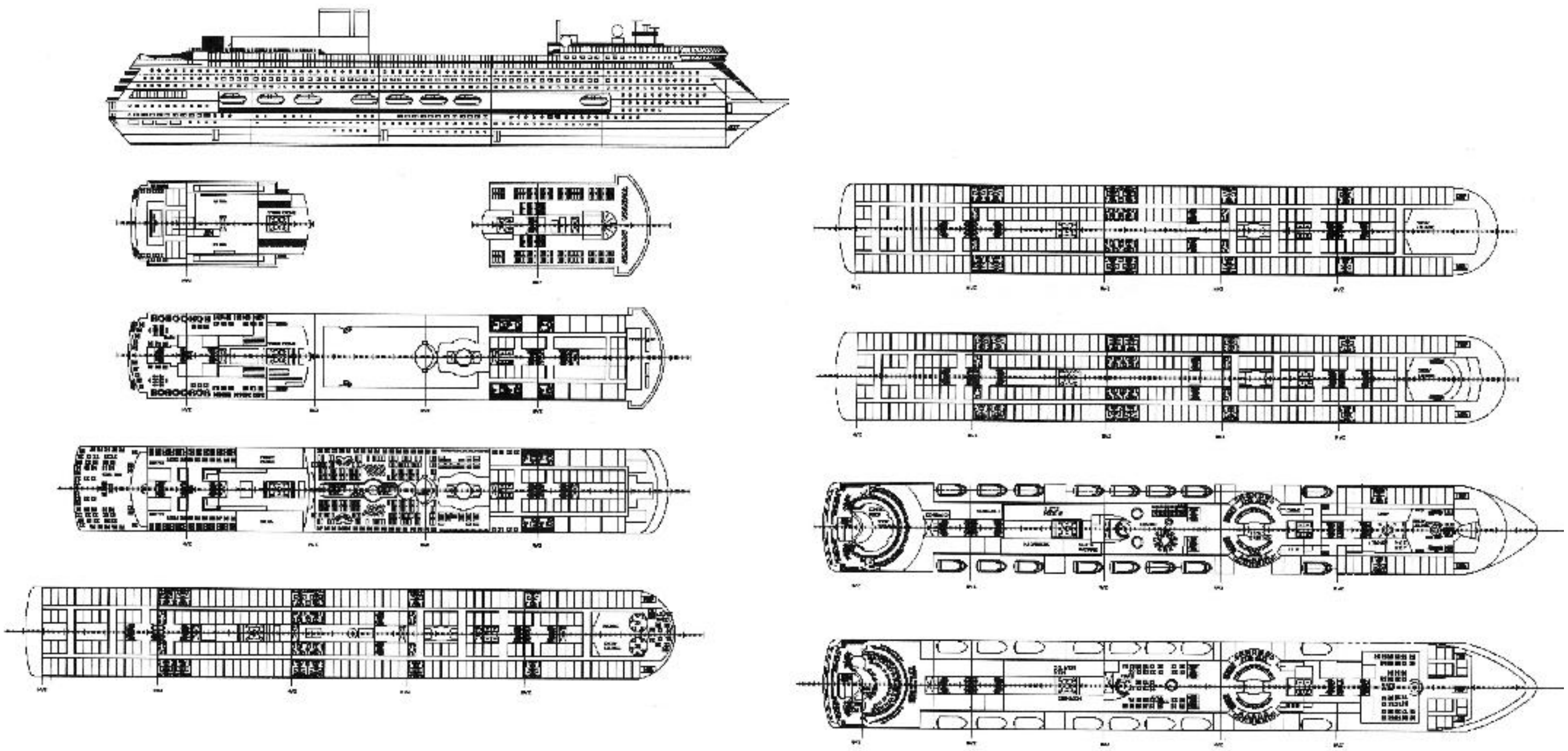
Moreover, container carrier loads containers on deck, and that causes **stability to be the ultimate criterion.**

➢Cruise ship

Cruise ship is a kind of **volume carrier because it has many decks and larger space for passengers.**
And the *KG* **is higher** which becomes the critical criterion on cruise ship.


Cruise ship

An Example of General Arrangement(GA) of a Cruise Ship(Volume Carrier)



4-2 Procedure of the Determination of Principal Dimensions for Deadweight Carrier & Volume Carrier

Procedure of the Determination of principal dimensions for a deadweight carrier

Deadweight Carrier

1	•**At first**, the principal dimensions such as *L, B, T,* C_B *are determined according to the* **weight equation**.	**Weight Equation(Physical constraint)** $\boxed{\rho \cdot L \cdot B \cdot T \cdot C_B \cdot (1+\alpha) = DWT + LWT}$ ✓ **Given:** DWT **(owner's requirements)** ✓ **Find:** L, B, T, C_B
2	• **Next,** depth is determined considering the required cargo hold capacity according to the **volume equation**.	**Volume Equation(Economical constraints)** $\boxed{V_{CH} = f(L, B, D)}$ ✓ **Given:** L, B, V_{CH} **(owner's requirements)** ✓ **Find:** D
3	•Then, it should be checked **lastly** that whether the depth and draft satisfy the **freeboard regulation**.	**Freeboard calculation(Regulatory constraints)** $\boxed{D \geq T + Fb(L, B, D, C_B)}$ ✓ **Given:** L, B, D, T, C_B ✓ **Check: Whether the chosen depth is equal or greater than the draft plus required freeboard or not.**

Procedure of the Determination of principal dimensions for a volume carrier

Volume Carrier

1	•**At first**, principal dimensions such as L, B, D are determined to provide the required cargo hold capacity according to the **volume equation.**	**Volume Equation(Economical constraints)** $\boxed{V_{CH} = f(L, B, D)}$ ✓ **Given:** V_{CH} **(owner's requirements)** ✓ **Find:** L, B, D
2	•**Next**, the principal dimensions such as T, C_B *are determined according to the* **weight equation.**	**Weight Equation(Physical constraint)** $\boxed{\rho \cdot L \cdot B \cdot T \cdot C_B \cdot (1+\alpha) = DWT + LWT}$ ✓ **Given:** L, B, DWT **(owner's requirements)** ✓ **Find:** T, C_B
3	•Then, it should be checked **lastly** that whether the depth and draft satisfy the **freeboard regulation**.	**Freeboard calculation(Regulatory constraints)** $\boxed{D \geq T + Fb(L, B, D, C_B)}$ ✓ **Given:** L, B, D, T, C_B ✓ **Check: Whether the chosen depth is equal or greater than the draft plus required freeboard or not.**

4-3 Determination of the Principal Dimensions of a 297,000 ton Deadweight VLCC (Very Large Crude Oil Carrier) based on a 279,500 ton Deadweight VLCC (Deadweight Carrier)

Example of the Principal Particulars of the Basis Ship of 279,500 ton Deadweight VLCC And Owner's Requirements of the Design Ship of 297,000 ton Deadweight VLCC

Design Ship: 297,000 Ton Deadweight VLCC(Very Large Crude Carrier)

		Basis Ship	Owner's Requirements	
Main Dimensions	Loa	abt. 330.30 m		
	Lbp	314.00 m		
	B,mld	58.00 m		
	Depth,mld	31.00 m		
	Td(design)	20.90 m	**21.50 m**	
	Ts(scant.)	22.20 m	**22.84 m**	
Deadweight(scant)		301,000 Ton	**320,000 Ton**	
Deadweight(design)		279,500 Ton	**297,000 Ton**	
Speed (at design draft 90% MCR(with 15% Sea Margin))		15.0 Knots	**16.0 Knots**	
M/E	TYPE	B&W 7S80MC		
	MCR	32,000 PS x 74.0 RPM		
	NCR	28,800 PS x 71.4 RPM		
FOC	SFOC	122.1 g/BHP.h		Based on NCR
	TON/DAY	84.4 (HFO)		
Cruising range		26,000 N/M	**26,500 N/M**	
Shape of Midship Section		Double side / Double bottom	Double side / Double bottom	
Capacity	Cargo Hold	abt. 345,500 m³	**abt. 360,000 m³**	
	H.F.O.	abt. 7,350 m³		
	D.O.	abt. 490 m³		
	Fresh Water	abt. 460 m³		
	Ballast	abt. 103,000 m³		Including Peak Tanks

Basis Ship

- **Dimensional Ratios**

$$L/B = 5.41,$$
$$B/T_d = 2.77,$$
$$B/D = 1.87,$$
$$L/D = 10.12$$

- **Hull form coefficient**

$$C_{B\_d} = 0.82$$

- **Lightweight(=41,000ton)**
  - Structural weight $\approx 36{,}400$ ton (88%)
  - Outfit weight $\approx 2{,}700$ ton (6.6%)
  - Machinery weight $\approx 1{,}900$ ton (4.5%)

$$\text{Cargo density} = \frac{\text{Deadweight}_{scant}}{\text{Cargo hold capacity}} = \frac{301{,}000}{345{,}500} = 0.87\,[ton/m^3] > 0.77$$

Deadweight Carrier

Step 1: Weight equation

Step 1: Weight Equation → Step 2: Volume Equation → Step 3: Freeboard Calculation

Step 1) The principal dimensions such as L, B, T_d, $C_{B,d}$ are determined by the weight equation.

$$\boxed{\rho \cdot L \cdot B \cdot T_d \cdot C_{B,d} \cdot (1+\alpha) = DWT_d + LWT}$$

ρ: density of sea water = 1.025 ton/m^3
α: a fraction of the shell appendage allowance = 0.0023

$$\left(1+\alpha = \frac{Displacement}{Moulded\ \ Displaced\ Volume}\bigg|_{basis} = \frac{313,007}{312,269} = 1.0023 \right)$$

✓ **Given**: DWT_d = **297,000** [ton], T_d = **21.5**[m],

V_s = **16**[$knots$]

✓ **Find**: L, B, $C_{B,d}$

***Subscript d: at design draft**

Method 1
for the Totalweight estimation

Step 1) Weight Equation → Step 2) Volume Equation → Step 3) Freeboard Calculation

$$\rho \cdot L \cdot B \cdot T_d \cdot C_{B,d} \cdot (1+\alpha) = DWT_d + LWT$$

Given: DWT_d = 297,000 [ton], T_d = 21.5[m]
Find: L, B, $C_{B,d}$

Method 1 : Assume that the total weight(W) is proportional to the deadweight.

$$W = \frac{W_{Basis}}{DWT_{d,Basis}} \cdot DWT_d$$

$$L \cdot B \cdot T_d \cdot C_{B,d} \cdot \rho \cdot (1+\alpha) = W$$

Design ship and **basis ship** are **assumed** to have the **same ratio** of the deadweight to the total weight.

Therefore, the total weight of the design ship can be estimated by the ratio of the deadweight to the total weight of the basis ship.

$$\frac{DWT_{d,Basis}}{W_{Basis}} = \frac{DWT_d}{W} \Rightarrow W = \frac{W_{Basis}}{DWT_{d,Basis}} \cdot DWT_d$$

$$= \frac{320,500}{279,500} \cdot 297,000$$

$$= 340,567\ [ton]$$

Step 1) Weight Equation → Step 2) Volume Equation → Step 3) Freeboard Calculation

$$\rho \cdot L \cdot B \cdot T_d \cdot C_{B,d} \cdot (1+\alpha) = DWT_d + LWT$$

Given: DWT_d = 297,000 [ton], T_d = 21.5[m]
Find: L, B, $C_{B,d}$

Method 1: $W = \frac{W_{Basis}}{DWT_{d,Basis}} \cdot DWT_d$

$$L \cdot B \cdot T_d \cdot C_{B,d} \cdot \rho \cdot (1+\alpha) = W$$

$$L \cdot B \cdot 21.5 \cdot C_{B,d} \cdot 1.025 \cdot (1+0.002) = 340,567$$

$$\boxed{L \cdot B \cdot C_{B,d} \cdot 22.08 = 340,567 \cdots (5.2)}$$

There are 3 unknown variables (L, B, $C_{B,d}$) with one given equation.
→ Nonlinear indeterminate equation!

Therefore, we have to assume two variables to solve this indeterminate equation.
The values of the dimensional ratio L/B and $C_{B,d}$ can be obtained from the basis ship.

$$L/B = L_{Basis} / B_{Basis} = 314/58 = 5.413$$

$$C_{B,d} = C_{B,d,Basis} = 0.8213$$

$$L \cdot B \cdot C_{B,d} \cdot 22.08 = 340,567 \cdots (5.2)$$

$$L / B = 5.413,\ C_{B,d} = 0.8213$$

Substituting the ratio obtained from the basis ship into the equation (5.2), the equation can be converted to a quadratic equation in L.

$$L \cdot \left(L / \left(L / B\right)\right) \cdot C_{B,d} \cdot 22.08 = 340,567$$

$$L(L / 5.143) \cdot 0.8213 \cdot 22.08 = 340,567$$

$$L^2 \cdot 3.349 = 340,567$$

$$\therefore L = 318.85[m]$$

$L = 318.85[m]$

We can obtain B from the ratio L/B of the basis ship.

$$B = L/(L/B)$$
$$= 318.85/5.413$$
$$= 58.90\ [m]$$

$$\therefore L = 318.85[m],\ \ B = 58.90[m],\ C_{B,d} = 0.8213$$

Then, depth is determined considering the required cargo hold capacity by the volume equation.
And it should be checked lastly that whether the depth and draft satisfy the freeboard regulation.

- Step 2: Volume equation

Step 2) Next, depth is determined considering the required cargo hold capacity by the volume equation.

$$\boxed{V_{CH} = f(L, B, D)}$$

- ✓ **Given:** L=318.85[m], B=58.90[m], $V_{CH,Req}$ = 360,000[m^3]
- ✓ **Find:** D

$$V_{CH} = f(L, B, D)$$

Given: L=318.85[m], B=58.90[m], V_{CH} = 360,000[m^3]
Find: D

Assume that the cargo hold capacity is proportional to $L \cdot B \cdot D$.

$$f(L, B, D) = C_{CH} \cdot L \cdot B \cdot D$$

$$V_{CH} = C_{CH} \cdot L \cdot B \cdot D$$

Coefficient C_{CH} can be obtained from the basis ship.

$$C_{CH} = \left.\frac{V_{CH}}{L \cdot B \cdot D}\right|_{Basis} = \frac{345,500}{314 \cdot 58 \cdot 31} = 0.612$$

We use the same coefficient C_{CH} for the determination of depth

$$V_{CH} = C_{CH} \cdot L \cdot B \cdot D$$

$$360,000 = 0.612 \times 318.85 \times 58.90 \times D$$

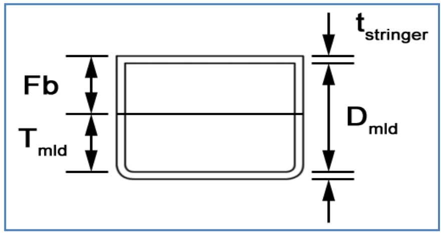
$$\therefore D = 31.32[m]$$

Step 3: Freeboard calculation

Step 1: Weight Equation → Step 2: Volume Equation → **Step 3: Freeboard Calculation**

Step 3: Then, it should be checked whether the depth and draft satisfy the freeboard regulation.

$$\boxed{D_{Fb} \geq T_s + Fb(L, B, D_{mld}, C_{B,d})}$$



$$\left(D_{Fb} = D_{mld} + t_{stringer}\right)$$

✓ **Given**: L=318.85[m], B=58.90[m], D (=D_{mld})= 31.32 [m], $T_{s,Req.}$=22.84[m], $C_{B,d,Basis}$ =0.8213, $t_{stringer,Basis}$ = 0.02[m]

✓ **Check**: The freeboard of the ship should be larger than the required freeboard.

Step 1) Weight Equation → Step 2) Volume Equation → Step 3) Freeboard Calculation

$$D_{Fb} \geq T_s + Fb(L, B, D_{mld}, C_{B,d})$$

Given: L=318.85[m], B=58.90[m], D (=D_{mld})= 31.32[m], T_s = 22.84[m], $C_{B,d}$=0.8213, $t_{stringer}$ = 0.02[m]

Check: Freeboard of the ship should be larger than that in accordance with the freeboard regulation.

At the early design stage, there are few data available for estimation of required freeboard. Thus, the required freeboard can be estimated from the basis ship.

Assume that the freeboard is proportional to the depth.

$$Fb(L, B, D_{mld}, C_{B,d}) = C_{Fb} \cdot D_{mld}$$

$$D_{Fb} \geq T_s + C_{Fb} \cdot D_{mld}$$

Coefficient C_{Fb} can be obtained from the basis ship.

$$C_{Fb} = \left.\frac{Fb}{D_{mld}}\right|_{Basis} = \frac{7.84}{31} = 0.253$$

Check: Freeboard of the design ship

$$D_{Fb} \geq T_s + C_{Fb} \cdot D_{mld}$$

$$D_{mld} + t_{stringer} \geq T_s + C_{Fb} \cdot D_{mld}$$

$$31.32 + 0.02 \geq 22.84 + 0.253 \cdot 31.32$$

$$31.34 \geq 30.76$$ **: Satisfied**

It is satisfied. However, this method is used for a rough estimation. Thus, after the principal dimensions are determined more accurately, freeboard needs to be calculated more accurately in accordance with ICLL 1966.

Method 2
for the Lightweight estimation

Step 1) Weight Equation → Step 2) Volume Equation → Step 3) Freeboard Calculation

$$\rho \cdot L \cdot B \cdot T_d \cdot C_{B,d} \cdot (1+\alpha) = DWT_d + LWT$$

Given: $DWT_d = 297{,}000$ [ton], $T_d = 21.5$[m]
Find: $L, B, C_{B,d}$

Method 2 : Assume that the lightweight could vary as the volume of the vessel represented by $L \cdot B \cdot D$.

$$LWT = C_{LWT} L \cdot B \cdot D$$

$$L \cdot B \cdot T_d \cdot C_{B,d} \cdot \rho \cdot (1+\alpha) = DWT_d + C_{LWT} \cdot L \cdot B \cdot D$$

Coefficient C_{LWT} can be obtained from the basis ship.

$$C_{LWT} = \left.\frac{LWT}{L \cdot B \cdot D}\right|_{Basis} = \frac{41{,}000}{314 \cdot 58 \cdot 31} = 0.072$$

$$L \cdot B \cdot T_d \cdot C_{B,d} \cdot \rho \cdot (1+\alpha) = DWT_d + C_{LWT} \cdot L \cdot B \cdot D$$

$$L \cdot B \cdot 21.5 \cdot C_{B,d} \cdot 1.025 \cdot (1+0.002) = 297{,}000 + 0.072 \cdot L \cdot B \cdot D$$

$$L \cdot B \cdot C_{B,d} \cdot 22.08 = 297{,}000 + 0.072 \cdot L \cdot B \cdot D \cdots (5.3)$$

There are 4 unknown variables (L, B, D, $C_{B,d}$) with one given equation.

→ Nonlinear indeterminate equation!

Step 1: Weight equation

Step 1) Weight Equation → Step 2) Volume Equation → Step 3) Freeboard Calculation

$$\rho \cdot L \cdot B \cdot T_d \cdot C_{B,d} \cdot (1+\alpha) = DWT_d + LWT$$

Given: DWT_d = 297,000 [ton], T_d = 21.5[m]
Find: L, B, $C_{B,d}$

Method 2: $LWT = C_{LWT} L \cdot B \cdot D$

$$L \cdot B \cdot C_{B,d} \cdot 22.08 = 297{,}000 + 0.072 \cdot L \cdot B \cdot D \cdots (5.3)$$

Therefore, we have to assume three variables to solve this indeterminate equation.

The values of the dimensional ratios L/B, B/D and $C_{B,d}$ can be obtained from the basis ship.

$$L/B = L_{Basis}/B_{Basis} = 314/58 = 5.413$$

$$B/D = B_{Basis}/D_{Basis} = 58/31 = 1.871$$

$$C_{B,d} = C_{B,d,Basis} = 0.8213$$

Substituting the ratios obtained from the basis ship into the equation (5.3), the equation can be converted to a cubic equation in L.

$$L \cdot \left(L/\left(L/B\right)\right) \cdot C_{B,d} \cdot 22.08 = 297{,}000 + 0.072 \cdot L \cdot \left(L/\left(L/B\right)\right) \cdot \left(L/\left(L/B\right)/\left(B/D\right)\right)$$

$$L \cdot \left(L / \left(L / B \right) \right) \cdot C_{B,d} \cdot 22.08 = 297,000 + 0.072 \cdot L \cdot \left(L / \left(L / B \right) \right) \cdot \left(L / \left(L / B \right) / \left(B / D \right) \right)$$

$$L(L / 5.143) \cdot 0.8213 \cdot 22.08 = 297,000 + 0.072 \cdot L \cdot \left(L / 5.413 \right) \cdot \left(\left(L / 5.413 \right) / 1.871 \right)$$

$$L^2 \cdot 3.349 = 297,000 + L^3 \cdot 0.0013$$

$$\therefore L = 318.48 \ [m]$$

Then B is calculated from the ratio L/B of the basis ship.

$$B = L / (L / B)$$
$$= 318.48 / 5.413$$
$$= 58.82 \ [m] \qquad \therefore L = 318.48[m], \quad B = 58.82[m], \quad C_{B,d} = 0.8213$$

Then, depth is determined considering the required cargo hold capacity by the volume equation.
And it should be checked lastly whether the depth and draft satisfy the freeboard regulation.

Method 3
for the Lightweight estimation in components

Step 1) Weight Equation → Step 2) Volume Equation → Step 3) Freeboard Calculation

$$\rho \cdot L \cdot B \cdot T_d \cdot C_{B,d} \cdot (1+\alpha) = DWT_d + LWT$$

Given: DWT_d = 297,000 [*ton*], T_d = 21.5[*m*], V_S = 16 [knots]
Find: $L, B, C_{B,d}$

Method 3 : Estimate the structural weight(W_s), outfit weight(W_o) and machinery weight(W_m) in components.

$$LWT = W_s + W_o + W_m$$

$$L \cdot B \cdot T_d \cdot C_{B,d} \cdot \rho \cdot (1+\alpha) = DWT_d + W_s + W_o + W_m$$

Structural weight(W_s) is estimated as follows:

$$W_s = C_s \cdot L^{1.6} \cdot (B+D)$$

Coefficient C_s can be obtained from the basis ship.

$$C_s = \left. \frac{W_s}{L^{1.6} \cdot (B+D)} \right|_{Basis} = \frac{36{,}400}{314^{1.6} \cdot (58+31)} = 0.0414$$

Step 1) Weight Equation → Step 2) Volume Equation → Step 3) Freeboard Calculation

$$\rho \cdot L \cdot B \cdot T_d \cdot C_{B,d} \cdot (1+\alpha) = DWT_d + LWT$$

Given: DWT_d = 297,000 [ton], T_d = 21.5[m], V_S = 16 [knots]
Find: L, B, $C_{B,d}$

Method 3: $LWT = W_s + W_o + W_m$

Outfit weight(W_o) is estimated as follows:

$$W_o = C_o \cdot L \cdot B$$

Coefficient C_o can be obtained from the basis ship.

$$C_o = \left.\frac{W_o}{L \cdot B}\right|_{Basis} = \frac{2,700}{314 \cdot 58} = 0.1483$$

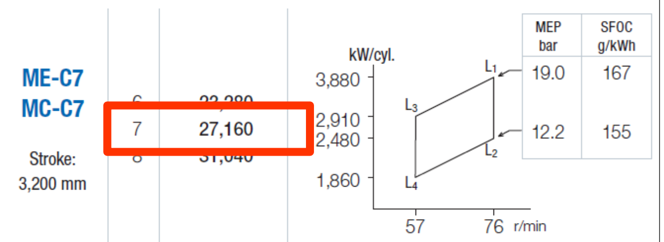
Machinery weight(W_o) is estimated as follows:

$$W_m = C_m \cdot NMCR$$

Coefficient C_m can be obtained from the basis ship.

$$C_m = \left.\frac{W_m}{NMCR}\right|_{Basis} = \frac{1,900}{36,952} = 0.0514$$

Main engine of basis ship
: 7S80MC-C



$$NMCR = 27,160[kW]$$
$$= 36,952[PS]$$

$NMCR$ can be estimated based on the resistance estimation, power prediction , and main engine selection. However, there are few data available for estimation of the $NMCR$ at the early design stage. Thus, $NMCR$ can be estimated using 'Admiralty formula'

Step 1) Weight Equation → Step 2) Volume Equation → Step 3) Freeboard Calculation

$$\rho \cdot L \cdot B \cdot T_d \cdot C_{B,d} \cdot (1+\alpha) = DWT_d + \boxed{LWT}$$

Given: DWT_d = 297,000 [ton], T_d = 21.5[m], V_S = 16 [knots]
Find: $L, B, C_{B,d}$

Method 3: $LWT = W_s + W_o + \boxed{W_m}$

$$W_m = C_m \cdot \boxed{NMCR}$$

$$NMCR = \frac{1}{\text{Engine Margin}} \cdot \frac{1}{\text{Derating ratio}} \cdot NCR$$

$$(\text{Engine Margin} = 0.9,\ \textit{Assumed}\ \text{Derating ratio} = 0.9)$$

$$NMCR = 1.265 \cdot NCR$$

By **applying the** **'Admiralty formula'** **to the** NCR, **the** $NMCR$ **can be estimated:**

$$NCR = \frac{\Delta^{2/3} \cdot V_s^{\,3}}{C_{ad}}$$

Coefficient C_{ad} **can be obtained from the basis ship.**

$$C_{ad} = \left.\frac{\Delta^{2/3} \cdot V_s^{\,3}}{NCR}\right|_{Basis} = \frac{320{,}500^{2/3} \cdot 15^3}{28{,}800} = 548.82 \qquad (V_{s,\ at\ design\ draft} = 15[knots])$$

$$NCR = \frac{\Delta^{2/3} \cdot V_s^{\,3}}{548.82}$$

$$NMCR = 1.265 \cdot \frac{\Delta^{2/3} \cdot V_s^{\,3}}{548.82}$$

$$= 0.0022 \cdot \Delta^{2/3} \cdot V_s^{\,3}$$

$$W_s = C_s \cdot L^{1.6} \cdot (B+D) \qquad C_s = 0.0414$$
$$W_o = C_o \cdot L \cdot B \qquad C_o = 0.1483$$
$$W_m = C_m \cdot NMCR \qquad C_m = 0.0514$$
$$NMCR = 0.0022 \cdot \Delta^{2/3} \cdot V_s^3$$

$$L \cdot B \cdot T_d \cdot C_{B,d} \cdot \rho \cdot (1+\alpha) = DWT_d + W_s + W_o + W_m$$

$$L \cdot B \cdot T_d \cdot C_{B,d} \cdot \rho \cdot (1+\alpha) = DWT_d + C_s \cdot L^{1.6} \cdot (B+D) + C_o \cdot L \cdot B + C_m \cdot NMCR$$

$$L \cdot B \cdot T_d \cdot C_{B,d} \cdot \rho \cdot (1+\alpha) = DWT_d + C_s \cdot L^{1.6} \cdot (B+D) + C_o \cdot L \cdot B + C_m \cdot (0.0022 \cdot \Delta^{2/3} \cdot V_s^{\,3})$$

$$L \cdot B \cdot T_d \cdot C_{B,d} \cdot \rho \cdot (1+\alpha) = DWT_d + C_s \cdot L^{1.6} \cdot (B+D) + C_o \cdot L \cdot B + C_m \cdot \left(0.0022 \cdot \left(L \cdot B \cdot T_d \cdot C_{B,d} \cdot \rho \cdot (1+\alpha)\right)^{2/3} \cdot V_s^{\,3}\right)$$

$$L \cdot B \cdot 21.5 \cdot C_{B,d} \cdot 1.025 \cdot (1+0.002) = 297,000 + 0.0414 \cdot L^{1.6} \cdot (B+D) + 0.1483 \cdot L \cdot B + 0.0514 \cdot \left(0.0022 \cdot \left(L \cdot B \cdot 21.5 \cdot C_{B,d} \cdot 1.025 \cdot (1+0.002)\right)^{2/3} \cdot 16^3\right)$$

$$L \cdot B \cdot 21.5 \cdot C_{B,d} \cdot 1.025 \cdot (1+0.002) = 297,000 + 0.0414 \cdot L^{1.6} \cdot (B+D) + 0.1483 \cdot L \cdot B$$
$$+0.0514 \cdot (0.0022 \cdot \left(L \cdot B \cdot 21.5 \cdot C_{B,d} \cdot 1.025 \cdot (1+0.002)\right)^{2/3} \cdot 16^3)$$

$$\boxed{\begin{aligned} L \cdot B \cdot C_{B,d} \cdot 22.08 &= 297,000 + 0.0414 \cdot L^{1.6} \cdot (B+D) + 0.1483 \cdot L \cdot B \\ &+0.00012 \cdot (L \cdot B \cdot C_{B,d} \cdot 22.08)^{2/3} \cdot 16^3 \cdots (5.4) \end{aligned}}$$

There are 4 unknown variables (L, B, D, $C_{B,d}$) with one equation.

→ Nonlinear indeterminate equation!

Therefore, we have to assume three variables to solve this indeterminate equation.

The values of the dimensional ratios L/B, B/D and $C_{B,d}$ can be obtained from the basis ship.

$$L/B = L_{Basis}/B_{Basis} = 314/58 = 5.413$$

$$B/D = B_{Basis}/D_{Basis} = 58/31 = 1.871$$

$$C_{B,d} = C_{B,d,Basis} = 0.8213$$

$$L \cdot B \cdot C_{B,d} \cdot 22.08 = 297,000 + 0.0414 \cdot L^{1.6} \cdot (B + D) + 0.1494 \cdot L \cdot B$$
$$+0.00012 \cdot (L \cdot B \cdot C_{B,d} \cdot 22.08)^{2/3} \cdot 16^3 \cdots (5.4)$$

$$L / B = 5.413,\ B / D = 1.871, C_{B,d} = 0.8213$$

Substituting the ratios obtained from the basis ship into the equation (5.4), the equation can be converted to a cubic equation in L.

$$L \cdot (L / (L / B)) \cdot C_{B,d} \cdot 22.08 = 297,000 + 0.0414 \cdot L^{1.6} \cdot ((L / (L / B)) + (L / (L / B) / (B / D))$$
$$+0.1483 \cdot L \cdot (L / (L / B))$$
$$+0.00012 \cdot (L \cdot (L / (L / B)) \cdot C_{B,d} \cdot 22.08)^{2/3} \cdot 16^3$$

$$L \cdot (L / 5.413) \cdot 0.8213 \cdot 22.08 = 297,000 + 0.0414 \cdot L^{1.6} \cdot ((L / 5.413) + (L / 5.413 / 1.871))$$
$$+0.1483 \cdot L \cdot (L / 5.413)$$
$$+0.00012 \cdot (L \cdot (L / 5.413) \cdot 0.8213 \cdot 22.08)^{2/3} \cdot 16^3$$

$$L^2 \cdot 3.349 = 297,000 + 0.0414 \cdot L^{1.6} \left(0.185 \cdot L + 0.099 \cdot L\right)$$
$$+0.0274 \cdot L^2 + 0.00012 \cdot (L^2 \cdot 3.349)^{2/3} \cdot 16^3$$

$$\therefore L = 318.57\ [m]$$

$$L = 318.57 \ [m]$$

Then, B *is calculated* **from the ratio** L/B **of the basis ship.**

$$B = L/(L/B)$$
$$= 318.57/5.413$$
$$= 58.84 \ [m]$$

$$\therefore L = 318.57[m], \ B = 58.84[m], \ C_{B,d} = 0.8213$$

Then, depth is determined considering the required cargo hold capacity by the volume equation.
And it should be checked lastly whether the depth and draft satisfy the freeboard regulation.

4-4 Determination of Main Dimensions and Block Coefficient of a 160,000 m^3 LNG Carrier based on a 138,000 m^3 LNG Carrier(Volume Carrier)

Example of the Principal Particulars of a Basis Ship of 138,000 m³ LNG Carrier and Owner's Requirements of a 160,000 m³ LNG Carrier

160,000 m³ LNG Carrier

		Basis Ship	Owner's Requirements	
Main Dimensions (m)	L_{OA}	277.0		
	L_{BP}	266.0		
	B_{mld}	43.4		
	D_{mld}	26.0		
	T_d (design)	11.4	11.4	
	T_s (scantling)	12.1	12.1	
Cargo Hold Capacity(m³)		138,000	160,000	
Service speed (knots)		19.5	19.5	
Main Engine	Type	Steam Turbine	2 Stroke Diesel Engine (×2)	
	DMCR	36000 PS, 88 RPM		With engine margin 10%
	NCR	32400 PS, 85 RPM		With sea margin 21%
SFOC (Ton/day)		180.64		
Deadweight (ton)		69,000	80,000	
DFOC (ton/day)		154.75		
Cruising Range (N.M)		13,000	11,400	

Basis Ship

- **Dimensional Ratios**

$$L/B = 6.31,$$
$$B/T_d = 3.81,$$
$$B/D = 1.67,$$
$$L/D = 10.23$$

- **Hull form coefficient**

$$※ C_{B\_d} = 0.742$$

- **Lightweight(=31,000ton)**
  - Structural weight ≈ 21,600 ton (≈70%)
  - Outfit weight ≈ 6,200 ton (≈ 20%)
  - Machinery weight ≈ 3,200 ton (≈ 10%)

$$\text{Cargo density} = \frac{\text{Deadweight}}{\text{Cargo hold capacity}} = \frac{69,000}{138,000} = 0.5\ [ton/m^3] < 0.77$$

Volume Carrier

- Step 1: Volume equation

Step 1: Volume Equation → Step 2: Weight Equation → Step 3: Freeboard Calculation

Step 1: The principal dimensions such as L, B, D are determined considering the required cargo hold capacity by the volume equation.

$$\boxed{V_{CH} = f(L, B, D)}$$

✓ **Given:** V_{CH} = 160,000[m^3]

✓ **Find:** L, B, D

Step 1) Volume Equation → Step 2) Weight Equation → Step 3) Freeboard Calculation

$$V_{CH} = f(L, B, D)$$

Given: $V_{CH} = 160{,}000[m^3]$

Find: L, B, D

Assume that the cargo hold capacity is proportional to $L \cdot B \cdot D$.

$$f(L, B, D) = C_{CH} \cdot L \cdot B \cdot D$$

$$V_{CH} = C_{CH} \cdot L \cdot B \cdot D$$

Coefficient C_{CH} can be obtained from the basis ship.

$$C_{CH} = \left. \frac{V_{CH}}{L \cdot B \cdot D} \right|_{Basis} = \frac{138{,}000}{266 \cdot 43.4 \cdot 26} = 0.460$$

$$V_{CH} = C_{CH} \cdot L \cdot B \cdot D$$

$$\boxed{160{,}000 = 0.460 \cdot L \cdot B \cdot D \cdots (6.1)}$$

There are 3 unknown variables (L, B, D) with one equation.

→ Nonlinear indeterminate equation!

Step 1) Volume Equation → Step 2) Weight Equation → Step 3) Freeboard Calculation

$$160,000 = 0.460 \cdot L \cdot B \cdot D \cdots (6.1)$$

$$V_{CH} = \boxed{f(L, B, D)}$$

Given: V_{CH} = 160,000[m^3]
Find: L, B, D

$$f(L, B, D) = C_{CH} \cdot L \cdot B \cdot D$$

Therefore, we have to assume two variables to solve this indeterminate equation.
The values of the dimensional ratios L/B and B/D can be obtained from the basis ship.

$$L / B = L_{Basis} / B_{Basis} = 266 / 43.4 = 6.129$$

$$B / D = B_{Basis} / D_{Basis} = 43.4 / 26 = 1.670$$

Substituting the ratios obtained from basis ship into the equation (6.1), the equation can be converted to a cubic equation in L.

$$160,000 = 0.460 \cdot L \cdot (L / (L / B)) \cdot (L / (L / B) / (B / D))$$

$$160,000 = 0.460 \cdot L \cdot (L / 6.129) \cdot (L / 6.129 / 1.670)$$

$$160,000 = 0.007 \cdot L^3$$

$$\therefore L = 279.4[m]$$

$L = 279.4\ [m]$

We can obtain B **and** D **from the ratios** L/B **and** B/D **of the basis ship.**

$$
\begin{aligned}
B &= L/(L/B) \\
&= 279.4/6.129 \\
&= 45.6\ [m]
\end{aligned}
$$

$$
\begin{aligned}
D &= L/(L/B)/(B/D) \\
&= 279.4/6.129/1.669 \\
&= 27.3\ [m]
\end{aligned}
$$

$$\therefore L = 279.4[m], \quad B = 45.6[m], \quad D = 27.3[m]$$

- Step 2: Weight equation

Step 1: Volume Equation → **Step 2: Weight Equation** → Step 3: Freeboard Calculation

Step 2: Then, block coefficient($C_{B,d}$) is determined by the weight equation.

$$\boxed{\rho \cdot L \cdot B \cdot T_d \cdot C_{B,d} \cdot (1+\alpha) = DWT_d + LWT}$$

ρ: density of sea water = 1.025 ton/m³
α: a fraction of the shell appendage allowance = 0.002

✓ **Given**: L = **279.4**[m], B = **45.6**[m], D = **27.3**[m], T_d = **11.4**[m], DWT_d = **80,000**[ton], V_s=**19.5**[$knots$]

✓ **Find**: $C_{B,d}$

*Subscript d: at design draft

Step 2: Lightweight estimation in components by the method 3

Step 1) Volume Equation → **Step 2) Weight Equation** → Step 3) Freeboard Calculation

$$\rho \cdot L \cdot B \cdot T_d \cdot C_{B,d} \cdot (1+\alpha) = DWT_d + LWT$$

Given: L = 279.4[m], B = 45.6[m], D = 27.3[m], T_d = 11.4[m], DWT_d = 80,000[ton], V=19.5[$knots$]

Find: $C_{B,d}$

Method 3 : Estimate the structural weight(W_s), outfit weight(W_o), and machinery weight(W_m) in components.

$$LWT = W_s + W_o + W_m$$

$$L \cdot B \cdot T_d \cdot C_{B,d} \cdot \rho \cdot (1+\alpha) = DWT_d + W_s + W_o + W_m$$

Structural weight(W_s) is estimated as follows:

$$W_s = C_s \cdot L^{1.6} \cdot (B+D)$$

Coefficient C_s can be obtained from the basis ship.

$$C_s = \left. \frac{W_s}{L^{1.6} \cdot (B+D)} \right|_{Basis} = \frac{21{,}600}{266^{1.6} \cdot (43.4+26)} = 0.0410$$

Step 1) Volume Equation → **Step 2) Weight Equation** → Step 3) Freeboard Calculation

$$\rho \cdot L \cdot B \cdot T_d \cdot C_{B,d} \cdot (1+\alpha) = DWT_d + LWT$$

Given: L = 279.4[m], B = 45.6[m], D = 27.3[m], T_d = 11.4[m], DWT_d = 80,000[ton], V=19.5[$knots$]

Find: $C_{B,d}$

Method 3: $LWT = W_s + W_o + W_m$

Outfit weight(W_o) is estimated as follows:

$$W_o = C_o \cdot L \cdot B$$

Coefficient C_o can be obtained from the basis ship.

$$C_o = \left.\frac{W_o}{L \cdot B}\right|_{Basis} = \frac{6{,}200}{266 \cdot 43.4} = 0.5371$$

Machinery weight(W_o) is estimated as follows:

$$W_m = C_m \cdot NMCR$$

Coefficient C_m can be obtained from the basis ship.

$$C_m = \left.\frac{W_m}{NMCR}\right|_{Basis} = \frac{3{,}200}{36{,}000} = 0.089$$

Because the main engine of the basis ship is steam turbine, NMCR of the basis ship is equal to MCR of that.

$$NMCR_{basis} = MCR_{basis}$$
$$= 36{,}000[PS]$$

At the early design stage, $NMCR$ can be estimated by 'Admiralty formula'

$$NMCR = \frac{1}{\text{Engine Margin}} \cdot \frac{1}{\text{Derating ratio}} \cdot NCR$$

$$(\text{Engine Margin} = 0.9,\ \text{Derating ratio} = 0.9)$$

$$NMCR = 1.265 \cdot NCR$$

By applying the 'Admiralty formula' to the NCR, the $NMCR$ also can be estimated.

$$NCR = \frac{\Delta^{2/3} \cdot V_s^{\ 3}}{C_{ad}}$$

Coefficient C_{ad} can be obtained from the basis ship.

$$C_{ad} = \left.\frac{\Delta^{2/3} \cdot V_s^{\ 3}}{NCR}\right|_{Basis} = \frac{100,000^{2/3} \cdot 19.5^3}{32,400} = 493.05 \quad (V_{s,\,at\ design\ draft} = 19.5[knots])$$

$$NCR = \frac{\Delta^{2/3} \cdot V_s^{\ 3}}{493.05}$$

$$NMCR = 1.265 \cdot \frac{\Delta^{2/3} \cdot V_s^{\ 3}}{493.05}$$

$$= 0.0025 \cdot \Delta^{2/3} \cdot V_s^{\ 3}$$

$$W_s = C_s \cdot L^{1.6} \cdot (B+D) \qquad C_s = 0.0410$$
$$W_o = C_o \cdot L \cdot B \qquad C_o = 0.5371$$
$$W_m = C_m \cdot NMCR \qquad C_m = 0.089$$
$$NMCR = 0.0025 \cdot \Delta^{2/3} \cdot V_s^3$$

$$L \cdot B \cdot T_d \cdot C_{B,d} \cdot \rho \cdot (1+\alpha) = DWT_d + W_s + W_o + W_m$$

$$L \cdot B \cdot T_d \cdot C_{B,d} \cdot \rho \cdot (1+\alpha) = DWT_d + C_s \cdot L^{1.6} \cdot (B+D) + C_o \cdot L \cdot B + C_m \cdot NMCR$$

$$L \cdot B \cdot T_d \cdot C_{B,d} \cdot \rho \cdot (1+\alpha) = DWT_d + C_s \cdot L^{1.6} \cdot (B+D) + C_o \cdot L \cdot B$$
$$+ C_m \cdot (0.0025 \cdot \Delta^{2/3} \cdot V_s^3)$$

$$L \cdot B \cdot T_d \cdot C_{B,d} \cdot \rho \cdot (1+\alpha) = DWT_d + C_s \cdot L^{1.6} \cdot (B+D) + C_o \cdot L \cdot B$$
$$+ C_m \cdot (0.0025 \cdot \left(L \cdot B \cdot T_d \cdot C_{B,d} \cdot \rho \cdot (1+\alpha)\right)^{2/3} \cdot V_s^3)$$

$$279.4 \cdot 45.6 \cdot 11.4 \cdot C_{B,d} \cdot 1.025 \cdot (1+0.002) = 80,000 + 0.0410 \cdot 279.4^{1.6} \cdot (45.6+27.3) + 0.5371 \cdot 279.4 \cdot 45.6$$
$$+0.089 \cdot (0.0025 \cdot \left(279.4 \cdot 45.6 \cdot 11.4 \cdot C_{B,d} \cdot 1.025 \cdot (1+0.002)\right)^{2/3} \cdot 19.5^3)$$

$$149,175 \cdot C_{B,d} = 80,000 + 24,554 + 6,843$$
$$+0.089 \cdot (0.0025 \cdot \left(149,175 \cdot C_{B,d}\right)^{2/3} \cdot 19.5^3)$$

$$149{,}175 \cdot C_{B,d} = 80{,}000 + 24{,}554 + 6{,}843 + 0.089 \cdot (0.0025 \cdot \left(149{,}175 \cdot C_{B,d}\right)^{2/3} \cdot 19.5^3)$$

$$149{,}175 \cdot C_{B,d} = 80{,}000 + 24{,}554 + 6{,}843 + 4{,}634 \cdot C_{B,d}{}^{2/3}$$

$$149{,}175 \cdot C_{B,d} = 111{,}397 + 4{,}634 \cdot C_{B,d}{}^{2/3}$$

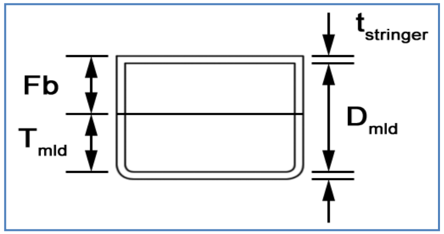
$$\therefore C_{B,d} = 0.773$$

- Step 3: Freeboard calculation

Step 1) Volume Equation → Step 2) Weight Equation → Step 3) Freeboard Calculation

Step 3: Then, it should be checked lastly whether the depth and draft satisfy the freeboard regulation.

$$\boxed{D_{Fb} \geq T_s + Fb(L, B, D_{mld}, C_{B,d})}$$



$$\left(D_{Fb} = D_{mld} + t_{stringer}\right)$$

✓ **Given**: L=279.4[m], B=45.6[m], D (=D_{mld})= 27.3 [m], T_s = 12.1[m], $C_{B,d}$ =0.773, $t_{stringer}$ = 0.02[m]

✓ **Check**: The freeboard of the ship should be larger than the required freeboard.

Step 1) Volume Equation → Step 2) Weight Equation → Step 3) Freeboard Calculation

$$D_{Fb} \geq T_s + Fb(L, B, D_{mld}, C_{B,d})$$

Given: L = 279.4[m], B = 45.6[m], $D(=D_{mld})$= 27.3[m], T_s = 12.1[m], $C_{B,d}$ = 0.773, $t_{stringer}$ = 0.02[m]

Check: Freeboard of the ship should be larger than that in accordance with the freeboard regulation.

At the early design stage, the required freeboard can be estimated from the basis ship.

Assume that the freeboard is proportional to the depth.

$$Fb(L, B, D_{mld}, C_{B,d}) = C_{Fb} \cdot D_{mld}$$

$$D_{Fb} \geq T_s + C_{Fb} \cdot D_{mld}$$

Coefficient C_{Fb} can be obtained from the basis ship.

$$C_{Fb} = \left.\frac{Fb}{D_{mld}}\right|_{Basis} = \frac{6.68}{26} = 0.257$$

Check: Freeboard of the design ship

$$D_{Fb} \geq T_s + C_{Fb} \cdot D_{mld}$$

$$D_{mld} + t_{stringer} \geq T_s + C_{Fb} \cdot D_{mld}$$

$$27.3 + 0.02 \geq 12.1 + 0.257 \cdot 27.3$$

$$27.32 \geq 19.11$$ **: Satisfied**

It is satisfied. However, this method is used for a rough estimation. So, after the main dimensions are determined more accurately, freeboard needs to be calculated more accurately through the freeboard regulation.

4-5 Determination of Principal Dimensions and Block Coefficient of a 4,100 TEU Container Carrier based on a 3,700 TEU Container Carrier(Volume Carrier)

Example of the Principal particulars of a Basis Ship of 3,700 TEU Container Carrier and Owner's Requirements of a 4,100 TEU Container Carrier

*TEU : twenty-foot equivalent units

3,700 TEU Container Carrier | **4,100 TEU Container Carrier**

	Basis Ship	Owner's requirements	
Main Dimension LOA LBP Bmld Dmld Td / Ts (design / scantling)	 257.4 m 245.24 m 32.2 m 19.3 m 10.1 / 12.5 m	 Less than **260.0 m** Less than **32.25 m** Abt. **11.0 / 12.6 m**	
Deadweight (design / scantling)	34,400 / 50,200 MT(metric ton)	**40,050/49,000 ~ 51,000 MT**	
Capacity			
Container on deck / in hold	2,174 TEU / 1,565 TEU	**Abt. 4,100TEU**	
Ballast water Heavy fuel oil	13,800 m3 6,200 m3	**Abt. 11,500 m3**	
Main Engine & Speed M / E type MCR (BHP × rpm) NCR (BHP × rpm) Service speed at NCR (Td, 15% SM) DFOC at NCR Cruising range	 Sulzer 7RTA84C 38,570 × 102 34,710 × 98.5 22.5 knots (at 11.5m) at 30,185 BHP 103.2 MT 20,000 N.M	 **24.5 knots (at 11.0m)** Abt. 20,000 N.M	
Others Complement	30 P.	**30 P.**	

Basis Ship

- **Dimensional Ratios**

$$L/B = 7.62$$
$$B/T_d = 3.19$$
$$B/D = 1.67$$
$$L/D = 12.71$$

- **Hull form coefficient**

$$※ C_{B\_d} = 0.62$$

- **Lightweight(=16,000ton)**
  - Structural weight $\approx$ 11,000 ton ($\approx$68%)
  - Outfit weight $\approx$ 3,200 ton ($\approx$ 20%)
  - Machinery weight $\approx$ 1,800 ton ($\approx$ 12%)

$$\text{Cargo density} = \frac{\text{Deadweight}_{scant}}{\text{Cargo hold capacity}} = \frac{\text{Deadweight}_{scant}}{V_{container} \times N_{container\,in\,cargo\,hold}} = \frac{50,200}{46.9 \cdot 1,565} = 0.68\,[ton/m^3] < 0.77$$

⇩

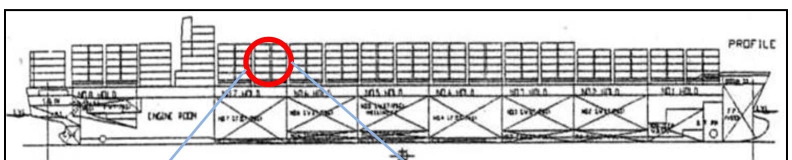
Volume Carrier

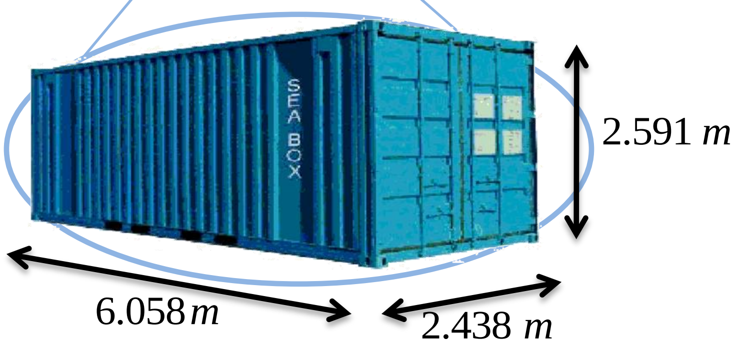
4,100 TEU Container Carrier Design based on the 3,700 TEU Container Carrier

N_L : Number of bays
N_B : Number of rows
N_D : Number of tiers

Example 2: 160,000 m³ LNG Carrier Design based on 138,000 m³ LNG Carrier	Example 3: 4,100TEU Container Carrier Design based on 3,700TEU Container Carrier
$\boxed{V_{CH} = f(L, B, D)}$	$\boxed{V_{CH} = f(L, B, D)}$
✓ **Given:** V_{CH} = **160,000[m^3]**	✓ **Given:** N_{C_req} = **4,100** TEU
✓ **Find:** L, B, D	✓ **Find:** L, B, D

Containers are arranged in bays in lengthwise, rows in beam wise, tiers in depth wise. It means that the main dimensions are determined discontinuously.





Example) 20' ISO Container size

Therefore, length, breadth and depth of container carrier vary stepwise according to the number and size of containers in cargo hold.

$$L = f(N_L) \quad B = f(N_B) \quad D = f(N_D)$$

$$L = L_H + L_{APT} + L_{ER} + L_{FPT} = \left(L_{clear,con} + 2L_{container} + L_{clear,hold}\right)\cdot\frac{N_L}{2} + L_{APT} + L_{ER} + L_{FPT}$$

$$B = B_H + B_{D.S} = \left(B_{clearance} + B_{container}\right)\cdot N_B - B_{clearance} + 2\cdot(B_{D.S} + B_{clearance,D.S})$$

$$D = D_H + D_{D.B} - D_{H.C} = \left(D_{clearance} + D_{container}\right)\cdot N_D + D_{D.B} - D_{H.C}$$

*Size of Container($L \times B \times D$)

20' ISO Container size : $6.058\,m \times 2.438\,m \times 2.591\,m$

40' ISO Container size : $12.192\,m \times 2.438\,m \times 2.591\,m$

- **Length**, breadth and depth of container carrier vary stepwise according to the number and size of containers.

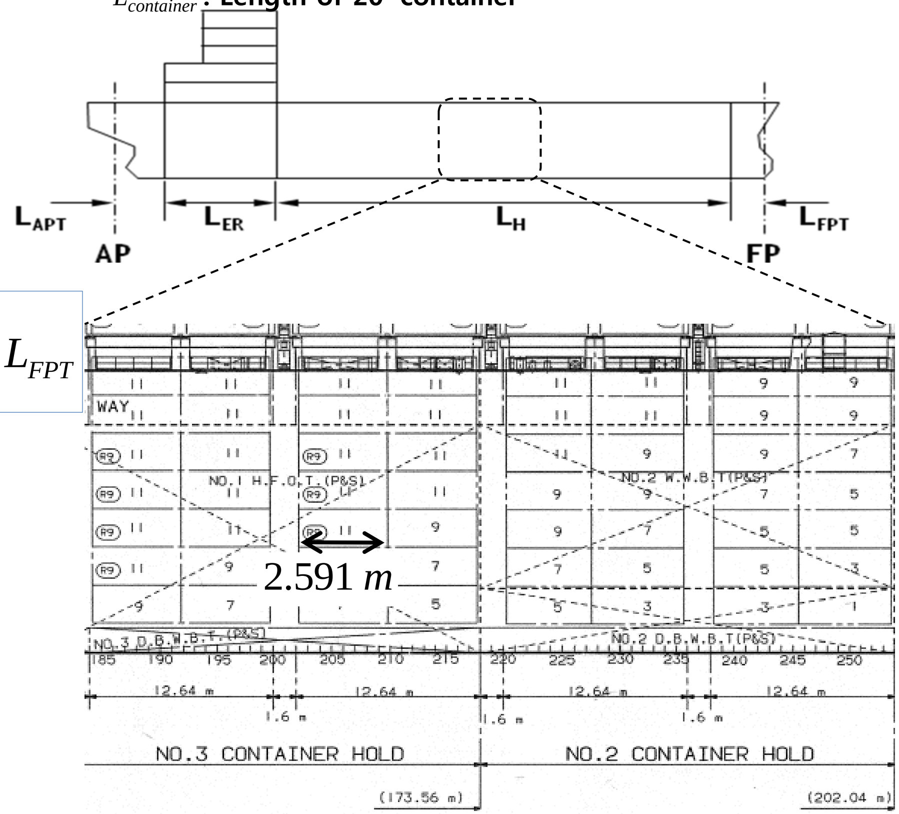
L_H : Length of cargo hold
L_{APT} : Length between aft perpendicular to aft bulkhead
L_{ER} : Length of engine room
L_{FPT} : Length between forward perpendicular to collision bulkhead
N_L : Number of bays
$L_{clear,con}$: Clearance between containers
$L_{clear,hold}$: Clearance between cargo holds
$L_{container}$: Length of 20' container

1) Length

$$\boxed{L = L_H + L_{APT} + L_{ER} + L_{FPT}}$$

$$L_H = \left(L_{clear,con} + 2L_{container} + L_{clear,hold}\right)\cdot\frac{N_L}{2}$$

$$L = \left(L_{clear,con} + 2L_{container} + L_{clear,hold}\right)\cdot\frac{N_L}{2} + L_{APT} + L_{ER} + L_{FPT}$$



Example)

$L_{clear,con} = 0.564[m]$, $L_{clear,hold} = 1.6[m]$,

$L_{APT} = 11.2[m]$, $L_{ER} = 30.4[m]$,

$L_{FPT} = 12.92[m]$, $L_{container} = 6.058[m]$

$$\rightarrow L = 7.14 \cdot N_L + 54.52$$

*Size of Container($L \times B \times D$)

20' ISO Container size : $6.058\,m \times 2.438\,m \times 2.591\,m$

40' ISO Container size : $12.192\,m \times 2.438\,m \times 2.591\,m$

- Length, breadth and depth of container carrier vary stepwise according to the number and size of containers.

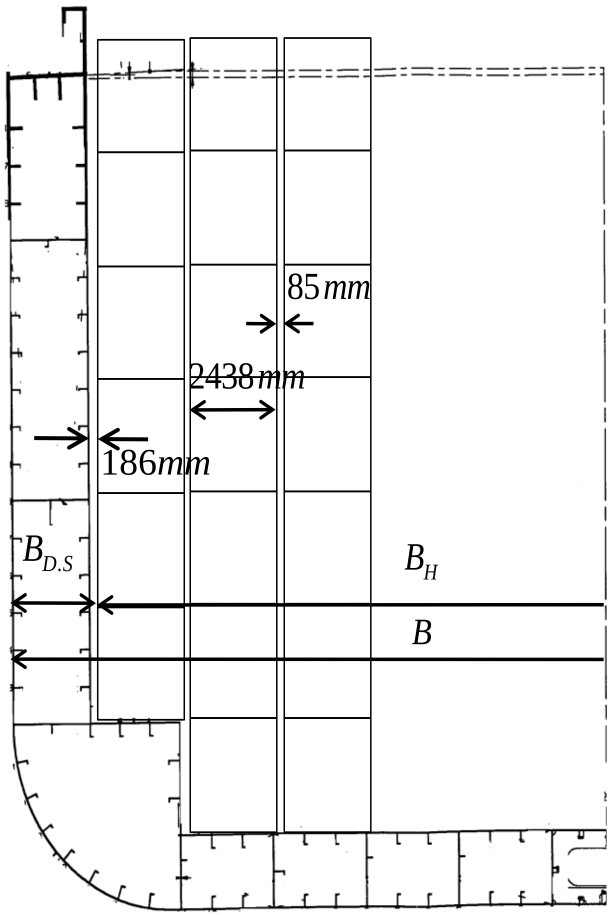
B_H : Breadth of cargo hold
$B_{D.S}$: Breadth of double side wing tank = 2.08m
N_B: Number of rows
$B_{clearance}$: Clearance between containers
$B_{clearance,D.S}$: Clearance between container and double side wing tank
$B_{container}$: Breadth of 20' container

2) Breadth

$$\boxed{B = B_H + 2 \cdot (B_{D.S} + B_{clearance,D.S})}$$

$$B_H = \left(B_{clearance} + B_{container}\right) \cdot N_B - B_{clearance}$$

$$B = \left(B_{clearance} + B_{container}\right) \cdot N_B - B_{clearance} + 2 \cdot (B_{D.S} + B_{clearance,D.S})$$



Example)

$B_{clearance} = 0.085[m]$, $B_{container} = 2.438[m]$

$B_{D.S} = 2.08[m]$, $B_{clearance,D.S} = 0.186\ [m]$

$$\rightarrow B = 2.523 \cdot N_B + 4.447$$

*Size of Container($L \times B \times D$)

20' ISO Container size : $6.058\,m \times 2.438\,m \times 2.591\,m$

40' ISO Container size : $12.192\,m \times 2.438\,m \times 2.591\,m$

- Length, breadth and depth of container carrier vary stepwise according to the number and size of containers.

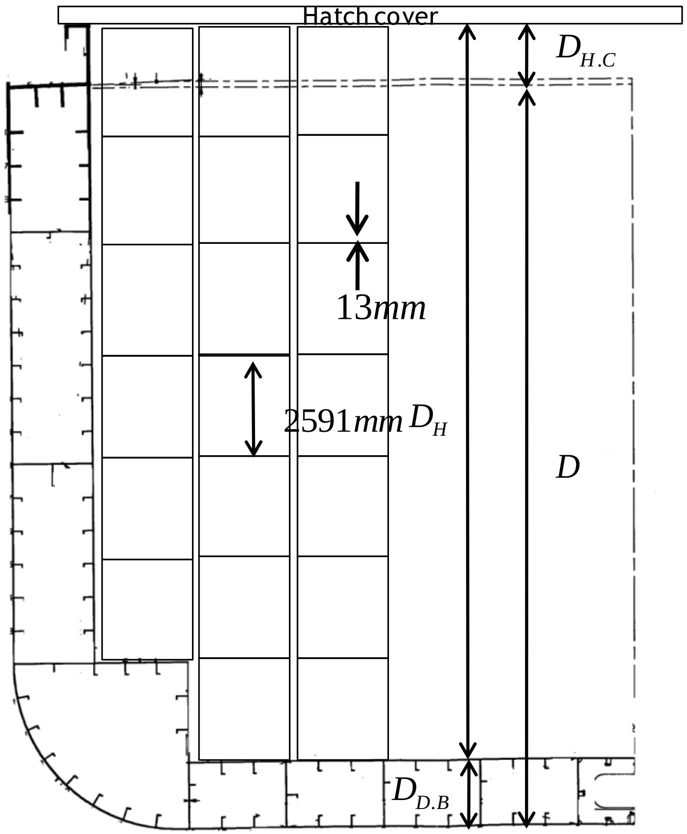
3) Depth

$$D = D_H + D_{D.B} - D_{H.C}$$

$$D_H = (D_{clearance} + D_{container}) \cdot N_D$$

$$D = (D_{clearance} + D_{container}) \cdot N_D + D_{D.B} - D_{H.C}$$

D_H : Depth of cargo hold
$D_{D.B}$: Depth of double bottom
$D_{H.C}$: Hatch coaming height
N_D: Number of tiers
$D_{clearance}$: Clearance between containers
$D_{container}$: Depth of 20' container



Example)

$D_{clearance} = 0.013[m]$, $D_{container} = 2.591[m]$

$D_{D.B} = 1.7[m]$, $D_{H.C} = 0.628[m]$

$$\rightarrow D = 2.604 \cdot N_D + 1.072$$

Step 1: Volume Equation → Step 2: Weight Equation → Step 3: Freeboard Calculation

Step 1: **The length, breadth and depth of container carrier are determined to a great extent by the arrangement of containers in cargo hold.**

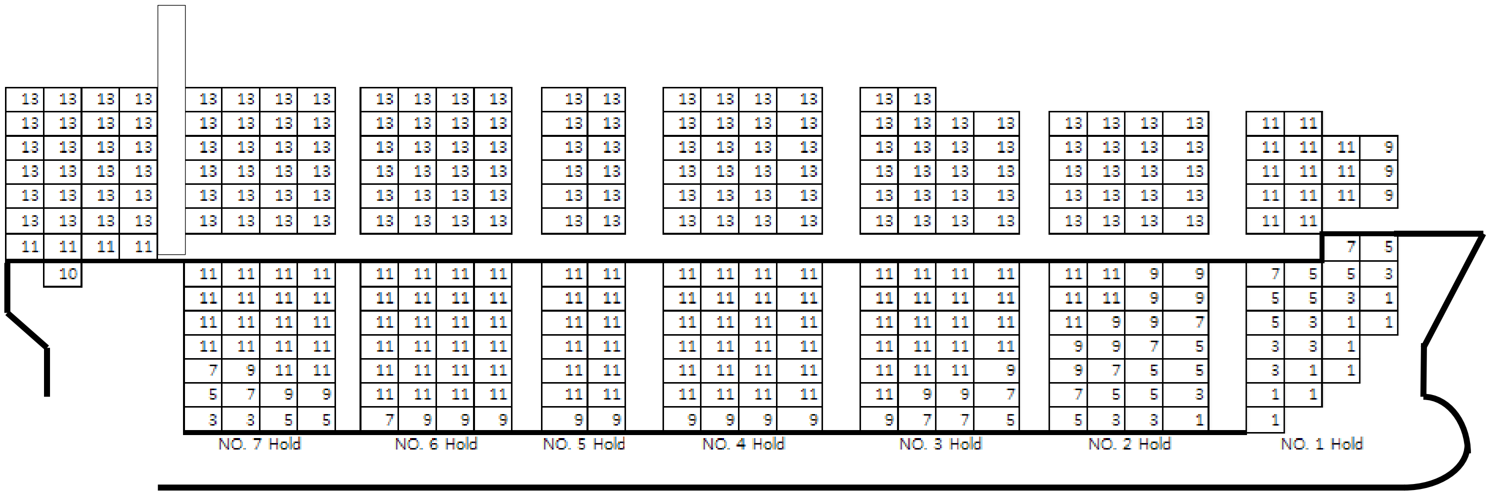
$$\boxed{N_{C\_req} = f(N_L, N_B, N_D)}$$

✓ **Given: The number of containers to be required = 4,100 [TEU]**

✓ **Find:** N_L, N_B, N_D

1. The number of additional containers to satisfy owner's requirement (4,100 *TEU*)

Basis ship(3,700 TEU Container Carrier)



In Hold: 1,565 *TEU*
On Deck: 2,174 *TEU*
Total : 3,739 *TEU*

→ The number of additional containers to be required: 361 *TEU*

Step 1) Volume Equation → Step 2) Weight Equation → Step 3) Freeboard Calculation

Given: Number of Container = 4,100 [TEU]
Find: N_L, N_B, N_D
Number of additional containers to be required : 361 TEU

$$B = 2.523 \cdot N_B + 4.447$$

2. Increase of the number of rows

1) Available breadth of the design ship

	Basis Ship	Owner's requirements
Bmld	32.2 m	Less than 32.25 m

Main dimensions for ships in Panama Canal

L_{max}	289.5 m
B_{max}	32.3 m
T_{max}	12.04 m

$$B_{available} = B_{\text{limit}} - B_{basis}$$
$$= 32.25 - 32.2$$
$$= 0.05[m]$$

$B_{avialable}$: Available breadth of design ship
B_{limit} : Breadth limited by owner's requirement
B_{basis}: Breadth of basis ship

Because 2.523 m is needed to increase 1 row in hold, it is not possible to increase the breadth.

$$\rightarrow \boxed{N_B = N_{B,basis} = 11 \; [TEU]}$$

Step 1) Volume Equation → Step 2) Weight Equation → Step 3) Freeboard Calculation

Given: Number of Container = 4,100 [TEU]
Find: N_L, N_B, N_D
Number of additional containers to be required : 361 TEU

$$L = 7.14 \cdot N_L + 54.52$$

3. Increase of the number of bays

1) Available length of the design ship

	Basis Ship	Owner's requirements
LOA LBP	257.4 m 245.24 m	Less than 260.0 m

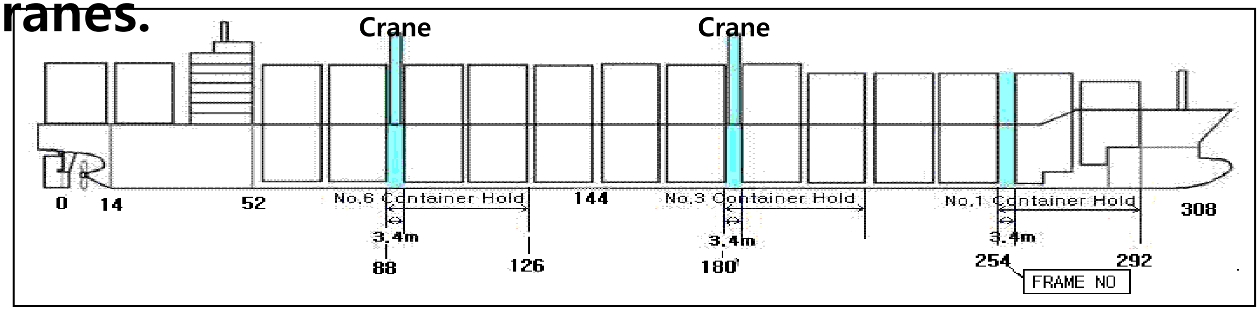
Main dimensions for ships in Panama Canal

L_{max}	289.5 m
B_{max}	32.3 m
T_{max}	12.04 m

$$\begin{aligned} L_{OA,available} &= L_{OA,\text{limit}} - L_{OA,basis} \\ &= 260 - 257.4 \\ &= 2.6[m] \end{aligned}$$

$L_{OA,available}$: Available LOA of design ship
$L_{OA,limit}$: L_{OA} limited by owner's requirement
$L_{OA,basis}$: L_{OA} of basis ship

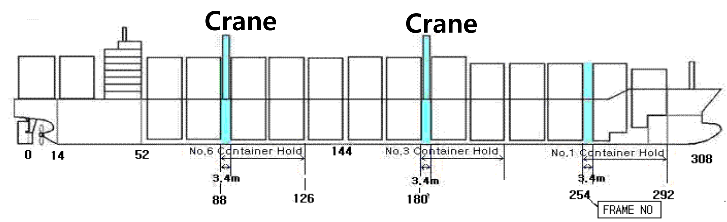
Because 7.14 m is needed to increase 1 bay in hold, it is not possible to increase the length.
However, because there is no requirement of cranes in the design ship, we can increase 1 bay in hold by utilizing the space of the three occupied cranes.



Basis ship(3,700 TEU Container Carrier)

2) Available length of the design ship by utilizing the space of cranes.

$$\boxed{L = 7.14 \cdot N_L + 54.52}$$



$$L_{crane} = (L_{space\ of\ crane} - L_{lashing\ bridge}) \cdot N_{space\ of\ crane}$$

$$= (3.4 - 1.6) \cdot 3$$

$$= 5.4[m]$$

L_{crane} : Available length of design ship by utilizing the space of crane

$L_{space\ of\ crane}$: Space of crane

$L_{lashing\ bridge}$: Space of lashing bridge(Clearance between holds)

$N_{space\ of\ crane}$: Number of space of crane

3) Total available length of design ship in lengthwise

$$= L_{OA,available} + L_{crane}$$

$$= 2.6 + 5.4$$

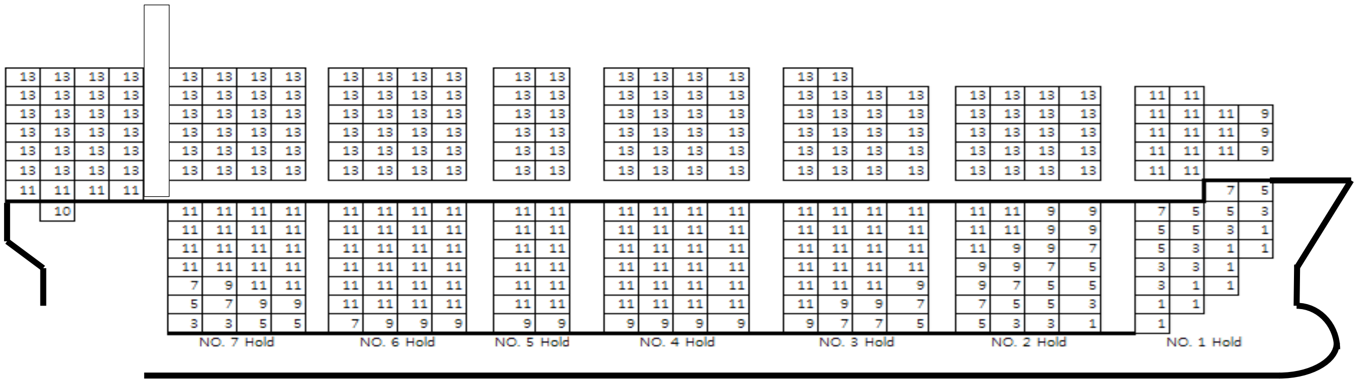
$$= 8[m] > 7.14[m] \rightarrow$$ **It is possible to increase 1 bay in hold.**

$$N_L = N_{L,basis} + 1$$

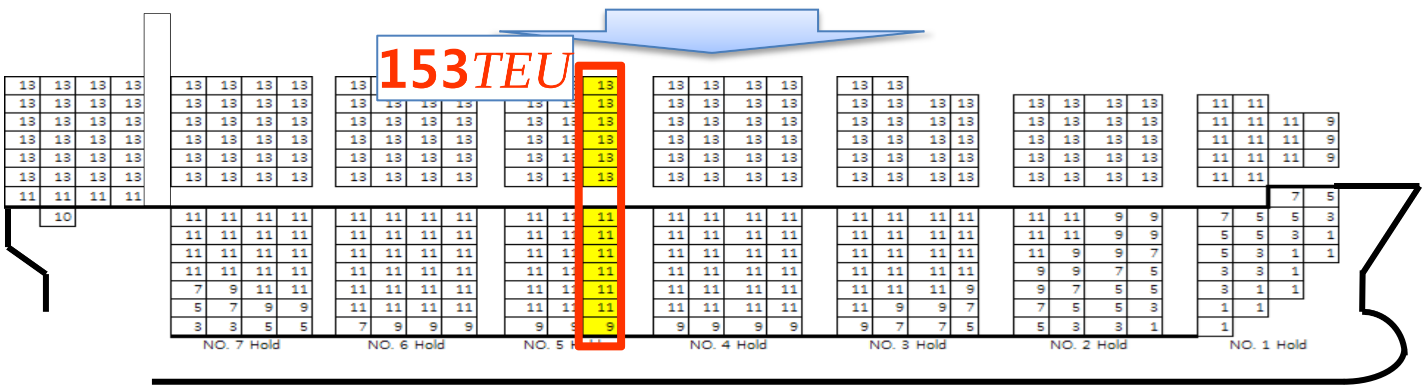
$$= 26 + 1$$

$$= 27 \quad [TEU]$$

4) Number of additional containers by increasing 1 bay.



Basis ship(3,700 *TEU* Container Carrier)



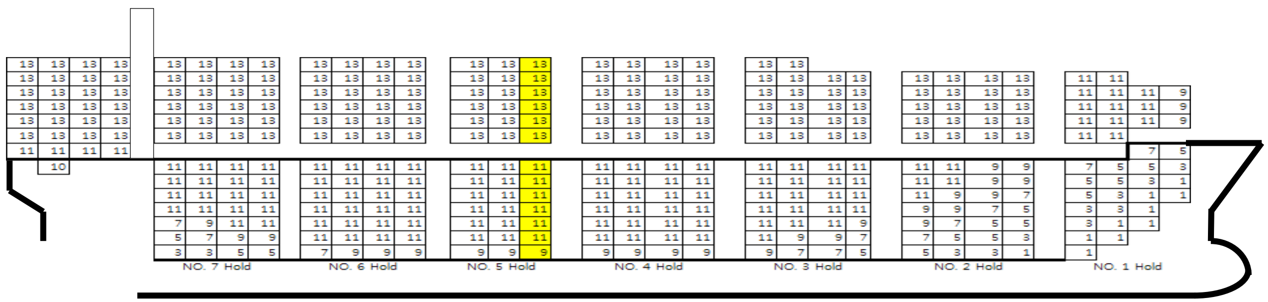
Basis ship + 1 bay

→ **Number of additional containers: 153 *TEU***

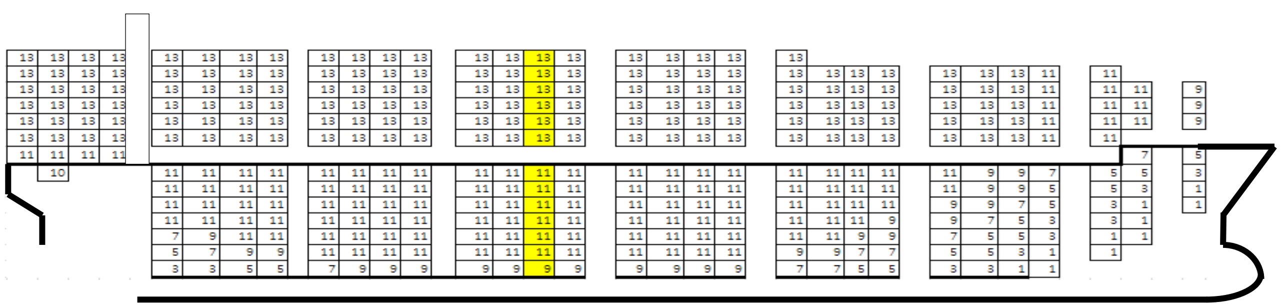
→ **Number of total containers: 3,892 *TEU***

→ **Number of additional containers to be required : 208 *TEU***

In general, the container carriers load two 40 ft containers in a hold. So, the containers of the design ship are arranged as follows:



Basis ship + 1 bay

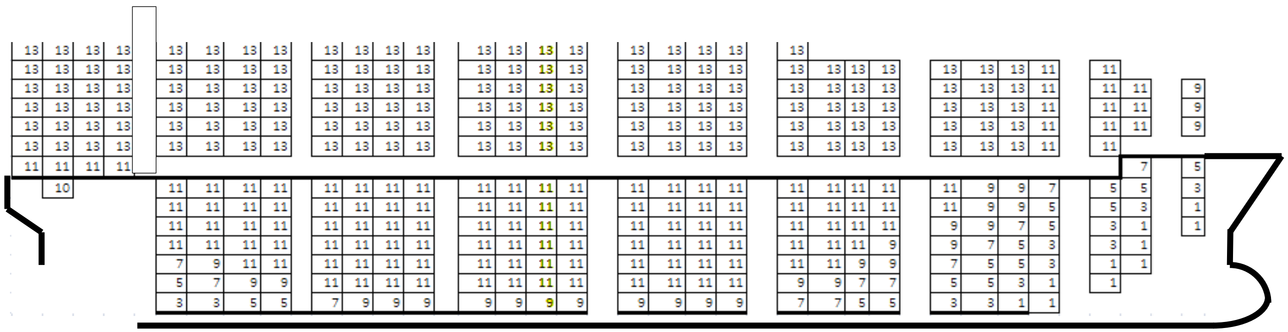


Basis ship + 1 bay

4. Increase of the number of tiers

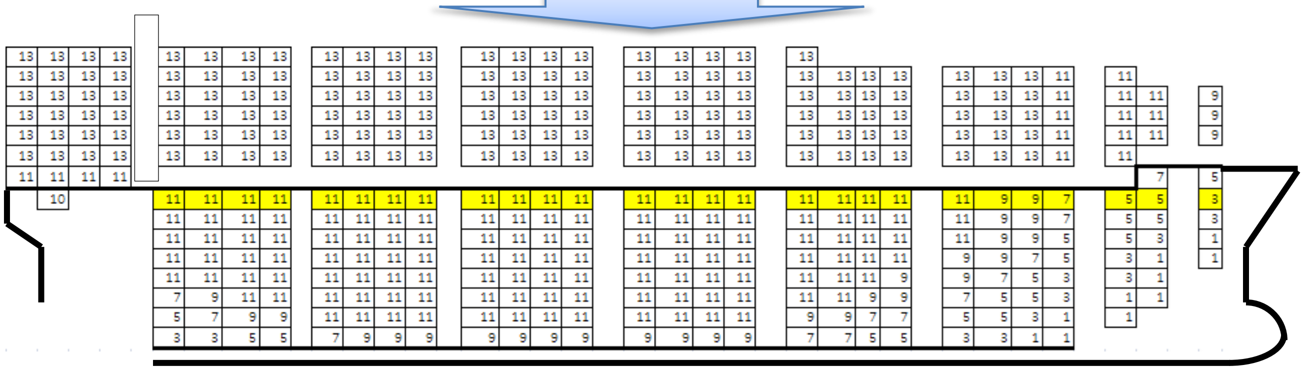
- There are two methods for increasing the tiers.

Method 1) Increase of the tiers in hold

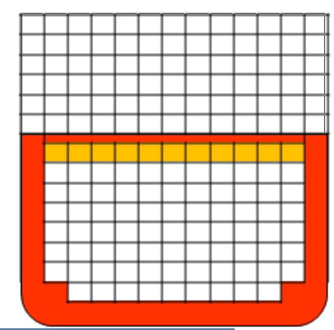


Basis ship + 1 bay (3,892 TEU)

$$
\begin{aligned}
N_D &= N_{D,basis} + 1 \\
&= 7 + 1 \\
&= 8 \quad [TEU]
\end{aligned}
$$



Basis ship + 1 bay + 1 tier in hold (4,161 TEU)

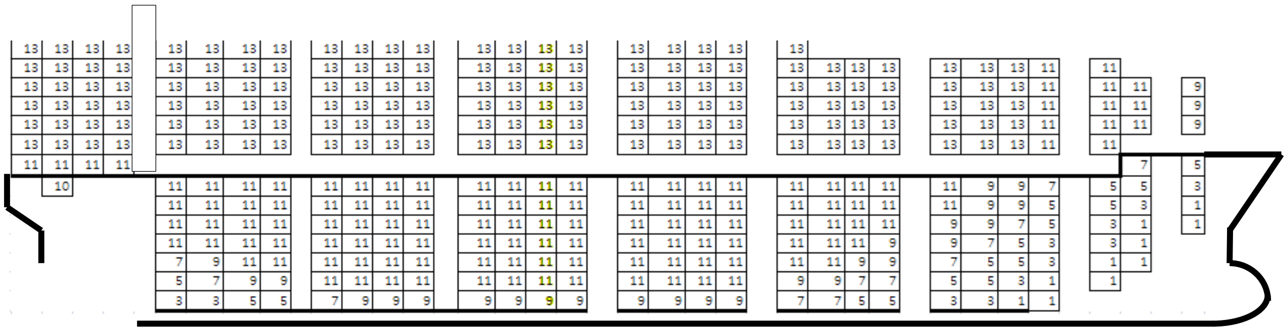


→ **Number of additional containers: 269** TEU

→ **Number of total containers: 4,161** TEU

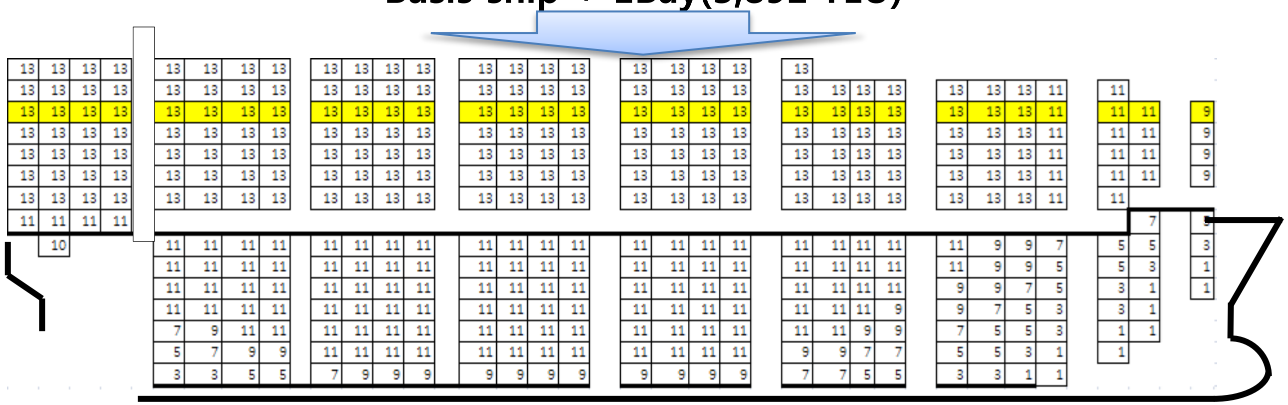
→ **Number of containers to be exceeded : 61** TEU

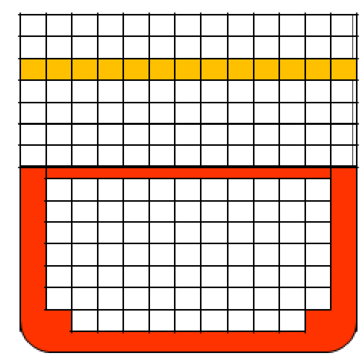
Method 2) Increase of the tiers on deck



$$N_D = N_{D,basis} = 7 \ [TEU]$$

Basis ship + 1Bay(3,892 TEU)





Basis ship + 1 bay + 1 tier on deck (4,285 TEU)

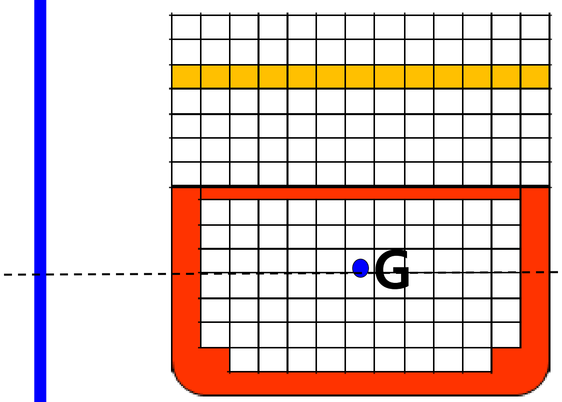
→ **Number of additional containers: 393** TEU

→ **Number of total containers: 4,285** TEU

→ **Number of containers to be exceeded : 185** TEU

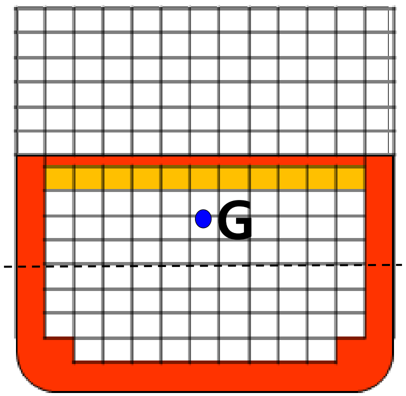
Comparison between two methods:

Method 1) Increase of the tiers on deck



$N_D = 7\ [TEU]$

Method 2) Increase of the tiers in hold



$N_D = 8\ [TEU]$

Center of mass of the containers in the method 1 method 2 are almost same.
However, the center of lightweight in the method 2 is higher than that in the method 1. So, <u>center of total mass in the method 2 is higher than that in method 1.</u>

$$\rightarrow KG_{method\,1} < KG_{method\,2}$$

$$GM = KB + BM - KG$$

$$GM_{method\,1} > GM_{method\,2}$$

$$GZ = GM \sin\phi$$

$$\boxed{GZ_{method\,1} > GZ_{method\,2}}$$

KG : **Distance from keel to vertical center of mass of container carrier**
GM : **Distance from vertical center of mass of container carrier to metacenter**
KB: **Distance from keel to center of buoyancy**
BM: **Distance from center of buoyancy to metacenter**
GZ: **Righting Arm**

<u>Therefore, for giving the ship better stability, method 1 is selected.</u>

5. Principal dimensions (L, B, D) determined by the arrangement of containers in cargo hold(N_L, N_D, N_B):

$$N_L = 27 \ [TEU]$$

$$\begin{aligned} L &= 7.14 \cdot N_L + 54.52 \\ &= 7.14 \cdot 27 + 54.52 \\ &= 247.76[m] \end{aligned}$$

$$N_B = 11 \ [TEU]$$

$$\begin{aligned} B &= 2.523 \cdot N_B + 4.447 \\ &= 2.523 \cdot 11 + 4.447 \\ &= 32.2[m] \end{aligned}$$

$$N_D = 7 \ [TEU]$$

$$\begin{aligned} D &= 2.604 \cdot N_D + 1.072 \\ &= 2.604 \cdot 7 + 1.072 \\ &= 19.3[m] \end{aligned}$$

$$\therefore L = 247.76[m], \quad B = 32.2[m], \quad D = 19.3[m]$$

Step 2: Block coefficient($C_{B,d}$) is determined by the weight equation:

$$\boxed{\rho \cdot L \cdot B \cdot T_d \cdot C_{B,d} \cdot (1+\alpha) = DWT_d + LWT}$$

ρ: density of sea water = 1.025 ton/m^3
α: a fraction of the shell appendage allowance
= 0.0029

$$\left(1+\alpha = \left.\frac{Displacement}{Moulded\ \ Displaced\ Volume}\right|_{basis} = \frac{49,848.7}{49,652.7} = 1.0039 \right)$$

✓ **Given:** L = **247.76**[m], B = **32.2**[m], D = **19.3**[m], T_d = **11.0**[m], DWT_d = **40,050**[ton], V_s=**24.5**[$knots$]

✓ **Find:** $C_{B,d}$

*Subscript d: at design draft

Lightweight estimation in components by the method 3

Step 1) Volume Equation → **Step 2) Weight Equation** → Step 3) Freeboard Calculation

$\rho \cdot L \cdot B \cdot T_d \cdot C_{B,d} \cdot (1+\alpha) = DWT_d + \boxed{LWT}$

Given: $L = 247.76[m]$, $B = 32.2[m]$, $D = 19.3[m]$, $T_d = 11.0[m]$, $DWT_d = 40{,}050[ton]$, $V = 24.5[knots]$

Find: $C_{B,d}$

Method 3 : Estimate the structural weight(W_s), outfit weight(W_o), and machinery weight(W_m) in components.

$$LWT = W_s + W_o + W_m$$

$$L \cdot B \cdot T_d \cdot C_{B,d} \cdot \rho \cdot (1+\alpha) = DWT_d + W_s + W_o + W_m$$

Structural weight(W_s) is estimated as follows:

$$W_s = C_s \cdot L^{1.6} \cdot (B+D)$$

Coefficient C_s can be obtained from the basis ship.

$$C_s = \left.\frac{W_s}{L^{1.6} \cdot (B+D)}\right|_{Basis} = \frac{11{,}000}{245.24^{1.6} \cdot (32.2+19.3)} = 0.032$$

Step 1) Volume Equation	→	Step 2) Weight Equation	→	Step 3) Freeboard Calculation

$$\rho \cdot L \cdot B \cdot T_d \cdot C_{B,d} \cdot (1+\alpha) = DWT_d + LWT$$

Given: L = **247.76**[m], B = **32.2**[m], D = **19.3**[m], T_d = **11.0**[m], DWT_d = **40,050**[ton], V=**24.5**[$knots$]

Find: $C_{B,d}$

Method 3: $LWT = W_s + W_o + W_m$

Outfit weight(W_o) is estimated as follows:

$$W_o = C_o \cdot L \cdot B$$

Coefficient C_o can be obtained from the basis ship.

$$C_o = \left.\frac{W_o}{L \cdot B}\right|_{Basis} = \frac{3,200}{245.24 \cdot 32.2} = 0.405$$

Machinery weight(W_o) is estimated as follows:

$$W_m = C_m \cdot NMCR$$

Coefficient C_m can be obtained from the basis ship.

$$C_m = \left.\frac{W_m}{NMCR}\right|_{Basis} = \frac{1,800}{38,570} = 0.047$$

Main engine of basis ship

: Sulzer 7RTA84C

Rated power: Propulsion Engines

Cyl.	Output in kW/bhp at							
	102 rpm				82 rpm			
	R1		R2		R3		R4	
	kW	bhp	kW	bhp	kW	bhp	kW	bhp
6	[illegible]	[illegible]	17 040	23 160	19 500	26 520	17 040	23 160
7	28 350	38 570	19 880	27 020	22 750	30 940	19 880	27 020
8	[illegible]	[illegible]	22 720	30 880	26 000	35 360	22 720	30 880
9	36 450	49 590	25 560	34 740	29 250	39 780	25 560	34 740
10	40 500	55 100	28 400	38 600	32 500	44 200	28 400	38 600
11	44 550	60 610	31 240	42 460	35 750	48 620	31 240	42 460
12	48 600	66 120	34 080	46 320	39 000	53 040	34 080	46 320

$$NMCR = 38,570[PS]$$

At the early design stage, $NMCR$ can be estimated by 'Admiralty formula'

$$NMCR = \frac{1}{\text{Engine Margin}} \cdot \frac{1}{\text{Derating ratio}} \cdot NCR$$

$$(\text{Engine Margin} = 0.9,\ \text{Derating ratio} = 0.9)$$

$$NMCR = 1.265 \cdot NCR$$

$$NCR = \frac{\Delta^{2/3} \cdot V_s^{\,3}}{C_{ad}}$$

Coefficient C_{ad} can be obtained from the basis ship.

$$C_{ad} = \left.\frac{\Delta^{2/3} \cdot V_s^{\,3}}{NCR}\right|_{Basis} = \frac{50,400^{2/3} \cdot 23.17^3}{34,710} = 488.96 \quad (V_{s,\ at\ design\ draft} = 23.17[knots])$$

$$NCR = \frac{\Delta^{2/3} \cdot V_s^{\,3}}{488.96}$$

$$NMCR = 1.265 \cdot \frac{\Delta^{2/3} \cdot V_s^{\,3}}{488.96}$$

$$= 0.0025 \cdot \Delta^{2/3} \cdot V_s^{\,3}$$

$$W_s = C_s \cdot L^{1.6} \cdot (B+D) \qquad C_s = 0.032$$

$$W_o = C_o \cdot L \cdot B \qquad C_o = 0.405$$

$$W_m = C_m \cdot NMCR \qquad C_m = 0.047$$

$$NMCR = 0.0025 \cdot \Delta^{2/3} \cdot V_s^3$$

$$L \cdot B \cdot T_d \cdot C_{B,d} \cdot \rho \cdot (1+\alpha) = DWT_d + W_s + W_o + W_m$$

$$L \cdot B \cdot T_d \cdot C_{B,d} \cdot \rho \cdot (1+\alpha) = DWT_d + C_s \cdot L^{1.6} \cdot (B+D) + C_o \cdot L \cdot B + C_m \cdot NMCR$$

$$L \cdot B \cdot T_d \cdot C_{B,d} \cdot \rho \cdot (1+\alpha) = DWT_d + C_s \cdot L^{1.6} \cdot (B+D) + C_o \cdot L \cdot B + C_m \cdot (0.0025 \cdot \Delta^{2/3} \cdot V_s^3)$$

$$L \cdot B \cdot T_d \cdot C_{B,d} \cdot \rho \cdot (1+\alpha) = DWT_d + C_s \cdot L^{1.6} \cdot (B+D) + C_o \cdot L \cdot B + C_m \cdot \left(0.0025 \cdot \left(L \cdot B \cdot T_d \cdot C_{B,d} \cdot \rho \cdot (1+\alpha)\right)^{2/3} \cdot V_s^3\right)$$

$$247.76 \cdot 32.2 \cdot 11.0 \cdot C_{B,d} \cdot 1.025 \cdot (1+0.0039) = 40,050 + 0.032 \cdot 247.76^{1.6} \cdot (32.2+19.3) + 0.405 \cdot 247.76 \cdot 32.2 + 0.047 \cdot \left(0.0025 \cdot \left(247.76 \cdot 32.2 \cdot 11.0 \cdot C_{B,d} \cdot 1.025 \cdot (1+0.0039)\right)^{2/3} \cdot 24.5^3\right)$$

$$90,306 \cdot C_{B,d} = 40,050 + 11,181 + 3,233 + 0.047 \cdot \left(0.0025 \cdot \left(90,306 \cdot C_{B,d}\right)^{2/3} \cdot 24.5^3\right)$$

$$90,306 \cdot C_{B,d} = 40,050 + 11,181 + 3,233 + 0.047 \cdot (0.0025 \cdot \left(90,306 \cdot C_{B,d}\right)^{2/3} \cdot 24.5^3)$$

$$90,306 \cdot C_{B,d} = 40,050 + 11,181 + 3,233 + 3,488 \cdot C_{B,d}{}^{2/3}$$

$$90,306 \cdot C_{B,d} = 54,464 + 3,488 \cdot C_{B,d}{}^{2/3}$$

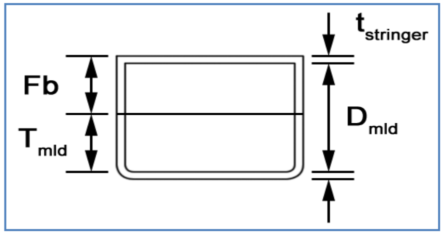
$$\therefore C_{B,d} = 0.632$$

- Step 3: Freeboard calculation

Step 1) Volume Equation → Step 2) Weight Equation → Step 3) Freeboard Calculation

Step 3: **Then, it should be checked lastly whether the depth and draft satisfy the freeboard regulation:**

$$\boxed{D_{Fb} \geq T_s + Fb(L, B, D_{mld}, C_{B,d})}$$



$$\left(D_{Fb} = D_{mld} + t_{stringer}\right)$$

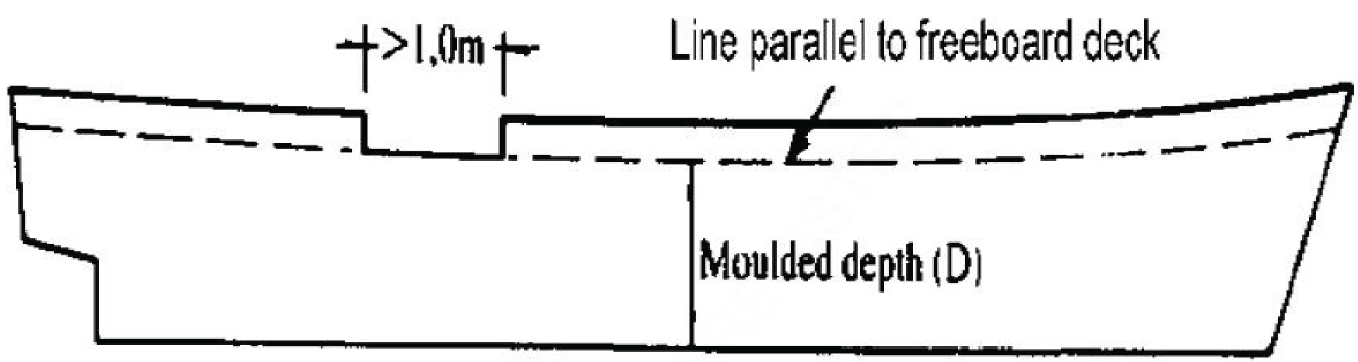
✓ **Given**: L=247.76[m], B=32.2[m], D (=D_{mld})= 19.3 [m], T_s = 11.0[m], $C_{B,d}$ =0.632, $t_{stringer}$ = 0.05[m]

✓ **Check**: **The freeboard of the ship should be larger than the required freeboard.**

Definition of freeboard deck[1]:

(a) The freeboard deck is normally the uppermost complete deck exposed to weather and sea, which has permanent means of closing all openings in the weather part thereof, and below which all openings in the sides of the ship are fitted with permanent means of watertight closing.

(b) Where a recess in the freeboard deck extends to the sides of the ship and is in excess of one meter in length, the lowest line of the exposed deck and the continuation of that line parallel to the upper part of the deck is taken as the freeboard deck.

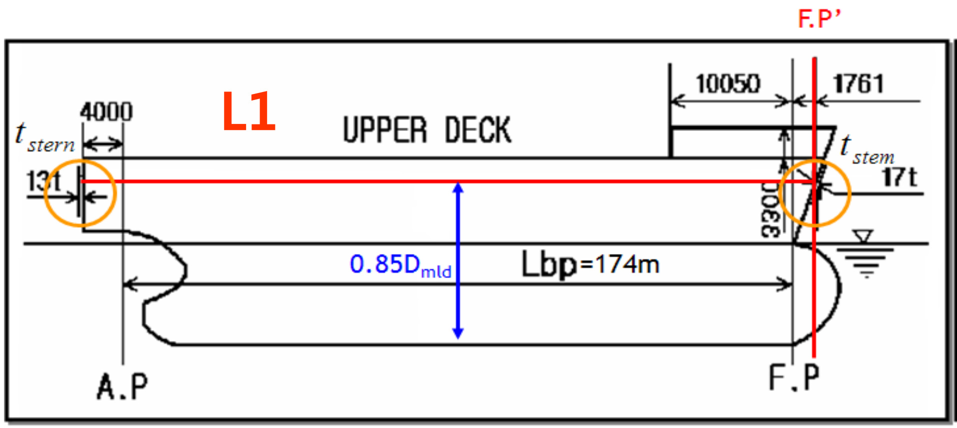


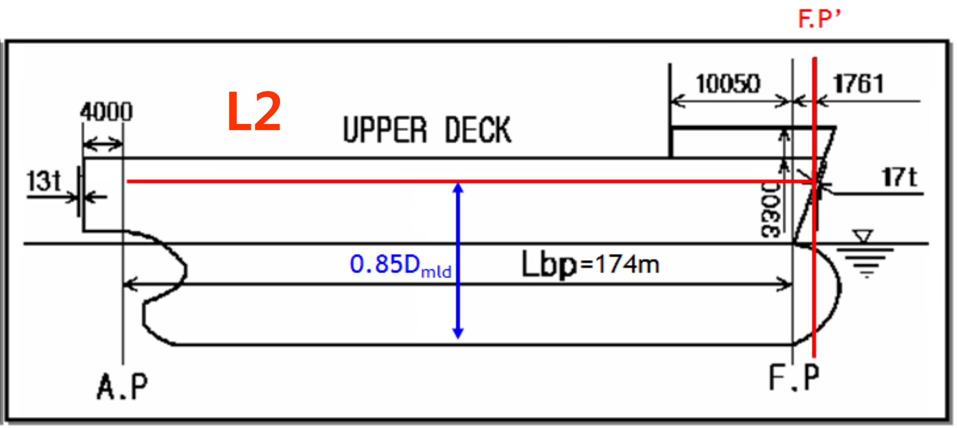
1) International Conference on Load Lines 1966, ANNEX1 Chapter 1, Reg.3-(9), 2003

Definition of freeboard Length(L_f)[2]:

(a) The length shall be taken as 96% of the total length on a waterline at 85% of the least moulded depth measured from the top of the keel(L1), or as the length from the fore side of the stem to the axis of the rudder stock on that waterline(L2), if that be greater.

(b) For ships without a rudder stock, the length (L) is to be taken as 96% of the waterline at 85% of the least moulded depth.



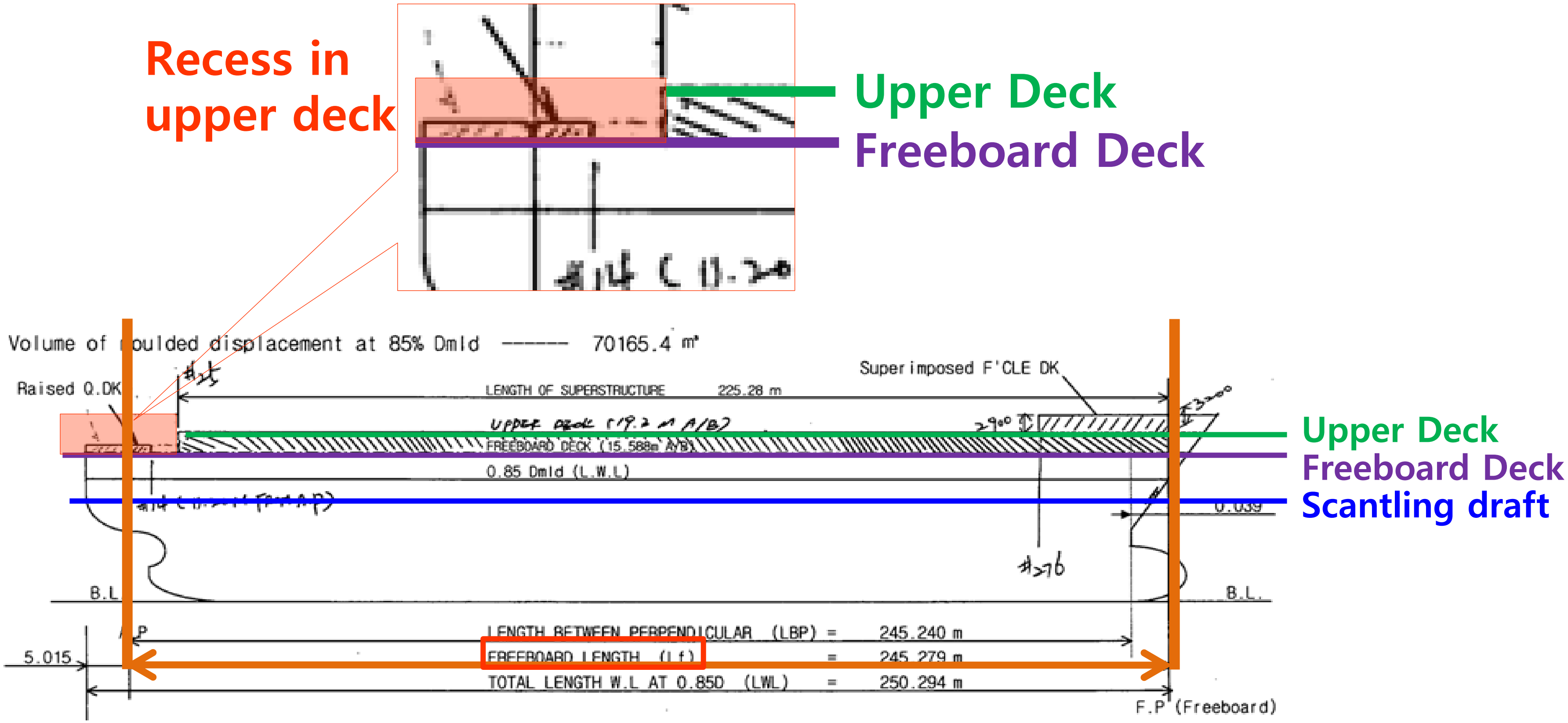


$$\boxed{L_f = \max(L_1, L_2)}$$

2) International Conference on Load Lines 1966, ANNEX1 Chapter 1, Reg.3-(1), 2003

The freeboard deck of the container carrier:

-Because there is a recess in the upper deck of the container carrier, the upper deck is discontinuous.



Therefore, the freeboard deck of the container carrier is the second deck.

Determination of the main dimensions of the 4,100 TEU Container Carrier
- Step 3: Freeboard calculation

Step 1) Volume Equation	→	Step 2) Weight Equation	→	Step 3) Freeboard Calculation

$$D_{Fb} \geq T_s + Fb(L, B, D_{mld}, C_{B,d})$$

Given: L=247.76[m], B=32.2[m], D (=D_{mld})= 19.3 [m], T_d = 11.0[m], $C_{B,d}$ =0.632, $t_{stringer}$ = 0.013[m]

Check: Freeboard of the ship should be larger than that in accordance with the freeboard regulation.

At the early design stage, the required freeboard can be estimated from the basis ship.

Assume that the freeboard is proportional to the depth.

$$Fb(L, B, D_{mld}, C_{B,d}) = C_{Fb} \cdot D_{mld}$$

$$D_{Fb} \geq T_s + C_{Fb} \cdot D_{mld}$$

Coefficient C_{Fb} can be obtained from the basis ship.

$$C_{Fb} = \left.\frac{Fb}{D_{mld}}\right|_{Basis} = \frac{3.101}{19.3} = 0.161$$

Check: Freeboard of the design ship

$$D_{Fb} \geq T_s + C_{Fb} \cdot D_{mld}$$

$$D_{\text{second deck}} + t_{stringer} \geq T_s + C_{Fb} \cdot D_{mld}$$

$$15.588 + 0.013 \geq 12.6 + 0.161 \cdot 19.3$$

$$15.601 \not\geq 15.701$$ **: Not satisfied**

$D_{second\ deck}$: Depth of the second deck
t_{string} : Thickness of second deck

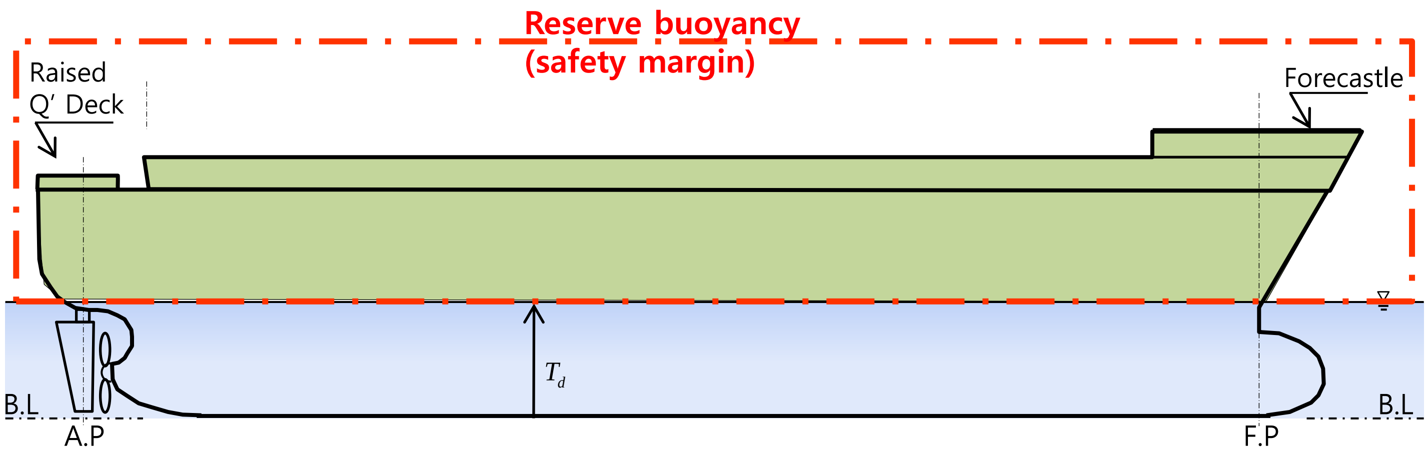
It is not satisfied. However, this method is used for a rough estimation. So, after the main dimensions are determined more accurately, freeboard needs to be calculated more accurately through the freeboard regulation.

Chapter 5. Freeboard Calculation

5-1. Concept

Purpose

- **The purpose of the freeboard**



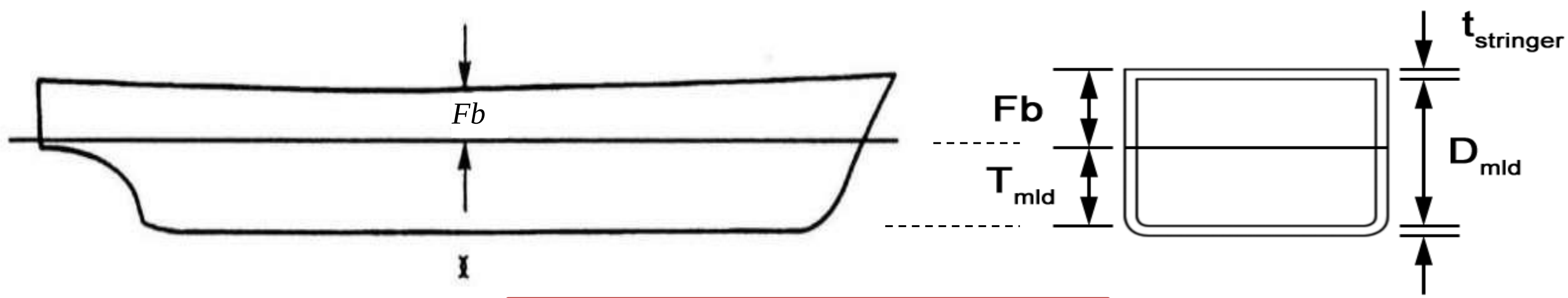
- **The ship needs an <u>additional safety margin</u> <u>to maintain buoyancy and stability while operating at sea</u>.**
  This safety margin is provided by <u>the reserve of buoyancy</u> of the hull located above the water surface(freeboard).

- **The regulation of the freeboard**

- **International Convention on Load Lines 1966 (<u>ICLL 1966</u>)**

Definition

▪ Freeboard(Fb)



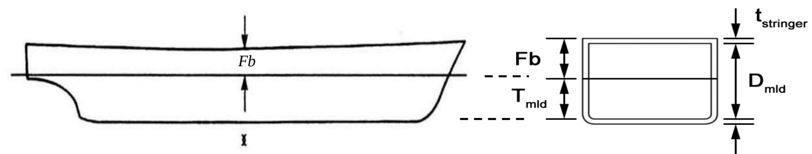
$$Fb = D_{mld} + t_{stringer} - T_{mld}$$

- **Definition** : The freeboard is the height of the freeboard deck above the load line measured at the deck edge at the mid-length between the perpendiculars. It includes the thickness of stringer plate.[1)]
- In other word, the distance between the water surface and the top of the deck at the side(at the deck line). It includes the thickness of stringer.

- **Molded Depth(D_{mld})** : The molded depth is the vertical distance measured from the top of the keel to the top of the freeboard deck beam at side.
- **Depth for freeboard(D_f)** : The depth for freeboard is the molded depth amdiships, plus the stringer thickness at side.

$D_f = D_{mld} + t_{stringer}$, $t_{stringer}$: Thickness of the stringer

1) International Convention on Load Lines 1966, ANNEX1 Chapter 1, Reg.3-(9), 2003



$$Fb = D_{mld} + t_{stringer} - T_{mld}$$

▪ Freeboard(Fb)

$$D_f - T_{mld} \geq Fb_{req.}$$

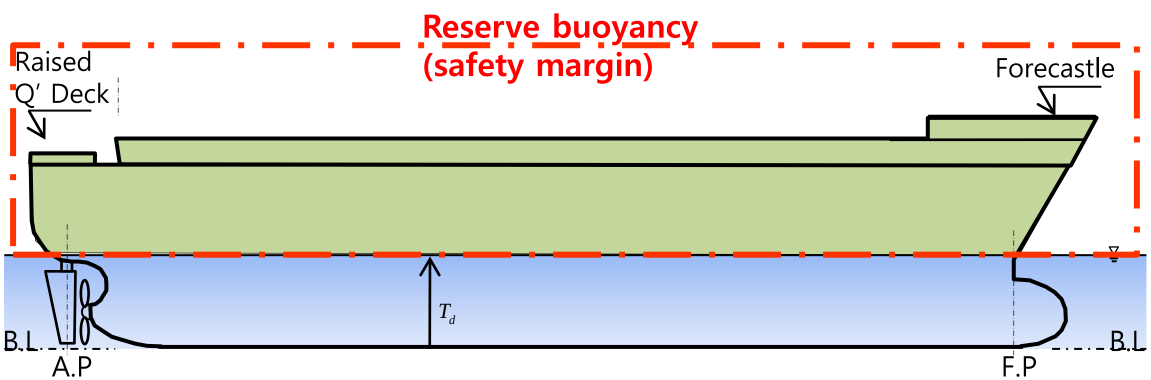
- **Requirement**

: **Actual freeboard should not be less than the required freeboard of ICLL 1966.**

Effect of freeboard on ships' characteristics

: The freeboard influences the following ship's characteristics

1. Dryness of deck. A dry deck is desirable
   (a) because walking on wet deck can be dangerous
   (b) as a safety measure against water entering through deck openings
   (c) to prevent violent seas destroying the superstructure
2. Reserve buoyancy in damaged condition
3. Intact stability (characteristics of righting arm curve).
4. Damaged stability.



Effect of freeboard on ships' characteristics

- **Large Freeboard**

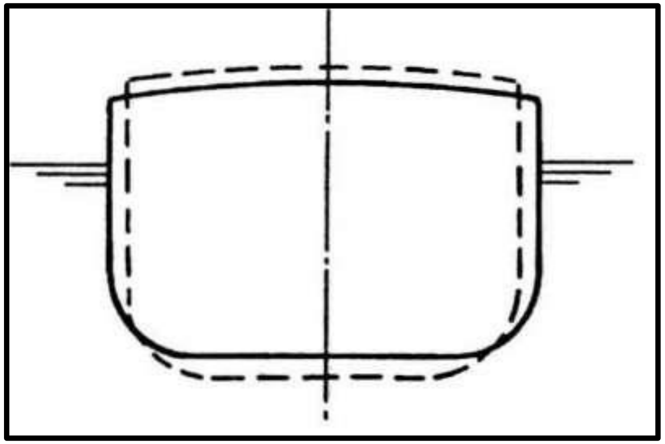


Fig. : Greater freeboard at the expense of width decreases stability

- A large freeboard improves stability. It is difficult to consider this factor in the design.

  Since for reasons of cost the necessary minimum underdeck volume should not be exceeded and the length is based on economic considerations, only a decrease in width would compensate for an increase in freeboard and depth.

Effect of freeboard on ships' characteristics

- **Increasing Freeboard**

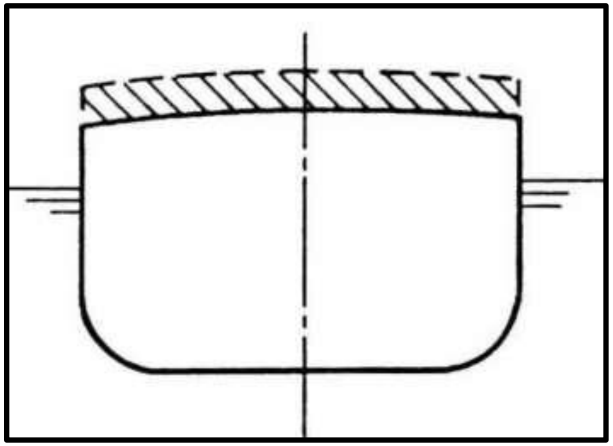


Fig. : Freeboard increased by additional superstructure

- Increasing depth and decreasing width would decrease both the initial stability and the righting arm curve.
- The stability would only be improved if the underwater form of the ship and the height of the centre of gravity remained unchanged and the freeboard were increased.

Effect of Sheer

- **Advantages and disadvantages of a construction 'without sheer'**

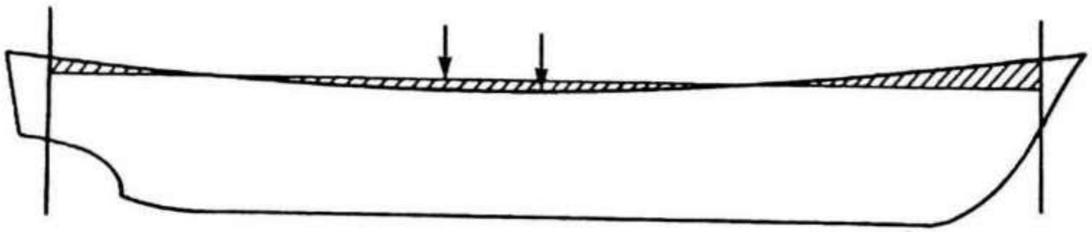


Fig. : Ship with and without sheer with same underdeck volume
(the differences in freeboard are exaggerated in the diagram)

Advantages of a construction 'without sheer'

+ Better stowage of containers in holds and on deck

+ Cheaper construction method, easier to manufacture

+ Greater carrying capacity with constant underdeck volume

Disadvantages of a construction 'without sheer'

- If the forecastle is not sufficiently high, reduced seakeeping ability

- Less aesthetic in appearance

Freeboard and sheer

- **Compensation for a lack of sheer**

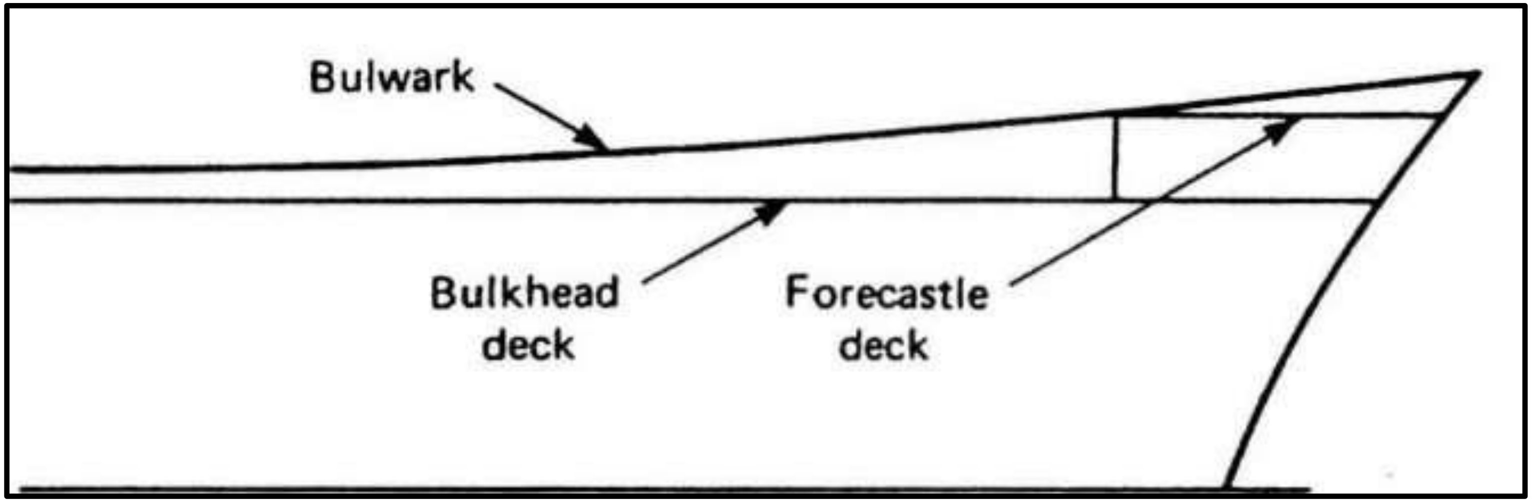


Fig. : Visual sheer effect using the line of the bulwark

The 'upper edge of bulwark` line can be extended to **give the appearance of sheer**

5-2 International Convention on Load Lines(ICLL) 1966

Regulation of the International Convention on Load Lines (ICLL)1966

- **The ICLL 1966 is structured as follows:**

Chapter I – General

- **Terms and concepts** are defined.
All the definitions of terms and concepts associated with freeboard and the freeboard calculation, and a description of how the freeboard is marked.

Chapter II – Conditions for the assignment of freeboard

- **Structural requirements** are defined.
Conditions for the assignment of freeboard structural requirements under which freeboard is assigned.

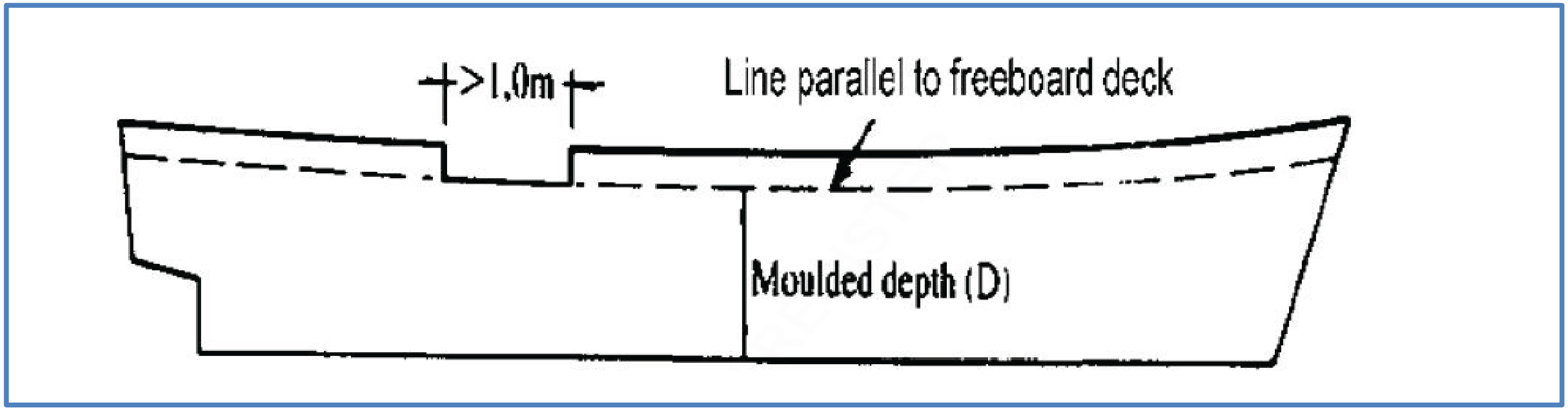
Chapter III – Freeboards

- **Procedure of freeboard calculation** is described.
The freeboard tables and the regulations for correcting the basis values given by the tables. This is **the central part** of the freeboard regulations.

The agreement is valid for cargo ships over 24 m in length and for non-cargo-carrying vessels, e.g. floating dredgers.
Warships are not subject to the freeboard regulations.

1. General Definitions

- **Freeboard deck**[1)]

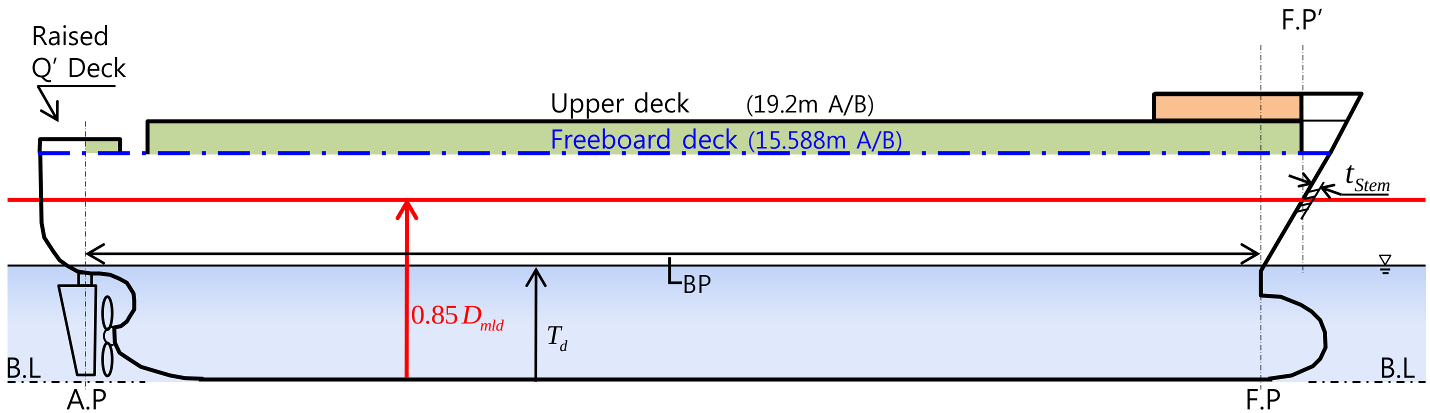


(a) The freeboard deck is normally the uppermost complete deck exposed to weather and sea, which has permanent means of closing all openings in the weather part thereof, and below which all openings in the sides of the ship are fitted with permanent means of watertight closing.

(b) Where a recess in the freeboard deck extends to the sides of the ship and is in excess of one meter in length, the lowest line of the exposed deck and the continuation of that line parallel to the upper part of the deck is taken as the freeboard deck.

1) International Convention on Load Lines 1966, ANNEX1 Chapter 1, Reg.3-(9), 2003

▪ Ex) Freeboard of 3,700 TEU Container Carrier

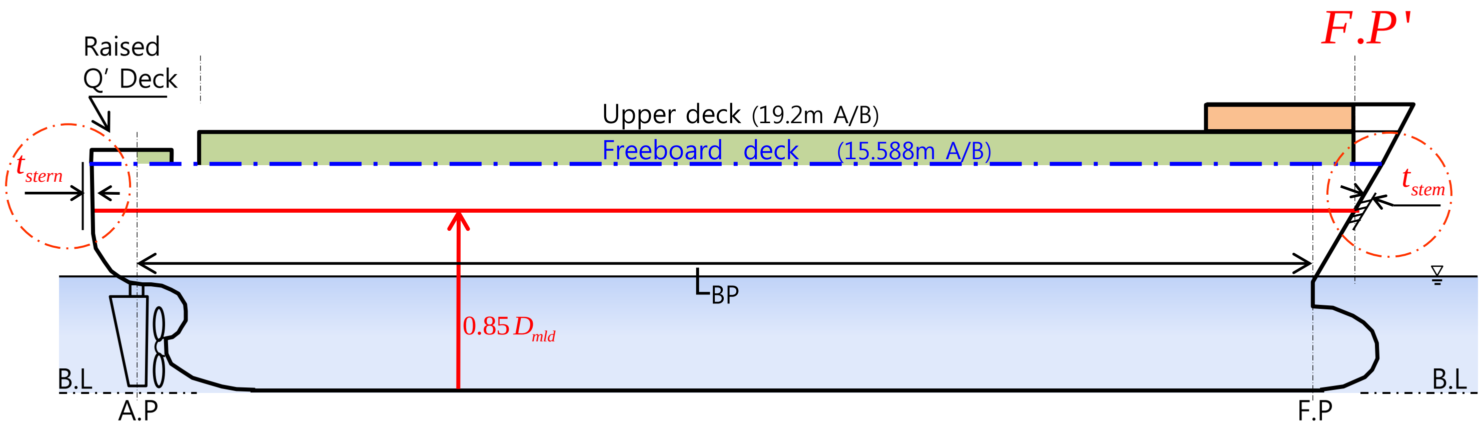


- There is a **recess in the upper deck** of the container carrier. In other words, the upper deck is **discontinuous**.

- **This 3,700 TEU container carrier is designed to assign 2nd deck as freeboard deck considering other design factors.**

- Quarter deck: deck at after part, in general, at ¼ of the ship's length after

▪ **Freeboard Length(L_f):** $\boxed{L_f = \max(L_1, L_2)}$



L_1 **: 96% of the total length(<u>including thickness of stem and stern</u>) on a <u>waterline at 85% of the molded depth</u> measured from the top of the keel(L_1)**

※ **Perpendicular : In the freeboard regulation, the forward perpendicular <u>is located at the point of the intersection of the waterline at 85% depth with the forward edge of the stem.</u>**

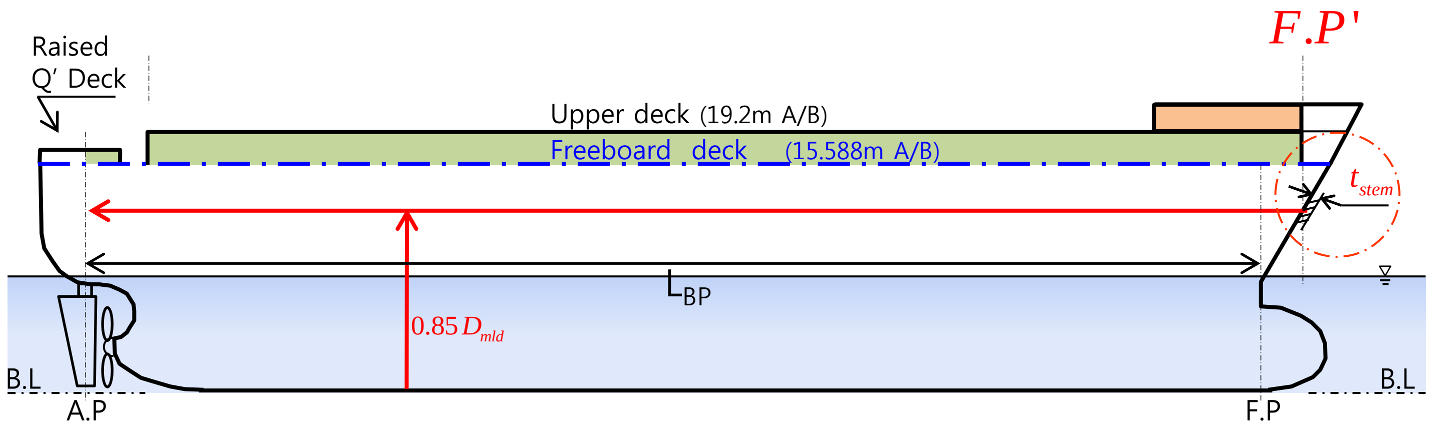
Example) L_1 of 3,700 TEU container carrier

$$L_1 = (t_{stern} + L_{Aft,0.85D} + L_{BP} + L_{Forward,0.85D} + t_{stem}) \times 0.96$$
$$= (0.015 + 5.0 + 245.24 + 0.024 + 0.015) \times 0.96$$
$$= 250.294 \times 0.96 = 240.282[m]$$

$L_{Aft,0.85D}$ **: 5.0 m**
$L_{Foward,0.85D}$ **: 0.024 m**
t_{stern} **: 0.015m**
t_{stem} **: 0.015m**
L_{BP}**: 245.24 m**

International Convention on Load Lines 1966, ANNEX1 Chapter 1, Reg.3-(1), 2003

▪ Freeboard Length(L_f): $\boxed{L_f = \max(L_1, L_2)}$



L_2 : **The length on a waterline at 85% of the molded depth from the fore side of the stem to the axis of the rudder stock**

Example) L_2 **of 3,700 TEU container carrier**

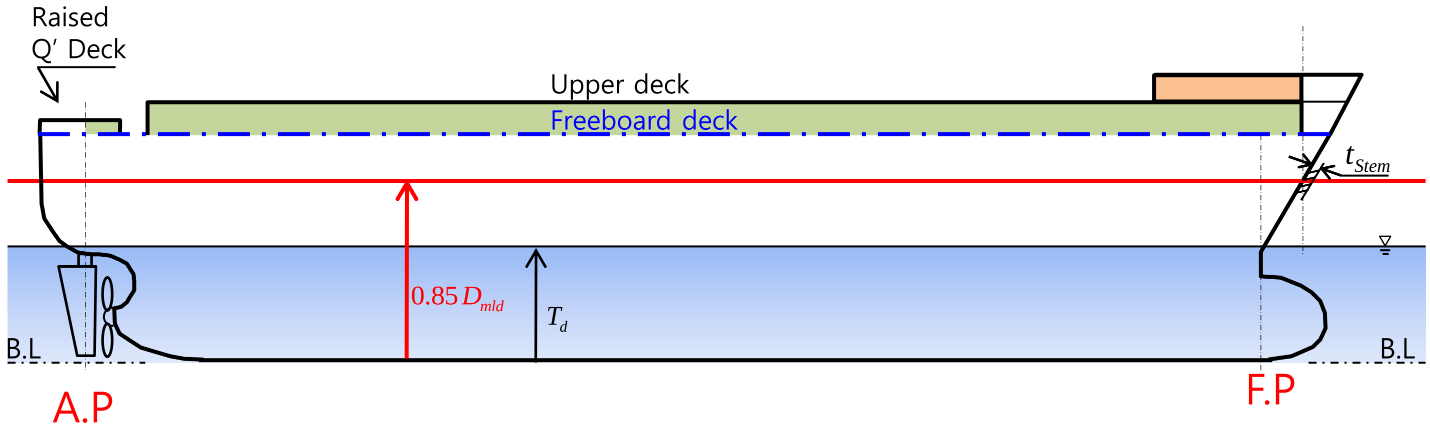
$$L_2 = L_{BP} + L_{Forward,0.85D} + t_{stem}$$
$$= 245.24 + 0.024 + 0.015 = 245.279[m]$$

$L_{Aft,0.85D}$: 5.0 m
$L_{Foward,0.85D}$: 0.024 m
t_{stern} : 0.015m
t_{stem} : 0.015m
L_{BP}: 245.24 m

$$L_f = \max(L_1, L_2)$$
$$= \max(240.282, 245.279) = 245.279\ (L_2)$$

International Convention on Load Lines 1966, ANNEX1 Chapter 1, Reg.3-(1), 2003

Perpendiculars



The aft perpendicular is established using the rudder axis. This somewhat anomalous approach due to the forward perpendicular makes sense, because the draught (to which usually the length is related) is not available as an input value.

The draught is only known after the freeboard calculation is finished.

2. Structural requirements

▪ **The requirement for the assignment of freeboard is that the ship is sufficiently safe and has adequate strength. The requirements in detail are:**

- **The particular structural requirements of the freeboard regulation must be satisfied. Particular attention should be given to : external doors, sill heights and ventilator heights, hatches and openings of every kind plus their sealing arrangements on decks and sides.**

(e.g. engine room openings, side windows, scuppers[1], freeing ports[2] and pipe outlets.)

1) Scupper: Openings in the shell plating just above deck plating to allow water to run overboard.
2) Freeing ports: An opening in the bulwark or rail for discharging large quantities of water, when thrown by the sea upon the ship's deck.

(http://www.libertyship.com/html/glossary/glosbody.htm:Project Liberty Ship - Glossary of Nautical and Shipbuilding Terms)

3. Required data for the Calculation of Freeboards

To calculate the freeboard of a ship in accordance with ICLL 1966, some data and plans are required as follows:

- -Lines or Offset Table (Fared Lines)
- -General Arrangement Plan (G/A)
- -Hydrostatic Table
- -Midship Section Plan(M/S)
- -Shell Expansion Plan
- -Construction Profile & Decks Plan
- -Superstructure Construction Plan,
- -Aft body Construction, Fore body Construction Plans

5-3. Freeboard calculation procedure

Types of ships

For the purpose of freeboard calculation, ships shall be divided into type ‘A’ and type ‘B’.

- **Type ‘A’ ships**
  - A type ‘A’ ship is designed to carry only liquid cargoes in bulk.

  Example) Crude Oil Carrier, LNG Carrier, etc.

  - The type ‘A’ ship has a high integrity of the exposed deck with only small access openings to cargo compartments, closed by watertight gasketed covers of steel or equivalent material.
  - The type ‘A’ ship has low permeability of loaded cargo compartments.

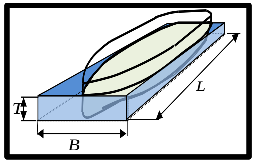
- **Type ‘B’ ships**
  - All ships which do not come within the provisions regarding type ‘A’ ships shall be considered as type ‘B’ ships.

  Example) Container Carrier, Bulk Carrier, Ore Carrier, etc.

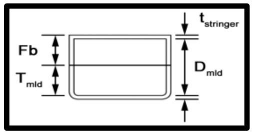
*3,700 TEU container carrier is a type ‘B’ ship.

Freeboard calculation procedure

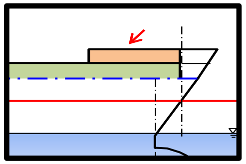
1. Tabular freeboard(F_t) calculation
2. Correction for block coefficient ($C_{B,0.85Dmld} \neq 0.68$)



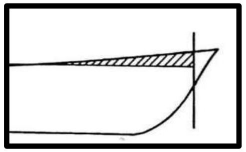
3. Correction for depth ($D_f \neq L_f/15$)



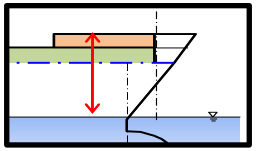
4. Deduction for superstructure and trunks



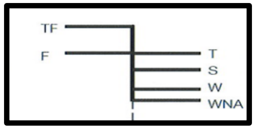
5. Correction for shear (shear $\neq$ standard shear)



6. Minimum bow height



7. Maximum molded summer draft



1) Tabular freeboard(F_t) calculation

1. Tabular freeboard(Ft) calculation
2. Correction for block coefficient ($C_{B,0.85Dmld} \neq 0.68$)
3. Correction for depth ($D_f \neq L_f/15$)
4. Deduction for superstructure and trunks
5. Correction for shear (shear ≠ standard shear)
6. Minimum bow height
7. Maximum moulded summer draft

$$L_f = \max(L_1, L_2) = 245.279[m]$$

⋮

Length of ship (m)	Freeboard (mm)
240	3690
241	3705
242	3720
243	3735
244	3750
245	3765
246	3780
247	3795
248	3808
249	3821
250	3835

⋮

[Table 1] Freeboard table for type 'B' ships

☑ The tabular freeboard for type 'B' ships shall be **determined from freeboard table** for type 'B' ships.

☑ Freeboards at intermediate lengths of ship shall be obtained by linear interpolation.

Example 3.700 TEU CTN Carrier)

$$L_f = 245.279[m]$$

$$\therefore F_t = \frac{3,765 \cdot (246 - 245.279) + 3,780 \cdot (245.279 - 245)}{(245.279 - 245) + (246 - 245.279)}$$

$$= 3,770[mm]$$

2) Correction for block coefficient(C_b)

1. Tabular freeboard(F_t) calculation
2. Correction for block coefficient ($C_{B,0.85Dmld} \neq 0.68$)
3. Correction for depth ($D_f \neq L_f/15$)
4. Deduction for superstructure and trunks
5. Correction for shear (shear ≠ standard shear)
6. Minimum bow height
7. Maximum moulded summer draft

▪ Block coefficient(C_B) at 0.85 Dmld

$$C_{B,0.85\cdot Dmld} = \frac{\nabla}{L_f \cdot B \cdot 0.85 D_{mld}}$$

-The volume of the molded displacement of the ship is <u>taken at a molded draught of $0.85D_{mld}$.</u> (∇)

3,700 TEU Container Carrier

D_{mld}	13.250 m
$C_{B,0.85Dmld}$	0.6705

☑ **If the block coefficient exceeds 0.68, the tabular freeboard specified in regulation 28 shall be multiplied by the factor.**

$$C_{B,0.85\cdot D_{mld}} \quad \text{Correction for block coefficient} = F_t \cdot \frac{(C_{B,0.85\cdot D_{mld}} + 0.68)}{1.36} - F_t$$

$$C_{B,0.85\cdot D_{mld}} \quad \text{There is no correction for block coefficient.}$$

Example 3.700 TEU CTN Carrier)

$$C_{B,0.85\cdot Dmld} = 0.6705 < 0.68$$

$$\rightarrow \text{There is no correction for block coefficient.}$$

3) Correction for depth(D_f)

$L_f = \max(L_1, L_2) = 245.279[m]$

$D_f = D_{mld} + t_{stringer}$

1. Tabular freeboard(F_t) calculation
2. Correction for block coefficient ($C_{B,0.85Dmld} \neq 0.68$)
3. Correction for depth (Df ≠Lf/15)
4. Deduction for superstructure and trunks
5. Correction for shear (shear ≠ standard shear)
6. Minimum bow height
7. Maximum moulded summer draft

▪ Depth for freeboard(D_f)

$$\boxed{D_f = D_{mld} + t_{stringer}}$$

$t_{stringer}$ **: Thickness of the freeboard deck**

3,700 TEU Container Carrier

D_{mld}	13.250 m
$t_{stringer}$	0.013m
$C_{B,0.85Dmld}$	0.6705

$$D_f \le L_f / 15$$

There is no correction for depth.

$$D_f > L_f / 15$$

$$\text{Correction for depth} = (D_f - L_f / 15) \cdot R$$

$$R = L_f / 0.48 : L_f < 120\, m$$

$$R = 250 : L_f \ge 120\, m$$

Example 3.700 TEU CTN Carrier)

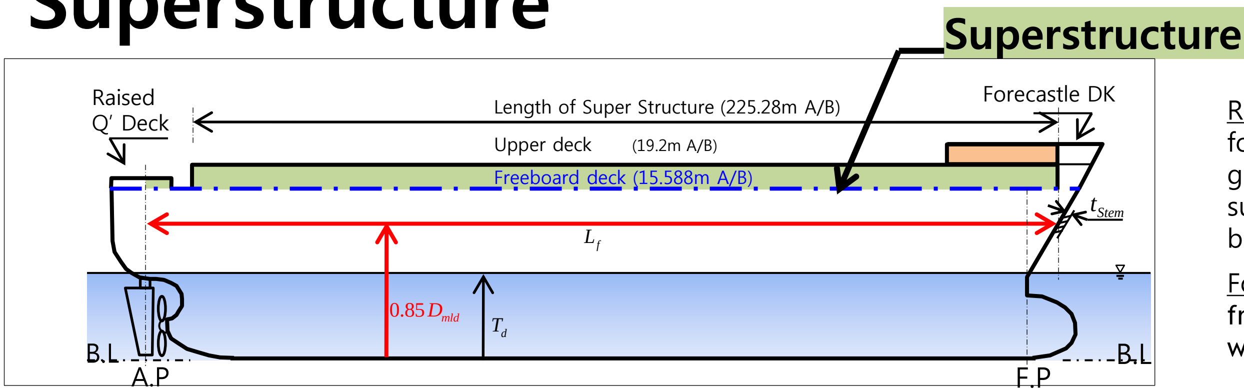
$$D_f = 15.601[m], \; L_f / 15 = 245.279 / 15 = 16.352[m]$$

$$\therefore D_f < L_f / 15$$

There is no correction for depth.

4) Deduction for superstructure and trunks

▪ Superstructure



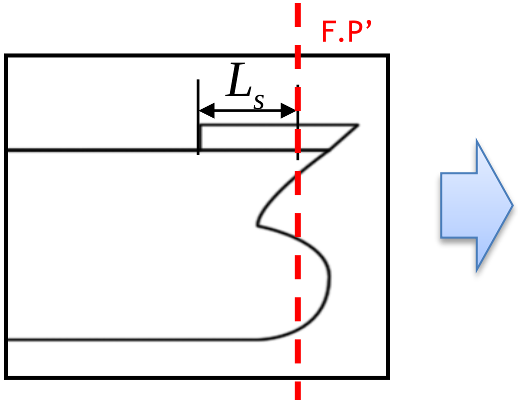
Raised Q' Deck: superstructure which extends forward from the after perpendicular, generally has a height less than a normal superstructure, and has an intact front bulkhead

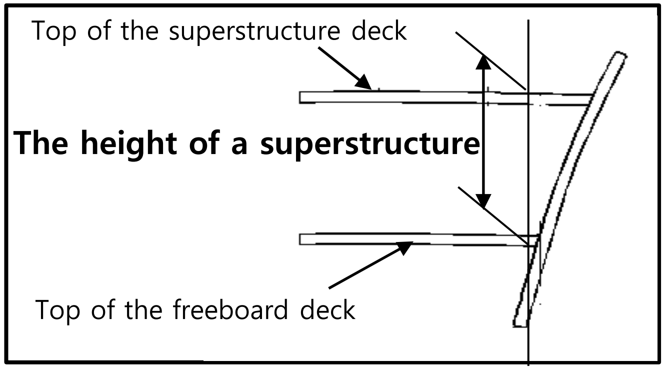
Forecastle DK: Superstructure which extends from the forward perpendicular aft to a point which is forward of the after perpendicular

- **A superstructure is a decked structure on the freeboard deck, extending from side to side of the ship or with the side plating not being inboard of the shell plating more than 4% of the breadth.**

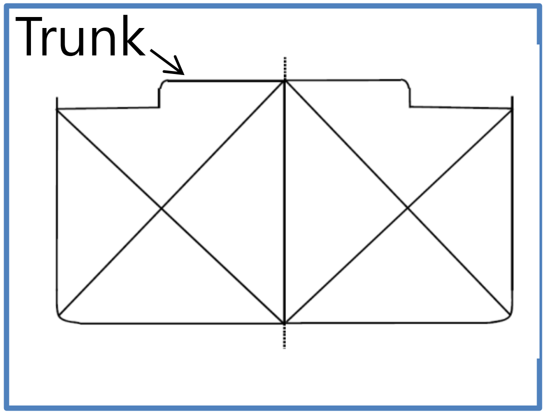
-**The height of a superstructure : The least vertical height measured at side from the top of the superstructure deck beams to the top of the freeboard deck beams.**

-**The length of a superstructure(L_s): The mean length of the part of the superstructure which lies within the freeboard length.**





Regulations for superstructure, trunk, raised quarter deck



- There are special regulations for trunks (Reg. 36) which are not covered here. E = S for an enclosed superstructure of standard height.
- S is the superstructure's length within L.

- If the superstructure is set in from the sides of the ship, E is modified by a factor b/B_s, where b is the superstructure width and B_s the ship width, both at the middle of the superstructure length (Reg. 35).

- For superstructures ending in curved bulkheads, S is specially defined by Reg. 34. If the superstructure height d_v is less than standard height d_s (Table 1.5a), E is modified by a factor d_v/d_s.
- The effective length of a raised quarter deck (if fitted with an intact front bulkead) is its length up to a maximum of 0.6L.
- Otherwise the raised quarterdeck is treated as a poop of less than standard height.

1	Tabular freeboard(F_t) calculation
2	Correction for block coefficient ($C_{B,0.85Dmld}$≠0.68)
3	Correction for depth (D_f ≠L_f/15)
4	Deduction for superstructure and trunks
5	Correction for shear (shear ≠ standard shear)
6	Minimum bow height
7	Maximum moulded summer draft

3,700 TEU Container Carrier

Item	Mean length (m)	Heig ht
Superstructure	225.28	3.71
Raised Q' Deck	11.20	1.24

☑ L_E : Effective length of superstructure(L_E)

L_E = mean length X [min(Standard Height, Actual Height)] / Standard Height

-If the height of an enclosed superstructure is

①higher than the standard height, the effective length of an enclosed superstructure of standard height shall be its length.

② less than the standard height, the effective length shall be its length reduced in the ratio of the actual height to the standard height.

*** The standard height of a superstructure shall be as given in the following table:**

L_f(m)	Raised quarterdeck(m)	All other superstructures(m)
30 or less	0.90	1.80
75	1.20	1.80
125 or more	1.80	2.30

The standard heights at intermediate lengths of the ship shall be obtained by linear interpolation.

L_s : **Length of a superstructure**

1	Tabular freeboard(F_t) calculation
2	Correction for block coefficient ($C_{B,0.85Dmld} \neq 0.68$)
3	Correction for depth ($D_f \neq L_f/15$)
4	Deduction for superstructure and trunks
5	Correction for shear (shear ≠ standard shear)
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7	Maximum moulded summer draft

3,700 TEU Container Carrier

3,700 TEU Container Carrier

Item	Mean length (m)	Height
Superstructure	225.28	3.71
Raised Q' Deck	11.20	1.24

L_f(m)	Raised quarterdeck(m)	All other superstructures(m)
30 or less	0.90	1.80
75	1.20	1.80
125 or more	1.80	2.30

Example 3.700 TEU CTN Carrier)

$$L_{\text{E,superstructure}} = 225.28\ [m]$$

$$L_{E,Rasied\ Q'deck} = L_{s,Rasied\ Q'deck} \cdot H_{Rasied\ Q'deck} / H_{\text{standard}}$$

$$= 11.20 \cdot 1.24 / 1.80 \quad (\because 1.24 < 1.80)$$

$$= 7.72\ [m]$$

$$\boxed{\therefore L_E = L_{E,Rasied\ Q'deck} + L_{\text{E,superstructure}} = 7.72 + 225.28 = 233.00[m]}$$

$L_f = \max(L_1, L_2) = 245.279[m]$
$L_E = 233.00[m]$

1. Tabular freeboard(F_t) calculation
2. Correction for block coefficient ($C_{B,0.85Dmld} \neq 0.68$)
3. Correction for depth ($D_f \neq L_f/15$)
4. Deduction for superstructure and trunks
5. Correction for shear (shear ≠ standard shear)
6. Minimum bow height
7. Maximum moulded summer draft

▪ Superstructure

☑ Deduction from the freeboard

-Where the effective length(L_E) of superstructures and trunk is
① 1.0 L_f

Deduction from the freeboard=
$$\begin{cases} 350mm & : L_f = 24m \\ 860mm & : L_f = 85m \\ 1{,}070mm & : L_f \geq 122m \end{cases}$$

② less than 1.0 L_f ,the deduction shall be a percentage obtained from the following table:
Percentage of deduction for type 'A' and 'B' ships

	Total Effective Length Superstructures and Trunks										
	0	0.1 L	0.2 L	0.3 L	0.4 L	0.5 L	0.6 L	0.7 L	0.8 L	0.9 L	1.0 L
Percentage of deduction for all types of superstructures	0	7	14	21	31	41	52	63	75.3	87.7	100

Percentages at intermediate lengths of superstructures and trunks shall be obtained by linear interpolation.

$L_f = \max(L_1, L_2) = 245.279[m]$

$L_E = 33.00[m]$

1. Tabular freeboard(F_t) calculation
2. Correction for block coefficient ($C_{B,0.85Dmld} \neq 0.68$)
3. Correction for depth ($D_f \neq L_f/15$)
4. Deduction for superstructure and trunks
5. Correction for shear (shear ≠ standard shear)
6. Minimum bow height
7. Maximum moulded summer draft

☑ Deduction from the freeboard

Example 3.700 TEU CTN Carrier)

$$L_f = 245.279[m]$$

$$L_E = 233.00[m]$$

$$\therefore L_E < L_f$$

Where the effective length(L_E) of superstructures and trunk is less than 1.0 L_f ,the deduction shall be a percentage obtained form the following table:

$$L_E / L_f = 0.95$$

	Total Effective Length Superstructures and Trunks										
	0	0.1 L	0.2 L	0.3 L	0.4 L	0.5 L	0.6 L	0.7 L	0.8 L	0.9 L	1.0 L
Percentage of deduction for all types of superstructures	0	7	14	21	31	41	52	63	75.3	87.7	100

Percentage of deduction for superstructures

$$\boxed{= 87.7 + (100 - 87.7)\ X\ (0.05/0.1) = 93.85\%}$$

$L_f = \max(L_1, L_2) = 245.279[m]$

Percentage of deduction for superstructures $= 93.85\%$

1	Tabular freeboard(F_t) calculation
2	Correction for block coefficient ($C_{B,0.85Dmld} \neq 0.68$)
3	Correction for depth ($D_f \neq L_f/15$)
4	Deduction for superstructure and trunks
5	Correction for shear (shear ≠ standard shear)
6	Minimum bow height
7	Maximum moulded summer draft

☑ Deduction from the freeboard

Example 3.700 TEU CTN Carrier)

$$L_f = 245.279[m]$$

Deduction from the freeboard= $\begin{cases} 350mm & : L_f = 24m \\ 860mm & : L_f = 85m \\ 1{,}070mm & : L_f \geq 122m \end{cases}$

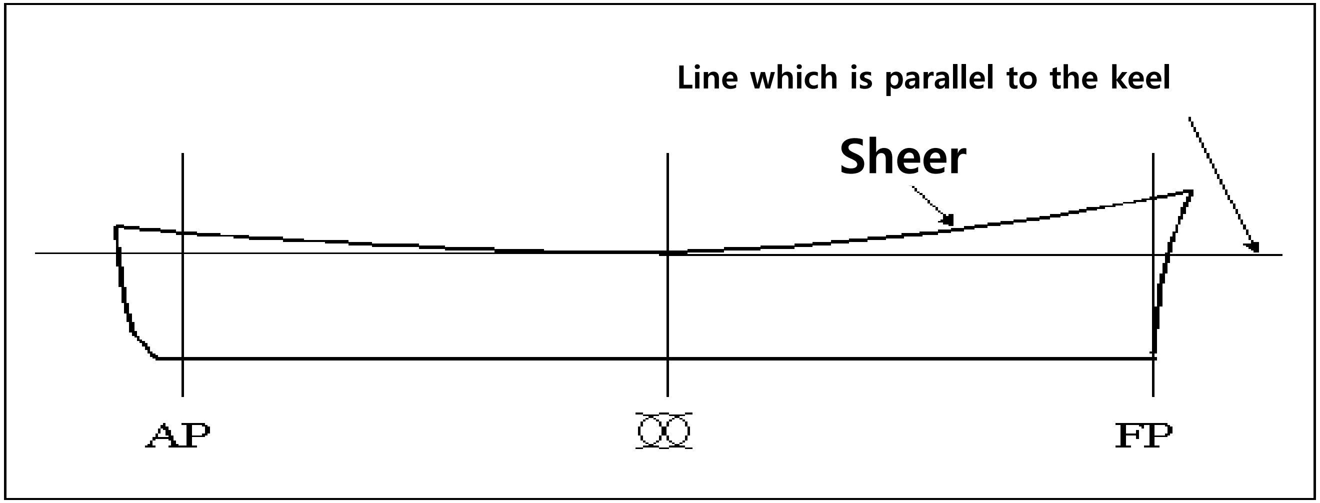
The deduction from the freeboard is multiplied by the percentage of deduction for superstructure.

deduction from the freeboard $= 1{,}070 \cdot 0.9385 = 1{,}004[mm]$

5) Correction for Sheer

1	Tabular freeboard(F_t) calculation
2	Correction for block coefficient ($C_{B,0.85Dmld}$≠0.68)
3	Correction for depth (D_f ≠L_f/15)
4	Deduction for superstructure and trunks
5	Correction for shear (shear ≠ standard shear)
6	Minimum bow height
7	Maximum moulded summer draft

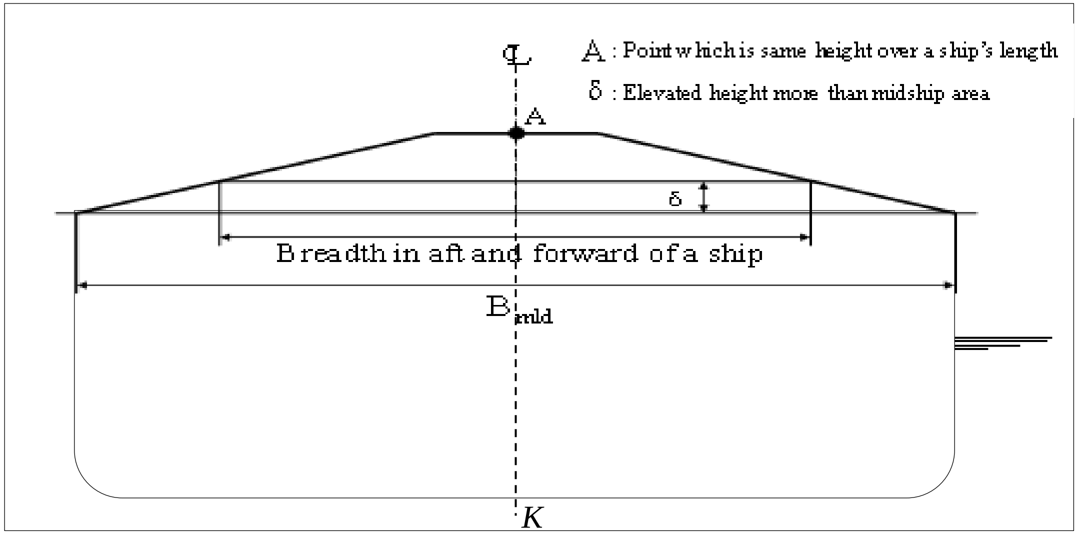
☑ Sheer



- Sheer is the upward rise of a ship's deck from mid length towards the bow and stern.

-The sheer gives the vessel extra reserve buoyancy at the stem and the stern.

☑ Camber



- Camber is the **transverse curvature of the weather deck**.

- The curvature helps to ensure sufficient **drainage** of any water on deck.

- For ships with camber of beam, care must be taken that the deck without sheer do not become too humped at the ends as a result of the deck beam. In other words, the deck 'centre-line' **should have no sheer and the deck edge line should be raised**.

☑ Correction for sheer

$$\text{Correction for shear} = (S_o - S)\cdot(0.75 - 0.5 r_1)$$

- S_o: **Standard height of Sheer (mm)**
- S : **Mean height of actual Sheer(mm)**
- r_1: **The effective length(L_E) of superstructures divided by freeboard length(L_f)**

$$r_1 = L_E / L_f$$

- When $S_0 > S$, **the tubular freeboard is added to the correction for Sheer.**

- When $S_0 < S$, **the tubular freeboard is subtracted to the correction for Sheer.**

1. Tabular freeboard(F_t) calculation
2. Correction for block coefficient ($C_{B,0.85Dmld} \neq 0.68$)
3. Correction for depth ($D_f \neq L_f/15$)
4. Deduction for superstructure and trunks
5. **Correction for shear (shear ≠ standard shear)**
6. Minimum bow height
7. Maximum moulded summer draft

a) Excess or deficiency of sheer. Design ship has no sheer

Station		Standard* Height(mm)	Standard* Ordinate	Standard* Factor	Standard* Product	Actual Height(mm)	Actual Ordinate	Actual Factor	Actual Product
After half	A.P	$25.0(L_f/3+10)$	2,294	1	2,294	S1	0	1	0
	$L_f/6$(from A.P)	$11.1(L_f/3+10)$	1,019	3	3,057	S2	0	3	0
	$L_f/3$(from A.P)	$2.8(L_f/3+10)$	257	3	771	S3	0	3	0
	Amidiship	0	0	1	0	S4	0	1	0
	Mean height	$S_A = 8.34(L_f/3 + 10)$			765	S_a			0
Forward half	Amidship	0	0	1	0	S4	0	1	0
	$L_f/3$(from F.P)	$5.6(L_f/3+10)$	514	3	1,542	S5	0	3	0
	$L_f/6$(from F.P)	$22.2(L_f/3+10)$	2,037	3	6,111	S6	0	3	0
	F.P	$50.0(L_f/3+10)$	4,588	1	4,588	S7	0	1	0
	Mean height	$S_F = 16.64(Lf/3 + 10)$			1,526	S_f			0

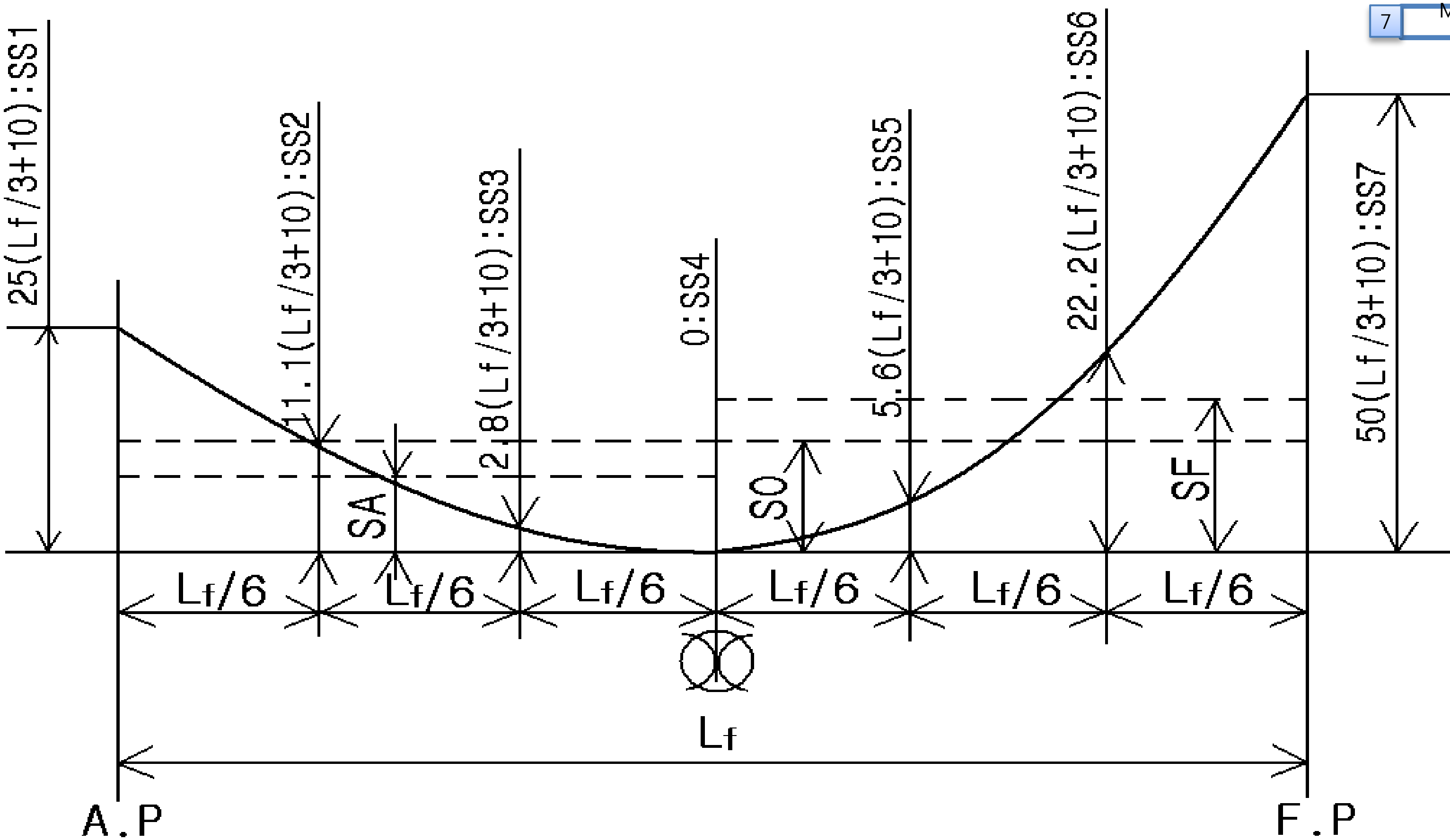
$$L_f = 245.279[m]$$

Standard height of sheer(S_o): $(S_A+S_F)/2$ = 1,146 mm

Mean height of actual sheer(S): $(S_a+S_f)/2$ = 0 mm

1 Tabular freeboard(F_t) calculation
2 Correction for block coefficient ($C_{B,0.85Dmld} \neq 0.68$)
3 Correction for depth ($D_f \neq L_f/15$)
4 Deduction for superstructure and trunks
5 Correction for shear (shear ≠ standard shear)
6 Minimum bow height
7 Maximum moulded summer draft

* Standard height of sheer



1	Tabular freeboard(F_t) calculation
2	Correction for block coefficient ($C_{B,0.85Dmld} \neq 0.68$)
3	Correction for depth ($D_f \neq L_f/15$)
4	Deduction for superstructure and trunks
5	Correction for shear (shear ≠ standard shear)
6	Minimum bow height
7	Maximum moulded summer draft

b) Sheer credit for superstructure

- **If the forward half of sheer profile or the after half of sheer profile are greater than the standard, sheer credit is given for a poop or forecastle. The sheer credit is the following:**

$$s = \frac{Y}{3} \cdot \frac{L'}{L_f}$$

s : the Sheer credit

Y : the difference between actual and standard height of superstructure at the after or forward perpendicular

L' : the mean enclosed length of poop or forecastle up to a maximum length of 0.5L

① Sheer credit for forecastle

$$s_f = \frac{Y_f}{3} \cdot \frac{L'}{L_f} = \frac{h_a - h_s}{3} \cdot \frac{L'}{L_f} = \frac{3,200 - 2,300}{3} \cdot \frac{25.3}{245.279} = 31$$

$$\rightarrow S'_f = S_f + s_f = 0 + 31 = 31\ [mm]$$

S' : Actual height of sheer corrected by sheer credit

$L_f = 245.279 [m] \quad Y_p = 0$

$h_a (actual\ height\ of\ forecastle) = 3,200\ [m$

$h_s = 2,300\ [m]$

$L'\ (length\ of\ forecastle) = 25.3\ [m]$

$S_a = 0$

$S_f = 0$

L_f(m)	Raised quarterdeck (m)	All other superstructures (m)
30 or less	0.90	1.80
75	1.20	1.80
125 or more	1.80	2.30

② Sheer credit for poop

$$s_p = \frac{Y_p}{3} \cdot \frac{L'}{L_f} = \frac{0 - 2,300}{3} \cdot \frac{0}{245.279} = 0$$

$$\rightarrow S'_a = S_a + s_p = 0 + 0 = 0\ [mm]$$

1 Tabular freeboard(F_t) calculation
2 Correction for block coefficient ($C_{B,0.85Dmld} \neq 0.68$)
3 Correction for depth ($D_f \neq L_f/15$)
4 Deduction for superstructure and trunks
5 Correction for shear (shear ≠ standard shear)
6 Minimum bow height
7 Maximum moulded summer draft

c) Correction for Sheer

Mean height of actual sheer(S):

$$S = \frac{\left(S'_a + S'_f\right)}{2} = \frac{(0+31)}{2} = 15.5\,[mm]$$

Standard height of Sheer(S_o)

:1,146 mm

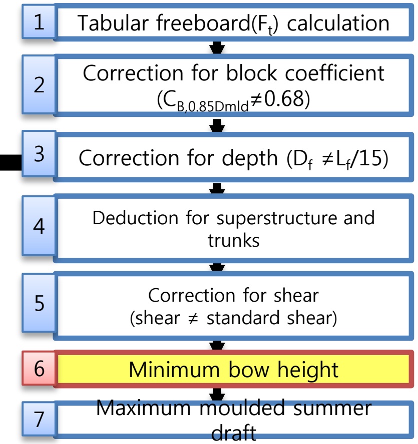
$S'_f = 31\,[mm]$

$S'_a = 0\,[mm]$

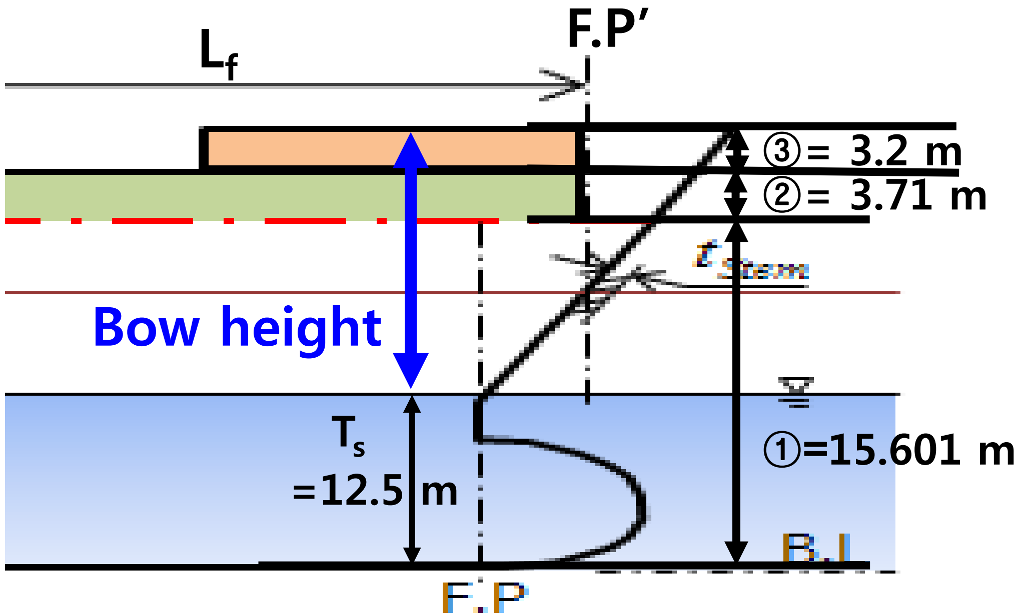
$r_1 = L_E / L_f = 0.95$

$$\text{Correction for shear} = (S_o - S)\cdot(0.75 - 0.5r_1)$$
$$= (1{,}146 - 15.5)\cdot(0.75 - 0.5\cdot 0.95)$$
$$= 311\,[mm]$$

6) Minimum bow height



☑ Bow height



- Bow height (Hb) is defined as the **vertical distance at the forward perpendicular** between the watersurface corresponding to the assigned **summer freeboard** and the designed trim and **the top of the exposed deck at side.**

Actual bow height = D_f(①) + Superstructure height(②) +Forecastle at F.P(③)-Ts
= 15.601 + 3.71 + 3.2 – 12.5
= 10.011 $[m]$

☑ Minimum bow height

① When $L_f < 250\,m$

$$Minimum\ bow\ height = 56 \cdot L_f \cdot \left(1 - \frac{L_f}{500}\right) \cdot \frac{1.36}{C_{B,0.85D} + 0.68}$$

② When $L_f \geq 250\,m$

$$Minimum\ bow\ height = 7000 \cdot \frac{1.36}{C_{B,0.85D} + 0.68}$$

- $C_{B,0.85D}$ **is the block coefficient which is to be taken as not less than 0.68.**
- **Actual bow height should be larger than minimum bow height.**

$$Minimum\ bow\ height = 56 \cdot L_f \cdot \left(1 - \frac{L_f}{500}\right) \cdot \frac{1.36}{C_{B,0.85D} + 0.68}$$

$$= 56 \cdot 245.279 \cdot \left(1 - \frac{245.279}{500}\right) \cdot \frac{1.36}{0.68 + 0.68}$$

$$= 6998\ [mm]$$

$$L_f = 245.279[m]$$
$$C_{B,0.85D} = 0.6705$$
$$D_f = 15.601[m]$$
$$T_s = 12.5\ [m]$$
$$Actual\ bow\ height = 10.011[m]$$

$$\therefore Actual\ bow\ height > Minimum\ bow\ height$$

☑ Correction for bow height

- **If actual bow height**

① is larger than minimum bow height.

$$Correction\ \ for\ bow\ height = 0$$

② is less than minimum bow height

$$Correction\ \ for\ bow\ height = Minimum\ bow\ height - Actual\ bow\ height$$

7) Maximum molded summer draft

1. Tabular freeboard(F_t) calculation
2. Correction for block coefficient ($C_{B,0.85Dmld} \neq 0.68$)
3. Correction for depth ($D_f \neq L_f/15$)
4. Deduction for superstructure and trunks
5. Correction for shear (shear ≠ standard shear)
6. Minimum bow height
7. Maximum moulded summer draft

☑ Maximum molded summer draft(d_s)

$$d_s = D_f - fs$$

*fs (Calculated summer freeboard)
= Correction for block coefficient + Correction for depth – Deduction for superstructure ± Correction for Sheer + Correction for minimum bow height

= 3,770 + 0 – 1,004 + 324 +0

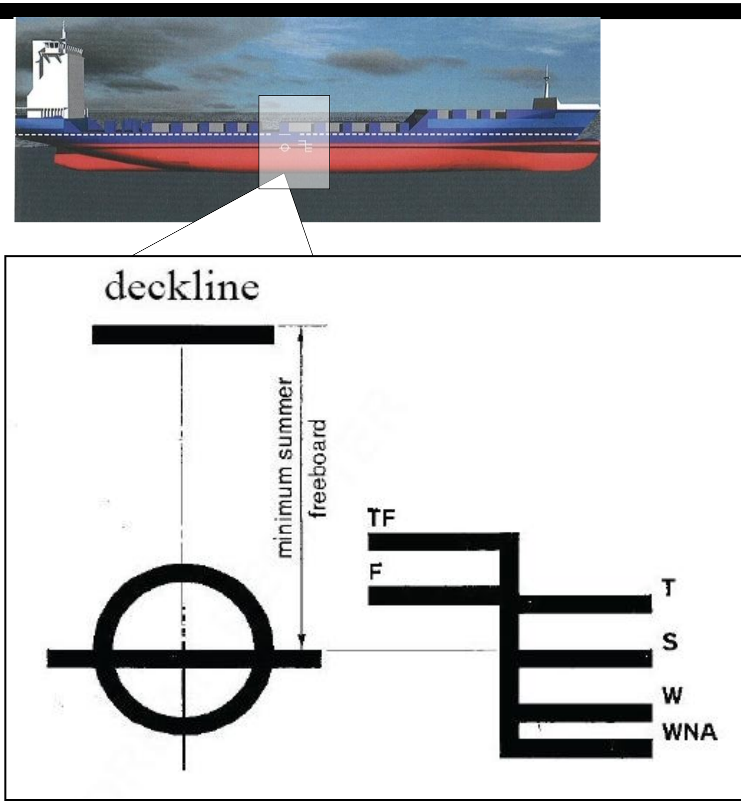
= 3,090 [mm]

Correction for block coefficient	3,770	mm
Correction for depth(D_f)	0	mm
Deduction for superstructure and trunks	-1,004	mm
Correction for Sheer	324	mm
Correction for minimum bow height	0	mm
Depth for freeboard(D_f)	8.625	m
Molded summer draft required by owner(Ts)	12.50	m

$$d_s = 15.601 - 3.090$$

$$= 12.511\,[m] > 12.5\,[m]$$

1. Tabular freeboard(F_t) calculation
2. Correction for block coefficient ($C_{B,0.85Dmld} \neq 0.68$)
3. Correction for depth ($D_f \neq L_f/15$)
4. Deduction for superstructure and trunks
5. Correction for shear (shear ≠ standard shear)
6. Minimum bow height
7. Maximum moulded summer draft



The Plimsoll[1] mark or Freeboard Mark is a symbol indicating the maximal mmersion of the ship in the water, leaving a minimal freeboard for safety.

The freeboard is marked according to the result of the freeboard calculation, where the summer freeboard in salt water is established.

Explanation of abbreviations used on the mark:

TF:	Tropical Fresh (for water with a density of 1.000 t/m³)
F:	Fresh (ditto)
T:	Tropical (for water with a density of 1.025 t/m³)
S:	Summer freeboard (ditto)
W:	Winter (ditto)
WNA :	Winter North Atlantic (ditto), only for ships, less than 100 meter
GL/NK/ LR:	Germanischer Lloyd / Nippon Kaiji Kyokai / Lloyd's Register

☑ **Tropical draft**

$$d_T = d_S + d_S / 48$$

☑ **Winter draft**

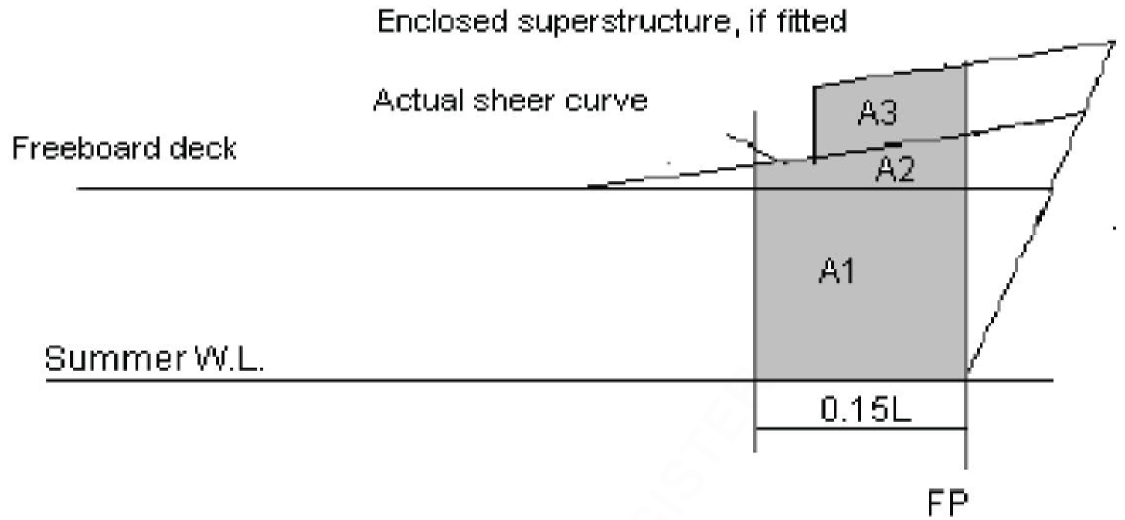
$$d_W = d_S - d_S / 48$$

Samuel Plimsoll (10 February 1825 – 3 June 1898) was a British politician and social reformer, now best remembered for having devised the Plimsoll line.

8. Reserve Buoyancy[1)]

F_0 : the tabular freeboard [mm]
f_1 : the correction for block coefficient [mm]
f_2 : the correction for depth [mm]

$$F_{\min} = F_0 \cdot f_1 + f_2$$



All ships assigned a type 'B' freeboard, other than oil tankers*, chemical tankers* and gas carriers*, shall have additional reserve buoyancy in the fore end.

The regulation is satisfied as follows:

$$A_1 + A_2 \geq (0.15 \cdot F_{\min} + 4 \cdot (L/3 + 10)) \cdot L/1000$$

and

$$A_3 \geq (0.15 \cdot F_{\min} + 4 \cdot (L/3 + 10)) \cdot L/1000$$

1) International Convention on Load Lines 1966, ANNEX1 Chapter 1, Reg.39-(5), 2003

Summary

Correction for block coefficient	3,953	mm
Correction for depth(D_f)	0	mm
Deduction for superstructure and trunks	-1,004	mm
Correction for Sheer	324	mm
Correction for minimum bow height	0	mm
Calculated summer freeboard(fs)	3,082	mm
Depth for freeboard(D_f)	15.601	m
Maximum molded summer draft(ds)	12.519	m
Molded summer draft required by owner(Ts)	12.500	m
Margin	19	mm

$$*d_s = D_f - fs$$

$$*\text{Margin} = d_s - T_s$$

Chapter 6. Resistance Prediction

6-1. Object of Resistance Prediction

1. Object of Resistance Prediction

Review) Weight Estimation : Method 4 $LWT = W_s + W_o + W_m$

$$L \cdot B \cdot T \cdot C_B \cdot \rho \cdot (1+\alpha) = DWT + C_s \cdot L^{1.6} \cdot (B+D) + C_o \cdot L \cdot B$$
$$+ C_m \cdot NMCR$$

There are few data available for estimation of the *NMCR* at the early design stage. Thus, *NMCR* can be roughly estimated by 'Admiralty formula'

C_{ad} : Admiralty coefficient
V_s : Speed of ship [knots]
Δ: Displacement [ton]
NCR: Required power for service speed

Admiralty formula :

$$NCR = f(\Delta, V_s)$$

$$NCR = C_{NCR} \cdot \Delta^{2/3} \cdot V_s^{\ 3}$$

$$\boxed{NCR = \frac{\Delta^{2/3} \cdot V_s^{\ 3}}{C_{ad}}}$$

Define $C_{ad} \equiv \frac{1}{C_{NCR}}$

C_{ad} **is called "Admiralty coefficient".**

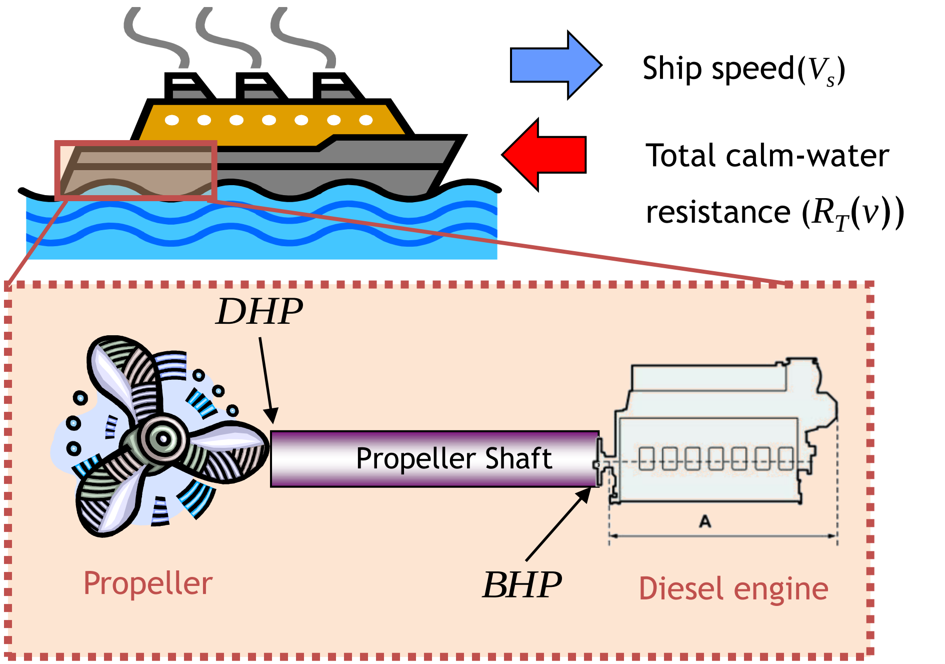
However, *NMCR* should be estimated more accurately based on the prediction of resistance and propulsion power.

☑Goal : NMCR Estimation.

At first, we have to predict **total calm-water resistance of a ship**

$$EHP = \boxed{R_T(v)} \cdot V_s$$

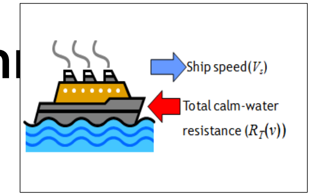
Then, by using the propulsive efficiency, shaft, and sea margin, required propulsive power can be estimated.



① EHP (Effective Horse Power)

$$EHP = \boxed{R_T(v) \cdot} V_s$$ (In calm water)

Resistance prediction



② DHP (Delivered Horse Power)

$$DHP = \frac{EHP}{\boxed{\eta_D}}$$

(η_D: Propulsive efficiency)

$\eta_D = \eta_O \cdot \eta_H \cdot \eta_R$

η_O : Open water efficiency

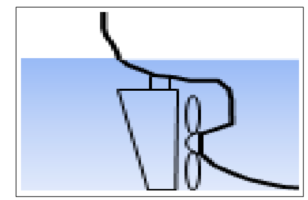
η_H : Hull efficiency

η_R : Relative rotative efficiency

Propeller efficiency

Thrust deduction and wake (due to additional resistance by propeller)

Hull-propeller interaction



③ BHP (Brake Horse Power)

$$BHP = \frac{DHP}{\eta_T}$$

(η_T : Transmission efficiency)

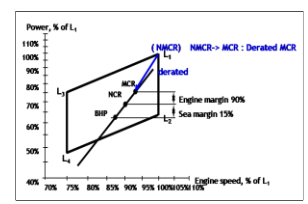
④ NCR (Normal Continuous Rating)

$$NCR = BHP(1 + \frac{\text{Sea Margine}}{100})$$

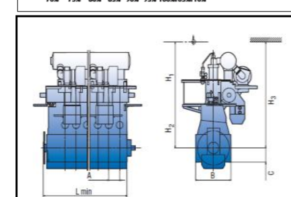
⑤ DMCR (Derated Maximum Continuous Rating)

$$\boxed{DMCR = \frac{NCR}{\text{Engine Margin}}}$$

Engine Selection



Engine Data

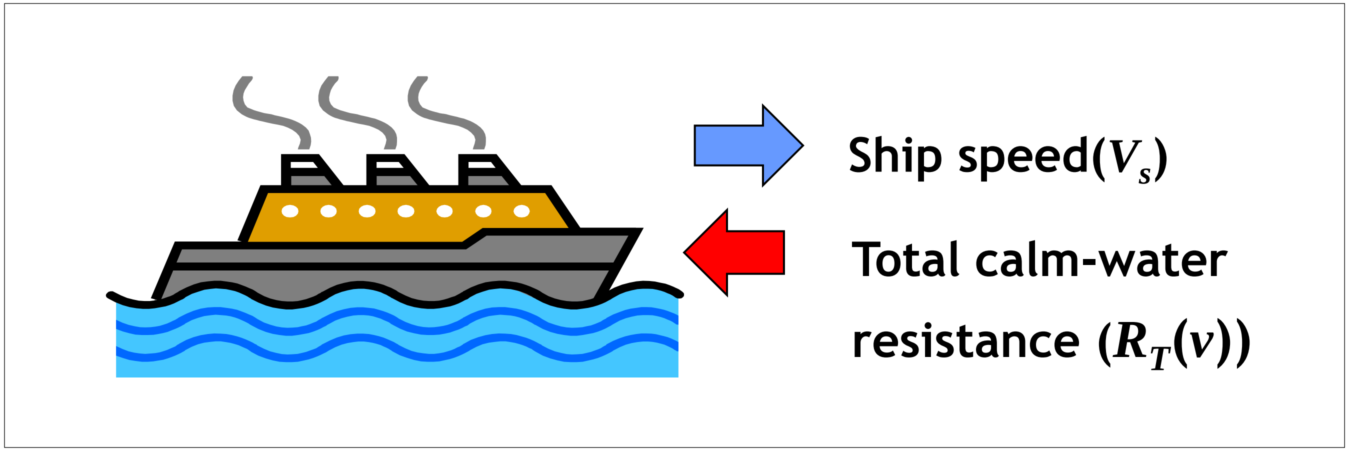


⑥ NMCR (Nominal Maximum Continuous Rating)

$$NMCR = \frac{DMCR}{\text{Derating rate}}$$

6-2. Decomposition of Resistance and Methods of Resistance Prediction

- Definition of resistance



☑ Resistance

- The resistance of a ship at a given speed is <u>the force required to tow the ship at that speed</u> in smooth water, assuming no interference from the towing ship.
- This total resistance is made up of a number of different components, which are caused by a variety of factors and which interact one with the other in an extremely complicated way.

1) PNA

- Types of resistance

In order to deal with the question more simply, it is usual to consider the total calm water resistance as being made up of four main components.

(a) The frictional resistance, due to the motion of the hull through a viscous fluid.

(b) The wave-making resistance, due to the energy that must be supplied continuously by the ship to the wave system created on the surface of the water.

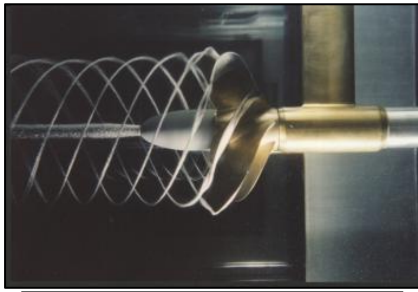
(c) Eddy resistance, due to the energy carried away by eddies shed from the hull or appendages. Local eddying will occur behind appendages such as bossings, shafts and shaft struts, and from stern frames and rudders if these items are not properly streamlined and aligned with the flow.

(d) Air resistance experienced by the above-water part of the main hull and the superstructures due to the motion of the ship through the air.

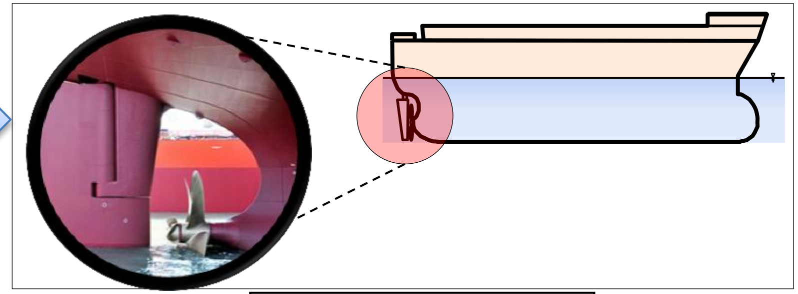
1) PNA

-Dimensional Analysis

Example) Model propeller test



A model propeller test



Real-ship propeller

▪ Dimensional Analysis

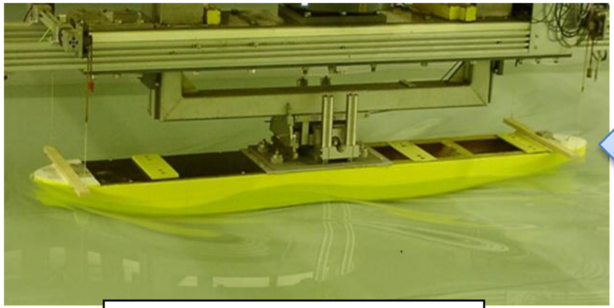
Dimensional analysis is essentially a means of utilizing a partial knowledge of a problem <u>when the details are too obscure to permit an exact analysis</u>.

It has the enormous advantage of requiring for its application a <u>knowledge only of the variables which govern the result</u>.

Dimensional solutions do not yield numerical answers, but they provide the <u>form of the answer</u> so that <u>every experiment can be used to the fullest advantage in determining a general empirical solution</u>.

1) PNA

Example) Model test in a towing tank



Model test

Design ship

▪ Application of dimensional analysis to a ship

To apply it to the flow around ships and the corresponding resistance, it is necessary to know only upon what variables the latter depends.

Applying dimensional analysis to the ship resistance problem, the resistance R could depend upon the following:

(a) Speed, V
(b) Size of body, which may be represented by the linear dimension, L.
(c) Density of fluid, ρ (mass per unit volume)
(d) Viscosity of fluid, μ
(e) Acceleration due to gravity, g
(f) Pressure per unit area in fluid, p

1) PNA

It is assumed that the resistance R can now be written in terms of unknown powers of these variables:

$$\frac{R}{1/2 \cdot \rho V^2 L^2} = f\left[\frac{\rho V L}{\mu}, \sqrt{\frac{V}{gL}}, \frac{V}{a}, \frac{\rho V^2 L}{\sigma}, \frac{p}{\rho V^2}\right]$$

The 1st term is the Reynold number R_n
The 2nd term is the Froude number F_n
The 3rd term is the Mach number M_a
The 4th term is the Weber number W_e
The 5th term is the Cavitation number σ_o

For the purpose of ship propulsion the 3rd and 4th term are not generally significant and can, therefore, be neglected. Hence equation reduces to the following for all practical ship purpose:

$$\Rightarrow \frac{R}{1/2 \rho S V^2} = f\left[\frac{VL}{\nu}, \frac{gL}{V^2}, \frac{p}{\rho V^2}\right]$$

- **Dimensionless number derived by dimensional analysis to a ship**

$$\frac{R}{1/2\rho SV^2} = f\left[\frac{VL}{\nu}, \frac{gL}{V^2}, \frac{p}{\rho V^2}\right]$$

***<u>Dimensional Homogeneity</u>**

Dimensional analysis rests on the basic principle that every equation which expresses a physical relationship must be <u>dimensionally homogeneous</u>.

Dimensionless Number:

<u>Rn (Reynolds Number)</u>: A dimensionless number that gives a measure of <u>**the ratio of inertial forces to viscous forces**</u>

$$R_n = \frac{VL}{\nu}$$

V : characteristic velocity of the ship

L : length of the ship at the waterline level

ν: kinematic viscosity

ν :In 10 degree seawater : 1.35×10^{-6}

In 15 degree seawater : 10^{-6}

<u>Fn (Froude Number)</u> : A dimensionless number comparing <u>**inertial and gravitational forces**</u>.

$$F_n = \frac{V}{\sqrt{gL}}$$

V : characteristic velocity of the ship

L : length of the ship at the waterline level

g : acceleration due to gravity

1) PNA

- Decomposition of resistance

Rn (Reynolds Number) : $R_n = \frac{VL}{\nu}$ Fn (Froude Number) : $F_n = \frac{V}{\sqrt{gL}}$

The concept of resistance decomposition helps in designing the hull form as the designer can focus on how to influence individual resistance components.

Resistance decomposition by Froude

Total resistance(R_T) = Frictional resistance(R_F) + Residual resistance(R_R) + Model-ship correlation resistance(ΔR_F)

Resistance decomposition by Hughes

Total resistance(R_T) = Viscous resistance(R_V) + Wave resistance(R_W)

▪ Frictional resistance prediction method

Frictional resistance is assumed to be a function of the Reynolds number.

Frictional resistance(R_F) :

The frictional resistance is usually predicted taking the resistance of an 'equivalent' flat plate of the same area and length as follows:

$$R_F = 1/2\rho \cdot C_F \cdot S \cdot V^2$$

ρ : density of sea water= 1.025(Mg/m^3)
C_F : frictional resistance coefficient
$V[m/s]$: characteristic velocity of the ship
$S[m^2]$: wetted surface

The 1957 ITTC(International Towing Tank Committee) line is expressed by the formula:

$$C_F = \frac{0.075}{(\log R_n - 2)^2}$$

R_n (Reynolds Number) : $\frac{VL}{\nu}$

***** Form factor 에 의해 3차원화 한게 아래식임을 설명***

Viscous resistance(R_V) : $R_V = (1+k)R_F + \Delta R_F$

k : form factor
ΔR_F : model-ship correlation factor

▪ Wave resistance prediction method

The ship creates a typical wave system which contributes to the total resistance. For fast, **slender ships** this component dominates.
In addition, there are breaking waves at the bow which dominate for slow, full hulls, but may also be considerable for **fast ships**.

The interaction of various wave systems is complicated leading to non-monotonous function of the wave resistance coefficient C_w.

The wave resistance depends strongly on the **local shape**.

$$R_W = f\,(L/B, B/T, C_b, F_n, LCB)$$

Example) Wave resistance formula in the method of Holtrop-Mennen

$$R_W = \rho g \nabla C_1 C_2 C_5 \exp\{m_1 F_n^d + m_4 \cos(\lambda F_n^{-2})\}$$

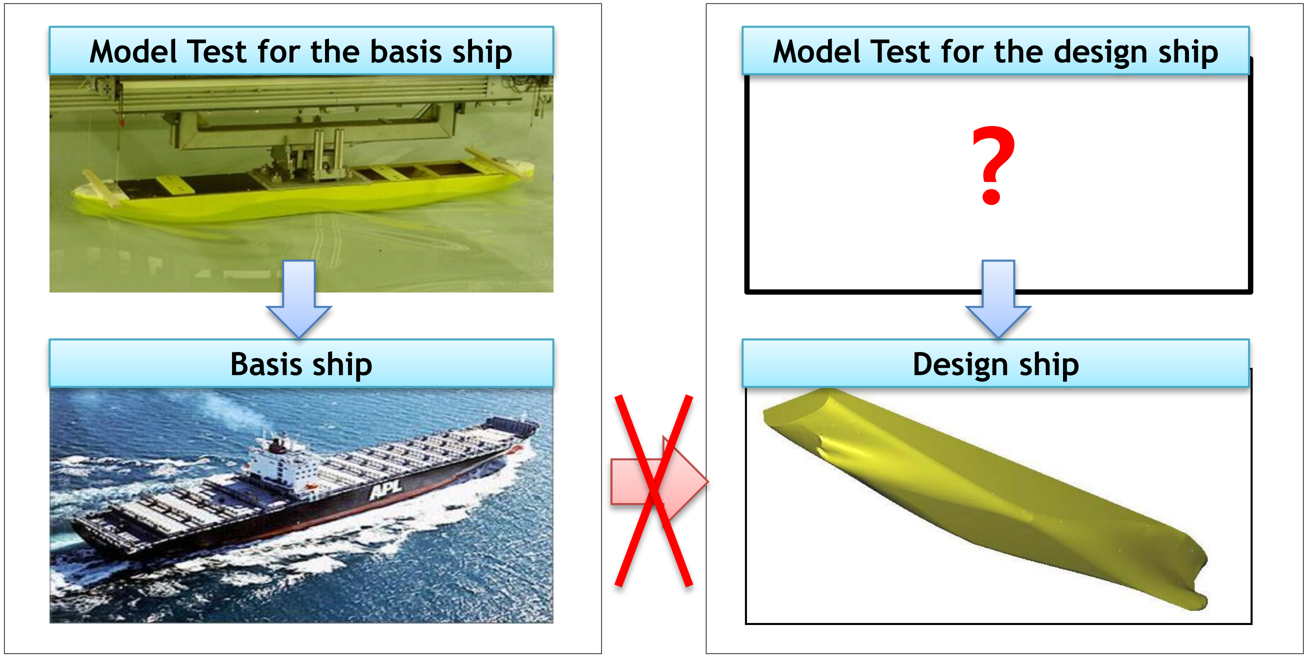
6-3. Resistance prediction by Holtrop-Mennen's method

Ship resistance evaluation methods

- **Traditional and standard series methods**
  - Taylor
  - Ayre
  - Lap
  - Auf'm Keller
  - Harvald
  - BSRA series
  - SSPA series
  - Series 60
  - Coaster series
  - ⋮
- **Regression based methods**
  - Scott
  - Holtrop
  - ⋮
- **Direct model test**
  - 2D extrapolation
  - 3D extrapolation
  - ⋮
- **Computational fluid dynamics (CFD)**
  - Advanced navier-Stokes Solution capabilities For 3D flow around ships
  - ⋮

1) Carlton

Resistance estimation by Holtrop-Mennen's method
- Reason why a statistical method is presented at the initial design stage of a ship



As the resistance of a full-scale ship cannot be measured directly, our knowledge about the resistance of ships comes from model tests.

However, at the initial design stage of a ship, the model for the design ship is not provided. Furthermore, the design ship and the basis ship are not preserved geometrical similarity.

Therefore, a statistical method was presented for the determination of the required propulsive power in the initial design stage.
This method was developed through a regression analysis of random model experiments and full-scale data.

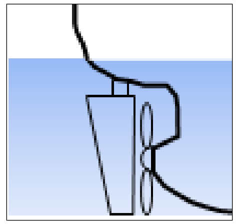
Many naval architects use the method, generally in the form presented in 1984 and find it gives acceptable results although it has to said that a number of the formula seem very complicated and the physics behind them are not at all clear, (a not infrequent corollary of regression analysis).

*Holtrop and Mennen's method, which was originally presented in the *Journal of International Shipbuilding Progress*, Vol. 25 (Oct. 1978), revised in Vol. 29 (July 1982) and again in N.S.M.B. Publication 769 (1984) and in a paper presented to SMSSH'88 (October 1988), meets all criteria with formulae derived by regression analysis from the considerable data bank of the Netherlands Ship Model Basin being provided for every variable.

1) watson

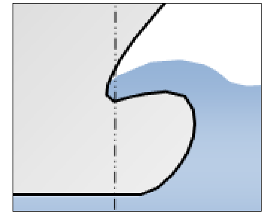
Formula

☑ Formula proposed by Holtrop & Mennen

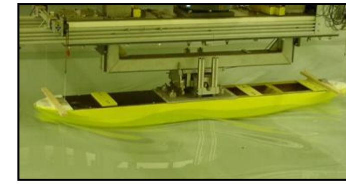


① Frictional resistance

③ Appendage resistance



⑤ Additional pressure resistance of bulbous bow near the water surface

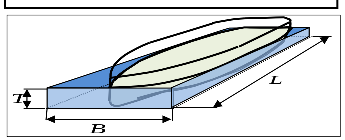


⑦ Model-ship correlation resistance

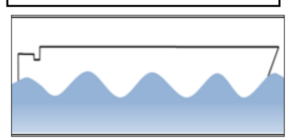
$$R_T = R_F(1+k_1) + R_{APP} + R_W + R_B + R_{TR} + R_A$$

Total resistance

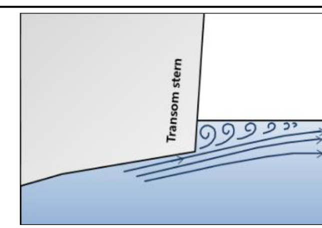
② Form factor of the hull



④ Wave resistance



⑥ Additional pressure resistance due to immersed transom immersion



1) holtrop-mennen

6-4. Resistance prediction by Holtrop-Mennen's method

for a 3,700TEU Container Carrier

Resistance prediction by Holtrop and Mennen's method
Example) 3,700TEU Container Carrier

	3,700TEU Container Carrier
Main Dimension L_{OA} L_{BP} B_{mld} D_{mld} Td / Ts (design / scantling)	 257.4 m 245.24 m 32.2 m 19.3 m 10.1 / 12.5 m
Deadweight (design / scantling)	 34,400 / 50,200 MT(metric ton)
Capacity Container on deck / in hold Ballast water Heavy fuel oil	 2,174 TEU / 1,565 TEU 13,800 m3 6,200 m3
Main Engine & Speed M / E type MCR (BHP × rpm) NCR (BHP × rpm) Service speed at NCR (Td, 15% SM) DFOC at NCR Cruising range	 Sulzer 7RTA84C 38,570 × 102 34,710 × 98.5 22.5 knots (at 11.5m) at 30,185 BHP 103.2 MT 20,000 N.M
Others Complement	30 P.

* 계산결과 수정중

	3,700TEU Container Carrier
Length on waterline (L_{WL})	239.26m
Length between perpendiculars(L_{BP})	245.24 m
Breadth moulded (B_{mld})	32.2 m
Draught moulded on F.P. (T_F)	10.1m
Draft moulded on A.P. (T_A)	10.1m
Displacement volume moulded ()	49652.7m^3
Longitudinal center of buoyancy	-0.531% aft of 1/2L_{BP}
Transverse bulb area	m^2
Center of bulb area above keel	m
line	0.9761
Midship section coefficient	0.7734
Waterplane area coefficient	
Transom area	
Wetted area appendages	
Stern shape parameter	
Propeller diameter	
number of propeller blades	
Clearance propeller with keel line	
Ship speed	

① Frictional resistance

$$R_T = \boxed{R_F}(1+k_1) + R_{APP} + R_W + R_B + R_{TR} + R_A$$

$$R_F = \frac{1}{2}\rho V^2 \boxed{C_F} S_{bh}$$

3,700 TEU Container Carrier

Item	Value
V	22.5 knots
LWL	239.26m
ν	

C_F : Coefficient of frictional resistance(ITTC 1957 friction formula)

$$C_F = \frac{0.075}{(\log R_n - 2)^2}$$

$$R_n = \frac{V \cdot L}{\nu}$$

R_n is based on the **waterline length(L_{WL})**

Example 3.700 TEU CTN Carrier)

$$R_n = \frac{V \cdot LWL}{\nu} = \frac{11.8312 \times 249.9}{1.19 \times 10^{-6}} = 2,378,767,153$$

$$C_F = \frac{0.075}{(\log R_n - 2)^2} = \frac{0.075}{(\log 2378767153 - 2)^2} = 0.001378$$

① Frictional resistance

$$R_T = \boxed{R_F}(1+k_1) + R_{APP} + R_W + R_B + R_{TR} + R_A$$

$$R_F = \frac{1}{2}\rho V^2 C_F \boxed{S_{bh}}$$

C_F =
ρ =

3,700 TEU Container Carrier

Item	Value
L	239.26m
T	10.1
B	
C_M	
C_{WP}	
A_{BT}	
C_B	

S_{bh}: **The wetted area of the bare hull**

$$S_{bh} = L(2T+B)\sqrt{C_M}(0.4530 + 0.4425C_B - 0.2862C_M - 0.003467B/T + 0.3696C_{WP}) + 2.38A_{BT}/C_B$$

In this formula, the hull form coefficients are based on the waterline length(L_{WL}).

Example 3.700 TEU CTN Carrier)

$$S_{bh} = L(2T+B)\sqrt{C_M}(0.4530 + 0.4425C_B - 0.2862C_M - 0.003467B/T + 0.3696C_{WP}) + 2.38A_{BT}/C_B$$
$$= L(2T+B)\sqrt{C_M}(0.4530 + 0.4425C_B - 0.2862C_M - 0.003467B/T + 0.3696C_{WP}) + 2.38A_{BT}/C_B$$

$$\therefore R_F = \frac{1}{2}\rho V^2 C_F S_{bh} = \frac{1}{2}\times 1.025 \times 11.8312^2 \times 0.001378 \times 9408 = 1083.952$$

$$R_F = \frac{1}{2}\rho V^2 C_F \boxed{S_{bh}}$$

S_{bh}: **The wetted area of the bare hull**

A_{BT} : **Transverse bulb area**

② Form factor of the bare hull

$$R_T = R_F \boxed{(1+k_1)} + R_{APP} + R_W + R_B + R_{TR} + R_A$$

$$1+k_1 = 0.93 + 0.487118 \cdot C_{14} (B/L)^{1.06806} (T/L)^{0.46106} (L/L_R)^{0.121563} \times (L^3/\nabla)^{0.36486} \cdot (1-C_P)^{-0.60247}$$

3,700 TEU Container Carrier

Item	Value
Afterbody from	

C_{14}: **The prismatic coefficient based on the waterline length**

$$C_{14} = 1 + 0.011 C_{stern}$$

C_{stern} $= -25$ Pram with gondola
$= -10$ V-shaped sections
$= 0$ Normal section shape
$= 10$ U-shaped sections

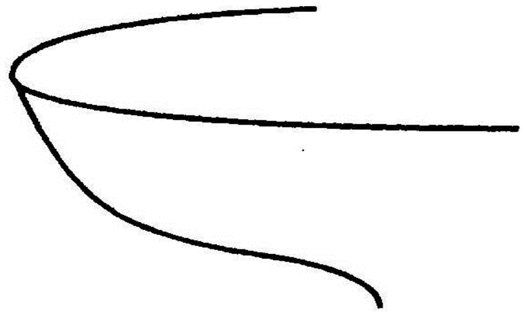
Example 3.700 TEU CTN Carrier)

C_{stern}

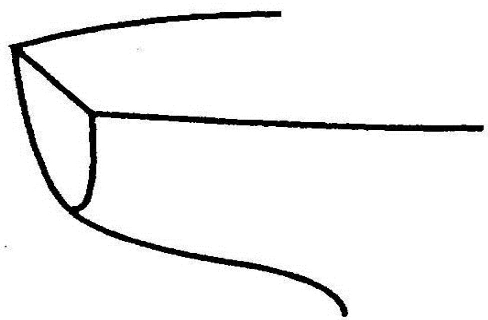
$$C_{14} = 1 + 0.011 C_{stern}$$
$$= 1 + 0.011 C_{stern}$$
$$=$$

② Form factor of the bare hull

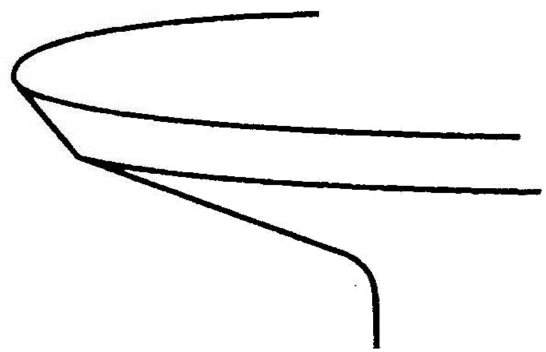
Types of stern



(A) Cruiser sterned vessels



(B) Transom sterned vessels



(C) Counter sterned vessels

② Form factor of the bare hull

$$R_T = R_F \boxed{(1+k_1)} + R_{APP} + R_W + R_B + R_{TR} + R_A$$

$$1+k_1 = 0.93 + 0.487118 \cdot C_{14}(B/L)^{1.06806}(T/L)^{0.46106}(L/\boxed{L_R})^{0.121563} \times (L^3/\nabla)^{0.36486} \cdot (1-C_P)^{-0.60247}$$

C_{14} :

3,700 TEU Container Carrier

Item	Value
L	
B	
T	
∇	
C_p	
L_{CB}	

L_R : Length of run

At early design stage, If L_R is unknown, it can be obtained by following formula:

$$L_R / L = 1 - C_P + 0.06 C_P \cdot L_{CB} / (4C_P - 1)$$

L_{CB} **: The longitudinal position of the centre of buoyancy forward of 0.5L as a percentage (%) of L.**

forward: (+) aft : (-)

Example 3.700 TEU CTN Carrier)

$$\begin{aligned} L_R &= L(1 - C_P + 0.06 C_P \cdot L_{CB} / (4C_P - 1)) \\ &= 239.26(1 - 0.6394 + 0.06 \times 0.6394 \times (-0.68) / (4 \times 0.6394 - 1)) \\ &= 82.255 \end{aligned}$$

$$\begin{aligned} 1+k_1 &= 0.93 + 0.487118 \cdot C_{14}(B/L)^{1.06806}(T/L)^{0.46106}(L/L_R)^{0.121563} \times (L^3/\nabla)^{0.36486} \cdot (1-C_P)^{-0.60247} \\ &= 0.93 + 0.487118 \times 0.89(32.2/239.26)^{1.06806}(10.1/239.26)^{0.46106}(239.26/82.255)^{0.121563} \times (239.26^3/49778)^{0.36486} \cdot (1-0.6394)^{-0.60247} \\ &= 1.123 \end{aligned}$$

③ Resistance of appendages

$$R_T = R_F(1+k_1) + \boxed{R_{APP}} + R_W + R_B + R_{TR} + R_A$$

$$R_{APP} = 1/2 \rho V^2 S_{APP} (1+k_2)_{eq} C_F$$

C_F =
ρ =

3,700 TEU Container Carrier

Item	Value
V	
Appendages(S_{APP})	

S_{APP} : The wetted area of the appendages

$(1+k_2)$: The appendage resistance factor

- Rudder behind skeg: 1.5~2.0
- Rudder of single screw ship: 1.3~1.5
- Twin-screw balance rudders: 2.8
- Shaft brackets: 3.0
- Skeg: 1.5~2.0
- Strut bossings: 3.0
- Hull bossings: 2.0
- Shafts: 2.0~4.0
- Stabilizer fins: 2.8
- Dome: 2.7
- Bilge keels: 1.4

The equivalent 1 + k_2 value for a combination of appendages is determined from:

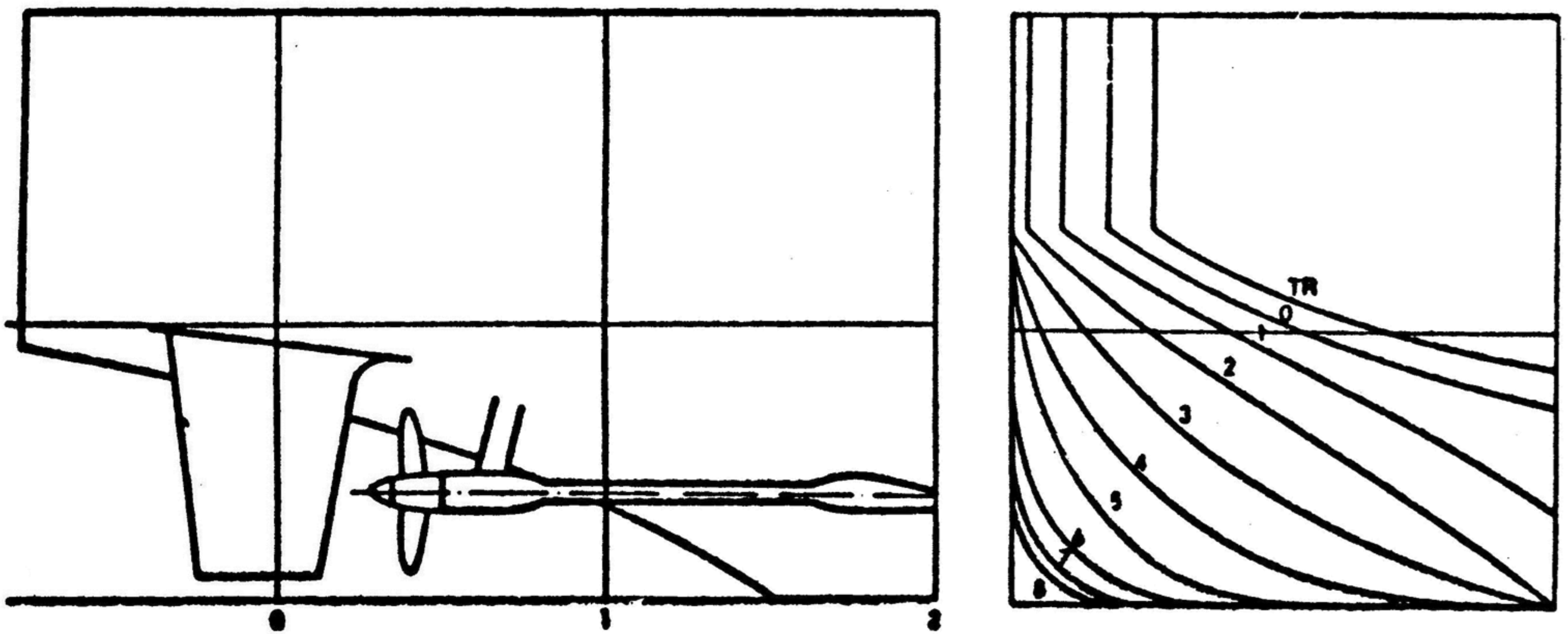
$$(1+k_2)_{eq} = \frac{\sum S_i (1+k_2)_i}{\sum S_i}$$

S_i and $(1+k_2)_i$ is the wetted area of the appendages and the appendage resistance factor for the i^th time.

$$(1+k_2)_{eq} = \frac{\sum S_i (1+k_2)_i}{\sum S_i} = \frac{82.74(1.4)+135(1.4)}{82.74+135} = 1.4$$

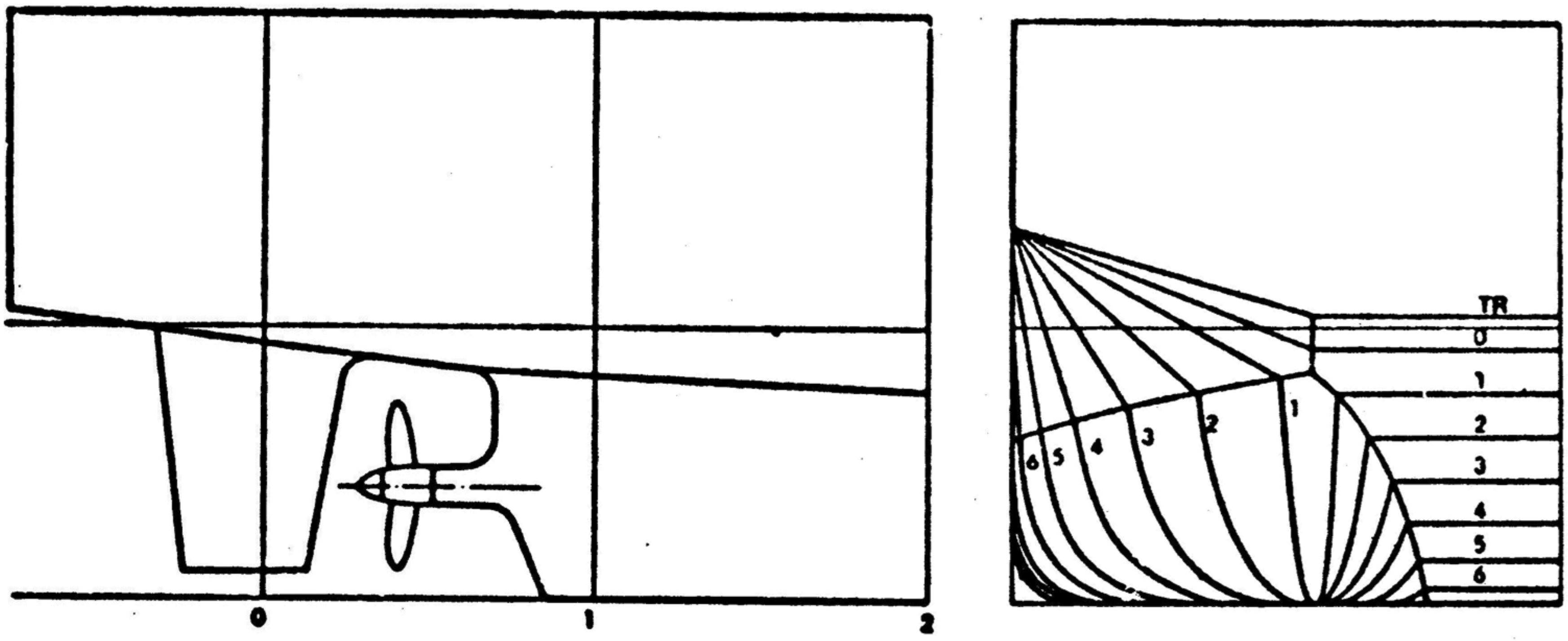
$$R_{APP} = \frac{1}{2}\rho V^2 S_{APP}(1+k_2)_{eq} C_F = \frac{1}{2}\times 1.025\times 11.831^2 \times (82.74+135)\times 1.4\times 0.001378 = 30.144$$

Hull appendage



Conventional twin-screw after body hull form

Hull appendage



Twin-screw twin-skeg after body hull form

④ Wave resistance(Low-speed range)

$$R_T = R_F(1+k_1) + R_{APP} + \boxed{R_W} + R_B + R_{TR} + R_A$$

- Low speed range: $F_n \le 0.4$

$L_R =$

3,700 TEU Container Carrier

Item	Value
L	
B	
T	
C_{WP}	
C_P	
L_{CB}	
∇	

$$R_W = \rho g \nabla \boxed{C_1} C_2 C_5 \exp\{m_1 F_n^d + m_4 \cos(\lambda F_n^{-2})\}$$

$$C_1 = 2223105 C_7^{3.78613} (T/B)^{1.07961} (90 - i_E)^{-1.37565}$$

$C_7 = 0.229577(B/L)^{0.33333}$: when B/L≤0.11

$C_7 = B/L$: when 0.11≤B/L≤0.25

$C_7 = 0.5 - 0.0625B/L$: when 0.25≤B/L

i_E: The half angle of entrance

At early design stage, If i_E is unknown, it can be obtained by following formula:

$$i_E = 1 + 89e^{\left\{\begin{array}{l} -(L/B)^{0.80856}(1-C_{WP})^{0.30484}(1-C_P-0.0225L_{CB})^{0.6367} \\ \times (L_R/B)^{0.34574}(100\nabla/L^3)^{0.16302} \end{array}\right\}}$$

$$B/L = 0.135$$

$$C_7 = B/L = 32.2/239.26 = 0.135$$

$$i_E = 1 + 89e^{\{-(L/B)^{0.80856}(1-C_{WP})^{0.30484}(1-C_P-0.0225L_{CB})^{0.6367} \times (L_R/B)^{0.34574}(100\nabla/L^3)^{0.16302}}$$

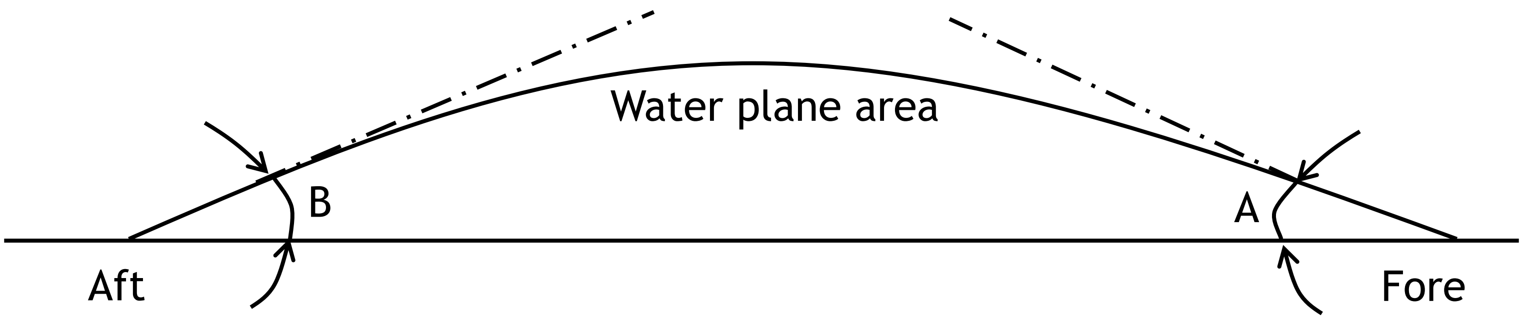
$$= 12.491$$

$$\therefore C_1 = 2223105 C_7^{3.78613} (T/B)^{1.07961} (90 - i_E)^{-1.37565}$$

$$= 2223105 \times 0.135^{3.78613} (10.1/32.2)^{1.07961} (90 - 12.491)^{-1.37565}$$

$$= 0.806$$

Meaning of a entrance angle



B : Angle of run of waterline

A : Angle of entrance of waterline

i_E : The half angle of entrance is the angle of the waterline at the bow in degrees with reference to the center plane but neglecting the local shape at the stem.

④ Wave resistance(Low-speed range)

$$R_T = R_F(1+k_1) + R_{APP} + \boxed{R_W} + R_B + R_{TR} + R_A$$

- Low speed range: $F_n \le 0.4$

$$R_W = \rho g \nabla C_1 \boxed{C_2} \boxed{C_5} \exp\{m_1 F_n^d + m_4 \cos(\lambda F_n^{-2})\}$$

3,700 TEU Container Carrier

Item	Value
B	
T	
T_F	
A_{BT}	
h_B	
A_T	
C_M	

C_2: A parameter which accounts for the reduction of the wave resistance due to the action of a bulbous bow

$$C_2 = e^{-1.89\sqrt{C_3}}$$

If there is not bulb, C_2 is 1.

$$C_3 = 0.56 A_{BT}^{1.5} / \{B \cdot T(0.31\sqrt{A_{BT}} + T_F - h_B)\}$$

A_{BT} : Transverse bulb area

h_B : The position of the centre of the transverse are A_{BT} above the keel line

T_F : The forward draft of the ship

C_5: A parameter which accounts for the reduction of the wave resistance due to the action of a transom stern

$$C_5 = 1 - 0.8 A_T / (B \cdot T \cdot C_M)$$

A_T : The immersed part of the transverse area of the transom at zero speed

$$\begin{aligned}
C_3 &= 0.56 A_{BT}^{1.5} / \{B \cdot T(0.31\sqrt{A_{BT}} + T_F - h_B)\} \\
&= 0.56 A_{BT}^{1.5} / \{B \cdot T(0.31\sqrt{A_{BT}} + T_F - h_B)\} \\
&= 0.06
\end{aligned}$$

$$\begin{aligned}
C_2 &= e^{-1.89\sqrt{C_3}} = e^{-1.89\sqrt{0.06}} \\
&= e^{-1.89\sqrt{C_3}} = e^{-1.89\sqrt{0.06}} \\
&= 0.629
\end{aligned}$$

$$\begin{aligned}
C_5 &= 1 - 0.8 A_T / (B \cdot T \cdot C_M) \\
&= 1 - 0.8 A_T / (B \cdot T \cdot C_M) \\
&=
\end{aligned}$$

$$R_T = R_F(1+k_1) + R_{APP} + \boxed{R_W} + R_B + R_{TR} + R_A$$

- Low speed range: $F_n \le 0.4$

$$R_W = \rho g \nabla C_1 C_2 C_5 \exp\{\boxed{m_1} F_n^{\boxed{d}} + \boxed{m_4}\cos(\lambda F_n^{-2})\}$$

3,700 TEU Container Carrier

Item	Value
L	
T	
C_p	
∇	
F_n	

$$m_1 = 0.0140407 L/T - 1.75254 \nabla^{1/3}/L - 4.79323 B/L - C_{16}$$

$C_{16} = 8.07981 C_P - 13.8673 C_P^{\,2} + 6.984388 C_P^{\,3}$: when $C_P \le 0.8$

$C_{16} = 1.73014 - 0.7067 C_P$: when $0.8 \le C_P$

$$d = -0.9$$

$$m_4 = C_{15} 0.4 e^{-0.034 F_n^{-3.29}}$$

$C_{15} = -1.69385$: when $L^3/\nabla \le 512$

$C_{15} = -1.69385 + (L/\nabla^{1/3} - 8.0)/2.36$: when $512 \le L^3/\nabla \le 1726.91$

$C_{15} = 0.0$: when $1726.91 \le L^3/\nabla$

$$C_{16} = 8.07981 C_P - 13.8673 C_P^{\,2} + 6.984388 C_P^{\,3} = 1.285$$

$$\begin{aligned} m_1 &= 0.0140407 L/T - 1.75254 \nabla^{1/3}/L - 4.79323 B/L - C_{16} \\ &= 0.0140407 \times 239.26/10.1 - 1.75254 \times 49778^{1/3}/239.26 \\ &\quad - 4.79323 \times 32.2/239.26 - 1.323 \\ &= -1.904 \end{aligned}$$

$$L^3/\nabla = 275.152 \le 512$$

$$\rightarrow C_{15} = -1.694$$

$$\begin{aligned} m_4 &= C_{15} 0.4 e^{-0.034 F_n^{-3.29}} \\ &= -1.694 \cdot 0.4 e^{-0.034 F_n^{-3.29}} \\ &= -0.02 \end{aligned}$$

$$R_T = R_F(1+k_1) + R_{APP} + \boxed{R_W} + R_B + R_{TR} + R_A$$

- Low speed range: $F_n \leq 0.4$

$$R_W = \rho g \nabla C_1 C_2 C_5 \exp\{\boxed{m_1} F_n^{\boxed{d}} + \boxed{m_4} \cos(\lambda F_n^{-2})\}$$

L_R =
C_1 =
C_2 =
C_5 =
m_1 =
d =
m_4 =

$$\lambda = 1.446 C_P - 0.03 L/B$$: when L/B≤12

$$\lambda = 1.446 C_P - 0.36$$: when 12≤L/B

3,700 TEU Container Carrier

Item	Value
L	
B	
C_p	
∇	
F_n	

$$L/B = 7.43 \leq 12$$

$$\rightarrow \lambda = 1.446 C_P - 0.03 L/B$$
$$= 1.446 \times 0.6733 - 0.03 \times 239.26 / 32.2$$
$$= 0.751$$

$$\therefore R_w = \rho g \nabla C_1 C_2 C_5 \exp\{m_1 F_n^d + m_4 \cos(\lambda F_n^{-2})\}$$
$$= 1.025 \times 9.81 \times 49778 \times 0.806 \times 0.629 \times 1 \times \exp\{-1.904 \times 0.2442^{-9} - 0.02 \times \cos(0.702 \times 0.2442^{-2}) = 286.364$$

④ Wave resistance(High-speed range)

$$R_T = R_F(1+k_1) + R_{APP} + \boxed{R_W} + R_B + R_{TR} + R_A$$

– High-speed range: $0.55 \le F_n$

$$R_W = \rho g \nabla \boxed{C_1} C_2 C_5 \exp\{\boxed{m_1} F_n^d + m_4 \cos(\lambda F_n^{-2})\}$$

L_R =
C_2 =
C_5 =
d =
m_4 =

3,700 TEU Container Carrier

Item	Value
L	
B	
C_p	
∇	
F_n	

In the high speed, the coefficients C1 and m1 are changed

$$C_1 = 6919.3 C_M^{-1.3346} (\nabla / L^3)^{2.00977} (L/B-2)^{1.40692}$$

$$m_1 = -7.2035 (B/L)^{0.326869} (T/B)^{0.605375}$$

$$C_1 = 6919.3 C_M^{-1.3346} (\nabla / L^3)^{2.00977} (L/B-2)^{1.40692}$$
$$= 6919.3 C_M^{-1.3346} (\nabla / L^3)^{2.00977} (L/B-2)^{1.40692}$$
$$=$$

$$m_1 = -7.2035 (B/L)^{0.326869} (T/B)^{0.605375}$$
$$= -7.2035 (B/L)^{0.326869} (T/B)^{0.605375}$$
$$=$$

$$\therefore R_w = \rho g \nabla C_1 C_2 C_5 \exp\{m_1 F_n^d + m_4 \cos(\lambda F_n^{-2})\}$$
$$= 1.025 \times 9.81 \times 49778 \times 0.806 \times 0.629 \times 1 \times \exp\{-1.904 \times 0.2442^{-9} - 0.02 \times \cos(0.702 \times 0.2442^{-2})$$
$$= 286.364$$

④ Wave resistance(Middle-speed range)

$$R_T = R_F(1+k_1) + R_{APP} + \boxed{R_W} + R_B + R_{TR} + R_A$$

– Middle-speed range: $0.4 \le F_n \le 0.55$

$$R_W = (R_W)_{at\,F_n=0.4} + (10F_n - 4)\cdot\{(R_W)_{at\,F_n=0.55} - (R_W)_{at\,F_n=0.4}\}/1.5$$

3,700 TEU Container Carrier

Item	Value
$(R_W)_{at\,F_n=0.4}$	
$(R_W)_{at\,F_n=0.55}$	
F_n	

$(R_W)_{at\,F_n=0.4}$: The wave resistance prediction for F_N = 0.4 according to the formula in low speed range

$(R_W)_{at\,F_n=0.55}$: The wave resistance prediction for F_N = 0.55 according to the formula in high speed range

$(R_W)_{at\,F_n=0.4} =$

$(R_W)_{at\,F_n=0.55} =$

$$\begin{aligned} \therefore R_W &= (R_W)_{at\,F_n=0.4} + (10F_n - 4)\cdot\{(R_W)_{at\,F_n=0.55} - (R_W)_{at\,F_n=0.4}\}/1.5 \\ &= (R_W)_{at\,F_n=0.4} + (10F_n - 4)\cdot\{(R_W)_{at\,F_n=0.55} - (R_W)_{at\,F_n=0.4}\}/1.5 \\ &= 286.364 \end{aligned}$$

⑤ Additional pressure resistance of bulbous bow near the water surface

$$R_T = R_F(1+k_1) + R_{APP} + R_W + \boxed{R_B} + R_{TR} + R_A$$

$$\boxed{R_B = 0.11 e^{(-3P_B^{-2})} \cdot F_{ni}^{\ 3} A_{BT}^{\ 1.5} \rho g / (1+F_{ni}^{\ 2})}$$

3,700 TEU Container Carrier

Item	Value
A_{BT}	
T_F	
h_B	
V	
g	
ρ	

P_B : A measure for the emergence of the bow

$$P_B = 0.56\sqrt{A_{BT}} / (T_F - 1.5h_B)$$

F_{ni} : The Froude number based on immersion of bulbous bow

$$F_{ni} = V / \sqrt{g(T_F - h_B - 0.25\sqrt{A_{BT}}) + 0.15V^2}$$

$$P_B = 0.56\sqrt{A_{BT}} / (T_F - 1.5h_B)$$
$$= 0.56\sqrt{A_{BT}} / (T_F - 1.5h_B)$$
$$=$$

$$F_{ni} = V / \sqrt{g(T_F - h_B - 0.25\sqrt{A_{BT}}) + 0.15V^2}$$
$$= V / \sqrt{g(T_F - h_B - 0.25\sqrt{A_{BT}}) + 0.15V^2}$$
$$=$$

$$\therefore R_B = 0.11 e^{(-3P_B^{-2})} \cdot F_{ni}^{\ 3} A_{BT}^{\ 1.5} \rho g / (1+F_{ni}^{\ 2})$$
$$= 0.11 e^{(-3P_B^{-2})} \cdot F_{ni}^{\ 3} A_{BT}^{\ 1.5} \rho g / (1+F_{ni}^{\ 2})$$
$$=$$

In the resent research, R_B=0.

⑥ Additional pressure resistance of immersed transom immersion

$$R_T = R_F(1+k_1) + R_{APP} + R_W + R_B + \boxed{R_{TR}} + R_A$$

$$R_{TR} = 1/2 \rho V^2 A_T C_6$$

3,700 TEU Container Carrier

Item	Value
A_T	
g	
B	
C_{WP}	
V	
ρ	

$C_6 = 0.2(1-0.2F_{nT})$: when $F_{nT} \le 5$

$C_6 = 0$: when $5 \le F_{nT}$

$$F_{nT} = V / \sqrt{2gA_T / (B + B \cdot C_{WP})}$$

$$F_{nT} = V / \sqrt{2gA_T / (B + B \cdot C_{WP})}$$
$$= V / \sqrt{2gA_T / (B + B \cdot C_{WP})}$$
$$=$$

Because $5 \le F_{nT}$

$$C_6 = 0.2(1-0.2F_{nT})$$
$$= 0.2(1-0.2F_{nT})$$
$$=$$

$$\therefore R_{TR} = 1/2 \rho V^2 A_T C_6$$
$$= 1/2 \rho V^2 A_T C_6$$
$$=$$

⑦ Model-ship correlation resistance

$$R_T = R_F(1+k_1) + R_{APP} + R_W + R_B + R_{TR} + \boxed{R_A}$$

$$R_A = 1/2 \rho V^2 S_{total} C_A$$

The model-ship correlation resistance R_A is supposed to describe primarily the effect of the hull roughness and the still-air resistance.

3,700 TEU Container Carrier

Item	Value
L	
C_B	
T_F	
V	
ρ	
S_{total}	

$$C_A = 0.006(L+100)^{-0.16} - 0.00205 + 0.003\sqrt{L/7.5}C_B{}^4 C_2(0.04 - C_4)$$

$C_4 = T_F / L$: when $T_F / L \le 0.04$

$C_4 = 0.04$: when $0.04 < T_F / L$

$$T_F / L = T_F / L$$

$$=$$

Because $0.04 \le T_F/L$

$$C_A = 0.006(L+100)^{-0.16} - 0.00205 + 0.003\sqrt{L/7.5}C_B{}^4 C_2(0.04 - C_4)$$

$$= 0.006(239.26+100)^{-0.16} - 0.00205 + 0.003\sqrt{239.26/7.5}0.6241^4 \times 0.629(0.04 - 0.04) \times 0.000312$$

$$\therefore R_A = \frac{1}{2}\rho V^2 S_{total} C_A = \frac{1}{2} \times 1.025 \times 11.8312^2 \times 9625.74 \times 0.000312 = 215.3948$$

Resistance Prediction for a Ship

Model test of the basis ship

Model test of the design ship

Given

Find

$$\frac{R_{T,basis,modeltest}}{R_{T,basis,Holtrop\ \&\ Mennen}} = \frac{R_{T,design,modeltest}}{R_{T,design,Holtrop\ \&\ Mennen}}$$

Calculate

Calculate

Resistance of basis ship estimated by Holtrop-Mennen method

Resistance of design ship estimated by Holtrop-Mennen method

Resistance estimation by Holtrop and Mennen's method
- Approximation formula of the propeller efficiency

$$\eta_D = \overset{①}{\eta_O} \cdot \overset{②}{\eta_H} \cdot \overset{③}{\eta_R}$$

MCR: Maximum Continuous Rating
NCR: Normal Continuous Rating)
BHP: Brake Horse Power
DHP: Delivered Horse Power
EHP: Effective Horse Power
R_T: Total Resistance
η_T: Transmission Efficiency
η_D: Propulsive Efficiency
η_O: Propeller Efficiency
η_H: Hull Efficiency
η_R: Relative Rotative Efficiency
t: Thrust Deduction Fraction
w: Wake Fraction

①

$$\eta_O = [1/(0.97 + 0.14\sqrt{B_P})] \cdot k$$

$$k = [1.11 - 0.11((A_E / A_O)/0.6)]$$

$$B_P = \frac{n(NCR\eta_T\eta_R)^{0.5}}{V(1-w)}$$

$$w = 0.3095 \cdot C_B + 10 \cdot C_V \cdot C_B - 0.23 \cdot \frac{D_P}{\sqrt{B \cdot T}}, \quad D_P = 15.4 \cdot \left(\frac{MCR}{n_{MCR}^3}\right)^{0.2} \cdot c_1,$$

Blade=5 : C_1=1,
Blade=4 : C_1=1.05

$$C_V = C_F \cdot (1+k) + C_A$$

$$C_F = \frac{0.075}{(\log R_n - 2)^2}, \quad R_n = \frac{V \cdot L}{\nu}$$

②

$$\eta_H = \frac{1-t}{1-w}, \qquad t = \frac{2}{3}w + 0.01$$

③

$$\eta_R = 0.98 \sim 1.03$$

If the ship has large Cb>0.8,

$$\eta_R = 0.88 + 0.02 \cdot (L/B)$$

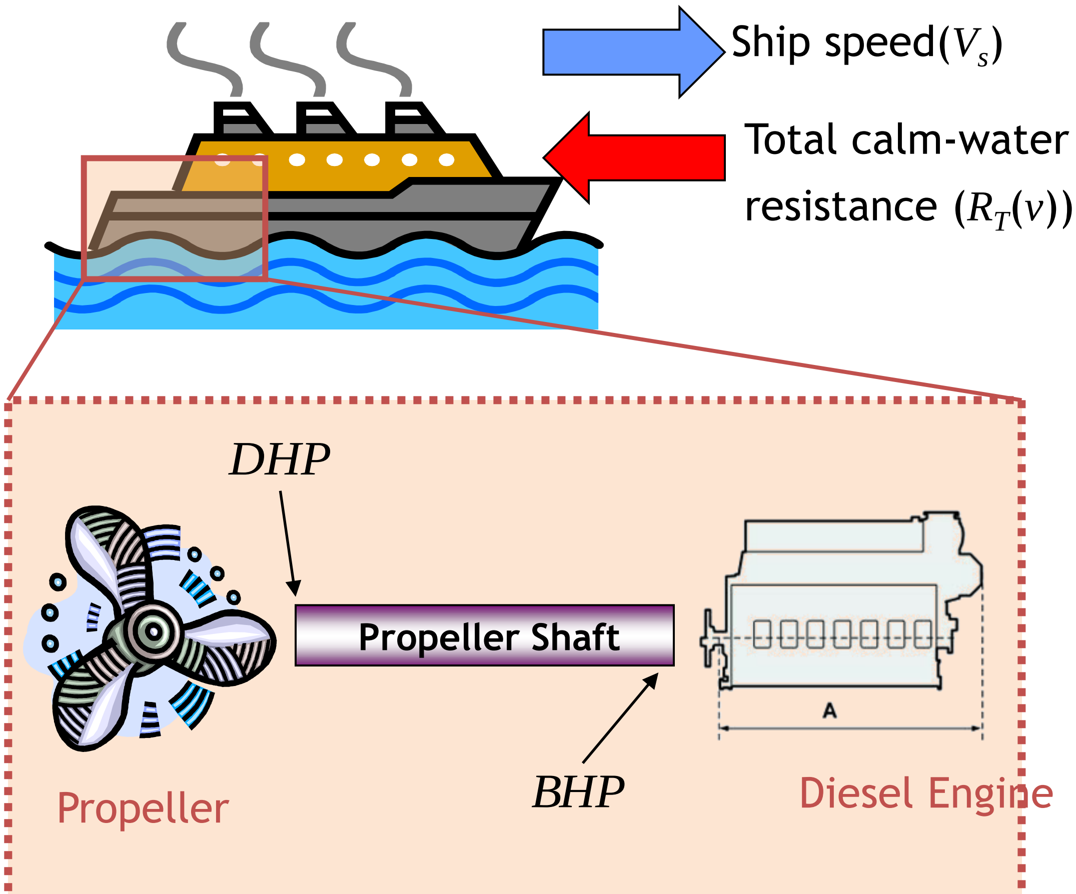
1) 이규열, "창의적 선박설계 8판",서울대학교 조선해양공학과,2007
2) 이창섭외 5인, "프로펠러 설계",문운당,2008

Chapter 7. Propeller & Main Engine Selection

Determination of Principal Dimension and Basic Performance Prediction

1. Owner's Requirements(Owner's Specification)
2. Similar Ship Data Search
3. Investigation of Rules & Regulation Requirements
4. Determination of Principal Dimensions
   - Displacement Estimation-
5. Lightweight Estimation
6. Speed-Power Prediction
   - Resistance Estimation/Main Engine Selection /Propeller Selection-
7. Cargo Capacity Estimation
8. Freeboard Calculation
9. Stability Calculation
10. Outline of General Arrangement Drawing
11. Preliminary Lines, Ship Re-Calculation
12. Setting of the List of Materials , Specification and Ship Price
13. Technical Documents (Specification)

(Review) Resistance, Power Estimation
- Power Prediction of Main Engine



① EHP (Effective Horse Power)

$$EHP = R_T(v) \cdot v$$ (In Calm Water)

② DHP (Delivered Horse Power)

$$DHP = \frac{EHP}{\eta_D}$$

(η_D : Propulsive efficiency)

$\eta_D = \eta_O \cdot \eta_H \cdot \eta_R$

η_O : Open water efficiency

η_H : Hull efficiency

η_R : Relative rotative efficiency

③ BHP (Brake Horse Power)

$$BHP = \frac{DHP}{\eta_T}$$

(η_T : Transmission efficiency)

④ NCR (Normal Continuous Rating)

$$NCR = BHP(1 + \frac{\text{Sea Margine}}{100})$$

⑤ DMCR (Derated Maximum Continuous Rating)

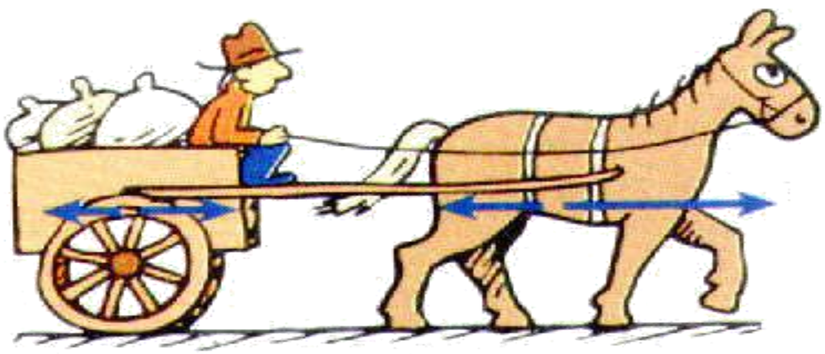
$$DMCR = \frac{NCR}{\text{Engine Margin}}$$

⑥ NMCR (Nominal Maximum Continuous Rating)

$$NMCR = \frac{DMCR}{\text{Derating rate}}$$

7-1 Propeller

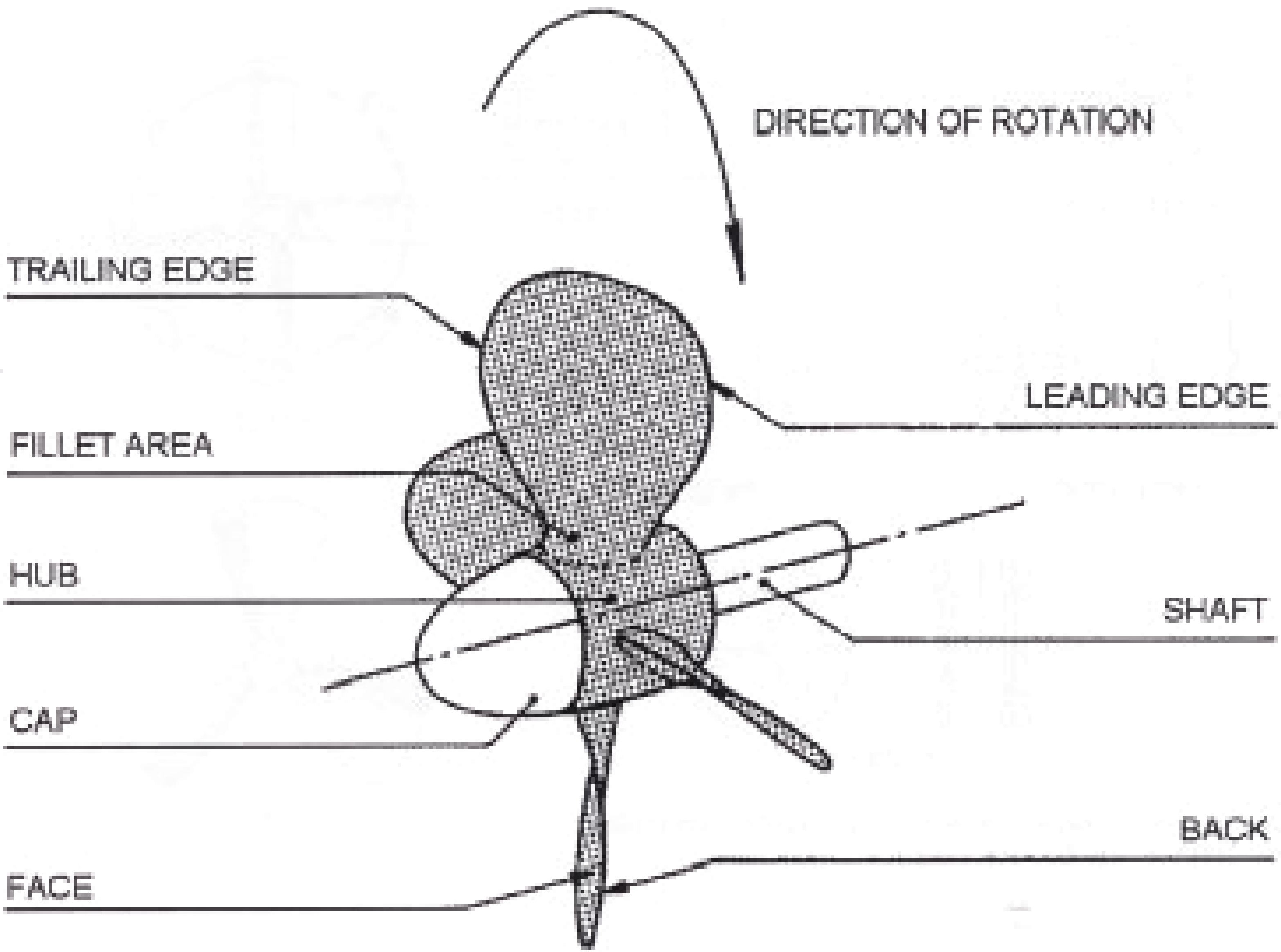
Concept of Propeller Main Dimensions Determination



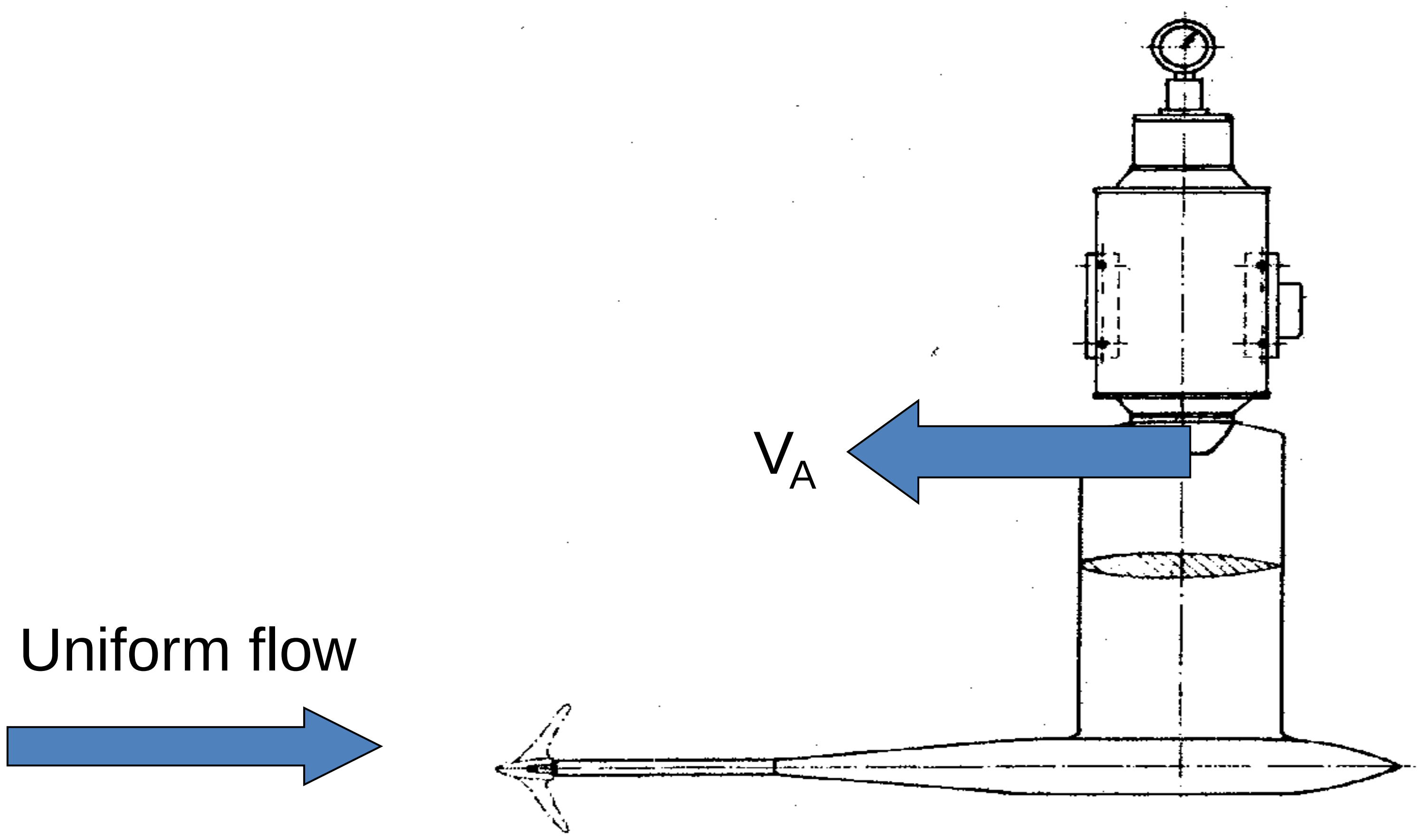
Wheel design to draw the carriage with cargo by one horse for maximum speed

Given	Find
•One Horse = Main Engine •Friction Power = Resistance of a Ship	•Wheel Design = Propeller Design •Maximum Speed = Maximum Speed of a Ship •Wheel Diameter = Main Dimensions of a Propeller

Propeller - Components



Propeller - Open Water Test



- Main non-dimensional coefficients of Propeller

From dimensional analysis :

① Thrust coefficient : $$\frac{T}{\rho \cdot n^2 \cdot D_P{}^4} = K_T$$

② Torque coefficient: $$\frac{Q}{\rho \cdot n^2 \cdot D_P{}^5} = K_Q$$

③ Advance ratio: $$J = \frac{v_A}{n \cdot D_P}$$

$$v_A = v \cdot (1 - w)$$

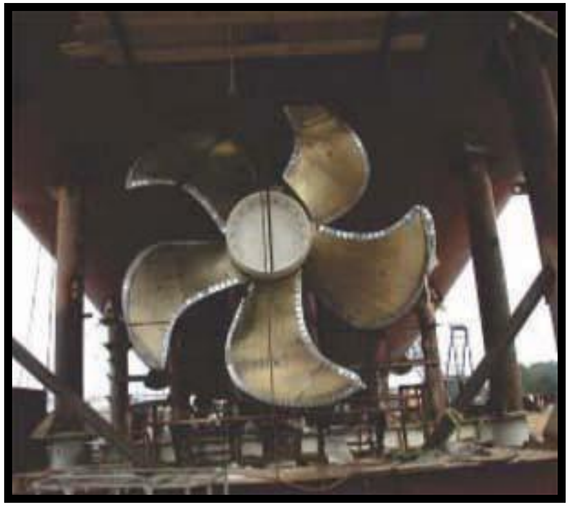
④ Propeller efficiency: $$\eta_o = \frac{J}{2\pi} \cdot \frac{K_T}{K_Q}$$

(in open water)

v	: Ship Speed[m/s]
w	: Wake fraction
T	: Thrust of the propeller[kN]
Q	: Torque absorbed by propeller [kN·m]
n	: Number of Revolutions[1/s]
D_P	: Propeller Diameter [m]
P_i	: Propeller Pitch [m]
V_A	: Speed of Advance [m/s]

Propeller Open Water Curve – POW propeller model

Actual Propeller



Geometric Similarity

Model Propeller

$$\frac{T}{\rho \cdot n^2 \cdot D_P{}^4} = K_T$$

$$\frac{Q}{\rho \cdot n^2 \cdot D_P{}^5} = K_Q$$

$$J = \frac{v_A}{n \cdot D_P}$$

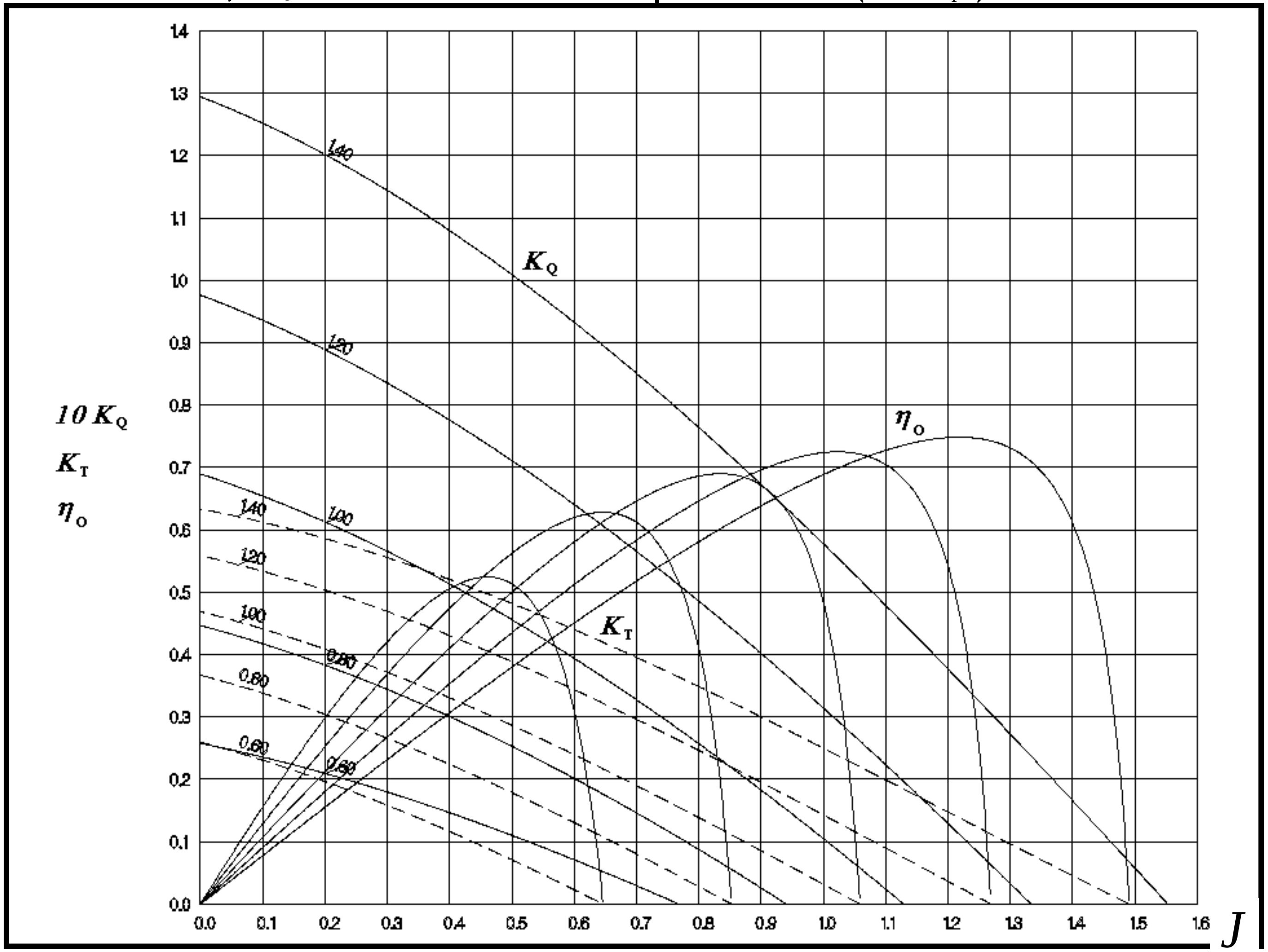
$$v_A = v \cdot (1 - w)$$

Same non-dimensional coefficient

$$\left(K_T \,, K_Q \,, J \right)$$

Propeller Open Water Curve – POW curve

- Values of K_T, K_Q and η_O at different pitch ratio (P_i / D_p).



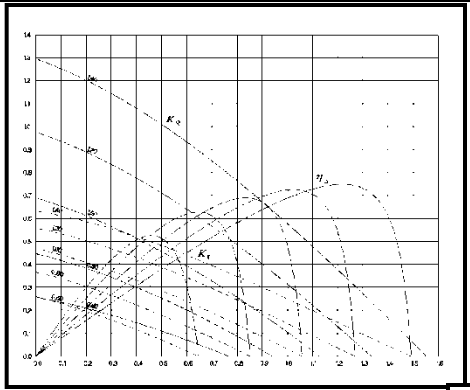
$$K_T = \frac{T}{\rho \cdot n^2 \cdot D_P{}^4}$$

$$K_Q = \frac{Q}{\rho \cdot n^2 \cdot D_P{}^5}$$

$$J = \frac{v_A}{n \cdot D_P}$$

$$\eta_o = \frac{J}{2\pi} \cdot \frac{K_T}{K_Q}$$

Propeller - Regression Polynomials for Propeller Open Water Curve



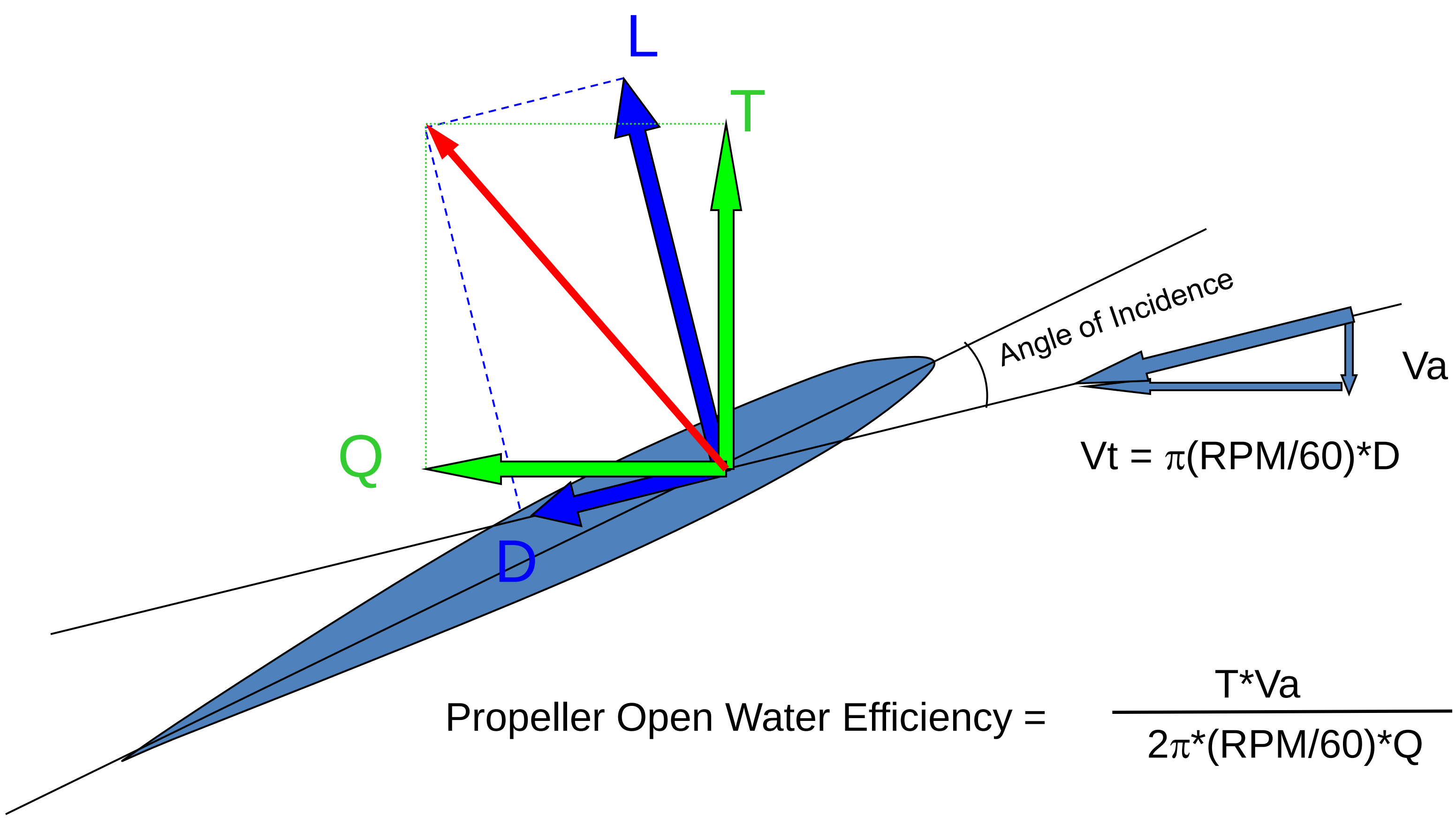
- Regression Polynomial

  :Function of Advance coefficient, Pitch ratio, Developed Area Ratio, No. of blades)

$$K_T \text{ and } K_Q = \sum C_{s,t,u,v} (J)^s (P_i / D_P)^t (A_E / A_O)^u z^v$$

K_T					K_Q				
$C_{s,t,u,v}$	s (J)	t (P/D_P)	u (A_E / A_o)	v (z)	$C_{s,t,u,v}$	s (J)	t (P/D_P)	u (A_E / A_o)	v (z)
+0.00880496	0	0	0	0	+0.00379368	0	0	0	0
-0.204554	1	0	0	0	+0.00886523	2	0	0	0
+0.166351	0	1	0	0	-0.032241	1	1	0	0
+0.158114	0	2	0	0	+0.00344778	0	2	0	0
-0.147581	2	0	1	0	-0.0408811	0	1	1	0
-0.481497	0	1	1	0	-0.108009	1	1	1	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

Propeller – Forces acting on



$$\text{Propeller Open Water Efficiency} = \frac{T*Va}{2\pi*(RPM/60)*Q}$$

7-2 Characteristics of Diesel Engine

Characteristics of Diesel Engine

☑ Brake Horse Power(BHP) of Diesel Engine

$$BHP = P_{me} \cdot L \cdot A \cdot n \cdot Z$$

Where, BHP: Brake Horse Power(kW)

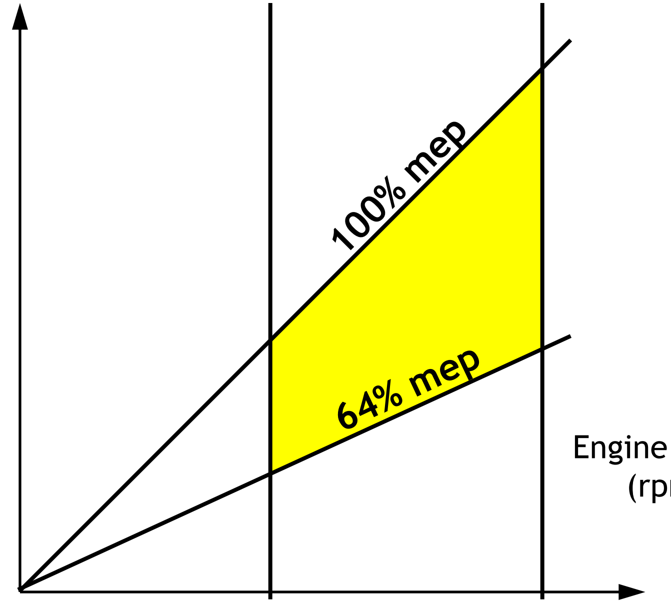
P_{me} : Mean Effective Presure $(\mathrm{k}N/m^2)$

L : Piston Stroke (m)

A : Piston Cross-sectional Area (m^2)

n : Number of Revolutions $(1/s)$

Z : Number of Cylinder



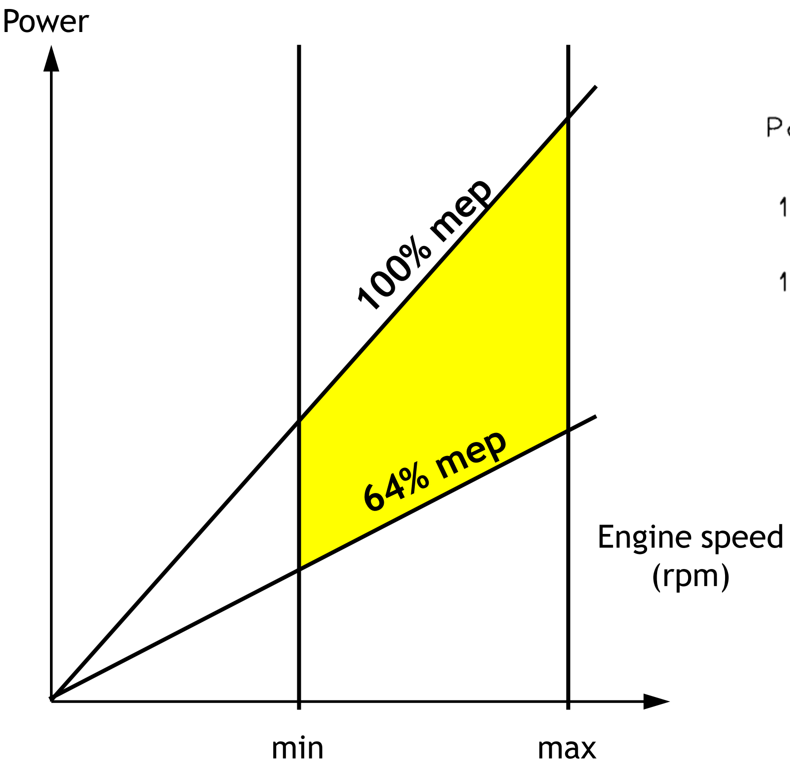
If A and Z are constant,

$$BHP = C_{DE} \cdot P_{me} \cdot n$$

Therefore, brake horse power “BHP” of a diesel engine is proportional to the rpm “n” and mean effective pressure “P_{me}”.

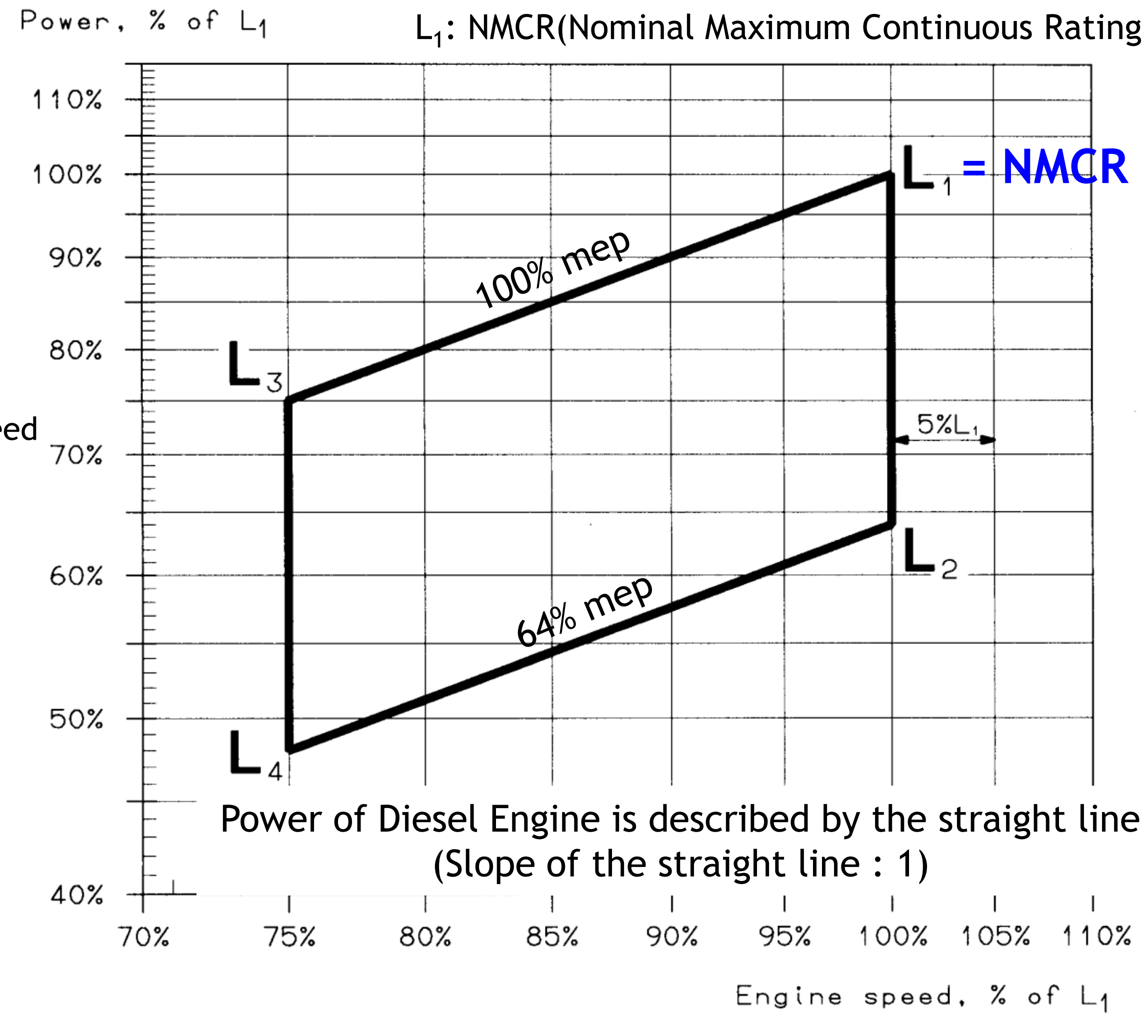
Characteristics of Diesel Engine(2)

-Diesel Engine Layout diagram



Logarithmic Scale

NMCR : Maximum power/speed combination of the chosen engine. This is the criterion of engine size, weight and price.



Power of Diesel Engine is described by the straight line
(Slope of the straight line : 1)

- The NMCR point is the upper right corner of the layout diagram.
- In the case of MAN/B&W engine, the NMCR point refers to L1.
- And in the case of Waertsilae(Sulzer) engine, R1 refers to this point

Engine Type Identification of MAN/B&W Diesel Engine

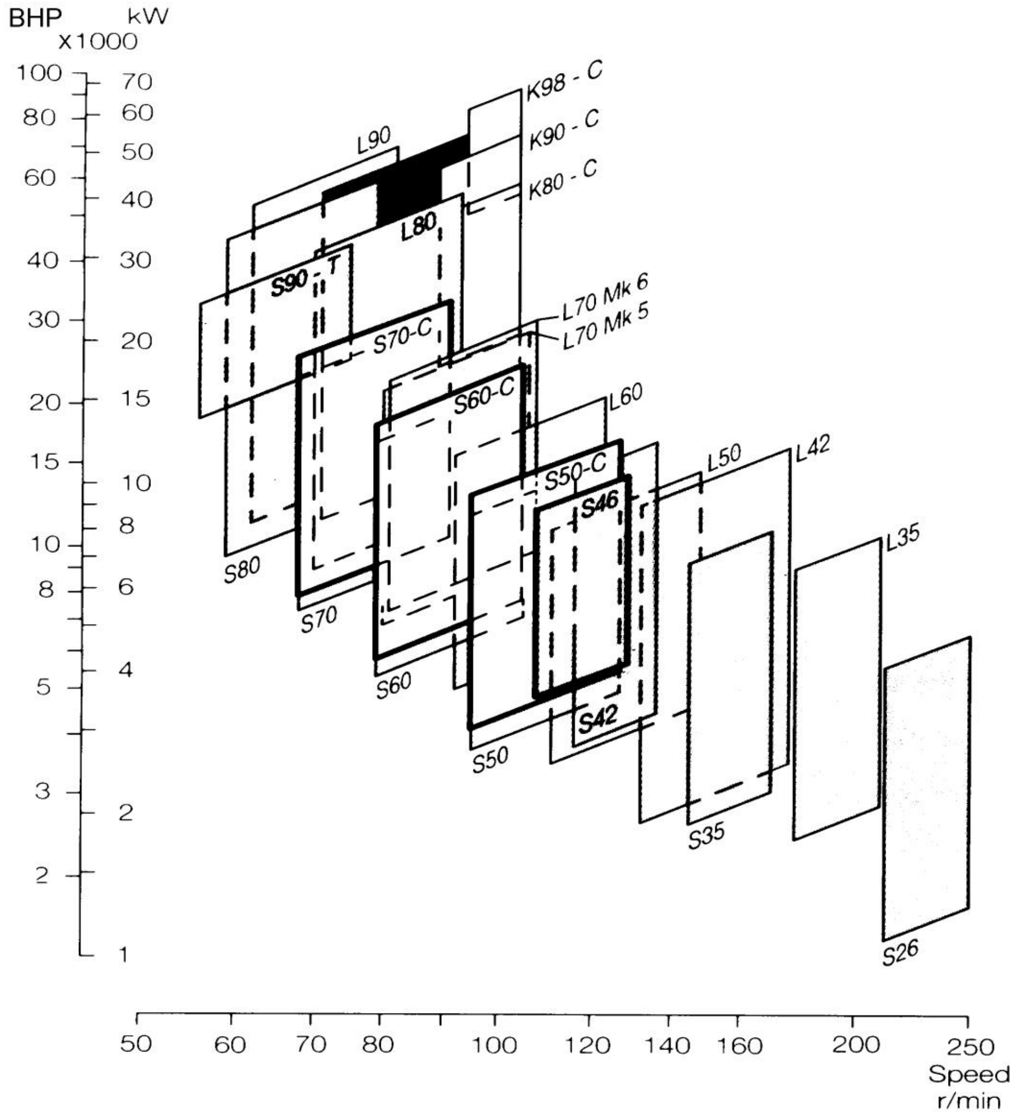
6 L 70 MC Mk V

- Mk V — Mark: Engine version
- MC — Engine programme
- 70 — Diameter of piston in cm : **bore**
- L — Stroke/bore ratio

S	Super long stroke	approximately 3.8
L	Long stroke	approximately 3.2
K	Short stroke	approximately 2.8

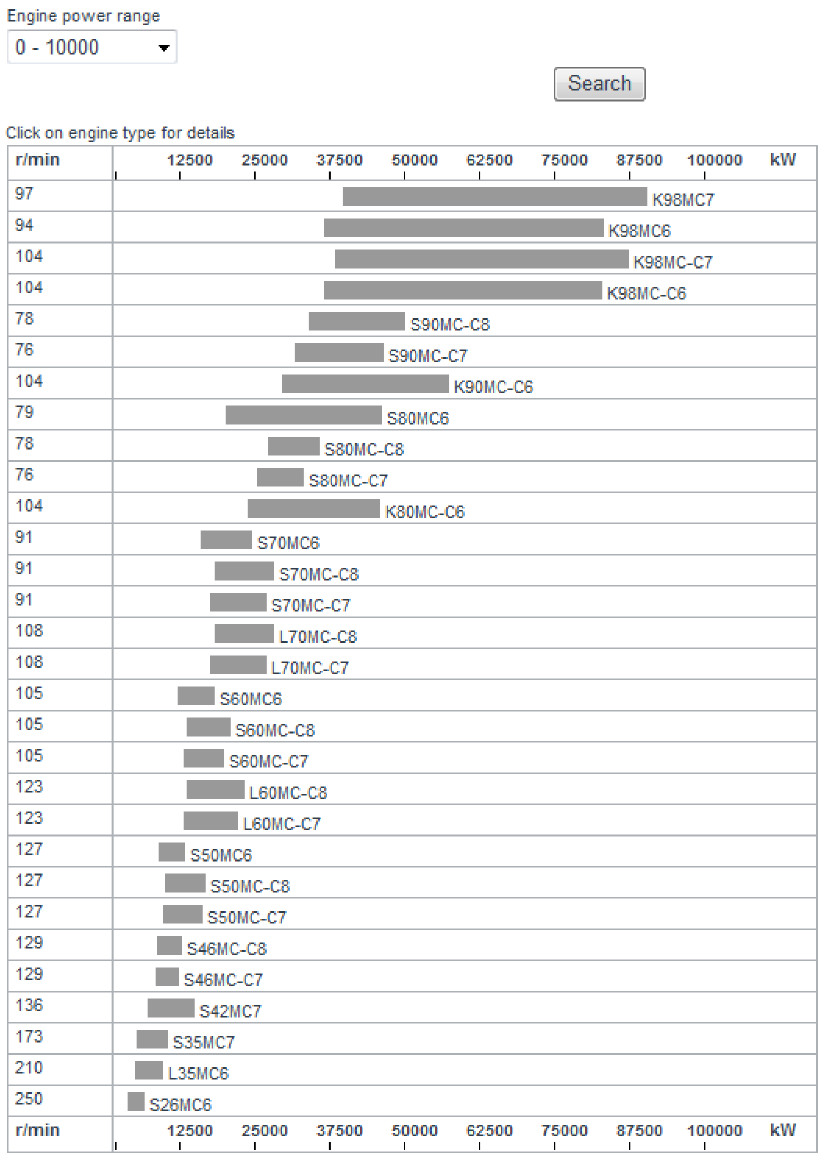
- 6 — Number of cylinders

MAN/B&W Diesel Engine



Ref: Two-stroke Engines MC programme 1996, MAN/B&W

MAN/B&W Diesel Engine



Ref: Two-stroke Engines MC Programme 2007,(MAN/B&W
http://www.manbw.com/engines/TwoStrokeLowSpeedPropEnginesProgram.asp)

MAN/B&W Two Stroke Low Speed Diesel Engine : S80MC6 Engine

- Example) S80MC6 Engine: Bore; 800mm, Stroke; 3,056mm

Bore: 800 mm, Stroke: 3056 mm

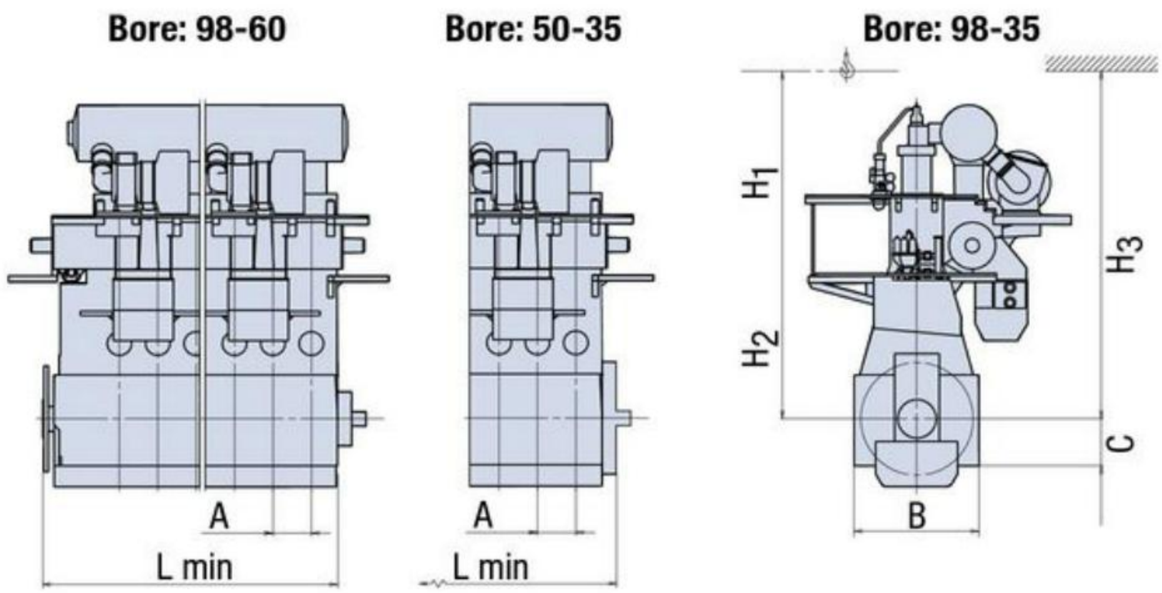
Main Data

Layout points		L_1	L_2	L_3	L_4
Speed	r/min	**79**	79	59	59
mep	bar	**18.0**	11.5	18.0	11.5
		kW	kW	kW	kW
5S80MC6		**18200**	11650	13600	8700
6S80MC6		**21840**	13980	16320	10440
7S80MC6		**25480**	16310	19040	12180
8S80MC6		**29120**	18640	21760	13920
9S80MC6		**32760**	20970	24480	15660
10S80MC6		**36400**	23300	27200	17400
11S80MC6		**40040**	25630	29920	19140
12S80MC6		**43680**	27960	32640	20880
Specific Fuel Oil Consumption (SFOC)					
g/kWh		**167**	155	167	155

Lubricating and Cylinder Oil Consumption

Lubricating oil	0.15 g/kWh
Cylinder oil	0.7 g/kWh

S80MC6 Engine: Bore; 800mm, Stroke; 3,056mm
-> Elevation View, Lmin, A,B



L_{min}: Minimum length of engine
A: Cylinder distance
B: Bedplate width
C: Crankshaft ⊕ to underside of foot flange
H_1: Normal lifting procedure
H_2: Reduced height lifting procedure
H_3: With electric double-jib crane

Main dimensions & weights

Cyl. No	5	6	7	8	9	10	11	12
L_{min} mm	9953	11377	12581	14005	16719	18143	19567	20991
H_1 mm	14125	14125	14125	14125	14125	14125	14125	14125
H_2 mm	13250	13250	13250	13250	13250	13250	13250	13250
H_3 mm	12925	12925	12925	12925	12925	12925	12925	12925
A mm	1736	1736	1736	1736	1736	1736	1736	1736
B mm	4824	4824	4824	4824	4824	4824	4824	4824
E mm	1424	1424	1424	1424	1424	1424	1424	1424
Dry Mass t*	777	885	996	1105	1223	1343	1458	1564

*The mass can vary up to 10% depending on the design and options chosen.

Matching the Powers and rpms of Propeller and Diesel Engine

Output Power of a Diesel Engine

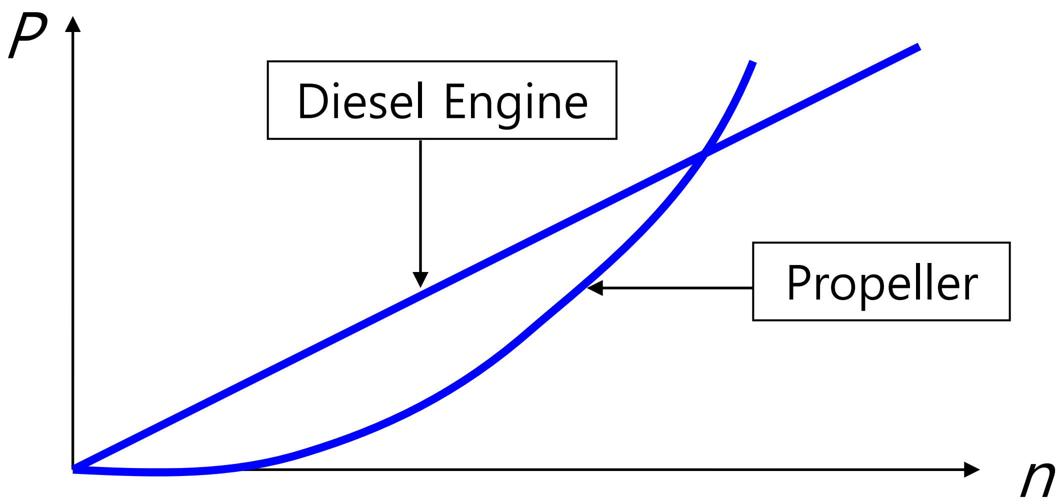
$$P_{D.E.} = P_{me} \cdot A \cdot L \cdot n \cdot Z$$

$$P_{D.E.} \propto n$$

Power absorbed by a propeller

$$P_{prop.} = 2\pi\rho \cdot n^3 \cdot D_P{}^5 \cdot K_Q = c_3 \cdot n^3$$

$$P_{prop.} \propto n^3$$



Sea Margin

- ☑ If the weather is bad, the resistance will increase compared to that at calm weather conditions. When the necessary engine power is to be determined, it is therefore normal to add an extra power margin.

- ☑ Sea margin is not an exact value, but usually expressed by the additional margin determined by shipyard or owner. The so-called sea margin is about 15% ~ 20 % of the power at calm water.

- ☑ Note: Light running propeller refers to the margin of propeller rpm.
  - Light running propeller margin(RPM margin)
    - MAN/B&W Engine : 2.5 ~ 5.0%
    - Sulzer Engine : 3.5 ~ 5.3%

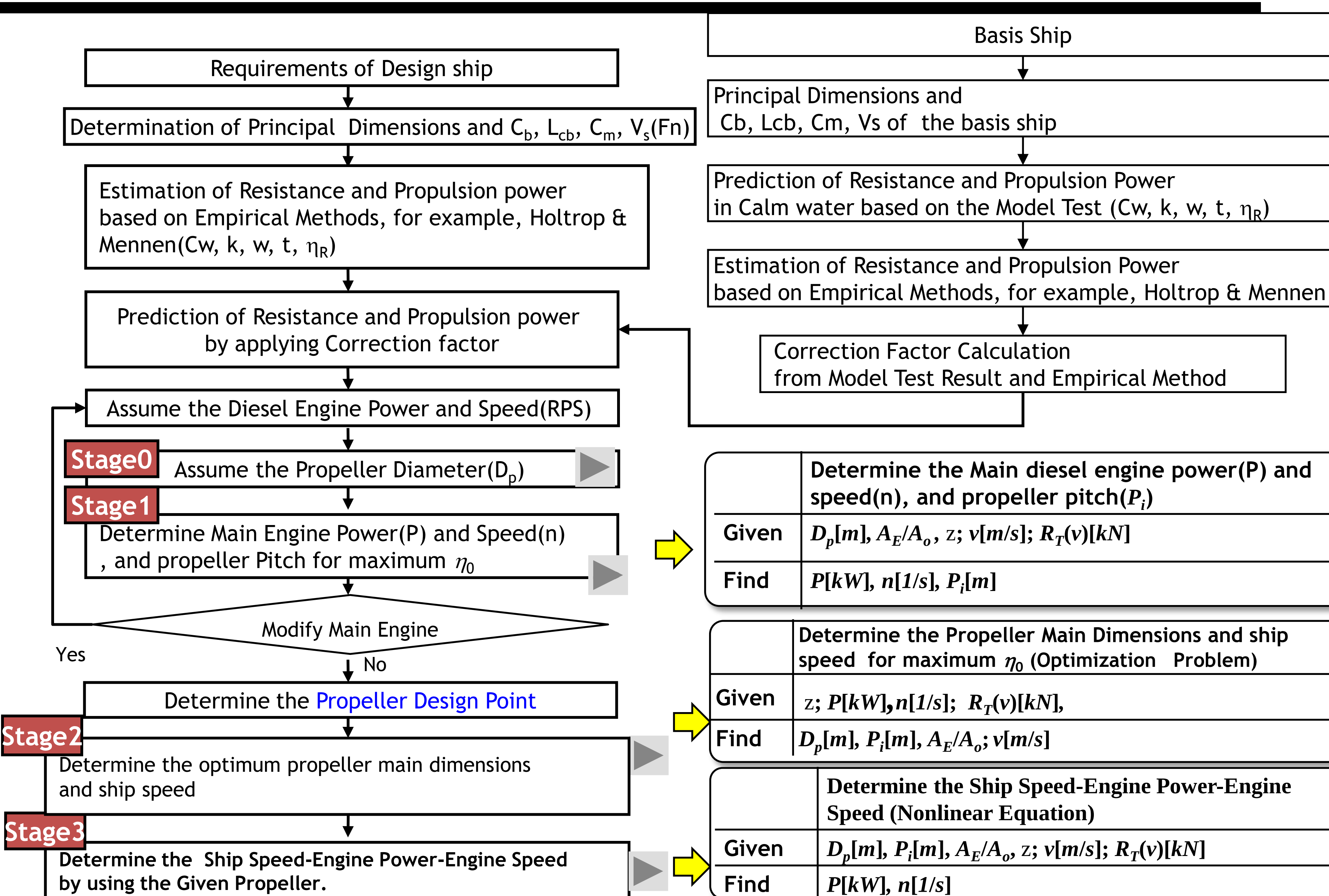
NCR, Engine Margin, MCR

☑ The normal continuous rating (NCR) is the power at which the engine is normally assumed to operate.

☑ The owner prefers that engine is operated continuously at maximum 85~ 90% of MCR to get the margin of speed.

7-3 Procedure of the Determination of Propeller Main Dimensions and Main Engine Selection

Determine the Power (BHP) and the RPM of the Main Engine



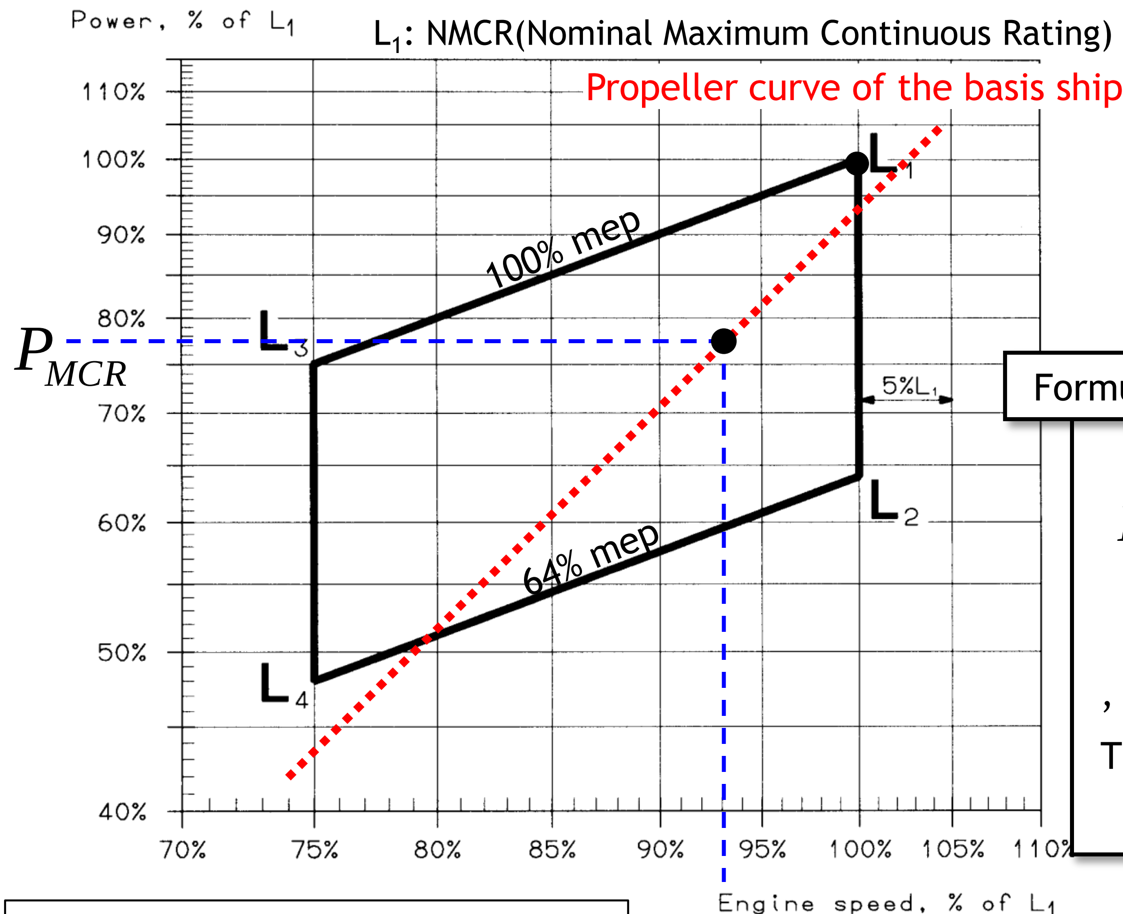
	Determine the Main diesel engine power(P) and speed(n), and propeller pitch(P_i)
Given	D_p[m], A_E/A_o, z; v[m/s]; R_T(v)[kN]
Find	P[kW], n[1/s], P_i[m]

	Determine the Propeller Main Dimensions and ship speed for maximum η_0 (Optimization Problem)
Given	z; P[kW], n[1/s]; R_T(v)[kN],
Find	D_p[m], P_i[m], A_E/A_o; v[m/s]

	Determine the Ship Speed-Engine Power-Engine Speed (Nonlinear Equation)
Given	D_p[m], P_i[m], A_E/A_o, z; v[m/s]; R_T(v)[kN]
Find	P[kW], n[1/s]

[Stage 0] Assume the Propeller Diameter

Propeller curve of the basis ship is drawn by connecting point(P_{MCR}, n_{MCR}) and origin(n=0, P=0).



- 1[st] Method: Use the diameter of the basis ship.
- 2[nd] Method: When the diameter of the basis ship is not given, estimate the diameter as follows.

Formula for estimation of propeller diameter(D_P)

$$D_p = 15.4 \times \left(\frac{P_{MCR}}{n_{MCR}^{3}} \right)_{basis\ ship}^{0.2} \times c_1$$

, where $c_1 = 1$ for 5 blades, 1.05 for 4 blades.

The MCR(P_{MCR}) and rpm(n_{MCR}) of the basis ship are used.

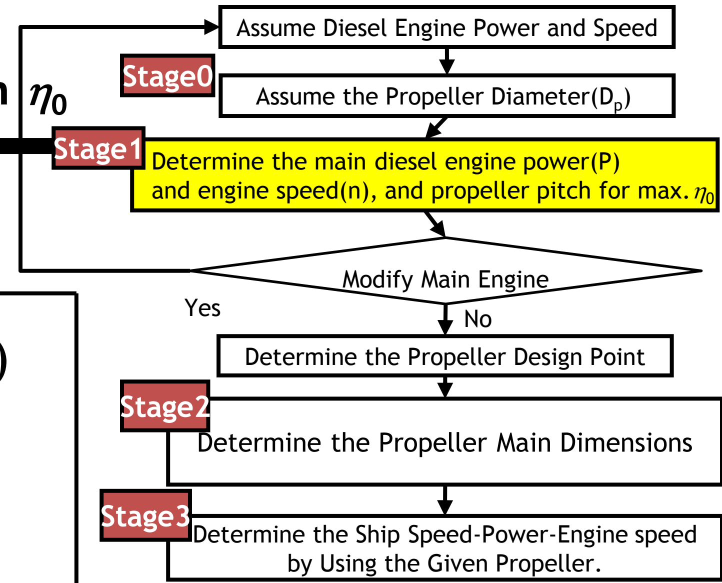
✓Delivered Power of the diesel engine :

$$P_{D.E.} \propto n$$

✓ Power absorbed by the propeller :

$$P_{prop.} \propto n^3$$

[Stage 1] Determine the main diesel engine power(P) and engine speed(n), and propeller pitch(P_i)for maximum η_0



Given	D_P [m] : Propeller diameter (from basis ship) A_E/A_O : Expanded area ratio z : Number of blades v [m/s] : Ship speed $R_T(v)$ [kN] : Resistance as function of Ship speed
Find	P_i [m] : Propeller pitch P [kW] : Delivered power to the propeller from the main diesel engine n [1/s] : Engine speed

Given	D_P [m], A_E/A_O, z ; v [m/s]; $R_T(v)$ [kN]
Find	P_i [m] ; P [kW] , n [1/s]

- Condition 1 : The propeller absorbs the torque delivered by the Main Diesel Engine

$$\boxed{\frac{P}{2\pi n}} = \boxed{\rho \cdot n^2 \cdot D_P^{\ 5} \cdot K_Q} \cdots (1)$$

Torque delivered by the Main Diesel Engine **=** Torque absorbed by the propeller

Given	D_P [m], A_E/A_O, z ; v [m/s]; $R_T(v)$ [kN]
Find	P_i [m] ; P [kW] , n [1/s]

- Condition 2 : The propeller should produce the required thrust at a given ship speed

$$\boxed{\frac{R_T}{1-t}} = \boxed{\rho \cdot n^2 \cdot D_P{}^4 \cdot K_T} \cdots (2)$$

The thrust which is required to propel the ship for the given speed = The thrust which is produced by the propeller

By using optimization method

Given	D_P [m], A_E/A_O, z ; v [m/s]; $R_T(v)$ [kN]
Find	P_i [m] ; P [kW] , n [1/s]

3 Unknowns

2 Equations

- **Condition 1** : **The propeller absorbs the torque delivered by the Diesel Engine**

$$\frac{P}{2\pi n} = \rho \cdot n^2 \cdot D_P{}^5 \cdot K_Q$$

- **Condition 2** : **The propeller should produce the required thrust at a given ship speed**

$$\frac{R_T}{1-t} = \rho \cdot n^2 \cdot D_P{}^4 \cdot K_T$$

Nonlinear Optimization Problem

Objective Function: Maximum η_0

$$\eta_o = \frac{T \cdot V_A}{DHP} = \frac{J}{2\pi} \cdot \frac{K_T}{K_Q}$$

Solve the nonlinear optimization problem.
Then the main engine power (P) and the speed of diesel engine (n) of the design ship are determined.

Ex: By using Lagrange Multiplier method

$$\frac{P}{2\pi n} = \rho \cdot n^2 \cdot D_P{}^5 \cdot K_Q$$

$$G_1(P_i, n, P) = \frac{P}{2\pi n} - \rho \cdot n^2 \cdot D_P{}^5 \cdot K_Q = 0 \quad \cdots\cdots \quad \text{(a)}$$

$$\frac{R_T}{1-t} = \rho \cdot n^2 \cdot D_P{}^4 \cdot K_T$$

$$G_2(P_i, n) = \frac{R_T}{1-t} - \rho \cdot n^2 \cdot D_P{}^4 \cdot K_T = 0 \cdots\cdots (b)$$

$$F(P_i, n) = \eta_0 = \frac{J}{2\pi} \cdot \frac{K_T}{K_Q} \quad \cdots\cdots \quad \text{(c)}$$

Lagrange Function

$$G_1(P_i,n,P)=\frac{P}{2\pi n}-\rho\cdot n^2\cdot D_P{}^5\cdot K_Q \quad \text{(a)}$$

$$G_2(P_i,n)=\frac{R_T}{1-t}-\rho\cdot n^2\cdot D_P{}^4\cdot K_T \quad (b)$$

$$F(P_i,n)=\eta_0=\frac{J}{2\pi}\cdot\frac{K_T}{K_Q} \quad \text{(c)}$$

- **Lagrange function:**

$$H(P_i,n,P)=F(P_i,n)+\lambda_1\cdot G_1(P_i,n,P)+\lambda_2\cdot G_2(P_i,n) \quad \cdots\cdots \quad \text{(d)}$$

- **Stationary point of H:** $\nabla H\left(P_i,n,P,\lambda_1,\lambda_2\right)=0$

$$\frac{\partial H}{\partial P_i}=\frac{J}{2\pi}\cdot\frac{\{(\frac{\partial K_T}{\partial P_i})\cdot K_Q-(\frac{\partial K_Q}{\partial P_i})\cdot K_T\}}{K_Q{}^2}+\lambda_1\cdot(-\rho\cdot n^2\cdot D_P^5\cdot\frac{\partial K_Q}{\partial P_i})+\lambda_2\cdot(-\rho\cdot n^2\cdot D_P^4\cdot\frac{\partial K_T}{\partial P_i}) \quad \cdots\cdots \quad \text{(e)}$$

$$\frac{\partial H}{\partial n}=\frac{1}{2\pi}\cdot\frac{\partial J}{\partial n}\cdot\frac{K_T}{K_Q}+\frac{J}{2\pi}\cdot\frac{\{(\frac{\partial K_T}{\partial n})\cdot K_Q-(\frac{\partial K_Q}{\partial n})\cdot K_T\}}{K_Q{}^2}+\lambda_1\cdot(-\frac{P}{2\cdot\pi\cdot n^2}-\rho\cdot 2\cdot n\cdot D_P^5\cdot K_Q-\rho\cdot n^2\cdot D_P^5\cdot\frac{\partial K_Q}{\partial n})$$

$$+\lambda_2\cdot(-\rho\cdot 2\cdot n\cdot D_P^4\cdot K_T-\rho\cdot n^2\cdot D_P^5\cdot\frac{\partial K_T}{\partial n})=0 \quad \cdots\cdots \quad \text{(f)}$$

$$\frac{\partial H}{\partial P}=\lambda_1\cdot\frac{1}{2\cdot\pi\cdot n}=0 \quad \cdots\cdots \quad \text{(g)}$$

$$\frac{\partial H}{\partial \lambda_1}=\frac{P}{2\pi n}-\rho\cdot n^2\cdot D_P{}^5\cdot K_Q=0 \quad \cdots\cdots \quad \text{(h)}$$

$$\frac{\partial H}{\partial \lambda_2}=\frac{R_T}{1-t}-\rho\cdot n^2\cdot D_P{}^4\cdot K_T=0 \quad \cdots\cdots \quad \text{(i)}$$

5 Equations: (e),(f),(g),(h),(i)

5 Unknowns: $P_i, n, P, \lambda_1, \lambda_2$

=> Can be solved by using numerical method, for example, Newton-Raphson Method

Given	D_P [m], A_E/A_O, z ; v [m/s]; $R_T(v)$ [kN]
Find	P_i [m] ; P [kW] , n [1/s]

3 Unknowns

2 Equality constraints

- **Condition 1** : **The propeller absorbs the torque delivered by the Diesel Engine**

$$\frac{P}{2\pi n} = \rho \cdot n^2 \cdot D_P^{\;5} \cdot K_Q \cdots (1)$$

- **Condition 2** : **The propeller should produce the required thrust at a given ship speed**

$$\frac{R_T}{1-t} = \rho \cdot n^2 \cdot D_P^{\;4} \cdot K_T \;\cdots (2)$$

Nonlinear indeterminate equation

Objective Function : **Find** Maximum η_0

$$\eta_o = \frac{J}{2\pi} \cdot \frac{K_T}{K_Q}$$

First assume the initial value and then determine the diesel main engine power(P) and the speed of diesel engine(n) by iteration to satisfy the conditions (1) and (2).

1

Given	D_P [m], A_E/A_O, z ; v [m/s]; $R_T(v)$ [kN]
Find	P_i [m] ; P [kW] , n [1/s]

2

Express the Condition 2 as $K_T = C_2 J^2$

Condition 2: $\dfrac{R_T}{1-t} = \rho \cdot n^2 \cdot D_P{}^4 \cdot K_T,$

Advance Ratio: $J = \dfrac{v_A}{n \cdot D_P} \Rightarrow n = \dfrac{v_A}{J \cdot D_P}$

$$K_T = \frac{R_T}{(1-t)\rho D_P{}^4} \cdot \frac{1}{n^2} \Rightarrow \frac{R_T}{(1-t)\rho D_P{}^4} \cdot \left(\frac{J \cdot D_P}{v_A}\right)^2$$

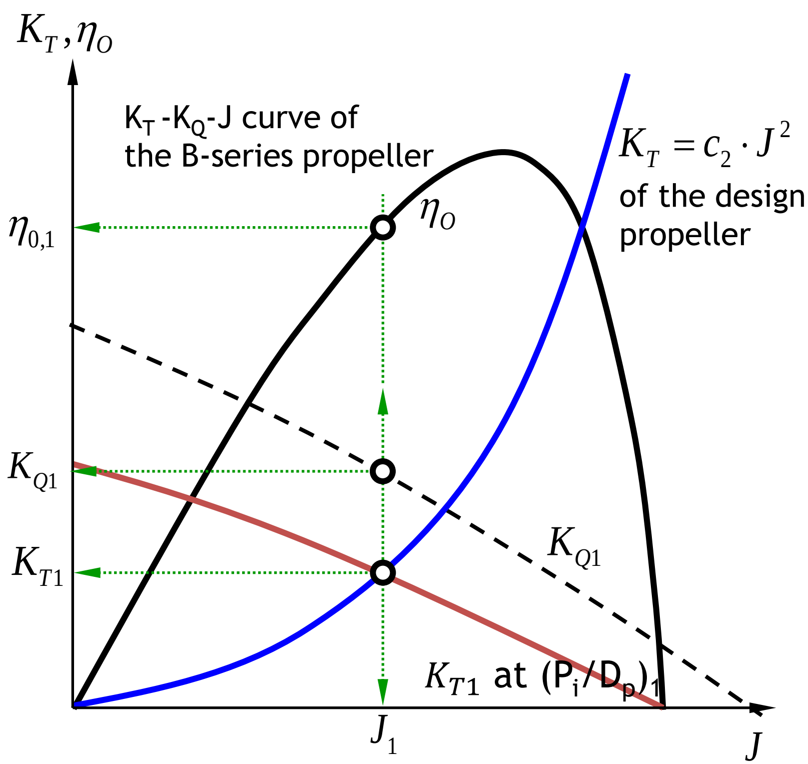
$$K_T = \frac{R_T}{(1-t)\rho D_P{}^2 v_A{}^2} J^2$$

$$\boxed{K_T = C_2 J^2} \quad , C_2 = \frac{R_T}{(1-t)\rho D_P{}^2 v_A{}^2}$$

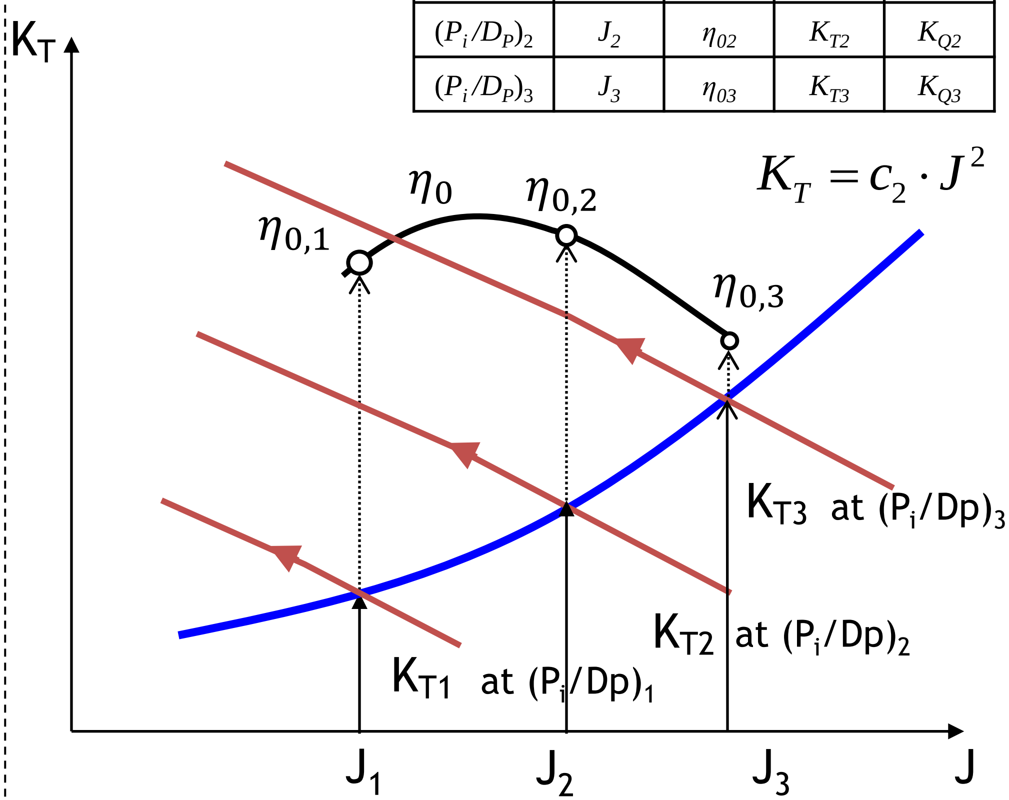
3

By using the POW-Curve(K_T-K_Q-J) of the series propeller data, for example, B-series propeller data, calculate the intersection point (J_1,K_{T1}) between the $K_T = c_2 \cdot J^2$ of the design propeller and the K_T-K_Q-J curve of the B-series propeller at a given pitch/diameter ratio $(P_i/D_p)_1$. And read the K_{Q1} and η_{01} at J_1.

Repeat this procedure by varying pitch/diameter ratio



P_i/D_P	J	η_0	K_T	K_Q
$(P_i/D_P)_1$	J_1	η_{01}	K_{T1}	K_{Q1}
$(P_i/D_P)_2$	J_2	η_{02}	K_{T2}	K_{Q2}
$(P_i/D_P)_3$	J_3	η_{03}	K_{T3}	K_{Q3}

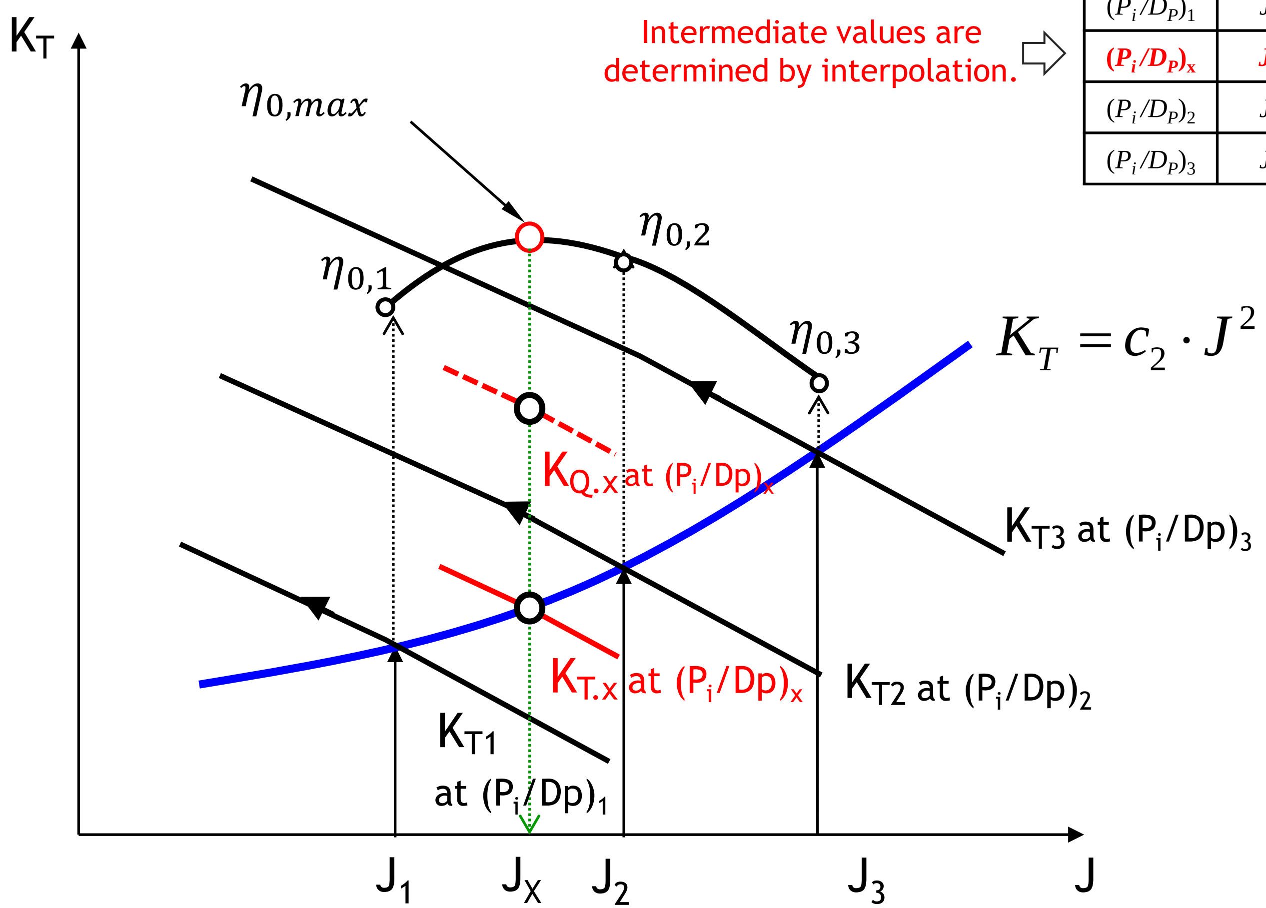


4

By using the set of K_T, K_Q, η_0 (varied with pitch ratio), determine J_x to maximize η_0 and pitch/diameter ratio $(P_i/D_p)_x$ at J_x.

Intermediate values are determined by interpolation.

P_i/D_P	J	η_0	K_T	K_Q
$(P_i/D_P)_1$	J_1	η_{01}	K_{T1}	K_{Q1}
$(P_i/D_P)_x$	J_x	η_{0x}	K_{Tx}	K_{Qx}
$(P_i/D_P)_2$	J_2	η_{02}	K_{T2}	K_{Q2}
$(P_i/D_P)_3$	J_3	η_{03}	K_{T3}	K_{Q3}



5

By using J_x from step 4, calculate n_x

$$J_x = \frac{v_A}{n_x \cdot D_P} \Rightarrow n_x = \frac{v_A}{D_P \cdot J_x}$$

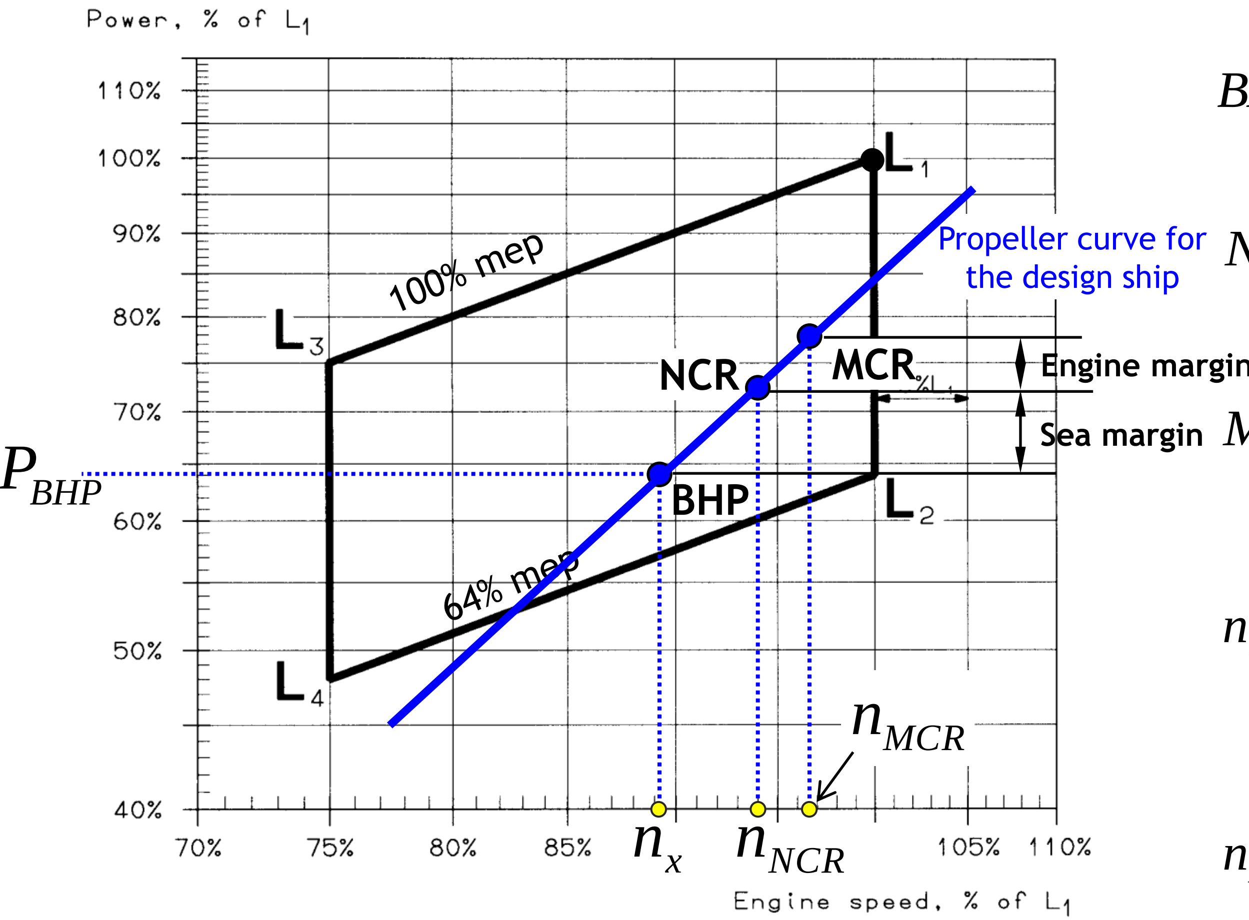
6

By using $K_{Q,x}$ from condition 1 and step 4, calculate P_x

$$\frac{P_x}{2\pi n} = \rho \cdot {n_x}^2 \cdot {D_P}^5 \cdot K_{Q,x} \Rightarrow P_x = 2\pi \cdot \rho \cdot {n_x}^3 \cdot {D_P}^5 \cdot K_{Q.x}$$

[Stage 1] Determine the main diesel engine power(P) and speed(n) for maximum η_0

Propeller curve for the design ship is drawn by connecting BHP and origin(n=0, P=0).



P_x(DHP) is determined from optimization.

$$BHP = \frac{DHP}{\eta_T}$$

η_T : Transmission efficiency

$$NCR = BHP(1 + S.M/100)$$

$$MCR = NCR / E.M$$

E.M: Engine Margin

$$n_{NCR} = \sqrt[3]{\frac{NCR}{c_3}} \quad , (P = c_3 \cdot n^3)$$

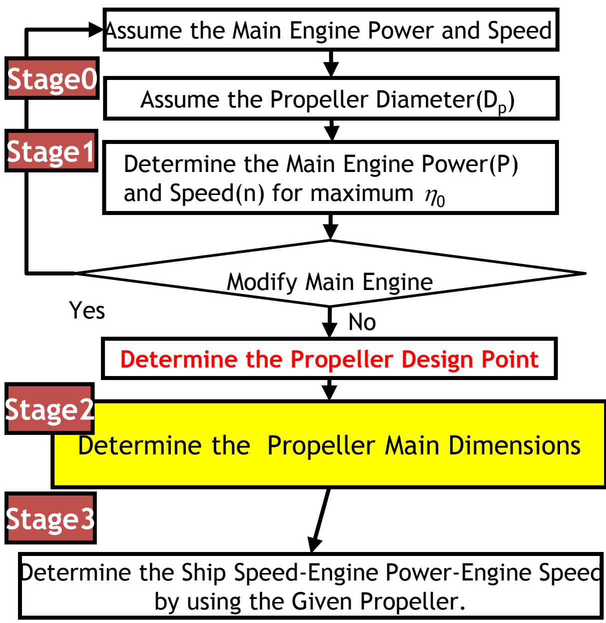
$$n_{MCR} = \sqrt[3]{\frac{MCR}{c_3}}$$

By using P and n required by the propeller, determine the NCR and MCR of the design ship. Check whether the NCR and MCR are in the layout diagram of the Diesel Engine.

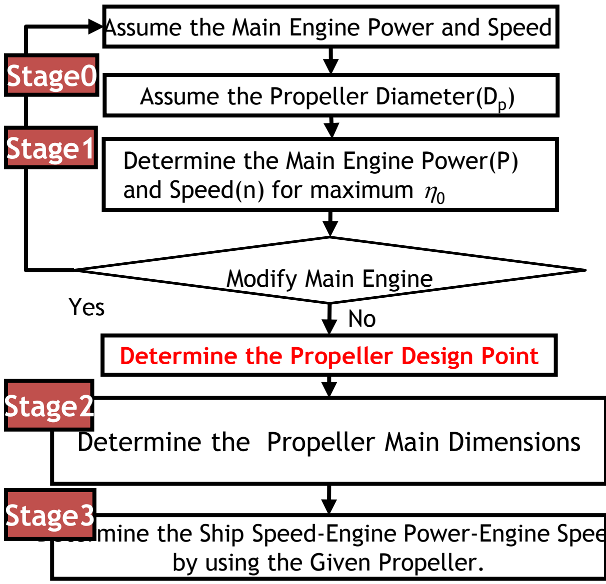
If the NCR and MCR are not in the layout diagram of the diesel engine, select another main engine.

[Stage 2] Determine the Propeller Main Dimensions for maximum η_0

Given	z : Number of Blades Propeller Design Point P $[kW]$: Delivered Power to Propeller from Main Engine (NCR) n [1/s] : RPS at MCR $R_T(v)$ $[kN]$: Resistance varied with Ship Speed
Find	D_P [m] : Propeller Diameter P_i [m] : Propeller Pitch A_E/A_O : Expanded Area Ratio v [m/s] : Ship Speed



Propeller Design Point:
Matching the Propeller and Diesel Engine

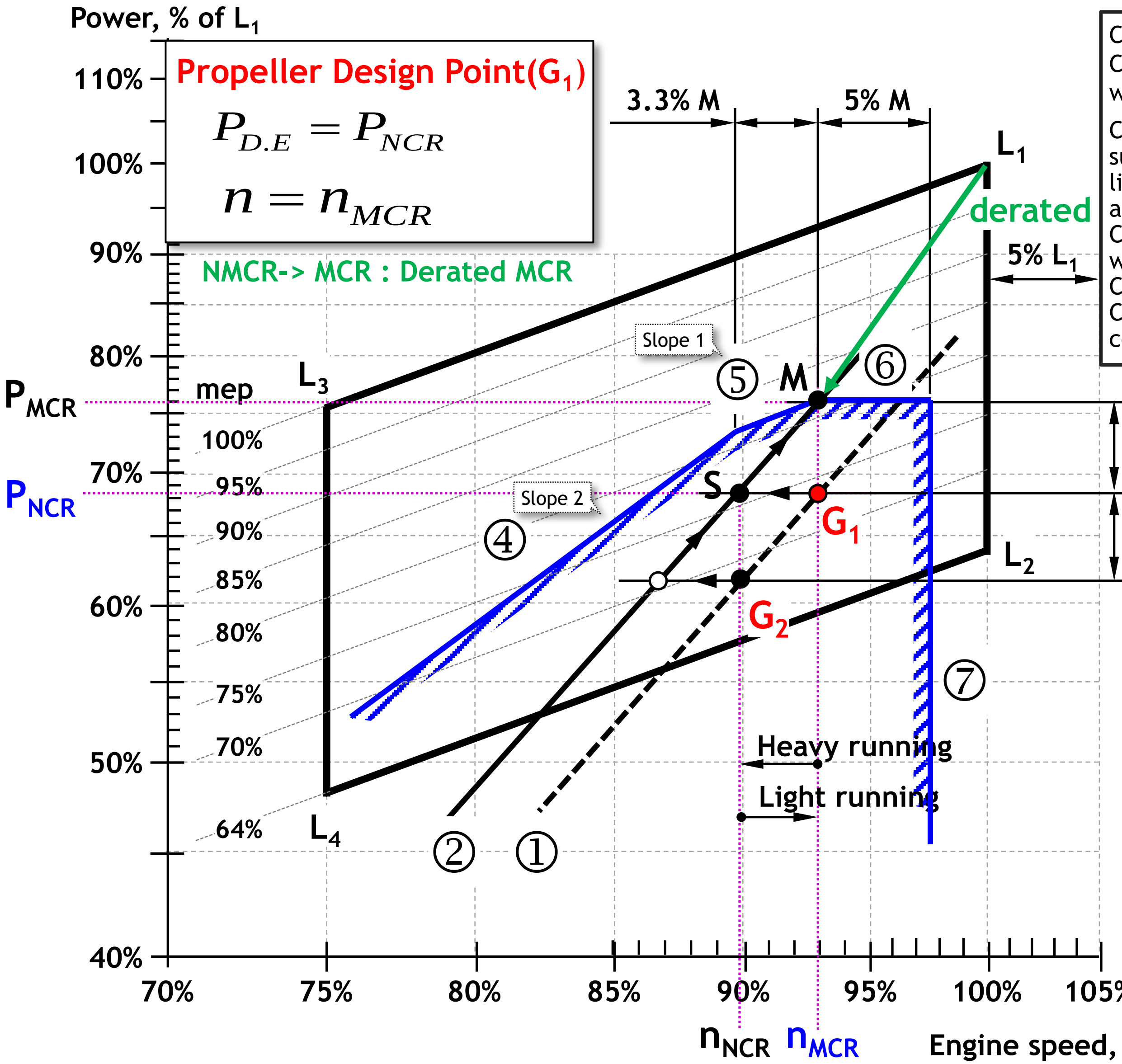


Matching the Propeller and Diesel Engine in the Diesel Engine Load Diagram in logarithmic scales

Continuous operation range of diesel engine

M : Specified MCR for propulsion (DMCR or MCR)

S : Normal continuous rating for propulsion (NCR)



Engine margin: NCR = 90% of MCR

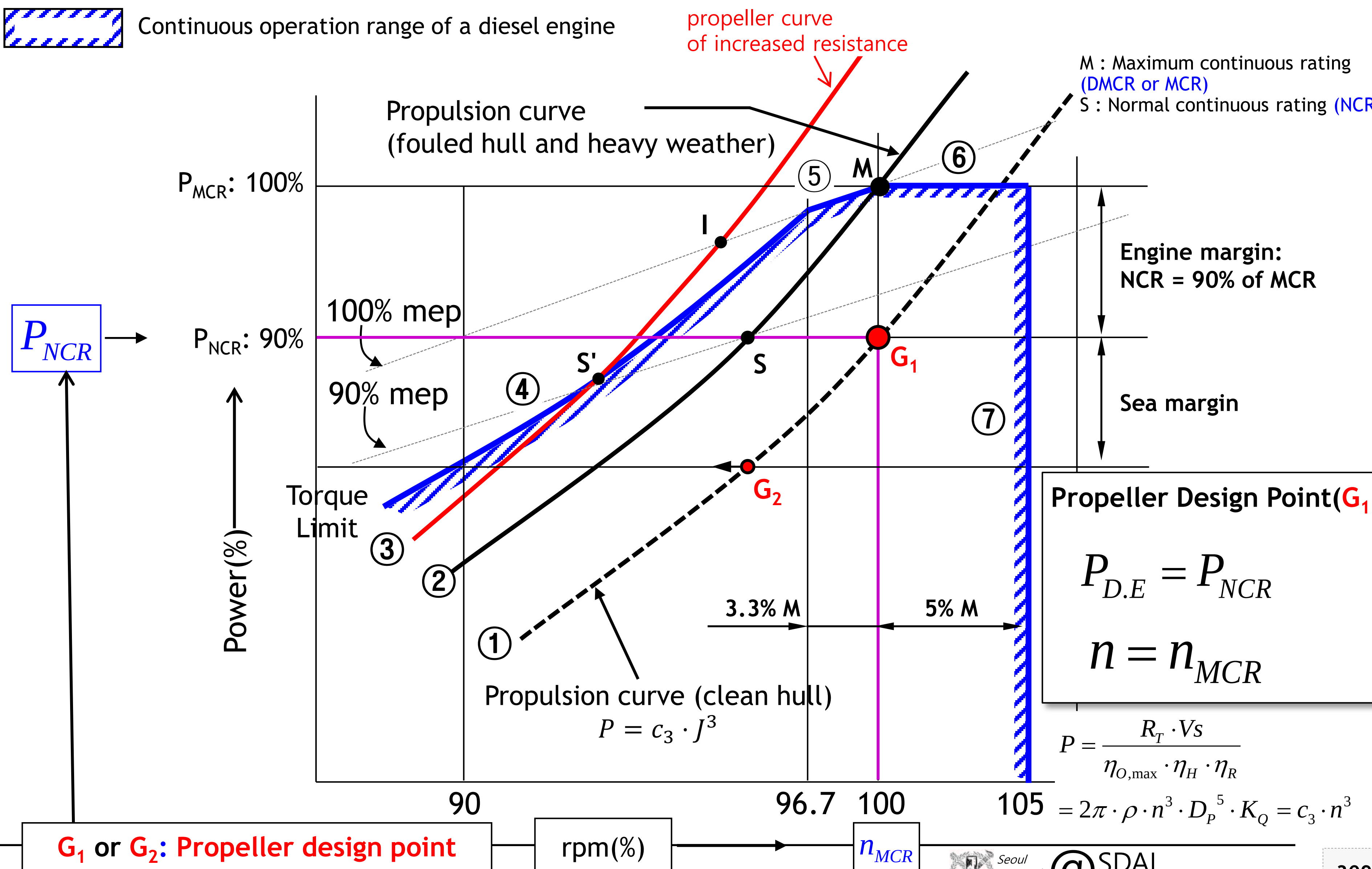
Sea margin

Curve ① : Propulsion curve, clean hull (light running)
Curve ② : Propulsion curve, fouled hull and heavy weather (heavy running)
Curve ④ :Torque/speed limit at which an ample air supply is available for combustion and imposes a limitation on the maximum combination of torque and speed. (proportional to n^2)
Curve ⑤ : Maximum mean effective pressure level, which can be accepted for continuous operation.
Curve ⑥ :Maximum power for continuous operation.
Curve ⑦ : Maximum speed which can be accepted for continuous operation.

G_2 is the intersection point of the curve ① with the reduced power considering the Sea margin and also propeller design point.

- MCR point can be any point within the layout diagram. When choosing the MCR, we have to consider the revolution of propeller, fuel consumption, and propeller operation layout and others all together.
- Once the MCR point has been chosen, and provided that the shaft line and auxiliary equipment are dimensioned accordingly, the specified MCR point is now the maximum rating. That is, the standard of all power and number of revolutions becomes MCR.

Propeller design point: Matching the Propeller and Diesel Engine in the Diesel Engine Load Diagram



Propeller Design Point(G_1)

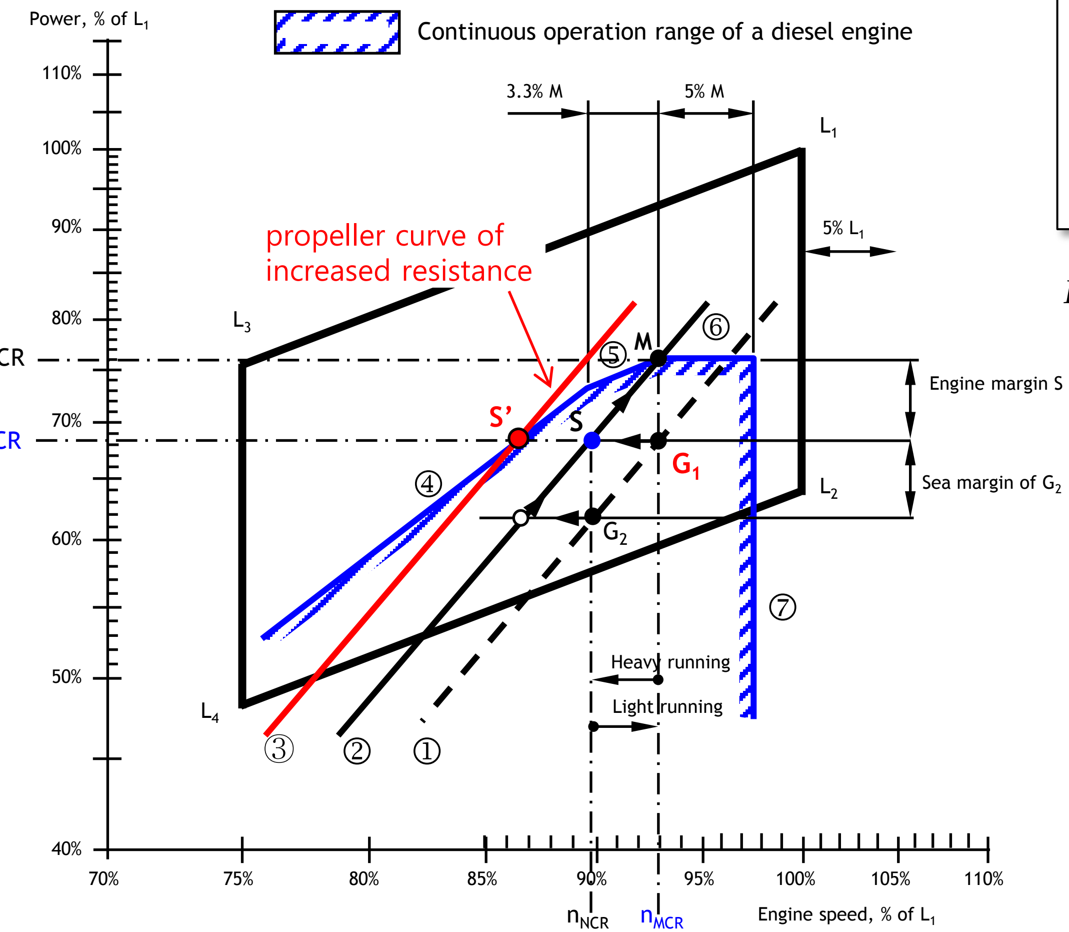
$$P_{D.E} = P_{NCR}$$

$$n = n_{MCR}$$

$$P = \frac{R_T \cdot Vs}{\eta_{O,\max} \cdot \eta_H \cdot \eta_R}$$

$$= 2\pi \cdot \rho \cdot n^3 \cdot D_P^{\,5} \cdot K_Q = c_3 \cdot n^3$$

Propeller design point: Matching the Propeller and Diesel Engine in logarithmic scales



Propeller Design Point(G_1)

$$P_{D.E} = P_{NCR}$$

$$n = n_{MCR}$$

$$P = \frac{R_T \cdot Vs}{\eta_{O,\max} \cdot \eta_H \cdot \eta_R} = 2\pi \cdot \rho \cdot n^3 \cdot D_P{}^5 \cdot K_Q = c_3 \cdot n^3$$

The reason why the propeller design point should be G_1.

- If the propeller is designed at the point S,

the propeller curve is the curve ②. When the resistance of ship increases with time, then the propeller curve will move to the curve ③. Thus the propeller and engine match at the point S' which is not NCR. This means the engine power of NCR cannot be delivered to the propeller, which results in reduction of ship speed.

- If the propeller is designed at the point G_1

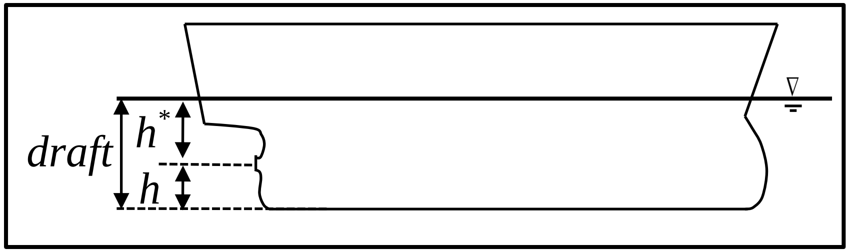
the propeller curve is the curve ①. When the resistance of ship becomes larger with time, the propeller curve ① will move to the curve ② so that the propeller operates at the point S(NCR).

$$J = \frac{v_A}{n \cdot D_P} \Rightarrow v_A = n \cdot D_P \cdot J$$

If J is constant, engine speed is proportional to ship speed.

[Stage 2] Determine the Propeller Main Dimensions
- Given : Engine Power, Engine Speed

Given	z ; $P_{NCR}[kW]$, n_{MCR} [1/s]; $R_T(v)$ [kN]
Find	D_p [m] , P_i [m], A_E/A_O ; v [m/s]



- Condition 3 : Required minimum expanded blade area ratio for non-cavitating criterion can be calculated by using one of the two formulas.

① Formula given by Keller

$$A_E / A_O \geq K + \frac{(1.3+0.3z)\cdot T}{D_P{}^2 \cdot (p_0 + \rho g h^* - p_v)}$$

K : single screw= 0.2, double screw = 0.1

P_0-P_v = 99.047 [kN/m^2]at 15℃ Sea water

h* : Shaft immersion depth [m]

h : Shaft Center Height (height from the baseline) [m]

T : Propeller Thrust [kN]

or ② Formula given by Burrill

$$A_E / A_O \geq F \cdot (\eta_0 /(1/J)^2)/[\{1+4.826(1/J)^2\}\cdot(1.067-0.229\cdot P_i / D_p)]$$

$$F = \frac{\eta_R \cdot B_P^2 \cdot v_A^{1.25}}{287.4(10.18+h)^{0.625}}$$

$$\boxed{B_P = n \cdot P^{0.5} / v_A^{2.5}}$$

$$\boxed{v_A = v\cdot(1-w)[knots]}$$

$P = DHP \cdot \eta_R [HP]$

$n[rpm]$

[Stage 2] Determination of Propeller Main Dimensions - Given : Engine Power, Engine Speed

Optimization Method

Given	z ; $P_{NCR}[kW]$, n_{MCR} [1/s]; $R_T(v)$ [kN]
Find	D_p [m] , P_i [m], A_E/A_O ; v [m/s]

4 Unknowns

2 Equality constraints

1 Inequality constraint

- Condition 1 : The propeller absorbs the torque delivered by the Diesel Engine

$$\frac{P}{2\pi n} = \rho \cdot n^2 \cdot D_P{}^5 \cdot K_Q$$

- Condition 2 : The propeller should produce the required thrust at a given ship's speed

$$\frac{R_T}{1-t} = \rho \cdot n^2 \cdot D_P{}^4 \cdot K_T$$

- Condition 3 : Required minimum expanded blade area ratio for non-cavitating criterion.

$$A_E / A_O \geq K + \frac{(1.3+0.3z)\cdot T}{D_P{}^2 \cdot (p_0 + \rho g h^* - p_v)}$$

Nonlinear indeterminate equation

Objective Function: Maximum η_0

$$\eta_o = \frac{J}{2\pi} \cdot \frac{K_T}{K_Q}$$

Propeller diameter(D_p), pitch(P_i), expanded blade area ratio(A_E/A_O) , and ship speed are determined to maximize the objective function by iteration.

A_O : Disc area($\pi D_P^2/4$)

A_E : Expanded propeller area

1 Assume the Expanded Area Ratio (A_E / A_o)

Assume that the expanded area ratio of the propeller of the design ship is the same as that of the basis ship.

2 Assume the ship speed v

3 Express the Condition 1 as $K_Q = C_1 J^5$

Condition 1: $\dfrac{P}{2\pi n} = \rho \cdot n^2 \cdot D_P^{\,5} \cdot K_Q,$

$$J = \frac{v_A}{n \cdot D_P} \Rightarrow \frac{nJ}{v_A} = \frac{1}{D_P}$$

$$K_Q = \frac{P}{2\pi n^3 \rho} \cdot \frac{1}{D_P^{\,5}} = \frac{P}{2\pi n^3 \rho} \cdot \left(\frac{nJ}{v_A}\right)^5$$

$$= \frac{P \cdot n^2}{2\pi \rho v_A^5} J^5 = C_1 J^5,$$

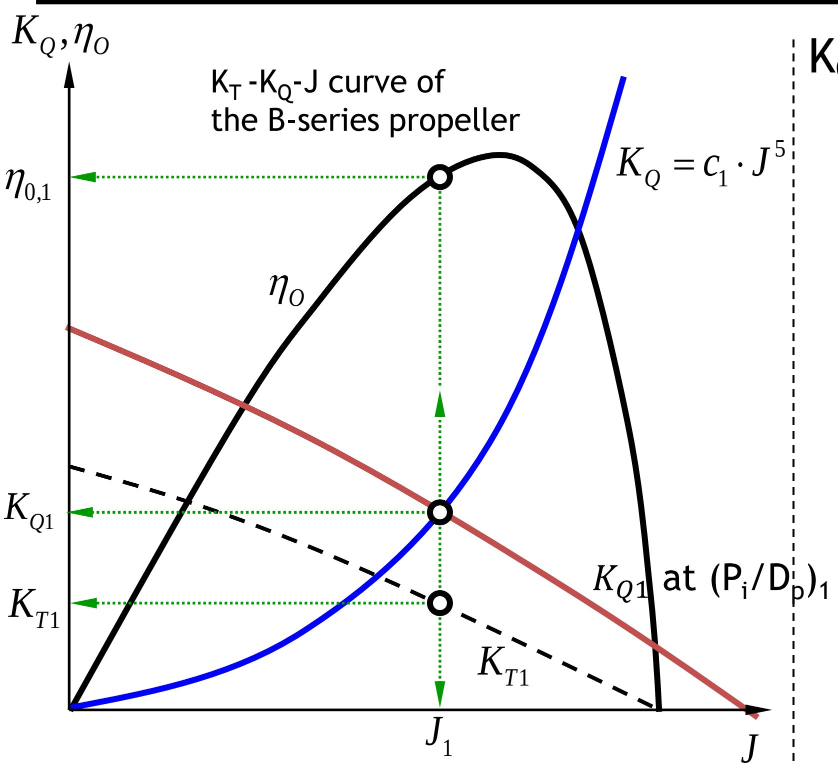
$$\boxed{K_Q = C_1 J^5}$$

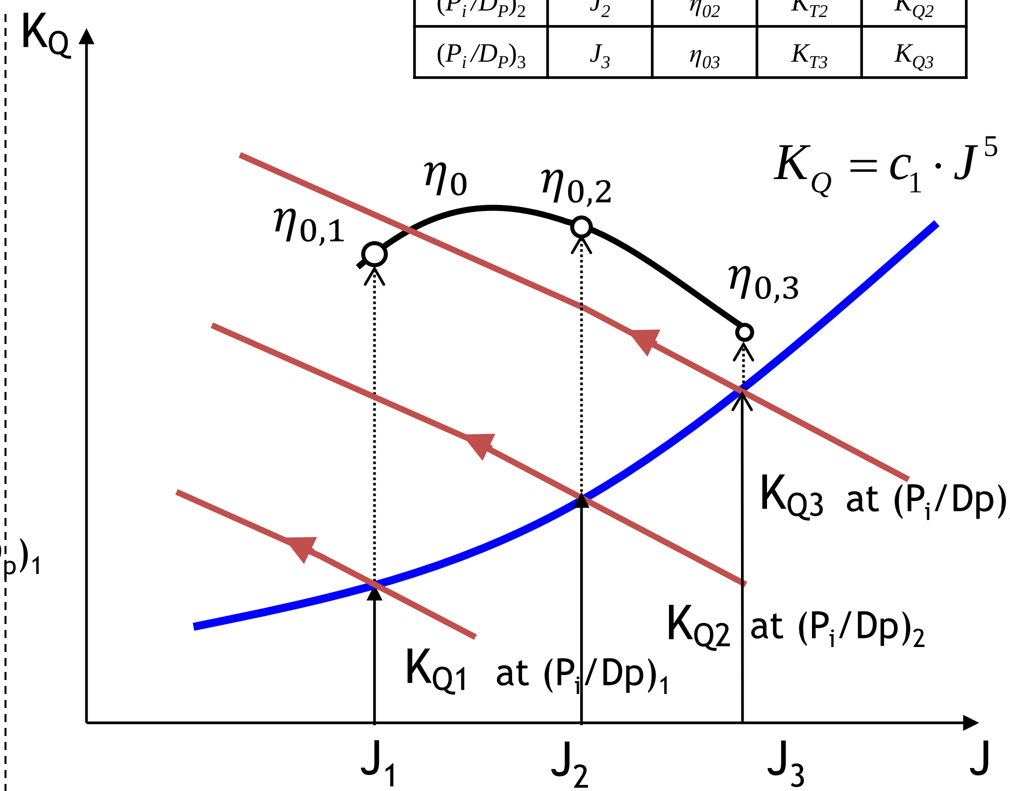
4

By using the POW-Curve(K_T-K_Q-J) of the series propeller data, for example, B-series propeller data, calculate the intersection point (J_1,K_{Q1}) between the $K_Q = c_1 \cdot J^5$ of the design propeller and the K_T-K_Q-J curve of the B-series propeller at a given pitch/diameter ratio $(P_i/D_p)_1$.
And read the K_{T1} and η_{01} at J_1.

Repeat this procedure by varying pitch/diameter ratio

P_i/D_P	J	η_0	K_T	K_Q
$(P_i/D_P)_1$	J_1	η_{01}	K_{T1}	K_{Q1}
$(P_i/D_P)_2$	J_2	η_{02}	K_{T2}	K_{Q2}
$(P_i/D_P)_3$	J_3	η_{03}	K_{T3}	K_{Q3}



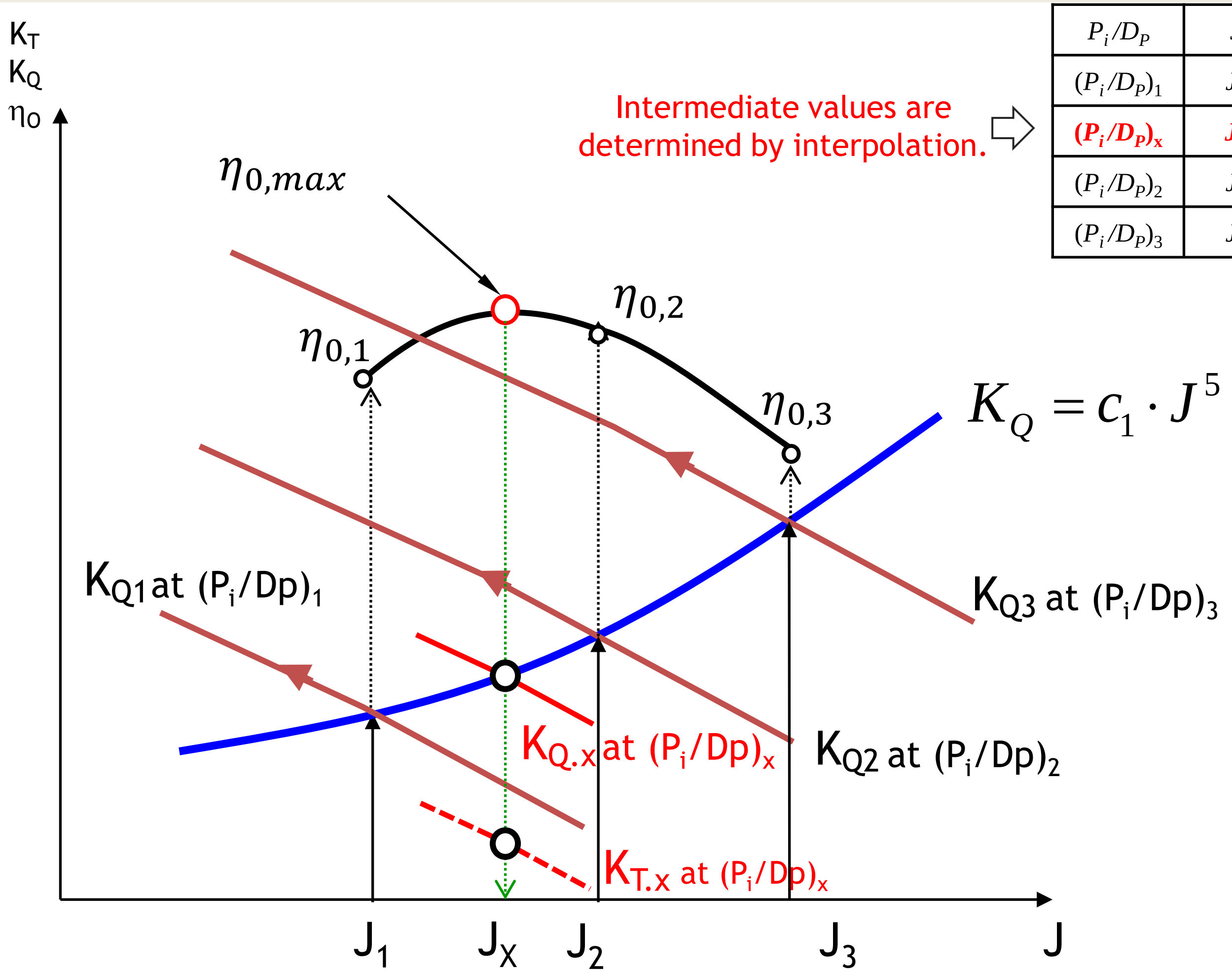


5

By using the set of K_T, K_Q, η_0 (varied with pitch ratio), determine J_x to maximize η_0 and pitch/diameter ratio $(P_i/D_p)_x$ at J_x .

Intermediate values are determined by interpolation.

P_i/D_P	J	η_0	K_T	K_Q
$(P_i/D_P)_1$	J_1	η_{01}	K_{T1}	K_{Q1}
$(P_i/D_P)_x$	J_x	η_{0x}	K_{Tx}	K_{Qx}
$(P_i/D_P)_2$	J_2	η_{02}	K_{T2}	K_{Q2}
$(P_i/D_P)_3$	J_3	η_{03}	K_{T3}	K_{Q3}



Calculation By Hand

[Stage 2] Determination of Propeller Main Dimensions

5 Calculate $D_{P.x}$ by using J_x at step 4

$$J = \frac{v_A}{n \cdot D_P} \Rightarrow D_{P.x} = \frac{v_A}{n \cdot J_x}$$

6 Do the values of D_{Px} and $K_{T.x}$ satisfy the condition 2 ?

No!! Go to **2** Assume another speed v

Check the condition 2: $\frac{R_T}{1-t} = \rho \cdot n^2 \cdot D_{P.x}{}^4 \cdot K_{T.x}$

Yes!!

7 Does the expanded area ratio (A_E / A_o) satisfy the condition 3 ?

No!! Go to **1** Assume another expanded area ratio (A_E / A_o)

Check the condition 3: $A_E / A_O \geq K + \frac{(1.3+0.3z) \cdot T}{D_P{}^2 \cdot (p_0 + \rho g h^* - p_v)}$

Yes!!

STOP

[Stage 2] Determination of the Propeller Main Dimensions (Summary)

Calculation By Hand

1. Assume the Expanded Area Ratio (A_E / A_o)
2. Assume the speed v
3. Express the Condition1 as $K_Q = C_1 J^5$
4. By using the set of K_T, K_Q, η_0 (varied with pitch ratio) determine J_x to maximize η_0 and pitch ratio $(P_i/D_p)_x$ at J_x .
5. Calculate $D_{P.x}$ by using J_x at step 4: $J_x = \dfrac{v_A}{n \cdot D_{P.x}}$
6. Do D_{Px} and K_{Tx} satisfy condition 2 ?
7. Does the expanded area ratio (A_E / A_o) satisfy condition 3 ?

1st Loop

2nd Loop

3rd Loop

Condition 1:

$$\frac{P}{2\pi n} = \rho \cdot n^2 \cdot D_P^{\ 5} \cdot K_Q$$

Condition 2:

$$\frac{R_T}{1-t} = \rho \cdot n^2 \cdot D_{P.x}^{\ 4} \cdot K_{T.x}$$

Relations between Propeller and Diesel Engine

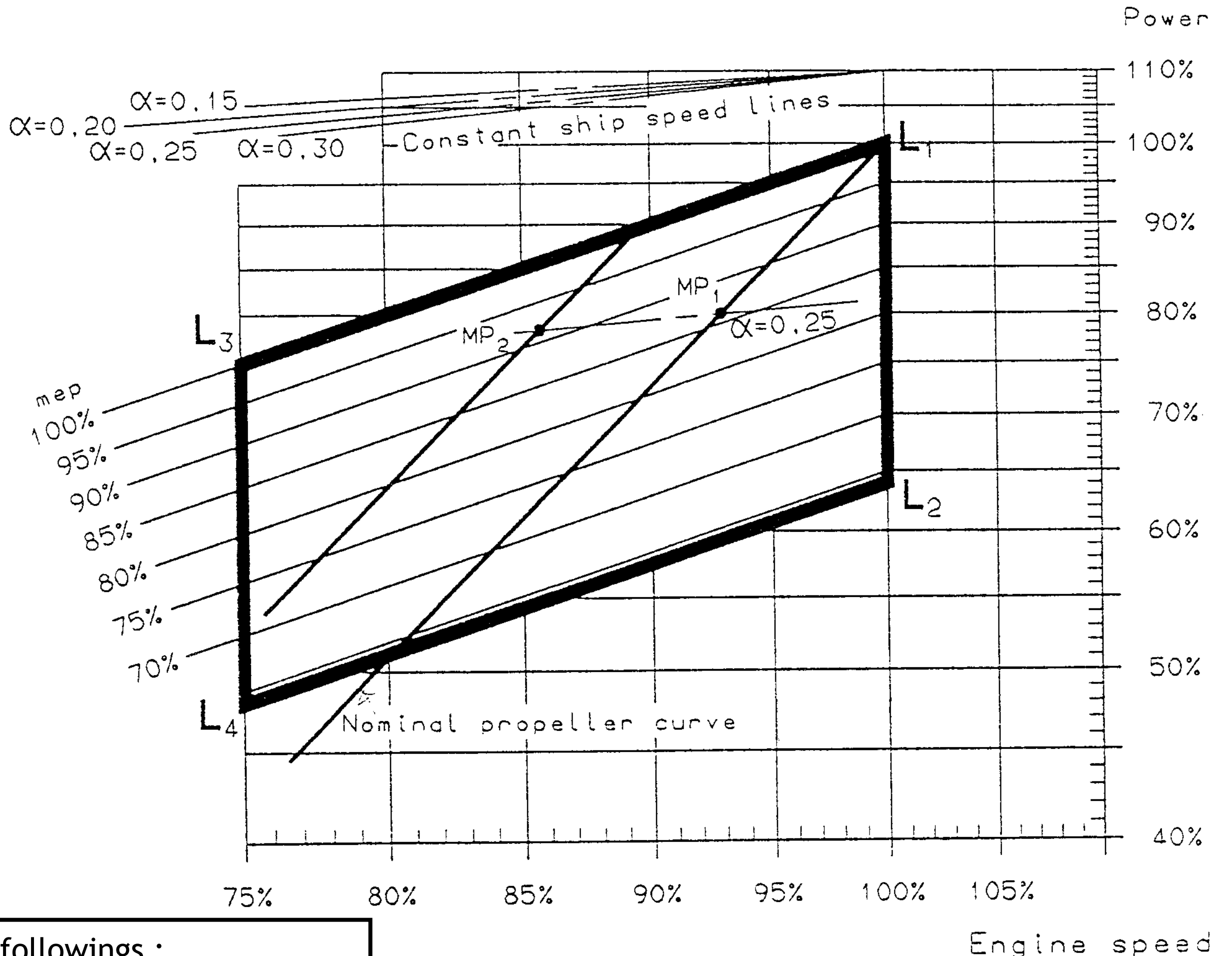
☑ The relations between rpm, efficiency of the propeller, and size of the diesel engine.

- If the rpm of a propeller increases, the optimum diameter of the propeller becomes smaller, and the efficiency of the propeller decreases.
- However, if the rpm of the propeller increases, we can select smaller diesel engine.

☑ Factors considered for Selecting the Diesel Engine

- Efficiency of the propeller
- Weight of the engine
- Arrangement of the engine room
- Initial investment cost (for large and low-speed diesel engine: about 180$/PS (at 1998))
- Operation cost

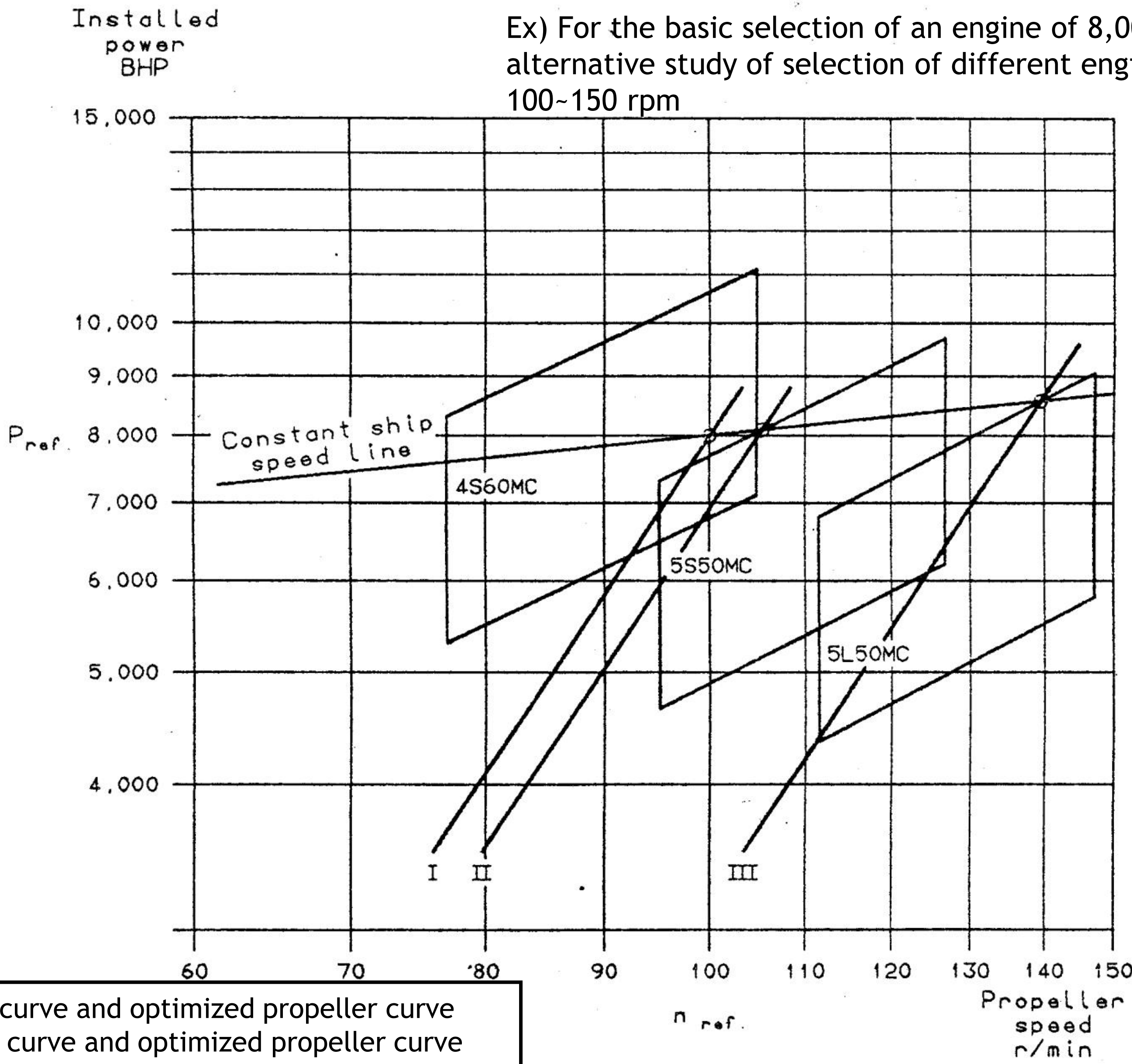
Selection of alternative MCR by using constant ship speed lines



To select MCR, consider followings :
- Smaller engine power and lower engine speed
- Derated power: NMCR-> DMCR
- Fuel oil consumption
- Propeller operating range

Selection of alternative engine type by using constant ship speed lines

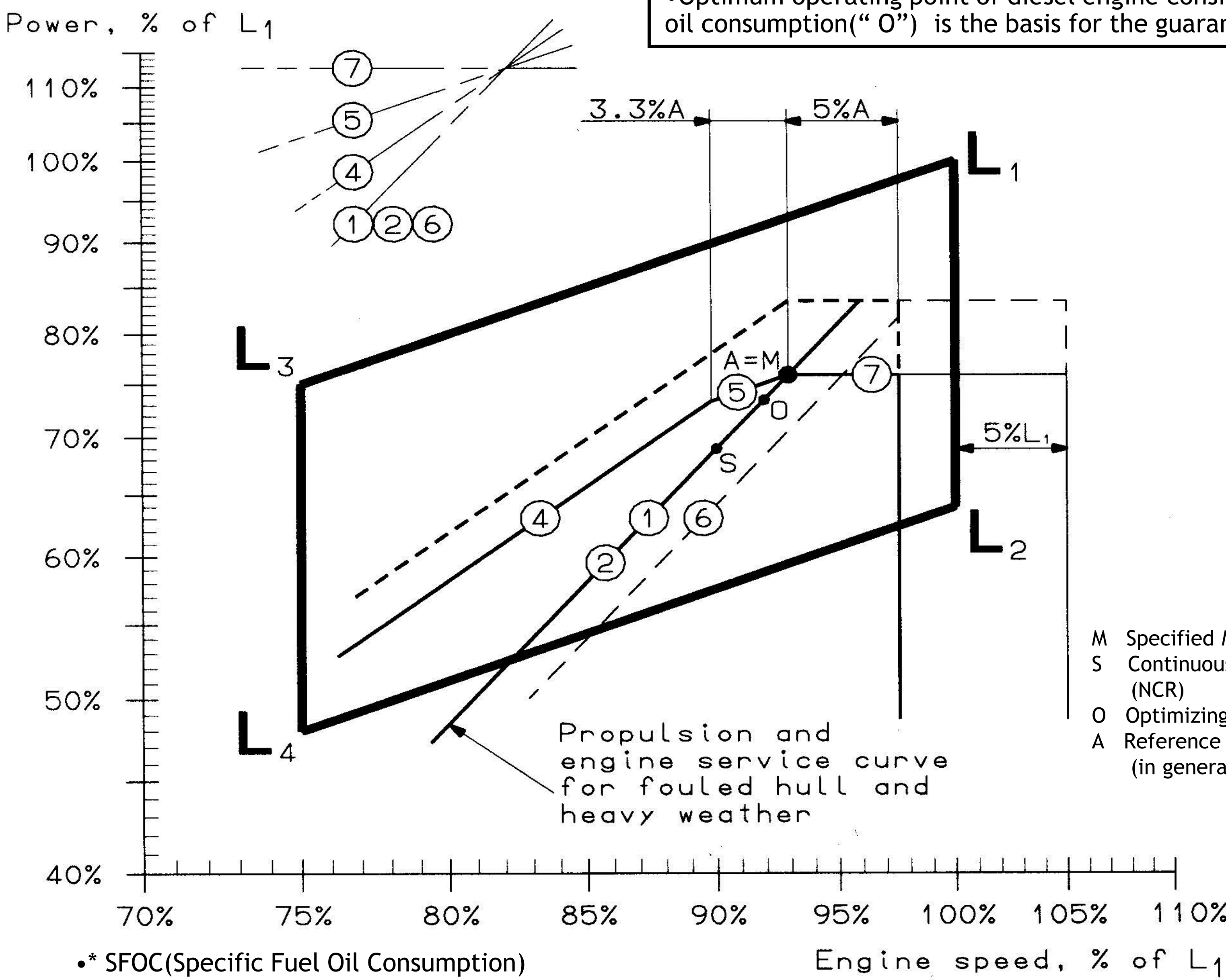
Ex) For the basic selection of an engine of 8,000BHP, 100rpm, alternative study of selection of different engines in the range of 100~150 rpm



- I: 4S60MC engine curve and optimized propeller curve
- II: 5S50MC engine curve and optimized propeller curve
- III: 5L50MC engine curve and optimized propeller curve

Optimum operating point of diesel engine considering the fuel oil consumption

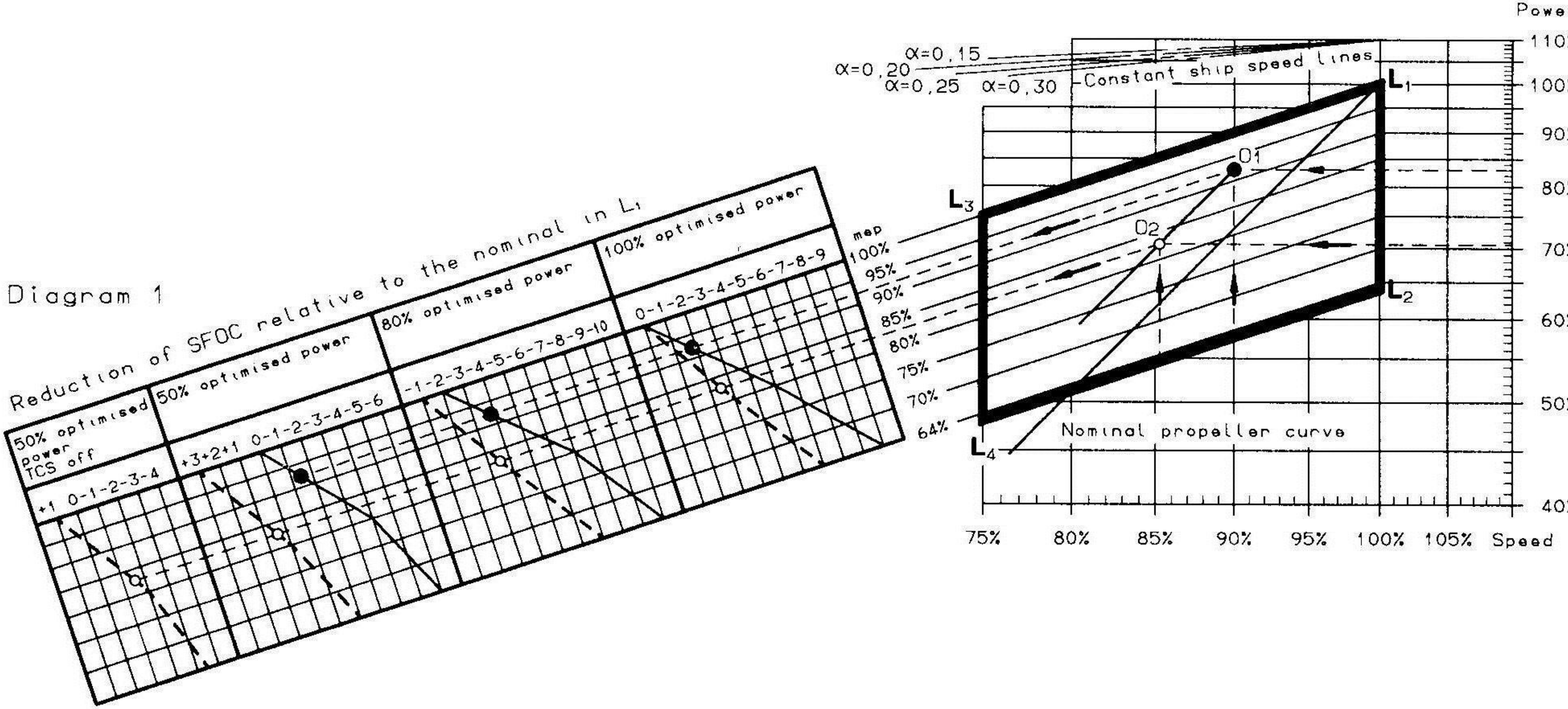
- Optimum operating point of diesel engine considering the fuel oil consumption(" O") is the basis for the guarantee SFOC



M Specified MCR of engine (MCR)
S Continuous service rating of engine (NCR)
O Optimizing point of engine (O)
A Reference point of load diagram (in general, A = M)

- * SFOC(Specific Fuel Oil Consumption)

Example of selection of optimum point " O" considering SFOC



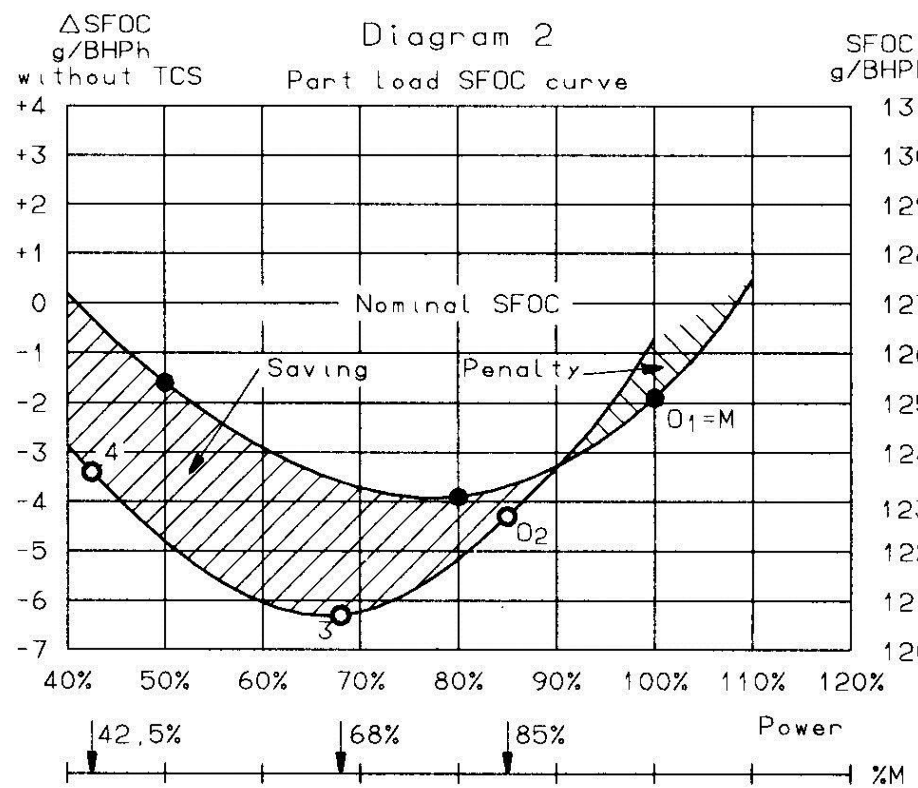
Example of selection of optimum point " O" considering SFOC

Data at nominal MCR (L_1):	
Power: 100% (L_1)	21,360 BHP
Speed: 100% (L_1)	106 r/min
Nominal SFOC	127 g/BHPh

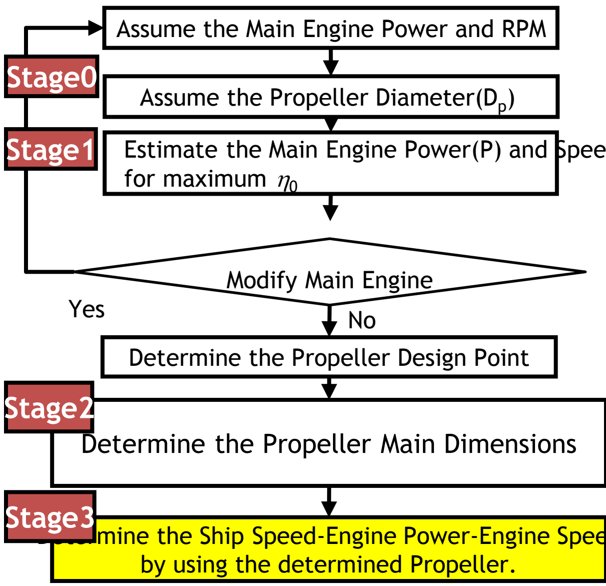
Data of optimising point (O):	O_1	O_2
Power: 100% of (O)	17,730 BHP	15,100 BHP
Speed: 100% of (O)	95.4 r/min	90.1 r/min
SFOC found:	125 g/BHPh	122.6 g/BHPh

L70MC	Nominal SFOC in g/BHPh at nominal MCR (L_1)
Conventional turbochargers	127
High efficiency turbochargers	125
High efficiency turbochargers and TCS	min. 122

O_1: Optimised in M
O_2: Optimised at 85% of power in M
Point 3: is 80% of O_2 = 0.80 x 0.85 of M = 68% M
Point 4: is 50% of O_2 = 0.50 x 0.85 of M = 42.5% M



[Stage 3] Determination of Ship Speed-Engine Power-Engine Speed by using the determined Propeller : The Propeller Main Dimensions have been determined.



Given	D_P [m] : Propeller Diameter P_i [m] : Propeller Pitch z : Number of Blades A_E/A_O : Expanded Area Ratio --- v [m/s] : Varied Ship Speed --- $R_T(v)$ [kN] : Resistance as function of Ship Speed
Find	P [kW] : Power of the main diesel engine, which is delivered to the propeller n [1/s] : Speed of the main diesel engine

[Stage 3] Determination of Ship Speed-Engine Power-Engine Speed by using the determined Propeller : The Propeller Main Dimensions have been determined.

Given	D_P , P_i, z, A_E/A_O; v [m/s]; $R_T(v)$ [kN]
Find	P [kW] , n [1/s]

- Condition 1 : The propeller absorbs the torque delivered by the Diesel Engine

$$\frac{P}{2\pi n} = \rho \cdot n^2 \cdot D_P{}^5 \cdot K_Q \ \cdots (1)$$

- Condition 2 : The propeller should produce the required thrust for a given ship speed

$$\frac{R_T}{1-t} = \rho \cdot n^2 \cdot D_P{}^4 \cdot K_T \cdots (2)$$

2 Unknowns

2 Equations

$\Downarrow$

Nonlinear equation

$\Rightarrow$ Not an optimization problem

1 Express the Condition 2 as $K_T = C_2 J^2$

Condition 2: $\dfrac{R_T}{1-t} = \rho \cdot n^2 \cdot D_P{}^4 \cdot K_T$, Advance Ratio: $J = \dfrac{v_A}{n \cdot D_P} \Rightarrow n = \dfrac{v_A}{J \cdot D_P}$

$$K_T = \frac{R_T}{(1-t)\rho D_P{}^4} \cdot \frac{1}{n^2} \Rightarrow \frac{R_T}{(1-t)\rho D_P{}^4} \cdot \left(\frac{J \cdot D_P}{v_A}\right)^2$$

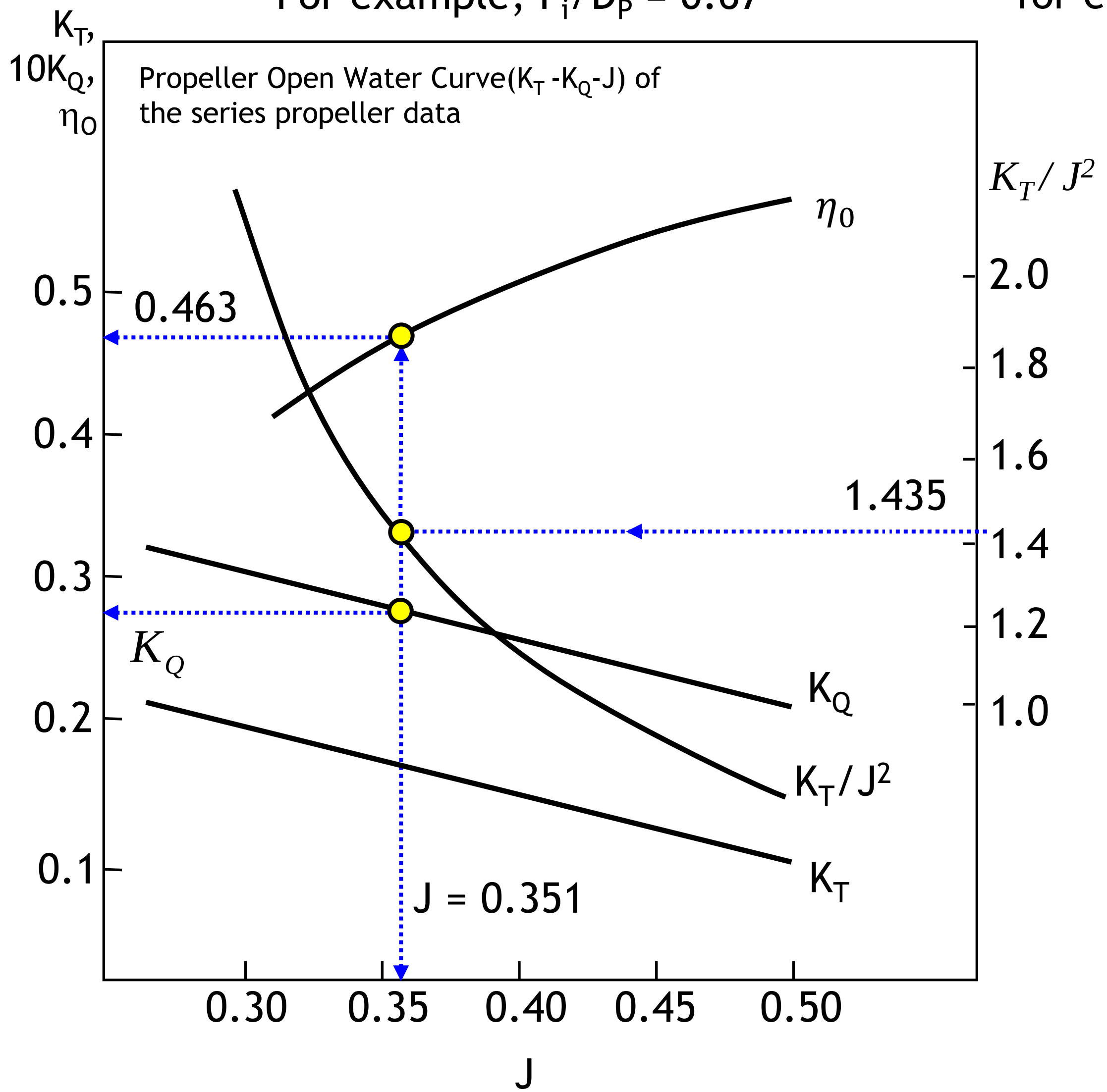
$$K_T = \frac{R_T}{(1-t)\rho D_P{}^2 v_A{}^2} J^2$$

$$\boxed{K_T = C_2 J^2} \qquad C_2 = \frac{R_T}{(1-t)\rho D_P{}^2 v_A{}^2}$$

Determine the power and speed of the diesel engine for various ship speeds.

Calculation By Hand

For example, $P_i/D_P = 0.67$



Propeller Open Water Curve(K_T -K_Q-J) of the series propeller data,
for example, B-series propeller data

$$\frac{K_T}{J^2} = C_2, \quad C_2 = \frac{R_T}{(1-t)\rho D_P^{\,2} v_A^{\,2}}$$

$$P = 2\pi \cdot \rho \cdot n^3 \cdot D_P^{\,5} \cdot K_Q$$

$$= \frac{R_T \cdot Vs}{\eta_{O,\max} \cdot \eta_H \cdot \eta_R}$$

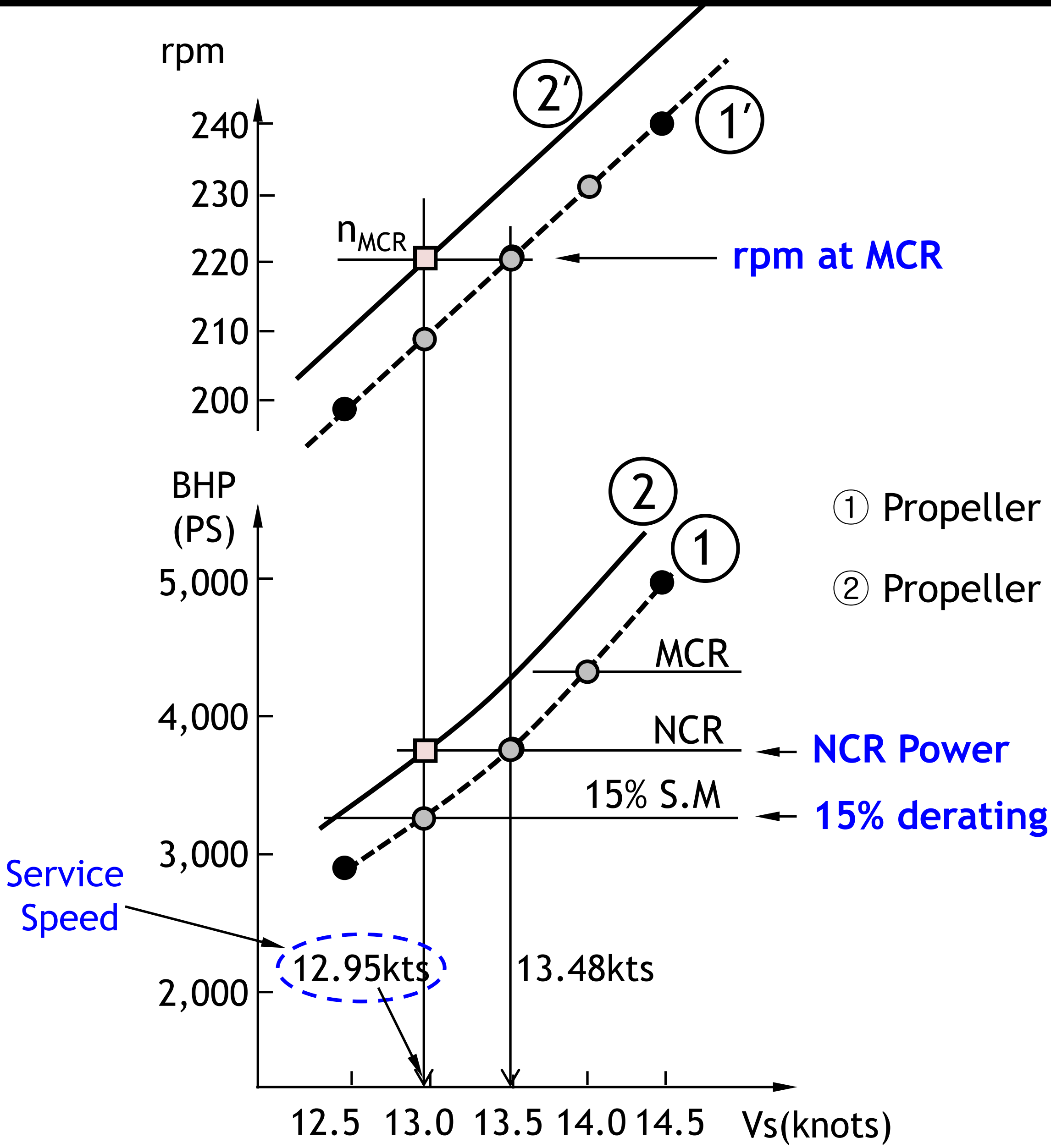
$$BHP = \frac{DHP}{\eta_T}$$

η_T : Transmission efficiency

V_S [kts]	③ EHP in calm water [PS]	w	t	① K_T/J^2	② J	n [RPM]	④ η_0	③ BHP in calm water
12.5	1686	0.381	0.224	1.374	0.355	202	0.470	2867
13.0	1965	0.380	0.223	1.418	0.352	212	0.465	3367
13.5	2240	0.379	0.221	1.435	0.351	221	0.463	3844
14.0	2536	0.377	0.219	1.443	0.348	232	0.460	4376
14.5	2898	0.375	0.216	1.470	0.345	243	0.457	5020

After the propeller main dimensions have been determined, calculate the power and Speed of diesel engine for various ship speeds.

Calculation By Hand



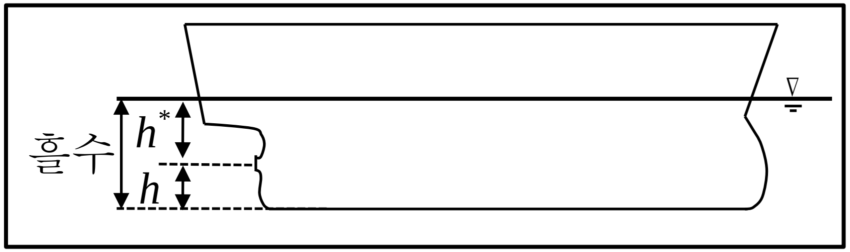
V_S [kts]	EHP in calm water [PS]	w	t	K_T/J^2	J	n [RPM]	η_0	BHP in calm water
12.5	1686	0.381	0.224	1.374	0.355	202	0.470	2867
13.0	1965	0.380	0.223	1.418	0.352	212	0.465	3367
13.5	2240	0.379	0.221	1.435	0.351	221	0.463	3844
14.0	2536	0.377	0.219	1.443	0.348	232	0.460	4376
14.5	2898	0.375	0.216	1.470	0.345	243	0.457	5020

프로펠러 최적 주요치수 결정 예제

참고자료

프로펠러 최적 주요치수 결정 예제

- 주기관 마력과 프로펠러 회전수가 주어진 경우



(Question) DWT 7,400 ton/400TEU 급 세미 컨테이너선의 프로펠러 주요 치수를 결정하시오.

Given Data

- 디젤엔진 마력과 회전수
  - MCR = 4,500PS , at 220rpm : 연속최대출력
  - NCR = 85%MCR , at 208rpm : 상용출력
  - 프로펠러 회전수 : 220rpm
  - 날개 수(z) : 4

- 기타
  - h (Shaft Center Height) : 2.35 [m]
  - h^* (축 침수 깊이) : 4.15 [m]
  - 흘수 : 6.5 [m]
  - Sea Margin = 15%

- Model Test

Ship Speed V [kts]	정수중 EHP [PS]	T [kN]	R [kN]	w	t	η_R	$\eta_H = \frac{1-t}{1-w}$
12.5	1686	248	192.5	0.381	0.224	1.018	1.254
13.0	1965	278	216.0	0.380	0.223	1.022	1.253
13.5	2240	304	236.8	0.379	0.221	1.024	1.254
14.0	2536	331	258.5	0.377	0.219	1.026	1.253

프로펠러 최적 주요치수 결정 예제

- Propeller 설계점 : NCR의 마력, MCR의 회전수

  NCR = 3825 [PS]

  N_{MCR} = 220/60 [1/s]

프로펠러 최적 주요치수 결정 예제

1 전개 면적비 (A_E / A_o) 가정

$(A_E / A_o) = 0.55$ 로 가정

2 속력 v 가정

$v = 13.5[kts]$ 로 가정

$v = 13.5[kts] = 13.5 \times 0.5144 = 6.945[m/s]$

Ship Speed V [kts]	**정수중** EHP [PS]	T [kN]	R [kN]	w	t	η_R	$\eta_H = \frac{1-t}{1-w}$
13.5	2240	304	236.8	0.379	0.221	1.024	1.254

※ 속력이 바뀔 경우 나머지 계수는 선형 보간으로 구함

$$P = DHP = \frac{NCR(S.W.)}{1.025} \times 0.736 \times \eta_T \times \eta_R = \frac{3,825}{1.025} \times 0.736 \times 0.98 \times 1.024 = 2.758 \text{ [kW]}$$

※ 단독 프로펠러를 기준으로 만든 곡선이므로 선미에서 전달되는 마력을 단독적으로 작동하는 마력으로 변환하기 위하여 η_R을 고려해야 함

프로펠러 최적 주요치수 결정 예제

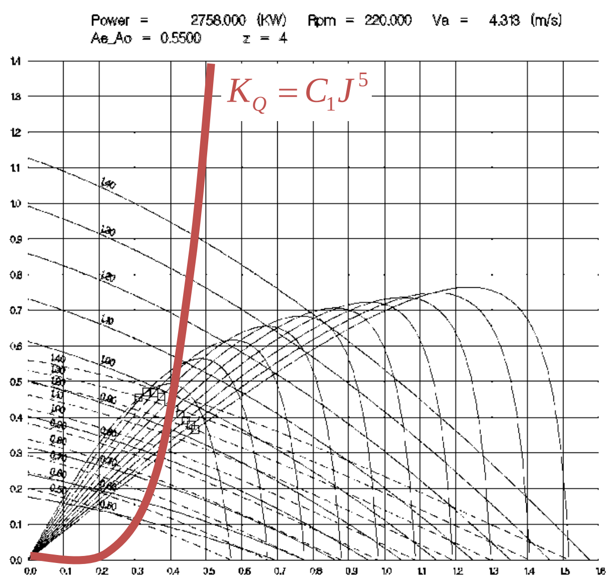
3 조건식1을 $K_Q = C_1 J^5$ 의 형태로 표현

$$v_A = v(1-w) = 6.945 \times (1-0.379) = 4.313$$

$$C_1 = \frac{P \cdot n^2}{2\pi \rho v_A^5} = \frac{2{,}758 \times (220/60)^2}{2\pi \times 1.0 \times 4.313^5} = 3.9551$$

$$\therefore K_Q = C_1 J^5 = 3.9551 J^5$$

4 프로펠러 단독성능 곡선을 이용하여 여러 피치비에서 최대 효율(η_0)을 내는 J와 그 때의 K_T 를 구함



프로펠러 최적 주요치수 결정 예제

-

$$P = 2758(KW),\ n = 220,\ v_A = 4.313(m/s),$$
$$A_E/A_O = 0.55,\ z = 4$$

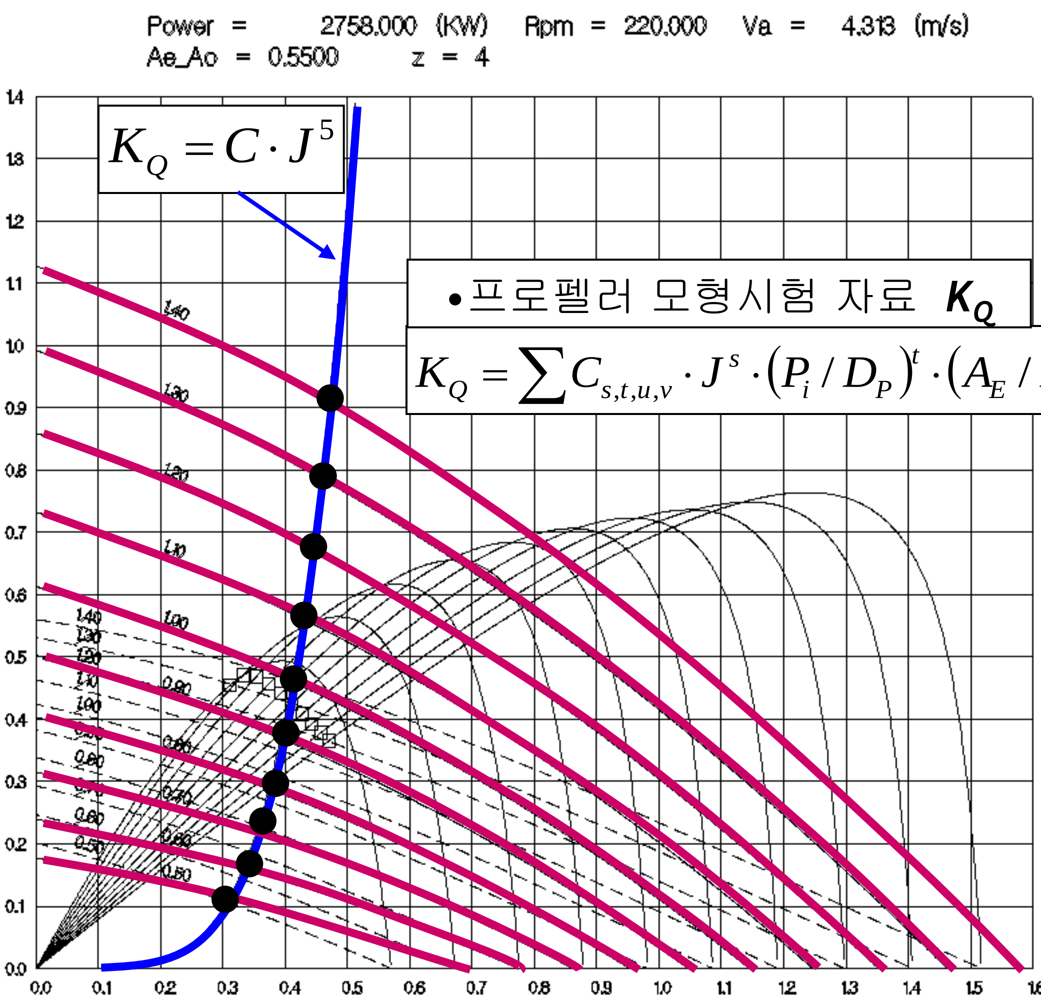
- 그래프의 교점

↓

- J 결정

↓

η_0 • 가 최대가 되는 $J,\ P_i/D$ 선정



- 프로펠러 모형시험 자료 K_Q

$$K_Q = \sum C_{s,t,u,v} \cdot J^s \cdot (P_i/D_P)^t \cdot (A_E/A_O)^u \cdot z^v$$

P_i/D_P	J	η_0	K_T
0.50	0.3105	0.4542	0.1049
0.60	**0.3323**	**0.4711**	**0.1428**
0.70	0.3535	0.4684	0.1818
0.80	0.3737	0.4570	0.2215
0.90	0.3927	0.4418	0.2610
1.00	0.4105	0.4252	0.2999
1.10	0.4271	0.4086	0.3378

프로펠러 최적 주요치수 결정 예제

5 단계 4에서 구한 J를 이용해 D_P를 구함

$$J = \frac{v_A}{n \cdot D_P} \Rightarrow D_P = \frac{v_A}{n \cdot J} = \frac{4.313}{(220/60) \times 0.3323} = 3.5396[m]$$

6 구한 D_P와 K_T가 조건식2를 만족하는가?

선박이 요구하는 추력(T_S) = $\frac{R_T}{1-t} = \frac{236.8}{1-0.221} = 304[kN]$

프로펠러가 낼 수 있는 추력(T_P)

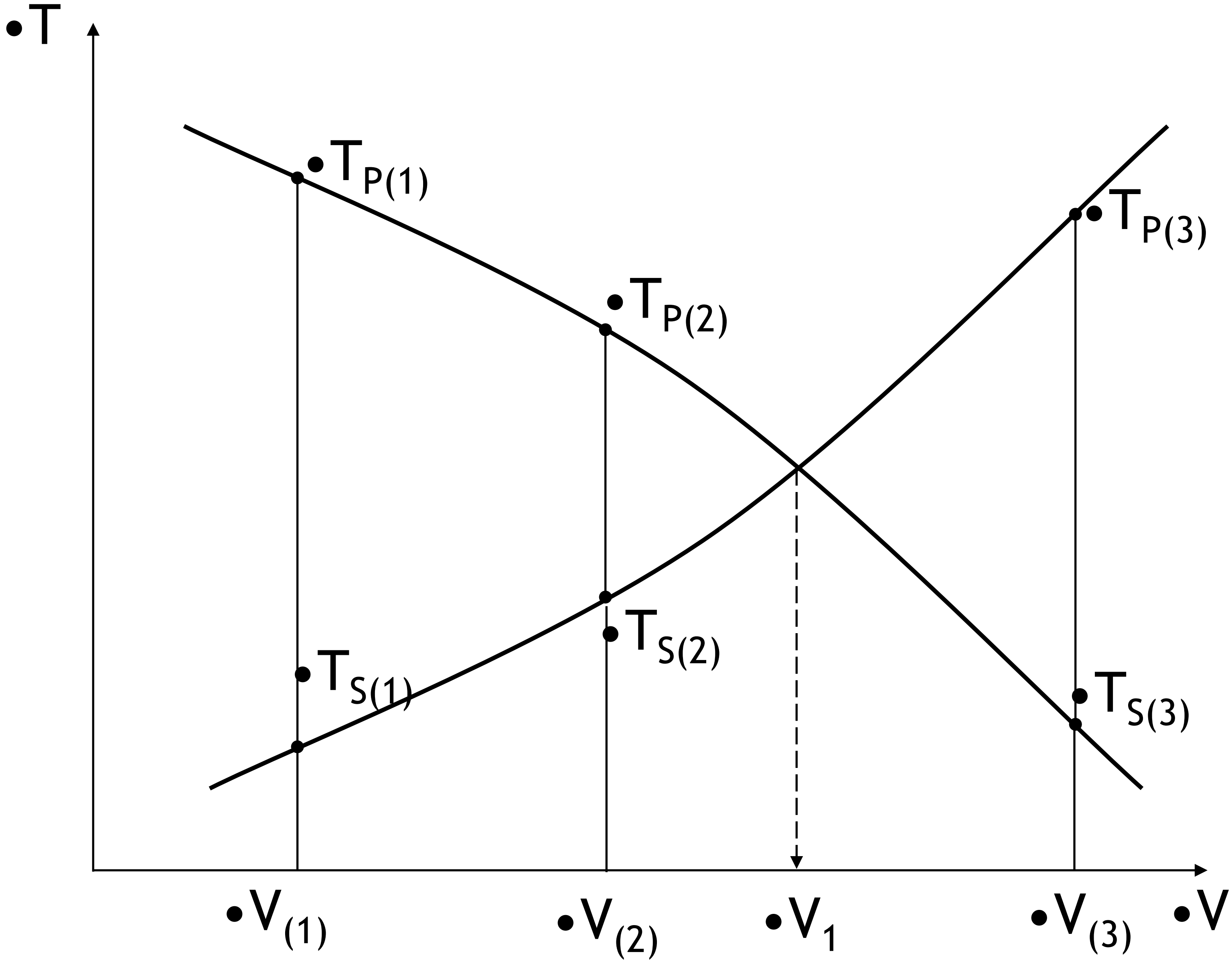
$$= \rho \cdot n^2 \cdot D_P{}^4 \cdot K_T = 1.025 \times (220/60)^2 \times 3.5396^4 \times 0.1428 = 308.8956[kN]$$

(선박이 요구하는 추력) < (프로펠러가 낼 수 있는 추력)

즉, 더 큰 속력을 낼 수 있음 -> 가정한 것 보다 더 큰 속력으로 가정하고 3~6의 과정 반복함

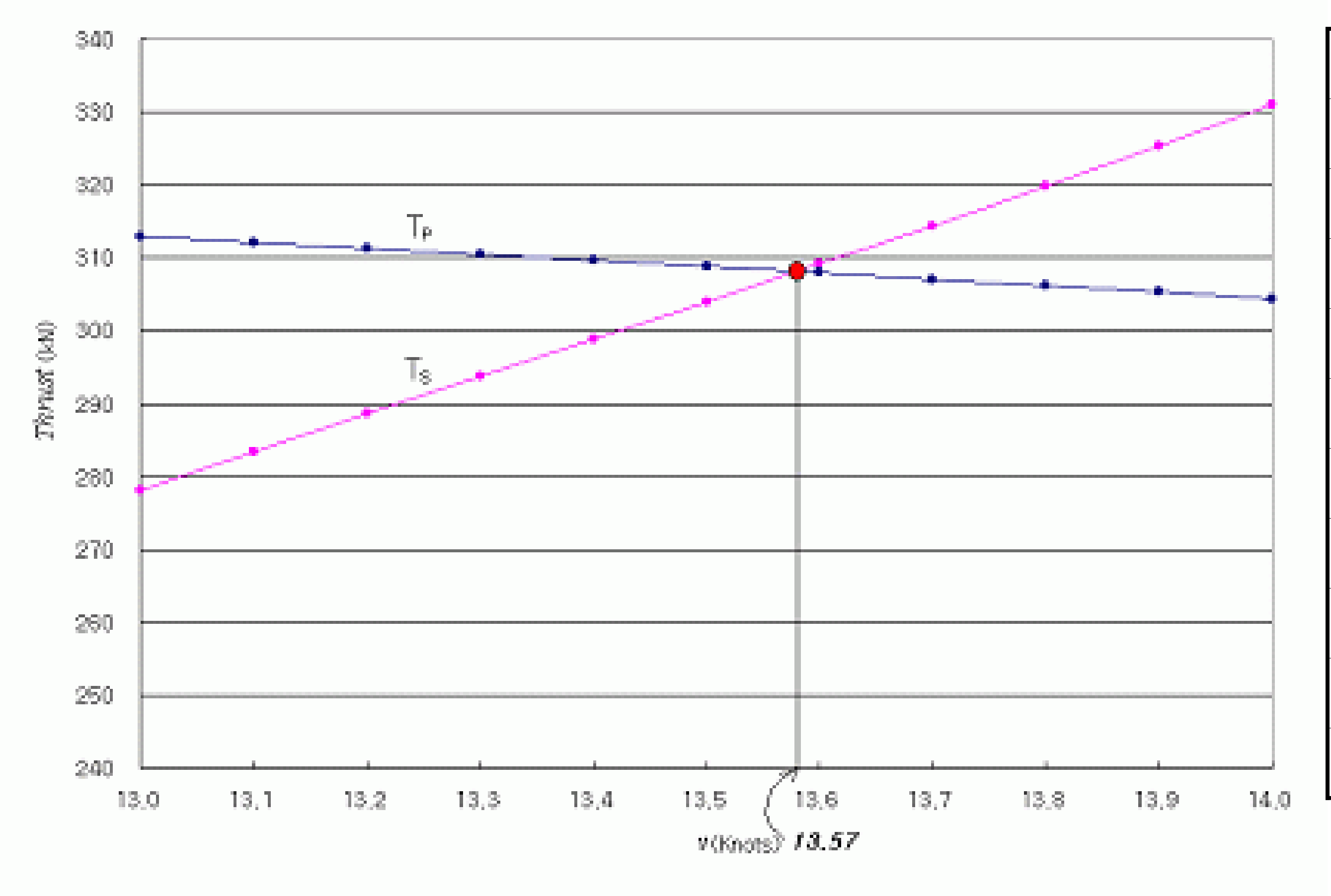
프로펠러 최적 주요치수 결정 예제

- 속력 v 결정을 위한 “추력 동일” 비선형 방정식($T_S = T_P$)의 해를 그래프를 이용하여 구하는 개념



프로펠러 최적 주요치수 결정 예제

- 전개 면적비 (A_E / A_o) 가 0.55일 때, 프로펠러 주요 요목



A_E/A_O	0.55
v (Knots)	13.57
w	0.3788
v_A (Knots)	8.4289
J	0.3339
η_O	0.4727
D_P (m)	3.5416
Pi/D_P	0.60
T_P (kN)	308.1892
T_S (kN)	307.6054
$(T_P - T_S)$	오차 = 0.5838

프로펠러 최적 주요치수 결정 예제

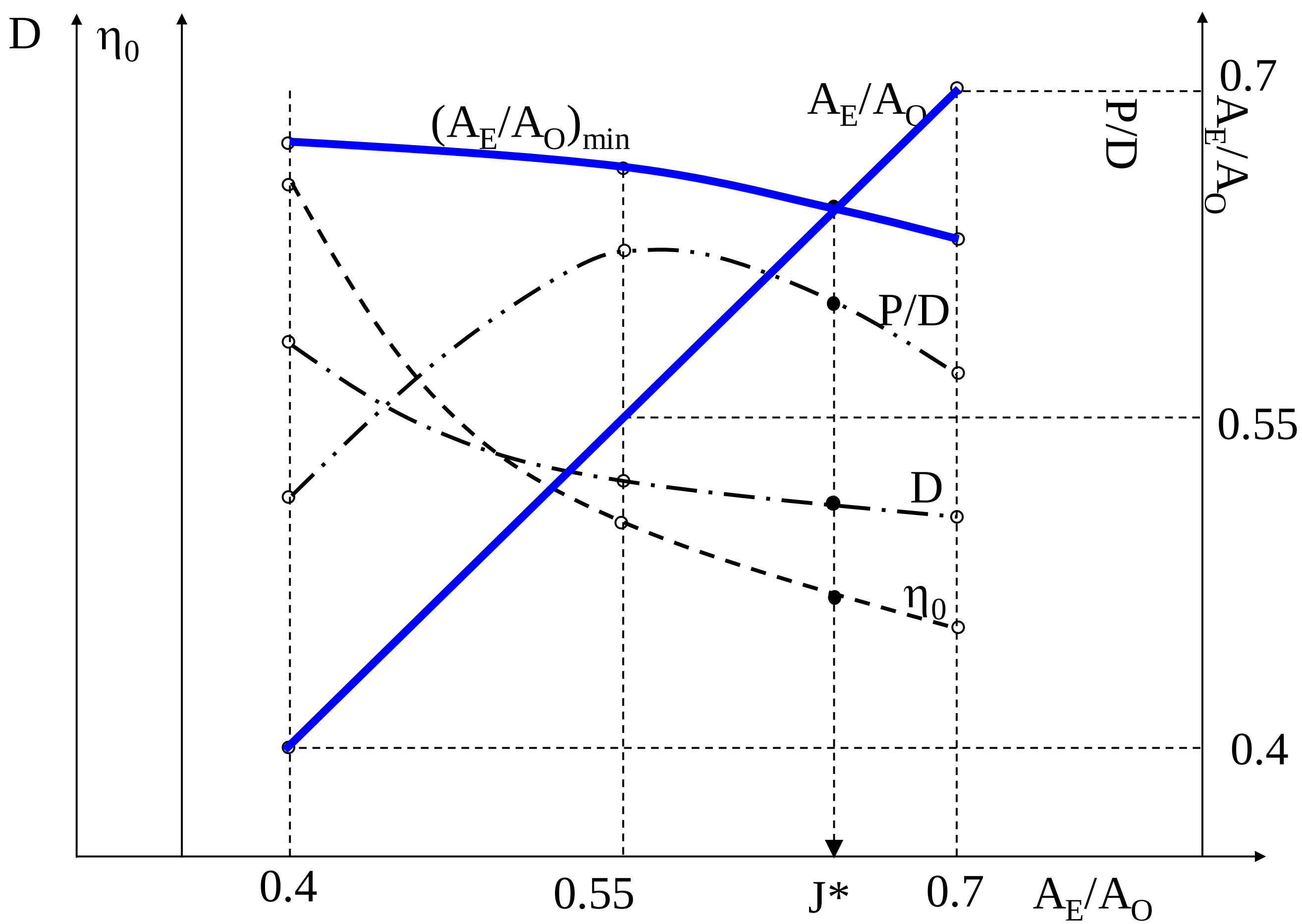
7 | 전개 면적비 (A_E / A_o)가 요구 전개면적비를 만족하는가?

$$A_E / A_O \geq K + \frac{(1.3 + 0.3z) \cdot T}{D_P{}^2 \cdot (p_0 + \rho g h^* - p_v)}$$

$$= 0.2 + \frac{(1.3 + 0.3 \times 4) \cdot 308.1892}{3.5416^2 \times (99.047 + 1.025 \times 9.81 \times 4.15} = 0.6363$$

새로운 전개 면적비를 가정하여 **2~7**의 과정을 반복함

프로펠러 최적 주요치수 결정 예제
- 최소 전개 면적비(A_E/A_O)의 검토



- Result

A_E/A_O	0.65
v (Knots)	13.48
w	0.3791
v_A (Knots)	8.3696
J	0.3329
η_O	0.4650
D_P (m)	3.5278
Pi/D_P	0.65
K_T	0.1411
K_Q	0.0161
T_P (kN)	301.1685
T_S (kN)	302.9741
$(T_P - T_S)$	-1.8056
A_E/A_O (Keller)	0.6401

Chapter 8. General Arrangement(G/A) Design

Arrangement Design (배치설계)

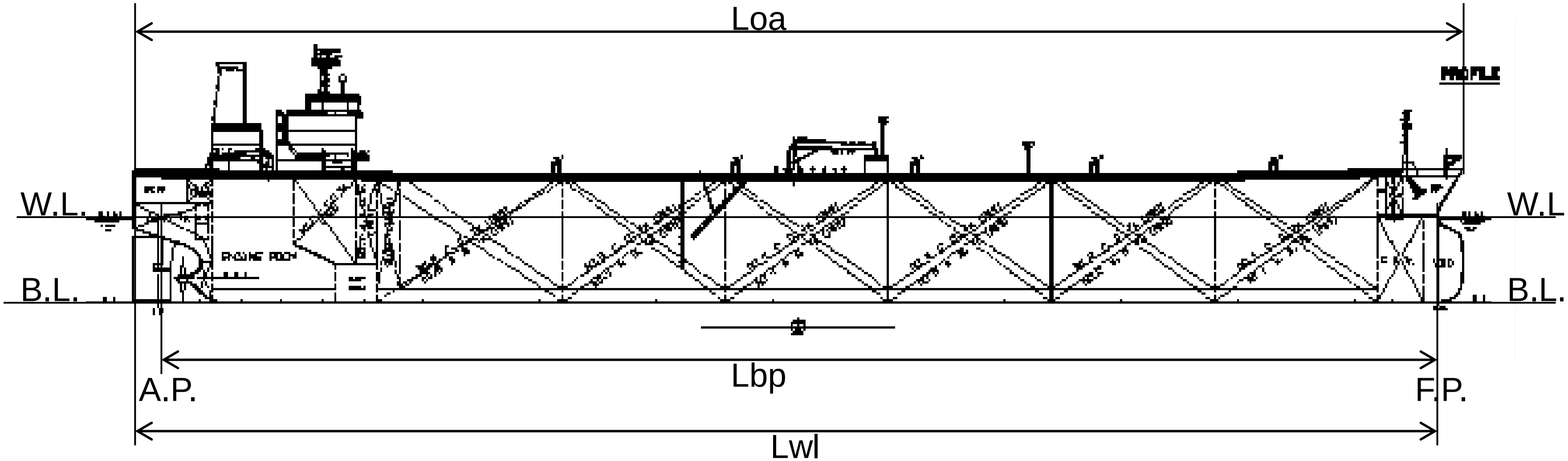
‘Design’ is a kind of ‘Arrangement’.

Arrangement design of a ship includes

- Compartment arrangement
- Equipment and piping arrangement
- Structural member arrangement

8-1 Main terminology

Main terminology

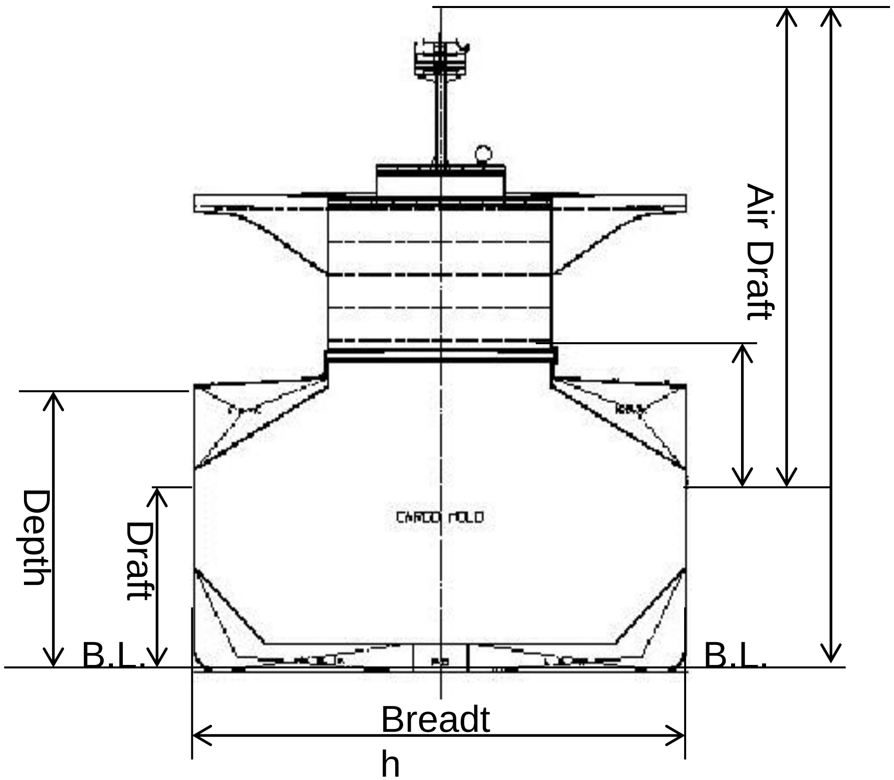


☑ Loa (Length OverAll) (m) : Maximum Length of Ship

☑ Lbp (Length Between Perpendiculars (A.P. ~ F.P.)) (m)
- A.P. : After Perpendicular (Normally, Center line of the Rudder Stock)
- F.P. : Inter-section line between Designed draft and fore side of the Stem, which is perpendicular to the baseline

☑ Lf (Freeboard Length) (m): Basis of Freeboard assignment, Damage Stability Calculation
- 96% of Lwl at 0.85 D or Lbp at 0.85 D, whichever is greater

☑ Rule Length (Scantling Length) (m): Basis of Structural Design and Equipment selection
- Intermediate one among (0.96 Lwl at Ts, 0.97 Lwl at Ts, Lbp at Ts)



- **B (Breadth) (m) :Maximum breadth of the ship, measured amidships**
  - **- B,molded: excluding shell plate thickness**
  - **- B,extreme: including shell plate thickness**
- **D (Depth) (m):Distance from the baseline to the deck side line**
  - **- D,molded: excluding keel plate thickness**
  - **- D,extreme: including keel plate thickness**
- **Td (Designed Draft) (m): Main operating Draft.**
  - **- In general, basis of Ship's Deadweight and Speed/Power performance**
- **Ts (Scantling Draft) (m): Basis of Ship's Structural Design**

- **Air Draft**

Distance(height above waterline only or including operating draft, see below for the detail) restricted by the port facilities, navigating route, etc.

- Air draft from baseline to the top of the mast

- Air draft from waterline to the top of the mast

- Air draft from waterline to the top of hatch cover

- …

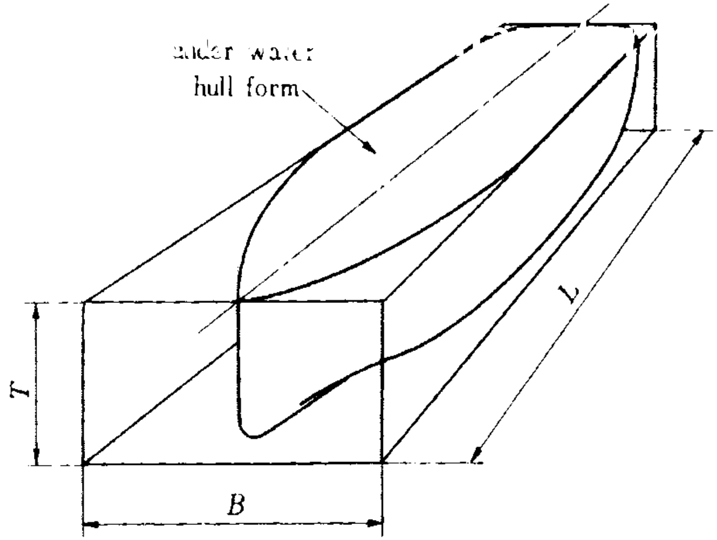
- ☑ Displacement (tonnes)
- ☑ Weight of water displaced by the Ship's submerged part

- ☑ Deadweight (tonnes): Cargo payload + Consumables (F.O., D.O., L.O., F.W., etc.) + DWT Constant
  : Displacement - Lightweight

- ☑ Cargo Payload (tonnes): Weight of loaded cargo at the loaded draft

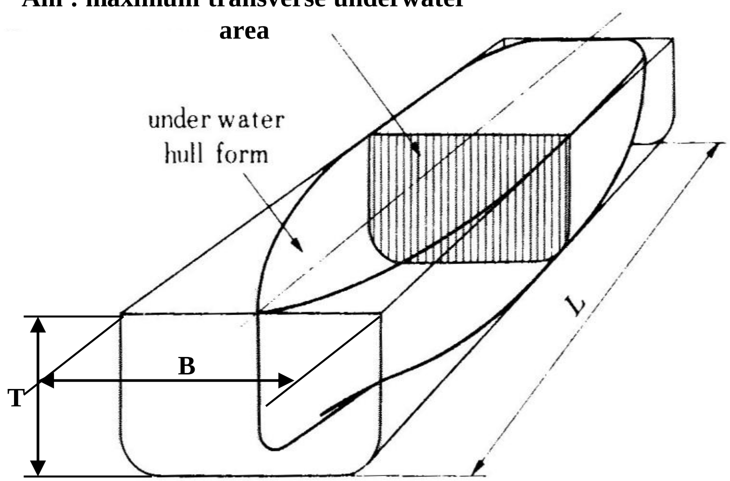
- ☑ DWT Constant (tonnes): Operational Liquid in the machinery and pipes, Provisions for crew, etc.

- ☑ LWT (tonnes): Total of hull steel weight and weight of equipment on board
- ☑ Trim: difference between draft at A.P. and F.P.
- ☑ Trim = {Displacement x (LCB - LCG)} / (MTC x 100)
- ☑ LCB : Longitudinal Center of Buoyancy
- ☑ LCG : Longitudinal Center of Gravity

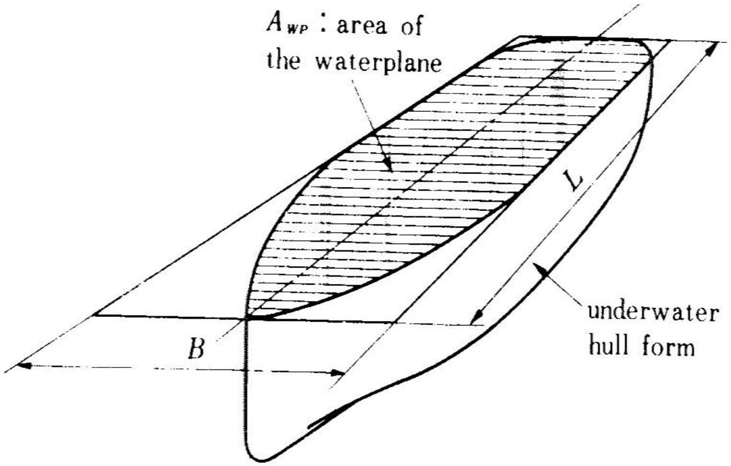


- **Cb (Block Coefficient)**
  = Displacement / (L x B x T x Density]
  , Density of Sea Water = 1.025 [Mg/m3]

Am : maximum transverse underwater area



- **Cm (Midship Section Coefficient)**
  = (Am) / (B x T)
- **Cp (Prismatic Coefficient)**
  = Displacement / {(Am) x L}



- **Cw (Water Plane Area Coefficient)**
  = Awp / (L x B)

- MCR (Maximum Continuous Rating) (PS x rpm)
  - NMCR (Nominal MCR)
  - DMCR (Derated MCR) / SMCR (Selected MCR)

- NCR (Normal Continuous Rating) (PS x rpm)

- Trial Power (PS x rpm) : Required power without Sea Margin at the service speed

- Sea Margin (%) : Power reserve for the influence of storm seas and wind including the effects of fouling and corrosion.

- Service Speed (knots) : Speed at NCR Power with the specific sea margin.

- DHP : Delivered Horse Power
  - Power actually delivered to the propeller with some power loss in the stern tube bearing and in any shaft tunnel bearings between the stern tube and the site of the torsion-meter

- EHP : Effective Horse Power
  - Required power to maintain intended speed of the ship

- η_D : quasi-propulsive coefficient = EHP / DHP

- RPM margin
  - To provide a sufficient torque reserve whenever full power must be attained under unfavorable weather conditions
  - To compensate for the expected future drop in revolutions for constant-power operation

Tonnage(톤수)

- Tonnage: normally, 100 ft^3 = 1 ton

  – Basis of various fee and tax

  – GT (Gross Tonnage): Total sum of the Volumes of every enclosed space

  – NT (Net Tonnage): Total sum of the Volumes of every cargo space
  – GT, NT should be calculated in accordance with " IMO 1969 Tonnage Measurement Regulation"

  – CGT (Compensated Gross Tonnage)

  – Panama and Suez canal have their own tonnage regulations.

GT-CGT 환산 계수

<table>
<tr><th>船型(DWT)
船種</th><th>100–
4,000</th><th>4,000–
10,000</th><th colspan="2">10,000–
30,000</th><th>30,000–
50,000</th><th>50,000–
80,000</th><th>80,000–
160,000</th><th>160,000–
250,000</th><th>250,000
이상</th><th>비 고</th></tr>
<tr><td rowspan="2">Crude oil carrier</td><td>1.70</td><td>1.15</td><td colspan="2">0.75</td><td>0.60</td><td>0.50</td><td>0.40</td><td>0.30</td><td>0.25</td><td>Single hull tanker</td></tr>
<tr><td>1.85</td><td>1.30</td><td colspan="2">0.85</td><td>0.70</td><td>0.55</td><td>0.45</td><td>0.35</td><td>0.30</td><td>Double hull tanker</td></tr>
<tr><td>Product carriers & Chemical carriers</td><td>2.30</td><td>1.60</td><td colspan="2">1.05</td><td>0.80</td><td>0.60</td><td colspan="3">0.55</td><td>Black product carrier
White product carrier</td></tr>
<tr><td>Bulk C+arriers</td><td>1.60</td><td>1.10</td><td colspan="2">0.70</td><td>0.60</td><td>0.50</td><td>0.40</td><td colspan="2">0.30</td><td>Chip carrier, Lumber Carrier, Car/bulk, Bulk/container, Open bulk</td></tr>
<tr><td>Combined carriers</td><td>1.60</td><td>1.10</td><td colspan="2">0.90</td><td>0.75</td><td>0.60</td><td>0.50</td><td colspan="2">0.40</td><td>Ore/bulk/oil</td></tr>
<tr><td>General Cargo Ships</td><td>1.85</td><td>1.35</td><td colspan="2">1.00</td><td>0.75</td><td>0.60</td><td>0.50</td><td colspan="2">0.40</td><td>Semi–container,
Multi–purpose cargo</td></tr>
<tr><td>Reefers</td><td>2.05</td><td>1.50</td><td colspan="7">1.25</td><td></td></tr>
<tr><td>Full container ships</td><td>1.85</td><td>1.20</td><td>10,000–
20,000
0.90</td><td>20,000–
30,000
0.80</td><td>0.75</td><td colspan="4">0.65</td><td></td></tr>
<tr><td>Ro–Ro vessels</td><td>1.50</td><td>1.05</td><td>0.80</td><td>0.70</td><td colspan="5">0.65</td><td></td></tr>
<tr><td>Car carriers</td><td>1.10</td><td>0.75</td><td>0.65</td><td>0.55</td><td colspan="5">0.45</td><td>Ro–Ro/Container</td></tr>
<tr><td>L.P.G. carriers</td><td>2.05</td><td>1.60</td><td>1.15</td><td>0.90</td><td>0.80</td><td colspan="4">0.70</td><td></td></tr>
<tr><td>L.N.G. carriers</td><td>2.05</td><td>1.60</td><td>1.25</td><td>1.15</td><td>1.00</td><td colspan="4">0.75</td><td></td></tr>
</table>

<table>
<tr><th>船型(GT)
船種</th><th>100–
1,000</th><th>1,000–
3,000</th><th>3,000–
10,000</th><th>10,000–
20,000</th><th>20,000–
40,000</th><th>40,000–
60,000</th><th>60,000
이 상</th><th>비 고</th></tr>
<tr><td>Ferries</td><td>3.00</td><td>2.25</td><td>1.65</td><td>1.15</td><td colspan="3">0.90</td><td></td></tr>
<tr><td>Passenger ships</td><td>6.00</td><td>4.00</td><td>3.00</td><td>2.00</td><td>1.60</td><td>1.40</td><td>1.25</td><td></td></tr>
<tr><td>Fishing vessels</td><td>4.00</td><td>3.00</td><td colspan="5">2.00</td><td>Fishing vessel & Fish factory ship</td></tr>
<tr><td>Other non–cargo vessels</td><td>5.00</td><td>3.20</td><td>2.00</td><td colspan="4">1.50</td><td>Tug & Supply vessel, Dredger, Ice breaker, Cable layer, Research ship, etc</td></tr>
</table>

註 : 100GT 이상 선박

Unit

- Unit
  - API (American Petroleum Institute) = (141.5 / S.G.) – 131.5
    - ex) API 40 → SG = 0.8251 tonnes/m3
    - cf] Actually, SG is density in SI system
  - LT (Long Ton) = 1.016 tonnes
  - SG (Specific Gravity) → tonnes/m3
  - SF (Stowage Factor) → ft3 / LT
    - ex) SF = 15 ft3 / LT → SG = 2.4 tonnes/m3
  - 1 knots = 1 nm/hr = 1.852 km/hr = 0.5144 m/sec
  - Barrel = 0.159 m3
    - ex) 1 mil. Barrels = 159,000 m3
  - PS = 0.7355 kW
    - ex) NMCR of B&W6S60MC : 12,240 kW = 16,680 PS

8-2 Concept of General Arrangement Design

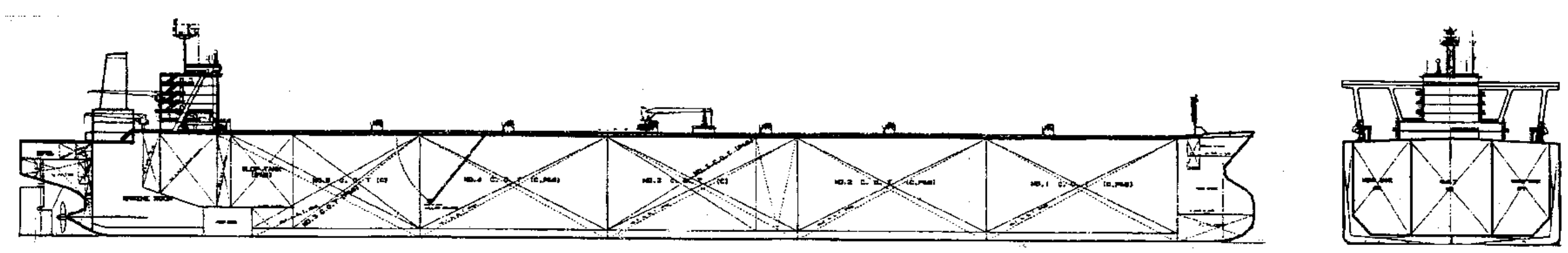
General Arrangement(일반배치)

☑ Sketch G/A(개략 일반 배치) : 선박의 구획과 탱크 배치.

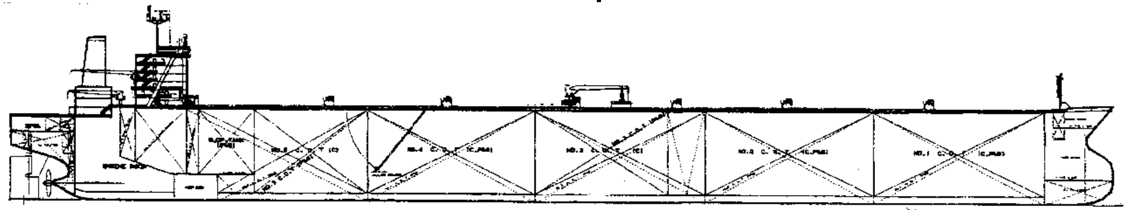
■Compartment arrangement(구획배치) : 화물창 및 탱크 용적을 주어진 여건 내에서 최대치를 얻는 것 ➔ 최적 구획배치 설계

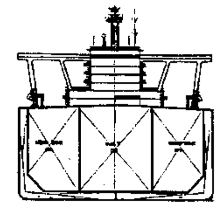
☑ Full General Arrangement(상세 일반배치):

선실배치, 하역장치, 계선장치, 교통장치 등의 상세 배치 포함

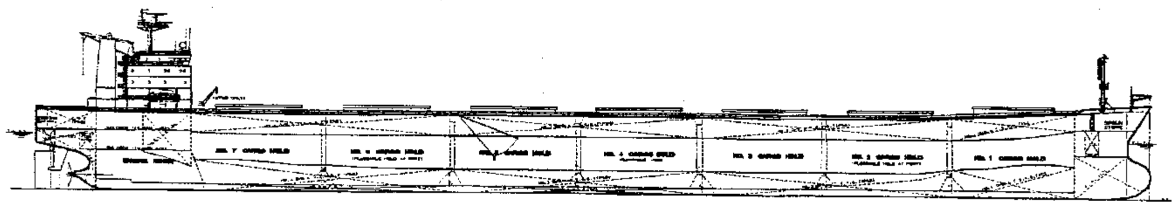


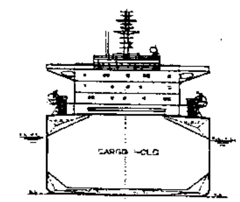
Tanker (VLCC) G/A(Profile, Midship Section)



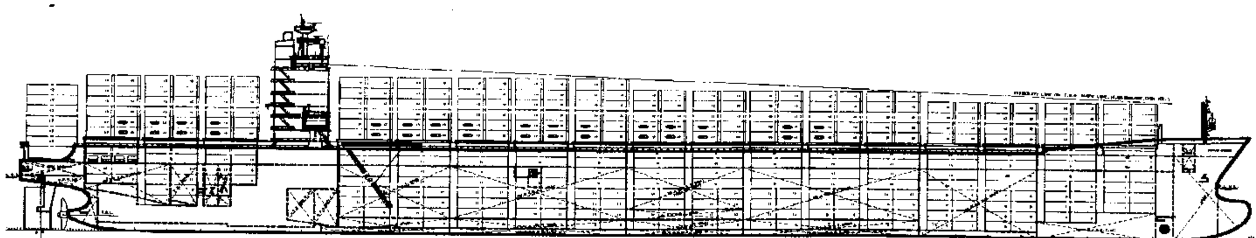


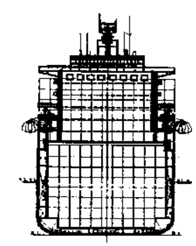
Bulker (Panamax) G/A(Profile, Midship Section)





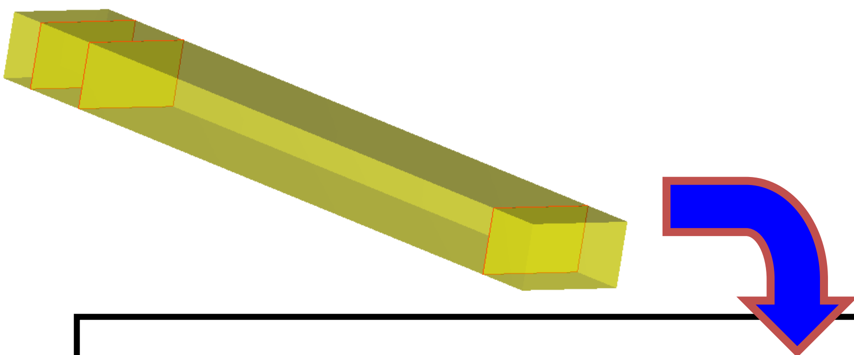
Container Carrier G/A(Profile, Midship Section)





Compartment Arrangement of Ship(선박 구획배치)

☑ 선박 구획배치는 LOA, B, D의 크기를 가지는 직육면체에 필요한 공간을 적절히 확보하는 과정



Concept of Compartment Arrangement

- 화물창 공간을 최대로 할 것 ➡ "선주 이익의 지표"
- Tanker 구획 배치 경우, MARPOL 규약에 따른 탱크 용량 및 배치, SBT(Segregated Ballast Tank), PL(Protective Location), 이중 선체 유조선의 이중저 높이 및 선측 탱크 폭 등에 대한 조건을 만족하면서 화물창 용적을 최대로 하는 배치
- 화물창 구획의 지원 기능들(기관실 구획, 거주구 구획, 연료유 구획, 밸러스트 탱크 구획)은 최소로 할 것 ➡ 기관실 길이 및 폭 최소
- 화물창 횡단면적(Cargo Hold Sectional Area)이 최대가 되도록 할 것 ➡ 중앙 횡단면(Midship Section), Double Bottom Height, FPT 길이 등 Rules & Regulation 만족 여부 검토
- 호퍼탱크 및 윙탱크의 적절한 배치
- Frame / Web / Longi. 간격 고려
- 계선 장치(anchoring), 계류 장치(mooring), 타(rudder) 등 고려
- 저항/ 추진, 조종성, 복원성, 진동 등을 동시 고려한 선형 선정

8-3 Cargo Hold Compartment Arrangement Design

☑ 화물창 구역의 구획 배치(Cargo Hold Compartment Arrangement Design)

- 수밀격벽
- 프레임 간격
- 이중저 높이
- Tanker 화물창 구획 배치
- Container Carrier 화물창 구획 배치
- Bulk Carrier 화물창 구획 배치
- Cofferdam 설치 기준

1. 수밀격벽 (Watertight bulkhead)

☑화물창의 개수, 길이 결정 요소

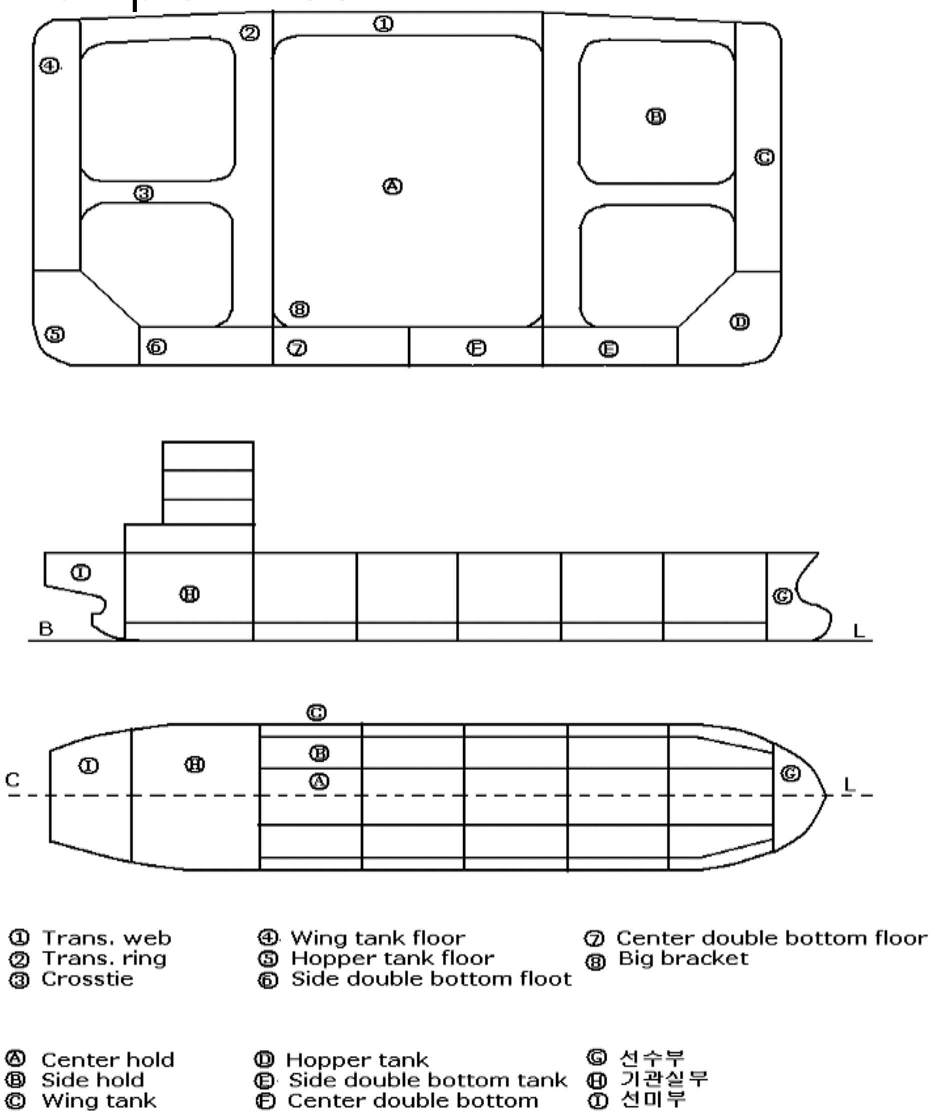
- 선박길이
- 손상 시 복원성
- 구조 강도

☑수밀격벽

- 화물창은 수밀격벽에 의하여 여러 개의 화물창으로 나누어진다.
- 수밀격벽(水密隔壁) : 수압을 가해도 물이 새지 않는 칸막이 벽.
- 선내에서 발생한 재해를 일부분에 국한시킴.
- 각 선급에서 규정

Compartment Arrangement
by watertight transverse & longitudinal bulkheads

Example: VLCC



- 주로 선박의 기본 성능에 큰 영향을 미치는 **구획을 배치하고 관련 규정을 검토**
- 화물창, 기관실, 선수창(FPT), 선미창(APT), 각종 탱크
- 화물창 용적 증감에 따른 조정

선급의 횡격벽 갯수 결정 기준

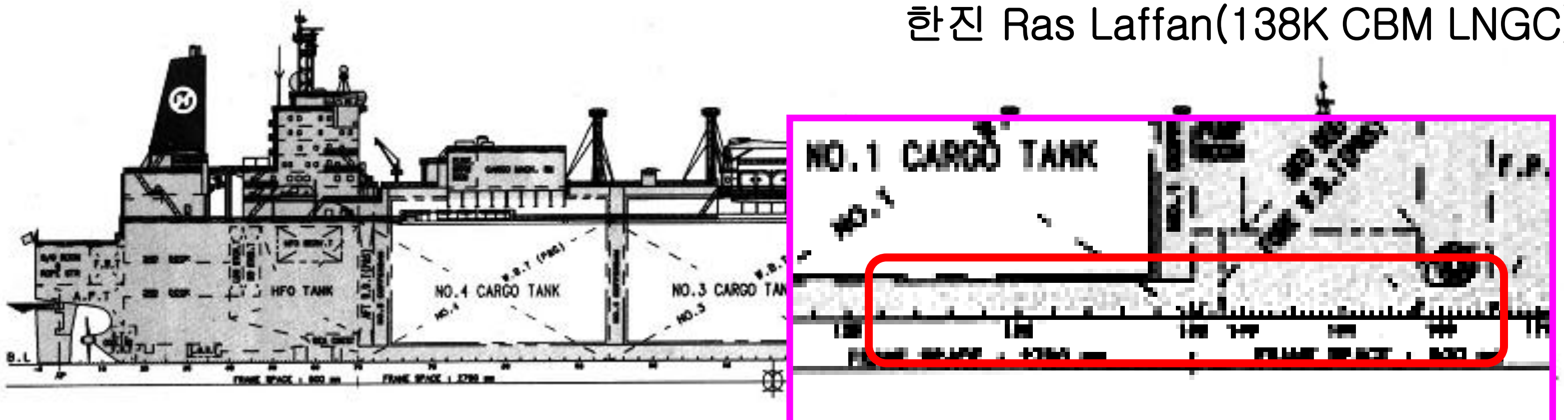
SHIP LENGTH (M)	ABS (HOLD BHD 수)		LR		DNV		BV	GL	KR	SHIP LENGTH (M)
	E/R AMID	AFT	E/R AMID	AFT	E/R AMID	AFT				
65			4	3	4	3	4	3		65
66			4	4	4	4		4		66
67 85									4	67 85
86			5	5	5	4				86
87 88 89 90 91	1	2					5	4 + 1/20 m	5	87 88 89 90 91
101	2	3					6		6	101
102 103 104 105										102 103 104 105
106 115			6	5	6	5				106 115
116 122			6	6						116 122
123 124 125							7		7	123 124 125
126 142			7	6	7	6				126 142
143										143
144 145							8		8	144 145
146 164			8	7	8	7				146 164
165							9		9	165
166 185			9	8	9	8				166 185
186 190									As determined by the society in each case	186 190
191 197					10	9				191 197
198 225	3	4								198 225
226										226

2. Frame spacing(늑골 간격)

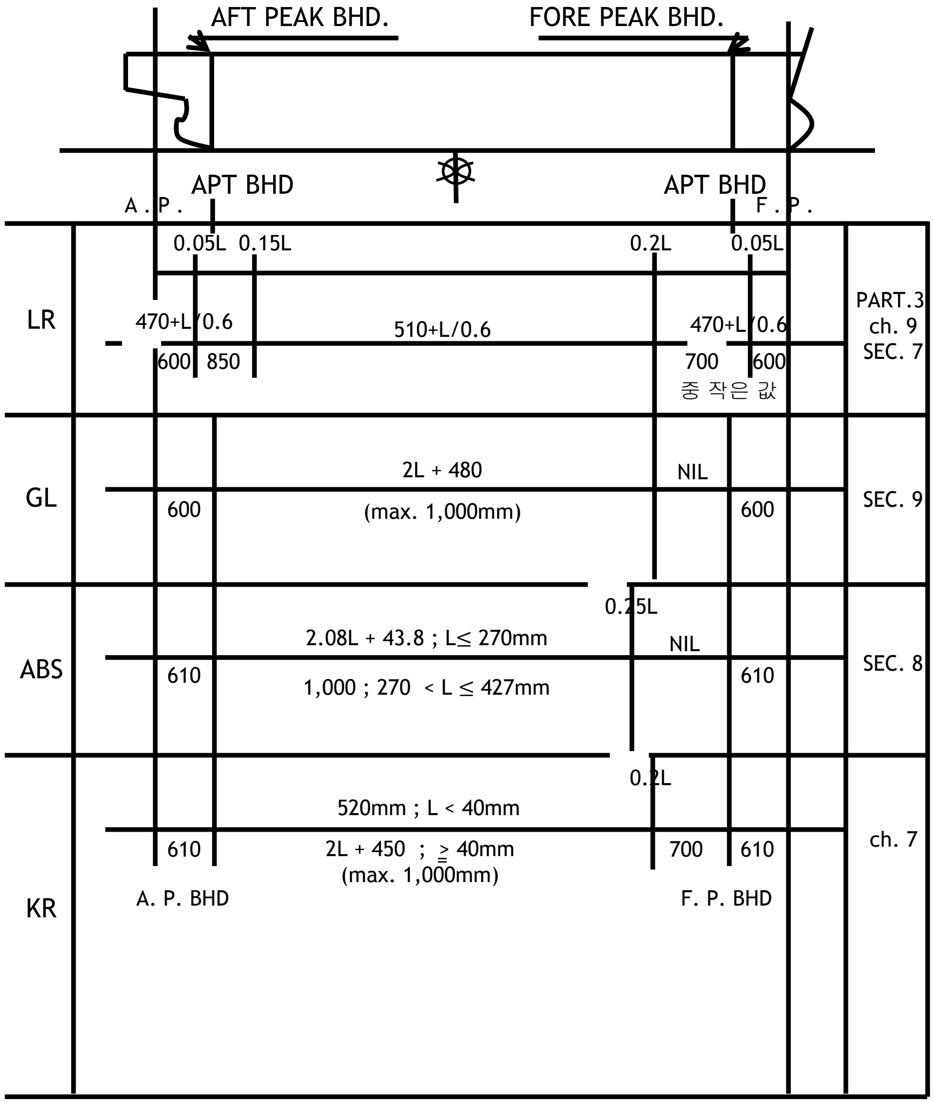
☑ 프레임 간격의 결정

- 선급규칙에서 규정하는 표준 프레임 간격
- 이중저의 늑판 배치
- 톱 사이드 및 갑판부의 트랜스버스 배치 등을 고려
- 가능한 한 등 간격으로 배치
- 화물창당 몇 개의 적정 프레임 개수를 결정할 것인가는 선체의 구조와 강도, lower stool 크기, 현장 작업성을 고려하여야 함.

한진 Ras Laffan(138K CBM LNGC)



각 선급에서 제안하는 프레임 간격



- Standard LONGITUDINAL FRAME SPACING ;
  2L + 550mm

3. 이중저 높이 (Double bottom height)

☑이중저 높이의 결정

- ■ 선체 강도
- ■ 화물창 용적
- ■ 밸러스트 용적
- ■ 현장 작업성

  이중저 탱크 내에서 족장설치 없이 작업 가능한 높이도 고려

8-4 Fore Part Compartment Arrangement Design

☑ 선수부 구획 배치(Fore Part Compartment Arrangement Design)

- General
- Collision Bulkhead(충돌 격벽)
- F.P.T. (Fore Peak Tank)
- F'cle (Fore Castle)Deck
- Bosun's Store
- Bulwark

개요

☑선수부 구획배치의 주요 고려사항

- 충돌 격벽(Collision Bulkhead)이 우선적으로 고려
- F.P.T.(Fore Peak Tank)의 용량
- 선수 계선(mooring)

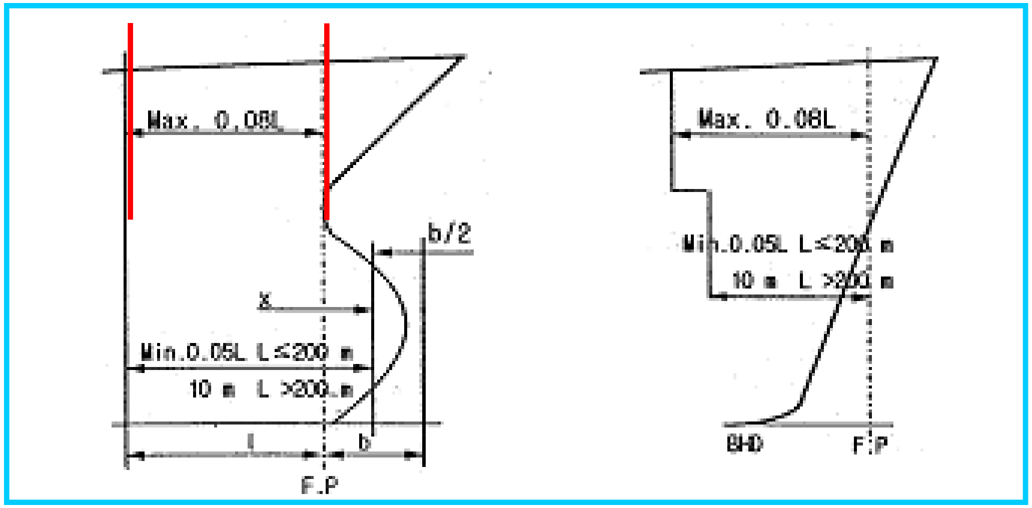
☑선수부의 프레임(Frame) 간격

- 일반적으로 선미부 및 기관실의 프레임 간격과 같게 적용

Collision Bulkhead

☑ F.P.T.와 화물유 탱크 사이의 선수 격벽

- 선급에서 최소거리/최대거리를 요구
- 화물창 용적을 크게 → 최소거리
- 선수 계선, anchor chain 적재 등을 고려
- 선수트림이 과도한 경우 선수트림을 줄이기 위해 선수부 구획길이를 길게 하는 경우도 있다.



☑설계 초기 단계의 경우 다음의 표를 이용하여 결정가능

Ship Type	LBP ≥ 250	LBP ≤ 250	Remark
Bulker	0.03 L + 3.0	0.02 L + 5.5	L : Rule Length
Tanker	0.03 L + 3.5	0.02 L + 6.0	
Container	0.03 L + 4.0	0.02 L + 6.5	

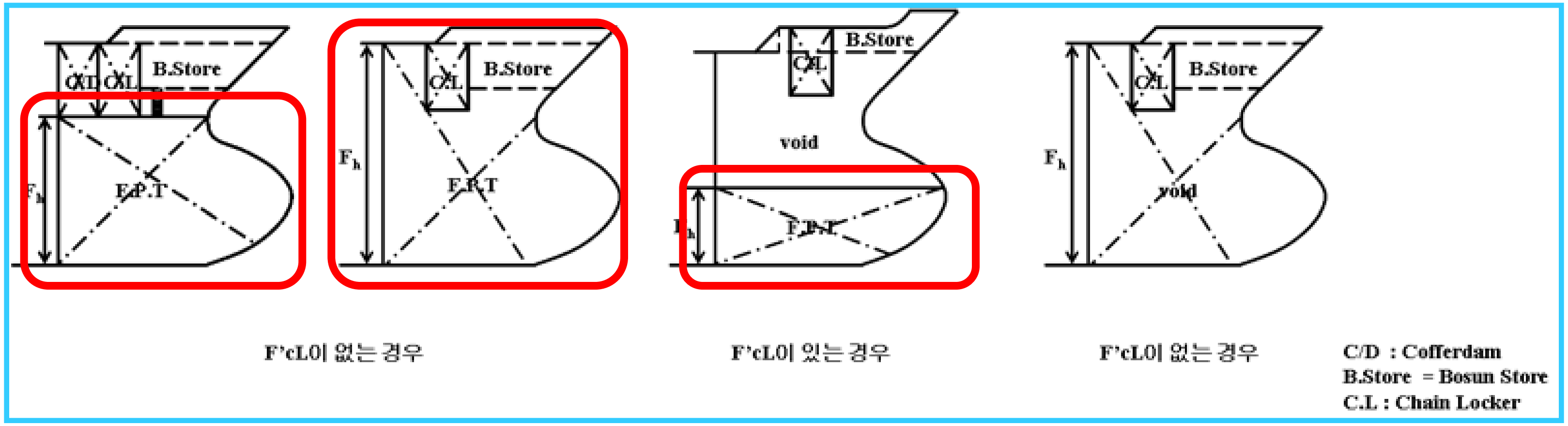
■ 실적선의 충돌격벽 위치

Ship Type	Pax Cont	Pax B/C	Aframax	Suezmax	VLCC
Coll. BHD～F.P	11.8	9.7	10.12	12.92	13.0

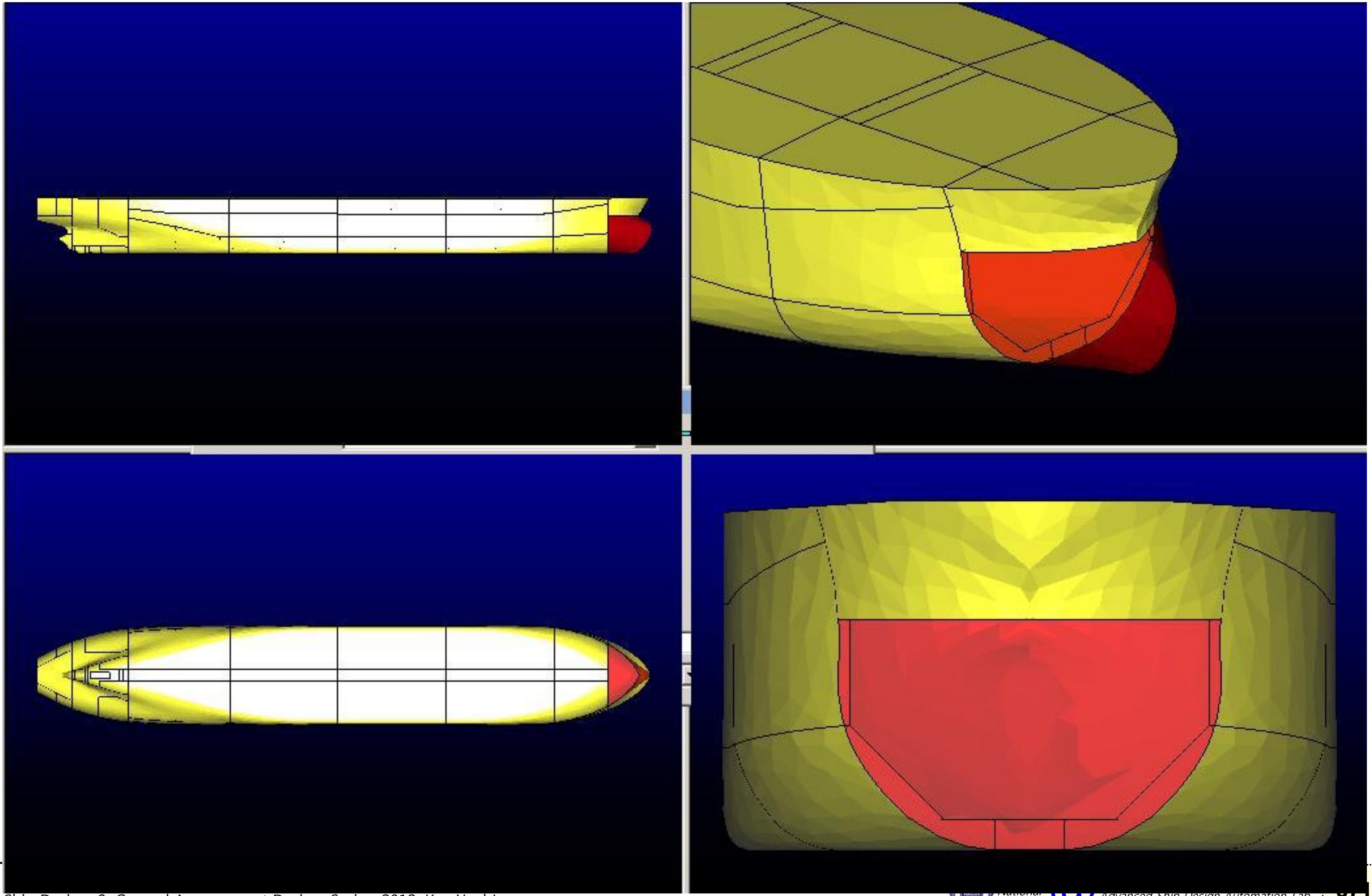
F.P.T. (Fore Peak Tank)

☑ F.P.T.의 용적은 Loading이 허용하는 한 최소로 하는 것이 유리하다.

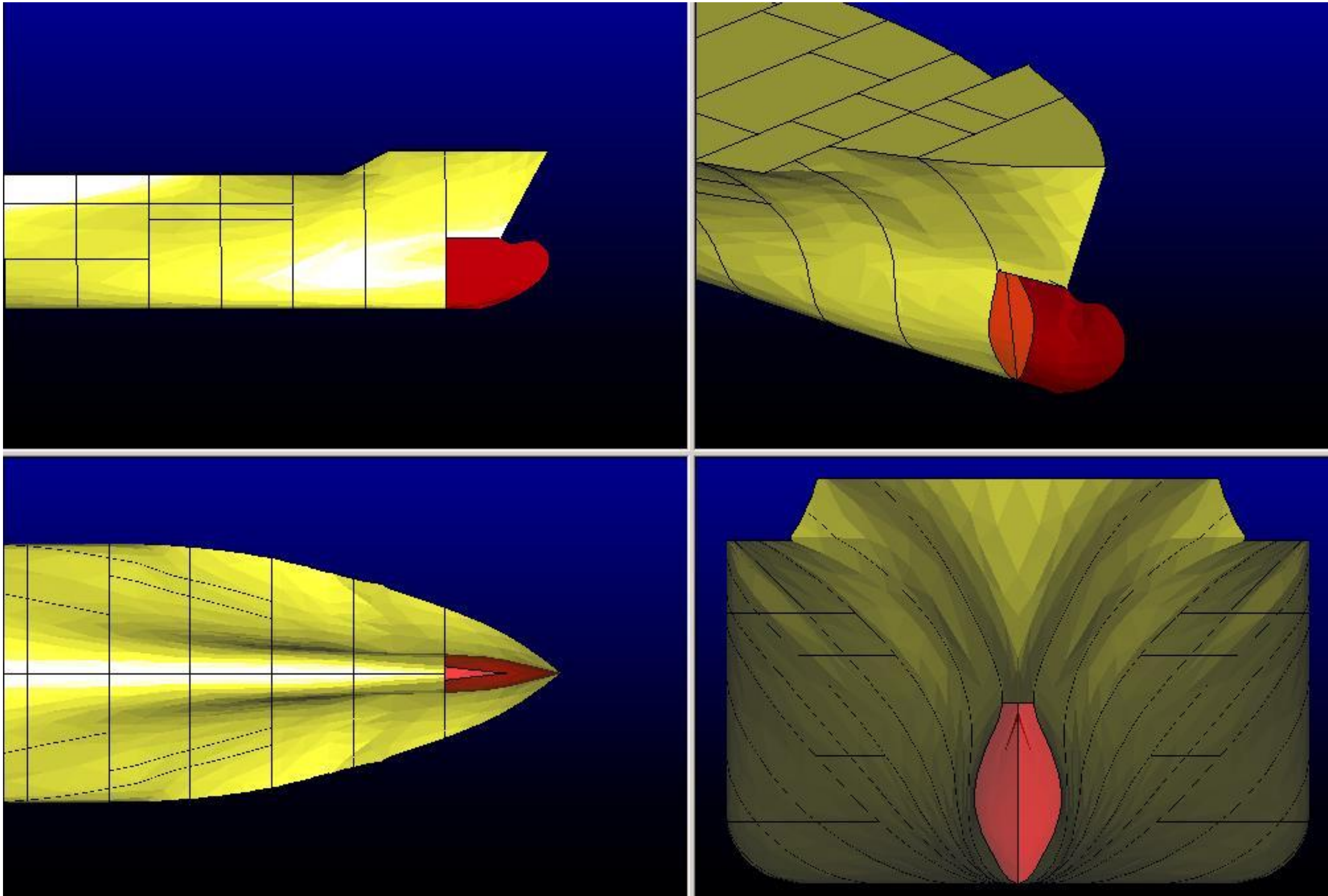
- 가능한 한 낮게 하는 것이 부재 최적화 측면에서 유리하고, 아울러 페인트 물량도 적어진다.



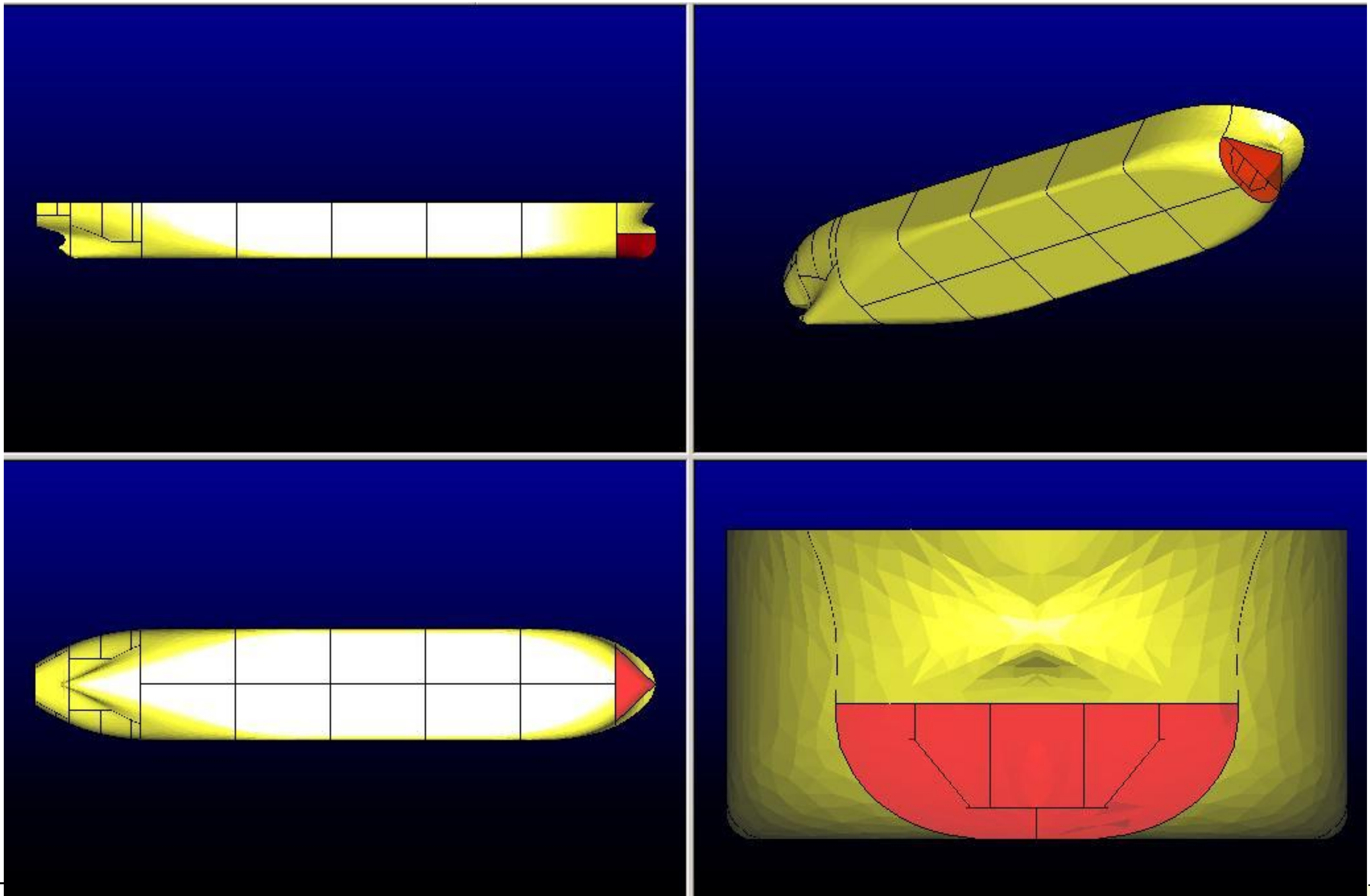
182K Bulk Carrier의 Fore Peak Tank



9000TEU Container Carrier의 Fore Peak Tank

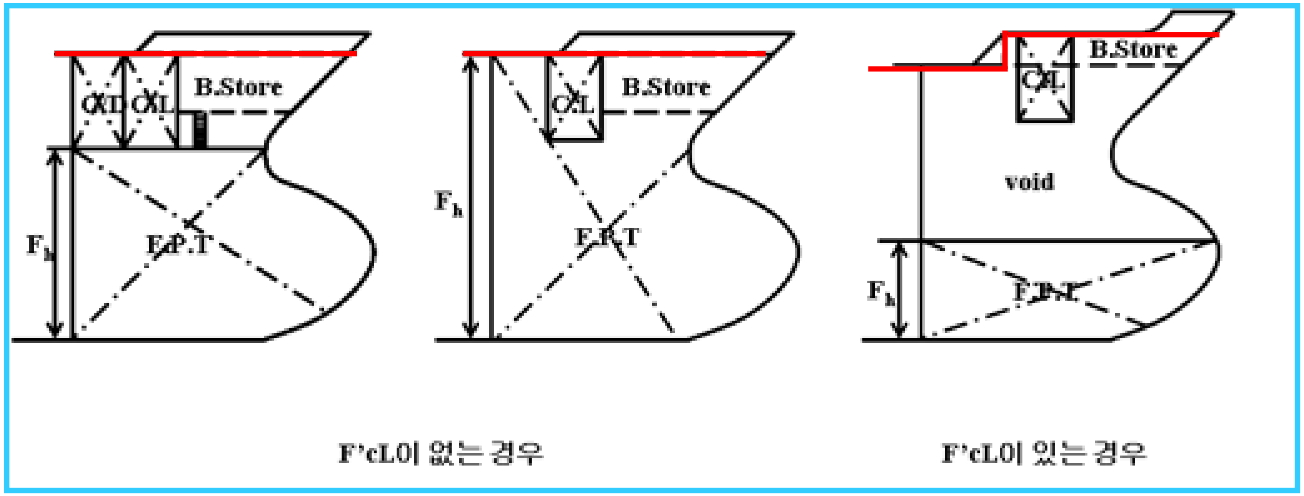


320K VLCC의 Fore Peak Tank



F'cle (Fore Castle)Deck

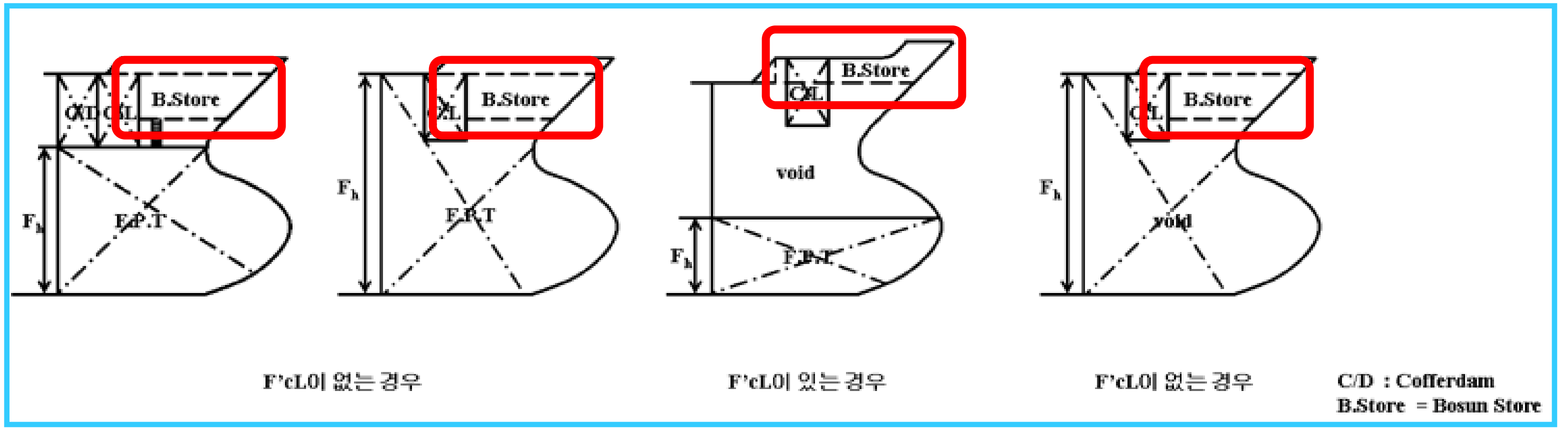
- **선수루(F'cle deck) 길이에 대한 건현 규정**
  - ✖ $f'cle\ length \geq 0.07L_f$ **(Lf : Load line length)**
  - ✖ **높이는 건현 규정상 125m 이상의 선박에서는 2.3m 이나, 통상 3.0m로 설계한다.**



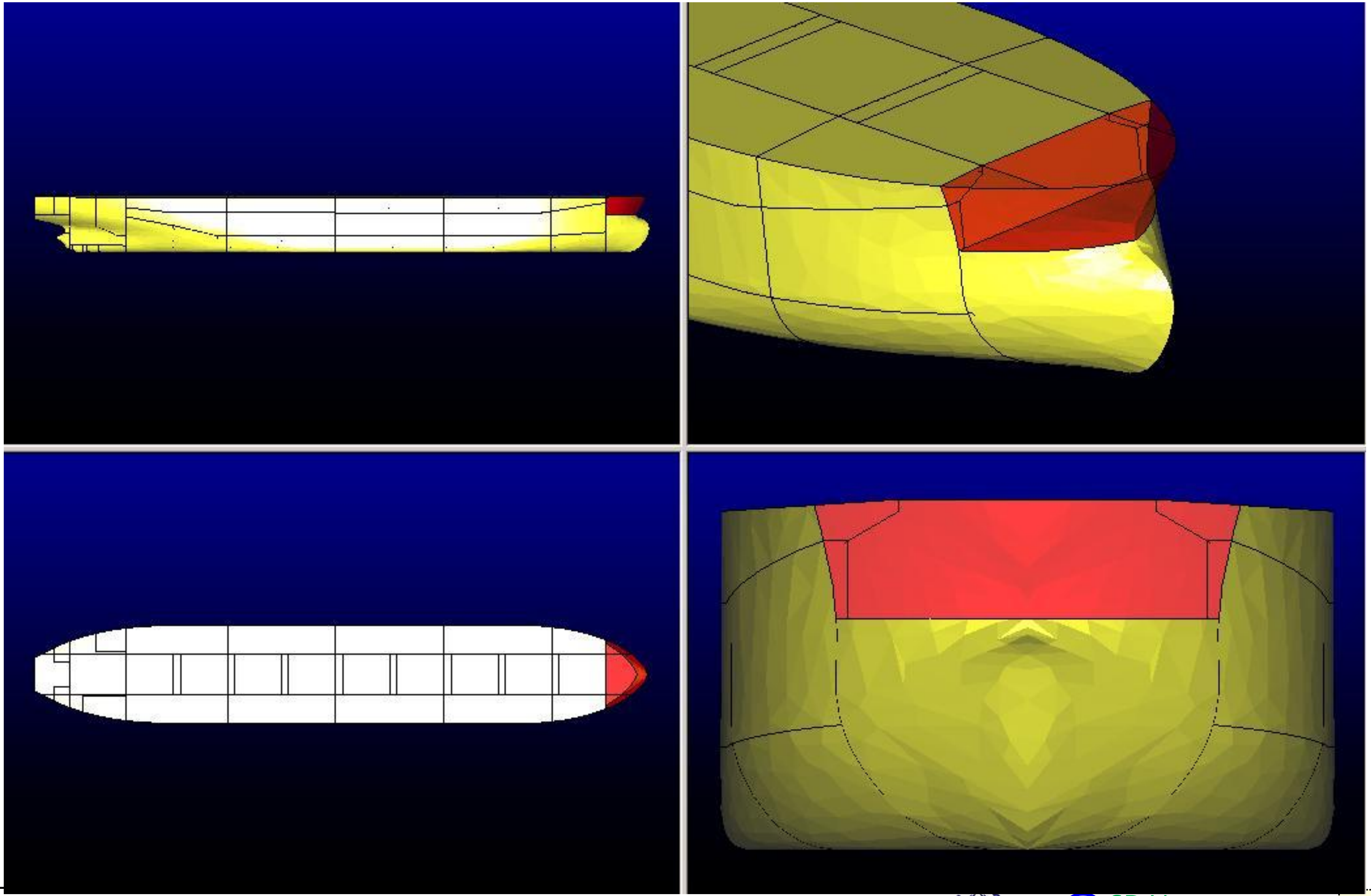
Bosun's Store

☑ **Bosun's Store : 선수부 창고, 갑판 창고**

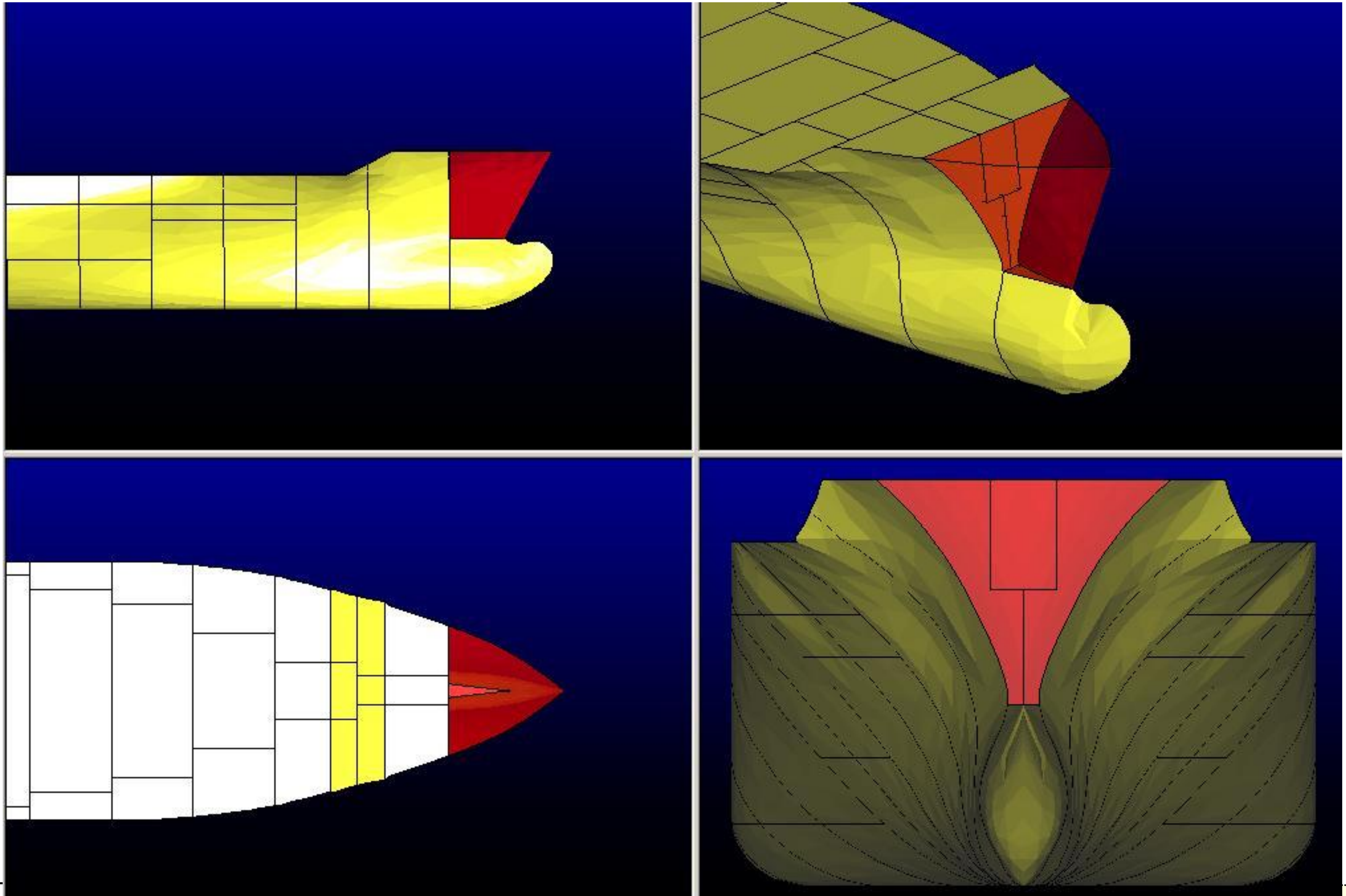
- F'cle을 가지는 선박은 F'cle에 설치
- F'cle을 가지지 않는 선박은 Upper Deck하부에 설치
- 통로는 좌현(port)에 배치하여 Mooring에 방해되지 않도록 한다.



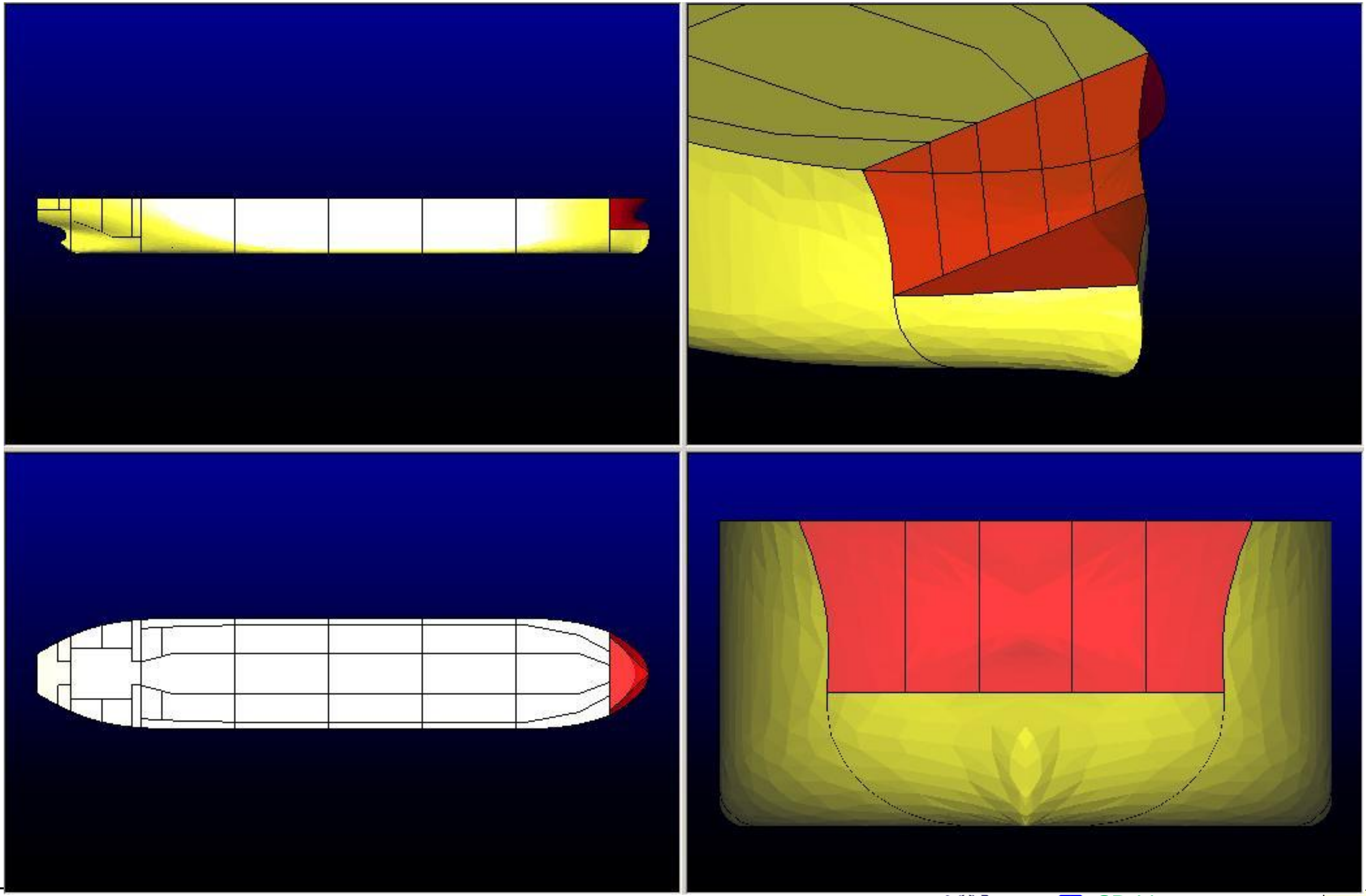
182K Bulk Carrier의 Bosun's Store



9000TEU Container Carrier의 Bosun's Store 및 Void Space

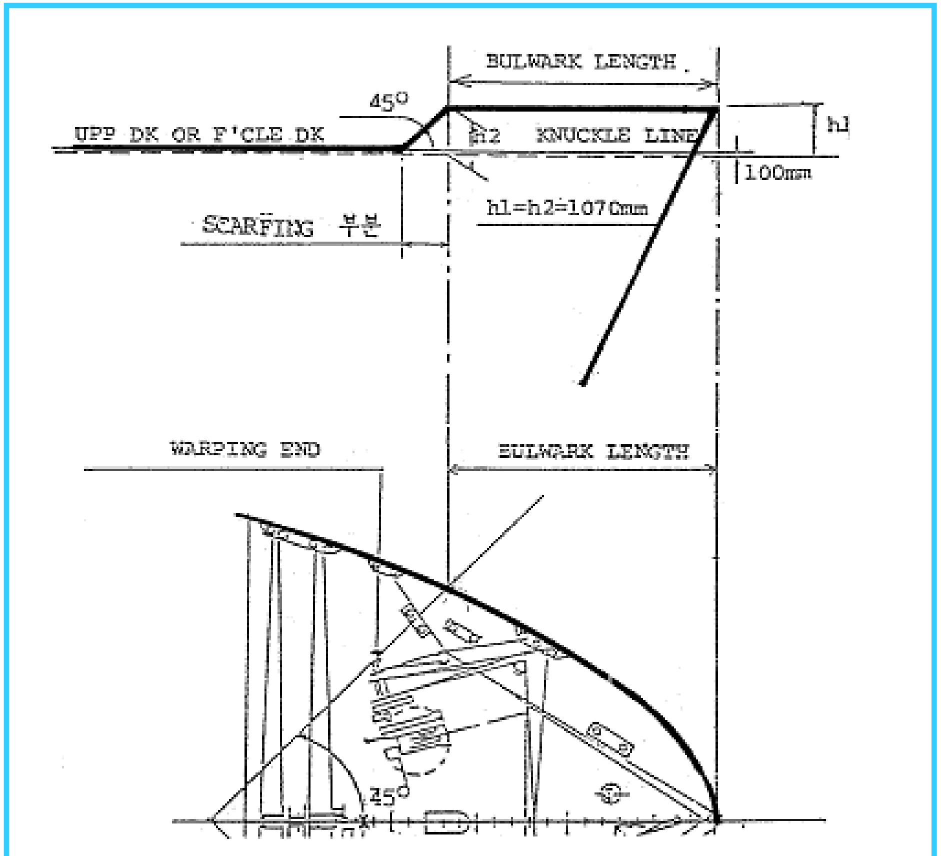


320K VLCC의 Bosun's Store 및 Void Space



Bulwark

☑Bulwark : 일종의 방파제, 선수 갑판 위의 장비 보호



- Windlass Warping End를 지나고 경사 부분은 45°가 되도록 한다.
- Bulwark의 높이는 1.1m로 한다.

8-5 Engine Room(E/R) Arrangement Design

☑ 기관실 배치(Engine Room Arrangement Design)

- 개요
- 기관실의 길이
- 주기관 설치 위치
- 기관실 내의 Hull Tank 배치
- 기관실의 높이
- 갑판 높이 결정 기준
- 각종 Room의 크기 결정
- Pump Room
- Deck House

개요

☑ 목표

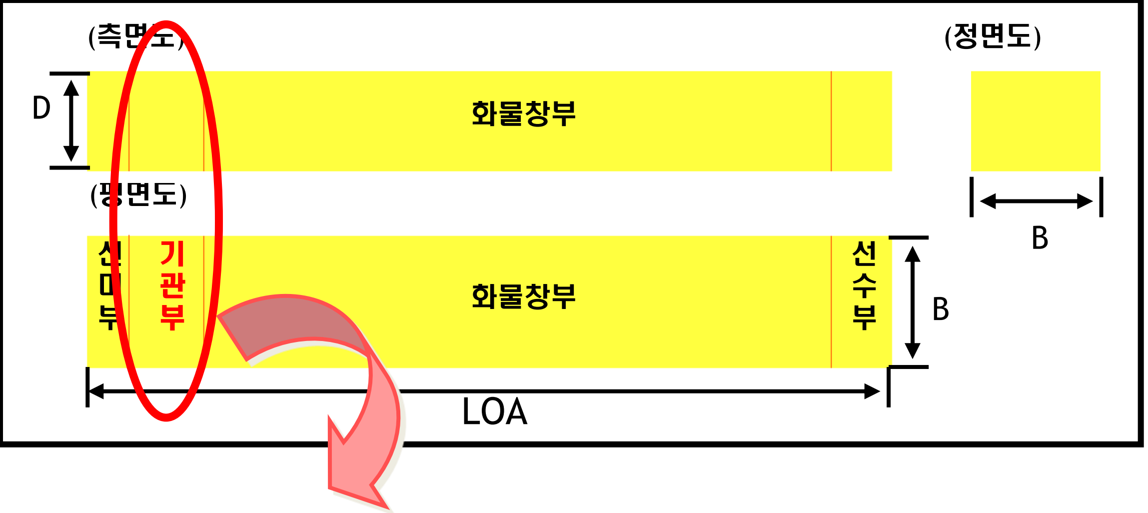
- 기관실 및 선실구획 등 비화물 적재구획은 **최소화**
- 화물 적재구획 **최대화**

☑ 기관실 배치와 선형

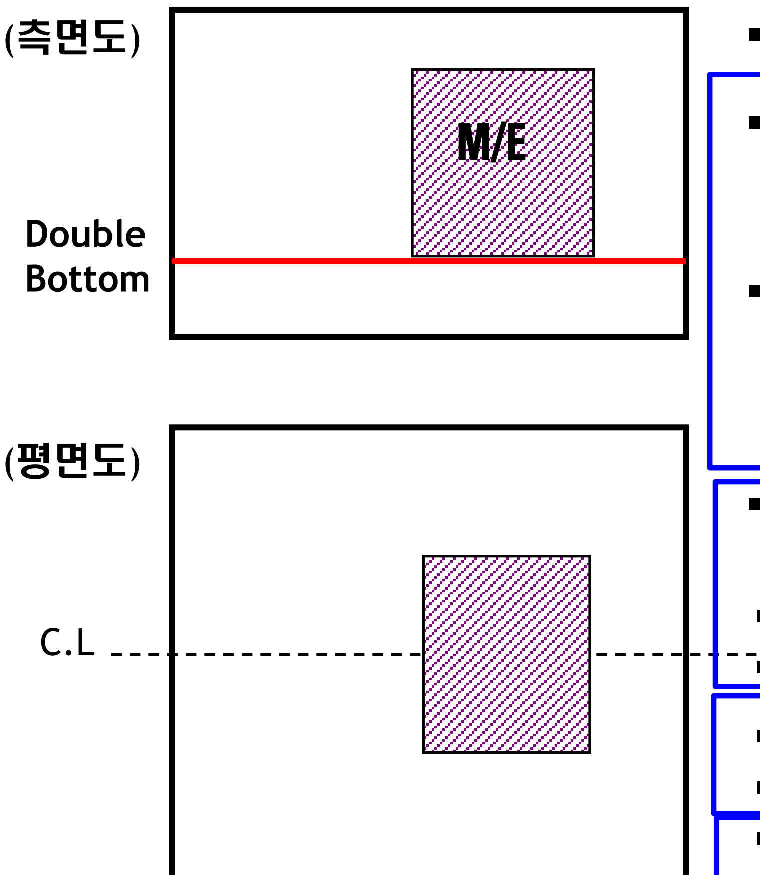
- 선속이 빨라지면
  → CB 작아짐
  → 기관실의 탱크 톱 면적 축소
  → 주기관을 설치 가능한 위치까지 앞으로 이동
  → 기관실 길이가 길어짐

☑ 기관실 프레임 간격

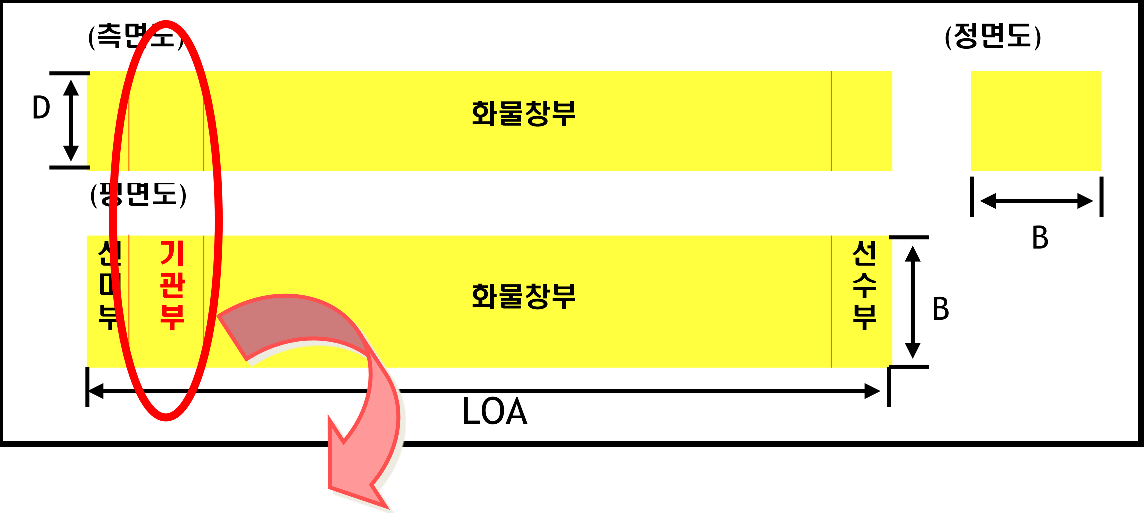
- 진동, 기관실 내 Web frame, Deck House 등과의 관계를 고려
- 재화중량 20,000ton 이상의 bulk carrier, tanker : 800~900mm



Tanks in Engine Room

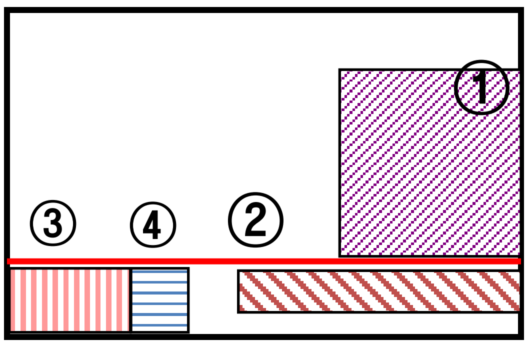


- Main Engine(M/E)
- H.F.O. SETT. TK (Heavy Fuel Oil Settling Tank)
  : Settling Tank는 연료가 사용되기 전에 연료에 존재하는 불순물(impurities)을 침전시키기 위한 Tank
- H.F.O. SERV. TK (Heavy Fuel Oil Service Tank)
  : Service Tank는 Settling Tank로부터 공급(supply,feed)된 연료를 저장하는 공간. Service Tank에 저장된 연료가 M/E로 공급됨.
- D.O. STOR. TK (Diesel Oil Store Tank) : Store Tank는 연료를 저장하는 공간
- D.O. SETT. TK (Diesel Oil Settling Tank)
- D.O. SERV. TK (Diesel Oil Service Tank)
- L.O. SETT. TK (Lubrication Oil Settling Tank)
- L.O. SERV. TK
- CYL.O. SETT. TK (Cylinder Oil Settling Tank)
- CYL.O. SERV. TK



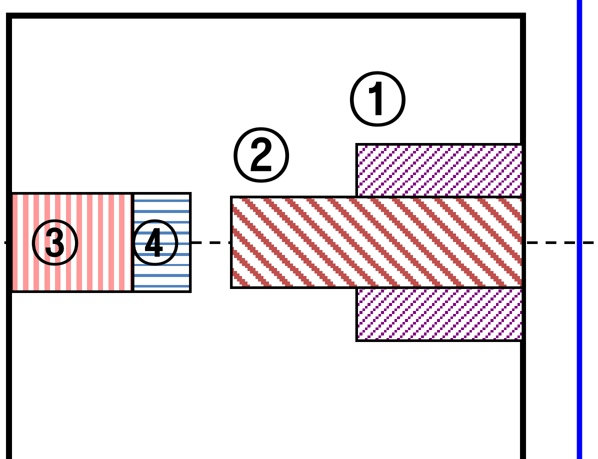
(측면도)

Double Bottom



(평면도)

C.L



- C.F.W DRAIN TK
  (Cooling Fresh Water Drain Tank)
  : **사용된 냉각수를 저장해 놓는 공간**
- HIGH S.C. (High Sea Chest):
  Water Ballast Tank**를 채우기 위한 해수를 공급하는 주입구가 설치된 공간, 선측면**(High)**에 위치.**
- LOW S.C. (Low Sea Chest):
  HIGH S.C.**와 동일한 목적으로 설치, 선저**(Low)**에 위치.**

① Main Engine

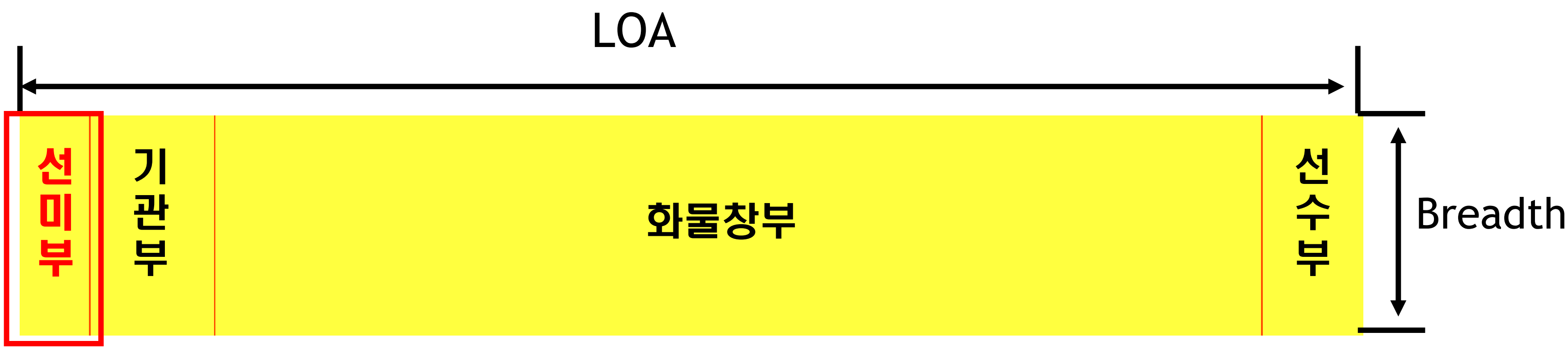
② L.O. SUMP TK : **주기관에서 사용한 윤활유를 저장할 공간**

③ BILGE HOLDING TK : Bilge Well**에 모인 오수**(dirty water)**를 저장하는 공간**

④ S/T L.O. DRAIN TK (Stern Tube Lubrication Oil Drain Tank)
: Stern Tube**에 사용된 윤활유를 저장하는 공간**

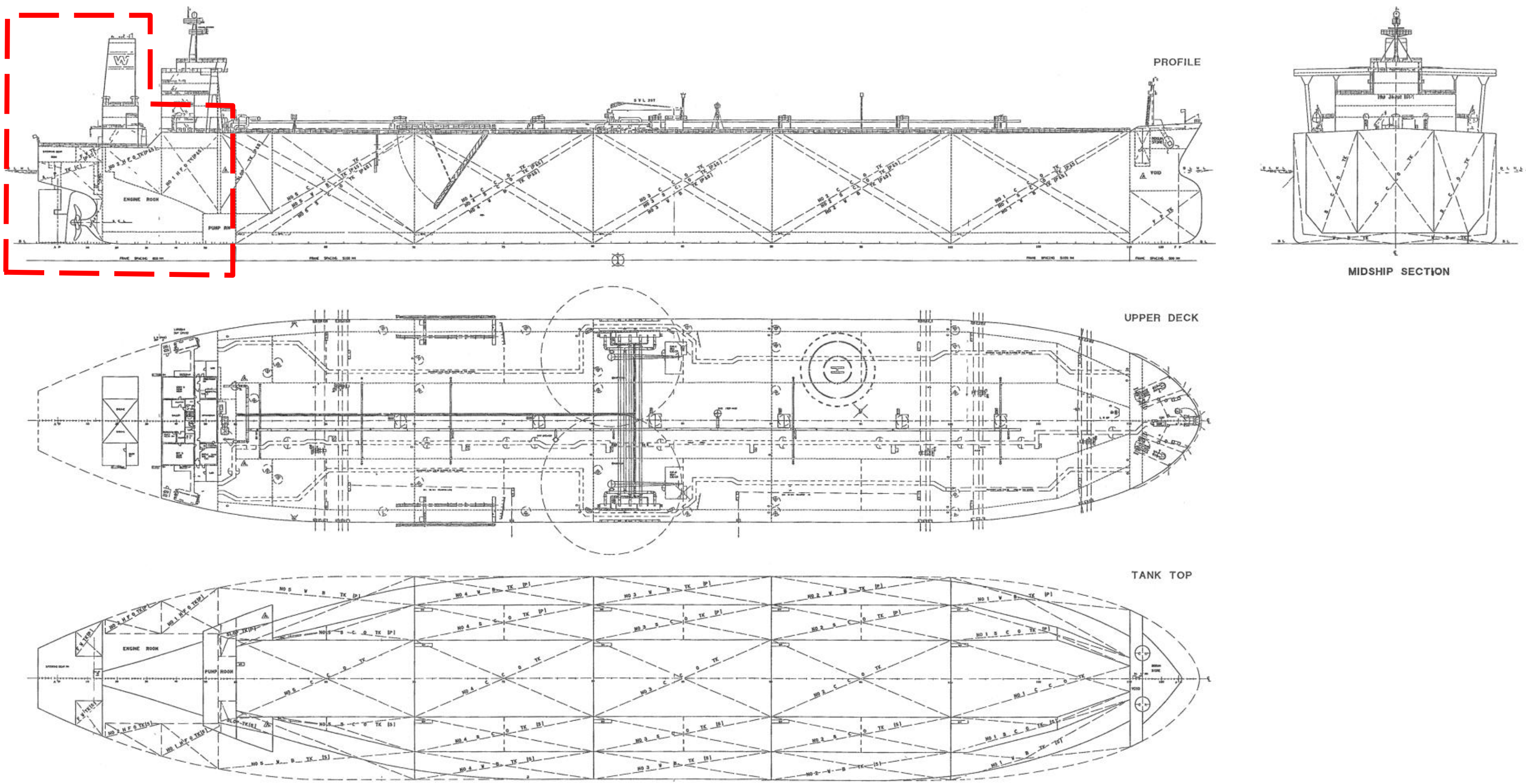
- F.O. OVERFLOW TK : **엔진에 연료 주입 시 흘러 넘친 연료를 저장하는 공간**
- B/W (Bilge Well) : **바닥으로 고인 각종 오수**(**물,연료 등**)**를 저장하는 공간**

Tanks and spaces in after body



- A.P. TK (After Peak Tank) : Trim 조절을 위해 Ballast Water를 채우는 공간
- Steering Gear Room : Rudder를 컨트롤하기 위한 모터 및 장비가 배치되는 공간
- F.W. TK (Fresh Water Tank) : 선상에서 생활하는 사람들이 사용할 청수를 저장
- Distilled F.W. TK : 보일러를 구동하는데 사용할 증류수를 저장하는 공간
- C.W.T (Cooling Water Tank) or S.T.C.W.T (Stern Tube C.W.T)
  : 엔진 냉각 또는 프로펠러 회전 시 Stern Tube에서 발생한 열을 식히기 위한 물을 넣어 둔 공간
- CO2 Room : 화재 시 사용할 CO2를 저장해 놓은 공간

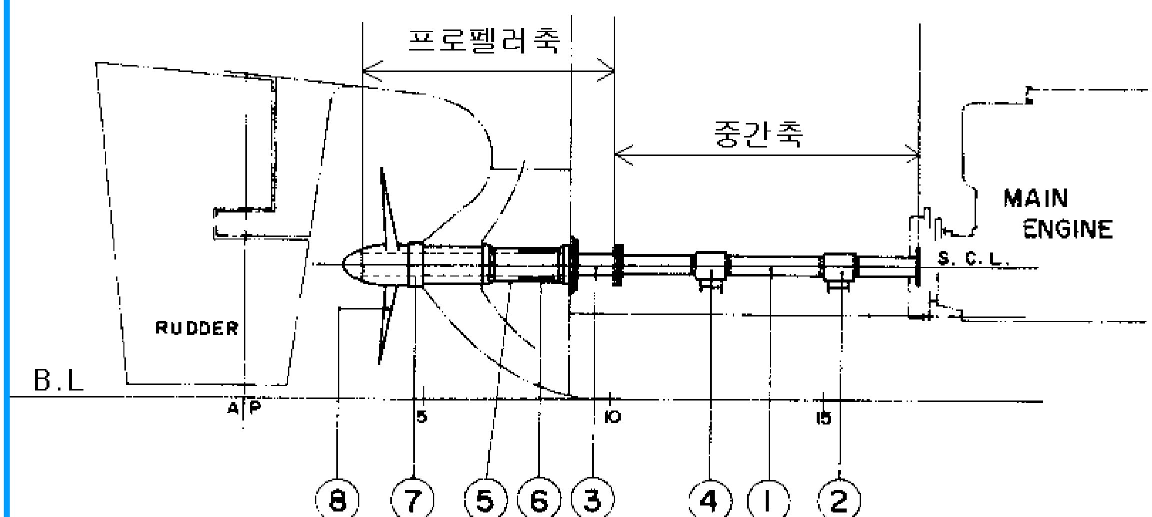
기관의장 구역의 정의



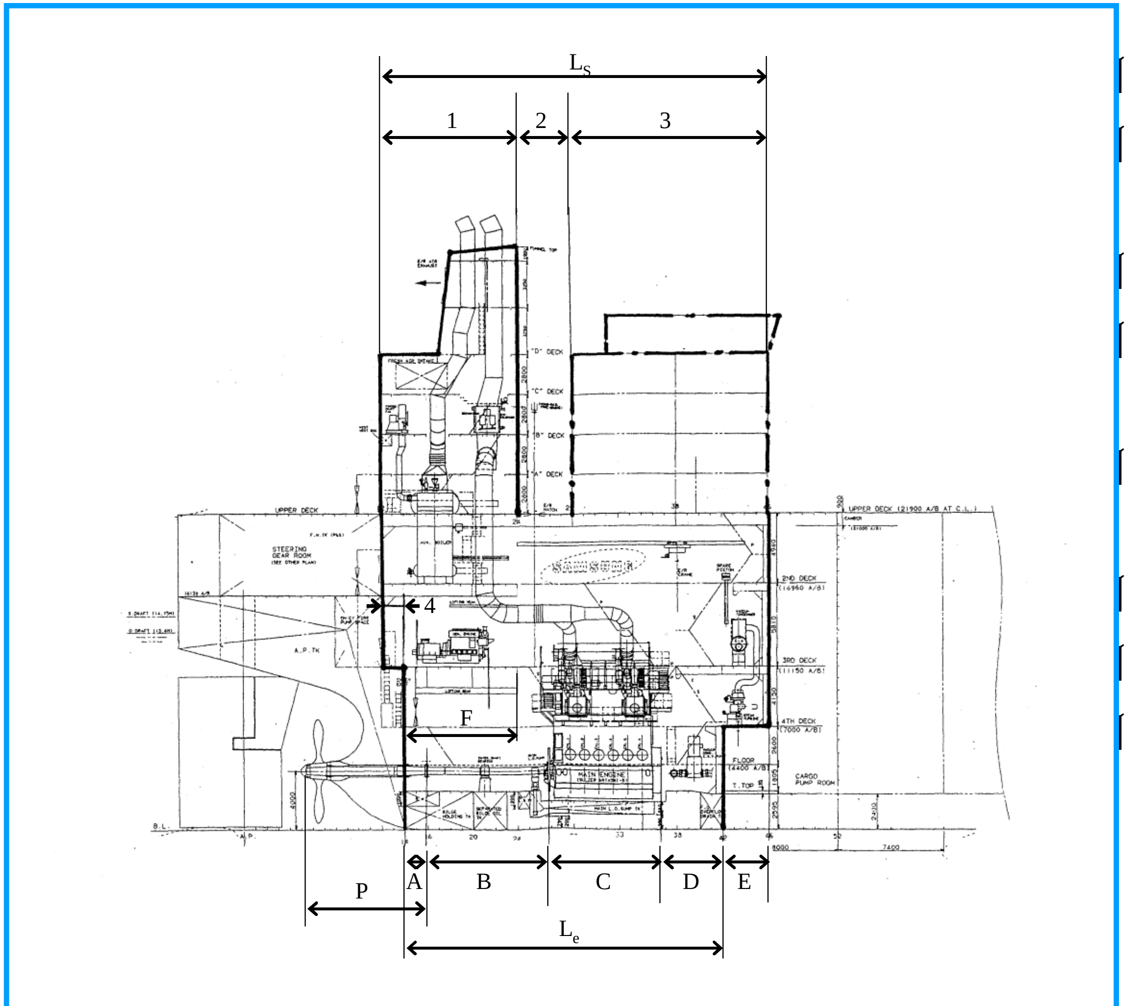
PROFILE
MIDSHIP SECTION
UPPER DECK
TANK TOP
ENGINE ROOM
PUMP ROOM

Shaft Arrangement (추진축계 배치)

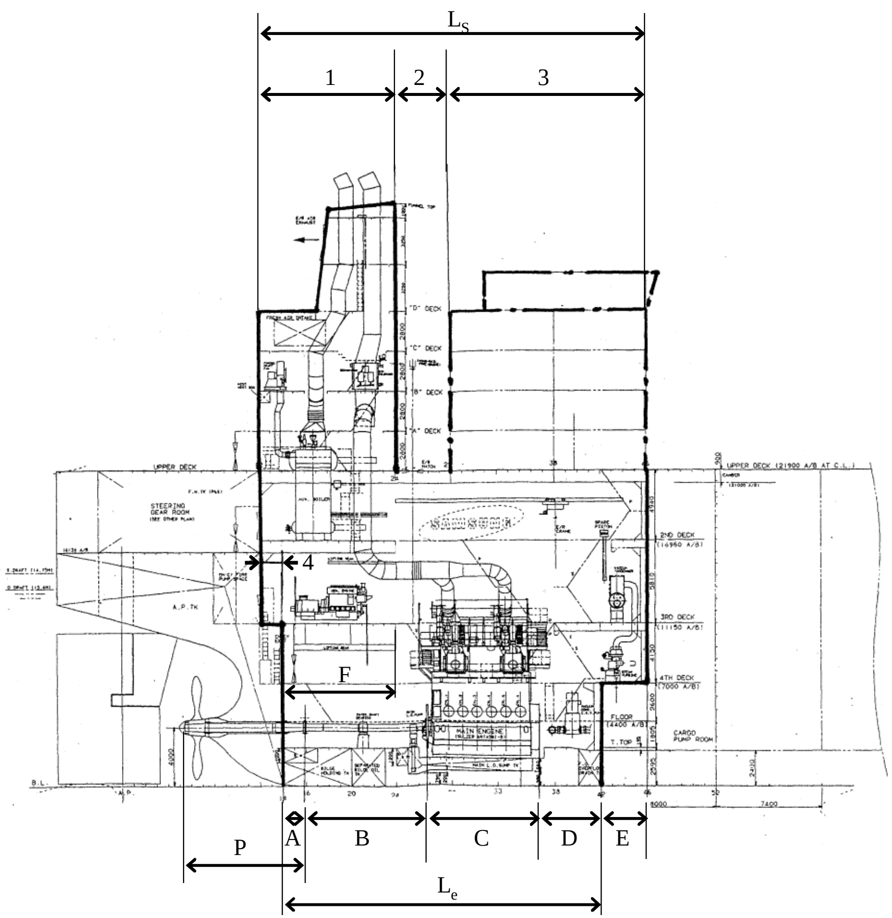
① 중간축(intermediate shaft)
② 중간축 베어링(intermediate shaft bearing)
③ 프로펠러축(propeller shaft)
④ Aftmost 베어링(Aftmost bearing)
⑤ 스턴튜브(stern tube)
⑥ 스턴튜브 베어링(stern tube bearing)
⑦ 로우프 가이드(rope guard)
⑧ 프로펠러(propeller)



E/R Length(기관실의 길이)



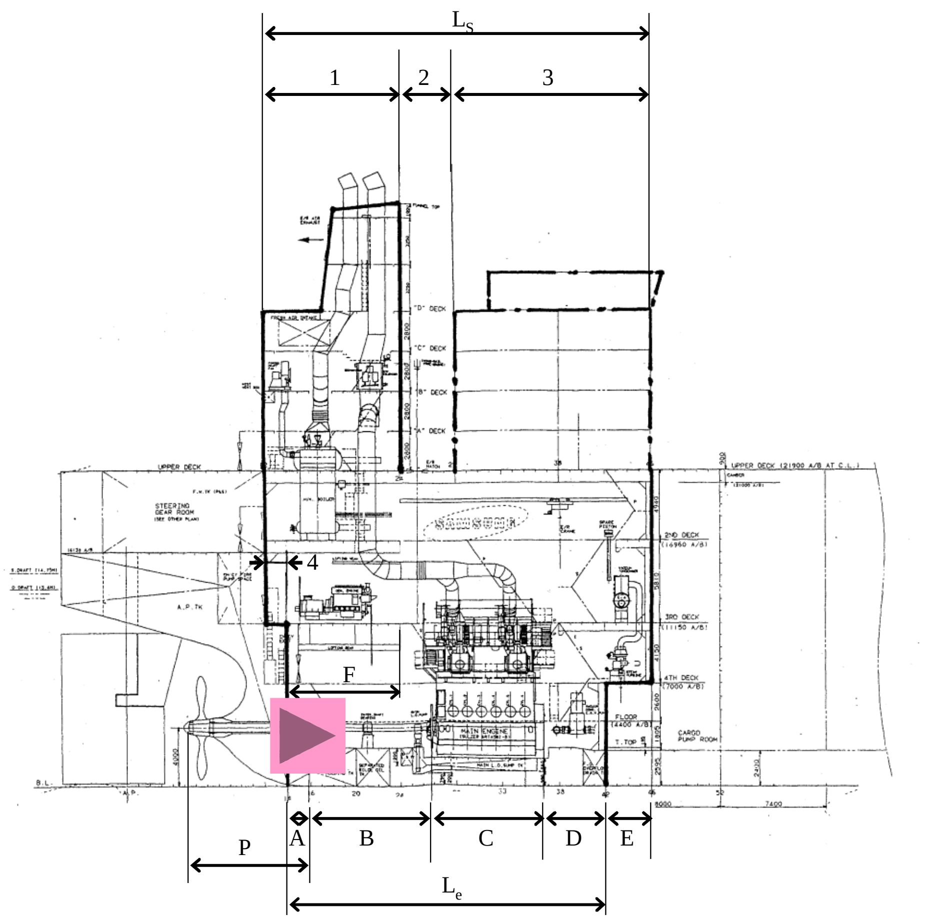
- P : 프로펠러 축 길이
- A : 기관실 선미 격벽 에서 추진 축 끝단까지의 길이
- B : 중간축 길이
- C : 주기관(Main Engine ; M/E) 의 길이
- D : 주기관과 기관실 전부 격벽 사이의 길이
- E : Pump Room Recess Space
- F : 디젤발전기 설치를 위한 길이
- La : A ~ D 까지의 길이



☑ A : 축 flange 연결작업 및 stern tube forward seal 설치를 위해 800~1000mm는 확보되어야 한다.

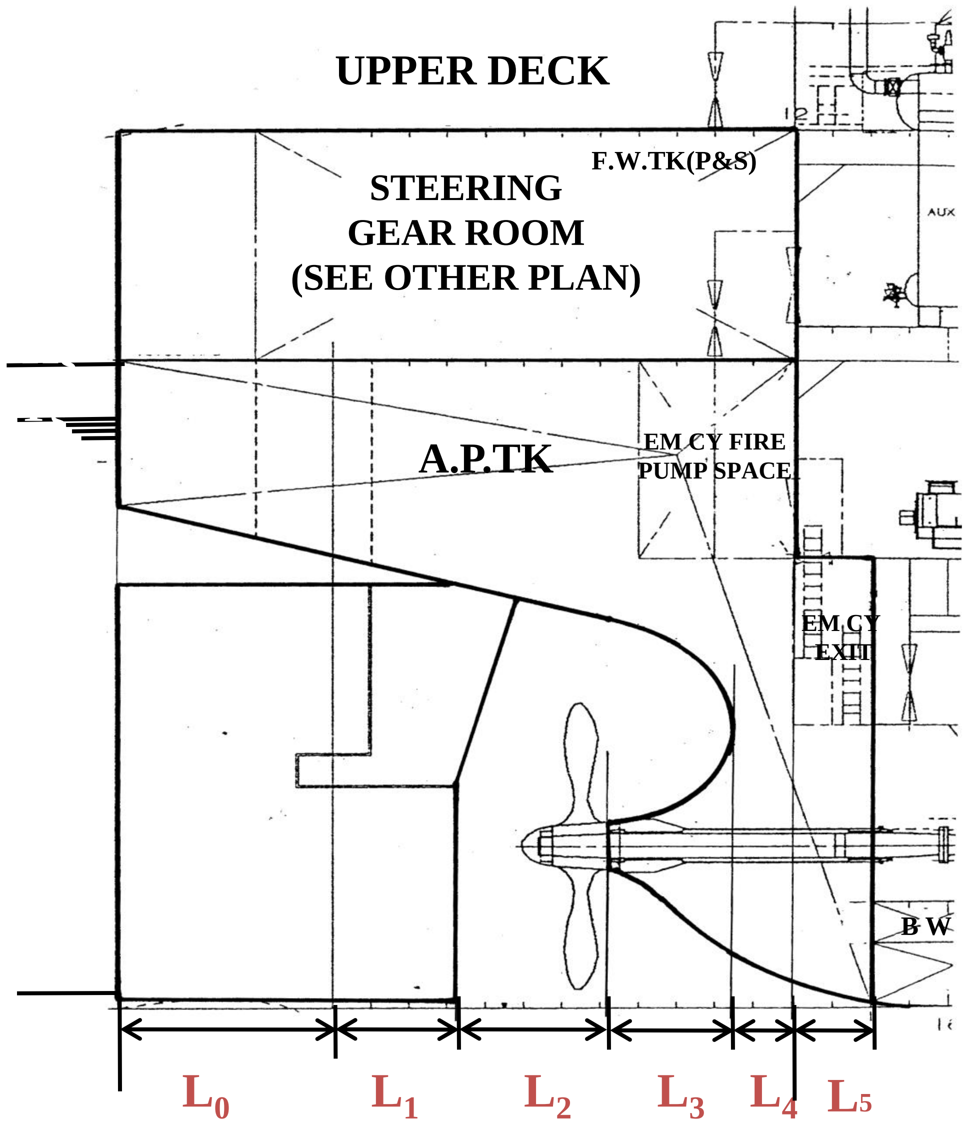
☑ B : 추진축을 기관실 내부로 빼낼 경우에 추진축의 길이, 주기관의 위치 등을 고려한다. 단, 추진축을 선박의 선미 방향으로 빼낼 경우는 추진축과 무관하게 결정되므로 매우 짧게 할 수 있다.

☑ A + B : 프로펠러 축 발출에 필요한 길이
추진축 stern tube의 보수, 유지, 관리 및 검사를 위한 공간. 일반적으로 이 길이는 추진축 길이보다 200~300mm정도 길어야 한다.

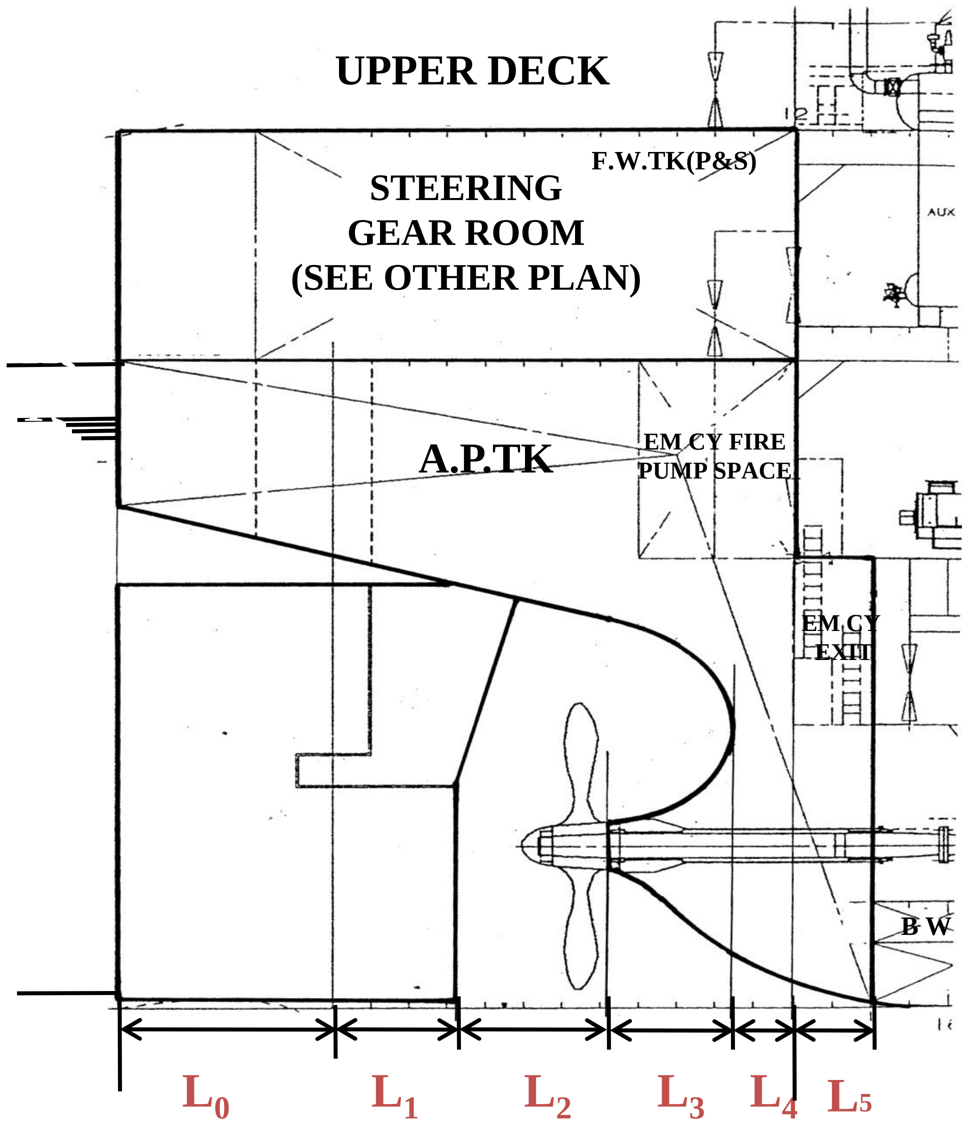


- C : 주기관에 따라서 결정된다.
- D : 주기관 앞쪽의 배관 및 펌프 (pump) 배치 공간으로서 선종에 따라 다르지만 일반적으로 최소 3 m는 되어야 한다.
- E : Bulker, Container는 이 구간이 존재하지 않는다.
- F : 디젤발전기 설치를 위한 길이
- 기타 고려사항
  - 구조의 연속성 확보를 통한 진동 예방
  - 비상 탈출구용 Trunk
  - FOT (FO Storage Tank) 설치
  - 축 발전기 설치 여부
  - 진동 감쇄기 설치 여부

Aft Length (선미부 길이)



- ☑ L_0 : Lines Design으로부터 얻어지게 됨
- ☑ L_1 : Rudder Balance Ratio로서 Rudder 설계로부터 얻어지게 됨
- ☑ L_2 : Propeller Removal 공간을 위한 거리 (프로펠러 수리를 위해 프로펠러를 빼내야 할 경우를 고려해야 함)
- ☑ L_3 : 프로펠러와 선체의 최소 거리로서 프로펠러에 의한 기진력, 진동 등의 감소를 위해 필요

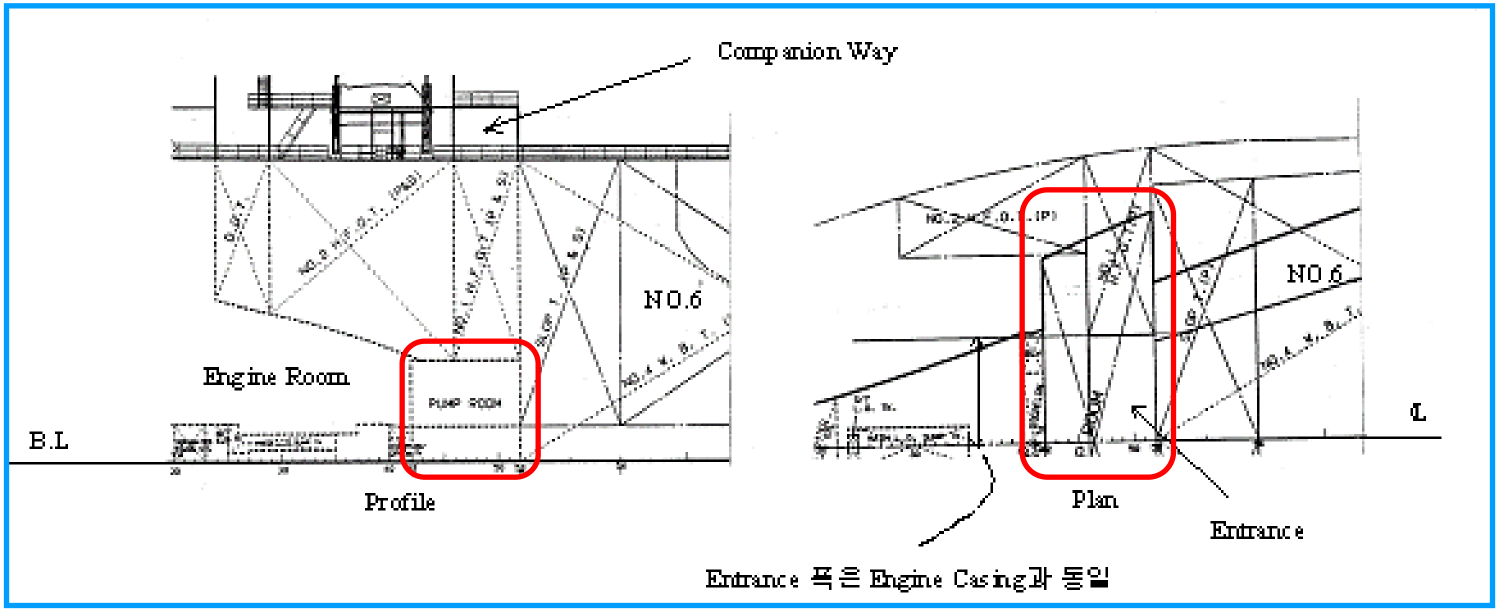


- ☑ L4 : 2 Frame Space 정도의 여유를 두어 용접성 등을 고려하여 결정 (G/A 측면)
- ☑ L5 : E/R으로부터의 비상 탈출구(Emergency Exit)를 위해 2 Frame Space 정도의 여유를 둠 (APT의 용량에 따라서 이 구간이 존재하지 않을 수도 있다.)
- ☑ La : AP ~ E/R Aft BHD 의 길이. (L1 ~ L4)
- ☑ Hs : Height for Steering Gear Floor
  Hs = Scantling Draft(Ts) + (0.6~1.2) m

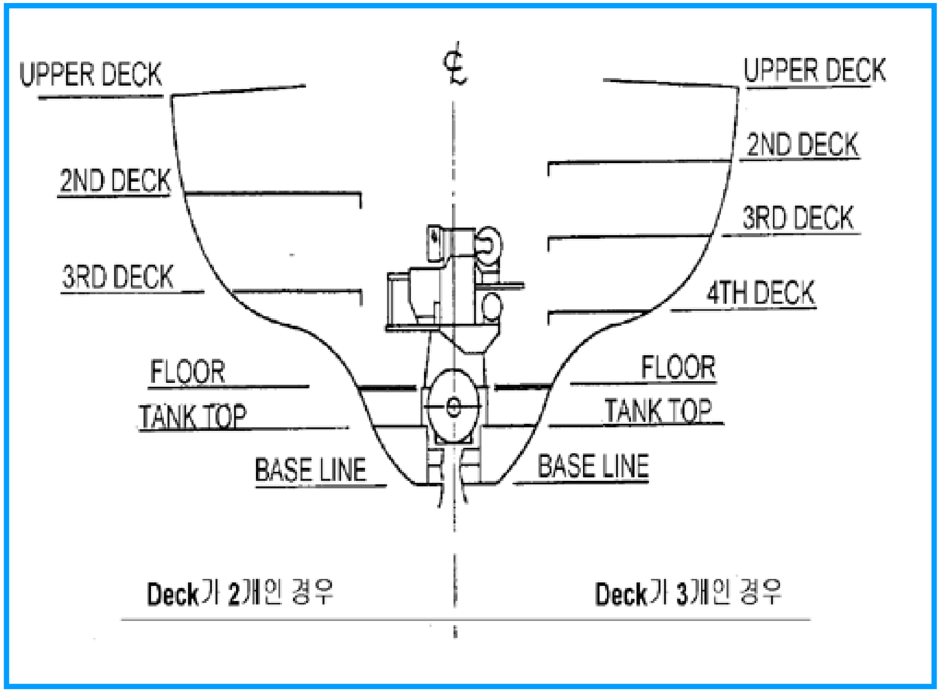
Pump Room

☑ Tanker: 기관실과 화물유 탱크 사이에 펌프실을 배치한다.

- 펌프실의 길이는 화물유 펌프 및 밸러스트 펌프의 크기, 파이프 배치, 액세스, 보수 유지 공간 확보 등을 고려하여 결정한다.
- Cargo Pump 3대, Ballast Pump 1 혹은 2대



기관실 내 갑판 높이 결정 기준



☑Tank top의 높이

- 프로펠러의 직경,주기관 타입, 윤활유 섬프탱크와 그 하부의 Cofferdam 등에 의해 결정

☑바닥(floor) 높이

- 일반적으로 DWT 30,000~ DWT 60,000급 선박의 바닥높이는 tank top에서 1500~1800mm가 적절

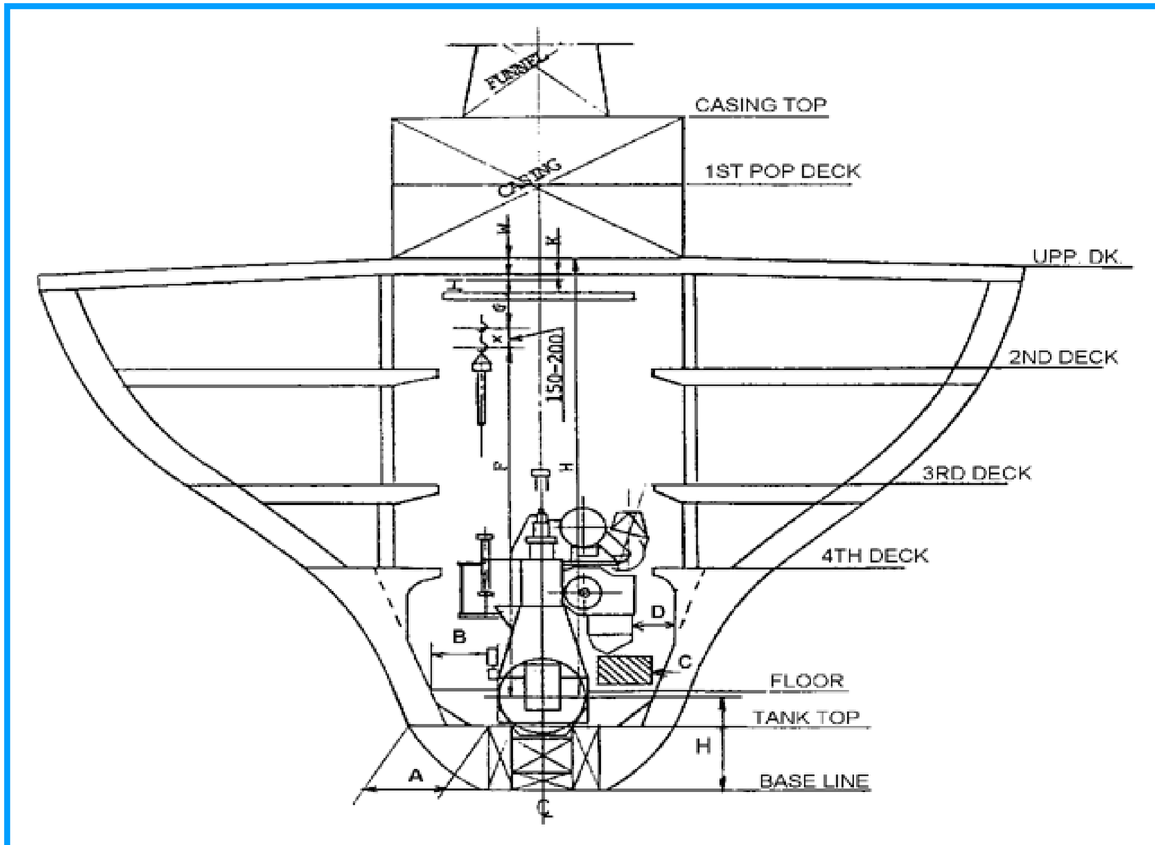
☑3층 갑판 높이

- 바닥에 설치된 기기, 덕트, 관 및 케이블 등의 의장품과 3층 갑판 하부 선체 구조의 크기를 고려하여 결정

- 2층 갑판 높이
  - ✖디젤발전기는 통상 3층 갑판에 설치되므로 디젤발전기의 피스톤 개방이 가능한지 검토
  - ✖3층 갑판과 2층 갑판 사이에는 관, 덕트 및 케이블 등의 의장품과 구조물이 가장 많이 설치되는 구간

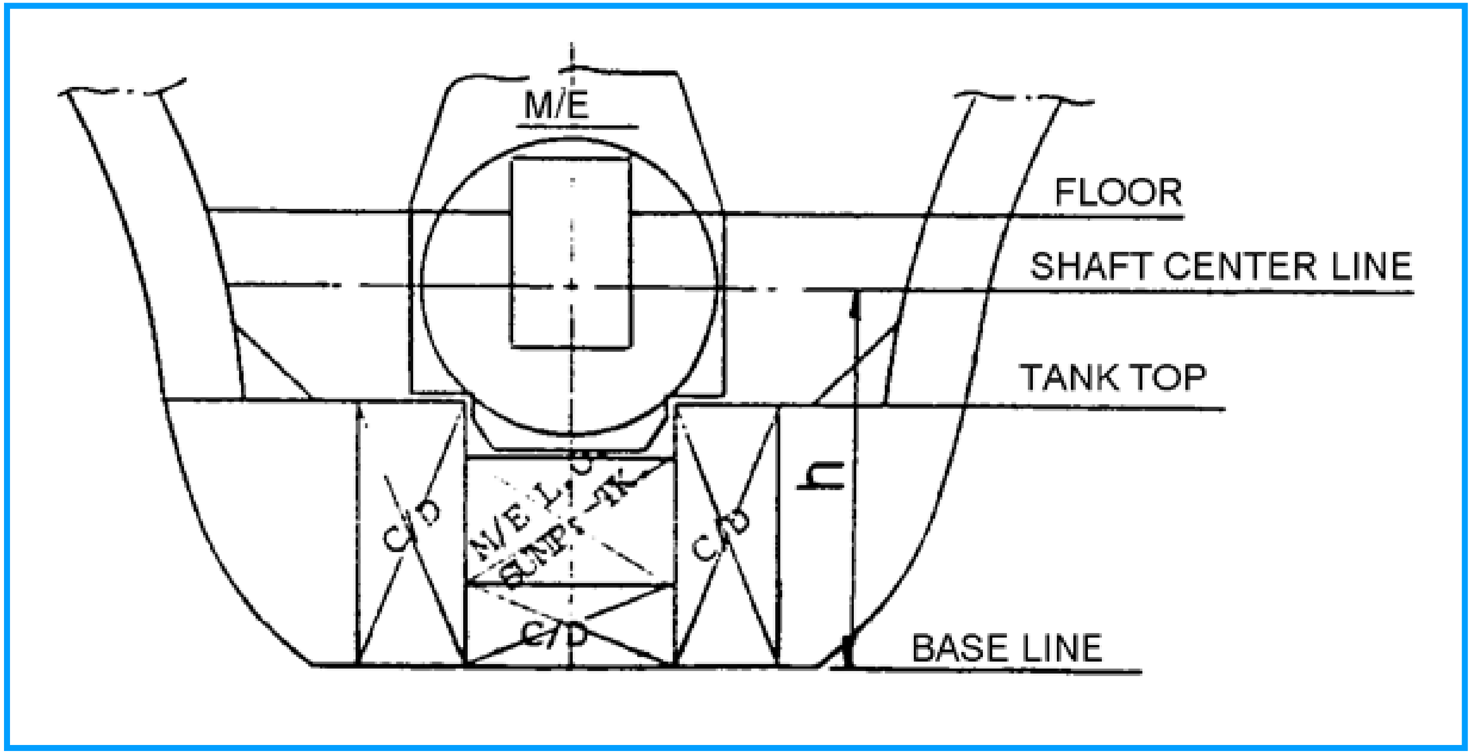
- 2층 갑판과 상갑판 사이의 높이
  - ✖DWT 40,000~60,000급 선박의 경우 2층 갑판에서 상갑판까지의 높이는 최소 4000mm 이상이면 적절하다.

주기관 설치 위치



- A : Side Stopper의 설치 및 파이프 설치를 위한 공간
- B : Turning Gear 측면의 통로(Passage Way)
  - 최소한 600mm의 통로가 확보되어야 한다. 이것이 불가능할 경우 turning gear를 상부로 설치할 경우도 있다.
- C : Air Cooler 하부의 통로
- D : 주기관 주위의 통로
- H : 축 중심 높이(Shaft Center Height)

축 중심 높이(Shaft Center Height)



☑ **프로펠러 직경과 주기관 타입이 결정된 후, 축 중심 높이는 프로펠러 잠김율(propeller immersion), 윤활유 섬프탱크(L.O Sump Tank) 및 섬프탱크 하부의 코퍼댐(cofferdam) 높이(최소한 600 mm)를 고려하여 결정하여야 한다.**

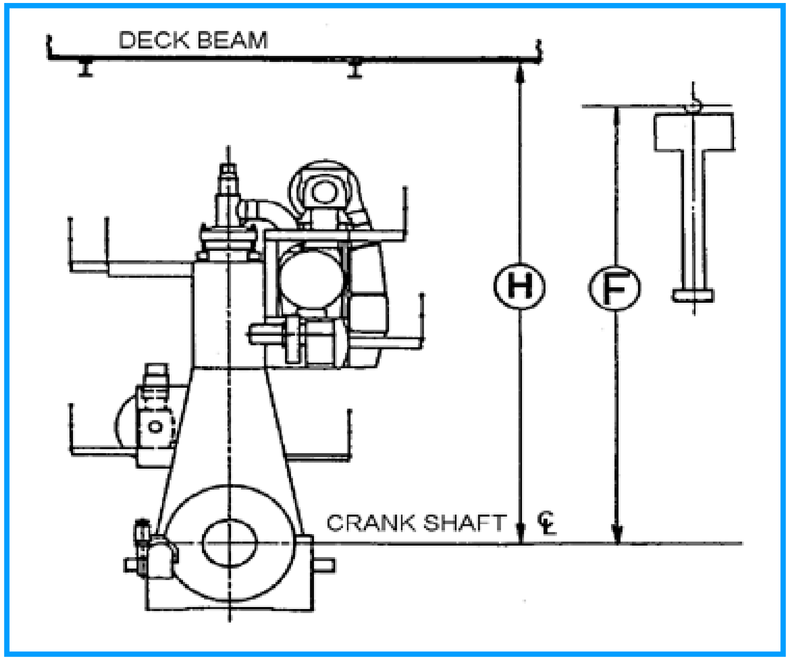
기관실의 높이

☑ 기관실 높이를 결정할 때 고려해야 할 요소

■ 주기관 피스톤개방 높이(M/E piston overhaul height)

■ 중간 갑판(대형선 : 3개, 중형선 : 2개)의 높이의 확보

■ 일반적으로 대형선인 경우 기관실의 높이는 별로 문제되지 않는다.



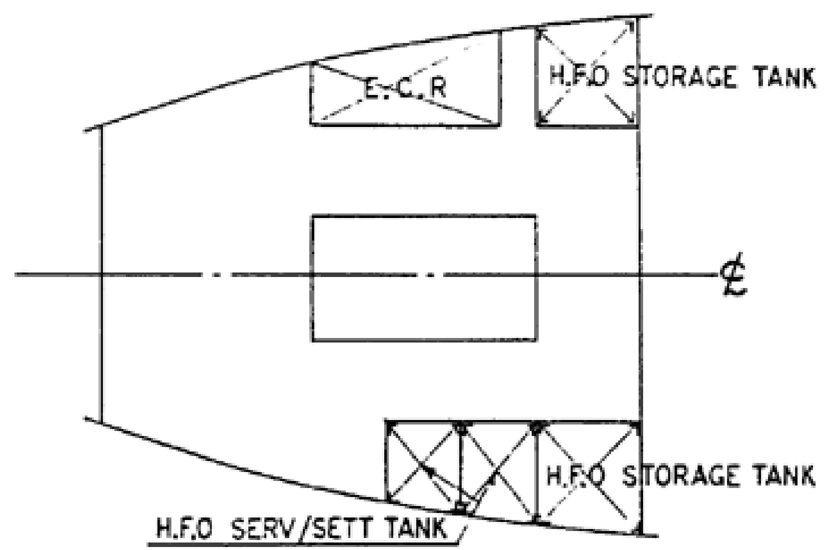
H >= F + G + W + K + X

- H : 기관실의 최상부 갑판과 크랭크샤프트 중심선 간의 높이
- F : 크랭크샤프트 중심선과 크레인 훅 간의 높이
- G : 크레인 및 I-beam 설치를 위한 높이
- W : 기관실의 최상부 갑판 웨브 깊이
- K : 크레인 상부 관 배치를 위한 높이 (250mm)
- X : clearance margin (150~200mm)

각종 Room의 크기 결정

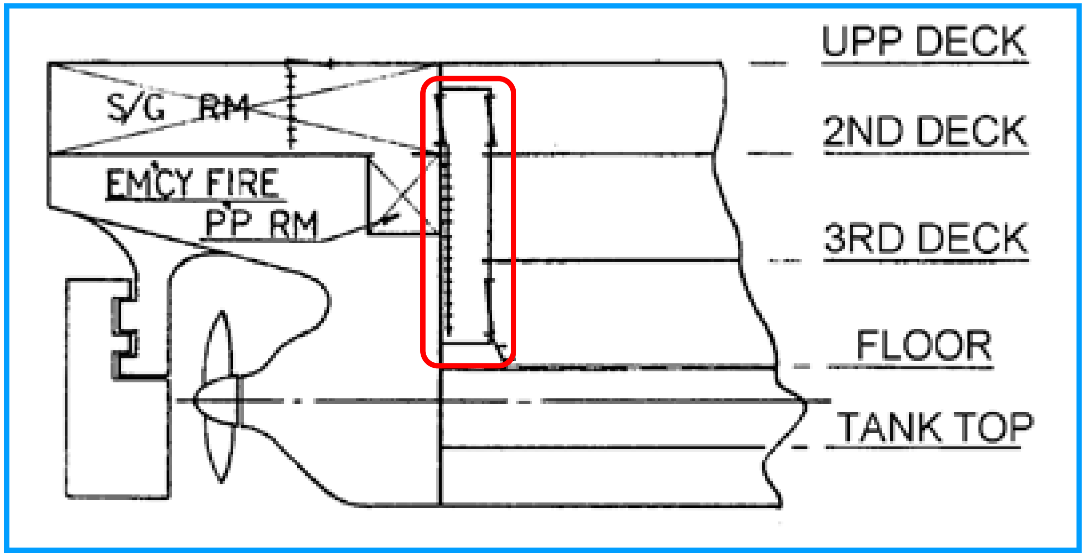
기관실에는 기관통제와 작업을 위하여 각종 room이 필요

☑Engine control room(ECR)



- ✖E.C.R은 주기관, 디젤발전기, 보일러 등의 주요기기를 감시하는데 가장 편리하고 용이하게 접근할 수 있는 위치에 배치한다.
- ✖통상 주기관보다 높게 설치된다.
- ✖E.C.R은 주기관 앞쪽 또는 좌현쪽에 위치하며 폭은 5~6m, 길이는 12~14m 정도로 한다.
- ✖H.F.O (Heavy Fuel Oil) Service/Settling tank는 E.C.R과 인접해서는 안 되고, 가능한 H.F.O storage tank (FOT)도 인접하지 않도록 한다.

☑Emergency escape trunk



✖기관실에는 화재 및 비상사태를 대비하여 하부 갑판 위치로부터 weather deck으로 유도되는 1개 이상의 비상탈출구를 설비해야 한다.

✖Trunk는 가능한 연속적이면서 emergency fire pump room이나 steering gear room 등을 이용하여 최단거리의 형태가 되도록 한다.

☑ **Engine room workshop**

- 주기관, 발전기, 보일러 및 제반 장치의 예비품 및 부품 등을 간단히 가공 또는 제작하는 공작기기 및 부품류들이 배치된다.

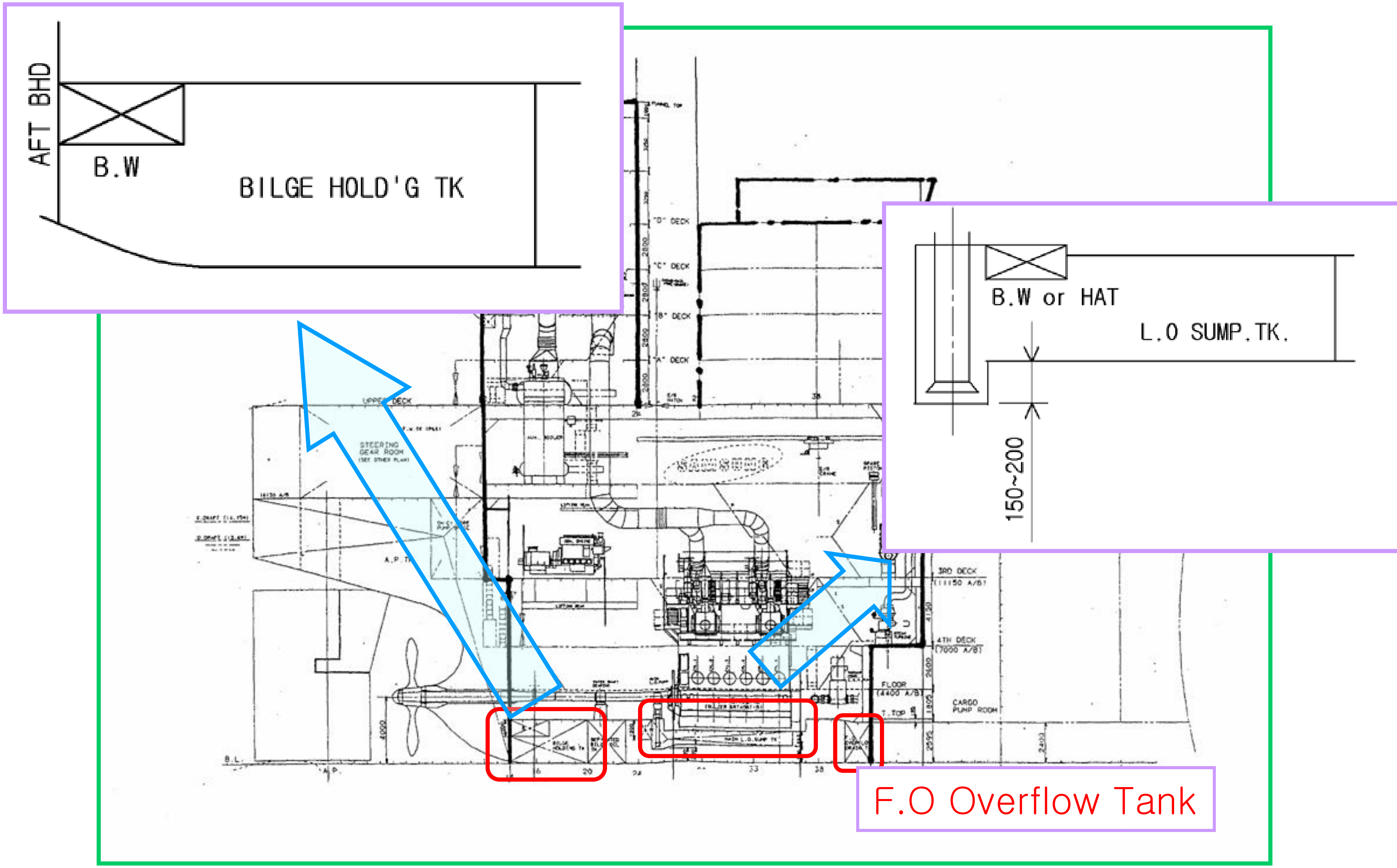
☑ **Engine room store**

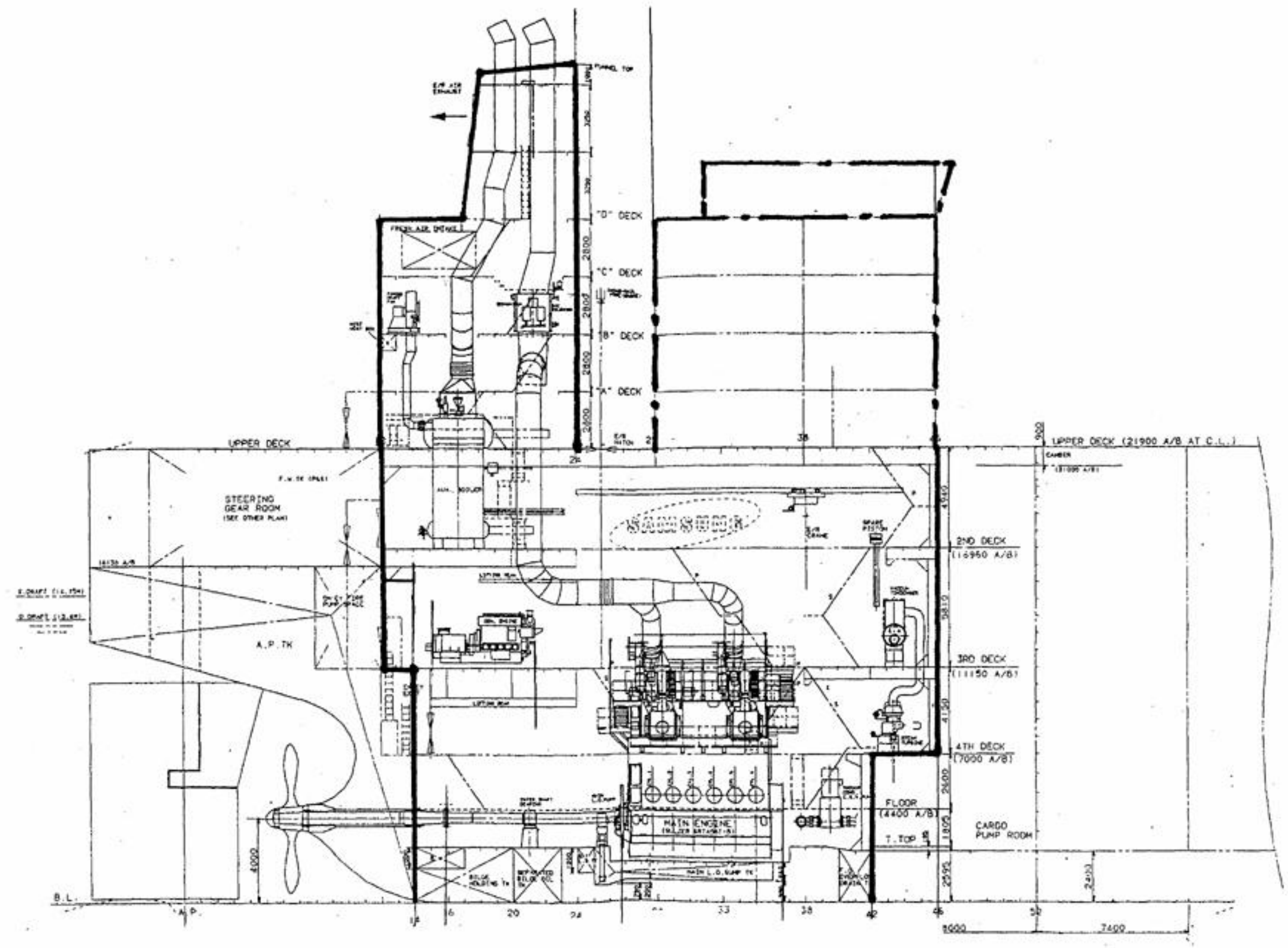
- Engine room내 store는 보조기기류의 예비품, 공구 및 부속품 등을 보관하는 장소

☑ **Purifier room**

- 선박의 운항에 필요한 fuel oil 및 lubrication oil을 청정하는 데 필요한 기기들이 설치되는 room
- Purifier room은 purifier, purifier용 heater, F.O purifier용 feed pump와 operating water tank가 설치되어야 한다.

기관실 내의 Hull Tank 배치




"D" DECK
"C" DECK
"B" DECK
"A" DECK
UPPER DECK
STEERING
GEAR ROOM
(SEE OTHER PLAN)
A.P.TK
UPPER DECK (21900 A/B AT C.L.)
2ND DECK
(16950 A/B)
3RD DECK
(11150 A/B)
4TH DECK
(7000 A/B)
FLOOR
(4400 A/B)
T.TOP
CARGO
PUMP ROOM
MAIN ENGINE
B.L.
A.P.

기관실 내의 Hull Tank 배치 [1]

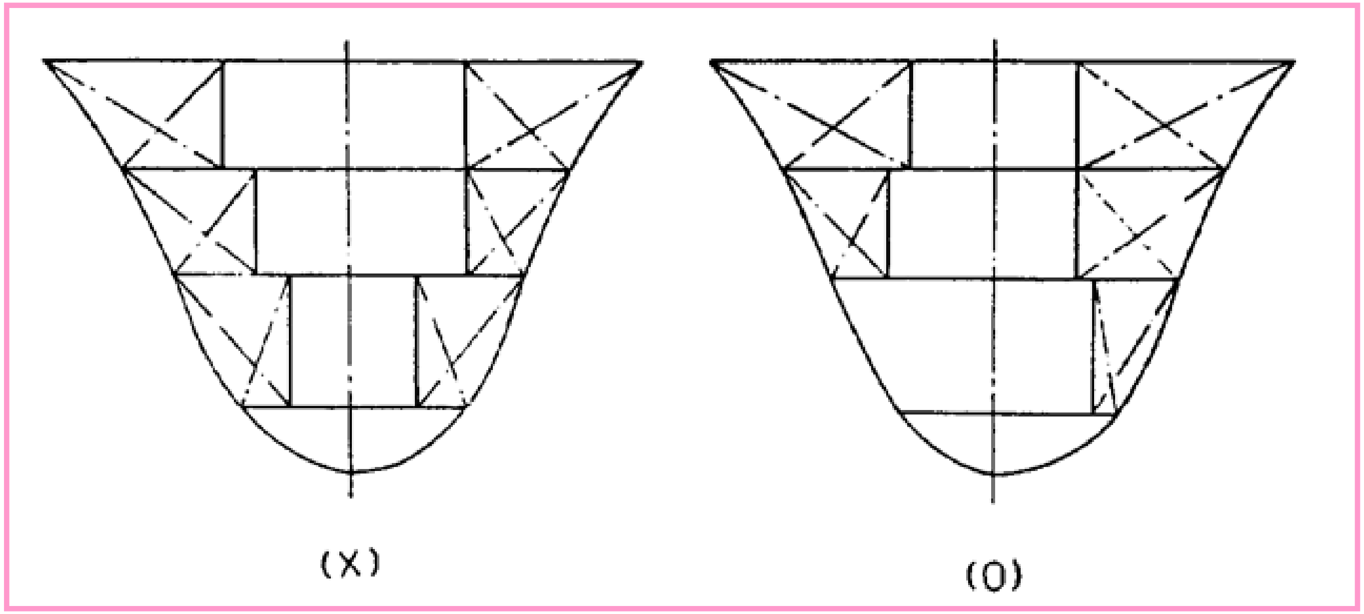
☑Cofferdam 설치

- L.O.T (lubrication oil tank)와 F.O.T (fuel oil tank) 사이
- Water tank와 oil tank 사이
- 가열되는 tank와 곡물저장고 사이
- F.O.T가 deck에서 끝나고 deck 하부가 다른 기기 공간 또는 E/R인 경우
- E/R과 emergency generator room 사이
- Main engine L.O sump tank 주위
- 기타 격리가 필요한 부분

☑손상시 복원성을 고려하여 Tank 배치

☑Room 및 탱크가 위 아래로 연결될 경우

- 가능 한 수직 방향으로 일치시키는 것이 좋다
- 그렇지 않을 경우 위에 있는 탱크가 배의 중심 쪽으로 더 들어가는 것이 바람직하다.
- 탱크 상부갑판(Tank Top)에 설치되는 기기의 배관이 탱크 내부로 설치되기 때문에 밑에 있는 탱크가 중심 쪽으로 더 들어가는 것은 좋지 않다.



☑기관실 내 Double Bottom Tank 배치

■Double bottom (D/B)에 위치하는 tank system 및 기기에서 자연적으로 유출되거나 계통상 최하단부에 배치되어야 할 탱크들로 구성된다.

1.Bilge Holding Tank

2.M/E L.O Sump Tank

3.F.O Overflow Tank

- 일반적으로 연료유 계통의 장비 및 배관이 선박의 좌현 쪽에 위치하므로 F.O Overflow Tank도 좌현 선수쪽에 위치

4.Oily Bilge Tank (or Waste Oil Tank)

- Oily bilge tank는 각종 dirty oil이 모이는 곳이므로 D/B의 좌현 선미쪽에 위치한다.

5.Bilge Well

- Bilge Well 은 선미쪽에 1개, 선수쪽의 좌,우현에 각 1개씩 배치한다.

6.그 외 각종 Drain Tank 및 D.O Storage Tank가 설치되기도 한다.

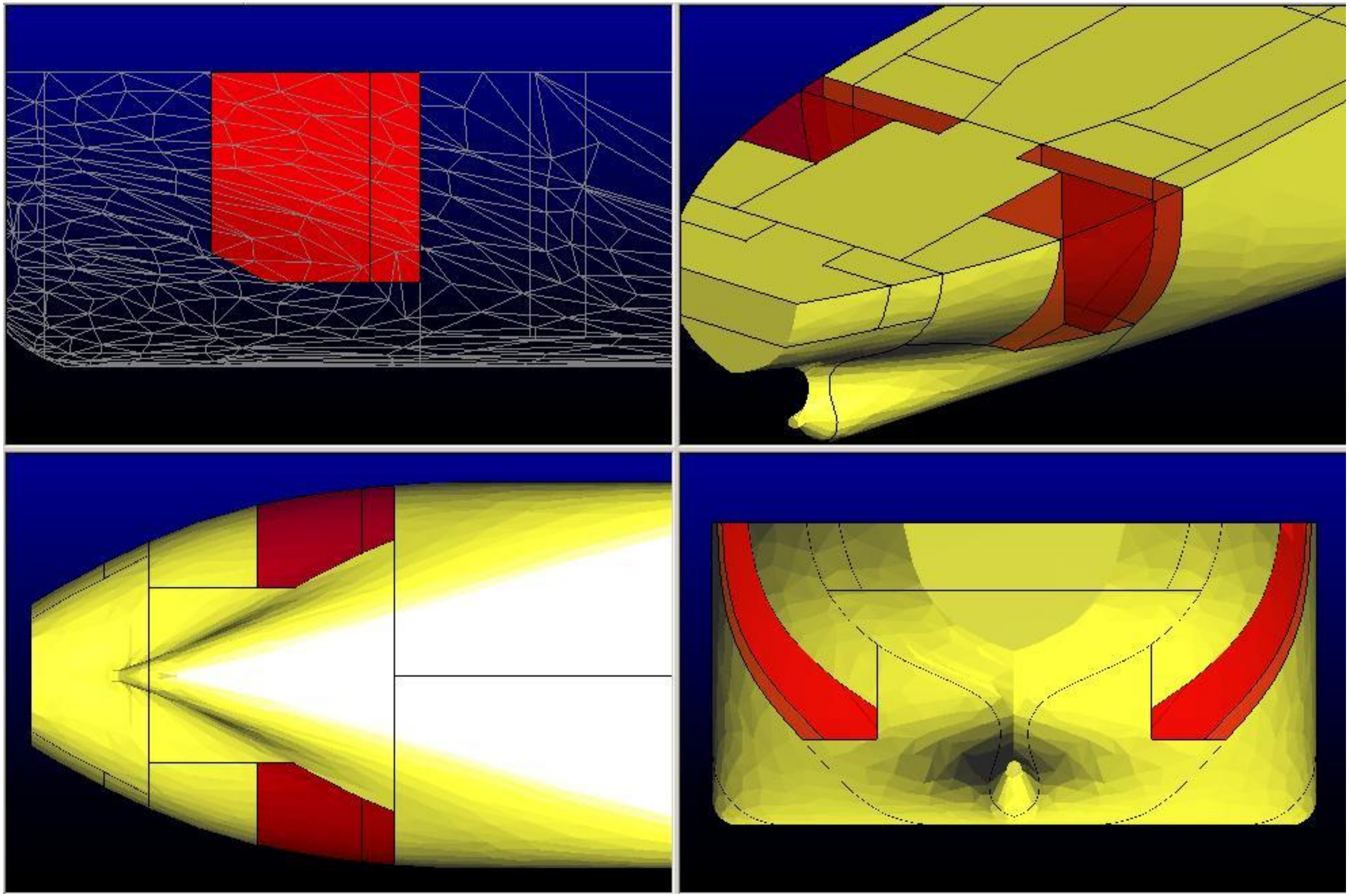
☑F.O.T (Fuel Oil Tank)의 배치

- 기본적으로 모든 F.O Tank는 hull tank로 배치.
  불가능할 시에는 적절한 drip tray를 설치한 portable tank로 만들기도 한다.
- F.O Tank는 그 밑면이 side shell 또는 double bottom top에 접하여야 한다. 그렇지 않을 때, 즉 deck에 접할 때는 deck 상부 또는 하부에 cofferdam을 설치해야 한다.
- F.O Tank는 전체를 묶어서 하나의 boundary로 구성하는 것이 바람직하며 기관실 Fwd Bulkhead와 접하여 배치한다.

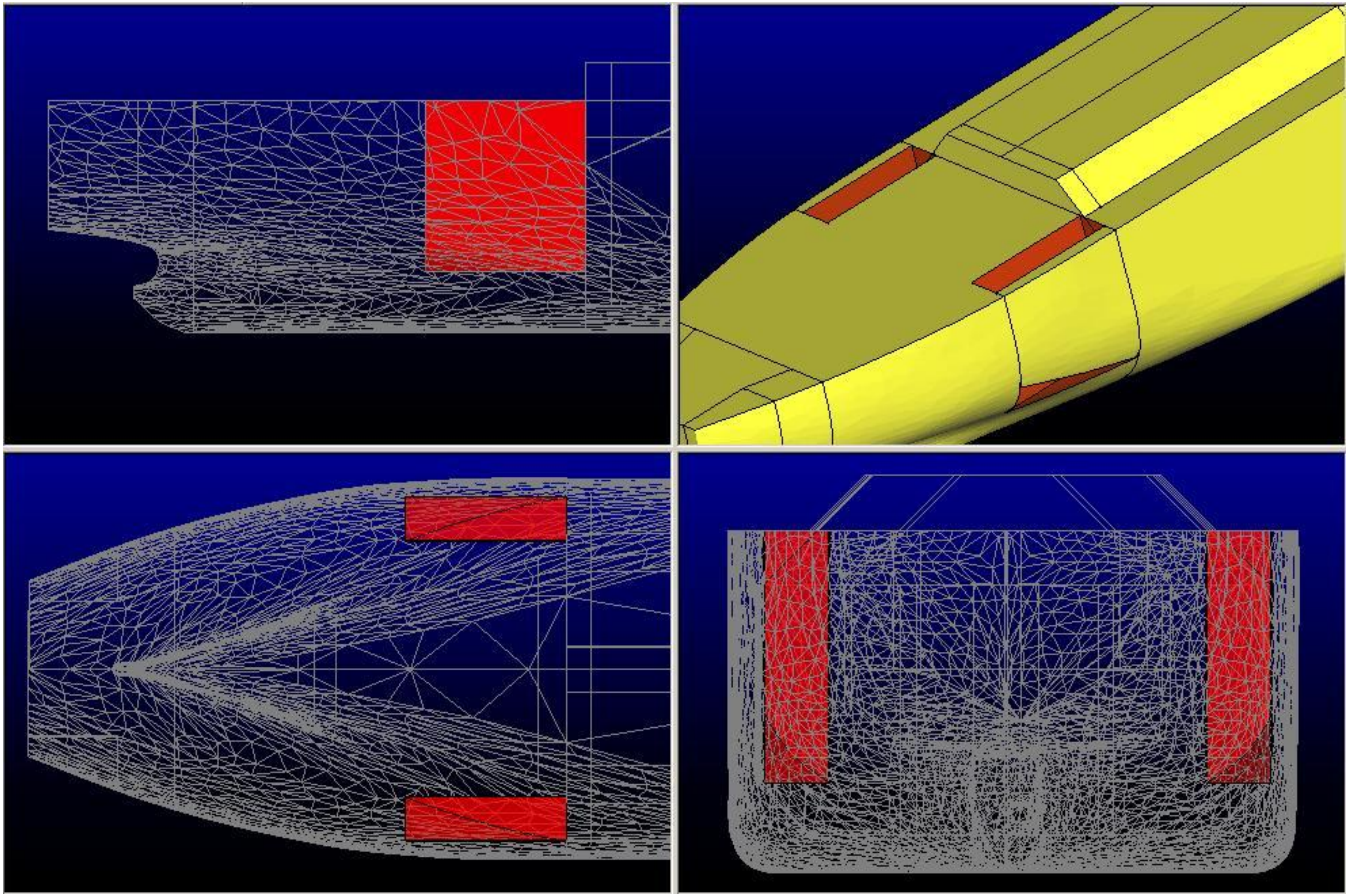
☑L.O.T (Lubrication Oil Tank)의 배치

- Lub.Oil Tank는 가능한 side shell과 접하지 않도록 배치한다.

320K VLCC의 F.O.T

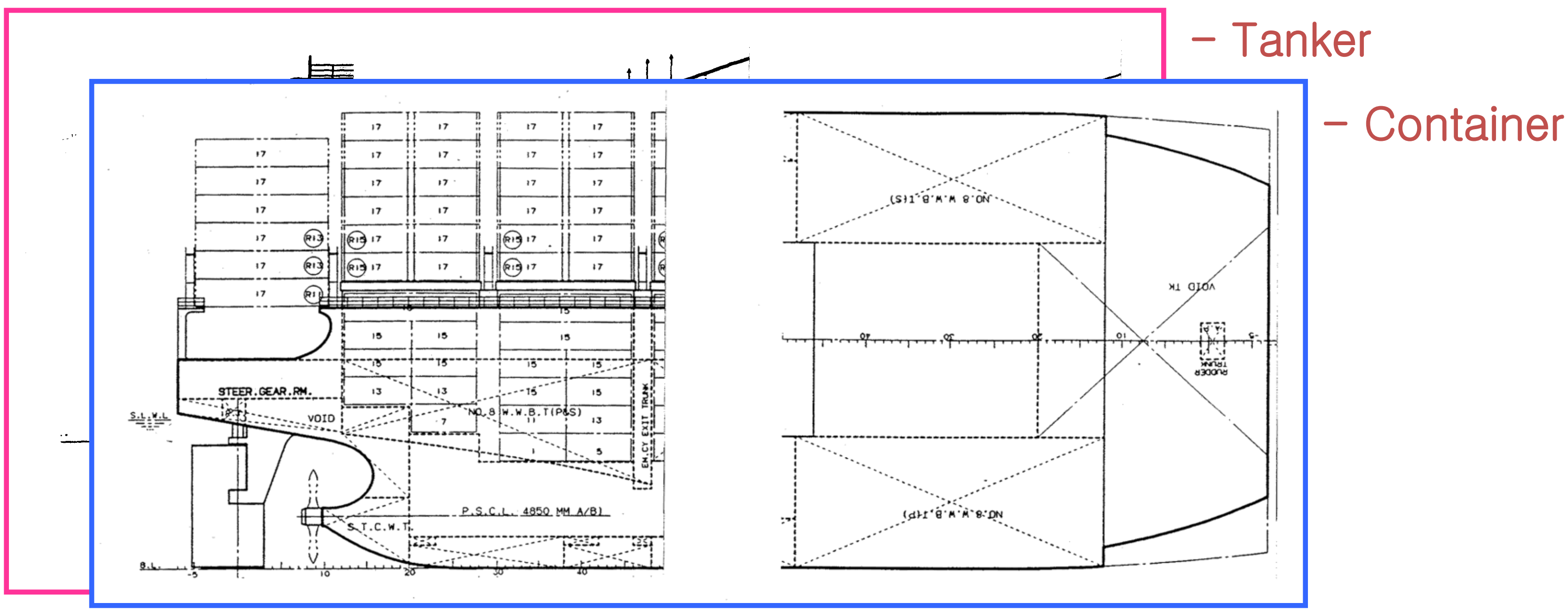


145K LNGC(LNG Carrier)의 F.O.T

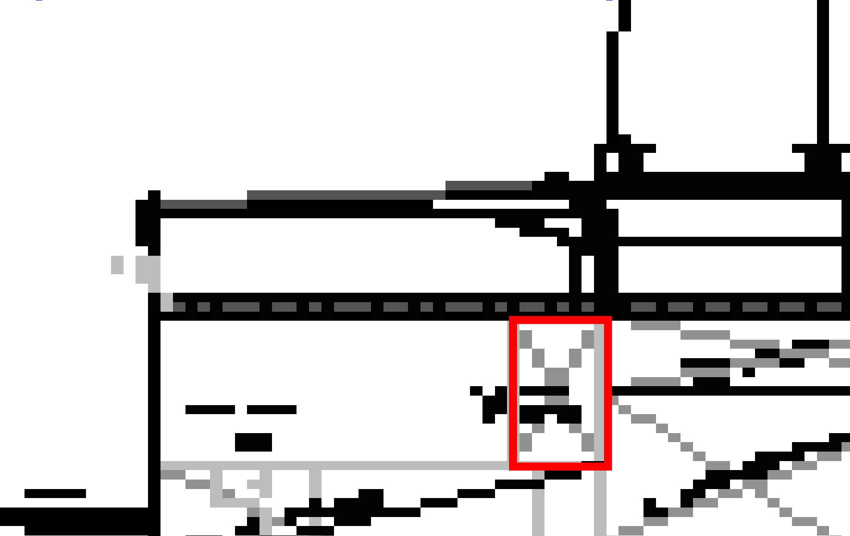


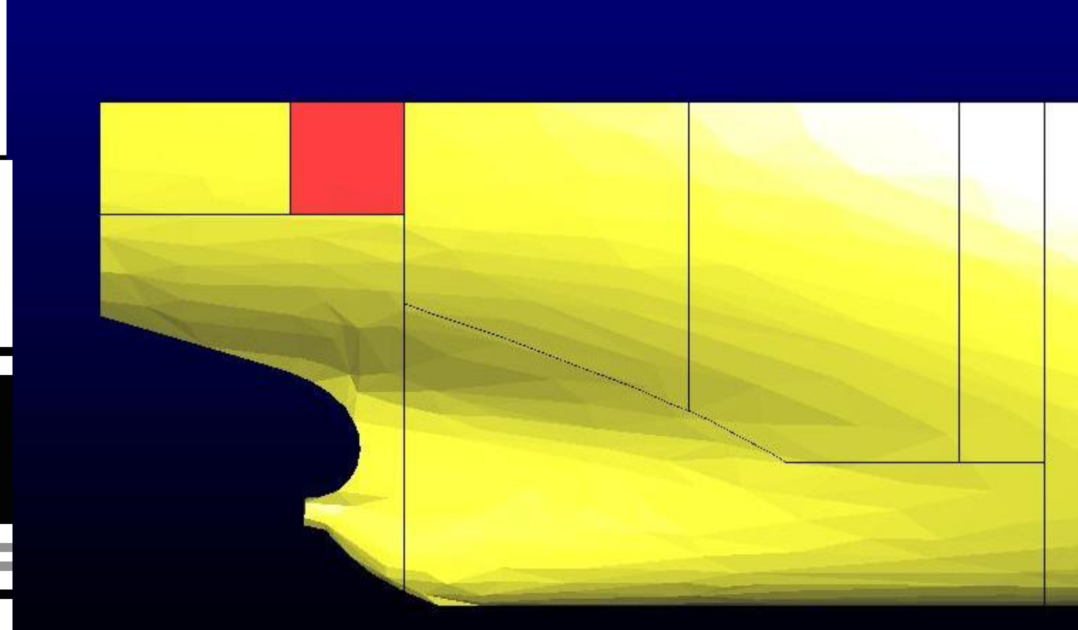
F.W.T. (Fresh Water Tank)

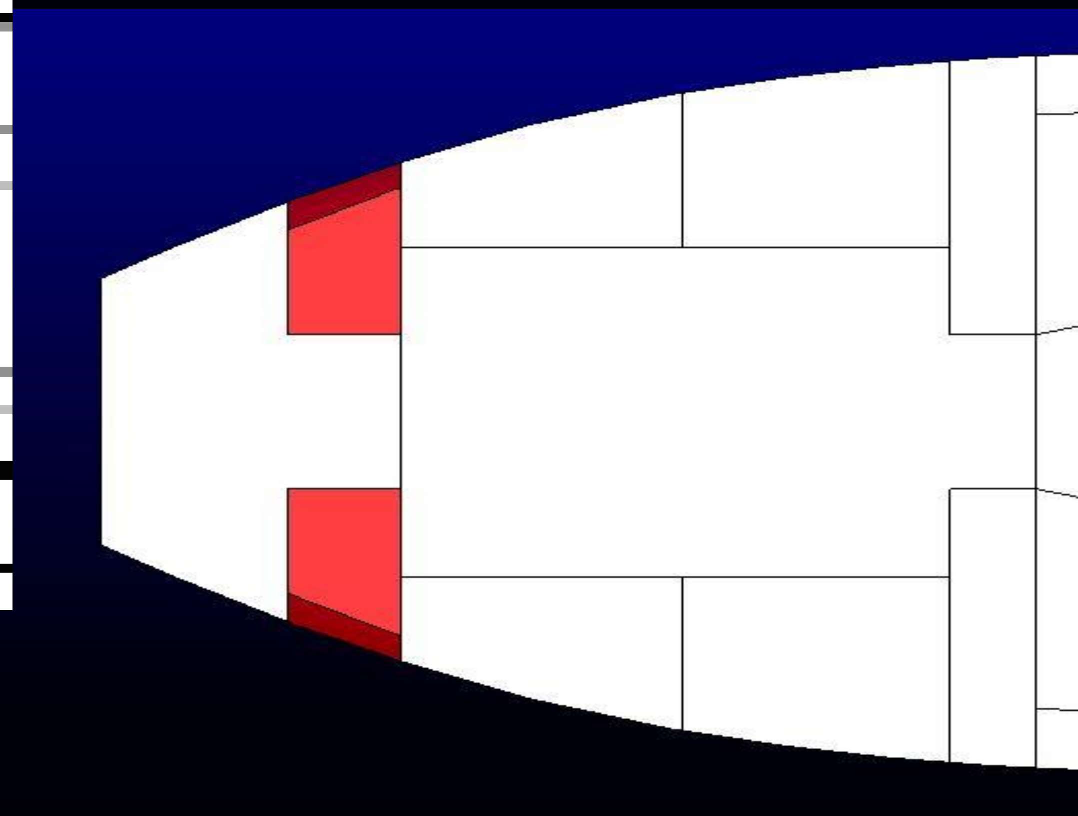
- ☑ Tanker & Bulker : Steering Gear Room 내 좌/우현
- ☑ Container : 기관실 앞 혹은 뒤쪽의 Passage Way 하부
- ☑ Distilled W.T와 Potable W.T로 구분하여 표시
- ☑ Greek Rule : Potable W.T와 Ballast T. 사이 void 설치

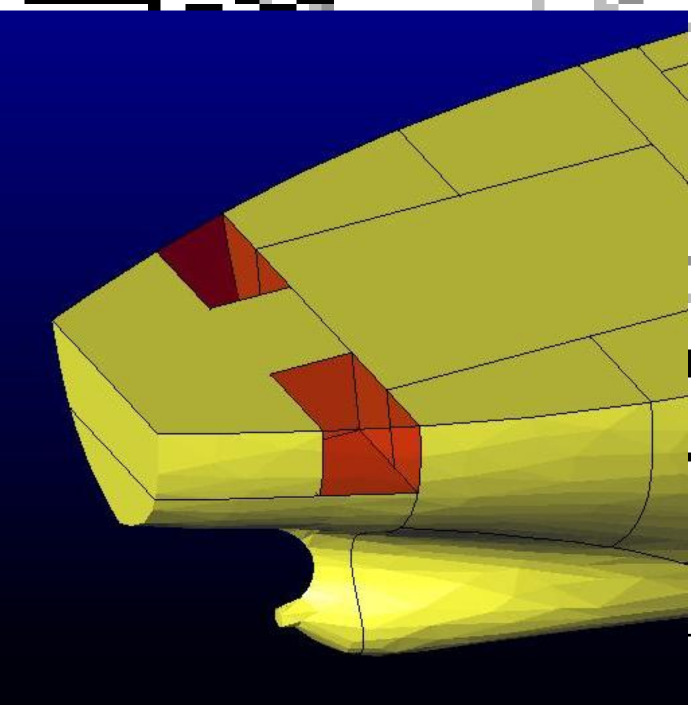


320K VLCC의 F.W.T (Fresh Water Tank)

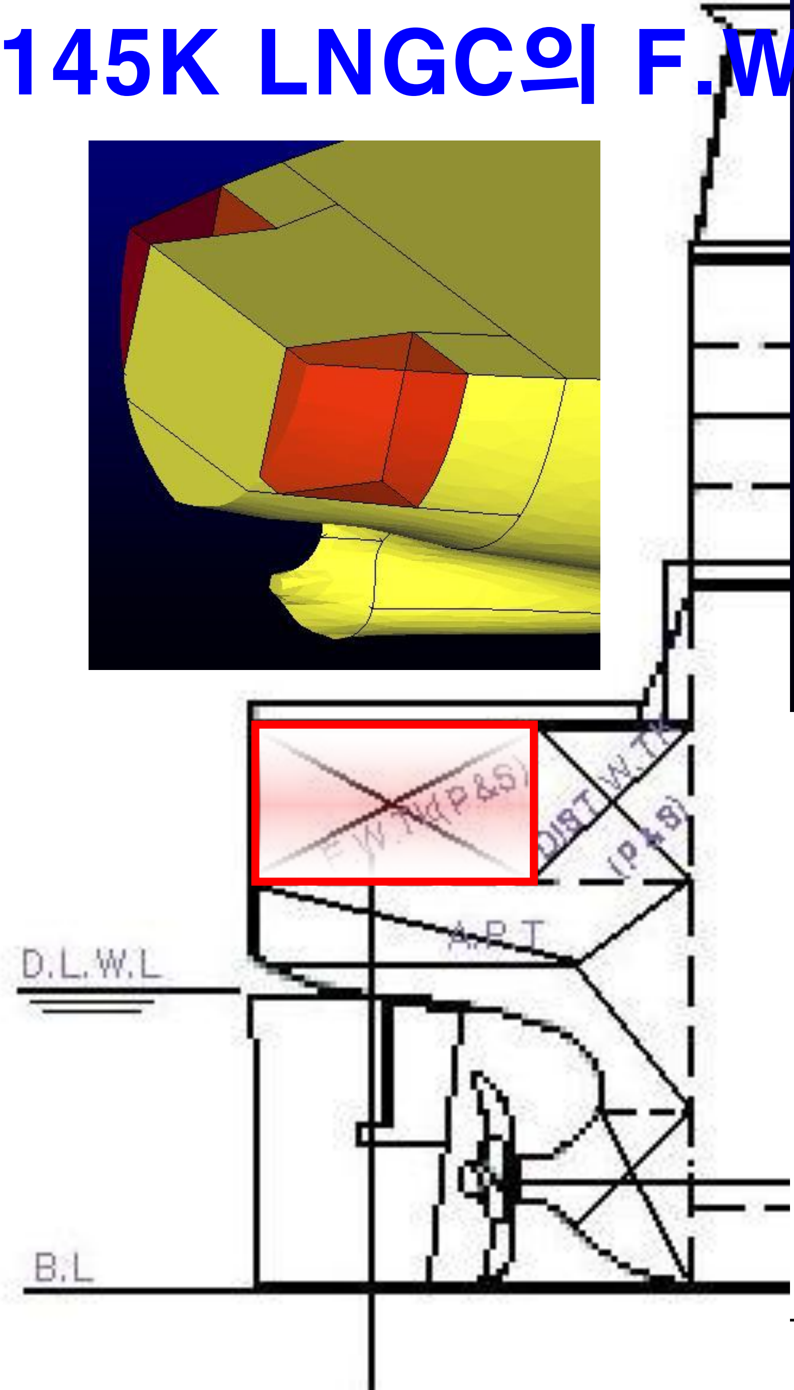


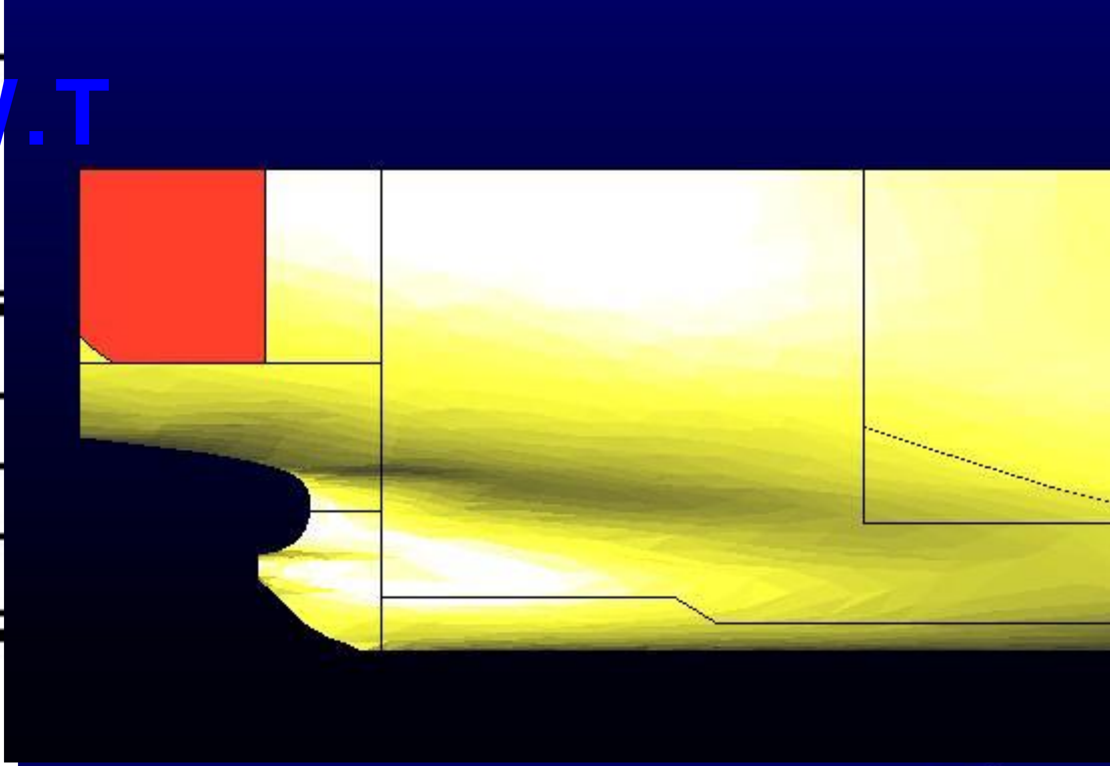


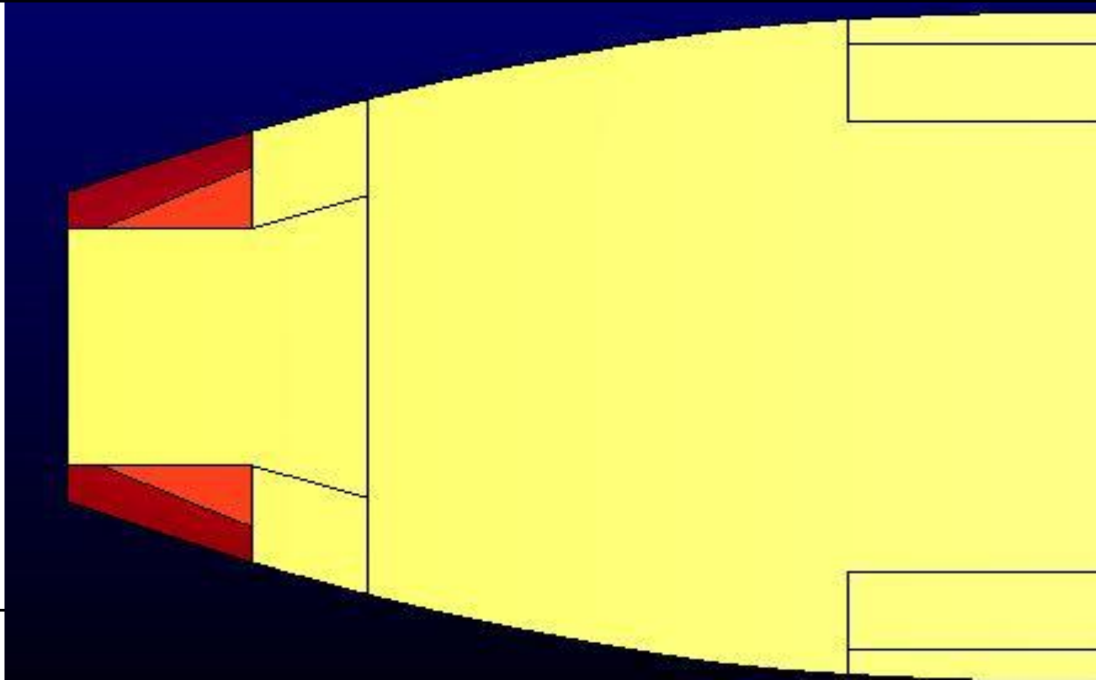




145K LNGC의 F.W.T



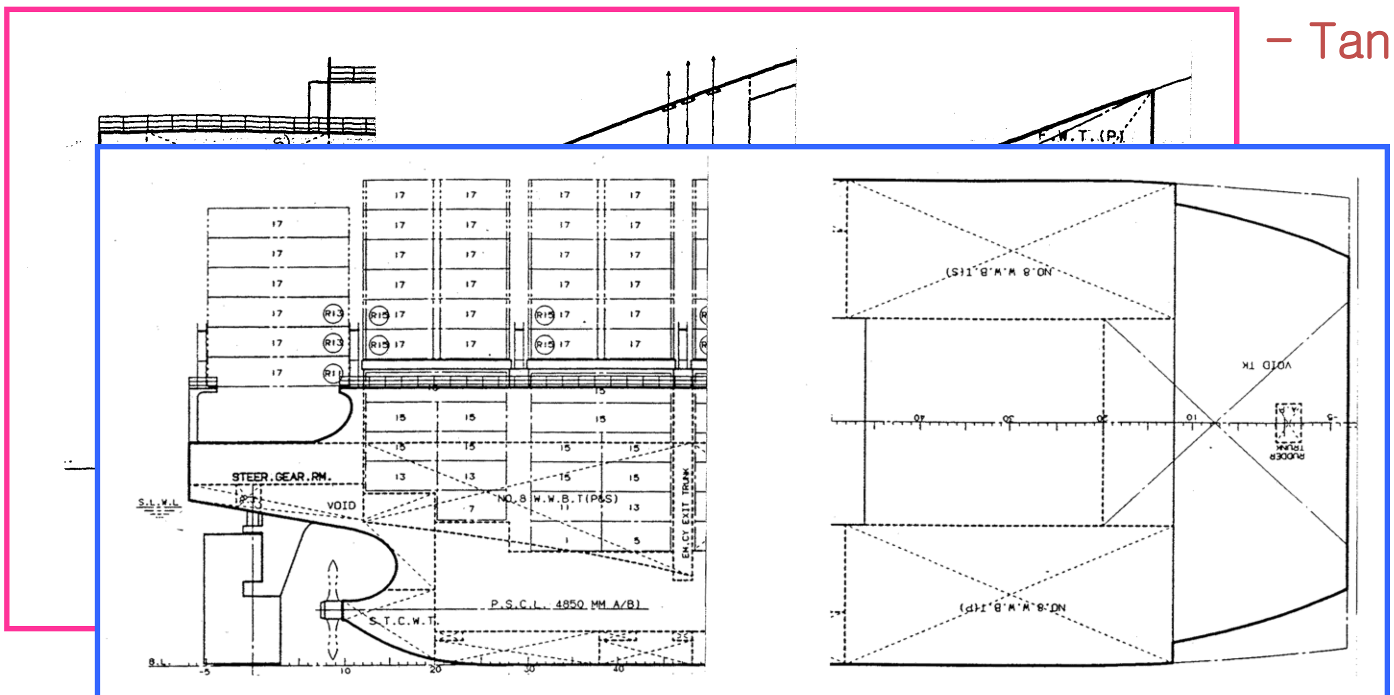




C.W.T. (Cooling Water Tank)

☑독립 Tank 혹은 APT와 일체형

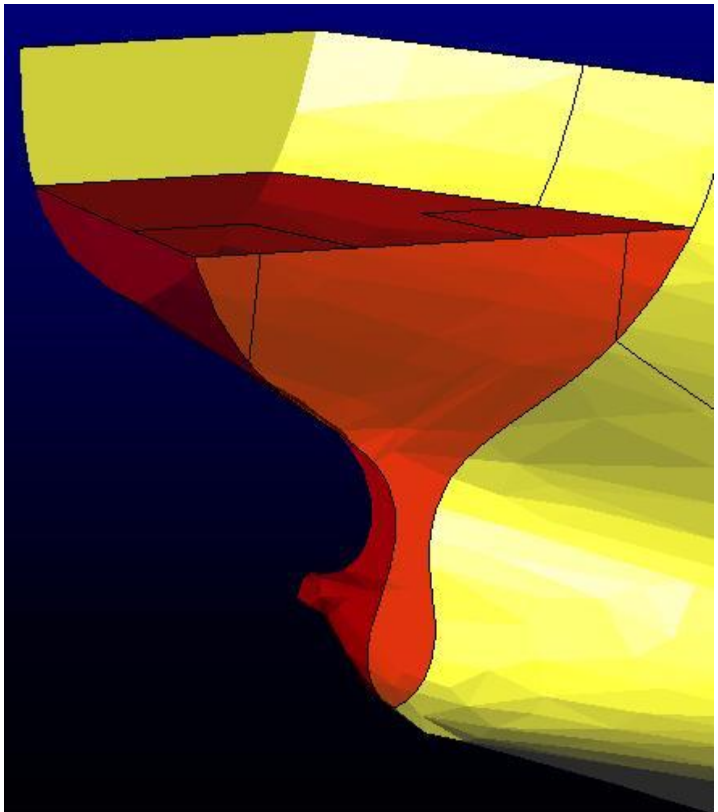
☑독립 Tank : Propeller Shaft 상방 0.3~0.5m로 하되 E/R 4th Floor 높이와 일치



– Tanker

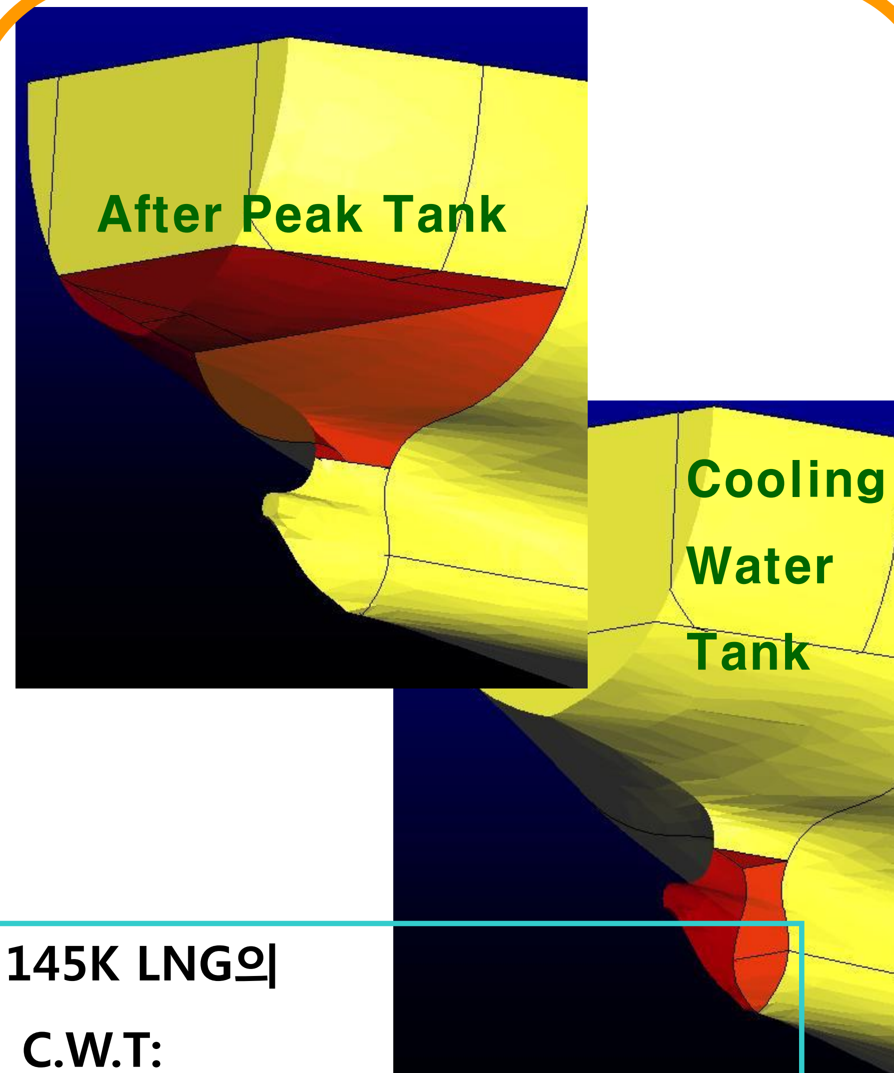
– Container

Cooling Water Tank



320K VLCC의 C.W.T:

A.P.T와 일체형



145K LNG의

C.W.T:

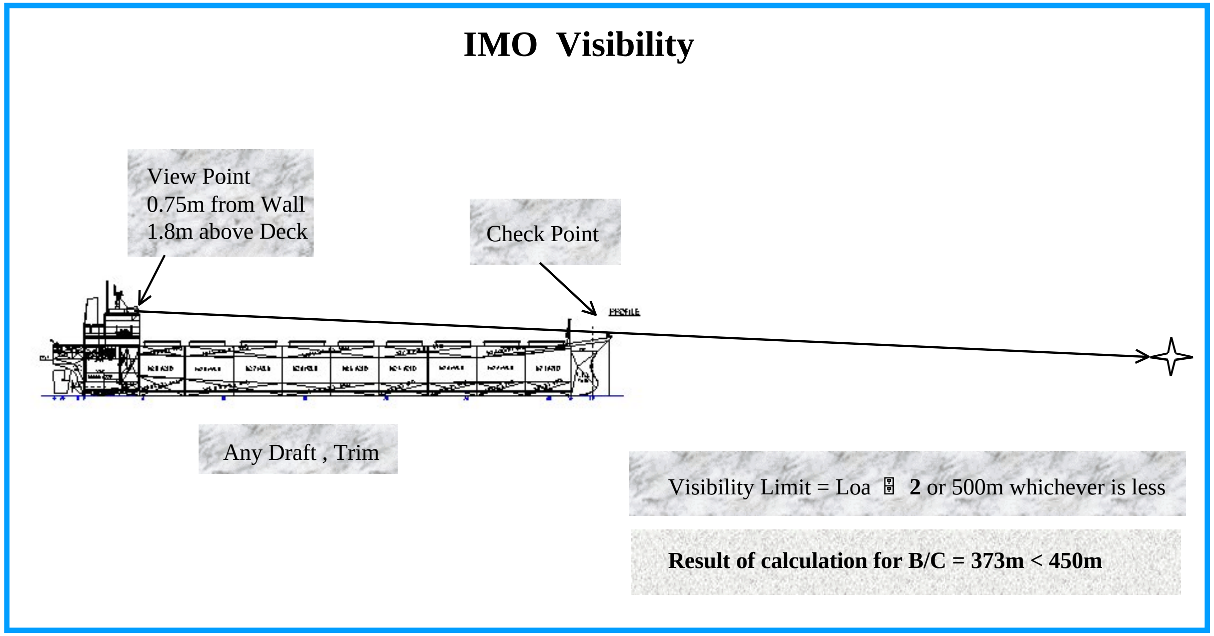
A.P.T와는 별개의 독립 Tank

Deck House

☑ Deck House의 설계는 선주 요구에 따른 거주 공간 확보가 무엇보다 중요하다.

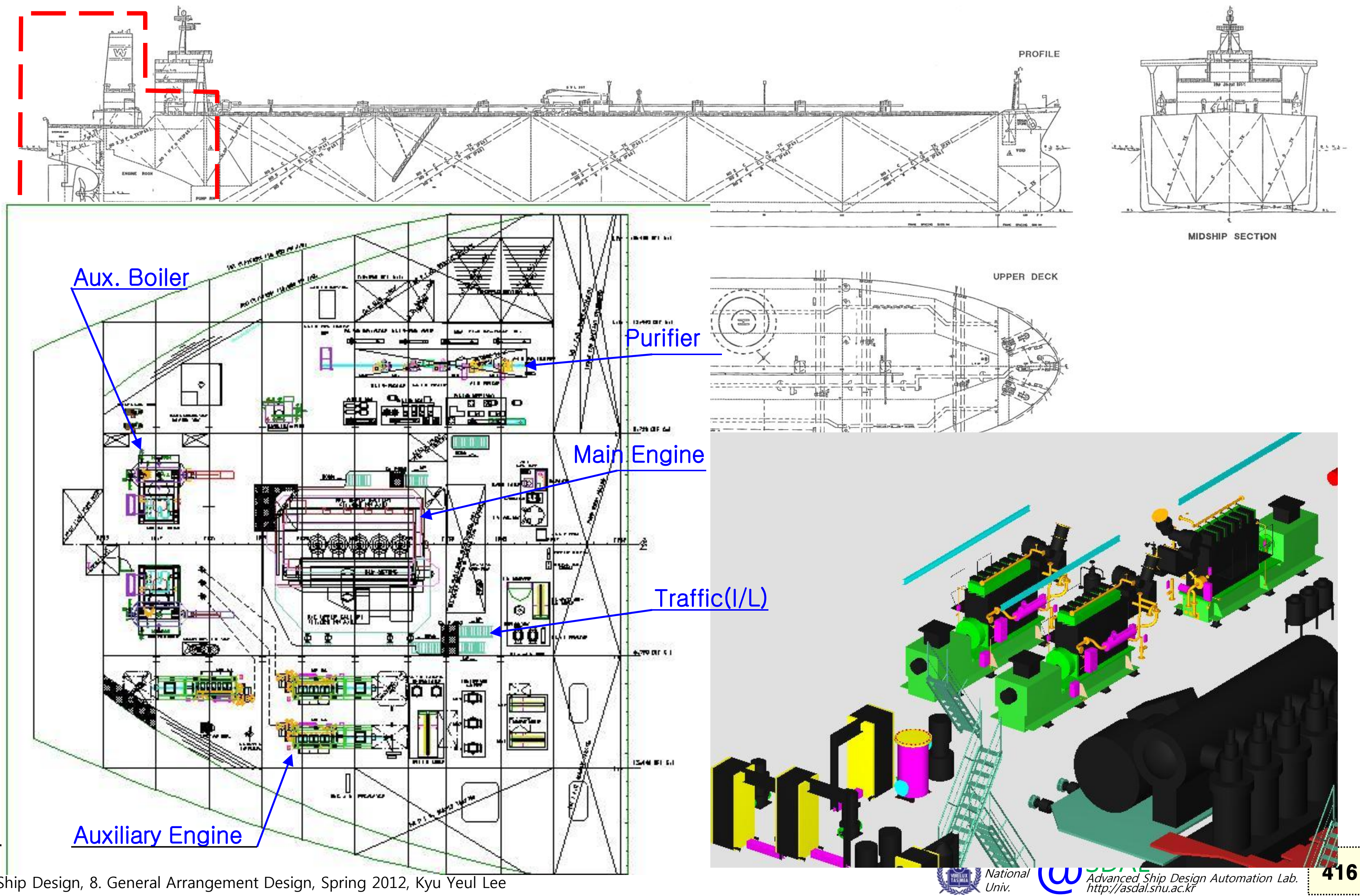
☑ IMO Visibility

- 배의 길이의 2배나 500m 중 작은 값



8-6 Major Equipment in Engine Room(E/R)

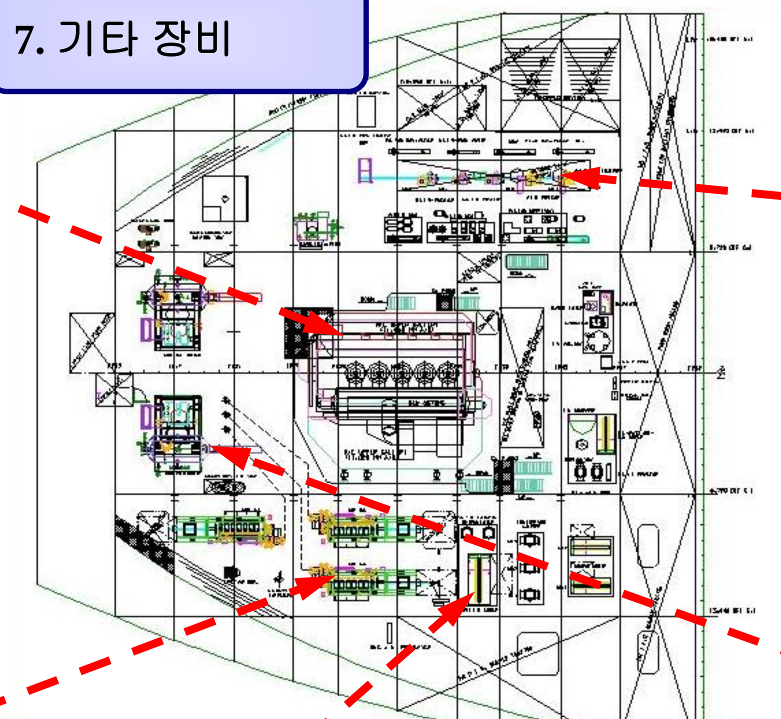
Engine Room(E/R)



Major Equipment in Engine Room(E/R)

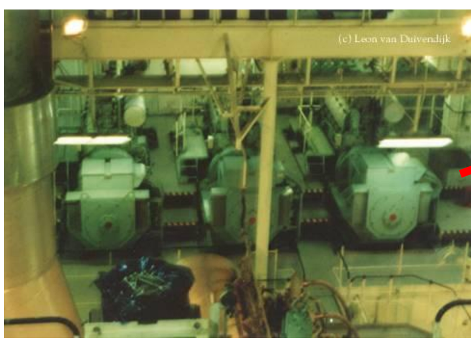
1. Main Engine

7. 기타 장비

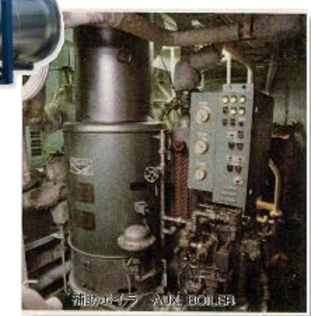


3. Purifier

2. Auxiliary Engine



4. Boiler



5. Fresh Water Generator

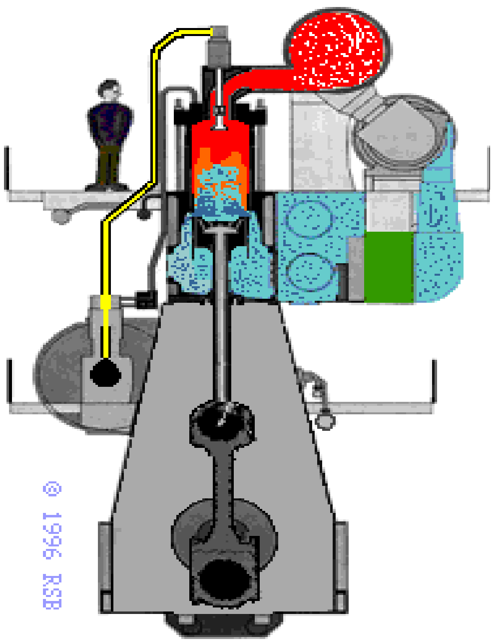
6. Main Air Compressor

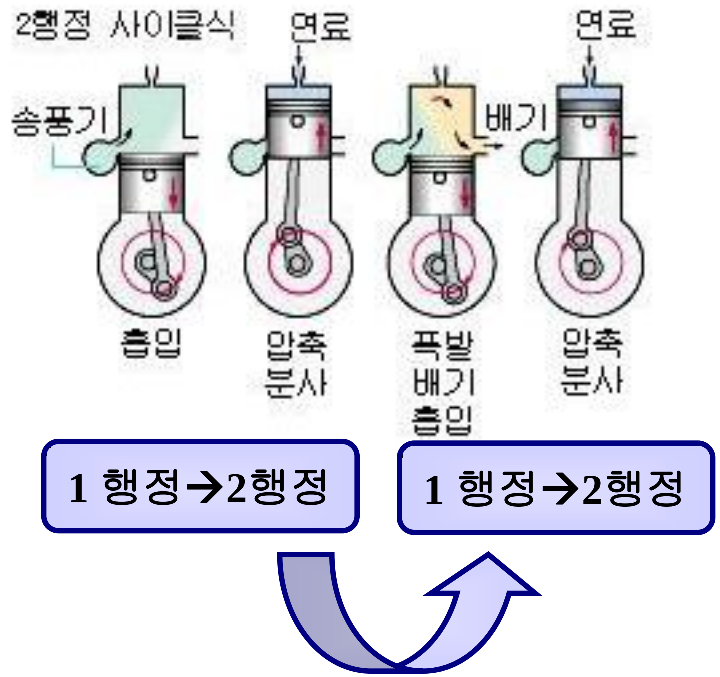
8. Compressed Air System

9. Cooling Water System

10. Fuel Oil System

1. Main Engine (주 추진기관)





- ☑ 700 cst/50˚C의 저질 중유(Heavy Oil)을 사용 → **Fuel Oil System** 구성을 위한 기기장비 및 Tank 설비(Circulation Pump, Viscosity, Purifier, Heater ..)
- ☑ Piston부위 마모 억제 → **L.O (Lubricate Oil) System** 구성을 위한 기기장비 및 Tank 설비
- ☑ Engine 냉각 → **Cooling System** 구성을 위한 기기장비
- ☑ 배기 가스(**Exhaust Gas System**)의 처리를 위한 설비

1 cst = 0.01 St = 0.000001 m2/s = 1 mm2/s

Viscosity(점도)

- **Absolute Viscosity(절대점도):점도란 액체 내의 전단속도가 있을 때 그 전단속도 방향의 수직면에서 속도의 방향으로 단위면적에 따라 생기는 전단응력의 크기로서 표시하는 유체의 내부저항이다. 점도의 차원은 질량×시간/면적이고 단위는 N.s/m2와 포아즈(Ps) 및 센티포아즈(cPs)를 쓴다. (1Ps =0.1N.s/m2, 1cPs = 1/100 Ps)**

- **Kinematics Viscosity(동점도):동점도란 점도를 그 액체의 동일상태(온도, 압력)에 있어서의 밀도로 나눈 값을 말하며, 그 차원은 (길이)2/시간이며, 단위로서는 m2/s와 보조단위로서 스톡크(St)및 센티스톡스(cSt)를 쓴다. (1St = 0.0001m2/s, 1cSt = 1/100 St)**

- 초당 일정지점에 머물러 있는 면적(점성이 높을수록 끈적함)

항목	디젤엔진	가솔린 엔진
연료	경유,석유	가솔린, LPG
연소사이클	사바테사이클	오토사이클
연료공급방식	분사펌프	기화기 혼합(가솔린은 실린더와 흡기매니폴드에 분사)
혼합기의 형성	압축공기에 연료를 안개상태로 분사(불균일 혼합)	흡입전에 연료와 공기가 혼합된 형태로 흡입(균일 혼합)
착화방법	압축열에 의한 자연착화	전기불꽃에 의한 점화
연소실형상	복잡	간단
압축비	16~23 : 1(공기만)	7~10 : 1(혼합기)
압축온도	500~550℃	120~140℃
폭발 압력	45~70kg/㎠	30~35kg/㎠
압축 압력	30~45kg/㎠	7~11kg/㎠
열효율	32~38%	25~32%
연료소비율	150~240g/Psh	230~300g/Psh
기관의 회전수	1600~4000rpm	2000~6500rpm
출력당 중량	5~8kg/㎠	3.5~4kg/㎠
시동 마력	5 Ps	1 Ps
용도	주로 지프, 버스, 트럭	주로 승용차
장점	·연료소비율이 적고 열효율이 높다. ·연료의 인화점이 높아서 화재의 위험성이 적다. ·전기점화장치가 없어 고장율이 적다 ·저질연료를 쓰므로 연료비가 싸다 ·배기가스는 유독성이 적다.	·회전수를 많이 높일 수 있다. ·마력당 무게가 적다. ·진동 소음이 적다. ·시동이 용이하다. ·보수와 정비가 용이하며 부속품 값이 싸다.

1 cst = 0.01 St = 0.000001 m2/s = 1 mm2/s

1.1 연료유의 분류

증류순서에 의한 일반적 분류

☑ **Gasoline (휘발유)**

- 원유를 정제하여 제일 먼저 증류되어 발생 (미국 : Gas, 영국: Spirit, 독일: Benzine)

☑ **Kerosene (등유)**

- Gasoline 다음으로 증류되는 유분으로 인화점 30~40°C, 비중 0.77~0.85 g/㎤, 탄소 85~86.5%, 수소 13.5~14.5%로 구성

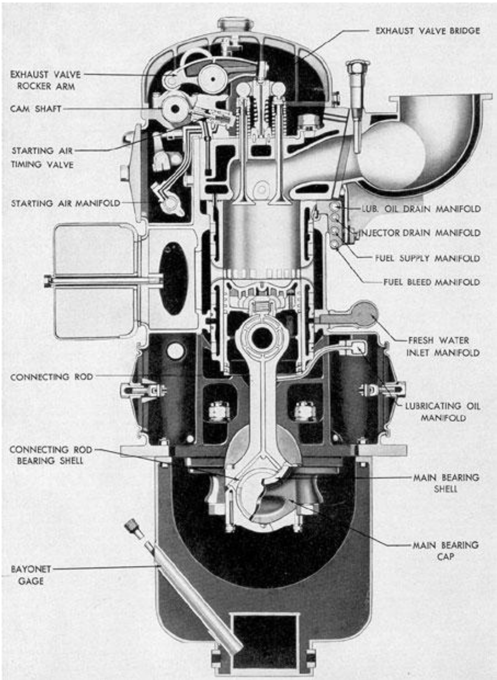
☑ **Light Oil or Gas Oil (경유)**

- 등유 다음의 유분으로 인화점 50~90°C, 비중 0.85~0.877
- 선박에 일반적으로 사용되는 Marine Diesel Oil(MDO)도 여기에 포함됨

☑ **Heavy Oil (중유)**

- 상기 유분을 뺀 마지막 뺀 잔유로서 잔사 연료유(Residual Fuel Oil) 혹은 연료유(Fuel Oil)이라 함
- 비중 0.9, 수분 0.2~1.0%, 회분 0.02~1.0%, 인화점 80~130°C
- 연소시 수분에 의해 소화(消火)되기 쉽고, 회분에 의해 Nozzle이 막히기 쉬우며 과도한 마모를 초래함
- 주로 디젤기관, 보일러 가열용, 화력발전용으로 사용되는 연료용 중유 (Heavy Fuel Oil)를 가리킨다

2. Auxiliary Engine (발전기 원동기)

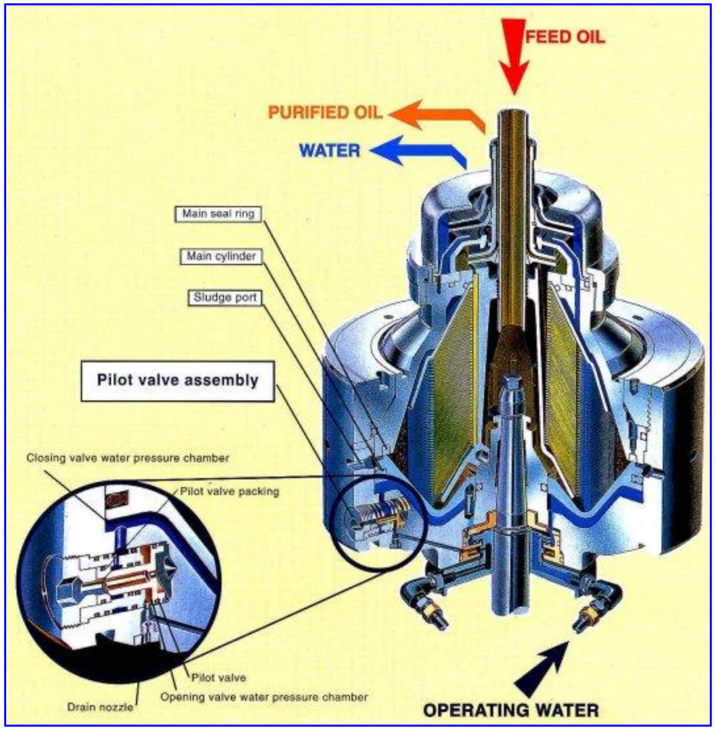


☑ 선내에 설치되는 모든 전기의 주 전원일 발전기를 구동하는 원동기로서 통상 3~4대가 설치됨

☑ 통상 중유(Heavy Fuel Oil)를 사용함

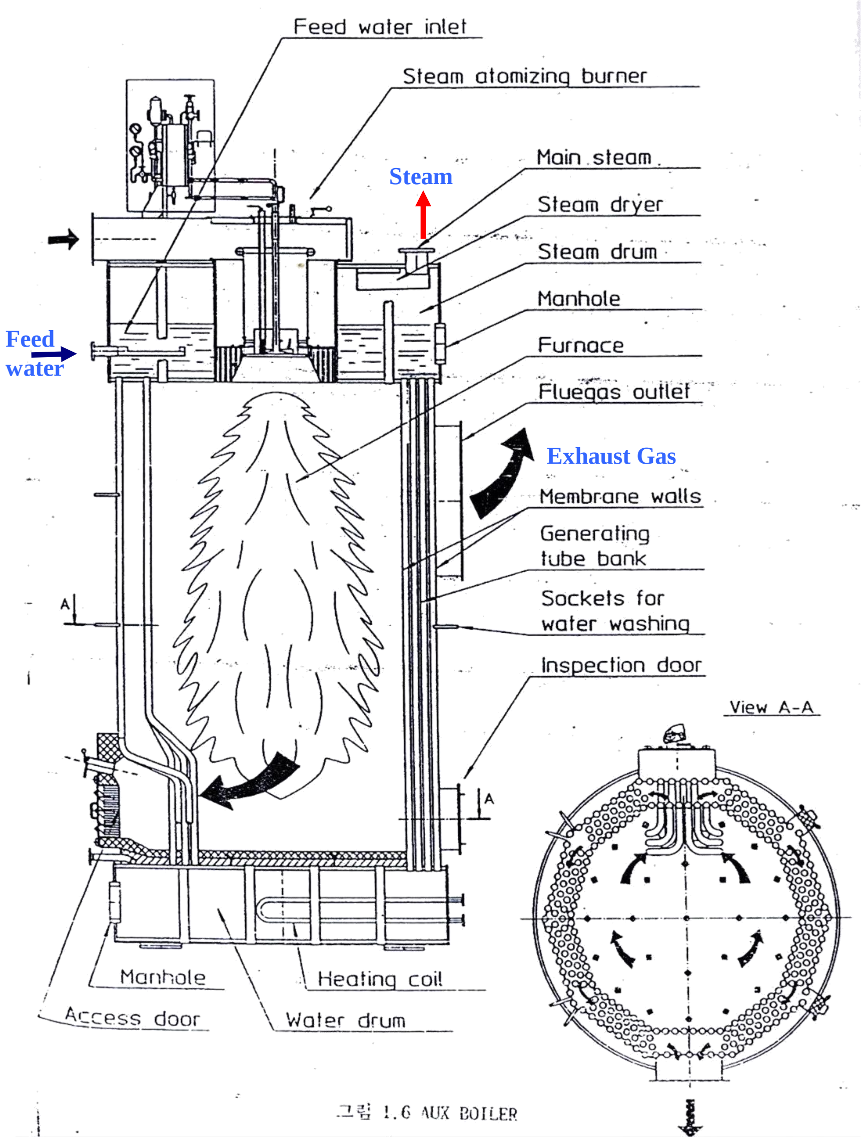
☑ Main Engine의 구동 방식과 거의 동일함으로 유사한 주변 설비 시스템을 구축하여야 함

3. Oil Purifier (유 청정기)

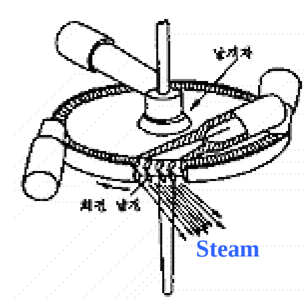


- ☑ Main Engine(주기관) 및 Aux. Engine(발전기), Boiler에 사용되는 연료유(H.F.O)에는 수분, 회분등의 불순물은 내연 기관의 연소상태를 나쁘게 하며 마모를 촉진시킨다. 또한, 윤활유도 장시간 사용함에 따라 수분과 불순물의 혼입이 발생한다.
- ☑ 이와 같은 연료유 또는 윤활유에 혼입된 수분 및 불순물을 제거하기 위해 Oil Purifier(유 청정기)를 사용한다.
- ☑ 작동원리는 Oil과 불순물 혹은 수분의 비중차를 고속 원심력을 이용 확대시켜 분리한다. (6000~8000 rpm)

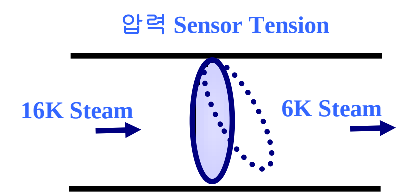
4. Boiler



- ☑ 선내의 난방, 취사 및 각종 가열장치에 소요되는 증기를 발생시키는 장치
- ☑ Oil Tanker의 경우 Cargo Oil Pump 및 Water Ballast Pump가 Steam 구동 Type일 경우 이를 위한 용량을 고려해야 함
- ☑ 일반 상선의 경우 주로 압력 7kg/cm2, 온도 169ºC정도의 저압 증기를 생산
- ☑ Tanker선의 경우 16Kg/cm2의 보일러로 부터 16K, 212ºC, 6K 168ºC, 4K 152ºC 등의 증기로 감압하여 사용

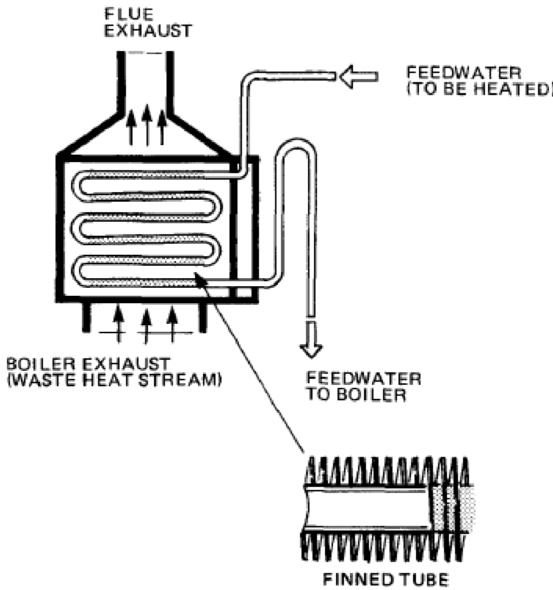


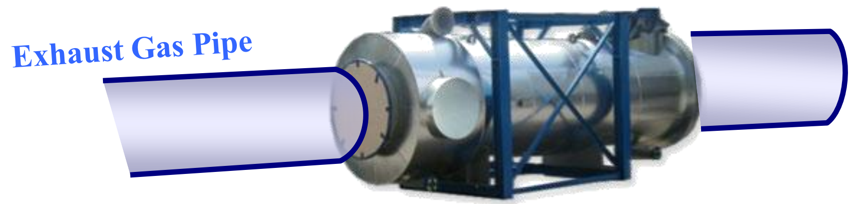
Cargo Oil Pump의 구동



감압밸브

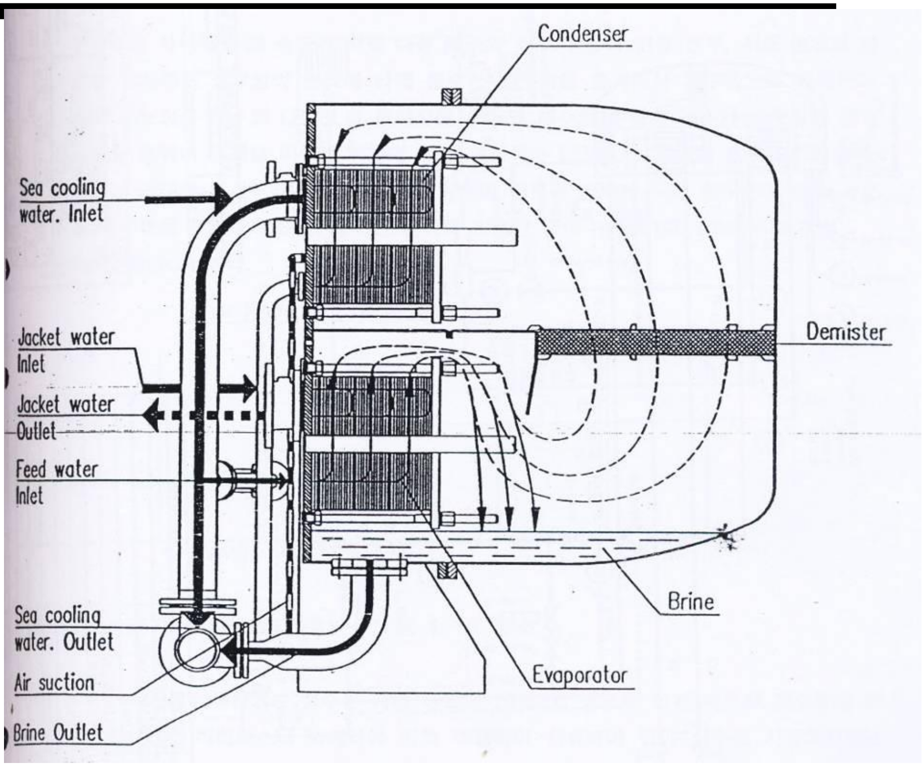
Exhaust Gas Boiler, Economizer (배기 보일러)



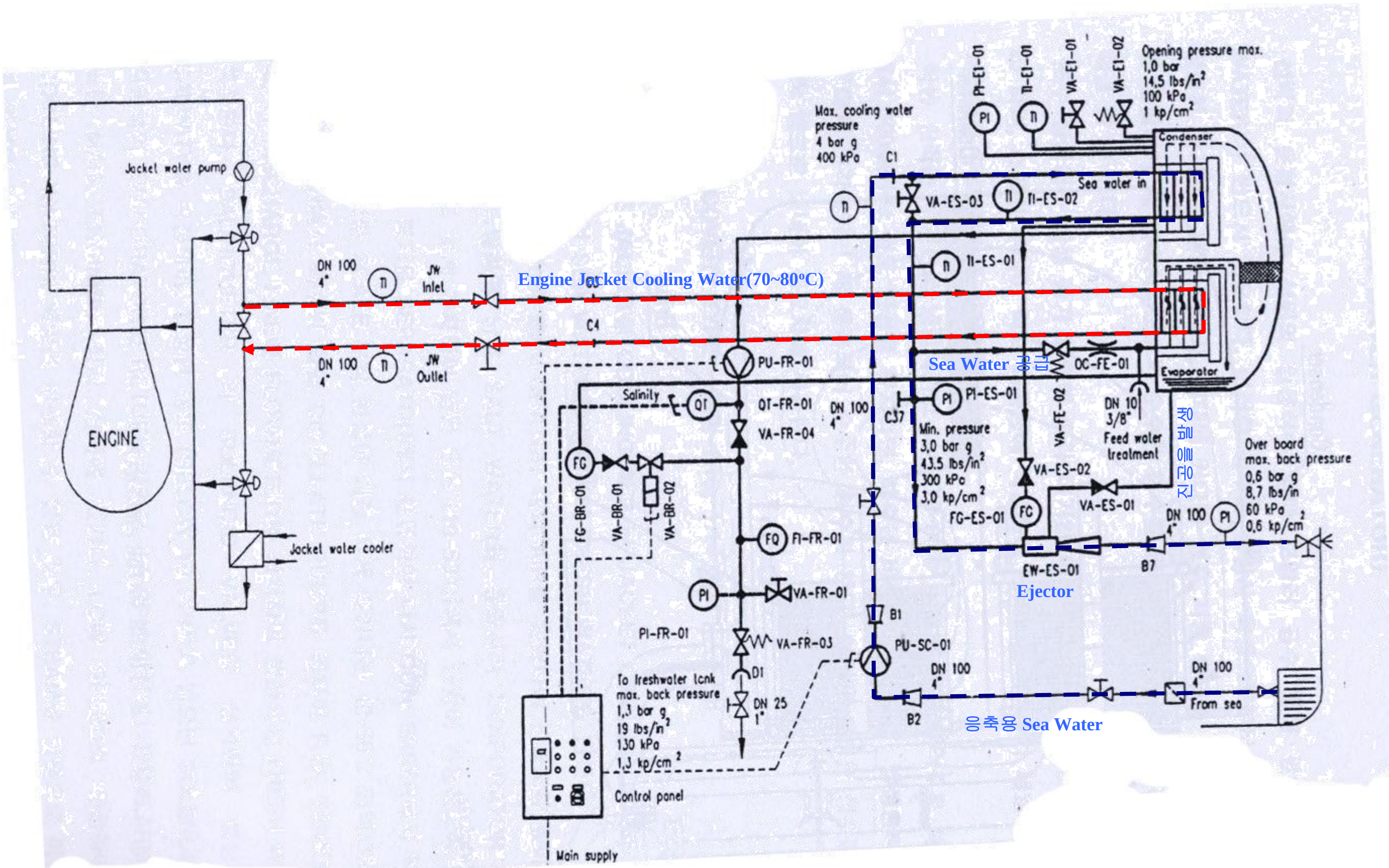


- ☑ 연료 절감을 목적으로 Main Engine의 배기온도가 약 250°C 정도인 것을 이용하여 증기를 발생시키는 장치
- ☑ Boiler내의 Boiler Water를 순환펌프로 Economizer를 통해 순환시켜 배기가스로 인해 가열하여 증기를 발생시킴
- ☑ Main Engine이 가동 중일 때만 증기발생이 가능함으로 항해시에만 운전됨

5. Fresh Water Generator (조수기)



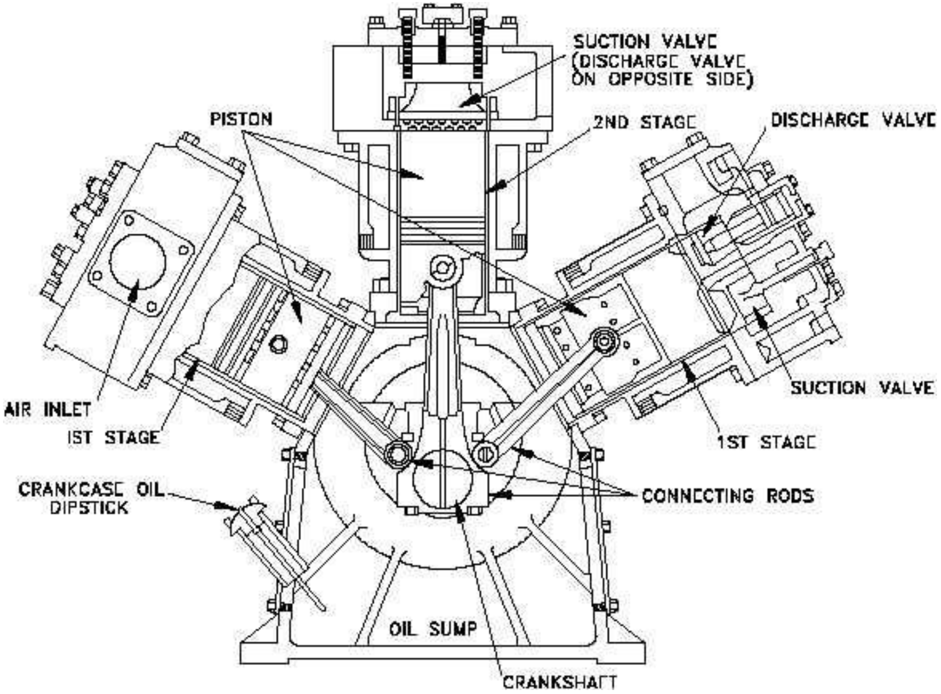
- ☑ 선원의 일용수 및 보일러 급수등에 필요한 청수를 생성하기 위한 장비로 해수를 증발시켜 재 응축시킴으로써 순수한 청수를 만든다
- ☑ 작동원리는 주기관을 냉각시키고 나온 70~80°C의 엔진냉각수로 해수를 가열/증발하여 Fresh Water를 얻는 Heat Recovery Type과 해수와 청수의 삼투압을 이용한 Reverse Osmosis Type이 있다



Engine Jacket Cooling Water(70~80ºC)
Sea Water 공급
진공을 발생
Ejector
응축용 Sea Water
Jacket water pump
ENGINE
Jacket water cooler
DN 100 4"
JW Inlet
JW Outlet
Max. cooling water pressure 4 bar g 400 kPa
Opening pressure max. 1,0 bar 14,5 lbs/in² 100 kPa 1 kp/cm²
Condenser
Evaporator
Sea water in
Salinity
Feed water treatment
Min. pressure 3,0 bar g 43,5 lbs/in² 300 kPa 3,0 kp/cm²
Over board max. back pressure 0,6 bar g 8,7 lbs/in 60 kPa 0,6 kp/cm²
From sea
To freshwater tank max. back pressure 1,3 bar g 19 lbs/in² 130 kPa 1,3 kp/cm²
Control panel
Main supply
PU-FR-01
QT-FR-01
VA-FR-04
FI-FR-01
VA-FR-01
PI-FR-01
VA-FR-03
PU-SC-01
EW-ES-01
VA-ES-01
VA-ES-02
VA-ES-03
TI-ES-01
TI-ES-02
PI-ES-01
FG-ES-01
OC-FE-01
VA-FE-02
PI-E1-01
TI-E1-01
VA-E1-01
VA-E1-02
FG-BR-01
VA-BR-01
VA-BR-02

- ☑ **Main Engine Jacket을 Cooling 하고 난 Jacket Cooling Water의 남은 열(70~80ºC)을 이용하여 Sea Water를 증류시키는 방식**
- ☑ **Air Ejector로써 Evaporator내의 Air를 흡출하여 진공도를 높여 낮은 온도(40~50ºC)에서 증발이 가능하도록 함**
- ☑ **여기서 발생된 증기를 다시 Condenser 에서 복수(復水)시킴 즉 Sea Water를 조수기 공급용으로도 사용하고, 증기의 응축을 위해 온도를 낮추는데도 사용함, 또한 Ejector를 이용하여 Evaporator내의 공기를 흡출하여 진공을 만드는데도 기여함 (기차가 빠르게 지나가고 나면 주위 공기가 빨려 들어가 순간적으로 진공이 이루어지는 원리)**

6. Air Compressor (공기 압축기)



☑ 압축공기는 Main Engine 및 Aux. Engine의 시동에 이용되며 각종 제어장치, 원격 조정장치, 계측, 경보 및 기계정비시 소제용으로 사용됨

☑ Main Engine 과 Aux. Engine의 시동에는 30kg/cm2의 고압공기가 이용됨으로 2대이상의 왕복동(Piston 방식) 공기 압축기로 압축공기를 생산하여 시동 공기조(Starting Air Reservoir)에 저장하고 이를 시동에 사용함

7. 기타 주요 기기장비(1)

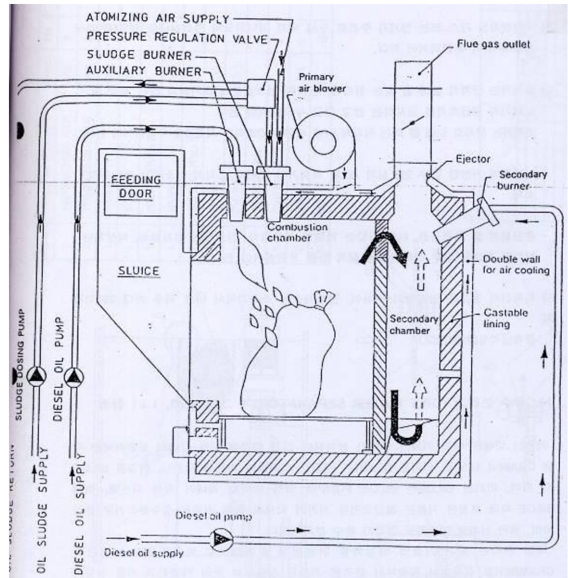
Sterilizer

Rehardening Filter

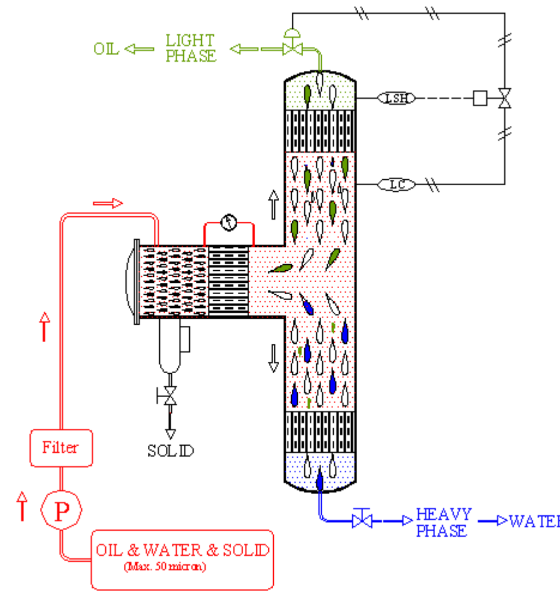
Hot Water Calorifier

- ☑ Sterilizer(살균기) : Fresh Water Generator(조수기)로부터 생산되는 청수(Fresh Water)는 가열 온도가 70~80oC로 낮기 때문에 해중에 포함되어있는 바이러스나 박테리아 등의 미생물이 함유되어 식용으로 사용하기 부적절함 → 자외선 살균식, 음이온전해식, 염소주입식등의 살균기를 설비함
- ☑ Rehardening Filter(경수화장치) : 조수기로부터 생산되는 청수(Fresh water)는 증류수(Distilled Water)로서 식용으로 사용할 수 없으며, 조수기 내부에서 과냉각 응축되어 공기중의 CO2를 흡수하여 산성으로 된다. 이러한 청수를 수산화 이온(OH-)을 발생시키는 화합물에 통과시켜 PH치를 상승시켜 약 알칼리수로 만들며 칼슘(Ca), 마그네슘(Mg)등도 녹아 들어가게 하여 경수화하여 자연수와 같은 음용수로 변환시키는 장치
- ☑ Hot Water Calorifier (온수 가열기) : 선내에 필요한 온수를 공급하기 위해 청수를 가열시키는 장치로서, 주로 증기 혹은 전기를 이용한다. (약 70~80°C)

Swage Treatment Plant



Incinerator



Oil Seperator

☑ **Sewage Treatment Plant (오수처리기)** : 선내에서 발생하는 오수 및 폐수를 해상오염을 방지하기 위해 미생물학적(Biological Type) 혹은 화학적(Chemical Type)으로 분해하여 배출하는 장비

☑ **Incinerator (소각기)**: 생활 폐기물, 연료유나 윤활유의 폐유, 기관실 Bilge로부터 분리해낸 기름등을 선내에서 소각하는 장비 (예열온도 : 650°C, 배기가스온도: 850~1200°C)

☑ **Oil Seperator (유수분리기)** : 기관실내의 장비의 운전시 발생되는 Drain은 최하부 Floor에 모이는데 이를 Bilge라 함. 통상 Bilge는 Bilge Pump에 의해 바다로 내보내게 되는데 이때 물과 기름을 분리하는 장치

$2\,NaCl + 2H_2O \rightarrow H_2 + Cl_2 + 2NaOH$

양극 (anode) : $2Cl$ --- $Cl_2 + 2e$

음극(Cathode) : $2Na + 2H_2O + 2e$ – $2\,NaOH + H_2$

↓

$2NaOH + Cl_2 \rightarrow NaClO + NaCl + H_2O$

↓

$NaClO + H_2O \rightarrow HClO + NaOH$

☑ **M.G.P.S (Marine Growth Preventing System**

선박의 Cooling System을 구성하고 있는 기기 및 Piping에 해수가 유입됨으로써, 해수에 의한 생성물 (Growth of Micro-Organisms, Shells, Slime, Seaweed etc.)이 고착하여 Piping 및 기기내부에 Damage를 입히고 나아가 원활한 Flow를 방해하고 관로를 차단하는 결과를 초래함

이러한 문제를 해결하기 위해 전기 화학적으로 여러 가지 해양 생성물들의 고착을 저지시키는 장치를 말함

일반적으로 해수에 15,000~2000 ppm의 염소가 ION상태로 존재하는데 이를 전기분해하여 결론적으로 일반 염소보다 수십배 내지 수백배의 강력한 살균작용을 가진 NaClO(차아염소산) 및 HClO(하이포아염소산)을 생성하여 투입하는 방법임

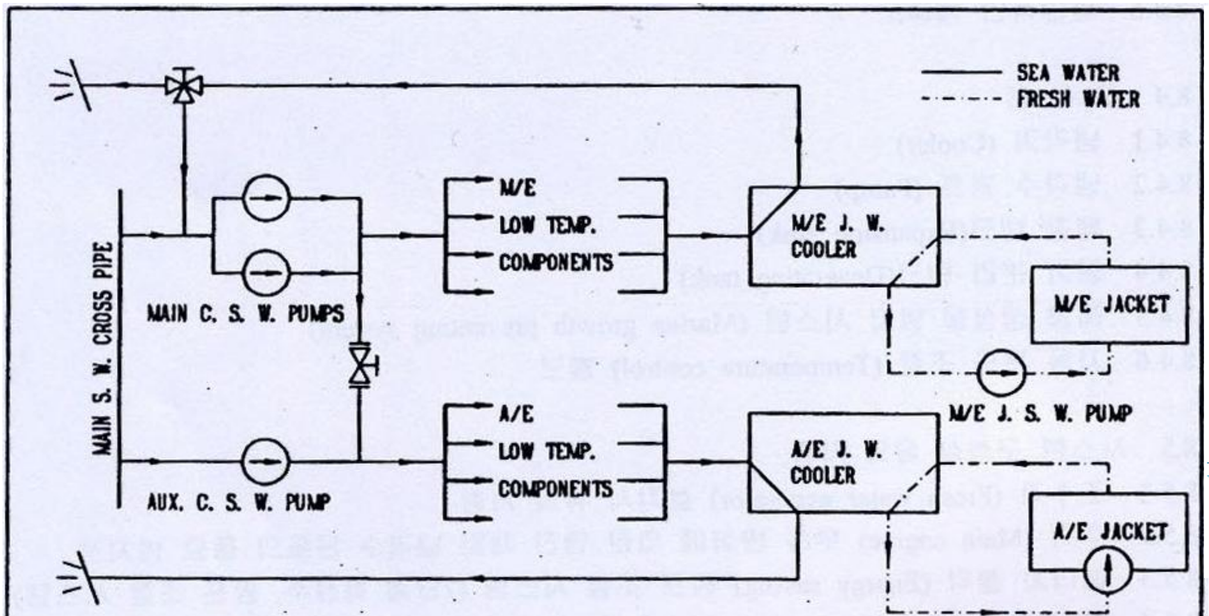
8. Compressed Air System의 분류

☑ **Control Air System** : Main Engine Maneuvering, Control Valve, Pneumatic Gauge, 각종 기기류의 Automatic Control등 자동제어장치 등의 작동용으로 사용되는데 Main Reservoir로부터 Pressure Reducing Valve를 통해 감압시켜 사용하거나, Control Air Compressor과 Reservoir를 전용으로 설치하기도 한다 (Control Air의 경우 System 내의 정밀한 부위를 통과해야 함으로 Control Air Dryer로 먼지,수분, 유분 등을 제거해 주어야 한다)

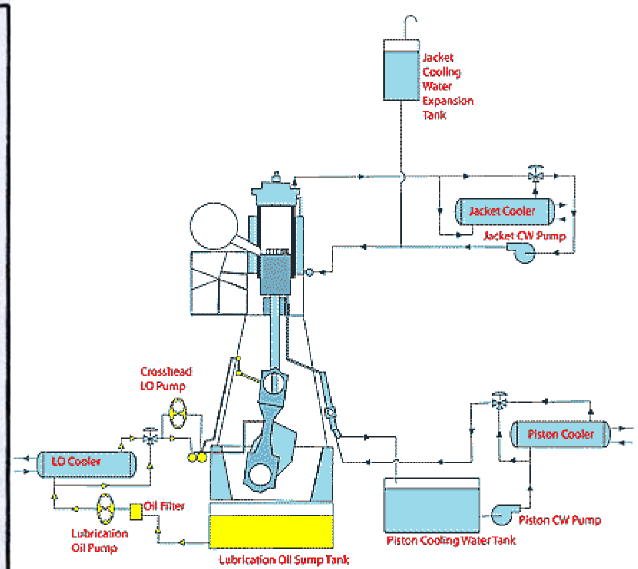
☑ **Service Air System** : Radar Mast 와 Funnel Top의 Air Horn, Fire Alarm 및 주요 장비 근처의 청소용으로 사용되며 주로 Main Air Reservoir의 고압공기를 감압시켜 Service Air Reservoir를 채우거나 별도로 Compressor를 설치하기도 한다

☑ **Quick Closing Air System** : 선박의 화재 발생시 인화물질인 Oil이 F.O or L.O Tank로부터 유출되어 화재가 확산되는 것을 미연에 방지하고 Tank Outlet Pipe Line이 Damage를 입었을 때 Oil 누출을 방지하기 위해 Engine Room Outside에서 주요 Valve류등을 Remote Shut-off 시키는 System

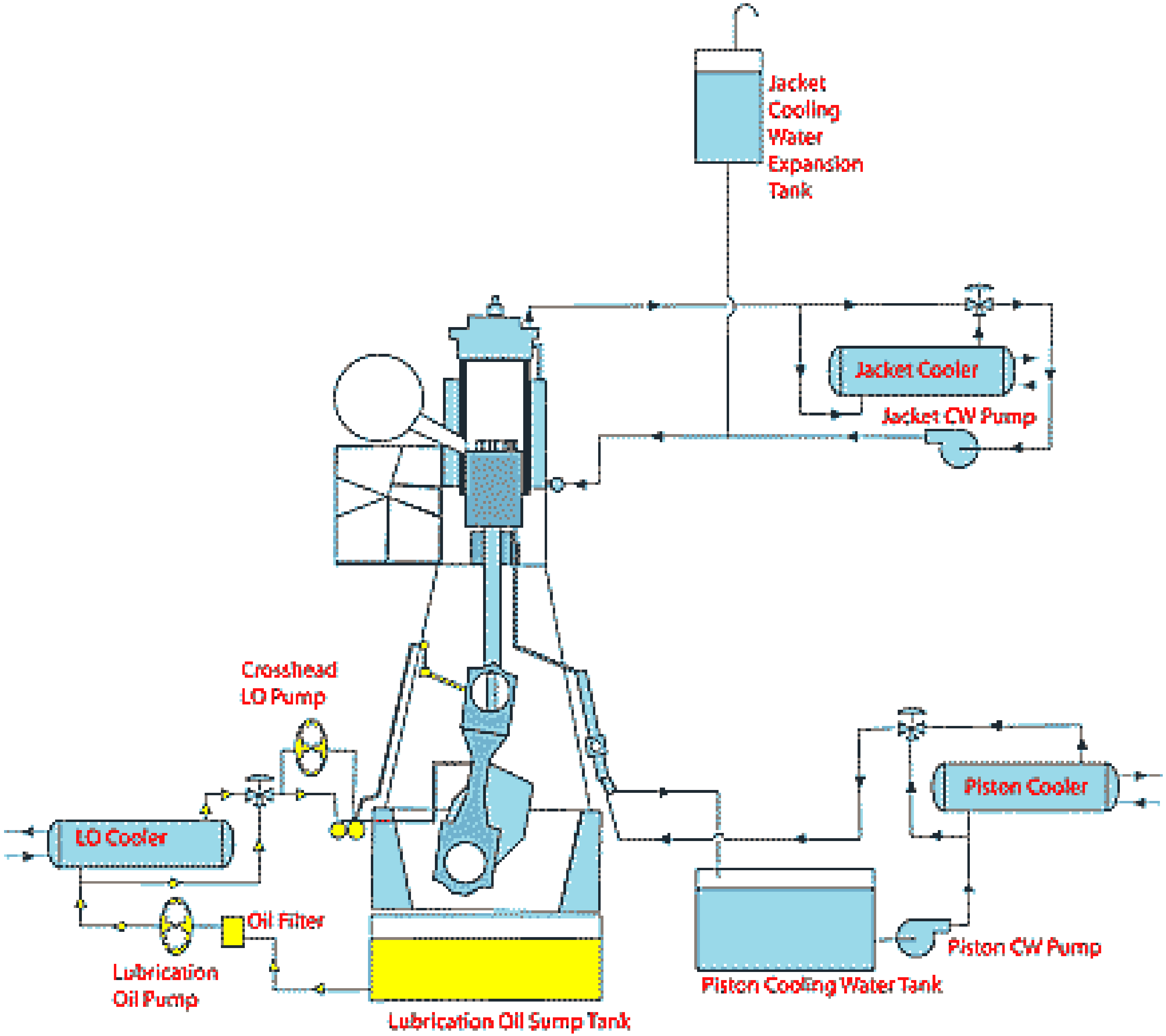
9. Cooling Water System- Conventional Cooling System



Conventional Cooling System



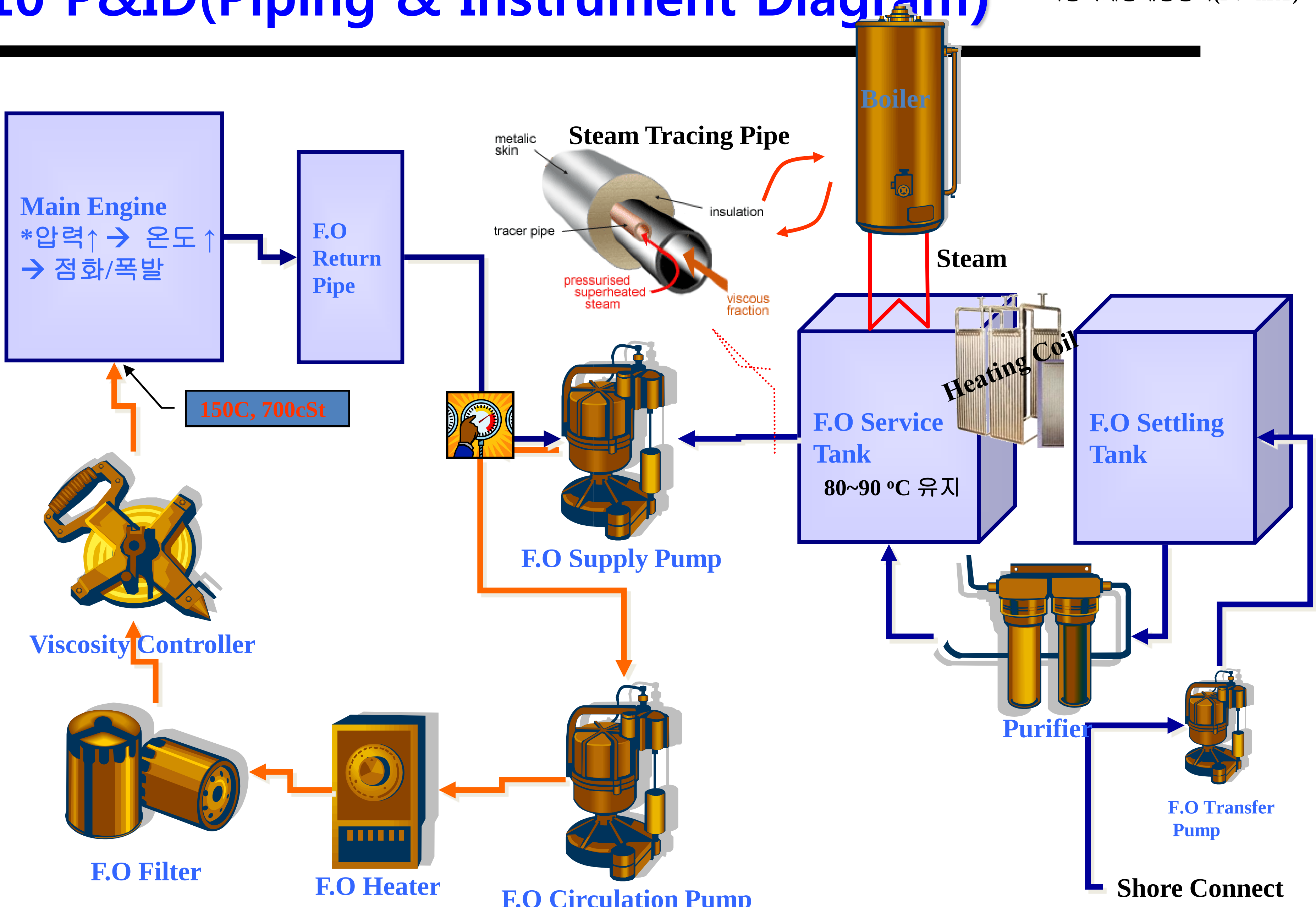
- Cooling Water System : 기관실내 Main Engine(주기), Aux. Engine(보기)등의 내연기관의 냉각, Exhaust(폐기) Steam의 응축 및 냉각을 위해 구성하는 시스템
- Conventional Cooling System
  Main Engine 및 Aux. Engine의 Cylinder Jacket을 Fresh Water(청수)로 냉각시키고, 그 외의 장비는 Sea Water(해수)로 냉각시키는 시스템으로 해수 냉각 시스템은 크게 2개 그룹, 주 해수 냉각 Pump에 의해 해수 냉각수가 공급되는 주기관련 장치 그룹과 보기 해수 냉각 펌프에 의해 해수 냉각수가 공급되는 보기 관련 장치 그룹으로 나눌 수 있다.
  장비 기능별로 독립적인 해수 냉각 시스템을 형성함으로써 펌프 구동비 절감 및 시스템 작동에 장점이 있으나 대부분 관이 해수로 이루어져 관부식에 대한 결점이 있다.

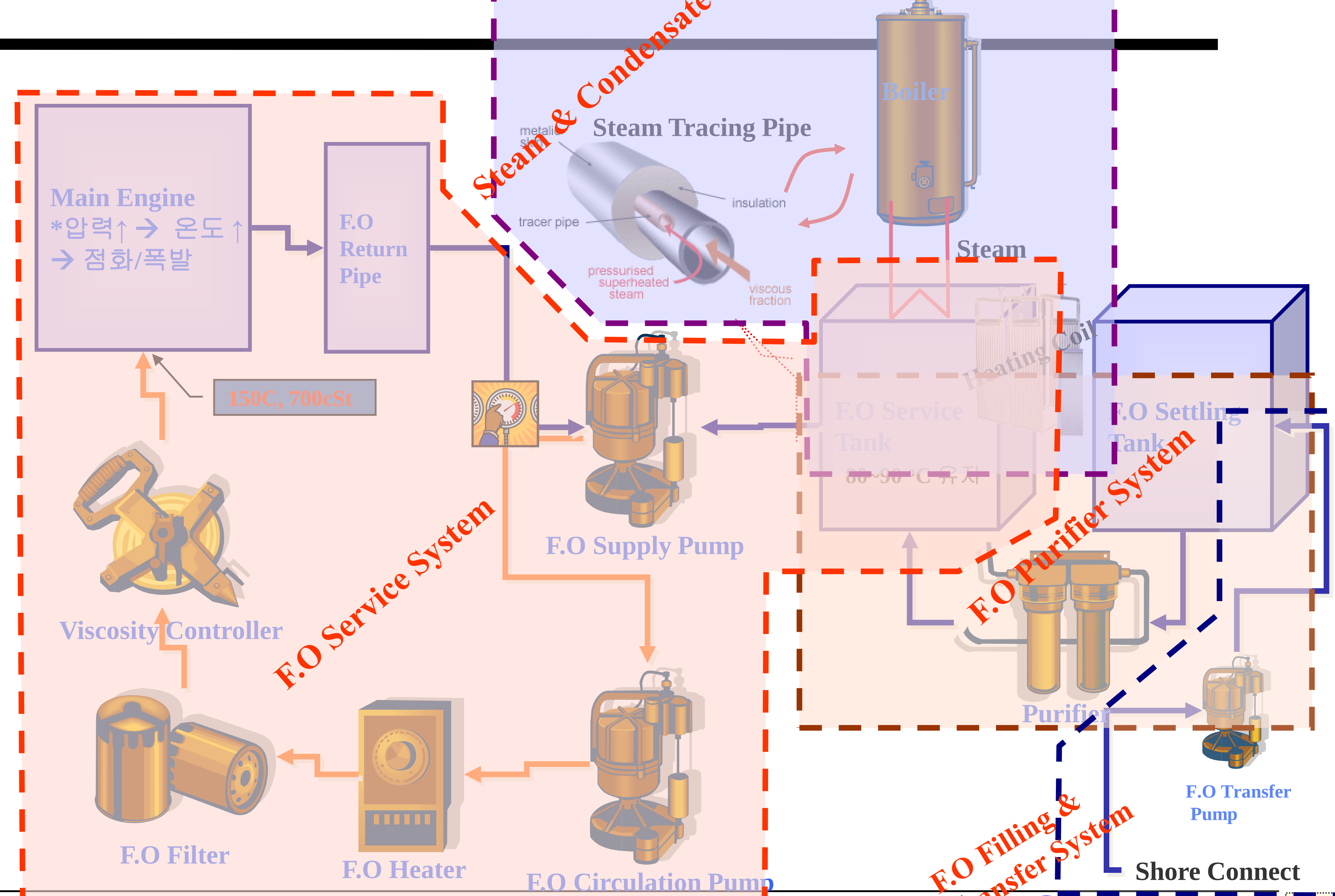


Jacket
Cooling
Water
Expansion
Tank
Jacket Cooler
Jacket CW Pump
Crosshead
LO Pump
LO Cooler
Oil Filter
Lubrication
Oil Pump
Lubrication Oil Sump Tank
Piston Cooler
Piston CW Pump
Piston Cooling Water Tank

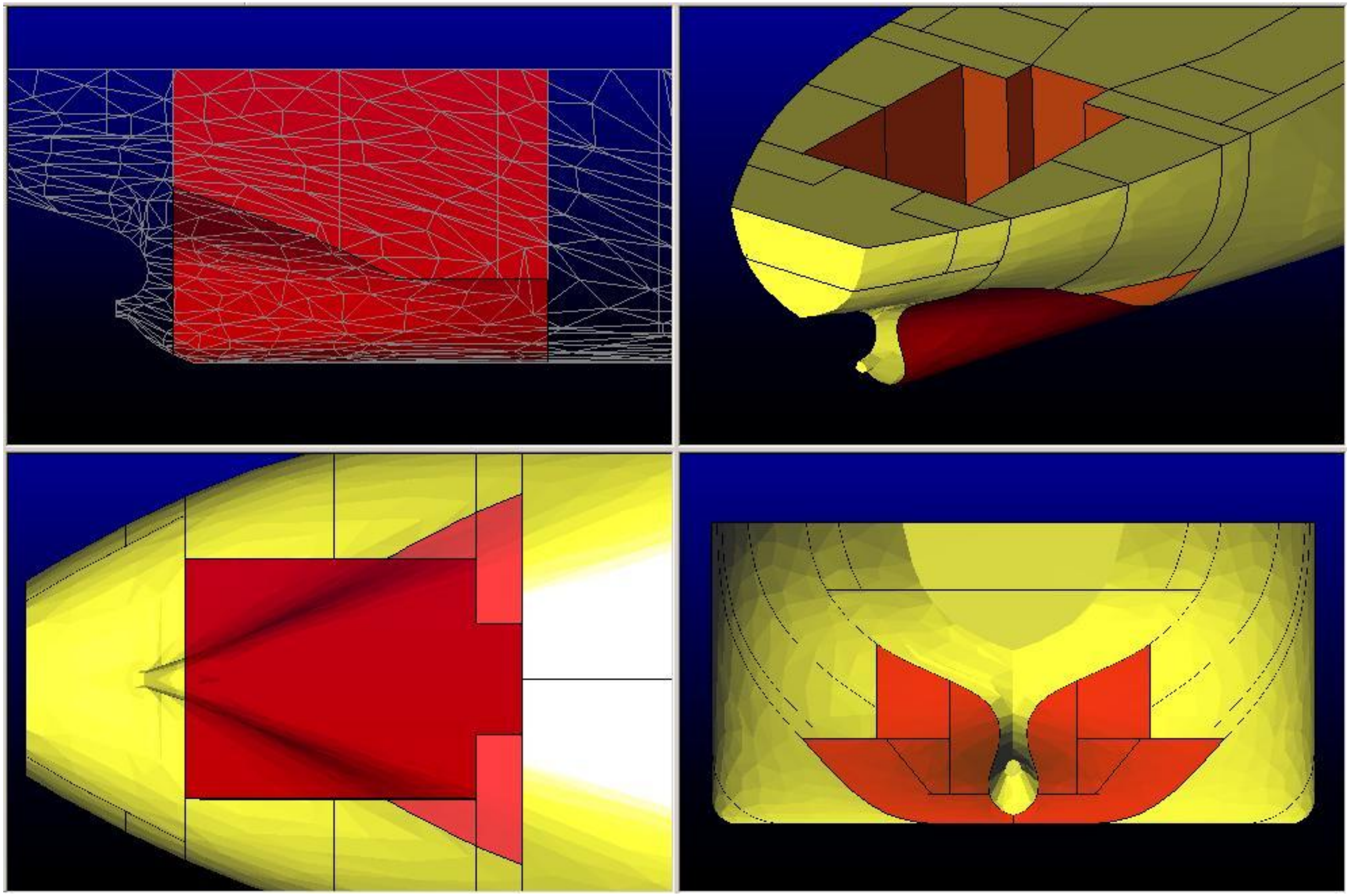
10 P&ID(Piping & Instrument Diagram)

* 보일/샤를의 법칙(PV/T= const)
이상기체상태방정식(PV=nRT)

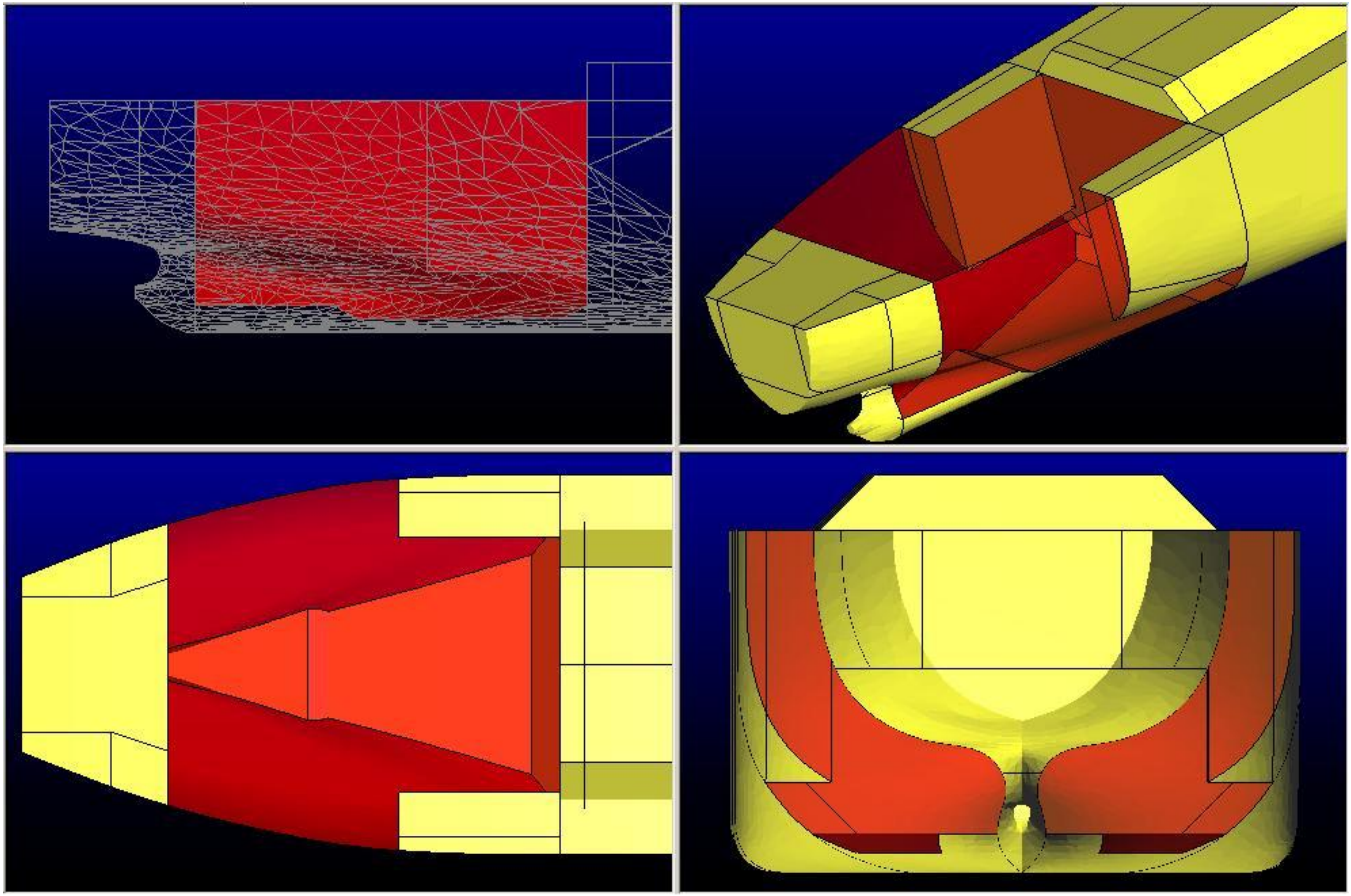



Steam & Condensate System
Steam Tracing Pipe
Boiler
insulation
tracer pipe
pressurised superheated steam
viscous fraction
Steam
Main Engine
*압력↑ → 온도↑
→ 점화/폭발
F.O Return Pipe
150C, 700cSt
Heating Coil
F.O Service Tank
80~90°C 유지
F.O Settling Tank
F.O Supply Pump
F.O Service System
F.O Purifier System
Viscosity Controller
Purifier
F.O Transfer Pump
F.O Filtering & Transfer System
Shore Connect
F.O Filter
F.O Heater
F.O Circulation Pump

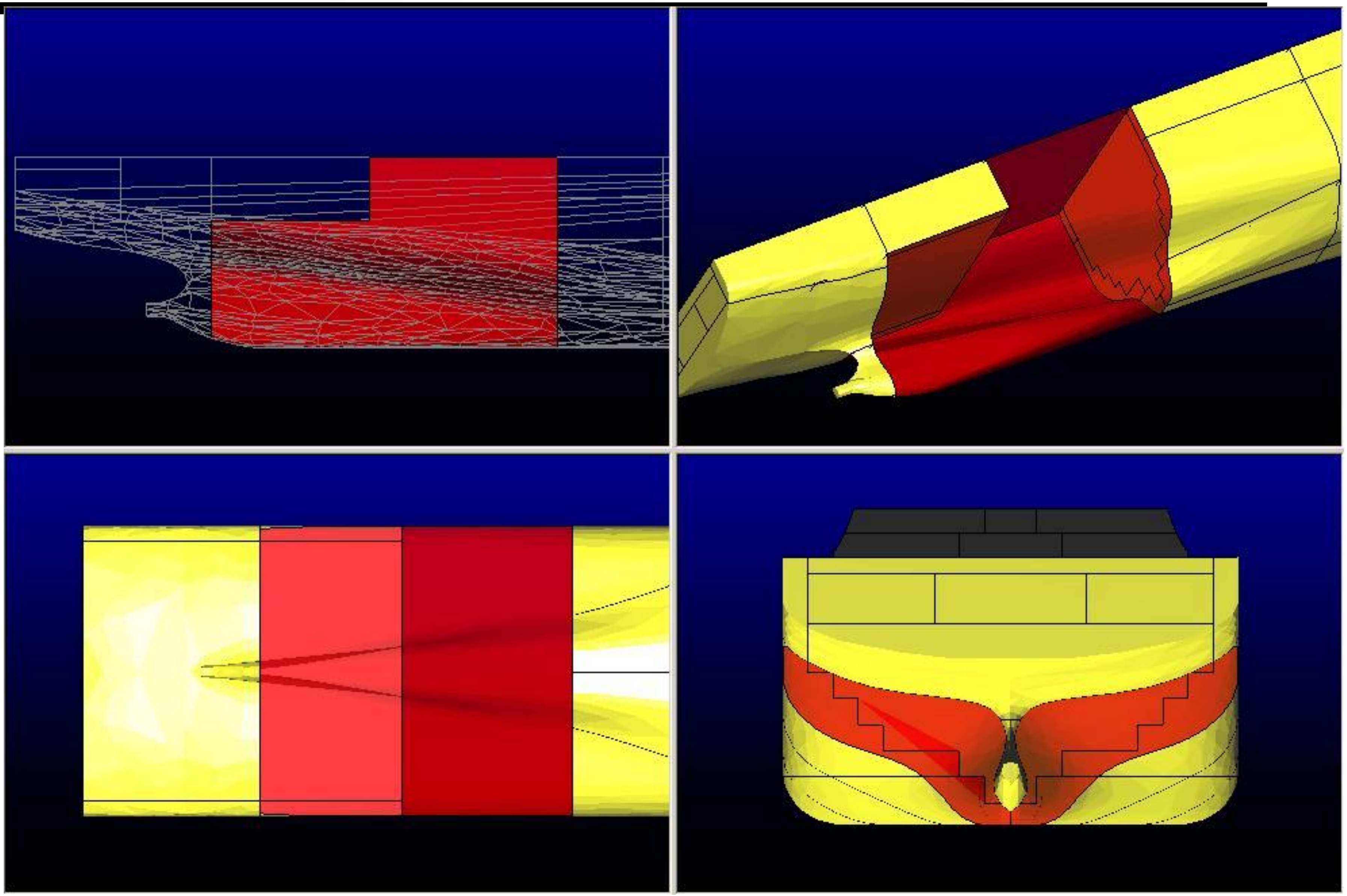
320K VLCC의 E/R



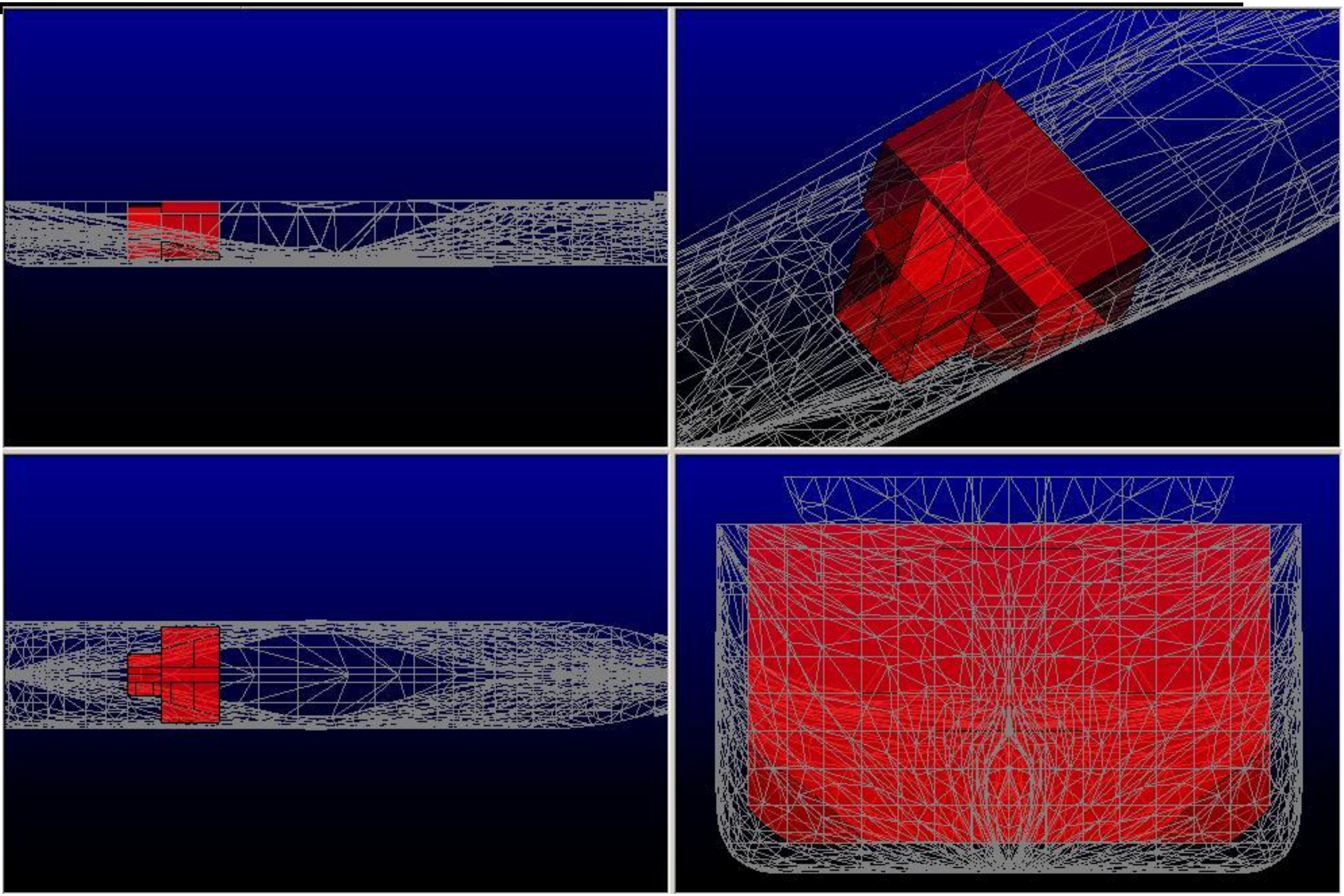
145K LNGC의 E/R



9,000TEU Container Carrier의 E/R

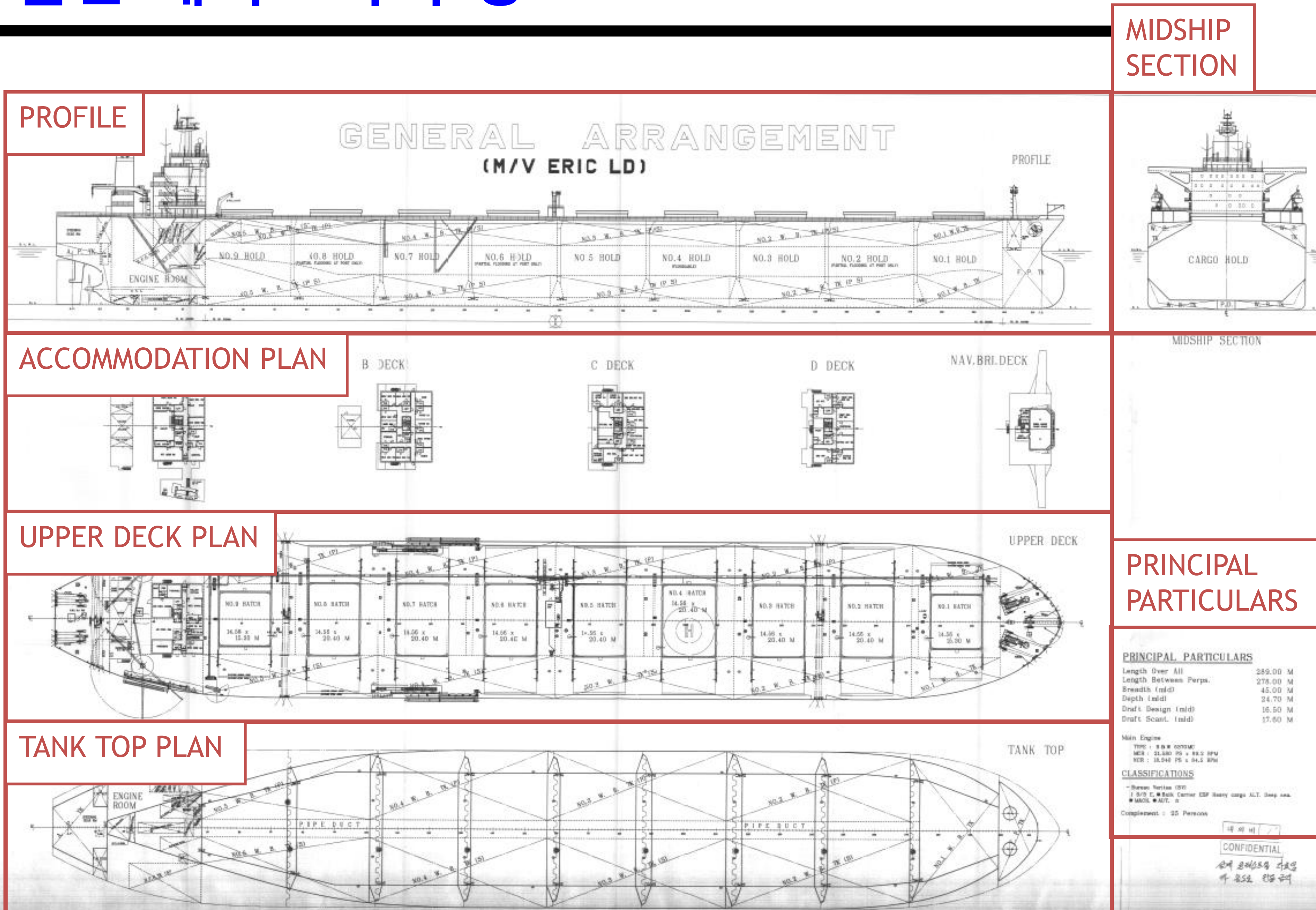


4,500TEU Container Carrier의 E/R



8-7 Reading the General Arrangement Plan

일반 배치도의 구성



MIDSHIP SECTION
PROFILE
ACCOMMODATION PLAN
UPPER DECK PLAN
TANK TOP PLAN
PRINCIPAL PARTICULARS
GENERAL ARRANGEMENT
(M/V ERIC LD)
PROFILE
ENGINE ROOM
NO.9 HOLD
NO.8 HOLD
NO.7 HOLD
NO.6 HOLD
NO.5 HOLD
NO.4 HOLD
NO.3 HOLD
NO.2 HOLD
NO.1 HOLD
CARGO HOLD
MIDSHIP SECTION
B DECK
C DECK
D DECK
NAV.BRI.DECK
UPPER DECK
NO.9 HATCH
NO.8 HATCH
NO.7 HATCH
NO.6 HATCH
NO.5 HATCH
NO.4 HATCH
NO.3 HATCH
NO.2 HATCH
NO.1 HATCH
TANK TOP
ENGINE ROOM
PIPE DUCT
PIPE DUCT
PRINCIPAL PARTICULARS
Length Over All 289.00 M
Length Between Perps. 278.00 M
Breadth (mld) 45.00 M
Depth (mld) 24.70 M
Draft Design (mld) 16.50 M
Draft Scant. (mld) 17.60 M
Main Engine
CLASSIFICATIONS
Complement : 25 Persons
CONFIDENTIAL

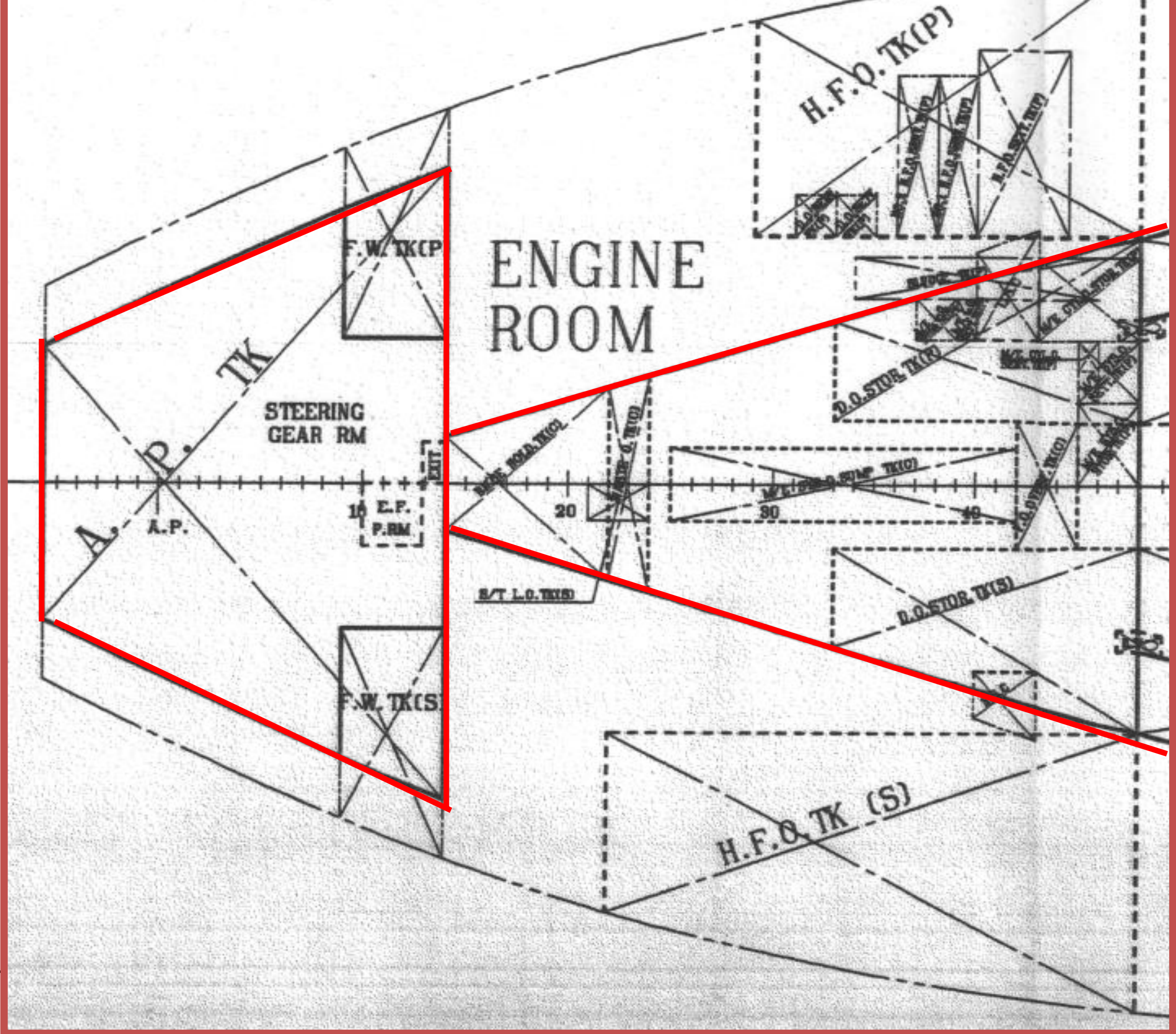
Reading the General Arrangement Plan

- 일반배치도에 사용되는 선종류에 따른 의미

① ———

(실선) : 자른 단면의 외곽선

ex) Tank Top Plan의 선미부



- **일반배치도에 사용되는 선종류에 따른 의미**

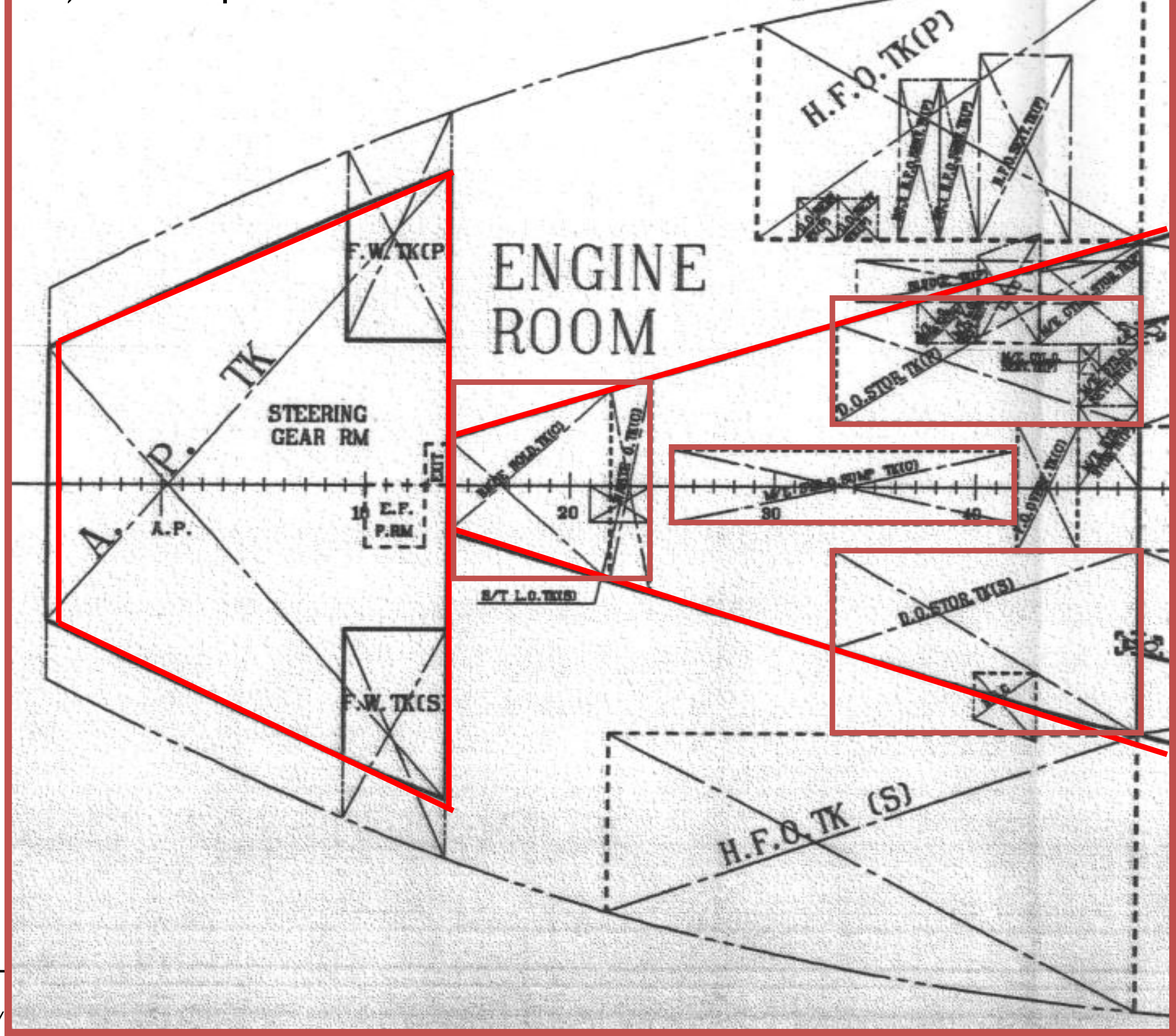
① ————————

(실선) : 자른 단면의 외곽선

② – – – – – – – –

(점선) : 잘린 단면보다 아래에 위치

ex) Tank Top Plan**의 선미부**



- **일반배치도에 사용되는 선종류에 따른 의미**

① ――――――――

(실선) : 자른 단면의 외곽선

② – – – – – – – –

(점선) : 잘린 단면보다 아래에 위치

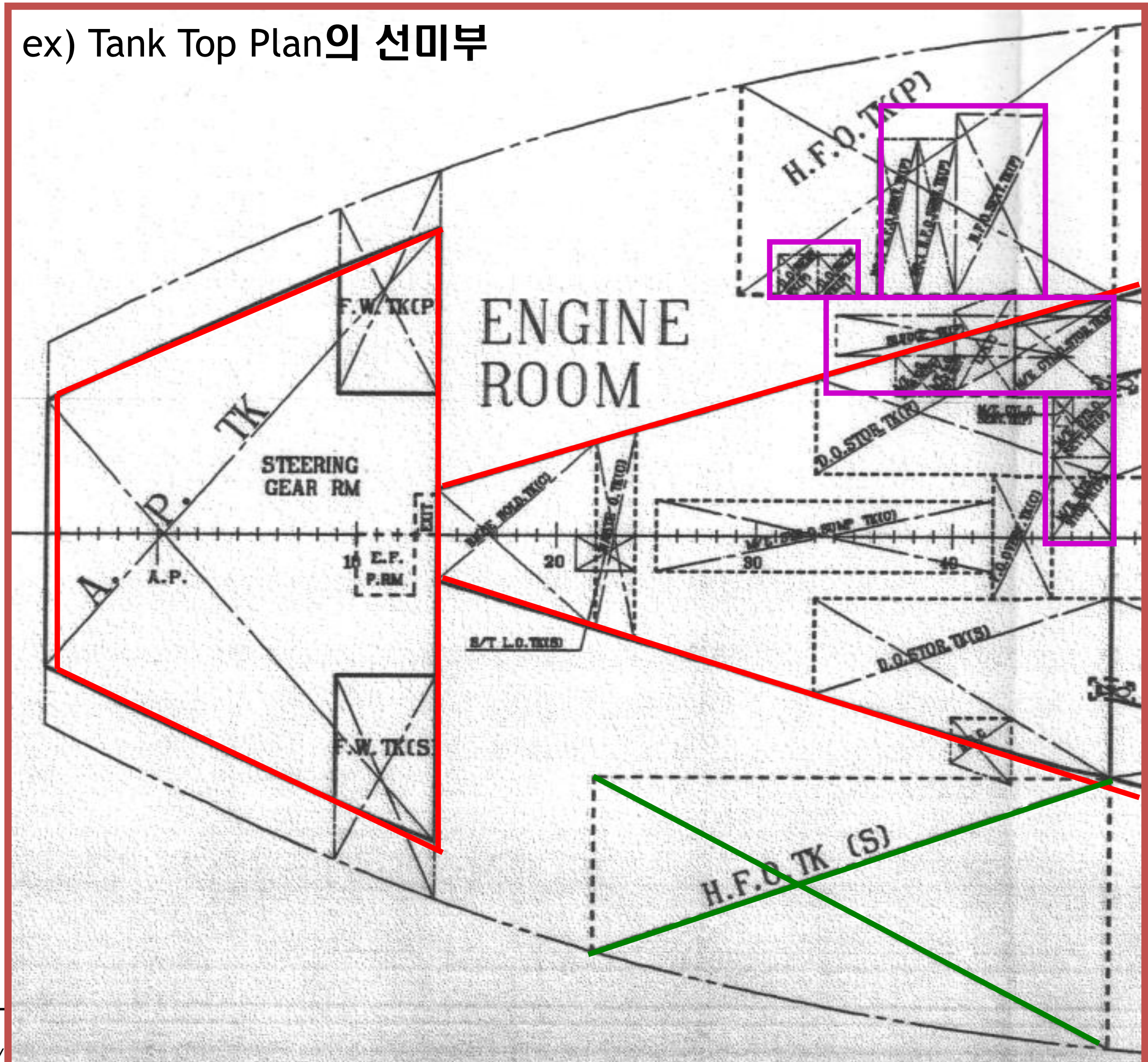
③ – · – · – · – · –

(일점쇄선) : 탱크 구획 표시

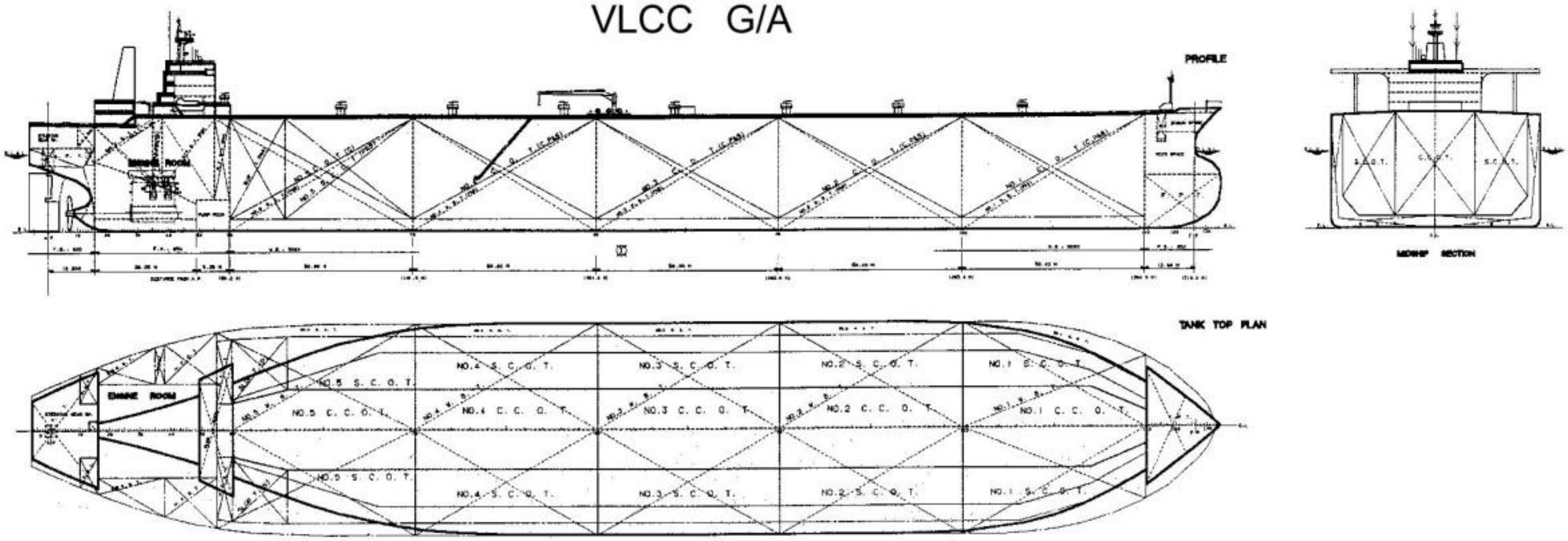
④ — · · — · · —

(이점쇄선) : 잘린 단면보다 위에 위치

ex) Tank Top Plan**의 선미부**



General arrangement(G/A) of a VLCC



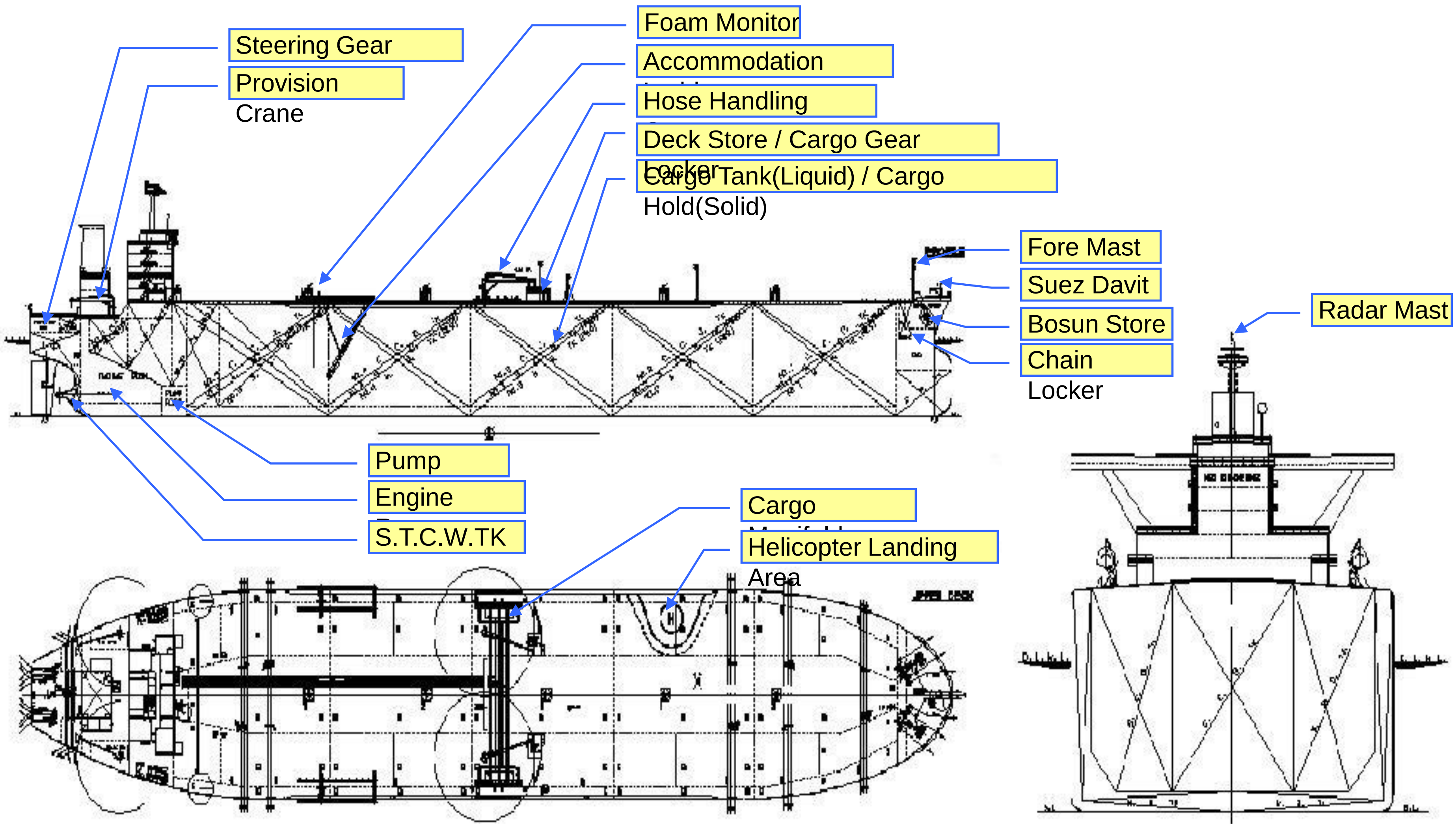
MAIN PARTICULARS

-Length over all: apprx. 330.50m
-Length betw. Perpendicular : 318.00m
-Bredth(moulded) : 58.00m
-Depth(moulded) : 31.25m

-Draught(Designed, moulded) : 21.40m
-Draught(Scantling, moulded) : 22.60m
-Deadweight at Td : apprx. 288,000MT
at Ts : apprx. 308,500MT

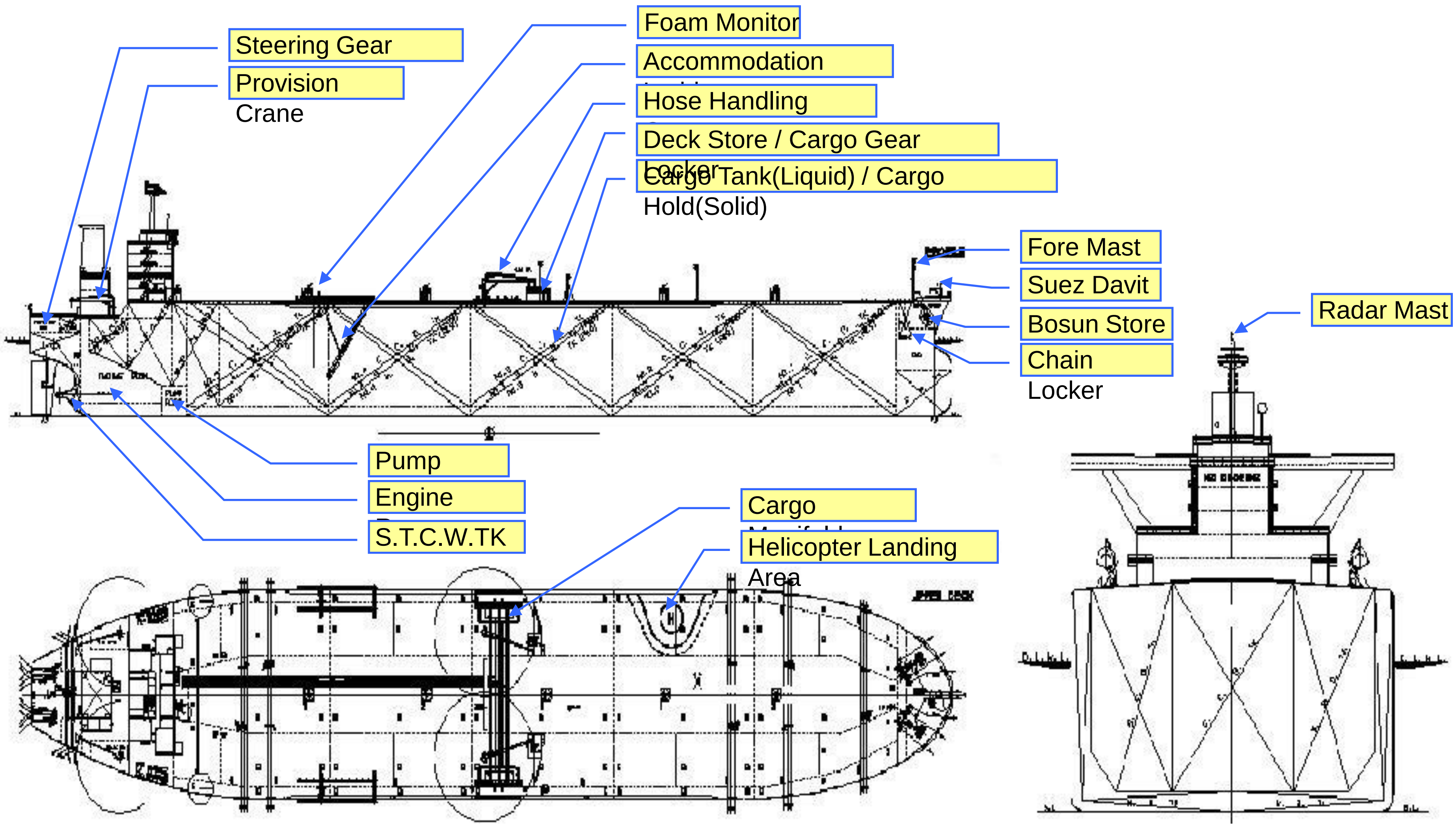
-Crusing Range : apprx. 30,000NM
-Service Speed : apprx. 15.3knots
(Designed draught, 90% MCR, 15% Sea margin)
-Class : DNV or ABS or LR equivalent
-Gross Tonnage : apprx. 160,480 tons
-Complements : 30 persons + 6suez crews

▪ General Arrangement of a VLCC



* 자료출처:대우조선해양

▪ General Arrangement of a VLCC



* 자료출처:대우조선해양

8-8 Arrangement Design of Tanker

Arrangement Design of Tanker

'Design' is a kind of 'Arrangement'.

Arrangement design of a ship includes

- **Compartment arrangement**
- **Equipment and piping arrangement**
- **Structural member arrangement**

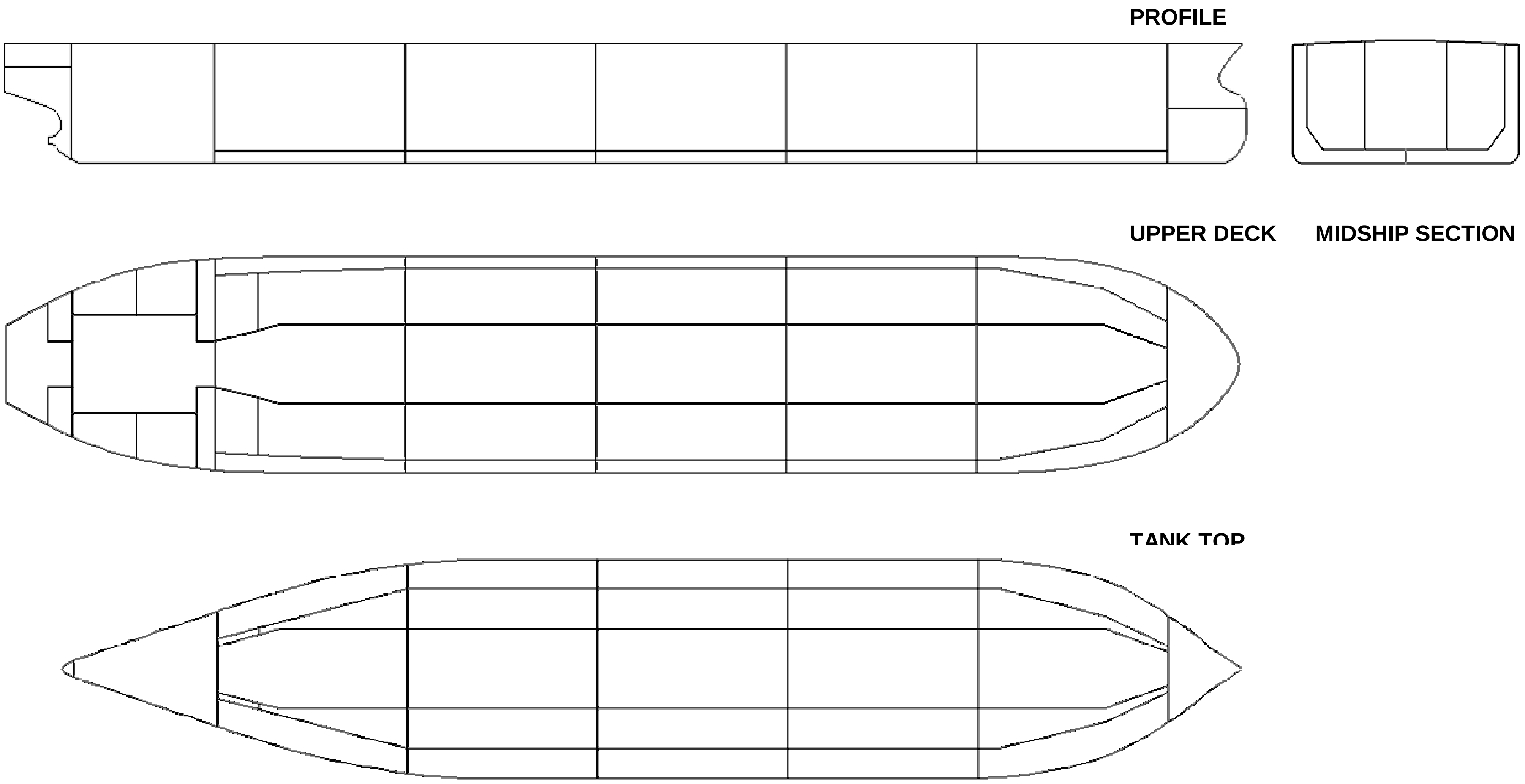
VLCC(Very Large Crude oil Carrier)

- ☑ 종류: 원유 운반선(Crude Oil Tanker), 석유 제품 운반선(Product Carrier), 화학 제품 운반선(Chemical Tanker)
- ☑ 속력: 14 ~ 15knots(약 26 ~ 27km/h)
- ☑ VLCC(Very Large Crude Oil Carrier)급: DWT 280,000 ~ 310,000톤
- ☑ 페르시아만 ~ 한국 사이의 1항차당 약 40일 소요(속력 15 ~ 16knots 기준)

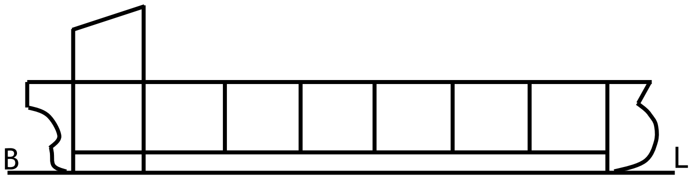
442,000ton DWT
ULCC(Ultra Large Crude Oil Carrier)

300,000ton DWT VLCC

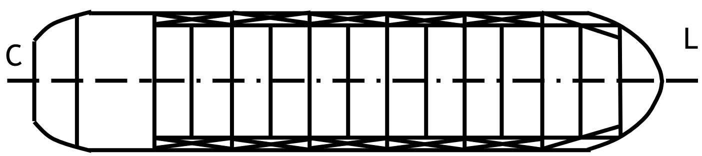
Compartment Arrangement of a VLCC



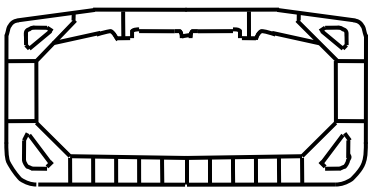
Various Types of Compartment Arrangements of Tankers



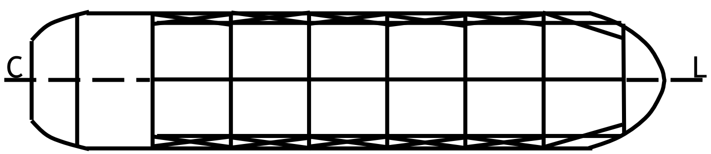
TYPE 1



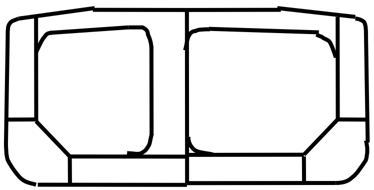
TYPE1



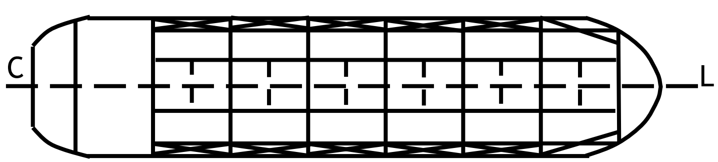
TYPE2



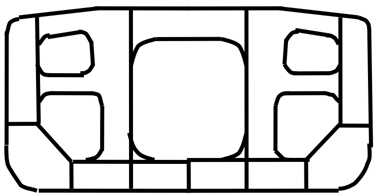
TYPE2



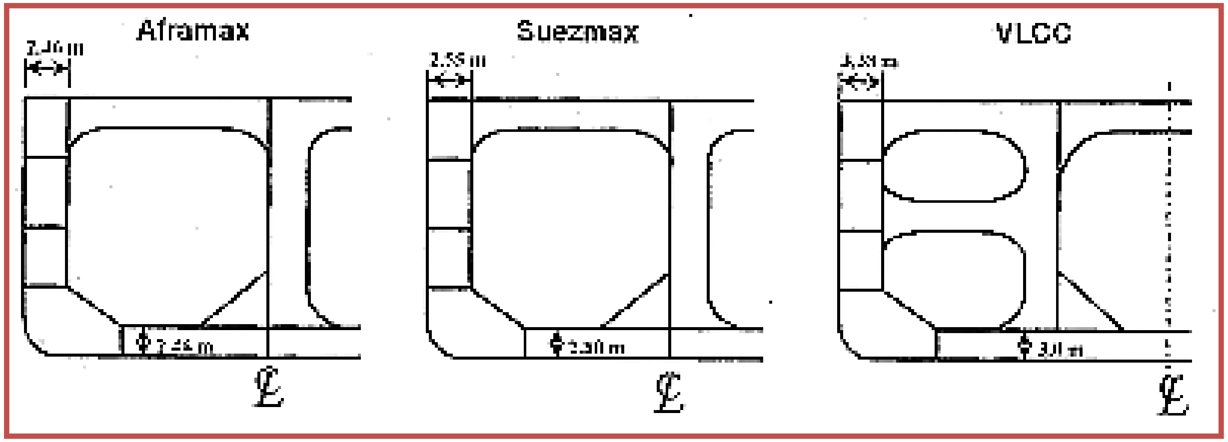
TYPE3



TYPE3

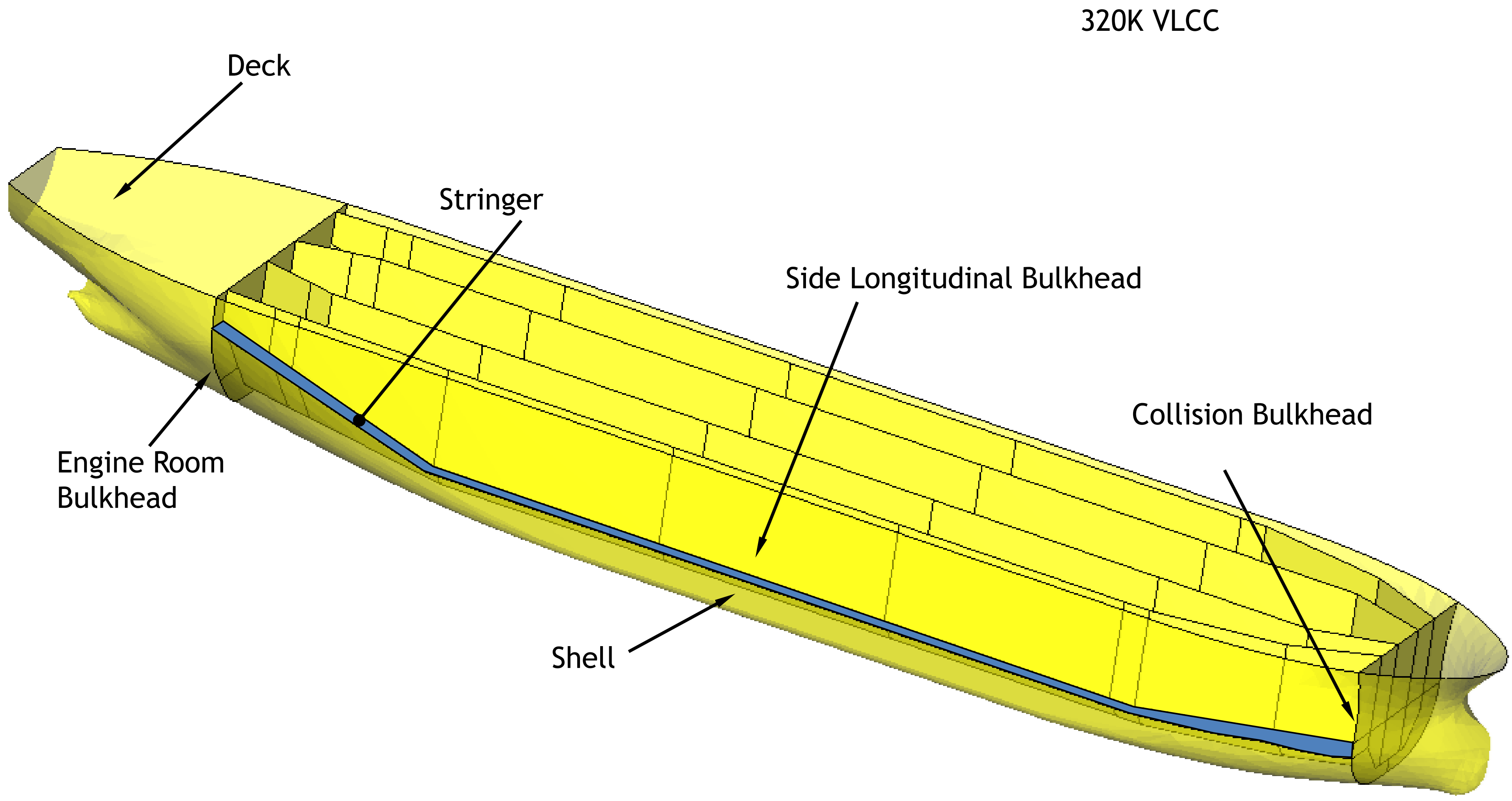


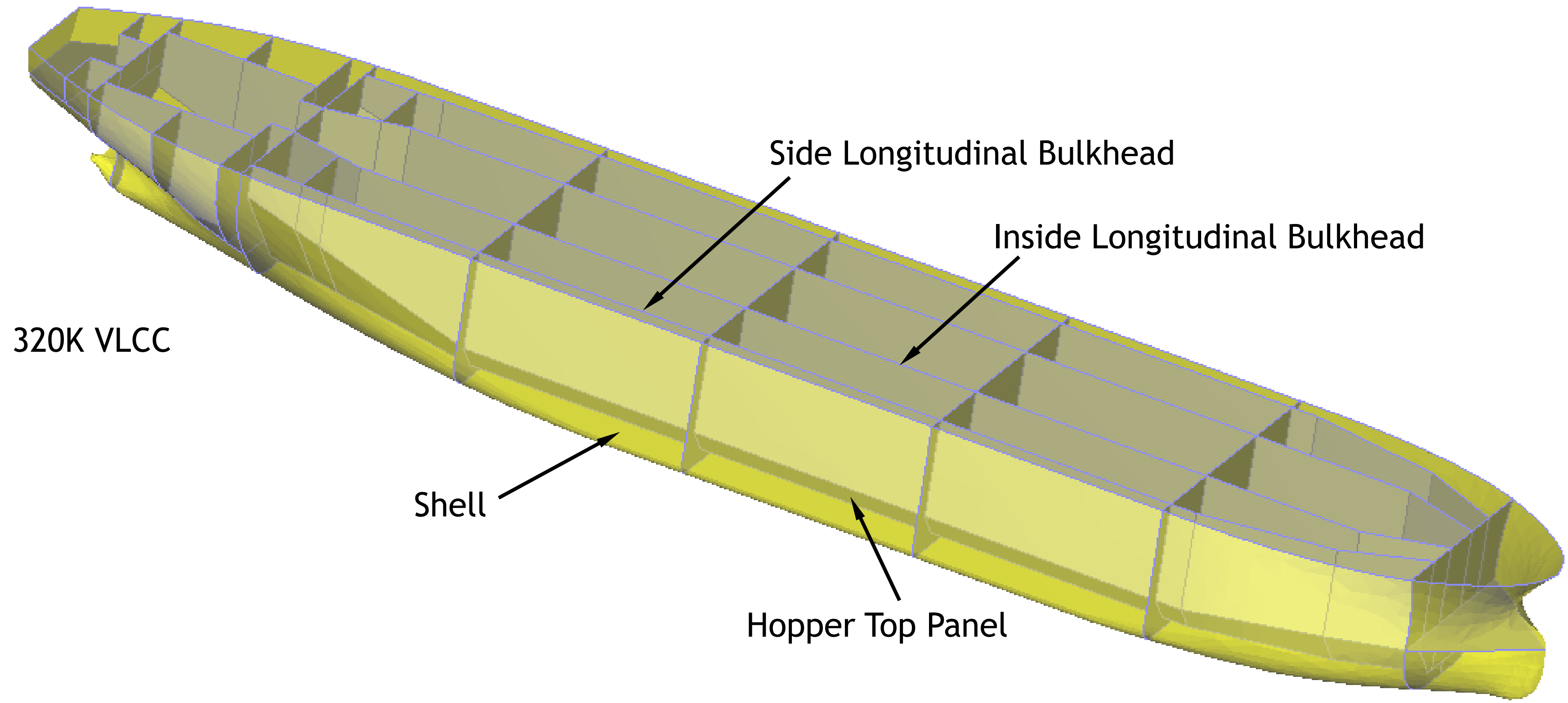
Double bottom height & Wing tank width of Various Types of Tankers



Ship Size	D/B Height	Wing Tank Width
Aframax	2.46 m	2.46 m
Suezmax	2.80 m	2.55 m
VLCC	3.0 m	3.38 m

Compartment Arrangement Model of a VLCC




Side Longitudinal Bulkhead
Inside Longitudinal Bulkhead
320K VLCC
Shell
Hopper Top Panel

Check point for compartment arrangement of tanker

- **Double Hull 요구 사항 (MARPOL 73/78)**
  - Slop Tank를 포함하여 Inner Hull은 외판과 min. 2.0m를 유지

- **Cargo Tank의 크기 제한 (MARPOL 73/78)**
  - PL & SBT를 계산한 후 요구사항을 만족하는지 Check
  - PL : Protective Location ; 방호적 배치
  - SBT: Segregated Ballast Tank ; 분리 밸러스트 탱크

- **Slop Tank (MARPOL 73/78)**
  - 화물창의 유수 분리용
  - 비상 상황으로 인해 Ballast 상태의 빈 화물창에 해수를 채웠을 경우, Tank Washing 등으로부터 발생되는 오수에서 화물유를 분리 저장
  - 용량 : Total Cargo의 2~3% 이상

Cofferdam 설치 기준

☑ Cofferdam

- 화물창과 기관실 사이 등에 설치함으로써 화재 예방을 하기 위한 공간

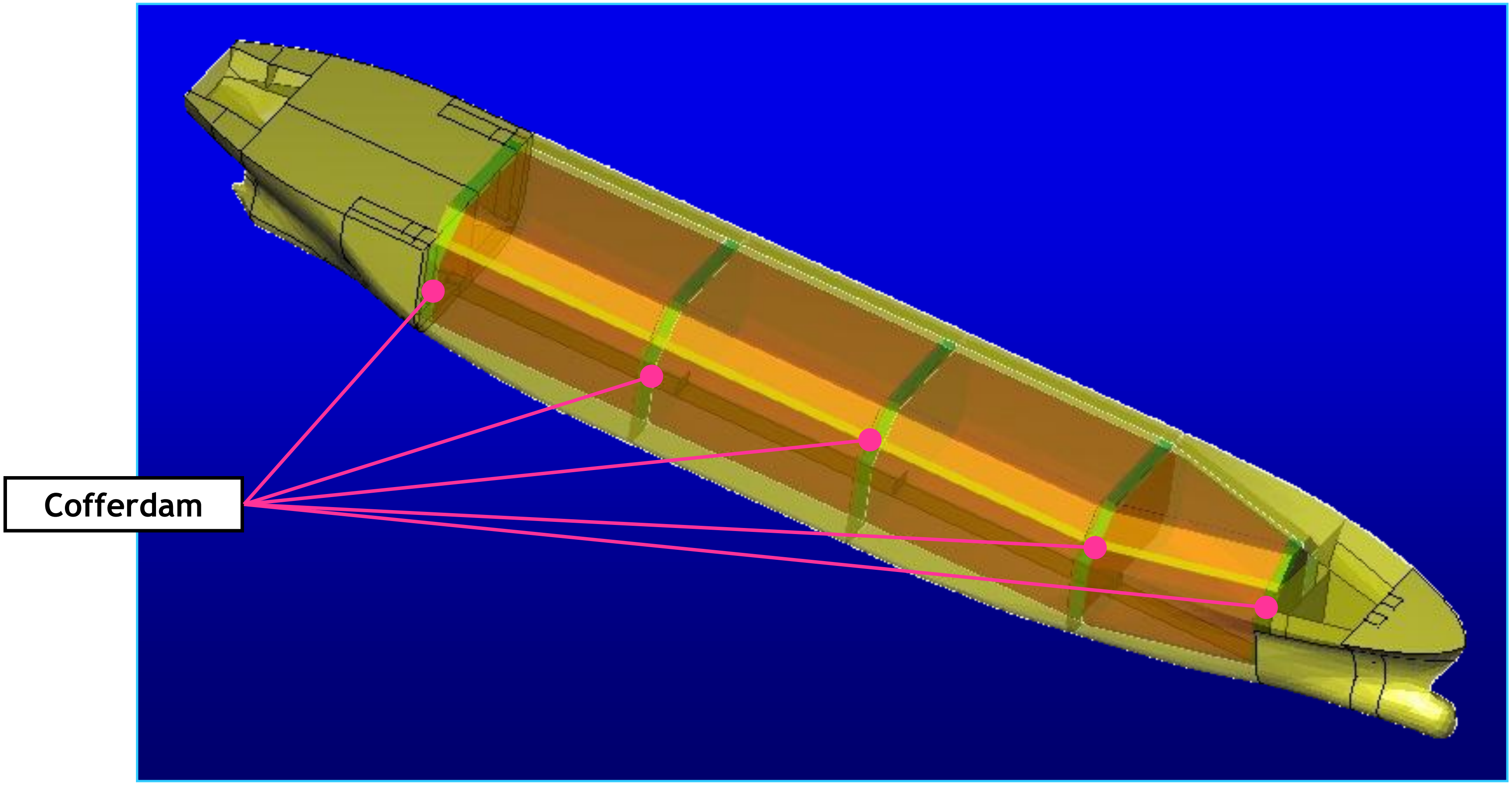
☑ Cofferdam이 설치되는 위치

- L.O.T(lubrication oil tank)와 F.O.T(fuel oil tank) 사이
- Water tank와 oil tank 사이
- 가열되는 tank와 곡물 저장고 사이
- F.O.T가 deck에서 끝나고 deck 하부가 다른 기기 공간 또는 기관실 (E/R; Engine Room)인 경우
- E/R과 emergency generator room 사이
- Main engine L.O sump tank 주위
- 기타 격리가 필요한 부분

☑ Cofferdam 설치와 관련된 각 선급의 규정

- 영국 LR(Lloyd) 선급
  - Oil Cargo Space의 Forward와 AFT End에 Cofferdam이 배치되어야 한다. Cargo Space의 End BHD의 전체 Area를 커버해야 한다.
  - Pump Room, Oil Fuel Bunker 또는 Water Ballast Tank는 Cofferdam 대신으로 적용이 가능하다.
  - Cofferdam은 Cargo Oil Tanker와 편의 공간 사이, Cargo Oil Tanker와 전기 장비를 설치한 사이에도 배치해야 한다.
- 독일 GL(Germanischer Lloyd) 선급
  - Product Tanker는 Cargo Tank와 Oil Fuel Tank 사이에 Cofferdam을 설치해야 한다. 그러나 발화점 60 ℃ 이상인 Non-Dangerous Liquid를 운반할 목적인 선박은 Cofferdam이 없어도 된다. 이 경우 Certificate에 명기된다.
- 선급 규정상 Cofferdam의 최소 규정치는 LR와 BV(프랑스 선급, Bureau Veritas)는 760mm 이상, GL과 DNV(노르웨이 선급, Det Norske Veritas)는 600mm 이상이며, ABS(미국 선급, American Bureau of Shipping)는 특별한 규정이 없다.

재화 용적 160,000CBM LNG선에서 화물창 탱크 사이에 장착된 Cofferdam의 예



8-9 Arrangement Design of Container Carrier

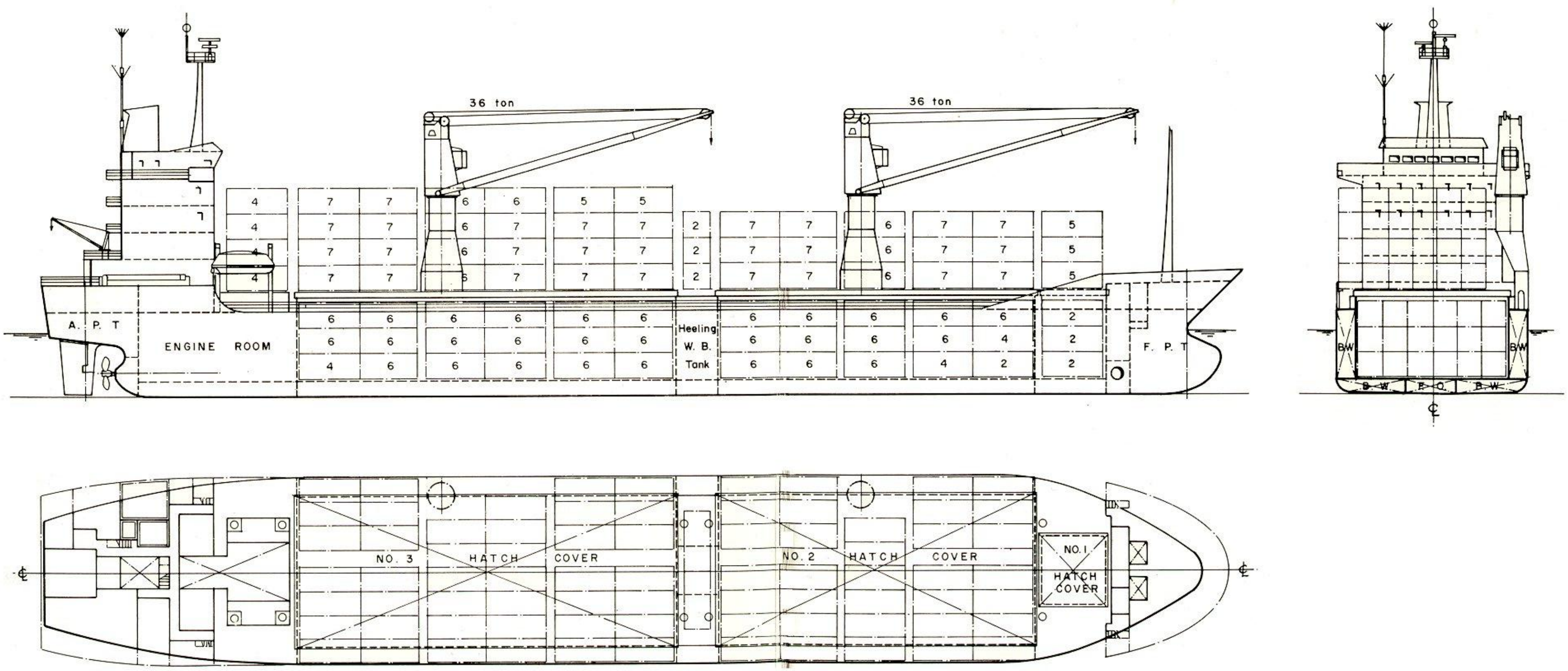
Arrangement Design of Container Carrier

'Design' is a kind of 'Arrangement'.

Arrangement design of a ship includes

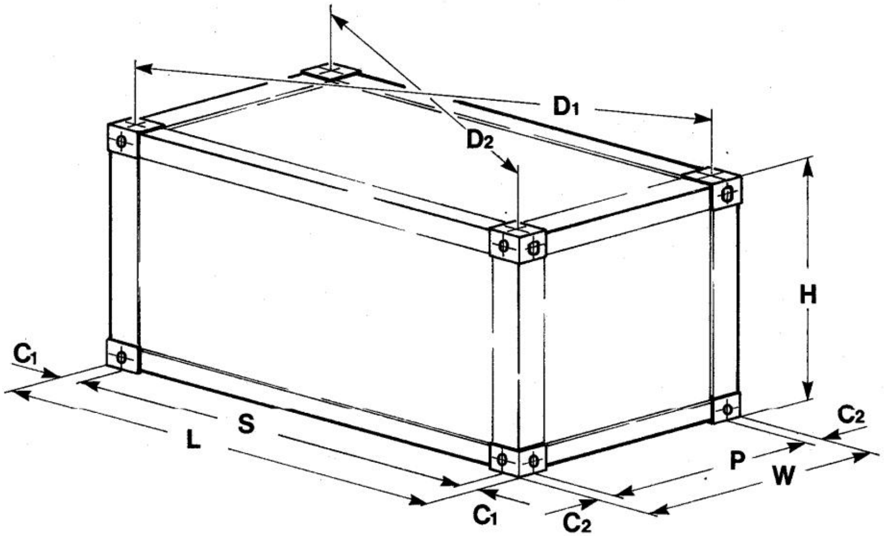
- **Compartment arrangement**
- **Equipment and piping arrangement**
- **Structural member arrangement**

400TEU Semi-Container Ship(Multi Purpose Container Vessel)



Length o.a	121.50m	Depth mld	8.50m	Deadweight at designed draft	7418tonnes
Length b.p	111.70m	Draft designed	6.45m	Service Speed(85%MCR, 15%S.M)	13.35knots
Breadth mld	19.20m	Drath scantling	6.50m	Complement	22persons

Size and weight of different container types

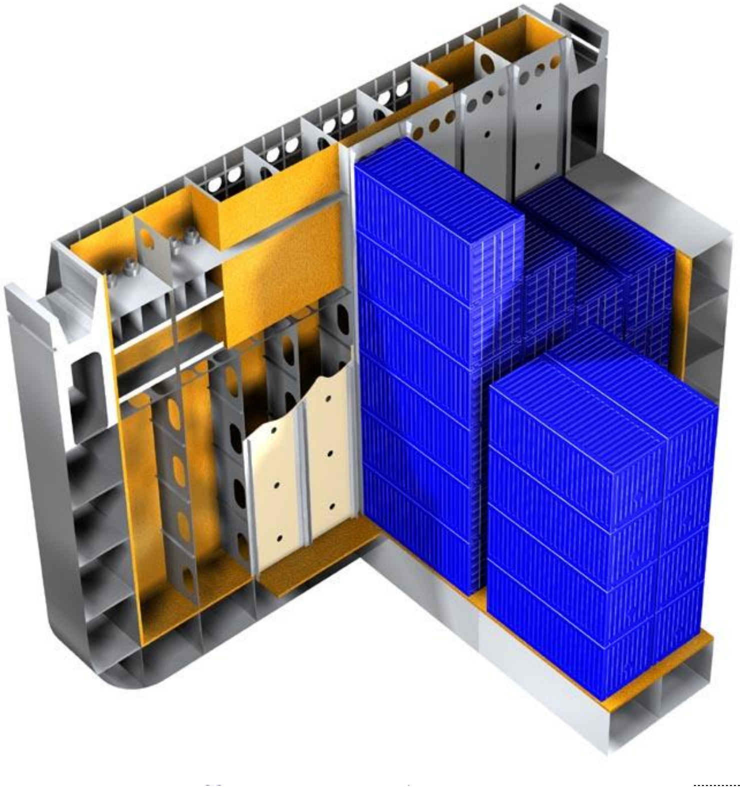


Type	Height(H)		Width(W)		Length(L)		Max. weight(kg)
	mm	ft-in	mm	ft-in	mm	ft-in	
1A	2,438	8'	2,438	8'	12,192	40'	30,480
1AA	2,591	**8'-6"**	2,438	**8'**	12,192	**40'**	30,480
1B	2,438	8'	2,438	8'	9,152	29'-11 1/4"	25,400
1C	2,438	8'	2,438	8'	6,058	19'-10 1/2"	20,320
1CC	2,591	**8'-6"**	2,438	**8'**	6,058	**19'-10 1/2"**	20,320
1D	2,438	8'	2,438	8'	2,991	9'-9 3/4"	10,160

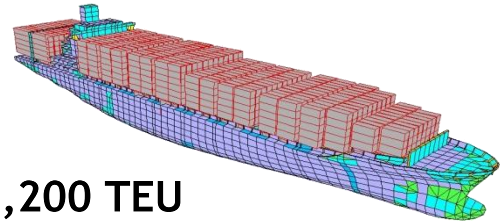
Large Container Carrier

- 5,000 ~ 8,500TEU(Post Panamax)

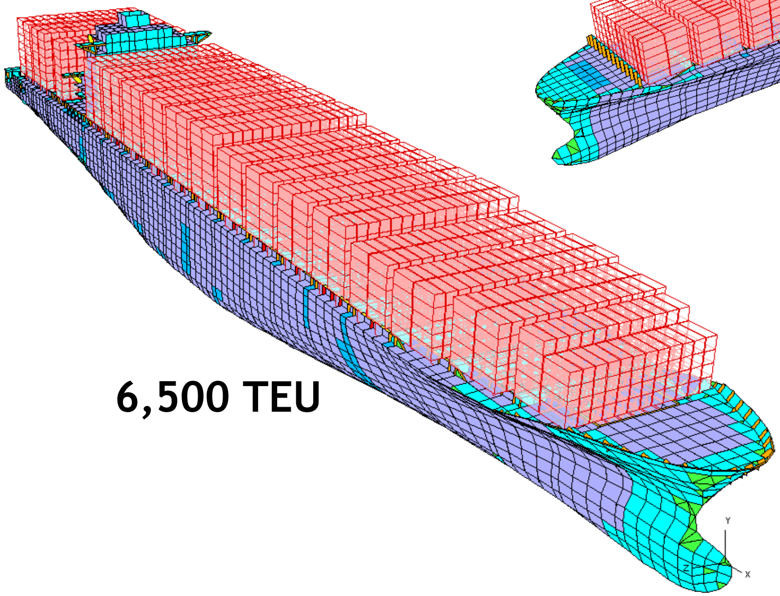
☑ 국내 조선소가 시장 점유율 대형화에서 우위
- 국내에서는 9,000TEU급의 건조를 넘어서 18,600 TEU급 건조 중, 20,000TEU급 개발
- 국내에서는 추진 시스템으로 12 Cycle 엔진을 적용하고 있으며 Pod 추진의 적용을 검토 중



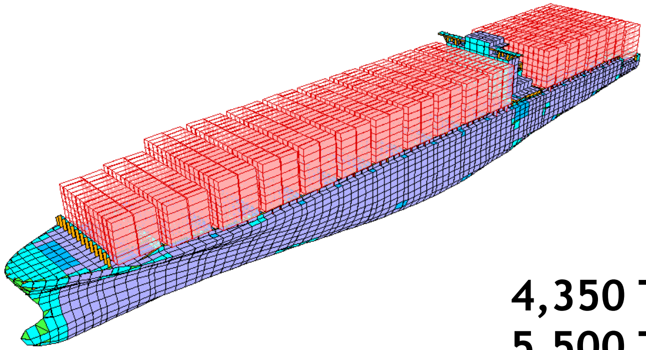
Size of Container Carrier



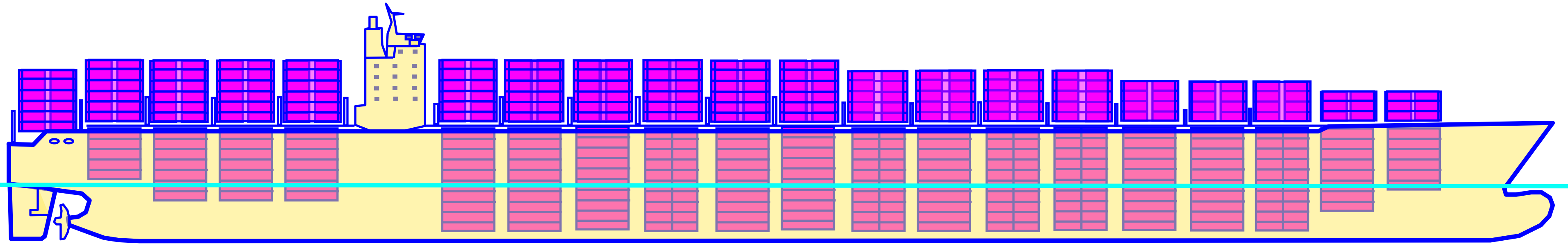
2,200 TEU



6,500 TEU



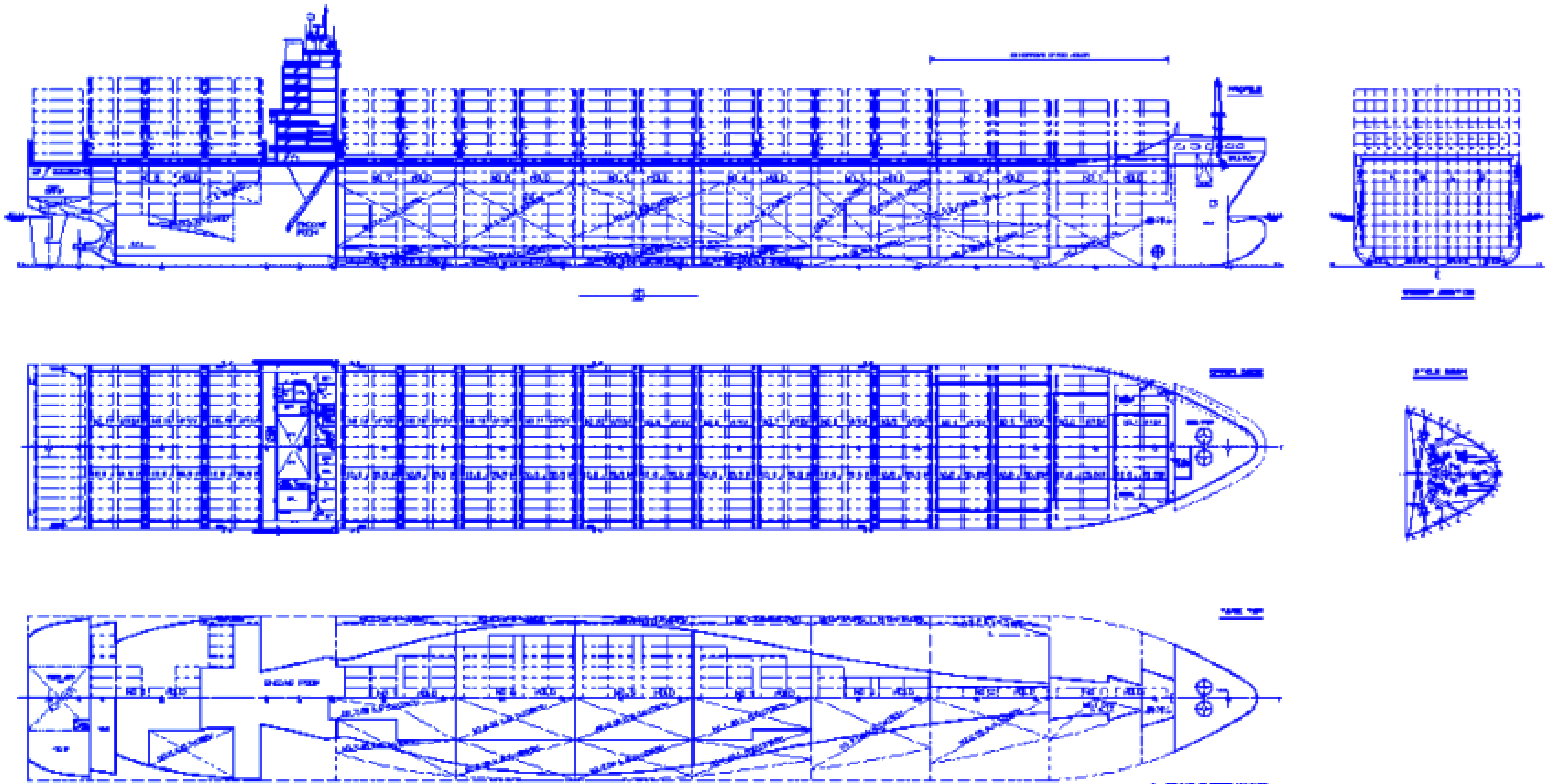
4,350 TEU
5,500 TEU



9,100 TEU

* 자료출처:대우조선해양

G/A of a 6,500TEU Container Carrier



Principal Dimensions

LOA	300.0 m
LBP	286.56 m
B	40.0 m
D	24.2 m
Td / Ts	12.0 / 14.5 m
Deadweight at Ts	: 78,100 ton
Service Speed at Td	: 27.0 knots
	at NCR with 15% sea margin

Container Capacities

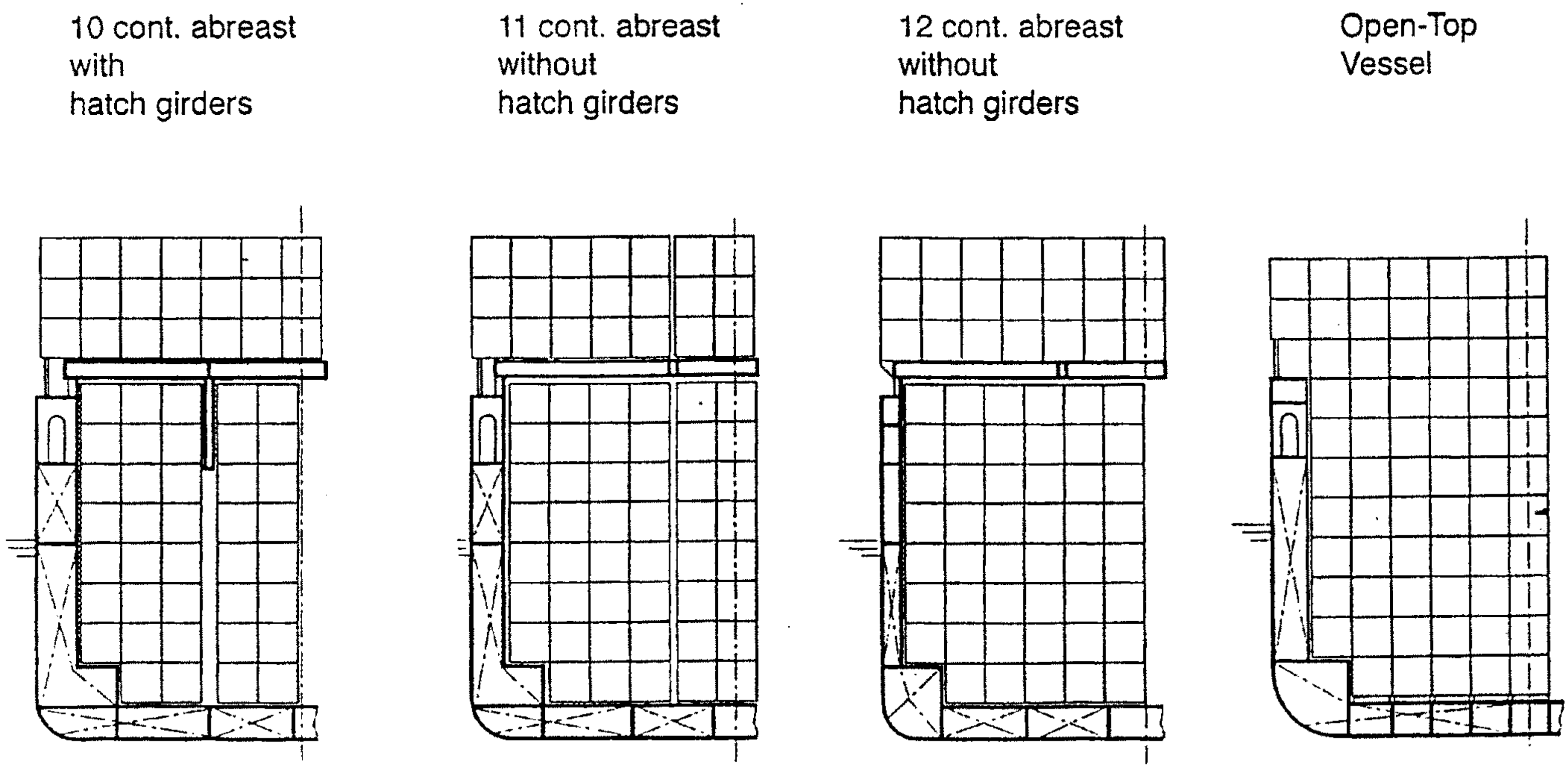
Total	6,456 TEU
On Deck	3,398 TEU
In Hold	3,058 TEU
Reefer Container(on deck)	500 FEU

Main Engine B&W 12K98MC-C

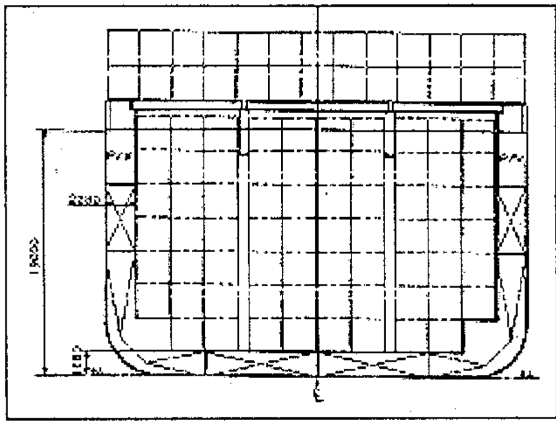
MCR	93,120 PS x 104.0 rpm
NCR	83,810 PS x 100.4 rpm
Cruising Range	: 23,500 N.M.

* 자료 출처: 대우조선해양

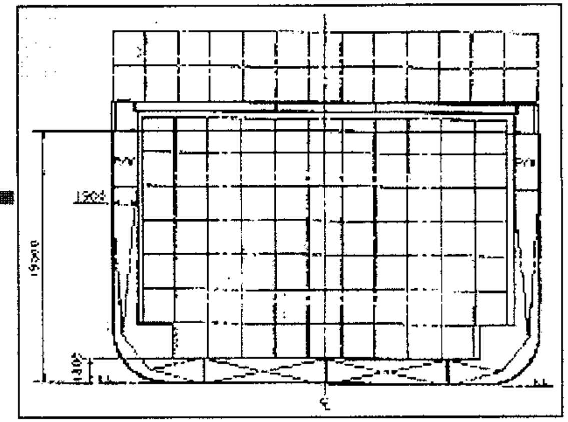
Various container arrangement in Midship section



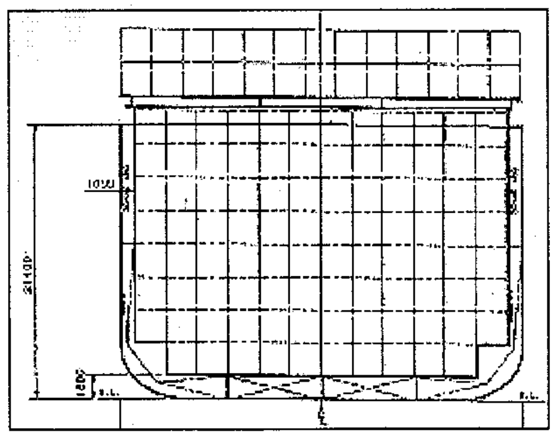
Increased Rows of a PAX(Panamax) Beam Container Carrier



10 ROWS
YEAR OF 1984
2,200TEU x 4 VESSELS

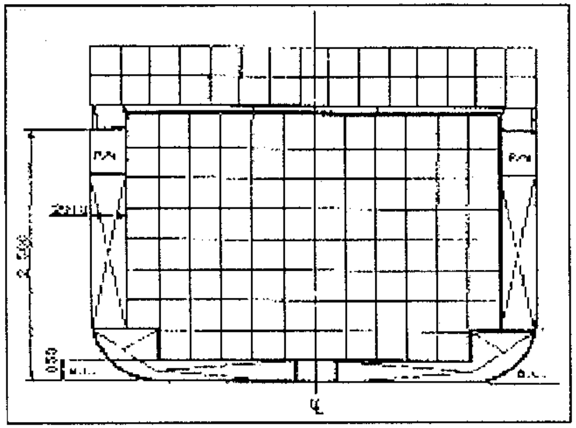


11 ROWS
YEAR OF 1990
4,400TEU x 12 VESSELS

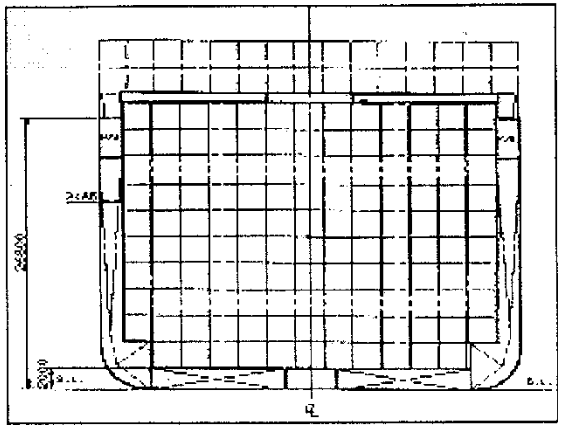


12 ROWS
YEAR OF 1995
2,700TEU x 3 VESSELS

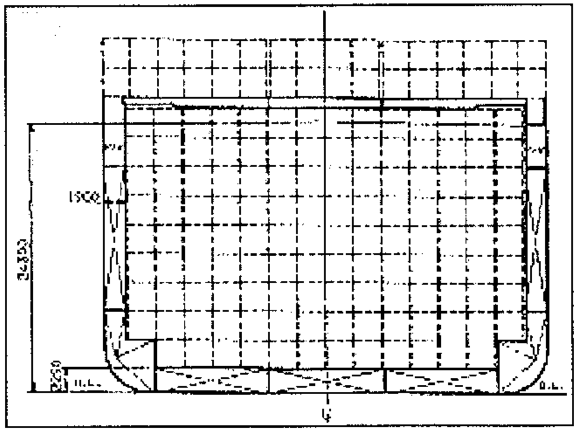
Increased Rows of a POSTPAX(Post Panamax) Beam Container Carrier



12 ROWS
YEAR OF 1990
B = 37.1 m



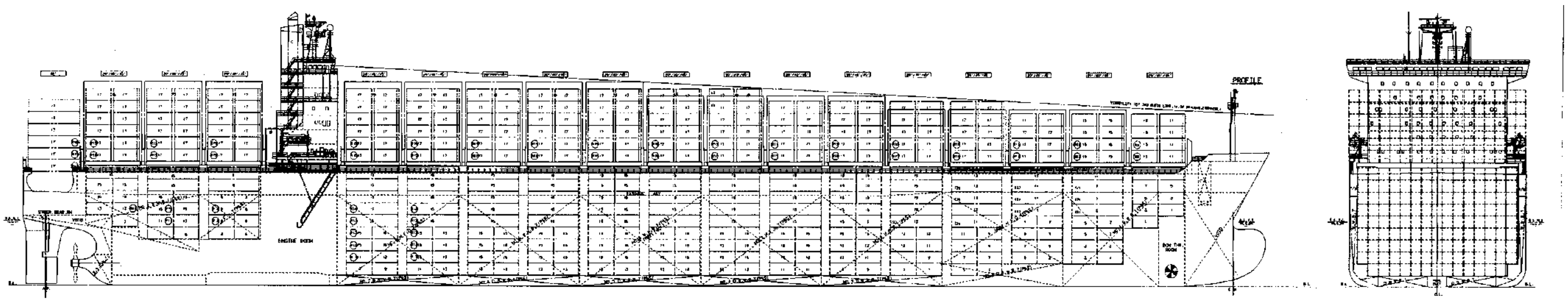
13 ROWS
YEAR OF 1992
B = 38.0 m



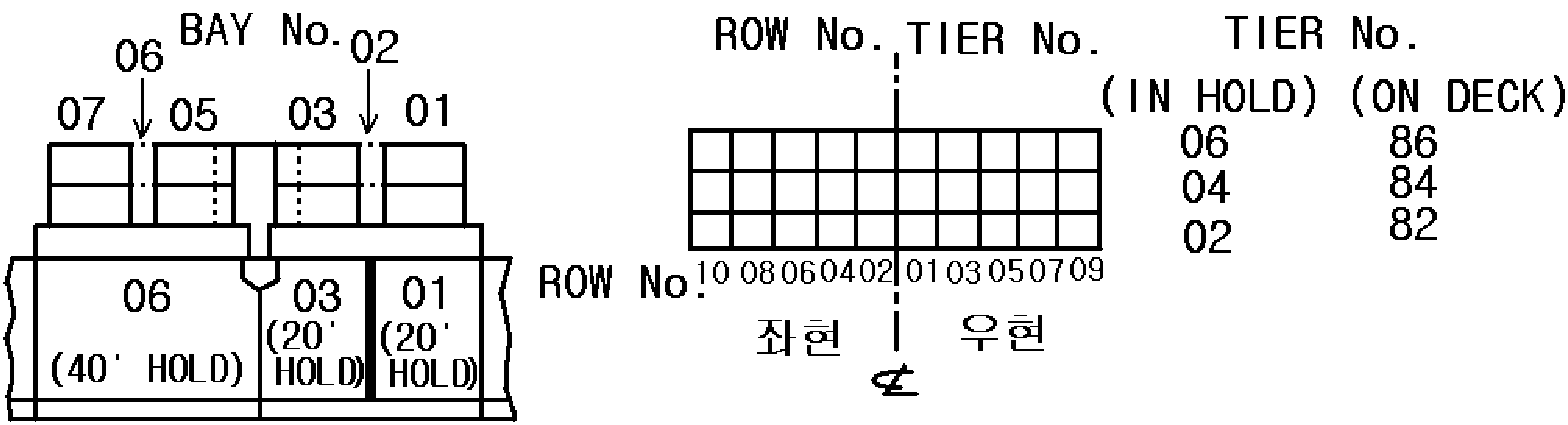
14 ROWS
YEAR OF 1995
B = 40.0 m

15 ROWS
YEAR OF 1995
B = 42.8 m

G/A (Profile & Midship) of a POSTPAX Beam Container Carrier



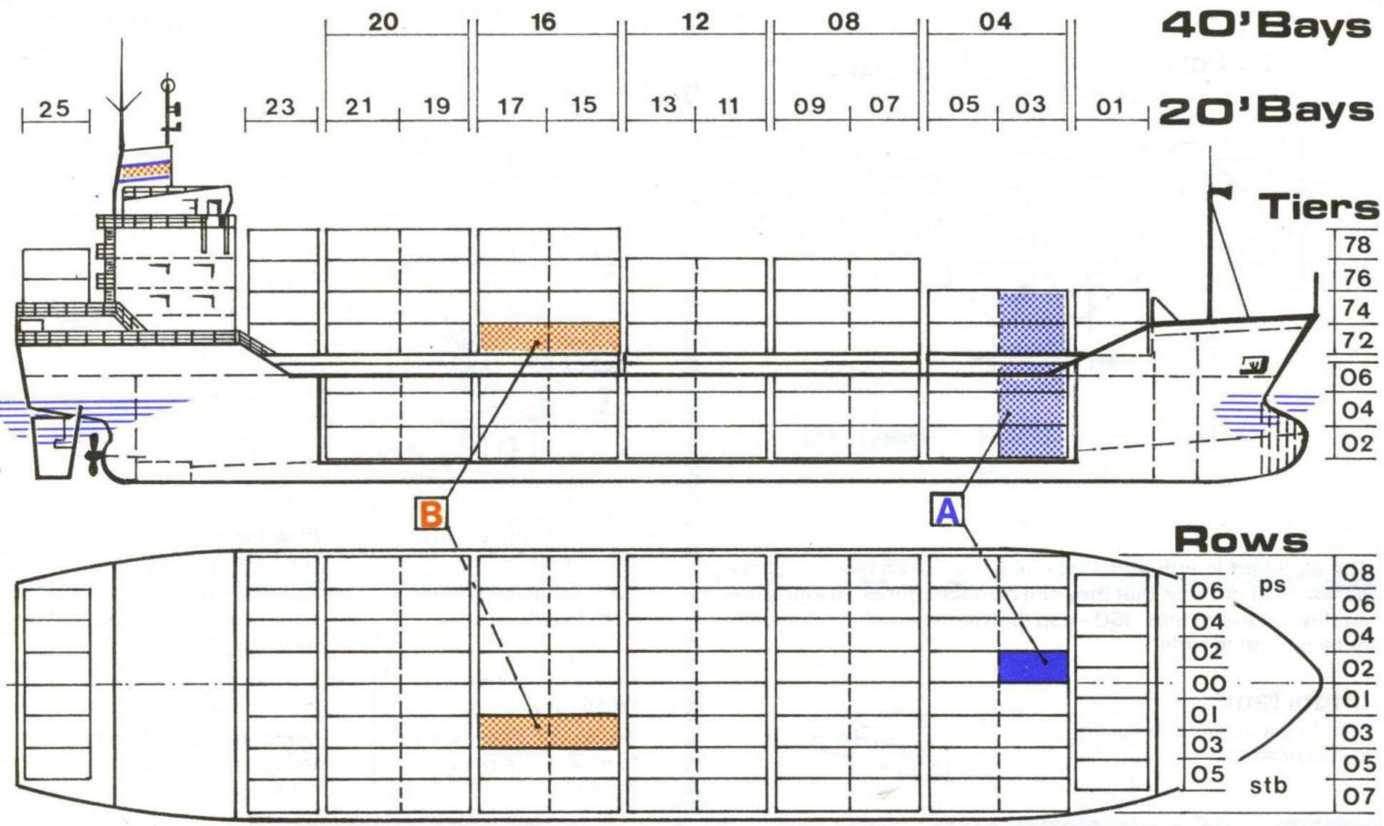
Code of the Container Position(컨테이너 적재 위치의 표시 방법)

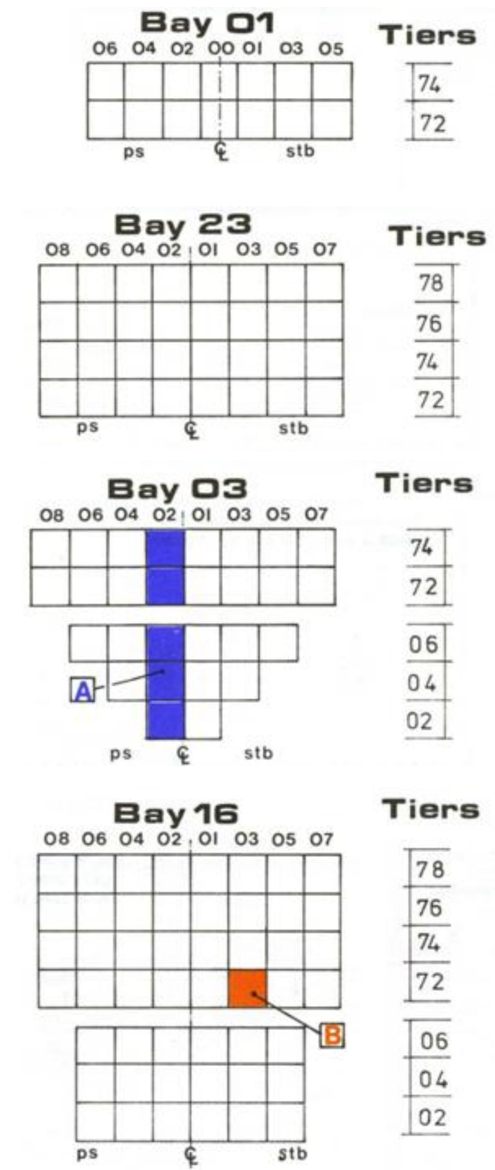


20′ 컨테이너용에 대하여는 선수로부터 홀수 번호를 붙이고,
40′ 컨테이너용에 대하여는 그 다음의 짝수 번호를 붙인다.
Tier No.는 짝수 번호로 한다. 갑판 위는 82로부터 시작한다.

컨테이너선에 탑재되는 컨테이너는 각기 놓여질 자리가 정해져 있으므로, 하역의 편의를 위하여 적재 장소에 번지를 붙여둔다. 그 번지는 선박의 앞뒤 방향(행(bay)), 가로 방향(열(row)) 및 위아래 방향(단(tier))의 위치로 표시된다. 번지의 표시 방법은 해운사 마다 일정하지는 않지만, 한 예를 들면 그림에 보인 바와 같다. 번지를 나타내는 숫자는 그림에 보인 것과 같이 선창이나 창구 덮개 적당한 곳에 표시된다. 셀 가이드는 일반적으로 고정식이지만, 20′ 컨테이너용으로 설계된 자리에는 40′ 컨테이너를 넣을 수 없으므로, 셀 가이드의 일부를 떼어내고 40′ 컨테이너도 실을 수 있게 한 방식도 있다.

Code of the Container Position

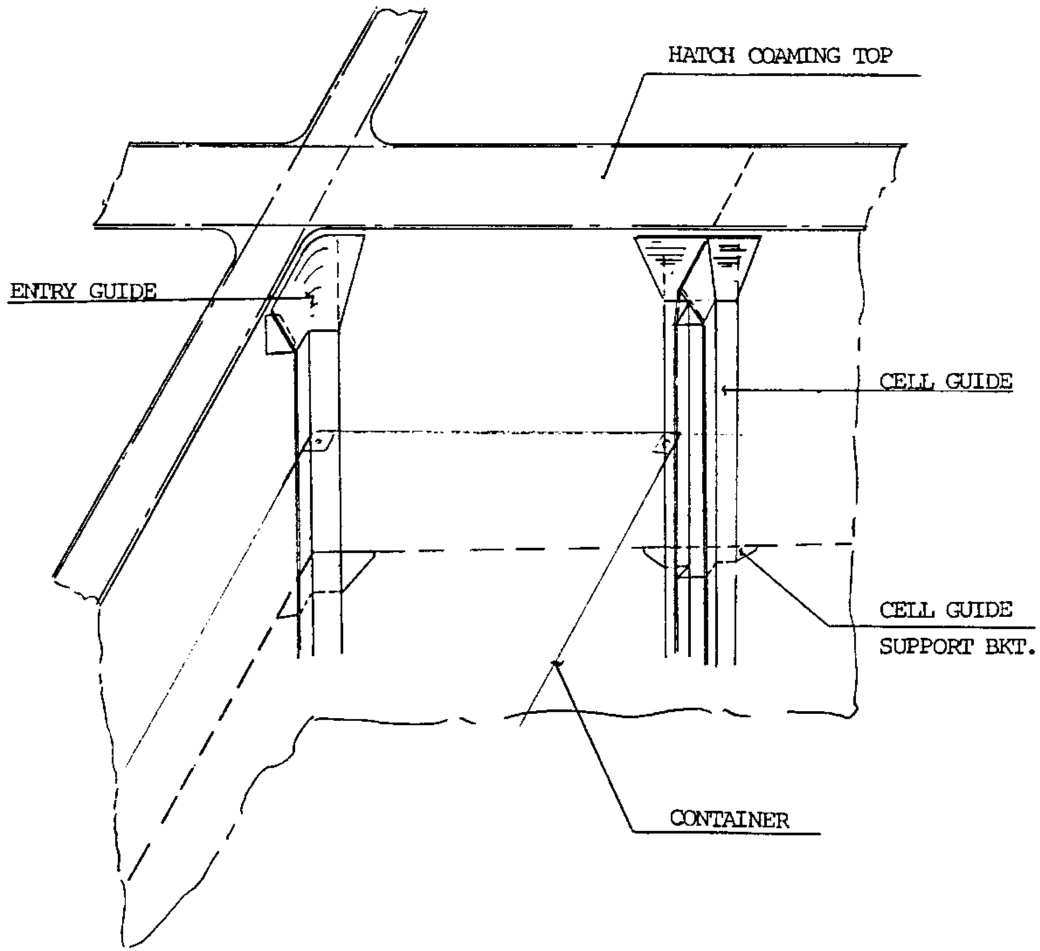




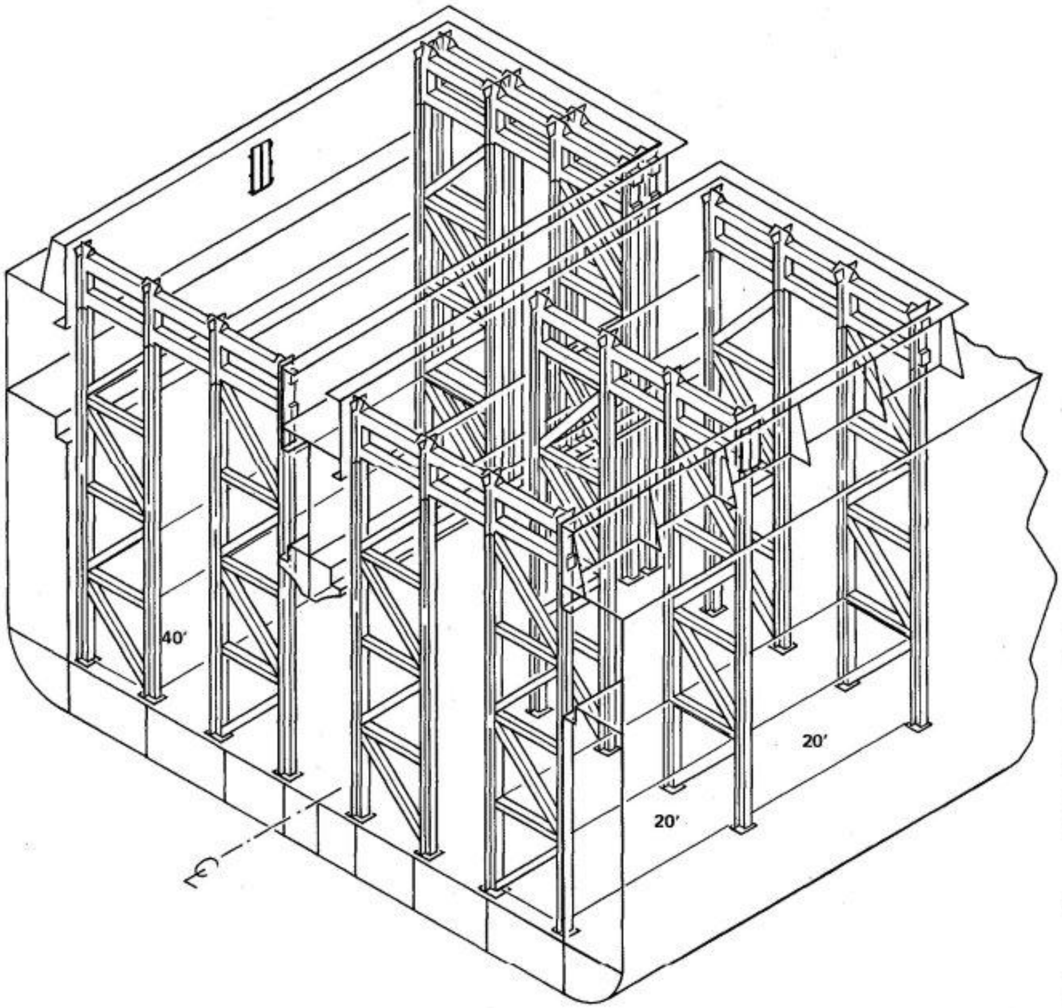
DEFINITION

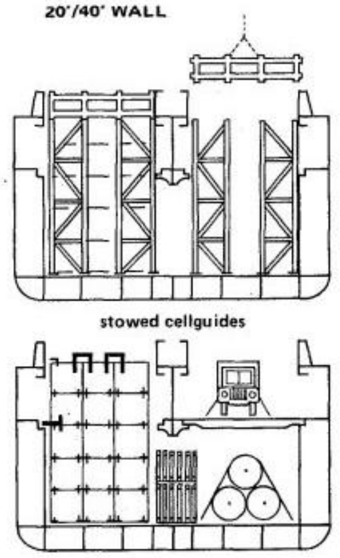
A Position of a stack	BAY No.	ROW No.		
Example 20'stack	03	– 02		
B Position of a container	BAY No.	ROW No.	TIER No.	
Example 40'container	16	– 03	– 72	7 tiers above basis (max. tiers on board)

Cell Guide System of Container Carrier



Cell Guide System of Container Carrier





CHARACTERISTICS

- adjustable cellguide system for 20'/40' Cont.
- dismantable and to be stowed in containers to be located under longitudinal bulkhead in centre-line
- combined cell/blockstowage for 20' Cont.

POSSIBLE ALTERATIONS

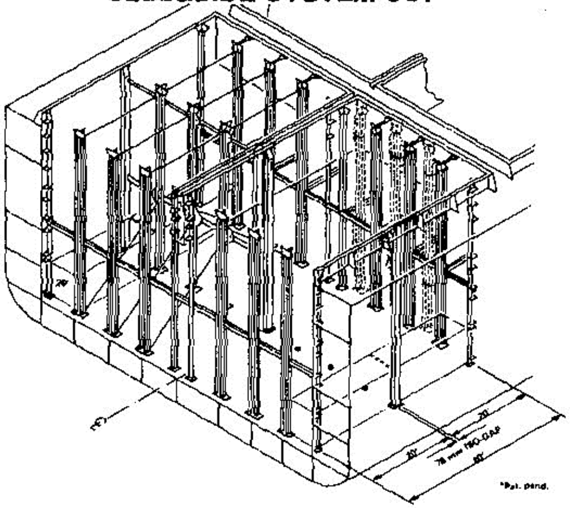
- 35' WALLS
- cellguide system fixed welded at 20' or 40' area (35')

SPECIFICATION

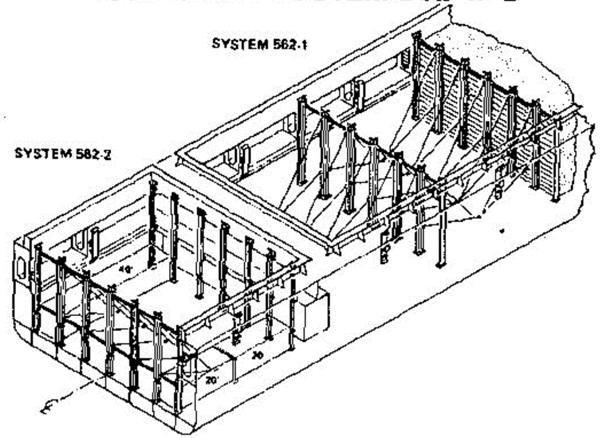
- Material : in accordance with the Classification Society
- Finish : upon client's request
- Class. approval : All items can be supplied with the approval of any Classification Society upon client's request

Various Cell Guide Systems

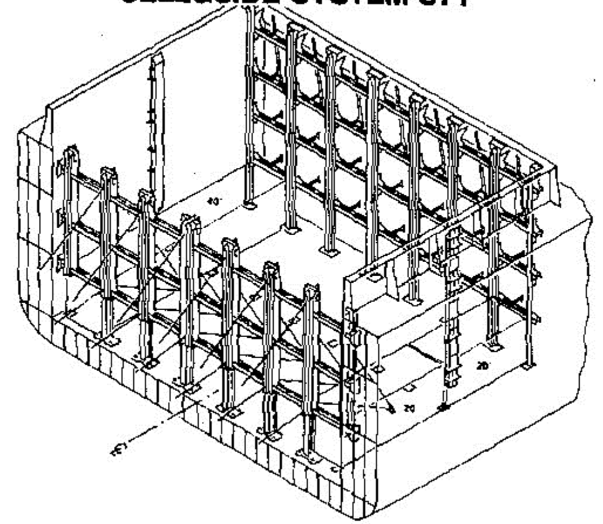
CELLGUIDE SYSTEM 561



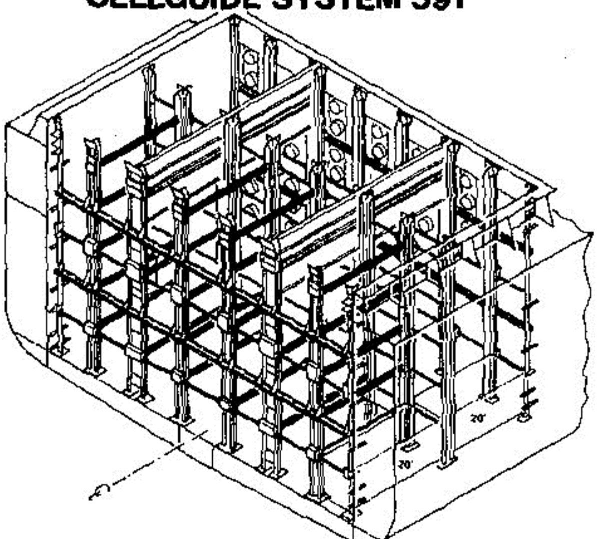
CELLGUIDE SYSTEM 562-1/-2



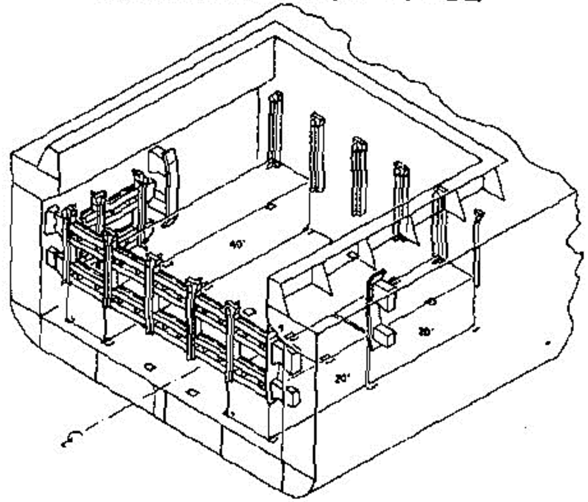
CELLGUIDE SYSTEM 571



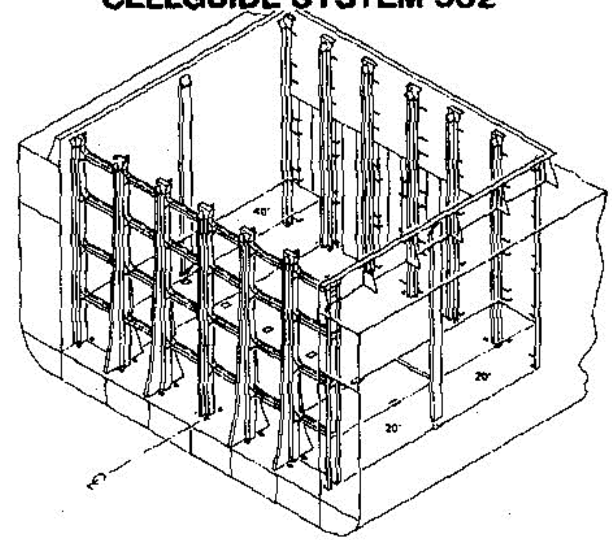
CELLGUIDE SYSTEM 591



CELLGUIDE SYSTEM 592



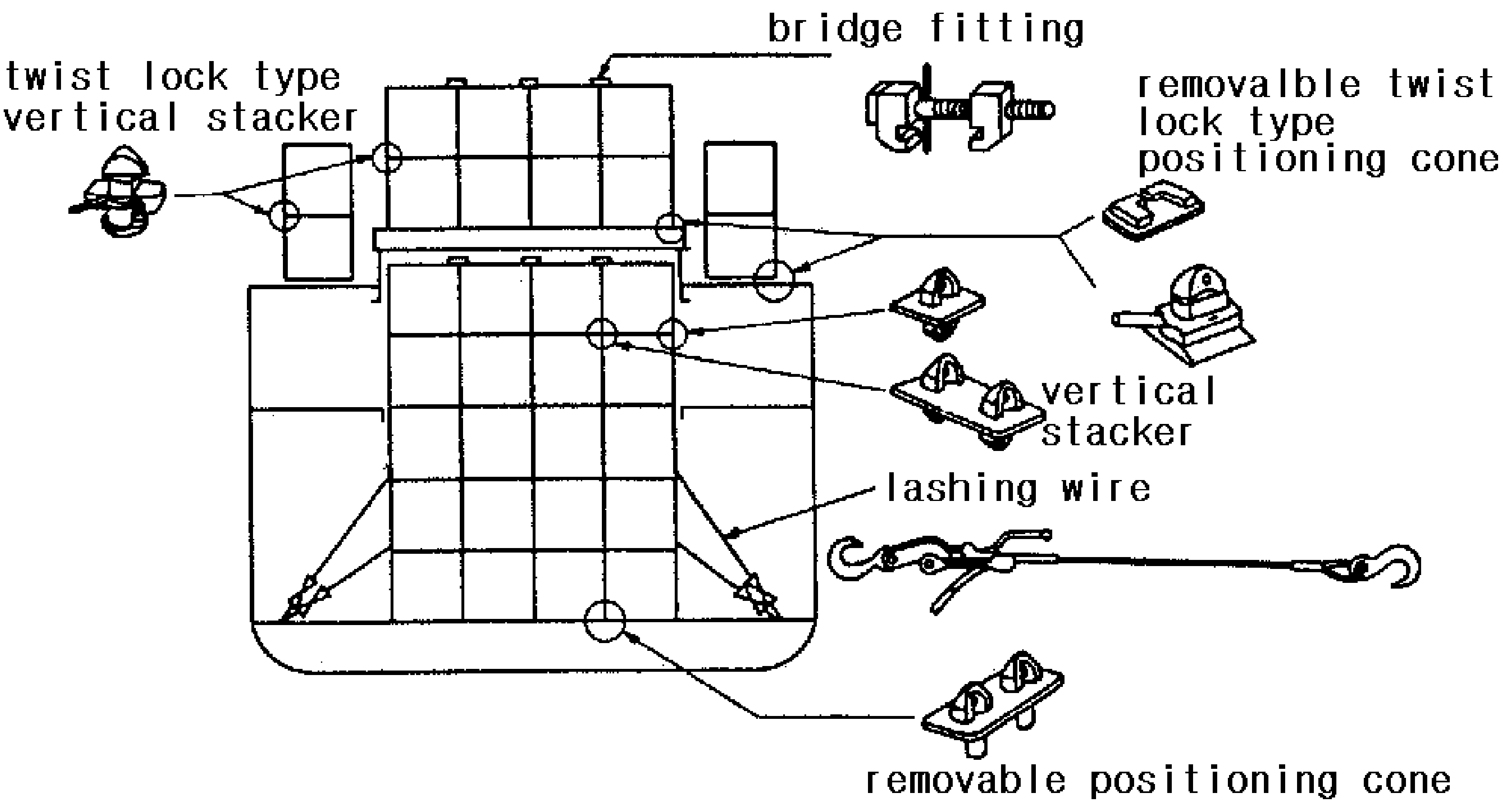
CELLGUIDE SYSTEM 582



DESIGN-MANUFACTURE-ASSEMBLY

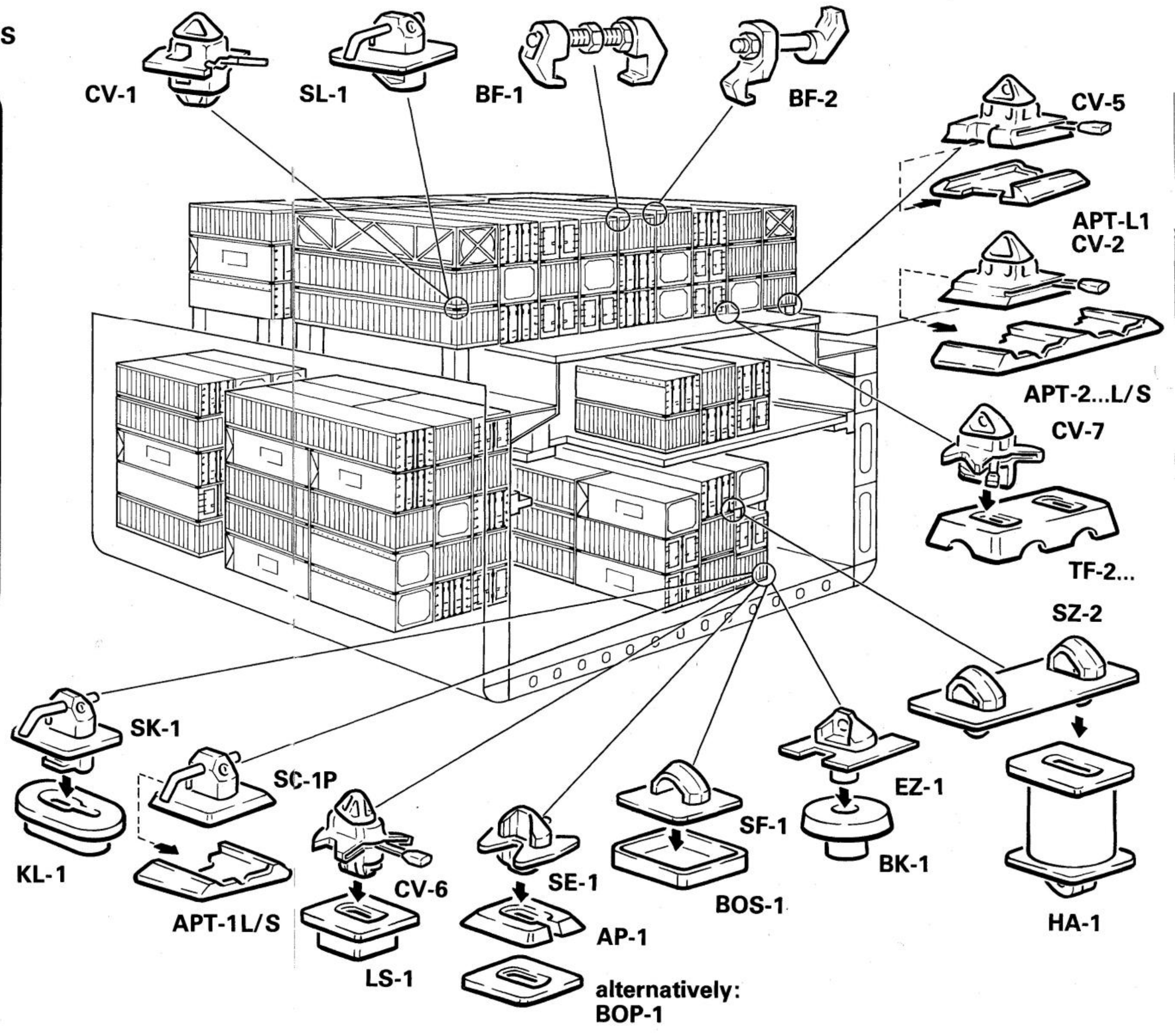
(자료 출처 : CONVER)

Container **Fittings** for **General Cargo Ship**



3 INTERMEDIATE & BOTTOM STACKERS TWISTLOCKS/BRIDGE FITTINGS

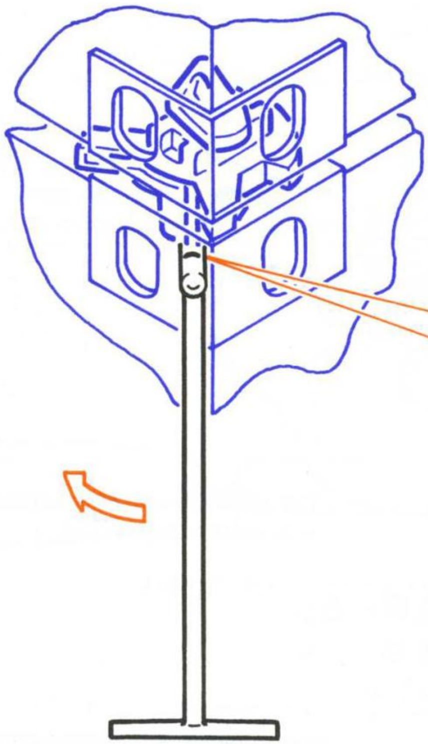
	DESCRIPTION	TYPE
3.1	INTERBRIDGE STACKING CONES SPECIAL STACKING CONES	SZ-... SZ-2-Y/-V/-CV
3.2	LEVELLING TYPES OF STACKING CONES	HA-1 HA-V/-CV
3.3	REMOVABLE CONE PLATES	EPZ-... EZ-1
3.4	BOTTOM STACKING CONES LOCKABLE STACKING CONES	SF-1/SFP-1/SE-1 SC-1/SL-1/SK-1
3.5	'SLIDE'-LOCKS SEPARATE CONES/ISO-SEA-LAND'	AC-1/AL-1 I/III/AK-P/L/IS-1/-2
3.6	BOTTOM TWISTLOCKS	CV-2 CV-5
3.7	TWISTLOCKS	CV-1 CV-1A
3.8	TWISTLOCKS FIXED BASE TWISTLOCKS	CV-3 CV-7/CV-7R
3.9	TWISTLOCKS	CV-6 CV-6-35'
3.10	TWISTLOCK ADAPTERS TWISTLOCK OPERATING RODS	LP-.../PA-... TYP I/II/III
3.11	TWISTLOCK OPERATIONS	
3.12	BRIDGE FITTINGS	BF-1/-2/-4 BF-3/BF-SR

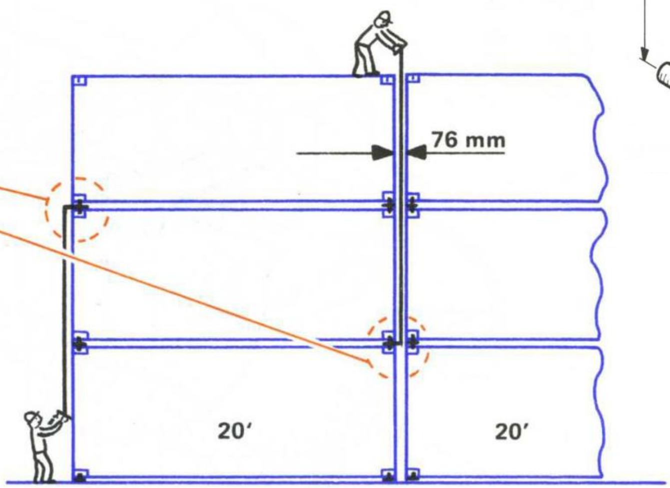


(자료 출처 : Conver)

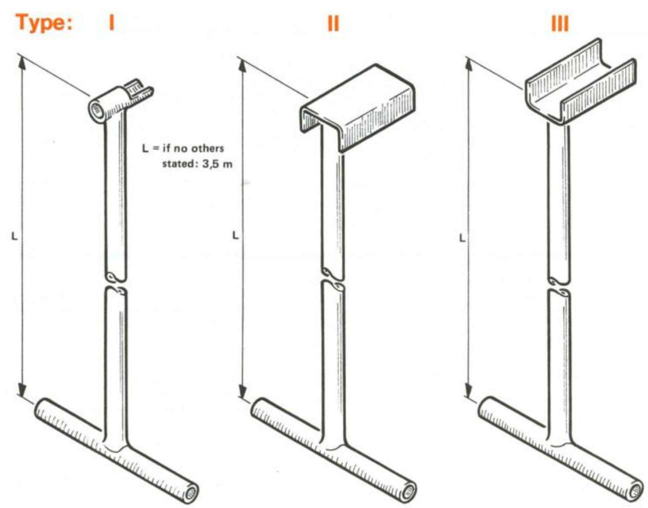
Remark:
Corresponding bottom foundations please see Sect. 2

Twistlock Operating Rods





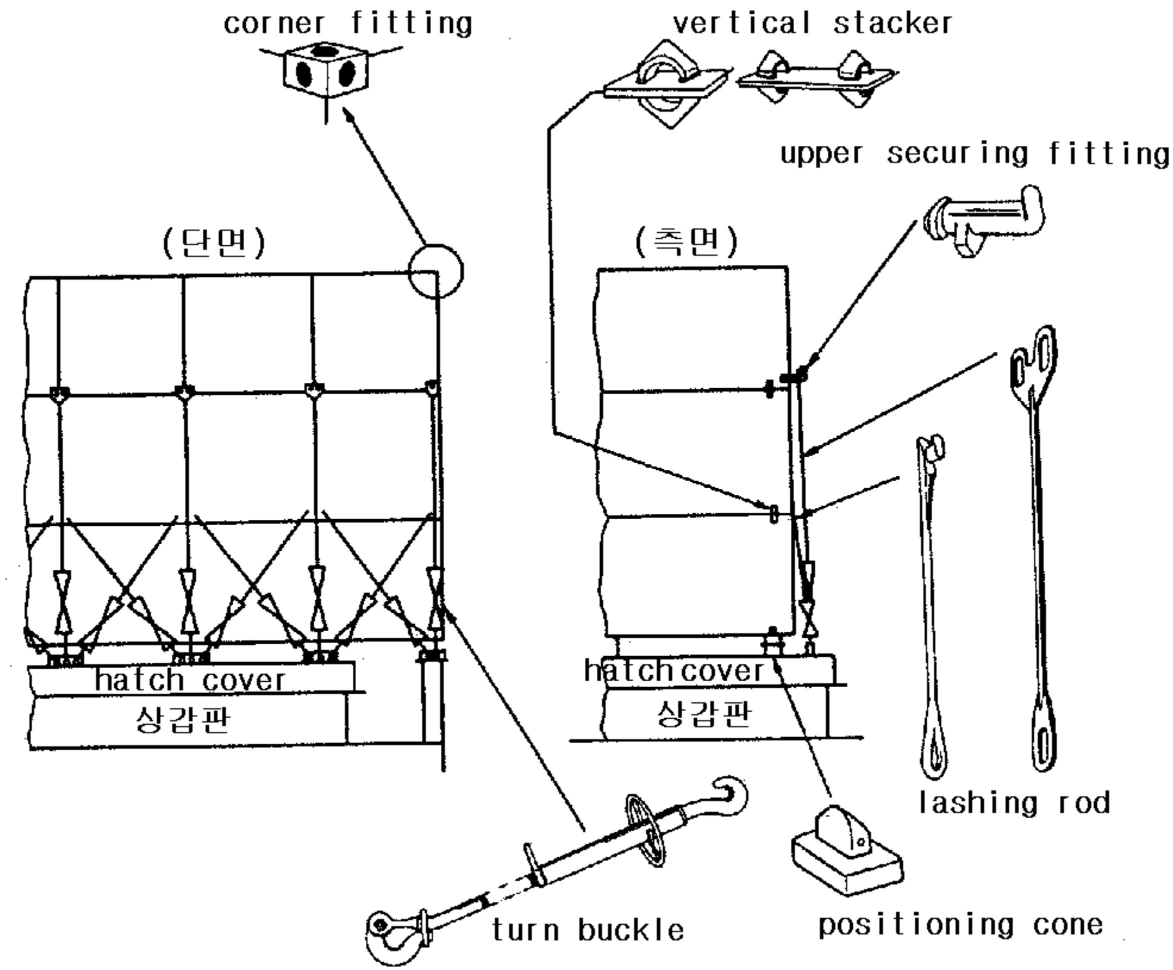
The twistlock operating rods can be used both for handling from deck or from top of containers.



NOTE: These are three types of actuator poles. Special types can be designed and manufactured on request. When ordering, please state the length of the actuator pole and the types of twistlocks (with or without plastic cap). Material: Steel-tube, on request: Al-tube.

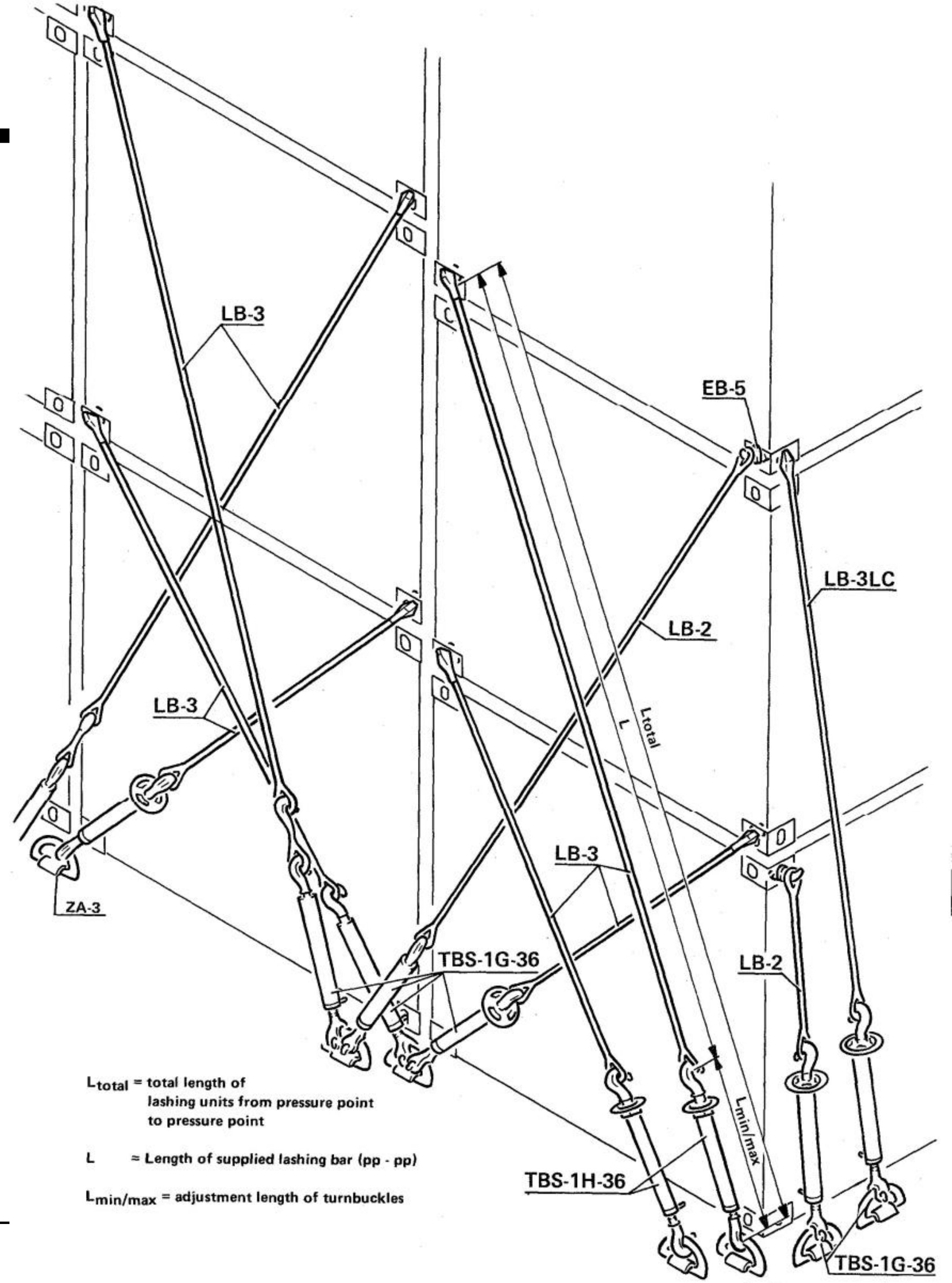
Container **lashing** equipment **on deck**

노출부 컨테이너의 결박 장치의 예



Container lashing equipment on deck

LB: Lashing Bar
TBS: Turn Buckle

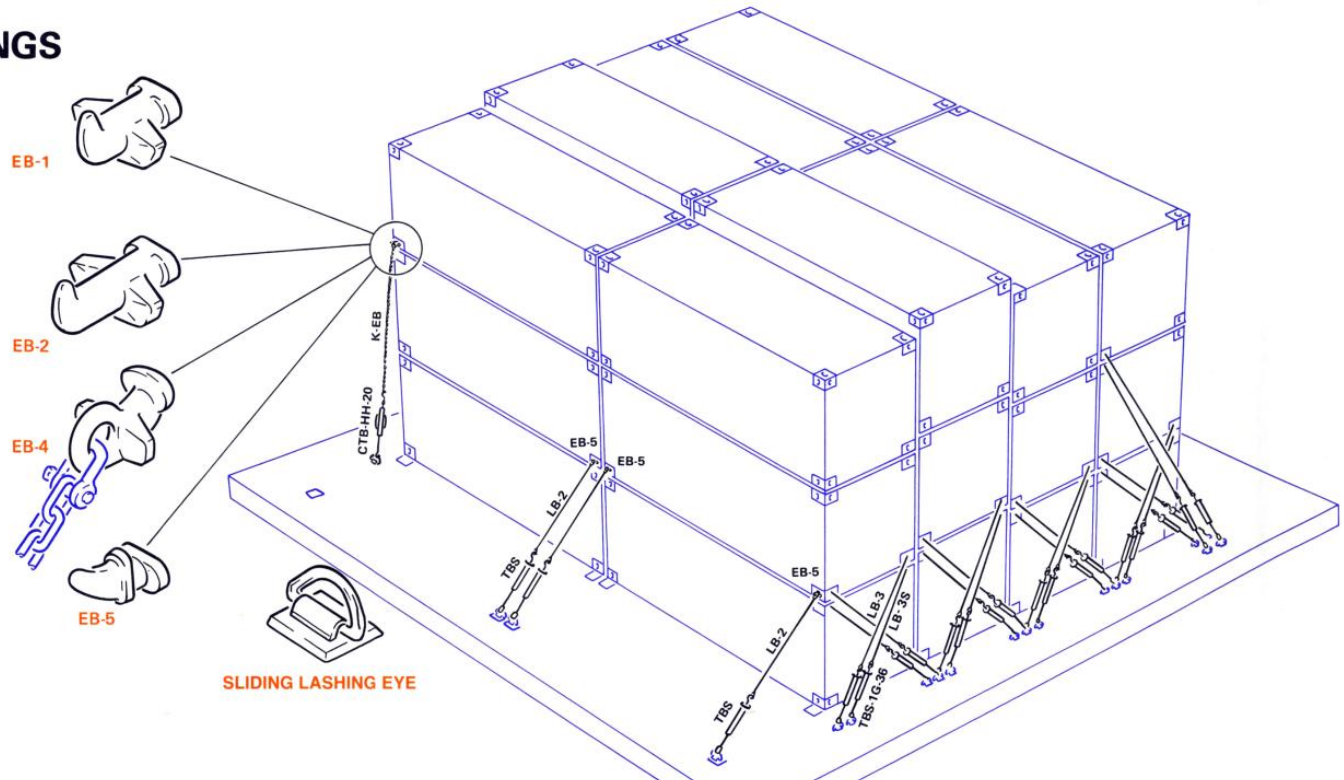


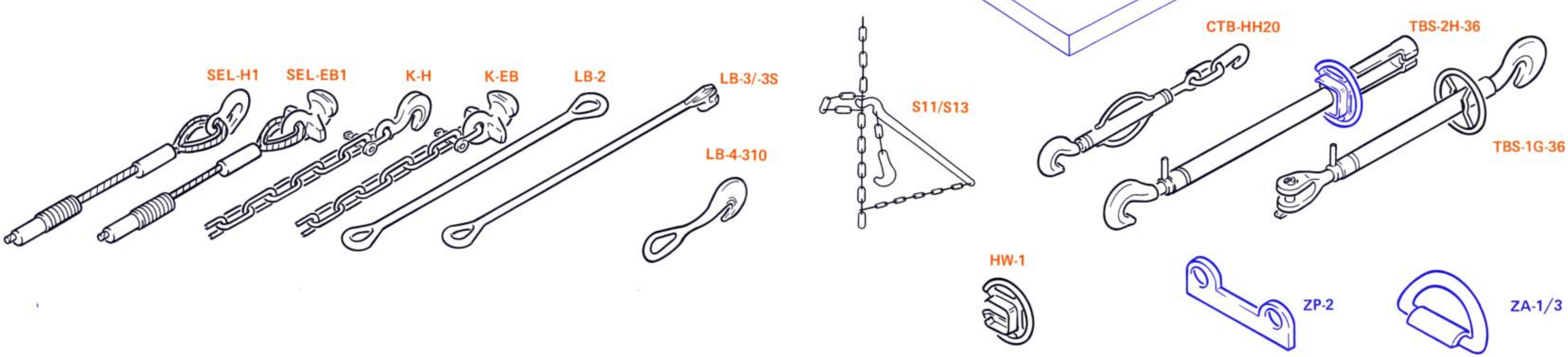
(자료 출처 : CONVER)

Lashing Units & Fittings

4 LASHING UNITS & FITTINGS

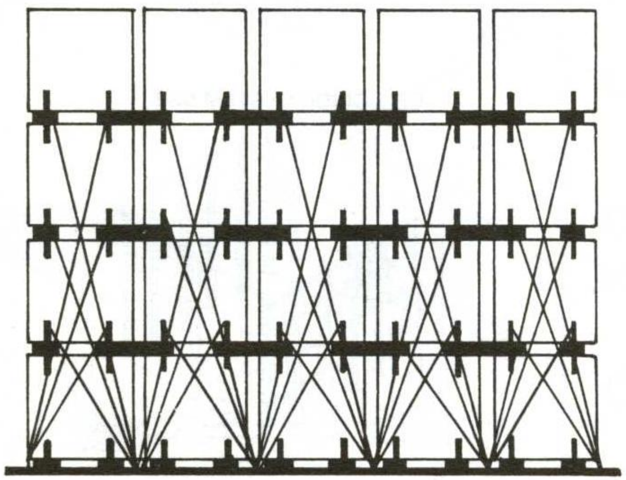
	DESCRIPTION
4.1	PARALASH ®-SYSTEM
4.2	PARALASH ®-SYSTEM
4.3	CONVENTIONAL LASH-SYSTEM
4.4	SEA-LAND CONTAINER-LASHING
4.5	WIRE-LASHING
4.6	CHAIN-LASHING
4.7	LASHING-BARS LASHING-WIRES
4.8	LASHING-CHAINS TURNBUCKLES
4.9	QUICK-RELEASE-LASHING TENSION LEVER
4.10	SECURING PADS HOOKS
4.11	SHACKLES
4.12	



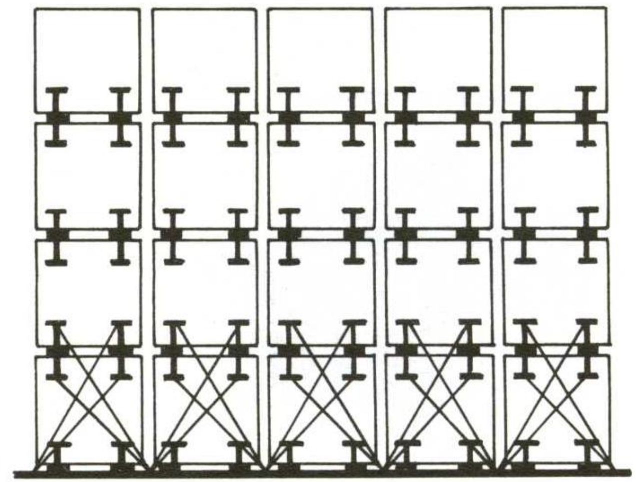


TWO GENERATIONS OF LASHINGS

CONVENTIONAL LASH-SYSTEM



CONVER PARALASH®-SYSTEM



Advantage of PARALASH®-SYSTEM

1.) 30-50% less lashing bars and turn-buckles

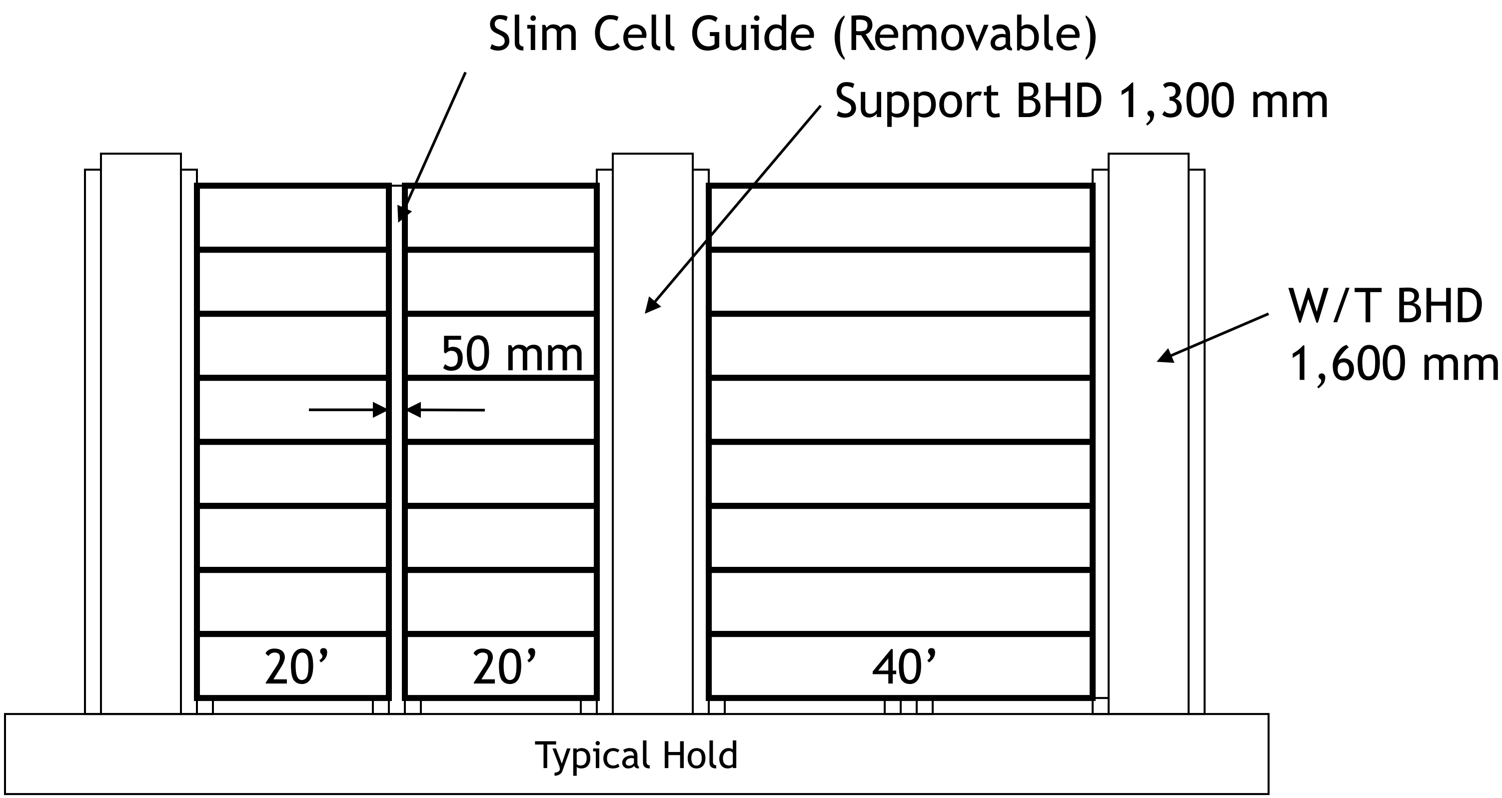
2.) same and partly higher stack load

3.) shorter lashing bars and thus better handling (weight)

4.) higher flexibility for stowage of 8′ and 8′6″ containers

5.) less investment and servicing costs

Container Arrangement in Hold



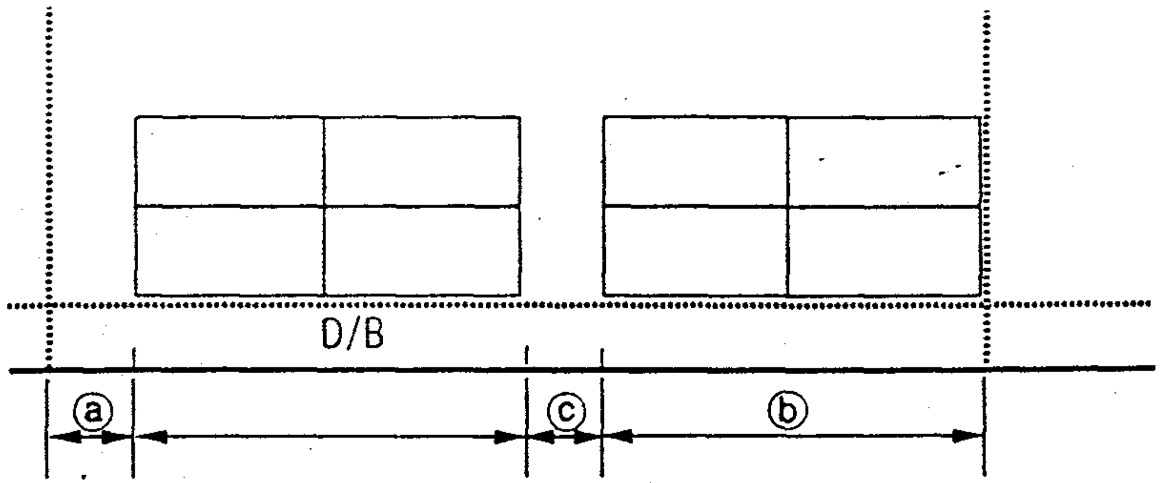
※ Note

. 20 ft Container 사이에는 50 mm의 Slim Cell Guide를 설치한다.

. Support BHD는 사람이 Access 가능하도록 통상 1.4 m의 공간을 둔다.

. 20 ft Container 전용인 경우 Slim Cell Guide를 설치하나 20 ft, 40 ft 겸용인 경우는 Slim Cell Guide를 설치하지 않는다.

Container의 배치 기준(In Hold)



- In Hold에는 주로 40 ft와 20 ft Container만 실리므로 ⓑ 길이는 아래의 기준으로 정하는 것을 표준으로 한다.

  4,000 TEU 이상일 때 → 12.72 m
  4,000 TEU 미만일 때 → 12.64 m

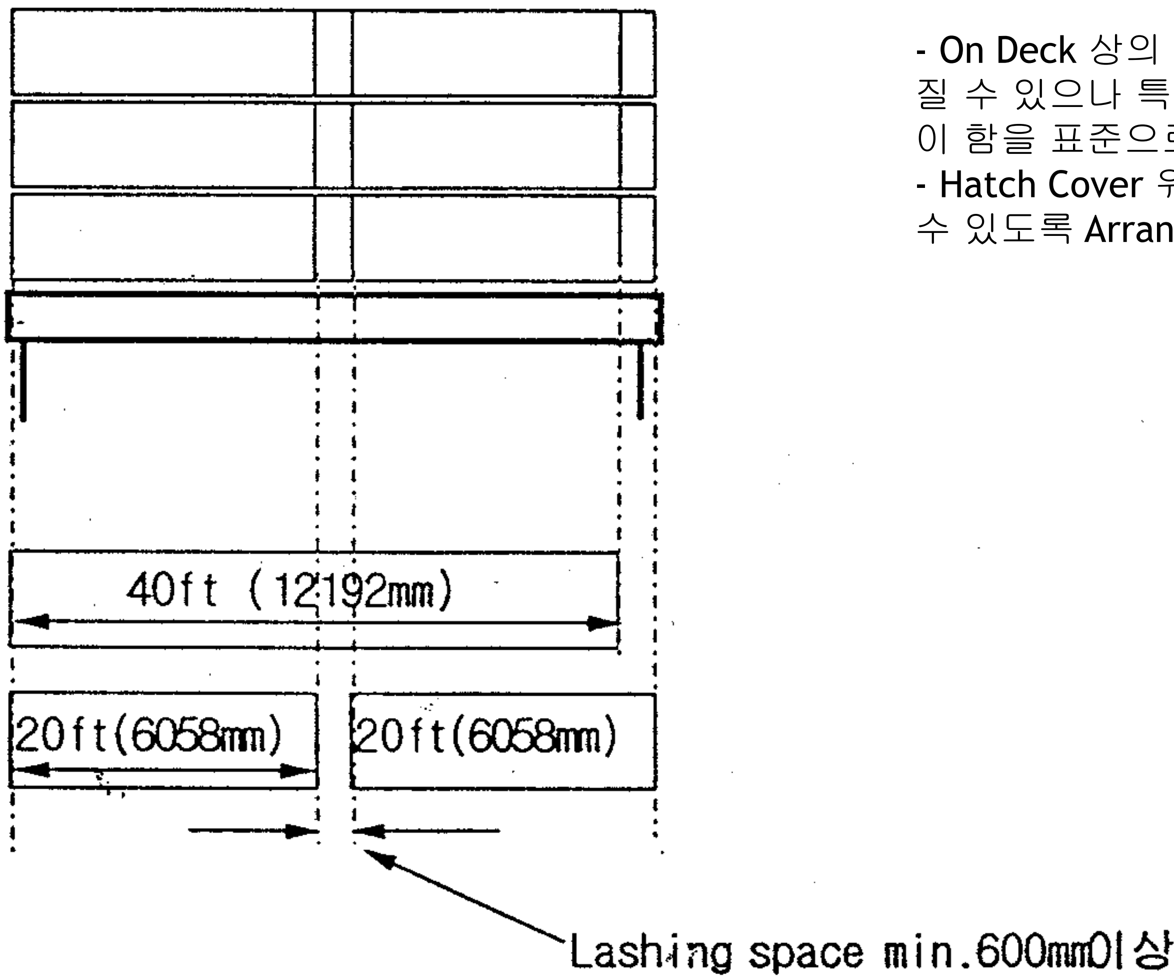
- Hold 사이 공간인 ⓐ와 ⓒ는 Hold Access Space로 통상 ⓐ는 1.60 m ⓒ는 1.4 m를 표준으로 한다.

- 단, Reefer Container Hold인 경우 ⓐ, ⓒ는 환기 공간을 고려하여 Reefer Socket이 설치되는 부분은 1.8 m로 하나 특별한 선주의 요구가 있을 시 선장과 협의하여 반영한다.

- On Deck에 Cargo Crane이 설치되는 경우는 ⓐ 또는 ⓒ를 3.4 m로 한다.

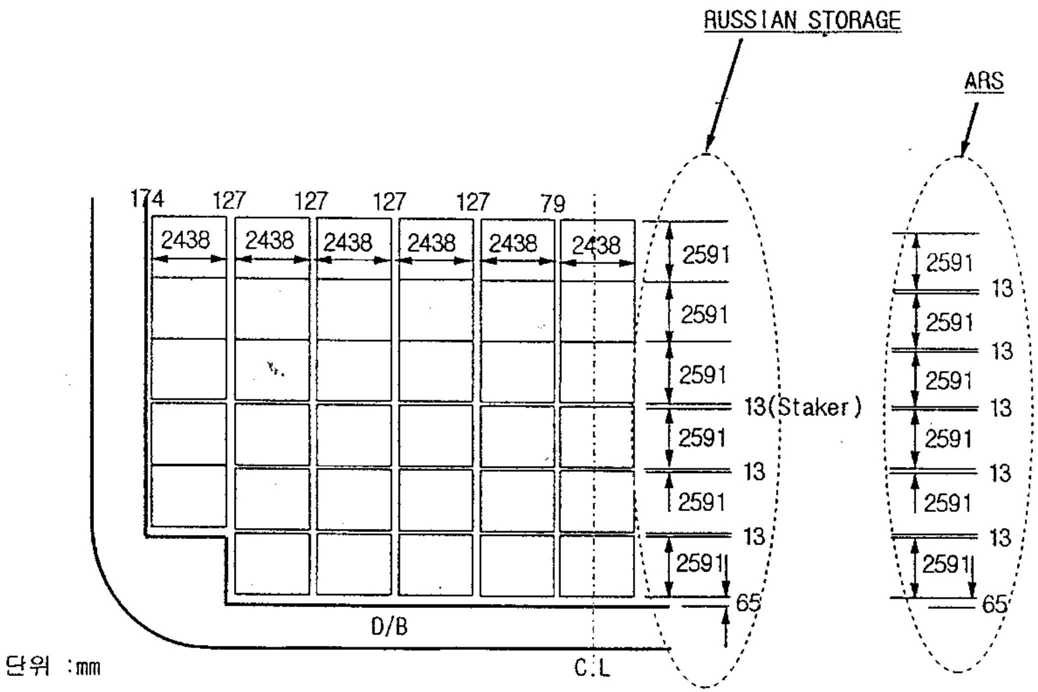
- 새로운 Design인 경우는 위의 원칙을 따르나 기존의 실적선을 사용하는 경우 Hold Space를 다르게 가져가는 경우도 있다.

Container의 배치 기준(On Deck)



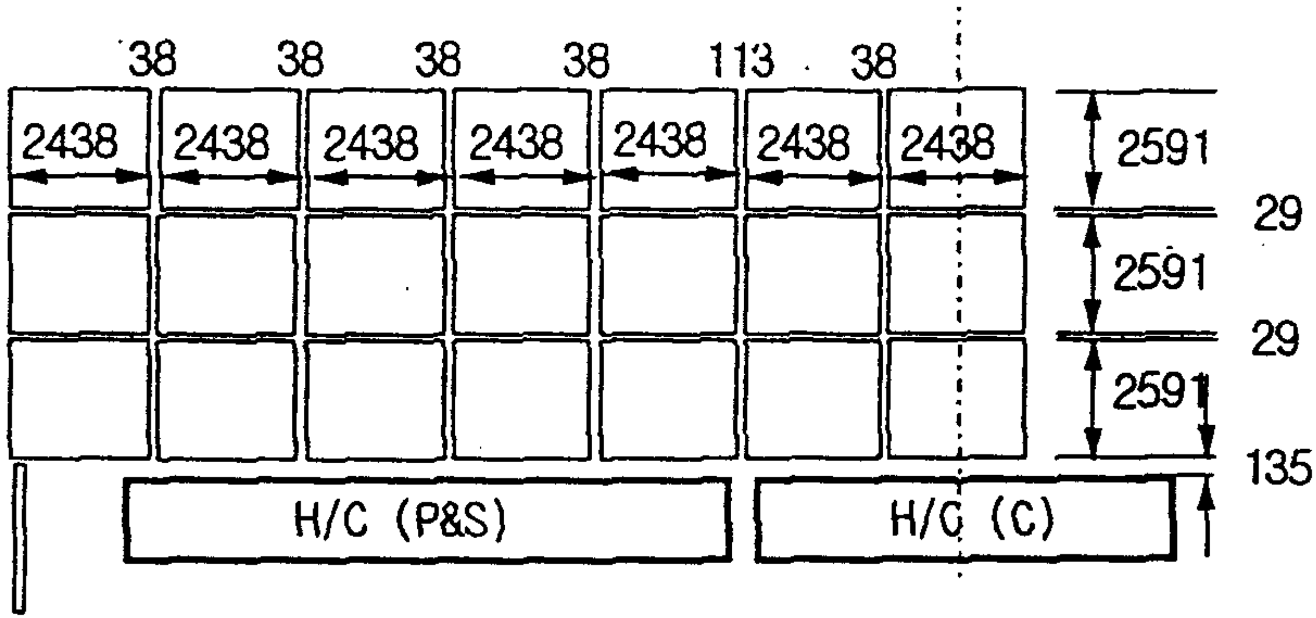
- **On Deck** 상의 **Container Arrange**는 선주에 따라 달라질 수 있으나 특별한 요구가 없을 때에는 위 그림과 같이 함을 표준으로 한다.
- **Hatch Cover** 위에 **20 ft**와 **40 ft Container**를 같이 실을 수 있도록 **Arrange**하여야 한다.

Container의 적재 기준(In Hold)



- Hold 내의 Container Arrange는 단순히 G/A 뿐만 아니라 Trim & Stability 계산 시 Cargo VCG 계산의 근거 및 In Hold Cargo Check Point 가 된다. 따라서 초기에 정확히 확인할 필요가 있다.
- 통상 Russian Storage (In Hold에 20 ft Container를 Slim Cell Guide 없이 싣고 그 위에 40 ft Container로 눌러서 싣는 방식)일 때 4단까지 실리므로 그 사이에 Stacker 13 mm를 고려하여야 하고 그 상방은 Staker가 없이 그냥 싣는 방식이다.
- ARS 방식일 때에는 Top Tier까지 Stacker 13 mm를 고려하여야 한다.
- 20 ft 전용 Hold (Slim Cell Guide)일 때에는 Stacker가 필요 없다.
- 또한 8 ft 6 inch Height가 Base이나 Top Tier는 9 ft 6 inch를 실을 수 있도록 고려하여야 한다. (Deck 및 Hatch Coaming Height 결정 시 고려)

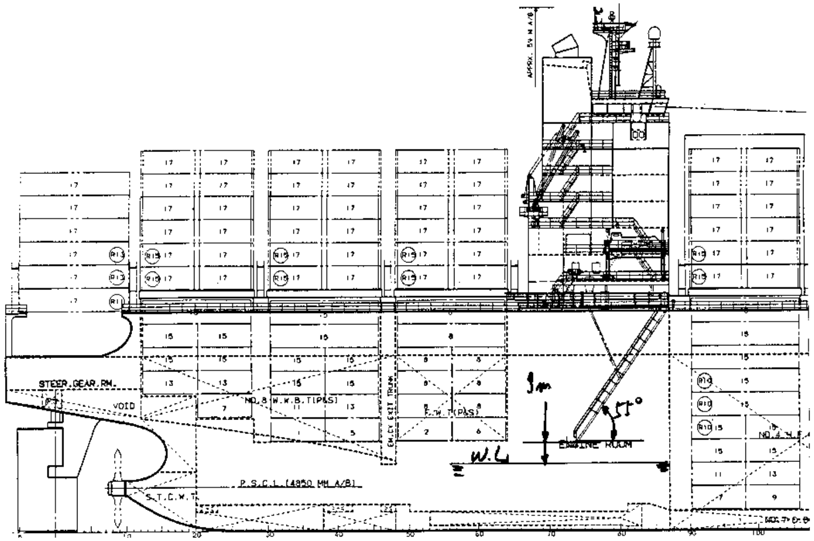
Container의 적재 기준(On Deck)



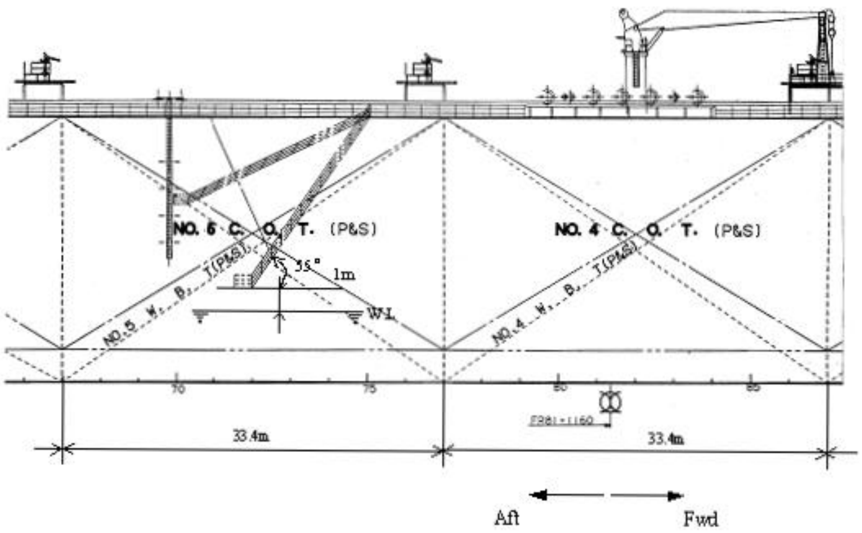
- On Deck Cargo Arrange는 기본적으로 위 그림과 같이 Loading한다. 단 Hatch Cover Height는 On Deck의 Cargo Arrange와 관련이 있으므로 선장의 확인을 받을 필요가 있다.
- On Deck Reefer Container Arrange의 경우 초기 Scheme을 확인하여 선장과 협의하여 그린다.

Accomodation Ladder(선측 사다리)

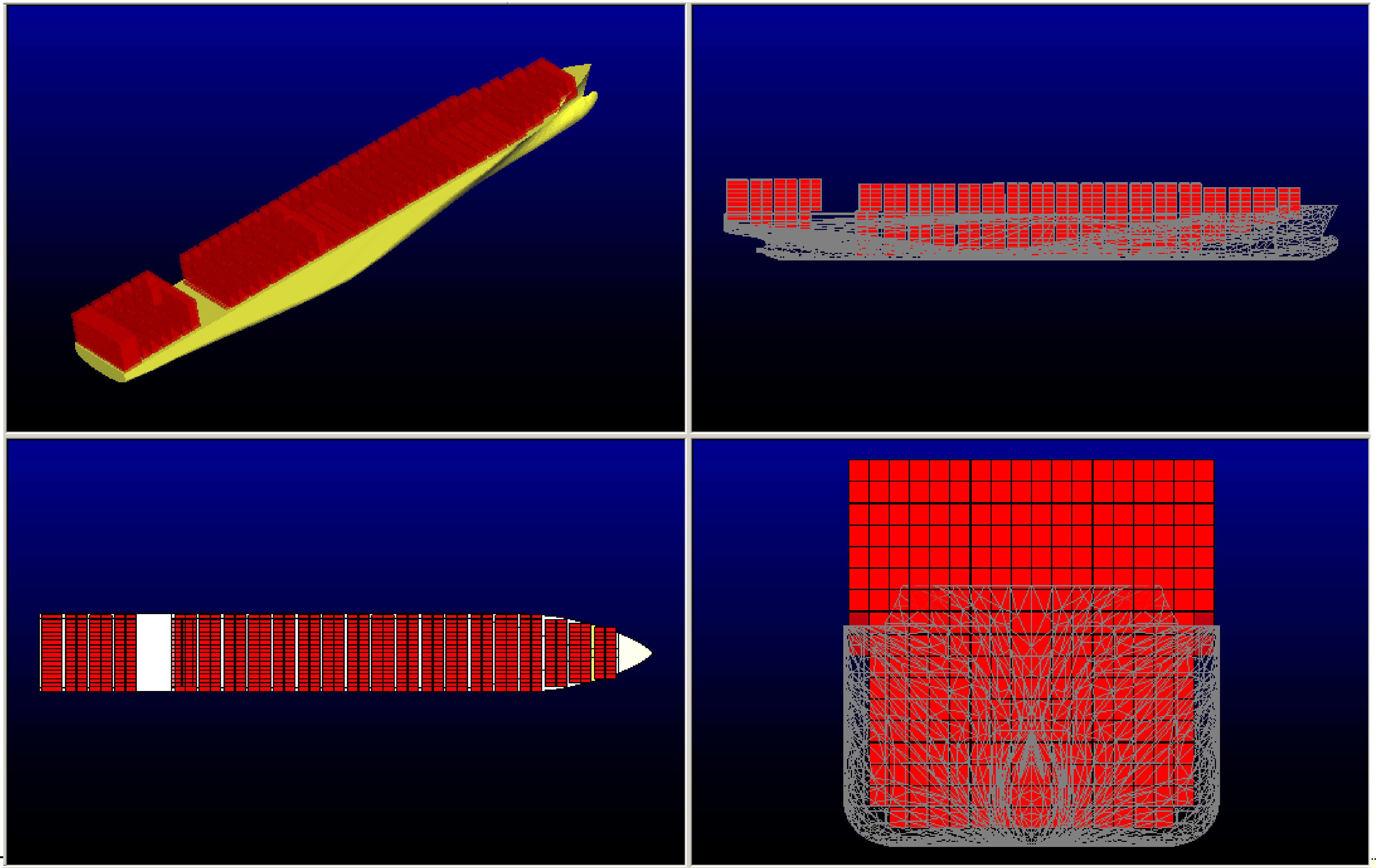
Container의 선측 사다리 장치 배치



Tanker의 선측 사다리 장치 배치



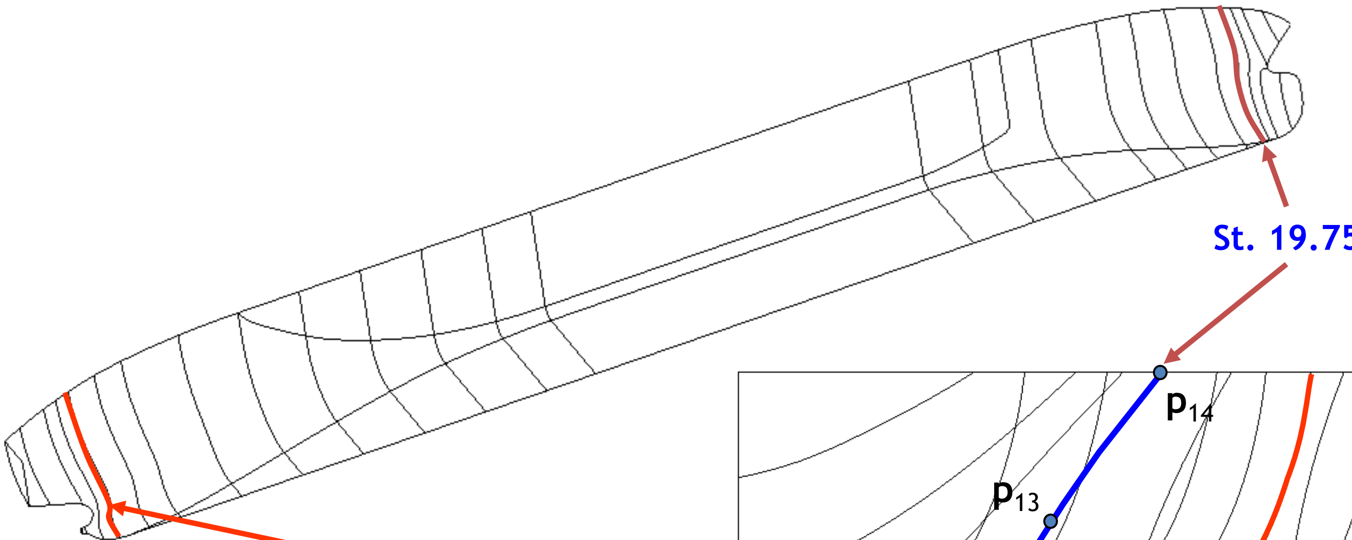
Example of the Container Loading of a 9,000TEU Container Carrier

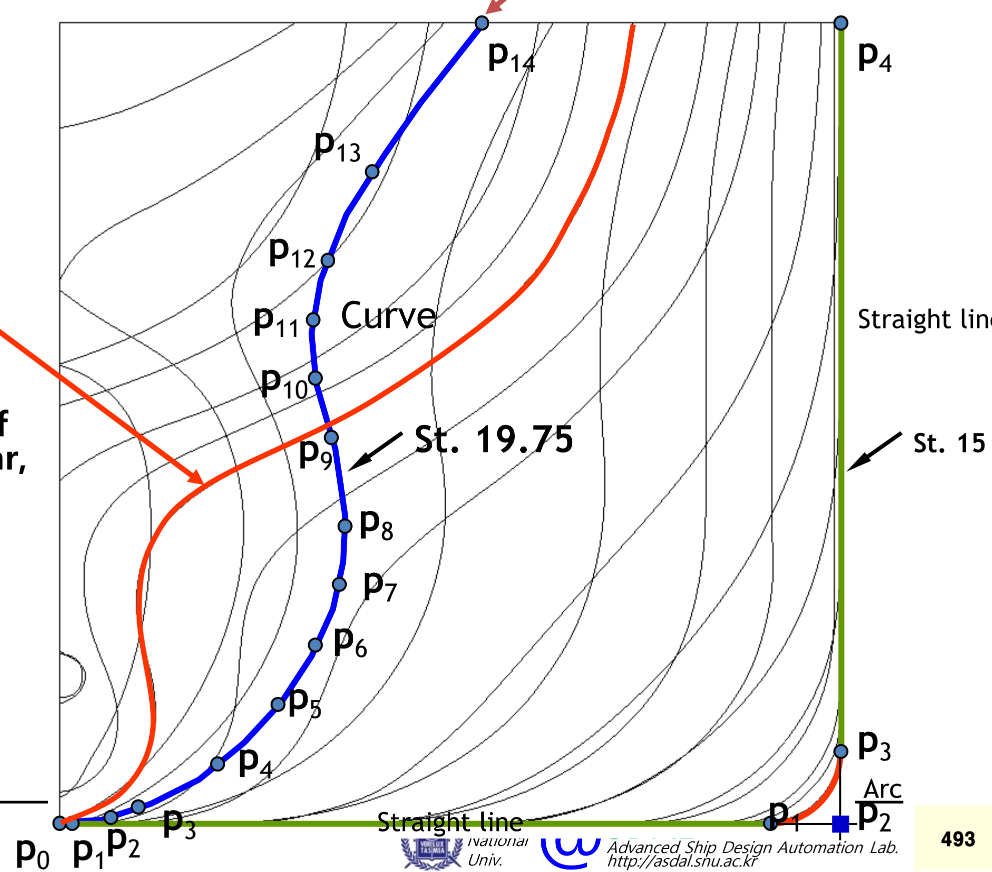


Chapter 9. Hull Form Design

9-1 Hull Form and Form Coefficients

Section Line & Body Plan

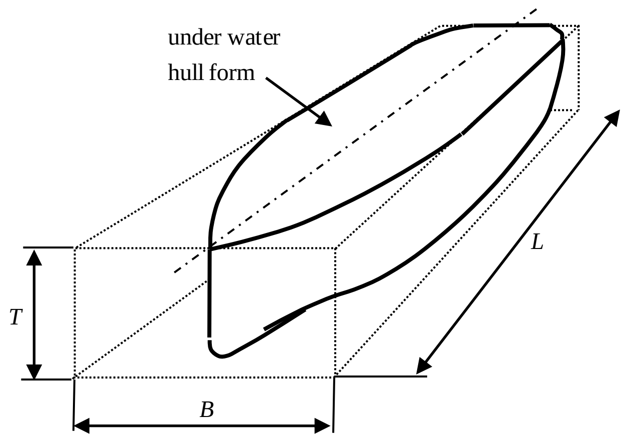




- ☑ **Section line is a curve located on a cross section.**
- ☑ **Stations are ship hull cross section at a spacing of $L_{BP}/20$, station 0 is located at the aft perpendicular, station number 20 at the forward perpendicular. Station number 10 therefore represents the midship section.**
- ☑ **In generally, because the section lines are located at the stations, they are called station line.**
- ☑ **Section lines make up the lines plan(Body plan).**

Form coefficients
- C_B(Block coeff.) and C_P(Prismatic coeff.)

C_B (Block coefficient)



$$C_B = \frac{\nabla}{L \cdot B \cdot T}$$

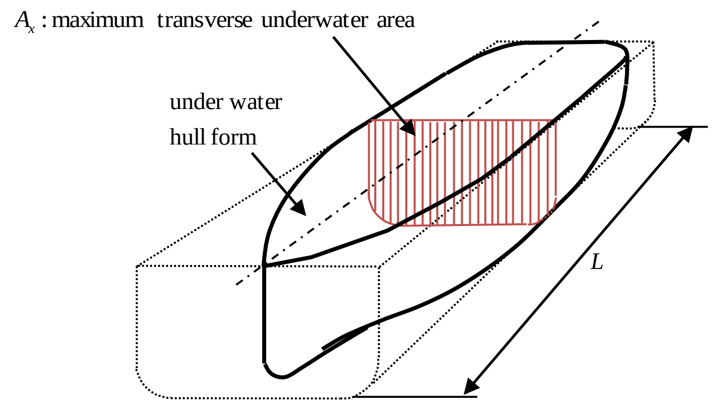
∇ = moulded volumd of displacement

L = length of the ship (LWL or LBP)

B = moulded breadth

T = moulded draft

C_P (Prismatic coefficient)



$$C_P = \frac{\nabla}{L \cdot A_M} = \frac{C_B}{C_M}$$

∇ = moulded volumd of displacement

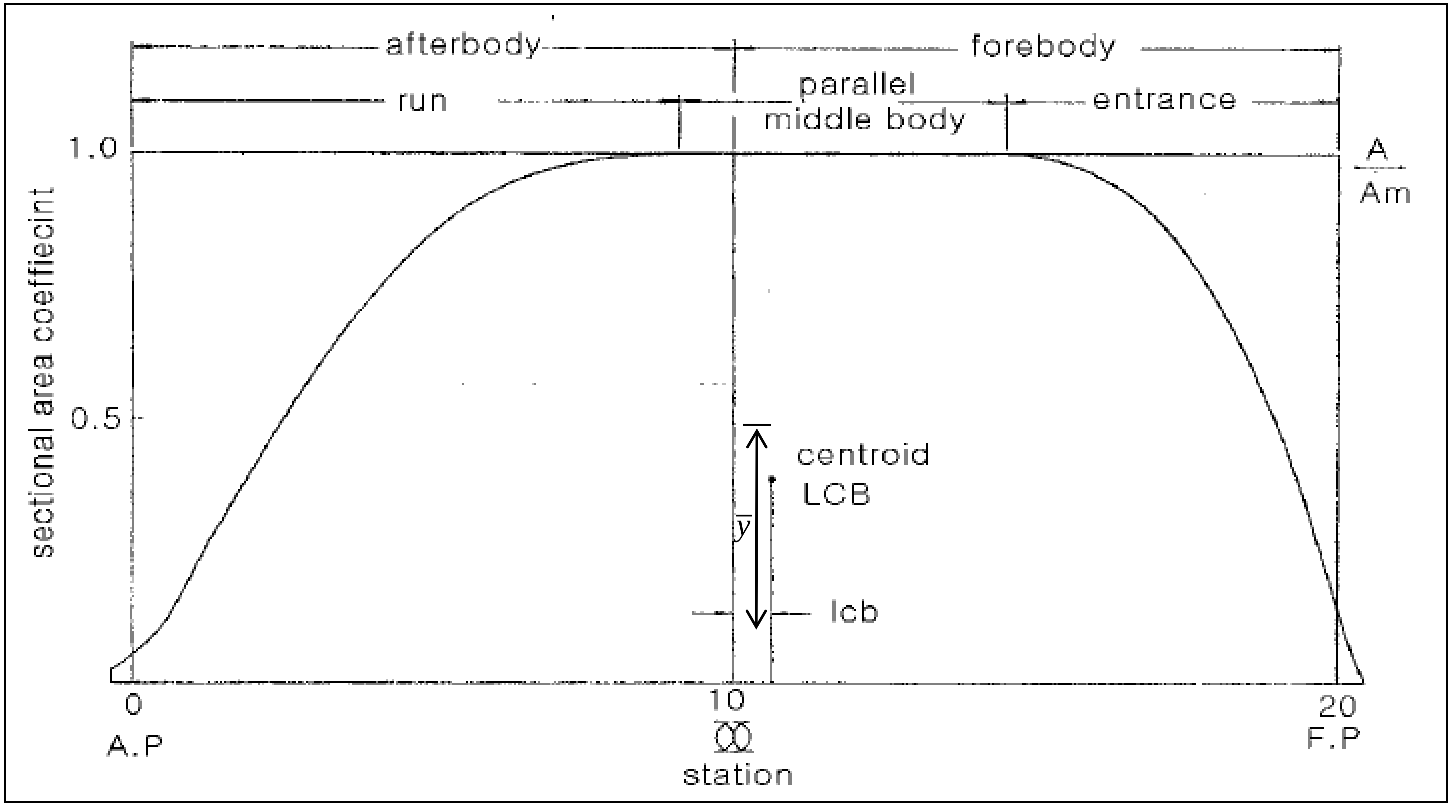
L = length of the ship(LWL or LBP)

A_M = area of the Midship Section

C_M = midship coefficient

C_P-curve(or Sectional area curve)

- C_P curve(or sectional area curve) is a diagram of transverse section areas up to the designed waterline plotted on a base on length.
- This diagram may be made dimensionless by plotting each ordinate as the ratio of the area A of any section to the area of the maximum section .
- This diagram represents the distribution of underwater volume along the length of a ship.



Sectional area curve or C_P-curve & LCB

9-2 Hull Form Variation Method

Hull form variation method - C_P Variation method

☑ In shipyard, the hull form of a similar basis ship is chosen and modified to the correct the main dimensions for the new design ship.

→ The hull form of the design ship can maintain the hydrodynamic property of the basis ship.

☑ C_P Variation method :

In deriving the lines for a new design from the a similar basis ship, it is usual to correct for displacement and LCB(Longitudinal Center of Buoyancy) by adjusting the longitudinal spacing of the transverse sections to suit the new CP curve.

– 1-C_P Variation method
– Lackenby Variation method

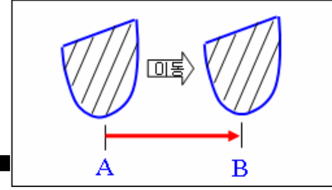
Correction for displacement

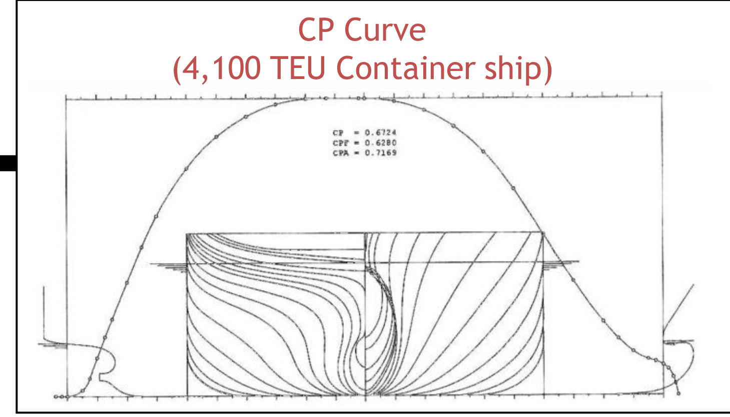
– Swing station method

Correction for LCB

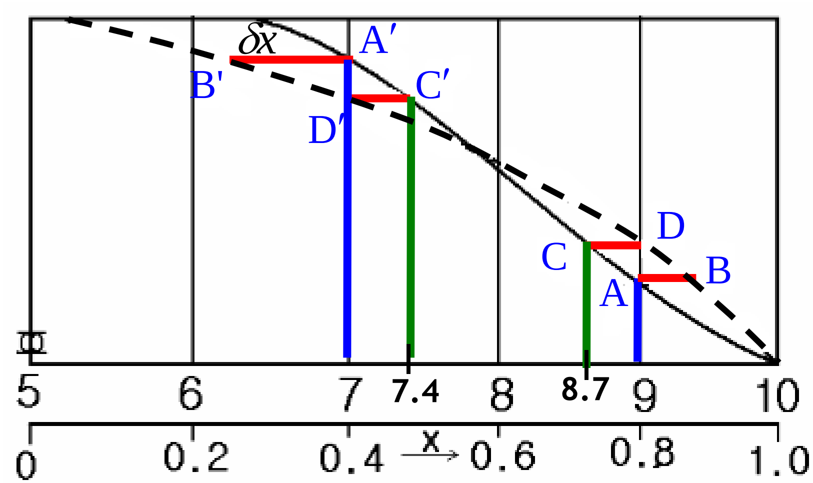
– Weighted modified swing method

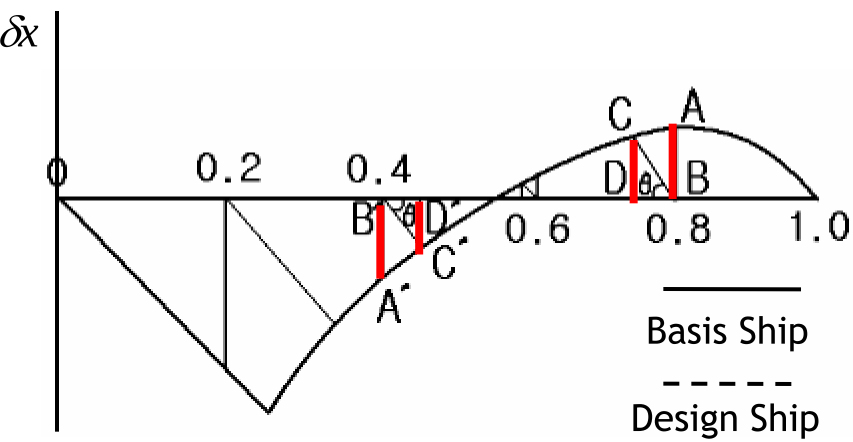
Hull form variation method
- C_P Variation method





- By adjusting the longitudinal spacing of the transverse sections to suit the new CP curve





*The L_{BP} is normalized in terms of two. (from Midship: ±1)

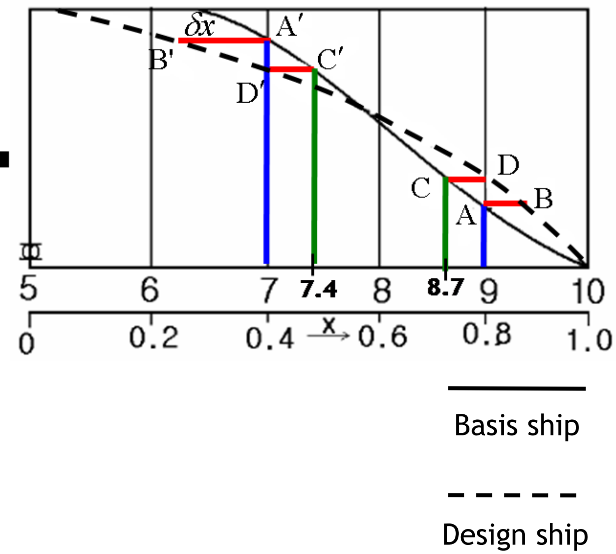
① The transverse section of the basis ship located at station 9(x=0.8)
→ In the design ship, the transverse section of the basis ship located at station 9 is moved through distance **AB**.

② The transverse section of the design ship located at station 9 is obtained from that of the basis ship located at station 8.7.

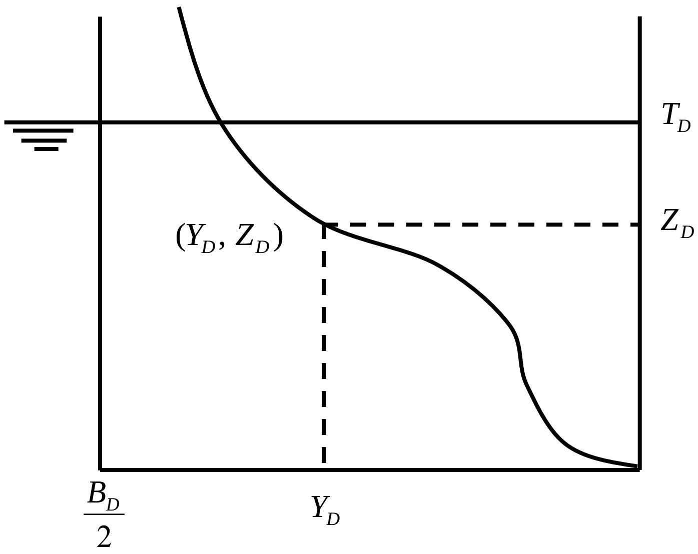
③ The transverse section of the basis ship located at station 7(x=0.4)
→ In the design ship, the transverse section of the basis ship located at station 7 is moved through distance **A′B′**.

④ The transverse section of the design ship located at station 7 is obtained from that of the basis ship located at station 7.4.

Design of a Body plan



The body plan of design ship is design by adjusting B(Breadth) and T(Draft).



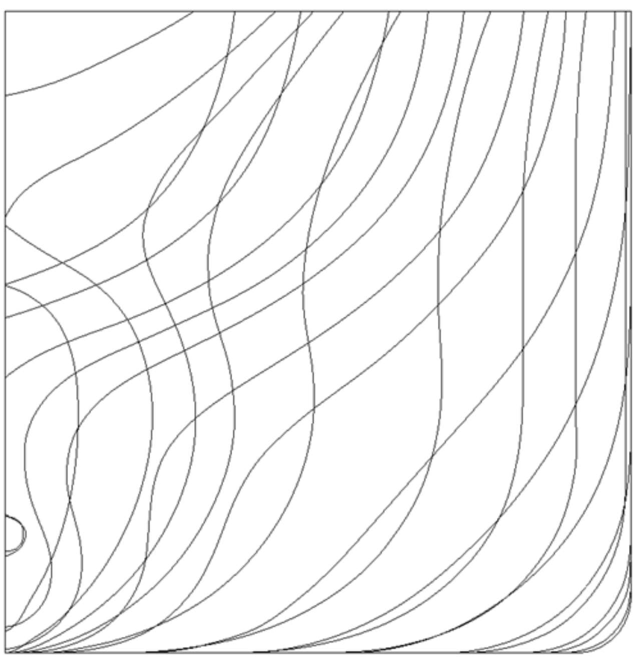
$$Z_D = Z_E \cdot \frac{T_D}{T_E}$$

$$Y_D = Y_E \cdot \frac{B_D}{B_E}$$

E : for existing

D : for desired

Body plan

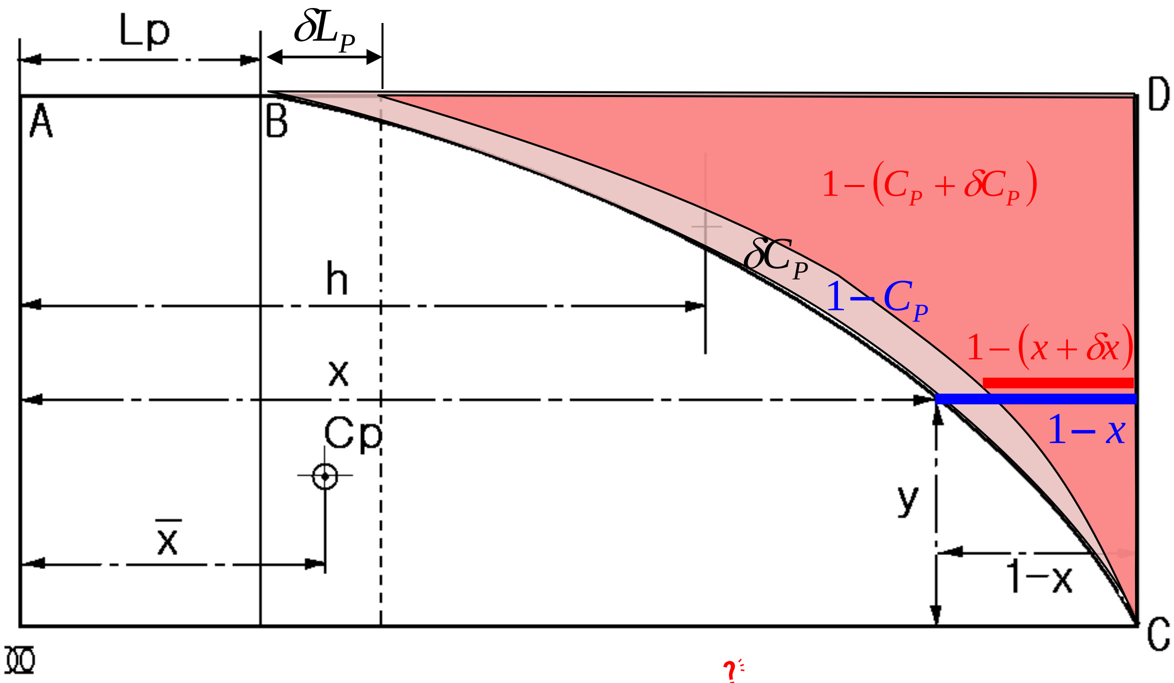


C_P Variation method
- "1-C_P" Variation method

Reference: Lackenby, On The Systematic Geometrical Variation of ship forms, 1950, RINA , p.290

Given : **The forebody and afterbody prismatic coefficient of the basis ship($C_{P_{a,f}}$),**
The required change in forebody and afterbody prismatic coefficient($\delta C_{P_{a,f}}$)

Find : $\delta x_{a,f}$



✓ **Assumption** : "The new spacing of the sections from the end of the body is made proportional to the difference between the respective prismatics and unity"

$$1-\left(x_{f,a}+\delta x_{f,a}\right):1-x_{f,a}$$

$$=1-\left(C_{P_{f,a}}+\delta C_{P_{f,a}}\right):1-C_{P_{f,a}}$$

$$\delta x_{f,a}=\frac{\boxed{\delta C_{P_{f,a}}}}{1-C_{P_{f,a}}}\left(1-x_{f,a}\right)$$

: "1-Cp" Variation method

How to get the value of $\delta C_{P_{f,a}}$?

$C_P = C_b / C_m$

δC_P: the required change in prismatic coefficient of the half-body

x : the fractional distance of any transverse section from midships

δx : the necessary longitudinal shift of the section at x to produce the required change in prismatic coefficient

h : the fractional distance from midships of the centroid of the added "sliver" of area represented by δC_P

L_P : the fractional parallel middle of the half-body

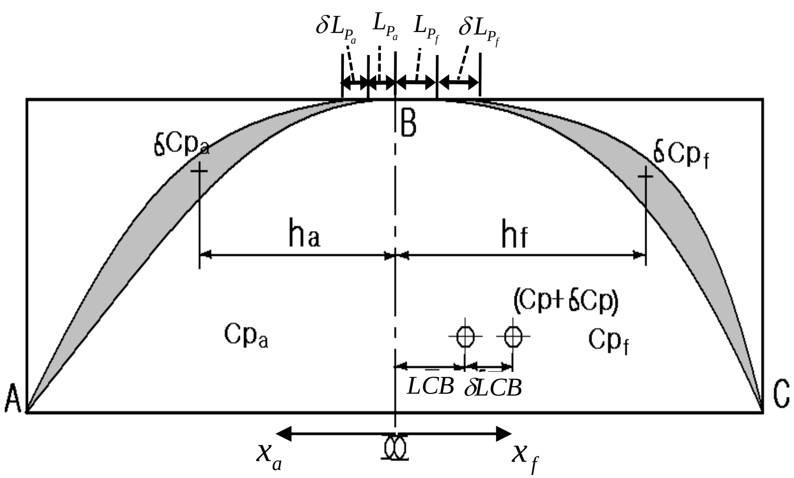
δL_P : the consequent change in parallel middle body

$\bar{x}$: the fractional distance from midships of the centroid of the half body

y : the area of the transverse section at x expressed as a fraction of a fraction of the maximum ordinate

C_P Variation method
- "1-C_P" Variation method

Reference : Lackenby, On The Systematic Geometrical Variation of ship forms, 1950, RINA , p.295



The + sign indicates movement away from midships.(x_a, x_f)

$$C_P = C_b / C_m$$

δC_P: the required change in prismatic coefficient of the half-body

x : the fractional distance of any transverse section from midships

δx : the necessary longitudinal shift of the section at x to produce the required change in prismatic coefficient

h : the fractional distance from midships of the centroid of the added "sliver" of area represented by δC_P

$\bar{x}$: the fractional distance from midships of the centroid of the half body

LCB : the distance of the LCB in the basis ship from midships expressed as a fraction of the half-length

δLCB : the required fractional shift of the LCB in the derived form

✓ "1-Cp"Variation method

$$\delta x_{f,a} = \frac{\boxed{\delta C_{P_{f,a}}}}{1-C_{P_{f,a}}}\left(1-x_{f,a}\right)$$

How to get the value of $\delta C_{P_{f,a}}$?

Method 1. Using the following formula

Given: C_P, δC_P, $\boxed{h_{a,f}}$, $\boxed{LCB}$, δLCB

Find: $\delta C_{P_{f,a}}$

$$\delta C_{Pf} = \frac{2\left[\delta C_P\left(h_a + LCB\right) + \delta LCB\left(C_P + \delta C_P\right)\right]}{h_f + h_a}$$

$$\delta C_{Pa} = \frac{2\left[\delta C_P\left(h_f - LCB\right) - \delta LCB\left(C_P + \delta C_P\right)\right]}{h_f + h_a}$$

The sing of LCB and δLCB are positive for forward of midships and negative for aft of midships.

❖ The derivation of the above formula can refer to the above reference.

$h_{a,f}$ **Calculation:**

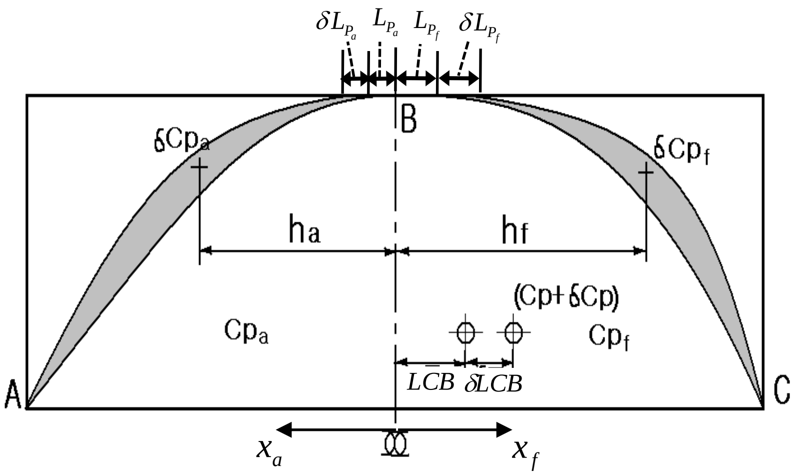
LCB **Calculation :**

C_P Variation method
- "1-C_P" Variation method

✓ "1-Cp"Variation method

$$\delta x_{f,a} = \frac{\boxed{\delta C_{P_{f,a}}}}{1-C_{P_{f,a}}}\left(1-x_{f,a}\right)$$

How to get the value of $\delta C_{P_{f,a}}$?



The + sign indicates movement away from midships.(x_a, x_f)

δC_P: the required change in prismatic coefficient of the half-body

x : the fractional distance of any transverse section from midships

δx : the necessary longitudinal shift of the section at x to produce the required change in prismatic coefficient

h : the fractional distance from midships of the centroid of the added "sliver" of area represented by δC_P

L_P: the fractional parallel middle of the half-body

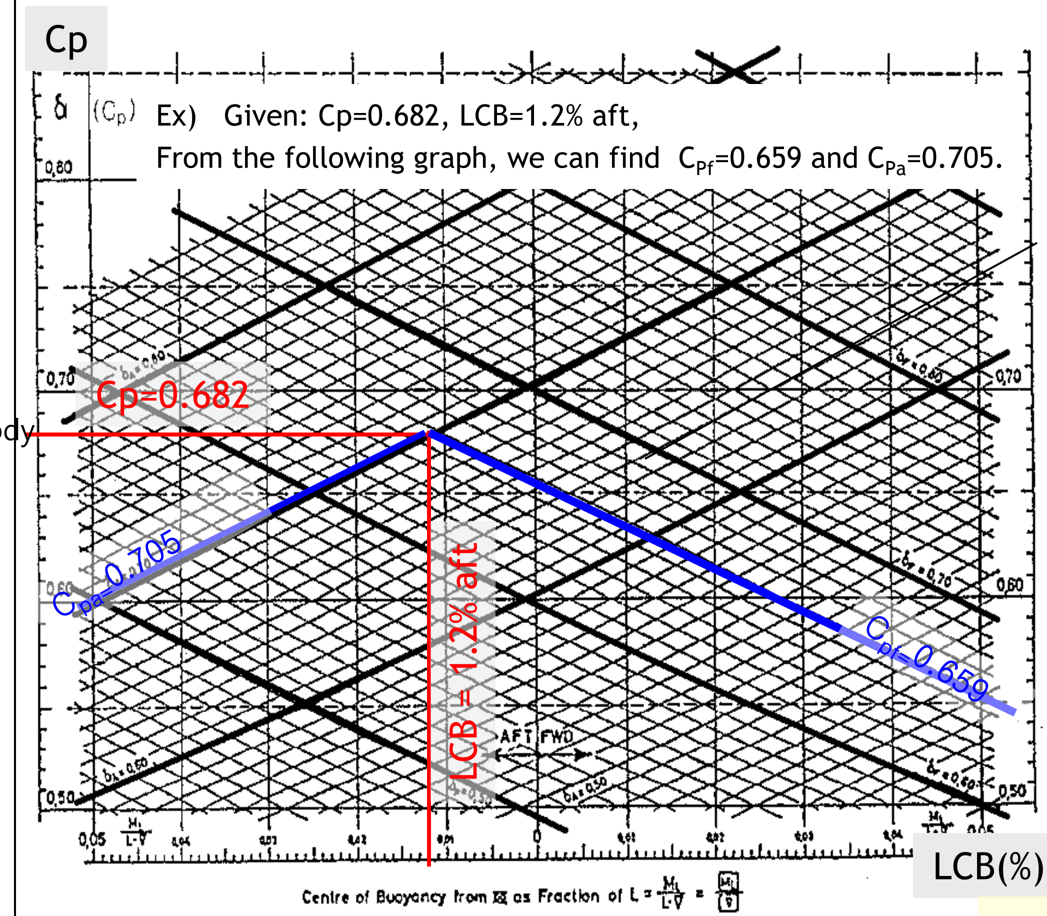
δL_P: the consequent change in parallel middle body

$\bar{x}$: the fractional distance from midships of the centroid of the half body

y : the area of the transverse section at x expressed as a fraction of a fraction of the maximum ordinate

Method 2. Using the statistical data

From the "Form Data IV" of Guldhammer,
we can find C_{P_a} and C_{P_f} according to the C_P and LCB.

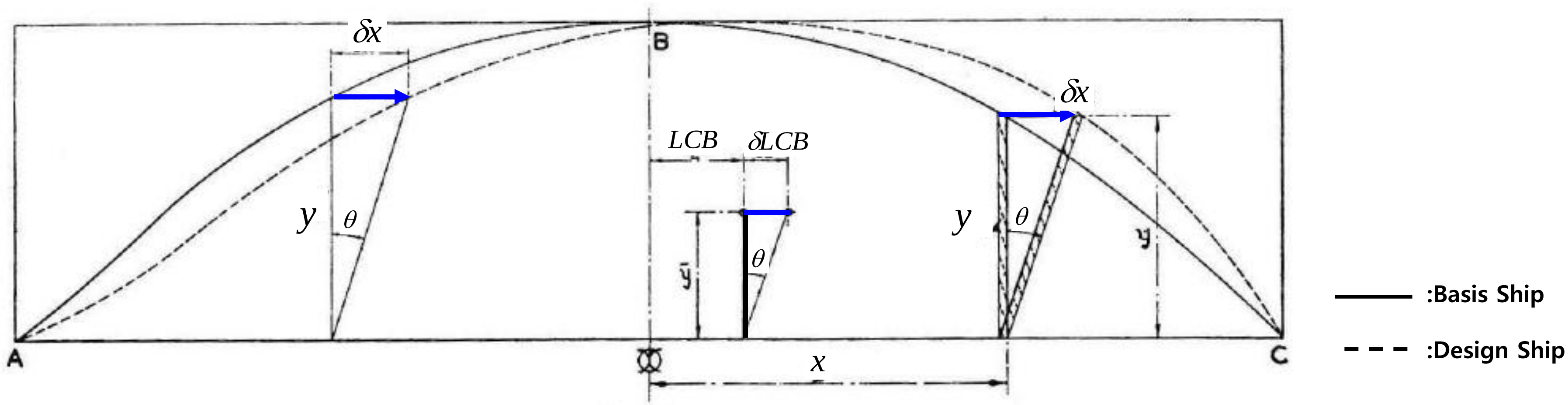


C_P Variation method
- "Swing station method"

Swing station method: Changing the LCB position of a ship's form by "swinging" the C_P curve

→This method is proposed only to change the LCB, the displacement being maintained constant.

Each transverse section of the basis ship is "swung" through the same angle θ as shown.



δLCB : the required change in LCB position

$\bar{y}$: the position of the vertical centroid of area above the base(VCB)

$$\bar{y} = \frac{\int_0^T z \cdot A_{wp}(z)dz}{\nabla}$$

→ $\bar{y}$: It can be obtained from the first moment of the submerged volume of the ship about x axis divided by the displacement.(KB, VCB) (In this method, this value has to be normalized)

Given: $\delta LCB,\ \bar{y},\ \ y(x)$

Find: $\delta x_{at\,x}$

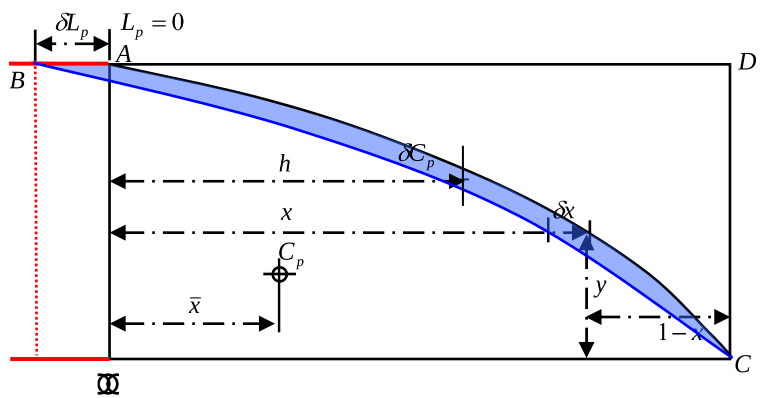
$$\tan\theta = \frac{\delta LCB}{\bar{y}} = \frac{\delta x}{y}$$

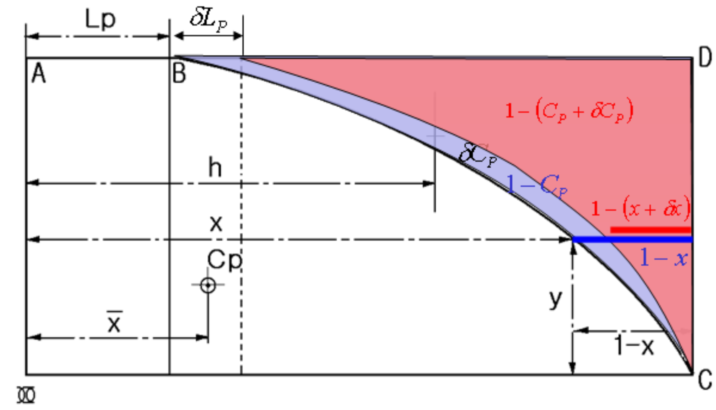
$$\boxed{\delta x = \frac{\delta LCB}{\bar{y}} \cdot y}$$

C_P Variation method

- Disadvantages of "1-C_p" Variation method

(1) This method cannot be used to reduce the displacement of a basis ship which has no parallel middle body.





$$\delta x_{f,a} = \frac{\delta C_{P_{f,a}}}{1 - C_{P_{f,a}}}\left(1 - x_{f,a}\right) \qquad \delta L_{P_{f,a}} = \frac{\delta C_{P_{f,a}}}{1 - C_{P_{f,a}}}\left(1 - L_{P_{f,a}}\right)$$

$C_P = C_b / C_m$

L_P : the fractional parallel middle of the half-body
x : the fractional distance of any transverse section from midships
$\bar{x}$: the fractional distance from midships of the centroid of the half body
y : the area of the transverse section at x expressed as a fraction of a fraction of the maximum ordinate
δC_P : the required change in prismatic coefficient of the half-body
δL_P : the consequent change in parallel middle body
δx : the necessary longitudinal shift of the section at x to produce the required change in prismatic coefficient
h : the fractional distance from midships of the centroid of the added "sliver" of area represented by δC_P

(2) There is no control over the extent of the parallel middle body in this method. That is, L_P and C_P cannot be varied independently.

(3) A basis ship form having no parallel middle body cannot be increased in displacement which has no parallel middle body.

(4) For a given change in C_P curve, the longitudinal distribution of the displacement added (or removed) cannot be controlled.

9-3 Lackenby's Hull Form Variation Method

C_P Variation method - "Lackenby" method- General Case

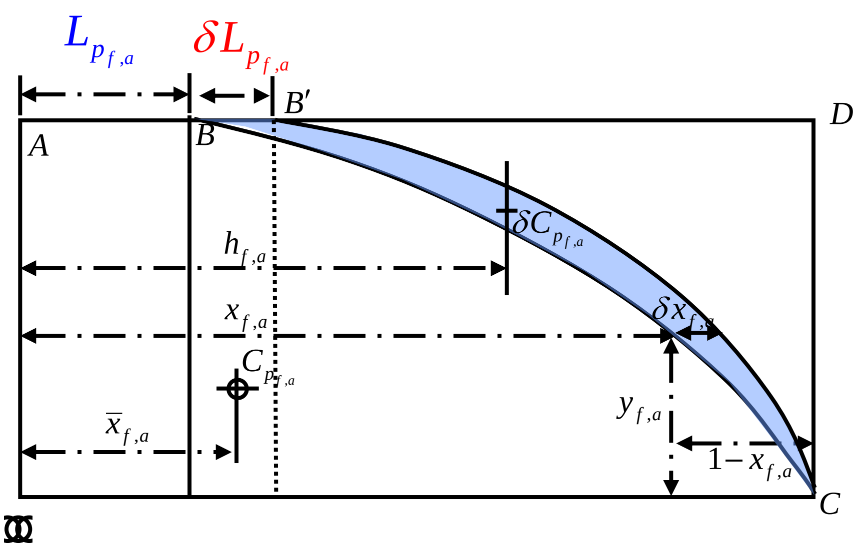
Reference : Lackenby, On The Systematic Geometrical Variation of ship forms, 1950, RINA , p.294, 308

"1-Cp" Variation Method

$$\delta x_{f,a} = \frac{\delta C_{P_{f,a}}}{1 - C_{P_{f,a}}}\left(1 - x_{f,a}\right)$$

Given: $C_{P_{f,a}},\ \delta C_{P_{f,a}},\ L_{P_{f,a}},\ \delta L_{P_{f,a}},\ \bar{x}_{f,a}, x_{f,a}$

Find: $\delta x_{f,a}$



<General Case>

Basis Form: Any extent of parallel middle body

Derived From: Any required change in prismatic coefficient and extent of parallel middle body

1

$$\delta x_{f,a} = (1 - x_{f,a})\left\{\frac{\delta L_{p_{f,a}}}{1 - L_{p_{f,a}}} + \frac{x_{f,a} - L_{p_{f,a}}}{A_{f,a}}\left[\delta C_{P_{f,a}} - \delta L_{p_{f,a}}\frac{(1 - C_{P_{f,a}})}{(1 - L_{p_{f,a}})}\right]\right\}$$

$$,(A_{f,a} = C_{P_{f,a}}(1 - 2\bar{x}_{f,a}) - L_{p_{f,a}}(1 - C_{P_{f,a}}))$$

→ In this formula, the change in the parallel middle body($\delta L_{pf,a}$) is included.

<Advantages of "Lackenby method">

1) The parallel middle body($L_{pf,a}$) can be controlled.

2) Because δx is proportional to $x(1-x)$, this method can be applied to the any case of the simple variation.

3) The required adjustments to the fore and after body prismatics to give any desired change in LCB position and total prismatic coefficient can be determined.

$C_P = C_b / C_m$

δC_P : the required change in prismatic coefficient of the half-body

x : the fractional distance of any transverse section from midships

δx : the necessary longitudinal shift of the section at x to produce the required change in prismatic coefficient

h : the fractional distance from midships of the centroid of the added "sliver" of area represented by δC_P

L_P : the fractional parallel middle of the half-body

δL_P : the consequent change in parallel middle body

$\bar{x}$: the fractional distance from midships of the centroid of the half body

y : the area of the transverse section at x expressed as a fraction of a fraction of the maximum ordinate

C_P Variation method

- "Lackenby" method- General Case

Reference : Lackenby, On The Systematic Geometrical Variation of ship forms, 1950, RINA , p.294,306,308,309

① "Lackenby" method <General Case>

Given: $C_{P_{f,a}}, \delta C_{P_{f,a}}, L_{P_{f,a}}, \delta L_{P_{f,a}}, \bar{x}_{f,a}, x_{f,a}$

Find: $\delta x_{f,a}$

$$\delta x_{f,a} = (1-x_{f,a})\left\{ \frac{\delta L_{p_{f,a}}}{1-L_{p_{f,a}}} + \frac{x_{f,a} - L_{p_{f,a}}}{A_{f,a}} \left[\delta C_{P_{f,a}} - \delta L_{p_{f,a}} \frac{(1-C_{P_{f,a}})}{(1-L_{p_{f,a}})} \right] \right\}$$

$$,(A_{f,a} = C_{P_{f,a}}(1-2\bar{x}_{f,a}) - L_{p_{f,a}}(1-C_{P_{f,a}}))$$

② Find: $\delta C_{P_{f,a}}$

Given: $C_P, \delta C_P, h_{a,f}, LCB, \delta LCB$

Find: $\delta C_{P_{f,a}}$

$$\delta C_{Pf} = \frac{2[\delta C_P(h_a + LCB) + \delta LCB(C_P + \delta C_P)]}{h_f + h_a}$$

$$\delta C_{Pa} = \frac{2[\delta C_P(h_f - LCB) - \delta LCB(C_P + \delta C_P)]}{h_f + h_a}$$

③ Find: $h_{f,a}$

Given: $C_{p_{f,a}}, \delta C_{p_{f,a}}, L_{p_{f,a}}, \delta L_{p_{f,a}}, \bar{x}_{f,a}, k_{f,a}$

Find: $h_{f,a}$

$$h_{f,a} = C_{p_{f,a}} \cdot \left(\frac{B_{f,a}}{C_{p_{f,a}}} \left[1 - \frac{\delta L_{p_{f,a}} \cdot (1-C_{p_{f,a}})}{\delta C_{p_{f,a}} \cdot (1-L_{p_{f,a}})} \right] + \frac{\delta L_{p_{f,a}} \cdot (1-2\bar{x}_{f,a})}{\delta C_{p_{f,a}} \cdot (1-L_{p_{f,a}})} \right)$$

$$(A_{f,a} = C_{P_{f,a}}(1-2\bar{x}_{f,a}) - L_{p_{f,a}}(1-C_{P_{f,a}})), \left(B_{f,a} = \frac{C_{p_{f,a}} \cdot \left[2\bar{x}_{f,a} - 3k_{f,a}{}^2 - L_{p_{f,a}} \cdot (1-2\bar{x}_{f,a}) \right]}{A_{f,a}} \right)$$

To obtain $h_{f,a}$, $\delta C_{P_{f,a}}$ have to be given!

Substituting equation ③ into equation ② and rearranging for $\delta C_{P_{f,a}}$,

Given: $C_P, \delta C_P, L_{p_{f,a}}, \delta L_{p_{f,a}}, LCB, \delta LCB, k_{f,a}$

Find: $\delta C_{P_{f,a}}$

$$\delta C_{p_f} = \frac{2\left[\delta C_p \cdot (B_a + LCB) + \delta LCB \cdot (C_P + \delta C_P) \right] + C_f \cdot \delta L_{p_f} - C_a \cdot \delta L_{p_a}}{B_f + B_a}$$

$$\delta C_{p_a} = \frac{2\left[\delta C_p \cdot (B_f - LCB) - \delta LCB \cdot (C_P + \delta C_P) \right] - C_f \cdot \delta L_{p_f} + C_a \cdot \delta L_{p_a}}{B_f + B_a}$$

$$(A_{f,a} = C_{P_{f,a}}(1-2\bar{x}_{f,a}) - L_{p_{f,a}}(1-C_{P_{f,a}}))$$

$$\left(B_{f,a} = \frac{C_{p_{f,a}} \cdot \left[2\bar{x}_{f,a} - 3k_{f,a}{}^2 - L_{p_{f,a}} \cdot (1-2\bar{x}_{f,a}) \right]}{A_{f,a}} \right), \left(C_{f,a} = \frac{B_{f,a}(1-C_{P_{f,a}}) - C_{P_{f,a}}(1-2\bar{x}_{f,a})}{1-L_{p_{f,a}}} \right)$$

$C_P = C_b / C_m$

δC_P : the required change in prismatic coefficient of the half-body

x : the fractional distance of any transverse section from midships

δx : the necessary longitudinal shift of the section at x to produce the required change in prismatic coefficient

h : the fractional distance from midships of the centroid of the added "sliver" of area represented by δC_P

L_P : the fractional parallel middle of the half-body

δL_P : the consequent change in parallel middle body

$\bar{x}$: the fractional distance from midships of the centroid of the half body

y : the area of the transverse section at x expressed as a fraction of a fraction of the maximum ordinate

① "Lackenby" method <General Case>

Given: $C_{P_{f,a}}, \delta C_{P_{f,a}}, L_{P_{f,a}}, \delta L_{P_{f,a}}, \bar{x}_{f,a}$

Find: $\delta x_{f,a}$

$$\delta x_{f,a} = (1-x_{f,a})\left\{\frac{\delta L_{p_{f,a}}}{1-L_{p_{f,a}}} + \frac{x_{f,a}-L_{p_{f,a}}}{A_{f,a}}\left[\delta C_{P_{f,a}} - \delta L_{p_{f,a}}\frac{(1-C_{P_{f,a}})}{(1-L_{p_{f,a}})}\right]\right\}$$

$$,(A_{f,a} = C_{P_{f,a}}(1-2\bar{x}_{f,a}) - L_{p_{f,a}}(1-C_{P_{f,a}}))$$

③ Find: $h_{f,a}$

Given: $C_{P_{f,a}}, \delta C_{P_{f,a}}, L_{P_{f,a}}, \delta L_{P_{f,a}}, \bar{x}_{f,a}, k_{f,a}$

Find: $h_{f,a}$

$$h_{f,a} = C_{P_{f,a}} \cdot \left(\frac{B_{f,a}}{C_{P_{f,a}}}\left[1 - \frac{\delta L_{P_{f,a}} \cdot (1-C_{P_{f,a}})}{\delta C_{P_{f,a}} \cdot (1-L_{P_{f,a}})}\right] + \frac{\delta L_{P_{f,a}} \cdot (1-2\bar{x}_{f,a})}{\delta C_{P_{f,a}} \cdot (1-L_{P_{f,a}})}\right)$$

$$(A_{f,a} = C_{P_{f,a}}(1-2\bar{x}_{f,a}) - L_{P_{f,a}}(1-C_{P_{f,a}})), \left(B_{f,a} = \frac{C_{P_{f,a}} \cdot \left[2\bar{x}_{f,a} - 3k_{f,a}^{2} - L_{P_{f,a}} \cdot (1-2\bar{x}_{f,a})\right]}{A_{f,a}}\right)$$

② Find: $\delta C_{P_{f,a}}$

Given: $C_P, \delta C_P, h_{a,f}, LCB, \delta LCB$

Find: $\delta C_{P_{f,a}}$

$$\delta C_{Pf} = \frac{2[\delta C_P(h_a + LCB) + \delta LCB(C_P + \delta C_P)]}{h_f + h_a}$$

$$\delta C_{Pa} = \frac{2[\delta C_P(h_f - LCB) - \delta LCB(C_P + \delta C_P)]}{h_f + h_a}$$

To obtain $h_{f,a}$, $\delta C_{P_{f,a}}$ have to be given!

Substituting equation ③ into equation ② and rearranging for $\delta C_{P_{f,a}}$,

Given: $C_P, \delta C_P, L_{P_{f,a}}, \delta L_{P_{f,a}}, LCB, \delta LCB, k_{f,a}$

Find: $\delta C_{P_{f,a}}$

$$\delta C_{P_f} = \frac{2\left[\delta C_p \cdot (B_a + LCB) + \delta LCB \cdot (C_P + \delta C_P)\right] + C_f \cdot \delta L_{P_f} - C_a \cdot \delta L_{P_a}}{B_f + B_a}$$

$$\delta C_{P_a} = \frac{2\left[\delta C_p \cdot (B_f - LCB) - \delta LCB \cdot (C_P + \delta C_P)\right] - C_f \cdot \delta L_{P_f} + C_a \cdot \delta L_{P_a}}{B_f + B_a}$$

$$(A_{f,a} = C_{P_{f,a}}(1-2\bar{x}_{f,a}) - L_{P_{f,a}}(1-C_{P_{f,a}}))$$

$$\left(B_{f,a} = \frac{C_{P_{f,a}} \cdot \left[2\bar{x}_{f,a} - 3k_{f,a}^{2} - L_{P_{f,a}} \cdot (1-2\bar{x}_{f,a})\right]}{A_{f,a}}\right) \left(C_{f,a} = \frac{B_{f,a}(1-C_{P_{f,a}}) - C_{P_{f,a}}(1-2\bar{x}_{f,a})}{1-L_{P_{f,a}}}\right)$$

$C_P = C_b / C_m$

δC_P : the required change in prismatic coefficient of the half-body

x : the fractional distance of any transverse section from midships

δx : the necessary longitudinal shift of the section at x to produce the required change in prismatic coefficient

h : the fractional distance from midships of the centroid of the added "sliver" of area represented by δC_P

L_P : the fractional parallel middle of the half-body

③ Find: $k_{f,a}$

$$k_{f,a} = \frac{I_{f,a}}{S_{f,a}}$$

$k_{f,a}$: the lever of the second moment (i.e. the radius of gyration) about midships expressed as a fraction of the length of the half body

$I_{f,a}$: the second moment about midships expressed as a fraction of the length of the half body

$S_{f,a}$: the area of the half body

C_P Variation method
- Relation between "1-CP" Variation method and "Lackenby" method

"1-C_P" variation method

Given : $C_{P_{a,f}}$, $\delta C_{P_{a,f}}$

Find : $\delta x_{a,f}$

$$\boxed{\delta x_{f,a} = \frac{\delta C_{P_{f,a}}}{1-C_{P_{f,a}}}\left(1-x_{f,a}\right)}$$

The change in parallel middle body by "1-C_P" variation method

Given : $C_{P_{f,a}}$, $\delta C_{P_{f,a}}$, $L_{P_{f,a}}$

Find : $\delta L_{P_{f,a}}$

$$\boxed{\delta L_{P_{f,a}} = \frac{\delta C_{P_{f,a}}}{1-C_{P_{f,a}}}\left(1-L_{P_{f,a}}\right)}$$

→ The consequent change in parallel middle body($\delta L_{pf,a}$) is calculated by "1-C_P" variation method.

"Lackenby "method <General Case>

Given: $C_{P_{f,a}}$, $\delta C_{P_{f,a}}$, $L_{P_{f,a}}$, $\delta L_{P_{f,a}}$, $\bar{x}_{f,a}$

Find: $\delta x_{f,a}$

$$\delta x_{f,a} = (1-x_{f,a})\left\{\frac{\delta L_{p_{f,a}}}{1-L_{p_{f,a}}} + \frac{x_{f,a}-L_{p_{f,a}}}{A_{f,a}}\left[\delta C_{P_{f,a}} - \delta L_{p_{f,a}}\frac{(1-C_{P_{f,a}})}{(1-L_{p_{f,a}})}\right]\right\}$$

$$,\left(A_{f,a} = C_{P_{f,a}}(1-2\bar{x}_{f,a}) - L_{p_{f,a}}(1-C_{P_{f,a}})\right)$$

→The change in the parallel middle body($\delta L_{pf,a}$) can be controlled.

(Substituting into the Lackenby method)

$$\delta x_{f,a} = (1-x_{f,a})\left\{\frac{\delta L_{p_{f,a}}}{1-L_{p_{f,a}}} + \frac{x_{f,a}-L_{p_{f,a}}}{A_{f,a}}\left[\delta C_{P_{f,a}} - \delta L_{p_{f,a}}\frac{(1-C_{P_{f,a}})}{(1-L_{p_{f,a}})}\right]\right\}$$

$$\delta x_{f,a} = (1-x_{f,a})\left\{\frac{\left\{\frac{\delta C_{p_{f,a}}}{1-C_{p_{f,a}}}\cancel{(1-L_{p_{f,a}})}\right\}}{\cancel{1-L_{p_{f,a}}}} + \frac{x_{f,a}-L_{p_{f,a}}}{A_{f,a}}\left[\delta C_{P_{f,a}} - \left\{\frac{\delta C_{p_{f,a}}}{\cancel{1-C_{p_{f,a}}}}\cancel{(1-L_{p_{f,a}})}\right\}\frac{\cancel{(1-C_{P_{f,a}})}}{\cancel{(1-L_{p_{f,a}})}}\right]\right\}$$

$$\boxed{\therefore \delta x_{f,a} = \frac{\delta C_{P_{f,a}}}{1-C_{P_{f,a}}}\left(1-x_{f,a}\right)}$$

→ This result is equal to the result of "1-C_P" variation method.

→ That is, "1-C_P" variation method is a special case of Lackenby method.

$C_P = C_b / C_m$

δC_P : the required change in prismatic coefficient of the half-body

x : the fractional distance of any transverse section from midships

δx : the necessary longitudinal shift of the section at x to produce the required change in prismatic coefficient

h : the fractional distance from midships of the centroid of the added "sliver" of area represented by δC_P

L_P : the fractional parallel middle of the half-body

δL_P :

$\bar{x}$:

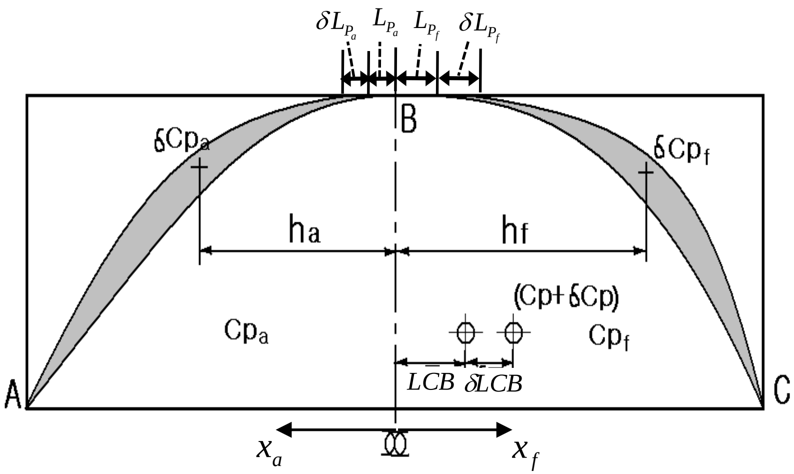
y : fraction of the maximum ordinate

Procedure of the Hull Form Variation

1. When C_P and LCB are obtained, calculate or estimate the C_P at forebody and aftbody.
2. By using the '$1-C_P$ Variation method', correct for the difference in C_B between the basis ship and design ship.
3. By using the 'Swing station method', correct for the difference in LCB Between the basis ship and design ship.
4. By correcting for the local part of C_P-curve, determine the Cp-curve of design ship.
5. Design of a Body plan
6. Design of a profile of the forebody
7. Design of a profile of the aftbody
8. Fairing

C_P Variation method
- "1-C_P" Variation method

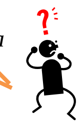
Reference: Lackenby, On The Systematic Geometrical Variation of ship forms, 1950, RINA , p.290



The + sign indicates movement away from midships.(x_a, x_f)

$$\delta C_{Pf} = \frac{2\left[\delta C_P\left(\boxed{h_a} + LCB\right) + \delta LCB\left(C_P + \delta C_P\right)\right]}{h_f + h_a}$$

$$\delta C_{Pa} = \frac{2\left[\delta C_P\left(\boxed{h_f} - LCB\right) - \delta LCB\left(C_P + \delta C_P\right)\right]}{h_f + h_a}$$



How to obtain $h_{f,a}$?

✓ Find: $h_{f,a}$

Given: $C_{P_{f,a}}$, $\bar{x}_{f,a}$

Find: $h_{f,a}$

$$h_{f,a} = \frac{C_{P_{f,a}}\left(1 - 2\bar{x}_{f,a}\right)}{1 - C_{P_{f,a}}} + \cancel{\frac{\delta C_{P_{f,a}}}{1 - C_{P_{f,a}}}\left[1 - 2C_{P_{f,a}}\left(1 - \bar{x}_{f,a}\right)\right]}$$

$$\boxed{h_{f,a} \approx \frac{C_{P_{f,a}}\left(1 - 2\bar{x}_{f,a}\right)}{1 - C_{P_{f,a}}}}$$

$C_P = C_b / C_m$

δC_P : the required change in prismatic coefficient of the half-body

x : the fractional distance of any transverse section from midships

δx : the necessary longitudinal shift of the section at x to produce the required change in prismatic coefficient

h : the fractional distance from midships of the centroid of the added "sliver" of area represented by δC_P

L_P : the fractional parallel middle of the half-body

δL_P : the consequent change in parallel middle body

$\bar{x}$: the fractional distance from midships of the centroid of the half body

y : the area of the transverse section at x expressed as a fraction of a fraction of the maximum ordinate

Formula for estimating the LCB

- LCB represents the balance of the displacement between forebody and aftbody.
  (So, it determines the distribution of the displacement of a ship)
  - Block coefficient of aftbody(C_{BA}) has an effect on the maneuverability of a ship
    (Recommending that C_{BA} is less than 0.76.)
  - Hull form of the forebody usually has effect on the wave resistance.
  - Hull form of the aftbody usually has effect on the friction resistance and propulsion ability.

⇨ Ponderous ship: LCB to be located at forebody
Slender ship: LCB to be located at midship or aftbody

- Formula for the LCB when C_{BA} is less than 0.76

$$C_{PA} = C_P - 0.0215 \cdot LCB$$

- When the C_B of the ship is 0.8~0.85(Ponderous ship):

LCB : 3.5~4.0 %

- Lap/Keller formula:

$$LCB[\%L] = 13.33 C_B - 9.0$$

When the LCB is estimated, apply the correction factor obtained from basis ship.

$$\frac{LCB_{\text{Basis, actual}}}{LCB_{\text{Basis}, estimate}} = C_{corr.}$$

$$LCB_{\text{design}} = C_{corr.} \cdot LCB_{\text{design,estimate}}$$

$LCB_{Basis,estimate}$: LCB of the basis ship to be estimated by the formula

$LCB_{Basis,actual}$: actual LCB of the basis ship

C_{corr}: correction factor

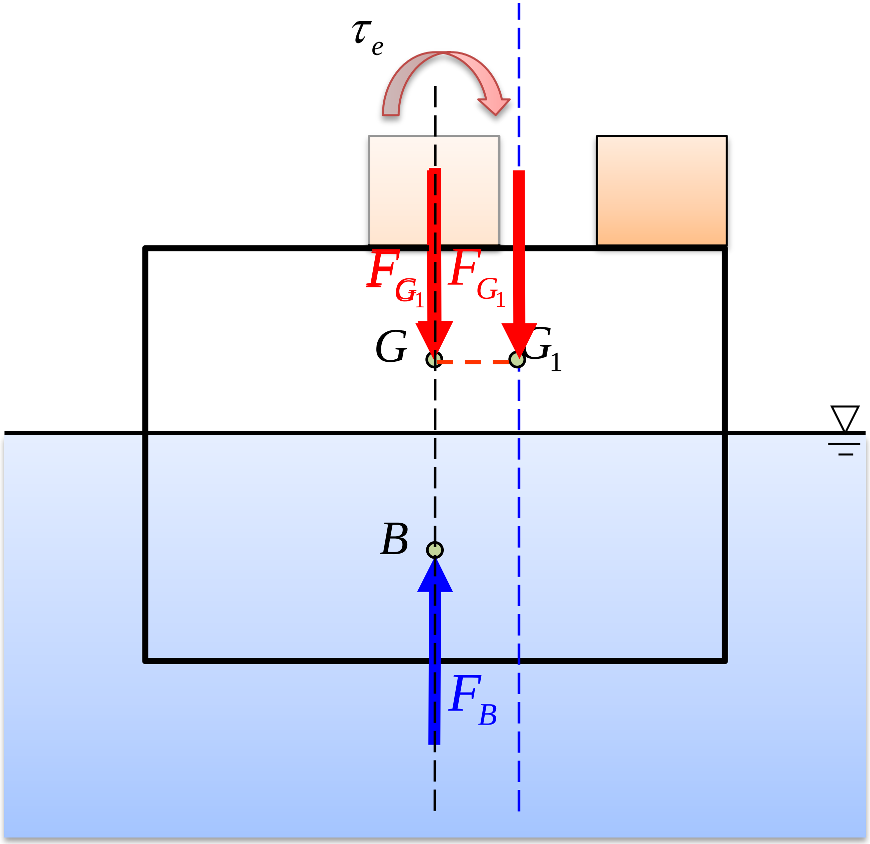
$LCB_{Design,estimate}$: LCB of the design ship to be estimated by the formula

LCB_{Design}: $LCB_{Design,estimate}$ multiplied by correction factor

Chapter 10. Computational Ship Stability

10-1 Concept of Ship Stability

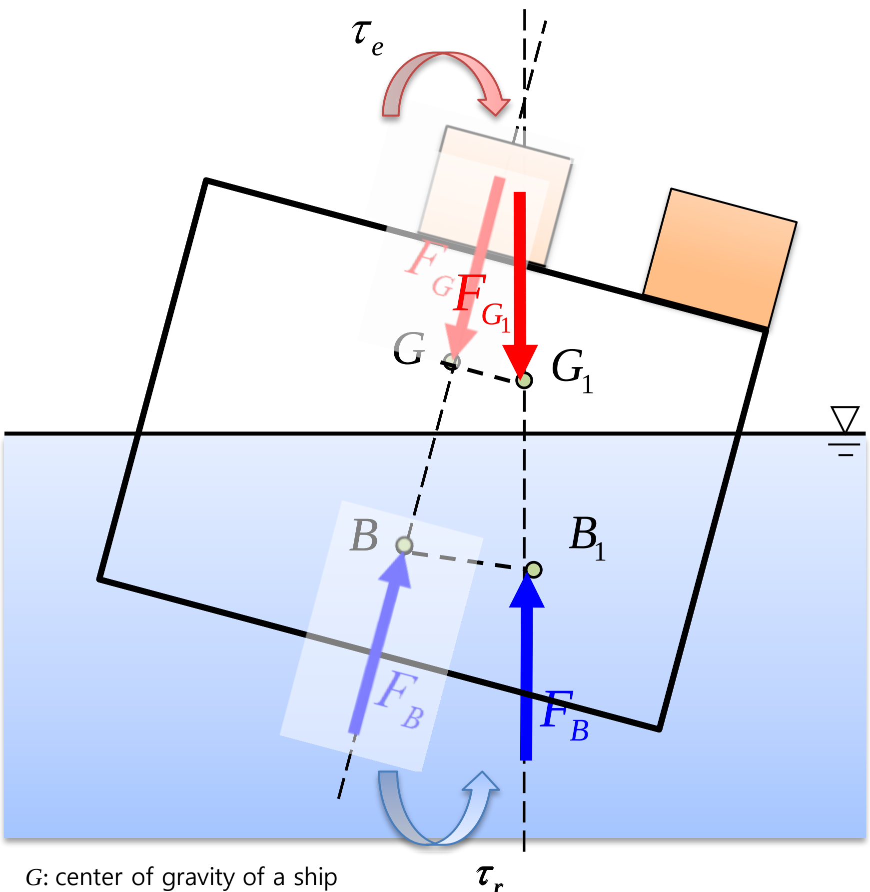
Concept of Stability of a ship



G: center of gravity of a ship
G_1: changed position of the center of gravity after moving a cargo
F_G : gravitational force of a ship
B: center of buoyancy at initial position
F_B : buoyant force
B_1: changed position of the center of buoyancy after the ship has been inclined

- When a cargo on the deck moves to the right side of a ship, the center of gravity of the ship moves to the point G_1, off the centerline.
- Because the buoyant force and the gravitational force are not on one line, the forces produce a moment to incline the ship.

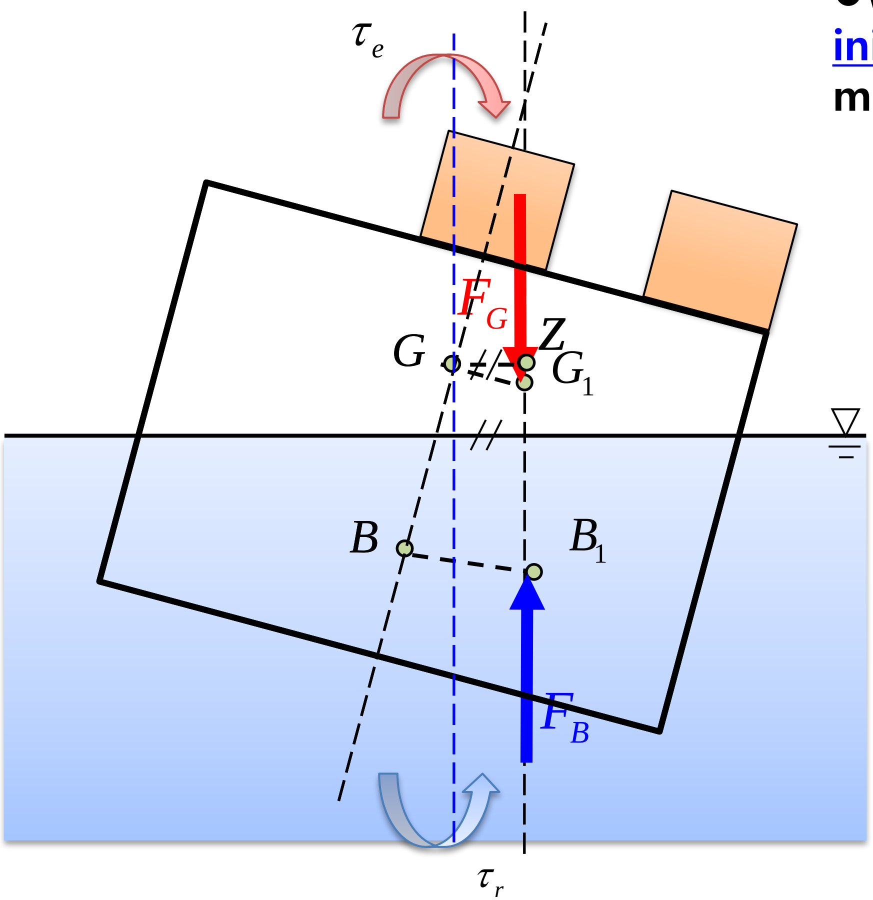
*You have a moment on the ship relative to any point that you choose. It does not matter where you pick a point.



G: center of gravity of a ship
G_1: changed position of the center of gravity after moving a cargo
F_G : gravitational force of a ship
B: center of buoyancy at initial position
F_B : buoyant force
B_1: changed position of the center of buoyancy after the ship has been inclined

- **The resultant moment becomes zero when the buoyant force and the gravitational force are on one line.**

Transverse Righting Moment (GZ)



- **Transverse Righting Moment**

$$\tau_{righting} = F_B \cdot \underline{GZ}$$

●When the cargo on the deck returns to the initial position, the center of gravity of the ship moves to the initial position G.

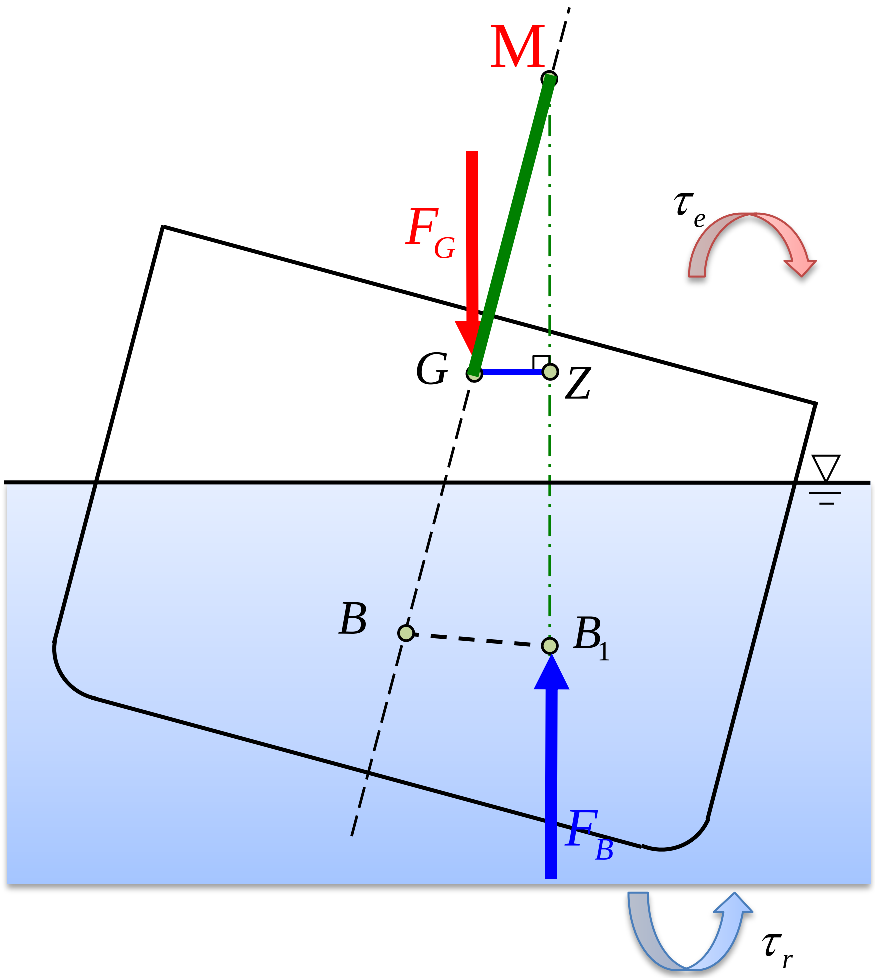
●Then, because the buoyant force and the gravitational force are not on one line, the forces produces a restoring moment to return the ship to the initial position.

※Naval architects refer to the restoring moment as "righting moment"

●The resultant moment arm of the buoyant force and gravitational force about longitudinal axis through point G is expressed by GZ, where Z is defined as the intersection of the line of buoyant force(F_B) through the new position of the center of buoyancy (B_1) with the transversely parallel line to the water surface through the center of gravity of the ship(G)

Definition of Metacenter (M), Metacentric Height (GM)

• **Righting Moment**
$$\tau_{righting} = F_B \cdot GZ$$



- Concept of Righting Moment

Definition of M (Metacenter)

• Intersection of the vertical line through the changed center of buoyancy(B_1) with the vertical line through the **previous** center of buoyancy(B)

• The term **meta** was selected as a prefix for center because its Greek meaning implies **movement**. The **metacenter** therefore is a **moving center**.

• From geometrical figure, $\underline{GZ}$ can be obtained with assumption that M does not change within a **small angle of inclination** (about 7°~10°)

$$GZ \approx GM \cdot \sin\phi$$

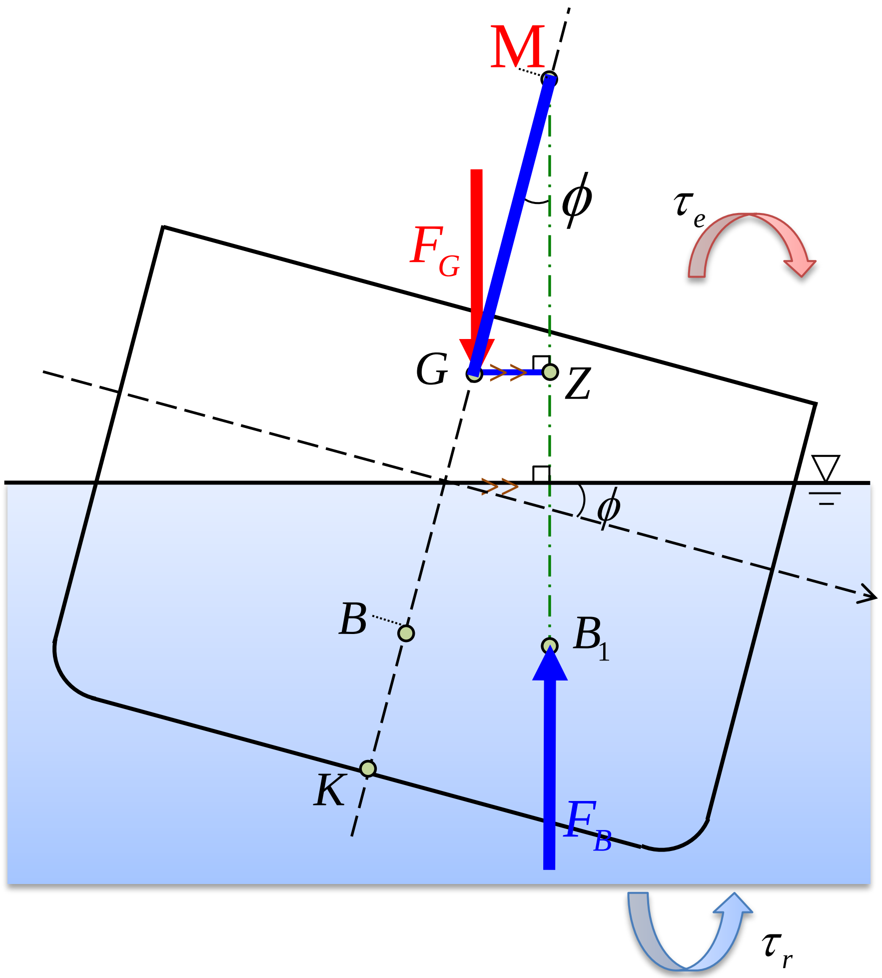
• **GM : Metacentric height,**

• **called as "Initial Stability"**

Component of GM

• Righting Moment

$$\tau_{righting} = F_B \cdot GZ$$



M : Metacenter
GM : Metacentric height
θ : Angle of heel
K : Intersection of three planes, i.e., centerplane , baseplane, and section plane

- Concept of Righting Moment

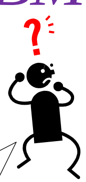
Righting arm for small angle of heel

$$GZ \approx GM \cdot \sin\phi$$

• From the geometrical configuration of the ship, GM is composed of as follows:

$$GM = \boxed{KB} + BM - KG$$

KB ≒ 51~52% draft

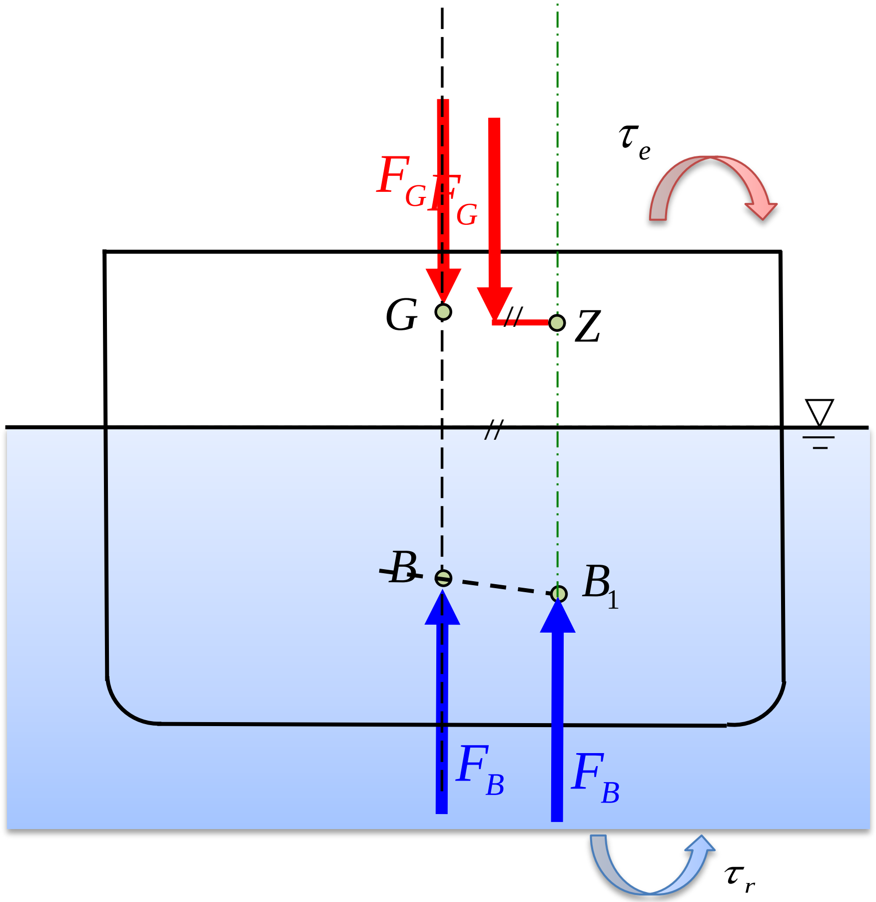


Center of gravity of the ship

$$BM = \frac{I_T}{\nabla}$$

I_T : moment of inertia of the waterplane area about longitudinal axis through COF

Righting Arm (GZ) at Large Angle of Inclination



① Produce a large heeling angle to the ship by applying an external moment

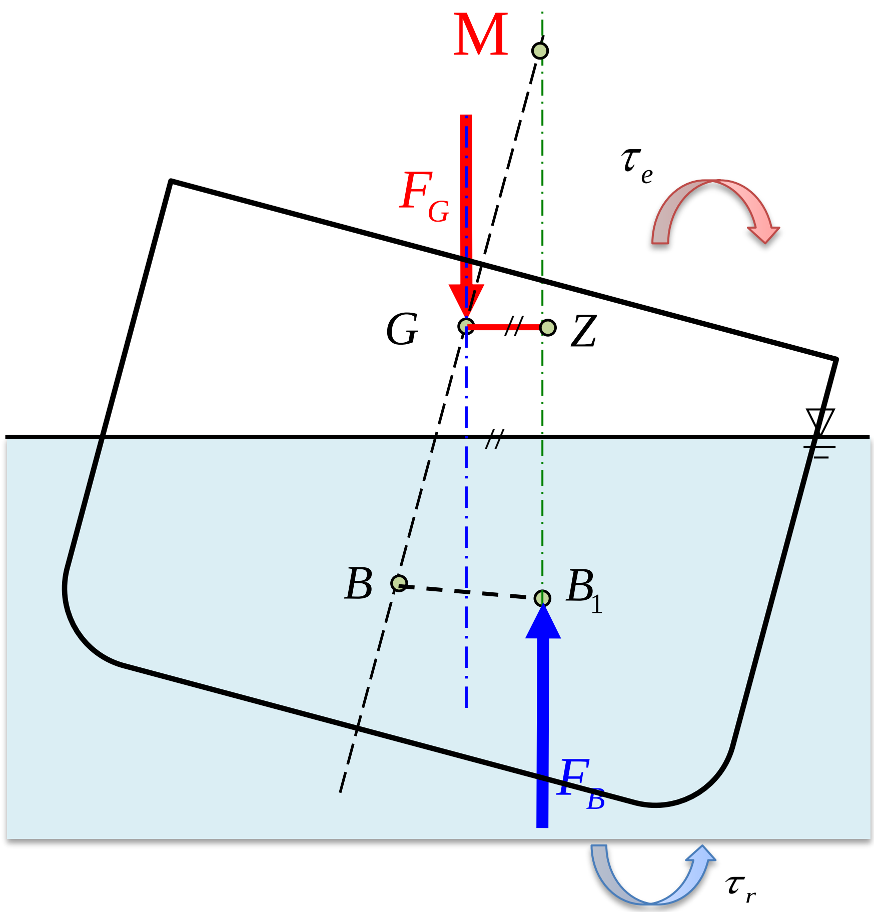
- **Transverse Righting Moment**

$$\tau_{righting} = F_B \cdot \underline{GZ}$$

- Use of metacentric height (***GM***) as righting arm is **not valid** for a ship at large angles of heel

To determine the righting arm of the ship at large angles of heel, it is necessary to know the position of the center of gravity(*G*) and the new position of the buoyancy(B_1).

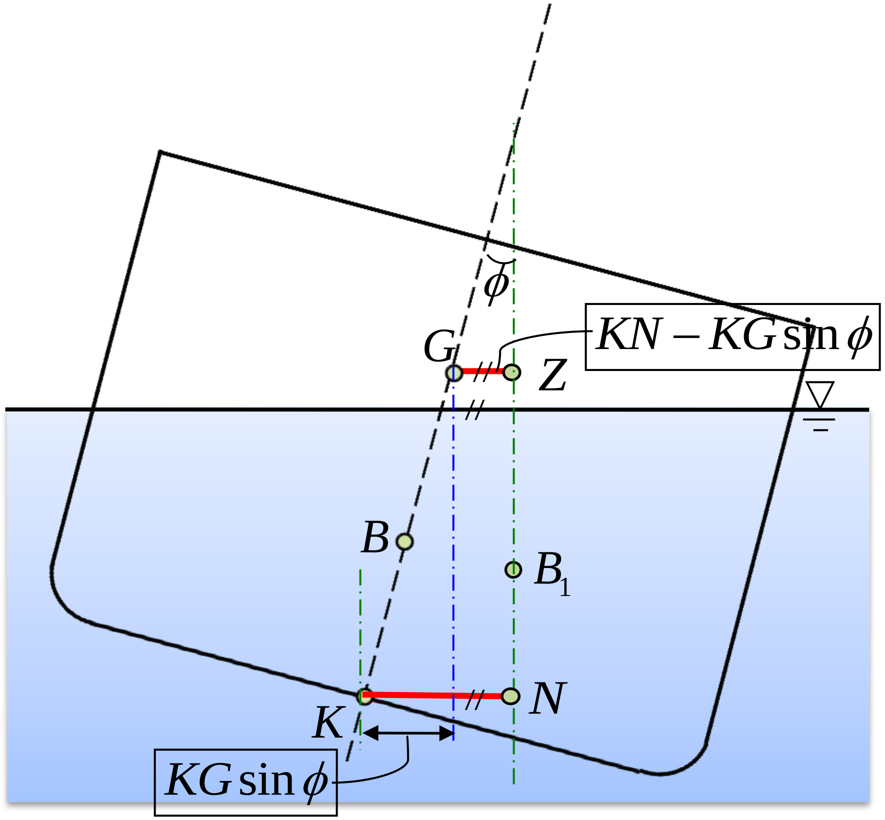
Righting Moment at Large Angle of Inclination



- The use of metacentric height(GM) as the righting arm is not valid for a ship at a large angle of inclination.

To determine the righting arm "GZ", it is necessary to know the position of the center of gravity(G) and the new position of the center of buoyancy(B_1).

$GZ = KN - KG\sin\phi$



K: Intersection of three plane, i.e., centerplane , baseplane, and section plane

N: Intersection of the vertical line through the changed center of buoyancy(B_1) with the transversely parallel line to the water surface through K

Suppose the center of gravity is on *K*.

Then *KN* represents the righting arm.

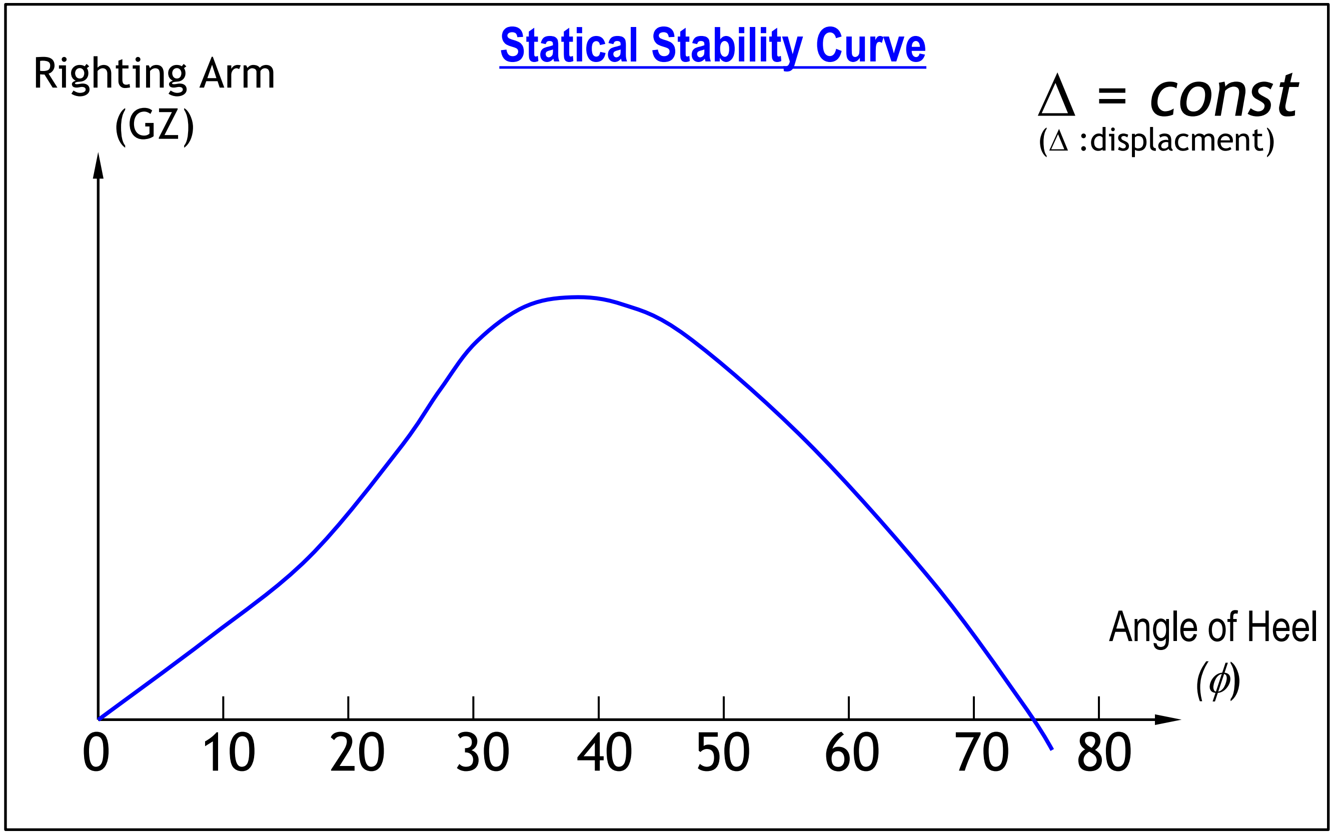
1) *KN* depends only on the geometry of the ship and can be calculated for various angles of heel and displacements without referring to a particular loading condition.

2) KG depends only on the weight distribution of the ship

If the location of center of gravity(G) is not on K, then, actual values of righting arm(GZ):

$$GZ = KN - KG\sin\phi$$

Definition and Purpose of the Statical Stability Curves (GZ Curves)



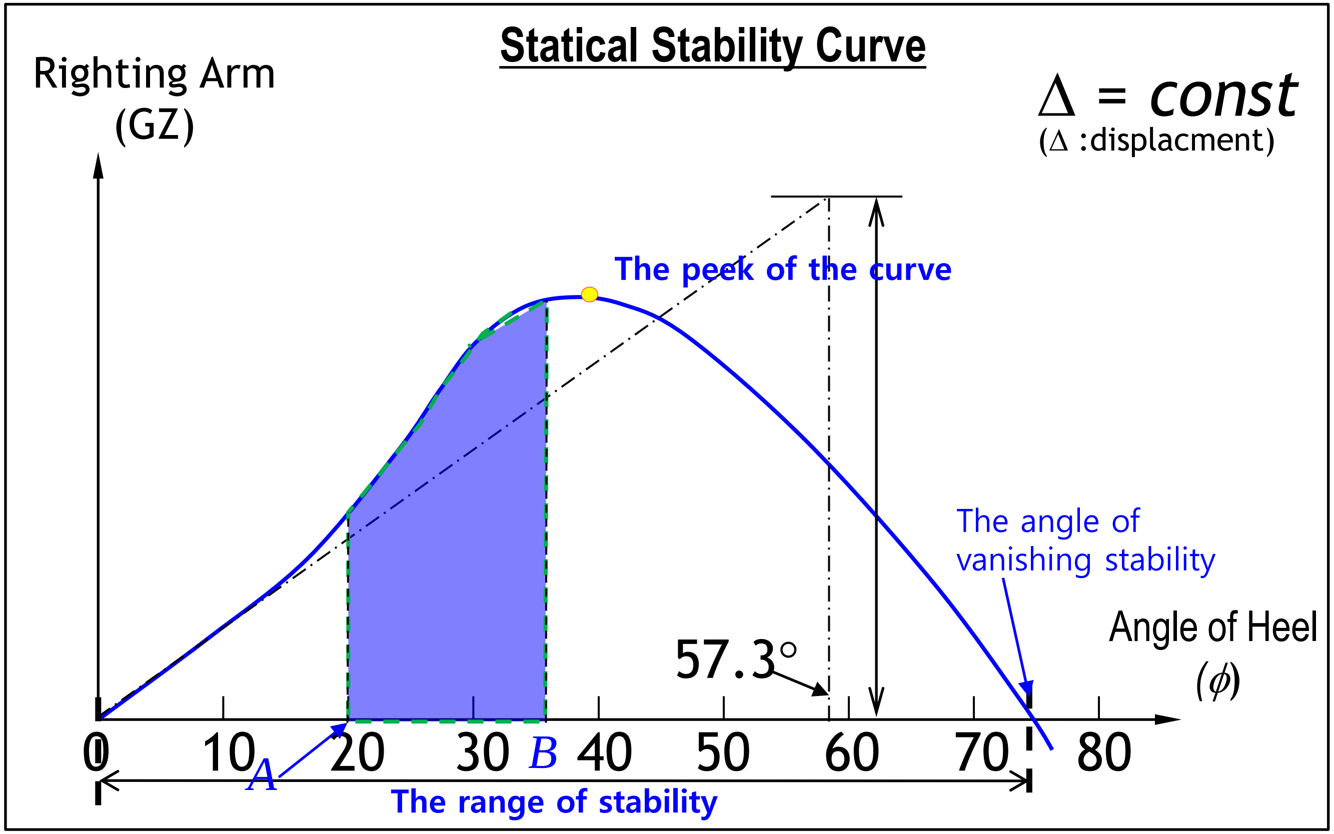
The statical stability curve is a plot of righting arm against the angle of heel for a given loading condition.

As far as the intact ship is concerned, the statical stability curve provides useful data for evaluating the ship's stability for the given loading condition.

Features of the Statical Stability Curves (GZ Curves)

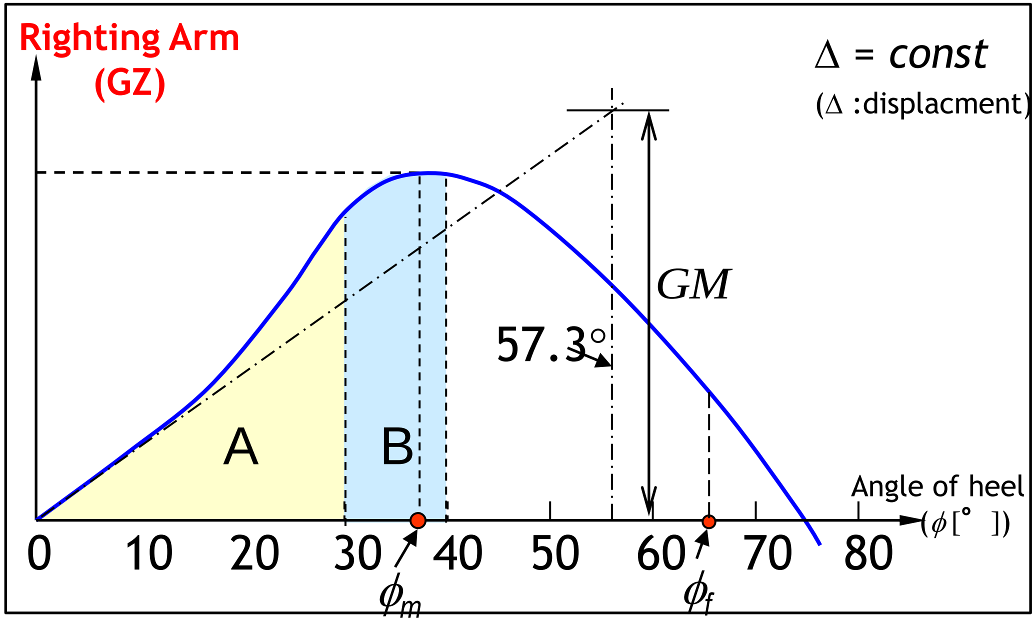
The statical stability curve has a number of features that are significant in the analysis of the ship's stability.

- The slope of the curve at zero degrees, the peak of the curve, the range of stability and the angle of vanishing stability, the area under the curve



IMO Regulations for Intact Stability

☑ **IMO recommendation on intact stability for passenger and cargo ships.**



Area A : Area under the righting arm curve between the heel angle of 0° and 30°

Area B : Area under the righting arm curve between the heel angle of 30° and min(40°, ϕ_f)

※ ϕ_f : Heel angle at which openings in the hull

ϕ_m : Heel angle of maximum righting arm

※ After receiving approval of calculation of IMO regulation from Owner and Classification Society, ship construction can proceed.

IMO Regulations for Intact Stability

(a) Area A ≥ 0.055 m-rad

(b) Area A + B ≥ 0.09 m-rad

(c) Area B ≥ 0.030 m-rad

(d) GZ ≥ 0.20 m at an angle of heel equal to or greater than 30°

(e) GZ_{max} should occur at an angle of heel preferably exceeding 30° but not less than 25° .

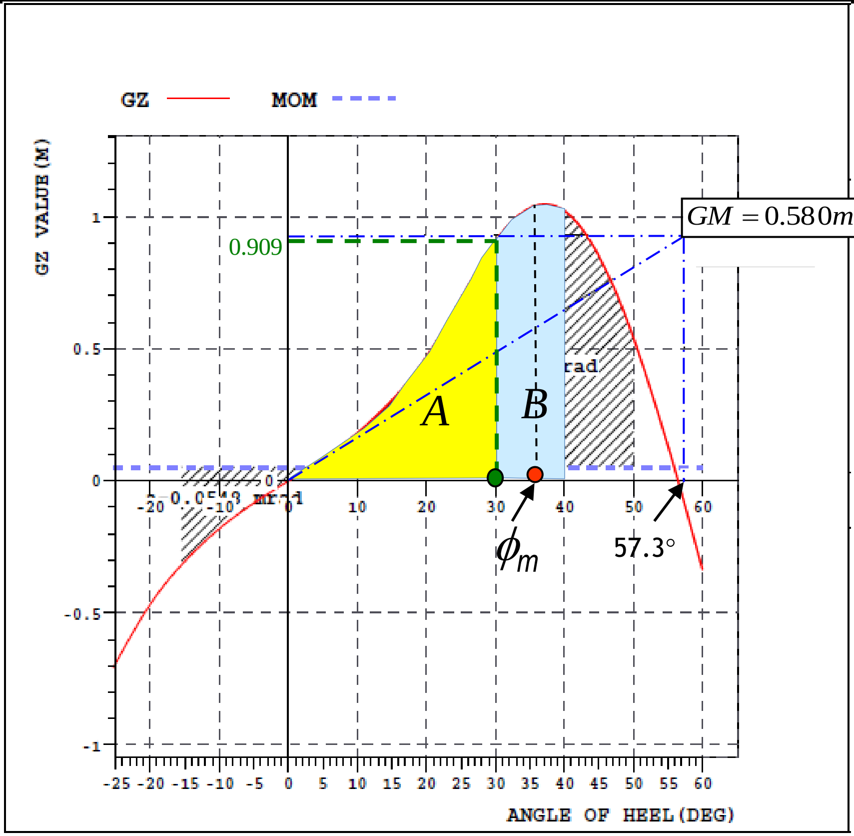
(f) The initial metacentric height GM_o should not be less than 0.15 m.

Static considerations

The work and energy considerations (dynamic stability)

IMO Regulations for Intact Stability Criteria

Ex) 7,000 TEU Container Carrier at Homogeneous Scantling Draft Arrival Condition



Area A : Area under the righting arm curve between the heel angle of 0° and 30°

Area B : Area under the righting arm curve between the heel angle of 30° and min(40°, ϕ_f)

※ ϕ_f : Heel angle at which openings in the hull

ϕ_f = 40°

Area B : Area under the righting arm curve between the heel angle of 30° and 40°

ϕ_m : Heel angle of maximum righting arm = 36.8°

(a) Area A ≥ 0.055 m-rad

$$Area\ A = 0.148\ m \cdot rad\ \geq 0.055\, m \cdot rad$$

(b) Area A + B ≥ 0.09 m-rad

$$Area\ A + B = 0.301\ m \cdot rad\ \geq 0.090\ m \cdot rad$$

(c) Area B ≥ 0.030 m-rad

$$Area\ B = 0.153\ m \cdot rad\ \geq 0.030\ m \cdot rad$$

(d) GZ ≥ 0.20 m at an angle of heel **equal to or greater** than 30°

$$GZ_{at\,angle\ of\ heel = 30°} = 0.909\ m \geq 0.200\ m$$

(e) GZ_{max} should occur at an angle of heel preferably exceeding 30° but not less than 25°.

$$\phi_m = 36.8° \geq 25°$$

(f) The initial metacentric height G_0M should **not be less** than 0.15 m.

$$GM = 0.58m\ \geq 0.15\ m$$

$$GG_0 = \frac{\sum \rho_F \cdot i_T}{\rho_{SW} \nabla}$$: Free surface moment

i_T : Moment of inertia of liquid plane area in tank about longitudinal axis

WEIGHT ITEMS	FILL. (%)	S.G	WEIGHT (MT)	L.C.G (M)	V.C.G (M)	F.S.M (MT-M)
NO2 DB WBT(P)	100.00	1.0250	560.1	228.280	2.640	0.0
NO2 DB WBT(S)	100.00	1.0250	560.1	228.280	2.640	0.0
NO3 DB WBT(P)	100.00	1.0250	940.7	200.357	2.015	0.0
NO3 DB WBT(S)	100.00	1.0250	940.7	200.357	2.015	0.0
NO3 WWBT(P)	100.00	1.0250	1070.1	201.907	11.873	0.0
NO3 WWBT(S)	100.00	1.0250	1070.1	201.907	11.873	0.0
NO4 DB WBT(P)	100.00	1.0250	1266.8	173.078	1.923	0.0
NO4 DB WBT(S)	100.00	1.0250	1266.8	173.078	1.923	0.0
NO5 DB WBT(P)	100.00	1.0250	1145.4	143.534	1.690	0.0
NO5 DB WBT(S)	100.00	1.0250	1145.4	143.534	1.690	0.0
NO5 WWBT(P)	100.00	1.0250	977.8	143.500	12.369	24.3
NO5 WWBT(S)	100.00	1.0250	977.8	143.500	12.369	24.3
NO6 DB WBT(P)	100.00	1.0250	1143.6	114.585	1.690	0.0
NO6 DB WBT(S)	100.00	1.0250	1143.6	114.585	1.690	0.0
NO7 DB WBT(P)	100.00	1.0250	1031.2	85.978	1.778	0.0
NO7 DB WBT(S)	100.00	1.0250	1031.2	85.978	1.778	0.0
TOTAL WATER BALLAST			16271.3	156.848	4.463	48.7
FRESH WATER			41.6	45.600	12.757	20.7
HEAVY FUEL OIL			800.0	71.121	12.188	7109.2
DIESEL OIL			40.0	66.300	11.175	60.5
LUBRICATING OIL			47.4	66.318	7.861	14.1
DEADWEIGHT CONSTANT			900.0	73.100	24.200	0.0
TOTAL DEADWEIGHT			92328	143.449	18.408	7253.3
LIGHT SHIP			27710	122.656	16.000	
TOTAL DISPLACEMENT			120038	138.649	17.852	7253.3

$$GG_0 = \frac{\sum \rho_F \cdot i_T}{\rho_{SW} \nabla} = \frac{7,253.3}{120,038} = 0.06m$$

Correction for effect of free surface of liquid in tanks is as follows:

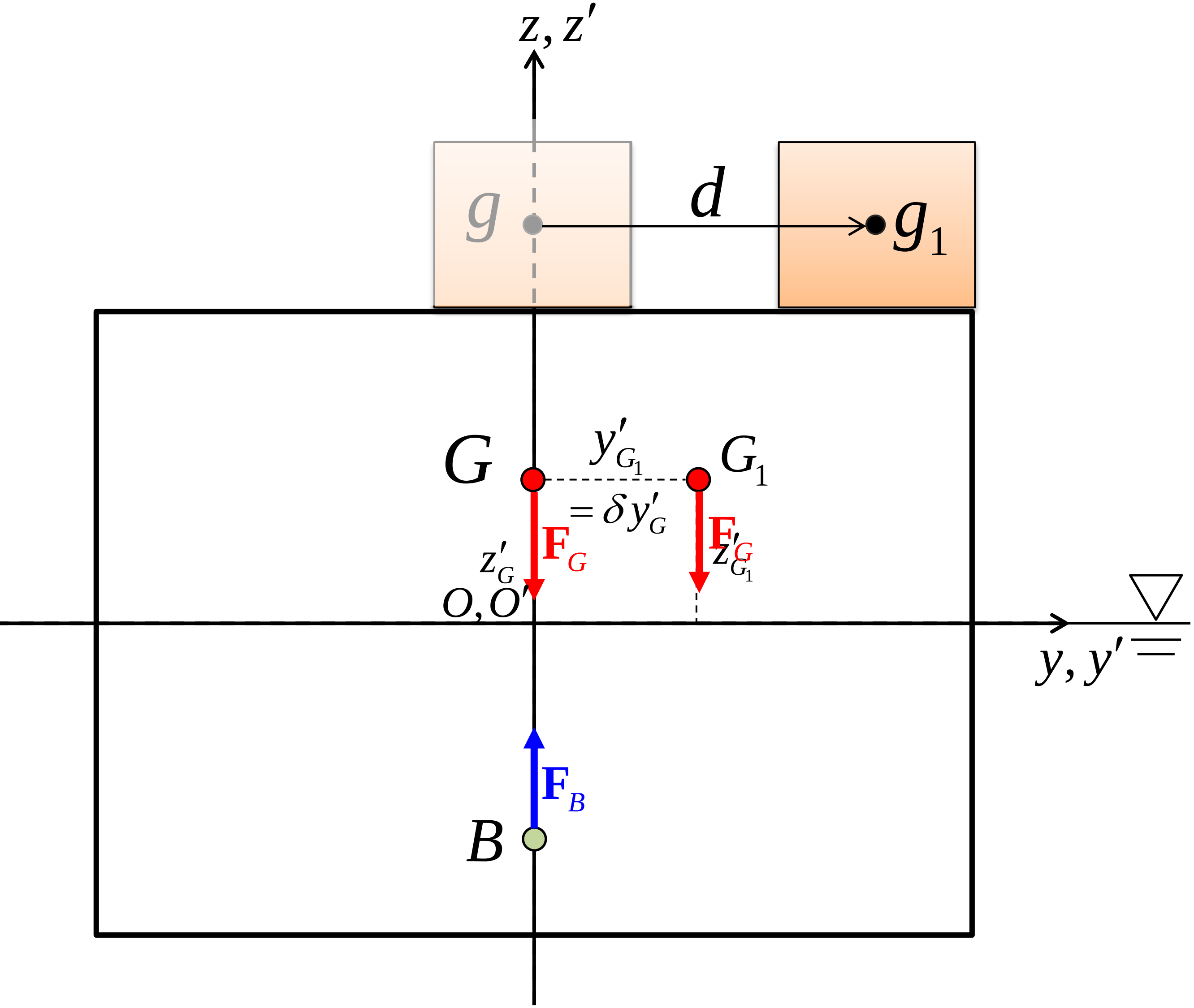
$$G_0M = GM - GG_0$$ ← Initial metacentric height(GM) at this loading condition = 0.64

$$= 0.64 - 0.06 = 0.58(m)$$

10-2 Determination of the Inclination Angle Caused by Moving a Load

1. move a load of weight "w" with distance "d" from "g" to "g_1"
2. Center of mass is then changed from G to G1
3. Because the point G1 and the point B are not on one line, the body will be inclined up to an angle " $-\phi$" so that the point B1 and G1 are on one line. We call this state as "static equilibrium"

$$\begin{bmatrix} y_P \\ z_P \end{bmatrix} = \begin{bmatrix} \cos(-\phi) & -\sin(-\phi) \\ \sin(-\phi) & \cos(-\phi) \end{bmatrix} \begin{bmatrix} y'_P \\ z'_P \end{bmatrix}$$



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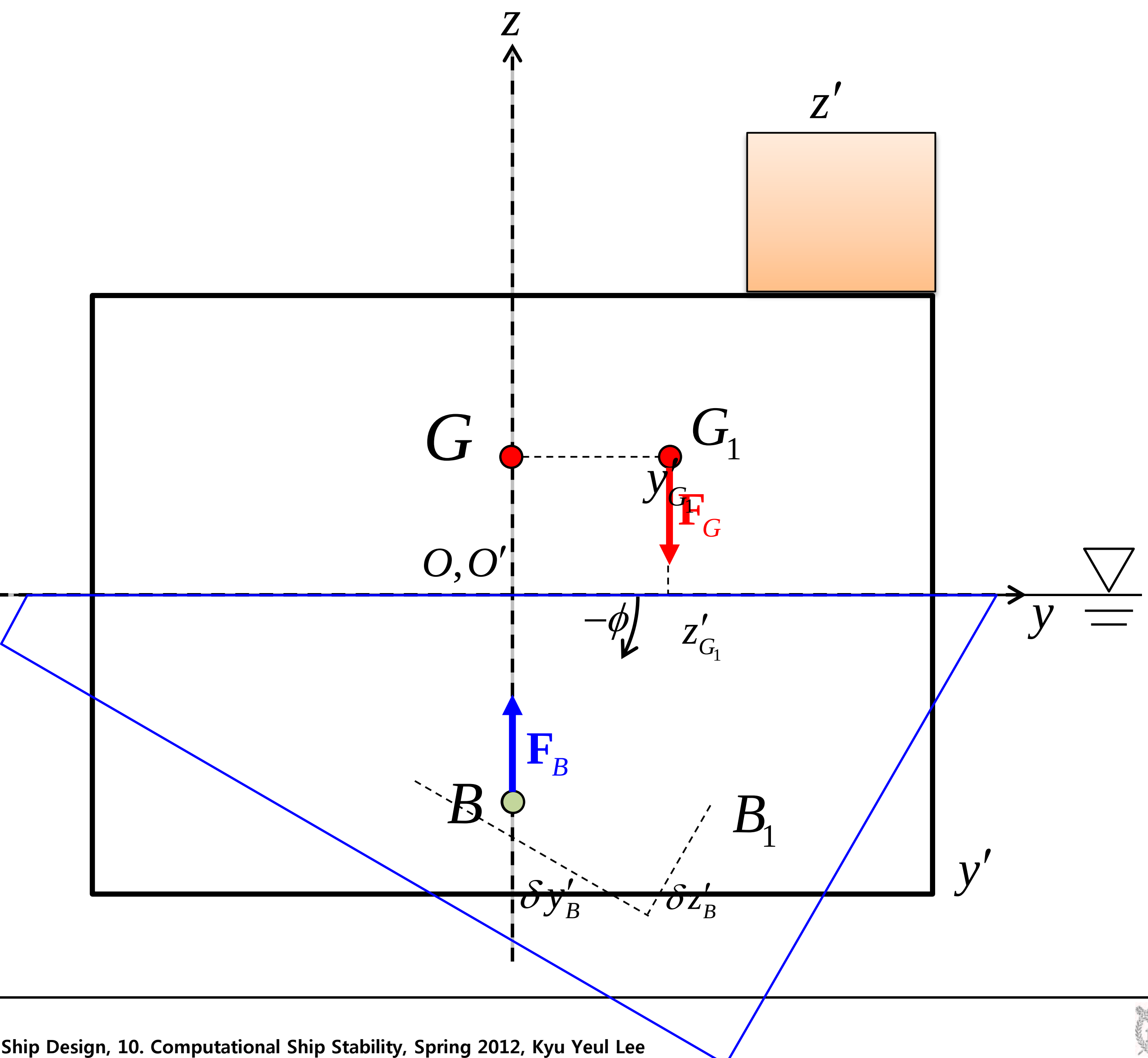

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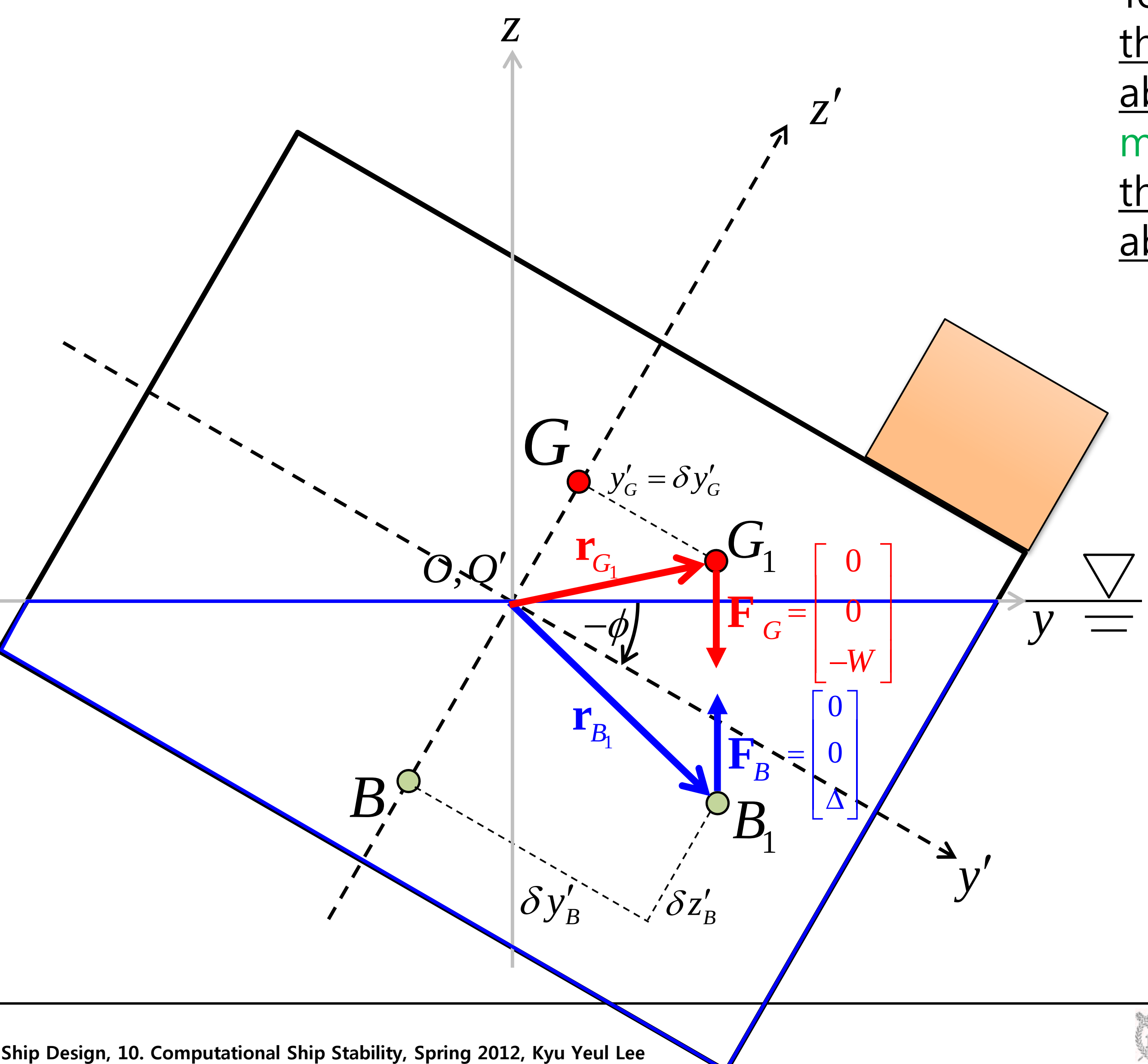


To be equilibrium state, the heeling moment of $\mathbf{F}_G$ about x-axis must be equal to the restoring moment of $\mathbf{F}_B$ about x-axis

Resultant external moment $\boldsymbol{\tau}^e$ about x-axis through point O must **be zero.**

$$\boldsymbol{\tau}^e = \mathbf{r}_{G_1} \times \mathbf{F}_G + \mathbf{r}_{B_1} \times \mathbf{F}_B$$

$$= \mathbf{i}(y_{G_1} \cdot F_{G,z} - z_{G_1} \cdot \cancel{F_{G,y}}) + \mathbf{i}(y_{B_1} \cdot F_{B,z} - z_{B_1} \cdot \cancel{F_{B,y}})$$

$$= \mathbf{i}(y_{G_1} \cdot (-W) + y_{B_1} \cdot \Delta)$$

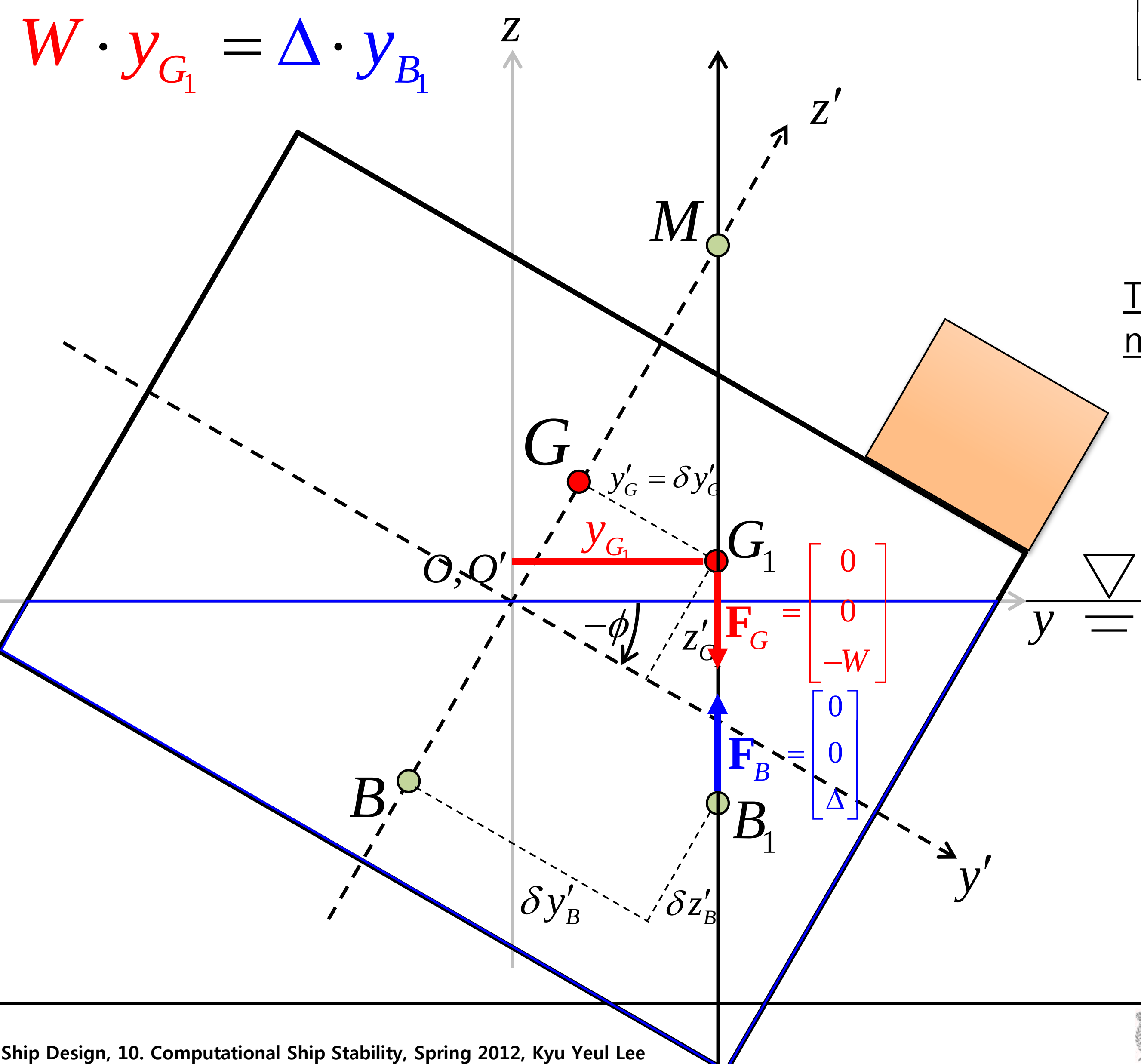
$$= 0$$

$$\Rightarrow W \cdot y_{G_1} = \Delta \cdot y_{B_1}$$

Determination of the inclining (heeling) angle - Inclining (Heeling) moment

$$\begin{bmatrix} y_P \\ z_P \end{bmatrix} = \begin{bmatrix} \cos(-\phi) & -\sin(-\phi) \\ \sin(-\phi) & \cos(-\phi) \end{bmatrix} \begin{bmatrix} y'_P \\ z'_P \end{bmatrix}$$

$$W \cdot y_{G_1} = \Delta \cdot y_{B_1}$$



$$\begin{bmatrix} y_{G_1} \\ z_{G_1} \end{bmatrix} = \begin{bmatrix} \cos(-\phi) & -\sin(-\phi) \\ \sin(-\phi) & \cos(-\phi) \end{bmatrix} \begin{bmatrix} y'_{G_1} \\ z'_{G_1} \end{bmatrix}$$

$$y_{G_1} = y'_{G_1} \cos\phi + z'_{G_1} \sin\phi$$

$$z_{G_1} = -y'_{G_1} \sin\phi + z'_{G_1} \cos\phi$$

The Inclining (Heeling) moment of $\mathbf{F}_G$ about x-axis

$$= W \cdot \left(y_{G_1} \right)$$

$$= W \cdot \left(y'_{G_1} \cos\phi + z'_{G_1} \sin\phi \right)$$

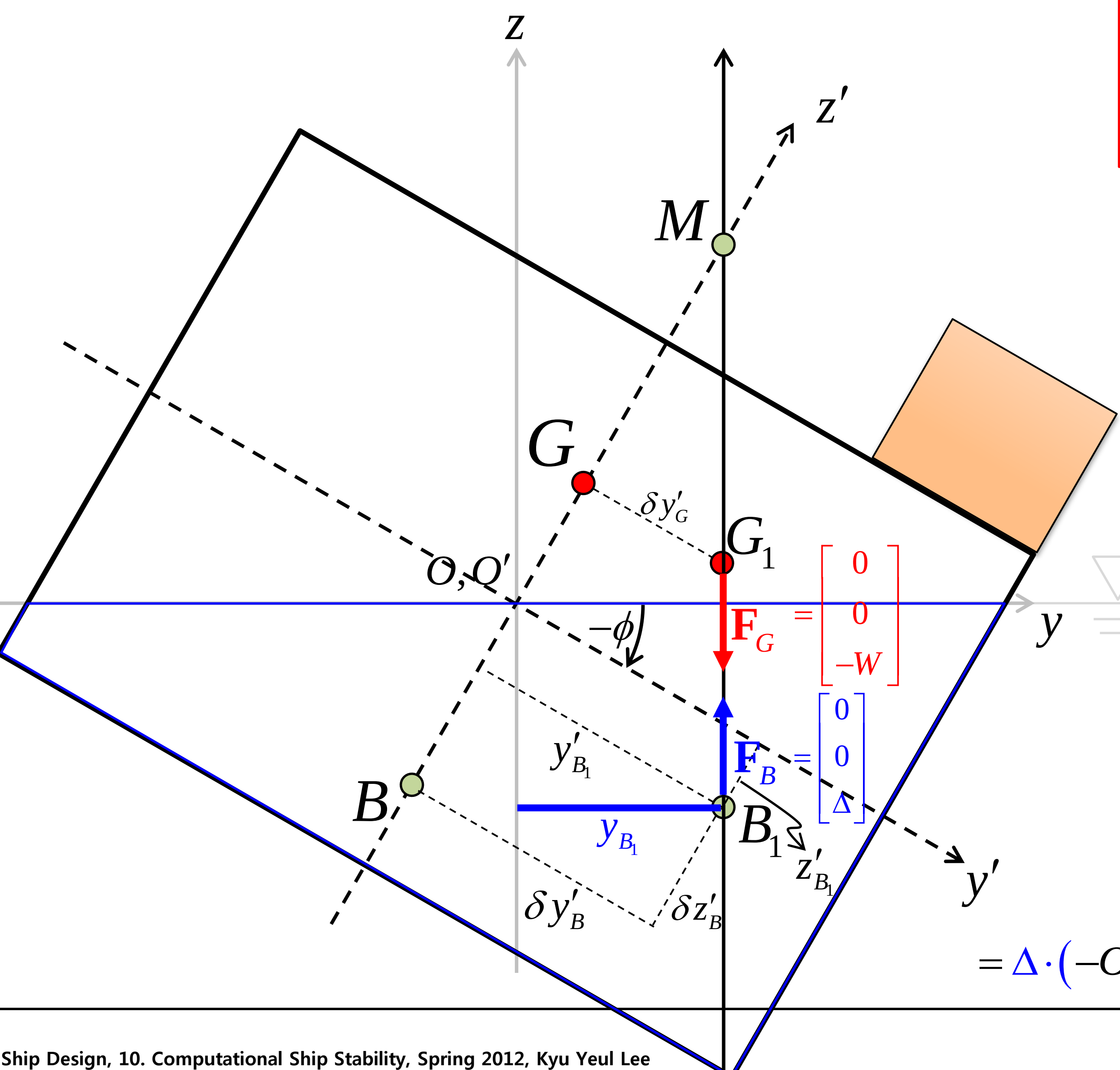
$$y'_{G_1} = \delta y'_G$$

$$z'_{G_1} = O'G$$

$$= W \cdot \left(\delta y'_G \cos\phi + O'G \sin\phi \right)$$

Determination of the heeling angle
- Restoring moment

$$\begin{bmatrix} y_P \\ z_P \end{bmatrix} = \begin{bmatrix} \cos(-\phi) & -\sin(-\phi) \\ \sin(-\phi) & \cos(-\phi) \end{bmatrix} \begin{bmatrix} y'_P \\ z'_P \end{bmatrix}$$



The heeling moment of $\mathbf{F}_G$ about x-axis

$$= W \cdot (\delta y'_G \cos\phi + O'G \sin\phi)$$

$$\begin{bmatrix} y_{B_1} \\ z_{B_1} \end{bmatrix} = \begin{bmatrix} \cos(-\phi) & -\sin(-\phi) \\ \sin(-\phi) & \cos(-\phi) \end{bmatrix} \begin{bmatrix} y'_{B_1} \\ z'_{B_1} \end{bmatrix}$$

$$y_{B_1} = y'_{B_1} \cos\phi + z'_{B_1} \sin\phi$$

$$z_{B_1} = -y'_{B_1} \sin\phi + z'_{B_1} \cos\phi$$

The restoring moment of $\mathbf{F}_B$ about x-axis

$$= \Delta \cdot (y_{B_1})$$

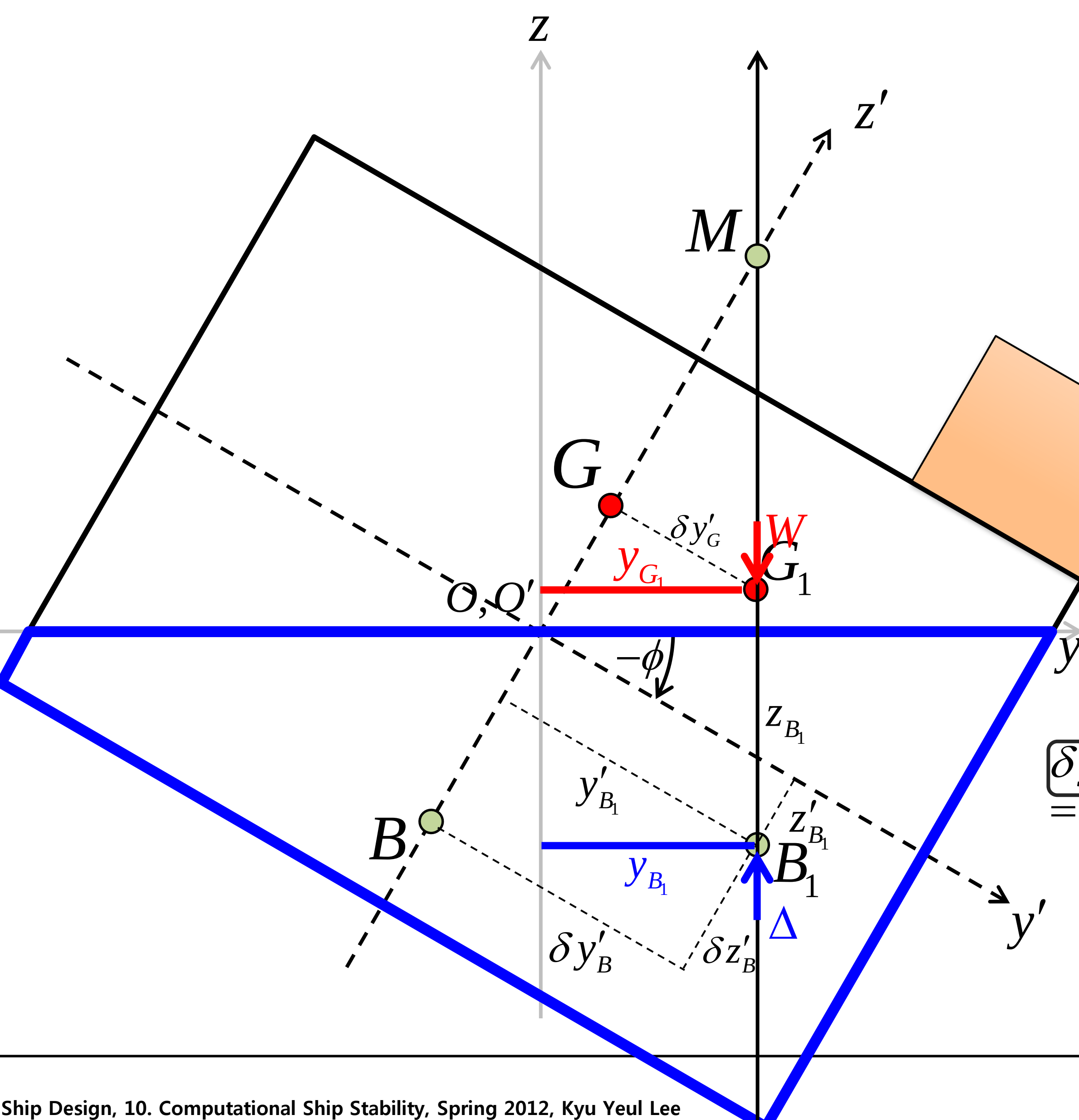
$$= \Delta \cdot (y'_{B_1} \cos\phi + z'_{B_1} \sin\phi)$$

$$y'_{B_1} = \delta y'_B, \; z'_{B_1} = -O'B + \delta z'_B$$

$$= \Delta \cdot (-O'B \sin\phi + \delta y'_B \cos\phi + \delta z'_B \sin\phi)$$

The Heeling angle can be calculated by equating the heeling moment and the restoring moment in Static Equilibrium

$$\begin{bmatrix} y_P \\ z_P \end{bmatrix} = \begin{bmatrix} \cos(-\phi) & -\sin(-\phi) \\ \sin(-\phi) & \cos(-\phi) \end{bmatrix} \begin{bmatrix} y'_P \\ z'_P \end{bmatrix}$$



The heeling moment of $\mathbf{F}_G$ about x-axis

$$= W \cdot \left(\delta y'_G \cos\phi + O'G \sin\phi \right)$$

The restoring moment of $\mathbf{F}_G$ about x-axis

$$= \Delta \cdot \left(-O'B \sin\phi + \delta y'_B \cos\phi + \delta z'_B \sin\phi \right)$$

Therefore

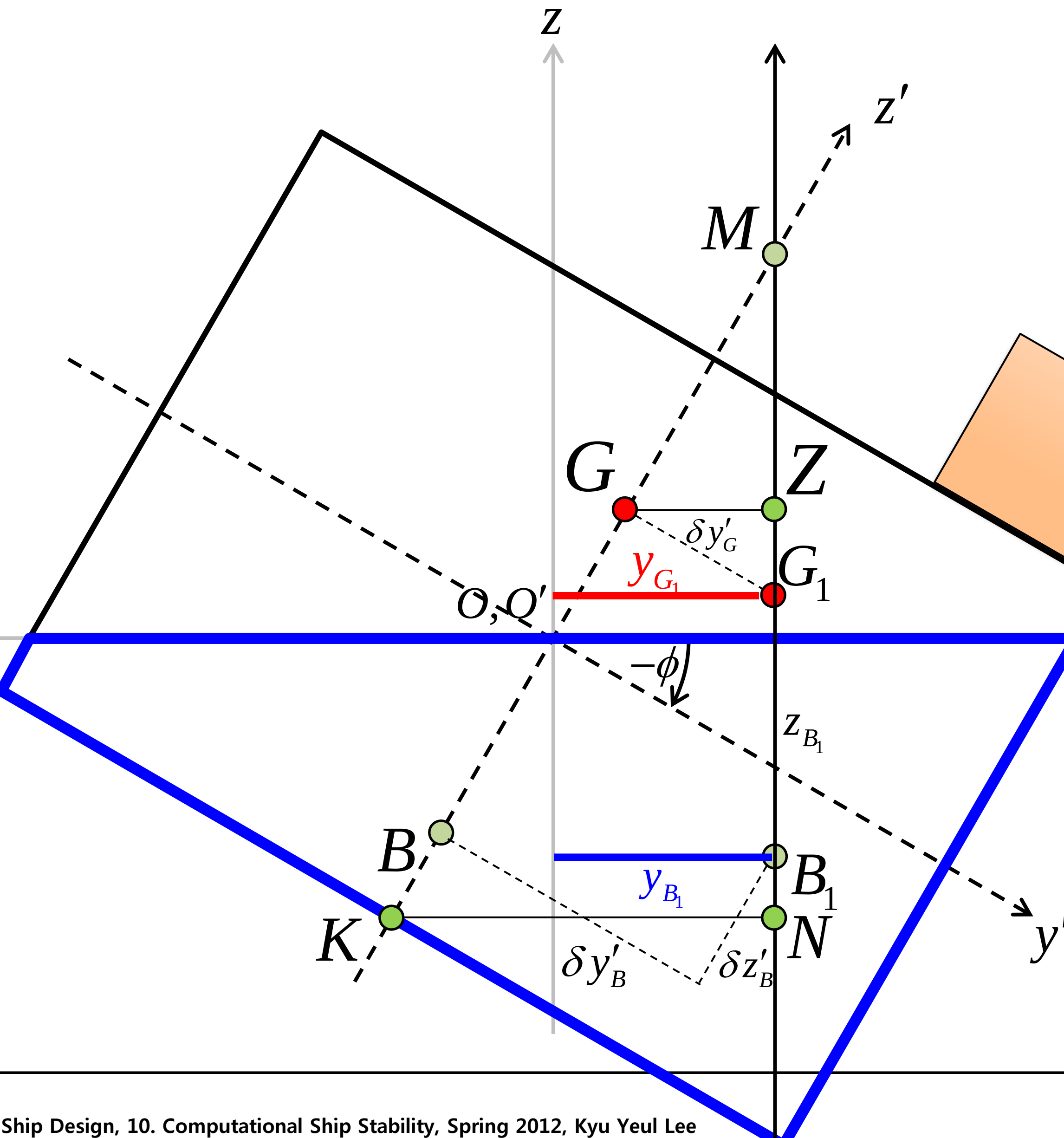
$$W \cdot \left(\delta y'_G \cos\phi + O'G \sin\phi \right) = \Delta \cdot \left(-O'B \sin\phi + \delta y'_B \cos\phi + \delta z'_B \sin\phi \right)$$

If the body floats in water, then $W = \Delta$

$$\boxed{\delta y'_G} \cos\phi + \boxed{O'G} \sin\phi = \boxed{-O'B} \sin\phi + \boxed{\delta y'_B} \cos\phi + \boxed{\delta z'_B} \sin\phi$$

To calculate the angle of inclination, $\delta y'_G, O'G, O'B, \delta y'_B$, and $\delta z'_B$ should be known.

$$\underbrace{\boxed{\delta y'_G \cos\phi + O'G\sin\phi}}_{y_{G_1}} = \underbrace{\boxed{-O'B\sin\phi + \delta y'_B \cos\phi + \delta z'_B \sin\phi}}_{y_{B_1}}$$



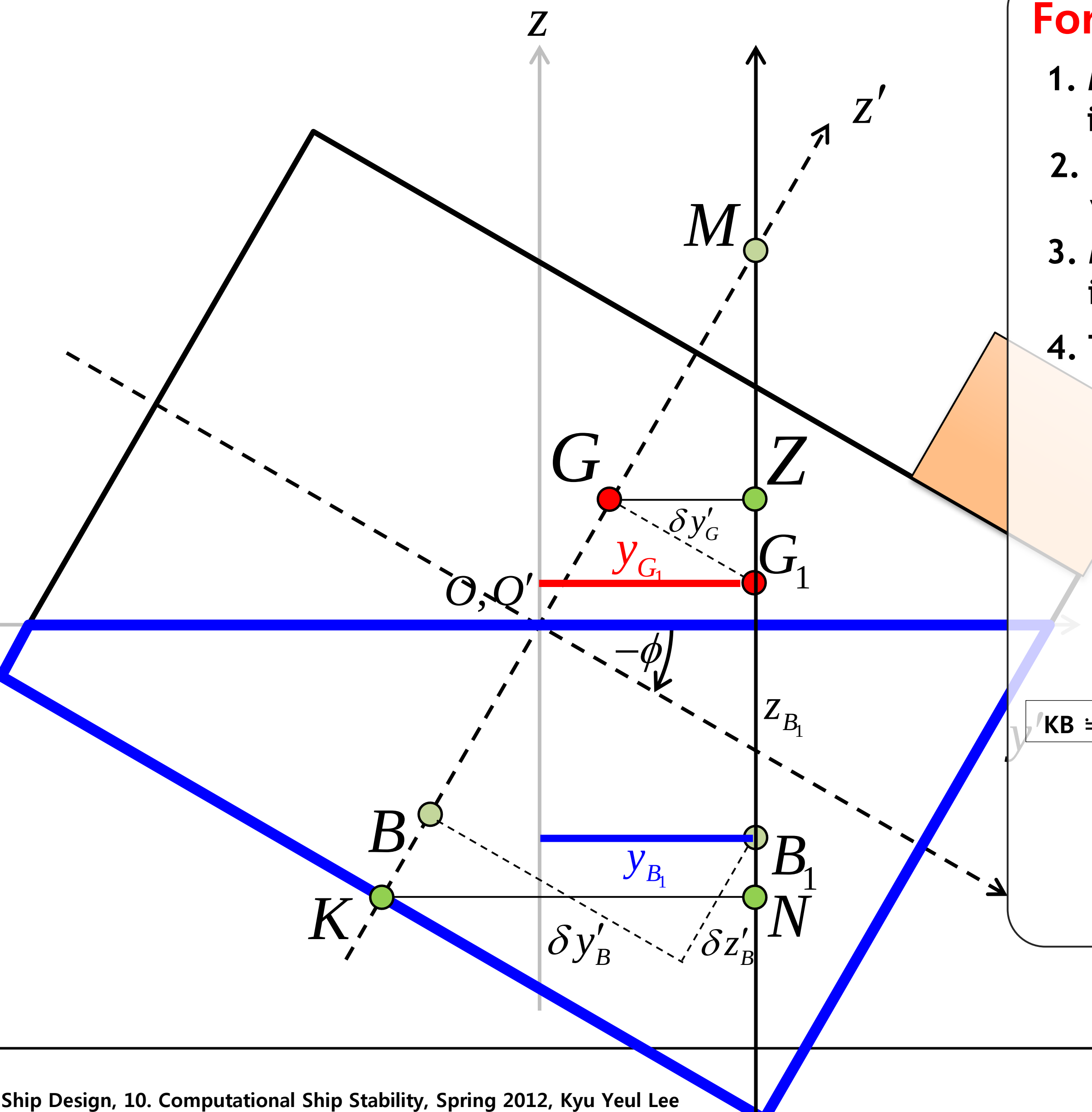
Restoring moment arm: GZ

$$GZ = \underbrace{\boxed{KN}}_{???} - \underbrace{\boxed{KG}}_{\text{known}}\sin\phi$$

$$KN = KO\sin\phi + y_{B_1}$$

To calculate GZ, y_{B_1} , i.e., $O'B$, $\delta y'_B$ and $\delta z'_B$ should be known

$\delta y'_B, \delta z'_B$ are related to the changed volume under the water surface



For small angle of inclination,

1. M does not change at small angle of inclination (about 7° to 10°)
2. For wall sided ship, the submerged volume and emerged volume are same.
3. Main deck is not submerged and bottom is not emerged.
4. Then GZ can be assumed as

$$GZ \approx GM \cdot \sin\phi$$

where

$$GM = KB + BM - KG$$

KB ≒ 51~52% draft

Center of mass of the body

How can you find the value of the BM?

To calculate GZ, BM should be obtained!!

Another approach to determine the heeling angle:
Assume that the body is not inclined whereas "the space-fixed reference frame" is inclined. Then the gravitational force(W) through point G_1 and buoyant force(Δ) through point B_1 are acting normal to "the water surface"

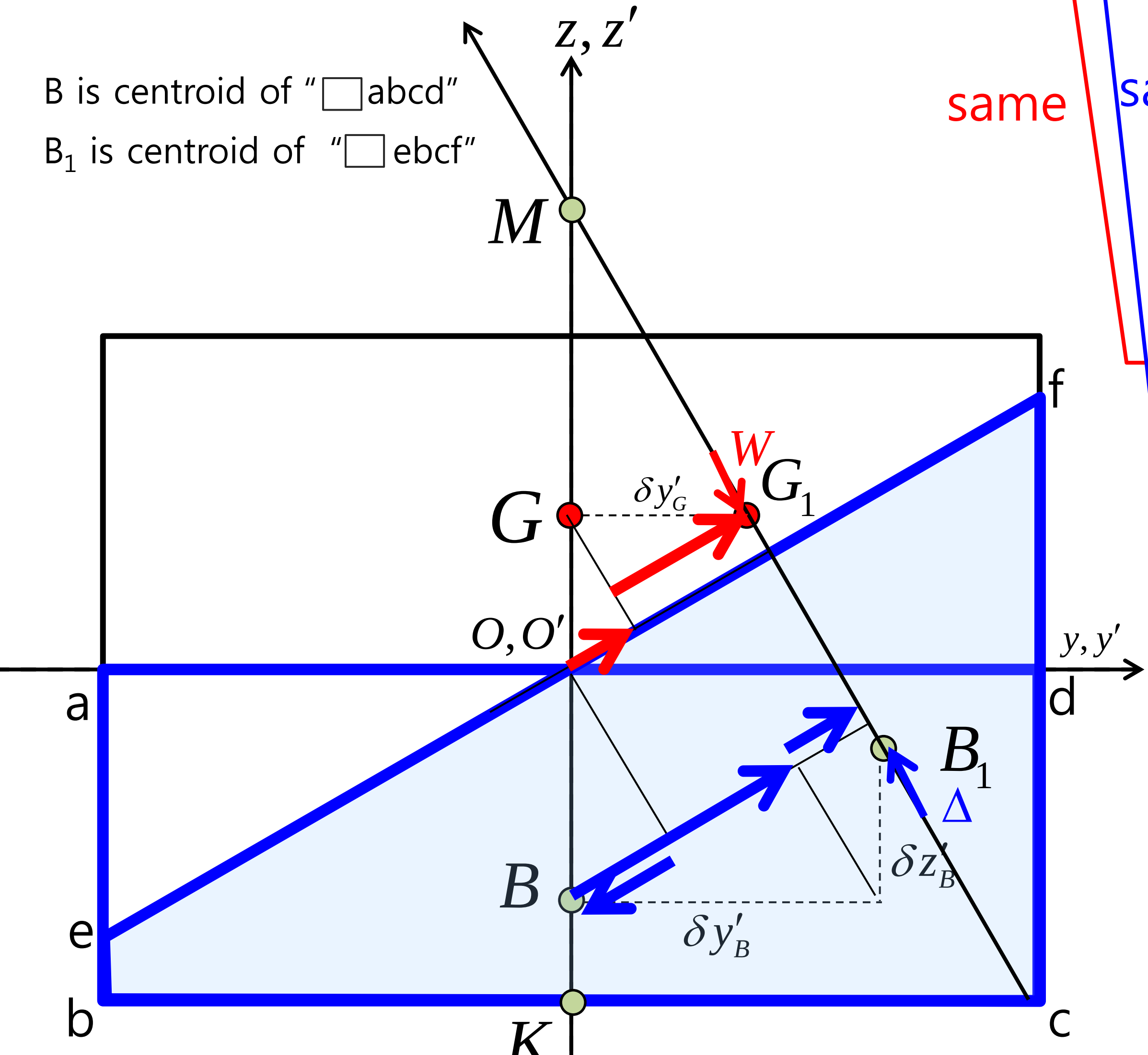
$$\boxed{W\cdot(\delta y'_G\cos\phi+OG\sin\phi)}$$
$$=\boxed{\Delta\cdot(-OB\sin\phi+\delta y'_B\cos\phi+\delta z'_B\sin\phi)}$$

same

same

B is centroid of "□abcd"

B_1 is centroid of "□ebcf"



<u>The heeling moment of W about longitudinal axis through point O</u>

$$=W\cdot(\delta y'_G\cos\phi+O'G\sin\phi)$$

<u>The righting moment of Δ about longitudinal axis through point O</u>

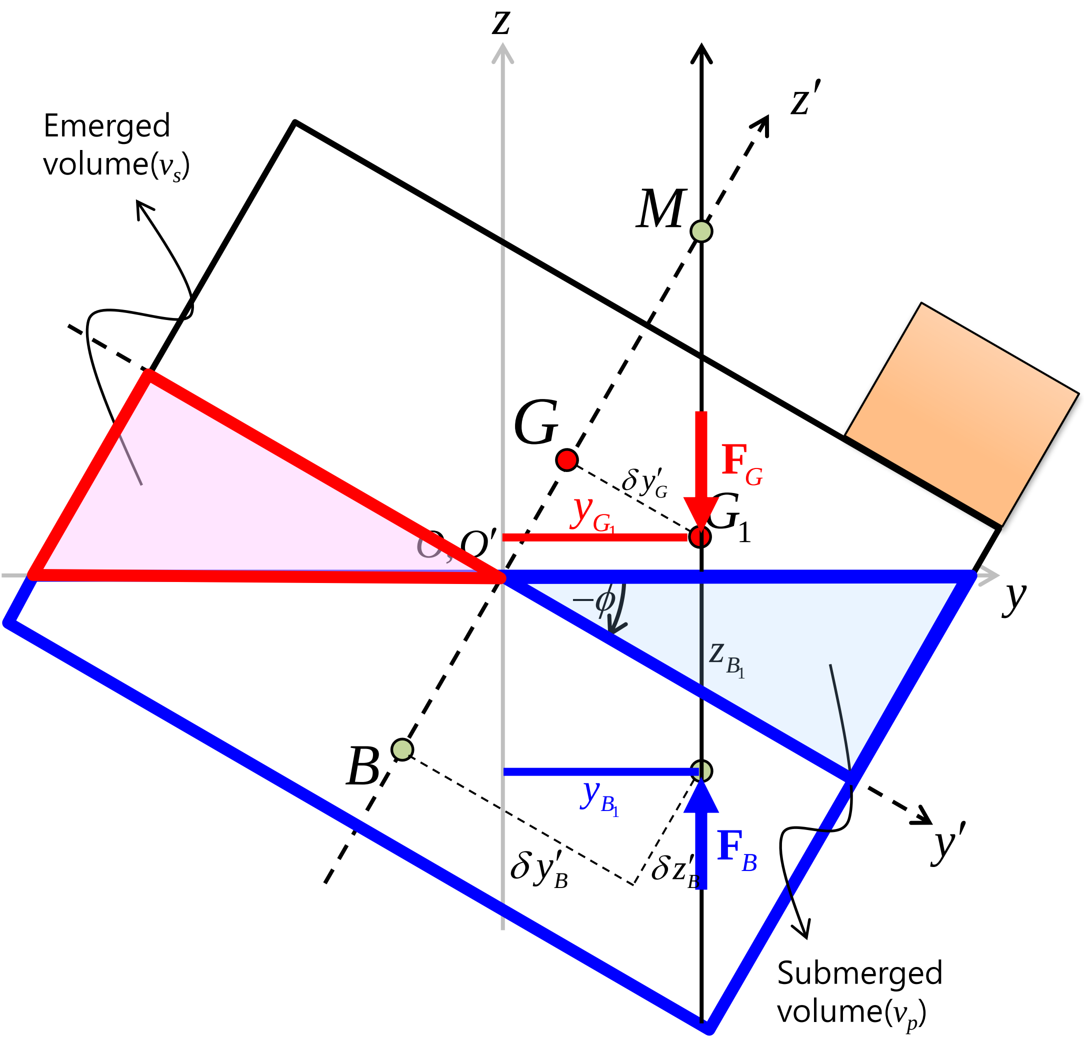
$$=\Delta\cdot(-O'B\sin\phi+\delta y'_B\cos\phi+\delta z'_B\sin\phi)$$

$O'x'y'z'$: Body-fixed reference frame

$Oxyz$: Space-fixed reference frame

Heeling angle in Static Equilibrium

$$W\cdot(\delta y'_G\cos\phi+O'G\sin\phi)$$
$$-\Delta\cdot(O'B\sin\phi+\delta y'_B\cos\phi+\delta z'_B\sin\phi)=0$$



Static Equilibrium

① Newton's 2nd law

$$ma=\sum F$$
$$=\mathbf{F}_G+\mathbf{F}_B=0 \quad ,(\because a=0)$$

In case of barge with rectangular section, the submerged volume and immerged volume are same at the heeling angle Φ where the main deck is assumed not to be submerged.

② Euler equation

$$I\dot{\omega}=\sum\tau=\mathbf{M}_{T,G}+\mathbf{M}_{T,B}=0,(\because \dot{\omega}=0)$$

$$W\cdot(\delta y'_G\cos\phi+O'G\sin\phi)$$
$$-\Delta\cdot(O'B\sin\phi+\delta y'_B\cos\phi+\delta z'_B\sin\phi)=0$$

$$\delta y'_B=\delta y'_B(\phi),\delta z'_B=\delta z'_B(\phi)$$

Moment is the function of Φ :

$$\mathbf{M}_{T,G}(\phi)+\mathbf{M}_{T,B}(\phi)=0$$

$\mathbf{M}_T(\phi)=0$ → **Nonlinear equation for** ϕ

Numerical method for solving nonlinear equation

$$\mathbf{M}_T(\phi)=0 \quad \Rightarrow \quad f_1(x)=0$$

$f_1(x)=0$: **Nonlinear equation of one variable x**

Given: $x^{(0)}, f_1\left(x^{(0)}\right)$

Find: x^* **, subject to:** $f_1\left(x^*\right)=0$

Taylor series expansion of f_1 **at** $x=x^{(0)}$

$$f_1\left(x^{(0)}+\delta x^{(0)}\right)=f_1\left(x^{(0)}\right)+\left.\frac{\partial f_1}{\partial x}\right|_{x^{(0)}}\delta x^{(0)}+\cancel{\frac{1}{2!}\left.\frac{\partial^2 f_1}{\partial x^2}\right|_{x^{(0)}}\left(\delta x^{(0)}\right)^2}+\cdots$$

Linearization

$$f_1\left(x^{(0)}+\delta x^{(0)}\right)\approx f_1\left(x^{(0)}\right)+\left.\frac{\partial f_1}{\partial x}\right|_{x^{(0)}}\delta x^{(0)}$$

Assume : $f_1\left(x^*\right)=0$

$$at,\ x^*=x^{(0)}+\delta x^{(0)}$$

$f_1(x)=0$: Nonlinear equation of one variable x

Given: $x^{(0)},\ f_1\left(x^{(0)}\right)$

Find: x^*

$$assume:\ f_1\left(x^*\right)=0$$

$$at\quad x^* = x^{(0)} + \delta x^{(0)}$$

$$f_1\left(x^{(0)}+\delta x^{(0)}\right) = f_1\left(x^{(0)}\right) + \left.\frac{\partial f_1}{\partial x}\right|_{x^{(0)}} \delta x^{(0)}$$

L.H.S $f_1\left(x^{(0)}+\delta x^{(0)}\right) = f_1\left(x^*\right)=0$

$$0 = f_1\left(x^{(0)}\right) + \left.\frac{\partial f_1}{\partial x}\right|_{x^{(0)}} \delta x^{(0)}$$

$$\underbrace{-f_1\left(x^{(0)}\right)}_{\text{Known}} = \underbrace{\left.\frac{\partial f_1}{\partial x}\right|_{x^{(0)}}}_{\text{Known}} \underbrace{\delta x^{(0)}}_{\text{Find}}$$

$f_1(x) = 0$: Nonlinear equation of one variable x

$$\underbrace{-f_1\left(x^{(0)}\right)}_{\text{Known}} = \underbrace{\left.\frac{\partial f_1}{\partial x}\right|_{x^{(0)}}}_{\text{Known}} \underbrace{\delta x^{(0)}}_{\text{Find}}$$

Iteration

$$x^{*} = x^{(0)} + \delta x^{(0)}$$

$$f_1\left(x^{*}\right) \overset{!}{=} 0 \Rightarrow \text{Check}$$

No

$$x^{(1)} = x^{(0)} + \delta x^{(0)}$$

$$x^{*} = x^{(1)} + \delta x^{(1)}$$

Yes

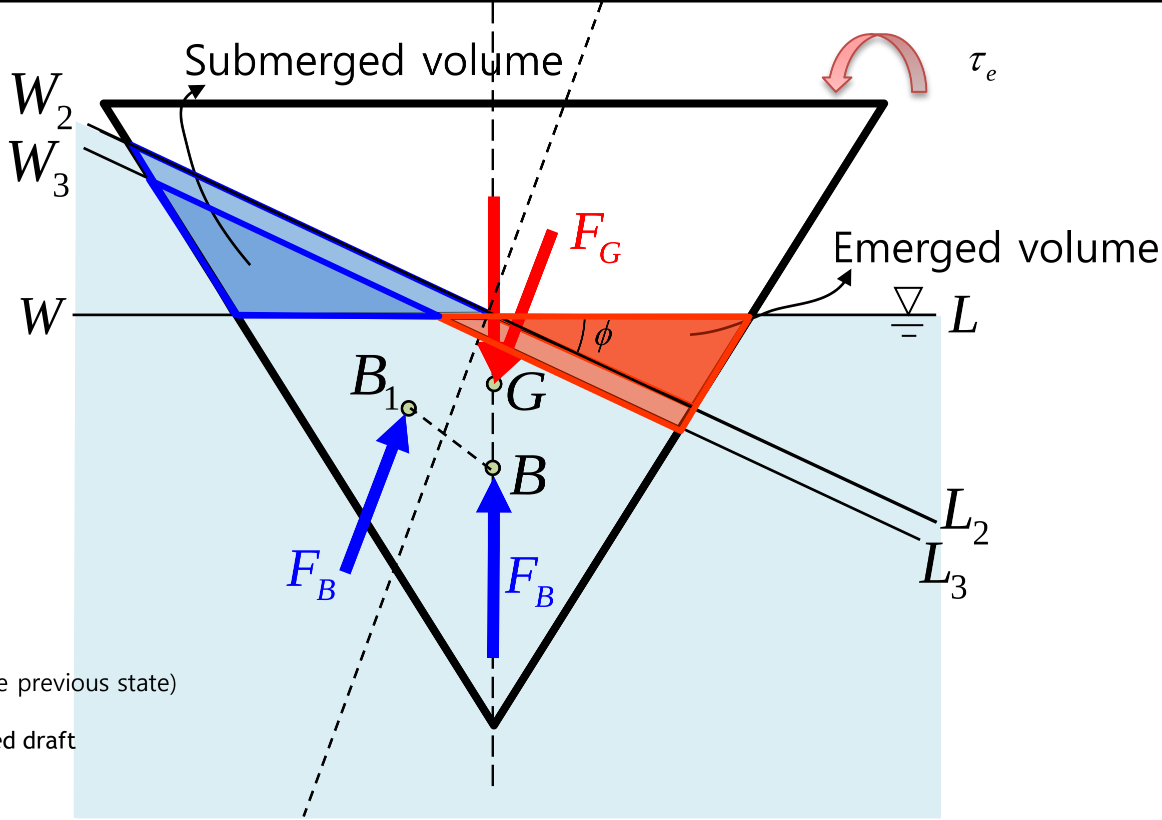
We find the solution, x^{*}.

10-3 Determination of the Inclination Angle Caused by Moving a Load for a Cylindrical Ship with Triangular Section Shape

THE DRAFT AND INCLINATION ANGLE MUST BE ADJUSTED FOR UNBALANCED DISPLACEMENT

If the submerged volume and the emerged volume are not same, the draft and inclination angle should be adjusted repeatedly

① Apply an external heeling moment to the ship
② Then release the external moment
③ Test whether the ship returns to its initial equilibrium position



WL: Waterplane before inclination(in the previous state)
W_2L_2: Waterplane after inclination
W_3L_3: Changed waterplane after adjusted draft

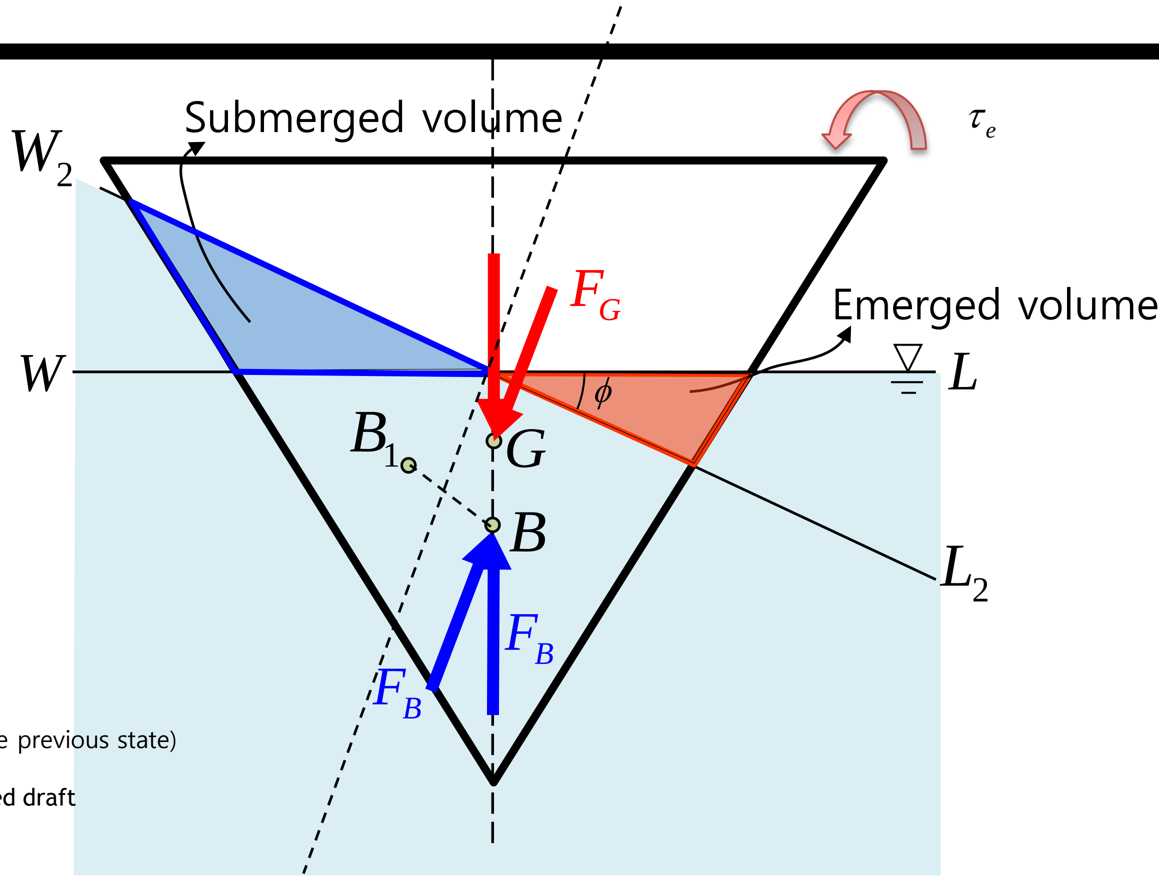
B_1: new position of center of buoyancy after the draft is adjusted

For the ship to be in static equilibrium, the magnitude of the buoyant force has to be equal to the magnitude of the weight : Force equilibrium

① A ship with triangular section shape is inclined with an angle of ϕ

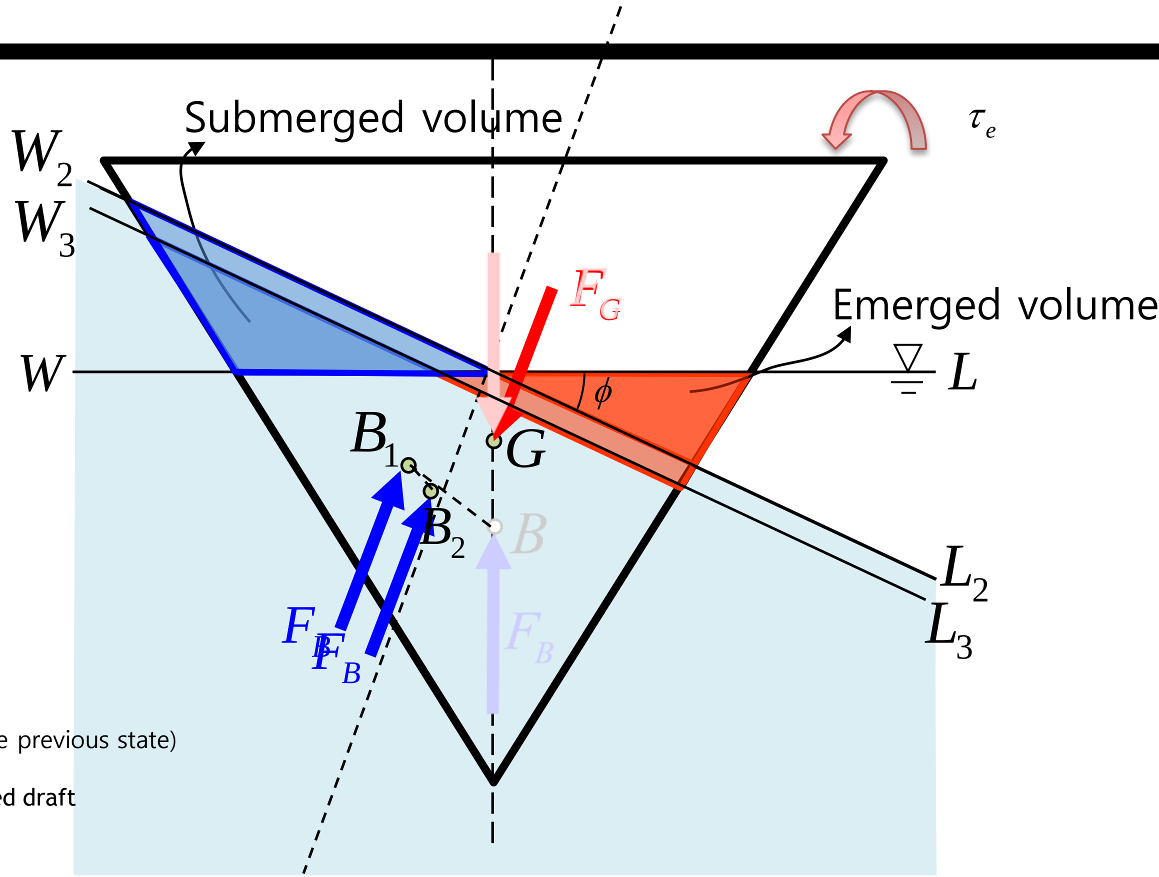
② In this case, the submerged volume and the emerged volume are not same.

③ Thus, the draft and inclination angle should be adjusted to maintain same displacement.



WL: Waterplane before inclination(in the previous state)
W_2L_2: Waterplane after inclination
W_3L_3: Changed waterplane after adjusted draft

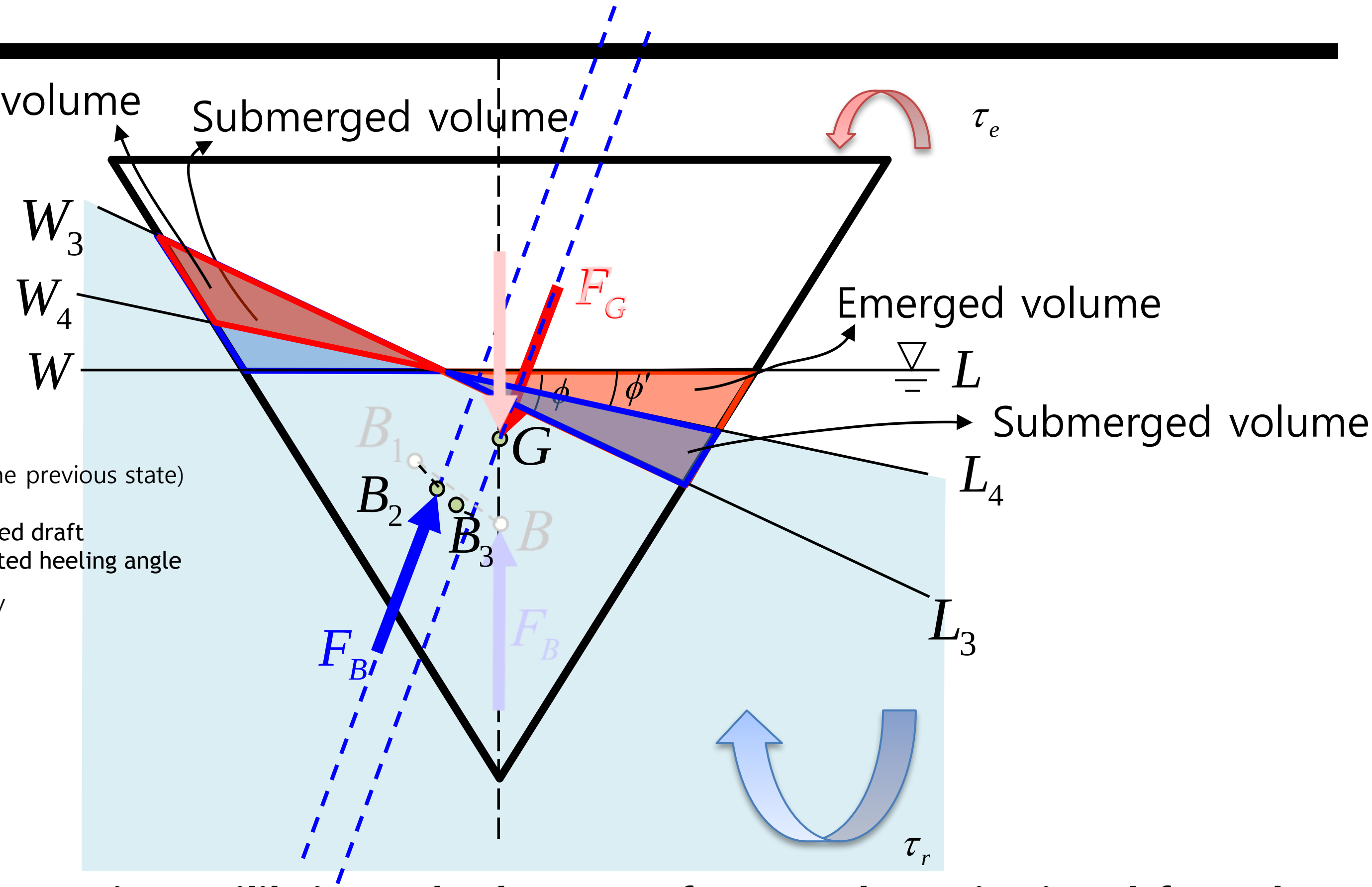
B_1: new position of center of buoyancy
after the draft is adjusted



WL: Waterplane before inclination(in the previous state)
W_2L_2: Waterplane after inclination
W_3L_3: Changed waterplane after adjusted draft

B_1: new position of center of buoyancy after the draft is adjusted

Need of **repeated calculation** of ship position in static equilibrium - **Iteration**



WL: Waterplane before inclination(in the previous state)
W_2L_2: Waterplane after inclination
W_3L_3: Changed waterplane after adjusted draft
$W4L4$: Changed waterplane after adjusted heeling angle

B_1: new position of center of buoyancy after the draft is adjusted

For the ship to be in static equilibrium, the buoyant force and gravitational force have to be on one line, so that the total moment about the transverse axis through any point becomes 0. : Moment equilibrium

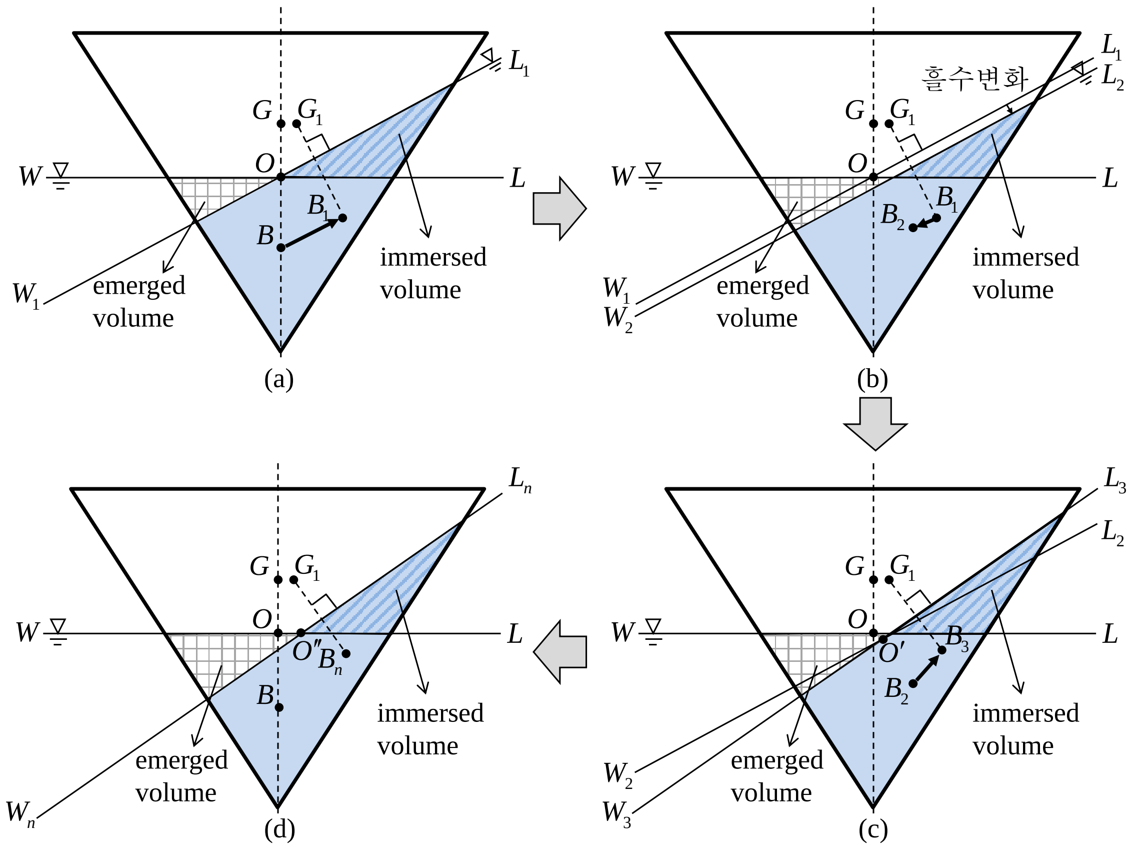
④ Due to the restoring moment, the angle of inclination decreases.

⑤ The submerged volume and the emerged volume are not same.

⑥ Thus, the draft and inclination angle should be adjusted again to maintain the same displacement.

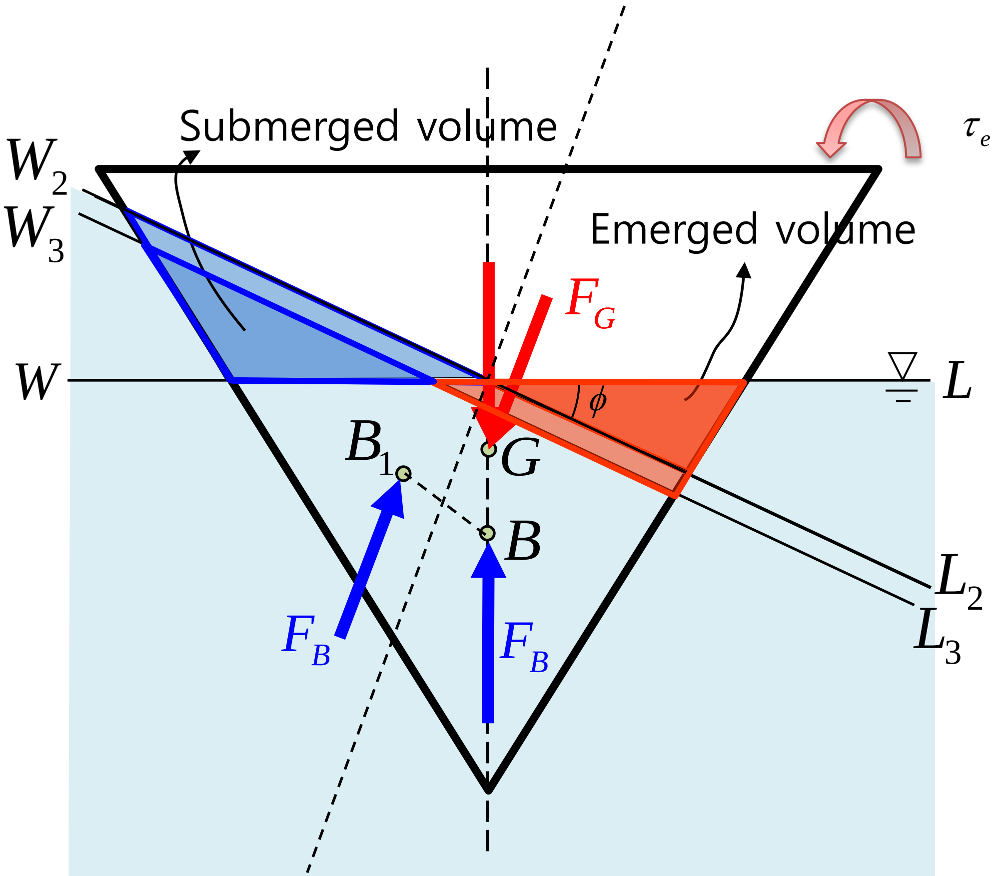
The angle of inclination and the draft are coupled. Iteration is required!

ROTATION OF CYLINDRICAL SHIP WITH TRIANGULAR SECTION SHAPE



(a)**와 같이 선박이 횡 경사모멘트로 인해 부면심**(center of floation)(**수선면적의 도심**)**인** O**를 중심으로 회전하게 된다. 이 경우, 수면 위로 노출된 용적**(emerged volume)**보다 수면 아래로 잠기는 용적**(immersed volume)**이 더 큰 것을 확인할 수 있다. 본 예제의 경우 횡 경사 전에는 부력과 중력의 크기가 서로 같아서 정적 평형상태를 이루고 있었는데, 횡 경사 후에는 부력의 크기가 커지게 되어** (b)**와 같이 중력과 크기가 같아질 때까지 수면** W1L1 **에서 수면** W2L2**로 선박이 떠오르게 된다. 그러나 선박이 떠오른 뒤에는 부력 중심이** B1**에서** B2**로 이동하게 되어 모멘트가 평형을 이루지 못한다. 따라서** (c)**와 같이 선박이 새로운 회전 중심** O'**을 기준으로 더 경사하여 수면** W3L3**과 같은 상태가 되고, 부력중심은** B2**에서** B3**으로 이동하게 된다. 이와 같이 흘수 변화와 회전을 반복하여 최종적으로는** (d)**와 같은 정적 평형상태에 도달하게 된다. 이 때의 최종적인 정적 평형상태의 선박은** (a)**에서 어떤 점을 중심으로 회전한 것일까? 정적 평형상태가 되기 위해서는** (d)**에서 경사 전의 수면 아래에 잠긴 용적과 경사 후의 수면 아래 잠긴 용적이 같아야 한다. 이를 이용하여 회전 중심** O"**을 계산해 보면, 결과적으로 삼각형 횡단면을 갖는 선박은 경사 전의 부심 가 아닌 다른 점을 기준으로 회전하였음을 알 수 있다.**

Nonlinear equations for determining the angle of inclination and draft in static equilibrium



Static Equilibrium

① **Newton's 2nd law**

$$ma = \sum F$$

$$= \mathbf{F}_G + \mathbf{F}_B = 0 \quad ,(\because a = 0)$$

② **Euler equation**

$$I\dot{\omega} = \sum \tau$$

$$= \mathbf{M}_{T,G} + \mathbf{M}_{T,B} = 0 \quad ,(\because \dot{\omega} = 0)$$

In this case, the submerged volume and emerged volume are not same at the heeling angle "Φ". Thus the draft "z" and also heeling angle "Φ" should be adjusted to maintain the same displacement.

It means that the following nonlinear equations should be satisfied.

$$\mathbf{F}_G + \mathbf{F}_B(z, \phi) = 0$$

$$\mathbf{M}_{T,G}(z, \phi) + \mathbf{M}_{T,B}(z, \phi) = 0$$

$$\mathbf{F}(z, \phi) = 0$$

$$\mathbf{M}_T(z, \phi) = 0$$

: Nonlinear equations of two variables

Numerical method for solving nonlinear equations of two variables

$$\mathbf{F}(z,\phi)=0$$
$$\mathbf{M}_T(z,\phi)=0$$

$\Rightarrow$

$$f_1(x_1,x_2)=0$$
$$f_2(x_1,x_2)=0$$

$f_1(x_1,x_2)=0,\quad f_2(x_1,x_2)=0$ **: Nonlinear equations of two variables**

Given: $x_1^{(0)}, x_2^{(0)}, f_1\left(x_1^{(0)}, x_2^{(0)}\right), f_2\left(x_1^{(0)}, x_2^{(0)}\right)$

Find: x_1^{*}, x_2^{*} **, subject to:** $f_1\left(x_1^{*}, x_2^{*}\right)=0,\ f_2\left(x_1^{*}, x_2^{*}\right)=0$

Taylor series expansion of f_1, f_2 **at** $x_1=x_1^{(0)}, x_2=x_2^{(0)}$

$$f_1\left(x_1^{(0)}+\delta x_1^{(0)}, x_2^{(0)}+\delta x_2^{(0)}\right)= f_1\left(x_1^{(0)}, x_2^{(0)}\right)+\left.\frac{\partial f_1}{\partial x_1}\right|_{x_1^{(0)},x_2^{(0)}}\delta x_1^{(0)}+\left.\frac{\partial f_1}{\partial x_2}\right|_{x_1^{(0)},x_2^{(0)}}\delta x_2^{(0)}+\cdots$$

$$f_2\left(x_1^{(0)}+\delta x_1^{(0)}, x_2^{(0)}+\delta x_2^{(0)}\right)= f_2\left(x_1^{(0)}, x_2^{(0)}\right)+\left.\frac{\partial f_2}{\partial x_1}\right|_{x_1^{(0)},x_2^{(0)}}\delta x_1^{(0)}+\left.\frac{\partial f_2}{\partial x_2}\right|_{x_1^{(0)},x_2^{(0)}}\delta x_2^{(0)}+\cdots$$

$$f_1(x_1, x_2) = 0, \quad f_2(x_1, x_2) = 0$$ **: Nonlinear equations of two variables**

Linearization by neglecting second and higher order terms

$$f_1\left(x_1^{(0)} + \delta x_1^{(0)}, x_2^{(0)} + \delta x_2^{(0)}\right) = f_1\left(x_1^{(0)}, x_2^{(0)}\right) + \left.\frac{\partial f_1}{\partial x_1}\right|_{x_1^{(0)}, x_2^{(0)}} \delta x_1^{(0)} + \left.\frac{\partial f_1}{\partial x_2}\right|_{x_1^{(0)}, x_2^{(0)}} \delta x_2^{(0)}$$

$$f_2\left(x_1^{(0)} + \delta x_1^{(0)}, x_2^{(0)} + \delta x_2^{(0)}\right) = f_2\left(x_1^{(0)}, x_2^{(0)}\right) + \left.\frac{\partial f_2}{\partial x_1}\right|_{x_1^{(0)}, x_2^{(0)}} \delta x_1^{(0)} + \left.\frac{\partial f_2}{\partial x_2}\right|_{x_1^{(0)}, x_2^{(0)}} \delta x_2^{(0)}$$

Assume :

$$f_1(x_1^*, x_2^*) = 0, \quad f_2(x_1^*, x_2^*) = 0$$

$$at \quad x_1^* = x_1^{(0)} + \delta x_1^{(0)}, \; x_2^* = x_2^{(0)} + \delta x_2^{(0)}$$

L.H.S

$$f_1\left(x_1^{(0)} + \delta x_1^{(0)}, x_2^{(0)} + \delta x_2^{(0)}\right) = f_1\left(x_1^*, x_2^*\right) = 0$$

$$f_2\left(x_1^{(0)} + \delta x_1^{(0)}, x_2^{(0)} + \delta x_2^{(0)}\right) = f_2\left(x_1^*, x_2^*\right) = 0$$

$$0 = f_1\left(x_1^{(0)}, x_2^{(0)}\right) + \left.\frac{\partial f_1}{\partial x_1}\right|_{x_1^{(0)}, x_2^{(0)}} \delta x_1^{(0)} + \left.\frac{\partial f_1}{\partial x_2}\right|_{x_1^{(0)}, x_2^{(0)}} \delta x_2^{(0)}$$

$$0 = f_2\left(x_1^{(0)}, x_2^{(0)}\right) + \left.\frac{\partial f_2}{\partial x_1}\right|_{x_1^{(0)}, x_2^{(0)}} \delta x_1^{(0)} + \left.\frac{\partial f_2}{\partial x_2}\right|_{x_1^{(0)}, x_2^{(0)}} \delta x_2^{(0)}$$

$f_1(x_1, x_2) = 0, \quad f_2(x_1, x_2) = 0$ **: Nonlinear equations of two variables**

$$0 = f_1\left(x_1^{(0)}, x_2^{(0)}\right) + \left.\frac{\partial f_1}{\partial x_1}\right|_{x_1^{(0)}, x_2^{(0)}} \delta x_1^{(0)} + \left.\frac{\partial f_1}{\partial x_2}\right|_{x_1^{(0)}, x_2^{(0)}} \delta x_2^{(0)}$$

$$0 = f_2\left(x_1^{(0)}, x_2^{(0)}\right) + \left.\frac{\partial f_2}{\partial x_1}\right|_{x_1^{(0)}, x_2^{(0)}} \delta x_1^{(0)} + \left.\frac{\partial f_2}{\partial x_2}\right|_{x_1^{(0)}, x_2^{(0)}} \delta x_2^{(0)}$$

Matrix form

$$\begin{bmatrix} -f_1\left(x_1^{(0)}, x_2^{(0)}\right) \\ -f_2\left(x_1^{(0)}, x_2^{(0)}\right) \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f_1}{\partial x_1} & \dfrac{\partial f_1}{\partial x_2} \\ \dfrac{\partial f_2}{\partial x_1} & \dfrac{\partial f_2}{\partial x_2} \end{bmatrix}_{x_1^{(0)}, x_2^{(0)}} \begin{bmatrix} \delta x_1^{(0)} \\ \delta x_2^{(0)} \end{bmatrix}$$

$f_1(x_1, x_2) = 0, \quad f_2(x_1, x_2) = 0$ **: Nonlinear equations of two variables**

$$\underbrace{\begin{bmatrix} -f_1\left(x_1^{(0)}, x_2^{(0)}\right) \\ -f_2\left(x_1^{(0)}, x_2^{(0)}\right) \end{bmatrix}}_{\text{Known}} = \underbrace{\begin{bmatrix} \dfrac{\partial f_1}{\partial x_1} & \dfrac{\partial f_1}{\partial x_2} \\ \dfrac{\partial f_2}{\partial x_1} & \dfrac{\partial f_2}{\partial x_2} \end{bmatrix}_{x_1^{(0)}, x_2^{(0)}}}_{\text{Known}} \underbrace{\begin{bmatrix} \delta x_1^{(0)} \\ \delta x_2^{(0)} \end{bmatrix}}_{\text{Find}}$$

Iteration

$$x_1^* = x_1^{(0)} + \delta x_1^{(0)}, \quad x_2^* = x_2^{(0)} + \delta x_2^{(0)}$$

$$f_1(x_1^*, x_2^*) \overset{!}{=} 0, \quad f_2(x_1^*, x_2^*) \overset{!}{=} 0$$

➔ **Check**

No

$$x_1^{(1)} = x_1^{(0)} + \delta x_1^{(0)}$$
$$x_2^{(1)} = x_2^{(0)} + \delta x_2^{(0)}$$

Yes

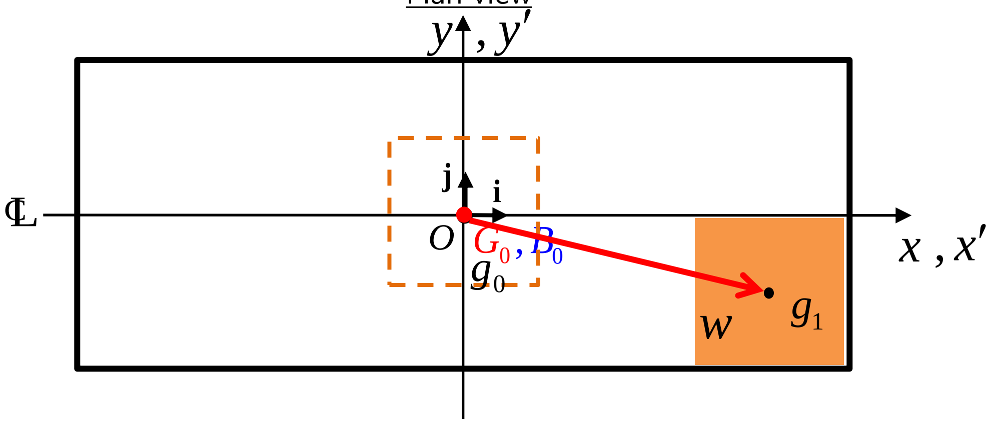
We find the solution, x_1^*, x_2^* .

10-4 Determination of heel angle, trim angle, and draft of a box-shaped barge when a cargo is loaded and then moved in transverse and longitudinal directions

Determination of heel angle and trim angle of a box-shaped barge when a cargo is loaded and then moved in transverse and longitudinal directions

A cargo, which is loaded on a barge, is loaded and then moved from g_0 to g_1 in -y direction and +x direction as in the figure. Determine **the final position and orientation (trim and heel) of the barge.**

Plan view



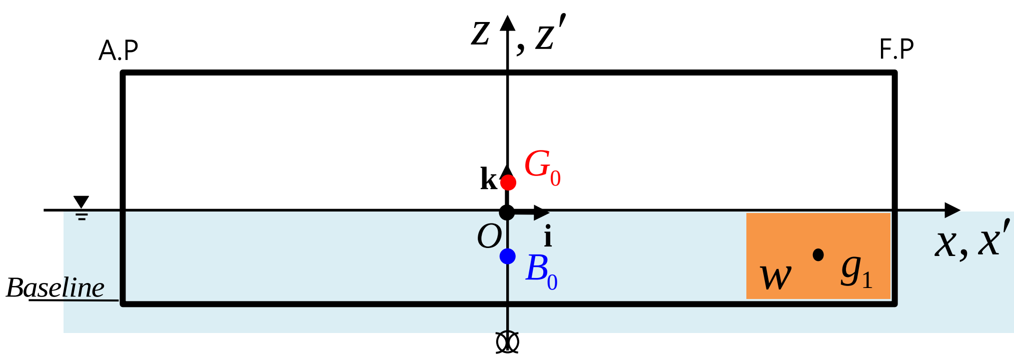
$O-xyz$: Space fixed coordinate system(reference frame)

$O-x'y'z'$: Body fixed coordinate system

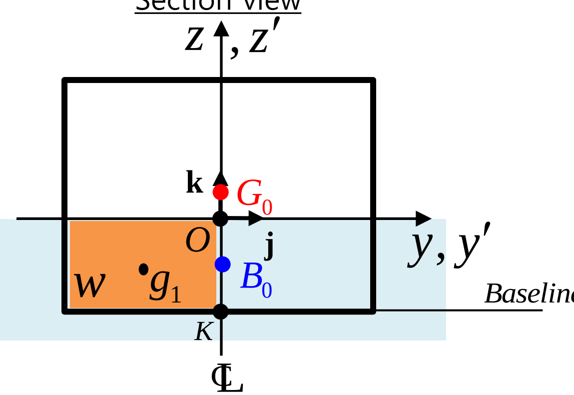
G_0: center of gravity of the barge before loading
B_0: center of buoyancy of the barge before loading

g_0, g_1 : center of gravity of the weight
w : weight of the cargo

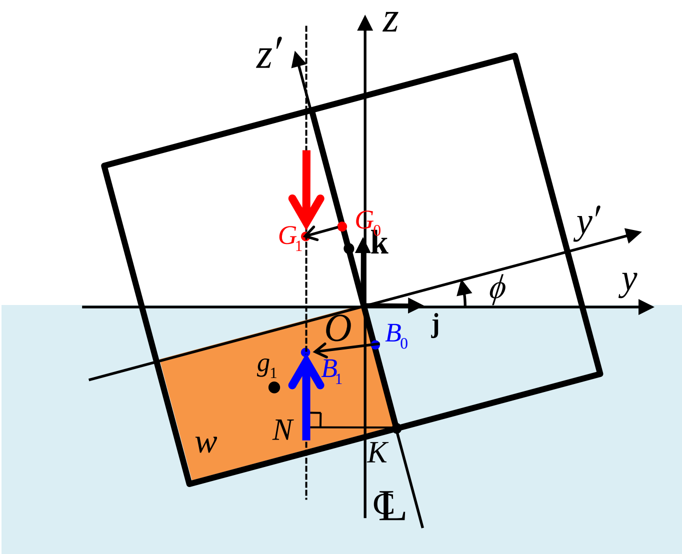
Elevation view



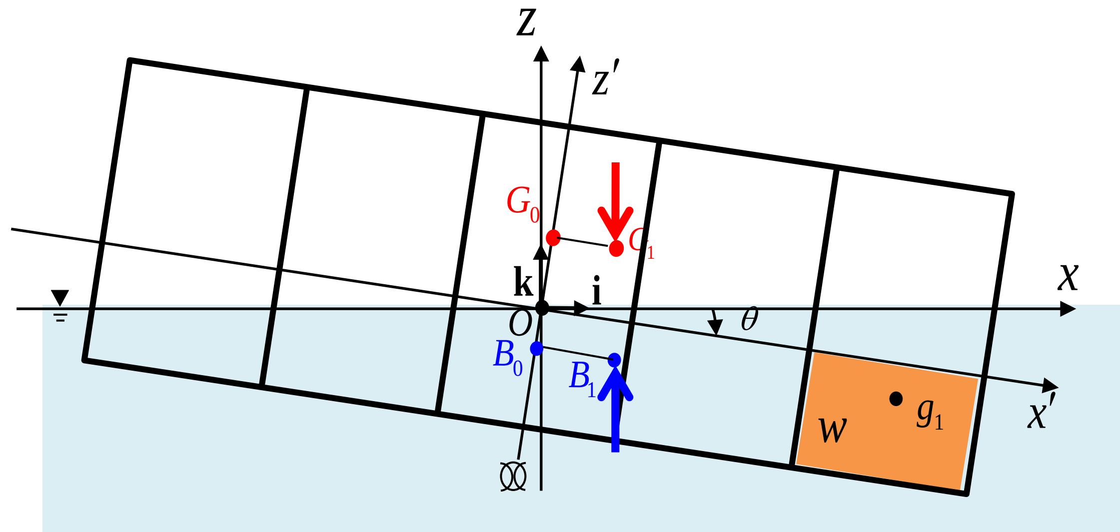
Section view



Section view



Elevation view



Static Equilibrium

① **Newton's 2nd law**

$$ma = \sum F = \mathbf{F}_G + \mathbf{F}_B = 0 \quad ,(\because a = 0)$$

② **Euler equation**

$$I_{xx}\dot{\omega}_x = \sum \tau_x = \mathbf{M}_{T,G} + \mathbf{M}_{T,B} = 0, (\because \dot{\omega}_x = 0)$$

$$I_{yy}\dot{\omega}_y = \sum \tau_y = \mathbf{M}_{L,G} + \mathbf{M}_{L,B} = 0, (\because \dot{\omega}_x = 0)$$

In this case, the submerged volume and immerged volume are same at the heeling angle "Φ" and trim angle "θ" *where* the main deck is assumed not to be submerged. It means that the following equations should be satisfied:

$$\mathbf{F}_G + \mathbf{F}_B = 0$$

$$\mathbf{M}_{T,G}(\phi,\theta) + \mathbf{M}_{T,B}(\phi,\theta) = 0$$

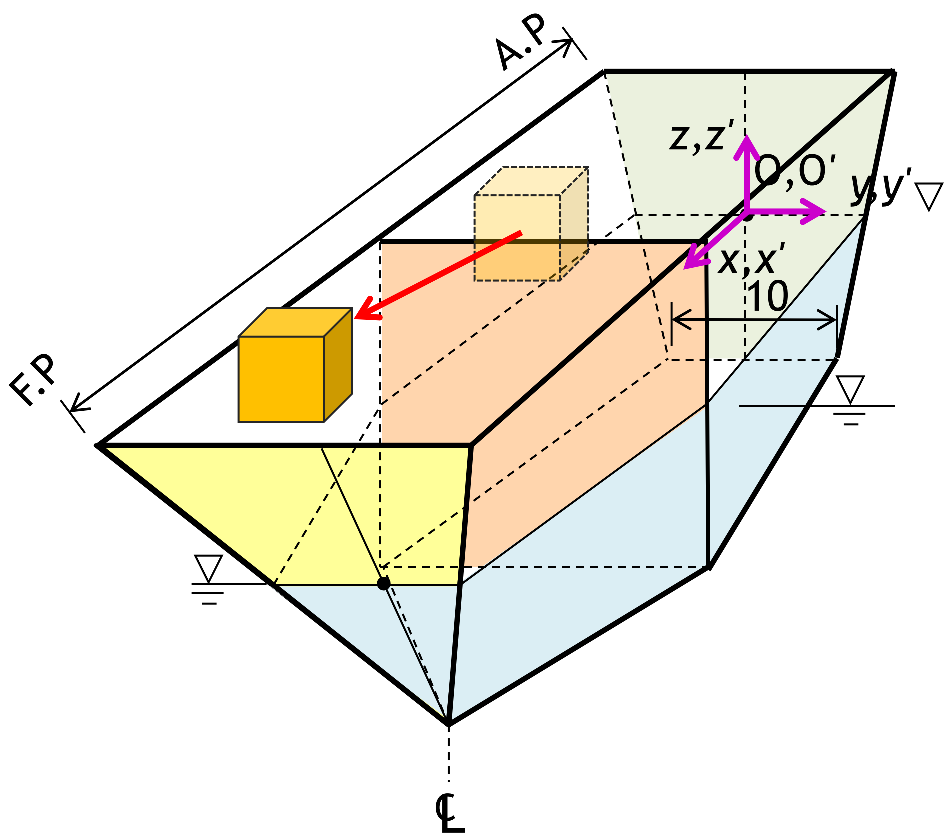
$$\mathbf{M}_{L,G}(\phi,\theta) + \mathbf{M}_{L,B}(\phi,\theta) = 0$$

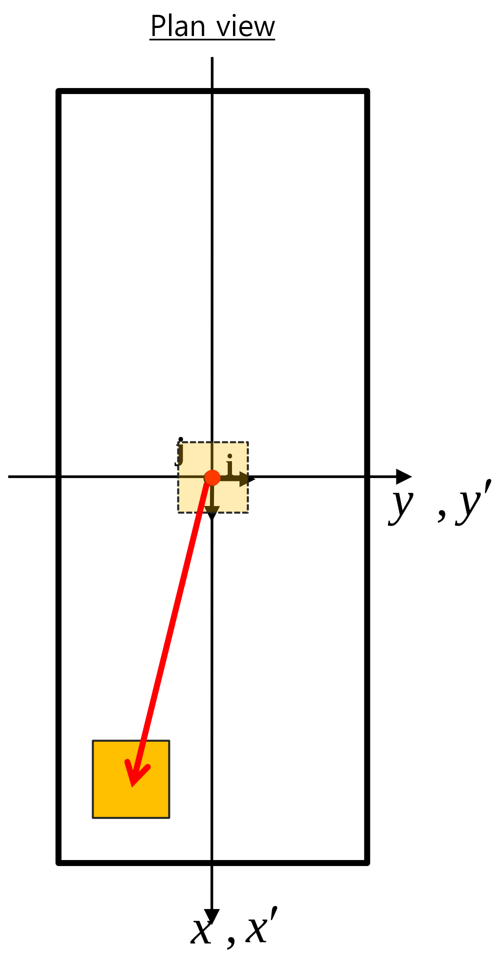
$$\Rightarrow \boxed{\begin{array}{l}\mathbf{M}_T(\phi,\theta) = 0 \\ \mathbf{M}_L(\phi,\theta) = 0\end{array}}$$

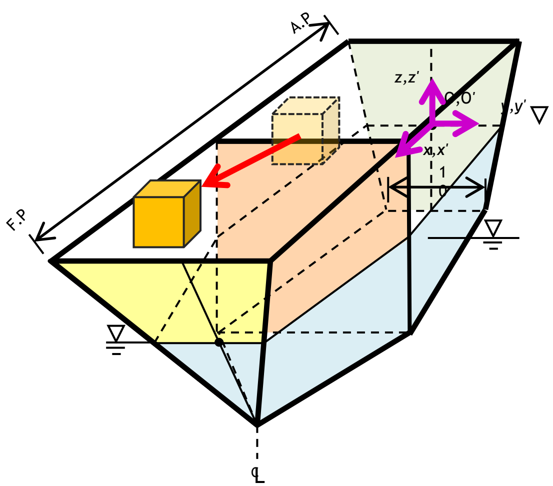
10-5 Determination of the heel angle, trim angle, and draft of a ship when a cargo is loaded and then moved in transverse and longitudinal directions

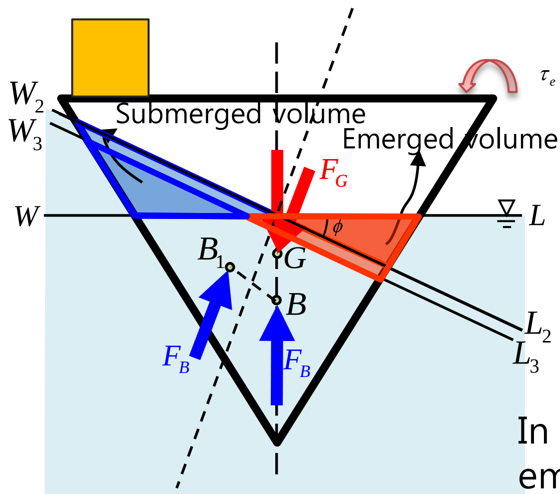
Determination of position and orientation of a ship when a cargo is loaded and then moved

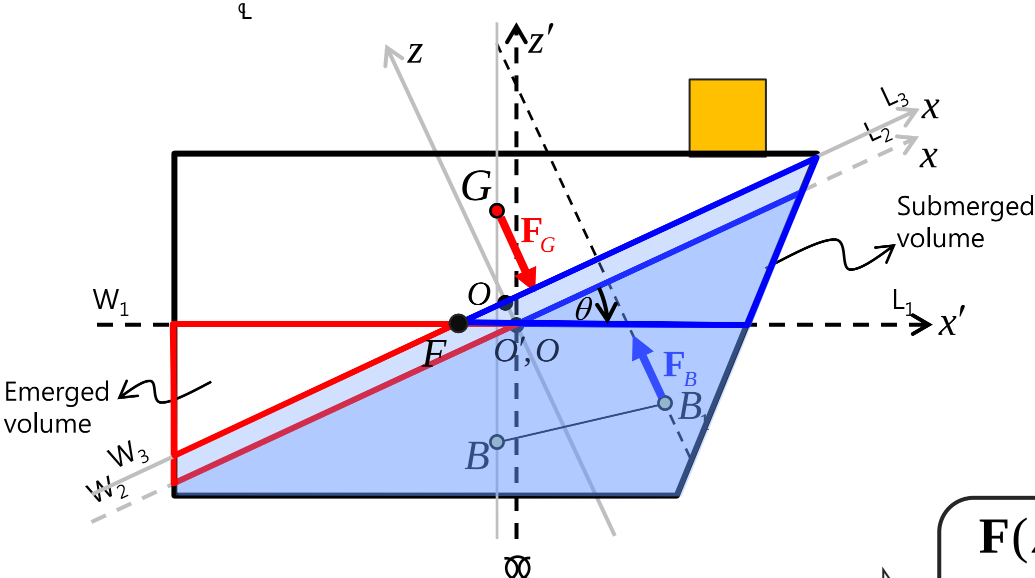
A cargo, which is loaded on a ship, is moved in -y direction and +x direction as in the figure. Determine the change of position and orientation (Trim and Heel) of the ship.











Static Equilibrium

① **Newton's 2nd law**

$$ma = \sum F = \mathbf{F}_G + \mathbf{F}_B = 0 \quad ,(\because a = 0)$$

② **Euler equation**

$$I_{xx}\dot{\omega}_x = \sum \tau_x = \mathbf{M}_{T,G} + \mathbf{M}_{T,B} = 0, (\because \dot{\omega}_x = 0)$$

$$I_{yy}\dot{\omega}_y = \sum \tau_y = \mathbf{M}_{L,G} + \mathbf{M}_{L,B} = 0, (\because \dot{\omega}_x = 0)$$

In this case, the submerged volume and emerged volume are **not same** at the heeling angle "Φ" and trim angle "θ". Thus draft, heeling angle, and trim angle should be **adjusted simultaneously** to maintain the same displacement. It means that the following equations should be satisfied:

$$\mathbf{F}_G + \mathbf{F}_B(z,\phi,\theta) = 0$$

$$\mathbf{M}_{T,G}(z,\phi,\theta) + \mathbf{M}_{T,B}(z,\phi,\theta) = 0$$

$$\mathbf{M}_{L,G}(z,\phi,\theta) + \mathbf{M}_{L,B}(z,\phi,\theta) = 0$$

$$\boxed{\begin{aligned}\mathbf{F}(z,\phi,\theta) &= 0\\ \mathbf{M}_T(z,\phi,\theta) &= 0\\ \mathbf{M}_L(z,\phi,\theta) &= 0\end{aligned}}$$

Nonlinear equations of three variables

Numerical method for solving nonlinear equations

$$\begin{aligned}\mathbf{F}(z,\phi,\theta)&=0\\ \mathbf{M}_T(z,\phi,\theta)&=0\\ \mathbf{M}_L(z,\phi,\theta)&=0\end{aligned}$$

$\Rightarrow$

$$\begin{bmatrix} F(\mathbf{q}_{HS}) \\ M_T(\mathbf{q}_{HS}) \\ M_L(\mathbf{q}_{HS}) \end{bmatrix} = \mathbf{0} \quad \textit{where } \mathbf{q}_{HS} = [z \quad \phi \quad \theta]^T$$

Given: $F\left(\mathbf{q}_{HS}{}^{(0)}\right), M_T\left(\mathbf{q}_{HS}{}^{(0)}\right), M_L\left(\mathbf{q}_{HS}{}^{(0)}\right)$, where $\mathbf{q}_{HS}^{(0)} = \left[z^{(0)} \quad \phi^{(0)} \quad \theta^{(0)}\right]^T$

Find: $\mathbf{q}_{HS}^* = \left[z^* \quad \phi^* \quad \theta^*\right]^T$, **subject to:** $F\left(\mathbf{q}_{HS}^*\right)=0,\ M_T\left(\mathbf{q}_{HS}^*\right)=0,\ M_L\left(\mathbf{q}_{HS}^*\right)=0$

Taylor series expansion of $F,\ M_T,\ M_L$ at $z^{(0)},\ \phi^{(0)},\ \theta^{(0)}$

And assume : $F\left(\mathbf{q}_{HS}^*\right)=0,\ M_T\left(\mathbf{q}_{HS}^*\right)=0,\ M_L\left(\mathbf{q}_{HS}^*\right)=0$ at $\mathbf{q}_{HS}^* = \left[z^* \quad \phi^* \quad \theta^*\right]^T$

$$\begin{bmatrix} -F\left(z^{(0)},\phi^{(0)},\theta^{(0)}\right) \\ -M_T\left(z^{(0)},\phi^{(0)},\theta^{(0)}\right) \\ -M_L\left(z^{(0)},\phi^{(0)},\theta^{(0)}\right) \end{bmatrix} = \begin{bmatrix} \frac{\partial F}{\partial z} & \frac{\partial F}{\partial \phi} & \frac{\partial F}{\partial \theta} \\ \frac{\partial M_T}{\partial z} & \frac{\partial M_T}{\partial \phi} & \frac{\partial M_T}{\partial \theta} \\ \frac{\partial M_L}{\partial z} & \frac{\partial M_L}{\partial \phi} & \frac{\partial M_L}{\partial \theta} \end{bmatrix}_{z^{(0)},\phi^{(0)},\theta^{(0)}} \begin{bmatrix} \delta z^{(0)} \\ \delta \phi^{(0)} \\ \delta \theta^{(0)} \end{bmatrix}$$

✓ Governing equation of a ship in hydrostatic equilibrium state

$$\begin{bmatrix} F(\mathbf{q}_{HS}) \\ M_T(\mathbf{q}_{HS}) \\ M_L(\mathbf{q}_{HS}) \end{bmatrix} = \mathbf{0} \quad \text{where} \begin{bmatrix} F(\mathbf{q}_{HS}) \\ M_T(\mathbf{q}_{HS}) \\ M_L(\mathbf{q}_{HS}) \end{bmatrix} = \begin{bmatrix} F_B(\mathbf{q}_{HS}) + F_G(\mathbf{q}_{HS}) + F_{ext}(\mathbf{q}_{HS}) \\ M_{BT}(\mathbf{q}_{HS}) + M_{GT}(\mathbf{q}_{HS}) + M_{extT}(\mathbf{q}_{HS}) \\ M_{BL}(\mathbf{q}_{HS}) + M_{GL}(\mathbf{q}_{HS}) + M_{extL}(\mathbf{q}_{HS}) \end{bmatrix},$$

$$\mathbf{q}_{HS} = [z \quad \phi \quad \theta]^T$$

Given: $\mathbf{q}_{HS}^{(0)} = \left[z^{(0)} \quad \phi^{(0)} \quad \theta^{(0)}\right]^T$,

$F\left(\mathbf{q}_{HS}^{(0)}\right), M_T\left(\mathbf{q}_{HS}^{(0)}\right), M_L\left(\mathbf{q}_{HS}^{(0)}\right)$

Find: $\mathbf{q}_{HS}^{*} = \left[z^{*} \quad \phi^{*} \quad \theta^{*}\right]^T$

, where $F\left(\mathbf{q}_{HS}^{*}\right) = 0, M_T\left(\mathbf{q}_{HS}^{*}\right) = 0, M_L\left(\mathbf{q}_{HS}^{*}\right) = 0$

Assumption

$z^{*} = z^{(0)} + \delta z^{(0)}, \ \phi^{*} = \phi^{(0)} + \delta\phi^{(0)},$

$\theta^{*} = \theta^{(0)} + \delta\theta^{(0)}$

$$\underbrace{\begin{bmatrix} -F\left(z^{(i)}, \phi^{(i)}, \theta^{(i)}\right) \\ -M_T\left(z^{(i)}, \phi^{(i)}, \theta^{(i)}\right) \\ -M_L\left(z^{(i)}, \phi^{(i)}, \theta^{(i)}\right) \end{bmatrix}}_{\text{Known}} = \underbrace{\begin{bmatrix} \frac{\partial F}{\partial z} & \frac{\partial F}{\partial \phi} & \frac{\partial F}{\partial \theta} \\ \frac{\partial M_T}{\partial z} & \frac{\partial M_T}{\partial \phi} & \frac{\partial M_T}{\partial \theta} \\ \frac{\partial M_L}{\partial z} & \frac{\partial M_L}{\partial \phi} & \frac{\partial M_L}{\partial \theta} \end{bmatrix}_{z^{(i)}, \phi^{(i)}, \theta^{(i)}}}_{\text{To be known}} \underbrace{\begin{bmatrix} \delta z^{(i)} \\ \delta\phi^{(i)} \\ \delta\theta^{(i)} \end{bmatrix}}_{\text{Find}}$$

$i = 1$

Iteration

$z^{(i+1)} = z^{(i)} + \delta z^{(i)}, \ \phi^{(i+1)} = \phi^{(i)} + \delta\phi^{(i)}, \ \theta^{(i+1)} = \theta^{(i)} + \delta\theta^{(i)}$

$F\left(\mathbf{q}_{HS}^{(i+1)}\right) \overset{!}{=} 0, M_T\left(\mathbf{q}_{HS}^{(i+1)}\right) \overset{!}{=} 0, M_L\left(\mathbf{q}_{HS}^{(i+1)}\right) \overset{!}{=} 0$ ➔ **Check**

No

$i \leftarrow i+1$

$z^{*} = z^{(i)} + \delta z^{(i)}$

$\phi^{*} = \phi^{(i)} + \delta\phi^{(i)}$

$\theta^{*} = \theta^{(i)} + \delta\theta^{(i)}$

Yes

$z^{(i+1)}, \phi^{(i+1)}, \theta^{(i+1)}$ **are the solution,** $z^{*}, \phi^{*}, \theta^{*}$.

$$\underbrace{\begin{bmatrix} -F\left(z^{(0)},\phi^{(0)},\theta^{(0)}\right) \\ -M_T\left(z^{(0)},\phi^{(0)},\theta^{(0)}\right) \\ -M_L\left(z^{(0)},\phi^{(0)},\theta^{(0)}\right) \end{bmatrix}}_{\text{Known}} = \underbrace{\begin{bmatrix} \frac{\partial F}{\partial z} & \frac{\partial F}{\partial \phi} & \frac{\partial F}{\partial \theta} \\ \frac{\partial M_T}{\partial z} & \frac{\partial M_T}{\partial \phi} & \frac{\partial M_T}{\partial \theta} \\ \frac{\partial M_L}{\partial z} & \frac{\partial M_L}{\partial \phi} & \frac{\partial M_L}{\partial \theta} \end{bmatrix}_{z^{(0)},\phi^{(0)},\theta^{(0)}}}_{\text{To be known}} \underbrace{\begin{bmatrix} \delta z^{(0)} \\ \delta\phi^{(0)} \\ \delta\theta^{(0)} \end{bmatrix}}_{\text{Find}}$$

$$\boxed{\begin{bmatrix} \frac{\partial F_B}{\partial z}+\frac{\partial F_G}{\partial z}+\frac{\partial F_{ext}}{\partial z} & \frac{\partial F_B}{\partial \phi}+\frac{\partial F_G}{\partial \phi}+\frac{\partial F_{ext}}{\partial \phi} & \frac{\partial F_B}{\partial \theta}+\frac{\partial F_G}{\partial \theta}+\frac{\partial F_{ext}}{\partial \theta} \\ \frac{\partial M_{BT}}{\partial z}+\frac{\partial M_{GT}}{\partial z}+\frac{\partial M_{extT}}{\partial z} & \frac{\partial M_{BT}}{\partial \phi}+\frac{\partial M_{GT}}{\partial \phi}+\frac{\partial M_{extT}}{\partial \phi} & \frac{\partial M_{BT}}{\partial \theta}+\frac{\partial M_{GT}}{\partial \theta}+\frac{\partial M_{extT}}{\partial \theta} \\ \frac{\partial M_{BL}}{\partial z}+\frac{\partial M_{GL}}{\partial z}+\frac{\partial M_{extL}}{\partial z} & \frac{\partial M_{BL}}{\partial \phi}+\frac{\partial M_{GL}}{\partial \phi}+\frac{\partial M_{extL}}{\partial \phi} & \frac{\partial M_{BL}}{\partial \theta}+\frac{\partial M_{GL}}{\partial \theta}+\frac{\partial M_{extL}}{\partial \theta} \end{bmatrix}}$$

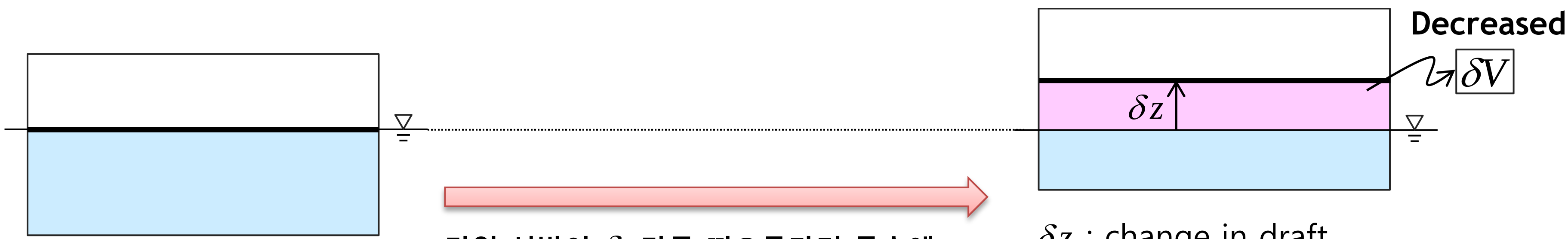
We have to know the partial derivatives associated with hydrostatic equilibrium.

These values are related to the position and orientation of the ship.

➔ Nonlinear

10-6 Sign Convention of the Changed Position and Orientation of a Ship in Static Equilibrium

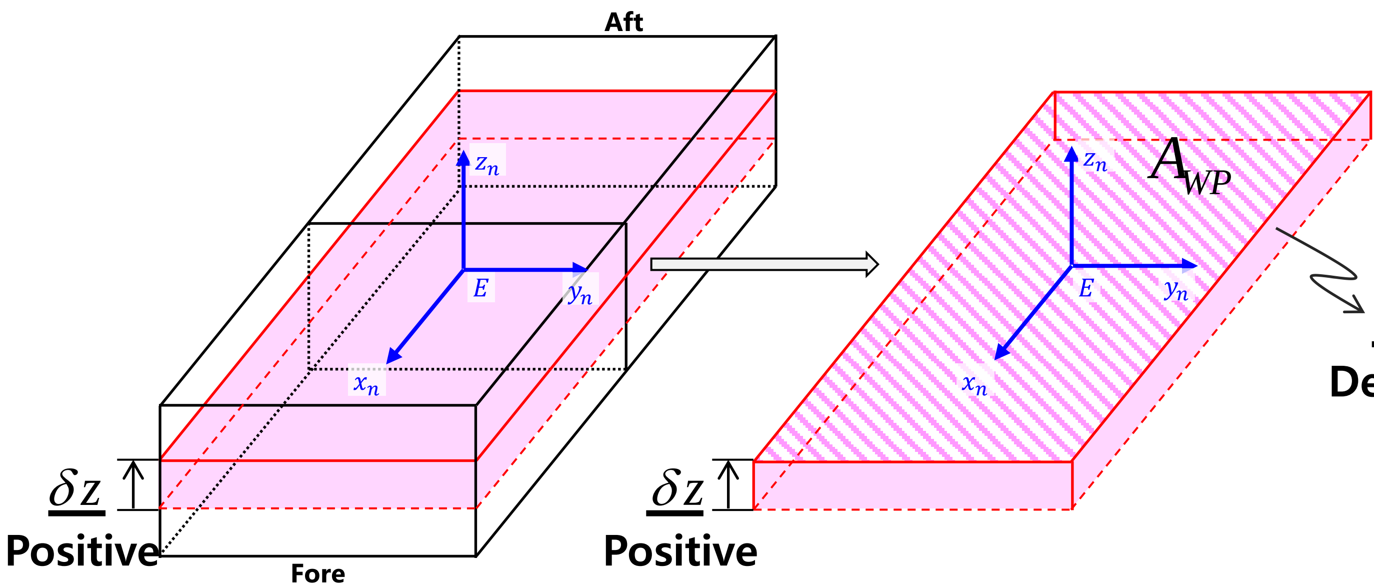
Change in displaced volume with respect to emersion



만일 선박이 δz 만큼 떠오른다면 물속에 잠긴 부피는 어떻게 변화될까?

δz : change in draft

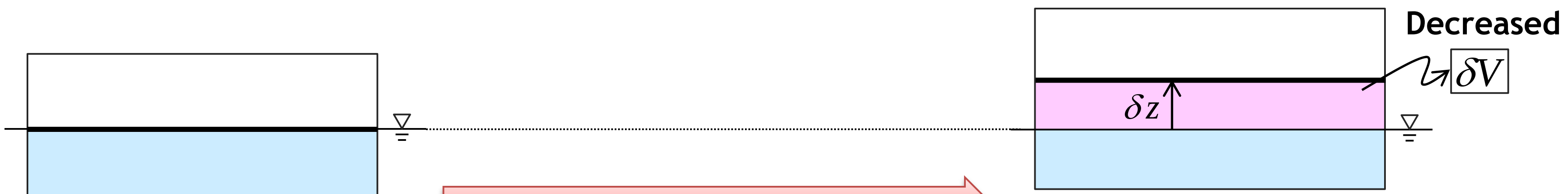
δV : change in submerged volume



$$\underline{\delta V} = -A_{WP}\,\delta z$$

Decreased $(+)(+)$

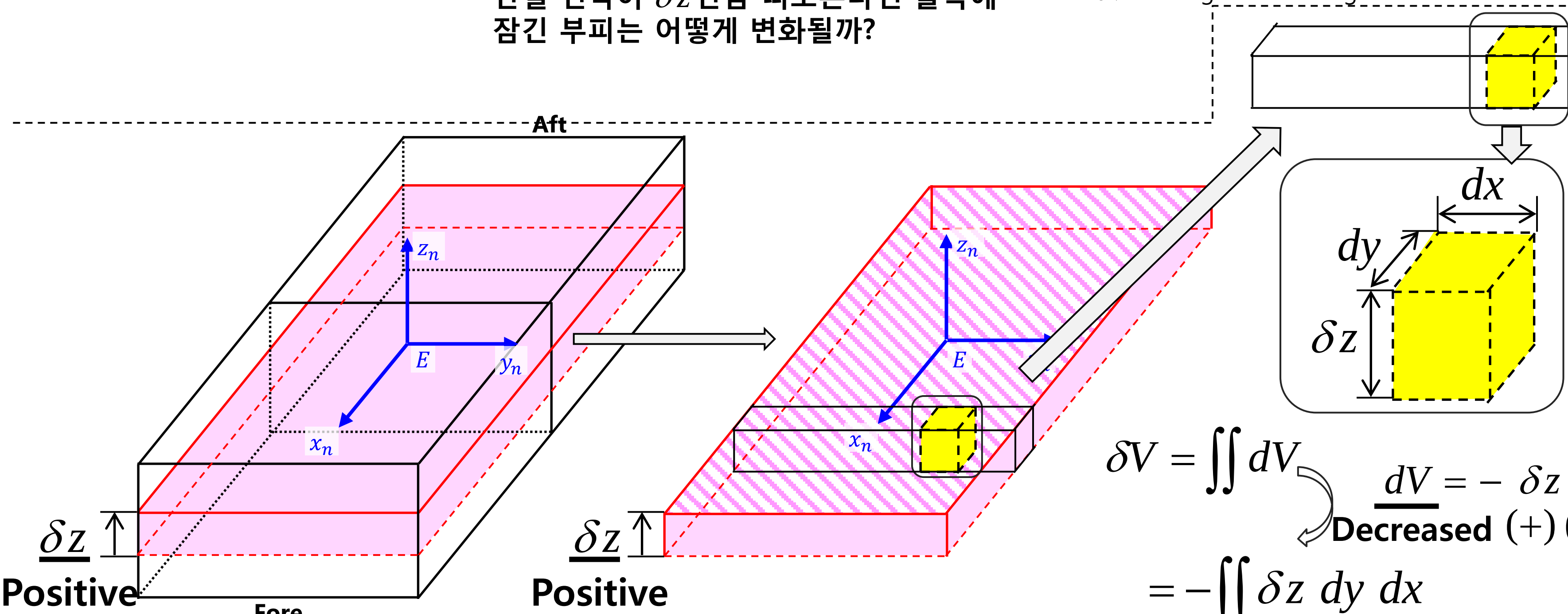
1. 좌변의 부피변화 δV는 음수이다.
2. 우변의 A_{WP} , δz가 모두 양수이다.
3. 따라서 우변에 "-"부호를 붙여야 한다.



δz : change in draft

δV : change in submerged volume

만일 선박이 δz만큼 떠오른다면 물속에 잠긴 부피는 어떻게 변화될까?



$$dV = -\delta z\ dy\ dx$$

$$\delta V = \iint dV$$

$$\underline{dV} = -\ \delta z\ dy\ dx$$

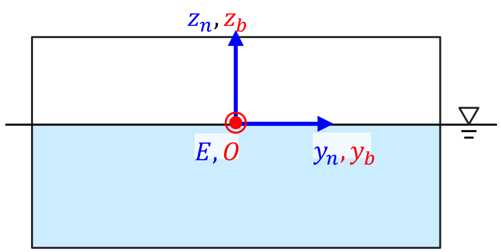
Decreased $(+)\,(+)(+)$

$$= -\iint \delta z\ dy\ dx$$

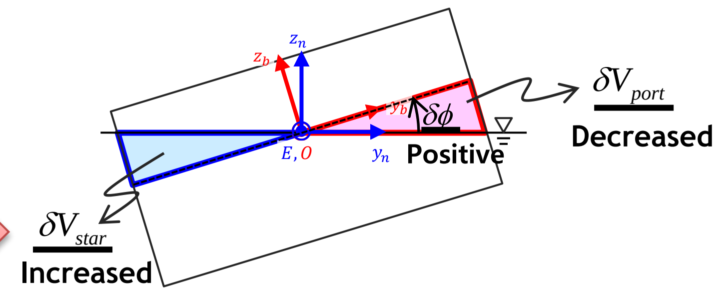
$$= -\int_{x_{aft}}^{x_{fore}} \int_{y_{star}}^{y_{port}} \delta z\ dy\ dx$$

1. 좌변의 부피변화 δV는 음수이다.
2. 우변의 $\delta z, dy, dx$가 모두 양수이다.
3. 따라서 우변에 "-"부호를 붙여야 한다.
4. dy, dx는 적분 방향에 따라 부호가 달라지는데, 여기서는 적분 구간이 음에서 양이므로 부호가 양수가 된다.
5. δz는 선박이 z_n방향으로 운동한 변위이다.

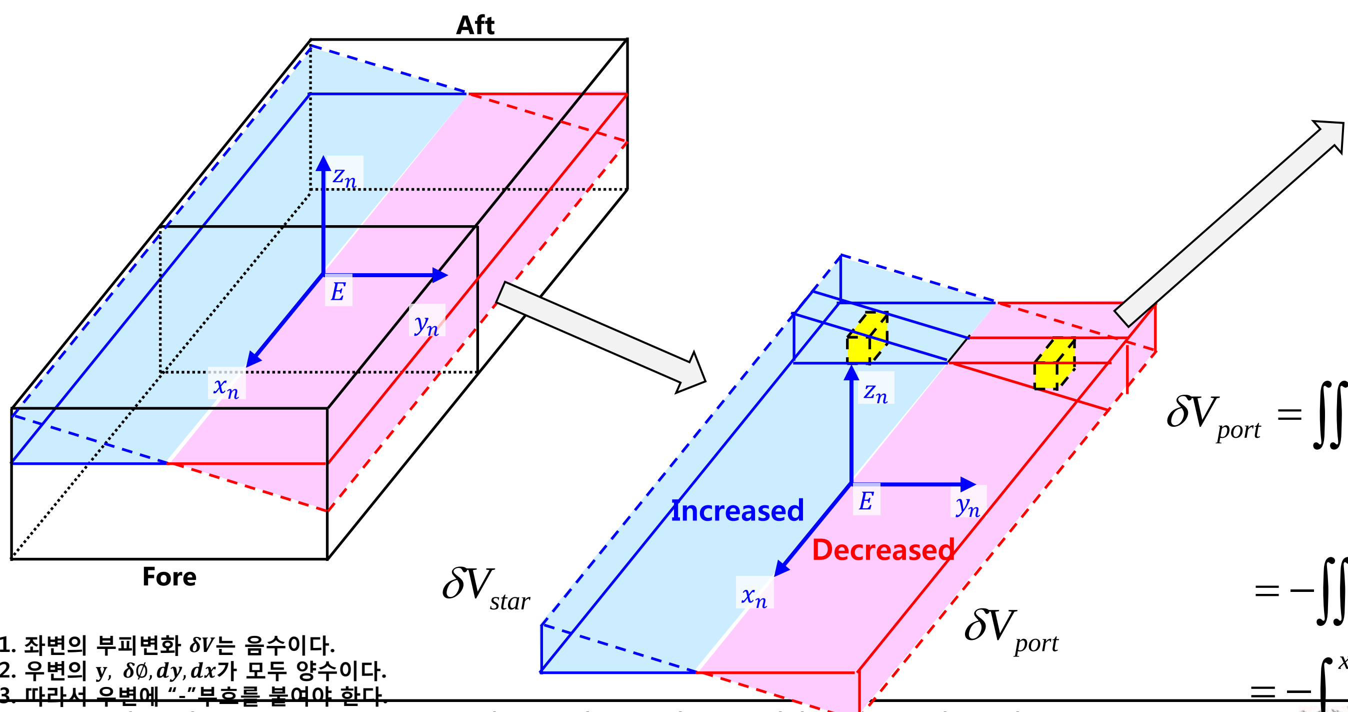
Change in displaced volume with respect to **heel**

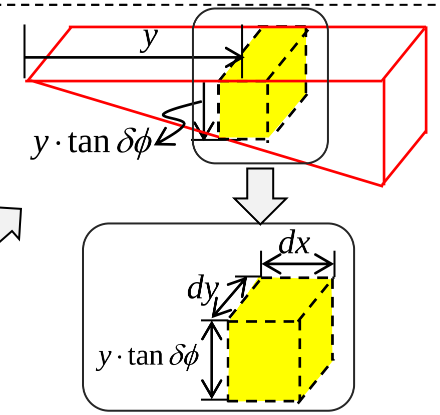


만일 선박이 x_n축을 중심으로 $\delta\phi$ 만큼 회전 한다면 물속에 잠긴 부피는 어떻게 변화될까?



δV : change in submerged volume





$$\delta V_{port} = \iint dV_{port}$$

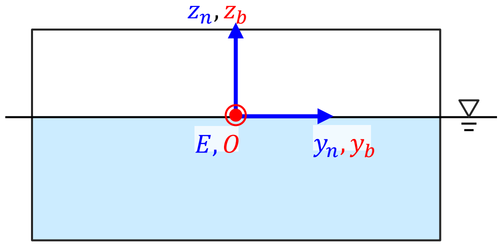
$$\underline{dV_{port}} = - \; y \cdot \tan\delta\phi \; dy \; dx$$

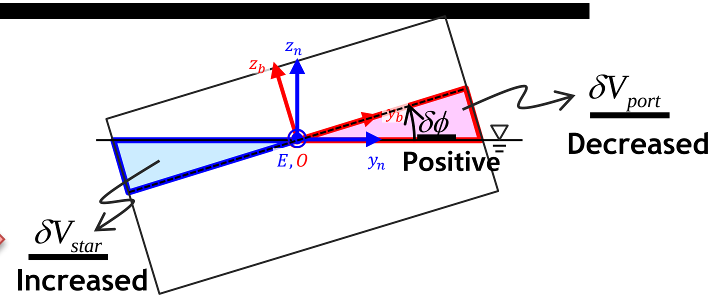
Decreased $(+) \quad (+)(+)(+)$

$$= -\iint y \cdot \tan\delta\phi \; dy \; dx$$

$$= -\int_{x_{aft}}^{x_{fore}} \int_{0}^{y_{port}} y \cdot \tan\delta\phi \; dy \; dx$$

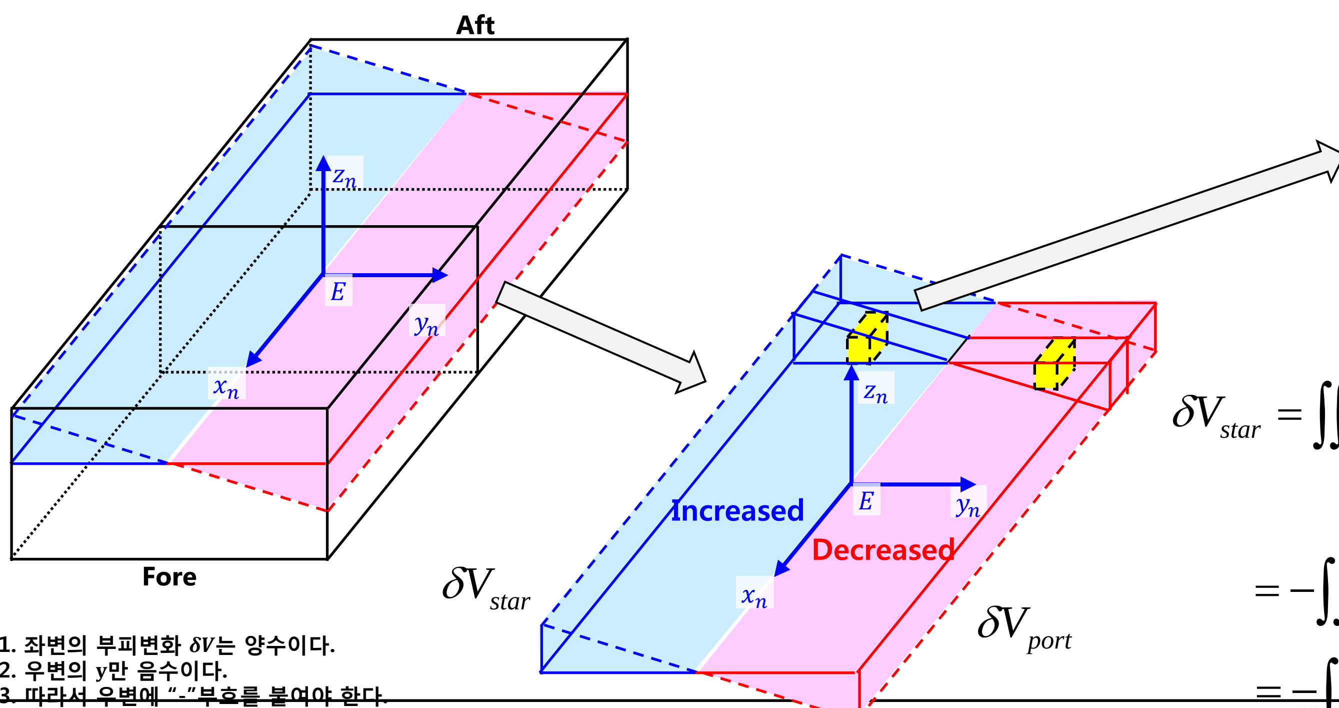
1. 좌변의 부피변화 δV는 음수이다.
2. 우변의 y, $\delta\emptyset, dy, dx$가 모두 양수이다.
3. 따라서 우변에 "-"부호를 붙여야 한다.
4. dy, dx는 적분 방향에 따라 부호가 달라지는데, 여기서는 적분 구간이 음에서 양이므로 부호가 양수가 된다.
5. $\delta\emptyset$는 선박이 x_n을 중심으로 회전한 각도이다.

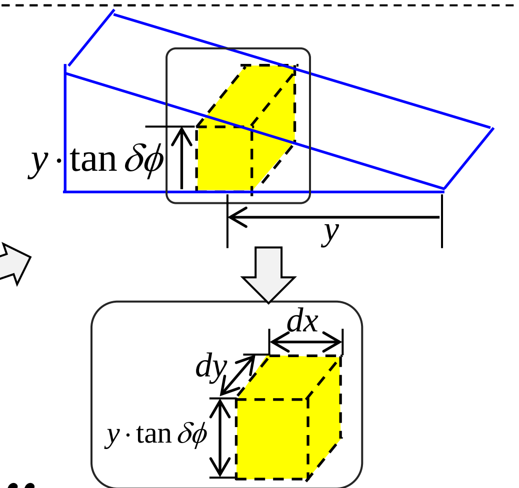




만일 선박이 x축을 중심으로 $\delta\phi$만큼 회전 한다면 물속에 잠긴 부피는 어떻게 변화될까?

δV : change in submerged volume





$$\delta V_{star} = \iint dV_{star}$$

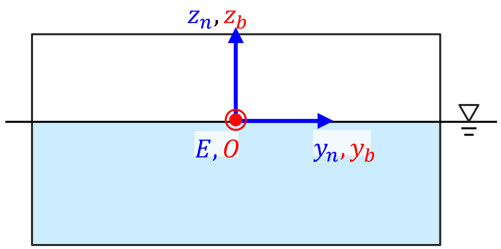
$$\underline{dV_{star}} = -\ y \cdot \tan\delta\phi\ dy\ dx$$

Increased $(-)\quad(+)(+)(+)$

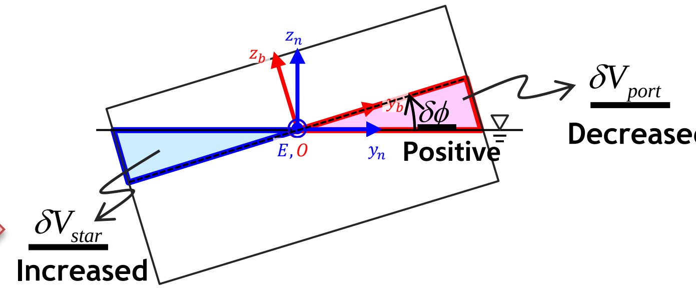
$$= -\iint y \cdot \tan\delta\phi\ dy\ dx$$

$$= -\int_{x_{aft}}^{x_{fore}} \int_{y_{star}}^{0} y \cdot \tan\delta\phi\ dy\ dx$$

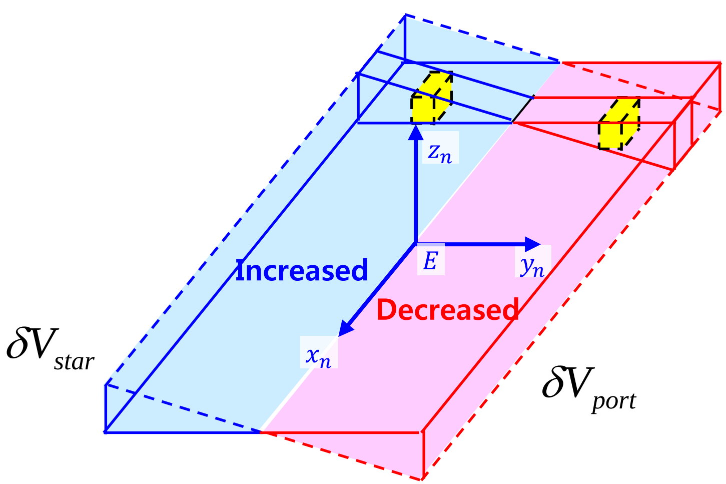
1. 좌변의 부피변화 δV는 양수이다.
2. 우변의 y만 음수이다.
3. 따라서 우변에 "-"부호를 붙여야 한다.
4. dy, dx는 적분 방향에 따라 부호가 달라지는데, 여기서는 적분 구간이 음에서 양이므로 부호가 양수가 된다.
5. $\delta\phi$는 선박이 x_n을 중심으로 회전한 각도이다.



만일 선박이 x축을 중심으로 $\delta\phi$만큼 회전 한다면 물속에 잠긴 부피는 어떻게 변화될까?



δV : change in submerged volume



$$dV = -y \cdot \tan\delta\phi \; dx \; dy$$

$$\delta V_{port} = -\int_{x_{aft}}^{x_{fore}} \int_{0}^{y_{port}} y \cdot \tan\delta\phi \; dy \; dx$$

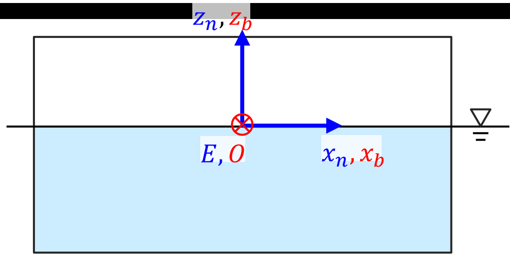
$$\delta V_{star} = -\int_{x_{aft}}^{x_{fore}} \int_{y_{star}}^{0} y \cdot \tan\delta\phi \; dy \; dx$$

$$\Rightarrow \delta V = \delta V_{star} + \delta V_{port}$$

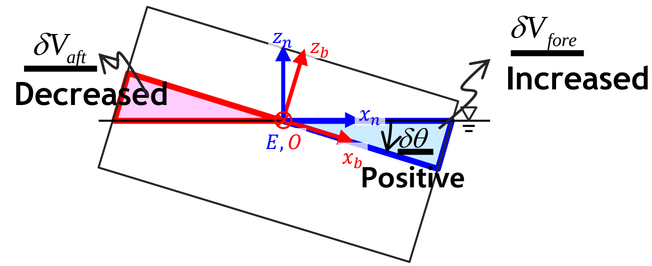
$$= -\int_{x_{aft}}^{x_{fore}} \int_{y_{star}}^{0} y \cdot \tan\delta\phi \; dy \; dx - \int_{x_{aft}}^{x_{fore}} \int_{0}^{y_{port}} y \cdot \tan\delta\phi \; dy \; dx$$

$$= -\int_{x_{aft}}^{x_{fore}} \int_{y_{star}}^{y_{port}} y \cdot \tan\delta\phi \; dy \; dx$$

Change in displaced volume with respect to trim

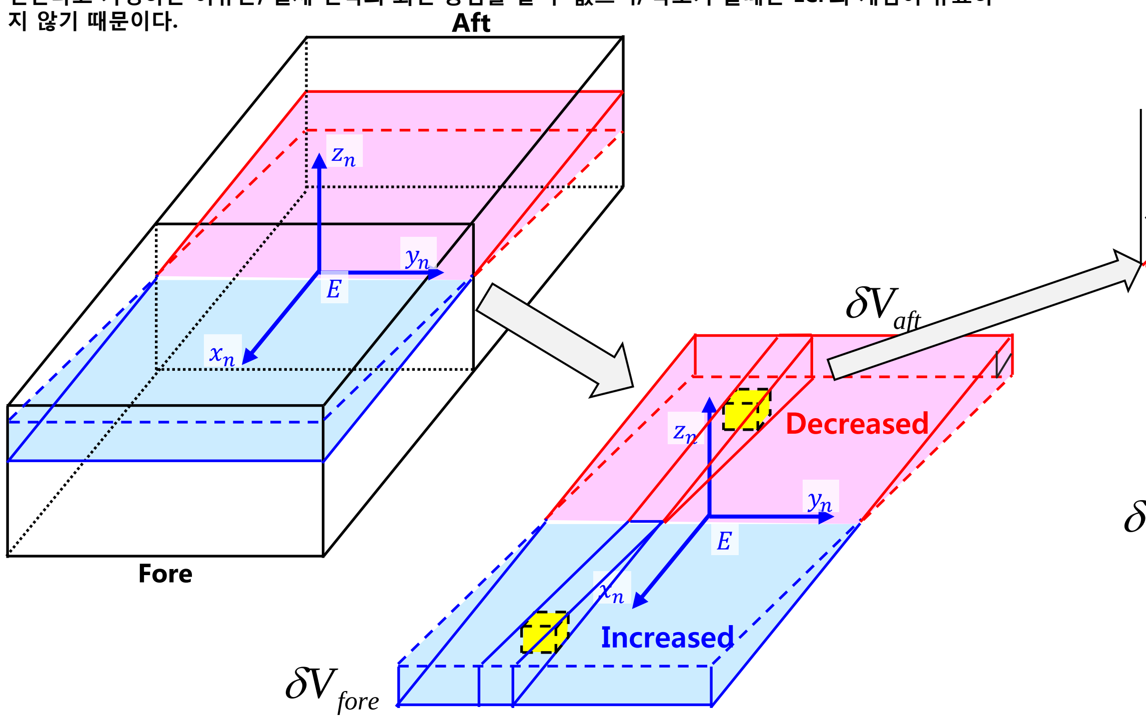


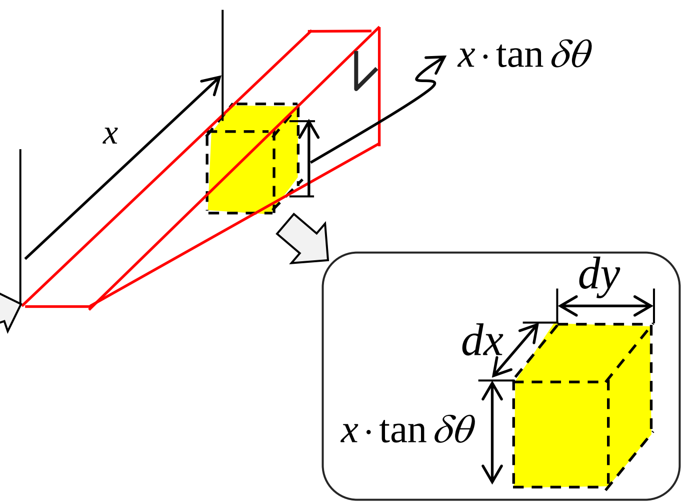
만일 선박이 y축을 중심으로 $\delta\theta$ 만큼 회전 한다면 물속에 잠긴 부피는 어떻게 변화될까?



δV : change in submerged volume

Classical Hydrostatic에서는 LCF를 기준으로 회전한다고 가정하였으나, 여기에서는 y_n축을 기준으로 회전한다고 가정하는 이유는, 실제 선박의 회전 중심을 알 수 없으며, 각도가 클때는 LCF의 개념이 유효하지 않기 때문이다.





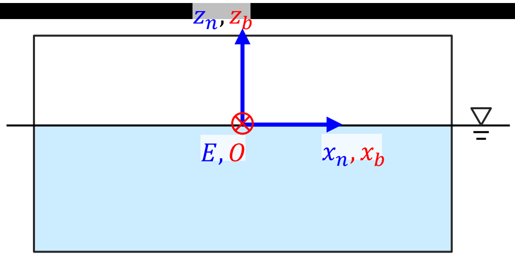
$$\delta V_{aft} = \iint dV_{aft}$$

$$\underset{\text{Decreased}}{dV_{aft}} = \underset{(-)}{x} \cdot \underset{(+)}{\tan\delta\theta}\ \underset{(+)}{dx}\ \underset{(+)}{dy}$$

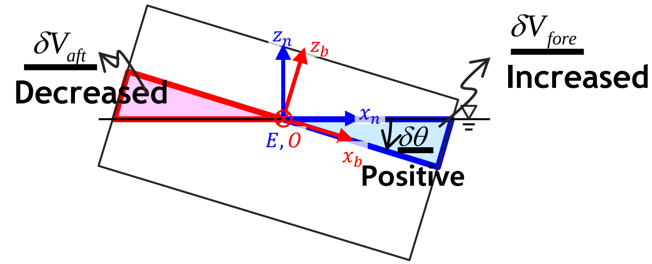
$$= \iint x \cdot \tan\delta\theta \, dx \, dy$$

$$= \int_{y_{star}}^{y_{port}} \int_{x_{aft}}^{0} x \cdot \tan\delta\theta \, dx \, dy$$

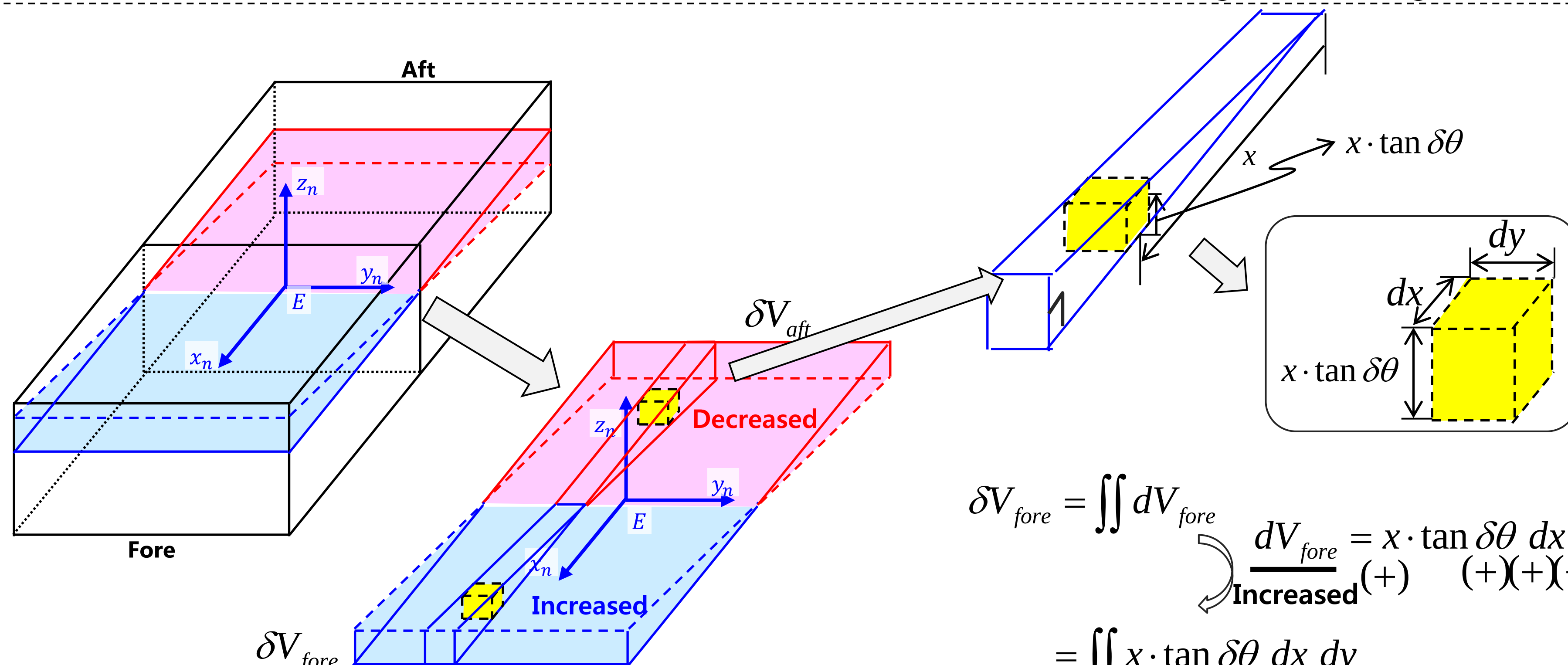
1. 좌변의 부피변화 δV는 음수이다.
2. 우변의 x만 음수이다.
3. 따라서 우변의 부호를 변경할 필요가 없다.
4. dy, dx는 적분 방향에 따라 부호가 달라지는데, 여기서는 적분 구간이 음에서 양이므로 부호가 양수가 된다.
5. $\delta\theta$는 선박이 y_n 을 중심으로 회전한 각도이다.



만일 선박이 y축을 중심으로 $\delta\theta$ 만큼 회전 한다면 물속에 잠긴 부피는 어떻게 변화될까?



δV : change in submerged volume



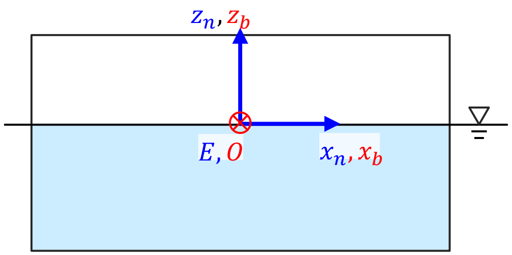
$$\delta V_{fore} = \iint dV_{fore}$$

$$\underset{\text{Increased}}{\underline{dV_{fore}}} = \underset{(+)}{x} \cdot \underset{(+)}{\tan\delta\theta}\ \underset{(+)}{dx}\ \underset{(+)}{dy}$$

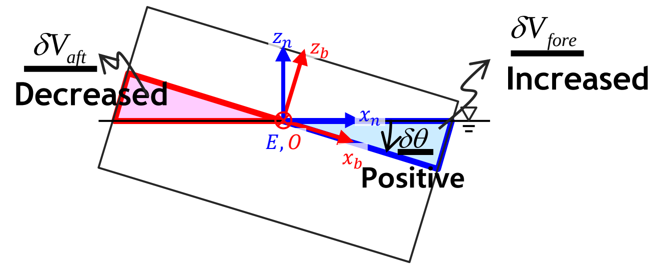
$$= \iint x \cdot \tan\delta\theta\ dx\ dy$$

$$= \int_{y_{star}}^{y_{port}} \int_{0}^{x_{fore}} x \cdot \tan\delta\theta\ dx\ dy$$

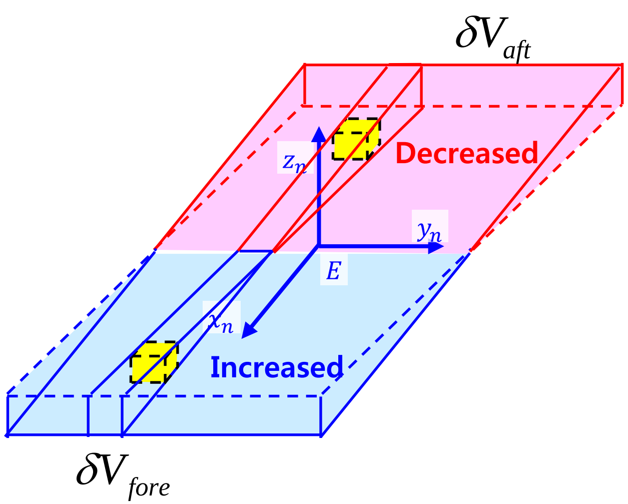
1. 좌변의 부피변화 δV는 음수이다.
2. 우변의 x만 음수이다.
3. 따라서 우변의 부호를 변경할 필요가 없다.
4. dy, dx는 적분 방향에 따라 부호가 달라지는데, 여기서는 적분 구간이 음에서 양이므로 부호가 양수가 된다.
5. $\delta\theta$는 선박이 y_n을 중심으로 회전한 각도이다.



만일 선박이 y축을 중심으로 $\delta\theta$ 만큼 회전 한다면 물속에 잠긴 부피는 어떻게 변화될까?



δV : change in submerged volume



$$dV = x \cdot \tan\delta\theta \; dx \; dy$$

$$\delta V_{aft} = \int_{y_{star}}^{y_{port}} \int_{x_{aft}}^{0} x \cdot \tan\delta\theta \; dx \; dy$$

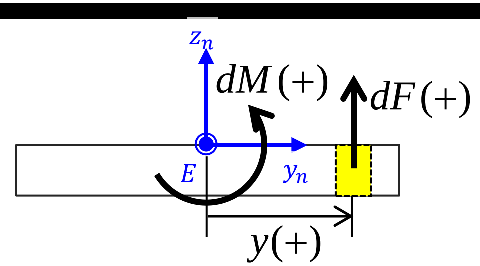
$$\delta V_{fore} = \int_{y_{star}}^{y_{port}} \int_{0}^{x_{fore}} x \cdot \tan\delta\theta \; dx \; dy$$

$$\Rightarrow \delta V = \delta V_{aft} + \delta V_{fore}$$

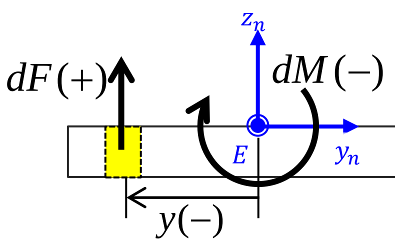
$$= \int_{y_{star}}^{y_{port}} \int_{x_{aft}}^{0} x \cdot \tan\delta\theta \; dx \; dy + \int_{y_{star}}^{y_{port}} \int_{0}^{x_{fore}} x \cdot \tan\delta\theta \; dx \; dy$$

$$= \int_{y_{star}}^{y_{port}} \int_{x_{aft}}^{x_{fore}} x \cdot \tan\delta\theta \; dx \; dy$$

Sign Convention for **Moment**



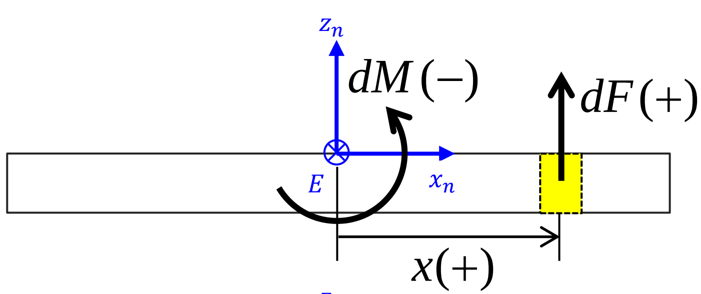
$$dM_{x_n} = y \cdot dF$$
$$(+) \qquad (+)(+)$$



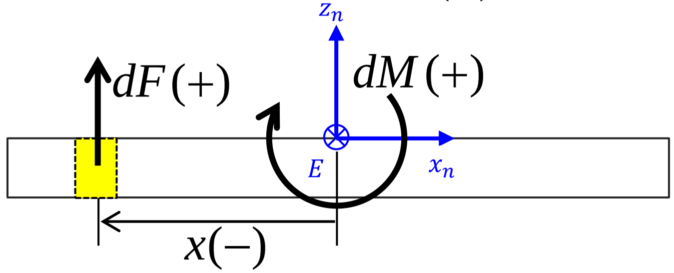
$$dM_{x_n} = y \cdot dF$$
$$(-) \qquad (-)(+)$$

$$dM_{x_n} = y \cdot dF$$

Moment about the x_n-axis



$$dM_{y_n} = -\,x \cdot dF$$
$$(-) \qquad (+)(+)$$



$$dM_{y_n} = -\,x \cdot dF$$
$$(+) \qquad (-)(+)$$

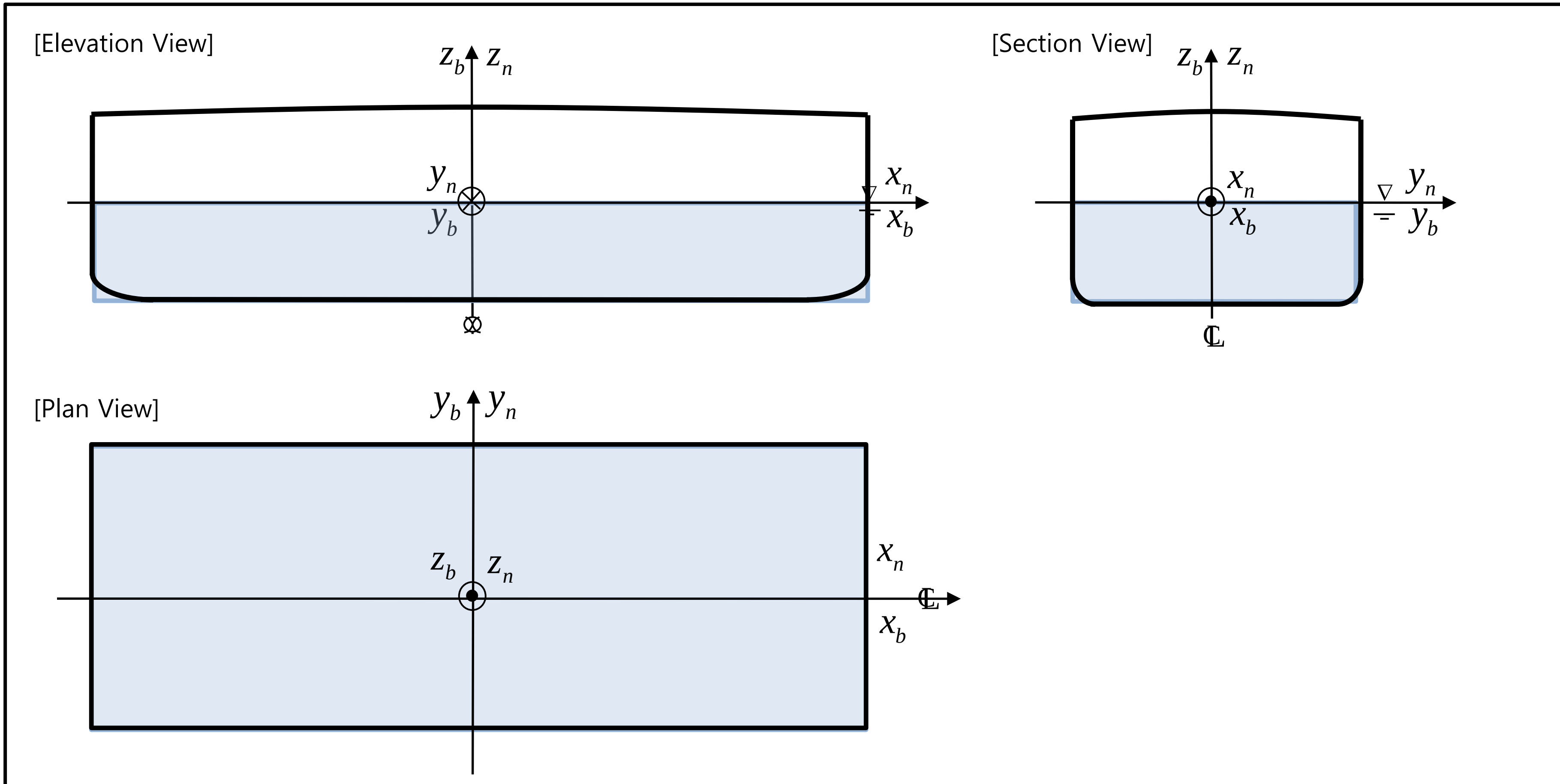
$$dM_{y_n} = -x \cdot dF$$

Moment about the y_n-axis

10-7 Governing Equation of Computational Ship Stability

- Derivation of Partial Derivatives Associated with Hydrostatic Equilibrium In the case that Initial Condition is Upright Position

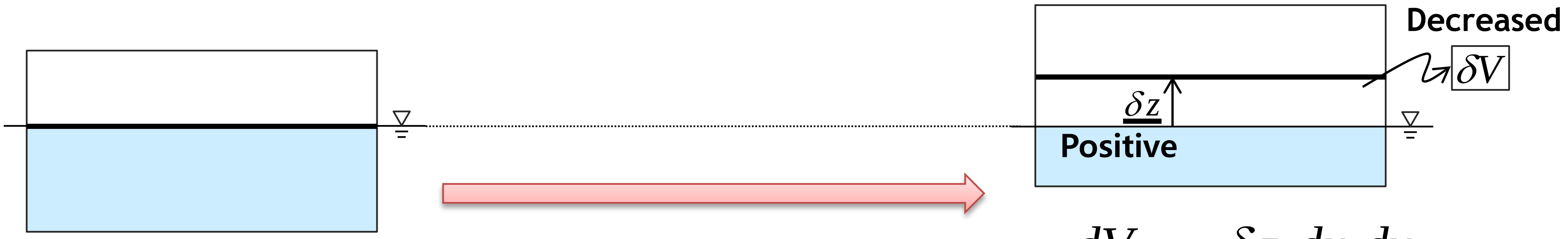
Shape of the Ship



Change in Buoyant Force with respect to Emersion (Translation along the positive z_n axis)

$$\begin{bmatrix} \boxed{\frac{\partial F_B}{\partial z}} & \frac{\partial F_B}{\partial \phi} & \frac{\partial F_B}{\partial \theta} \\ \frac{\partial M_{BT}}{\partial z} & \frac{\partial M_{BT}}{\partial \phi} & \frac{\partial M_{BT}}{\partial \theta} \\ \frac{\partial M_{BL}}{\partial z} & \frac{\partial M_{BL}}{\partial \phi} & \frac{\partial M_{BL}}{\partial \theta} \end{bmatrix}$$

[Section View] [Section View]



$$dV = -\delta z \; dx \; dy$$

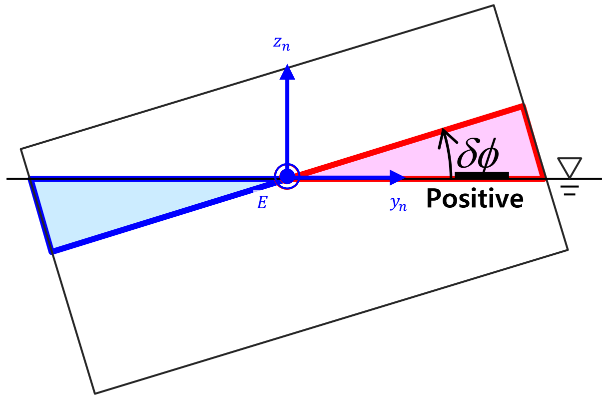
Change in buoyant force with respect to emersion

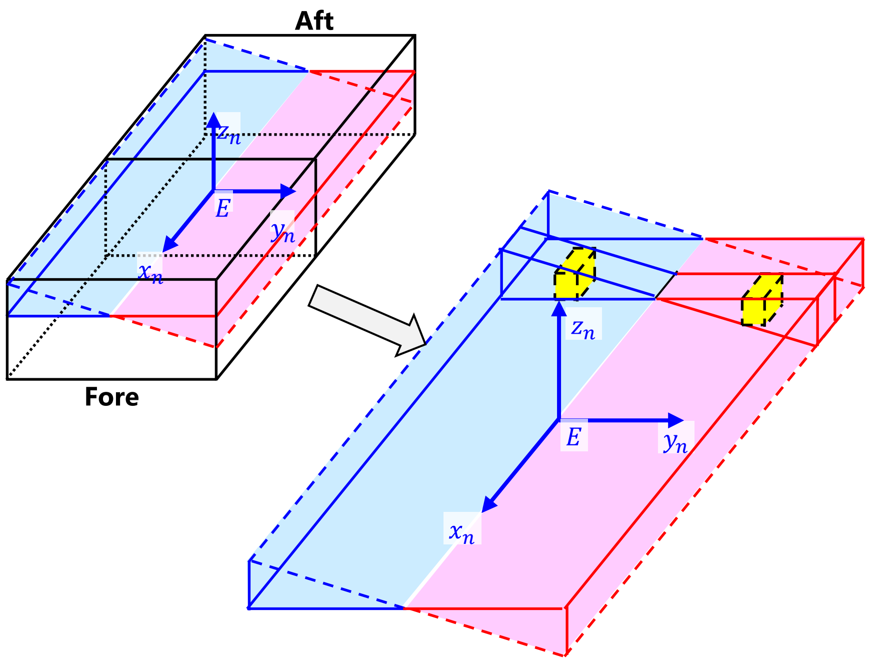
$$\begin{aligned} \delta F_B &= \iint dF_B \\ &= \iint \rho g dV \\ &= -\rho g \cdot \iint \delta z \; dx \; dy \qquad \delta z\text{는 xy의 함수가 아님} \\ &= -\rho g \delta z \cdot \iint dx \; dy \\ &= -\rho g \delta z \cdot A_{WP} \end{aligned}$$

$$\boxed{\therefore \frac{\partial F_B}{\partial z} = -\rho g \cdot A_{WP}}$$

Change in Buoyant Force with respect to Heel (Rotation about the x_n axis, $\delta\phi$)

$$\begin{bmatrix} \boxed{\frac{\partial F_B}{\partial z}} & \boxed{\frac{\partial F_B}{\partial \phi}} & \frac{\partial F_B}{\partial \theta} \\ \frac{\partial M_{BT}}{\partial z} & \frac{\partial M_{BT}}{\partial \phi} & \frac{\partial M_{BT}}{\partial \theta} \\ \frac{\partial M_{BL}}{\partial z} & \frac{\partial M_{BL}}{\partial \phi} & \frac{\partial M_{BL}}{\partial \theta} \end{bmatrix}$$





$$dV = -y \cdot \tan\delta\phi \; dx \; dy$$

Change in buoyant force with respect to heel

$$\delta F_B = \iint dF_B$$

$$= \iint \rho g dV$$

$$= -\rho g \iint y \cdot \tan\delta\phi \; dx \; dy$$

δϕ는 xy의 함수가 아님

$$= -\rho g \tan\delta\phi \iint y \cdot \; dx \; dy$$

$$= -\rho g \cdot \tan\delta\phi \cdot T_{x_n}$$

If $\delta\phi \ll 1$,

$$\delta F_B = -\rho g \cdot \delta\phi \cdot T_{x_n}$$

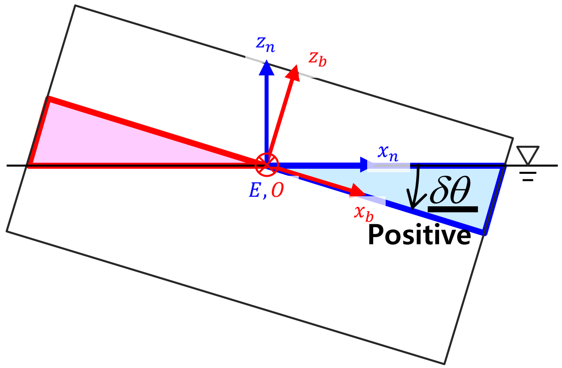
$$T_{x_n} \rightarrow T_{WP}$$

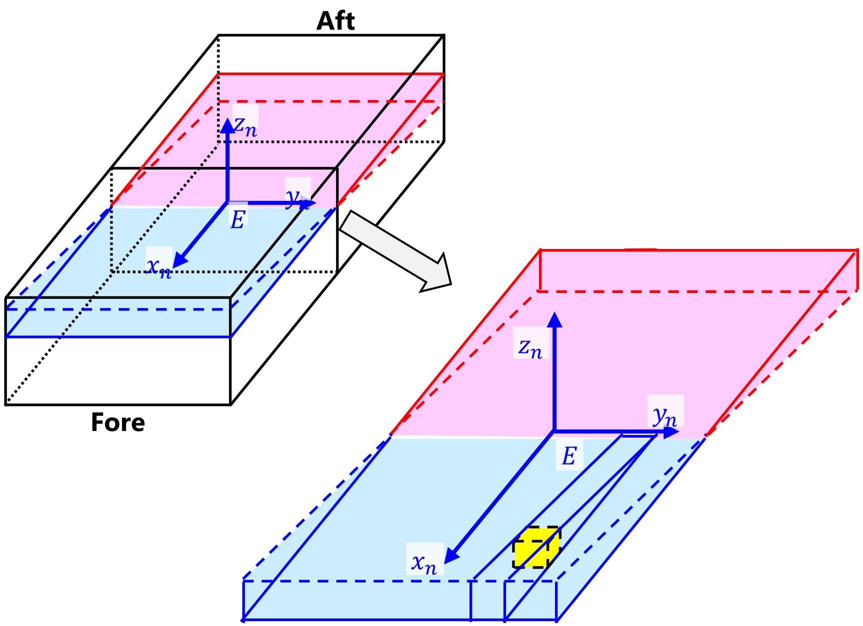
$$\boxed{\therefore \frac{\partial F_B}{\partial \phi} = -\rho g \cdot \underline{T_{WP}}}$$

: Transverse moment of waterplane area about x_n axis

Change in Buoyant Force with respect to Trim
(Rotation about the y_n axis, $\delta\theta$)

$$\begin{bmatrix} \boxed{\dfrac{\partial F_B}{\partial z}} & \boxed{\dfrac{\partial F_B}{\partial \phi}} & \boxed{\dfrac{\partial F_B}{\partial \theta}} \\ \dfrac{\partial M_{BT}}{\partial z} & \dfrac{\partial M_{BT}}{\partial \phi} & \dfrac{\partial M_{BT}}{\partial \theta} \\ \dfrac{\partial M_{BL}}{\partial z} & \dfrac{\partial M_{BL}}{\partial \phi} & \dfrac{\partial M_{BL}}{\partial \theta} \end{bmatrix}$$





$$dV = x \cdot \tan\delta\theta \; dx \, dy$$

Change in buoyant force with respect to trim

$$\delta F_B = \iint dF_B$$

$$= \iint \rho g dV$$

$$= \rho g \iint x \cdot \tan\delta\theta \; dx \, dy$$

$\delta\theta$는 xy의 함수가 아님

$$= \rho g \tan\delta\theta \iint x \cdot \; dx \, dy$$

$$= \rho g \cdot \tan\delta\theta \cdot L_{y_n}$$

If $\delta\theta \ll 1$,

$$\delta F_B = \rho g \cdot \delta\theta \cdot L_{y_n}$$

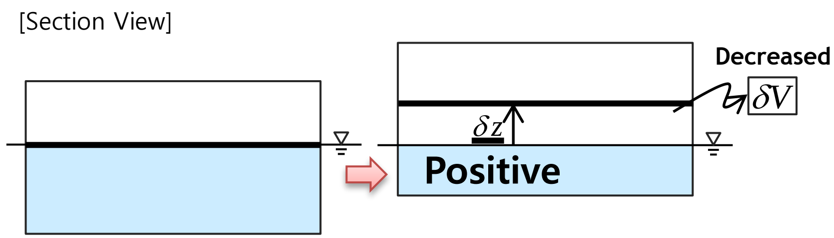
$$L_{y_n} \rightarrow L_{WP}$$

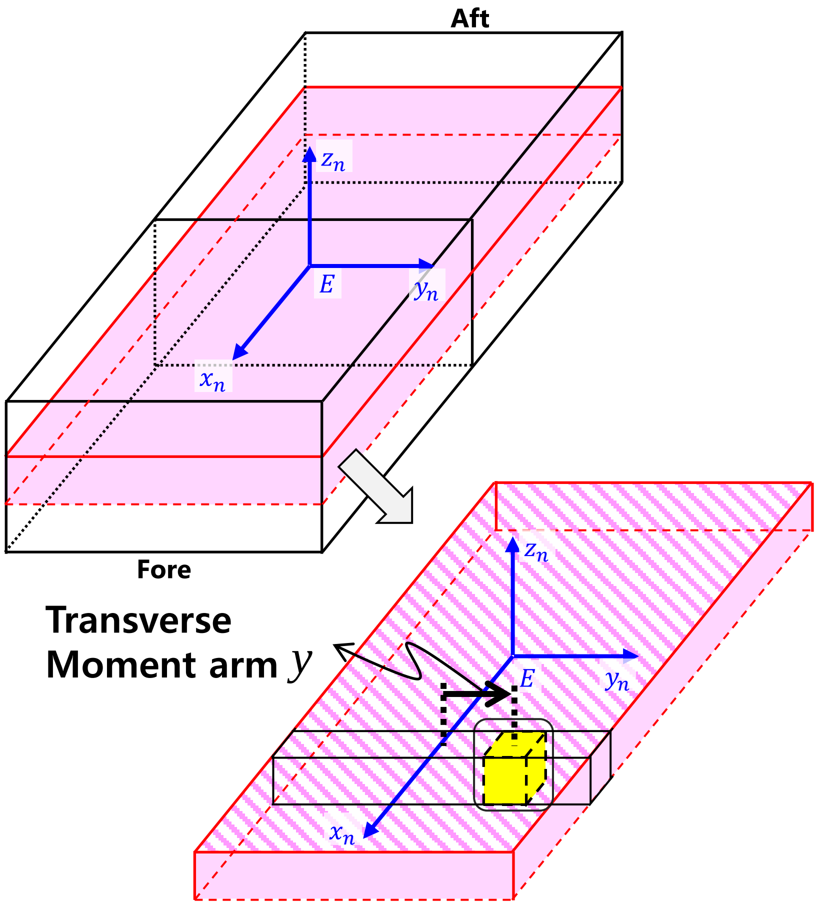
$$\boxed{\therefore \frac{\partial F_B}{\partial \theta} = \rho g \cdot \underline{L_{WP}}}$$

: Longitudinal moment of waterplane area about y_n axis

Change in Transverse Moment due to Buoyant Force with respect to Emersion (Translation along the positive z_n axis)

$$\begin{bmatrix} \frac{\partial F_B}{\partial z} & \frac{\partial F_B}{\partial \phi} & \frac{\partial F_B}{\partial \theta} \\ \boxed{\frac{\partial M_{BT}}{\partial z}} & \frac{\partial M_{BT}}{\partial \phi} & \frac{\partial M_{BT}}{\partial \theta} \\ \frac{\partial M_{BL}}{\partial z} & \frac{\partial M_{BL}}{\partial \phi} & \frac{\partial M_{BL}}{\partial \theta} \end{bmatrix}$$





$$dV = -\delta z\ dy\ dx$$

$$dM_{x_n} = y \cdot dF$$

$$\begin{aligned} \delta M_{x_n} &= \iint dM_{x_n} \\ &= \iint y \cdot dF \\ &= \iint y \cdot \rho g \cdot dV \\ &= -\iint y \cdot \rho g \cdot \delta z \cdot dx\ dy \\ &= -\rho g \delta z \cdot \iint y \cdot dx\ dy \\ &= -\rho g \delta z \cdot T_{x_n} \end{aligned}$$

$$T_{x_n} \rightarrow T_{WP}\ , M_{x_n} \rightarrow M_{BT}$$

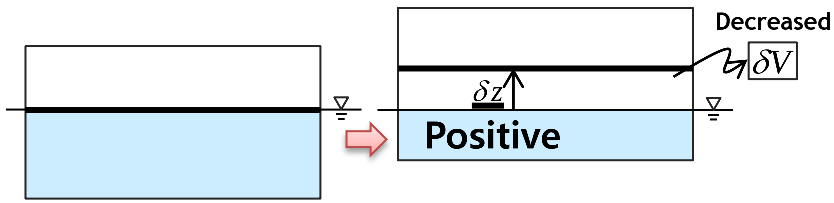
$$\boxed{\therefore \frac{\partial M_{BT}}{\partial z} = -\rho g \cdot \underline{T_{WP}}}$$

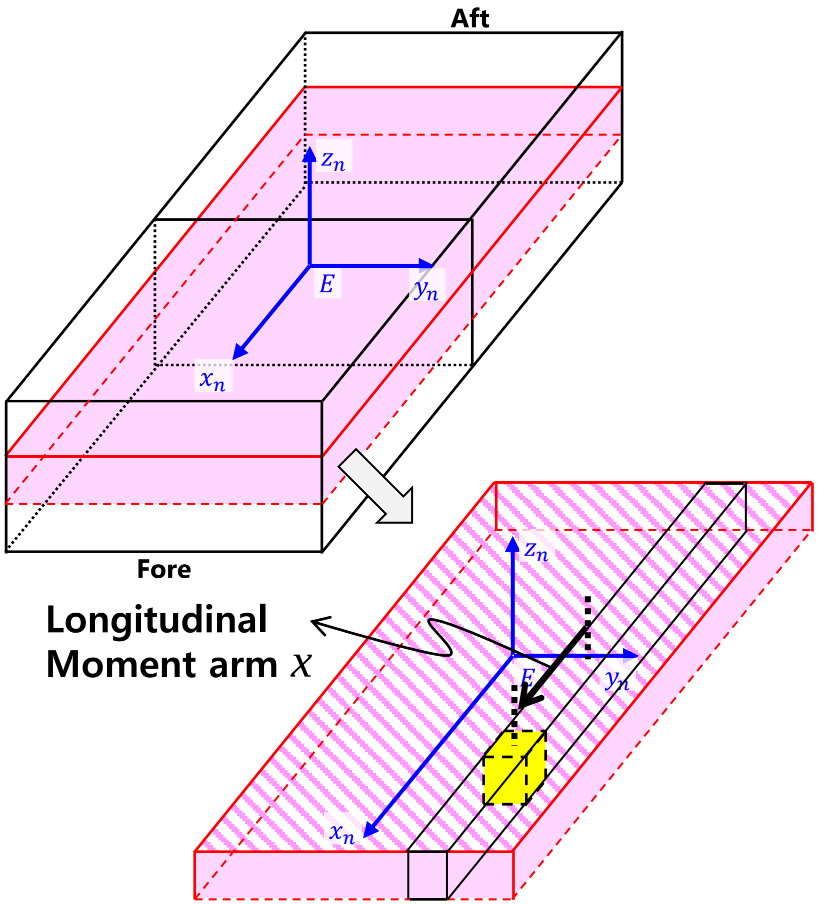
: Transverse moment of waterplane area about x_n axis

Change in Longitudinal Moment due to Buoyant Force with respect to Emersion (Translation along the positive z_n axis)

$$\begin{bmatrix} \frac{\partial F_B}{\partial z} & \frac{\partial F_B}{\partial \phi} & \frac{\partial F_B}{\partial \theta} \\ \frac{\partial M_{BT}}{\partial z} & \frac{\partial M_{BT}}{\partial \phi} & \frac{\partial M_{BT}}{\partial \theta} \\ \frac{\partial M_{BL}}{\partial z} & \frac{\partial M_{BL}}{\partial \phi} & \frac{\partial M_{BL}}{\partial \theta} \end{bmatrix}$$

[Section View]





$$\delta V = -\iint \delta z \, dy \, dx$$

$$dM_{y_n} = -x \cdot dF$$

$$\begin{aligned} \delta M_{y_n} &= \iint dM_{y_n} \\ &= \iint -x \cdot dF = -\iint x \cdot dF \\ &= -\iint x \cdot \rho g \cdot dV \\ &= -\left(-\iint x \cdot \rho g \cdot \delta z \cdot dx \, dy\right) \\ &= \rho g \delta z \cdot \iint x \cdot dx \, dy \\ &= \rho g \delta z \cdot L_{y_n} \end{aligned}$$

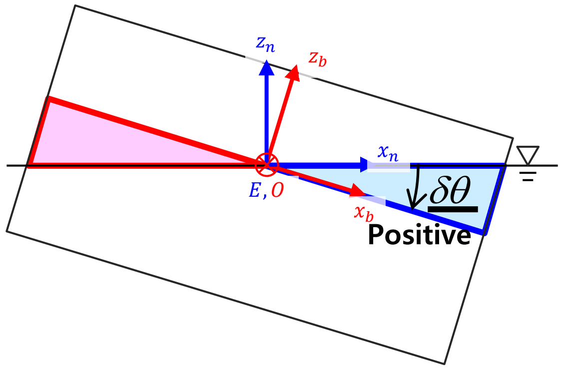
$$L_{y_n} \rightarrow L_{WP}, \; M_{x_n} \rightarrow M_{BL}$$

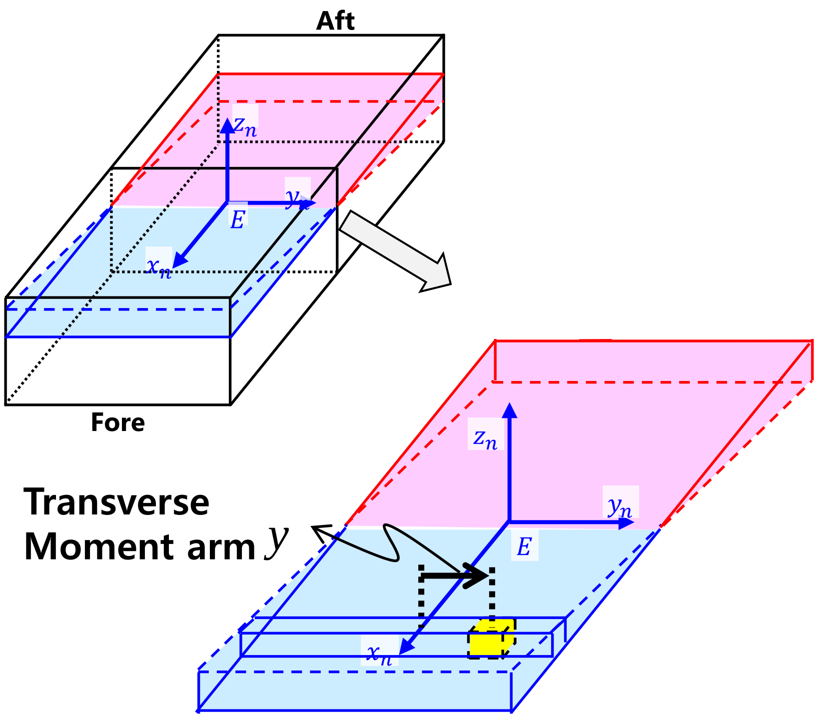
$$\boxed{\therefore \frac{\partial M_{BL}}{\partial z} = \rho g \cdot \underline{L_{WP}}}$$

: Longitudinal moment of waterplane area about y_n axis

Change in Transverse Moment due to Buoyant Force with respect to Trim(Rotation about the y_n axis, $\delta\theta$)

$$\begin{bmatrix} \frac{\partial F_B}{\partial z} & \frac{\partial F_B}{\partial \phi} & \frac{\partial F_B}{\partial \theta} \\ \frac{\partial M_{BT}}{\partial z} & \frac{\partial M_{BT}}{\partial \phi} & \boxed{\frac{\partial M_{BT}}{\partial \theta}} \\ \frac{\partial M_{BL}}{\partial z} & \frac{\partial M_{BL}}{\partial \phi} & \frac{\partial M_{BL}}{\partial \theta} \end{bmatrix}$$





$$dV = x \cdot \tan\delta\theta \; dx \; dy$$

$$dM_{x_n} = y \cdot dF$$

$$\begin{aligned} \delta M_{x_n} &= \iint dM_{x_n} \\ &= \iint y \cdot dF \\ &= \iint y \cdot \rho g \cdot dV \\ &= \iint y \cdot \rho g \cdot x \cdot \tan\delta\theta \cdot dx \; dy \\ &= \rho g \cdot \tan\delta\theta \cdot \iint xy \cdot dx \; dy \\ &= \rho g \cdot \tan\delta\theta \cdot I_P \end{aligned}$$

If $\delta\theta \ll 1$,

$$\delta M_{x_n} = \rho g \cdot \delta\theta \cdot I_P$$

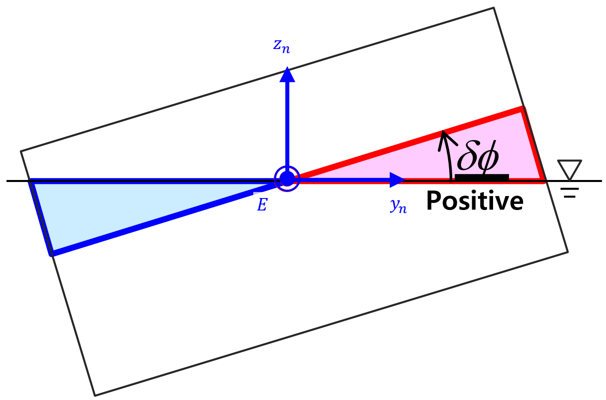
$$M_{x_n} \rightarrow M_{BT}$$

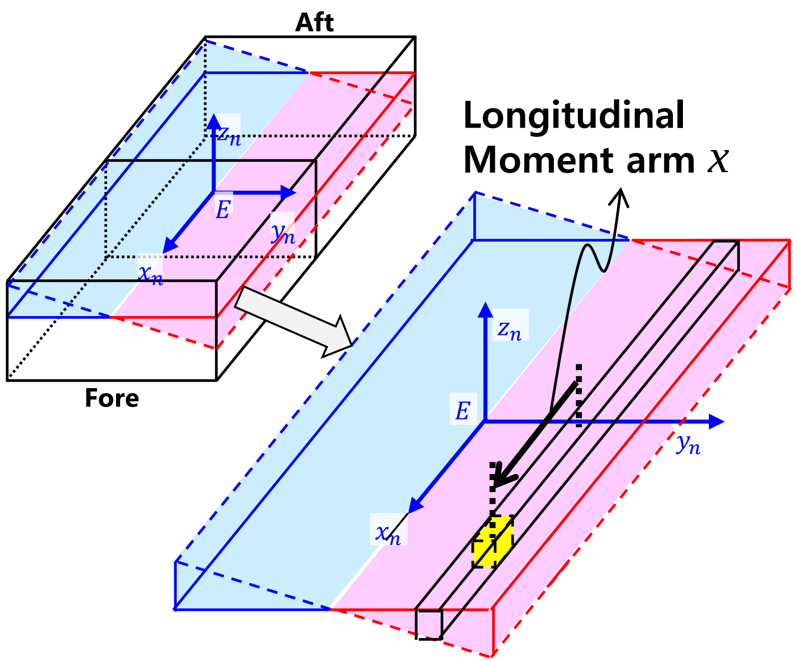
$$\boxed{\therefore \frac{\partial M_{BT}}{\partial \theta} = \rho g \cdot \underline{I_P}}$$

: Product of inertia of waterplane area about x_n and y_n axis

Change in Longitudinal Moment due to Buoyant Force with respect to Heel(Rotation about the x_n axis, $\delta\phi$)

$$\begin{bmatrix} \frac{\partial F_B}{\partial z} & \frac{\partial F_B}{\partial \phi} & \frac{\partial F_B}{\partial \theta} \\ \frac{\partial M_{BT}}{\partial z} & \frac{\partial M_{BT}}{\partial \phi} & \frac{\partial M_{BT}}{\partial \theta} \\ \frac{\partial M_{BL}}{\partial z} & \boxed{\frac{\partial M_{BL}}{\partial \phi}} & \frac{\partial M_{BL}}{\partial \theta} \end{bmatrix}$$





$$dV = -y \cdot \tan\delta\phi \; dx\,dy$$

$$dM_{y_n} = -x \cdot dF$$

$$\begin{aligned} \delta M_{y_n} &= \iint dM_{y_n} \\ &= \iint -x \cdot dF = -\iint x \cdot dF \\ &= -\iint x \cdot \rho g \cdot dV \\ &= -\left(-\iint x \cdot \rho g \cdot y \cdot \tan\delta\phi \cdot dx\,dy\right) \\ &= \rho g \cdot \tan\delta\phi \cdot \iint xy \cdot dx\,dy \\ &= \rho g \cdot \tan\delta\phi \cdot I_P \end{aligned}$$

$\Downarrow$ **If** $\delta\phi \ll 1$,

$$\delta M_{y_n} = \rho g \cdot \delta\phi \cdot I_P$$

$$M_{y_n} \rightarrow M_{BL}$$

$$\boxed{\therefore \frac{\partial M_{BL}}{\partial \phi} = \rho g \cdot \underline{I_P}}$$

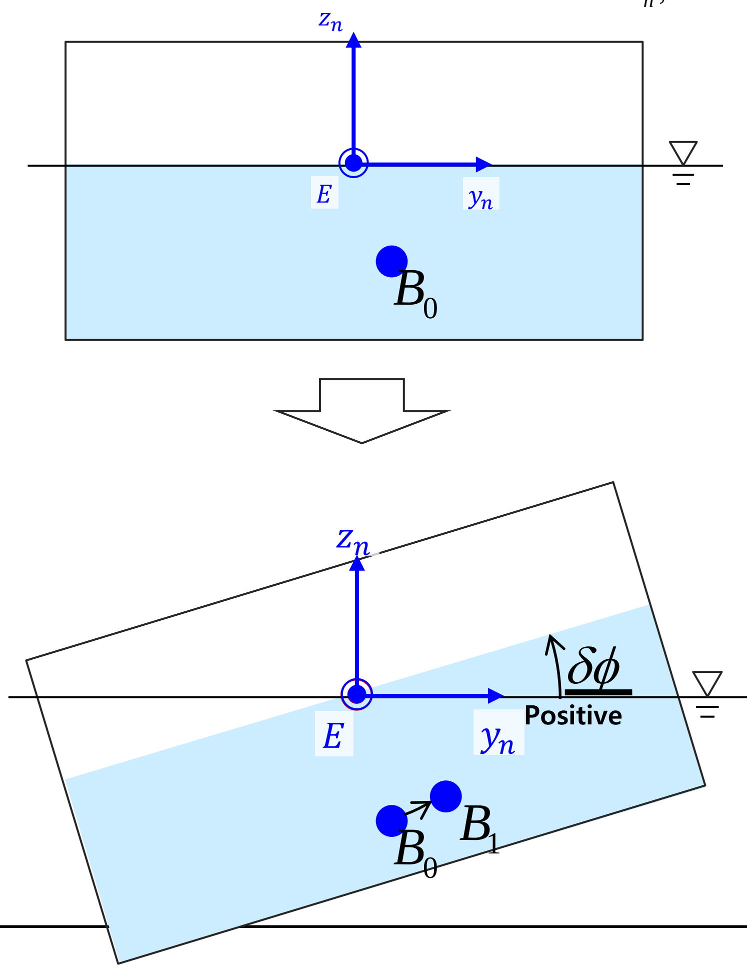
: Product of inertia of waterplane area about x_n and y_n axis

Change in **Transverse Moment** due to Buoyant Force with respect to **Heel** (Rotation about the x_n axis, $\delta\phi$)

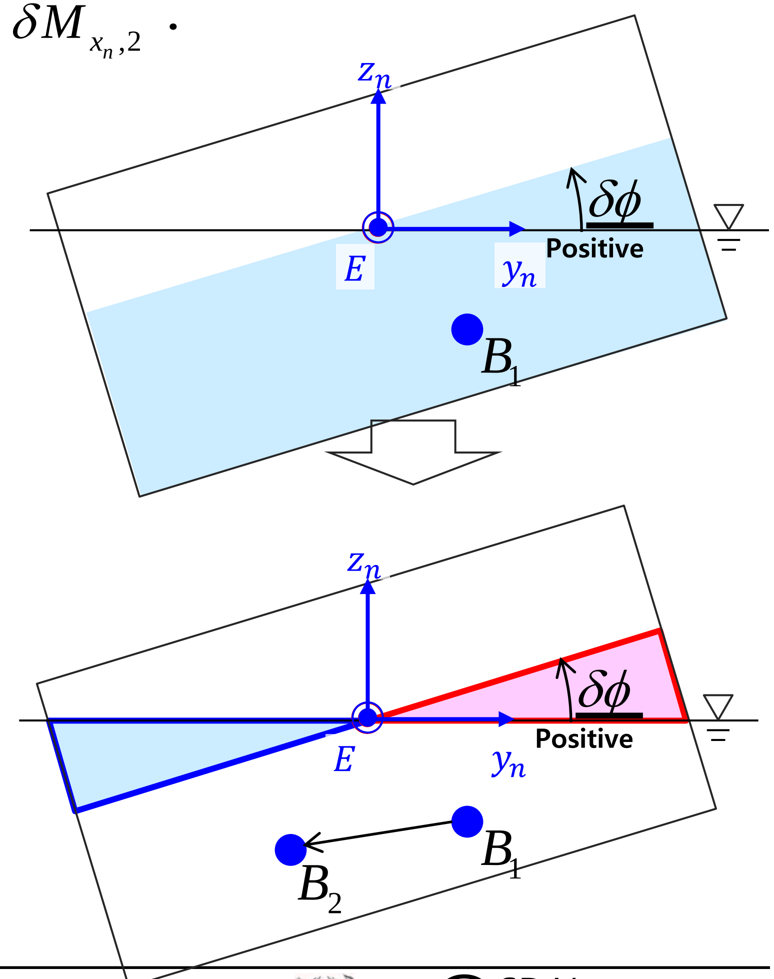
$$\begin{bmatrix} \frac{\partial F_B}{\partial z} & \frac{\partial F_B}{\partial \phi} & \frac{\partial F_B}{\partial \theta} \\ \frac{\partial M_{BT}}{\partial z} & \boxed{\frac{\partial M_{BT}}{\partial \phi}} & \frac{\partial M_{BT}}{\partial \theta} \\ \frac{\partial M_{BL}}{\partial z} & \frac{\partial M_{BL}}{\partial \phi} & \frac{\partial M_{BL}}{\partial \theta} \end{bmatrix}$$

The change in transverse moment, δM_{x_n}, due to buoyant force about x_n axis through point O is made up by two different components :

①The change in the moment of the current displaced volume , $\delta M_{x_n,1}$.

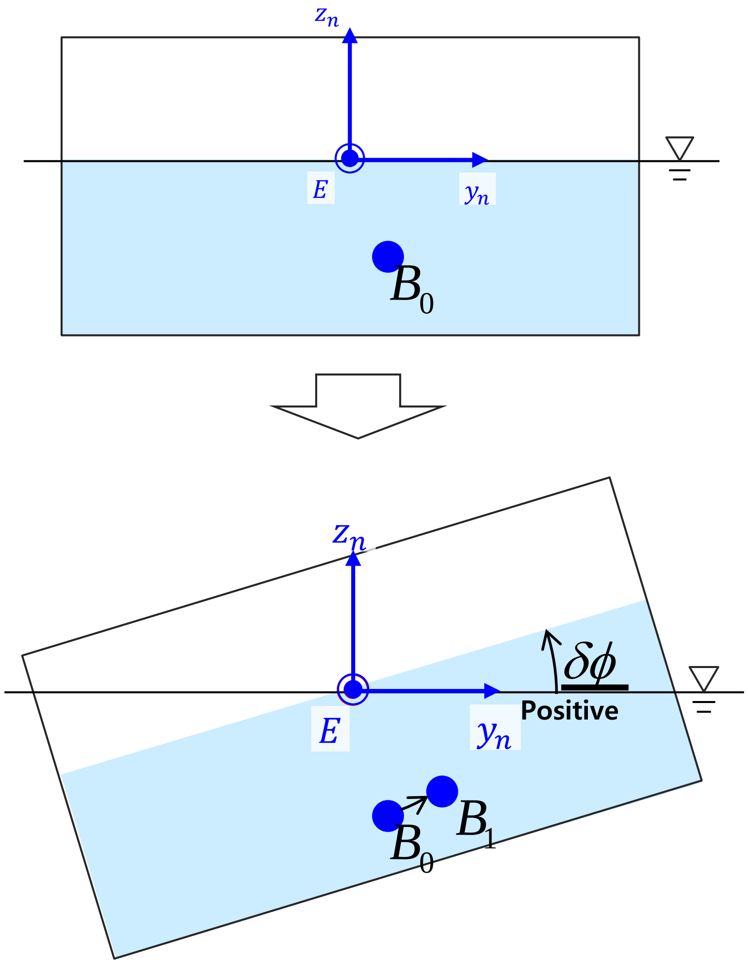


②The change in the displaced volume, $\delta M_{x_n,2}$.

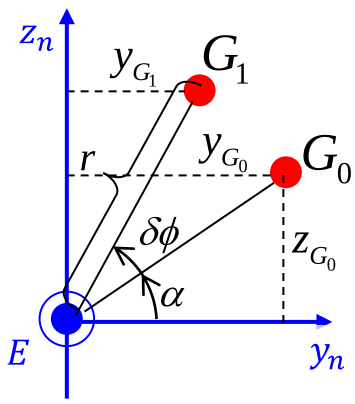


$$\begin{bmatrix} \frac{\partial F_B}{\partial z} & \frac{\partial F_B}{\partial \phi} & \frac{\partial F_B}{\partial \theta} \\ \frac{\partial M_{BT}}{\partial z} & \boxed{\frac{\partial M_{BT}}{\partial \phi}} & \frac{\partial M_{BT}}{\partial \theta} \\ \frac{\partial M_{BL}}{\partial z} & \frac{\partial M_{BL}}{\partial \phi} & \frac{\partial M_{BL}}{\partial \theta} \end{bmatrix}$$

①The change in the moment of the current displaced volume ,$\delta M_{x_n,1}$.

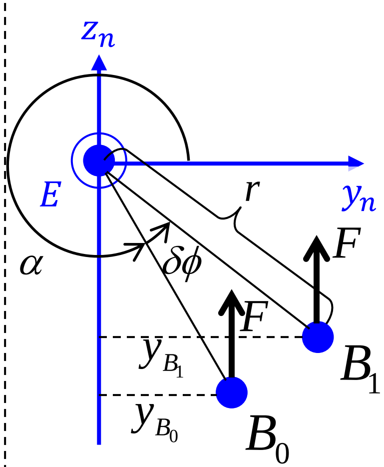


For easy understanding, consider the point located in the 1st quadrant(1사분면)



$$\begin{aligned} y_{G_1} &= r\cos(\alpha+\delta\phi) \\ &= r(\cos\alpha\cdot\cos\delta\phi - \sin\alpha\cdot\sin\delta\phi) \\ &= r\cos\alpha\cdot\cos\delta\phi - r\sin\alpha\cdot\sin\delta\phi \\ &= y_{G_0}\cdot\cos\delta\phi - z_{G_0}\cdot\sin\delta\phi \end{aligned}$$

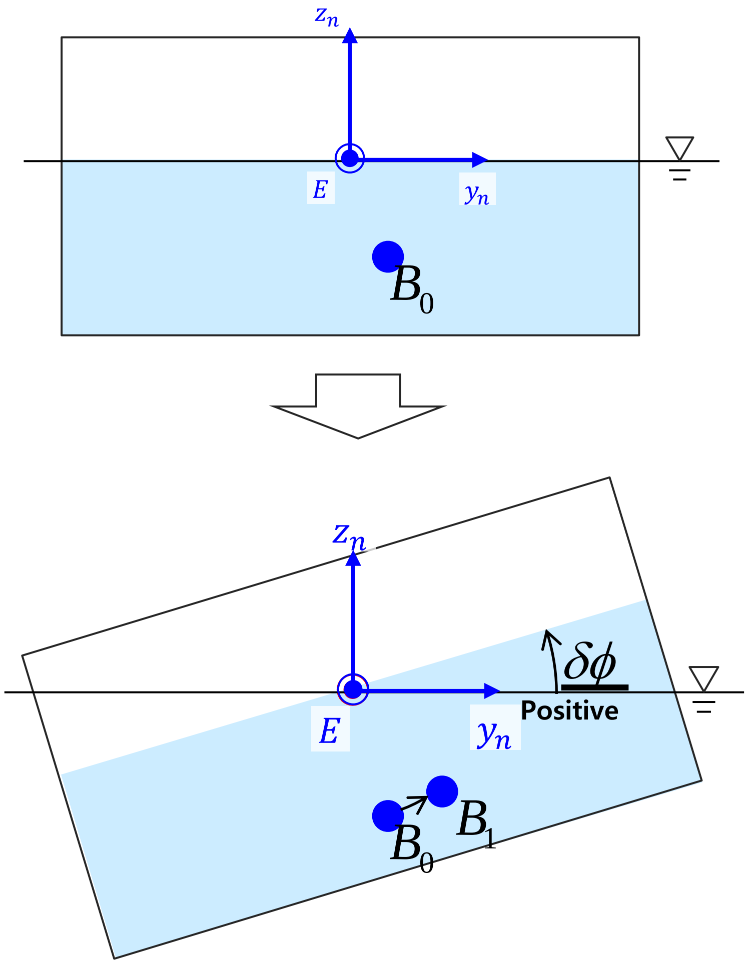
In the same manner

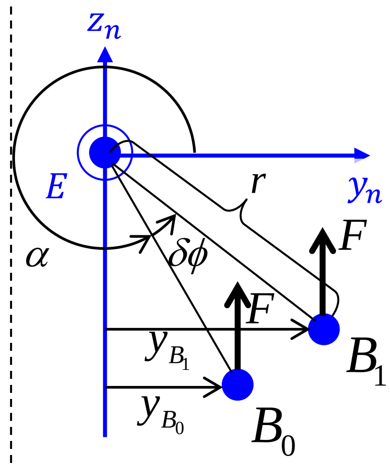


$$\begin{aligned} y_{B_1} &= r\cos(\alpha+\delta\phi) \\ &= r(\cos\alpha\cdot\cos\delta\phi - \sin\alpha\cdot\sin\delta\phi) \\ &= r\cos\alpha\cdot\cos\delta\phi - r\sin\alpha\cdot\sin\delta\phi \\ &= y_{B_0}\cdot\cos\delta\phi - z_{B_0}\cdot\sin\delta\phi \end{aligned}$$

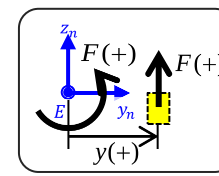
$$\begin{bmatrix} \frac{\partial F_B}{\partial z} & \frac{\partial F_B}{\partial \phi} & \frac{\partial F_B}{\partial \theta} \\ \frac{\partial M_{BT}}{\partial z} & \boxed{\frac{\partial M_{BT}}{\partial \phi}} & \frac{\partial M_{BT}}{\partial \theta} \\ \frac{\partial M_{BL}}{\partial z} & \frac{\partial M_{BL}}{\partial \phi} & \frac{\partial M_{BL}}{\partial \theta} \end{bmatrix}$$

①The change in the moment of the current displaced volume ,$\delta M_{x_n,1}$.



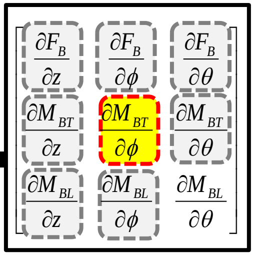


$$\begin{aligned} y_{B_1} &= r\cos(\alpha+\delta\phi) \\ &= r(\cos\alpha\cdot\cos\delta\phi-\sin\alpha\cdot\sin\delta\phi) \\ &= r\cos\alpha\cdot\cos\delta\phi-r\sin\alpha\cdot\sin\delta\phi \\ &= y_{B_0}\cdot\cos\delta\phi-z_{B_0}\cdot\sin\delta\phi \end{aligned}$$

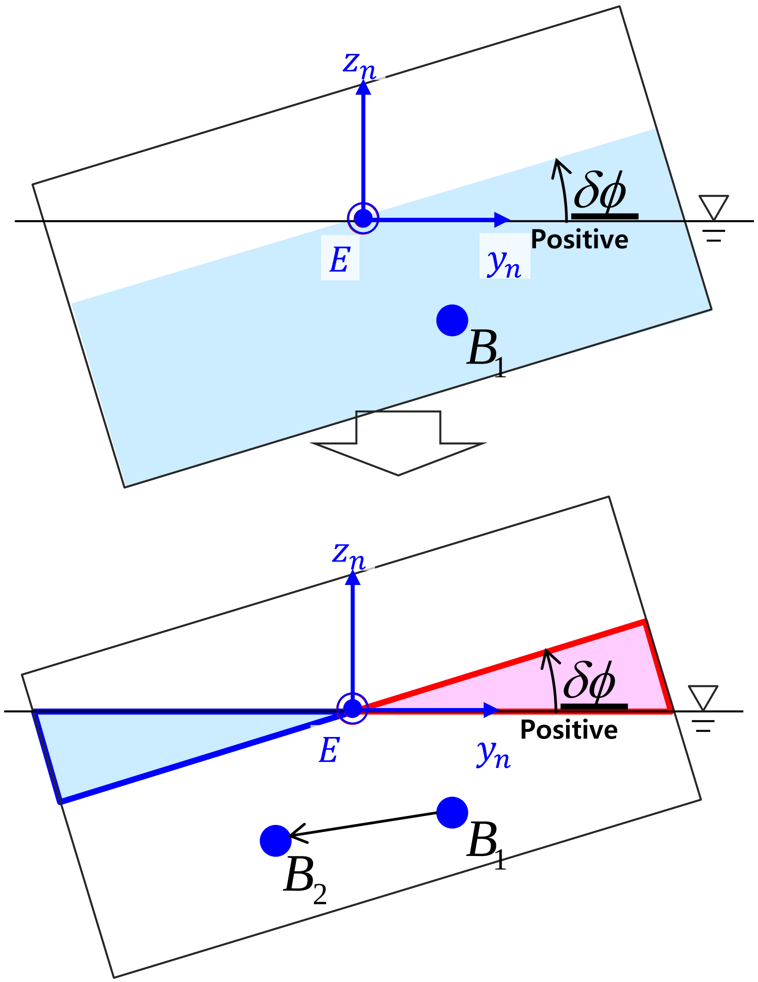


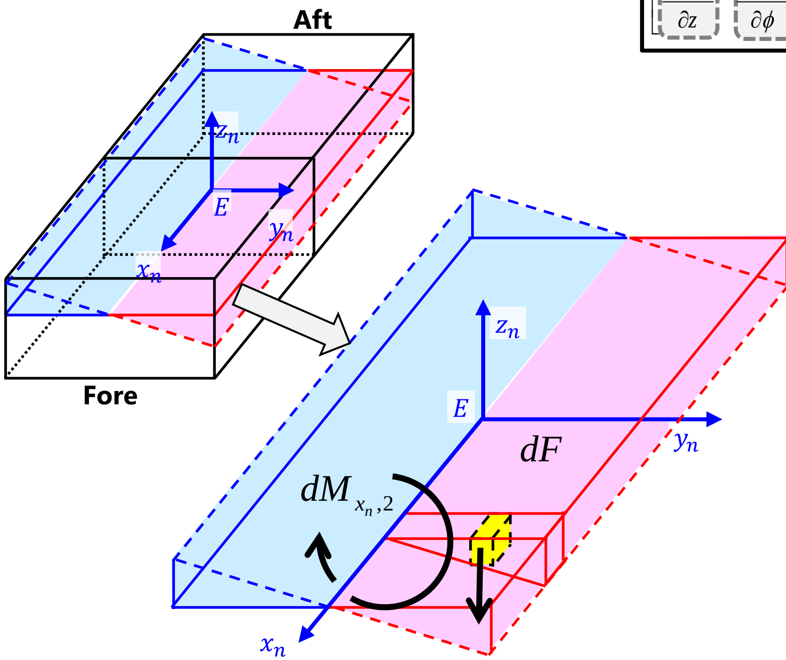
$$\begin{aligned} \delta M_{x_n,1} &= M_{B_1}-M_{B_0} \\ &= y_{B_1}\cdot F-y_{B_0}\cdot F \\ &= (y_{B_0}\cdot\cos\delta\phi-z_{B_0}\cdot\sin\delta\phi)\cdot F-y_{B_0}\cdot F \\ &= (y_{B_0}\cdot\cos\delta\phi-z_{B_0}\cdot\sin\delta\phi-y_{B_0})\cdot F \\ &= \left\{y_{B_0}(\cos\delta\phi-1)-z_{B_0}\sin\delta\phi\right\}\cdot F \\ &= \left\{y_{B_0}(\cos\delta\phi-1)-z_{B_0}\sin\delta\phi\right\}\cdot\rho g V \end{aligned}$$

$$\underset{(+)}{M_{x_n}} = \underset{(+)}{y}\cdot\underset{(+)}{F}$$



②The change in the displaced volume, $\delta M_{x_n,2}$.



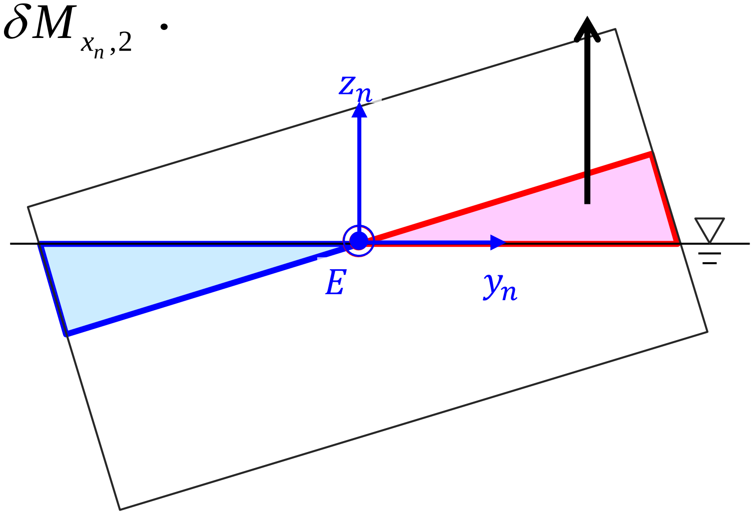


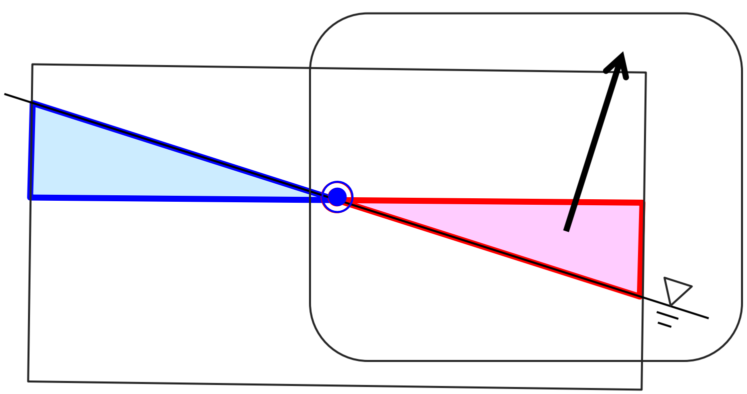
$$\delta M_{x_n,2} = \iint dM_{x_n,2}$$

$dM_{x_n,2}$: Infinitesimal moment due to the infinitesimal buoyant force dF about x_n-axis

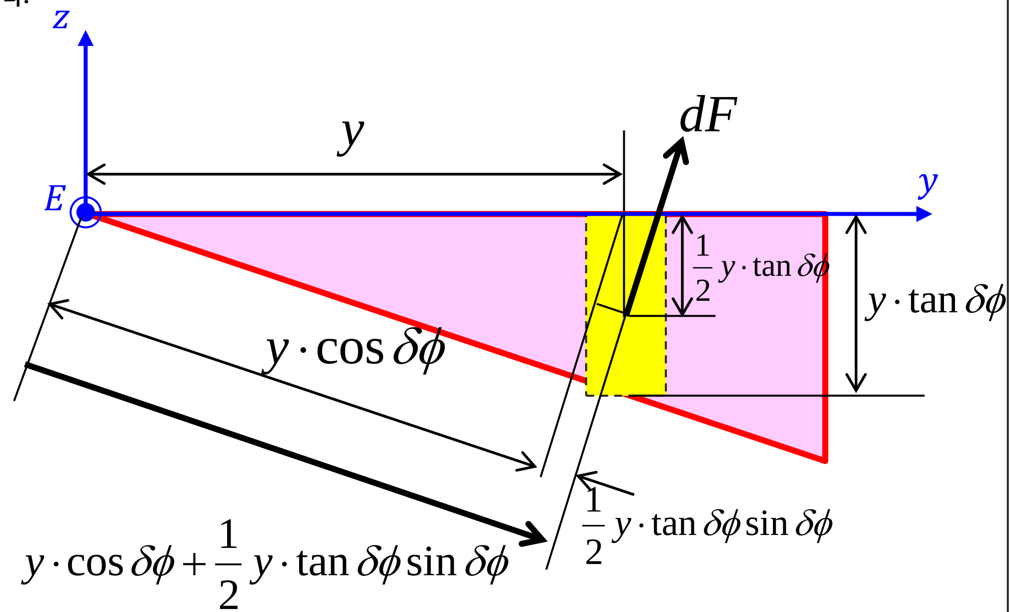
$$\begin{bmatrix} \frac{\partial F_B}{\partial z} & \frac{\partial F_B}{\partial \phi} & \frac{\partial F_B}{\partial \theta} \\ \frac{\partial M_{BT}}{\partial z} & \boxed{\frac{\partial M_{BT}}{\partial \phi}} & \frac{\partial M_{BT}}{\partial \theta} \\ \frac{\partial M_{BL}}{\partial z} & \frac{\partial M_{BL}}{\partial \phi} & \frac{\partial M_{BL}}{\partial \theta} \end{bmatrix}$$

②The change in the displaced volume, $\delta M_{x_n,2}$ ·





주의!!: 우리가 알고 있는 것은 현재의 수선면 정보이며, 다음 자세의 수선면 정보는 알기가 어렵다. 따라서 적분은 현재의 수선면에 따라서 수행한다. 하지만 다음 자세가 되었을 때 힘의 변화를 알아야 하며, 그때의 힘은 다음 자세의 수선면에 수직해야 한다. 따라서 아래 그림과 같이 표현하고 적분을 수행한다. 참고로 3 by 3 메트릭스를 유도하는 과정에서 나오는 모든 좌표는 수선면 고정 좌표계를 기준으로 한 것이다.



Transverse Moment arm

$$\delta M_{x_n,2} = \iint dM_{x_n,2}$$

$$= \iint \left(y \cdot \cos\delta\phi + \frac{1}{2} y \cdot \tan\delta\phi \sin\delta\phi \right) \cdot dF$$

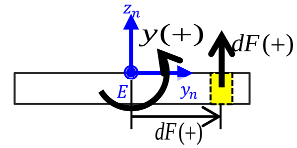
$$
\begin{bmatrix}
\frac{\partial F_B}{\partial z} & \frac{\partial F_B}{\partial \phi} & \frac{\partial F_B}{\partial \theta} \\
\frac{\partial M_{BT}}{\partial z} & \boxed{\frac{\partial M_{BT}}{\partial \phi}} & \frac{\partial M_{BT}}{\partial \theta} \\
\frac{\partial M_{BL}}{\partial z} & \frac{\partial M_{BL}}{\partial \phi} & \frac{\partial M_{BL}}{\partial \theta}
\end{bmatrix}
$$

②The change in the displaced volume, $\delta M_{x_n,2}$ ·

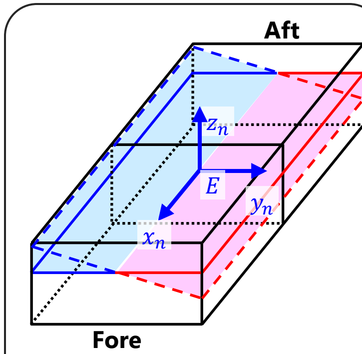
$$y \cdot \cos\delta\phi + \frac{1}{2} y \cdot \tan\delta\phi \sin\delta\phi$$

Transverse Moment arm

$$\underset{(+)}{dM_{x_n}} = \underset{(+)}{y} \cdot \underset{(+)}{dF}$$



$$
\begin{aligned}
\delta M_{x_n,2} &= \iint dM_{x_n,2} \\
&= \iint \left(y \cdot \cos\delta\phi + \frac{1}{2} y \cdot \tan\delta\phi \sin\delta\phi \right) \cdot dF \\
&= \iint \left(y \cdot \cos\delta\phi + \frac{1}{2} y \cdot \tan\delta\phi \sin\delta\phi \right) \cdot \rho g dV \\
&= -\rho g \iint \left(y \cdot \cos\delta\phi + \frac{1}{2} y \cdot \tan\delta\phi \sin\delta\phi \right) \cdot y \cdot \tan\delta\phi \; dx \, dy \\
&= -\rho g \left\{ \iint (y \cdot \cos\delta\phi) \cdot y \cdot \tan\delta\phi \; dx \, dy + \iint \left(\frac{1}{2} y \cdot \tan\delta\phi \sin\delta\phi \right) \cdot y \cdot \tan\delta\phi \; dx \, dy \right\} \\
&= -\rho g \left\{ \iint y^2 \cdot \sin\delta\phi \; dx \, dy + \iint \frac{1}{2} y^2 \cdot \tan^2\delta\phi \sin\delta\phi \; dx \, dy \right\} \\
&= -\rho g \left\{ \sin\delta\phi \iint y^2 \; dx \, dy + \frac{1}{2} \tan^2\delta\phi \sin\delta\phi \iint y^2 \; dx \, dy \right\} \\
&= -\rho g \left\{ \sin\delta\phi \left(1 + \frac{1}{2} \tan^2\delta\phi \right) \iint y^2 \; dx \, dy \right\} \\
&= -\rho g \left\{ \sin\delta\phi \left(1 + \frac{1}{2} \tan^2\delta\phi \right) I_{x_n} \right\}
\end{aligned}
$$



$$dV = -y \cdot \tan\delta\phi \; dx \, dy$$

Change in Transverse Moment due to Buoyant Force with respect to Heel (Rotation about the x_n axis, $\delta\phi$)

$$\begin{matrix} \frac{\partial F_B}{\partial z} & \frac{\partial F_B}{\partial \phi} & \frac{\partial F_B}{\partial \theta} \\ \frac{\partial M_{BT}}{\partial z} & \boxed{\frac{\partial M_{BT}}{\partial \phi}} & \frac{\partial M_{BT}}{\partial \theta} \\ \frac{\partial M_{BL}}{\partial z} & \frac{\partial M_{BL}}{\partial \phi} & \boxed{\frac{\partial M_{BL}}{\partial \theta}} \end{matrix}$$

①The change in the moment of the current displaced volume , $\delta M_{x_n,1}$.

$$\delta M_{x_n,1} = \left\{ y_{B_0} (\cos\delta\phi - 1) - z_{B_0} \sin\delta\phi \right\} \cdot \rho g V$$

②The change in the displaced volume, $\delta M_{x_n,2}$.

$$\delta M_{x_n,2} = -\rho g \left\{ \left(\sin\delta\phi + \frac{1}{2}\tan^2\delta\phi \sin\delta\phi \right) I_{x_n} \right\}$$

$$\delta M_{x_n} = \delta M_{x_n,1} + \delta M_{x_n,2}$$

$$= \left\{ y_{B_0} (\cos\delta\phi - 1) - z_{B_0} \sin\delta\phi \right\} \cdot \rho g V - \rho g \left\{ \sin\delta\phi \left(1 + \frac{1}{2}\tan^2\delta\phi \right) I_{x_n} \right\}$$

$$= \rho g V \left[\left\{ y_B (\cos\delta\phi - 1) - z_B \sin\delta\phi \right\} - \left\{ \sin\delta\phi \left(1 + \frac{1}{2}\tan^2\delta\phi \right) \frac{I_{x_n}}{V} \right\} \right]$$

If $\delta\phi \ll 1$,

$$= \rho g V \left[\left\{ y_B (1-1) - z_B \delta\phi \right\} - \left\{ \delta\phi (1+0) \frac{I_{x_n}}{V} \right\} \right]$$

Classical Hydrostatic에서는 고려했던 $1/2\tan^2\delta\phi$항을 작다고 무시하는 이유는 굳이 고려하지 않아도 iteration을 통하여 실제 해를 구할 수 있기 때문

$$= \rho g V \left[-z_B \delta\phi - \delta\phi \frac{I_{x_n}}{V} \right]$$

$I_{x_n} \rightarrow I_T, M_{x_n} \rightarrow M_{BT}$

$$\boxed{\frac{\partial M_{BT}}{\partial \phi} = -\rho g V \cdot \left(z_B + \frac{I_T}{V} \right)}$$

$*KM_T = KB + BM_T$: The components are corresponding to the terms in classical ship hydrostatics

In the same manner

$$\boxed{\frac{\partial M_{BL}}{\partial \theta} = -\rho g V \cdot \left(z_B + \frac{I_L}{V} \right)}$$

Change in Force and Moment due to Gravitational Force

Change in Force and Moment due to Gravitational Force

$$\begin{bmatrix} \boxed{\frac{\partial F_G}{\partial z}} & \boxed{\frac{\partial F_G}{\partial \phi}} & \boxed{\frac{\partial F_G}{\partial \theta}} \\ \boxed{\frac{\partial M_{GT}}{\partial z}} & \frac{\partial M_{GT}}{\partial \phi} & \boxed{\frac{\partial M_{GT}}{\partial \theta}} \\ \boxed{\frac{\partial M_{GL}}{\partial z}} & \boxed{\frac{\partial M_{GL}}{\partial \phi}} & \frac{\partial M_{GL}}{\partial \theta} \end{bmatrix}$$

$$\frac{\partial F_G}{\partial z} = \frac{\partial F_G}{\partial \phi} = \frac{\partial F_G}{\partial \theta} = 0$$

1. Elements (1,1), (1,2), (1,3) are zero, since the gravitational force does not change with respect to the immersion, heel, and trim.

$$\frac{\partial M_{GT}}{\partial z} = \frac{\partial M_{GL}}{\partial z} = 0$$

2. Elements (2,1), (3,1) are zero, since the transverse moment and longitudinal moment do not change with respect to the immersion.

$$\frac{\partial M_{GT}}{\partial \theta} = 0$$

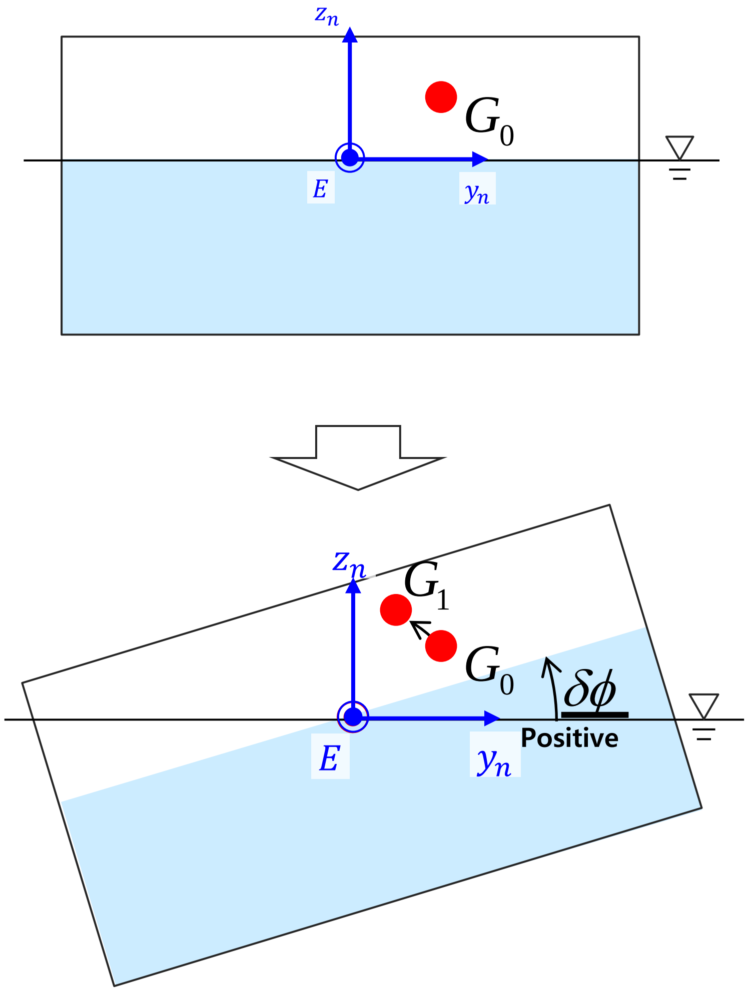
3. Elements (2,3) is zero, since the transverse moment does not change with respect to the trim.

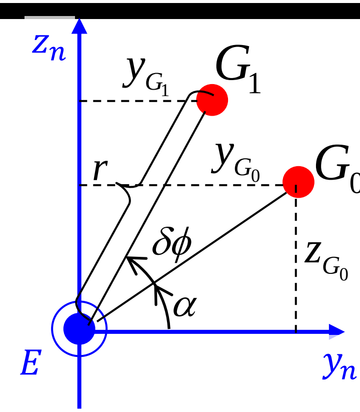
$$\frac{\partial M_{GL}}{\partial \phi} = 0$$

4. Elements (3,2) is zero, since the longitudinal moment does not change with respect to the heel.

Change in Transverse Moment due to Gravitational Force with respect to Heel (Rotation about the x_n axis, $\delta\phi$)

$$\begin{bmatrix} \frac{\partial F_G}{\partial z} & \frac{\partial F_G}{\partial \phi} & \frac{\partial F_G}{\partial \theta} \\ \frac{\partial M_{GT}}{\partial z} & \boxed{\frac{\partial M_{GT}}{\partial \phi}} & \frac{\partial M_{GT}}{\partial \theta} \\ \frac{\partial M_{GL}}{\partial z} & \frac{\partial M_{GL}}{\partial \phi} & \boxed{\frac{\partial M_{GL}}{\partial \theta}} \end{bmatrix}$$





$$\begin{aligned} y_{G_1} &= r\cos(\alpha+\delta\phi) \\ &= r(\cos\alpha\cdot\cos\delta\phi-\sin\alpha\cdot\sin\delta\phi) \\ &= r\cos\alpha\cdot\cos\delta\phi - r\sin\alpha\cdot\sin\delta\phi \\ &= y_{G_0}\cdot\cos\delta\phi - z_{G_0}\cdot\sin\delta\phi \end{aligned}$$

$$\begin{aligned} \delta M_{x_n} &= M_{G_1} - M_{G_0} \\ &= y_{G_1}\cdot F - y_{G_0}\cdot F \\ &= (y_{G_0}\cdot\cos\delta\phi - z_{G_0}\cdot\sin\delta\phi)\cdot F - y_{G_0}\cdot F \\ &= (y_{G_0}\cdot\cos\delta\phi - z_{G_0}\cdot\sin\delta\phi - y_{G_0})\cdot F \\ &= \left\{ y_{G_0}(\cos\delta\phi-1) - z_{G_0}\sin\delta\phi \right\}\cdot F \\ &= \left\{ y_{G_0}(\cos\delta\phi-1) - z_{G_0}\sin\delta\phi \right\}\cdot \rho g V \end{aligned}$$

$$\underset{(+)}{M_{x_n}} = \underset{(+)}{y}\cdot\underset{(+)}{F}$$

If $\delta\phi \ll 1$,

$$= \left\{ y_{G_0}\cancel{(1-1)} - z_{G_0}\delta\phi \right\}\cdot\rho g V$$

$$M_{x_n} \rightarrow M_{GT}$$

$$\boxed{\frac{\partial M_{GT}}{\partial \phi} = z_{G_0}\rho g V = -z_{G_0}F_G}$$

In the same manner $\boxed{\frac{\partial M_{GL}}{\partial \theta} = z_{G_0}\rho g V = -z_{G_0}F_G}$

GOVERNING EQUATION OF A SHIP IN HYDROSTATIC EQUILIBRIUM STATE(Governing Equations of Computational Ship Stability)

Governing Equations of Computational Ship Stability

$$
\begin{bmatrix} \cancel{F_z} - F_z\left(z_n^{(k)},\phi^{(k)},\theta^{(k)}\right) \\ \cancel{M_T} - M_T\left(z_n^{(k)},\phi^{(k)},\theta^{(k)}\right) \\ \cancel{M_L} - M_L\left(z_n^{(k)},\phi^{(k)},\theta^{(k)}\right) \end{bmatrix}
=
\begin{bmatrix}
\begin{array}{c} \frac{\partial F_B}{\partial z_n}+\frac{\partial F_G}{\partial z_n}+\frac{\partial F_{ext}}{\partial z_n} \\ \boxed{-\rho g A_{WP}{}^{(k)}} \end{array} &
\begin{array}{c} \frac{\partial F_B}{\partial \phi}+\frac{\partial F_G}{\partial \phi}+\frac{\partial F_{ext}}{\partial \phi} \\ \boxed{-\rho g A_{WP}{}^{(k)}\cdot {}^{n}y_{F^{(k)}/E}} \end{array} &
\begin{array}{c} \frac{\partial F_B}{\partial \theta}+\frac{\partial F_G}{\partial \theta}+\frac{\partial F_{ext}}{\partial \theta} \\ \boxed{\rho g A_{WP}{}^{(k)}\cdot {}^{n}x_{F^{(k)}/E}} \end{array} \\
\begin{array}{c} \frac{\partial M_{BT}}{\partial z_n}+\frac{\partial M_{GT}}{\partial z_n}+\frac{\partial M_{extT}}{\partial z_n} \\ \boxed{-\rho g A_{WP}{}^{(k)}\cdot {}^{n}y_{F^{(k)}/E}} \end{array} &
\begin{array}{c} \frac{\partial M_{BT}}{\partial \phi}+\frac{\partial M_{GT}}{\partial \phi}+\frac{\partial M_{extT}}{\partial \phi} \\ \boxed{\begin{array}{c} -\rho g\left({}^{n}z_{B^{(k)}/E}\nabla^{(k)}+I_T{}^{(k)}\right) \\ -{}^{n}z_{G^{(k)}/E}\cdot F_G - {}^{n}z_{G_{ext}^{(k)}/E}\cdot F_{ext}{}^{(k)} \end{array}} \end{array} &
\begin{array}{c} \frac{\partial M_{BT}}{\partial \theta}+\frac{\partial M_{GT}}{\partial \theta}+\frac{\partial M_{extT}}{\partial \theta} \\ \boxed{\rho g I_P{}^{(k)}} \end{array} \\
\begin{array}{c} \frac{\partial M_{BL}}{\partial z_n}+\frac{\partial M_{GL}}{\partial z_n}+\frac{\partial M_{extL}}{\partial z_n} \\ \boxed{\rho g A_{WP}{}^{(k)}\cdot {}^{n}x_{F^{(k)}/E}} \end{array} &
\begin{array}{c} \frac{\partial M_{BL}}{\partial \phi}+\frac{\partial M_{GL}}{\partial \phi}+\frac{\partial M_{extL}}{\partial \phi} \\ \boxed{\rho g I_P{}^{(k)}} \end{array} &
\begin{array}{c} \frac{\partial M_{BL}}{\partial \theta}+\frac{\partial M_{GL}}{\partial \theta}+\frac{\partial M_{extL}}{\partial \theta} \\ \boxed{\begin{array}{c} -\rho g\left({}^{n}z_{B^{(k)}/E}\nabla^{(k)}+I_L{}^{(k)}\right) \\ -{}^{n}z_{G^{(k)}/E}\cdot F_G - {}^{n}z_{G_{ext}^{(k)}/E}\cdot F_{ext}{}^{(k)} \end{array}} \end{array}
\end{bmatrix}_{\substack{z_n=z_n^{(k)} \\ \phi=\phi^{(k)} \\ \theta=\theta^{(k)}}}
\begin{bmatrix} \delta z_n{}^{(k)} \\ \delta\phi^{(k)} \\ \delta\theta^{(k)} \end{bmatrix}
$$

$= 0$

We want to find the static equilibrium position and orientation!

F_G: gravitational force exerted on a ship
M_T: transverse moment of a ship about x_n axis
M_L: longitudinal moment of a ship about y_n axis
$A_{WP}^{(k)}$: waterplane area of a ship at k^{th} step
$I_T^{(k)}$: transverse moment of inertia of the waterplane area of a ship about x_n axis at k^{th} step
$I_L^{(k)}$: longitudinal moment of inertia of the waterplane area of a ship about y_n axis at k^{th} step
$I_P^{(k)}$: centrifugal moment of the waterplane area of a ship about x_n and y_n axis at k^{th} step
F_B: buoyant force exerted on a ship
F_{ext}: external force exerted on a ship

${}^{n}x_{F^{(k)}/E}$: x_n coordinate of centroid of the waterplane area of a ship
${}^{n}y_{F^{(k)}/E}$: y_n coordinate of centroid of the waterplane area of a ship
${}^{n}z_{B^{(k)}/E}$: z_n coordinate of center of the displaced volume of a ship
${}^{n}z_{G^{(k)}/E}$: z_n coordinate of center of mass of the ship
$\delta z^{(k)}$: change in the draft at k^{th} step
$\delta\phi^{(k)}$: change in the angle of heel at k^{th} step
$\delta\theta^{(k)}$: change in the angle of trim at k^{th} step
μ_V: permeability of a compartment
μ_F: surface permeability of a compartment

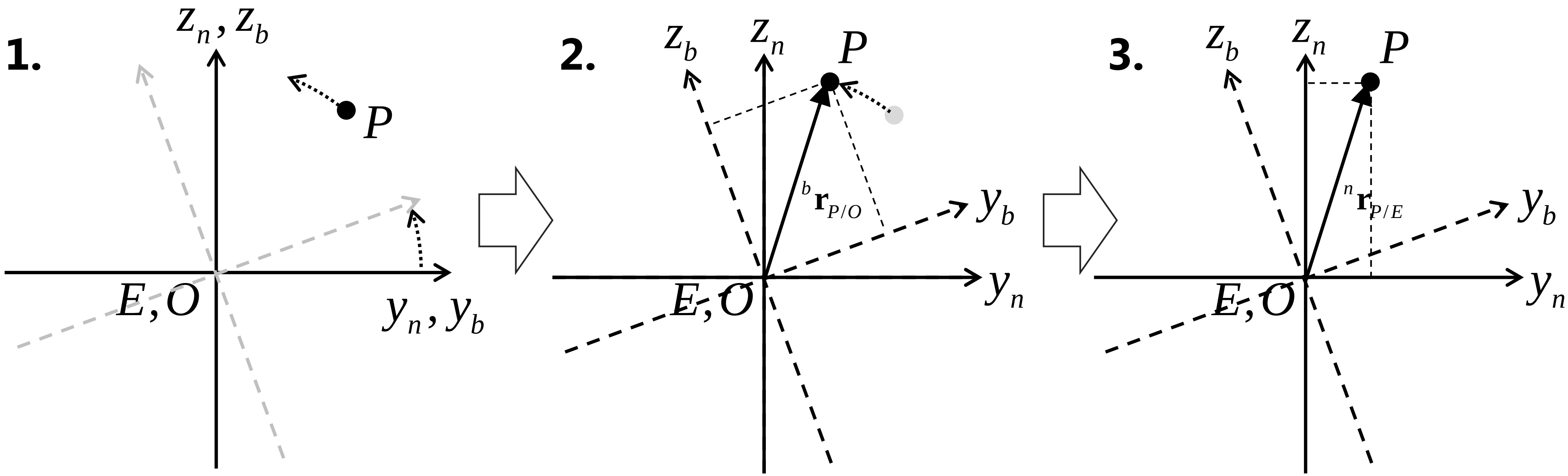
$a_{WP}^{(k)}$: waterplane area of a flooded compartment at k^{th} step
$i_T^{(k)}$: transverse moment of inertia of the waterplane area of a flooded compartment about x_n axis at k^{th} step
$i_L^{(k)}$: longitudinal moment of inertia of the waterplane area of a flooded compartment about y_n axis at k^{th} step
$i_P^{(k)}$: centrifugal moment of the waterplane area of a flooded compartment about x_n and y_n axis at k^{th} step
${}^{n}x_{f^{(k)}/E}$: x_n coordinate of centroid of the waterplane area of a flooded compartment at k^{th} step
${}^{n}y_{f^{(k)}/E}$: y_n coordinate of centroid of the waterplane area of a flooded compartment at k^{th} step
${}^{n}z_{G_{ext}^{(k)}/E}$: z_n coordinate of center of the submerged volume of a flooded compartment at k^{th} step

10-8 Derivation of Partial Derivatives Associated with Hydrostatic Equilibrium Considering Euler Angle

FORWARD AND INVERSE PROBLEM OF COORDINATE TRANSFORMATION

Coordinate Transformation: Forward problem

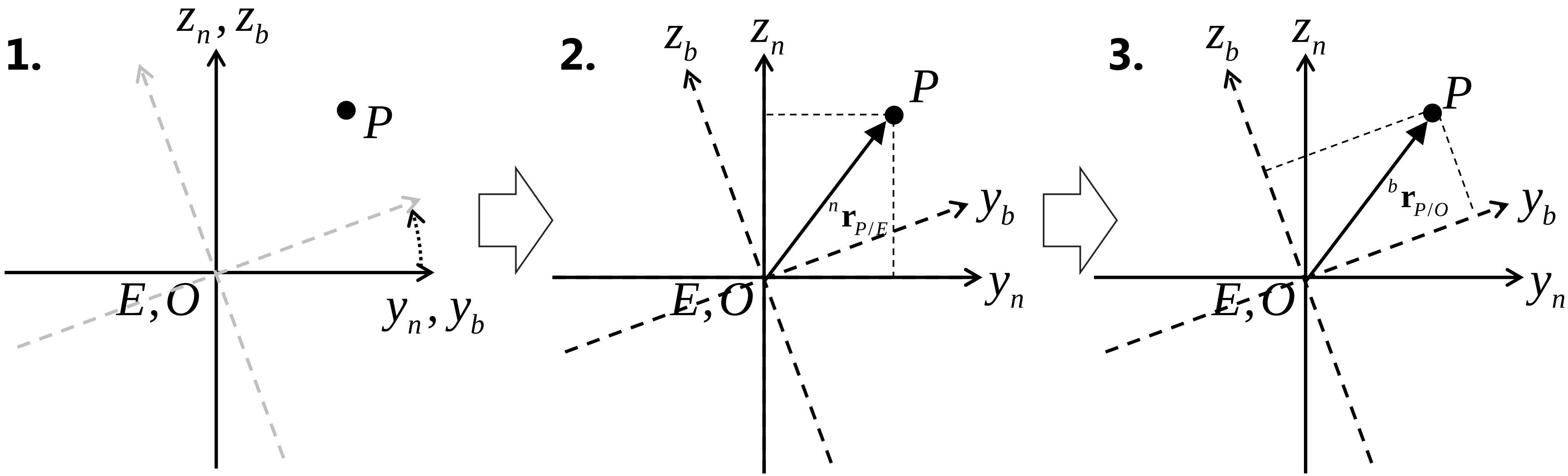
$$^{n}\mathbf{r}_{P/E} = {}^{n}\mathbf{R}_{b} \, {}^{b}\mathbf{r}_{P/O}$$



1. 문제정의: 점 **P**가 **b-frame**과 함께 회전하는 경우
2. 점 **P**가 **b-frame**과 함께 회전하였으므로, 알고 있는 벡터는 **b-frame**에서 기술한 점 **P**의 위치벡터 $^{b}\mathbf{r}_{P/O}$
3. 최종적으로 구하고자 하는 벡터는 **n-frame**에서 기술한 점 **P**의 위치벡터 $^{n}\mathbf{r}_{P/E}$

Coordinate Transformation: Inverse problem

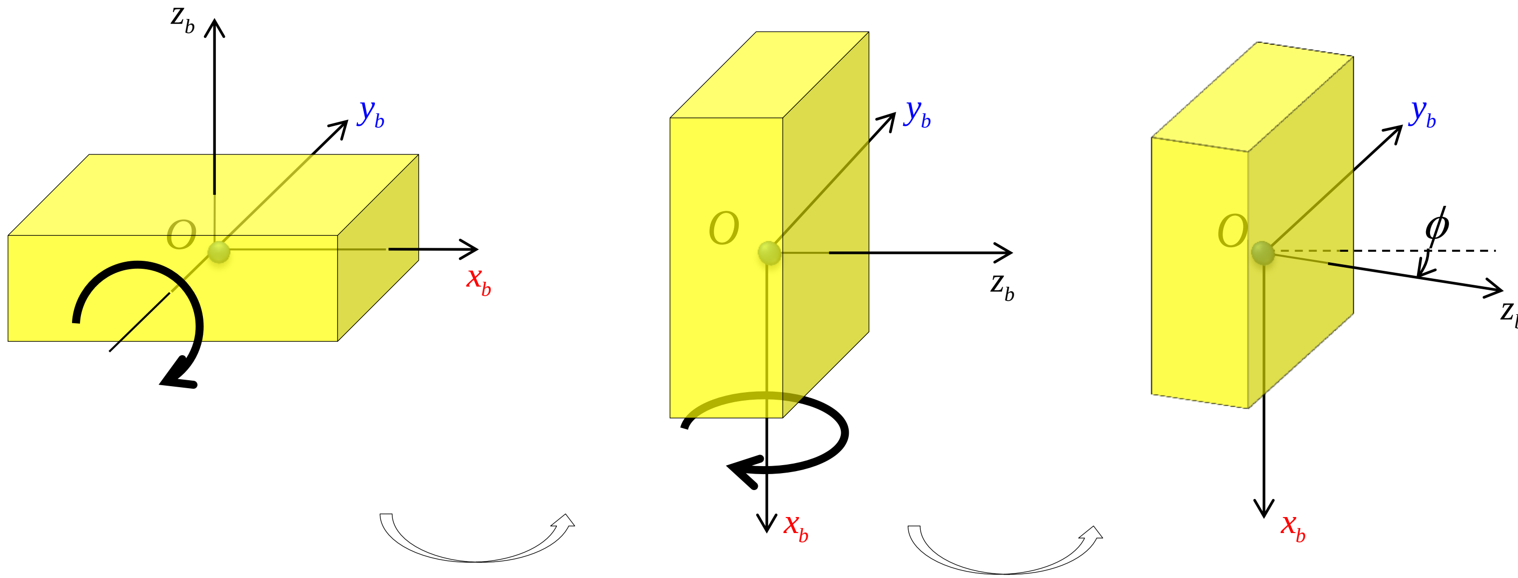
$$ {}^{n}\mathbf{r}_{P/E} = {}^{n}\mathbf{R}_{b}\,{}^{b}\mathbf{r}_{P/O} $$



1. 문제정의: 점 P는 n-frame과 함께 고정되어 있고 b-frame만 회전하는 경우
2. 점 P가 n-frame과 함께 고정되어 있으므로, 알고 있는 벡터는 n-frame에서 기술한 점 P의 위치벡터 ${}^{n}\mathbf{r}_{P/E}$
3. 최종적으로 구하고자 하는 벡터는 b-frame에서 기술한 점 P의 위치벡터 ${}^{b}\mathbf{r}_{P/O}$

Change in Transverse Moment due to Buoyant Force with respect to Heel after 90 degrees of Trim

Consider the ship, whose orientation is defined with ZYX Euler angle.



Trim of an angle 90°

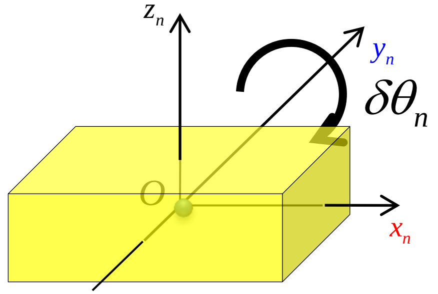
Heel of an angle ∅°

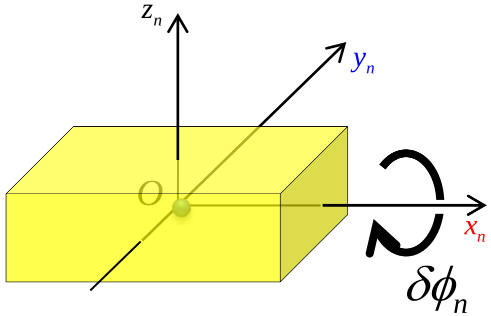
Heeling dose not generate any restoring force and moment after trim of an angle 90°.

선박이 90도만큼 Trim한 뒤에는 Heel이 선박의 복원력을 발생시키지 못한다.

Change in Transverse Moment due to Buoyant Force with respect to Heel

1. 이 전에 유도한 3 by 3 메트릭스는 아래와 같이 z_n축이 수선면에 수직인 좌표계(water surface fixed coordinates system)의 각 축을 중심으로한 회전에 대하여 유도한 것이다.

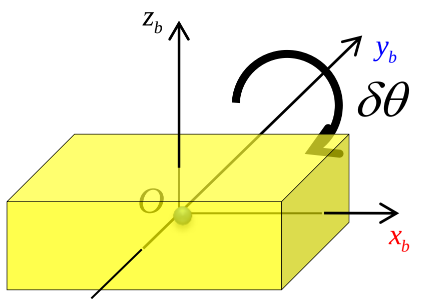


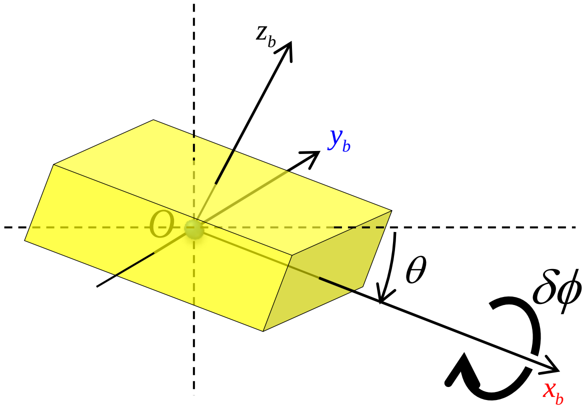


미소 회전을 아래와 같이 하나의 벡터로 표현하자

$$\begin{bmatrix} \delta\phi_n \\ \delta\theta_n \\ 0 \end{bmatrix}$$

2. 그러나 Euler angle을 사용하여 선박의 자세를 표현하면, 선박은 아래와 같이 회전한다.



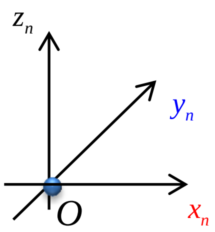


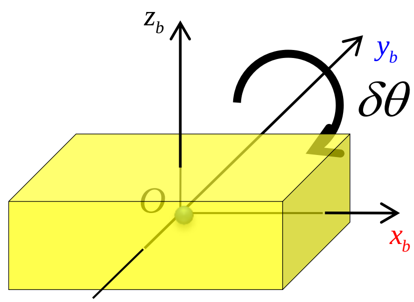
미소 회전을 아래와 같이 하나의 벡터로 표현하자

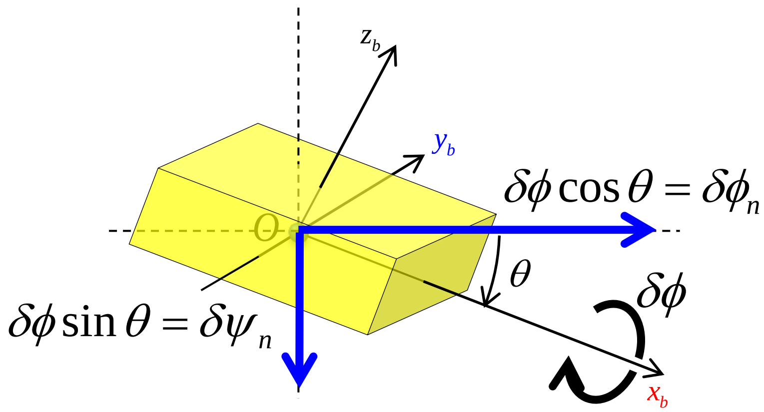
$$\begin{bmatrix} \delta\phi \\ \delta\theta \\ 0 \end{bmatrix}$$

위의 두 미소 회전 사이의 관계는 어떻게 될까?

Transformation between water surface fixed coordinates system and Euler angle







$$\delta\theta\mathbf{j}_b = \delta\theta\mathbf{j}_n \qquad \delta\phi\mathbf{i}_b = \delta\phi\cos\theta\mathbf{i}_n + \delta\phi\sin\theta\mathbf{k}_n$$

$$\delta\theta\mathbf{j}_b + \delta\phi\mathbf{i}_b = \boxed{\delta\phi\cos\theta}\mathbf{i}_n + \boxed{\delta\theta}\mathbf{j}_n + \delta\phi\sin\theta\mathbf{k}_n$$
$$= \boxed{\delta\phi_n}\mathbf{i}_n + \boxed{\delta\theta_n}\mathbf{j}_n + \delta\psi_n\mathbf{k}_n$$

$$\begin{bmatrix} \boxed{\delta\phi_n} \\ \boxed{\delta\theta_n} \end{bmatrix} = \begin{bmatrix} \boxed{\delta\phi\cos\theta} \\ \boxed{\delta\theta} \end{bmatrix} \Rightarrow \begin{bmatrix} \delta\phi_n \\ \delta\theta_n \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \delta\phi \\ \delta\theta \end{bmatrix}$$

Derivation of Partial Derivatives Associated with Hydrostatic Equilibrium
Considering Euler Angle

$$\begin{bmatrix} \delta\phi_n \\ \delta\theta_n \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \delta\phi \\ \delta\theta \end{bmatrix}$$

$$\begin{bmatrix} -\rho g A_{WP} & -\rho g T_{WP} & \rho g L_{WP} \\ -\rho g T_{WP} & -\rho g \left({}^{n}z_{B/E}\nabla + I_T \right) & \rho g I_P \\ \rho g L_{WP} & \rho g I_P & -\rho g \left({}^{n}z_{B/E}\nabla + I_L \right) \end{bmatrix} \begin{bmatrix} \delta z_n \\ \delta\phi_n \\ \delta\theta_n \end{bmatrix}$$

$$= \begin{bmatrix} -\rho g A_{WP} & -\rho g T_{WP} & \rho g L_{WP} \\ -\rho g T_{WP} & -\rho g \left({}^{n}z_{B/E}\nabla + I_T \right) & \rho g I_P \\ \rho g L_{WP} & \rho g I_P & -\rho g \left({}^{n}z_{B/E}\nabla + I_L \right) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \delta z \\ \delta\phi \\ \delta\theta \end{bmatrix}$$

$$= \begin{bmatrix} -\rho g A_{WP} & -\rho g T_{WP}\cos\theta & \rho g L_{WP} \\ -\rho g T_{WP} & -\rho g \left({}^{n}z_{B/E}\nabla + I_T \right)\cos\theta & \rho g I_P \\ \rho g L_{WP} & \rho g I_P \cos\theta & -\rho g \left({}^{n}z_{B/E}\nabla + I_L \right) \end{bmatrix} \begin{bmatrix} \delta z \\ \delta\phi \\ \delta\theta \end{bmatrix}$$

CHAPTER 11 COUPLED IMMERSION, HEEL, AND TRIM

Governing Equations of Computational Ship Stability

$$
\begin{bmatrix} \cancel{F_z} - F_z(z_{n0},\phi_0,\theta_0) \\ \cancel{M_T} - M_T(z_{n0},\phi_0,\theta_0) \\ \cancel{M_L} - M_L(z_{n0},\phi_0,\theta_0) \end{bmatrix}
=
\left[\begin{array}{c|c|c}
\dfrac{\partial F_B}{\partial z_n}+\dfrac{\partial F_G}{\partial z_n}+\dfrac{\partial F_{ext}}{\partial z_n} & \dfrac{\partial F_B}{\partial \phi}+\dfrac{\partial F_G}{\partial \phi}+\dfrac{\partial F_{ext}}{\partial \phi} & \dfrac{\partial F_B}{\partial \theta}+\dfrac{\partial F_G}{\partial \theta}+\dfrac{\partial F_{ext}}{\partial \theta} \\
\boxed{= -\rho g A_{WP}} & \boxed{= -\rho g A_{WP}\, {}^{n}y_{F/E}} & \boxed{= \rho g A_{WP}\, {}^{n}x_{F/E}} \\
\hline
\dfrac{\partial M_{BT}}{\partial z_n}+\dfrac{\partial M_{GT}}{\partial z_n}+\dfrac{\partial M_{extT}}{\partial z_n} & \dfrac{\partial M_{BT}}{\partial \phi}+\dfrac{\partial M_{GT}}{\partial \phi}+\dfrac{\partial M_{extT}}{\partial \phi} & \dfrac{\partial M_{BT}}{\partial \theta}+\dfrac{\partial M_{GT}}{\partial \theta}+\dfrac{\partial M_{extT}}{\partial \theta} \\
\boxed{= -\rho g A_{WP}\, {}^{n}y_{F/E}} & \boxed{\begin{array}{l} = -\rho g\left({}^{n}z_{B/E}\nabla + I_T\right) \\ -{}^{n}z_{G/E}\cdot F_G - {}^{n}z_{G_{ext}/E}\cdot F_{ext}\end{array}} & \boxed{= \rho g I_P} \\
\hline
\dfrac{\partial M_{BL}}{\partial z_n}+\dfrac{\partial M_{GL}}{\partial z_n}+\dfrac{\partial M_{extL}}{\partial z_n} & \dfrac{\partial M_{BL}}{\partial \phi}+\dfrac{\partial M_{GL}}{\partial \phi}+\dfrac{\partial M_{extL}}{\partial \phi} & \dfrac{\partial M_{BL}}{\partial \theta}+\dfrac{\partial M_{GL}}{\partial \theta}+\dfrac{\partial M_{extL}}{\partial \theta} \\
\boxed{= \rho g A_{WP}\, {}^{n}x_{F/E}} & \boxed{= \rho g I_P} & \boxed{\begin{array}{l} = -\rho g\left({}^{n}z_{B/E}\nabla + I_L\right) \\ -{}^{n}z_{G/E}\cdot F_G - {}^{n}z_{ext./E}\cdot F_{ext}\end{array}}
\end{array}\right]_{\substack{z_n=z_{n0}\\ \phi=\phi_0\\ \theta=\theta_0}}
\begin{bmatrix}\delta z_n \\ \delta\phi \\ \delta\theta\end{bmatrix}
$$

$$= 0$$

We want to find the static equilibrium position and orientation!

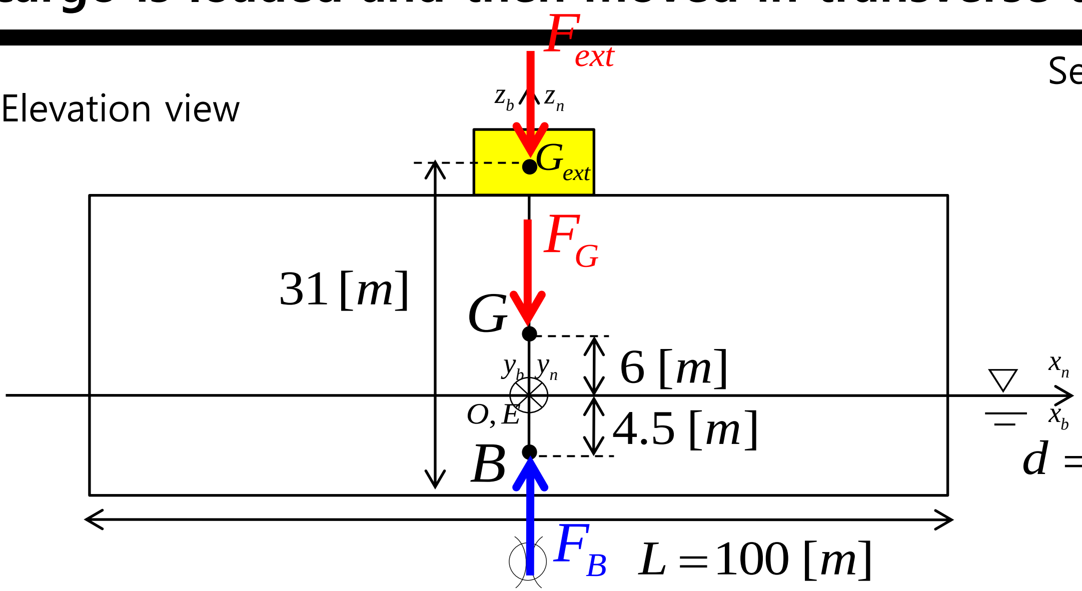
F_G : the weight of the ship
M_T : the transverse moment of the ship about x_n axis(through point E)
M_L : the longitudinal moment of the ship about y_n axis(through point E) z_G,
A_{WP} : the waterplane area of the ship **at current position**
I_T : the transverse moment of inertia of the waterplane area of the ship about x_n axis (through point E)
I_L : the longitudinal moment of inertia of the waterplane area of the ship about y_n axis (through point E)
I_P : the centrifugal moment of the waterplane area of the ship about x_n and y_n axis (through point E)
F_B : the buoyant force exerted on the ship
F_{ext} : the external force exerted on the ship

x_F, y_F : the x and y coordinates of the center of waterplane area of the ship in the x_n, y_n, z_n frame
z_G, z_B : the z coordinates of the center of gravity and the displacement volume of the ship in the x_n, y_n, z_n frame
δz_n : the change in draft
$\delta\phi$: the change in angle of heel
$\delta\theta$: the change in angle of trim

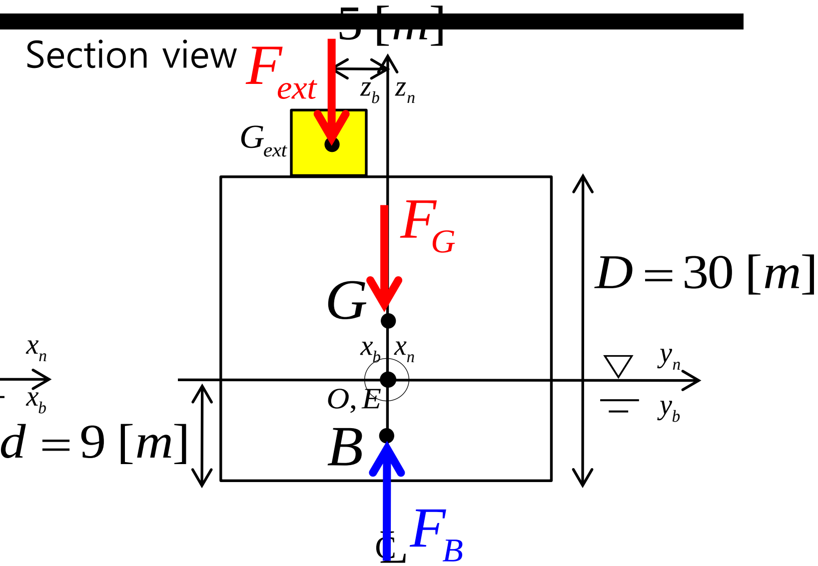
11-1 Coupled Immersion and Heel

Example of Finding Coupled Immersion and Heel of a box-shaped barge when a cargo is loaded and then moved in transverse direction

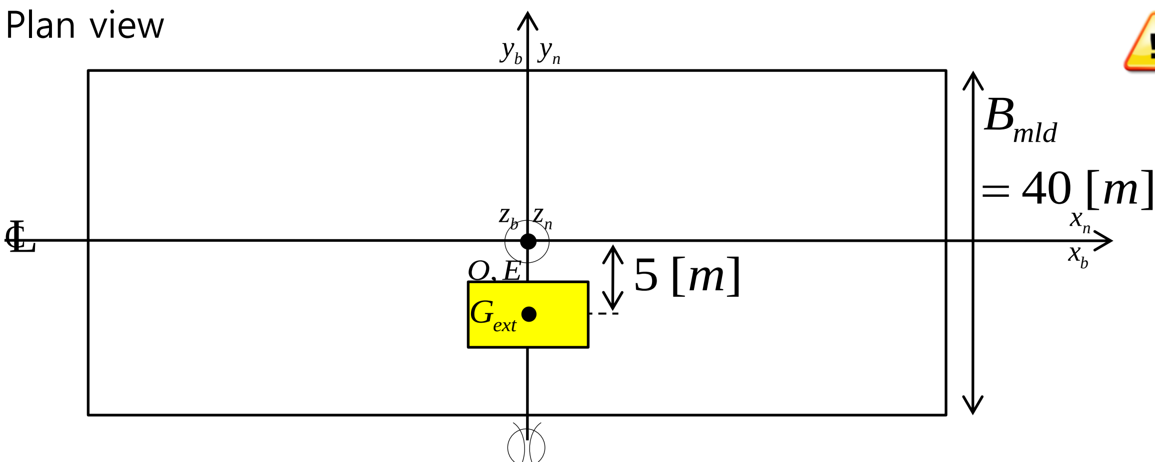
Elevation view



Section view



Plan view



In this case, **the external force** is given as a loaded cargo which is not included in the weight of the ship.

$L = 100\ [m]$	${}^{n}\mathbf{r}_{G/E} = [0 \quad 0 \quad 6]^T\ [m]$
$B_{mld} = 40\ [m]$	${}^{n}\mathbf{r}_{B/E} = [0 \quad 0 \quad -4.5]^T\ [m]$
$D = 30\ [m]$	${}^{n}\mathbf{r}_{G_{ext}/E} = [0 \quad -5 \quad 22]^T\ [m]$
$d = 9\ [m]$	

$$F_{G,z} = -3.6 \times 10^5\ [kN]$$

$$F_{ext,z} = -4.0 \times 10^4\ [kN]$$

$$\rho g = 10\ [Mg/m^2 s^2]$$

n-frame : **water surface-fixed** reference frame (Inertial reference frame), the x_n, y_n, z_n frame

b-frame : body-fixed reference frame, the x_b, y_b, z_b frame

F_G : the weight of the ship

F_B : the buoyant force exerted on the ship (initial state)

F_{ext} : the weight of the loaded cargo as an external force exerted on the ship

In this case, there is no force that causes the ship to rotate about y_n axis (through point E). Thus we use two equations in the governing equations.

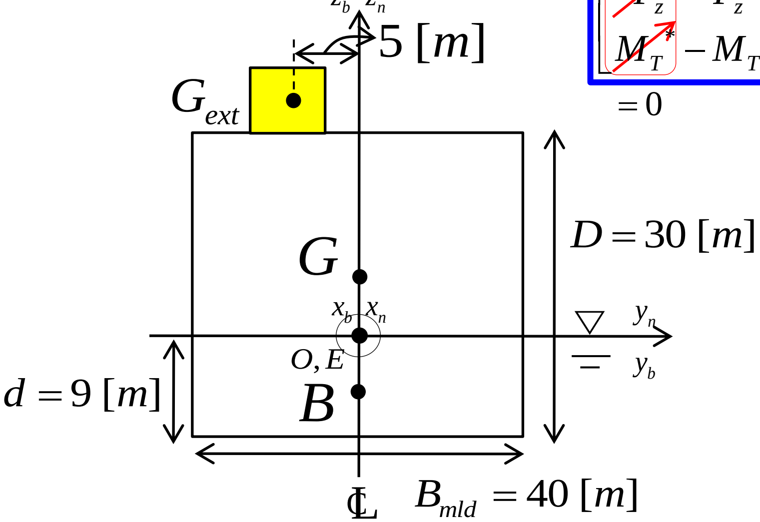
$$
\begin{bmatrix} \cancel{F_z^*} - F_z(z_{n0},\phi_0,\theta_0) \\ \cancel{M_T^*} - M_T(z_{n0},\phi_0,\theta_0) \\ \cancel{M_L^*} - M_L(z_{n0},\phi_0,\theta_0) \end{bmatrix}
=
\begin{bmatrix}
\begin{array}{c} \dfrac{\partial F_B}{\partial z_n}+\dfrac{\partial F_G}{\partial z_n}+\dfrac{\partial F_{ext}}{\partial z_n} \\ \boxed{=-\rho g A_{WP}} \end{array} &
\begin{array}{c} \dfrac{\partial F_B}{\partial \phi}+\dfrac{\partial F_G}{\partial \phi}+\dfrac{\partial F_{ext}}{\partial \phi} \\ \boxed{=-\rho g A_{WP}\,{}^{n}y_{F/E}} \end{array} &
\begin{array}{c} \dfrac{\partial F_B}{\partial \theta}+\dfrac{\partial F_G}{\partial \theta}+\dfrac{\partial F_{ext}}{\partial \theta} \\ \boxed{=\rho g A_{WP}\,{}^{n}x_{F/E}} \end{array} \\
\begin{array}{c} \dfrac{\partial M_{BT}}{\partial z_n}+\dfrac{\partial M_{GT}}{\partial z_n}+\dfrac{\partial M_{extT}}{\partial z_n} \\ \boxed{} \end{array} &
\begin{array}{c} \dfrac{\partial M_{BT}}{\partial \phi}+\dfrac{\partial M_{GT}}{\partial \phi}+\dfrac{\partial M_{extT}}{\partial \phi} \\ \boxed{\begin{array}{l} =-\rho g\left({}^{n}z_{B/E}\nabla+I_T\right) \\ \quad -{}^{n}z_{G/E}\cdot F_G-{}^{n}z_{G_{ext}/E}\cdot F_{ext} \end{array}} \end{array} &
\begin{array}{c} \dfrac{\partial M_{BT}}{\partial \theta}+\dfrac{\partial M_{GT}}{\partial \theta}+\dfrac{\partial M_{extT}}{\partial \theta} \\ \boxed{=\rho g I_P} \end{array} \\
\begin{array}{c} \dfrac{\partial M_{BL}}{\partial z_n}+\dfrac{\partial M_{GL}}{\partial z_n}+\dfrac{\partial M_{extL}}{\partial z_n} \\ \boxed{=\rho g A_{WP}\,{}^{n}x_{F/E}} \end{array} &
\begin{array}{c} \dfrac{\partial M_{BL}}{\partial \phi}+\dfrac{\partial M_{GL}}{\partial \phi}+\dfrac{\partial M_{extL}}{\partial \phi} \\ \boxed{=\rho g I_P} \end{array} &
\begin{array}{c} \dfrac{\partial M_{BL}}{\partial \theta}+\dfrac{\partial M_{GL}}{\partial \theta}+\dfrac{\partial M_{extL}}{\partial \theta} \\ \boxed{\begin{array}{l} =-\rho g\left({}^{n}z_{B/E}\nabla+I_L\right) \\ \quad -{}^{n}z_{G/E}\cdot F_G-{}^{n}z_{ext./E}\cdot F_{ext} \end{array}} \end{array}
\end{bmatrix}_{\substack{z_n=z_{n0}\\ \phi=\phi_0\\ \theta=\theta_0}}
\begin{bmatrix} \delta z_n \\ \delta\phi \\ \delta\theta \end{bmatrix}
$$

$= 0$

We want to find the position and orientation in the static equilibrium state.

1^{st} Iteration

1. Calculation of Force and Moments



$L = 100\ [m]$ $B_{mld} = 40\ [m]$ $D = 30\ [m]$ $d = 9\ [m]$	${}^{n}\mathbf{r}_{G/E} = [0 \quad 0 \quad 6]^T\ [m]$ ${}^{n}\mathbf{r}_{B/E} = [0 \quad 0 \quad -4.5]^T\ [m]$ ${}^{n}\mathbf{r}_{G_{ext}/E} = [0 \quad -5 \quad 22]^T\ [m]$

$F_{G,z} = -3.6\times10^5\ [kN]$

$F_{ext,z} = -4.0\times10^4\ [kN]$

$\rho g = 10\ [Mg/m^2s^2]$

We have to use the values of the current floating position!

$$\boxed{\begin{bmatrix} \cancel{F_z^*} - F_z \\ \cancel{M_T^*} - M_T \end{bmatrix}} = \begin{bmatrix} -\rho g A_{WP} & -\rho g A_{WP}\, {}^{n}y_{F/E} \\ -\rho g A_{WP}\, {}^{n}y_{F/E} & -\rho g\left({}^{n}z_{B/E}\nabla + I_T\right) - {}^{n}z_{G/E}\cdot F_{G,z} - {}^{n}z_{G_{ext}/E}\cdot F_{ext,z} \end{bmatrix} \begin{bmatrix} \delta z_n \\ \delta\phi \end{bmatrix}$$

$= 0$

$$F_z = F_{B,z} + F_{G,z} + F_{ext,z}$$

$$= \rho g L B_{mld} d + F_{G,z} + F_{ext,z}$$

$$= 10\cdot100\cdot40\cdot9 - 3.6\times10^5 - 4.0\times10^4$$

$$= \boxed{3.6\times10^5 - 3.6\times10^5} - 4.0\times10^4$$

$$= -4.0\times10^4\ [kN]$$

Because the ship is in static equilibrium, the buoyant force is equal and opposite to the weight of the ship.

$$M_T = M_{BT} + M_{GT} + M_{extT}$$

$$= {}^{n}y_{B/E}\cdot F_{B,z} + {}^{n}y_{G/E}\cdot F_{G,z} + {}^{n}y_{G_{ext}/E}\cdot F_{ext,z}$$

$$= \underline{0}\cdot(3.6\times10^5) + \underline{0}\cdot(-3.6\times10^5)$$

$$+ (-5)\cdot(-4.0\times10^4)$$

$$= +2.0\times10^5\ [kN\cdot m]$$

The initial center of buoyancy and the initial center of gravity lie on the center line of the ship.

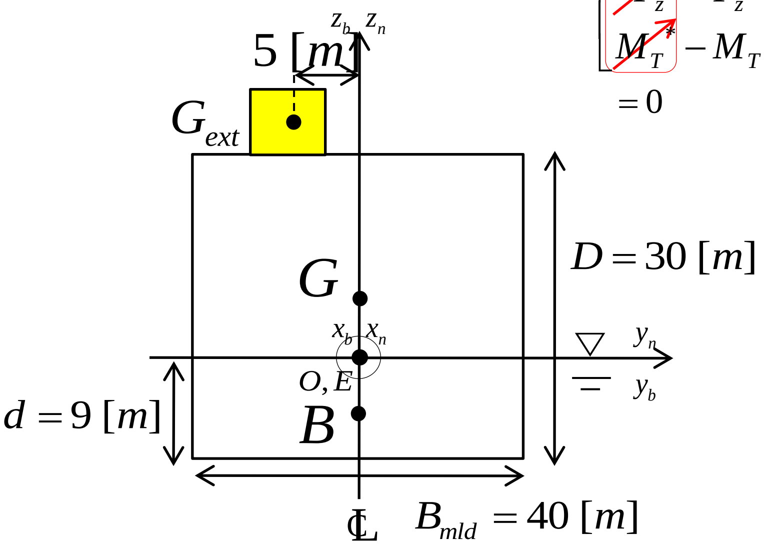
Calculation of the Waterplane area

(1st Iteration)

We use the values in current floating position!

$$\begin{bmatrix} \cancel{F_z^*} - F_z \\ \cancel{M_T^*} - M_T \end{bmatrix} = \boxed{\begin{bmatrix} -\rho g A_{WP} & -\rho g A_{WP}\,{}^n y_{F/E} \\ -\rho g A_{WP}\,{}^n y_{F/E} & -\rho g \left({}^n z_{B/E}\nabla + I_T\right) - {}^n z_{G/E}\cdot F_{G,z} - {}^n z_{G_{ext}/E}\cdot F_{ext,z} \end{bmatrix}} \begin{bmatrix} \delta z_n \\ \delta\phi \end{bmatrix}$$

$= 0$



$$-\rho g A_{WP} = -\rho g L B_{mld} = -10\cdot 100\cdot 40$$
$$= -4.0\times 10^4\ [kN/m]$$

$$\underbrace{\cancel{F_z^*}}_{=0} - F_z = -\rho g A_{WP}.\delta z_n - \rho g A_{WP}\,{}^n y_{F/E}\cdot\delta\phi$$

$$\frac{\partial F_B}{\partial z} = \frac{\partial F_B}{\partial T} = -\rho g\cdot A_{wp}(T,\phi,\theta)_{T=T_0}$$

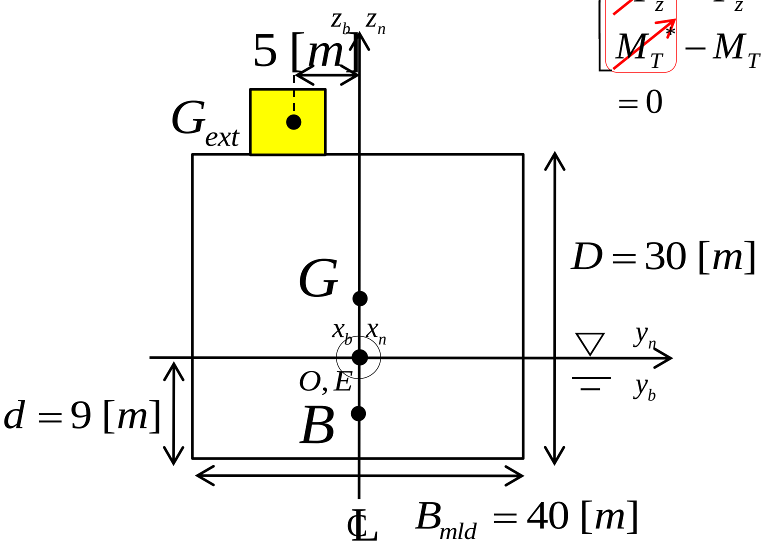
$L = 100\ [m]$	${}^n\mathbf{r}_{G/E} = [0\quad 0\quad 6]^T\ [m]$
$B_{mld} = 40\ [m]$	${}^n\mathbf{r}_{B/E} = [0\quad 0\quad -4.5]^T\ [m]$
$D = 30\ [m]$	${}^n\mathbf{r}_{G_{ext}/E} = [0\quad -5\quad 22]^T\ [m]$
$d = 9\ [m]$	

$$F_{G,z} = -3.6\times 10^5\ [kN]$$
$$F_{B,z} = 3.6\times 10^5\ [kN]$$
$$F_{ext,z} = -4.0\times 10^4\ [kN]$$
$$\rho g = 10\ [Mg/m^2 s^2]$$

We use the values in current floating position!

$$\begin{bmatrix} \cancel{F_z^*} - F_z \\ \cancel{M_T^*} - M_T \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP} & -\rho g A_{WP}\, {}^n y_{F/E} \\ -\rho g A_{WP}\, {}^n y_{F/E} & -\rho g \left({}^n z_{B/E} \nabla + I_T \right) - {}^n z_{G/E} \cdot F_{G,z} - {}^n z_{G_{ext}/E} \cdot F_{ext,z} \end{bmatrix} \begin{bmatrix} \delta z_n \\ \delta \phi \end{bmatrix}$$

$=0$



$$-\rho g A_{WP}\, {}^n y_{F/E} = -10 \cdot (4.0 \times 10^4) \cdot 0$$
$$= 0 [kN]$$

$$\cancel{F_z^*} - F_z = -\rho g A_{WP} \cdot \delta z_n \underline{- \rho g A_{WP}\, {}^n y_{F/E}} \cdot \delta \phi$$

$=0$

$$\frac{\partial F_B}{\partial \phi} = -\rho g \cdot A_{wp}(T, \phi, \theta)_{\phi=\phi_0} \cdot {}^n y_{F/E}$$

$L = 100\ [m]$

$B_{mld} = 40\ [m]$

$D = 30\ [m]$

$d = 9\ [m]$

${}^n \mathbf{r}_{G/E} = [0 \quad 0 \quad 6]^T\ [m]$

${}^n \mathbf{r}_{B/E} = [0 \quad 0 \quad -4.5]^T\ [m]$

${}^n \mathbf{r}_{G_{ext}/E} = [0 \quad -5 \quad 22]^T\ [m]$

$F_{G,z} = -3.6 \times 10^5\ [kN]$

$F_{B,z} = 3.6 \times 10^5\ [kN]$

$F_{ext,z} = -4.0 \times 10^4\ [kN]$

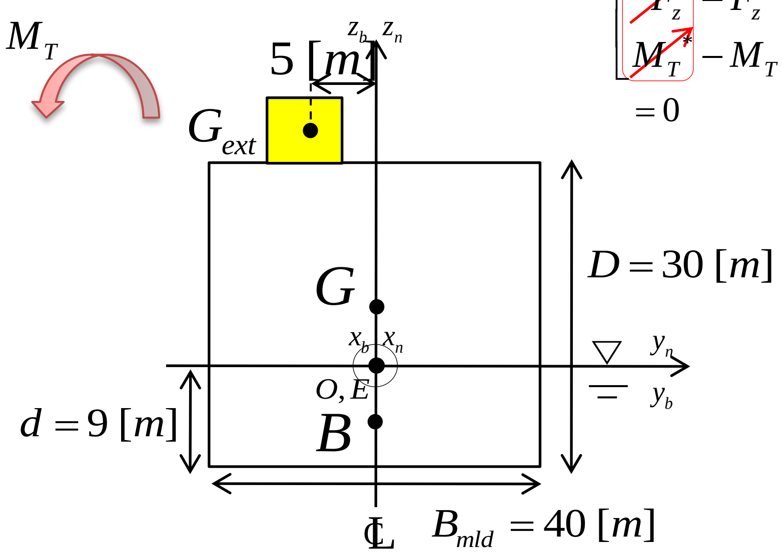
$\rho g = 10\ [Mg / m^2 s^2]$

(1st Iteration)

We use the values in current floating position!

$$\begin{bmatrix} \cancel{F_z^*} - F_z \\ \cancel{M_T^*} - M_T \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP} & -\rho g A_{WP}\, {}^n y_{F/E} \\ -\rho g A_{WP}\, {}^n y_{F/E} & -\rho g\left({}^n z_{B/E}\nabla + I_T\right) - {}^n z_{G/E}\cdot F_{G,z} - {}^n z_{G_{ext}/E}\cdot F_{ext,z} \end{bmatrix} \begin{bmatrix} \delta z_n \\ \delta\phi \end{bmatrix}$$

$= 0$



$$-\rho g\left({}^n z_{B/E}\nabla + I_T\right) = -\rho g\left({}^n z_{B/E}\cdot LB_{mld}d + \frac{LB_{mld}^3}{12}\right)$$

$$= -10\left((-4.5)100\cdot 40\cdot 9 + \frac{100\cdot 40^3}{12}\right) = -3.71\times 10^6\ [kN\cdot m]$$

$$-{}^n z_{G/E}\cdot F_{G,z} = -6\cdot(-3.6\times 10^5) = 2.16\times 10^6\ [kN\cdot m]$$

$$-{}^n z_{ext/E}\cdot F_{ext,z} = -22\cdot(-4.0\times 10^4) = 8.8\times 10^5\ [kN\cdot m]$$

$$-\rho g\left({}^n z_{B/E}\nabla + I_T\right) - {}^n z_{G/E}\cdot F_{G,z} - {}^n z_{ext/E}\cdot F_{ext,z}$$

$$= -3.71\times 10^6 + 2.16\times 10^6 + 8.8\times 10^5$$

$$= \underline{-6.7\times 10^5\ [kN\cdot m]}$$

$L = 100\ [m]$

$B_{mld} = 40\ [m]$

$D = 30\ [m]$

$d = 9\ [m]$

${}^n\mathbf{r}_{G/E} = [0 \quad 0 \quad 6]^T\ [m]$

${}^n\mathbf{r}_{B/E} = [0 \quad 0 \quad -4.5]^T\ [m]$

${}^n\mathbf{r}_{G_{ext}/E} = [0 \quad -5 \quad 22]^T\ [m]$

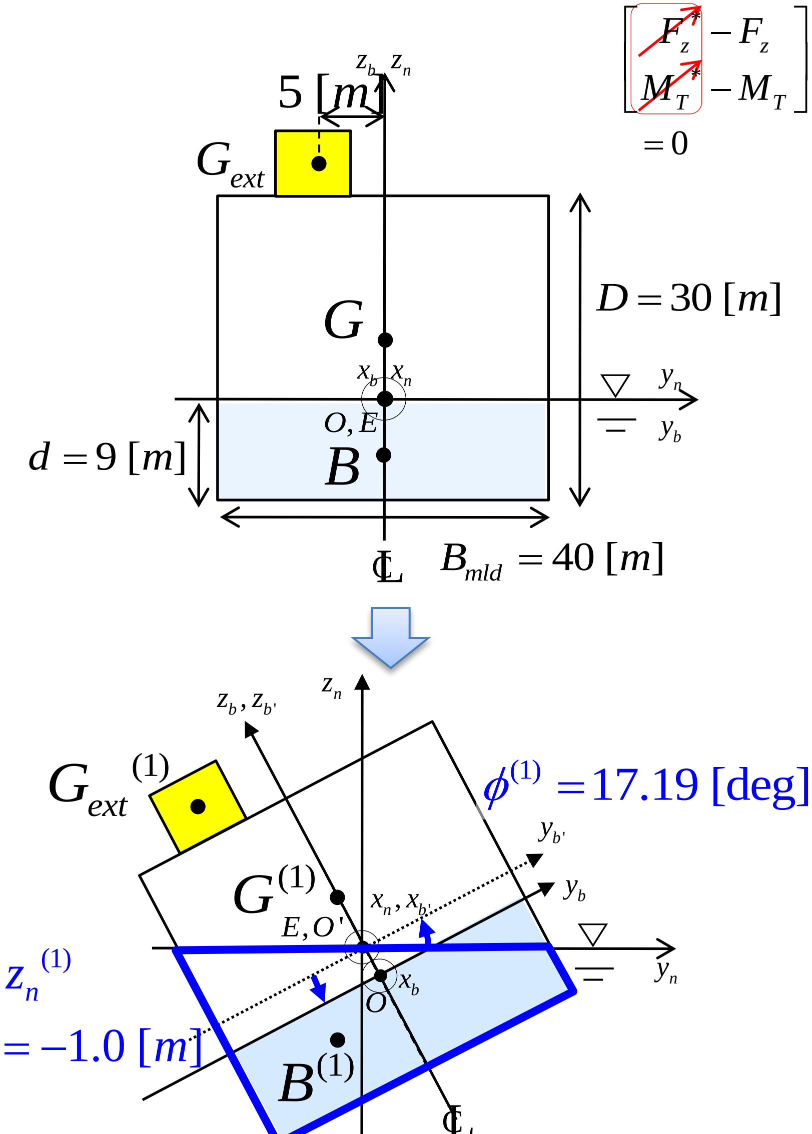
$F_{G,z} = -3.6\times 10^5\ [kN]$

$F_{B,z} = 3.6\times 10^5\ [kN]$

$F_{ext,z} = -4.0\times 10^4\ [kN]$

$\rho g = 10\ [Mg/m^2 s^2]$

(1st Iteration)



$$\begin{bmatrix} \cancel{F_z^*} - F_z \\ \cancel{M_T^*} - M_T \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP} & -\rho g A_{WP}\, {}^n y_{F/E} \\ -\rho g A_{WP}\, {}^n y_{F/E} & -\rho g \left({}^n z_{B/E} \nabla + I_T \right) - {}^n z_{G/E} \cdot F_{G,z} - {}^n z_{G_{ext}/E} \cdot F_{ext,z} \end{bmatrix} \boxed{\begin{bmatrix} \delta z_n \\ \delta \phi \end{bmatrix}}$$

$$=0$$

$$\begin{bmatrix} 4.0\times10^4 \\ -2.0\times10^5 \end{bmatrix} = \begin{bmatrix} -4.0\times10^4 & 0 \\ 0 & -6.7\times10^5 \end{bmatrix} \begin{bmatrix} \delta z_n \\ \delta\phi \end{bmatrix}$$

$$\begin{bmatrix} \delta z_n \\ \delta\phi \end{bmatrix} = \begin{bmatrix} -4.0\times10^4 & 0 \\ 0 & -6.7\times10^5 \end{bmatrix}^{-1} \begin{bmatrix} 4.0\times10^4 \\ -2.0\times10^5 \end{bmatrix}$$

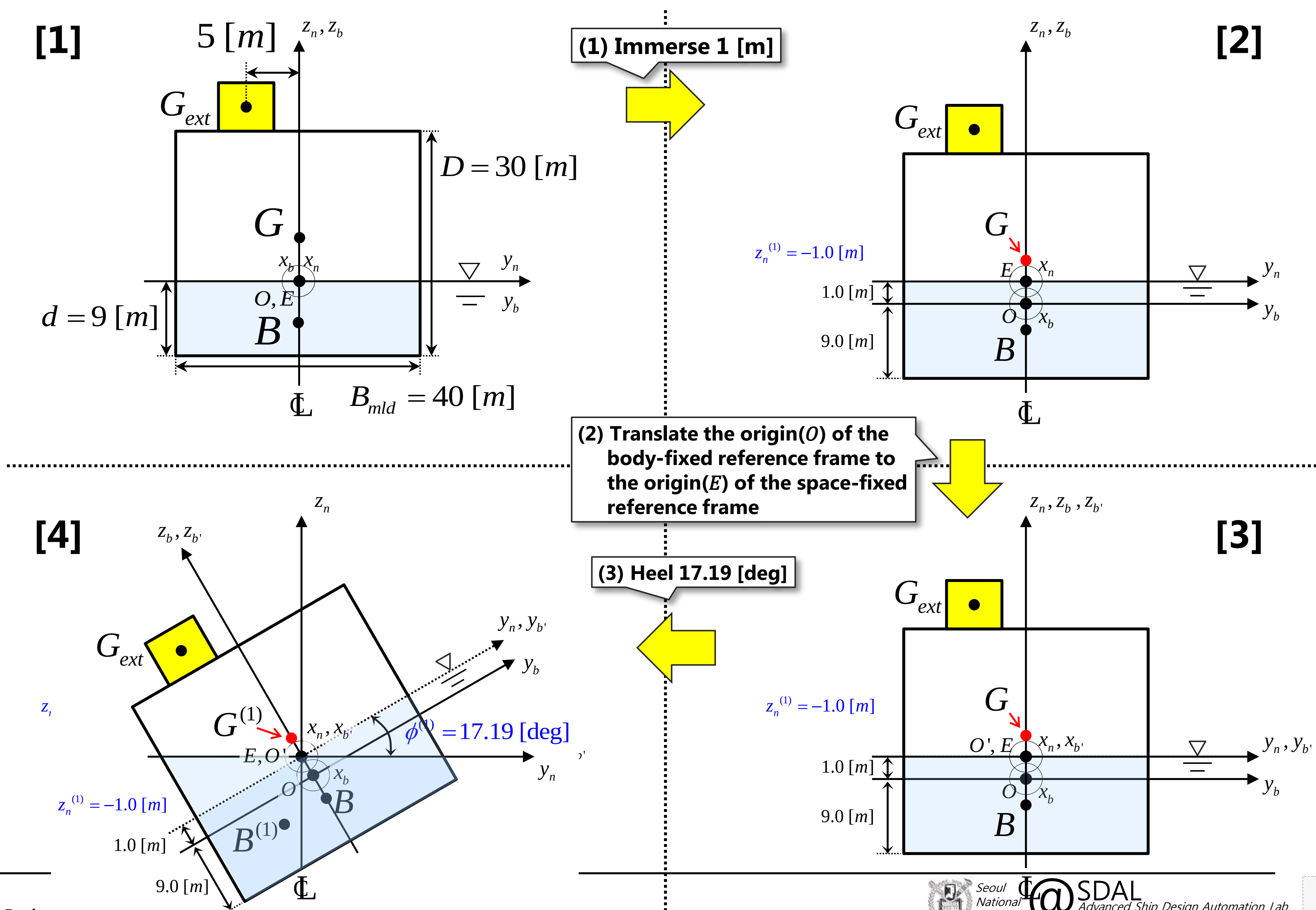
$$= \begin{bmatrix} -1.0\ [m] \\ 0.30\ [rad] \end{bmatrix} = \begin{bmatrix} -1.0\ [m] \\ 17.19\ [\deg] \end{bmatrix}$$

$$z_n^{(1)} = z_n^{(0)} + \delta z_n = 0 + (-1.0) = -1.0\ [m]$$

$$\phi^{(1)} = \phi^{(0)} + \delta\phi = 0 + (17.19) = 17.19\ [\deg]$$

To represent the rotational motion of a body, we translate the origin(o) of the body-fixed reference frame {b-frame; x_b, y_b, z_b} to the origin(E) of the space-fixed reference frame {E-frame; x_n, y_n, z_n}.
We define the translated body-fixed frame as {b'-frame}. Then we rotate the {b'-frame) w.r.t the space-fixed frame.

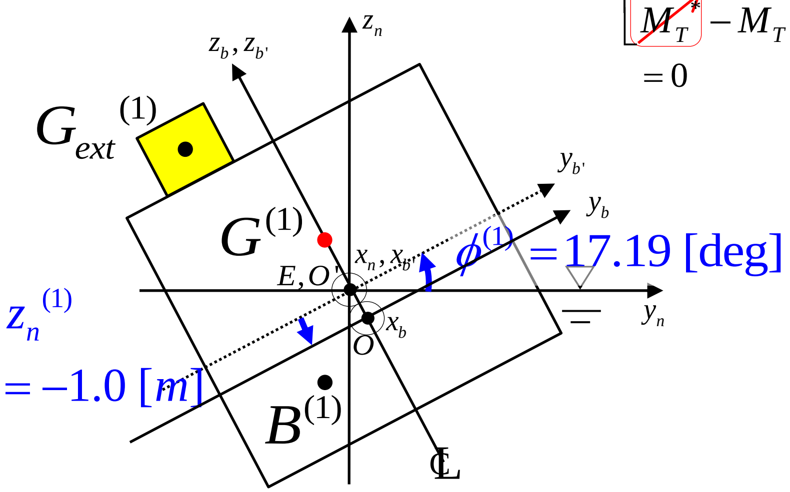
Calculation of Immersion and Heel

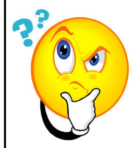


Check whether the Ship is in Static Equilibrium

(1st Iteration)

$$\underbrace{\begin{bmatrix} \cancel{F_z^*} - F_z \\ \cancel{M_T^*} - M_T \end{bmatrix}}_{=0} = \begin{bmatrix} -\rho g A_{WP} & -\rho g A_{WP} \, {}^n y_{F/E} \\ -\rho g A_{WP} \, {}^n y_{F/E} & -\rho g \left({}^n z_{B/E} \nabla + I_T \right) - {}^n z_{G/E} \cdot F_{G,z} - {}^n z_{G_{ext}/E} \cdot F_{ext,z} \end{bmatrix} \begin{bmatrix} \delta z_n \\ \delta \phi \end{bmatrix}$$





Is the ship in static equilibrium?

Let us check whether the ship is in static equilibrium!

$L = 100$ [m]

$B_{mld} = 40$ [m]

$D = 30$ [m]

$d = 9$ [m]

${}^n\mathbf{r}_{G/E} = [0 \quad 0 \quad 6]^T$ [m]

${}^n\mathbf{r}_{B/E} = [0 \quad 0 \quad -4.5]^T$ [m]

${}^n\mathbf{r}_{G_{ext}/E} = [0 \quad -5 \quad 22]^T$ [m]

$F_{G,z} = -3.6 \times 10^5$ [kN]

$F_{B,z} = 3.6 \times 10^5$ [kN]

$F_{ext,z} = -4.0 \times 10^4$ [kN]

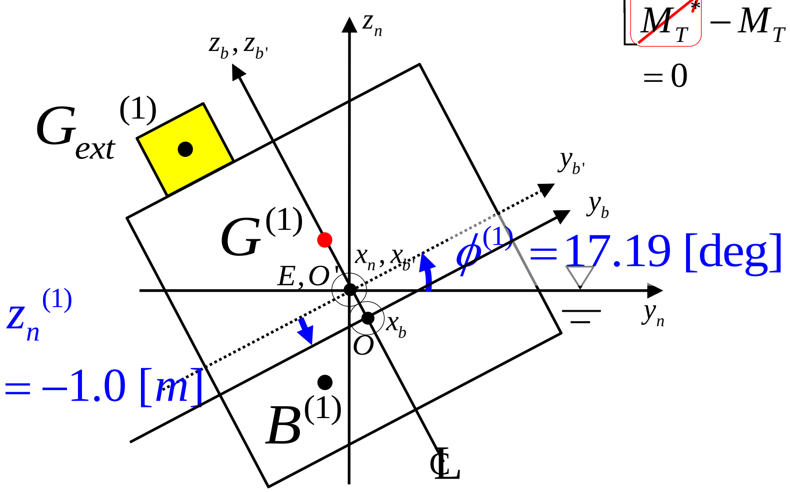
$\rho g = 10$ [Mg/m^2s^2]

Check for the force equilibrium

(1st Iteration)

$$\begin{bmatrix} \cancel{F_z^*} - F_z \\ \cancel{M_T^*} - M_T \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP} & -\rho g A_{WP}\,{}^n y_{F/E} \\ -\rho g A_{WP}\,{}^n y_{F/E} & -\rho g \left({}^n z_{B/E}\nabla + I_T\right) - {}^n z_{G/E}\cdot F_{G,z} - {}^n z_{G_{ext}/E}\cdot F_{ext,z} \end{bmatrix} \begin{bmatrix} \delta z_n \\ \delta\phi \end{bmatrix}$$

$$= 0$$



Is the ship in static equilibrium?

Force equilibrium :

For the convenience of the calculation, we rotate the b′-frame about xn-axis with an angle of $-\phi$ in clock wise direction, and calculate the center of buoyancy w.r.t. the b′-frame.

$L = 100\ [m]$, $B_{mld} = 40\ [m]$, $D = 30\ [m]$, $d = 9\ [m]$

${}^n\mathbf{r}_{G/E} = [0 \quad 0 \quad 6]^T\ [m]$

${}^n\mathbf{r}_{B/E} = [0 \quad 0 \quad -4.5]^T\ [m]$

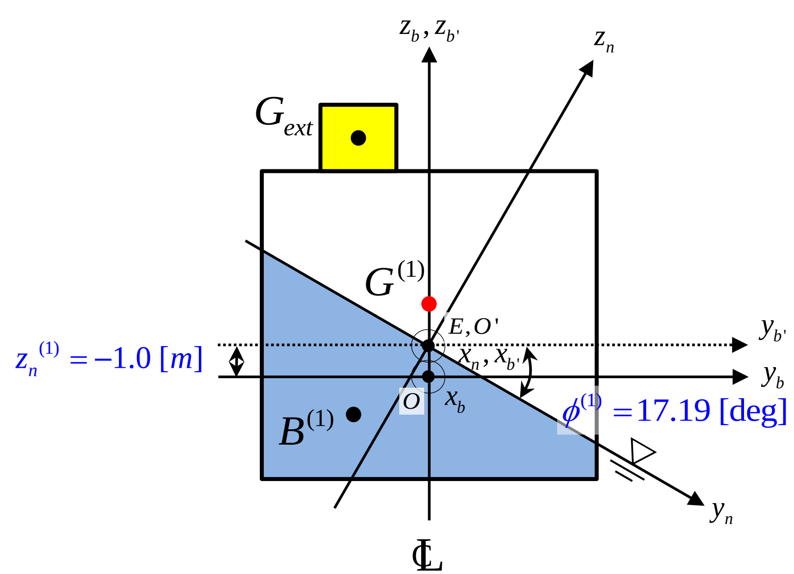
${}^n\mathbf{r}_{G_{ext}/E} = [0 \quad -5 \quad 22]^T\ [m]$

$F_{G,z} = -3.6\times10^5\ [kN]$

$F_{B,z}^{(1)} = 4.0\times10^5\ [kN]$

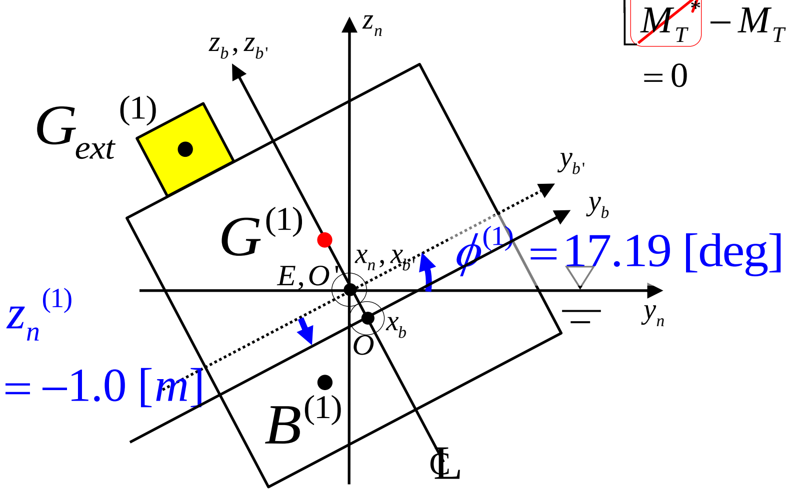
$F_{ext,z} = -4.0\times10^4\ [kN]$

$\rho g = 10\ [Mg/m^2s^2]$



$$\begin{bmatrix} \cancel{F_z^*} - F_z \\ \cancel{M_T^*} - M_T \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP} & -\rho g A_{WP} \, {}^n y_{F/E} \\ -\rho g A_{WP} \, {}^n y_{F/E} & -\rho g \left({}^n z_{B/E} \nabla + I_T \right) - {}^n z_{G/E} \cdot F_{G,z} - {}^n z_{G_{ext}/E} \cdot F_{ext,z} \end{bmatrix} \begin{bmatrix} \delta z_n \\ \delta \phi \end{bmatrix}$$

$= 0$



$L = 100$ [m], $B_{mld} = 40$ [m], $D = 30$ [m], $d = 9$ [m]

${}^n\mathbf{r}_{G/E} = [0 \quad 0 \quad 6]^T$ [m]

${}^n\mathbf{r}_{B/E} = [0 \quad 0 \quad -4.5]^T$ [m]

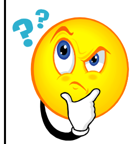
${}^n\mathbf{r}_{G_{ext}/E} = [0 \quad -5 \quad 22]^T$ [m]

$F_{G,z} = -3.6 \times 10^5$ [kN]

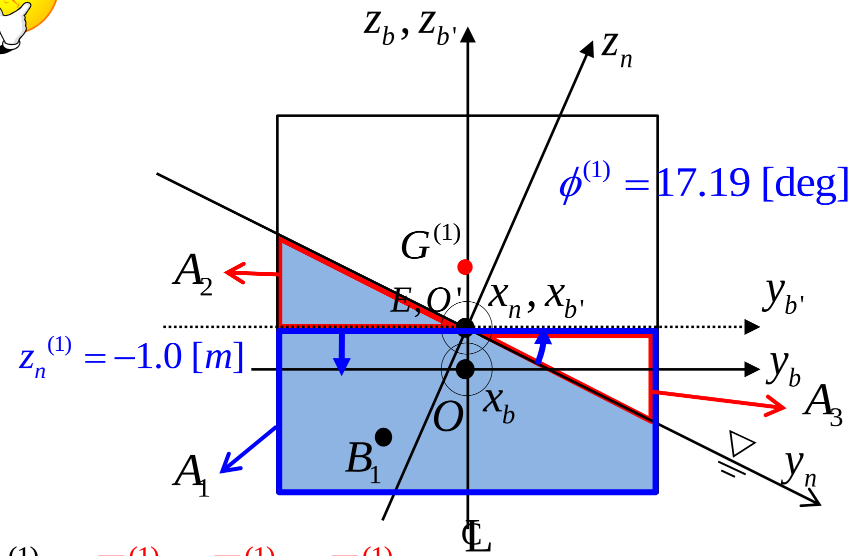
$F_{B,z}^{(1)} = 4.0 \times 10^5$ [kN]

$F_{ext,z} = -4.0 \times 10^4$ [kN]

$\rho g = 10$ [$Mg / m^2 s^2$]



Is the ship in static equilibrium?



$$F_z^{(1)} = F_{B,z}^{(1)} + F_{G,z}^{(1)} + F_{ext,z}^{(1)}$$

$$F_{B,z}^{(1)} = \rho g V = \rho g L A_{Section}$$

$$A_{Section} = A_1 + A_2 + A_3$$

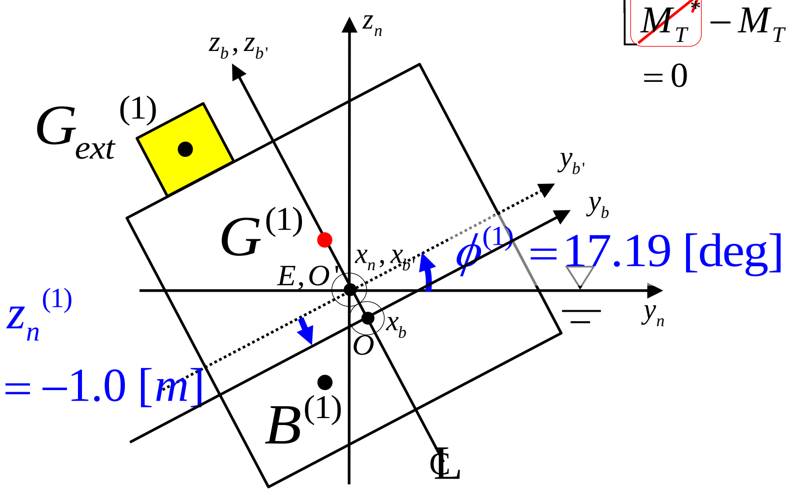
$$d_1 = \frac{B_{mld}}{2} \tan\left|\phi^{(1)}\right| = \frac{40}{2} \tan 17.19^\circ = 6.19 \text{ [m]}$$

$$A_{Section} = 400 + 61.9 + (-61.9) = 400 \text{ [m}^2\text{]}$$

$$F_{B,z}^{(1)} = \rho g V = 10 \cdot 100 \cdot 400 = 4.0 \times 10^5 \text{ [kN]}$$

$$\begin{bmatrix} \cancel{F_z^*} - F_z \\ \cancel{M_T^*} - M_T \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP} & -\rho g A_{WP}\, {}^n y_{F/E} \\ -\rho g A_{WP}\, {}^n y_{F/E} & -\rho g\left({}^n z_{B/E}\nabla + I_T\right) - {}^n z_{G/E}\cdot F_{G,z} - {}^n z_{G_{ext}/E}\cdot F_{ext,z} \end{bmatrix} \begin{bmatrix} \delta z_n \\ \delta\phi \end{bmatrix}$$

$= 0$



$L = 100$ [m]	${}^n\mathbf{r}_{G/E} = [0 \quad 0 \quad 6]^T$ [m]
$B_{mld} = 40$ [m]	${}^n\mathbf{r}_{B/E} = [0 \quad 0 \quad -4.5]^T$ [m]
$D = 30$ [m]	${}^n\mathbf{r}_{G_{ext}/E} = [0 \quad 5 \quad 22]^T$ [m]
$d = 9$ [m]	

$F_{G,z} = -3.6\times10^5$ [kN]

$F_{B,z}^{(1)} = 4.0\times10^5$ [kN]

$F_{ext,z} = -4.0\times10^4$ [kN]

$\rho g = 10$ [Mg / m²s²]

Is the ship in static equilibrium?

Force equilibrium :

$$F_z^{(1)} = F_{B,z}^{(1)} + F_{G,z}^{(1)} + F_{ext,z}^{(1)}$$

$$F_{B,z}^{(1)} = 4.0\times10^5 \ [kN]$$

$$F_{G,z}^{(1)} = F_{G,z} = -3.6\times10^5 \ [kN]$$

$$F_{ext,z}^{(1)} = F_{ext,z} = -4.0\times10^4 \ [kN]$$

$$F^{(1)} = 4.0\times10^5 - 3.6\times10^5 - 4.0\times10^4$$

$$= \boxed{0 \ [kN]} < e$$

Tolerance

where, e(epsilon) : an arbitrarily small positive quantity

The static equilibrium of force is satisfied!

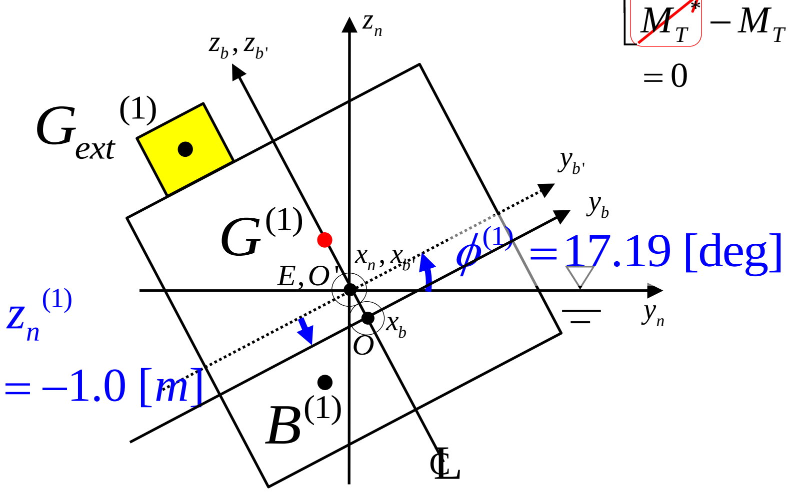
The immersion and heel are decoupled, since the section of the ship is rectangle.

Check for the moment equilibrium
-Calculation of the yb-coordinate of the submerged volume

(1st Iteration)

$$\begin{bmatrix} \cancel{F_z^*} - F_z \\ \cancel{M_T^*} - M_T \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP} & -\rho g A_{WP}\, {}^n y_{F/E} \\ -\rho g A_{WP}\, {}^n y_{F/E} & -\rho g \left({}^n z_{B/E} \nabla + I_T \right) - {}^n z_{G/E} \cdot F_{G,z} - {}^n z_{G_{ext}/E} \cdot F_{ext,z} \end{bmatrix} \begin{bmatrix} \delta z_n \\ \delta \phi \end{bmatrix}$$

$= 0$



$$M_T^{(1)} = M_{BT}^{(1)} + M_{GT}^{(1)} + M_{extT}^{(1)}$$

$$M_{BT}^{(1)} = {}^n y_{B^{(1)}/E} \cdot F_{B,z}^{(1)}$$

$${}^{b'} y_{B^{(1)}/O'} = \frac{{}^{b'} y_{c_1/O'} A_1 + {}^{b'} y_{c_2/O'} A_2 + {}^{b'} y_{c_3/O'} A_3}{A_1 + A_2 + A_3}$$

$$= \frac{0 \cdot 400 + (-13.33) \cdot (61.9) + (13.33) \cdot (-61.9)}{400 - 61.9 + 61.9}$$

$$= -4.13\ [m]$$

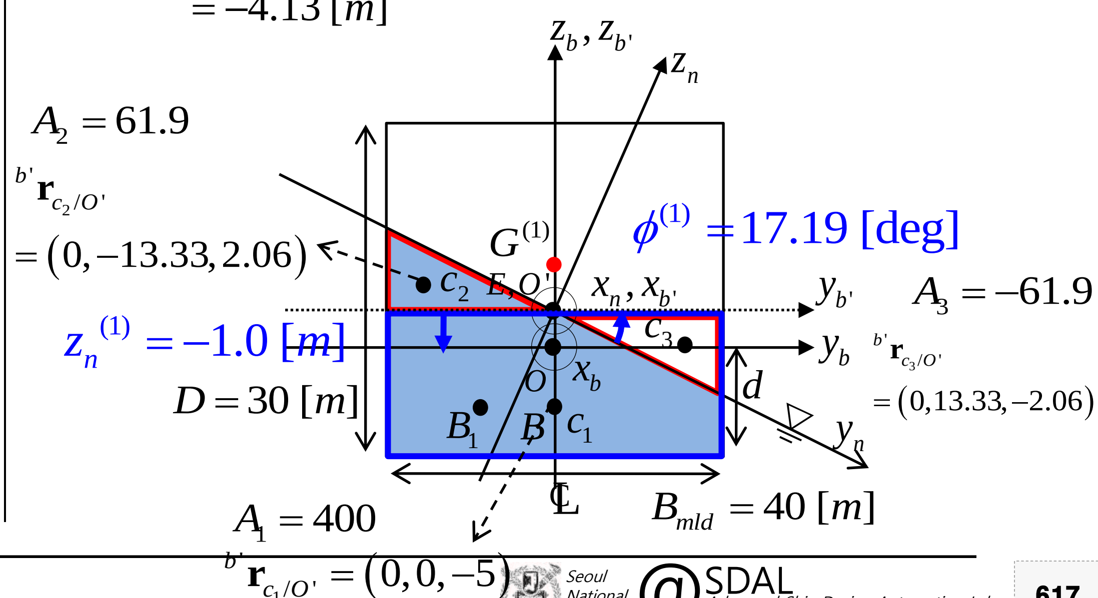
$L = 100\ [m]$	${}^n \mathbf{r}_{G/E} = [0 \quad 0 \quad 6]^T\ [m]$
$B_{mld} = 40\ [m]$	${}^n \mathbf{r}_{B/E} = [0 \quad 0 \quad -4.5]^T\ [m]$
$D = 30\ [m]$	${}^n \mathbf{r}_{G_{ext}/E} = [0 \quad 5 \quad 22]^T\ [m]$
$d = 9\ [m]$	

$F_G = -3.6 \times 10^5\ [kN]$

$F_B^{(1)} = 4.0 \times 10^5\ [kN]$

$F_{ext} = -4.0 \times 10^4\ [kN]$

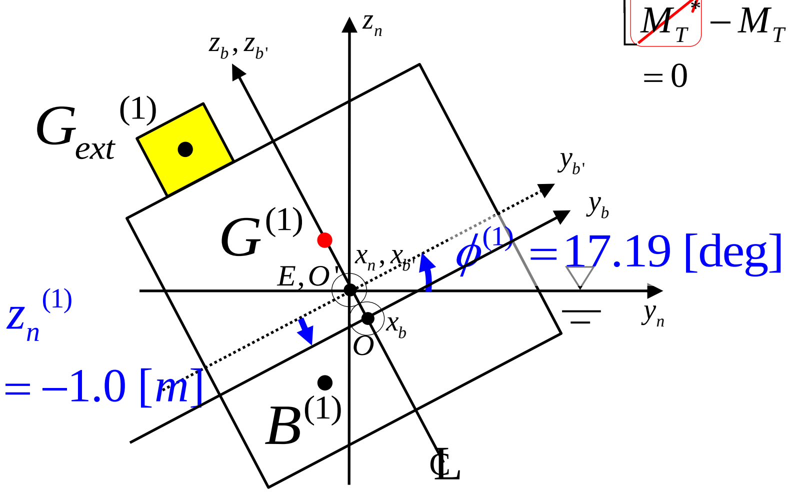
$\rho g = 10\ [Mg / m^2 s^2]$



Calculation of the zb-coordinate of the center of the submerged volume (1st Iteration)

$$\begin{bmatrix} \cancel{F_z^*} - F_z \\ \cancel{M_T^*} - M_T \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP} & -\rho g A_{WP}\,{}^{n}y_{F/E} \\ -\rho g A_{WP}\,{}^{n}y_{F/E} & -\rho g\left({}^{n}z_{B/E}\nabla + I_T\right) - {}^{n}z_{G/E}\cdot F_{G,z} - {}^{n}z_{G_{ext}/E}\cdot F_{ext,z} \end{bmatrix} \begin{bmatrix} \delta z_n \\ \delta\phi \end{bmatrix}$$

$$= 0$$



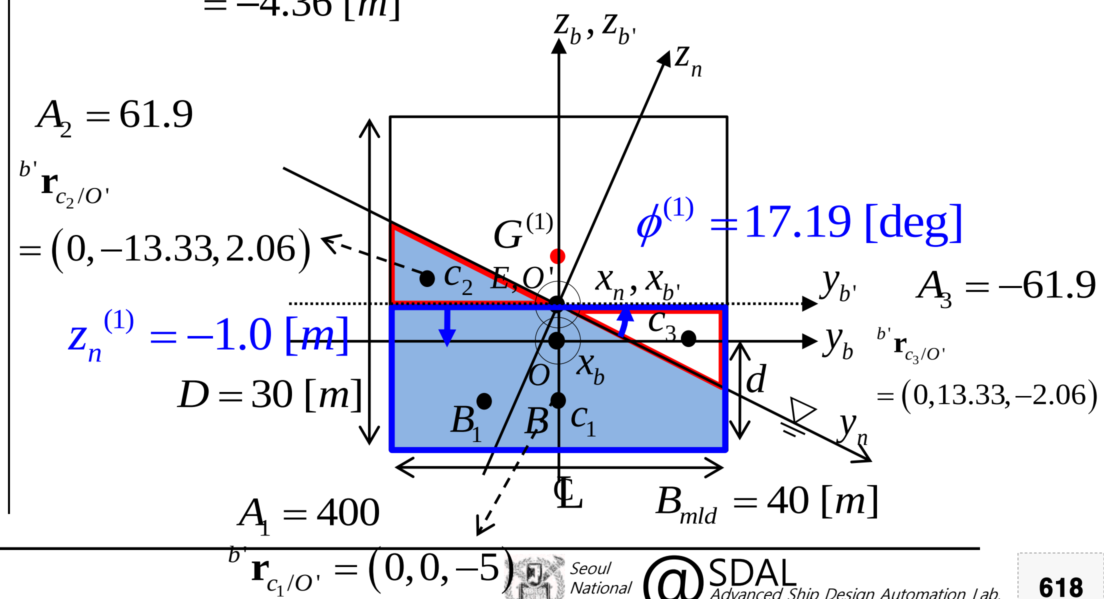
$$M_T^{(1)} = M_{BT}^{(1)} + M_{GT}^{(1)} + M_{extT}^{(1)} \qquad {}^{b'}y_{B^{(1)}/O'} = -4.13\ [m]$$

$$M_{BT}^{(1)} = {}^{n}y_{B^{(1)}/E}\cdot F_{B,z}^{(1)}$$

$$ {}^{b'}z_{B^{(1)}/O'} = \frac{{}^{b'}z_{c_1/O'}A_1 + {}^{b'}z_{c_2/O'}A_2 + {}^{b'}z_{c_3/O'}A_3}{A_1 + A_2 + A_3}$$

$$= \frac{(-5)\cdot 400 + 2.06\cdot 61.9 + (-2.06)\cdot(-61.9)}{400 - 61.9 + 61.9}$$

$$= -4.36\ [m]$$



$L = 100\ [m]$	${}^{n}\mathbf{r}_{G/E} = [0\ \ 0\ \ 6]^T\ [m]$
$B_{mld} = 40\ [m]$	${}^{n}\mathbf{r}_{B/E} = [0\ \ 0\ \ -4.5]^T\ [m]$
$D = 30\ [m]$	${}^{n}\mathbf{r}_{G_{ext}/E} = [0\ \ 5\ \ 22]^T\ [m]$
$d = 9\ [m]$	

$F_G = -3.6\times 10^5\ [kN]$

$F_B^{(1)} = 4.0\times 10^5\ [kN]$

$F_{ext} = -4.0\times 10^4\ [kN]$

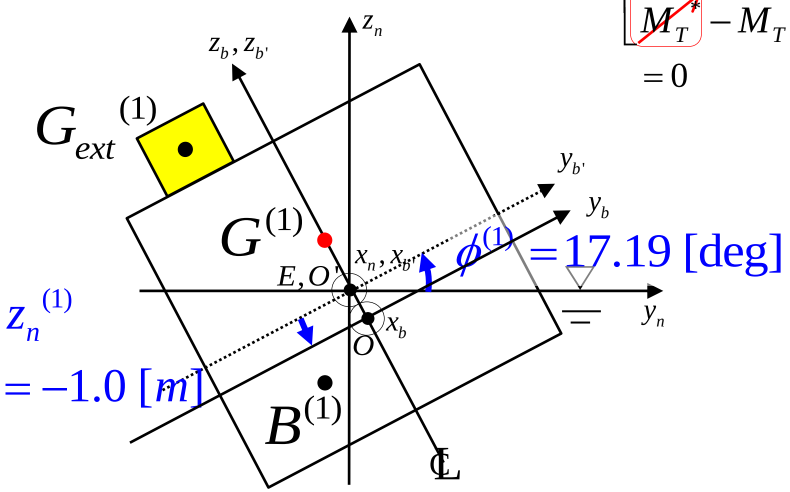
$\rho g = 10\ [Mg/m^2 s^2]$

Rotational Transformation

(1st Iteration)

$$\begin{bmatrix} \cancel{F_z^*} - F_z \\ \cancel{M_T^*} - M_T \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP} & -\rho g A_{WP} \, {}^n y_{F/E} \\ -\rho g A_{WP} \, {}^n y_{F/E} & -\rho g \left({}^n z_{B/E} \nabla + I_T \right) - {}^n z_{G/E} \cdot F_{G,z} - {}^n z_{G_{ext}/E} \cdot F_{ext,z} \end{bmatrix} \begin{bmatrix} \delta z_n \\ \delta \phi \end{bmatrix}$$

$= 0$



$$M_T^{(1)} = M_{BT}^{(1)} + M_{GT}^{(1)} + M_{extT}^{(1)}$$

$$M_{BT}^{(1)} = {}^n y_{B^{(1)}/E} \cdot F_{B,z}^{(1)}$$

$${}^{b'} y_{B^{(1)}/O'} = -4.13 \, [m]$$

$${}^{b'} z_{B^{(1)}/O'} = -4.36 \, [m]$$

${}^{b'}\mathbf{r}_{B^{(1)}/O'}$ is the center of buoyancy in the b'-frame.

Thus, to calculate the center of buoyancy in the water surface-fixed frame, ${}^n\mathbf{r}_{B^{(1)}/E}$, we have to rotate the b'-frame about xn -axis to the original position with the angle of ϕ in the counter-clock wise direction.

$L = 100 \, [m]$

$B_{mld} = 40 \, [m]$

$D = 30 \, [m]$

$d = 9 \, [m]$

${}^n\mathbf{r}_{G/E} = [0 \quad 0 \quad 6]^T \, [m]$

${}^n\mathbf{r}_{B/E} = [0 \quad 0 \quad -4.5]^T \, [m]$

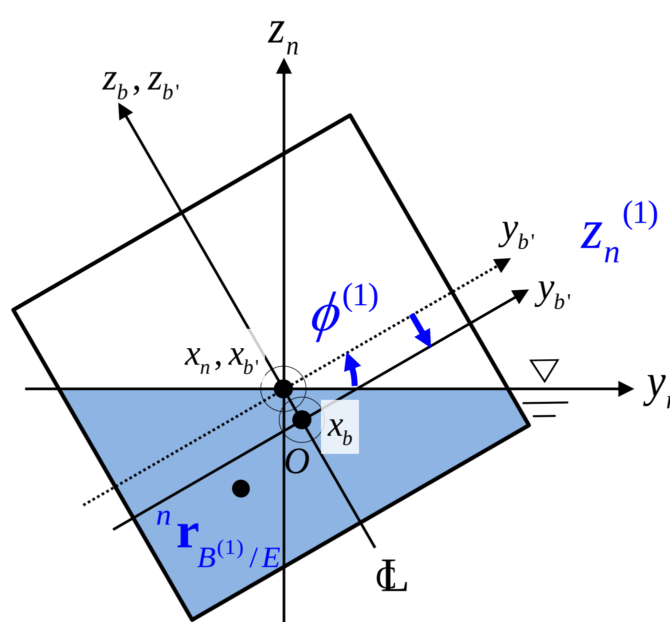
${}^n\mathbf{r}_{G_{ext}/E} = [0 \quad 5 \quad 22]^T \, [m]$

$F_G = -3.6 \times 10^5 \, [kN]$

$F_B^{(1)} = 4.0 \times 10^5 \, [kN]$

$F_{ext} = -4.0 \times 10^4 \, [kN]$

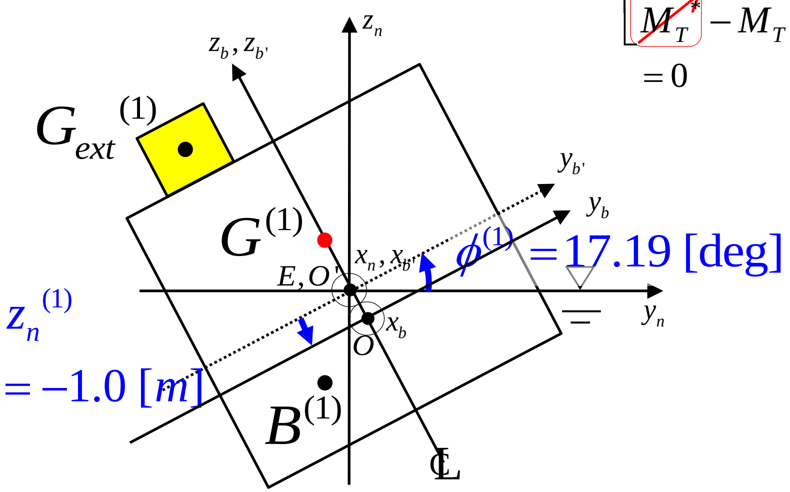
$\rho g = 10 \, [Mg / m^2 s^2]$



Rotational transformation of the yb and zb coordinates defined in the b-frame to the yn and zn coordinates in the n-frame (1st Iteration)

$$\begin{bmatrix} \cancel{F_z^*} - F_z \\ \cancel{M_T^*} - M_T \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP} & -\rho g A_{WP}\,{}^{n}y_{F/E} \\ -\rho g A_{WP}\,{}^{n}y_{F/E} & -\rho g\left({}^{n}z_{B/E}\nabla + I_T\right) - {}^{n}z_{G/E}\cdot F_{G,z} - {}^{n}z_{G_{ext}/E}\cdot F_{ext,z} \end{bmatrix} \begin{bmatrix} \delta z_n \\ \delta\phi \end{bmatrix}$$

$$=0$$



$$M_T^{(1)} = M_{BT}^{(1)} + M_{GT}^{(1)} + M_{extT}^{(1)}$$

$$M_{BT}^{(1)} = {}^{n}y_{B^{(1)}/E}\cdot F_{B,z}^{(1)}$$

$${}^{b'}y_{B^{(1)}/O'} = -4.13\ [m]$$

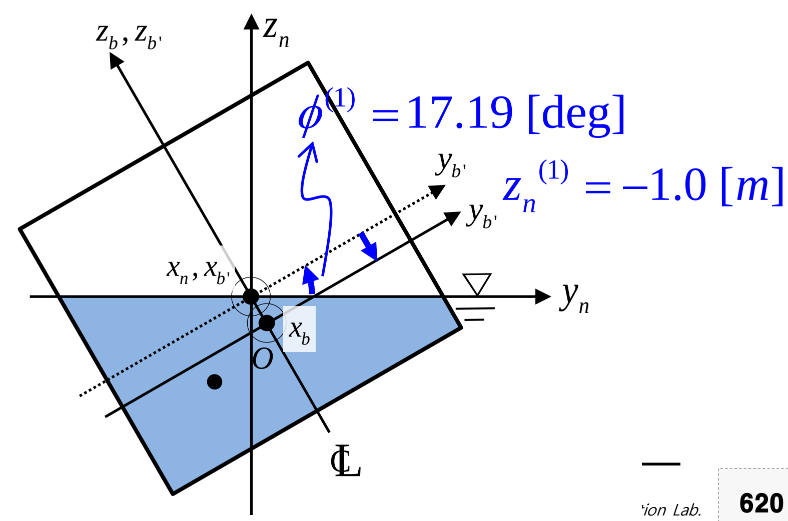
$${}^{b'}z_{B^{(1)}/O'} = -4.36\ [m]$$

$${}^{n}\mathbf{r}_{B^{(1)}/E} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi^{(1)} & -\sin\phi^{(1)} \\ 0 & \sin\phi^{(1)} & \cos\phi^{(1)} \end{bmatrix} {}^{b'}\mathbf{r}_{B^{(1)}/O'}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(17.19) & -\sin(17.19) \\ 0 & \sin(17.19) & \cos(17.19) \end{bmatrix} \begin{bmatrix} 0 \\ -4.13 \\ -4.36 \end{bmatrix}$$

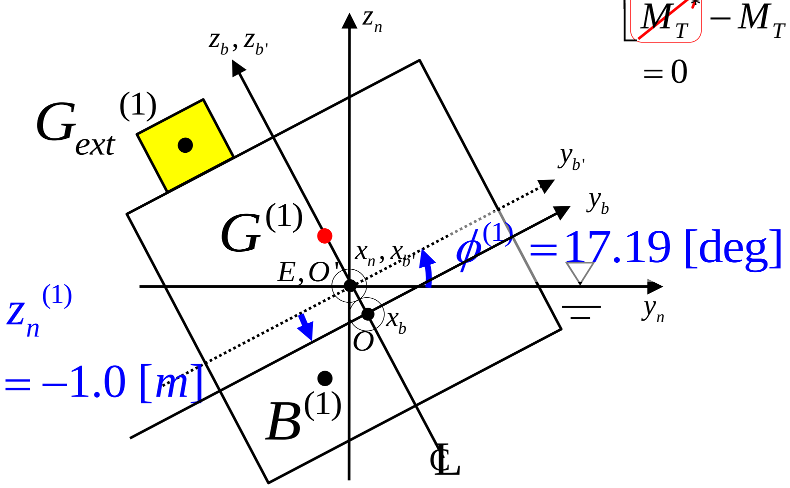
$L = 100\ [m]$	${}^{n}\mathbf{r}_{G/E} = [0\ \ 0\ \ 6]^T\ [m]$
$B_{mld} = 40\ [m]$	${}^{n}\mathbf{r}_{B/E} = [0\ \ 0\ \ -4.5]^T\ [m]$
$D = 30\ [m]$	${}^{n}\mathbf{r}_{G_{ext}/E} = [0\ \ 5\ \ 22]^T\ [m]$
$d = 9\ [m]$	

$$F_G = -3.6\times10^5\ [kN]$$
$$F_B^{(1)} = 4.0\times10^5\ [kN]$$
$$F_{ext} = -4.0\times10^4\ [kN]$$
$$\rho g = 10\ [Mg/m^2s^2]$$



$$\begin{bmatrix} \cancel{F_z^*} - F_z \\ \cancel{M_T^*} - M_T \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP} & -\rho g A_{WP}\, {}^{n}y_{F/E} \\ -\rho g A_{WP}\, {}^{n}y_{F/E} & -\rho g\left({}^{n}z_{B/E}\nabla + I_T\right) - {}^{n}z_{G/E}\cdot F_{G,z} - {}^{n}z_{G_{ext}/E}\cdot F_{ext,z} \end{bmatrix} \begin{bmatrix} \delta z_n \\ \delta\phi \end{bmatrix}$$

$$= 0$$



$$M_T^{(1)} = M_{BT}^{(1)} + M_{GT}^{(1)} + M_{extT}^{(1)}$$

$$M_{BT}^{(1)} = {}^{n}y_{B^{(1)}/E} \cdot F_{B,z}^{(1)}$$

$${}^{n}\mathbf{r}_{B^{(1)}/E} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi^{(1)} & -\sin\phi^{(1)} \\ 0 & \sin\phi^{(1)} & \cos\phi^{(1)} \end{bmatrix} {}^{b'}\mathbf{r}_{B^{(1)}/O'}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(17.19) & -\sin(17.19) \\ 0 & \sin(17.19) & \cos(17.19) \end{bmatrix} \begin{bmatrix} 0 \\ -4.13 \\ -4.36 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -2.66 \\ -5.39 \end{bmatrix}$$

$${}^{n}y_{B^{(1)}/E} = -2.66\ [m]$$

$$M_{BT}^{(1)} = (-2.66)\cdot(4.0\times 10^5)$$

$$= -1.06\times 10^6\ [kN\cdot m]$$

$L = 100\ [m]$	${}^{n}\mathbf{r}_{G/E} = [0\quad 0\quad 6]^T\ [m]$
$B_{mld} = 40\ [m]$	${}^{n}\mathbf{r}_{B^{(1)}/E} = [0\quad -2.66\quad -5.39]^T\ [m]$
$D = 30\ [m]$	${}^{n}\mathbf{r}_{G_{ext}/E} = [0\quad 5\quad 22]^T\ [m]$
$d = 9\ [m]$	

$F_G = -3.6\times 10^5\ [kN]$

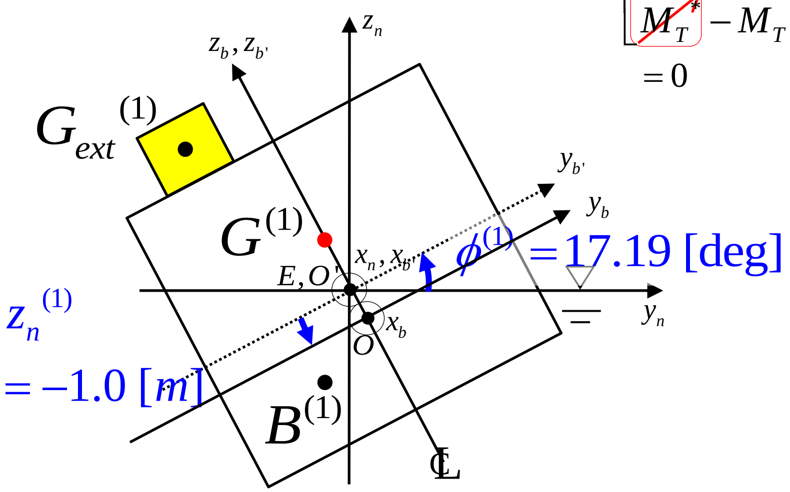
$F_B^{(1)} = 4.0\times 10^5\ [kN]$

$F_{ext} = -4.0\times 10^4\ [kN]$

$\rho g = 10\ [Mg/m^2 s^2]$

$$\begin{bmatrix} \cancel{F_z^*} - F_z \\ \cancel{M_T^*} - M_T \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP} & -\rho g A_{WP}\,{}^n y_{F/E} \\ -\rho g A_{WP}\,{}^n y_{F/E} & -\rho g\left({}^n z_{B/E}\nabla + I_T\right) - {}^n z_{G/E}\cdot F_{G,z} - {}^n z_{G_{ext}/E}\cdot F_{ext,z} \end{bmatrix}\begin{bmatrix}\delta z_n \\ \delta\phi\end{bmatrix}$$

$= 0$



$$M_T^{(1)} = M_{BT}^{(1)} + M_{GT}^{(1)} + M_{extT}^{(1)}$$

$$M_{BT}^{(1)} = -1.06\times10^6 \ [kN\cdot m$$

$$M_{GT}^{(1)} = {}^n y_{G^{(1)}/E}\cdot F_{G,z}^{(1)}$$

The center of mass, ${}^{b'}\mathbf{r}_{G^{(1)}/O'}$, with respect to the b'-frame does notchange. However, the center of gravity, ${}^{n}\mathbf{r}_{G^{(1)}/E'}$, with respect to the water surface-fixed frame changes during the rotation. The change in the center of gravity, ${}^{n}\mathbf{r}_{G^{(1)}/E}$ causes an additional heeling moment..

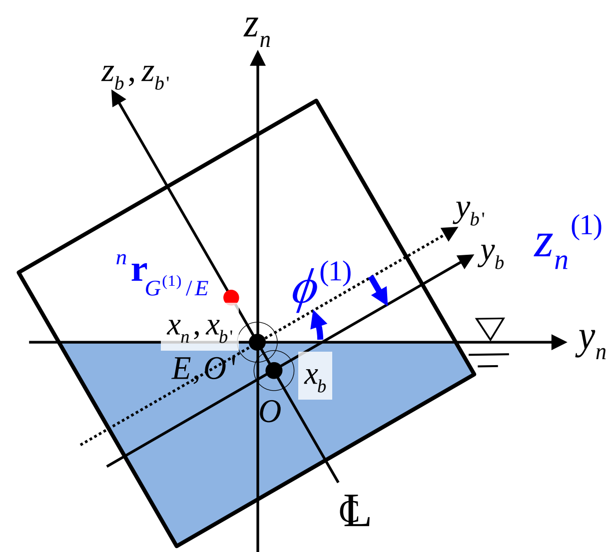
$L = 100\ [m]$	${}^n\mathbf{r}_{G/E} = [0 \quad 0 \quad 6]^T\ [m]$
$B_{mld} = 40\ [m]$	${}^n\mathbf{r}_{B^{(1)}/E} = [0 \quad -2.66 \quad -5.39]^T\ [m]$
$D = 30\ [m]$	${}^n\mathbf{r}_{G_{ext}/E} = [0 \quad 5 \quad 22]^T\ [m]$
$d = 9\ [m]$	

$F_G = -3.6\times10^5\ [kN]$

$F_B^{(1)} = 4.0\times10^5\ [kN]$

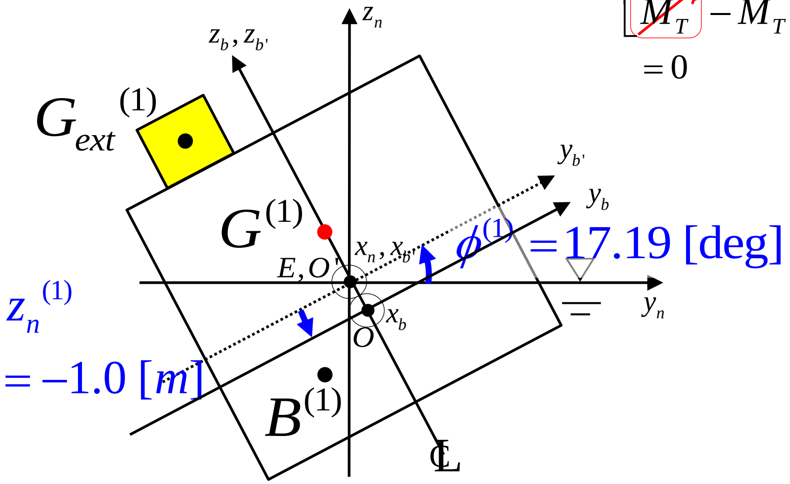
$F_{ext} = -4.0\times10^4\ [kN]$

$\rho g = 10\ [Mg/m^2 s^2]$



$$\begin{bmatrix} \cancel{F_z^*} - F_z \\ \cancel{M_T^*} - M_T \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP} & -\rho g A_{WP}\,{}^n y_{F/E} \\ -\rho g A_{WP}\,{}^n y_{F/E} & -\rho g\left({}^n z_{B/E}\nabla + I_T\right) - {}^n z_{G/E}\cdot F_{G,z} - {}^n z_{G_{ext}/E}\cdot F_{ext,z} \end{bmatrix} \begin{bmatrix} \delta z_n \\ \delta\phi \end{bmatrix}$$

$= 0$



$$M_T^{(1)} = M_{BT}^{(1)} + M_{GT}^{(1)} + M_{extT}^{(1)} \qquad M_{BT}^{(1)} = -1.06\times 10^6\ [kN\cdot m$$

$$M_{GT}^{(1)} = {}^n y_{G^{(1)}/E}\cdot F_{G,z}^{(1)}$$

2. Rotation with heel

1. Translation with immersio

$${}^n\mathbf{r}_{G^{(1)}/E} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi^{(1)} & -\sin\phi^{(1)} \\ 0 & \sin\phi^{(1)} & \cos\phi^{(1)} \end{bmatrix} {}^{b'}\mathbf{r}_{G^{(1)}/O'}, \quad {}^{b'}\mathbf{r}_{G^{(1)}/O'} = {}^{b}\mathbf{r}_{G/O} + \begin{bmatrix} 0 \\ 0 \\ z_n^{(1)} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(17.19) & -\sin(17.19) \\ 0 & \sin(17.19) & \cos(17.19) \end{bmatrix} \left(\begin{bmatrix} 0 \\ 0 \\ 6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -1.0 \end{bmatrix} \right)$$

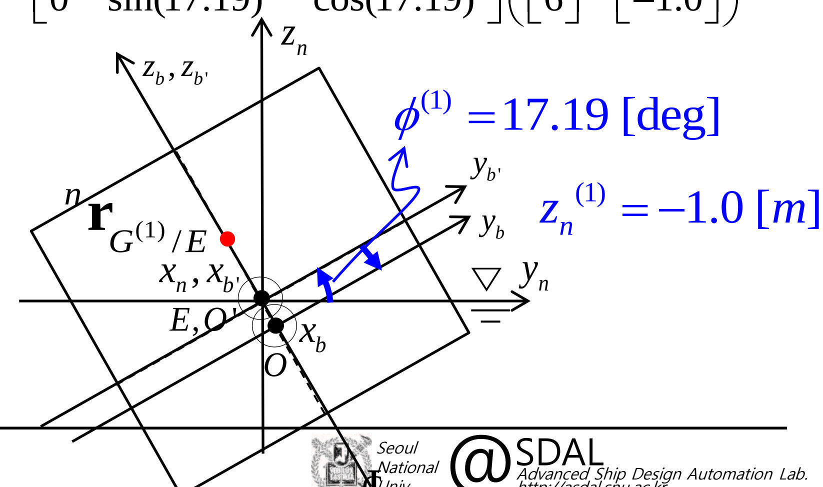
$L = 100\ [m]$	${}^n\mathbf{r}_{G/E} = [0\quad 0\quad 6]^T\ [m]$
$B_{mld} = 40\ [m]$	${}^n\mathbf{r}_{B^{(1)}/E} = [0\quad -2.66\quad -5.39]^T\ [m]$
$D = 30\ [m]$	${}^n\mathbf{r}_{G_{ext}/E} = [0\quad 5\quad 22]^T\ [m]$
$d = 9\ [m]$	

$F_G = -3.6\times 10^5\ [kN]$

$F_B^{(1)} = 4.0\times 10^5\ [kN]$

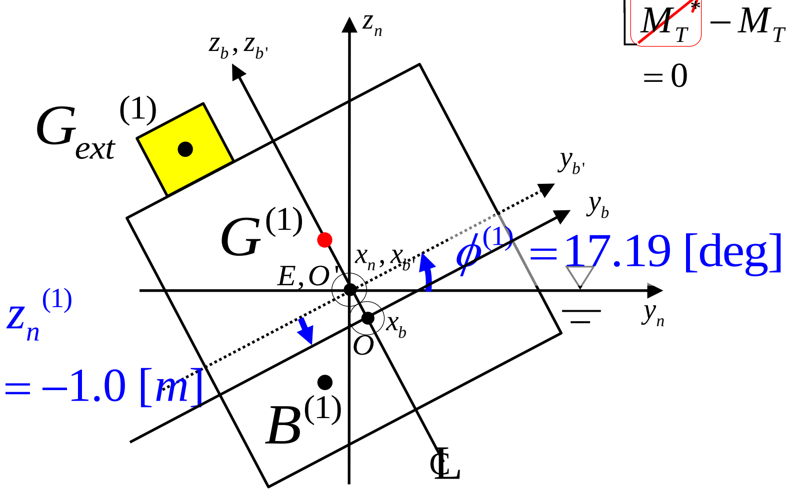
$F_{ext} = -4.0\times 10^4\ [kN]$

$\rho g = 10\ [Mg/m^2 s^2]$



$$\begin{bmatrix} \cancel{F_z^*} - F_z \\ \cancel{M_T^*} - M_T \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP} & -\rho g A_{WP}\,{}^n y_{F/E} \\ -\rho g A_{WP}\,{}^n y_{F/E} & -\rho g\left({}^n z_{B/E}\nabla + I_T\right) - {}^n z_{G/E}\cdot F_{G,z} - {}^n z_{G_{ext}/E}\cdot F_{ext,z} \end{bmatrix} \begin{bmatrix} \delta z_n \\ \delta\phi \end{bmatrix}$$

$= 0$



$$M_T^{(1)} = M_{BT}^{(1)} + M_{GT}^{(1)} + M_{extT}^{(1)}$$

$$M_{BT}^{(1)} = -1.06\times 10^6 \ [kN\cdot m$$

$$M_{GT}^{(1)} = {}^n y_{G^{(1)}/E}\cdot F_{G,z}^{(1)}$$

$$= \begin{bmatrix} 0 \\ -1.48 \\ 4.78 \end{bmatrix}$$

$${}^n y_{G^{(1)}/E} = -1.48 \ [m]$$

$$M_{GT}^{(1)} = (-1.48)\cdot(-3.6\times 10^5)$$

$$= 5.33\times 10^5 \ [kN\cdot m]$$

$L = 100 \ [m]$	${}^n\mathbf{r}_{G^{(1)}/E} = [0 \quad -1.48 \quad 4.78]^T \ [m]$
$B_{mld} = 40 \ [m]$	${}^n\mathbf{r}_{B^{(1)}/E} = [0 \quad -2.66 \quad -5.39]^T \ [m]$
$D = 30 \ [m]$	${}^n\mathbf{r}_{G_{ext}/E} = [0 \quad 5 \quad 22]^T \ [m]$
$d = 9 \ [m]$	

$F_G = -3.6\times 10^5 \ [kN]$

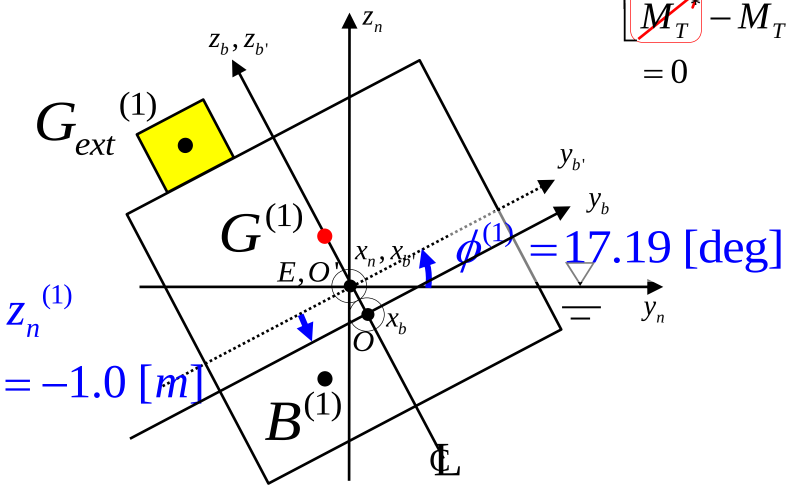
$F_B^{(1)} = 4.0\times 10^5 \ [kN]$

$F_{ext} = -4.0\times 10^4 \ [kN]$

$\rho g = 10 \ [Mg/m^2 s^2]$

$$\begin{bmatrix} \cancel{F_z^*} - F_z \\ \cancel{M_T^*} - M_T \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP} & -\rho g A_{WP}\, {}^{n}y_{F/E} \\ -\rho g A_{WP}\, {}^{n}y_{F/E} & -\rho g\left({}^{n}z_{B/E}\nabla + I_T\right) - {}^{n}z_{G/E}\cdot F_{G,z} - {}^{n}z_{G_{ext}/E}\cdot F_{ext,z} \end{bmatrix} \begin{bmatrix} \delta z_n \\ \delta\phi \end{bmatrix}$$

$$= 0$$



$$M_{BT}^{(1)} = -1.06\times10^{6}\ [kN\cdot m]$$
$$M_{GT}^{(1)} = 5.33\times10^{5}\ [kN\cdot m]$$

$$M_T^{(1)} = M_{BT}^{(1)} + M_{GT}^{(1)} + M_{extT}^{(1)}$$

$$M_{extT}^{(1)} = {}^{n}y_{G_{ext}^{(1)}/E}\cdot F_{ext,z}^{(1)}$$

$${}^{n}\mathbf{r}_{G_{ext}^{(1)}/E} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi^{(1)} & -\sin\phi^{(1)} \\ 0 & \sin\phi^{(1)} & \cos\phi^{(1)} \end{bmatrix} {}^{b'}\mathbf{r}_{G_{ext}^{(1)}/E},\quad {}^{b'}\mathbf{r}_{G_{ext}^{(1)}/E} = {}^{b'}\mathbf{r}_{G_{ext}/E} + \begin{bmatrix} 0 \\ 0 \\ z_n^{(1)} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(17.19) & -\sin(17.19) \\ 0 & \sin(17.19) & \cos(17.19) \end{bmatrix} \left(\begin{bmatrix} 0 \\ 5 \\ 22 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -1.0 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 0 \\ -10.98 \\ 18.58 \end{bmatrix}$$

$${}^{n}y_{G_{ext}^{(1)}/E} = -10.98\ [m]$$

$$M_{extT}^{(1)} = (-10.98)\cdot(-4.0\times10^{4})$$
$$= 4.39\times10^{5}\ [kN\cdot m]$$

$L = 100\ [m]$

$B_{mld} = 40\ [m]$

$D = 30\ [m]$

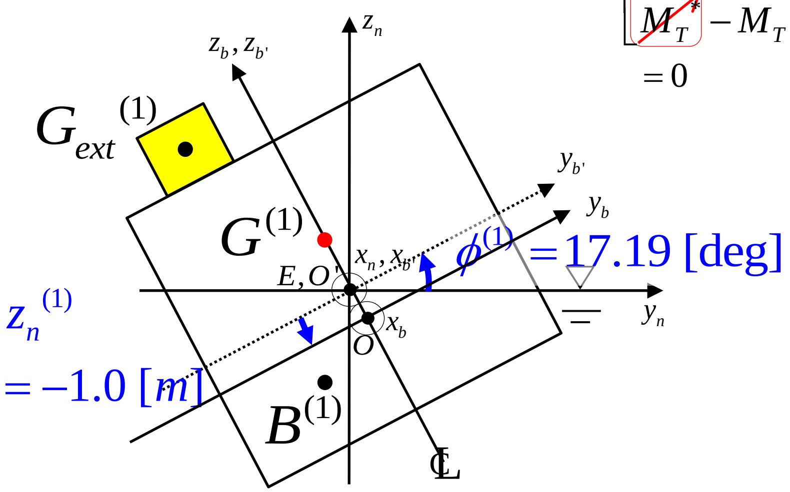
$d = 9\ [m]$

$${}^{n}\mathbf{r}_{G^{(1)}/E} = [0 \quad -1.48 \quad 4.78]^T\ [m]$$
$${}^{n}\mathbf{r}_{B^{(1)}/E} = [0 \quad -2.66 \quad -5.39]^T\ [m]$$
$${}^{n}\mathbf{r}_{G_{ext}^{(1)}/E} = [0 \quad -10.98 \quad 18.58]^T\ [m]$$

$$F_G = -3.6\times10^{5}\ [kN]$$
$$F_B^{(1)} = 4.0\times10^{5}\ [kN]$$
$$F_{ext} = -4.0\times10^{4}\ [kN]$$
$$\rho g = 10\ [Mg/m^2 s^2]$$

$$\begin{bmatrix} \cancel{F_z^*} - F_z \\ \cancel{M_T^*} - M_T \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP} & -\rho g A_{WP} \, {}^n y_{F/E} \\ -\rho g A_{WP} \, {}^n y_{F/E} & -\rho g \left({}^n z_{B/E} \nabla + I_T \right) - {}^n z_{G/E} \cdot F_{G,z} - {}^n z_{G_{ext}/E} \cdot F_{ext,z} \end{bmatrix} \begin{bmatrix} \delta z_n \\ \delta \phi \end{bmatrix}$$

$= 0$



$$M_T^{(1)} = M_{BT}^{(1)} + M_{GT}^{(1)} + M_{extT}^{(1)}$$

$$M_{BT}^{(1)} = -1.06 \times 10^6 \ [kN \cdot m]$$
$$M_{GT}^{(1)} = 5.33 \times 10^5 \ [kN \cdot m]$$
$$M_{extT}^{(1)} = 4.39 \times 10^5 \ [kN \cdot m]$$

$$M_T^{(1)} = -1.06 \times 10^6 + 5.33 \times 10^5 + 4.39 \times 10^5$$
$$= \boxed{-8.8 \times 10^4 \ [kN \cdot m]}$$

$L = 100$ [m]	${}^n\mathbf{r}_{G^{(1)}/E} = [0 \quad -1.48 \quad 4.78]^T$ [m]
$B_{mld} = 40$ [m]	${}^n\mathbf{r}_{B^{(1)}/E} = [0 \quad -2.66 \quad -5.39]^T$ [m]
$D = 30$ [m]	${}^n\mathbf{r}_{G_{ext}^{(1)}/E} = [0 \quad -10.98 \quad 18.58]^T$ [m]
$d = 9$ [m]	

$$F_G = -3.6 \times 10^5 \ [kN]$$
$$F_B^{(1)} = 4.0 \times 10^5 \ [kN]$$
$$F_{ext} = -4.0 \times 10^4 \ [kN]$$
$$\rho g = 10 \ [Mg / m^2 s^2]$$

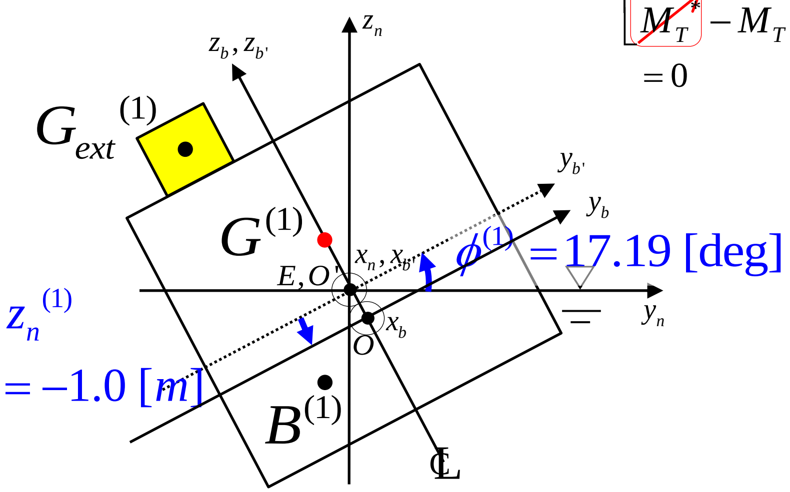
What does this positive moment mean?

- Because we considered only the horizontal movement of the center of buoyancy for small angle of inclination, the righting arm is decreased.
- To produce the righting moment with the decreased righting moment, the angle of heel increased.

n-frame : waterplane fixed reference frame(inertial frame) ($x_n y_n z_n$ axis)
b-frame : body-fixed reference frame ($x_b y_b z_b$ axis)

$$\begin{bmatrix} \cancel{F_z^*} - F_z \\ \cancel{M_T^*} - M_T \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP} & -\rho g A_{WP} \, {}^n y_{F/E} \\ -\rho g A_{WP} \, {}^n y_{F/E} & -\rho g \left({}^n z_{B/E} \nabla + I_T \right) - {}^n z_{G/E} \cdot F_{G,z} - {}^n z_{G_{ext}/E} \cdot F_{ext,z} \end{bmatrix} \begin{bmatrix} \delta z_n \\ \delta \phi \end{bmatrix}$$

$= 0$



$L = 100\ [m]$

$B_{mld} = 40\ [m]$

$D = 30\ [m]$

$d = 9\ [m]$

${}^n\mathbf{r}_{G^{(1)}/E} = [0 \quad -1.48 \quad 4.78]^T\ [m]$

${}^n\mathbf{r}_{B^{(1)}/E} = [0 \quad -2.66 \quad -5.39]^T\ [m]$

${}^n\mathbf{r}_{G_{ext}^{(1)}/E} = [0 \quad -10.98 \quad 18.58]^T\ [m]$

$F_G = -3.6 \times 10^5\ [kN]$

$F_B^{(1)} = 4.0 \times 10^5\ [kN]$

$F_{ext} = -4.0 \times 10^4\ [kN]$

$\rho g = 10\ [Mg/m^2 s^2]$

$$M_T^{(1)} = M_{BT}^{(1)} + M_{GT}^{(1)} + M_{extT}^{(1)}$$

$$M_{BT}^{(1)} = -1.06 \times 10^6\ [kN \cdot m]$$
$$M_{GT}^{(1)} = 5.33 \times 10^5\ [kN \cdot m]$$
$$M_{extT}^{(1)} = 4.39 \times 10^5\ [kN \cdot m]$$

$$M_T^{(1)} = -1.06 \times 10^6 + 5.33 \times 10^5 + 4.39 \times 10^5$$
$$= \boxed{-8.8 \times 10^4\ [kN \cdot m]}$$

$$\left| -8.8 \times 10^4 \right| [kN \cdot m] > e$$

Tolerance

where, e(epsilon) : an arbitrarily small positive quantity

The static equilibrium of moment is not satisfied!

We have to iterate!

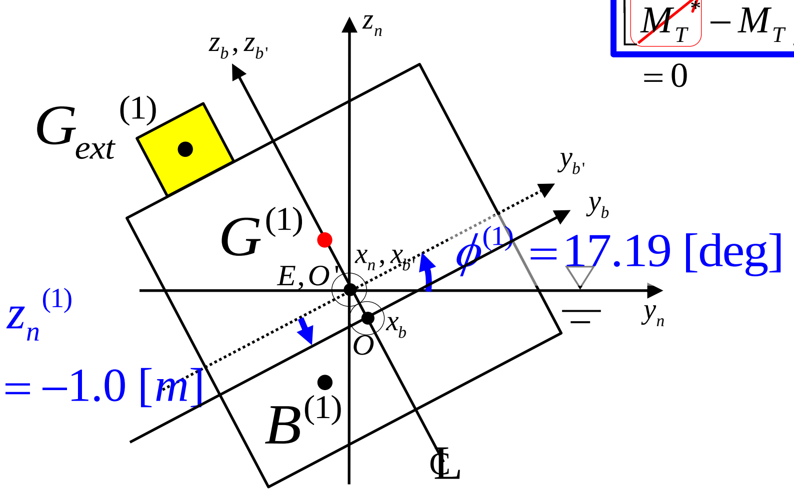
The immersion and heel are decoupled, since the section of the ship is rectangle.

(2nd Iteration)

We use the values in current floating position!

$$\boxed{\begin{bmatrix} \cancel{F_z^*} - F_z \\ \cancel{M_T^*} - M_T \end{bmatrix}} = \begin{bmatrix} -\rho g A_{WP} & -\rho g A_{WP} \, {}^n y_{F/E} \\ -\rho g A_{WP} \, {}^n y_{F/E} & -\rho g \left({}^n z_{B/E} \nabla + I_T \right) - {}^n z_{G/E} \cdot F_{G,z} - {}^n z_{G_{ext}/E} \cdot F_{ext,z} \end{bmatrix} \begin{bmatrix} \delta z_n \\ \delta \phi \end{bmatrix}$$

$= 0$



$$F_z^{(1)} = F_{B,z}^{(1)} + F_{G,z}^{(1)} + F_{ext,z}^{(1)}$$

$$= 4.0\times10^5 - 3.6\times10^5 - 4.0\times10^4$$

$$= \underline{0 \; [kN]}$$

$$M_T^{(1)} = M_{BT}^{(1)} + M_{GT}^{(1)} + M_{extT}^{(1)}$$

$$= {}^n y_{B^{(1)}/E} \cdot F_{B,z}^{(1)} + {}^n y_{G^{(1)}/E} \cdot F_{G,z}^{(1)} + {}^n y_{G_{ext}^{(1)}/E} \cdot F_{ext,z}^{(1)}$$

$$= (-2.66)\cdot(4.0\times10^5) + (-1.48)\times(-3.6\times10^5)$$

$$+ (-10.98)\times(-4.0\times10^4)$$

$$= \underline{-8.8\times10^4 \; [kN \cdot m]}$$

$L = 100\;[m]$

$B_{mld} = 40\;[m]$

$D = 30\;[m]$

$d = 9\;[m]$

${}^n\mathbf{r}_{G^{(1)}/E} = [0 \quad -1.48 \quad 4.78]^T\;[m]$

${}^n\mathbf{r}_{B^{(1)}/E} = [0 \quad -2.66 \quad -5.39]^T\;[m]$

${}^n\mathbf{r}_{G_{ext}^{(1)}/E} = [0 \quad -10.98 \quad 18.58]^T\;[m]$

$F_G = -3.6\times10^5\;[kN]$

$F_B^{(1)} = 4.0\times10^5\;[kN]$

$F_{ext} = -4.0\times10^4\;[kN]$

$\rho g = 10\;[Mg/m^2 s^2]$

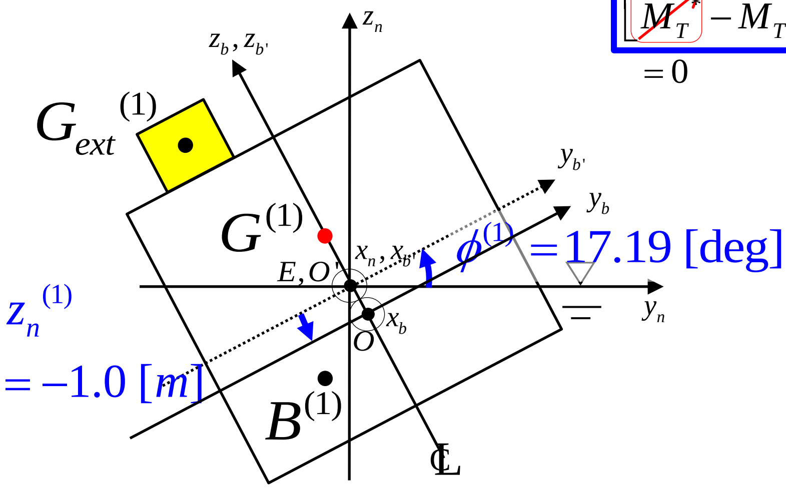
※In previous state :

$$F_z = F_{B,z} + F_{G,z} + F_{ext,z}$$
$$= -4.0\times10^4\;[kN]$$
$$M_T = M_{BT} + M_{GT} + M_{extT}$$
$$= +2.0\times10^5\;[kN \cdot m]$$

We use the values in current floating position!

$$\boxed{\begin{bmatrix} \cancel{F_z^*} - F_z \\ \cancel{M_T^*} - M_T \end{bmatrix}} = \begin{bmatrix} -\rho g A_{WP} & -\rho g A_{WP}\,{}^n y_{F/E} \\ -\rho g A_{WP}\,{}^n y_{F/E} & -\rho g\left({}^n z_{B/E}\nabla + I_T\right) - {}^n z_{G/E}\cdot F_{G,z} - {}^n z_{G_{ext}/E}\cdot F_{ext,z} \end{bmatrix}\begin{bmatrix} \delta z_n \\ \delta\phi \end{bmatrix}$$

$= 0$



$$-\rho g A_{WP} = -\rho g L B_{mld}^{(1)}, \quad B_{mld}^{(1)} = \frac{B_{mld}}{\cos\left|\phi^{(1)}\right|} = \frac{40}{\cos 17.19^\circ} = 41.87$$

$$= -10\cdot 100\cdot 41.87 \ = \underline{-4.19\times 10^4\ [kN/m]}$$

$$-\rho g A_{WP}\,{}^n y_{F/E} = -10\cdot(4.19\times 10^4)\cdot 0 = \underline{0\ [kN]}$$

$$-\rho g\left({}^n z_{B/E}\nabla + I_T\right) = -{}^n z_{B/E}F_B - \rho g\frac{L\left(B_{mld}^{(1)}\right)^3}{12}$$

$$= -(-5.39)\cdot(4.0\times 10^5) - 10\cdot\frac{100\cdot 41.87^3}{12}$$

$$= -3.96\times 10^6\ [kN\cdot m]$$

$$-{}^n z_{G/E}\cdot F_{G,z} = -4.78\cdot(-3.6\times 10^5) = 1.72\times 10^6\ [kN\cdot m]$$

$$-{}^n z_{G_{ext}/E}\cdot F_{ext,z} = -18.58\cdot(-4.0\times 10^4) = 7.43\times 10^5\ [kN\cdot m]$$

$$-\rho g\left({}^n z_{B/E}\nabla + I_T\right) - {}^n z_{G/E}\cdot F_{G,z} - {}^n z_{G_{ext}/E}\cdot F_{ext,z}$$

$$= -3.96\times 10^6 + 1.72\times 10^6 + 7.43\times 10^5$$

$$= \underline{-1.50\times 10^6\ [kN\cdot m]}$$

$L = 100\ [m]$

$B_{mld} = 40\ [m]$

$D = 30\ [m]$

$d = 9\ [m]$

${}^n\mathbf{r}_{G^{(1)}/E} = [0 \quad -1.48 \quad 4.78]^T\ [m]$

${}^n\mathbf{r}_{B^{(1)}/E} = [0 \quad -2.66 \quad -5.39]^T\ [m]$

${}^n\mathbf{r}_{G_{ext}^{(1)}/E} = [0 \quad -10.98 \quad 18.58]^T\ [m]$

$F_G = -3.6\times 10^5\ [kN]$

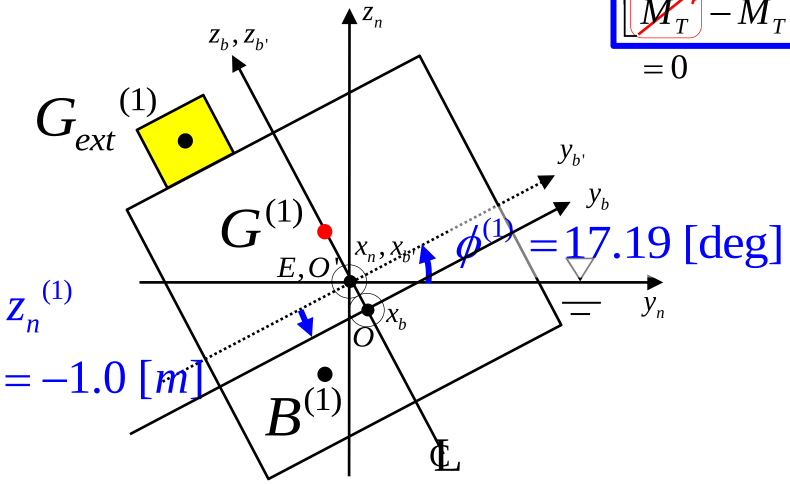
$F_B^{(1)} = 4.0\times 10^5\ [kN]$

$F_{ext} = -4.0\times 10^4\ [kN]$

$\rho g = 10\ [Mg/m^2 s^2]$

We use the values in current floating position!

$$\underbrace{\begin{bmatrix} \cancel{F_z^*} - F_z \\ \cancel{M_T^*} - M_T \end{bmatrix}}_{=0} = \begin{bmatrix} -\rho g A_{WP} & -\rho g A_{WP} \, {}^{n}y_{F/E} \\ -\rho g A_{WP} \, {}^{n}y_{F/E} & -\rho g \left({}^{n}z_{B/E} \nabla + I_T \right) - {}^{n}z_{G/E} \cdot F_{G,z} - {}^{n}z_{G_{ext}/E} \cdot F_{ext,z} \end{bmatrix} \begin{bmatrix} \delta z_n \\ \delta \phi \end{bmatrix}$$



$$-\rho g A_{WP} = -4.19 \times 10^4 \ [kN/m]$$

$$-\rho g A_{WP} \, {}^{n}y_{F/E} = 0 \ [kN]$$

$$-\rho g \left({}^{n}z_{B/E} \nabla + I_T \right) - {}^{n}z_{G/E} \cdot F_{G,z} - {}^{n}z_{G_{ext}/E} \cdot F_{ext,z} = -1.50 \times 10^6 \ [kN \cdot m]$$

$L = 100 \ [m]$

$B_{mld} = 40 \ [m]$

$D = 30 \ [m]$

$d = 9 \ [m]$

$${}^{n}\mathbf{r}_{G^{(1)}/E} = [0 \quad -1.48 \quad 4.78]^T \ [m]$$

$${}^{n}\mathbf{r}_{B^{(1)}/E} = [0 \quad -2.66 \quad -5.39]^T \ [m]$$

$${}^{n}\mathbf{r}_{G_{ext}^{(1)}/E} = [0 \quad -10.98 \quad 18.58]^T \ [m]$$

$$F_G = -3.6 \times 10^5 \ [kN]$$

$$F_B^{(1)} = 4.0 \times 10^5 \ [kN]$$

$$F_{ext} = -4.0 \times 10^4 \ [kN]$$

$$\rho g = 10 \ [Mg/m^2 s^2]$$

※In previous state :

$$-\rho g A_{WP} = -4.0 \times 10^4 \ [kN/m]$$

$$-\rho g A_{WP} \, {}^{n}y_{F/E} = 0 \ [kN]$$

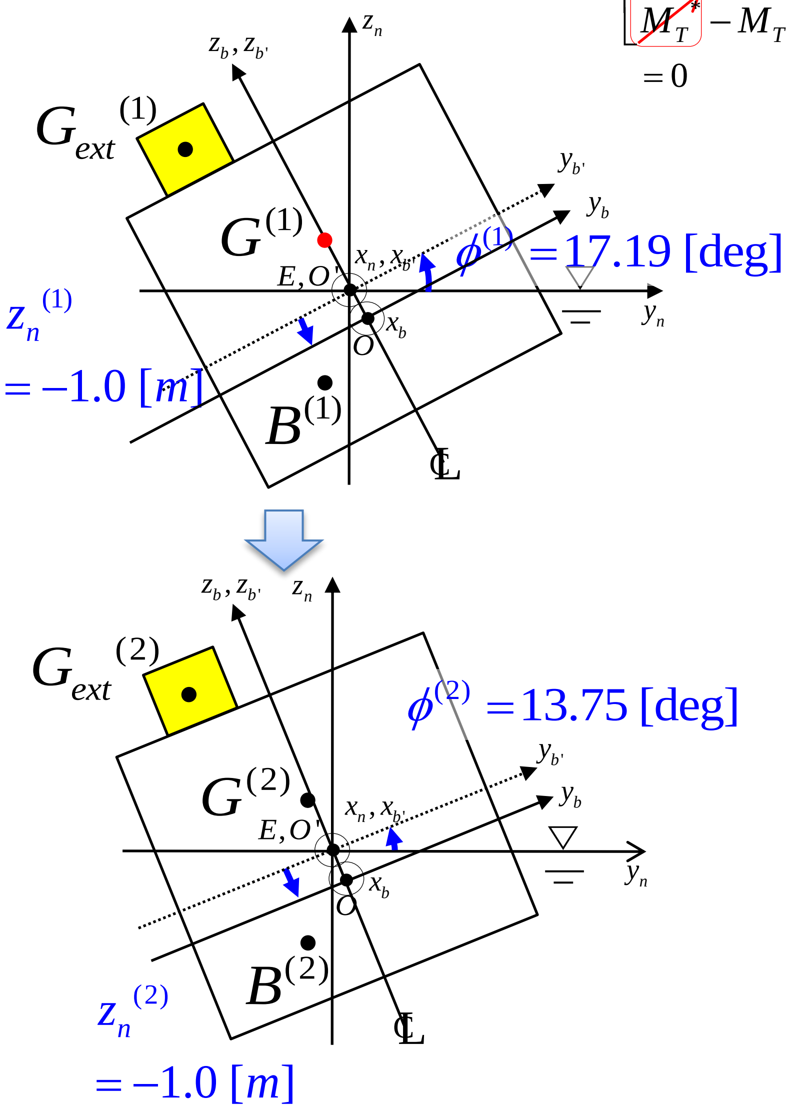
$$-\rho g \left({}^{n}z_{B/E} \nabla + I_T \right) - {}^{n}z_{G/E} \cdot F_{G,z} - {}^{n}z_{G_{ext}/E} \cdot F_{ext,z} = -6.7 \times 10^5 \ [kN \cdot m]$$

3. Calculation of Immersion and Heel

(2nd Iteration)

$$\begin{bmatrix} \cancel{F_z^*} - F_z \\ \cancel{M_T^*} - M_T \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP} & -\rho g A_{WP}\, {}^n y_{F/E} \\ -\rho g A_{WP}\, {}^n y_{F/E} & -\rho g\left({}^n z_{B/E}\nabla + I_T\right) - {}^n z_{G/E}\cdot F_{G,z} - {}^n z_{G_{ext}/E}\cdot F_{ext,z} \end{bmatrix} \boxed{\begin{bmatrix} \delta z_n \\ \delta\phi \end{bmatrix}}$$

$= 0$



$$\begin{bmatrix} 0 \\ -8.8\times10^4 \end{bmatrix} = \begin{bmatrix} -4.19\times10^4 & 0 \\ 0 & -1.50\times10^6 \end{bmatrix} \begin{bmatrix} \delta z_n \\ \delta\phi \end{bmatrix}$$

$$\begin{bmatrix} \delta z_n \\ \delta\phi \end{bmatrix} = \begin{bmatrix} -4.19\times10^4 & 0 \\ 0 & -1.50\times10^6 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ -8.8\times10^4 \end{bmatrix}$$

$$= \begin{bmatrix} 0\ [m] \\ -0.06\ [rad] \end{bmatrix}$$

$$= \begin{bmatrix} 0\ [m] \\ -3.44\ [\text{deg}] \end{bmatrix}$$

$$z_n^{(2)} = z_n^{(1)} + \delta z_n = -1.0 + 0 = -1.0\ [m]$$

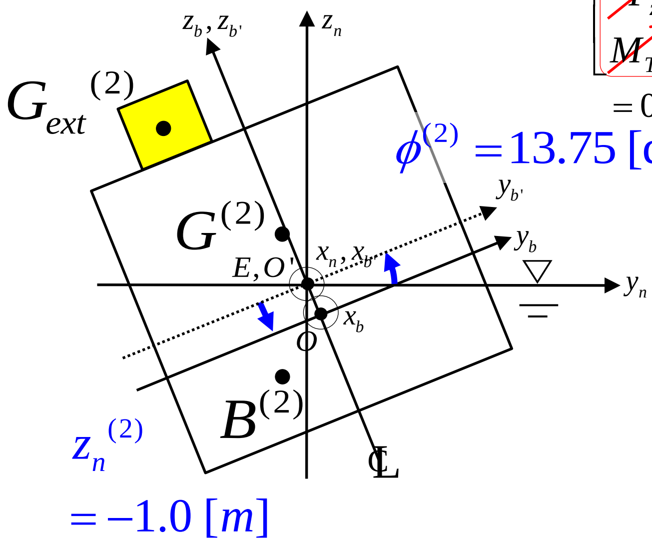
$$\phi^{(2)} = \phi^{(1)} + \delta\phi = 17.19 - 3.44 = 13.75\ [\text{deg}]$$

4. Check whether the Ship is in Static Equilibrium

(2nd Iteration)

$$\begin{bmatrix} \cancel{F_z^*} - F_z \\ \cancel{M_T^*} - M_T \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP} & -\rho g A_{WP}\,{}^n y_{F/E} \\ -\rho g A_{WP}\,{}^n y_{F/E} & -\rho g\left({}^n z_{B/E}\nabla + I_T\right) - {}^n z_{G/E}\cdot F_{G,z} - {}^n z_{G_{ext}/E}\cdot F_{ext,z} \end{bmatrix} \begin{bmatrix} \delta z_n \\ \delta\phi \end{bmatrix}$$

$= 0$



$$F_z^{(2)} = F_{B,z}^{(2)} + F_{G,z}^{(2)} + F_{ext,z}^{(2)}$$

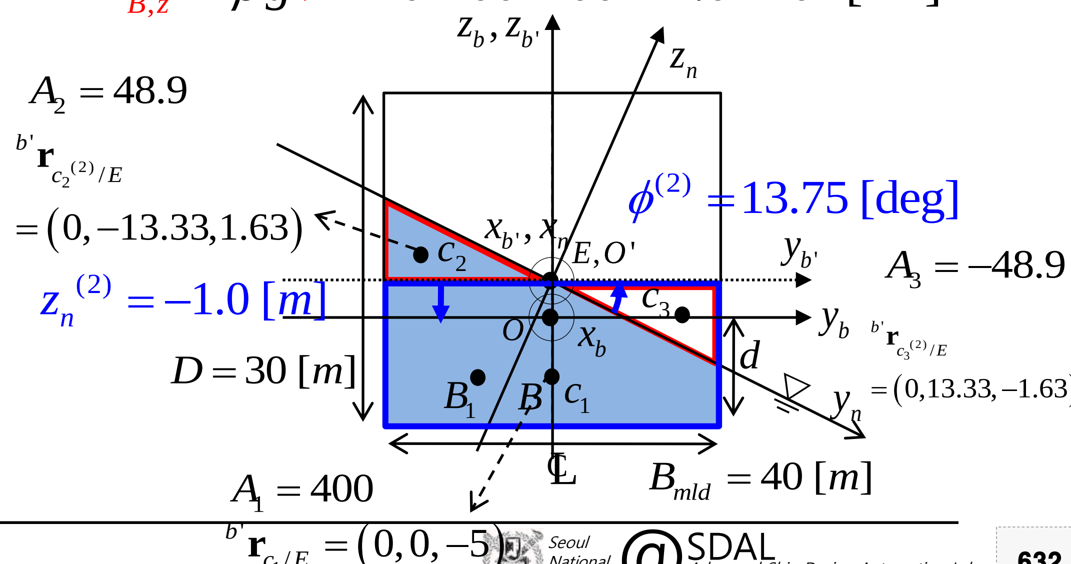
$$F_{B,z}^{(2)} = \rho g V = \rho g L A_{Section}$$

$$A_{Section} = A_1 + A_2 + A_3$$

$$d_1 = \frac{B_{mld}}{2}\tan\left|\phi^{(1)}\right| = \frac{40}{2}\tan 13.75^\circ = 4.89\ [m]$$

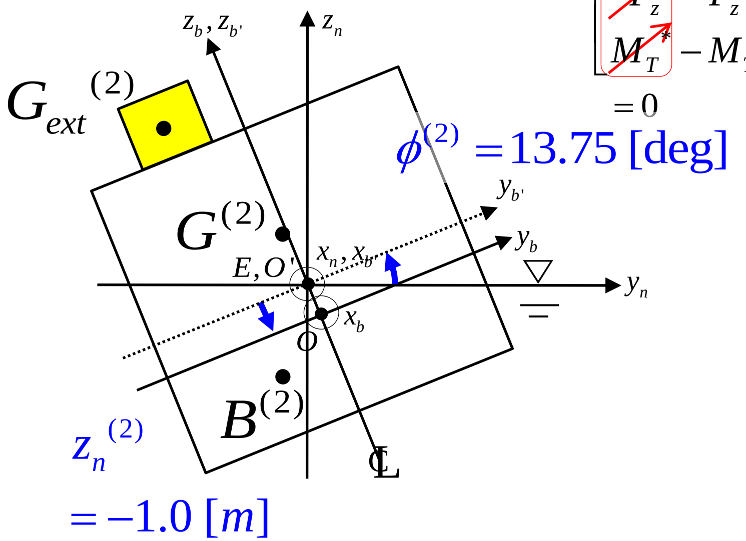
$$A_{Section} = 400 + (-48.9) + 48.9 = 400\ [m^2]$$

$$F_{B,z}^{(2)} = \rho g V = 10\cdot 100\cdot 400 = 4.0\times 10^5\ [kN]$$



$L = 100\ [m]$	$G^{(1)}(0, 1.48, 4.78)\ [m]$
$B_{mld} = 40\ [m]$	$B^{(1)}(0, 2.66, -5.39)\ [m]$
$D = 30\ [m]$	$g^{(1)}(0, 10.98, 18.58)\ [m]$
$d = 9\ [m]$	

$F_G^{(1)} = -3.6\times 10^5\ [kN]$
$F_B^{(1)} = 4.0\times 10^5\ [kN]$
$F_{ext}^{(1)} = -4.0\times 10^4\ [kN]$
$\rho g = 10\ [Mg/m^2 s^2]$



$$\begin{bmatrix} \cancel{F_z^*} - F_z \\ \cancel{M_T^*} - M_T \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP} & -\rho g A_{WP}\, {}^n y_{F/E} \\ -\rho g A_{WP}\, {}^n y_{F/E} & -\rho g \left({}^n z_{B/E} \nabla + I_T \right) - {}^n z_{G/E} \cdot F_{G,z} - {}^n z_{G_{ext}/E} \cdot F_{ext,z} \end{bmatrix} \begin{bmatrix} \delta z_n \\ \delta \phi \end{bmatrix}$$

$= 0$

$$F_z^{(2)} = F_{B,z}^{(2)} + F_{G,z}^{(2)} + F_{ext,z}^{(2)}$$

$$F_{B,z}^{(2)} = 4.0 \times 10^5 \ [kN]$$

$$F_{G,z}^{(2)} = F_G = -3.6 \times 10^5 \ [kN]$$

$$F_{ext,z}^{(2)} = F_{ext} = -4.0 \times 10^4 \ [kN]$$

$$F_z^{(2)} = 4.0 \times 10^5 - 3.6 \times 10^5 - 4.0 \times 10^4$$

$$= \boxed{0 \ [kN]} < e$$

Tolerance

where, e(epsilon) : an arbitrarily small positive quantity

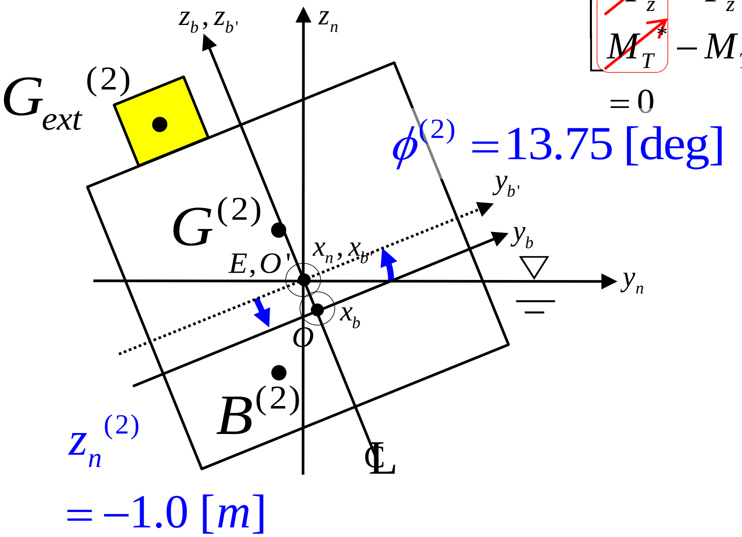
The static equilibrium of force is satisfied!

$L = 100$ [m]	$G^{(1)}(0, 1.48, 4.78)$ [m]
$B_{mld} = 40$ [m]	$B^{(1)}(0, 2.66, -5.39)$ [m]
$D = 30$ [m]	$g^{(1)}(0, 10.98, 18.58)$ [m]
$d = 9$ [m]	

$F_G^{(2)} = -3.6 \times 10^5$ [kN]
$F_B^{(2)} = 4.0 \times 10^5$ [kN]
$F_{ext}^{(2)} = -4.0 \times 10^4$ [kN]
$\rho g = 10$ [$Mg/m^2 s^2$]

$$\begin{bmatrix} \cancel{F_z^*} - F_z \\ \cancel{M_T^*} - M_T \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP} & -\rho g A_{WP}\,{}^n y_{F/E} \\ -\rho g A_{WP}\,{}^n y_{F/E} & -\rho g\left({}^n z_{B/E}\nabla + I_T\right) - {}^n z_{G/E}\cdot F_{G,z} - {}^n z_{G_{ext}/E}\cdot F_{ext,z} \end{bmatrix} \begin{bmatrix} \delta z_n \\ \delta\phi \end{bmatrix}$$

$=0$



$$M_T^{(2)} = M_{BT}^{(2)} + M_{GT}^{(2)} + M_{extT}^{(2)}$$

$$M_{BT}^{(2)} = {}^n y_{B^{(2)}/E} \cdot F_{B,z}^{(2)}$$

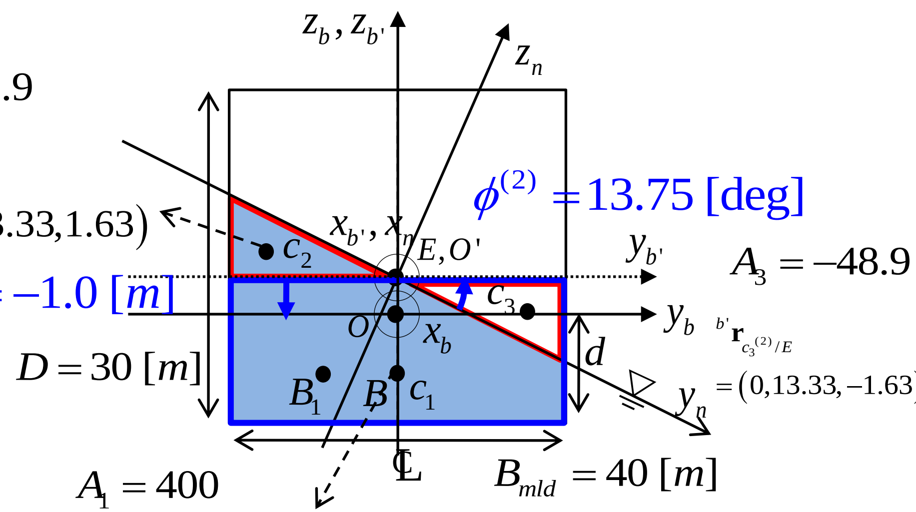
$${}^{b'} y_{B^{(2)}/E} = \frac{{}^{b'} y_{c_1^{(2)}/E} A_1 + {}^{b'} y_{c_2^{(2)}/E} A_2 + {}^{b'} y_{c_3^{(2)}/E} A_3}{A_1 + A_2 + A_3}$$

$$= \frac{0 \cdot 400 + (-13.33)\cdot(48.9) + (13.33)\cdot(-48.9)}{400 - 48.9 + 48.9}$$

$$= -3.26 \text{ [m]}$$

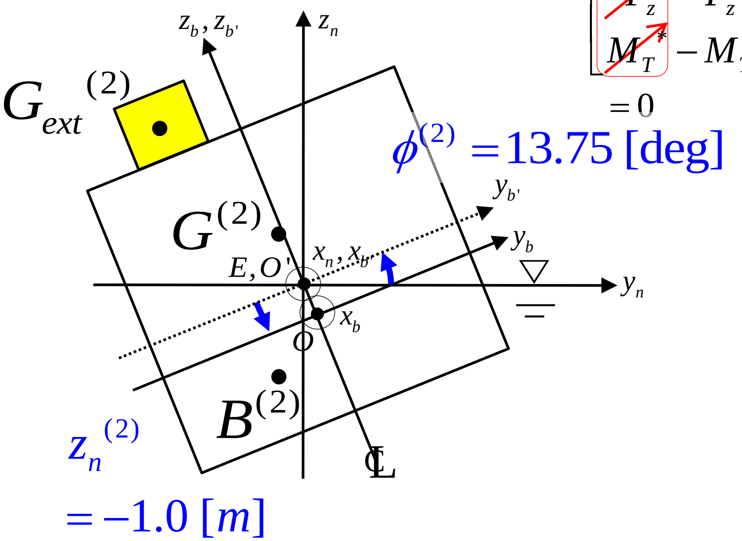
$L = 100$ [m]	$G^{(1)}(0, 1.48, 4.78)$ [m]
$B_{mld} = 40$ [m]	$B^{(1)}(0, 2.66, -5.39)$ [m]
$D = 30$ [m]	$g^{(1)}(0, 10.98, 18.58)$ [m]
$d = 9$ [m]	

$F_G^{(2)} = -3.6 \times 10^5$ [kN]
$F_B^{(2)} = 4.0 \times 10^5$ [kN]
$F_{ext}^{(2)} = -4.0 \times 10^4$ [kN]
$\rho g = 10$ [$Mg/m^2 s^2$]



$$\begin{bmatrix} \cancel{F_z^*} - F_z \\ \cancel{M_T^*} - M_T \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP} & -\rho g A_{WP}\,{}^{n}y_{F/E} \\ -\rho g A_{WP}\,{}^{n}y_{F/E} & -\rho g\left({}^{n}z_{B/E}\nabla + I_T\right) - {}^{n}z_{G/E}\cdot F_{G,z} - {}^{n}z_{G_{ext}/E}\cdot F_{ext,z} \end{bmatrix} \begin{bmatrix} \delta z_n \\ \delta\phi \end{bmatrix}$$

$=0$



$$M_T^{(2)} = M_{BT}^{(2)} + M_{GT}^{(2)} + M_{extT}^{(2)}$$

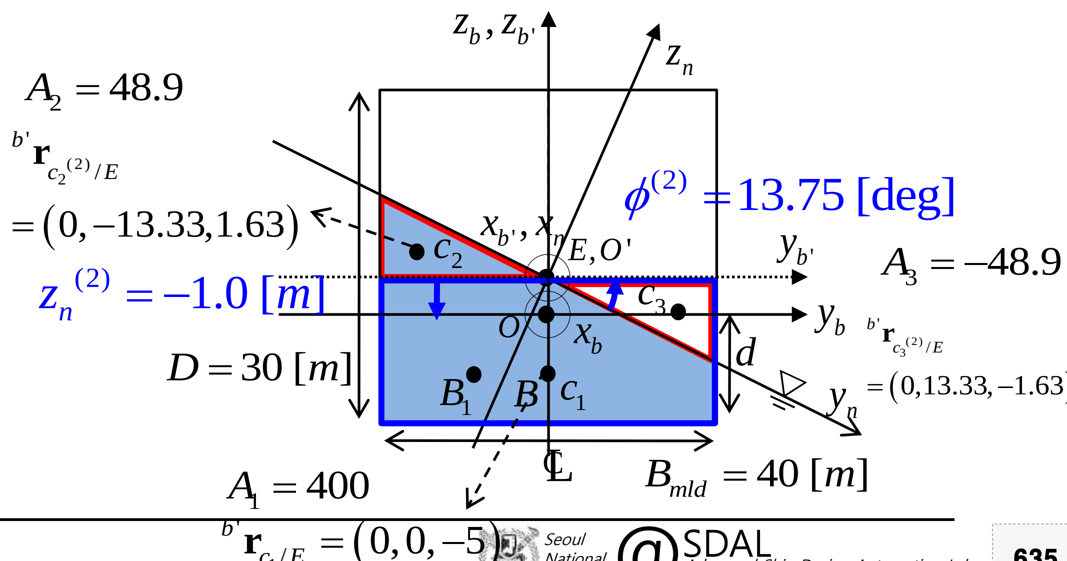
$${}^{b'}y_{B^{(2)}/E} = -3.26\ [m]$$

$$M_{BT}^{(2)} = {}^{n}y_{B^{(2)}/E}\cdot F_{B,z}^{(2)}$$

$${}^{b'}z_{B^{(2)}/E} = \frac{{}^{b'}z_{c_1^{(2)}/E}A_1 + {}^{b'}z_{c_2^{(2)}/E}A_2 + {}^{b'}z_{c_3^{(2)}/E}A_3}{A_1 + A_2 + A_3}$$

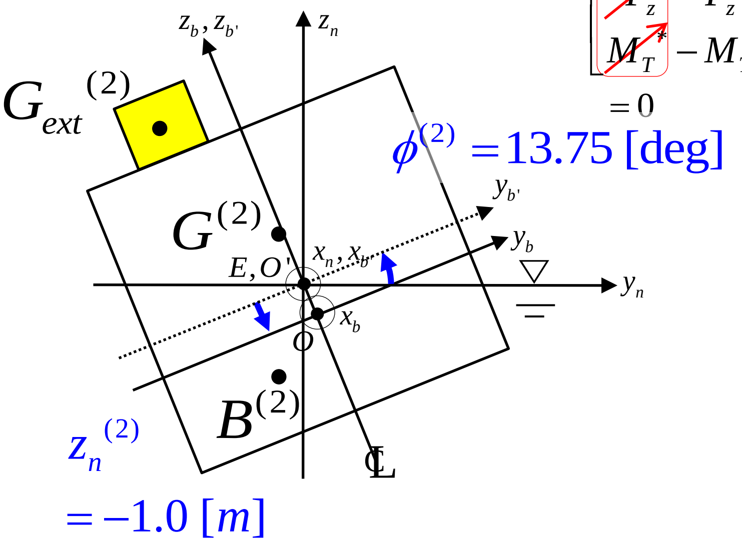
$$= \frac{(-5)\cdot 400 + (-1.63)\cdot(48.9) + 1.63\cdot(-48.9)}{400 - 48.9 + 48.9}$$

$$= -4.60\ [m]$$



$L = 100\ [m]$	$G^{(1)}(0, 1.48, 4.78)\ [m]$
$B_{mld} = 40\ [m]$	$B^{(1)}(0, 2.66, -5.39)\ [m]$
$D = 30\ [m]$	$g^{(1)}(0, 10.98, 18.58)\ [m]$
$d = 9\ [m]$	

$F_G^{(2)} = -3.6\times 10^5\ [kN]$
$F_B^{(2)} = 4.0\times 10^5\ [kN]$
$F_{ext}^{(2)} = -4.0\times 10^4\ [kN]$
$\rho g = 10\ [Mg/m^2 s^2]$



$$\begin{bmatrix} \cancel{F_z^*} - F_z \\ \cancel{M_T^*} - M_T \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP} & -\rho g A_{WP}\,{}^{n}y_{F/E} \\ -\rho g A_{WP}\,{}^{n}y_{F/E} & -\rho g\left({}^{n}z_{B/E}\nabla + I_T\right) - {}^{n}z_{G/E}\cdot F_{G,z} - {}^{n}z_{G_{ext}/E}\cdot F_{ext,z} \end{bmatrix} \begin{bmatrix} \delta z_n \\ \delta\phi \end{bmatrix}$$

$= 0$

$$M_T^{(2)} = M_{BT}^{(2)} + M_{GT}^{(2)} + M_{extT}^{(2)}$$

$$M_{BT}^{(2)} = {}^{n}y_{B^{(2)}/E} \cdot F_{B,z}^{(2)}$$

$${}^{b'}y_{B^{(2)}/E} = -3.26\ [m]$$

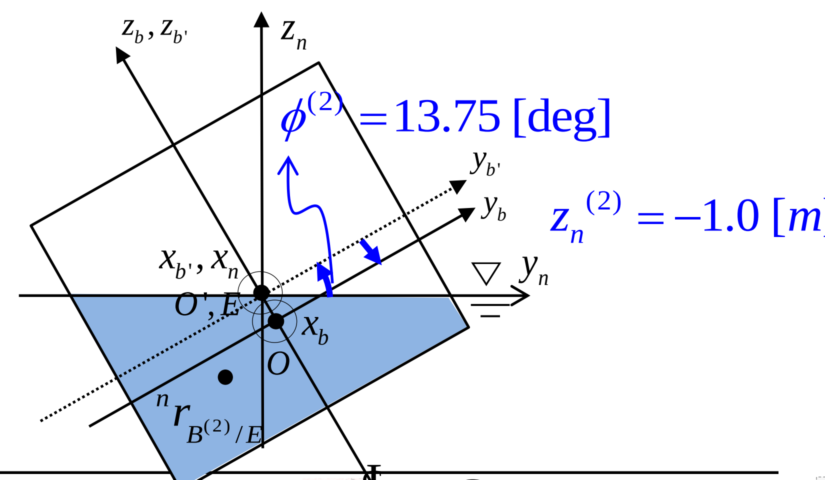
$${}^{b'}z_{B^{(2)}/E} = -4.60\ [m]$$

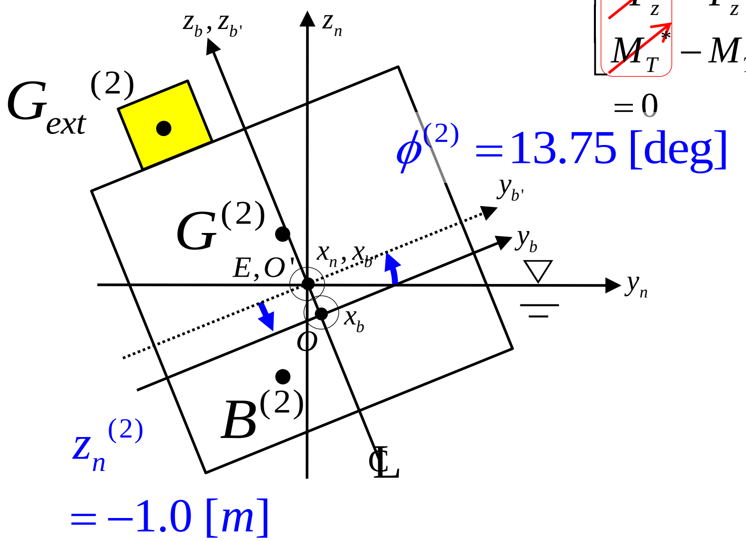
$${}^{n}\mathbf{r}_{B^{(2)}/E} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi^{(2)} & -\sin\phi^{(2)} \\ 0 & \sin\phi^{(2)} & \cos\phi^{(2)} \end{bmatrix} {}^{b'}\mathbf{r}_{B^{(2)}/E}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(13.75) & -\sin(13.75) \\ 0 & \sin(13.75) & \cos(13.75) \end{bmatrix} \begin{bmatrix} 0 \\ -3.26 \\ -4.60 \end{bmatrix}$$

$L = 100\ [m]$	$G^{(1)}(0, 1.48, 4.78)\ [m]$
$B_{mld} = 40\ [m]$	$B^{(1)}(0, 2.66, -5.39)\ [m]$
$D = 30\ [m]$	$g^{(1)}(0, 10.98, 18.58)\ [m]$
$d = 9\ [m]$	

$F_G^{(2)} = -3.6\times10^5\ [kN]$
$F_B^{(2)} = 4.0\times10^5\ [kN]$
$F_{ext}^{(2)} = -4.0\times10^4\ [kN]$
$\rho g = 10\ [Mg/m^2s^2]$





$$\begin{bmatrix} \cancel{F_z^*} - F_z \\ \cancel{M_T^*} - M_T \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP} & -\rho g A_{WP} \, {}^n y_{F/E} \\ -\rho g A_{WP} \, {}^n y_{F/E} & -\rho g \left({}^n z_{B/E} \nabla + I_T \right) - {}^n z_{G/E} \cdot F_{G,z} - {}^n z_{G_{ext}/E} \cdot F_{ext,z} \end{bmatrix} \begin{bmatrix} \delta z_n \\ \delta \phi \end{bmatrix}$$

$= 0$

$L = 100$ [m]	$G^{(1)}(0, 1.48, 4.78)$ [m]
$B_{mld} = 40$ [m]	$B^{(2)}(0, 2.07, -5.24)$ [m]
$D = 30$ [m]	$g^{(1)}(0, 10.98, 18.58)$ [m]
$d = 9$ [m]	

$F_G^{(2)} = -3.6 \times 10^5$ [kN]
$F_B^{(2)} = 4.0 \times 10^5$ [kN]
$F_{ext}^{(2)} = -4.0 \times 10^4$ [kN]
$\rho g = 10$ [$Mg/m^2 s^2$]

$$M_T^{(2)} = M_{BT}^{(2)} + M_{GT}^{(2)} + M_{extT}^{(2)}$$

$${}^{b'} y_{B^{(2)}/E} = -3.26 \text{ [m]}$$

$${}^{b'} z_{B^{(2)}/E} = -4.60 \text{ [m]}$$

$$M_{BT}^{(2)} = {}^n y_{B^{(2)}/E} \cdot F_{B,z}^{(2)}$$

$${}^n \mathbf{r}_{B^{(2)}/E} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi^{(2)} & -\sin\phi^{(2)} \\ 0 & \sin\phi^{(2)} & \cos\phi^{(2)} \end{bmatrix} {}^{b'} \mathbf{r}_{B^{(2)}/E}$$

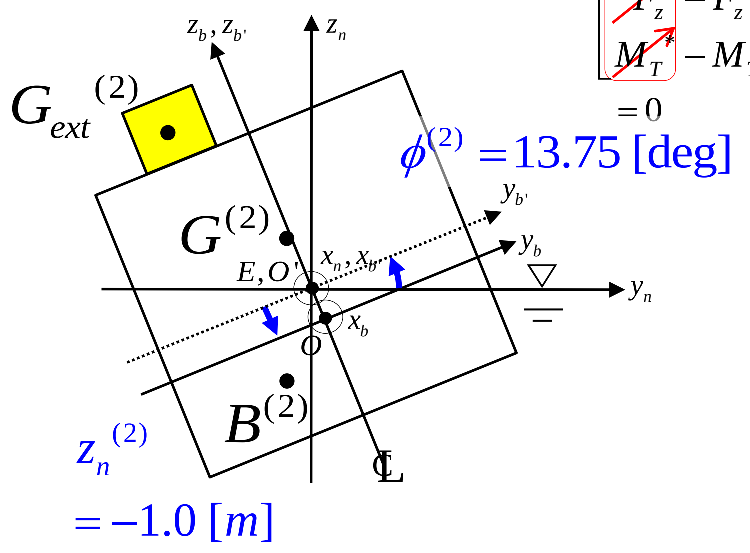
$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(13.75) & -\sin(13.75) \\ 0 & \sin(13.75) & \cos(13.75) \end{bmatrix} \begin{bmatrix} 0 \\ -3.26 \\ -4.60 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -2.07 \\ -5.24 \end{bmatrix}$$

$${}^n y_{B^{(2)}/E} = -2.07 \text{ [m]}$$

$$M_{BT}^{(2)} = (-2.07) \cdot (4.0 \times 10^5)$$

$$= -8.28 \times 10^5$$



$$\begin{bmatrix} \cancel{F_z^*} - F_z \\ \cancel{M_T^*} - M_T \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP} & -\rho g A_{WP} \, {}^n y_{F/E} \\ -\rho g A_{WP} \, {}^n y_{F/E} & -\rho g \left({}^n z_{B/E} \nabla + I_T \right) - {}^n z_{G/E} \cdot F_{G,z} - {}^n z_{G_{ext}/E} \cdot F_{ext,z} \end{bmatrix} \begin{bmatrix} \delta z_n \\ \delta \phi \end{bmatrix}$$

$= 0$

$$M_T^{(2)} = M_{BT}^{(2)} + M_{GT}^{(2)} + M_{extT}^{(2)}$$

$$M_{BT}^{(2)} = -8.28 \times 10^5$$

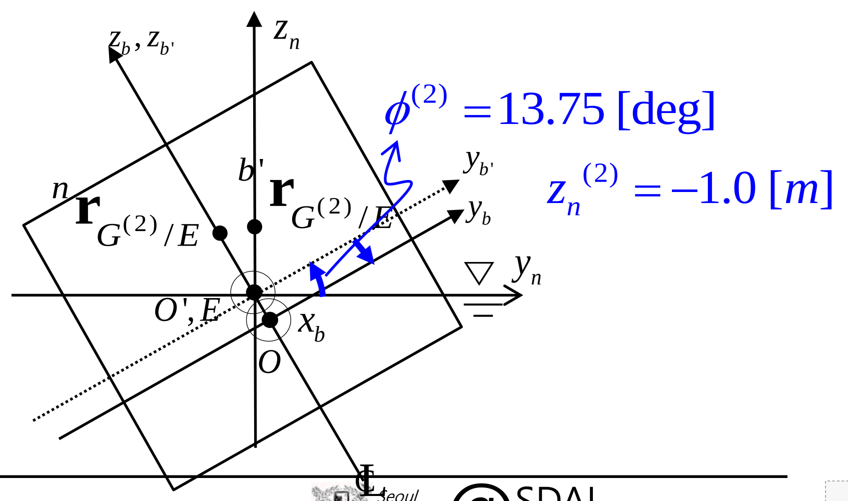
$$M_{GT}^{(2)} = {}^n y_{G^{(2)}/E} \cdot F_G^{(2)}$$

$${}^n \mathbf{r}_{G^{(2)}/E} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi^{(2)} & -\sin\phi^{(2)} \\ 0 & \sin\phi^{(2)} & \cos\phi^{(2)} \end{bmatrix} {}^b \mathbf{r}_{G^{(2)}/E}, \quad {}^{b'} \mathbf{r}_{G^{(2)}/E} = {}^{b'} \mathbf{r}_{G/E} + \begin{bmatrix} 0 \\ 0 \\ z_n^{(2)} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(13.75) & -\sin(13.75) \\ 0 & \sin(13.75) & \cos(13.75) \end{bmatrix} \left(\begin{bmatrix} 0 \\ 0 \\ 6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -1.0 \end{bmatrix} \right)$$

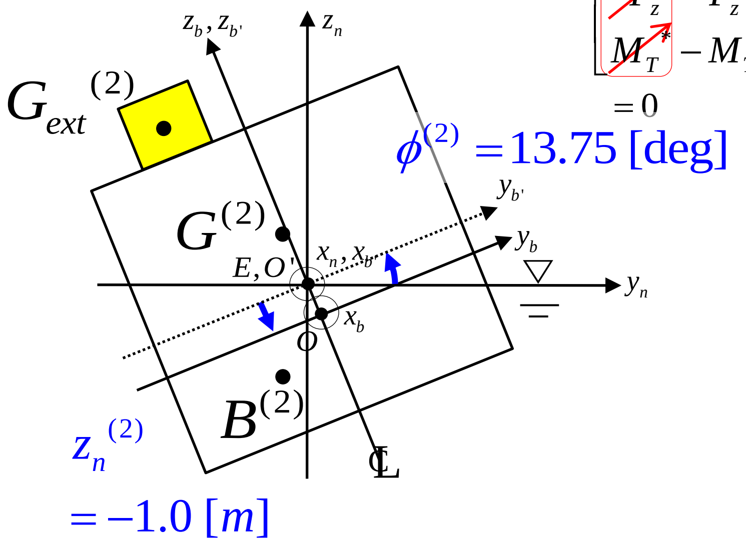
$L = 100$ [m]	$G^{(1)}(0, 1.48, 4.78)$ [m]
$B_{mld} = 40$ [m]	$B^{(2)}(0, 2.07, -5.24)$ [m]
$D = 30$ [m]	$g^{(1)}(0, 10.98, 18.58)$ [m]
$d = 9$ [m]	

$F_G^{(2)} = -3.6 \times 10^5$ [kN]
$F_B^{(2)} = 4.0 \times 10^5$ [kN]
$F_{ext}^{(2)} = -4.0 \times 10^4$ [kN]
$\rho g = 10$ [Mg / m²s²]



$$\begin{bmatrix} \cancel{F_z^*} - F_z \\ \cancel{M_T^*} - M_T \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP} & -\rho g A_{WP}\, {}^n y_{F/E} \\ -\rho g A_{WP}\, {}^n y_{F/E} & -\rho g\left({}^n z_{B/E}\nabla + I_T\right) - {}^n z_{G/E}\cdot F_{G,z} - {}^n z_{G_{ext}/E}\cdot F_{ext,z} \end{bmatrix} \begin{bmatrix} \delta z_n \\ \delta\phi \end{bmatrix}$$

$= 0$



$L = 100\ [m]$	$G^{(2)}(0, 1.19, 4.86)\ [m]$
$B_{mld} = 40\ [m]$	$B^{(2)}(0, 2.07, -5.24)\ [m]$
$D = 30\ [m]$	$g^{(1)}(0, 10.98, 18.58)\ [m]$
$d = 9\ [m]$	

$F_G^{(2)} = -3.6\times10^5\ [kN]$
$F_B^{(2)} = 4.0\times10^5\ [kN]$
$F_{ext}^{(2)} = -4.0\times10^4\ [kN]$
$\rho g = 10\ [Mg/m^2 s^2]$

$M_{BT}^{(2)} = -8.28\times10^5$

$$M_T^{(2)} = M_{BT}^{(2)} + M_{GT}^{(2)} + M_{extT}^{(2)}$$

$$M_{GT}^{(2)} = {}^n y_{G^{(2)}/E} \cdot F_G^{(2)}$$

$${}^n\mathbf{r}_{G^{(2)}/E} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi^{(2)} & -\sin\phi^{(2)} \\ 0 & \sin\phi^{(2)} & \cos\phi^{(2)} \end{bmatrix} {}^{b'}\mathbf{r}_{G^{(2)}/E}, \quad {}^{b'}\mathbf{r}_{G^{(2)}/E} = {}^{b'}\mathbf{r}_{G/E} + \begin{bmatrix} 0 \\ 0 \\ z_n^{(2)} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(13.75) & -\sin(13.75) \\ 0 & \sin(13.75) & \cos(13.75) \end{bmatrix}\left(\begin{bmatrix} 0 \\ 0 \\ 6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -1.0 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 0 \\ -1.19 \\ 4.86 \end{bmatrix}$$

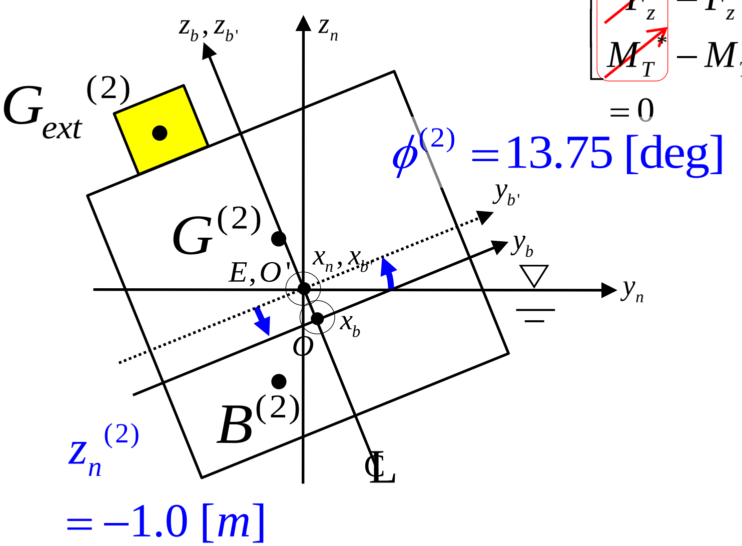
$${}^n y_{G^{(2)}/E} = -1.19\ [m]$$

$$M_{GT}^{(2)} = (-1.19)\cdot(-3.6\times10^5)$$

$$= 4.28\times10^5$$

$$\begin{bmatrix} F_z^* - F_z \\ M_T^* - M_T \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP} & -\rho g A_{WP}\,{}^n y_{F/E} \\ -\rho g A_{WP}\,{}^n y_{F/E} & -\rho g\left({}^n z_{B/E}\nabla + I_T\right) - {}^n z_{G/E}\cdot F_{G,z} - {}^n z_{G_{ext}/E}\cdot F_{ext,z} \end{bmatrix} \begin{bmatrix} \delta z_n \\ \delta\phi \end{bmatrix}$$

$$F_z^* = 0,\quad M_T^* = 0$$



$L = 100\ [m]$	$G^{(2)}(0, 1.19, 4.86)\ [m]$
$B_{mld} = 40\ [m]$	$B^{(2)}(0, 2.07, -5.24)\ [m]$
$D = 30\ [m]$	$g^{(2)}(0, 9.85, 19.21)\ [m]$
$d = 9\ [m]$	

$F_G^{(2)} = -3.6\times10^5\ [kN]$

$F_B^{(2)} = 4.0\times10^5\ [kN]$

$F_{ext}^{(2)} = -4.0\times10^4\ [kN]$

$\rho g = 10\ [Mg/m^2 s^2]$

$$M_T^{(2)} = M_{BT}^{(2)} + M_{GT}^{(2)} + M_{extT}^{(2)}$$

$$M_{BT}^{(2)} = -8.28\times10^5$$

$$M_{GT}^{(2)} = 4.28\times10^5$$

$$M_{extT}^{(2)} = {}^n y_{G_{ext}^{(2)}/E}\cdot F_{ext}^{(2)}$$

$${}^n\mathbf{r}_{G_{ext}^{(2)}/E} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi^{(2)} & -\sin\phi^{(2)} \\ 0 & \sin\phi^{(2)} & \cos\phi^{(2)} \end{bmatrix} {}^{b'}\mathbf{r}_{G_{ext}^{(2)}/E},\quad {}^{b'}\mathbf{r}_{G_{ext}^{(2)}/E} = {}^{b'}\mathbf{r}_{G_{ext}/E} + \begin{bmatrix} 0 \\ 0 \\ z_n^{(2)} \end{bmatrix}$$

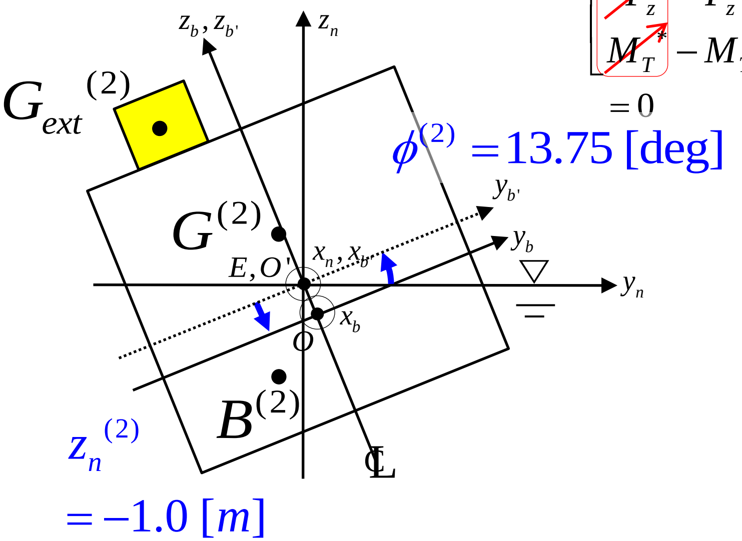
$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(13.75) & -\sin(13.75) \\ 0 & \sin(13.75) & \cos(13.75) \end{bmatrix} \left(\begin{bmatrix} 0 \\ 5 \\ 22 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -1.0 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 0 \\ -9.85 \\ 19.21 \end{bmatrix}$$

$${}^n y_{G_{ext}^{(2)}/E} = -9.85\ [m]$$

$$M_{extT}^{(2)} = (-9.85)\cdot(-4.0\times10^4)$$

$$= 3.94\times10^5$$



$$\begin{bmatrix} \underbrace{\begin{matrix} \cancel{F_z^*} \\ \cancel{M_T^*} \end{matrix}}_{=0} \begin{matrix} - F_z \\ - M_T \end{matrix} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP} & -\rho g A_{WP}\, {}^n y_{F/E} \\ -\rho g A_{WP}\, {}^n y_{F/E} & -\rho g\left({}^n z_{B/E}\nabla + I_T\right) - {}^n z_{G/E}\cdot F_{G,z} - {}^n z_{G_{ext}/E}\cdot F_{ext,z} \end{bmatrix}\begin{bmatrix} \delta z_n \\ \delta\phi \end{bmatrix}$$

$$M_T^{(2)} = M_{BT}^{(2)} + M_{GT}^{(2)} + M_{extT}^{(2)}$$

$$M_{BT}^{(2)} = -8.28\times10^5$$

$$M_{GT}^{(2)} = 4.28\times10^5$$

$$M_{extT}^{(2)} = 3.94\times10^5$$

$$M_T^{(2)} = -8.28\times10^5 + 4.28\times10^5 + 3.94\times10^5$$

$$= \boxed{-6.00\times10^3\ [kN\cdot m]}$$

$$\left|-6.00\times10^3\right|\ [kN\cdot m] > e$$

Tolerance

where, e(epsilon) : an arbitrarily small positive quantity

$L = 100\ [m]$
$B_{mld} = 40\ [m]$
$D = 30\ [m]$
$d = 9\ [m]$

$G^{(2)}(0, 1.19, 4.86)\ [m]$
$B^{(2)}(0, 2.07, -5.24)\ [m]$
$g^{(2)}(0, 9.85, 19.21)\ [m]$

$F_G^{(2)} = -3.6\times10^5\ [kN]$
$F_B^{(2)} = 4.0\times10^5\ [kN]$
$F_{ext}^{(2)} = -4.0\times10^4\ [kN]$
$\rho g = 10\ [Mg/m^2s^2]$

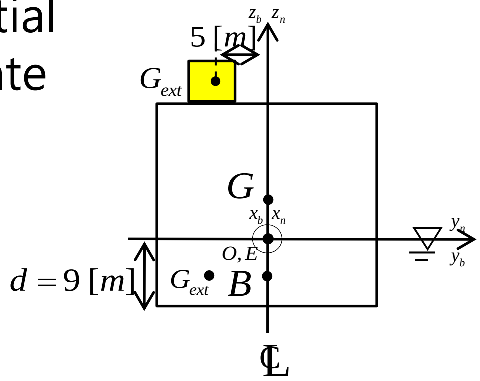
The static equilibrium of moment is not satisfied!

To obtain the accurate result, you have to iterate until the static equilibrium is satisfied.

An Example of Immersion and Heel of a Box-Shaped Ship with a Fixed Weight - Summary

Initial State

Section view



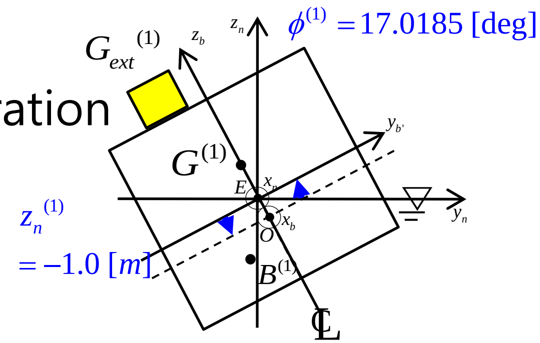
Force(KN)	Transverse Moment(KN·m)
F = -4.00x10⁴	**M_T = 2.00x10⁵**

$$\begin{bmatrix} -4.00\times10^4 \\ 2.00\times10^5 \end{bmatrix} = \begin{bmatrix} -4.00\times10^4 & 0.0 \\ 0.0 & 6.73\times10^5 \end{bmatrix} \begin{bmatrix} \delta z_n \\ \delta\phi \end{bmatrix}$$

$\delta z_n = -1.00\,[m]$

$\delta\phi = 17.0185\,[\deg]$

1st Iteration



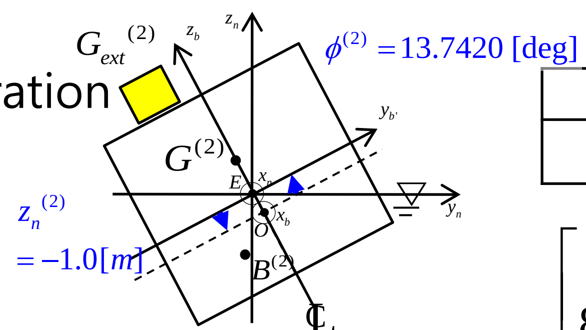
Force(KN)	Transverse Moment(KN·m)
F = 0.0	**M_T = -8.48x10⁴**

$$\begin{bmatrix} 0.0 \\ 8.48\times10^4 \end{bmatrix} = \begin{bmatrix} -4.18\times10^4 & 0.0 \\ 0.0 & -1.48\times10^6 \end{bmatrix} \begin{bmatrix} \delta z_n \\ \delta\phi \end{bmatrix}$$

$\delta z_n = 0.0\,[m]$

$\delta\phi = -3.2766\,[\deg]$

2nd Iteration



Force(KN)	Transverse Moment(KN·m)
F = 0.0	**M_T = -8.31x10³**

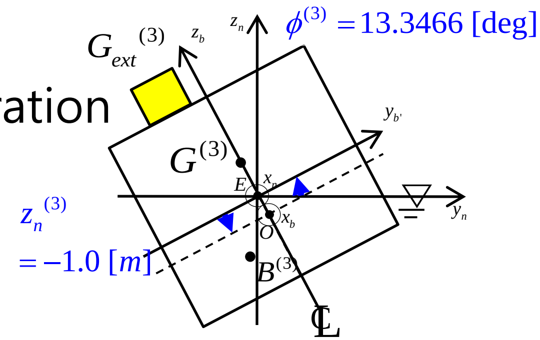
$$\begin{bmatrix} 0.0 \\ 8.31\times10^3 \end{bmatrix} = \begin{bmatrix} -4.12\times10^4 & 0.0 \\ 0.0 & -1.20\times10^6 \end{bmatrix} \begin{bmatrix} \delta z_n \\ \delta\phi \end{bmatrix}$$

$\delta z_n = 0.0\,[m]$

$\delta\phi = -0.3954\,[\deg]$

summary

3rd Iteration



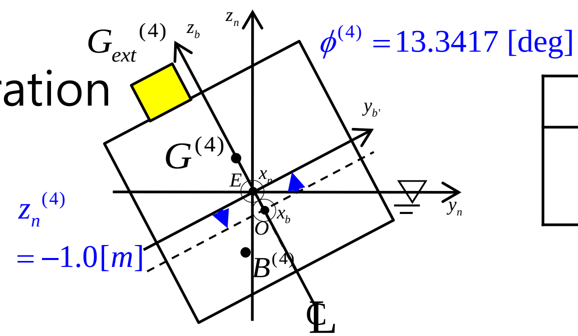
$\delta z_n = 0.0\ [m]$

$\delta\phi = -0.3954$ [deg]

Force(KN)	Transverse Moment(KN·m)
F = 0.0	M_T = -100.34

$$\begin{bmatrix} 0.0 \\ 100.34 \end{bmatrix} = \begin{bmatrix} -4.11\times10^4 & 0.0 \\ 0.0 & -1.18\times10^6 \end{bmatrix} \begin{bmatrix} \delta z_n \\ \delta\phi \end{bmatrix}$$

4th Iteration



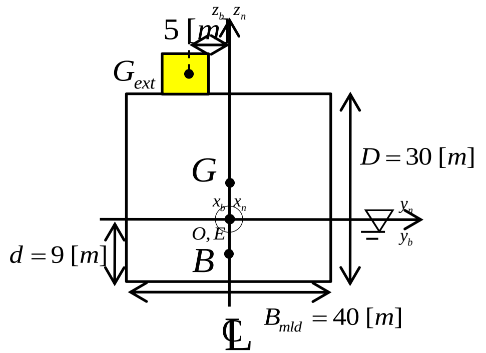
$\delta z_n = 0.0\ [m]$

$\delta\phi = -0.0049$ [deg]

Force(KN)	Transverse Moment(KN·m)
F = 0.0 (|F|<e(=1.0))	M_T = -0.15 (|M_T|<e(=1.0))

An Example of Immersion and Heel of a Box-Shaped Ship with a Fixed Weight - Summary

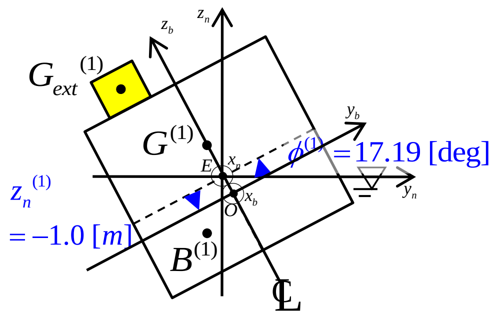
Initial State



Force(KN)			Transverse Moment Arm(m)			Transverse Moment(KN·m)		
F_B	F_G	F_{ext}	y_B	y_G	y_{ext}	M_{BT}	M_{GT}	M_{extT}
3.60×10^5	-3.60×10^5	-4.00×10^4	0	0	-5	0	0	-2.00×10^5
$F = -4.00 \times 10^4$						$M_T = 2.00 \times 10^5$		

$$\delta z_n = -1.0\ [m]$$
$$\delta\phi = 17.19\ [\text{deg}]$$

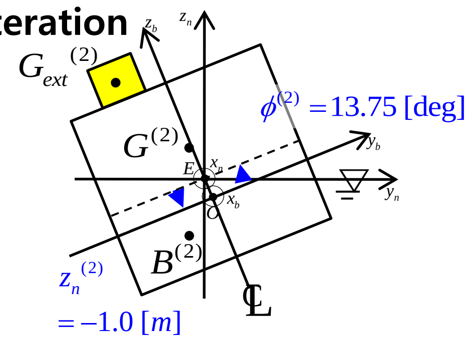
1st Iteration



Force(KN)			Transverse Moment Arm(m)			Transverse Moment(KN·m)		
F_B	F_G	F_{ext}	y_B	y_G	y_{ext}	M_{BT}	M_{GT}	M_{extT}
4.00×10^5	-3.60×10^5	-4.00×10^4	-2.66	-1.48	-10.98	1.06×10^6	-5.33×10^5	-4.39×10^5
$F = 0.00$						$M_T = -8.80 \times 10^4$		

$$\delta z_n = 0\ [m]$$
$$\delta\phi = -3.44\ [\text{deg}]$$

2nd Iteration



Force(KN)			Transverse Moment Arm(m)			Transverse Moment(KN·m)		
F_B	F_G	F_{ext}	y_B	y_G	y_{ext}	M_{BT}	M_{GT}	M_{extT}
4.00×10^5	-3.60×10^5	-4.00×10^4	-2.07	-1.19	-9.85	8.28×10^5	-4.28×10^5	-3.94×10^5
$F = 0.00$						$M_T = -6.00 \times 10^3$		

We have to iterate, until total transverse moment is smaller than epsilon(e).

11-2. Coupled Immersion, Heel, and Trim of a Box-shaped Ship

Governing Equations of Computational Ship Stability

$$
\begin{bmatrix} \cancel{F_z} - F_z\left(z_n^{(k)}, \phi^{(k)}, \theta^{(k)}\right) \\ \cancel{M_T} - M_T\left(z_n^{(k)}, \phi^{(k)}, \theta^{(k)}\right) \\ \cancel{M_L} - M_L\left(z_n^{(k)}, \phi^{(k)}, \theta^{(k)}\right) \end{bmatrix}
=
\left.\begin{bmatrix}
\frac{\partial F_B}{\partial z_n}+\frac{\partial F_G}{\partial z_n}+\frac{\partial F_{ext}}{\partial z_n} & \frac{\partial F_B}{\partial \phi}+\frac{\partial F_G}{\partial \phi}+\frac{\partial F_{ext}}{\partial \phi} & \frac{\partial F_B}{\partial \theta}+\frac{\partial F_G}{\partial \theta}+\frac{\partial F_{ext}}{\partial \theta} \\
\boxed{-\rho g A_{WP}^{(k)}} & \boxed{-\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E}} & \boxed{\rho g A_{WP}^{(k)} \cdot {}^{n}x_{F^{(k)}/E}} \\
\frac{\partial M_{BT}}{\partial z_n}+\frac{\partial M_{GT}}{\partial z_n}+\frac{\partial M_{extT}}{\partial z_n} & \frac{\partial M_{BT}}{\partial \phi}+\frac{\partial M_{GT}}{\partial \phi}+\frac{\partial M_{extT}}{\partial \phi} & \frac{\partial M_{BT}}{\partial \theta}+\frac{\partial M_{GT}}{\partial \theta}+\frac{\partial M_{extT}}{\partial \theta} \\
\boxed{-\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E}} & \boxed{\begin{matrix} -\rho g\left({}^{n}z_{B^{(k)}/E}\nabla^{(k)} + I_T^{(k)}\right) \\ -{}^{n}z_{G^{(k)}/E}\cdot F_G - {}^{n}z_{G_{ext}^{(k)}/E}\cdot F_{ext}^{(k)} \end{matrix}} & \boxed{\rho g I_P^{(k)}} \\
\frac{\partial M_{BL}}{\partial z_n}+\frac{\partial M_{GL}}{\partial z_n}+\frac{\partial M_{extL}}{\partial z_n} & \frac{\partial M_{BL}}{\partial \phi}+\frac{\partial M_{GL}}{\partial \phi}+\frac{\partial M_{extL}}{\partial \phi} & \frac{\partial M_{BL}}{\partial \theta}+\frac{\partial M_{GL}}{\partial \theta}+\frac{\partial M_{extL}}{\partial \theta} \\
\boxed{\rho g A_{WP}^{(k)} \cdot {}^{n}x_{F^{(k)}/E}} & \boxed{\rho g I_P^{(k)}} & \boxed{\begin{matrix} -\rho g\left({}^{n}z_{B^{(k)}/E}\nabla^{(k)} + I_L^{(k)}\right) \\ -{}^{n}z_{G^{(k)}/E}\cdot F_G - {}^{n}z_{G_{ext}^{(k)}/E}\cdot F_{ext}^{(k)} \end{matrix}}
\end{bmatrix}\right|_{\substack{z_n=z_n^{(k)} \\ \phi=\phi^{(k)} \\ \theta=\theta^{(k)}}}
\begin{bmatrix} \delta z_n^{(k)} \\ \delta\phi^{(k)} \\ \delta\theta^{(k)} \end{bmatrix}
$$

$= 0$

We want to find the static equilibrium position and orientation!

F_G: gravitational force exerted on a ship

M_T: transverse moment of a ship about x_n axis

M_L: longitudinal moment of a ship about y_n axis

$A_{WP}^{(k)}$: waterplane area of a ship at k^th step

$I_T^{(k)}$: transverse moment of inertia of the waterplane area of a ship about x_n axis at k^th step

$I_L^{(k)}$: longitudinal moment of inertia of the waterplane area of a ship about y_n axis at k^th step

$I_P^{(k)}$: centrifugal moment of the waterplane area of a ship about x_n and y_n axis at k^th step

F_B: buoyant force exerted on a ship

F_{ext}: external force exerted on a ship

${}^{n}x_{F^{(k)}/E}$: x_n coordinate of centroid of the waterplane area of a ship

${}^{n}y_{F^{(k)}/E}$: y_n coordinate of centroid of the waterplane area of a ship

${}^{n}z_{B^{(k)}/E}$: z_n coordinate of center of the displaced volume of a ship

${}^{n}z_{G^{(k)}/E}$: z_n coordinate of center of mass of the ship

$\delta z^{(k)}$: change in the draft at k^th step

$\delta\phi^{(k)}$: change in the angle of heel at k^th step

$\delta\theta^{(k)}$: change in the angle of trim at k^th step

μ_V: permeability of a compartment

μ_F: surface permeability of a compartment

$a_{WP}^{(k)}$: waterplane area of a flooded compartment at k^th step

$i_T^{(k)}$: transverse moment of inertia of the waterplane area of a flooded compartment about x_n axis at k^th step

$i_L^{(k)}$: longitudinal moment of inertia of the waterplane area of a flooded compartment about y_n axis at k^th step

$i_P^{(k)}$: centrifugal moment of the waterplane area of a flooded compartment about x_n and y_n axis at k^th step

${}^{n}x_{f^{(k)}/E}$: x_n coordinate of centroid of the waterplane area of a flooded compartment at k^th step

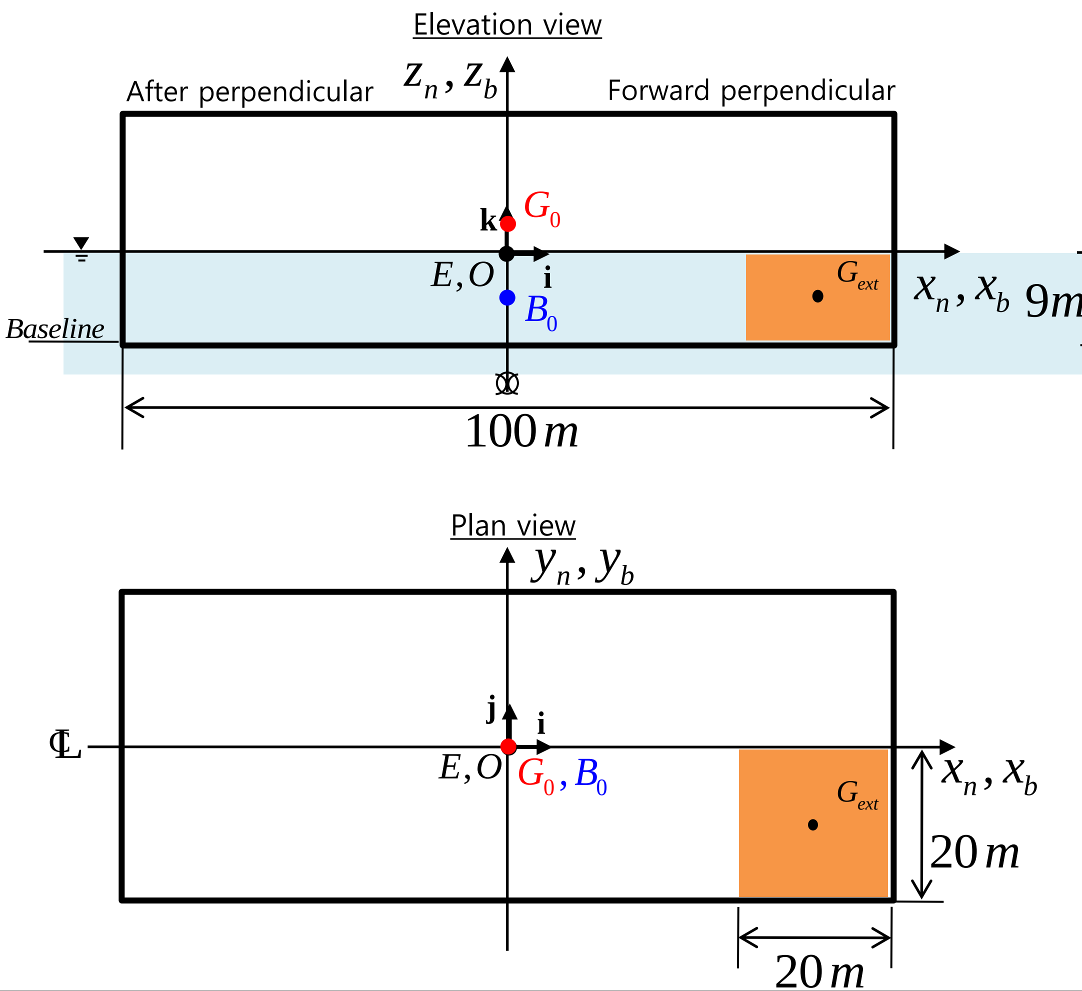
${}^{n}y_{f^{(k)}/E}$: y_n coordinate of centroid of the waterplane area of a flooded compartment at k^th step

${}^{n}z_{G_{ext}^{(k)}/E}$: z_n coordinate of center of the submerged volume of a flooded compartment at k^th step

Example of Coupled Immersion, Heel, and Trim of a Box-shaped Ship when a Cargo is Moved in Transverse and Longitudinal Directions

A ship is floating in the sea water with loading a cargo in the cargo hold located in the -y direction and +x direction. Calculate the change of the position and orientation(Immersion, Trim and Heel) of the ship.

$$L = 100\,m,\ \ B = 40\,m,\ \ D = 30m,\ \ T = 9m, KG_0 = 15\,m,\ Og_0 = (40, -10, -4), w = 40,000[kN]$$



✓The information of cargo (w):
- l(length) : 20 m
- b(breath) : 20m
- h(height) : 9m
- weight: 4 x 10^4 kN
- center of gravity w.r.t the body-fixed coordinate system: (40,-10,-4.5)

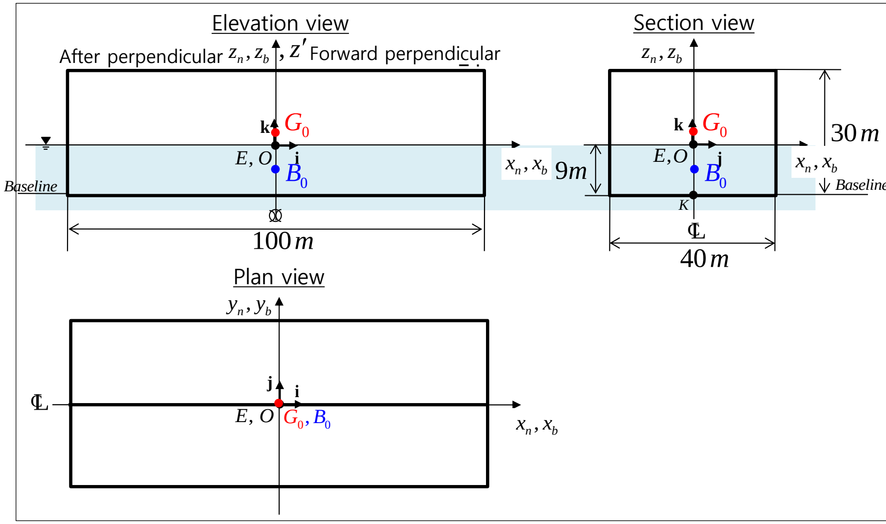
$E - x_n y_n z_n$: Space-fixed coordinate system

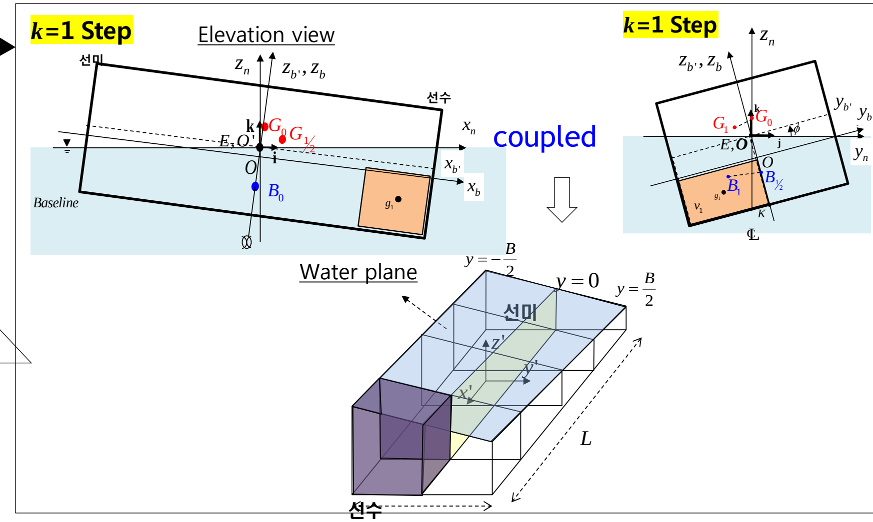
$O - x_b y_b z_b$: Body-fixed coordinate system

G : center of gravity of the ship

: center of buoyancy of the ship

G_{ext}: center of gravity of cargo





$$
\begin{bmatrix} F_z^* - F_z^{(k)} \\ M_T^* - M_T^{(k)} \\ M_L^* - M_L^{(k)} \end{bmatrix}
=
\begin{bmatrix}
-\rho g A_{WP}^{(k)} & -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} & \rho g A_{WP}^{(k)} \cdot {}^{n}x_{F^{(k)}/E} \\
-\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} & -\rho g \left({}^{n}z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)} \right) - {}^{n}z_{G^{(k)}/E} \cdot F_{G,z} - {}^{n}z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} & \rho g I_P^{(k)} \\
\rho g A_{WP}^{(k)} \cdot {}^{n}x_{F^{(k)}/E} & \rho g I_P^{(k)} & -\rho g \left({}^{n}z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)} \right) - {}^{n}z_{G^{(k)}/E} \cdot F_{G,z} - {}^{n}z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)}
\end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}}
\begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}
$$

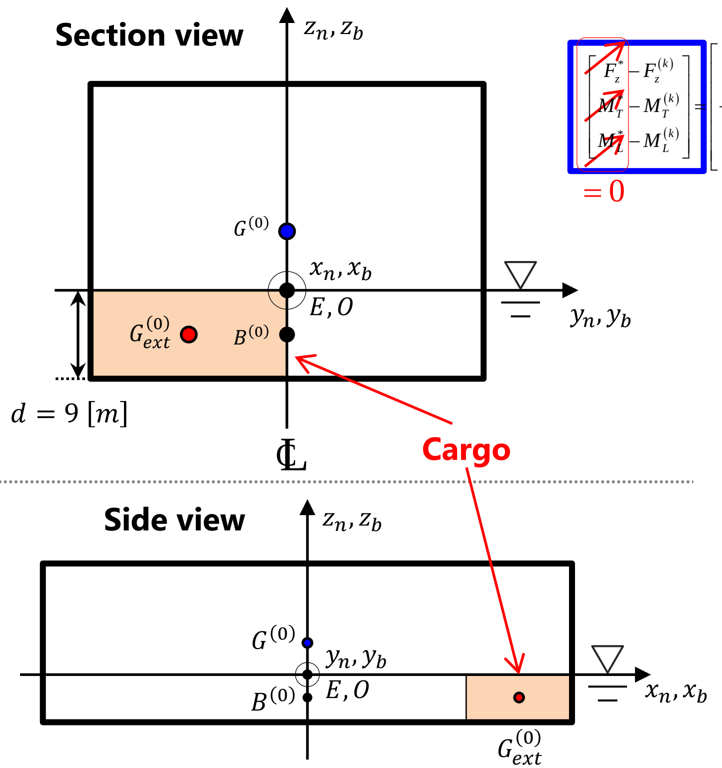
given

known

Find

Because the change of draft, heeling angle and trim angle are coupled, the equations have to be solved simultaneously.

1. Calculation of Force and Moments at k=0 step



$$\underbrace{\boxed{\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \\ \cancel{M_L^*} - M_L^{(k)} \end{bmatrix}}}_{=0} = \begin{bmatrix} -\rho g A_{WP}{}^{(k)} & -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} & \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T{}^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} & \rho g I_P{}^{(k)} \\ \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} & \rho g I_P{}^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L{}^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$L = 100\ [m]$	${}^n\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$
$B_{mld} = 40\ [m]$	${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [40 \quad -10 \quad -4.5]^T\ [m]$
$D = 30\ [m]$	${}^n\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T\ [m]$
$d = 9\ [m]$	${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2s^2]$	

$\nabla^{(0)} = 3.6 \times 10^4\ [m^3]$

$F_{G,z} = -3.6 \times 10^5\ [kN]$

$F_{G_{ext},z} = -4.0 \times 10^4\ [kN]$

$F_{B,z}^{(0)} = 3.6 \times 10^5\ [kN]$

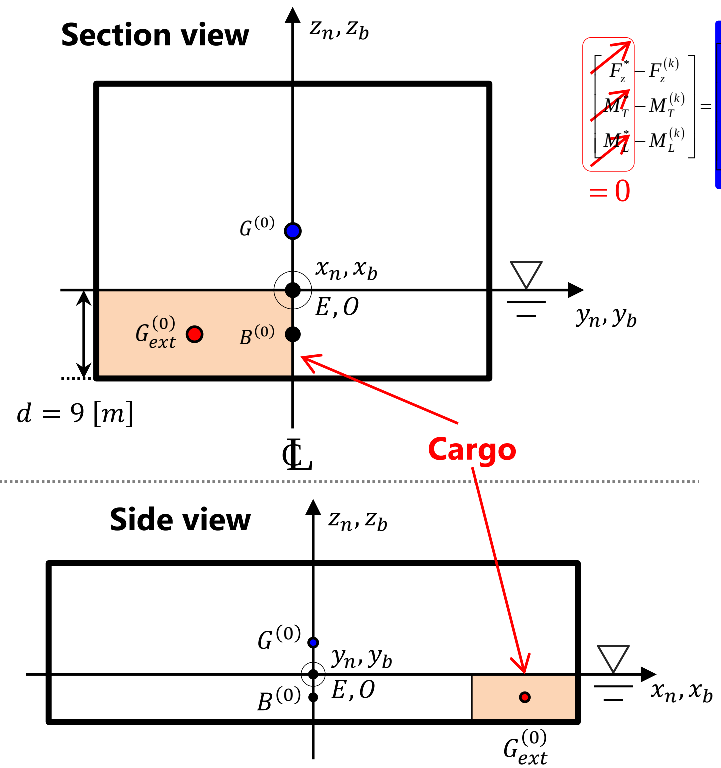
$$\begin{aligned} F_z^{(0)} &= F_{B,z}^{(0)} + F_{G,z}^{(0)} + F_{ext,z}^{(0)} \\ &= \rho g \nabla^{(0)} + F_{G,z} + F_{ext,z} \\ &= 10 \cdot (3.6 \times 10^4) + (-3.6 \times 10^5) + (-4.0 \times 10^4) \\ &= -4.0 \times 10^4\ [kN] \end{aligned}$$

$$\begin{aligned} \nabla^{(0)} &= L \cdot B_{mld} \cdot d = 100 \cdot 40 \cdot 30 \\ &= 3.6 \times 10^4\ [m^3] \end{aligned}$$

$$\begin{aligned} M_T^{(0)} &= M_{BT}^{(0)} + M_{GT}^{(0)} + M_{extT}^{(0)} \\ &= {}^n y_{B^{(0)}/E} \cdot F_{B,z}^{(0)} + {}^n y_{G^{(0)}/E} \cdot F_{G,z} + {}^n y_{G_{ext}^{(0)}/E} \cdot F_{ext,z} \\ &= 0 \cdot (3.6 \times 10^5) + 0 \cdot (-3.6 \times 10^5) + (-10) \cdot (-4.0 \times 10^4) \\ &= 4.0 \times 10^5\ [kN \cdot m] \end{aligned}$$

$$\begin{aligned} M_L^{(0)} &= M_{BL}^{(0)} + M_{GL}^{(0)} + M_{extL}^{(0)} \\ &= \left(-{}^n x_{B^{(0)}/E} \cdot F_{B,z}^{(0)} \right) + \left(-{}^n x_{G^{(0)}/E} \cdot F_{G,z} \right) + \left(-{}^n x_{G_{ext}^{(0)}/E} \cdot F_{ext,z} \right) \\ &= [-0 \cdot (3.6 \times 10^5)] + [-0 \cdot (-3.6 \times 10^5)] + [-40 \cdot (-4.0 \times 10^4)] \\ &= 1.6 \times 10^6\ [kN \cdot m] \end{aligned}$$

2. Calculation of the Properties of the Waterplane at k=0 step



$L = 100\ [m]$

$B_{mld} = 40\ [m]$

$D = 30\ [m]$

$d = 9\ [m]$

$\rho g = 10\ [Mg/m^2 s^2]$

${}^{n}\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$

${}^{n}\mathbf{r}_{G_{ext}^{(0)}/E} = [40 \quad -10 \quad -4.5]^T\ [m]$

${}^{n}\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T\ [m]$

${}^{n}\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$

$\nabla^{(0)} = 3.6 \times 10^4\ [m^3]$

$F_{G,z} = -3.6 \times 10^5\ [kN]$

$F_{G_{ext},z} = -4.0 \times 10^4\ [kN]$

$F_{B,z}^{(0)} = 3.6 \times 10^5\ [kN]$

$F_z^{(0)} = -4.0 \times 10^4\ [kN]$

$M_T^{(0)} = 4.0 \times 10^5\ [kN]$

$M_L^{(0)} = 1.6 \times 10^6\ [kN]$

$$\underbrace{\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \\ \cancel{M_L^*} - M_L^{(k)} \end{bmatrix}}_{=0} = \boxed{\begin{bmatrix} -\rho g A_{WP}^{(k)} & -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} & \rho g A_{WP}^{(k)} \cdot {}^{n}x_{F^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} & -\rho g \left({}^{n}z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)} \right) - {}^{n}z_{G^{(k)}/E} \cdot F_{G,z} - {}^{n}z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} & \rho g I_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^{n}x_{F^{(k)}/E} & \rho g I_P^{(k)} & -\rho g \left({}^{n}z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)} \right) - {}^{n}z_{G^{(k)}/E} \cdot F_{G,z} - {}^{n}z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$$-\rho g A_{WP}^{(0)} = -\rho g (L \cdot B_{mld}) = -10 \cdot (100 \cdot 40)$$
$$= -4.0 \times 10^4\ [kN/m]$$

$$-\rho g A_{WP}^{(0)} \cdot {}^{n}y_{F^{(0)}/E} = -(4.0 \times 10^4) \cdot 0 = 0\ [kN]$$

$$\rho g A_{WP}^{(0)} \cdot {}^{n}x_{F^{(0)}/E} = (4.0 \times 10^4) \cdot 0 = 0\ [kN]$$

$$I_T^{(0)} = \frac{1}{12} L \cdot B_{mld}^3 = \frac{1}{12} \cdot 100 \cdot 40^3 = 5.33 \times 10^5\ [m^4]$$

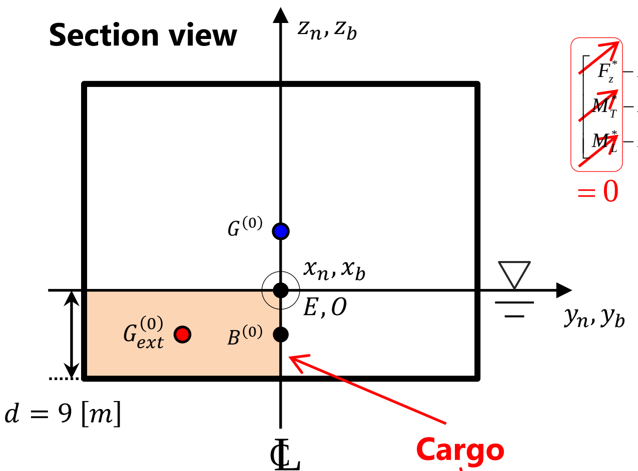
$$I_L^{(0)} = \frac{1}{12} L^3 \cdot B_{mld} = \frac{1}{12} \cdot 100^3 \cdot 40 = 3.33 \times 10^6\ [m^4]$$

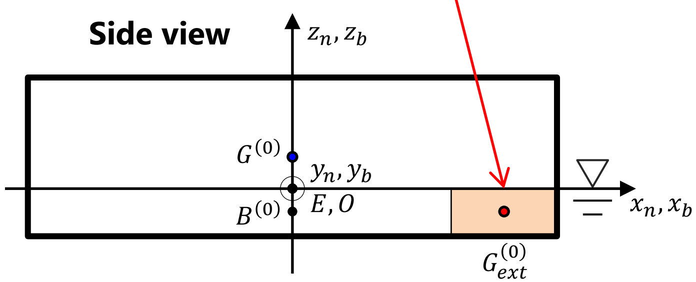
$$I_P^{(0)} = 0\ [m^4]$$

$$-\rho g \left({}^{n}z_{B^{(0)}/E} \nabla^{(0)} + I_T^{(0)} \right) - {}^{n}z_{G^{(0)}/E} \cdot F_{G,z} - {}^{n}z_{G_{ext}^{(0)}/E} \cdot F_{ext,z}^{(0)}$$
$$= -10 \cdot [-4.5 \cdot (3.6 \times 10^4) + (5.33 \times 10^5)]$$
$$-6 \cdot (-3.6 \times 10^5) - (-4.5) \cdot (-4.0 \times 10^4)$$
$$= -1.73 \times 10^6\ [kN \cdot m]$$

$$-\rho g \left({}^{n}z_{B^{(0)}/E} \nabla^{(0)} + I_L^{(0)} \right) - {}^{n}z_{G^{(0)}/E} \cdot F_{G,z} - {}^{n}z_{G_{ext}^{(0)}/E} \cdot F_{ext,z} \cdot F_{ext,z}^{(0)}$$
$$= -10 \cdot [-4.5 \cdot (3.6 \times 10^4) + (3.33 \times 10^6)]$$
$$-6 \cdot (-3.6 \times 10^5) - (-4.5) \cdot (-4.0 \times 10^4)$$
$$= -2.97 \times 10^7\ [kN \cdot m]$$

3. Calculation of Immersion, Trim, and Heel at k=0 step





$L = 100\ [m]$

$B_{mld} = 40\ [m]$

$D = 30\ [m]$

$d = 9\ [m]$

$\rho g = 10\ [Mg/m^2 s^2]$

${}^n\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$

${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [40 \quad -10 \quad -4.5]^T\ [m]$

${}^n\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T\ [m]$

${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$

$\nabla^{(0)} = 3.6 \times 10^4\ [m^3]$

$F_{G,z} = -3.6 \times 10^5\ [kN]$ $\qquad F_z^{(0)} = -4.0 \times 10^4\ [kN]$

$F_{G_{ext},z} = -4.0 \times 10^4\ [kN]$ $\qquad M_T^{(0)} = 4.0 \times 10^5\ [kN]$

$F_{B,z}^{(0)} = 3.6 \times 10^5\ [kN]$ $\qquad M_L^{(0)} = 1.6 \times 10^6\ [kN]$

$$\underbrace{\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \\ \cancel{M_L^*} - M_L^{(k)} \end{bmatrix}}_{=0} = \boxed{\left.\begin{bmatrix} -\rho g A_{WP}{}^{(k)} & -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} & \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g\left({}^n z_{B^{(k)}/E}\nabla^{(k)} + I_T{}^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} & \rho g I_P{}^{(k)} \\ \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} & \rho g I_P{}^{(k)} & -\rho g\left({}^n z_{B^{(k)}/E}\nabla^{(k)} + I_L{}^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} \end{bmatrix}\right|_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}}} \begin{bmatrix} \delta z^{(k)} \\ \delta\phi^{(k)} \\ \delta\theta^{(k)} \end{bmatrix}$$

$$\begin{bmatrix} 4.0\times10^4 \\ -4.0\times10^5 \\ -1.6\times10^6 \end{bmatrix} = \begin{bmatrix} -4.0\times10^4 & 0 & 0 \\ 0 & -1.73\times10^6 & 0 \\ 0 & 0 & -2.97\times10^7 \end{bmatrix} \begin{bmatrix} \delta z_n{}^{(0)} \\ \delta\phi^{(0)} \\ \delta\theta^{(0)} \end{bmatrix}$$

$$\boxed{\begin{bmatrix} \delta z_n{}^{(0)} \\ \delta\phi^{(0)} \\ \delta\theta^{(0)} \end{bmatrix}} = \begin{bmatrix} -4.0\times10^4 & 0 & 0 \\ 0 & -1.73\times10^6 & 0 \\ 0 & 0 & -2.97\times10^7 \end{bmatrix}^{-1} \begin{bmatrix} 4.0\times10^4 \\ -4.0\times10^5 \\ -1.6\times10^6 \end{bmatrix}$$

$$= \begin{bmatrix} -1.0\ [m] \\ 0.231[rad] \\ 0.054[rad] \end{bmatrix} = \boxed{\begin{bmatrix} -1.0\ [m] \\ 13.22[\deg] \\ 3.08[\deg] \end{bmatrix}}$$

$$z_n{}^{(1)} = z_n{}^{(0)} + \delta z_n{}^{(0)} = 0 + (-1.0) = -1.0\ [m]$$

$$\phi^{(1)} = \phi^{(0)} + \delta\phi^{(0)} = 0 + (13.22) = 13.22\ [\deg]$$

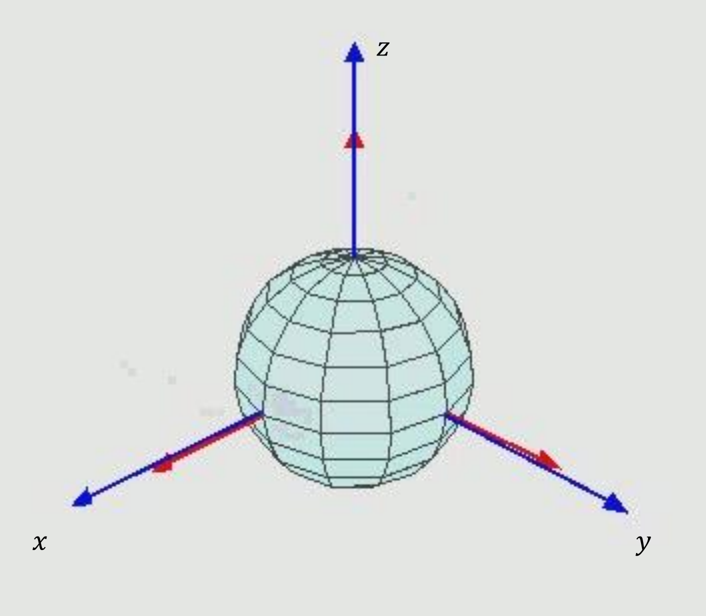
$$\theta^{(1)} = \theta^{(0)} + \delta\theta^{(0)} = 0 + (3.08) = 3.08\ [\deg]$$

회전 변환 정의: Trim, Heel 순서로 한다

Orientation of the rigid body in spatial motion - Euler angle

One of the most common and widely used parameters in describing reference orientations are the three independent Euler angle.
The transformation between two coordinate systems(Inertial frame and body fixed frame) can be carried out by means of three successive rotations performed in a given sequence.

Ahmed A. Shabana, Dynamics of multibody systems, third edition, Cambridge University Press, 2005, pp. 63



Orientation of the rigid body in spatial motion - Euler angle

One of the most common and widely used parameters in describing reference orientations are the three independent Euler angle.
The transformation between two coordinate systems(Inertial frame and body fixed frame) can be carried out by means of
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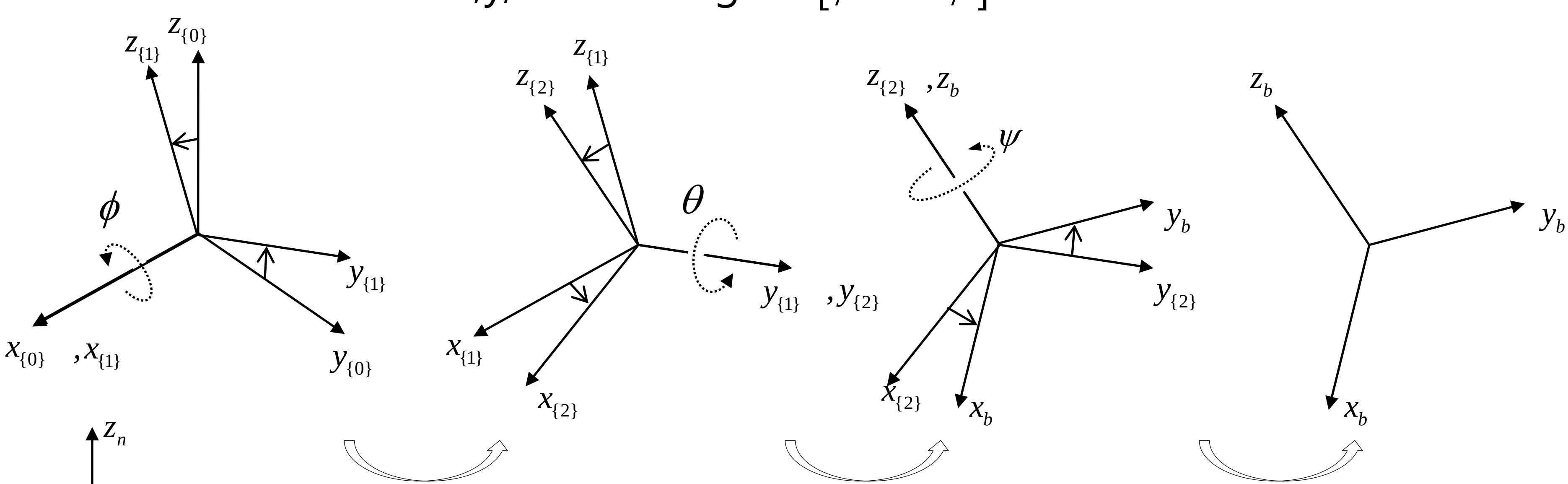
Ahmed A. Shabana, Dynamics of multibody systems, third edition, Cambridge University Press, 2005, pp. 63

x,y,z Euler angle $[\phi \quad \theta \quad \psi]^T$



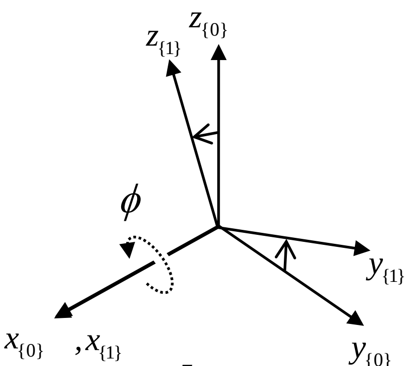
Rotation of an angle ϕ about the $x_{\{0\}}$ axis

Rotation of an angle θ about the $y_{\{1\}}$ axis

Rotation of an angle ψ about the $z_{\{2\}}$ axis

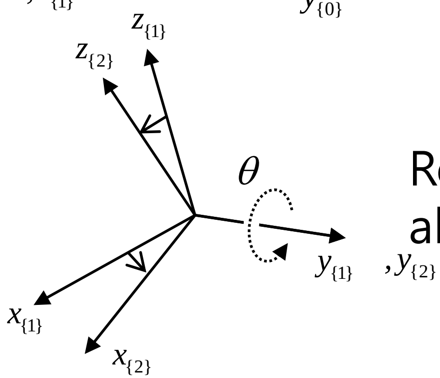
Jerry ginsberg, Engineering Dynamics, Georgia Institute of Technology, 2008, p.229~p.231

Rotation transformation in spatial motion



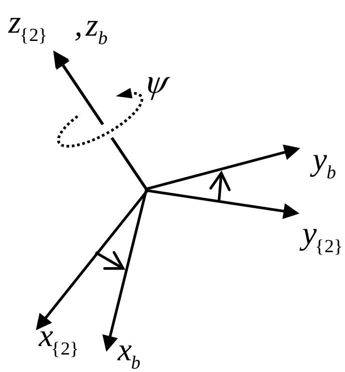
Rotation of an angle ϕ about the $x_{\{0\}}$-axis

$$ {}^{\{0\}}\mathbf{r}_{P/O} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix} {}^{\{1\}}\mathbf{r}_{P/O} $$



Rotation of an angle θ about the $y_{\{1\}}$-axis

$$ {}^{\{1\}}\mathbf{r}_{P/O} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} {}^{\{2\}}\mathbf{r}_{P/O} $$

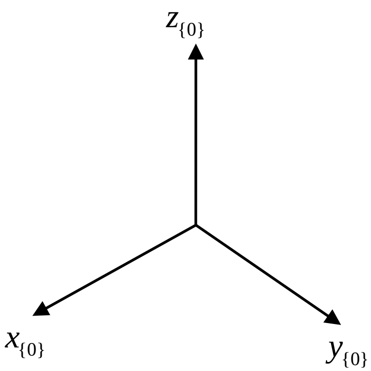


Rotation of an angle ψ about the $z_{\{2\}}$ axis

$$ {}^{\{2\}}\mathbf{r}_{P/O} = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} {}^{b}\mathbf{r}_{P/O} $$

Rotation transformation in spatial motion

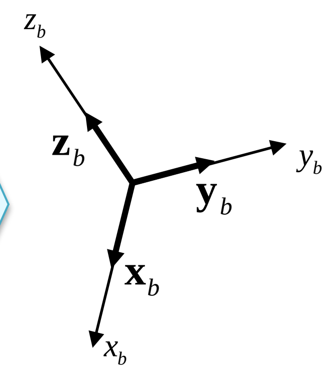
x,y,z Euler angle $[\phi \quad \theta \quad \psi]^T$



Rotation of an angle ϕ about the x axis

Rotation of an angle θ about the y axis

Rotation of an angle ψ about the z axis

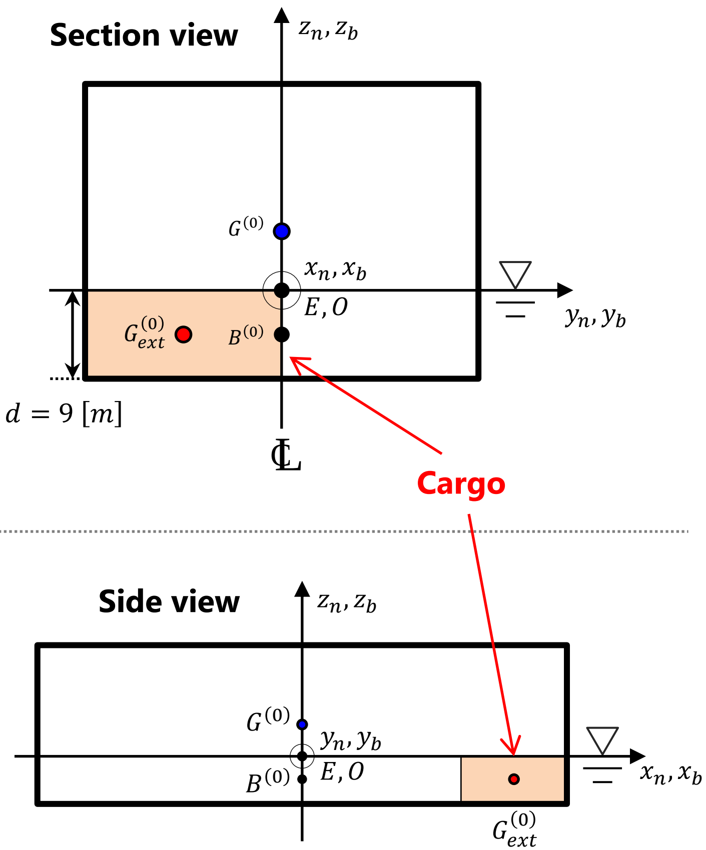


$$ {}^{\{0\}}\mathbf{r}_{P/O} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix} {}^{\{1\}}\mathbf{r}_{P/O} \qquad {}^{\{1\}}\mathbf{r}_{P/O} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} {}^{\{2\}}\mathbf{r}_{P/O} \qquad {}^{\{2\}}\mathbf{r}_{P/O} = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} {}^{b}\mathbf{r}_{P/O} $$

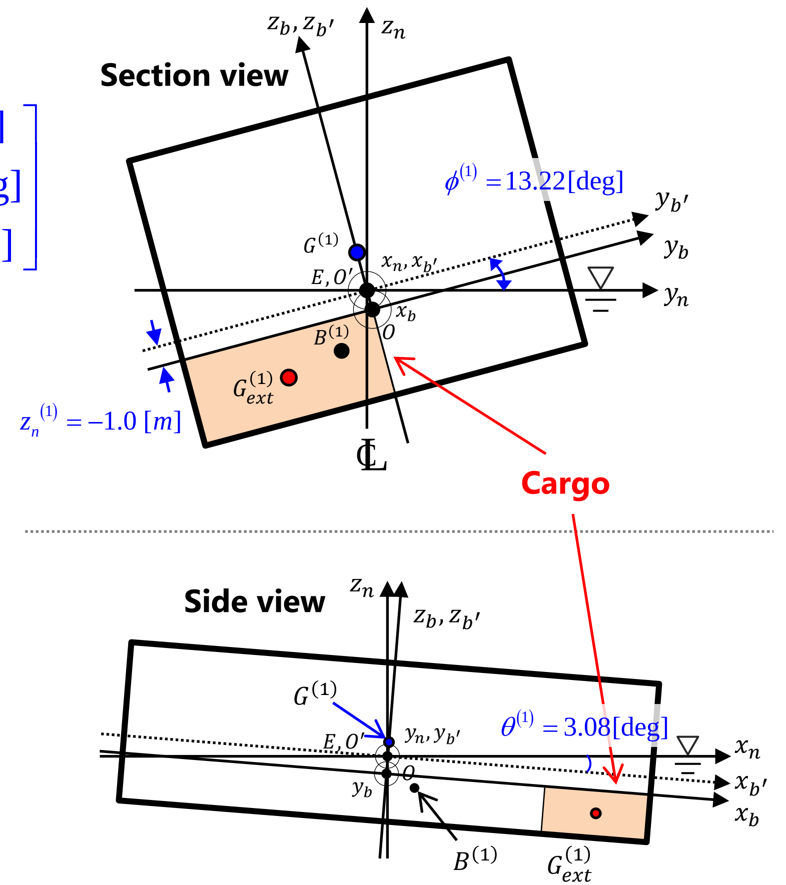
$$ {}^{\{0\}}\mathbf{r}_{P/O} = \begin{bmatrix} \cos\theta\cos\psi & -\cos\theta\sin\psi & \sin\theta \\ \sin\phi\sin\theta\cos\psi + \cos\phi\sin\psi & -\sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi & -\sin\phi\cos\theta \\ -\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi & \cos\phi\sin\theta\sin\psi + \sin\phi\cos\psi & \cos\phi\cos\theta \end{bmatrix} {}^{b}\mathbf{r}_{P/O} $$

$$ {}^{\{0\}}\mathbf{x}_b \qquad {}^{\{0\}}\mathbf{y}_b \qquad {}^{\{0\}}\mathbf{z}_b $$

3. Calculation of Immersion, Trim, and Heel at k=0 step

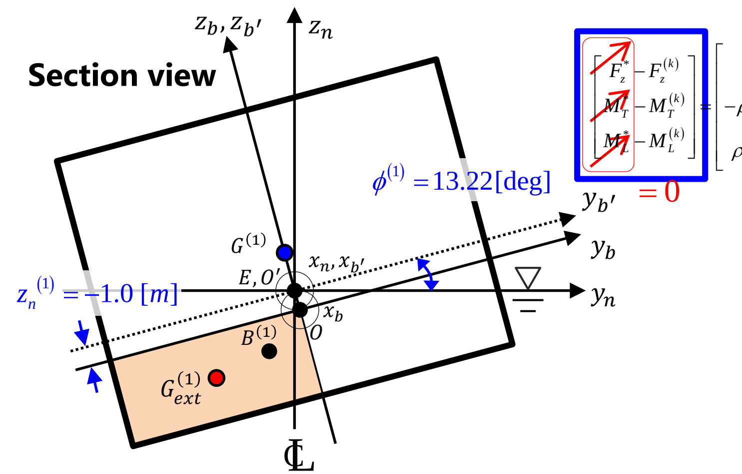


$$\begin{bmatrix} \delta z_n^{(0)} \\ \delta\phi^{(0)} \\ \delta\theta^{(0)} \end{bmatrix} = \begin{bmatrix} -1.0\ [m] \\ 13.22[\text{deg}] \\ 3.08[\text{deg}] \end{bmatrix}$$



b′-frame: b-frame을 n-frame의 원점 E으로 translation한 coordinate system

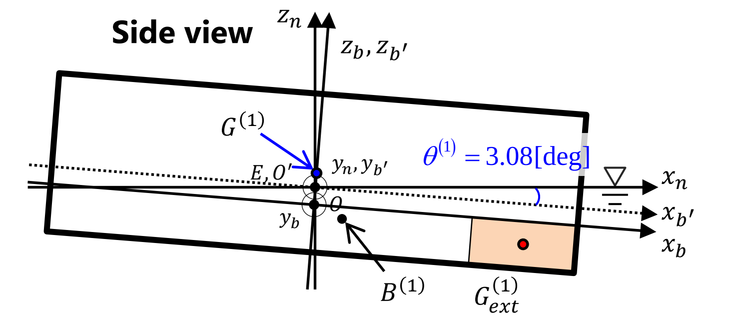
4. Check for the Ship to be in Static Equilibrium at k=0 step



$$\boxed{\begin{bmatrix} F_z^* - F_z^{(k)} \\ M_T^* - M_T^{(k)} \\ M_L^* - M_L^{(k)} \end{bmatrix}}_{=0} = \begin{bmatrix} -\rho g A_{WP}^{(k)} & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} & \rho g I_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} & \rho g I_P^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

Is the ship in static equilibrium?

Let us check for the ship to be in static equilibrium!



$L = 100\ [m]$
$B_{mld} = 40\ [m]$
$D = 30\ [m]$
$d = 9\ [m]$
$\rho g = 10\ [Mg/m^2 s^2]$

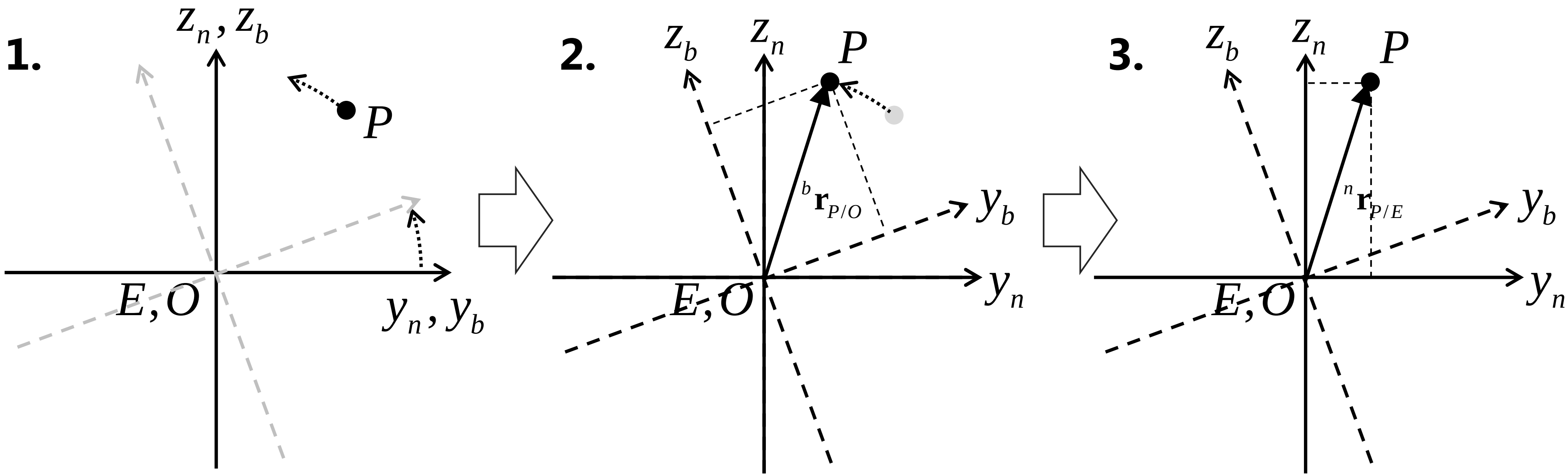
${}^n\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$
${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [40 \quad -10 \quad -4.5]^T\ [m]$
${}^n\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T\ [m]$
${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$

$\nabla^{(0)} = 3.6 \times 10^4\ [m^3]$

$F_{G,z} = -3.6 \times 10^5\ [kN]$ $\quad F_z^{(0)} = -4.0 \times 10^4\ [kN]$
$F_{G_{ext},z} = -4.0 \times 10^4\ [kN]$ $\quad M_T^{(0)} = 4.0 \times 10^5\ [kN]$
$F_{B,z}^{(0)} = 3.6 \times 10^5\ [kN]$ $\quad M_L^{(0)} = 1.6 \times 10^6\ [kN]$

Coordinate Transformation: Forward problem

$$ {}^{n}\mathbf{r}_{P/E} = {}^{n}\mathbf{R}_{b}\, {}^{b}\mathbf{r}_{P/O} $$



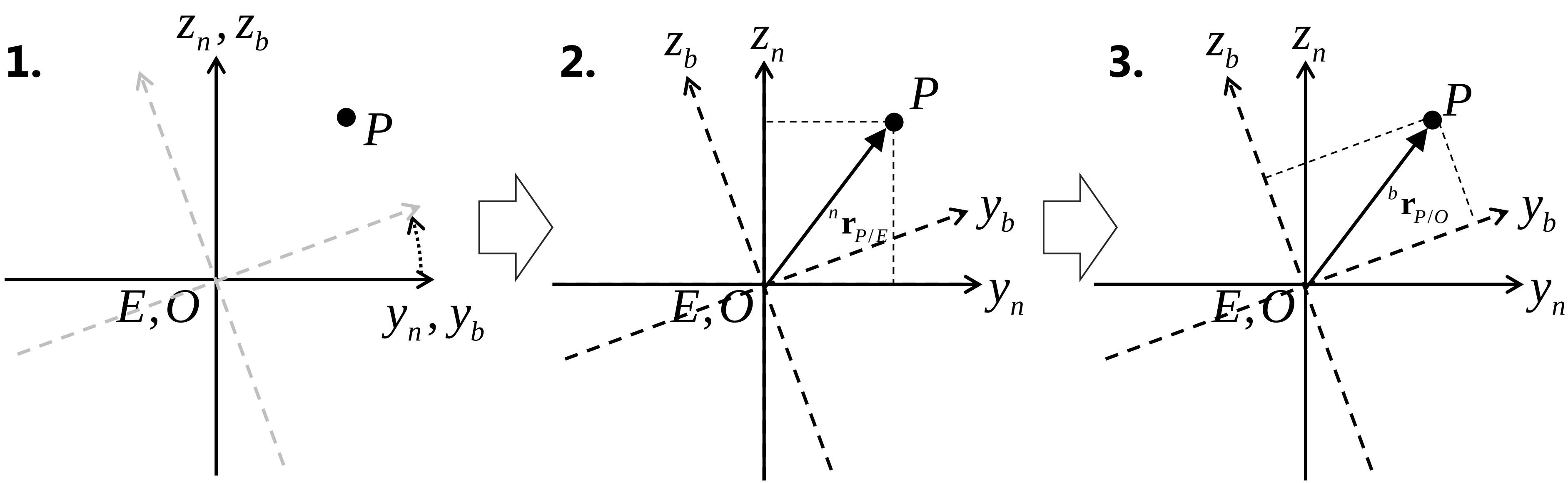
1. 문제정의: 점 **P**가 **b-frame**과 함께 회전하는 경우

2. 점 **P**가 **b-frame**과 함께 회전하였으므로, 알고 있는 벡터는 **b-frame**에서 기술한 점 **P**의 위치벡터 ${}^{b}\mathbf{r}_{P/O}$

3. 최종적으로 구하고자 하는 벡터는 **n-frame**에서 기술한 점 **P**의 위치벡터 ${}^{n}\mathbf{r}_{P/E}$

Coordinate Transformation: Inverse problem

$$ {}^{n}\mathbf{r}_{P/E} = {}^{n}\mathbf{R}_{b}\, {}^{b}\mathbf{r}_{P/O} $$

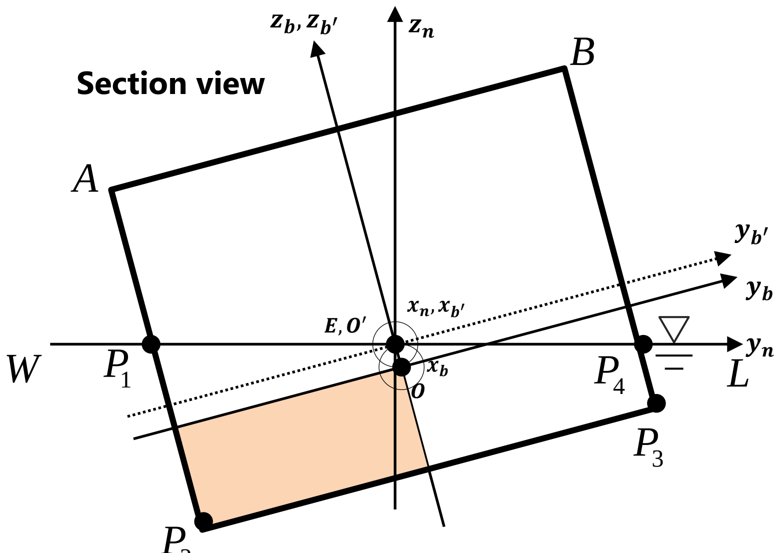


1. 문제정의: 점 P는 n-frame과 함께 고정되어 있고 b-frame만 회전하는 경우

2. 점 P가 n-frame과 함께 고정되어 있으므로, 알고 있는 벡터는 n-frame에서 기술한 점 P의 위치벡터 ${}^{n}\mathbf{r}_{P/E}$

3. 최종적으로 구하고자 하는 벡터는 b-frame에서 기술한 점 P의 위치벡터 ${}^{b}\mathbf{r}_{P/O}$

Calculation of Position and Orientation of a Barge Ship When a Cargo is Moved - 4. Check for the Ship to be in Static Equilibrium (1st Iteration)

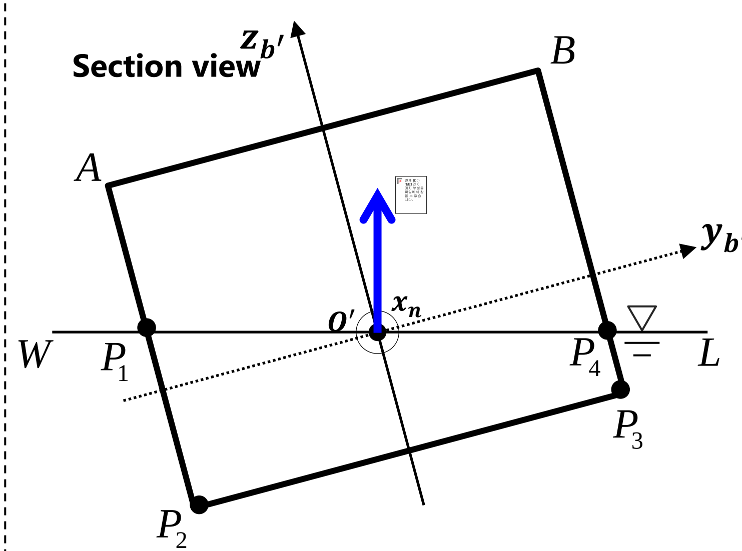


1. 문제: 점 P_1, P_2, P_3, P_4의 위치를 **n-frame**에서 계산하고자 함

2. **n-frame**에서 정의한 **A, B,** P_2, P_3 의 위치는 회전 변환을 통해 알 수 있음

3. 점 P_1, P_2의 위치는 직선들의 교차 계산을 통해 계산해야 함

4. 교차 계산을 위하여 아래 직선들의 식을 **n-frame**에서 정의해야 함
   - AP_2 , P_3B (직선 **WL**의 식은 이미 알고 있음)
   - 2번 절차에서 점 **A, B,** P_2, P_3 의 위치를 구하였으므로 쉽게 계산가능



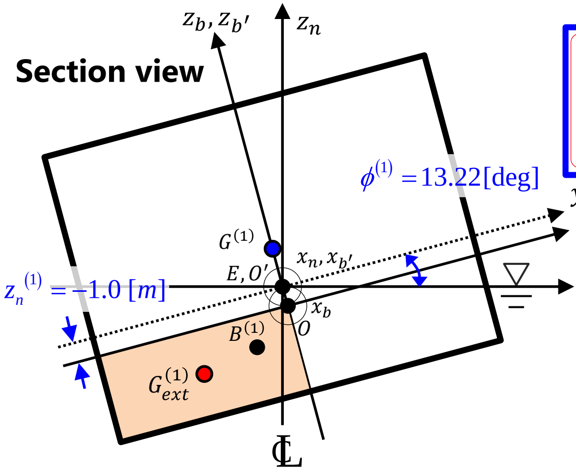
1. 문제: 점 P_1, P_2, P_3, P_4의 위치를 **b-frame**에서 계산하고자 함

2. **b-frame**에서 **A, B,** P_2, P_3 의 위치는 이미 알고 있음

3. 점 P_1, P_2의 위치는 직선들의 교차 계산을 통해 계산해야 함

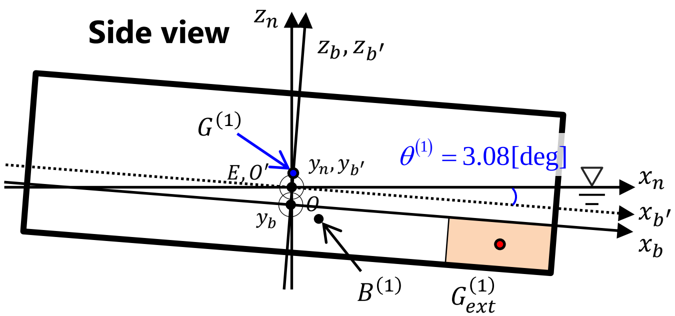
4. 교차 계산을 위하여 직선 **WL**의 식을 **b-frame**에서 정의해야 함(직선 AP_2 , P_3B의 식은 이미 알고 있음)
   - 직선**WL**의 식은 **b-frame**에서 정의하기 위하여 직선**WL**에 수직인 벡터 **"v"**를 **b-frame**의 단위벡터로 표현해야 함
   - 회전하지 않은 벡터**"v"**를 **n-frame**에서 표현한 것은 **(0,0,1)**로서 쉽게 알 수 있음
   - 즉, 이 문제는 회전하지 않은 벡터 **"v"**를 회전된 **b-frame**에서 기술하고자 하는 **inverse problem**임.

4. Check for the Ship to be in Static Equilibrium at k=0 step



$$\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \\ \cancel{M_L^*} - M_L^{(k)} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP}{}^{(k)} & -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} & \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T{}^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} & \rho g I_P{}^{(k)} \\ \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} & \rho g I_P{}^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L{}^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$



$L = 100\ [m]$	${}^n\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$
$B_{mld} = 40\ [m]$	${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [40 \quad -10 \quad -4.5]^T\ [m]$
$D = 30\ [m]$	${}^n\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T\ [m]$
$d = 9\ [m]$	${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2s^2]$	

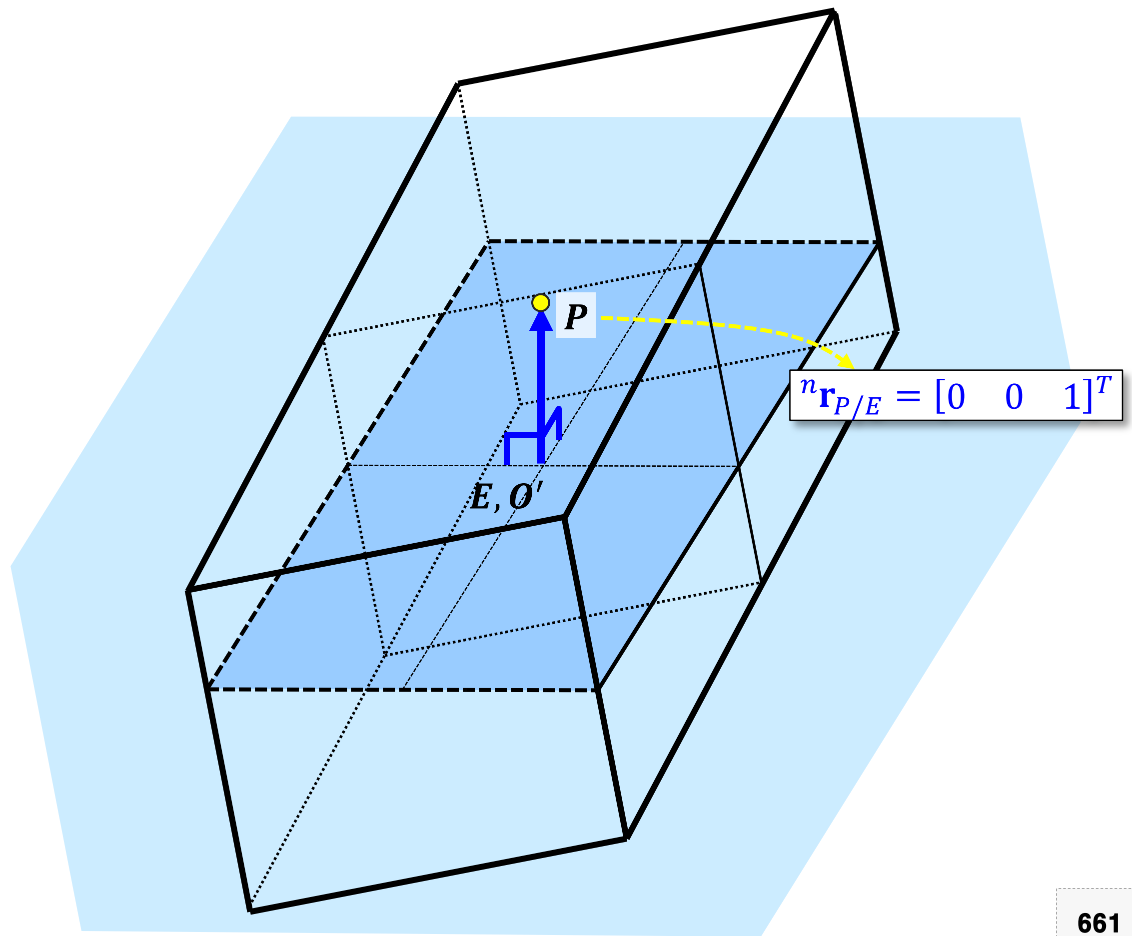
$\nabla^{(0)} = 3.6 \times 10^4\ [m^3]$

$F_{G,z} = -3.6 \times 10^5\ [kN]$ $\qquad F_z^{(0)} = -4.0 \times 10^4\ [kN]$

$F_{G_{ext},z} = -4.0 \times 10^4\ [kN]$ $\qquad M_T^{(0)} = 4.0 \times 10^5\ [kN]$

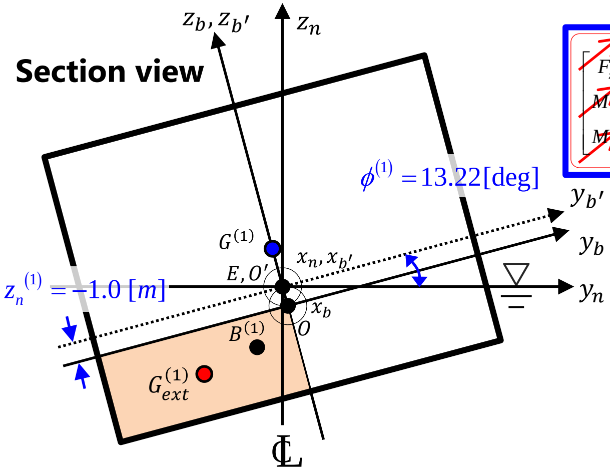
$F_{B,z}^{(0)} = 3.6 \times 10^5\ [kN]$ $\qquad M_L^{(0)} = 1.6 \times 10^6\ [kN]$

Normal vector of waterplane
decomposed in the n-frame: ${}^n\mathbf{r}_{P/E}$



4. Check for the Ship to be in Static Equilibrium at k=0 step

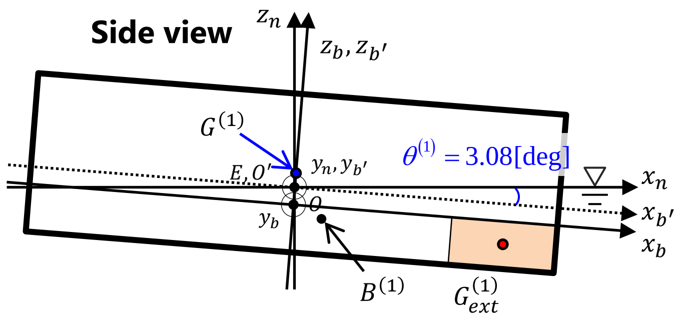
Section view



$$\boxed{\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \\ \cancel{M_L^*} - M_L^{(k)} \end{bmatrix}} = \begin{bmatrix} -\rho g A_{WP}{}^{(k)} & -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} & \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T{}^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} & \rho g I_P{}^{(k)} \\ \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} & \rho g I_P{}^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L{}^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$

Side view



$L = 100\ [m]$
$B_{mld} = 40\ [m]$
$D = 30\ [m]$
$d = 9\ [m]$
$\rho g = 10\ [Mg/m^2 s^2]$

${}^n\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$
${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [40 \quad -10 \quad -4.5]^T\ [m]$
${}^n\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T\ [m]$
${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$

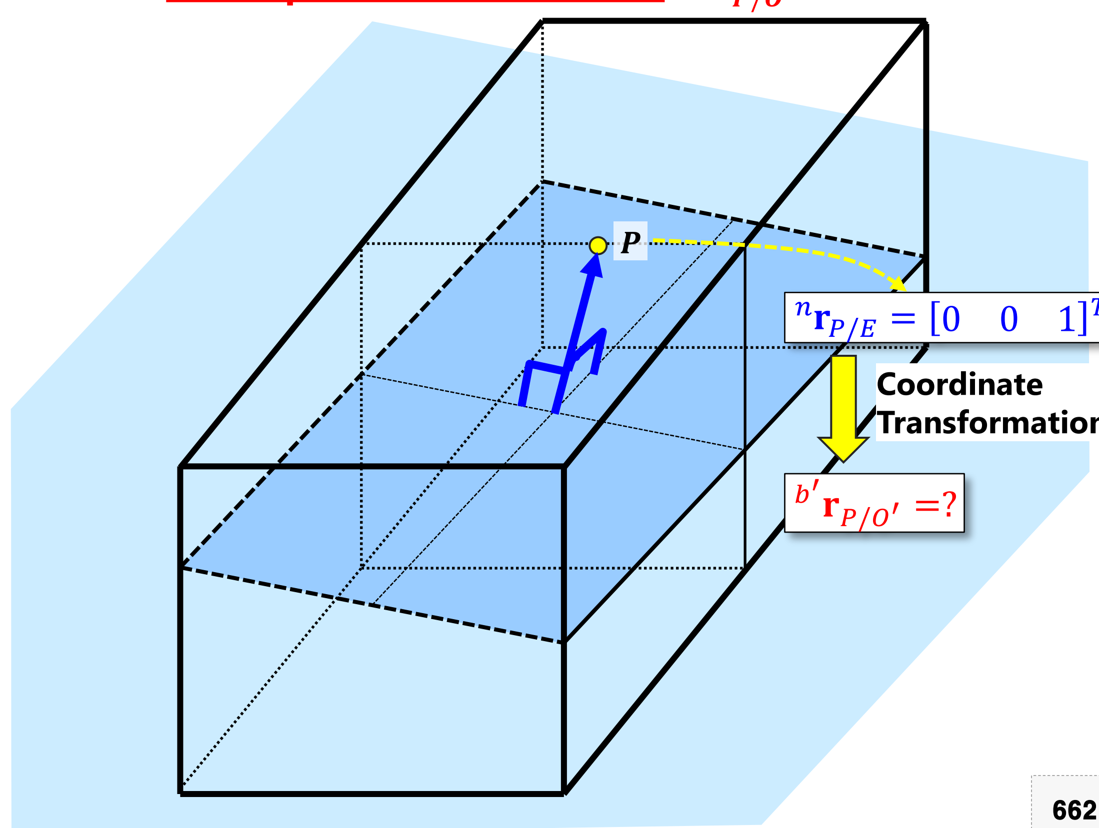
$\nabla^{(0)} = 3.6 \times 10^4\ [m^3]$
$F_{G,z} = -3.6 \times 10^5\ [kN]$
$F_{G_{ext},z} = -4.0 \times 10^4\ [kN]$
$F_{B,z}^{(0)} = 3.6 \times 10^5\ [kN]$

$F_z^{(0)} = -4.0 \times 10^4\ [kN]$
$M_T^{(0)} = 4.0 \times 10^5\ [kN]$
$M_L^{(0)} = 1.6 \times 10^6\ [kN]$

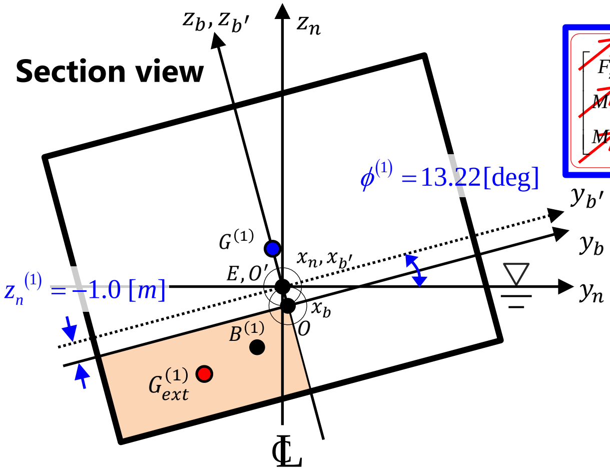
Normal vector of waterplane
decomposed in the n-frame: ${}^n\mathbf{r}_{P/E}$

Normal vector of waterplane
decomposed in the b'-frame: ${}^{b'}\mathbf{r}_{P/O'}$

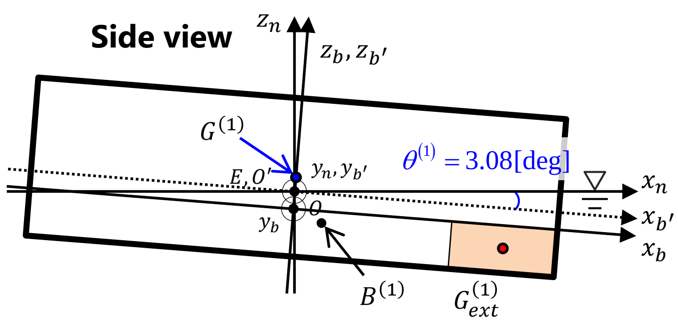
Coordinate Transformation



4. Check for the Ship to be in Static Equilibrium at k=0 step



$$\boxed{\begin{bmatrix} F_z^* - F_z^{(k)} \\ M_T^* - M_T^{(k)} \\ M_L^* - M_L^{(k)} \end{bmatrix}}_{=0} = \begin{bmatrix} -\rho g A_{WP}^{(k)} & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} & \rho g I_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} & \rho g I_P^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$



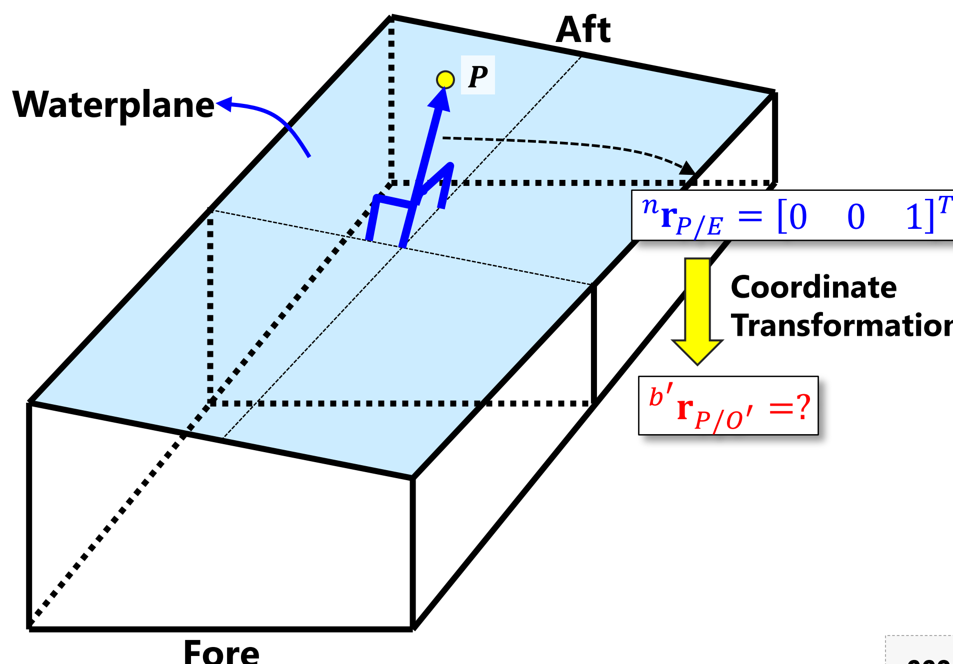
$L = 100\ [m]$	${}^n\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$
$B_{mld} = 40\ [m]$	${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [40 \quad -10 \quad -4.5]^T\ [m]$
$D = 30\ [m]$	${}^n\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T\ [m]$
$d = 9\ [m]$	${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2s^2]$	

$\nabla^{(0)} = 3.6 \times 10^4\ [m^3]$

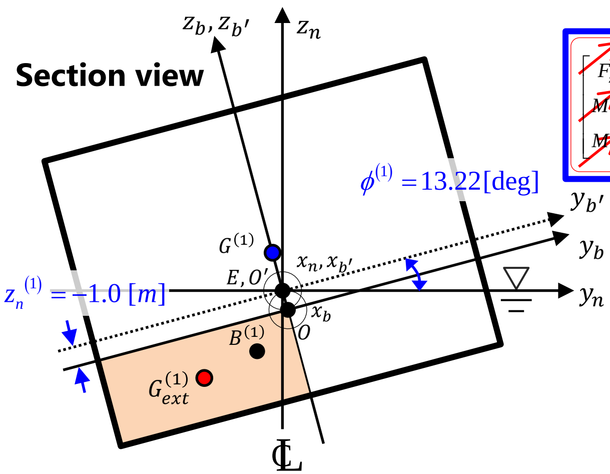
$F_{G,z} = -3.6 \times 10^5\ [kN]$	$F_z^{(0)} = -4.0 \times 10^4\ [kN]$
$F_{G_{ext},z} = -4.0 \times 10^4\ [kN]$	$M_T^{(0)} = 4.0 \times 10^5\ [kN]$
$F_{B,z}^{(0)} = 3.6 \times 10^5\ [kN]$	$M_L^{(0)} = 1.6 \times 10^6\ [kN]$

Normal vector of waterplane
decomposed in the b′-frame: ${}^{b'}\mathbf{r}_{P/O'}$

$$\underbrace{{}^n\mathbf{r}_{P/E}}_{\text{Given}} = \begin{bmatrix} \cos\theta^{(1)} & 0 & \sin\theta^{(1)} \\ 0 & 1 & 0 \\ -\sin\theta^{(1)} & 0 & \cos\theta^{(1)} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi^{(1)} & -\sin\phi^{(1)} \\ 0 & \sin\phi^{(1)} & \cos\phi^{(1)} \end{bmatrix} \underbrace{{}^{b'}\mathbf{r}_{P/O'}}_{\text{Find}}$$

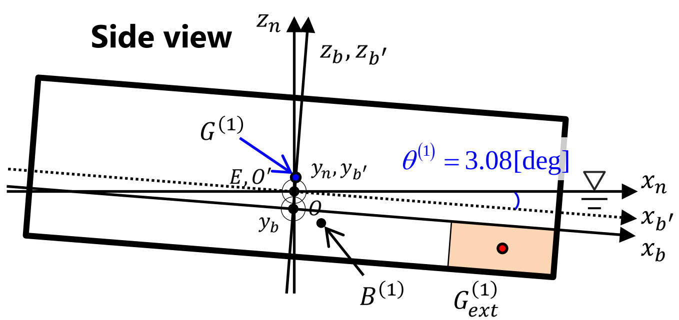


4. Check for the Ship to be in Static Equilibrium at k=0 step



$$\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \\ \cancel{M_L^*} - M_L^{(k)} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP}^{(k)} & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} & \rho g I_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} & \rho g I_P^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$



$L = 100\ [m]$	${}^n\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$
$B_{mld} = 40\ [m]$	${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [40 \quad -10 \quad -4.5]^T\ [m]$
$D = 30\ [m]$	${}^n\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T\ [m]$
$d = 9\ [m]$	${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2 s^2]$	

$\nabla^{(0)} = 3.6 \times 10^4\ [m^3]$

$F_{G,z} = -3.6 \times 10^5\ [kN]$ $\quad F_z^{(0)} = -4.0 \times 10^4\ [kN]$

$F_{G_{ext},z} = -4.0 \times 10^4\ [kN]$ $\quad M_T^{(0)} = 4.0 \times 10^5\ [kN]$

$F_{B,z}^{(0)} = 3.6 \times 10^5\ [kN]$ $\quad M_L^{(0)} = 1.6 \times 10^6\ [kN]$

Normal vector of waterplane

decomposed in the b'-frame: ${}^{b'}\mathbf{r}_{P/O'}$

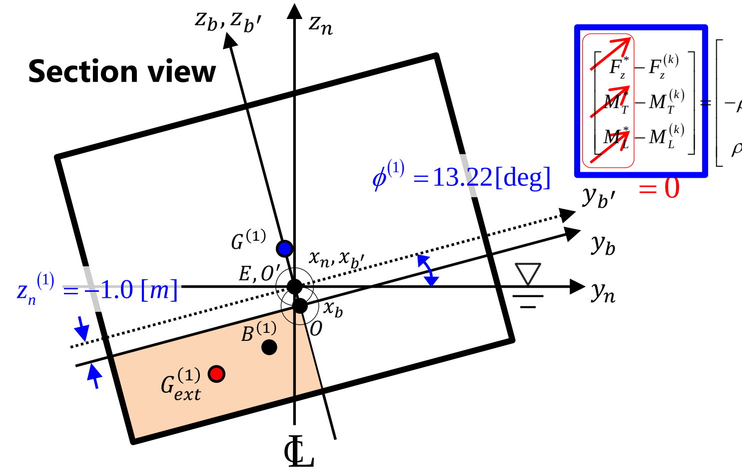
$${}^n\mathbf{r}_{P/E} = \begin{bmatrix} \cos\theta^{(1)} & 0 & \sin\theta^{(1)} \\ 0 & 1 & 0 \\ -\sin\theta^{(1)} & 0 & \cos\theta^{(1)} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi^{(1)} & -\sin\phi^{(1)} \\ 0 & \sin\phi^{(1)} & \cos\phi^{(1)} \end{bmatrix} {}^{b'}\mathbf{r}_{P/O'}$$

$${}^{b'}\mathbf{r}_{P/O'} = \left(\begin{bmatrix} \cos\theta^{(1)} & 0 & \sin\theta^{(1)} \\ 0 & 1 & 0 \\ -\sin\theta^{(1)} & 0 & \cos\theta^{(1)} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi^{(1)} & -\sin\phi^{(1)} \\ 0 & \sin\phi^{(1)} & \cos\phi^{(1)} \end{bmatrix} \right)^{-1} {}^n\mathbf{r}_{P/E}$$

$$= \left(\begin{bmatrix} \cos 3.08 & 0 & \sin 3.08 \\ 0 & 1 & 0 \\ -\sin 3.08 & 0 & \cos 3.08 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 13.22 & -\sin 13.22 \\ 0 & \sin 13.22 & \cos 13.22 \end{bmatrix} \right)^{-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -0.0538 \\ 0.2284 \\ 0.9721 \end{bmatrix}$$

4. Check for the Ship to be in Static Equilibrium at k=0 step



$$
\boxed{\begin{bmatrix} F_z^* - F_z^{(k)} \\ M_T^* - M_T^{(k)} \\ M_L^* - M_L^{(k)} \end{bmatrix}} = \begin{bmatrix} -\rho g A_{WP}^{(k)} & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} & \rho g I_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} & \rho g I_P^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}
$$

$= 0$

Normal vector of waterplane decomposed in the b′-frame:

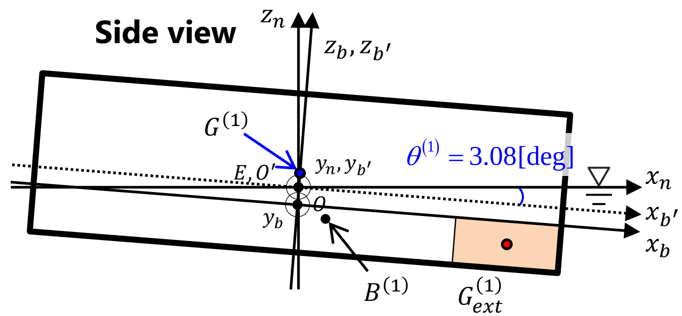
$${}^{b'}\mathbf{r}_{P/O'} = [-0.0538 \quad 0.2284 \quad 0.9721]^T$$

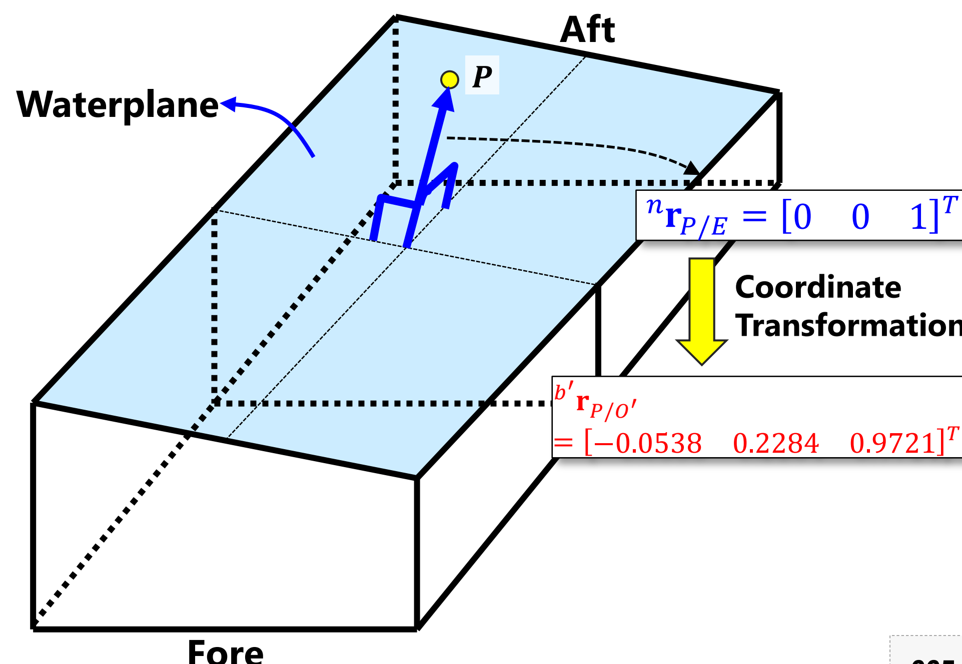
Equation of waterplane: ${}^{b'}\mathbf{r}_{P/O'} \cdot \left({}^{b'}\mathbf{r} - {}^{b'}\mathbf{r}_{E/O'} \right) = 0$

$$-0.0538x + 0.2284y + 0.9721z = 0$$

(x, y, z are defined in the b′-frame)

$$z = (0.0538x - 0.2284y)/0.9721$$





$L = 100\ [m]$	${}^n\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$
$B_{mld} = 40\ [m]$	${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [40 \quad -10 \quad -4.5]^T\ [m]$
$D = 30\ [m]$	${}^n\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T\ [m]$
$d = 9\ [m]$	${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2s^2]$	

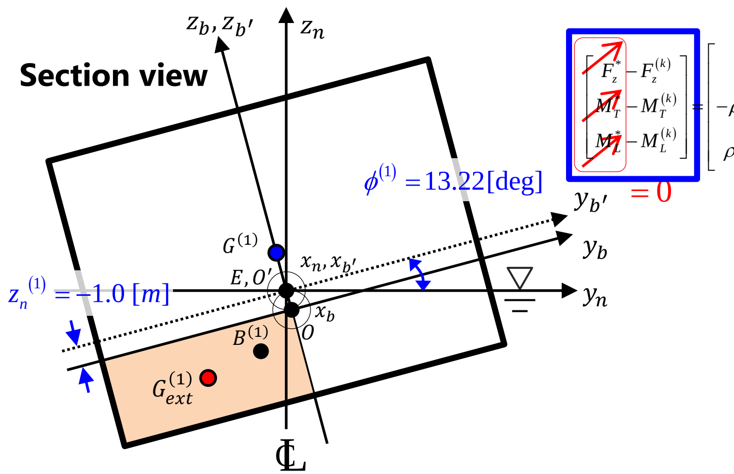
$\nabla^{(0)} = 3.6 \times 10^4\ [m^3]$

$F_{G,z} = -3.6 \times 10^5\ [kN]$ $\qquad F_z^{(0)} = -4.0 \times 10^4\ [kN]$

$F_{G_{ext},z} = -4.0 \times 10^4\ [kN]$ $\qquad M_T^{(0)} = 4.0 \times 10^5\ [kN]$

$F_{B,z}^{(0)} = 3.6 \times 10^5\ [kN]$ $\qquad M_L^{(0)} = 1.6 \times 10^6\ [kN]$

4. Check for the Ship to be in Static Equilibrium at k=0 step



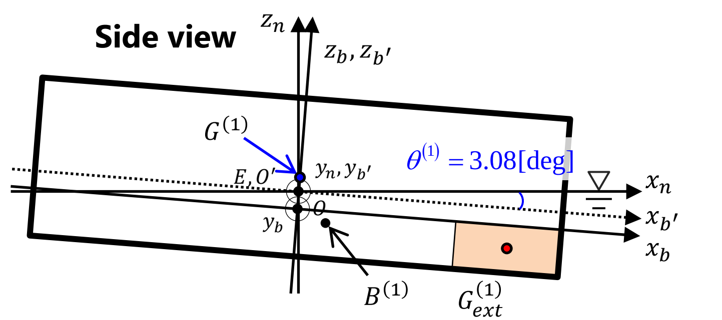
$$\boxed{\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \\ \cancel{M_L^*} - M_L^{(k)} \end{bmatrix}} = \begin{bmatrix} -\rho g A_{WP}{}^{(k)} & -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} & \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T{}^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} & \rho g I_P{}^{(k)} \\ \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} & \rho g I_P{}^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L{}^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

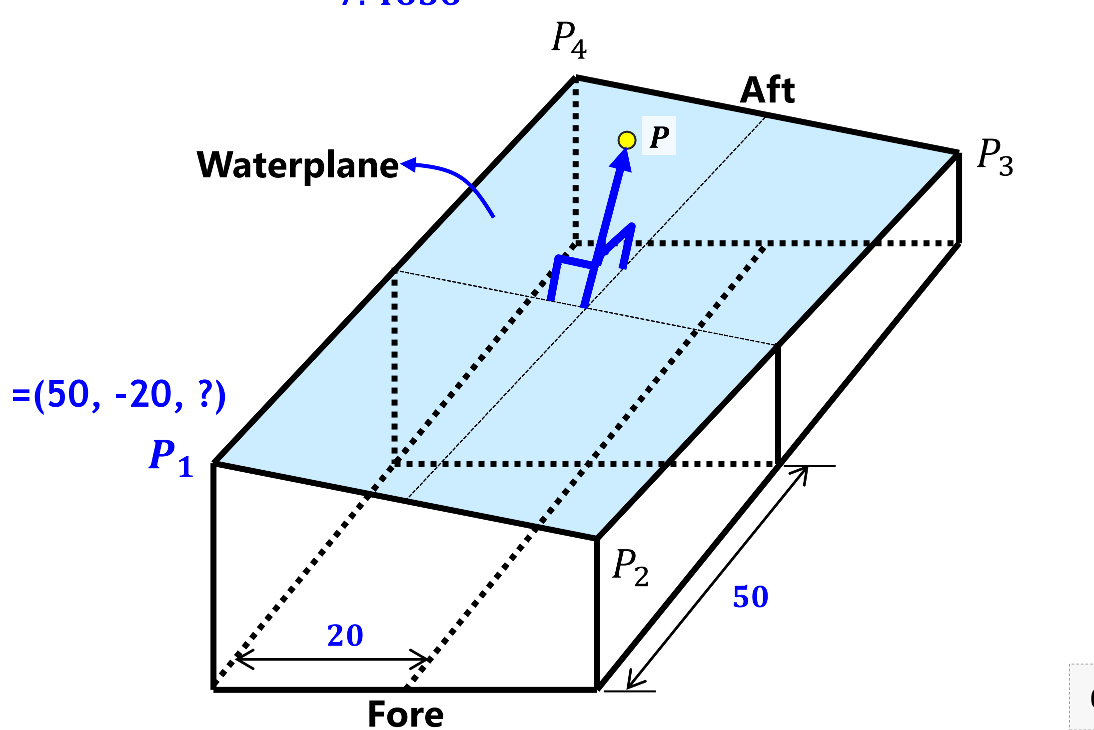
$= 0$

Equation of waterplane: $z = (0.0538x - 0.2284y)/0.9721$

$${}^{b'}\mathbf{r}_{P_1/O'}: {}^{b'}x_{P_1/O'} = \mathbf{50},\ {}^{b'}y_{P_1/O'} = \mathbf{-20}$$

$$\begin{aligned} {}^{b'}z_{P_1/O'} &= \left(0.0538 \cdot {}^{b'}x_{P_1/O'} - 0.2284 \cdot {}^{b'}y_{P_1/O'}\right)/0.9721 \\ &= (0.0538 \cdot 50 - 0.2284 \cdot (-20))/0.9721 \\ &= \mathbf{7.4656} \end{aligned}$$





$L = 100\ [m]$	${}^n\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$
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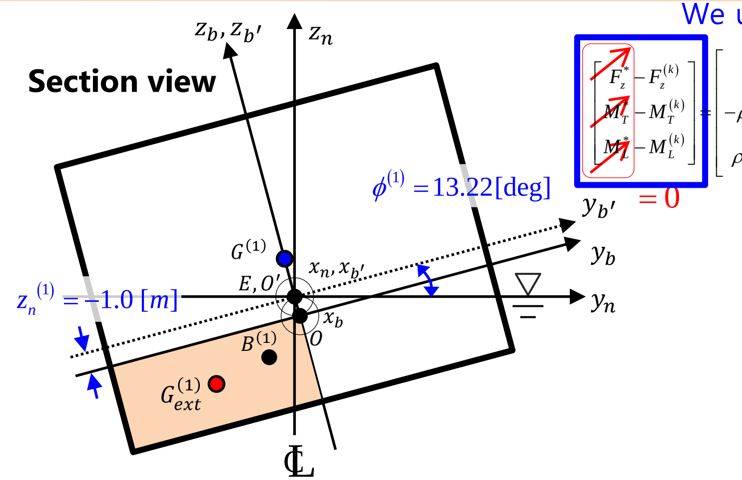
$D = 30\ [m]$	${}^n\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T\ [m]$
$d = 9\ [m]$	${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2 s^2]$	

$\nabla^{(0)} = 3.6 \times 10^4\ [m^3]$

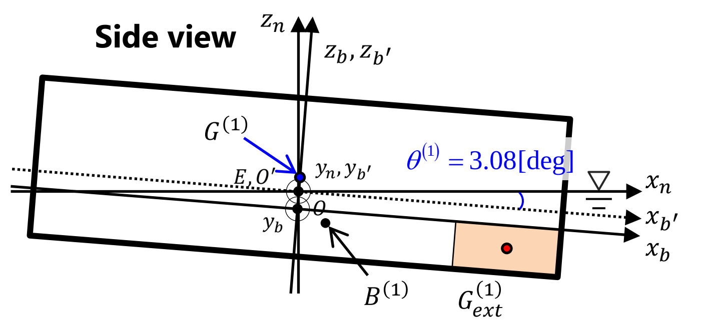
$F_{G,z} = -3.6 \times 10^5\ [kN]$	$F_z^{(0)} = -4.0 \times 10^4\ [kN]$
$F_{G_{ext},z} = -4.0 \times 10^4\ [kN]$	$M_T^{(0)} = 4.0 \times 10^5\ [kN]$
$F_{B,z}^{(0)} = 3.6 \times 10^5\ [kN]$	$M_L^{(0)} = 1.6 \times 10^6\ [kN]$

Calculation of Position and Orientation of a Barge Ship When a Cargo is Moved - 4. Check for the Ship to be in Static Equilibrium at k=0 step

Section view



Side view



$L = 100\ [m]$	${}^n\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$
$B_{mld} = 40\ [m]$	${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [40 \quad -10 \quad -4.5]^T\ [m]$
$D = 30\ [m]$	${}^n\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T\ [m]$
$d = 9\ [m]$	
$\rho g = 10\ [Mg/m^2s^2]$	${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$

$\nabla^{(0)} = 3.6 \times 10^4\ [m^3]$

$F_{G,z} = -3.6 \times 10^5\ [kN]$ $\quad F_z^{(0)} = -4.0 \times 10^4\ [kN]$

$F_{G_{ext},z} = -4.0 \times 10^4\ [kN]$ $\quad M_T^{(0)} = 4.0 \times 10^5\ [kN]$

$F_{B,z}^{(0)} = 3.6 \times 10^5\ [kN]$ $\quad M_L^{(0)} = 1.6 \times 10^6\ [kN]$

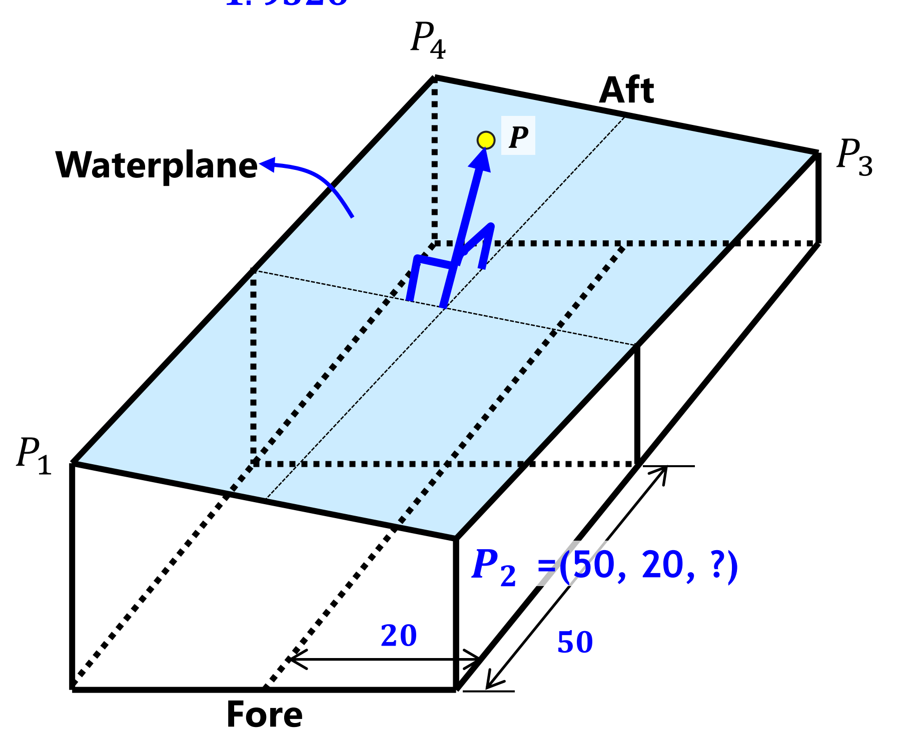
We use the values in current floating position!

$$\boxed{\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \\ \cancel{M_L^*} - M_L^{(k)} \end{bmatrix}}_{=0} = \begin{bmatrix} -\rho g A_{WP}{}^{(k)} & -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} & \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T{}^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} & \rho g I_P{}^{(k)} \\ \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} & \rho g I_P{}^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L{}^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

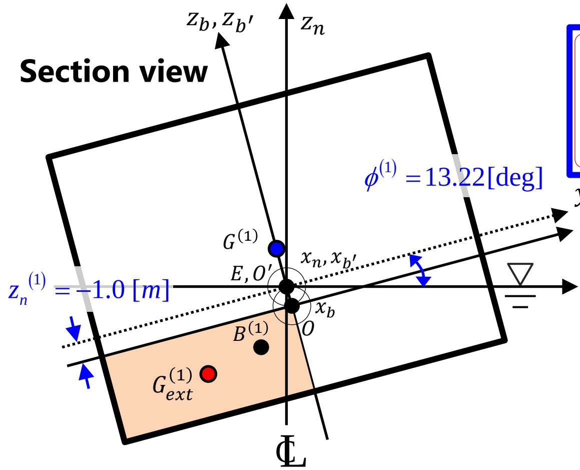
Equation of waterplane: $z = (0.0538x - 0.2284y)/0.9721$

$${}^{b'}\mathbf{r}_{P_2/O'}: {}^{b'}x_{P_2/O'} = \mathbf{50},\ {}^{b'}y_{P_2/O'} = \mathbf{20}$$

$$\begin{aligned} {}^{b'}z_{P_2/O'} &= (0.0538 \cdot {}^{b'}x_{P_2/O'} - 0.2284 \cdot {}^{b'}y_{P_2/O'})/0.9721 \\ &= (0.0538 \cdot 50 - 0.2284 \cdot 20\,)/0.9721 \\ &= \mathbf{-1.9326} \end{aligned}$$



4. Check for the Ship to be in Static Equilibrium at k=0 step



$$\boxed{\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \\ \cancel{M_L^*} - M_L^{(k)} \end{bmatrix}}_{=0} = \begin{bmatrix} -\rho g A_{WP}{}^{(k)} & -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} & \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T{}^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} & \rho g I_P{}^{(k)} \\ \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} & \rho g I_P{}^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L{}^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

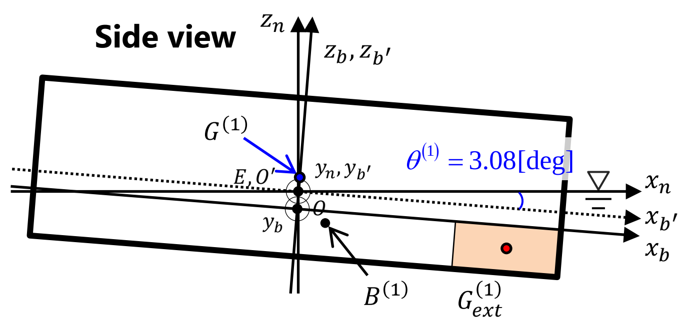
Force equilibrium:

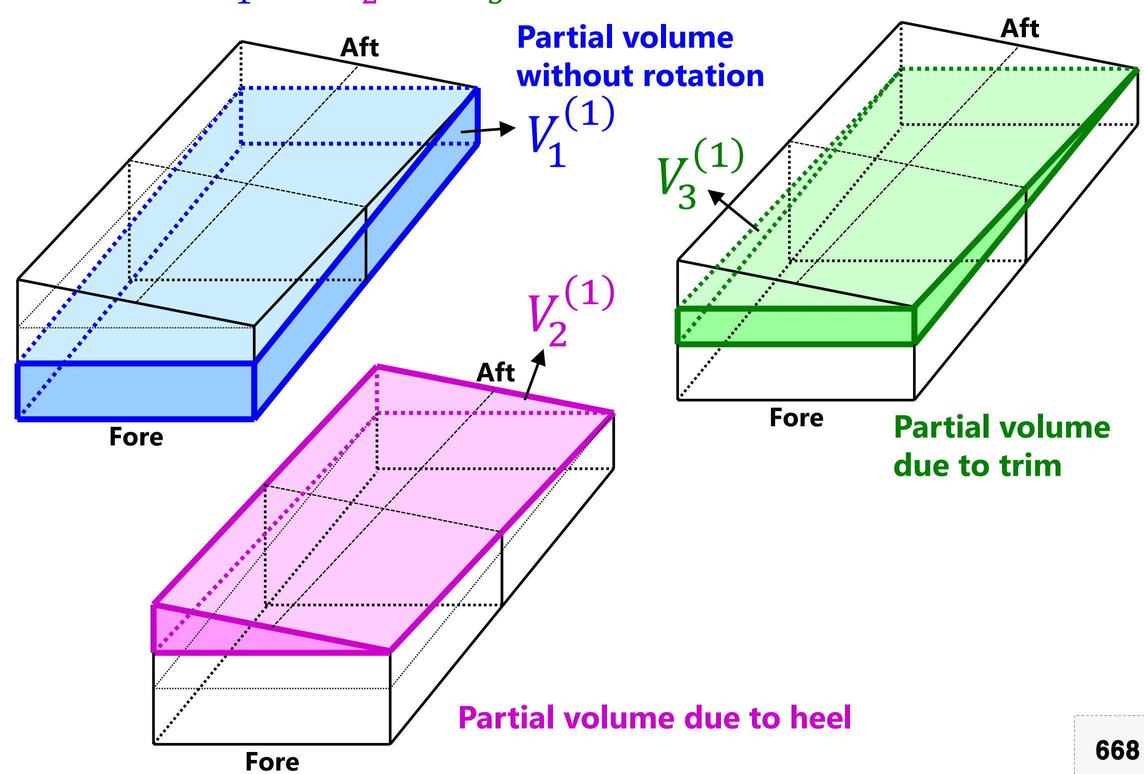
$$F_z^{(1)} = F_{B,z}^{(1)} + F_{G,z}^{(1)} + F_{ext,z}^{(1)}$$

$$F_{B,z}^{(1)} = \rho g \nabla^{(1)}$$

$$\nabla^{(1)} = V_1^{(1)} + V_2^{(1)} + V_3^{(1)}$$

$${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 7.4656]^T\ [m]$$
$${}^{b'}\mathbf{r}_{P_2/O'} = [50 \quad 20 \quad -1.9326]^T\ [m]$$
$${}^{b'}\mathbf{r}_{P_3/O'} = [-50 \quad 20 \quad -7.4656]^T\ [m]$$
$${}^{b'}\mathbf{r}_{P_4/O'} = [-50 \quad -20 \quad 1.9326]^T\ [m]$$





$L = 100\ [m]$
$B_{mld} = 40\ [m]$
$D = 30\ [m]$
$d = 9\ [m]$
$\rho g = 10\ [Mg/m^2 s^2]$

${}^n\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$
${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [40 \quad -10 \quad -4.5]^T\ [m]$
${}^n\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T\ [m]$
${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$

$\nabla^{(0)} = 3.6 \times 10^4\ [m^3]$

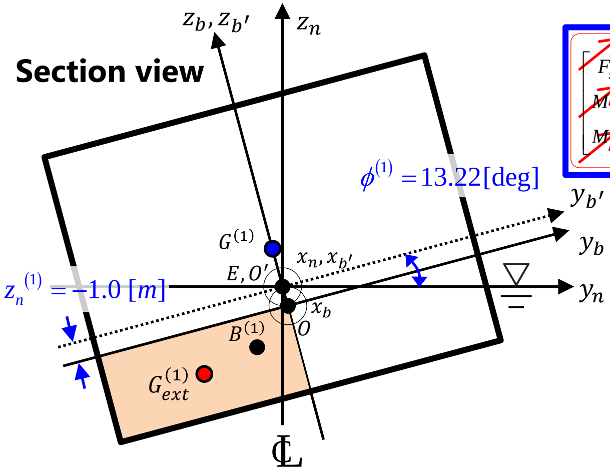
$F_{G,z} = -3.6 \times 10^5\ [kN]$ $\qquad F_z^{(0)} = -4.0 \times 10^4\ [kN]$

$F_{G_{ext},z} = -4.0 \times 10^4\ [kN]$ $\qquad M_T^{(0)} = 4.0 \times 10^5\ [kN]$

$F_{B,z}^{(0)} = 3.6 \times 10^5\ [kN]$ $\qquad M_L^{(0)} = 1.6 \times 10^6\ [kN]$

4. Check for the Ship to be in Static Equilibrium at k=0 step

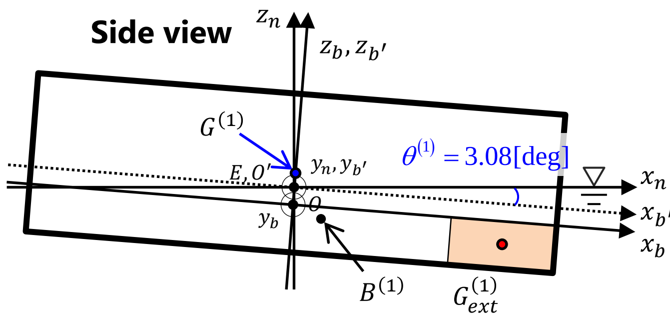
Section view



$$\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \\ \cancel{M_L^*} - M_L^{(k)} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP}^{(k)} & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} & \rho g I_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} & \rho g I_P^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$

Side view



$L = 100\ [m]$

$B_{mld} = 40\ [m]$

$D = 30\ [m]$

$d = 9\ [m]$

$\rho g = 10\ [Mg/m^2 s^2]$

${}^n\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$

${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [40 \quad -10 \quad -4.5]^T\ [m]$

${}^n\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T\ [m]$

${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$

$\nabla^{(0)} = 3.6 \times 10^4\ [m^3]$

$F_{G,z} = -3.6 \times 10^5\ [kN]$

$F_{G_{ext},z} = -4.0 \times 10^4\ [kN]$

$F_{B,z}^{(0)} = 3.6 \times 10^5\ [kN]$

$F_z^{(0)} = -4.0 \times 10^4\ [kN]$

$M_T^{(0)} = 4.0 \times 10^5\ [kN]$

$M_L^{(0)} = 1.6 \times 10^6\ [kN]$

Force equilibrium:

$$F_z^{(1)} = F_{B,z}^{(1)} + F_{G,z}^{(1)} + F_{ext,z}^{(1)}$$

$$F_{B,z}^{(1)} = \rho g \nabla^{(1)}$$

$$\nabla^{(1)} = V_1^{(1)} + V_2^{(1)} + V_3^{(1)}$$

$$V_1^{(1)} = L \cdot B_{mld} \cdot \overline{P_3 A_3} = 40 \cdot 100 \cdot (-7.4656 - (-10))$$

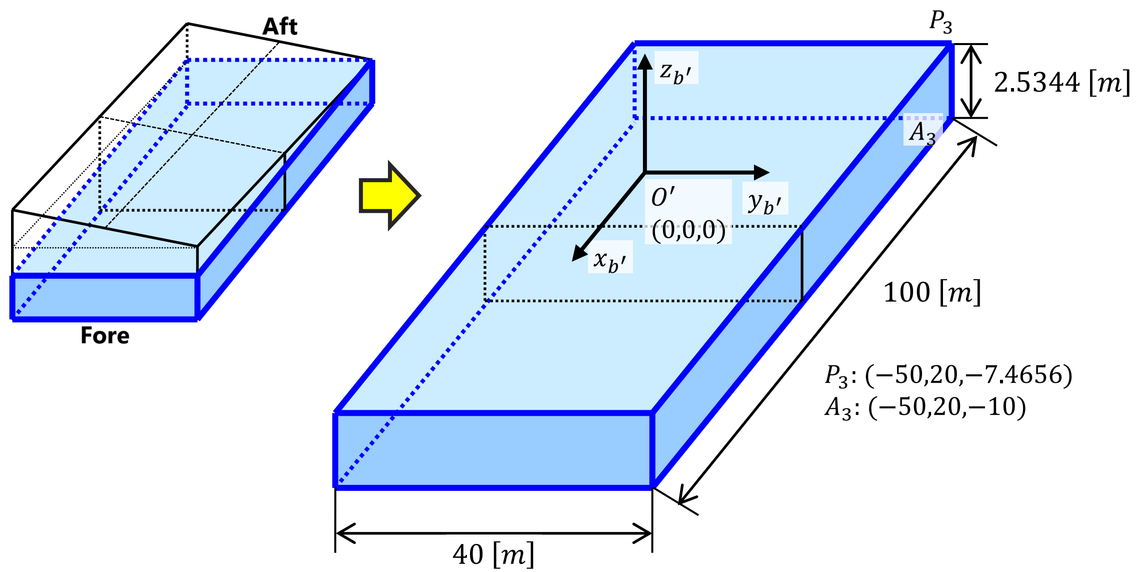
$$= 1.0137 \times 10^4\ [m^3]$$

${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 7.4656]^T\ [m]$

${}^{b'}\mathbf{r}_{P_2/O'} = [50 \quad 20 \quad -1.9326]^T\ [m]$

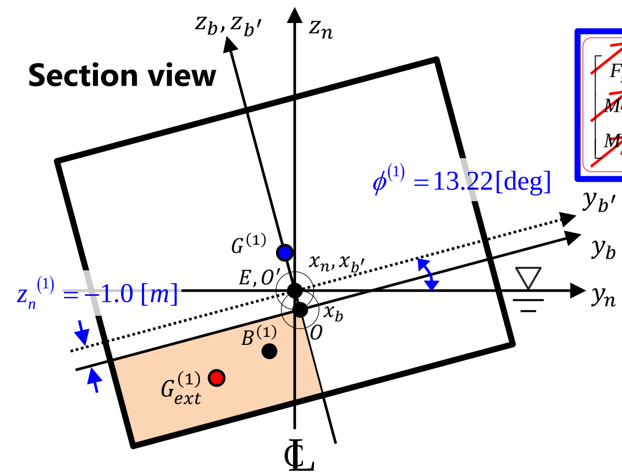
${}^{b'}\mathbf{r}_{P_3/O'} = [-50 \quad 20 \quad -7.4656]^T\ [m]$

${}^{b'}\mathbf{r}_{P_4/O'} = [-50 \quad -20 \quad 1.9326]^T\ [m]$



4. Check for the Ship to be in Static Equilibrium at k=0 step

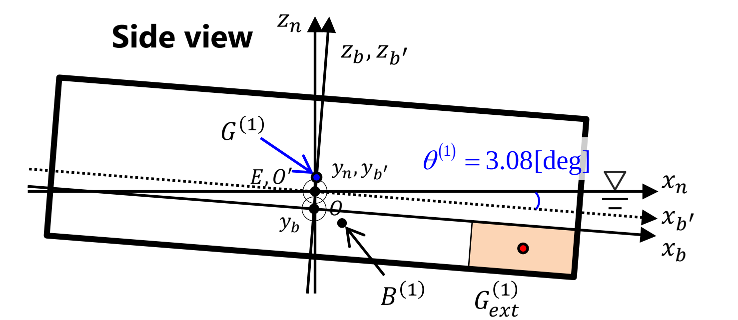
Section view



$$\boxed{\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \\ \cancel{M_L^*} - M_L^{(k)} \end{bmatrix}} = \begin{bmatrix} -\rho g A_{WP}^{(k)} & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} & \rho g I_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} & \rho g I_P^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$

Side view



$L = 100\ [m]$
$B_{mld} = 40\ [m]$
$D = 30\ [m]$
$d = 9\ [m]$
$\rho g = 10\ [Mg/m^2 s^2]$

${}^n\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$
${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [40 \quad -10 \quad -4.5]^T\ [m]$
${}^n\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T\ [m]$
${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$

$\nabla^{(0)} = 3.6 \times 10^4\ [m^3]$
$F_{G,z} = -3.6 \times 10^5\ [kN]$
$F_{G_{ext},z} = -4.0 \times 10^4\ [kN]$
$F_{B,z}^{(0)} = 3.6 \times 10^5\ [kN]$

$F_z^{(0)} = -4.0 \times 10^4\ [kN]$
$M_T^{(0)} = 4.0 \times 10^5\ [kN]$
$M_L^{(0)} = 1.6 \times 10^6\ [kN]$

Force equilibrium:

$$F_z^{(1)} = F_{B,z}^{(1)} + F_{G,z}^{(1)} + F_{ext,z}^{(1)}$$

$$F_{B,z}^{(1)} = \rho g \nabla^{(1)}$$

$$\nabla^{(1)} = V_1^{(1)} + V_2^{(1)} + V_3^{(1)}$$

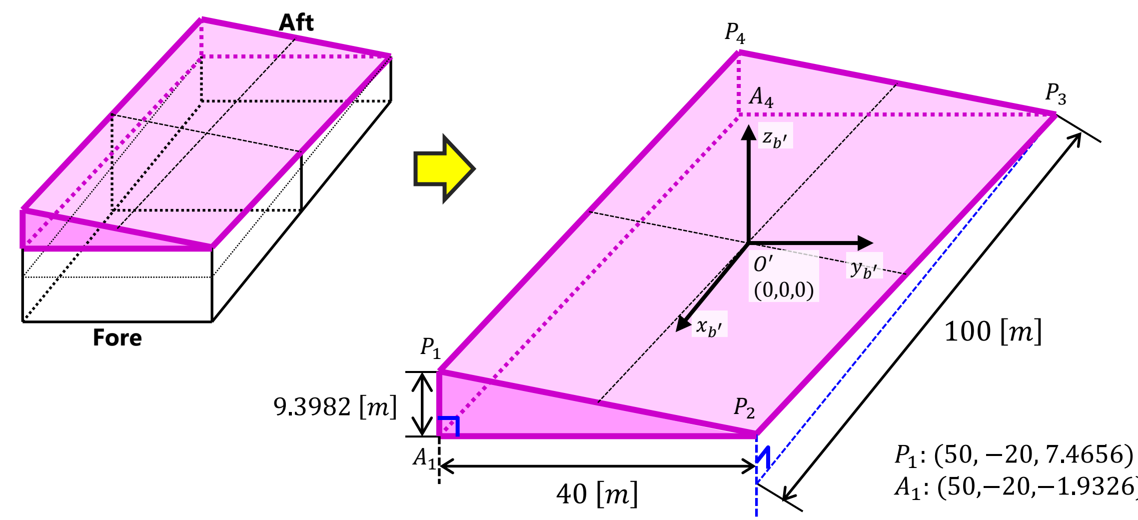
$$V_2^{(1)} = Triangular\ Area \times Length = \left(\frac{1}{2} \cdot B_{mld} \cdot \overline{P_1 A_1} \right) \cdot L$$

$$= [0.5 \cdot 40 \cdot \{7.4656 - (-1.9326)\}] \cdot 100$$

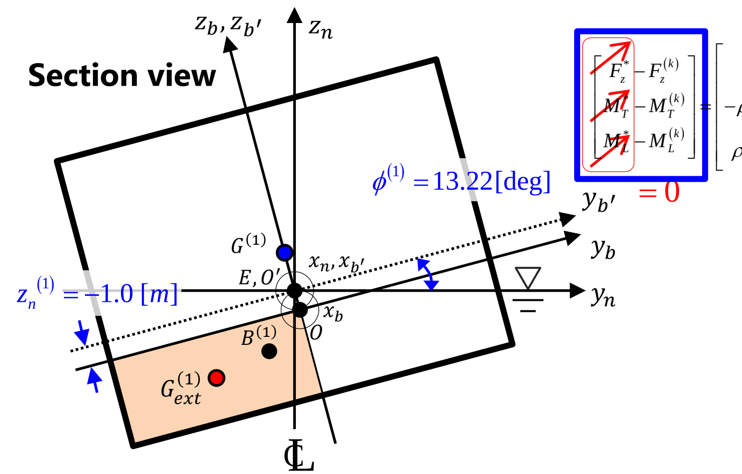
$$= 1.8796 \times 10^4\ [m^3]$$

${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 7.4656]^T\ [m]$
${}^{b'}\mathbf{r}_{P_2/O'} = [50 \quad 20 \quad -1.9326]^T\ [m]$
${}^{b'}\mathbf{r}_{P_3/O'} = [-50 \quad 20 \quad -7.4656]^T\ [m]$
${}^{b'}\mathbf{r}_{P_4/O'} = [-50 \quad -20 \quad 1.9326]^T\ [m]$

$V_1 = 1.0137 \times 10^4\ [m^3]$

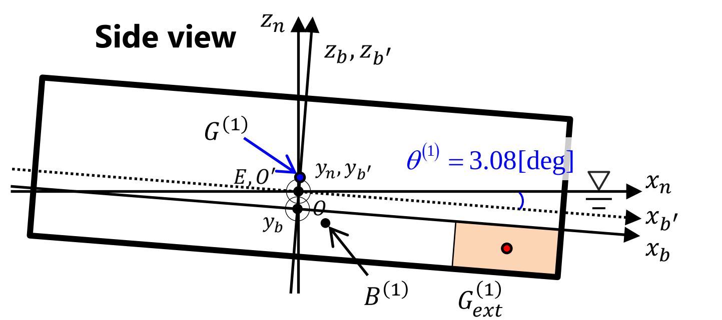


4. Check for the Ship to be in Static Equilibrium at k=0 step



$$\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \\ \cancel{M_L^*} - M_L^{(k)} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP}^{(k)} & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} & \rho g I_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} & \rho g I_P^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$



$L = 100\ [m]$	${}^n\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$
$B_{mld} = 40\ [m]$	${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [40 \quad -10 \quad -4.5]^T\ [m]$
$D = 30\ [m]$	${}^n\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T\ [m]$
$d = 9\ [m]$	${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2 s^2]$	

$\nabla^{(0)} = 3.6 \times 10^4\ [m^3]$

$F_{G,z} = -3.6 \times 10^5\ [kN]$ $\qquad F_z^{(0)} = -4.0 \times 10^4\ [kN]$

$F_{G_{ext},z} = -4.0 \times 10^4\ [kN]$ $\qquad M_T^{(0)} = 4.0 \times 10^5\ [kN]$

$F_{B,z}^{(0)} = 3.6 \times 10^5\ [kN]$ $\qquad M_L^{(0)} = 1.6 \times 10^6\ [kN]$

Force equilibrium:

$$F_z^{(1)} = F_{B,z}^{(1)} + F_{G,z}^{(1)} + F_{ext,z}^{(1)}$$

$$F_{B,z}^{(1)} = \rho g \nabla^{(1)}$$

$$\nabla^{(1)} = V_1^{(1)} + V_2^{(1)} + V_3^{(1)}$$

$$V_3^{(3)} = Triangular\ Area \times Height = \left(\frac{1}{2} \cdot L \cdot \overline{P_2 A_2} \right) \cdot B_{mld}$$

$$= [0.5 \cdot 100 \cdot \{-1.9326 - (-7.4656)\}] \cdot 40$$

$$= 1.1066 \times 10^4\ [m^3]$$

${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 7.4656]^T\ [m]$

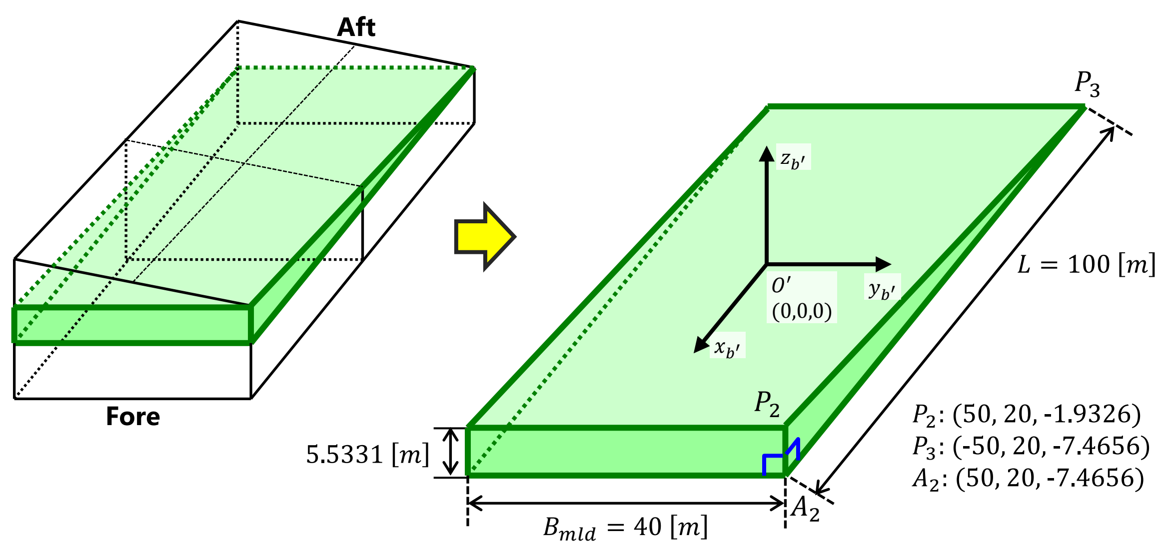
${}^{b'}\mathbf{r}_{P_2/O'} = [50 \quad 20 \quad -1.9326]^T\ [m]$

${}^{b'}\mathbf{r}_{P_3/O'} = [-50 \quad 20 \quad -7.4656]^T\ [m]$

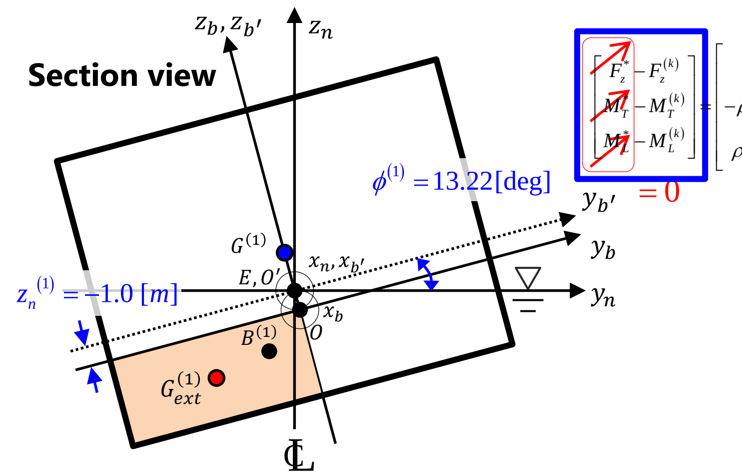
${}^{b'}\mathbf{r}_{P_4/O'} = [-50 \quad -20 \quad 1.9326]^T\ [m]$

$V_1 = 1.0137 \times 10^4\ [m^3]$

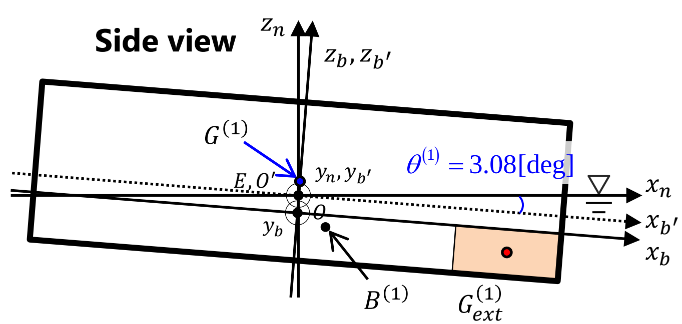
$V_2 = 1.8796 \times 10^4\ [m^3]$



4. Check for the Ship to be in Static Equilibrium at k=0 step



$$\underbrace{\boxed{\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \\ \cancel{M_L^*} - M_L^{(k)} \end{bmatrix}}}_{=0} = \begin{bmatrix} -\rho g A_{WP}{}^{(k)} & -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} & \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T{}^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} & \rho g I_P{}^{(k)} \\ \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} & \rho g I_P{}^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L{}^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$



$L = 100\ [m]$

$B_{mld} = 40\ [m]$

$D = 30\ [m]$

$d = 9\ [m]$

$\rho g = 10\ [Mg/m^2 s^2]$

${}^n\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$

${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [40 \quad -10 \quad -4.5]^T\ [m]$

${}^n\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T\ [m]$

${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$

$\nabla^{(0)} = 3.6 \times 10^4\ [m^3]$

$F_{G,z} = -3.6 \times 10^5\ [kN]$

$F_{G_{ext},z} = -4.0 \times 10^4\ [kN]$

$F_{B,z}^{(0)} = 3.6 \times 10^5\ [kN]$

$F_z^{(0)} = -4.0 \times 10^4\ [kN]$

$M_T^{(0)} = 4.0 \times 10^5\ [kN]$

$M_L^{(0)} = 1.6 \times 10^6\ [kN]$

Force equilibrium:

${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 7.4656]^T\ [m]$

${}^{b'}\mathbf{r}_{P_2/O'} = [50 \quad 20 \quad -1.9326]^T\ [m]$

${}^{b'}\mathbf{r}_{P_3/O'} = [-50 \quad 20 \quad -7.4656]^T\ [m]$

${}^{b'}\mathbf{r}_{P_4/O'} = [-50 \quad -20 \quad 1.9326]^T\ [m]$

$V_1 = 1.0137 \times 10^4\ [m^3]$

$V_2 = 1.8796 \times 10^4\ [m^3]$

$V_3 = 1.1066 \times 10^4\ [m^3]$

$$F_z^{(1)} = F_{B,z}^{(1)} + F_{G,z}^{(1)} + F_{ext,z}^{(1)}$$

$$F_{B,z}^{(1)} = \rho g \nabla^{(1)}$$

$$\begin{aligned} \nabla^{(1)} &= V_1^{(1)} + V_2^{(1)} + V_3^{(1)} \\ &= (1.0137 \times 10^4) + (1.8796 \times 10^4) \\ &\quad + (1.1066 \times 10^4) \\ &= 4.0 \times 10^4\ [m^3] \end{aligned}$$

$$F_{B,z}^{(1)} = \rho g \nabla^{(1)} = 10 \cdot (4.0 \times 10^4) = 4.0 \times 10^5\ [kN]$$

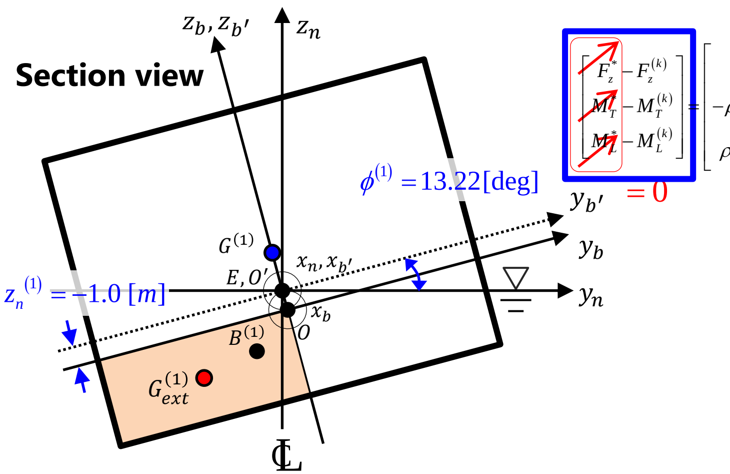
$$\begin{aligned} F_z^{(1)} &= F_{B,z}^{(1)} + F_{G,z}^{(1)} + F_{ext,z}^{(1)} \\ &= (4.0 \times 10^5) + (-3.6 \times 10^5) + (-4.0 \times 10^4) \\ &= \boxed{0\ [kN]} < e \end{aligned}$$

Tolerance

where, e(epsilon) : an arbitrarily small positive quantity

The static equilibrium of force is satisfied!

4. Check for the Ship to be in Static Equilibrium at k=0 step



$$\boxed{\begin{bmatrix} F_z^* - F_z^{(k)} \\ M_T^* - M_T^{(k)} \\ M_L^* - M_L^{(k)} \end{bmatrix}} = \begin{bmatrix} -\rho g A_{WP}^{(k)} & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} & \rho g I_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} & \rho g I_P^{(k)} & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$

Moment equilibrium:

First, we calculate center of volume for each volume V_1, V_2, and V_3.

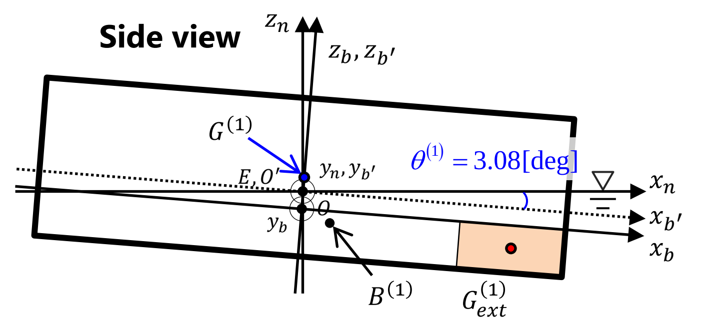
$${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 7.4656]^T\ [m]$$
$${}^{b'}\mathbf{r}_{P_2/O'} = [50 \quad 20 \quad -1.9326]^T\ [m]$$
$${}^{b'}\mathbf{r}_{P_3/O'} = [-50 \quad 20 \quad -7.4656]^T\ [m]$$
$${}^{b'}\mathbf{r}_{P_4/O'} = [-50 \quad -20 \quad 1.9326]^T\ [m]$$

Volume V_1: Box

C_1: located at centerline, midship and $T_0/2$

$${}^{b'}\mathbf{r}_{C_1/O'} = ({}^{b'}\mathbf{r}_{P_3/O'} + {}^{b'}\mathbf{r}_{B_1/O'})/2 = [\underbrace{0}_{\text{midship}} \quad \underbrace{0}_{\text{centerline}} \quad (-10 + (-7.4656))/2]^T$$

$$= [0 \quad 0 \quad -8.7328]^T$$



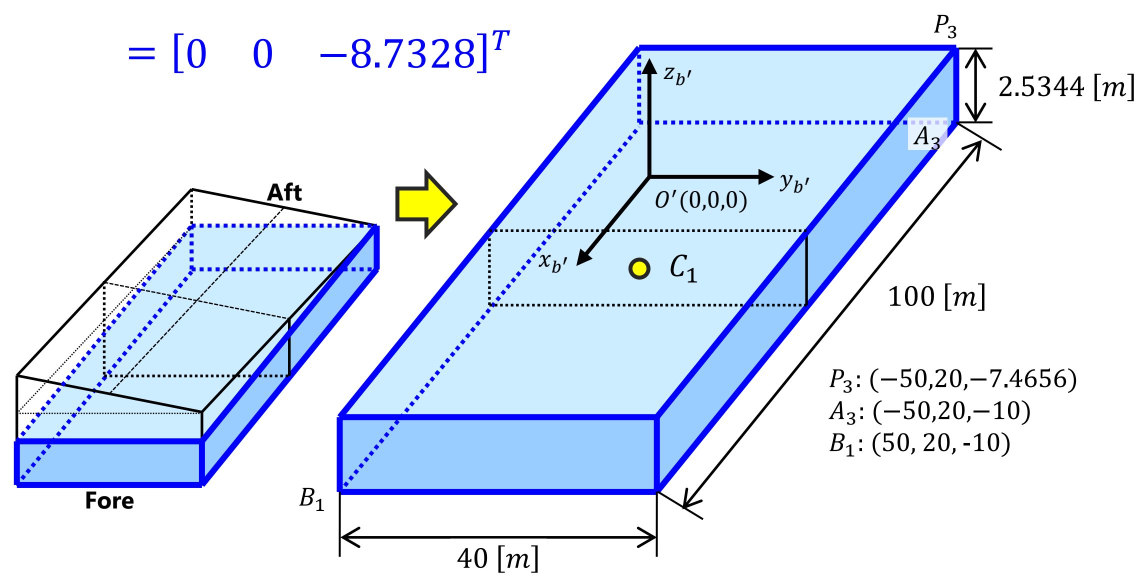
$L = 100\ [m]$	${}^n\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$
$B_{mld} = 40\ [m]$	${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [40 \quad -10 \quad -4.5]^T\ [m]$
$D = 30\ [m]$	
$d = 9\ [m]$	${}^n\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T\ [m]$
$\rho g = 10\ [Mg/m^2 s^2]$	${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$

$\nabla^{(1)} = 4.0 \times 10^4\ [m^3]$

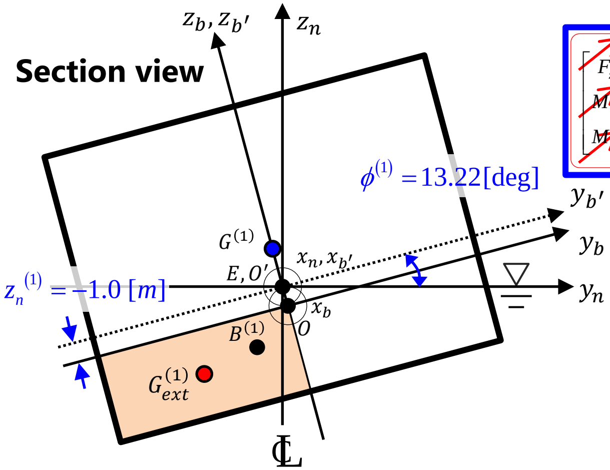
$F_{G,z} = -3.6 \times 10^5\ [kN]$ $\qquad F_z^{(1)} = 0\ [kN]$

$F_{G_{ext},z} = -4.0 \times 10^4\ [kN]$ $\qquad M_T^{(0)} = 4.0 \times 10^5\ [kN]$

$F_{B,z}^{(1)} = 4.0 \times 10^5\ [kN]$ $\qquad M_L^{(0)} = 1.6 \times 10^6\ [kN]$

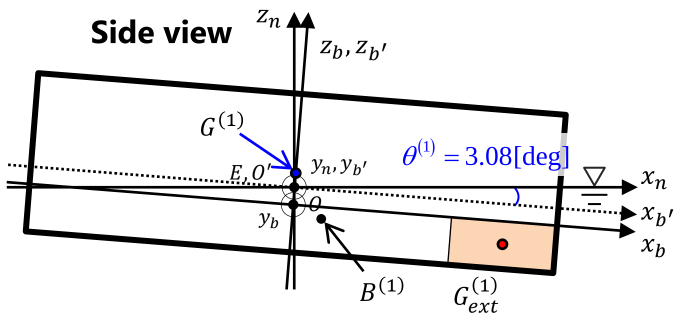


4. Check for the Ship to be in Static Equilibrium at k=0 step



$$\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \\ \cancel{M_L^*} - M_L^{(k)} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP}{}^{(k)} & -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} & \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T{}^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} & \rho g I_P{}^{(k)} \\ \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} & \rho g I_P{}^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L{}^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$



$L = 100\ [m]$	${}^n\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$
$B_{mld} = 40\ [m]$	${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [40 \quad -10 \quad -4.5]^T\ [m]$
$D = 30\ [m]$	${}^n\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T\ [m]$
$d = 9\ [m]$	
$\rho g = 10\ [Mg/m^2 s^2]$	${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$

$\nabla^{(1)} = 4.0 \times 10^4\ [m^3]$

$F_{G,z} = -3.6 \times 10^5\ [kN]$ $\quad F_z^{(1)} = 0\ [kN]$

$F_{G_{ext},z} = -4.0 \times 10^4\ [kN]$ $\quad M_T^{(0)} = 4.0 \times 10^5\ [kN]$

$F_{B,z}^{(1)} = 4.0 \times 10^5\ [kN]$ $\quad M_L^{(0)} = 1.6 \times 10^6\ [kN]$

Moment equilibrium:

First, we calculate center of volume for each volume V_1, V_2, and V_3.

$${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 7.4656]^T\ [m]$$
$${}^{b'}\mathbf{r}_{P_2/O'} = [50 \quad 20 \quad -1.9326]^T\ [m]$$
$${}^{b'}\mathbf{r}_{P_3/O'} = [-50 \quad 20 \quad -7.4656]^T\ [m]$$
$${}^{b'}\mathbf{r}_{P_4/O'} = [-50 \quad -20 \quad 1.9326]^T\ [m]$$
$${}^{b'}\mathbf{r}_{C_1/O'} = [0 \quad 0 \quad -8.7328]^T\ [m]$$

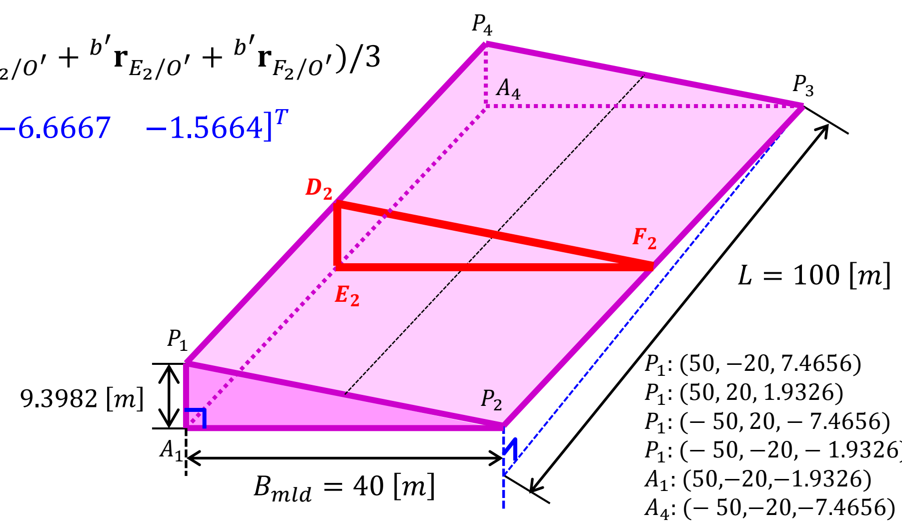
Volume V_2: Trigonal prism

Center of volume:

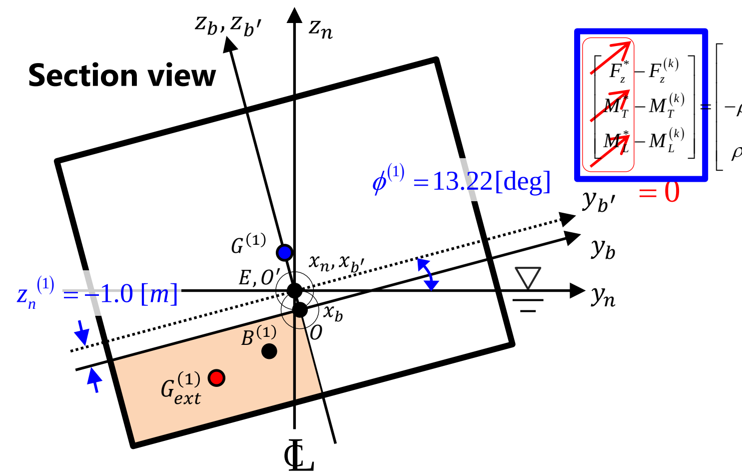
= Center of area of triangle $D_2E_2F_2$

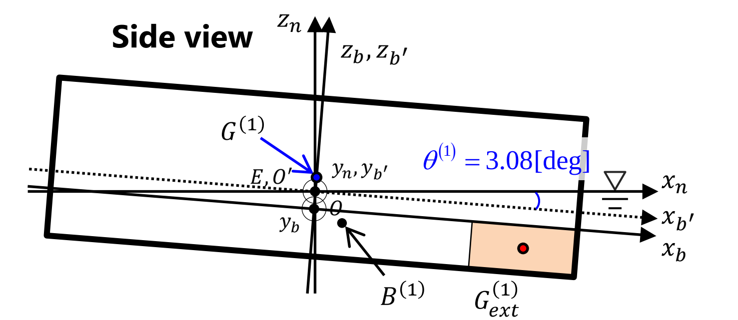
$${}^{b'}\mathbf{r}_{D_2/O'} = ({}^{b'}\mathbf{r}_{P_1/O'} + {}^{b'}\mathbf{r}_{P_4/O'})/2 = (0,\ -20,\ 4.6991)$$
$${}^{b'}\mathbf{r}_{E_2/O'} = ({}^{b'}\mathbf{r}_{A_1/O'} + {}^{b'}\mathbf{r}_{A_4/O'})/2 = (0,\ -20,\ -4.6991)$$
$${}^{b'}\mathbf{r}_{F_2/O'} = ({}^{b'}\mathbf{r}_{P_2/O'} + {}^{b'}\mathbf{r}_{P_3/O'})/2 = (0,\ 20,\ -4.6991)$$

$${}^{b'}\mathbf{r}_{C_2/O'} = ({}^{b'}\mathbf{r}_{D_2/O'} + {}^{b'}\mathbf{r}_{E_2/O'} + {}^{b'}\mathbf{r}_{F_2/O'})/3$$
$$= [0 \quad -6.6667 \quad -1.5664]^T$$



4. Check for the Ship to be in Static Equilibrium at k=0 step





$L = 100\ [m]$

$B_{mld} = 40\ [m]$

$D = 30\ [m]$

$d = 9\ [m]$

$\rho g = 10\ [Mg/m^2 s^2]$

${}^n\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$

${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [40 \quad -10 \quad -4.5]^T\ [m]$

${}^n\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T\ [m]$

${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$

$\nabla^{(1)} = 4.0 \times 10^4\ [m^3]$

$F_{G,z} = -3.6 \times 10^5\ [kN]$ $\qquad F_z^{(1)} = 0\ [kN]$

$F_{G_{ext},z} = -4.0 \times 10^4\ [kN]$ $\qquad M_T^{(0)} = 4.0 \times 10^5\ [kN]$

$F_{B,z}^{(1)} = 4.0 \times 10^5\ [kN]$ $\qquad M_L^{(0)} = 1.6 \times 10^6\ [kN]$

$$\boxed{\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \\ \cancel{M_L^*} - M_L^{(k)} \end{bmatrix}}_{=0} = \begin{bmatrix} -\rho g A_{WP}^{(k)} & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} & \rho g I_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} & \rho g I_P^{(k)} & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

Moment equilibrium:

First, we calculate center of volume for each volume V_1, V_2, and V_3.

${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 7.4656]^T\ [m]$

${}^{b'}\mathbf{r}_{P_2/O'} = [50 \quad 20 \quad -1.9326]^T\ [m]$

${}^{b'}\mathbf{r}_{P_3/O'} = [-50 \quad 20 \quad -7.4656]^T\ [m]$

${}^{b'}\mathbf{r}_{P_4/O'} = [-50 \quad -20 \quad 1.9326]^T\ [m]$

${}^{b'}\mathbf{r}_{C_1/O'} = [0 \quad 0 \quad -8.7332]^T\ [m]$

${}^{b'}\mathbf{r}_{C_2/O'} = [0 \quad -6.6667 \quad -1.5664]^T$

Volume V_3: Trigonal prism

Center of volume

= Center of area of triangle $D_3E_3F_3$

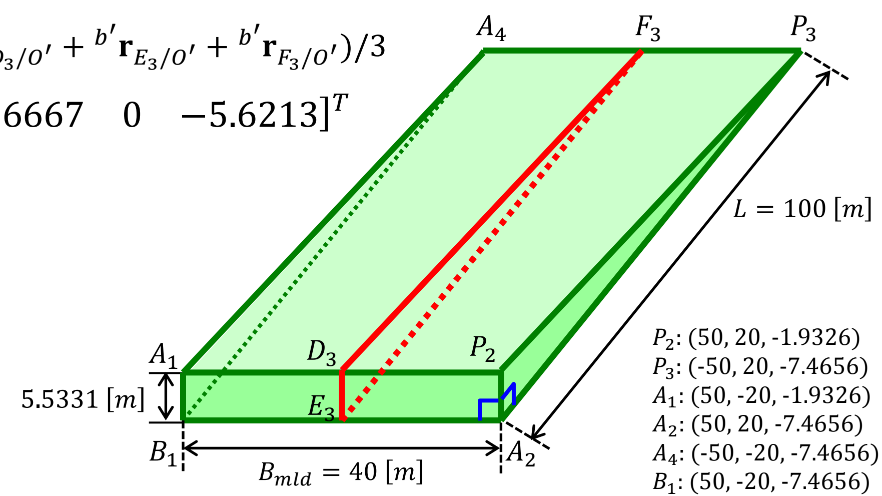
${}^{b'}\mathbf{r}_{D_3/O'} = ({}^{b'}\mathbf{r}_{A_1/O'} + {}^{b'}\mathbf{r}_{P_2/O'})/2$=(50, 0, -1.9326)

${}^{b'}\mathbf{r}_{E_3/O'} = ({}^{b'}\mathbf{r}_{B_1/O'} + {}^{b'}\mathbf{r}_{A_2/O'})/2$=(50, 0, -7.4656)

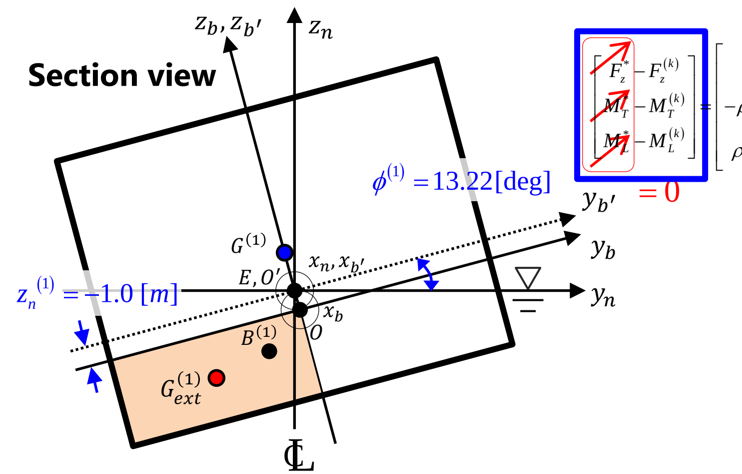
${}^{b'}\mathbf{r}_{F_3/O'} = ({}^{b'}\mathbf{r}_{P_3/O'} + {}^{b'}\mathbf{r}_{P_4/O'})/2$=(-50, 0, -7.4656)

$${}^{b'}\mathbf{r}_{C_3/O'} = ({}^{b'}\mathbf{r}_{D_3/O'} + {}^{b'}\mathbf{r}_{E_3/O'} + {}^{b'}\mathbf{r}_{F_3/O'})/3$$

$$= [16.6667 \quad 0 \quad -5.6213]^T$$

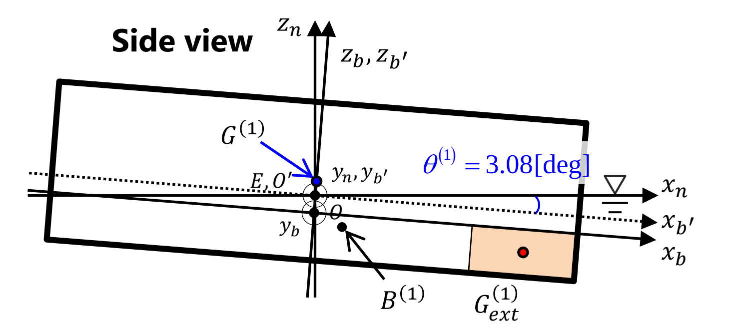


4. Check for the Ship to be in Static Equilibrium at k=0 step



$$\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \\ \cancel{M_L^*} - M_L^{(k)} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP}^{(k)} & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} & \rho g I_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} & \rho g I_P^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$



$L = 100\ [m]$

$B_{mld} = 40\ [m]$

$D = 30\ [m]$

$d = 9\ [m]$

$\rho g = 10\ [Mg/m^2 s^2]$

${}^n\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$

${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [40 \quad -10 \quad -4.5]^T\ [m]$

${}^n\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T\ [m]$

${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$

$\nabla^{(1)} = 4.0 \times 10^4\ [m^3]$

$F_{G,z} = -3.6 \times 10^5\ [kN]$

$F_z^{(1)} = 0\ [kN]$

$F_{G_{ext},z} = -4.0 \times 10^4\ [kN]$

$M_T^{(0)} = 4.0 \times 10^5\ [kN]$

$F_{B,z}^{(1)} = 4.0 \times 10^5\ [kN]$

$M_L^{(0)} = 1.6 \times 10^6\ [kN]$

Moment equilibrium:

${}^{b'}\mathbf{r}_{C_1/O'} = [0 \quad 0 \quad -8.7328]^T\ [m]$

${}^{b'}\mathbf{r}_{C_2/O'} = [0 \quad -6.6667 \quad -1.5664]^T\ [m]$

${}^{b'}\mathbf{r}_{C_3/O'E} = [16.6667 \quad 0 \quad -5.6213]^T\ [m]$

Center of Buoyancy:

$V_1 = 1.0137 \times 10^4\ [m^3]$

$V_2 = 1.8796 \times 10^4\ [m^3]$

$V_3 = 1.1066 \times 10^4\ [m^3]$

$${}^{b'}\mathbf{r}_{B^{(1)}/O'} = \frac{{}^{b'}\mathbf{r}_{C_1/O'} \cdot V_1 + {}^{b'}\mathbf{r}_{C_2/O'} \cdot V_2 + {}^{b'}\mathbf{r}_{C_3/O'} \cdot V_3}{V_1 + V_2 + V_3}$$

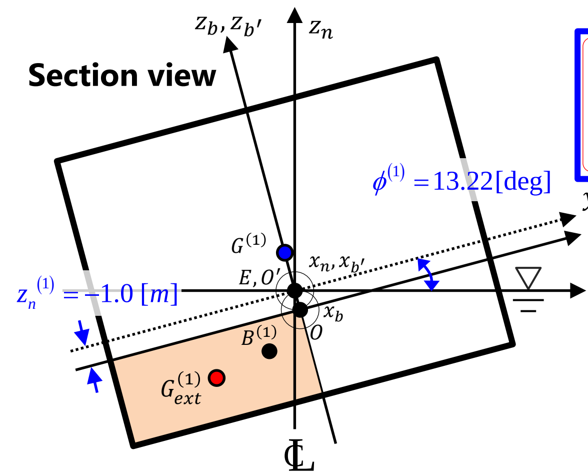
$$= [4.6109 \quad -3.1327 \quad -4.5044]^T\ [m]$$

In this case, for convenience of calculating the center of displaced volume ${}^{b'}\mathbf{r}_{B^{(1)}/O'}$ of the ship, we use b′-frame. The origin O′ of b′-frame coincides with the origin E of n-frame. And the orientation of b′-frame is the same as that of b-frame. So, to obtain the center of buoyancy with respect to n-frame, ${}^n\mathbf{r}_{B^{(1)}/E}$, we have to perform the rotational transformation.

$${}^n\mathbf{r}_{B^{(1)}/E} = \begin{bmatrix} \cos\theta^{(1)} & 0 & \sin\theta^{(1)} \\ 0 & 1 & 0 \\ -\sin\theta^{(1)} & 0 & \cos\theta^{(1)} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi^{(1)} & -\sin\phi^{(1)} \\ 0 & \sin\phi^{(1)} & \cos\phi^{(1)} \end{bmatrix} {}^{b'}\mathbf{r}_{B^{(1)}/O'}$$

$$= \begin{bmatrix} \cos 3.08 & 0 & \sin 3.08 \\ 0 & 1 & 0 \\ -\sin 3.08 & 0 & \cos 3.08 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 13.22 & -\sin 13.22 \\ 0 & \sin 13.22 & \cos 13.22 \end{bmatrix} \begin{bmatrix} 4.6109 \\ -3.1327 \\ -4.5044 \end{bmatrix} = \boxed{\begin{bmatrix} 4.3298 \\ -2.0194 \\ -5.3422 \end{bmatrix}}$$

4. Check for the Ship to be in Static Equilibrium at k=0 step



$$\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \\ \cancel{M_L^*} - M_L^{(k)} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP}{}^{(k)} & -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} & \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T{}^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} & \rho g I_P{}^{(k)} \\ \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} & \rho g I_P{}^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L{}^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$

Moment equilibrium:

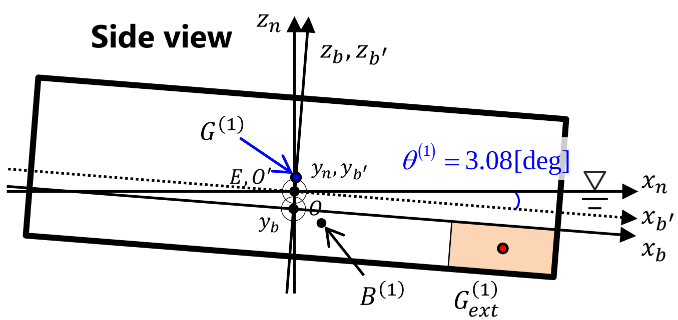
Center of Gravity:

The center of mass, ${}^b\mathbf{r}_{G^{(1)}/O}$, with respect to the body fixed frame is identical with respect to the floating position. But the center of mass, ${}^n\mathbf{r}_{G^{(1)}/E}$, with respect to the waterplane-fixed frame changes with respect to the rotation. The change in the center of mass, ${}^n\mathbf{r}_{G^{(1)}/E}$, with respect to the waterplane-fixed frame causes an additional heeling moment arm.

$${}^n\mathbf{r}_{G^{(1)}/E} = \begin{bmatrix} \cos\theta^{(1)} & 0 & \sin\theta^{(1)} \\ 0 & 1 & 0 \\ -\sin\theta^{(1)} & 0 & \cos\theta^{(1)} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi^{(1)} & -\sin\phi^{(1)} \\ 0 & \sin\phi^{(1)} & \cos\phi^{(1)} \end{bmatrix} {}^{b'}\mathbf{r}_{G^{(1)}/O'}, \quad {}^{b'}\mathbf{r}_{G^{(1)}/O'} = {}^b\mathbf{r}_{G^{(1)}/O} + \begin{bmatrix} 0 \\ 0 \\ z_n^{(1)} \end{bmatrix}$$

$$= \begin{bmatrix} \cos 3.08 & 0 & \sin 3.08 \\ 0 & 1 & 0 \\ -\sin 3.08 & 0 & \cos 3.08 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 13.22 & -\sin 13.22 \\ 0 & \sin 13.22 & \cos 13.22 \end{bmatrix} \left(\begin{bmatrix} 0 \\ 0 \\ 6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \right)$$

$$= \boxed{\begin{bmatrix} 0.2618 \\ -1.1436 \\ 4.8604 \end{bmatrix}}$$



$L = 100\ [m]$

$B_{mld} = 40\ [m]$

$D = 30\ [m]$

$d = 9\ [m]$

$\rho g = 10\ [Mg/m^2 s^2]$

${}^n\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$

${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [40 \quad -10 \quad -4.5]^T\ [m]$

${}^n\mathbf{r}_{B^{(1)}/E} = [4.33 \quad -2.02 \quad -5.34]^T\ [m]$

${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$

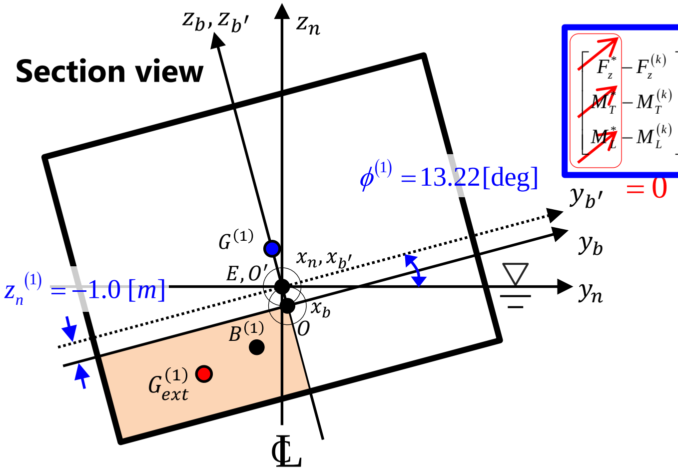
$\nabla^{(1)} = 4.0 \times 10^4\ [m^3]$

$F_{G,z} = -3.6 \times 10^5\ [kN]$ $\qquad F_z^{(1)} = 0\ [kN]$

$F_{G_{ext},z} = -4.0 \times 10^4\ [kN]$ $\qquad M_T^{(0)} = 4.0 \times 10^5\ [kN]$

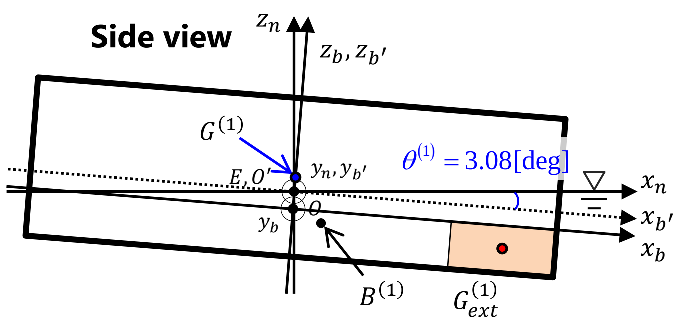
$F_{B,z}^{(1)} = 4.0 \times 10^5\ [kN]$ $\qquad M_L^{(0)} = 1.6 \times 10^6\ [kN]$

4. Check for the Ship to be in Static Equilibrium at k=0 step



$$\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \\ \cancel{M_L^*} - M_L^{(k)} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP}{}^{(k)} & -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} & \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T{}^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} & \rho g I_P{}^{(k)} \\ \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} & \rho g I_P{}^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L{}^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$



$L = 100\ [m]$
$B_{mld} = 40\ [m]$
$D = 30\ [m]$
$d = 9\ [m]$
$\rho g = 10\ [Mg/m^2 s^2]$

${}^n\mathbf{r}_{G^{(1)}/E} = [0.26 \quad -1.14 \quad 4.86]^T\ [m]$
${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [40 \quad -10 \quad -4.5]^T\ [m]$
${}^n\mathbf{r}_{B^{(1)}/E} = [4.33 \quad -2.02 \quad -5.34]^T\ [m]$
${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$

$\nabla^{(1)} = 4.0 \times 10^4\ [m^3]$
$F_{G,z} = -3.6 \times 10^5\ [kN]$
$F_{G_{ext},z} = -4.0 \times 10^4\ [kN]$
$F_{B,z}^{(1)} = 4.0 \times 10^5\ [kN]$

$F_z^{(1)} = 0\ [kN]$
$M_T^{(0)} = 4.0 \times 10^5\ [kN]$
$M_L^{(0)} = 1.6 \times 10^6\ [kN]$

Moment equilibrium:

Center of Gravity (Cargo):

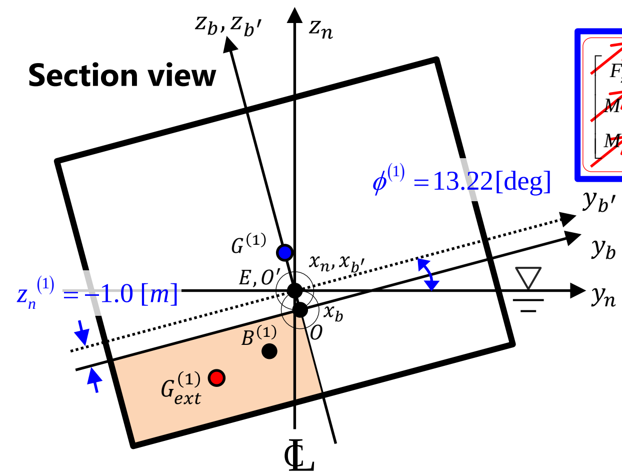
In this case, for convenience of calculating the center of external force, ${}^{b'}r_{G_{ext}^{(1)}/O'}$ of the ship, we use b'-frame. So, to obtain the center of external force with respect to n-frame, ${}^n r_{G_{ext}^{(1)}/E}$, we have to perform the rotational transformation.

$${}^n\mathbf{r}_{G_{ext}{}^{(1)}/E} = \begin{bmatrix} \cos\theta^{(1)} & 0 & \sin\theta^{(1)} \\ 0 & 1 & 0 \\ -\sin\theta^{(1)} & 0 & \cos\theta^{(1)} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi^{(1)} & -\sin\phi^{(1)} \\ 0 & \sin\phi^{(1)} & \cos\phi^{(1)} \end{bmatrix} {}^{b'}\mathbf{r}_{G_{ext}{}^{(1)}/O'}$$

$$= \begin{bmatrix} \cos 3.08 & 0 & \sin 3.08 \\ 0 & 1 & 0 \\ -\sin 3.08 & 0 & \cos 3.08 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 13.22 & -\sin 13.22 \\ 0 & \sin 13.22 & \cos 13.22 \end{bmatrix} \begin{bmatrix} 40 \\ -10 \\ -5.5 \end{bmatrix}$$

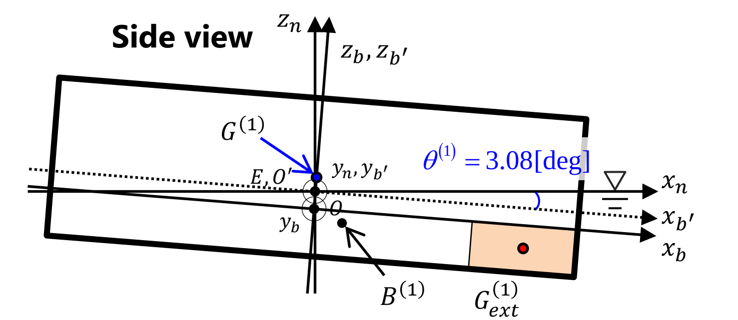
$$= \boxed{\begin{bmatrix} 39.5311 \\ -8.4769 \\ -9.7818 \end{bmatrix}}$$

4. Check for the Ship to be in Static Equilibrium at k=0 step



$$\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \\ \cancel{M_L^*} - M_L^{(k)} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP}^{(k)} & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} & \rho g I_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} & \rho g I_P^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$



$L = 100\ [m]$	${}^n\mathbf{r}_{G^{(1)}/E} = [0.26 \quad -1.14 \quad 4.86]^T\ [m]$
$B_{mld} = 40\ [m]$	${}^n\mathbf{r}_{G_{ext}^{(1)}/E} = [39.58 \quad -8.48 \quad -9.78]^T\ [m]$
$D = 30\ [m]$	${}^n\mathbf{r}_{B^{(1)}/E} = [4.33 \quad -2.02 \quad -5.34]^T\ [m]$
$d = 9\ [m]$	
$\rho g = 10\ [Mg/m^2 s^2]$	${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$

$\nabla^{(1)} = 4.0 \times 10^4\ [m^3]$

$F_{G,z} = -3.6 \times 10^5\ [kN]$	$F_z^{(1)} = 0\ [kN]$
$F_{G_{ext},z} = -4.0 \times 10^4\ [kN]$	$M_T^{(0)} = 4.0 \times 10^5\ [kN]$
$F_{B,z}^{(1)} = 4.0 \times 10^5\ [kN]$	$M_L^{(0)} = 1.6 \times 10^6\ [kN]$

Moment equilibrium:

$$M_T^{(1)} = M_{BT}^{(1)} + M_{GT}^{(1)} + M_{extT}^{(1)}$$

$$= {}^n y_{B^{(1)}/E} \cdot F_{B,z}^{(1)} + {}^n y_{G^{(1)}/E} \cdot F_{G,z} + {}^n y_{G_{ext}^{(1)}/E} \cdot F_{ext,z}$$

$$= -2.02 \cdot (4.0 \times 10^5) + (-1.14) \cdot (-3.6 \times 10^5) + (-8.48) \cdot (-4.0 \times 10^4)$$

$$= -5.70 \times 10^4\ [kN \cdot m]$$

Tolerance

$$\rightarrow |-5.70 \times 10^4| > e$$

where, e(epsilon) : an arbitrarily small positive quantity

$$M_L^{(1)} = M_{BL}^{(1)} + M_{GL}^{(1)} + M_{extL}^{(1)}$$

$$= \left(-{}^n x_{B^{(1)}/E} \cdot F_{B,z}^{(1)} \right) + \left(-{}^n x_{G^{(1)}/E} \cdot F_{G,z} \right) + \left(-{}^n x_{G_{ext}^{(1)}/E} \cdot F_{ext,z} \right)$$

$$= [-4.33 \cdot (4.0 \times 10^5)] + [-0.26 \cdot (-3.6 \times 10^5)] + [-39.58 \cdot (-4.0 \times 10^4)]$$

$$= -5.64 \times 10^4\ [kN \cdot m]$$

Tolerance

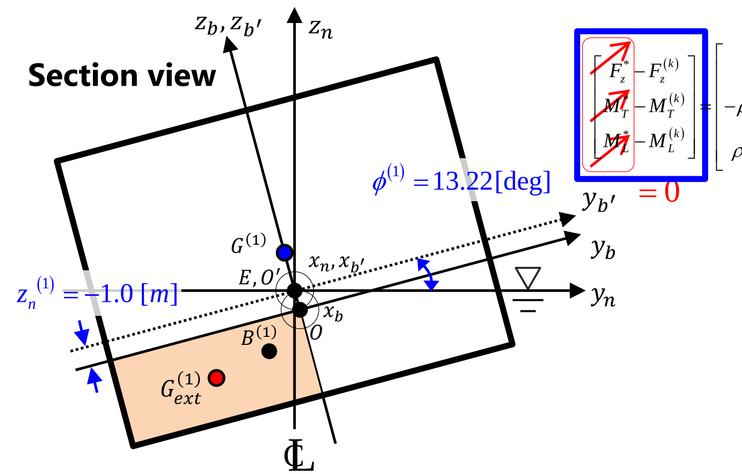
$$\rightarrow |-5.64 \times 10^4| > e$$

where, e(epsilon) : an arbitrarily small positive quantity

The static equilibrium of moment is not satisfied!
We have to iterate!

1. Calculation of Force and Moments at k=1 step

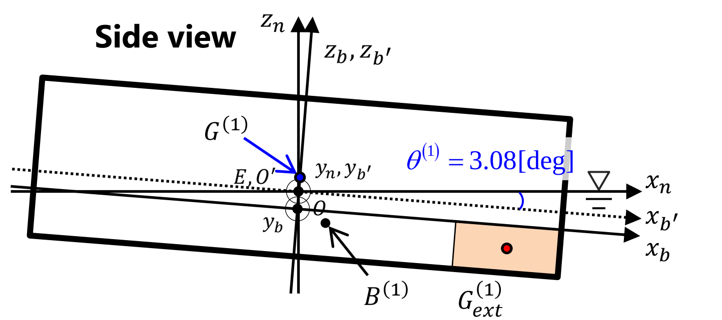
Section view



$$\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \\ \cancel{M_L^*} - M_L^{(k)} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP}{}^{(k)} & -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} & \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T{}^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} & \rho g I_P{}^{(k)} \\ \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} & \rho g I_P{}^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L{}^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$

Side view



$L = 100\ [m]$

$B_{mld} = 40\ [m]$

$D = 30\ [m]$

$d = 9\ [m]$

$\rho g = 10\ [Mg/m^2 s^2]$

${}^n\mathbf{r}_{G^{(1)}/E} = [0.26 \quad -1.14 \quad 4.86]^T\ [m]$

${}^n\mathbf{r}_{G_{ext}^{(1)}/E} = [39.58 \quad -8.48 \quad -9.78]^T\ [m]$

${}^n\mathbf{r}_{B^{(1)}/E} = [4.33 \quad -2.02 \quad -5.34]^T\ [m]$

${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$

$\nabla^{(1)} = 4.0 \times 10^4\ [m^3]$

$F_{G,z} = -3.6 \times 10^5\ [kN]$

$F_{G_{ext},z} = -4.0 \times 10^4\ [kN]$

$F_{B,z}^{(1)} = 4.0 \times 10^5\ [kN]$

$F_z^{(1)} = 0\ [kN]$

$M_T^{(1)} = -5.70 \times 10^4\ [kN]$

$M_L^{(1)} = -5.64 \times 10^6\ [kN]$

Force:

$$F_z^{(1)} = F_{B,z}^{(1)} + F_{G,z}^{(1)} + F_{ext,z}^{(1)}$$

$$= 0\ [kN]$$

Moment:

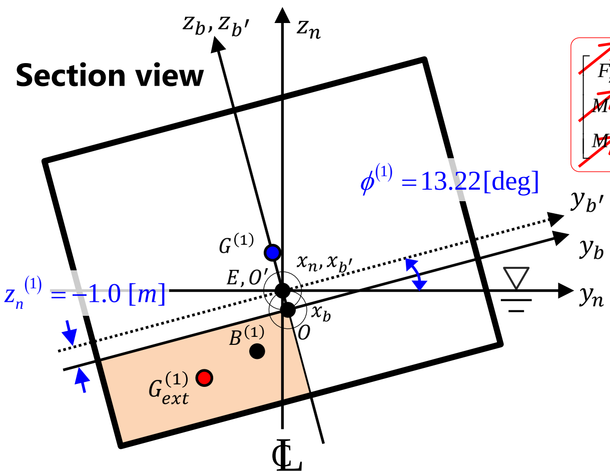
$$M_T^{(1)} = M_{BT}^{(1)} + M_{GT}^{(1)} + M_{extT}^{(1)}$$

$$= -5.70 \times 10^4\ [kN \cdot m]$$

$$M_L^{(1)} = M_{BL}^{(1)} + M_{GL}^{(1)} + M_{extL}^{(1)}$$

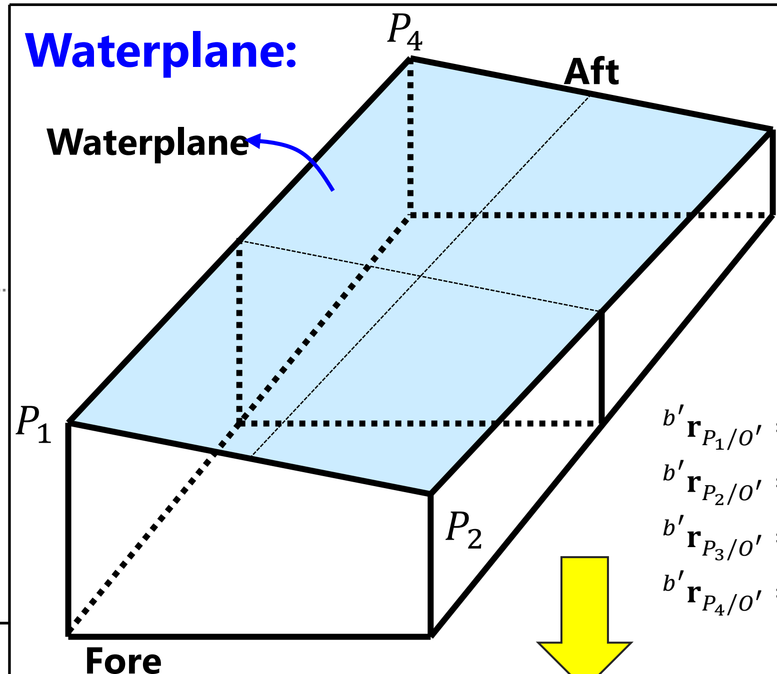
$$= -5.64 \times 10^4\ [kN \cdot m]$$

2. Calculation of the Values of the Waterplane at k=1 step

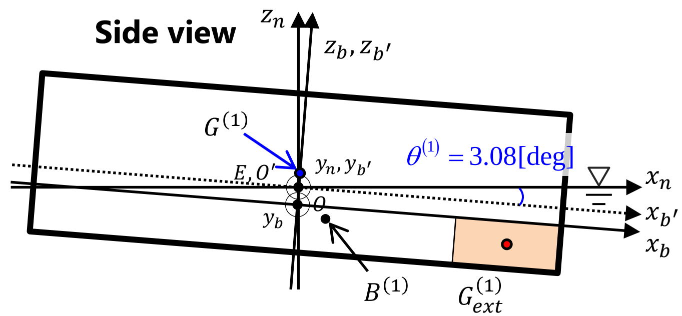


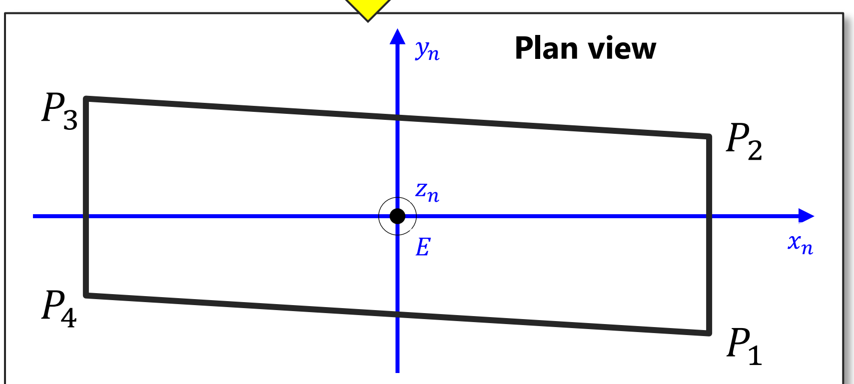
$$\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \\ \cancel{M_L^*} - M_L^{(k)} \end{bmatrix} = \boxed{\begin{bmatrix} -\rho g A_{WP}^{(k)} & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} & \rho g I_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} & \rho g I_P^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$



$${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 7.4656]^T\ [m]$$
$${}^{b'}\mathbf{r}_{P_2/O'} = [50 \quad 20 \quad -1.9326]^T\ [m]$$
$${}^{b'}\mathbf{r}_{P_3/O'} = [-50 \quad 20 \quad -7.4656]^T\ [m]$$
$${}^{b'}\mathbf{r}_{P_4/O'} = [-50 \quad -20 \quad 1.9326]^T\ [m]$$





$L = 100\ [m]$

$B_{mld} = 40\ [m]$

$D = 30\ [m]$

$d = 9\ [m]$

$\rho g = 10\ [Mg/m^2 s^2]$

${}^n\mathbf{r}_{G^{(1)}/E} = [0.26 \quad -1.14 \quad 4.86]^T\ [m]$

${}^n\mathbf{r}_{G_{ext}^{(1)}/E} = [39.58 \quad -8.48 \quad -9.78]^T\ [m]$

${}^n\mathbf{r}_{B^{(1)}/E} = [4.33 \quad -2.02 \quad -5.34]^T\ [m]$

${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$

$\nabla^{(1)} = 4.0 \times 10^4\ [m^3]$

$F_{G,z} = -3.6 \times 10^5\ [kN]$

$F_{G_{ext},z} = -4.0 \times 10^4\ [kN]$

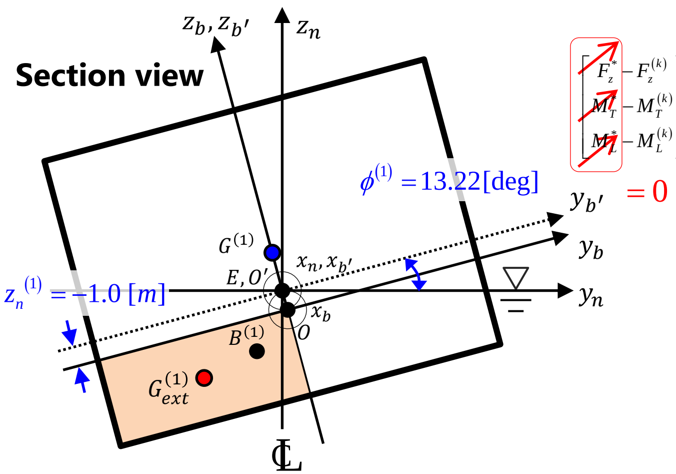
$F_{B,z}^{(1)} = 4.0 \times 10^5\ [kN]$

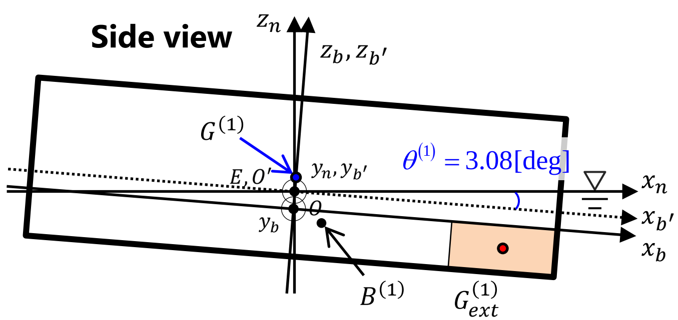
$F_z^{(1)} = 0\ [kN]$

$M_T^{(1)} = -5.70 \times 10^4\ [kN]$

$M_L^{(1)} = -5.64 \times 10^6\ [kN]$

2. Calculation of the Values of the Waterplane at k=1 step





$$
\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \\ \cancel{M_L^*} - M_L^{(k)} \end{bmatrix} = \boxed{\begin{bmatrix} -\rho g A_{WP}{}^{(k)} & -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} & \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T{}^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} & \rho g I_P{}^{(k)} \\ \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} & \rho g I_P{}^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L{}^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}
$$

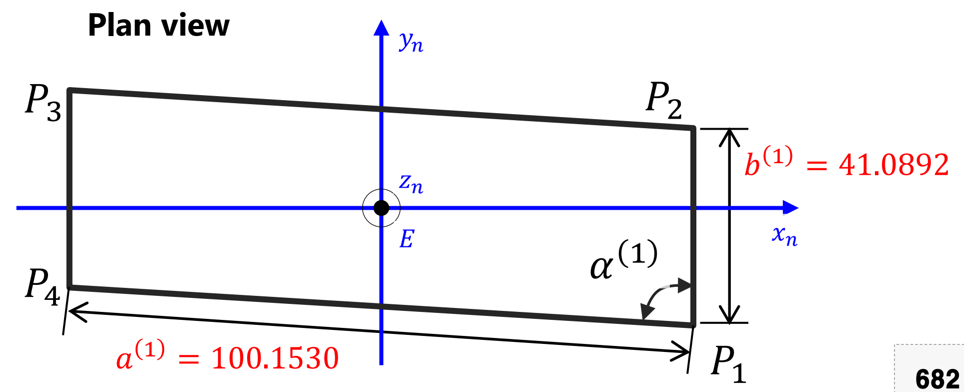
$= 0$

Waterplane:

$$ {}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 7.4656]^T\ [m] $$
$$ {}^{b'}\mathbf{r}_{P_2/O'} = [50 \quad 20 \quad -1.9326]^T\ [m] $$
$$ {}^{b'}\mathbf{r}_{P_3/O'} = [-50 \quad 20 \quad -7.4656]^T\ [m] $$
$$ {}^{b'}\mathbf{r}_{P_4/O'} = [-50 \quad -20 \quad 1.9326]^T\ [m] $$

$$ \overrightarrow{P_1P_2} = [50 \quad 20 \quad -1.9326]^T - [50 \quad -20 \quad 7.4656]^T $$
$$ = [0 \quad 40 \quad -9.3995]^T $$
$$ \left|\overrightarrow{P_1P_2}\right| = 41.0892 \ \left(= a^{(1)}\right) $$

$$ \overrightarrow{P_1P_4} = [50 \quad -20 \quad -1.9326]^T - [-50 \quad -20 \quad 1.9326]^T $$
$$ = [-100 \quad 0 \quad -5.5319]^T $$
$$ \left|\overrightarrow{P_1P_4}\right| = 100.1530 \ \left(= b^{(1)}\right) $$



$L = 100\ [m]$
$B_{mld} = 40\ [m]$
$D = 30\ [m]$
$d = 9\ [m]$
$\rho g = 10\ [Mg/m^2 s^2]$

$^n\mathbf{r}_{G^{(1)}/E} = [0.26 \quad -1.14 \quad 4.86]^T\ [m]$
$^n\mathbf{r}_{G_{ext}^{(1)}/E} = [39.58 \quad -8.48 \quad -9.78]^T\ [m]$
$^n\mathbf{r}_{B^{(1)}/E} = [4.33 \quad -2.02 \quad -5.34]^T\ [m]$
$^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$

$\nabla^{(1)} = 4.0 \times 10^4\ [m^3]$
$F_{G,z} = -3.6 \times 10^5\ [kN]$
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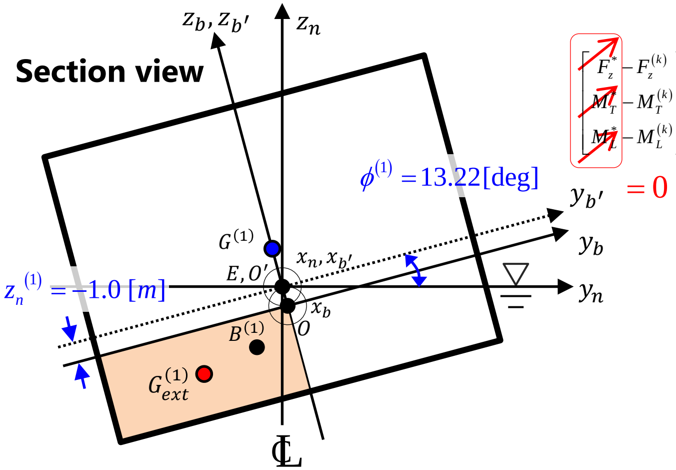
$F_{B,z}^{(1)} = 4.0 \times 10^5\ [kN]$

$F_z^{(1)} = 0\ [kN]$
$M_T^{(1)} = -5.70 \times 10^4\ [kN]$
$M_L^{(1)} = -5.64 \times 10^6\ [kN]$

2. Calculation of the Values of the Waterplane at k=1 step

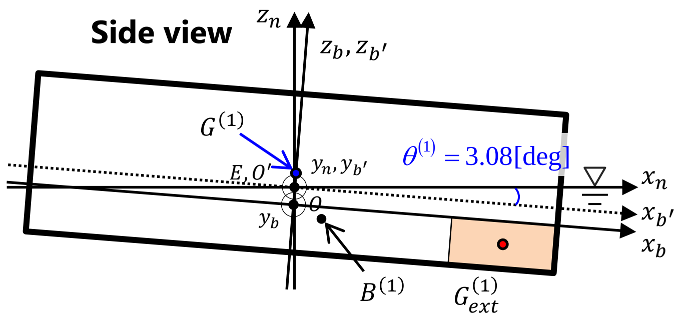
Section view



$$\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \\ \cancel{M_L^*} - M_L^{(k)} \end{bmatrix} = \boxed{\begin{bmatrix} -\rho g A_{WP}^{(k)} & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} & \rho g I_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} & \rho g I_P^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$

Side view



$L = 100\ [m]$
$B_{mld} = 40\ [m]$
$D = 30\ [m]$
$d = 9\ [m]$
$\rho g = 10\ [Mg/m^2 s^2]$

${}^n\mathbf{r}_{G^{(1)}/E} = [0.26 \quad -1.14 \quad 4.86]^T\ [m]$
${}^n\mathbf{r}_{G_{ext}^{(1)}/E} = [39.58 \quad -8.48 \quad -9.78]^T\ [m]$
${}^n\mathbf{r}_{B^{(1)}/E} = [4.33 \quad -2.02 \quad -5.34]^T\ [m]$
${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$

$\nabla^{(1)} = 4.0 \times 10^4\ [m^3]$
$F_{G,z} = -3.6 \times 10^5\ [kN]$
$F_{G_{ext},z} = -4.0 \times 10^4\ [kN]$
$F_{B,z}^{(1)} = 4.0 \times 10^5\ [kN]$

$F_z^{(1)} = 0\ [kN]$
$M_T^{(1)} = -5.70 \times 10^4\ [kN]$
$M_L^{(1)} = -5.64 \times 10^6\ [kN]$

Waterplane:

$$\alpha^{(1)} = \cos^{-1} \frac{\overrightarrow{P_1P_2} \cdot \overrightarrow{P_1P_4}}{\left|\overrightarrow{P_1P_2}\right| \cdot \left|\overrightarrow{P_1P_4}\right|}$$

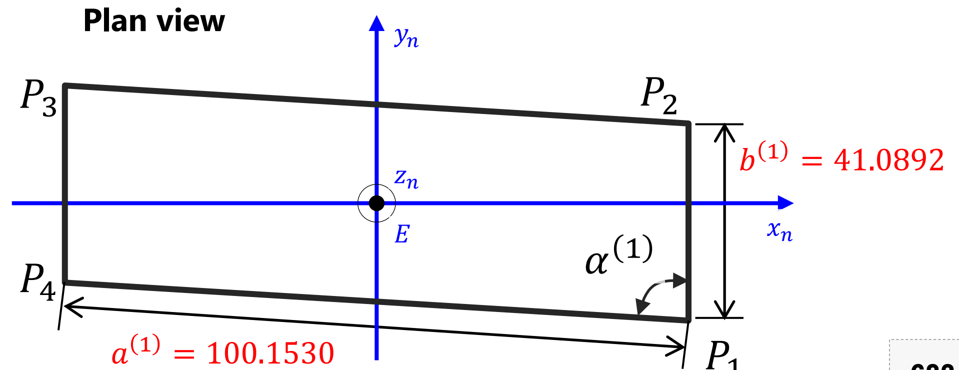
$$= \cos^{-1} \left[\frac{(0 \cdot (-100)) + 40 \cdot 0 + (-9.3995) \cdot (-5.5319)}{41.0892 \cdot 100.1530} \right]$$

$$= 89.2760\ [deg]$$

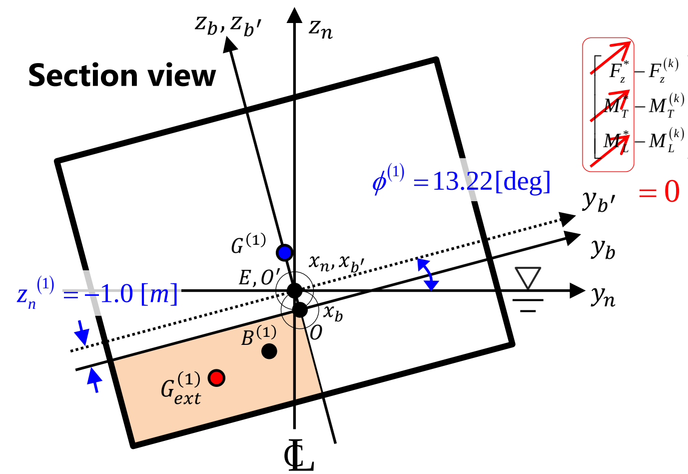
${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 7.4656]^T\ [m]$
${}^{b'}\mathbf{r}_{P_2/O'} = [50 \quad 20 \quad -1.9326]^T\ [m]$
${}^{b'}\mathbf{r}_{P_3/O'} = [-50 \quad 20 \quad -7.4656]^T\ [m]$
${}^{b'}\mathbf{r}_{P_4/O'} = [-50 \quad -20 \quad 1.9326]^T\ [m]$

$\overrightarrow{P_1P_2} = [0 \quad 40 \quad -9.3995]^T$
$\left|\overrightarrow{P_1P_2}\right| = 41.0892$ $(= a^{(1)})$
$\overrightarrow{P_1P_4} = [-100 \quad 0 \quad -5.5319]^T$
$\left|\overrightarrow{P_1P_4}\right| = 100.1530$ $(= b^{(1)})$

Plan view



2. Calculation of the Values of the Waterplane at k=1 step



$$\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \\ \cancel{M_L^*} - M_L^{(k)} \end{bmatrix} = \boxed{\begin{bmatrix} -\rho g A_{WP}{}^{(k)} & -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} & \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T{}^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} & \rho g I_P{}^{(k)} \\ \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} & \rho g I_P{}^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L{}^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} \end{bmatrix}}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$

Waterplane area:

$$A_{WP}^{(1)} = a^{(1)} \cdot b^{(1)} \cdot \sin\alpha^{(1)}$$

$$= 100.1530 \cdot 41.0892 \cdot \sin 89.2760$$

$$= 4.1149 \times 10^3\ [m^3]$$

$${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 7.4656]^T\ [m]$$

$${}^{b'}\mathbf{r}_{P_2/O'} = [50 \quad 20 \quad -1.9326]^T\ [m]$$

$${}^{b'}\mathbf{r}_{P_3/O'} = [-50 \quad 20 \quad -7.4656]^T\ [m]$$

$${}^{b'}\mathbf{r}_{P_4/O'} = [-50 \quad -20 \quad 1.9326]^T\ [m]$$

$$\overrightarrow{P_1P_2} = [0 \quad 40 \quad -9.3995]^T$$

$$|\overrightarrow{P_1P_2}| = 41.0892 \ (= a^{(1)})$$

$$\overrightarrow{P_1P_4} = [-100 \quad 0 \quad -5.5319]^T$$

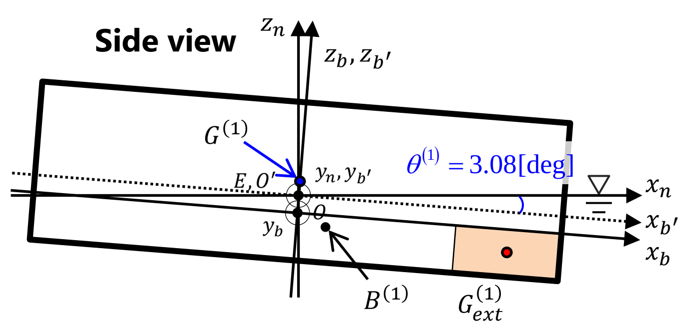
$$|\overrightarrow{P_1P_4}| = 100.1530 \ (= b^{(1)})$$

$$\alpha^{(1)} = 89.2760\ [deg]$$

Center of waterplane:

(equal to center of area)

$${}^n\mathbf{r}_{F^{(1)}/E} = (P_1 + P_3)/2 = [0 \quad 0 \quad 0]^T\ [m]$$



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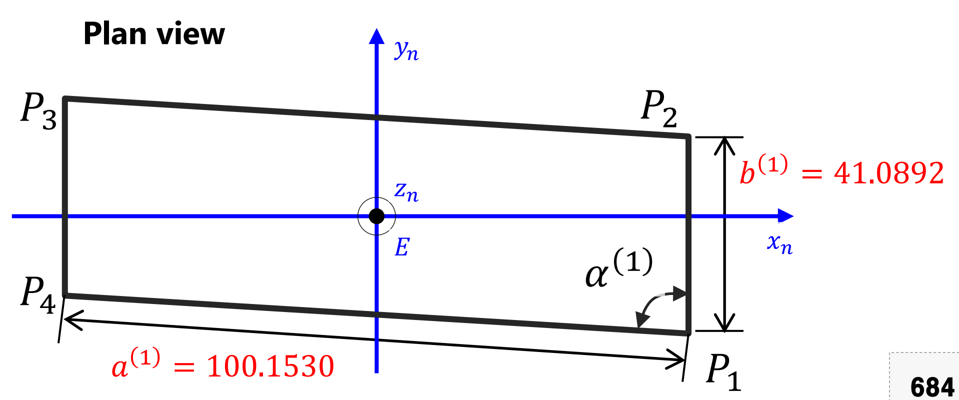
$F_{B,z}^{(1)} = 4.0 \times 10^5\ [kN]$

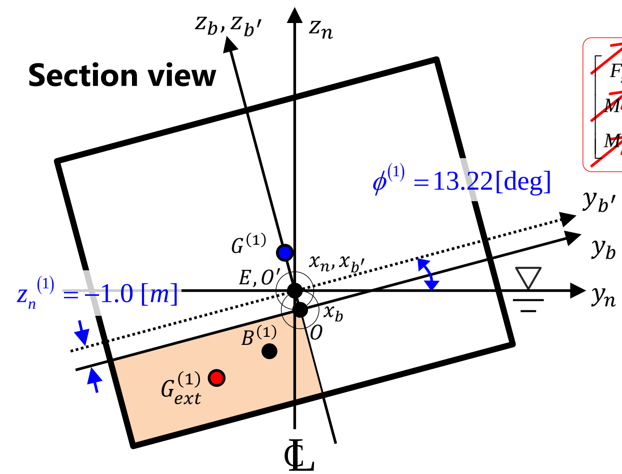
$F_z^{(1)} = 0\ [kN]$

$M_T^{(1)} = -5.70 \times 10^4\ [kN]$

$M_L^{(1)} = -5.64 \times 10^6\ [kN]$

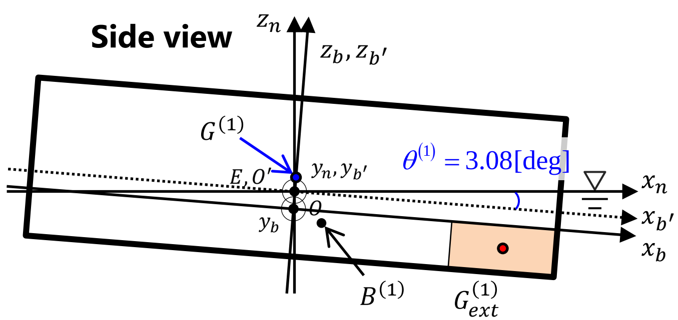


2. Calculation of the Values of the Waterplane at k=1 step



$$\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \\ \cancel{M_L^*} - M_L^{(k)} \end{bmatrix} = \boxed{\begin{bmatrix} -\rho g A_{WP}{}^{(k)} & -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} & \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T{}^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} & \rho g I_P{}^{(k)} \\ \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} & \rho g I_P{}^{(k)} & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L{}^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$



$L = 100\ [m]$
$B_{mld} = 40\ [m]$
$D = 30\ [m]$
$d = 9\ [m]$
$\rho g = 10\ [Mg/m^2 s^2]$

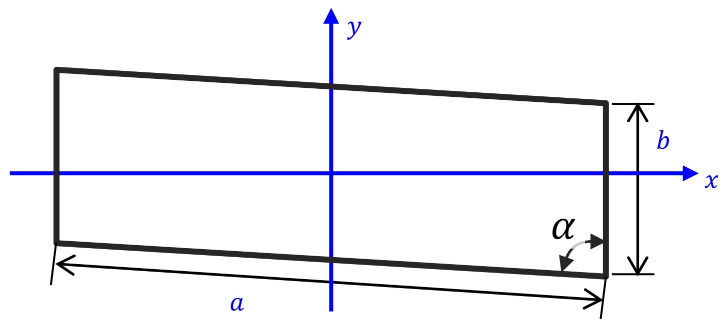
${}^n\mathbf{r}_{G^{(1)}/E} = [0.26 \quad -1.14 \quad 4.86]^T\ [m]$
${}^n\mathbf{r}_{G_{ext}^{(1)}/E} = [39.58 \quad -8.48 \quad -9.78]^T\ [m]$
${}^n\mathbf{r}_{B^{(1)}/E} = [4.33 \quad -2.02 \quad -5.34]^T\ [m]$
${}^n\mathbf{r}_{F^{(1)}/E} = [0 \quad 0 \quad 0]^T\ [m]$

$\nabla^{(1)} = 4.0 \times 10^4\ [m^3]$
$F_{G,z} = -3.6 \times 10^5\ [kN]$
$F_{G_{ext},z} = -4.0 \times 10^4\ [kN]$
$F_{B,z}^{(1)} = 4.0 \times 10^5\ [kN]$

$F_z^{(1)} = 0\ [kN]$
$M_T^{(1)} = -5.70 \times 10^4\ [kN]$
$M_L^{(1)} = -5.64 \times 10^6\ [kN]$

Waterplane:

Second moment of area of parallelogram:



$$I_{xx} = \frac{1}{12} ab \sin\theta\, (b^2 + a^2 \cos^2\theta)$$

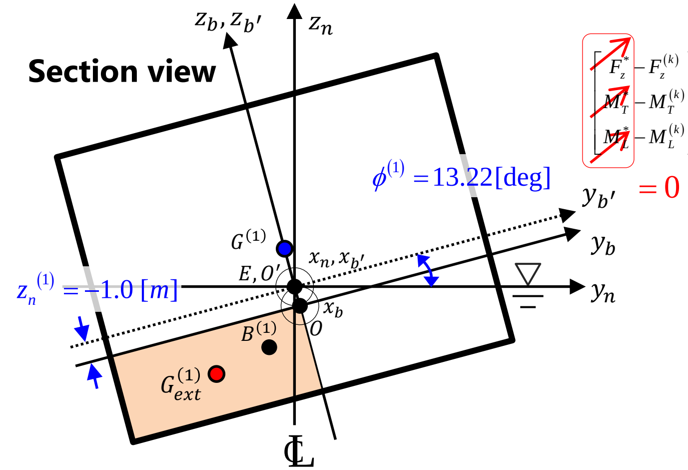
$$I_{yy} = \frac{1}{12} a^3 b \sin^3 \alpha$$

$$I_{xy} = 0$$

${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 7.4656]^T\ [m]$
${}^{b'}\mathbf{r}_{P_2/O'} = [50 \quad 20 \quad -1.9326]^T\ [m]$
${}^{b'}\mathbf{r}_{P_3/O'} = [-50 \quad 20 \quad -7.4656]^T\ [m]$
${}^{b'}\mathbf{r}_{P_4/O'} = [-50 \quad -20 \quad 1.9326]^T\ [m]$

$\overrightarrow{P_1P_2} = [0 \quad 40 \quad -9.3995]^T$
$|\overrightarrow{P_1P_2}| = 41.0892\ (= a^{(1)})$
$\overrightarrow{P_1P_4} = [-100 \quad 0 \quad -5.5319]^T$
$|\overrightarrow{P_1P_4}| = 100.1530\ (= b^{(1)})$
$\alpha^{(1)} = 89.2760\ [deg]$

2. Calculation of the Values of the Waterplane at k=1 step



$$\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \\ \cancel{M_L^*} - M_L^{(k)} \end{bmatrix} = \left.\begin{bmatrix} -\rho g A_{WP}{}^{(k)} & -\rho g A_{WP}{}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} & \rho g A_{WP}{}^{(k)} \cdot {}^{n}x_{F^{(k)}/E} \\ -\rho g A_{WP}{}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} & -\rho g\left({}^{n}z_{B^{(k)}/E}\nabla^{(k)} + I_T{}^{(k)}\right) - {}^{n}z_{G^{(k)}/E} \cdot F_{G,z} - {}^{n}z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} & \rho g I_P{}^{(k)} \\ \rho g A_{WP}{}^{(k)} \cdot {}^{n}x_{F^{(k)}/E} & \rho g I_P{}^{(k)} & -\rho g\left({}^{n}z_{B^{(k)}/E}\nabla^{(k)} + I_L{}^{(k)}\right) - {}^{n}z_{G^{(k)}/E} \cdot F_{G,z} - {}^{n}z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} \end{bmatrix}\right|_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta\phi^{(k)} \\ \delta\theta^{(k)} \end{bmatrix}$$

$= 0$

Waterplane:

$$I_T^{(1)} = I_{x_n x_n}$$

$$= \frac{1}{12} a^{(1)} b^{(1)} \sin\alpha^{(1)} \left[{b^{(1)}}^2 + {a^{(1)}}^2 \cos^2\alpha^{(1)} \right]$$

$$= \frac{1}{12} \cdot 100.1530 \cdot 41.0892 \cdot \sin 89.2760$$

$$\times \left[41.0892^2 + 100.1530^2 \cos^2 89.2760 \right]$$

$$= 5.7949 \times 10^5\ [m^4]$$

$${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 7.4656]^T\ [m]$$

$${}^{b'}\mathbf{r}_{P_2/O'} = [50 \quad 20 \quad -1.9326]^T\ [m]$$

$${}^{b'}\mathbf{r}_{P_3/O'} = [-50 \quad 20 \quad -7.4656]^T\ [m]$$

$${}^{b'}\mathbf{r}_{P_4/O'} = [-50 \quad -20 \quad 1.9326]^T\ [m]$$

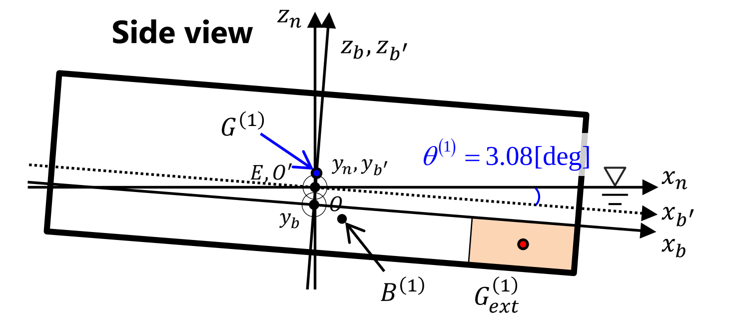
$$\overrightarrow{P_1P_2} = [0 \quad 40 \quad -9.3995]^T$$

$$\left|\overrightarrow{P_1P_2}\right| = 41.0892\ (= a^{(1)})$$

$$\overrightarrow{P_1P_4} = [-100 \quad 0 \quad -5.5319]^T$$

$$\left|\overrightarrow{P_1P_4}\right| = 100.1530\ (= b^{(1)})$$

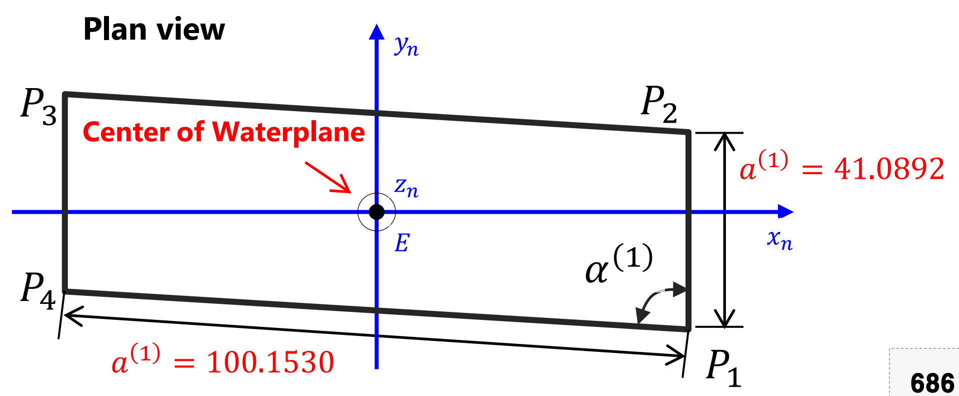
$$\alpha^{(1)} = 89.2760\ [deg]$$



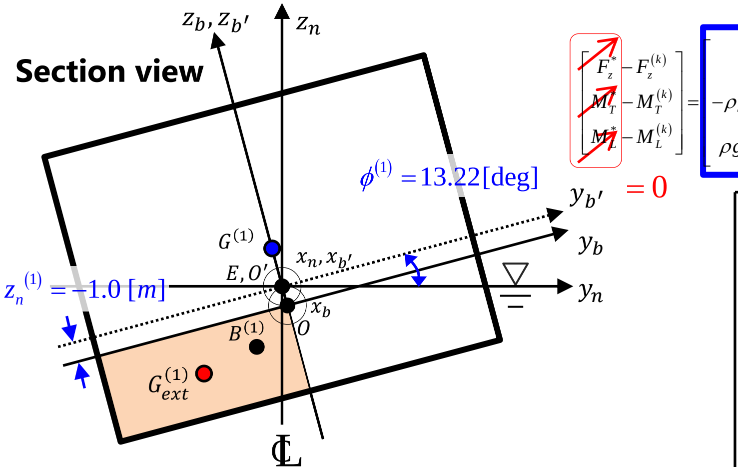
$L = 100\ [m]$	${}^{n}\mathbf{r}_{G^{(1)}/E} = [0.26 \quad -1.14 \quad 4.86]^T\ [m]$
$B_{mld} = 40\ [m]$	${}^{n}\mathbf{r}_{G_{ext}^{(1)}/E} = [39.58 \quad -8.48 \quad -9.78]^T\ [m]$
$D = 30\ [m]$	${}^{n}\mathbf{r}_{B^{(1)}/E} = [4.33 \quad -2.02 \quad -5.34]^T\ [m]$
$d = 9\ [m]$	${}^{n}\mathbf{r}_{F^{(1)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2s^2]$	

$\nabla^{(1)} = 4.0 \times 10^4\ [m^3]$

$F_{G,z} = -3.6 \times 10^5\ [kN]$	$F_z^{(1)} = 0\ [kN]$
$F_{G_{ext},z} = -4.0 \times 10^4\ [kN]$	$M_T^{(1)} = -5.70 \times 10^4\ [kN]$
$F_{B,z}^{(1)} = 4.0 \times 10^5\ [kN]$	$M_L^{(1)} = -5.64 \times 10^6\ [kN]$

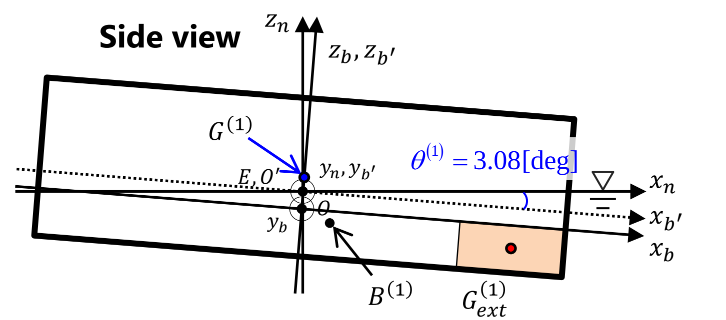


2. Calculation of the Values of the Waterplane at k=1 step



$$\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \\ \cancel{M_L^*} - M_L^{(k)} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP}{}^{(k)} & -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} & \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T{}^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} & \rho g I_P{}^{(k)} \\ \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} & \rho g I_P{}^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L{}^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$



$L = 100\ [m]$

$B_{mld} = 40\ [m]$

$D = 30\ [m]$

$d = 9\ [m]$

$\rho g = 10\ [Mg/m^2 s^2]$

${}^n\mathbf{r}_{G^{(1)}/E} = [0.26 \quad -1.14 \quad 4.86]^T\ [m]$

${}^n\mathbf{r}_{G_{ext}^{(1)}/E} = [39.58 \quad -8.48 \quad -9.78]^T\ [m]$

${}^n\mathbf{r}_{B^{(1)}/E} = [4.33 \quad -2.02 \quad -5.34]^T\ [m]$

${}^n\mathbf{r}_{F^{(1)}/E} = [0 \quad 0 \quad 0]^T\ [m]$

$\nabla^{(1)} = 4.0 \times 10^4\ [m^3]$

$F_{G,z} = -3.6 \times 10^5\ [kN]$

$F_{G_{ext},z} = -4.0 \times 10^4\ [kN]$

$F_{B,z}^{(1)} = 4.0 \times 10^5\ [kN]$

$F_z^{(1)} = 0\ [kN]$

$M_T^{(1)} = -5.70 \times 10^4\ [kN]$

$M_L^{(1)} = -5.64 \times 10^6\ [kN]$

Waterplane:

$$I_L^{(1)} = I_{y_n y_n}$$

$$= \frac{1}{12} a^{(1)3} b^{(1)} \sin^3 \alpha^{(1)}$$

$$= \frac{1}{12} \cdot 100.1530^3 \cdot 41.0892 \cdot \sin^3 89.2760$$

$$= 3.4390 \times 10^6\ [m^4]$$

$$I_P^{(1)} = I_{x_n y_n} = 0\ [m^4]$$

${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 7.4656]^T\ [m]$

${}^{b'}\mathbf{r}_{P_2/O'} = [50 \quad 20 \quad -1.9326]^T\ [m]$

${}^{b'}\mathbf{r}_{P_3/O'} = [-50 \quad 20 \quad -7.4656]^T\ [m]$

${}^{b'}\mathbf{r}_{P_4/O'} = [-50 \quad -20 \quad 1.9326]^T\ [m]$

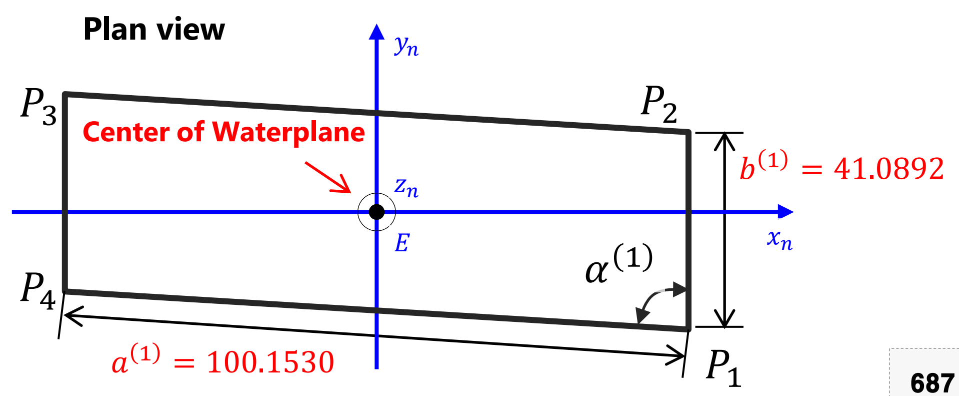
$\overrightarrow{P_1 P_2} = [0 \quad 40 \quad -9.3995]^T$

$|\overrightarrow{P_1 P_2}| = 41.0892 \ (= a^{(1)})$

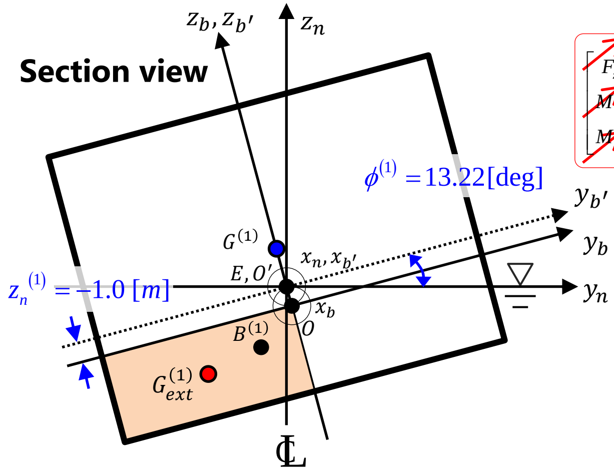
$\overrightarrow{P_1 P_4} = [-100 \quad 0 \quad -5.5319]^T$

$|\overrightarrow{P_1 P_4}| = 100.1530 \ (= b^{(1)})$

$\alpha^{(1)} = 89.2760\ [deg]$

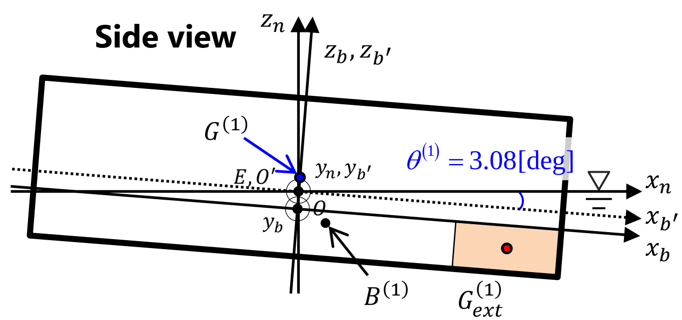


2. Calculation of the Values of the Waterplane at k=1 step



$$\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \\ \cancel{M_L^*} - M_L^{(k)} \end{bmatrix} = \boxed{\begin{bmatrix} -\rho g A_{WP}^{(k)} & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} & \rho g I_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} & \rho g I_P^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$



$L = 100\ [m]$
$B_{mld} = 40\ [m]$
$D = 30\ [m]$
$d = 9\ [m]$
$\rho g = 10\ [Mg/m^2 s^2]$

${}^n\mathbf{r}_{G^{(1)}/E} = [0.26 \quad -1.14 \quad 4.86]^T\ [m]$
${}^n\mathbf{r}_{G_{ext}^{(1)}/E} = [39.58 \quad -8.48 \quad -9.78]^T\ [m]$
${}^n\mathbf{r}_{B^{(1)}/E} = [4.33 \quad -2.02 \quad -5.34]^T\ [m]$
${}^n\mathbf{r}_{F^{(1)}/E} = [0 \quad 0 \quad 0]^T\ [m]$

$\nabla^{(1)} = 4.0 \times 10^4\ [m^3]$
$F_{G,z} = -3.6 \times 10^5\ [kN]$
$F_{G_{ext},z} = -4.0 \times 10^4\ [kN]$
$F_{B,z}^{(1)} = 4.0 \times 10^5\ [kN]$
$F_z^{(1)} = 0\ [kN]$
$M_T^{(1)} = -5.70 \times 10^4\ [kN]$
$M_L^{(1)} = -5.64 \times 10^6\ [kN]$

$$-\rho g A_{WP}^{(1)} = -10 \cdot (4.1149 \times 10^3)$$
$$= -4.1149 \times 10^4\ [kN/m]$$

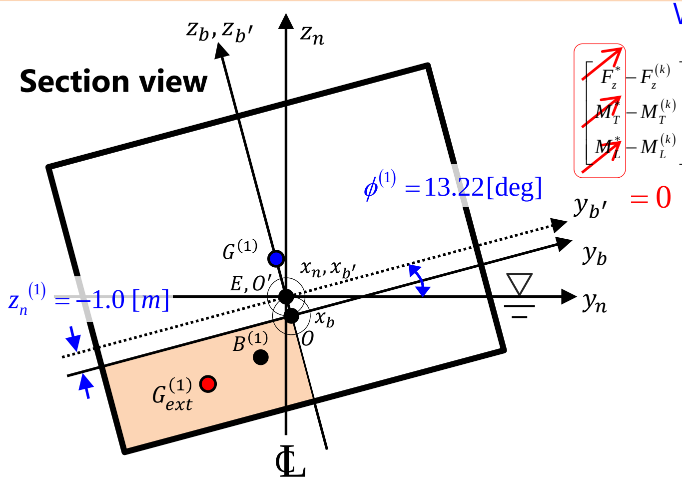
$a^{(1)} = 100.1530\ [m]$
$b^{(1)} = 41.0892\ [m]$
$\alpha^{(1)} = 89.2760\ [deg]$
$A_{WP}^{(1)} = 4.1149 \times 10^3\ [m^3]$
$I_T^{(1)} = 5.7950 \times 10^5\ [m^4]$
$I_L^{(1)} = 3.4390 \times 10^6\ [m^4]$
$I_P^{(1)} = 0\ [m^4]$

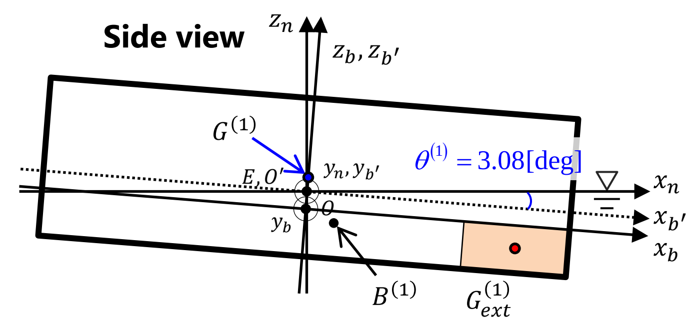
$$-\rho g A_{WP}^{(1)} \cdot {}^n y_{F^{(1)}/E} = -(4.1149 \times 10^4) \cdot 0 = 0\ [kN]$$
$$\rho g A_{WP}^{(1)} \cdot {}^n x_{F^{(1)}/E} = (4.1149 \times 10^4) \cdot 0 = 0\ [kN]$$

$$-\rho g \left({}^n z_{B^{(1)}/E} \nabla^{(1)} + I_T^{(1)} \right) - {}^n z_{G^{(1)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(1)}/E} \cdot F_{ext,z}$$
$$= -10 \cdot [-5.34 \cdot (4.1149 \times 10^4) + (5.7950 \times 10^5)]$$
$$-4.86 \cdot (-3.6 \times 10^5) - (-9.78) \cdot (-4.0 \times 10^4)$$
$$= -2.2995 \times 10^6\ [kN \cdot m]$$

$$-\rho g \left({}^n z_{B^{(1)}/E} \nabla^{(1)} + I_L^{(1)} \right) - {}^n z_{G^{(1)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(1)}/E} \cdot F_{ext,z}$$
$$= -10 \cdot [-5.34 \cdot (4.0 \times 10^4) + (3.4390 \times 10^6)]$$
$$-4.86 \cdot (-3.6 \times 10^5) - (-9.78) \cdot (-4.0 \times 10^4)$$
$$= -3.0895 \times 10^7\ [kN \cdot m]$$

3. Calculation of Immersion, Trim, and Heel at k=1 step





$L = 100\ [m]$

$B_{mld} = 40\ [m]$

$D = 30\ [m]$

$d = 9\ [m]$

$\rho g = 10\ [Mg/m^2 s^2]$

${}^n\mathbf{r}_{G^{(1)}/E} = [0.26 \quad -1.14 \quad 4.86]^T\ [m]$

${}^n\mathbf{r}_{G_{ext}^{(1)}/E} = [39.58 \quad -8.48 \quad -9.78]^T\ [m]$

${}^n\mathbf{r}_{B^{(1)}/E} = [4.33 \quad -2.02 \quad -5.34]^T\ [m]$

${}^n\mathbf{r}_{F^{(1)}/E} = [0 \quad 0 \quad 0]^T\ [m]$

$\nabla^{(1)} = 4.0 \times 10^4\ [m^3]$

$F_{G,z} = -3.6 \times 10^5\ [kN]$ $\qquad F_Z^{(1)} = 0\ [kN]$

$F_{G_{ext},z} = -4.0 \times 10^4\ [kN]$ $\qquad M_T^{(1)} = -5.70 \times 10^4\ [kN]$

$F_{B,z}^{(1)} = 4.0 \times 10^5\ [kN]$ $\qquad M_L^{(1)} = -5.64 \times 10^6\ [kN]$

We use the values in current floating position!

$$\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \\ \cancel{M_L^*} - M_L^{(k)} \end{bmatrix} = \boxed{\begin{bmatrix} -\rho g A_{WP}^{(k)} & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} & \rho g I_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} & \rho g I_P^{(k)} & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}}} \begin{bmatrix} \delta z^{(k)} \\ \delta\phi^{(k)} \\ \delta\theta^{(k)} \end{bmatrix}$$

$= 0$

$$\begin{bmatrix} 0 \\ -5.84\times10^5 \\ -5.52\times10^6 \end{bmatrix} = \begin{bmatrix} -4.12\times10^4 & 0 & 0 \\ 0 & -2.30\times10^6 & 0 \\ 0 & 0 & -3.09\times10^7 \end{bmatrix} \begin{bmatrix} \delta z_n^{(1)} \\ \delta\phi^{(1)} \\ \delta\theta^{(1)} \end{bmatrix}$$

$$\boxed{\begin{bmatrix} \delta z_n^{(1)} \\ \delta\phi^{(1)} \\ \delta\theta^{(1)} \end{bmatrix}} = \begin{bmatrix} -4.12\times10^4 & 0 & 0 \\ 0 & -2.30\times10^6 & 0 \\ 0 & 0 & -3.09\times10^7 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ -5.84\times10^5 \\ -5.52\times10^6 \end{bmatrix}$$

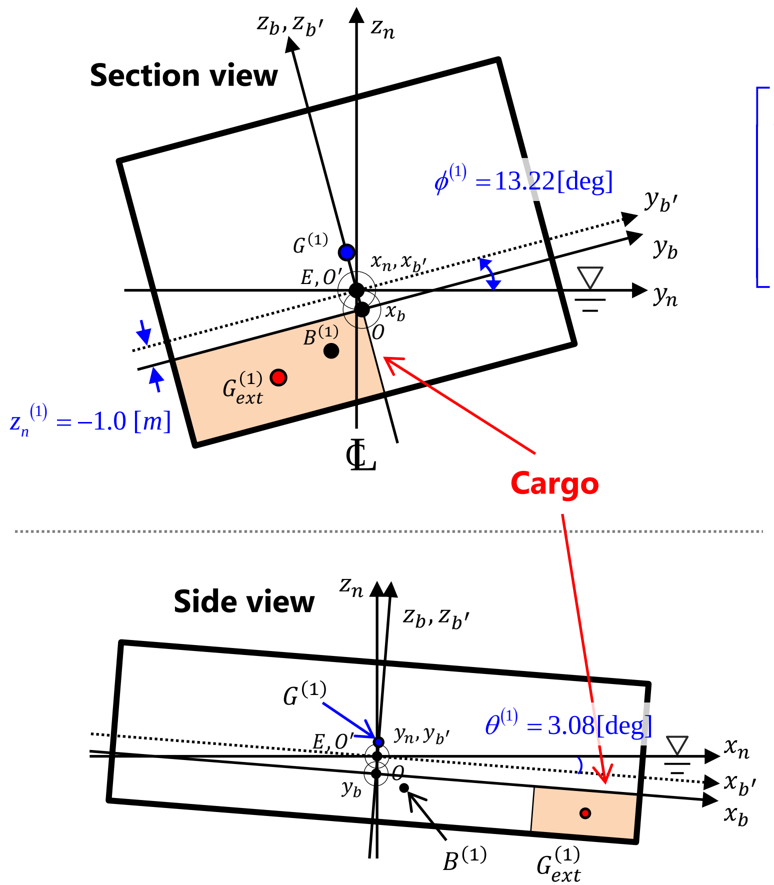
$$= \begin{bmatrix} 0\ [m] \\ -0.0245[rad] \\ -0.0015[rad] \end{bmatrix} = \boxed{\begin{bmatrix} 0\ [m] \\ -1.4036[\text{deg}] \\ -0.0849[\text{deg}] \end{bmatrix}}$$

$$z_n^{(2)} = z_n^{(1)} + \delta z_n^{(1)} = -1.0 + 0 = -1.0\ [m]$$

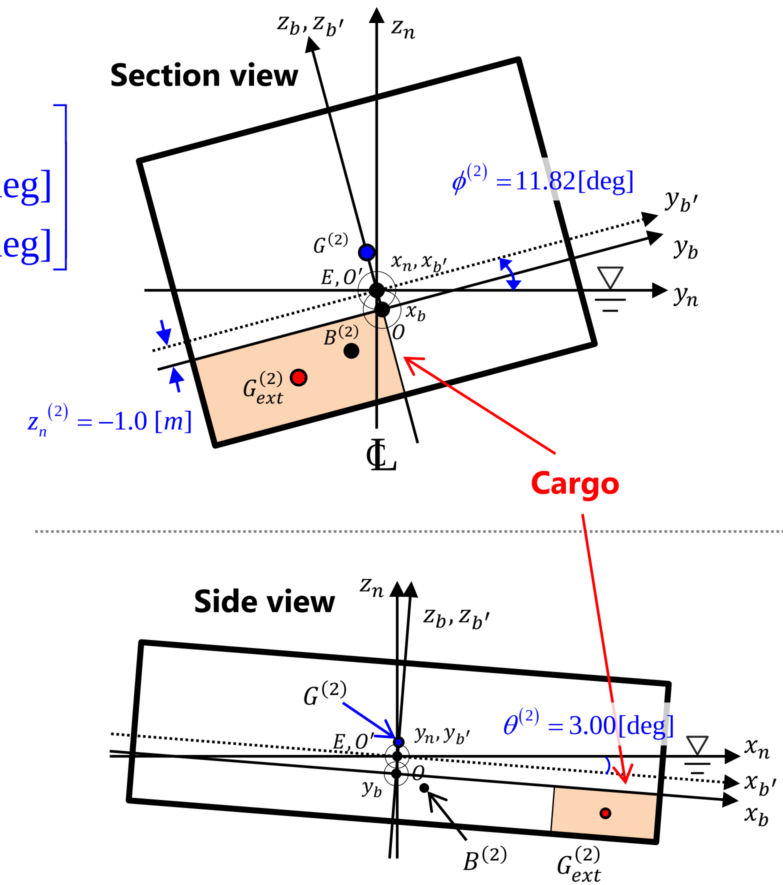
$$\phi^{(2)} = \phi^{(1)} + \delta\phi^{(1)} = 13.22 + (-1.40) = 11.82\ [\text{deg}]$$

$$\theta^{(2)} = \theta^{(1)} + \delta\theta^{(1)} = 3.08 + (-0.08) = 3.00\ [\text{deg}]$$

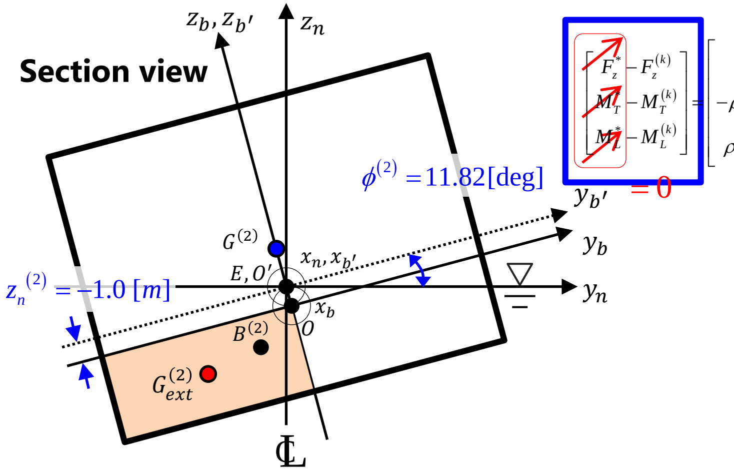
3. Calculation of Immersion, Trim, and Heel at k=1 step

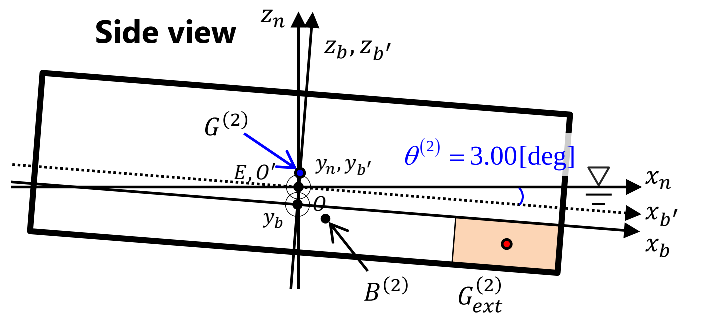


$$\begin{bmatrix} \delta z_n^{(1)} \\ \delta\phi^{(1)} \\ \delta\theta^{(1)} \end{bmatrix} = \begin{bmatrix} 0\ [m] \\ -1.4036[\text{deg}] \\ -0.0849[\text{deg}] \end{bmatrix}$$



4. Check for the Ship to be in Static Equilibrium at k=1 step





$$\underbrace{\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \\ \cancel{M_L^*} - M_L^{(k)} \end{bmatrix}}_{=0} = \begin{bmatrix} -\rho g A_{WP}{}^{(k)} & -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} & \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T{}^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} & \rho g I_P{}^{(k)} \\ \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} & \rho g I_P{}^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L{}^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

Is the ship in static equilibrium?

Let us check for the ship to be in static equilibrium!

$L = 100\ [m]$

$B_{mld} = 40\ [m]$

$D = 30\ [m]$

$d = 9\ [m]$

$\rho g = 10\ [Mg/m^2s^2]$

${}^n\mathbf{r}_{G^{(1)}/E} = [0.26 \quad -1.14 \quad 4.86]^T\ [m]$

${}^n\mathbf{r}_{G_{ext}^{(1)}/E} = [39.58 \quad -8.48 \quad -9.78]^T\ [m]$

${}^n\mathbf{r}_{B^{(1)}/E} = [4.33 \quad -2.02 \quad -5.34]^T\ [m]$

${}^n\mathbf{r}_{F^{(1)}/E} = [0 \quad 0 \quad 0]^T\ [m]$

$\nabla^{(1)} = 4.0 \times 10^4\ [m^3]$

$F_{G,z} = -3.6 \times 10^5\ [kN]$

$F_{G_{ext},z} = -4.0 \times 10^4\ [kN]$

$F_{B,z}^{(1)} = 4.0 \times 10^5\ [kN]$

$F_z^{(1)} = 0\ [kN]$

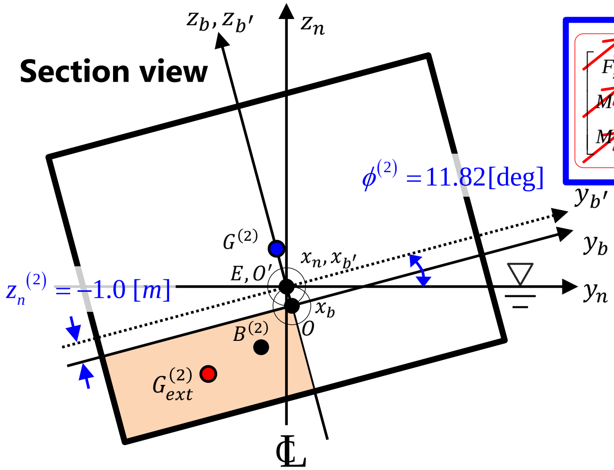
$M_T^{(1)} = -5.70 \times 10^4\ [kN]$

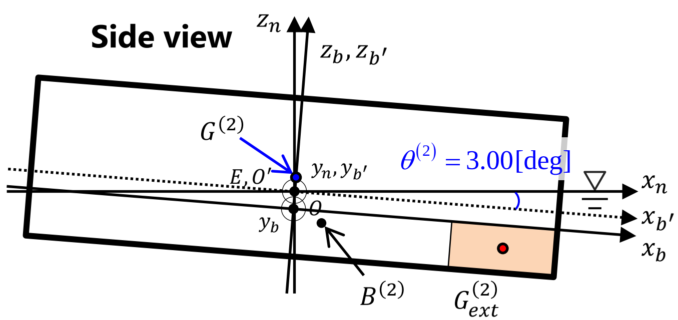
$M_L^{(1)} = -5.64 \times 10^6\ [kN]$

4. Check for the Ship to be in Static Equilibrium at k=1 step

$$\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \\ \cancel{M_L^*} - M_L^{(k)} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP}^{(k)} & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} & \rho g I_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} & \rho g I_P^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$





$L = 100$ [m]	${}^n\mathbf{r}_{G^{(1)}/E} = [0.26 \quad -1.14 \quad 4.86]^T$ [m]
$B_{mld} = 40$ [m]	${}^n\mathbf{r}_{G_{ext}^{(1)}/E} = [39.58 \quad -8.48 \quad -9.78]^T$ [m]
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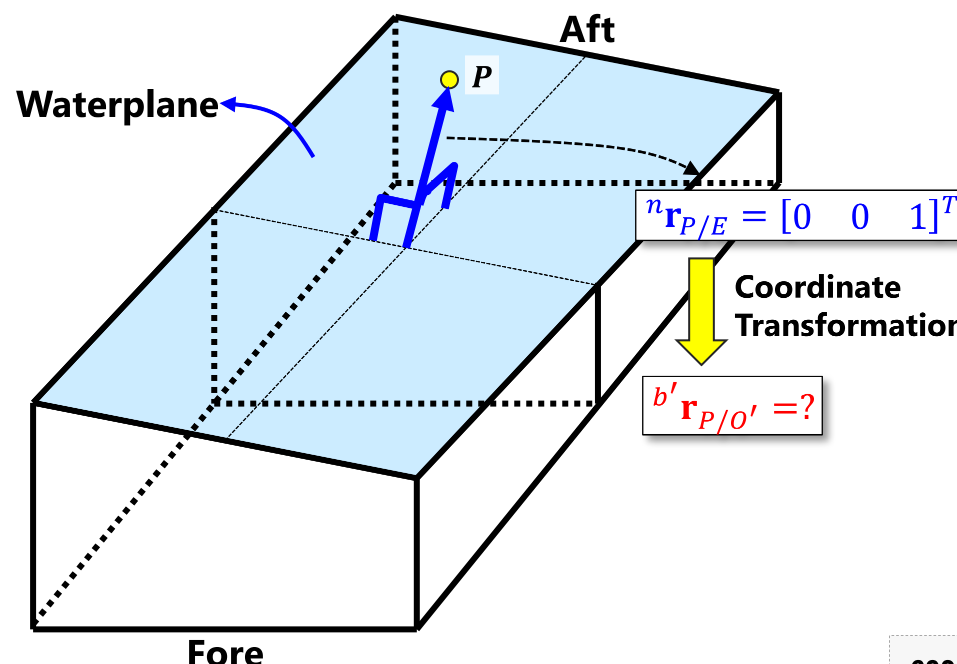
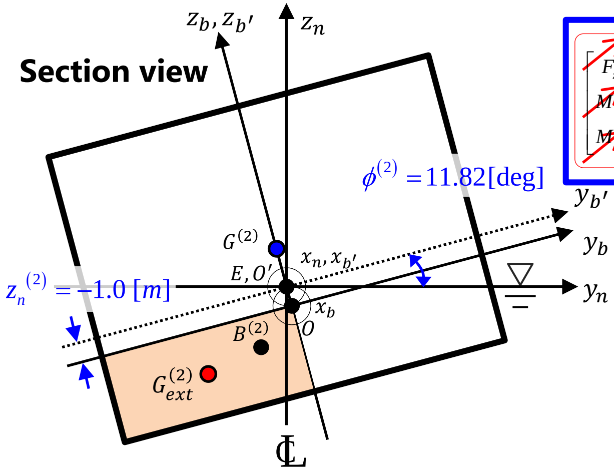
Normal vector of waterplane
decomposed in the b'-frame: $^{b'}\mathbf{r}_{P/O'}$

$$\underbrace{{}^n\mathbf{r}_{P/E}}_{\text{Given}} = \begin{bmatrix} \cos\theta^{(1)} & 0 & \sin\theta^{(1)} \\ 0 & 1 & 0 \\ -\sin\theta^{(1)} & 0 & \cos\theta^{(1)} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi^{(1)} & -\sin\phi^{(1)} \\ 0 & \sin\phi^{(1)} & \cos\phi^{(1)} \end{bmatrix} \underbrace{{}^{b'}\mathbf{r}_{P/O'}}_{\text{Find}}$$

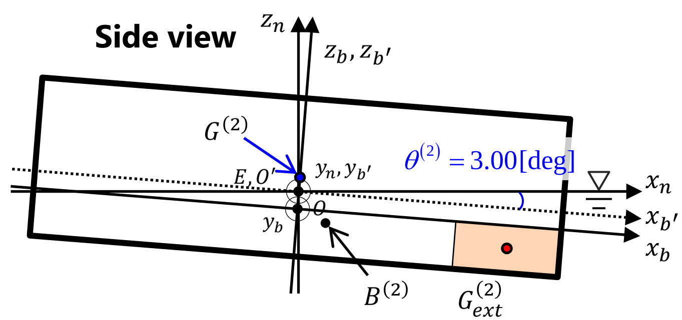


4. Check for the Ship to be in Static Equilibrium at k=1 step



$$\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \\ \cancel{M_L^*} - M_L^{(k)} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP}^{(k)} & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} & \rho g I_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} & \rho g I_P^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$



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$F_{B,z}^{(1)} = 4.0 \times 10^5\ [kN]$ $\qquad M_L^{(1)} = -5.64 \times 10^6\ [kN]$

Normal vector of waterplane decomposed in the b′-frame: ${}^{b'}\mathbf{r}_{P/O'}$

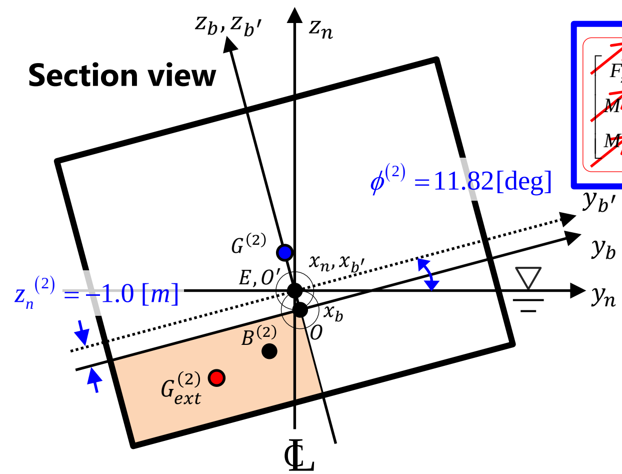
$${}^n\mathbf{r}_{P/E} = \begin{bmatrix} \cos\theta^{(1)} & 0 & \sin\theta^{(1)} \\ 0 & 1 & 0 \\ -\sin\theta^{(1)} & 0 & \cos\theta^{(1)} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi^{(1)} & -\sin\phi^{(1)} \\ 0 & \sin\phi^{(1)} & \cos\phi^{(1)} \end{bmatrix} {}^{b'}\mathbf{r}_{P/O'}$$

$${}^{b'}\mathbf{r}_{P/O'} = \left(\begin{bmatrix} \cos\theta^{(1)} & 0 & \sin\theta^{(1)} \\ 0 & 1 & 0 \\ -\sin\theta^{(1)} & 0 & \cos\theta^{(1)} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi^{(1)} & -\sin\phi^{(1)} \\ 0 & \sin\phi^{(1)} & \cos\phi^{(1)} \end{bmatrix} \right)^{-1} {}^n\mathbf{r}_{P/E}$$

$$= \left(\begin{bmatrix} \cos 3.00 & 0 & \sin 3.00 \\ 0 & 1 & 0 \\ -\sin 3.00 & 0 & \cos 3.00 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 11.82 & -\sin 11.82 \\ 0 & \sin 11.82 & \cos 11.82 \end{bmatrix} \right)^{-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -0.0523 \\ 0.2045 \\ 0.9775 \end{bmatrix}$$

4. Check for the Ship to be in Static Equilibrium at k=1 step



$$\boxed{\begin{bmatrix} F_z^* - F_z^{(k)} \\ M_T^* - M_T^{(k)} \\ M_L^* - M_L^{(k)} \end{bmatrix}} = \begin{bmatrix} -\rho g A_{WP}^{(k)} & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z} & \rho g I_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} & \rho g I_P^{(k)} & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$

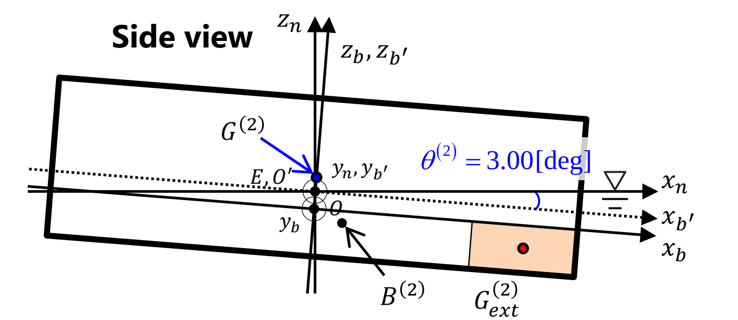
Normal vector of waterplane decomposed in the b'-frame:

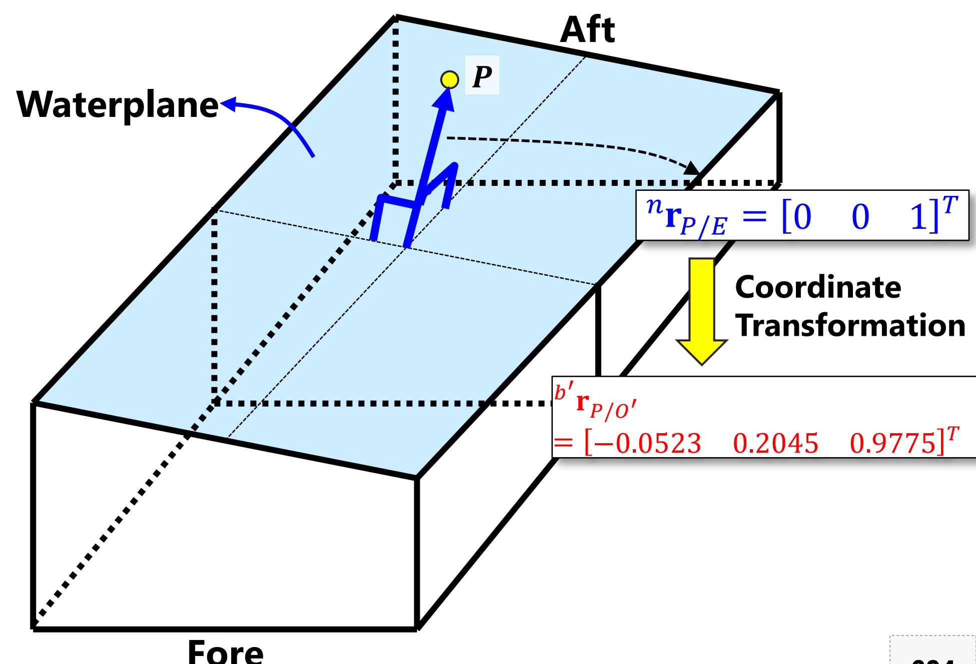
$${}^{b'}\mathbf{r}_{P/O'} = [-0.0523 \quad 0.2045 \quad 0.9775]^T$$

Equation of waterplane: ${}^{b'}\mathbf{r}_{P/O'} \cdot \left(\mathbf{r} - {}^n\mathbf{r}_{O'/E}\right) = 0$

$$-0.0523x + 0.2045y + 0.9775z = 0$$

$$z = (0.0523x - 0.2045y)/0.9775$$





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4. Check for the Ship to be in Static Equilibrium at k=1 step

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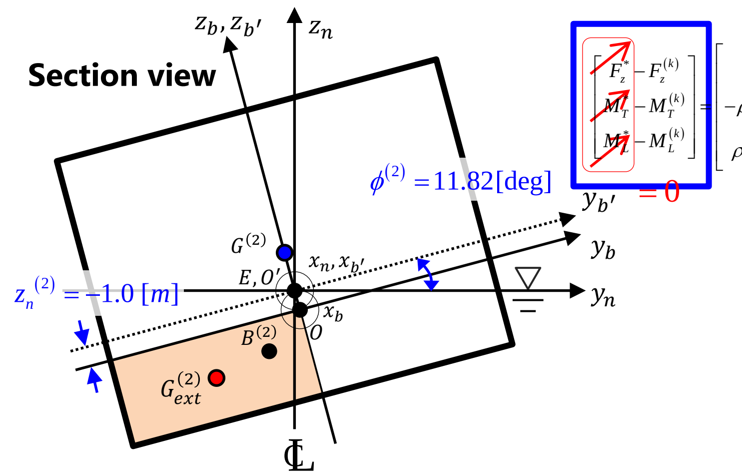
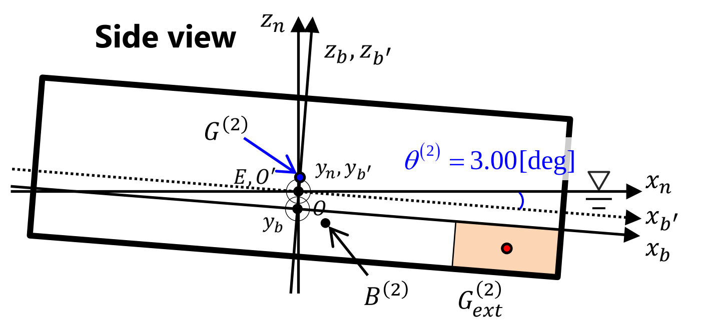
$= 0$





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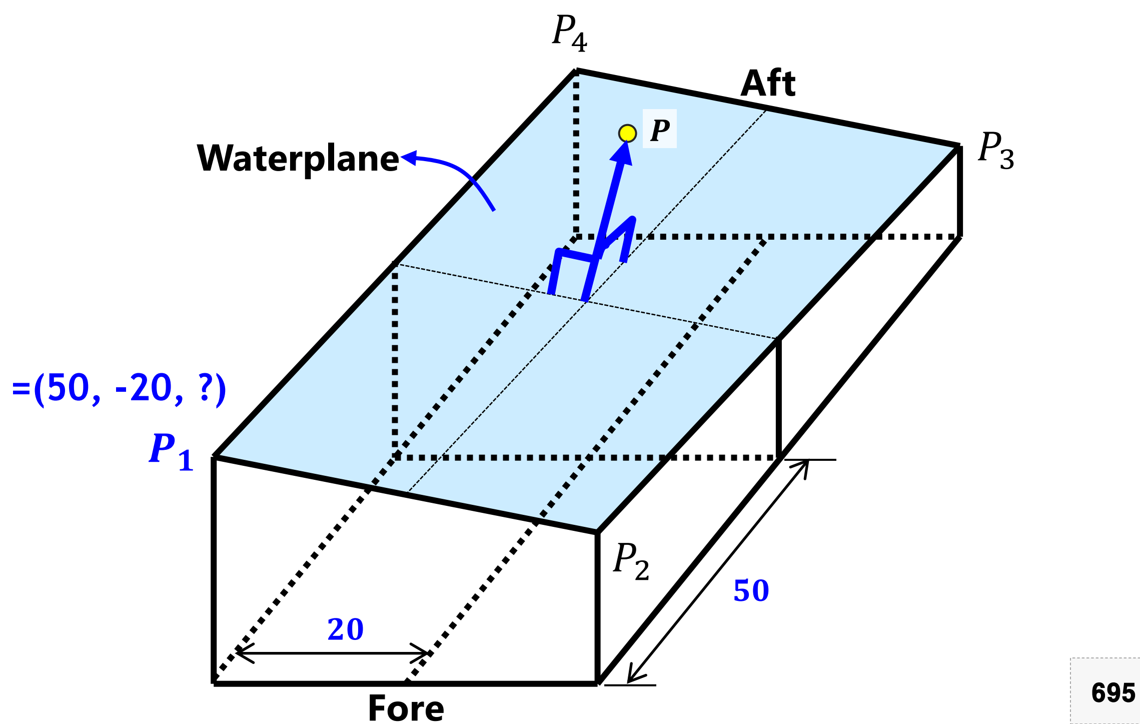
Equation of waterplane: $z = (0.0523x - 0.2045y)/0.9775$

$${}^{b'}\mathbf{r}_{P_1/O'}: {}^{b'}x_{P_1/O'} = \mathbf{50},\ {}^{b'}y_{P_1/O'} = \mathbf{-20}$$

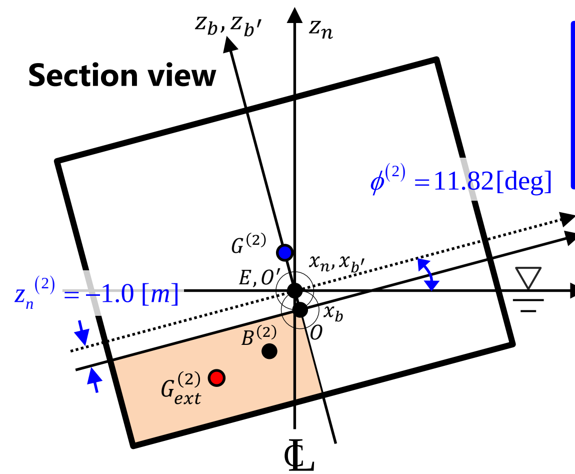
$${}^{b'}z_{P_1/O'} = \left(0.0523 \cdot {}^{b'}x_{P_1/O'} - 0.2045 \cdot {}^{b'}y_{P_1/O'}\right)/0.9775$$

$$= (0.0523 \cdot 50 - 0.2045 \cdot (-20))/0.9775$$

$$= \mathbf{6.8593}$$



4. Check for the Ship to be in Static Equilibrium at k=1 step



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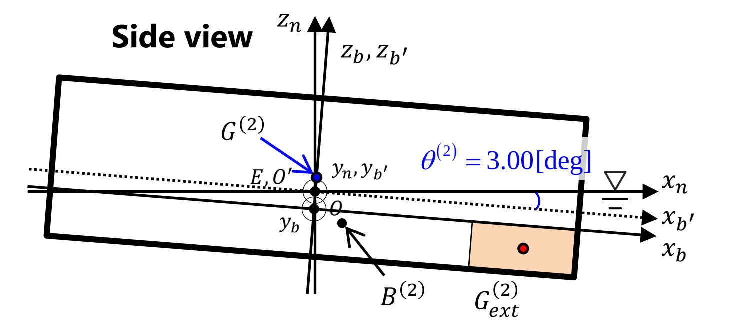
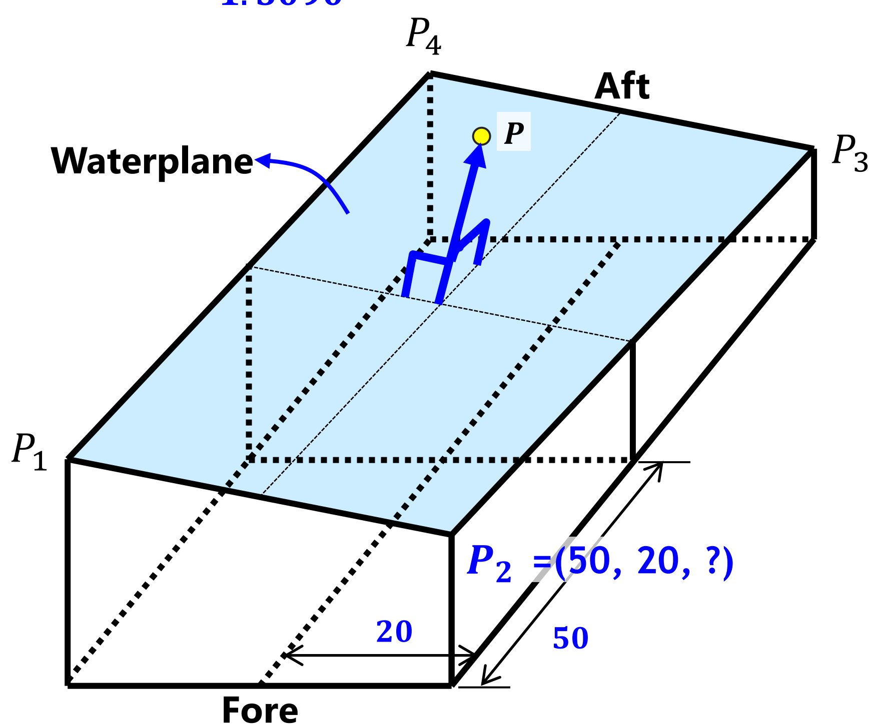
$= 0$

Equation of waterplane: $z = (0.0523x - 0.2045y)/0.9775$

$${}^{b'}\mathbf{r}_{P_2/O'}: {}^{b'}x_{P_2/O'} = \mathbf{50}, {}^{b'}y_{P_2/O'} = \mathbf{20}$$

$$\begin{aligned} {}^{b'}z_{P_2/O'} &= \left(0.0523 \cdot {}^{b'}x_{P_2/O'} - 0.2045 \cdot {}^{b'}y_{P_2/O'}\right)/0.9775 \\ &= (0.0523 \cdot 50 - 0.2045 \cdot 20)/0.9775 \\ &= \mathbf{-1.5090} \end{aligned}$$





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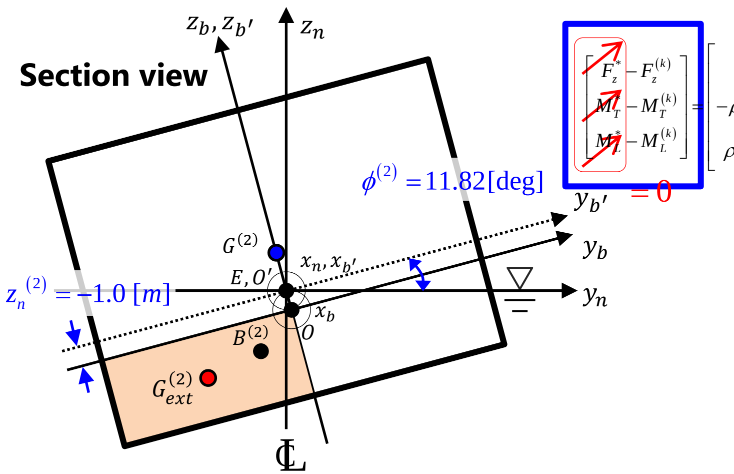
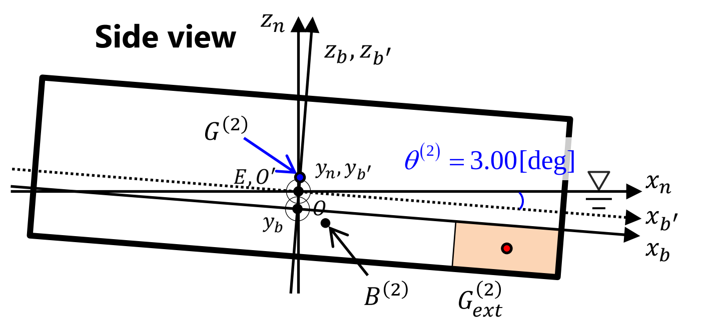
$$\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \\ \cancel{M_L^*} - M_L^{(k)} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP}{}^{(k)} & -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} & \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T{}^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} & \rho g I_P{}^{(k)} \\ \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} & \rho g I_P{}^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L{}^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$





$L = 100\ [m]$	${}^n\mathbf{r}_{G^{(1)}/E} = [0.26 \quad -1.14 \quad 4.86]^T\ [m]$
$B_{mld} = 40\ [m]$	${}^n\mathbf{r}_{G_{ext}^{(1)}/E} = [39.58 \quad -8.48 \quad -9.78]^T\ [m]$
$D = 30\ [m]$	${}^n\mathbf{r}_{B^{(1)}/E} = [4.33 \quad -2.02 \quad -5.34]^T\ [m]$
$d = 9\ [m]$	${}^n\mathbf{r}_{F^{(1)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2 s^2]$	

$\nabla^{(1)} = 4.0 \times 10^4\ [m^3]$

$F_{G,z} = -3.6 \times 10^5\ [kN]$ $\qquad F_z^{(1)} = 0\ [kN]$

$F_{G_{ext},z} = -4.0 \times 10^4\ [kN]$ $\qquad M_T^{(1)} = -5.70 \times 10^4\ [kN]$

$F_{B,z}^{(1)} = 4.0 \times 10^5\ [kN]$ $\qquad M_L^{(1)} = -5.64 \times 10^6\ [kN]$

Force equilibrium:

$$F_z^{(2)} = F_{B,z}^{(2)} + F_{G,z}^{(2)} + F_{ext,z}^{(2)}$$

$$F_{B,z}^{(2)} = \rho g \nabla^{(2)}$$

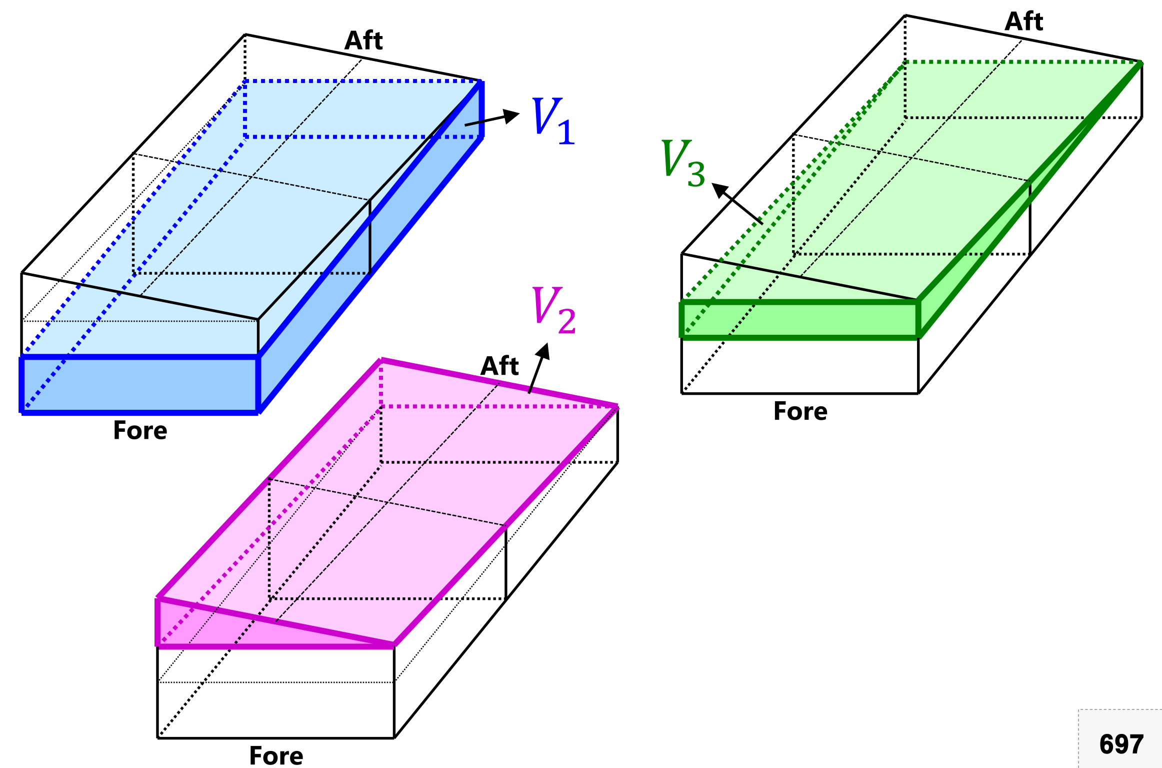
$$\nabla^{(2)} = V_1 + V_2 + V_3$$

$${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 6.8593]^T\ [m]$$

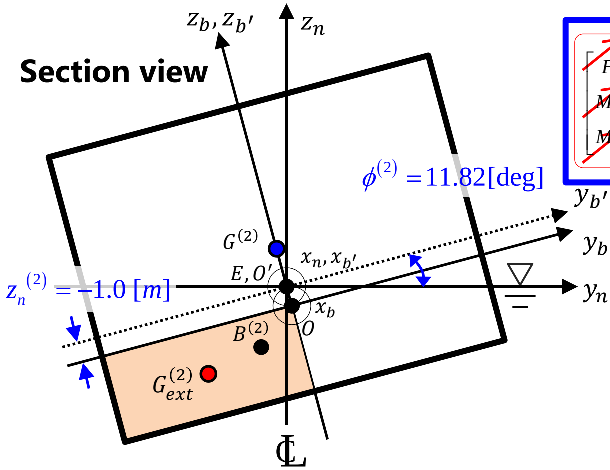
$${}^{b'}\mathbf{r}_{P_2/O'} = [50 \quad 20 \quad -1.5090]^T\ [m]$$

$${}^{b'}\mathbf{r}_{P_3/O'} = [-50 \quad 20 \quad -6.8593]^T\ [m]$$

$${}^{b'}\mathbf{r}_{P_4/O'} = [-50 \quad -20 \quad 1.5090]^T\ [m]$$

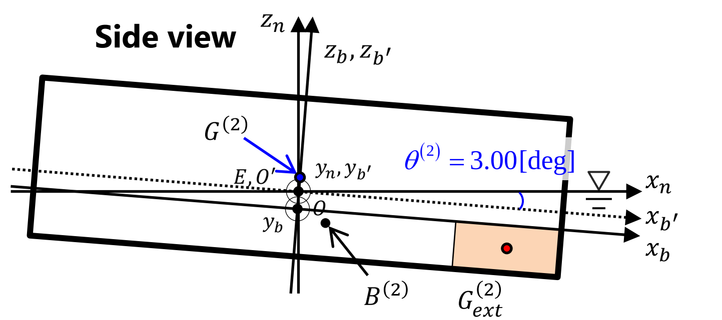


4. Check for the Ship to be in Static Equilibrium at k=1 step



$$\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \\ \cancel{M_L^*} - M_L^{(k)} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP}^{(k)} & -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} & \rho g A_{WP}^{(k)} \cdot {}^{n}x_{F^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} & -\rho g \left({}^{n}z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)} \right) - {}^{n}z_{G^{(k)}/E} \cdot F_{G,z} - {}^{n}z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} & \rho g I_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^{n}x_{F^{(k)}/E} & \rho g I_P^{(k)} & -\rho g \left({}^{n}z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)} \right) - {}^{n}z_{G^{(k)}/E} \cdot F_{G,z} - {}^{n}z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$



$L = 100\ [m]$

$B_{mld} = 40\ [m]$

$D = 30\ [m]$

$d = 9\ [m]$

$\rho g = 10\ [Mg/m^2 s^2]$

${}^{n}\mathbf{r}_{G^{(1)}/E} = [0.26 \quad -1.14 \quad 4.86]^T\ [m]$

${}^{n}\mathbf{r}_{G_{ext}^{(1)}/E} = [39.58 \quad -8.48 \quad -9.78]^T\ [m]$

${}^{n}\mathbf{r}_{B^{(1)}/E} = [4.33 \quad -2.02 \quad -5.34]^T\ [m]$

${}^{n}\mathbf{r}_{F^{(1)}/E} = [0 \quad 0 \quad 0]^T\ [m]$

$\nabla^{(1)} = 4.0 \times 10^4\ [m^3]$

$F_{G,z} = -3.6 \times 10^5\ [kN]$

$F_{G_{ext},z} = -4.0 \times 10^4\ [kN]$

$F_{B,z}^{(1)} = 4.0 \times 10^5\ [kN]$

$F_z^{(1)} = 0\ [kN]$

$M_T^{(1)} = -5.70 \times 10^4\ [kN]$

$M_L^{(1)} = -5.64 \times 10^6\ [kN]$

Force equilibrium:

$${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 6.8593]^T\ [m]$$

$${}^{b'}\mathbf{r}_{P_2/O'} = [50 \quad 20 \quad -1.5090]^T\ [m]$$

$${}^{b'}\mathbf{r}_{P_3/O'} = [-50 \quad 20 \quad -6.8593]^T\ [m]$$

$${}^{b'}\mathbf{r}_{P_4/O'} = [-50 \quad -20 \quad 1.5090]^T\ [m]$$

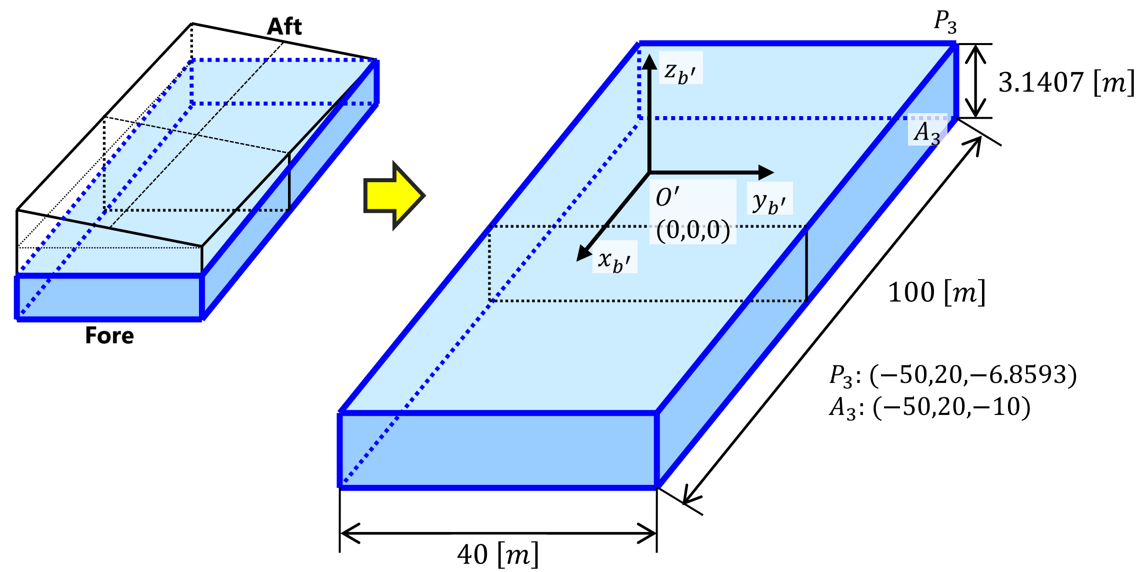
$$F_z^{(2)} = F_{B,z}^{(2)} + F_{G,z}^{(2)} + F_{ext,z}^{(2)}$$

$$F_{B,z}^{(2)} = \rho g \nabla^{(2)}$$

$$\nabla^{(2)} = V_1 + V_2 + V_3$$

$$V_1 = L \cdot B_{mld} \cdot \overline{P_3 A_3} = 40 \cdot 100 \cdot \left(-6.8593 - (-10) \right)$$

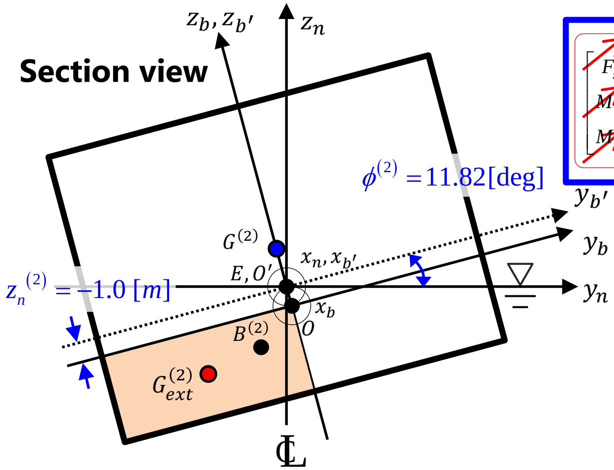
$$= 1.2563 \times 10^4\ [m^3]$$

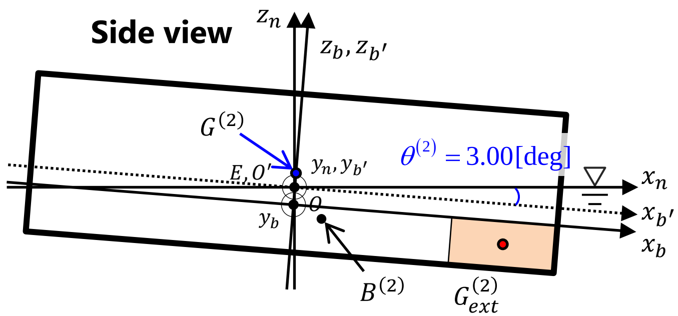


4. Check for the Ship to be in Static Equilibrium at k=1 step

$$\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \\ \cancel{M_L^*} - M_L^{(k)} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP}{}^{(k)} & -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} & \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T{}^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} & \rho g I_P{}^{(k)} \\ \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} & \rho g I_P{}^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L{}^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$





$L = 100\ [m]$	${}^n\mathbf{r}_{G^{(1)}/E} = [0.26 \quad -1.14 \quad 4.86]^T\ [m]$
$B_{mld} = 40\ [m]$	${}^n\mathbf{r}_{G_{ext}^{(1)}/E} = [39.58 \quad -8.48 \quad -9.78]^T\ [m]$
$D = 30\ [m]$	${}^n\mathbf{r}_{B^{(1)}/E} = [4.33 \quad -2.02 \quad -5.34]^T\ [m]$
$d = 9\ [m]$	${}^n\mathbf{r}_{F^{(1)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2 s^2]$	

$\nabla^{(1)} = 4.0 \times 10^4\ [m^3]$

$F_{G,z} = -3.6 \times 10^5\ [kN]$	$F_z^{(1)} = 0\ [kN]$
$F_{G_{ext},z} = -4.0 \times 10^4\ [kN]$	$M_T^{(1)} = -5.70 \times 10^4\ [kN]$
$F_{B,z}^{(1)} = 4.0 \times 10^5\ [kN]$	$M_L^{(1)} = -5.64 \times 10^6\ [kN]$

Force equilibrium:

$$F_z^{(2)} = F_{B,z}^{(2)} + F_{G,z}^{(2)} + F_{ext,z}^{(2)}$$

$$F_{B,z}^{(2)} = \rho g \nabla^{(2)}$$

$$\nabla^{(2)} = V_1 + V_2 + V_3$$

$$V_2 = (0.5 \cdot B_{mld} \cdot \overline{P_1 A_1}) \cdot L$$

$$= 0.5 \cdot 40 \cdot \{6.8593 - (-1.5090)\} \cdot 100$$

$$= 1.6737 \times 10^4\ [m^3]$$

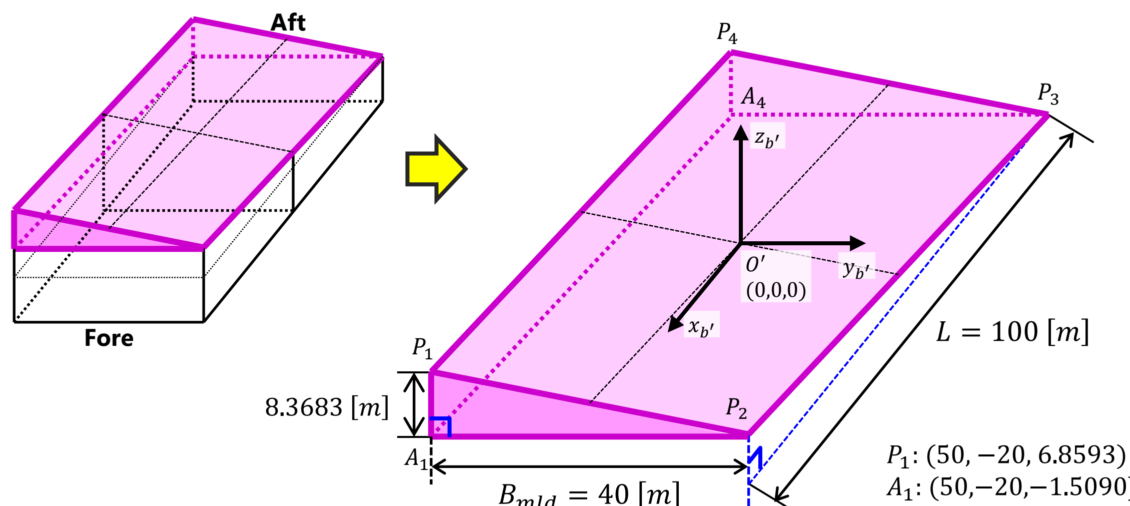
${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 6.8593]^T\ [m]$

${}^{b'}\mathbf{r}_{P_2/O'} = [50 \quad 20 \quad -1.5090]^T\ [m]$

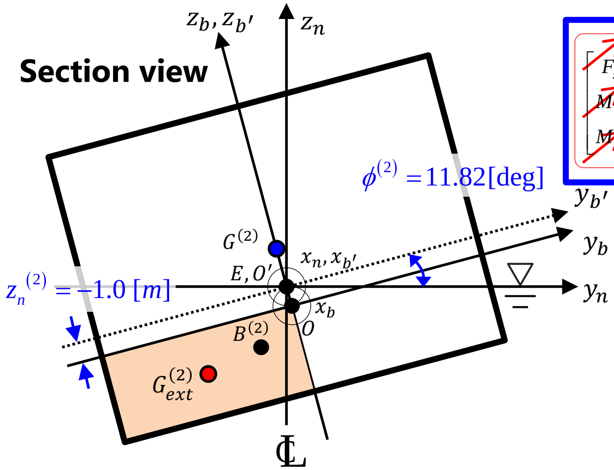
${}^{b'}\mathbf{r}_{P_3/O'} = [-50 \quad 20 \quad -6.8593]^T\ [m]$

${}^{b'}\mathbf{r}_{P_4/O'} = [-50 \quad -20 \quad 1.5090]^T\ [m]$

$V_1 = 1.2563 \times 10^4\ [m^3]$

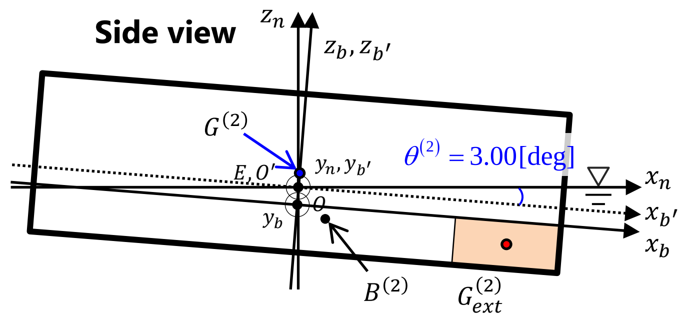


4. Check for the Ship to be in Static Equilibrium at k=1 step



$$\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \\ \cancel{M_L^*} - M_L^{(k)} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP}^{(k)} & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} & \rho g I_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} & \rho g I_P^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$



$L = 100\ [m]$

$B_{mld} = 40\ [m]$

$D = 30\ [m]$

$d = 9\ [m]$

$\rho g = 10\ [Mg/m^2 s^2]$

${}^n\mathbf{r}_{G^{(1)}/E} = [0.26 \quad -1.14 \quad 4.86]^T\ [m]$

${}^n\mathbf{r}_{G_{ext}^{(1)}/E} = [39.58 \quad -8.48 \quad -9.78]^T\ [m]$

${}^n\mathbf{r}_{B^{(1)}/E} = [4.33 \quad -2.02 \quad -5.34]^T\ [m]$

${}^n\mathbf{r}_{F^{(1)}/E} = [0 \quad 0 \quad 0]^T\ [m]$

$\nabla^{(1)} = 4.0 \times 10^4\ [m^3]$

$F_{G,z} = -3.6 \times 10^5\ [kN]$

$F_{G_{ext},z} = -4.0 \times 10^4\ [kN]$

$F_{B,z}^{(1)} = 4.0 \times 10^5\ [kN]$

$F_z^{(1)} = 0\ [kN]$

$M_T^{(1)} = -5.70 \times 10^4\ [kN]$

$M_L^{(1)} = -5.64 \times 10^6\ [kN]$

Force equilibrium:

$$F_z^{(2)} = F_{B,z}^{(2)} + F_{G,z}^{(2)} + F_{ext,z}^{(2)}$$

$$F_{B,z}^{(2)} = \rho g \nabla^{(2)}$$

$$\nabla^{(2)} = V_1 + V_2 + V_3$$

$$V_3 = (0.5 \cdot L \cdot \overline{P_2 A_2}) \cdot B_{mld}$$

$$= 0.5 \cdot 100 \cdot \{-1.5090 - (-6.8593)\} \cdot 40$$

$$= 1.0701 \times 10^4\ [m^3]$$

${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 6.8593]^T\ [m]$

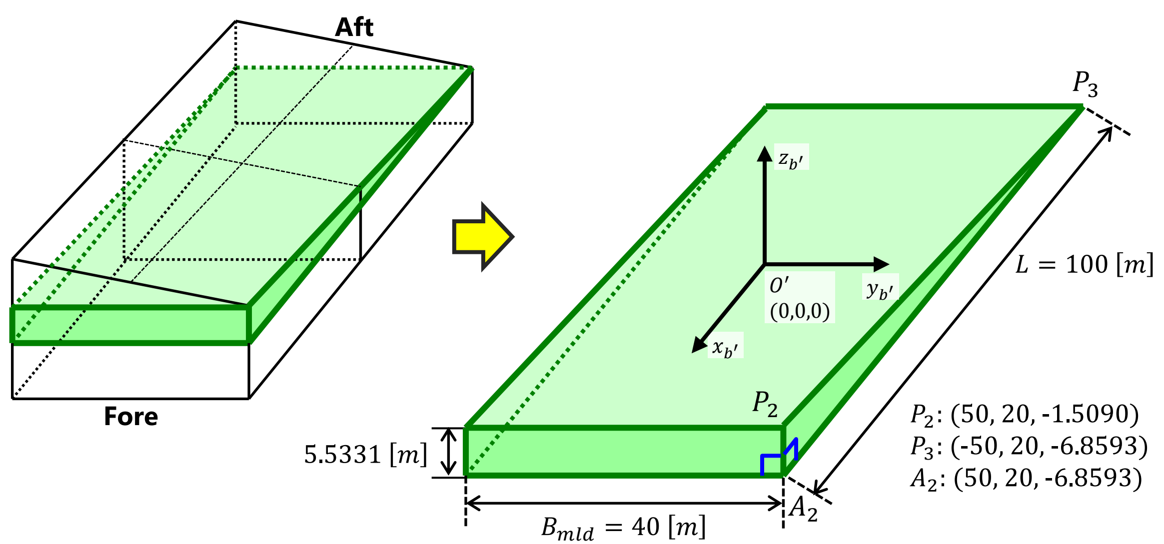
${}^{b'}\mathbf{r}_{P_2/O'} = [50 \quad 20 \quad -1.5090]^T\ [m]$

${}^{b'}\mathbf{r}_{P_3/O'} = [-50 \quad 20 \quad -6.8593]^T\ [m]$

${}^{b'}\mathbf{r}_{P_4/O'} = [-50 \quad -20 \quad 1.5090]^T\ [m]$

$V_1 = 1.2563 \times 10^4\ [m^3]$

$V_2 = 1.6737 \times 10^4\ [m^3]$

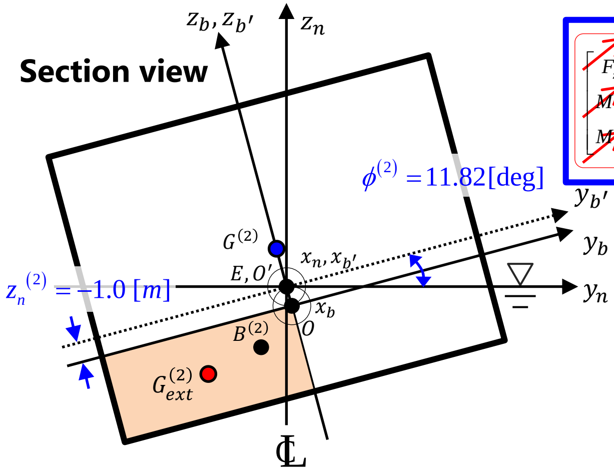


4. Check for the Ship to be in Static Equilibrium at k=1 step

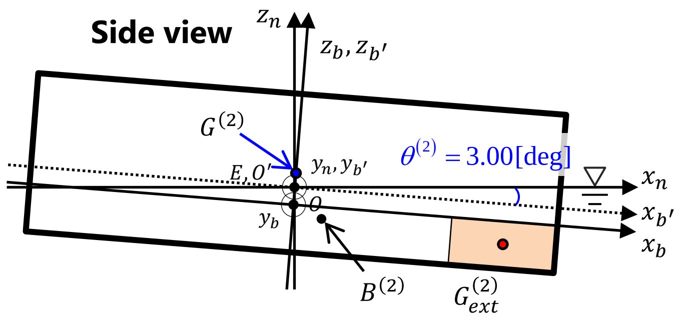
$$\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \\ \cancel{M_L^*} - M_L^{(k)} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP}^{(k)} & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} & \rho g I_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} & \rho g I_P^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$

Section view



Side view



$L = 100\ [m]$

$B_{mld} = 40\ [m]$

$D = 30\ [m]$

$d = 9\ [m]$

$\rho g = 10\ [Mg/m^2 s^2]$

${}^n\mathbf{r}_{G^{(1)}/E} = [0.26 \quad -1.14 \quad 4.86]^T\ [m]$

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${}^n\mathbf{r}_{B^{(1)}/E} = [4.33 \quad -2.02 \quad -5.34]^T\ [m]$

${}^n\mathbf{r}_{F^{(1)}/E} = [0 \quad 0 \quad 0]^T\ [m]$

$\nabla^{(1)} = 4.0 \times 10^4\ [m^3]$

$F_{G,z} = -3.6 \times 10^5\ [kN]$ $\qquad F_z^{(1)} = 0\ [kN]$

$F_{G_{ext},z} = -4.0 \times 10^4\ [kN]$ $\qquad M_T^{(1)} = -5.70 \times 10^4\ [kN]$

$F_{B,z}^{(1)} = 4.0 \times 10^5\ [kN]$ $\qquad M_L^{(1)} = -5.64 \times 10^6\ [kN]$

Force equilibrium:

${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 6.8593]^T\ [m]$

${}^{b'}\mathbf{r}_{P_2/O'} = [50 \quad 20 \quad -1.5090]^T\ [m]$

${}^{b'}\mathbf{r}_{P_3/O'} = [-50 \quad 20 \quad -6.8593]^T\ [m]$

${}^{b'}\mathbf{r}_{P_4/O'} = [-50 \quad -20 \quad 1.5090]^T\ [m]$

$$F_z^{(2)} = F_{B,z}^{(2)} + F_{G,z}^{(2)} + F_{ext,z}^{(2)}$$

$$F_{B,z}^{(2)} = \rho g \nabla^{(2)}$$

$$\nabla^{(2)} = V_1 + V_2 + V_3$$

$$= (1.2563 \times 10^4) + (1.6737 \times 10^4) + (1.0701 \times 10^4)$$

$$= 4.0 \times 10^4\ [m^3]$$

$V_1 = 1.2563 \times 10^4\ [m^3]$

$V_2 = 1.6737 \times 10^4\ [m^3]$

$V_3 = 1.0701 \times 10^4\ [m^3]$

$$F_{B,z}^{(2)} = \rho g \nabla^{(2)} = 10 \cdot (4.0 \times 10^4) = 4.0 \times 10^5\ [kN]$$

$$F_z^{(2)} = F_{B,z}^{(2)} + F_{G,z}^{(2)} + F_{ext,z}^{(2)}$$

$$= (4.0 \times 10^5) + (-3.6 \times 10^5) + (-4.0 \times 10^4)$$

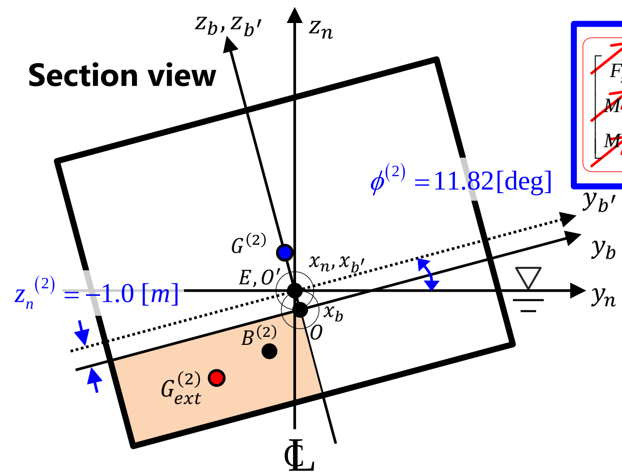
$$= \boxed{0\ [kN]} < e$$

Tolerance

where, e(epsilon) : an arbitrarily small positive quantity

The static equilibrium of force is satisfied!

4. Check for the Ship to be in Static Equilibrium at k=1 step



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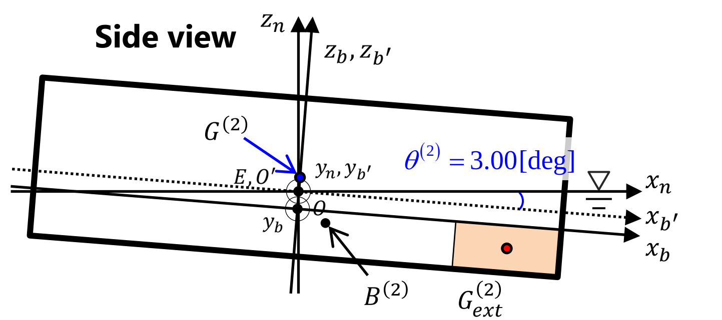
$= 0$



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${}^n\mathbf{r}_{B^{(1)}/E} = [4.33 \quad -2.02 \quad -5.34]^T\ [m]$

${}^n\mathbf{r}_{F^{(1)}/E} = [0 \quad 0 \quad 0]^T\ [m]$

$\nabla^{(2)} = 4.0 \times 10^4\ [m^3]$

$F_{G,z} = -3.6 \times 10^5\ [kN]$

$F_{G_{ext},z} = -4.0 \times 10^4\ [kN]$

$F_{B,z}^{(2)} = 4.0 \times 10^5\ [kN]$

$F_z^{(2)} = 0\ [kN]$

$M_T^{(1)} = -5.70 \times 10^4\ [kN]$

$M_L^{(1)} = -5.64 \times 10^6\ [kN]$

Moment equilibrium:

First, we calculate center of volume for each volume V_1, V_2, and V_3.

${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 6.8593]^T\ [m]$

${}^{b'}\mathbf{r}_{P_2/O'} = [50 \quad 20 \quad -1.5090]^T\ [m]$

${}^{b'}\mathbf{r}_{P_3/O'} = [-50 \quad 20 \quad -6.8593]^T\ [m]$

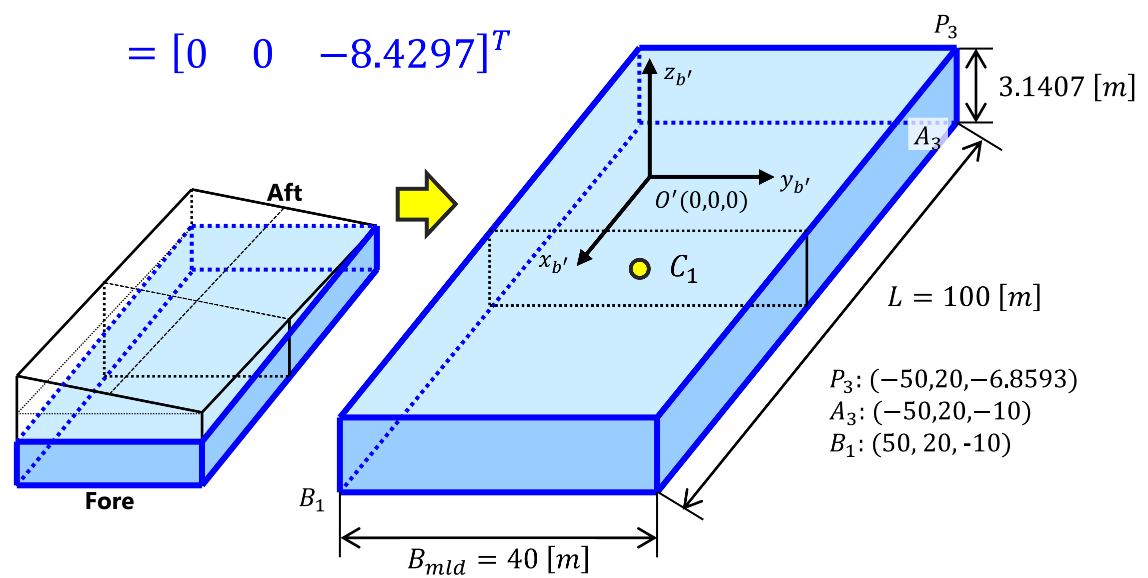
${}^{b'}\mathbf{r}_{P_4/O'} = [-50 \quad -20 \quad 1.5090]^T\ [m]$

Volume V_1: Cube

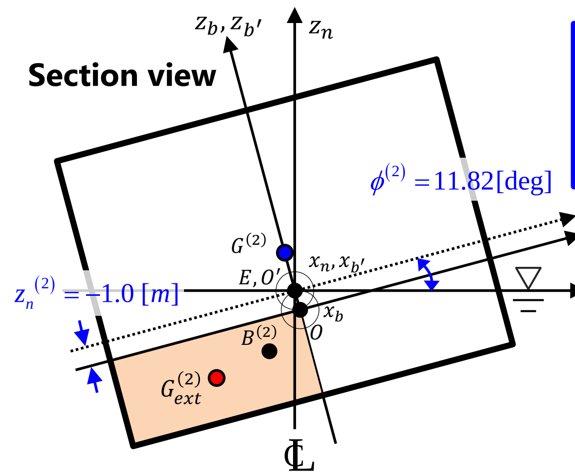
C_1 located at centerline, midship and $T_0/2$

$${}^{b'}\mathbf{r}_{C_1/O'} = ({}^{b'}\mathbf{r}_{P_3/O'} + {}^{b'}\mathbf{r}_{B_1/O'})/2 = [\underbrace{0}_{\text{midship}} \quad \underbrace{0}_{\text{centerline}} \quad (-10 + (-6.8593))/2]^T$$

$$= [0 \quad 0 \quad -8.4297]^T$$

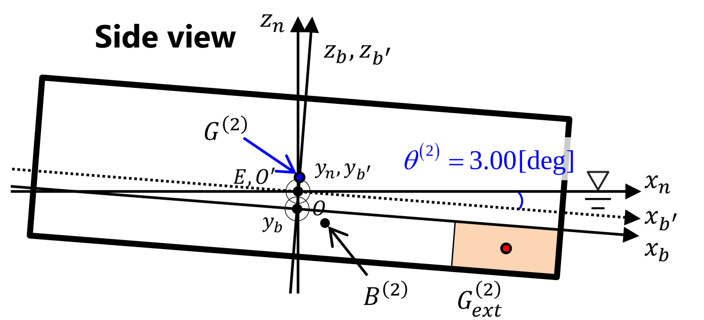


4. Check for the Ship to be in Static Equilibrium at k=1 step



$$\begin{bmatrix} F_z^* - F_z^{(k)} \\ M_T^* - M_T^{(k)} \\ M_L^* - M_L^{(k)} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP}^{(k)} & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} & \rho g I_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} & \rho g I_P^{(k)} & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$



$L = 100\ [m]$
$B_{mld} = 40\ [m]$
$D = 30\ [m]$
$d = 9\ [m]$
$\rho g = 10\ [Mg/m^2 s^2]$

${}^n\mathbf{r}_{G^{(1)}/E} = [0.26 \quad -1.14 \quad 4.86]^T\ [m]$
${}^n\mathbf{r}_{G_{ext}^{(1)}/E} = [39.58 \quad -8.48 \quad -9.78]^T\ [m]$
${}^n\mathbf{r}_{B^{(1)}/E} = [4.33 \quad -2.02 \quad -5.34]^T\ [m]$
${}^n\mathbf{r}_{F^{(1)}/E} = [0 \quad 0 \quad 0]^T\ [m]$

$\nabla^{(2)} = 4.0 \times 10^4\ [m^3]$
$F_{G,z} = -3.6 \times 10^5\ [kN]$
$F_{G_{ext},z} = -4.0 \times 10^4\ [kN]$
$F_{B,z}^{(2)} = 4.0 \times 10^5\ [kN]$

$F_z^{(2)} = 0\ [kN]$
$M_T^{(1)} = -5.70 \times 10^4\ [kN]$
$M_L^{(1)} = -5.64 \times 10^6\ [kN]$

Moment equilibrium:
First, we calculate center of volume for each volume V_1, V_2, and V_3.

${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 6.8593]^T\ [m]$
${}^{b'}\mathbf{r}_{P_2/O'} = [50 \quad 20 \quad -1.5090]^T\ [m]$
${}^{b'}\mathbf{r}_{P_3/O'} = [-50 \quad 20 \quad -6.8593]^T\ [m]$
${}^{b'}\mathbf{r}_{P_4/O'} = [-50 \quad -20 \quad 1.5090]^T\ [m]$
${}^{b'}\mathbf{r}_{C_1/O'} = [0 \quad 0 \quad -8.4297]^T\ [m]$

Volume V_2: Trigonal prism

Center of volume:
= Center of area of triangle $D_2E_2F_2$

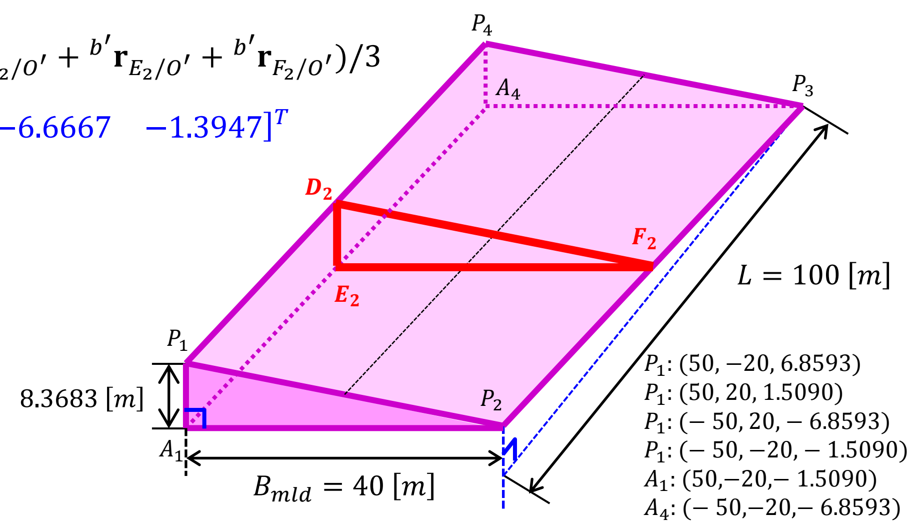
${}^{b'}\mathbf{r}_{D_2/O'} = ({}^{b'}\mathbf{r}_{P_1/O'} + {}^{b'}\mathbf{r}_{P_4/O'})/2$=(0, -20, 4.1841)

${}^{b'}\mathbf{r}_{E_2/O'} = ({}^{b'}\mathbf{r}_{A_1/O'} + {}^{b'}\mathbf{r}_{A_4/O'})/2$=(0, -20, -4.1841)

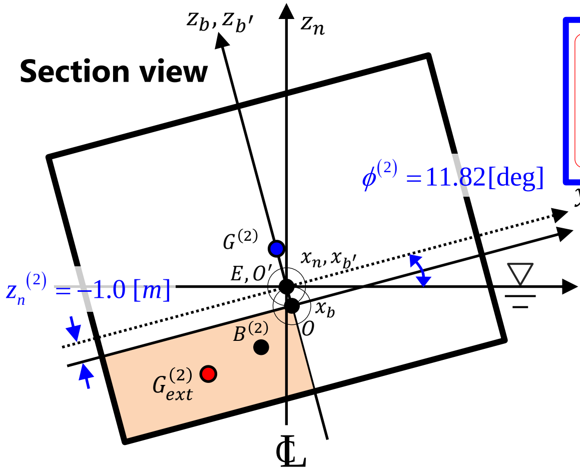
${}^{b'}\mathbf{r}_{F_2/O'} = ({}^{b'}\mathbf{r}_{P_2/O'} + {}^{b'}\mathbf{r}_{P_3/O'})/2$=(0, 20, -4.1841)

${}^{b'}\mathbf{r}_{C_2/O'} = ({}^{b'}\mathbf{r}_{D_2/O'} + {}^{b'}\mathbf{r}_{E_2/O'} + {}^{b'}\mathbf{r}_{F_2/O'})/3$

$= [0 \quad -6.6667 \quad -1.3947]^T$

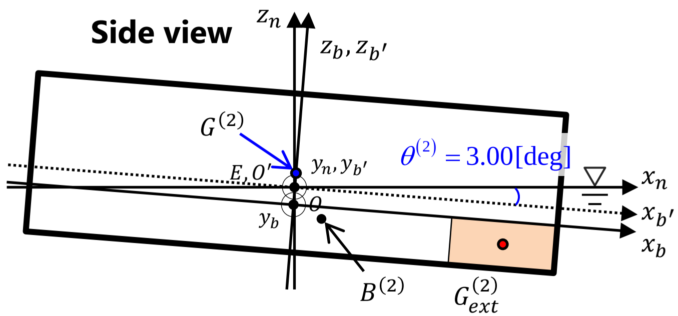


4. Check for the Ship to be in Static Equilibrium at k=1 step



$$\boxed{\begin{bmatrix} F_z^* - F_z^{(k)} \\ M_T^* - M_T^{(k)} \\ M_L^* - M_L^{(k)} \end{bmatrix}} = \begin{bmatrix} -\rho g A_{WP}{}^{(k)} & -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} & \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T{}^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} & \rho g I_P{}^{(k)} \\ \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} & \rho g I_P{}^{(k)} & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L{}^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$



$L = 100\ [m]$

$B_{mld} = 40\ [m]$

$D = 30\ [m]$

$d = 9\ [m]$

$\rho g = 10\ [Mg/m^2s^2]$

${}^n\mathbf{r}_{G^{(1)}/E} = [0.26 \quad -1.14 \quad 4.86]^T\ [m]$

${}^n\mathbf{r}_{G_{ext}^{(1)}/E} = [39.58 \quad -8.48 \quad -9.78]^T\ [m]$

${}^n\mathbf{r}_{B^{(1)}/E} = [4.33 \quad -2.02 \quad -5.34]^T\ [m]$

${}^n\mathbf{r}_{F^{(1)}/E} = [0 \quad 0 \quad 0]^T\ [m]$

$\nabla^{(2)} = 4.0 \times 10^4\ [m^3]$

$F_{G,z} = -3.6 \times 10^5\ [kN]$

$F_{G_{ext},z} = -4.0 \times 10^4\ [kN]$

$F_{B,z}^{(2)} = 4.0 \times 10^5\ [kN]$

$F_z^{(2)} = 0\ [kN]$

$M_T^{(1)} = -5.70 \times 10^4\ [kN]$

$M_L^{(1)} = -5.64 \times 10^6\ [kN]$

Moment equilibrium:

First, we calculate center of volume for each volume V_1, V_2, and V_3.

Volume V_3: Trigonal prism

Center of volume

= Center of area of triangle $D_3E_3F_3$

${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 6.8593]^T\ [m]$

${}^{b'}\mathbf{r}_{P_2/O'} = [50 \quad 20 \quad -1.5090]^T\ [m]$

${}^{b'}\mathbf{r}_{P_3/O'} = [-50 \quad 20 \quad -6.8593]^T\ [m]$

${}^{b'}\mathbf{r}_{P_4/O'} = [-50 \quad -20 \quad 1.5090]^T\ [m]$

${}^{b'}\mathbf{r}_{C_1/O'} = [0 \quad 0 \quad -8.4297]^T\ [m]$

${}^{b'}\mathbf{r}_{C_2/O'} = [0 \quad -6.6667 \quad -1.3947]^T$

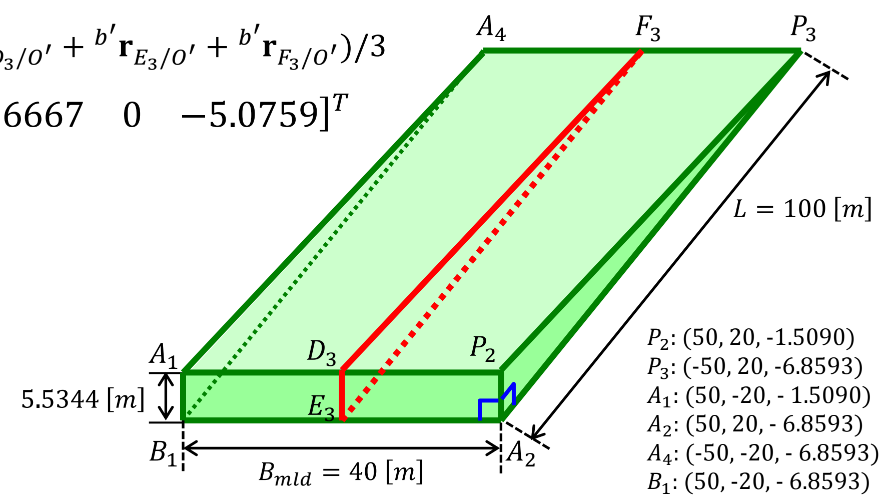
${}^{b'}\mathbf{r}_{D_3/O'} = ({}^{b'}\mathbf{r}_{A_1/O'} + {}^{b'}\mathbf{r}_{P_2/O'})/2 = (50,\ 0,\ -1.5090)$

${}^{b'}\mathbf{r}_{E_3/O'} = ({}^{b'}\mathbf{r}_{B_1/O'} + {}^{b'}\mathbf{r}_{A_2/O'})/2 = (50,\ 0,\ -6.8593)$

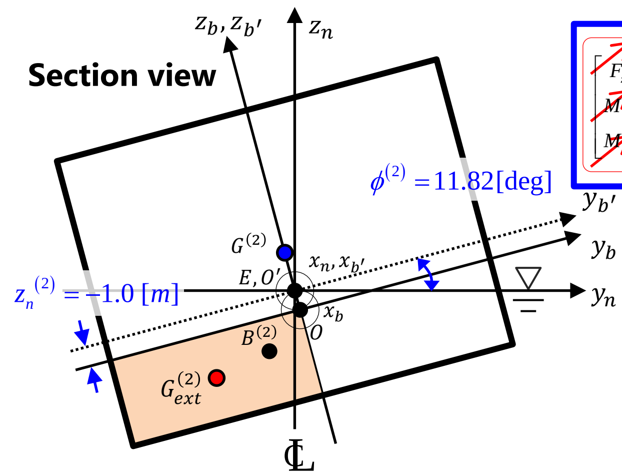
${}^{b'}\mathbf{r}_{F_2/O'} = ({}^{b'}\mathbf{r}_{P_3/O'} + {}^{b'}\mathbf{r}_{P_4/O'})/2 = (-50,\ 0,\ -6.8593)$

$${}^{b'}\mathbf{r}_{C_3/O'} = ({}^{b'}\mathbf{r}_{D_3/O'} + {}^{b'}\mathbf{r}_{E_3/O'} + {}^{b'}\mathbf{r}_{F_3/O'})/3$$

$$= [16.6667 \quad 0 \quad -5.0759]^T$$

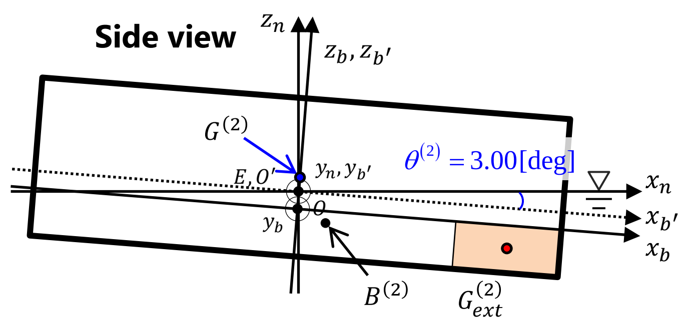


4. Check for the Ship to be in Static Equilibrium at k=1 step



$$\boxed{\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \\ \cancel{M_L^*} - M_L^{(k)} \end{bmatrix}} = \begin{bmatrix} -\rho g A_{WP}^{(k)} & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} & \rho g I_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} & \rho g I_P^{(k)} & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$



$L = 100\ [m]$

$B_{mld} = 40\ [m]$

$D = 30\ [m]$

$d = 9\ [m]$

$\rho g = 10\ [Mg/m^2 s^2]$

${}^n\mathbf{r}_{G^{(1)}/E} = [0.26 \quad -1.14 \quad 4.86]^T\ [m]$

${}^n\mathbf{r}_{G_{ext}^{(1)}/E} = [39.58 \quad -8.48 \quad -9.78]^T\ [m]$

${}^n\mathbf{r}_{B^{(1)}/E} = [4.33 \quad -2.02 \quad -5.34]^T\ [m]$

${}^n\mathbf{r}_{F^{(1)}/E} = [0 \quad 0 \quad 0]^T\ [m]$

$\nabla^{(2)} = 4.0 \times 10^4\ [m^3]$

$F_{G,z} = -3.6 \times 10^5\ [kN]$

$F_{G_{ext},z} = -4.0 \times 10^4\ [kN]$

$F_{B,z}^{(2)} = 4.0 \times 10^5\ [kN]$

$F_z^{(2)} = 0\ [kN]$

$M_T^{(1)} = -5.70 \times 10^4\ [kN]$

$M_L^{(1)} = -5.64 \times 10^6\ [kN]$

Moment equilibrium:

Center of Buoyancy:

${}^{b'}\mathbf{r}_{C_1/O'} = [0 \quad 0 \quad -8.4297]^T\ [m]$

${}^{b'}\mathbf{r}_{C_2/O'} = [0 \quad -6.6667 \quad -1.3947]^T\ [m]$

${}^{b'}\mathbf{r}_{C_3/O'} = [16.6667 \quad 0 \quad -5.0759]^T\ [m]$

$V_1 = 1.2563 \times 10^4\ [m^3]$

$V_2 = 1.6737 \times 10^4\ [m^3]$

$V_3 = 1.0701 \times 10^4\ [m^3]$

$${}^{b'}\mathbf{r}_{B^{(1)}/O'} = \frac{{}^{b'}\mathbf{r}_{C_1/O'} \cdot V_1 + {}^{b'}\mathbf{r}_{C_2/O'} \cdot V_2 + {}^{b'}\mathbf{r}_{C_3/O'} \cdot V_3}{V_1 + V_2 + V_3}$$

$$= [4.4586 \quad -2.7894 \quad -4.5890]^T\ [m]$$

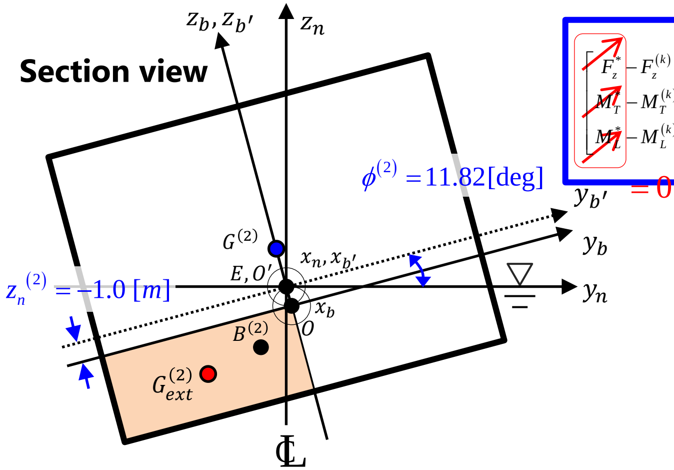
In this case, for convenience of calculating the center of displaced volume ${}^{b'}\mathbf{r}_{B^{(1)}/O'}$ of the ship, we use b′-frame. The origin O′ of b′-frame coincides with the origin E of n-frame. And the orientation of b′-frame is the same as that of b-frame. So, to obtain the center of buoyancy with respect to n-frame, ${}^n\mathbf{r}_{B^{(1)}/E}$, we have to perform the rotational transformation.

$${}^n\mathbf{r}_{B^{(2)}/E} = \begin{bmatrix} \cos\theta^{(2)} & 0 & \sin\theta^{(2)} \\ 0 & 1 & 0 \\ -\sin\theta^{(2)} & 0 & \cos\theta^{(2)} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi^{(2)} & -\sin\phi^{(2)} \\ 0 & \sin\phi^{(2)} & \cos\phi^{(2)} \end{bmatrix} {}^{b'}\mathbf{r}_{B^{(2)}/O'}$$

$$= \begin{bmatrix} \cos 3.0 & 0 & \sin 3.0 \\ 0 & 1 & 0 \\ -\sin 3.0 & 0 & \cos 3.0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 11.82 & -\sin 11.82 \\ 0 & \sin 11.82 & \cos 11.82 \end{bmatrix} \begin{bmatrix} 4.4586 \\ -2.7894 \\ -4.5890 \end{bmatrix} = \boxed{\begin{bmatrix} 4.1879 \\ -1.7903 \\ -5.2892 \end{bmatrix}}$$

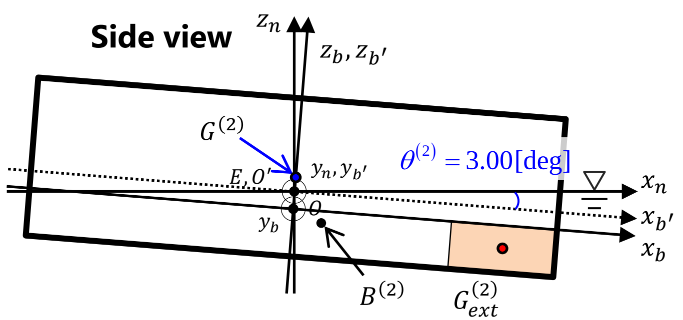
4. Check for the Ship to be in Static Equilibrium at k=1 step

Section view



$$\boxed{\begin{bmatrix} F_z^* - F_z^{(k)} \\ M_T^* - M_T^{(k)} \\ M_L^* - M_L^{(k)} \end{bmatrix}}_{=0} = \begin{bmatrix} -\rho g A_{WP}^{(k)} & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} & \rho g I_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} & \rho g I_P^{(k)} & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

Side view



$L = 100\ [m]$	${}^n\mathbf{r}_{G^{(1)}/E} = [0.26 \quad -1.14 \quad 4.86]^T\ [m]$
$B_{mld} = 40\ [m]$	${}^n\mathbf{r}_{G_{ext}^{(1)}/E} = [39.58 \quad -8.48 \quad -9.78]^T\ [m]$
$D = 30\ [m]$	${}^n\mathbf{r}_{B^{(2)}/E} = [4.46 \quad -2.79 \quad -4.59]^T\ [m]$
$d = 9\ [m]$	${}^n\mathbf{r}_{F^{(1)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2 s^2]$	

$\nabla^{(2)} = 4.0 \times 10^4\ [m^3]$

$F_{G,z} = -3.6 \times 10^5\ [kN]$ $\qquad F_z^{(2)} = 0\ [kN]$

$F_{G_{ext},z} = -4.0 \times 10^4\ [kN]$ $\qquad M_T^{(1)} = -5.70 \times 10^4\ [kN]$

$F_{B,z}^{(2)} = 4.0 \times 10^5\ [kN]$ $\qquad M_L^{(1)} = -5.64 \times 10^6\ [kN]$

Moment equilibrium:

Center of Gravity:

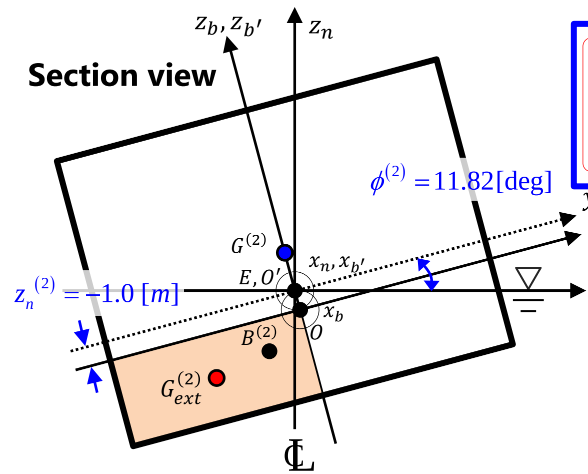
The center of mass, ${}^b\mathbf{r}_{G^{(1)}/O}$, with respect to the body fixed frame is identical with respect to the floating position. But the center of mass, ${}^n\mathbf{r}_{G^{(1)}/E}$, with respect to the waterplane-fixed frame changes with respect to the rotation. The change in the center of mass, ${}^n\mathbf{r}_{G^{(1)}/E}$, with respect to the waterplane-fixed frame causes an additional heeling moment arm.

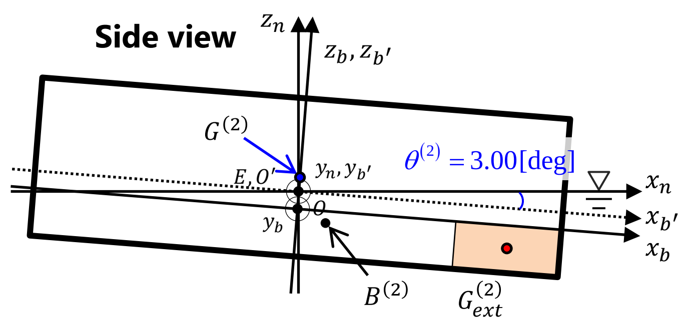
$${}^n\mathbf{r}_{G^{(2)}/E} = \begin{bmatrix} \cos\theta^{(2)} & 0 & \sin\theta^{(2)} \\ 0 & 1 & 0 \\ -\sin\theta^{(2)} & 0 & \cos\theta^{(2)} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi^{(2)} & -\sin\phi^{(2)} \\ 0 & \sin\phi^{(2)} & \cos\phi^{(2)} \end{bmatrix} {}^{b'}\mathbf{r}_{G^{(2)}/O'}, \quad {}^{b'}\mathbf{r}_{G^{(2)}/O'} = {}^b\mathbf{r}_{G^{(2)}/O} + \begin{bmatrix} 0 \\ 0 \\ z_n^{(2)} \end{bmatrix}$$

$$= \begin{bmatrix} \cos 3.0 & 0 & \sin 3.0 \\ 0 & 1 & 0 \\ -\sin 3.0 & 0 & \cos 3.0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 11.82 & -\sin 11.82 \\ 0 & \sin 11.82 & \cos 11.82 \end{bmatrix} \left(\begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right)$$

$$= \boxed{\begin{bmatrix} 0.2558 \\ -1.0242 \\ 4.8873 \end{bmatrix}}$$

4. Check for the Ship to be in Static Equilibrium at k=1 step





$L = 100\ [m]$	$^n\mathbf{r}_{G^{(2)}/E} = [0.26 \quad -1.02 \quad 4.89]^T\ [m]$
$B_{mld} = 40\ [m]$	$^n\mathbf{r}_{G_{ext}^{(1)}/E} = [39.58 \quad -8.48 \quad -9.78]^T\ [m]$
$D = 30\ [m]$	$^n\mathbf{r}_{B^{(2)}/E} = [4.46 \quad -2.79 \quad -4.59]^T\ [m]$
$d = 9\ [m]$	
$\rho g = 10\ [Mg/m^2s^2]$	$^n\mathbf{r}_{F^{(1)}/E} = [0 \quad 0 \quad 0]^T\ [m]$

$\nabla^{(2)} = 4.0 \times 10^4\ [m^3]$

$F_{G,z} = -3.6 \times 10^5\ [kN]$	$F_z^{(2)} = 0\ [kN]$
$F_{G_{ext},z} = -4.0 \times 10^4\ [kN]$	$M_T^{(1)} = -5.70 \times 10^4\ [kN]$
$F_{B,z}^{(2)} = 4.0 \times 10^5\ [kN]$	$M_L^{(1)} = -5.64 \times 10^6\ [kN]$

$$\boxed{\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \\ \cancel{M_L^*} - M_L^{(k)} \end{bmatrix}} = \begin{bmatrix} -\rho g A_{WP}{}^{(k)} & -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} & \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T{}^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} & \rho g I_P{}^{(k)} \\ \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} & \rho g I_P{}^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L{}^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$

Moment equilibrium:

Center of Gravity (Cargo):

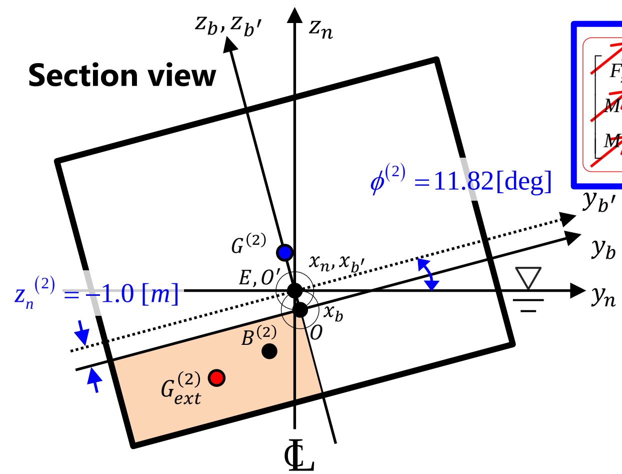
In this case, for convenience of calculating the center of external force, $^{b'}r_{G_{ext}^{(1)}/O'}$ of the ship, we use b'-frame. So, to obtain the center of external force with respect to n-frame, $^{n}r_{G_{ext}^{(1)}/E}$, we have to perform the rotational transformation.

$$^n\mathbf{r}_{G_{ext}{}^{(2)}/E} = \begin{bmatrix} \cos\theta^{(2)} & 0 & \sin\theta^{(2)} \\ 0 & 1 & 0 \\ -\sin\theta^{(2)} & 0 & \cos\theta^{(2)} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi^{(2)} & -\sin\phi^{(2)} \\ 0 & \sin\phi^{(2)} & \cos\phi^{(2)} \end{bmatrix} {}^{b'}\mathbf{r}_{G_{ext}{}^{(2)}/O'}$$

$$= \begin{bmatrix} \cos 3.0 & 0 & \sin 3.0 \\ 0 & 1 & 0 \\ -\sin 3.0 & 0 & \cos 3.0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 11.82 & -\sin 11.82 \\ 0 & \sin 11.82 & \cos 11.82 \end{bmatrix} \begin{bmatrix} 40 \\ -10 \\ -5.5 \end{bmatrix}$$

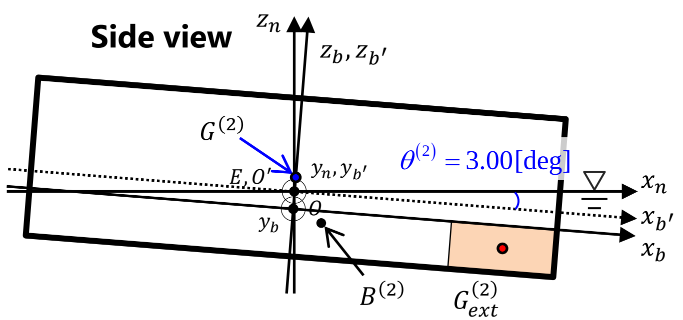
$$= \boxed{\begin{bmatrix} 39.5568 \\ -8.6613 \\ -9.5127 \end{bmatrix}}$$

4. Check for the Ship to be in Static Equilibrium at k=1 step



$$\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \\ \cancel{M_L^*} - M_L^{(k)} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP}^{(k)} & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} & \rho g I_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} & \rho g I_P^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)} \right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$



$L = 100\ [m]$

$B_{mld} = 40\ [m]$

$D = 30\ [m]$

$d = 9\ [m]$

$\rho g = 10\ [Mg/m^2 s^2]$

${}^n\mathbf{r}_{G^{(2)}/E} = [0.26 \quad -1.02 \quad 4.89]^T\ [m]$

${}^n\mathbf{r}_{G_{ext}^{(2)}/E} = [39.56 \quad -8.66 \quad -9.51]^T\ [m]$

${}^n\mathbf{r}_{B^{(2)}/E} = [4.46 \quad -2.79 \quad -4.59]^T\ [m]$

${}^n\mathbf{r}_{F^{(1)}/E} = [0 \quad 0 \quad 0]^T\ [m]$

$\nabla^{(2)} = 4.0 \times 10^4\ [m^3]$

$F_{G,z} = -3.6 \times 10^5\ [kN]$ $\qquad F_z^{(2)} = 0\ [kN]$

$F_{G_{ext},z} = -4.0 \times 10^4\ [kN]$ $\qquad M_T^{(1)} = -5.70 \times 10^4\ [kN]$

$F_{B,z}^{(2)} = 4.0 \times 10^5\ [kN]$ $\qquad M_L^{(1)} = -5.64 \times 10^6\ [kN]$

Moment equilibrium:

$$M_T^{(2)} = M_{BT}^{(2)} + M_{GT}^{(2)} + M_{extT}^{(2)}$$

$$= {}^n y_{B^{(2)}/E} \cdot F_{B,z}^{(2)} + {}^n y_{G^{(2)}/E} \cdot F_{G,z} + {}^n y_{G_{ext}^{(2)}/E} \cdot F_{ext,z}$$

$$= -2.79 \cdot (4.0 \times 10^5) + (-1.02) \cdot (-3.6 \times 10^5) + (-8.66) \cdot (-4.0 \times 10^4)$$

$$= -1.27 \times 10^3\ [kN \cdot m]$$

Tolerance

$\rightarrow |-1.27 \times 10^3| > e$

where, e(epsilon) : an arbitrarily small positive quantity

$$M_L^{(2)} = M_{BL}^{(2)} + M_{GL}^{(2)} + M_{extL}^{(2)}$$

$$= \left(-{}^n x_{B^{(2)}/E} \cdot F_{B,z}^{(2)} \right) + \left(-{}^n x_{G^{(2)}/E} \cdot F_{G,z} \right) + \left(-{}^n x_{G_{ext}^{(2)}/E} \cdot F_{ext,z} \right)$$

$$= [-4.46 \cdot (4.0 \times 10^5)] + [-0.26 \cdot (-3.6 \times 10^5)] + [-39.56 \cdot (-4.0 \times 10^4)]$$

$$= -9.45 \times 10^2\ [kN \cdot m]$$

Tolerance

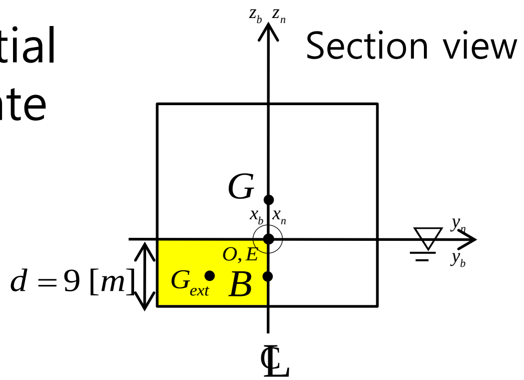
$\rightarrow |-9.45 \times 10^2| > e$

where, e(epsilon) : an arbitrarily small positive quantity

The static equilibrium of moment is not satisfied!
We have to iterate!

Calculation of Position and Orientation of a Barge Ship When a Cargo is Moved - Summary

Initial State

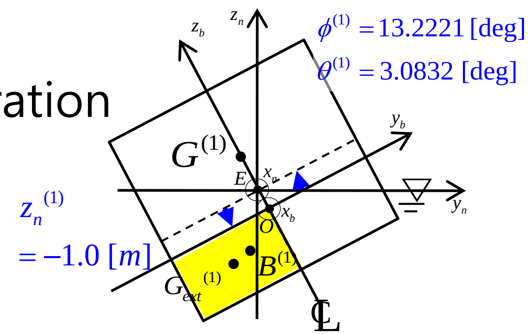


Force(KN)	Transverse Moment(KN·m)	Longitudinal Moment(KN·m)
F = -4.00x10⁴	**M_T = 4.00x10⁵**	**M_L = 1.60x10⁶**

$$\begin{bmatrix} 4.0\times10^4 \\ -4.0\times10^5 \\ -1.6\times10^6 \end{bmatrix} = \begin{bmatrix} -4.0\times10^4 & 0.0 & 0.0 \\ 0.0 & -1.73\times10^6 & 0.0 \\ 0.0 & 0.0 & -2.97\times10^7 \end{bmatrix} \begin{bmatrix} \delta z_n \\ \delta\phi \\ \delta\theta \end{bmatrix}$$

$\delta z_n = -1.0\ [m]$
$\delta\phi = 13.2221\ [\deg]$
$\delta\theta = 3.0832\ [\deg]$

1st Iteration

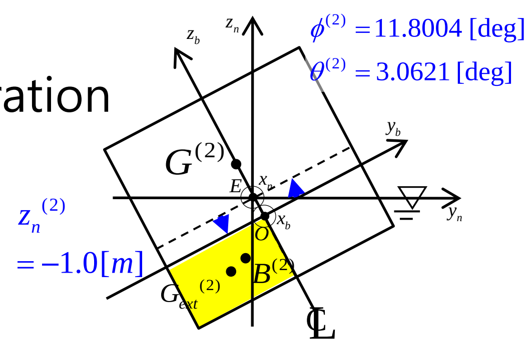


Force(KN)	Transverse Moment(KN·m)	Longitudinal Moment(KN·m)
F = 0.0	**M_T = -5.70x10⁴**	**M_L = -1.32x10⁴**

$$\begin{bmatrix} 0.0 \\ -5.70\times10^4 \\ -1.32\times10^4 \end{bmatrix} = \begin{bmatrix} -4.11\times10^4 & 0.0 & 0.0 \\ 0.0 & -2.29\times10^6 & -7.13\times10^4 \\ 0.0 & -7.13\times10^4 & -3.09\times10^7 \end{bmatrix} \begin{bmatrix} \delta z_n \\ \delta\phi \\ \delta\theta \end{bmatrix}$$

$\delta z_n = 0.0\ [m]$
$\delta\phi = -1.4217\ [\deg]$
$\delta\theta = -0.0211\ [\deg]$

2nd Iteration



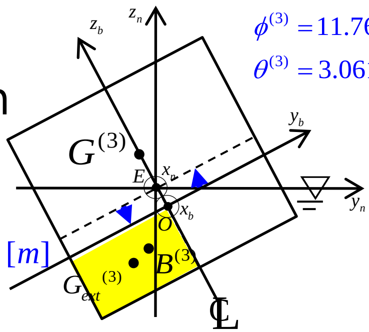
Force(KN)	Transverse Moment(KN·m)	Longitudinal Moment(KN·m)
F = 0.0	**M_T = -1.20x10³**	**M_L = -4.16x10²**

$$\begin{bmatrix} 0.0 \\ -1.20\times10^3 \\ -4.16\times10^2 \end{bmatrix} = \begin{bmatrix} -4.09\times10^4 & 0.0 & 0.0 \\ 0.0 & -2.20\times10^6 & -6.23\times10^4 \\ 0.0 & -6.23\times10^4 & -3.07\times10^7 \end{bmatrix} \begin{bmatrix} \delta z_n \\ \delta\phi \\ \delta\theta \end{bmatrix}$$

$\delta z_n = 0.0\ [m]$
$\delta\phi = -0.0311\ [\deg]$
$\delta\theta = -0.0007\ [\deg]$

Calculation of Position and Orientation of a Barge Ship When a Cargo is Moved - Summary

3rd Iteration



$\phi^{(3)} = 11.7693$ [deg]

$\theta^{(3)} = 3.0613$ [deg]

$z_n^{(3)} = -1.0$ [m]

$\delta z_n = 0.0$ [m]

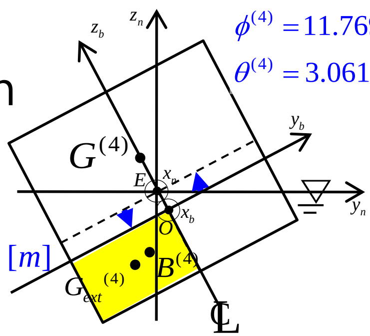
$\delta\phi = -0.0311$ [deg]

$\delta\theta = -0.0007$ [deg]

Force(KN)	Transverse Moment(KN·m)	Longitudinal Moment(KN·m)
F = 0.0	M_T = -0.5488	M_L = -8.1403

$$\begin{bmatrix} 0.0 \\ -0.5488 \\ -8.1403 \end{bmatrix} = \begin{bmatrix} -4.09\times10^4 & 0.0 & 0.0 \\ 0.0 & -2.20\times10^6 & -6.21\times10^5 \\ 0.0 & -6.21\times10^5 & -3.07\times10^7 \end{bmatrix} \begin{bmatrix} \delta z_n \\ \delta\phi \\ \delta\theta \end{bmatrix}$$

4th Iteration



$\phi^{(4)} = 11.7693$ [deg]

$\theta^{(4)} = 3.0613$ [deg]

$z_n^{(4)} = -1.0$ [m]

$\delta z_n = 0.0$ [m]

$\delta\phi = -0.0$ [deg]

$\delta\theta = 0.0$ [deg]

Force(KN)	Transverse Moment(KN·m)	Longitudinal Moment(KN·m)
F = 0.0 (|F|<e(=1.0))	M_T = -0.0003 ($\|M_T\|$<e(=1.0))	M_L = -0.1708 ($\|M_L\|$<e(=1.0))

Calculation of Position and Orientation of a Barge Ship When a Cargo is Moved - Comparison between different order of rotation

Rotation Order: Trim → Heel

No.	Immersion [m]	Heel [deg]	Trim [deg]
1	-1.0000	13.2221	3.0832
2	-1.0000	11.8185	2.9983
3	-1.0000	11.7857	2.9969
4	-1.0000	11.7856	2.9969
5	-1.0000	11.7856	2.9969

Final normal vector of waterplane w.r.t the virtual body-fixed frame (b′-frame):

$$^{b'}\mathbf{r}_{P/O'} = (0.0523, 0.2040, 0.9776)$$

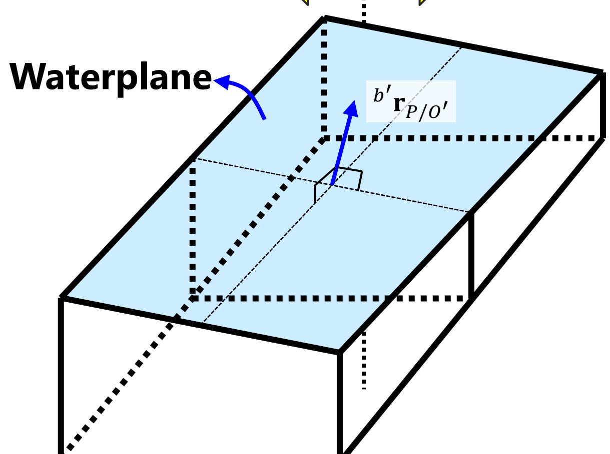
Rotation Order: Heel → Trim

No.	Immersion [m]	Heel [deg]	Trim [deg]
1	-1.0000	13.2221	3.0832
2	-1.0000	11.8004	3.0621
3	-1.0000	11.7693	3.0613
4	-1.0000	11.7693	3.0613

Final normal vector of waterplane w.r.t the virtual body-fixed frame (b′-frame):

$$^{b'}\mathbf{r}_{P/O'} = (0.0523, 0.2040, 0.9776)$$

Same



Topics in ship design automation

Prof. Kyu-Yeul Lee

Fall, 2010

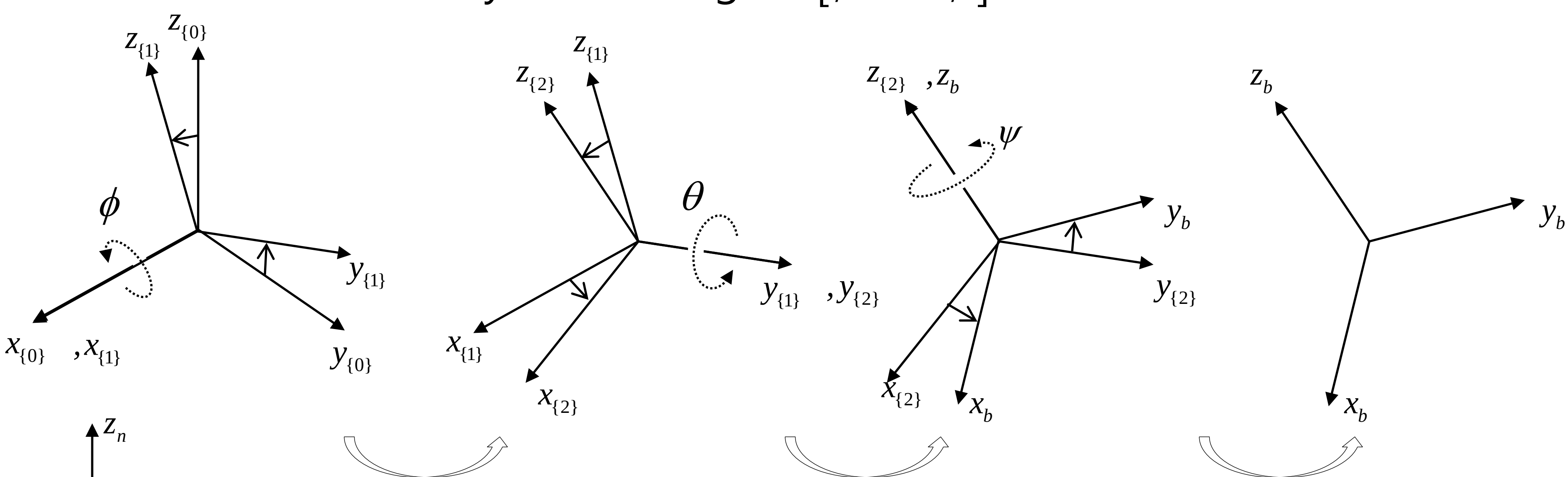
Department of Naval Architecture and Ocean Engineering,
Seoul National University College of Engineering

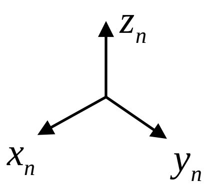
Orientation of the rigid body in spatial motion - Euler angle

One of the most common and widely used parameters in describing reference orientations are the three independent Euler angle.
The transformation between two coordinate systems(Inertial frame and body fixed frame) can be carried out by means of three successive rotations performed in a given sequence.

Ahmed A. Shabana, Dynamics of multibody systems, third edition, Cambridge University Press, 2005, pp. 63

x,y,z Euler angle $[\phi \quad \theta \quad \psi]^T$





Rotation of an angle ϕ about the $x_{\{0\}}$ axis

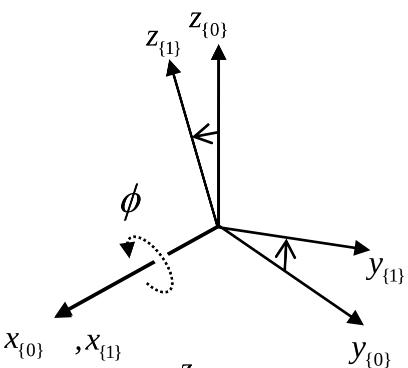
Rotation of an angle θ about the $y_{\{1\}}$ axis

Rotation of an angle ψ about the $z_{\{2\}}$ axis

- Axes of the E′ frame are parallel to axes of the E-frame.

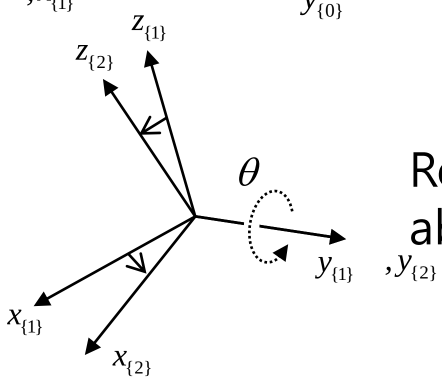
Jerry ginsberg, Engineering Dynamics, Georgia Institute of Technology, 2008, p.229~p.231

Rotation transformation in spatial motion



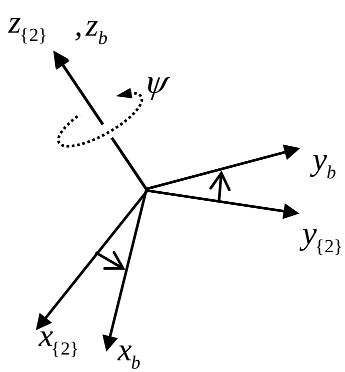
Rotation of an angle ϕ about the $x_{\{0\}}$-axis

$$ {}^{\{0\}}\mathbf{r}_{P/O} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix} {}^{\{1\}}\mathbf{r}_{P/O} $$



Rotation of an angle θ about the $y_{\{1\}}$-axis

$$ {}^{\{1\}}\mathbf{r}_{P/O} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} {}^{\{2\}}\mathbf{r}_{P/O} $$

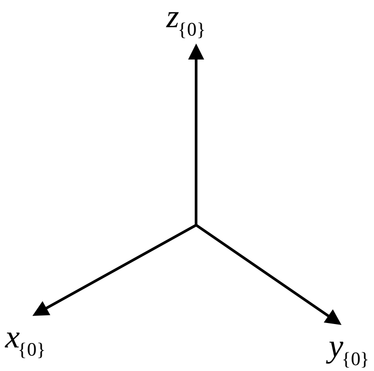


Rotation of an angle ψ about the $z_{\{2\}}$ axis

$$ {}^{\{2\}}\mathbf{r}_{P/O} = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} {}^{b}\mathbf{r}_{P/O} $$

Rotation transformation in spatial motion

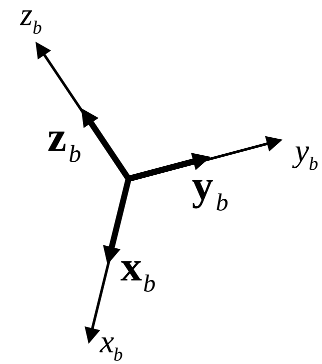
x,y,z Euler angle $[\phi \quad \theta \quad \psi]^T$



Rotation of an angle ϕ about the x axis

Rotation of an angle θ about the y axis

Rotation of an angle ψ about the z axis



$${}^{\{0\}}\mathbf{r}_{P/O} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix} {}^{\{1\}}\mathbf{r}_{P/O} \qquad {}^{\{1\}}\mathbf{r}_{P/O} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} {}^{\{2\}}\mathbf{r}_{P/O} \qquad {}^{\{2\}}\mathbf{r}_{P/O} = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} {}^{b}\mathbf{r}_{P/O}$$

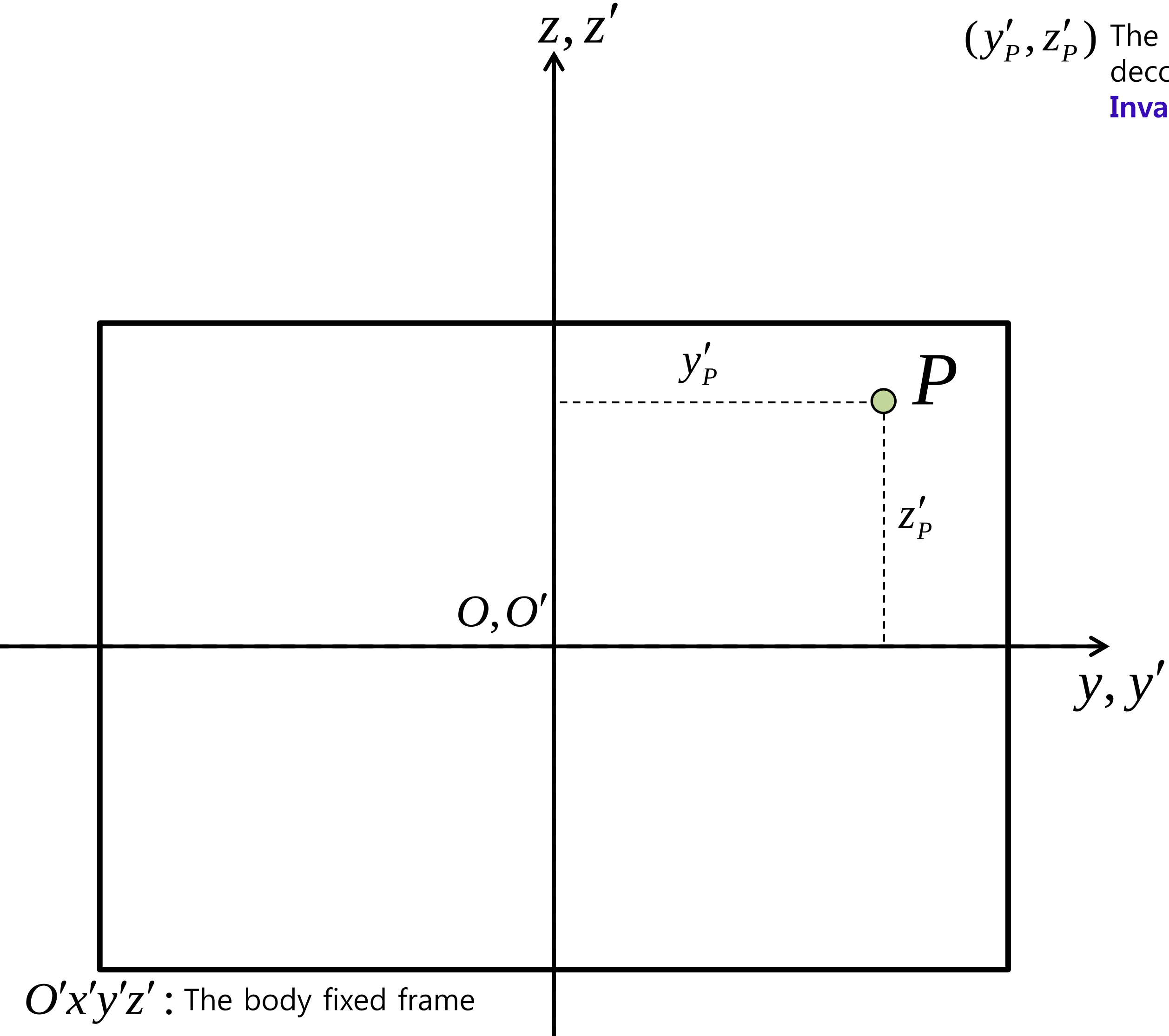
$${}^{\{0\}}\mathbf{r}_{P/O} = \begin{bmatrix} \cos\theta\cos\psi & -\cos\theta\sin\psi & \sin\theta \\ \sin\phi\sin\theta\cos\psi + \cos\phi\sin\psi & -\sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi & -\sin\phi\cos\theta \\ -\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi & \cos\phi\sin\theta\sin\psi + \sin\phi\cos\psi & \cos\phi\cos\theta \end{bmatrix} {}^{b}\mathbf{r}_{P/O}$$

$${}^{\{0\}}\mathbf{x}_b \qquad {}^{\{0\}}\mathbf{y}_b \qquad {}^{\{0\}}\mathbf{z}_b$$

Coordinate Transformation
참 고 자 료

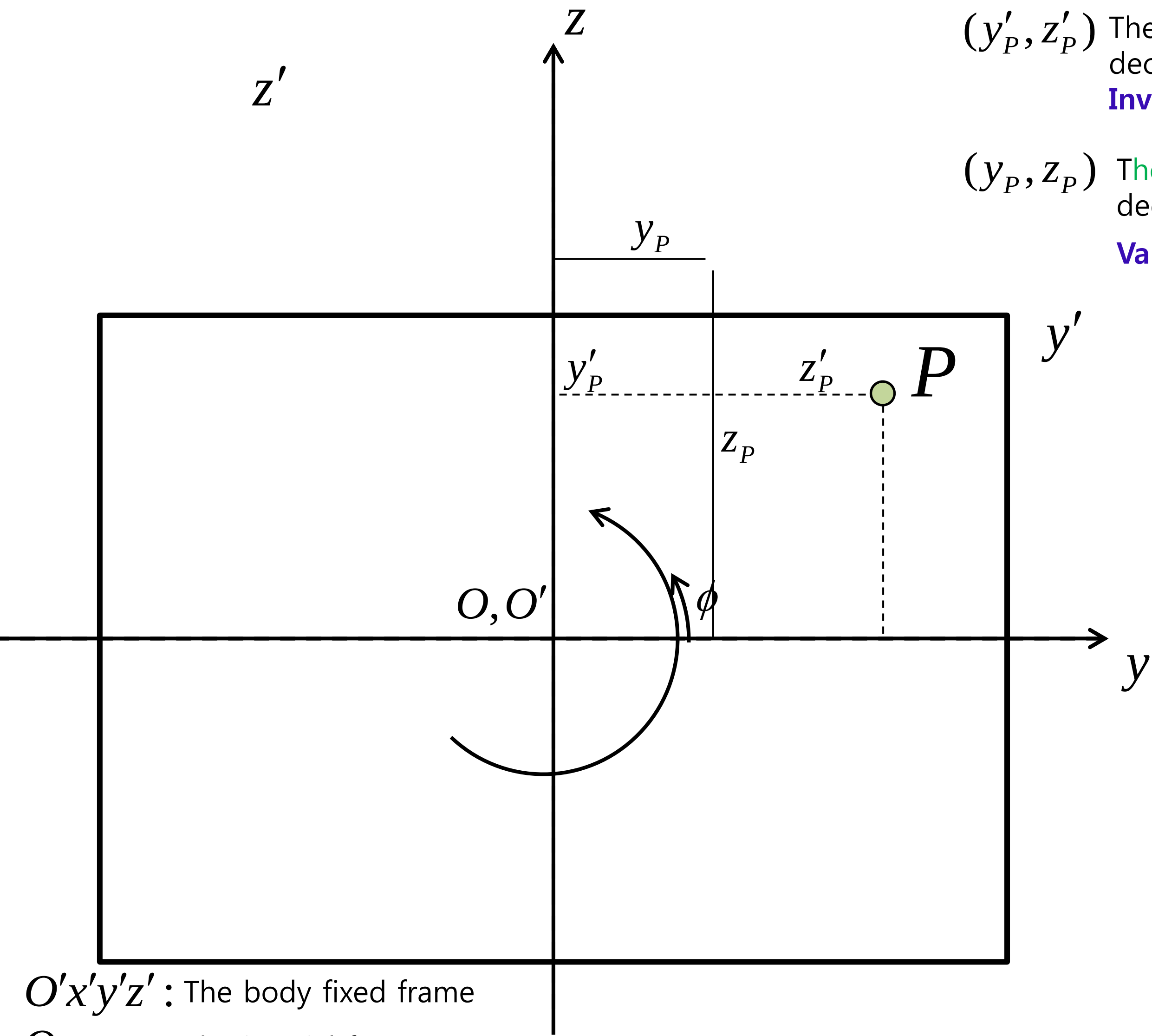
Representation of a Point "P" on an object with respect to the body fixed frame (decomposed in the body fixed frame)

(y'_P, z'_P) The Position vector of the point P decomposed in the body fixed frame
Invariant with respect to the body fixed frame



$O'x'y'z'$: The body fixed frame
$Oxyz$: The inertial frame

Rotate the object with an angle of ϕ and then represent the point "P" on the object with respect to the inertial frame



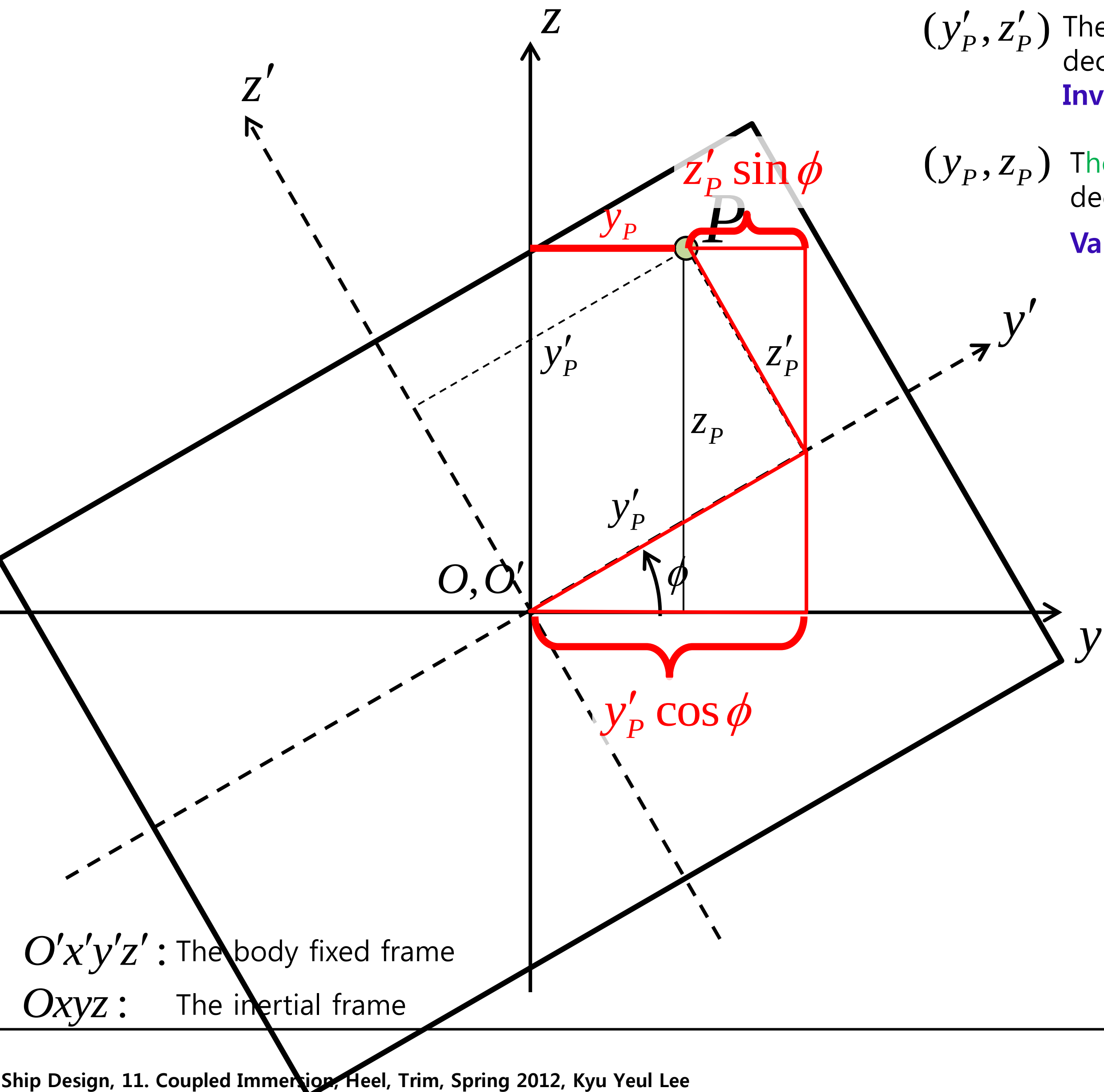
(y'_P, z'_P) The Position vector of the point P decomposed in the body fixed frame
Invariant with respect to the body fixed frame

(y_P, z_P) The Position vector of the point P decomposed in the initial frame
Variant with respect to the inertial frame.

$O'x'y'z'$: The body fixed frame

$Oxyz$: The inertial frame

Coordinate Transformation of a Position Vector



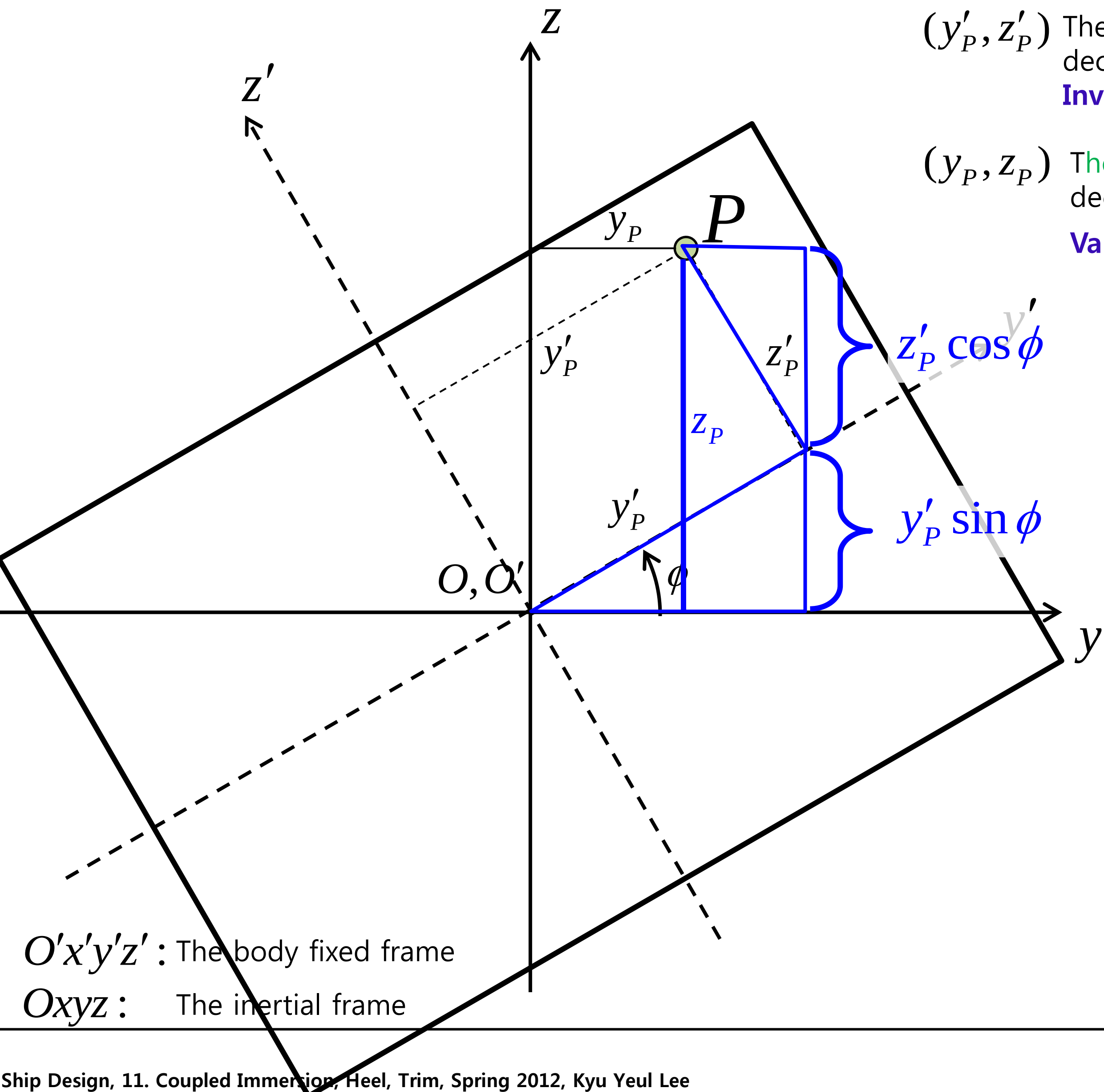
(y'_P, z'_P) The Position vector of the point P decomposed in the body fixed frame
Invariant with respect to the body fixed frame

(y_P, z_P) The Position vector of the point P decomposed in the initial frame
Variant with respect to the inertial frame.

$$y_P = y'_P \cos\phi - z'_P \sin\phi$$

$O'x'y'z'$: The body fixed frame
$Oxyz$: The inertial frame

Coordinate Transformation of a Position Vector



(y'_P, z'_P) The Position vector of the point P decomposed in the body fixed frame
Invariant with respect to the body fixed frame

(y_P, z_P) The Position vector of the point P decomposed in the initial frame
Variant with respect to the inertial frame.

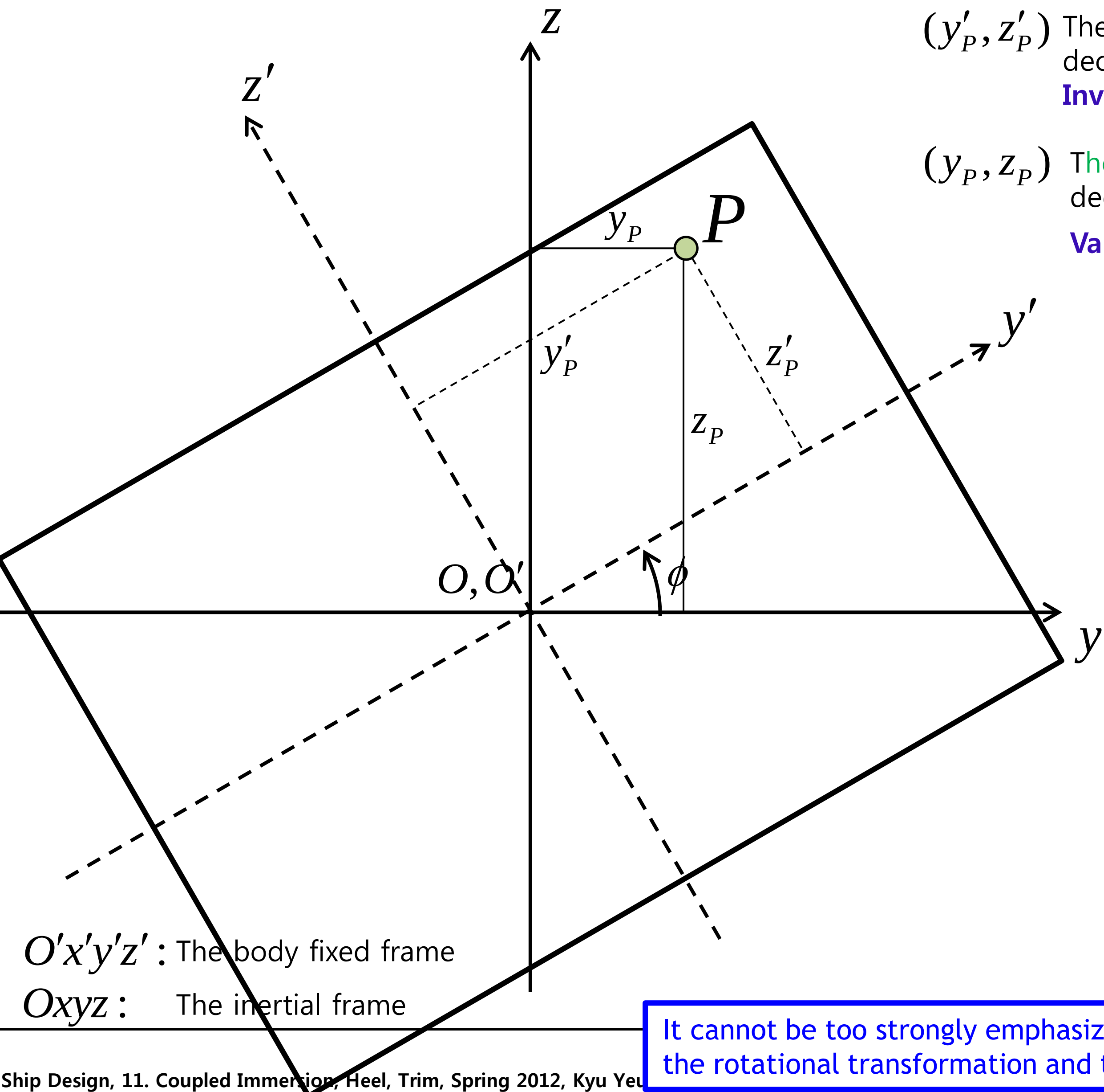
$$y_P = y'_P \cos\phi - z'_P \sin\phi$$

$$z_P = y'_P \sin\phi + z'_P \cos\phi$$

$O'x'y'z'$: The body fixed frame

$Oxyz$: The inertial frame

Coordinate Transformation of a Position Vector



(y'_P, z'_P) The Position vector of the point P decomposed in the body fixed frame
Invariant with respect to the body fixed frame

(y_P, z_P) The Position vector of the point P decomposed in the initial frame
Variant with respect to the inertial frame.

$$y_P = y'_P \cos\phi - z'_P \sin\phi$$
$$z_P = y'_P \sin\phi + z'_P \cos\phi$$

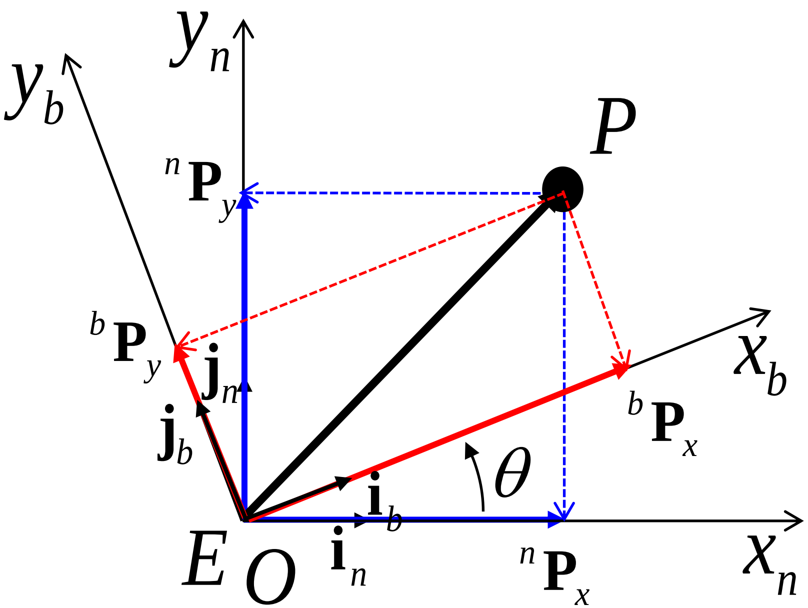
Matrix Form

$$\begin{bmatrix} y_P \\ z_P \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} y'_P \\ z'_P \end{bmatrix}$$
$$^n\mathbf{r}_P = {}^n\mathbf{R}_b\, {}^b\mathbf{r}_P$$

$O'x'y'z'$: The body fixed frame
$Oxyz$: The inertial frame

It cannot be too strongly emphasized that
the rotational transformation and the coordinate transformation are important.

Coordinates transformation of the vector



$$\mathbf{P} = {}^nP_x\,\mathbf{i}_n + {}^nP_y\,\mathbf{j}_n$$

$$\mathbf{P} = {}^bP_x\,\mathbf{i}_b + {}^bP_y\,\mathbf{j}_b$$

$${}^nP_x\,\mathbf{i}_n + {}^nP_y\,\mathbf{j}_n = {}^bP_x\,\mathbf{i}_b + {}^bP_y\,\mathbf{j}_b$$

$${}^nP_x\,\mathbf{i}_n + {}^nP_y\,\mathbf{j}_n = ({}^bP_x\cos\theta - {}^bP_y\sin\theta)\mathbf{i}_n + ({}^bP_x\sin\theta + {}^bP_y\cos\theta)\mathbf{j}_n$$

$$\begin{bmatrix} {}^nP_x \\ {}^nP_y \end{bmatrix}\begin{bmatrix}\mathbf{i}_n & \mathbf{j}_n\end{bmatrix} = \begin{bmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix} {}^bP_x \\ {}^bP_y \end{bmatrix}\begin{bmatrix}\mathbf{i}_n & \mathbf{j}_n\end{bmatrix}$$

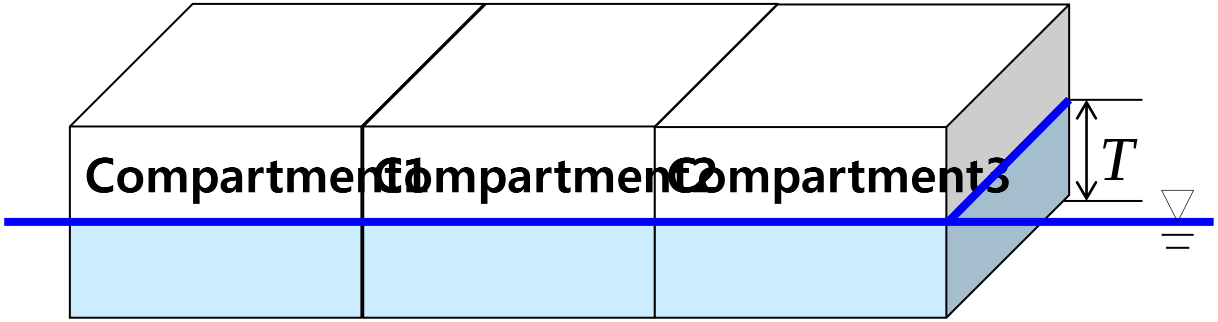
$$\begin{bmatrix} {}^nP_x \\ {}^nP_y \end{bmatrix} = \underbrace{\begin{bmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{bmatrix}}_{{}^n\mathbf{R}_b}\begin{bmatrix} {}^bP_x \\ {}^bP_y \end{bmatrix} \Rightarrow \begin{bmatrix} {}^nP_x \\ {}^nP_y \end{bmatrix} = {}^n\mathbf{R}_b\begin{bmatrix} {}^bP_x \\ {}^bP_y \end{bmatrix}$$

Chapter 12. Damage Stability

12-1 Concept of Damage Stability

Example) Box-Shaped Ship

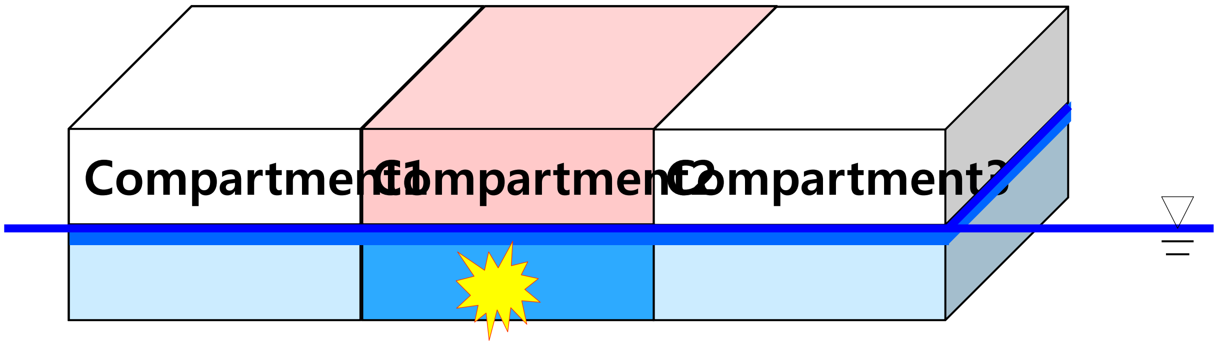
✓ **A ship is composed of three compartments.**



When a compartment of the ship is damaged, what is the new position of this ship?

Immersion due to the flooding

When the compartment in the midship is damaged, what is the new position of this ship?



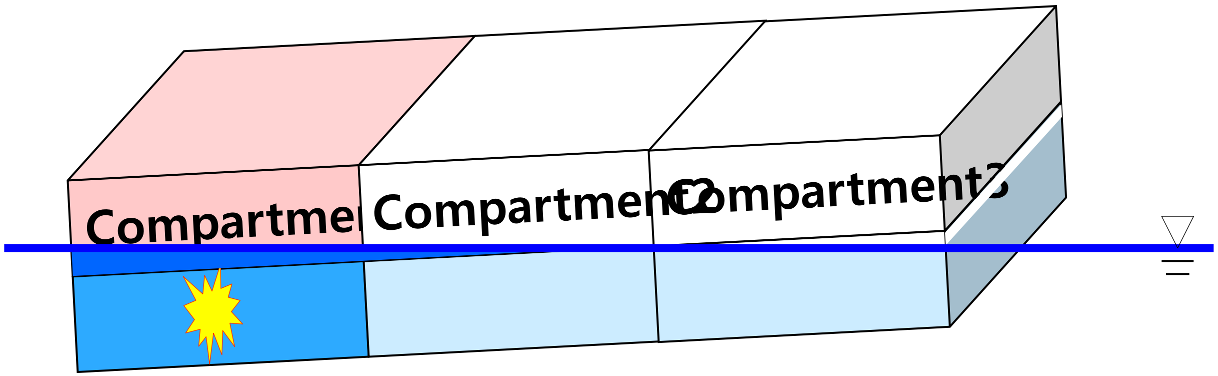
The position of the ship will be changed.

Immersion

*The new position of the ship can be calculated by **the method of added weight or lost buoyancy**.

Immersion and Trim due to the flooding

When the compartment at the aft of the ship is damaged, what is the new position of this ship?



The position of the ship will be changed.

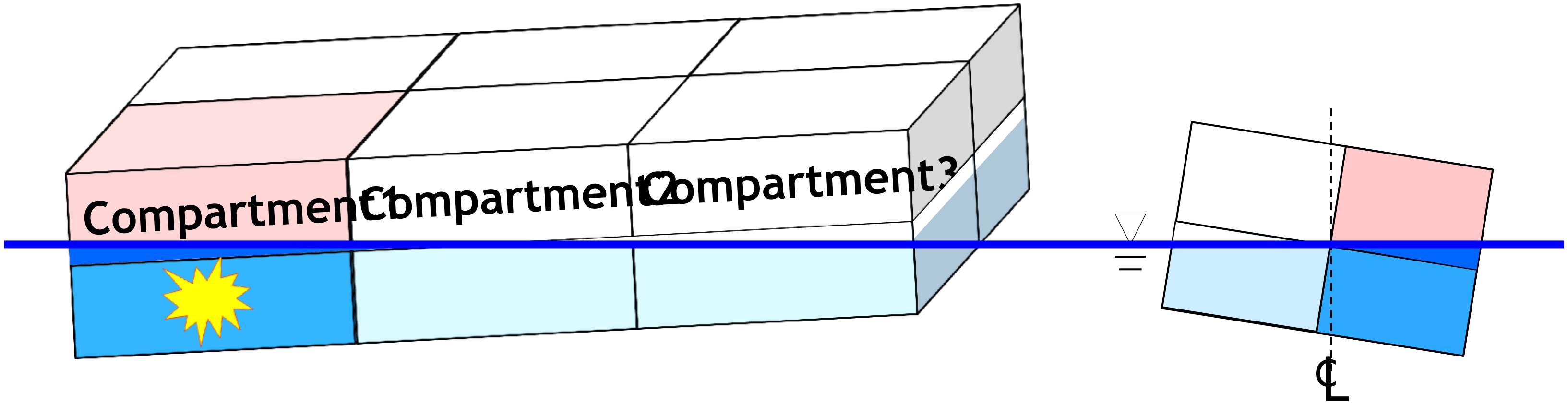
Immersion + Trim

*The new position of the ship can be calculated by **<u>the method of added weight or lost buoyancy</u>**.

Immersion, Trim, and Heel due to the flooding

✓ **When the ship is composed of "six" compartments.**

When the compartment at the aft and right side of the ship is damaged, what is the new position of the ship?

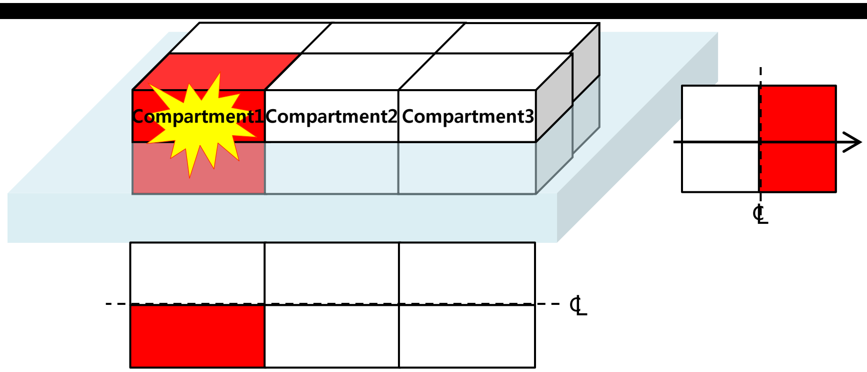


The position of the ship will be changed.

Immersion + Trim + Heel

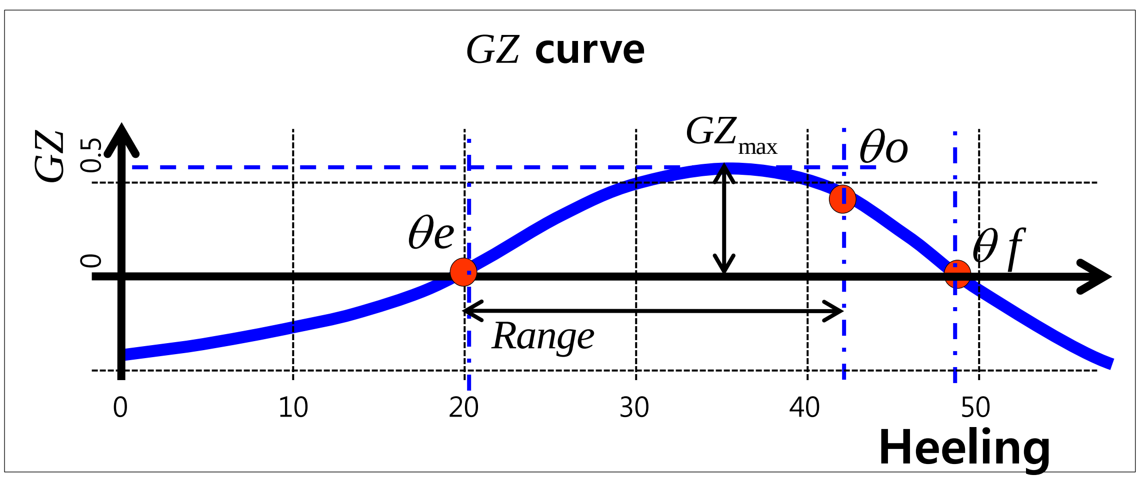
*The new position of the ship can be calculated by **the method of added weight or lost buoyancy**.

GZ curve after flooding



Immersion + Trim + Heel

✓ To measure the damage stability, calculate the GZ curve of this damage case by finding the new center of buoyancy and center of mass.

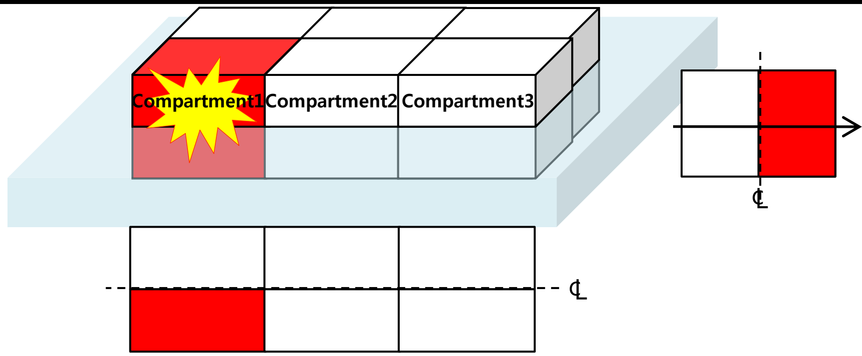


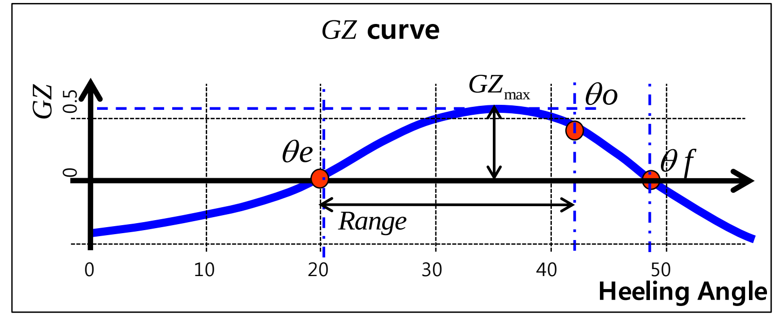
θe : Equilibrium heel angle.
θv : $\theta v = \text{minimum}(\theta f, \theta o)$
(in this case, θv equals to θo)
GZ_{max} : Maximum value of GZ.
Range : Range of positive righting arm.
Flooding stage : Discrete step during the flooding process.

θf : angle of flooding (righting arm becomes negative)

θo : angle at which an **"opening"** incapable of being closed weathertight becomes submerged.

Two Methods to Measure the Ship's Damage Stability





How to measure the ship's stability in a damaged condition?

Deterministic Method : **Calculation of survivability of a ship based on the position, stability and inclination in damaged conditions**

Probabilistic Method : **Calculation of survivability of a ship based on the probability of damage**

Damage Stability

It must be remembered that the resulting (virtual) displacement no only differ from the initial displacement, but varies with change in trim or heel.

Application

SOLAS 2006 Amend / Chapter II1 / Reg. 7
3 When determining the positive righting lever (GZ) of the residual stability curve, the displacement used should be that of the intact condition. That is, the **constant displacement method** of calculation should be used.

In constant displacement method, the GZ-curve related values are represented so that the displacement of the ship is assumed to be constant (=initial displacement).

This means that to get the correct uprighting moments from the GZ-values, GZ must be multiplied by the initial displacement.

Damage assumptions - MARPOL, IBC, IGC

Location of damage

		MARPOL	IBC	IGC
Draft		For any operating draft reflecting loading conditions		
Location of damage in lengthwise	Anywhere	Lf>225m	Type 1[1] Type 2[1] Lf>150m Type 3[1] Lf>225m	Type 1G[2] Type 2PG[2] Type 2G[2] Lf>150m Type 3G[2] Lf≥125m
	Anywhere (Engine room: 1 compartment)	150m<Lf<225m	Type 2 ≤150m Type 3 125m<Lf<225m	Type 2G Lf≤150m
	Anywhere (Engine room: exception)	Lf≤150m	Type 3 Lf<125m	Lf<125m Type 3G

Extent of damage

				MARPOL	IBC	IGC
Extent of Damage	Side Damage	Longitudinal Extent		$Lf^{2/3}/3$ or 14.5m, whichever is the lesser		
		Transverse Extent		B/5 or 11.5m, whichever is the lesser		
		Vertical Extent		No limit		
	Bottom Damage	Longitudinal Extent	FP'～0.3	$Lf^{2/3}/3$ or 14.5m, whichever is the lesser		
			0.3～Aft	$Lf^{2/3}/3$ or 5.0m, whichever is the lesser		Lf/10 or 5.0m, whichever is the lesser
		Transverse Extent	FP'～0.3	B/6 or 10.0m, whichever is the lesser		
			0.3 ～Aft	B/6 or 5.0m, whichever is the lesser		
		Vertical Extent		B/15 or 6.0m, whichever is the lesser		B/15 or 2m, whichever is the lesser

→ bottom raking damage[3], Reg. 28 of MARPOL 73/78

– Longitudinal Extent:	20,000t ≤ DWT ≤ 75,000t	: 0.4 Lf from FP'
	75,000t ≤ DWT	: 0.6 Lf from FP'
– Transverse Extent:	20,000t ≤ DWT	: B/3 anywhere
– Vertical Extent:	20,000t ≤ DWT	: breach of outer hull[6]

1) Type 1, Type 2, Type 3: Classification of chemical tanker according to the danger of the loaded cargo. The ship, which carries most dangerous cargo, is classified into Type 1.
2) Type 1G, Type 2G, Type 2PG, Type 3G: Classification of gas carrier according to the danger of the loaded cargo. The ship, which carries most dangerous cargo, is classified into Type 1G.
3) bottom raking damage is only considered in MARPOL

Damage assumptions - ICLL

Location of damage

<table>
<tr><td colspan="2"></td><td>ICLL</td></tr>
<tr><td colspan="2">Draft</td><td>Summer load line</td></tr>
<tr><td rowspan="2">Location of damage in lengthwise</td><td>Anywhere (Engine room: 1 compartment)</td><td>Lf>150m
ship type
A: 1 compartment / B-60: 1 compartment / B-100: 2 compartments</td></tr>
<tr><td>Anywhere (Engine room: exception)</td><td>100m<Lf≤150m
ship type
B-60: 1 compartment / B-100: 2 compartments</td></tr>
</table>

Extent of damage

<table>
<tr><td colspan="3"></td><td>ICLL</td></tr>
<tr><td rowspan="3">Extent of Damage</td><td rowspan="3">Side Damage</td><td>Longitudinal Extent</td><td>Type A: 1 compartment
Type B-60: 1 compartment
Type B-100: 2 compartments</td></tr>
<tr><td>Transverse Extent</td><td>/5 or 11.5m, whichever is the lesser</td></tr>
<tr><td>Vertical Extent</td><td>No limit</td></tr>
</table>

DAMAGE ASSUMPTION

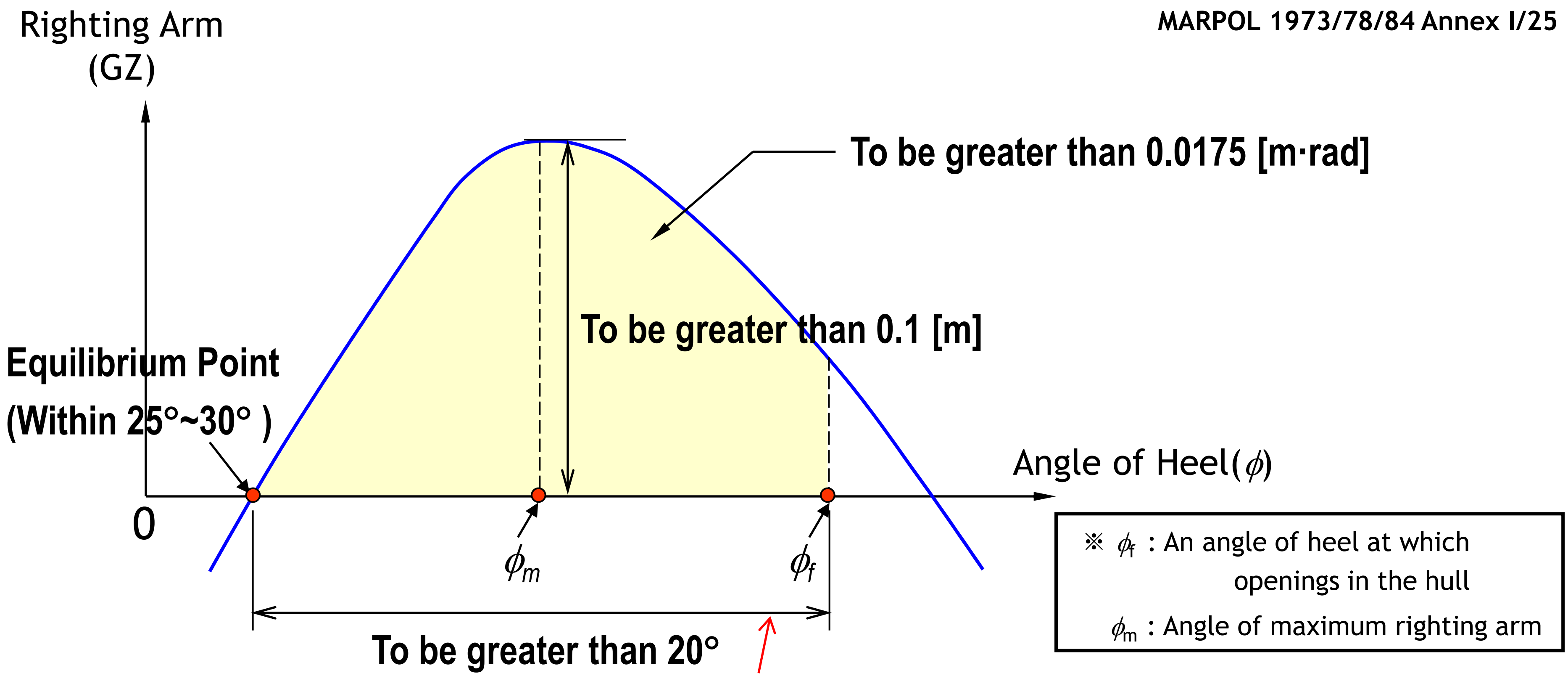
a) The vertical extent of damage in all cases is assumed to be from the base line upwards without limit.

b) The transverse extent of damage is equal to one-fifth (1/5) or 11.5 meters, whichever is the lesser of breadth inboard from the side of the ship perpendicularly to the centerline at the level of the summer load water line.

c) No main transverse bulkhead is damaged.

MARPOL Regulation for Damage Stability

MARPOL 1973/78/84 Annex I/25



a) The final waterline shall be below the lower edge of any opening through which progressive flooding may take place.

b) The angle of heel due to unsymmetrical flooding shall not exceed 25 degrees, provided that this angle may be increased up to 30 degrees if no deck edge immersion occurs.

c) Righting lever curve has at least a range of 20 degrees beyond the position of equilibrium in association with a maximum residual righting lever of at least 0.1 meter within the 20 degrees range

d) The area under the curve within this range shall not be less than 0.0175 meter-radians

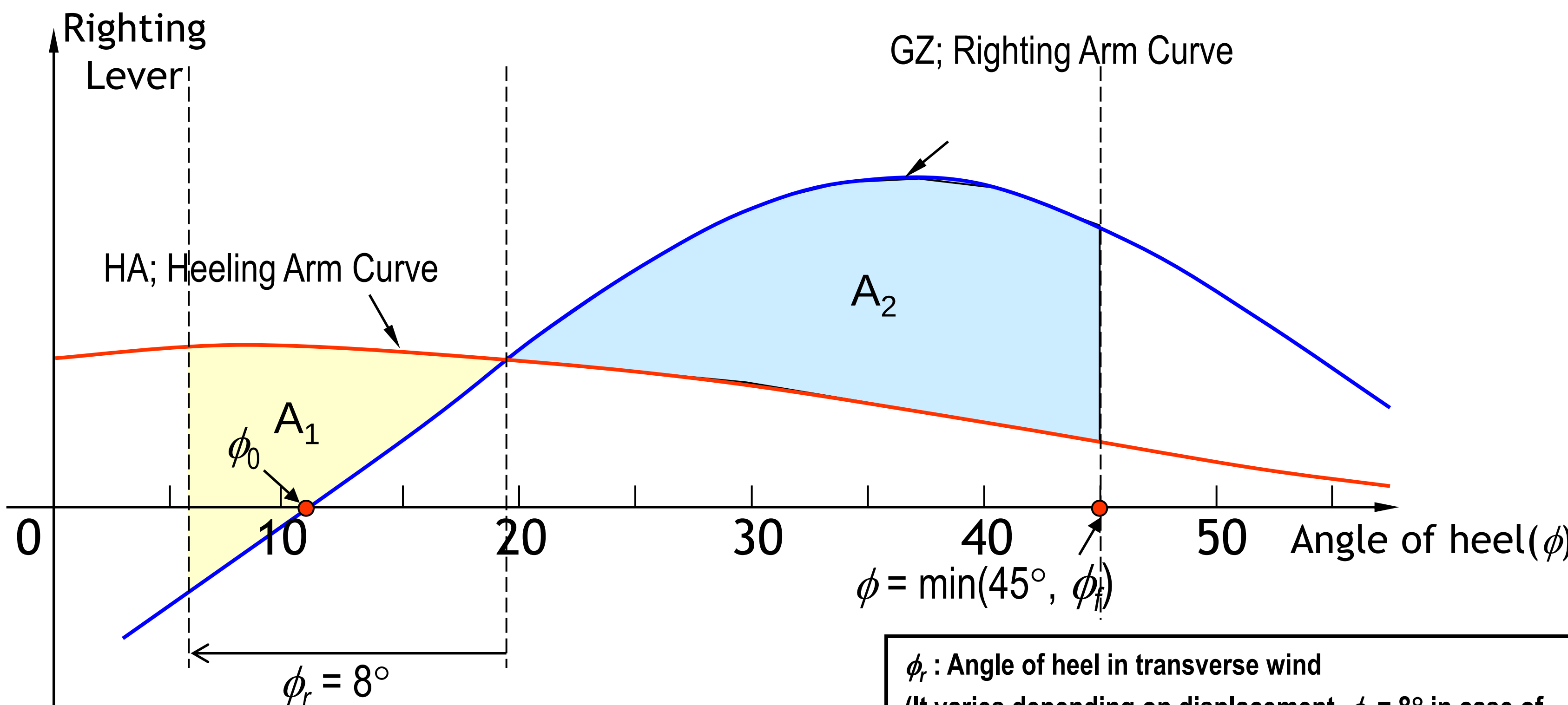
- Damage Stability

MARPOL 1973/78/84 Annex I/25

Damage Stability Criteria of Battleship*

Regulation

ϕ_0(Initial Angle of Heel) $\leq 15°$, $A_2 \geq 1.4 \cdot A_1$



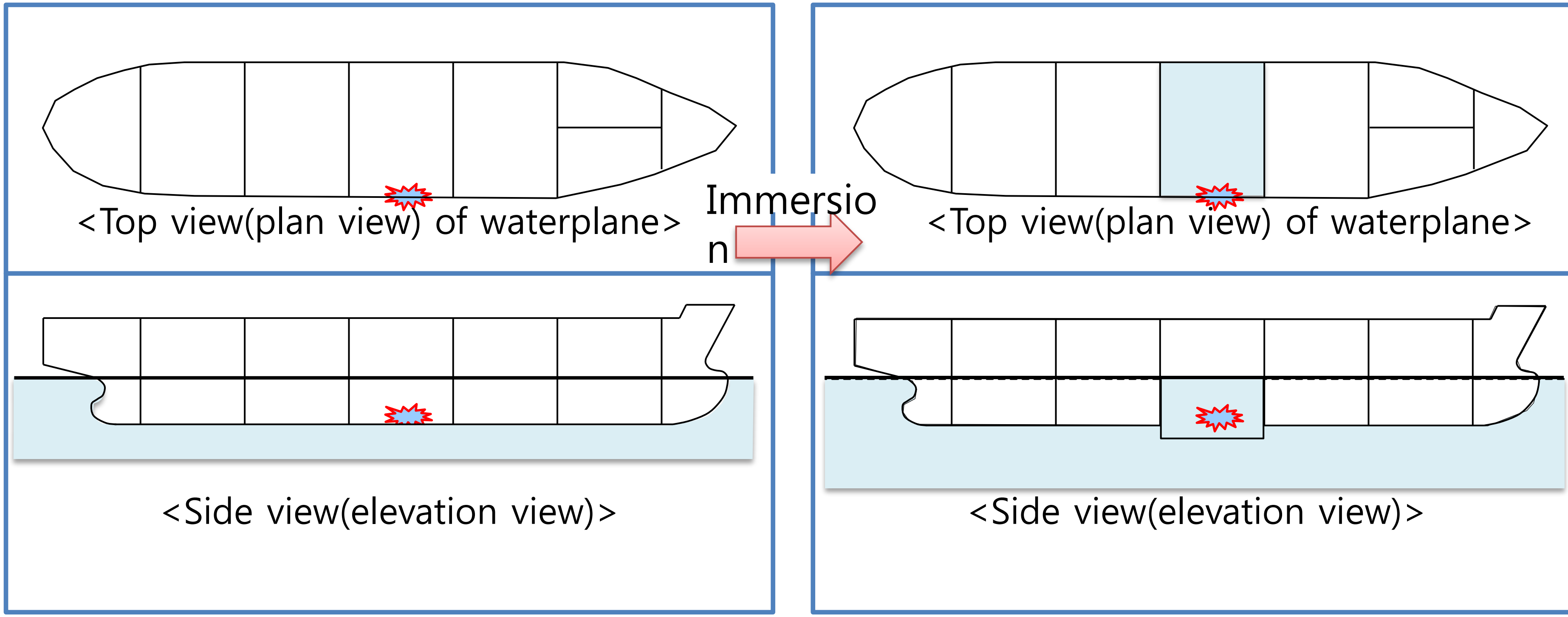
ϕ_r : Angle of heel in transverse wind
(It varies depending on displacement, ϕ_r = 8° in case of battleship with displacement of 9,000 ton)
ϕ_f : An angle of heel at which openings in the hull

* Surko, S.W., "An Assessment of Current Warship Damaged Stability Criteria, Naval Engineers Journal, 1994.

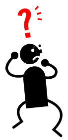
12-2 Two Methods to Measure the Ship's Damage Stability

Change in Position due to Flooding

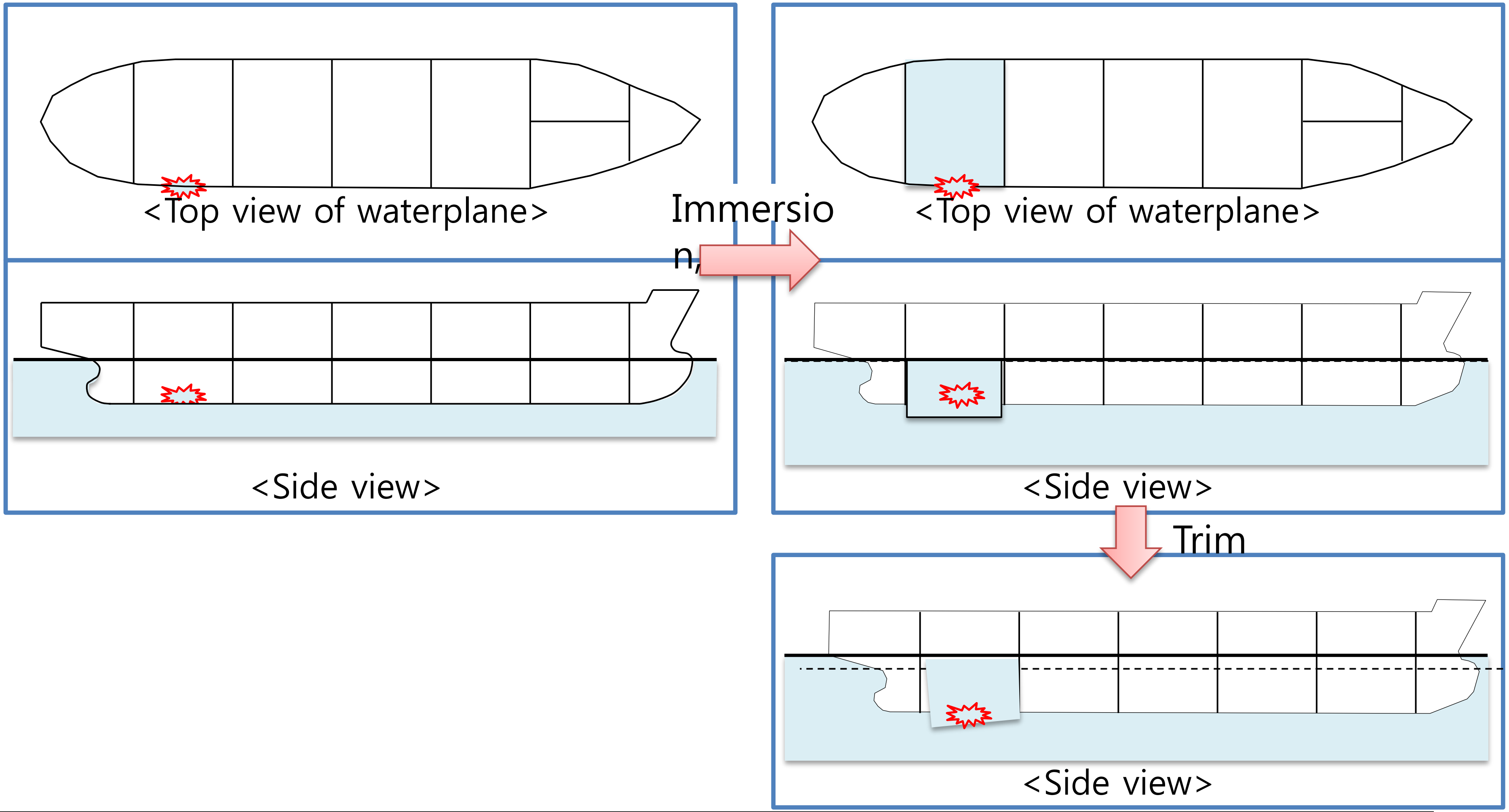
What happens if the compartment located in the center of a ship is damaged?



Change in Position due to Flooding

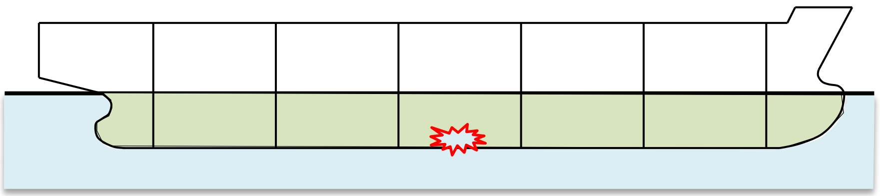


What happens if the compartment located in the aft part of a ship is damaged?



LOST BUOYANCY METHOD

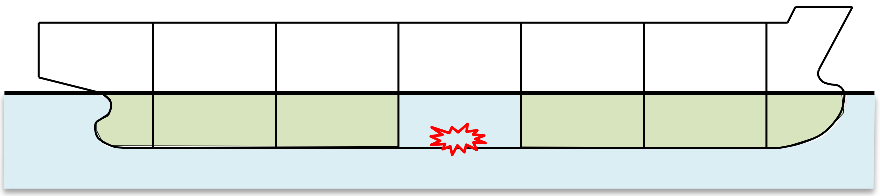
: Concept of Lost Buoyancy Method



<Side view>

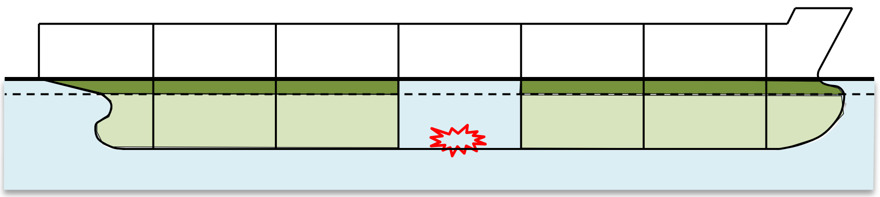
A damage occurs

: Volume which contribute buoyancy



<Side view>

The buoyancy of the flooded space is lost.



<Side view>

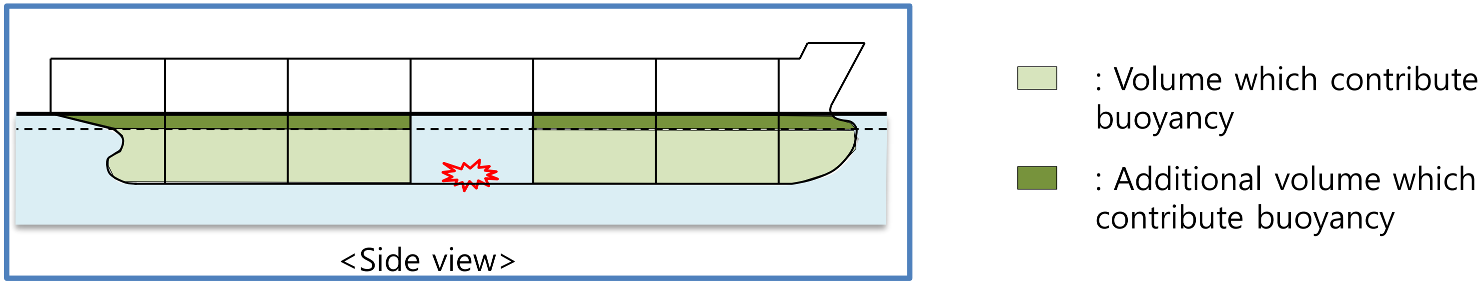
The lost buoyancy must be regained by an increase of draft

: Additional volume which contribute buoyancy (regained buoyancy)

Lost buoyancy method

"In the lost buoyancy method, the water that enters the ship is considered still part of the sea, and the buoyancy of the flooded space is lost"

Lost buoyancy method



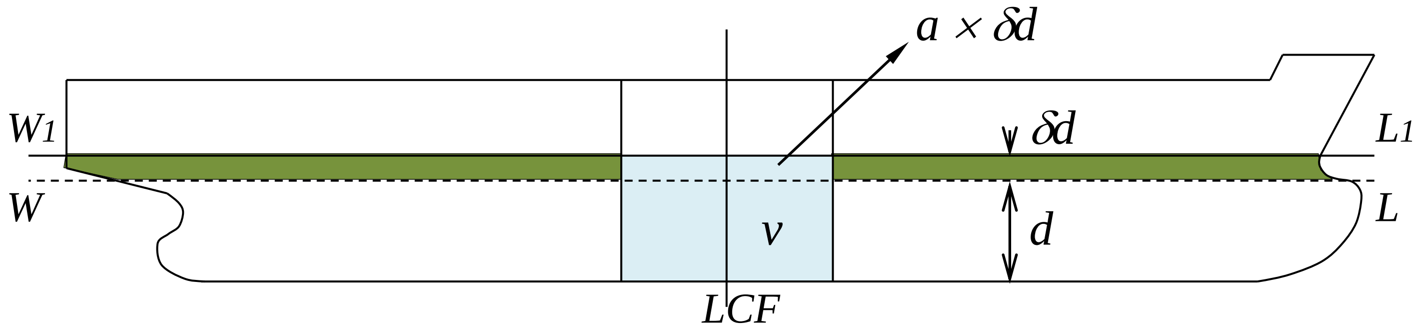
In this method, it is assumed that the flooded compartment has free communication with the sea.

The flooded compartment can be considered a sieve, and that offers no buoyancy to the ship. Only the intact portions of the ship on either side of the flooded compartment contribute to the buoyancy

Since buoyancy has been lost, it must be regained via an increase in the draft.

The ship will sink until the volume of the newly immersed portions equals the volume of the flooded compartment.

The water that enters damaged compartment is considered an still part of the sea, and the buoyancy of the flooded space is lost.
And the loss of buoyancy is regained by an increase of draft.



Loss of buoyancy
(Seawater flooded into damaged compartment is considered as part of the sea)

Loss of buoyancy = Regained buoyancy by the increase of draft

$$\rho \cdot g \cdot v = \rho \cdot g \cdot (A_{WP} - a) \cdot \delta d$$

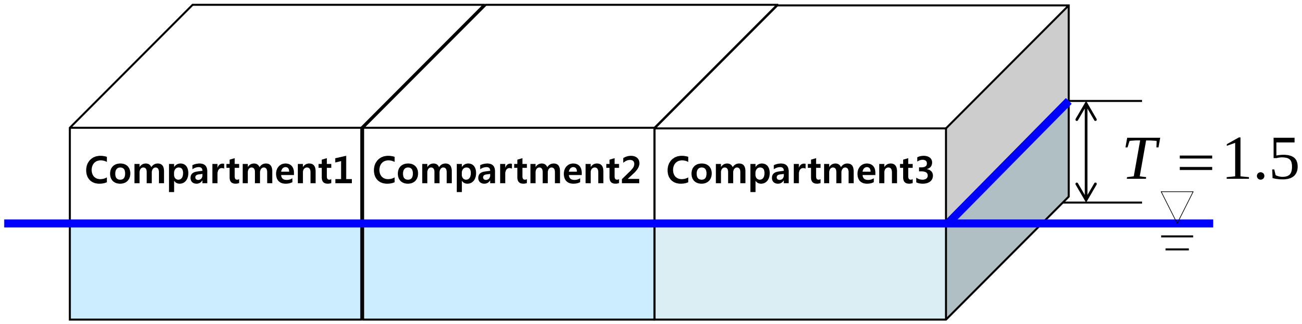
Changed draft due to lost buoyancy

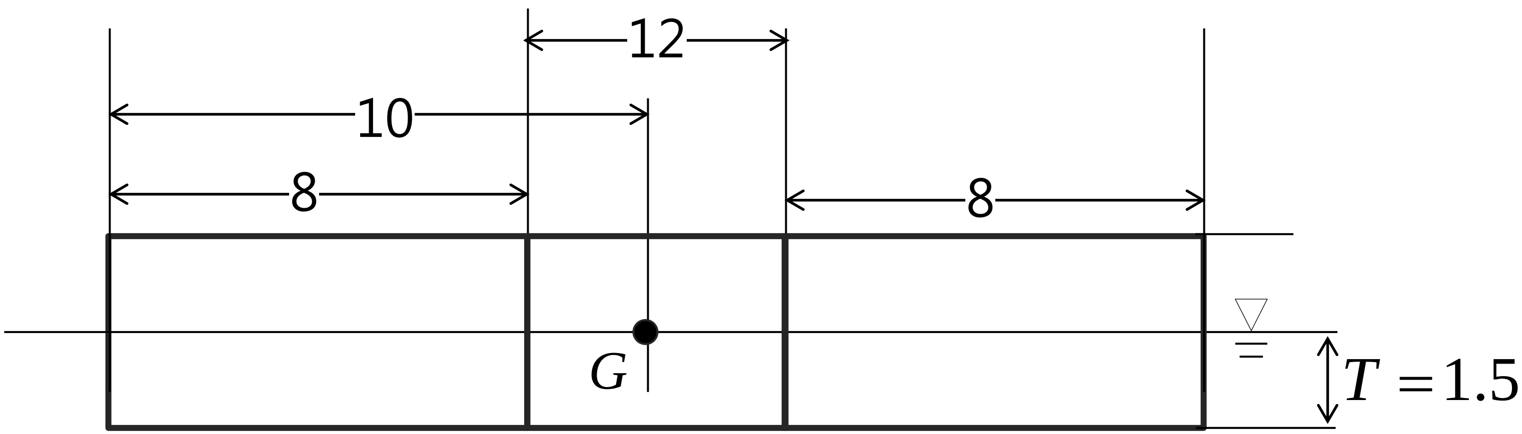
$$\delta d = \frac{v}{A_{WP} - a}$$

A_{WP} : **Waterplane area of the ship**
(Including waterplane area of the damaged compartment)
a : **Waterplane area of the damaged compartment**
d : **Draft before the compartment is not damaged**
δd : **Draft change due to damaged compartment**
v : **Volume of damaged compartment below waterplane**

Example) A compartment of a Box-Shaped Ship is Damaged

✓ **A ship is composed of three compartments.**



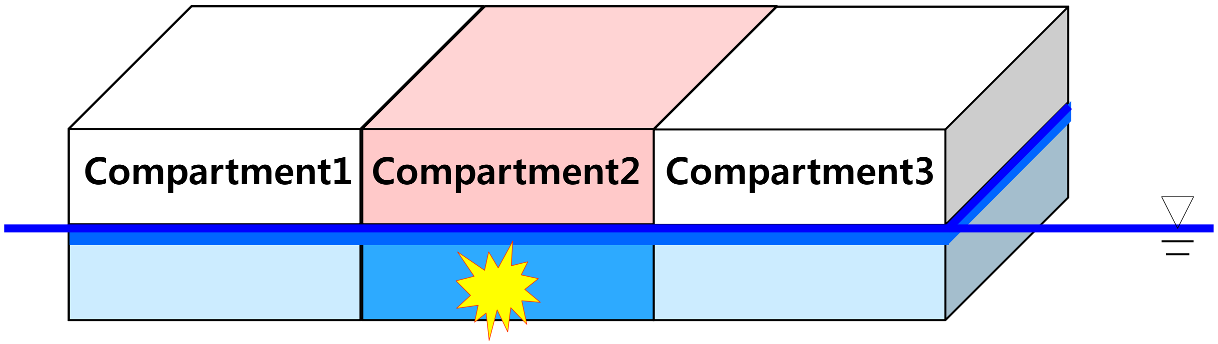


Initial Displacement: $\nabla_I = LBT = 20 \times 5 \times 1.5 = 150m^3$

When a compartment of the ship is damaged, what is the new position of the ship?

Immersion due to the flooding*

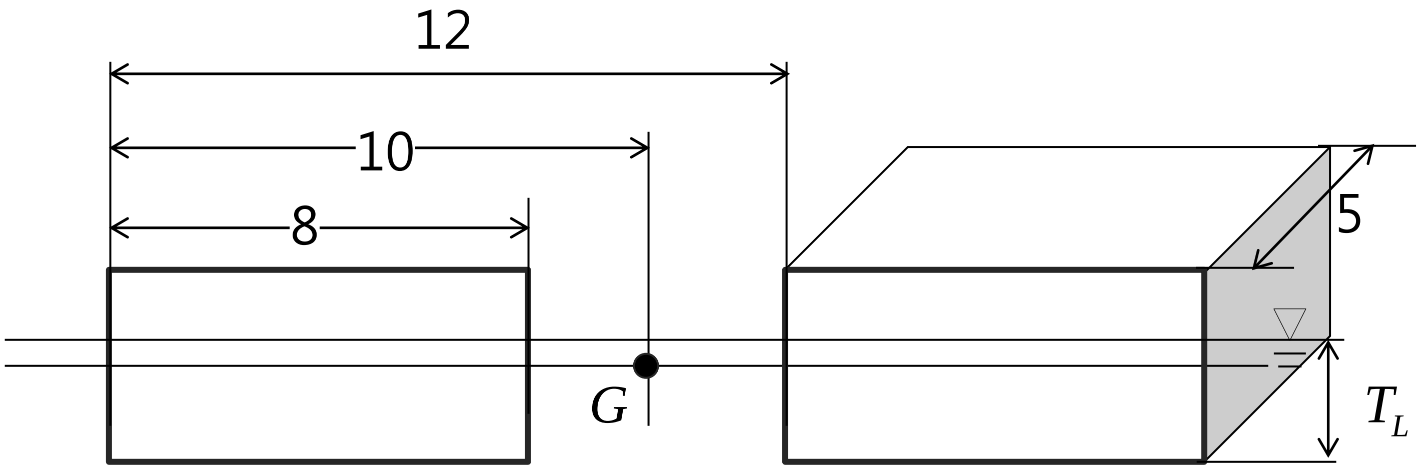
If the compartment in the midship is damaged, what is the new position of the ship?



The position of the ship will be changed.

Immersion

*The new position of the ship can be calculated by **the method of added weight or lost buoyancy**.



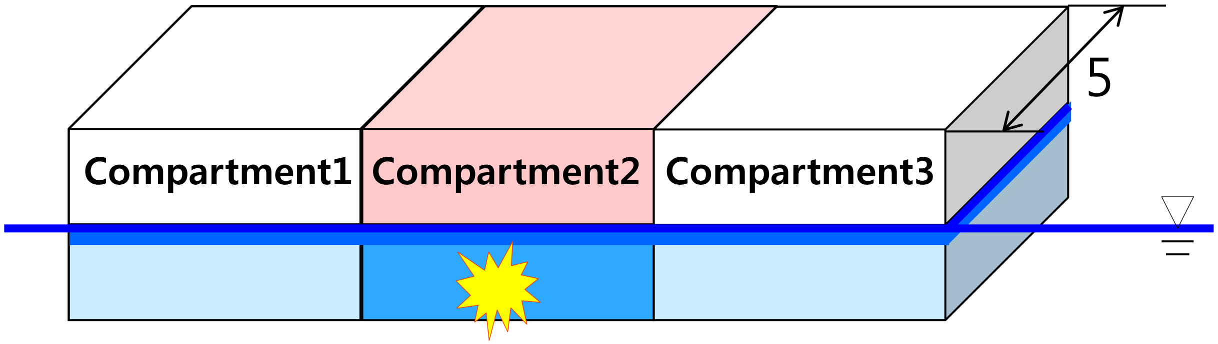
Metacentric radius: $BM_L = \frac{I_L}{\nabla_I} = \frac{166.6667}{150} = 1.111m$

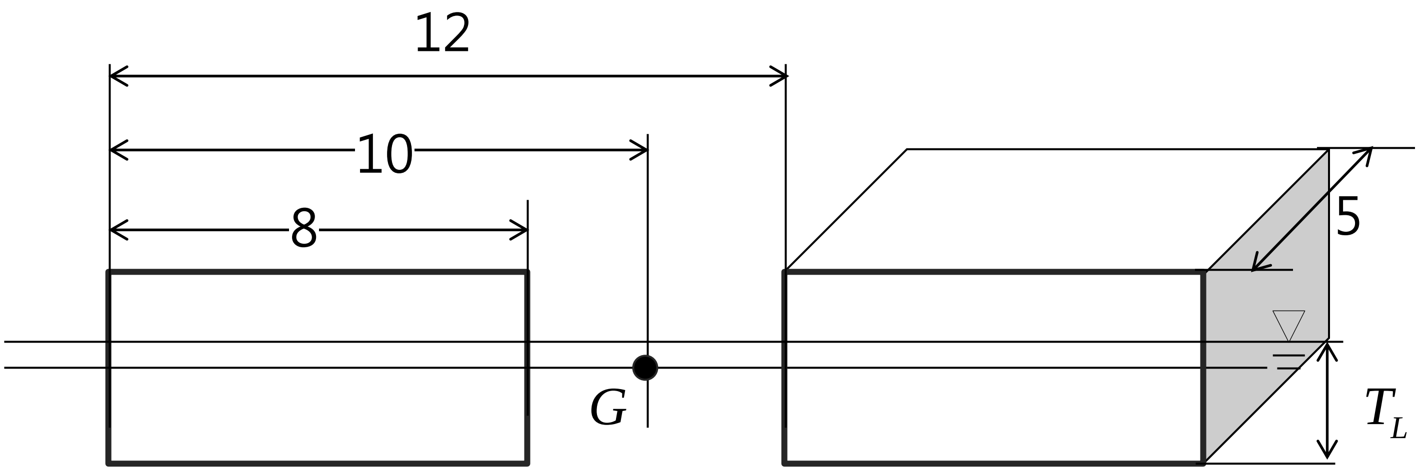
Metacentric Height: $GM_L = KB_L + BM_L - KG = 0.938 + 1.111 - 1.5 = 0.549m$

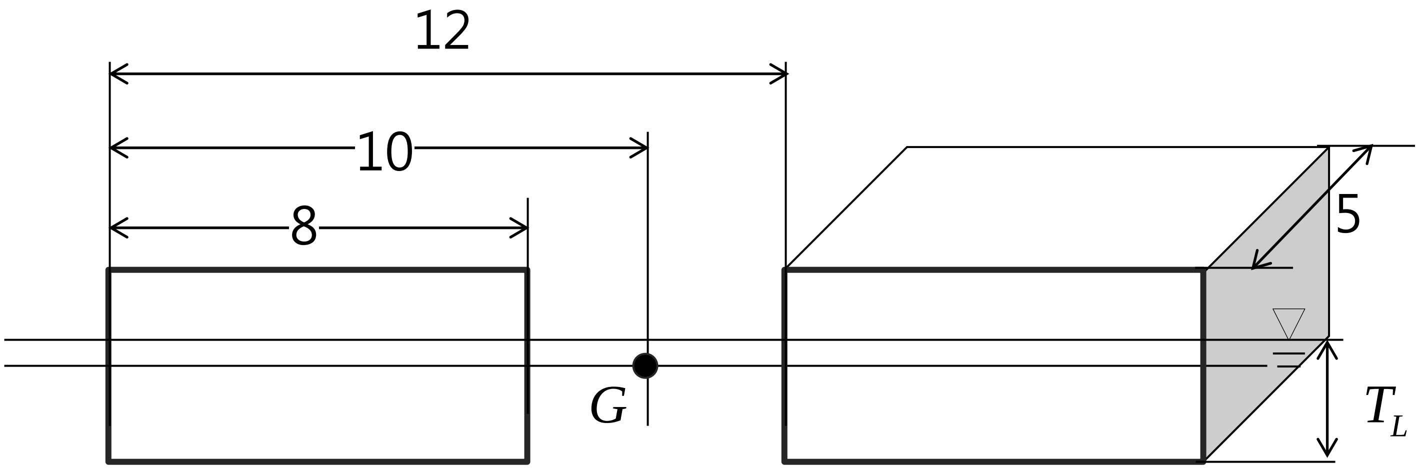
Righting moment for small heel angles, in lost-buoyancy method

$$M_{RL} = \Delta_I GM_L \sin\phi = 153.75 \times 0.549 \sin\phi = 84.349 \sin\phi \ (t \cdot m)$$

If the compartment in the midship is damaged, what is the new position of the ship?







Water plane area: $A_L = (L-l)B = (20-4)\times 5 = 80m^2$

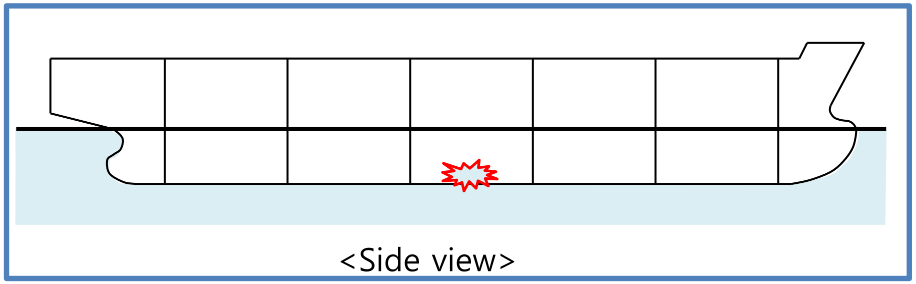
Draft after immersion: $T_L = \dfrac{\nabla_I}{A_L} = \dfrac{150}{80} = 1.875m$, where $\nabla_I = 150$

$$KB_L = \frac{T_L}{2} = \frac{1.875}{2} = 0.938m$$

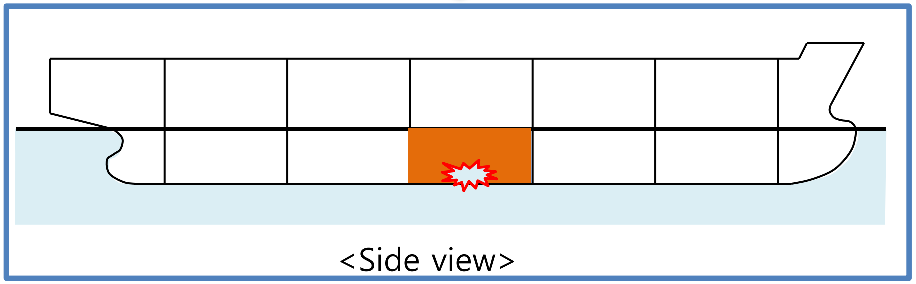
Moment of inertia of the waterplane area about the transverse axis through point G: $I_L = \dfrac{B^3(L-l)}{12} = \dfrac{5^3(20-4)}{12} = 166.6667m^4$

ADDED WEIGHT METHOD

Calculation Method 2.
: Added Weight Method

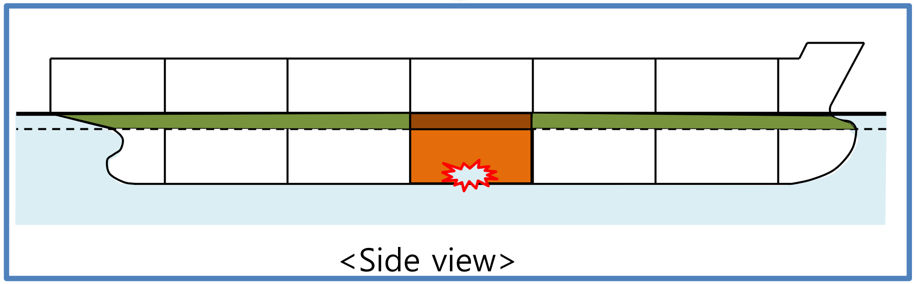


A damage occurs



Flooded water is considered as the added weight

: Added weight



Added weight will be equilibrium with the buoyancy regained by an increase of draft

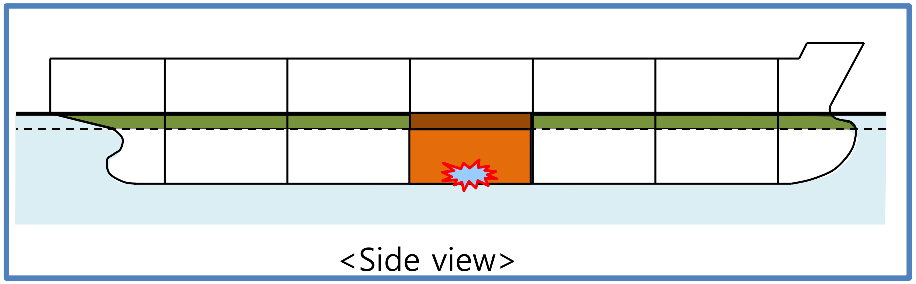
: Additional added weight

: Additional volume which contribute buoyancy

Added weight method

The water that enters damaged compartment is considered an **added weight** with no loss of buoyancy.

Added weight method



The water that enters damaged compartment is considered an **added weight** with no loss of buoyancy.

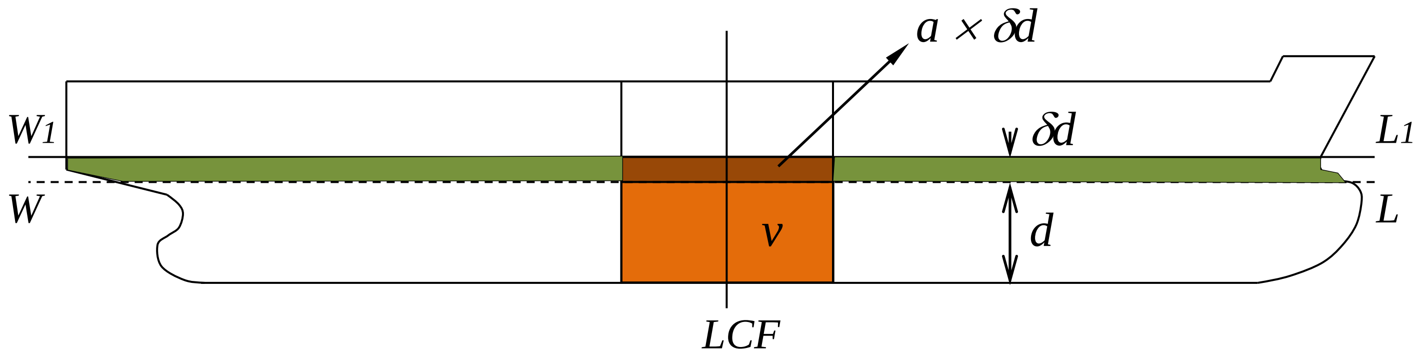
This is a **misnomer**, since water in space open to the sea and free to run in or out does not actually add to a ship's weight.

For calculation purposes, it is **convenient** to regard such flooding water as adding to the displacement.

However, it must be remembered that the resulting (virtual) displacement no only differ from the initial displacement, but varies with change in trim or heel.

Since the added weight method involves a direct integration of volumes up to the damaged condition waterplane, it is just as well adapted to dealing with complex flooding conditions as with simple ones.

"The water that enters damaged compartment is considered an added weight with no loss of buoyancy."



Weight of seawater due to damaged compartment $w = \rho \cdot g \cdot (v + a \cdot \delta d)$

Increased buoyancy due to the change in draft $b = \rho \cdot g \cdot (A_{WP} \cdot \delta d)$

$w = b$

$$\rho \cdot g \cdot (v + a \cdot \delta d) = \rho \cdot g \cdot (A_{WP} \cdot \delta d)$$

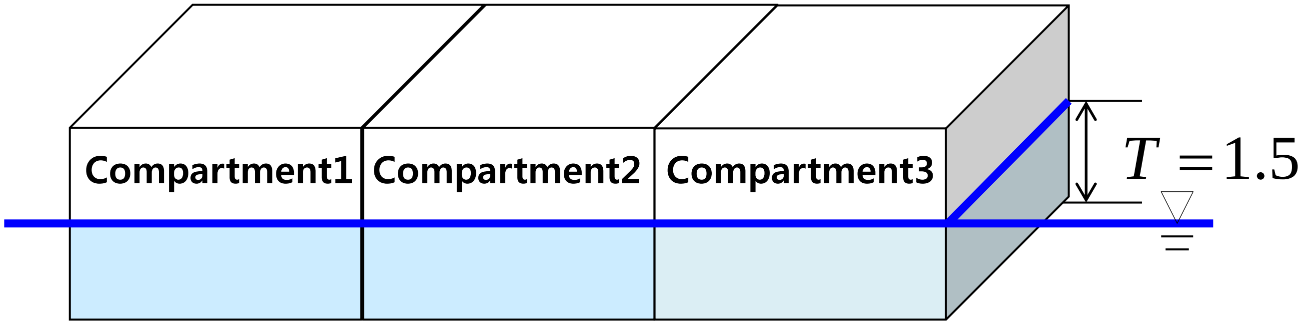
The changed draft due to compensate weight of damaged compartment

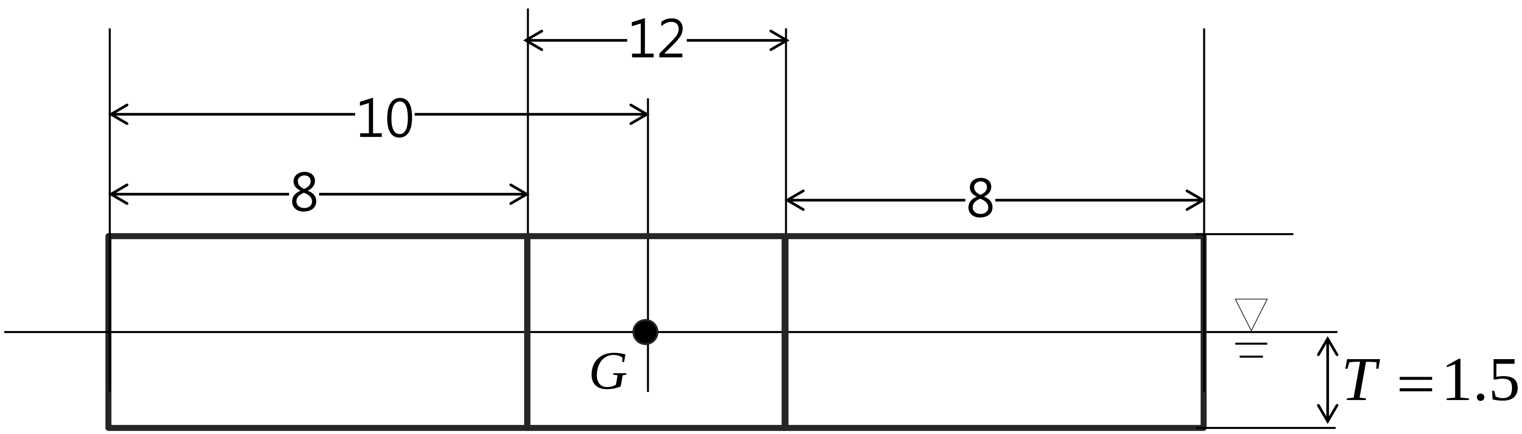
$$\delta d = \frac{v}{A_{WP} - a}$$

A_{WP} : Waterplane area of the ship (Including waterplane area of the damaged compartment)
a : Waterplane area of the damaged compartment
d : Draft before the compartment is not damaged
δd : Draft change due to damaged compartment
v : Volume of damaged compartment below waterplane

Example) A Compartment of a Box-Shaped Ship is damaged

- ✓ **A ship is composed of three compartments.**



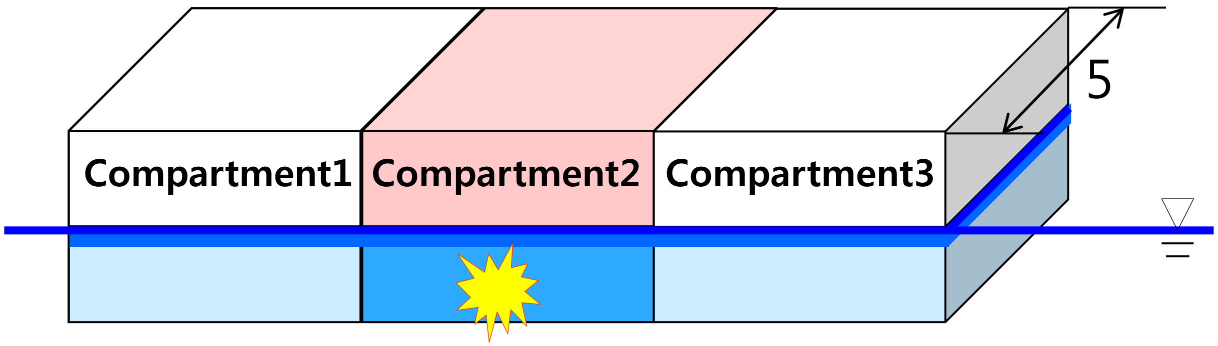


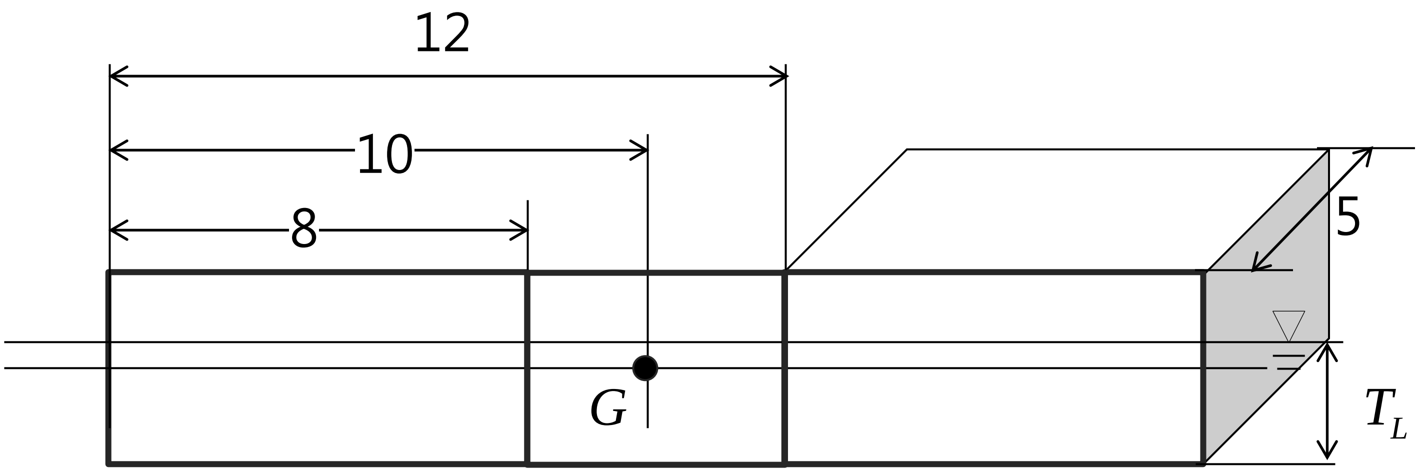
Initial Displacement: $\nabla_I = LBT = 20 \times 5 \times 1.5 = 150 m^3$

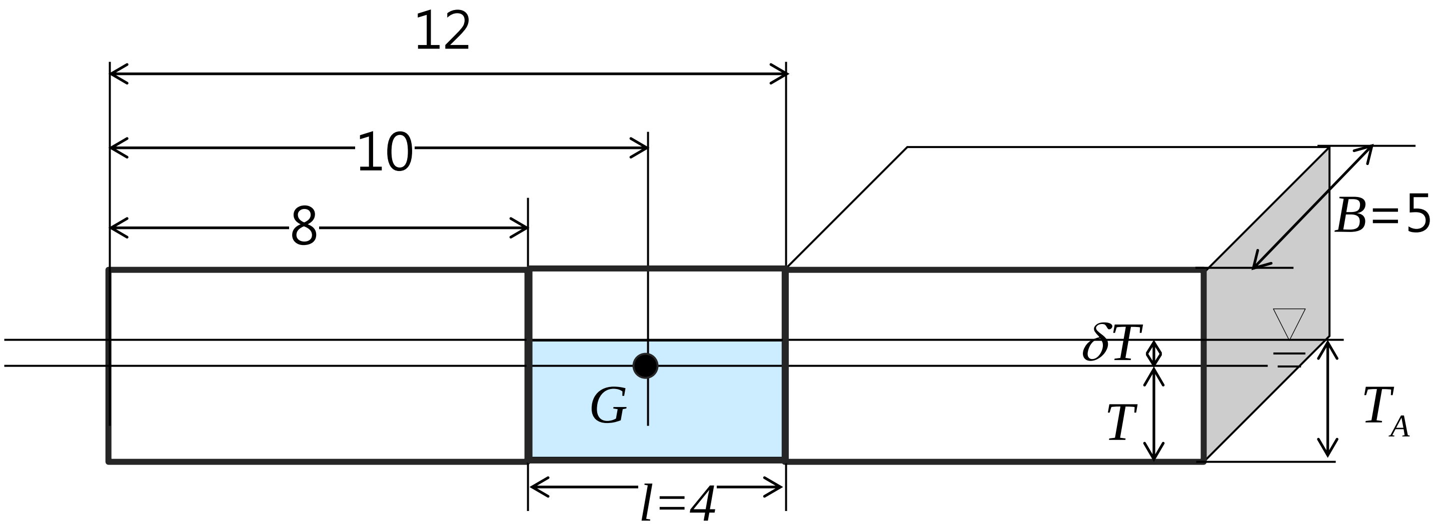
When a compartment of the ship is damaged, what is the new position of the ship?

Immersion due to the flooding

If the compartment in the midship is damaged, what is the new position of the ship?







The volume of flooding water: $v = lBT_A = lB(T + \delta T)$

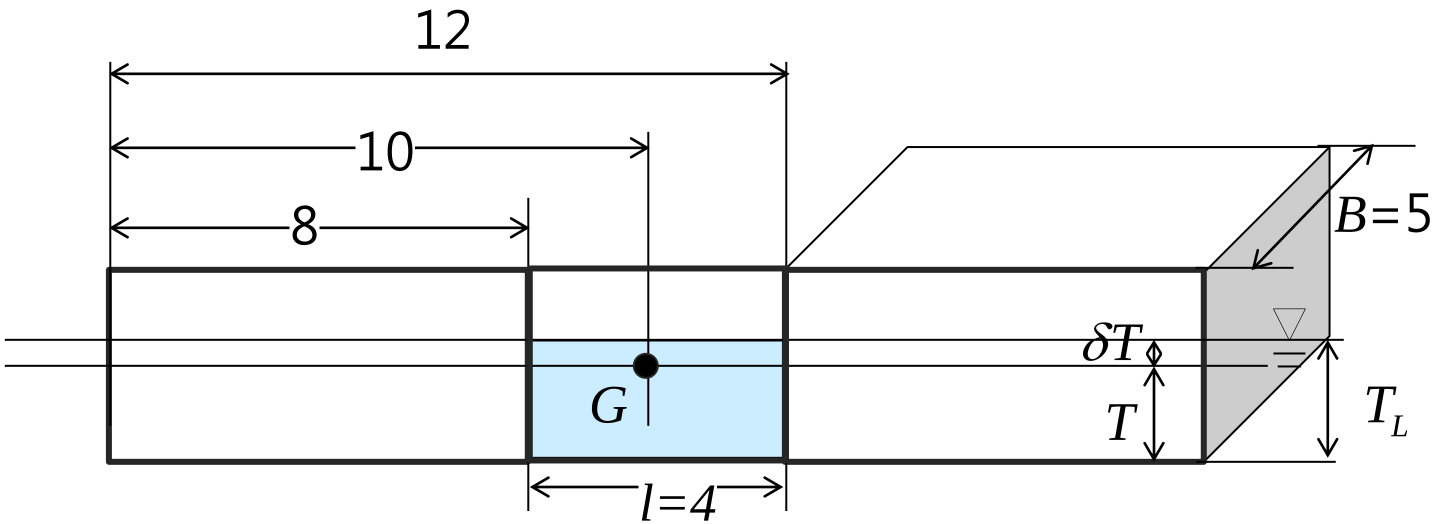
The additional buoyant volume: $\delta\nabla = LB\delta T$

Because $v = \delta\nabla$,

$$lB(T + \delta T) = LB\delta T$$

$$l(T + \delta T) = L\delta T$$

$$lT = (L - l)\delta T \qquad \delta T = \frac{lT}{L - l} = \frac{4 \times 1.5}{20 - 4} = 0.375\ m$$

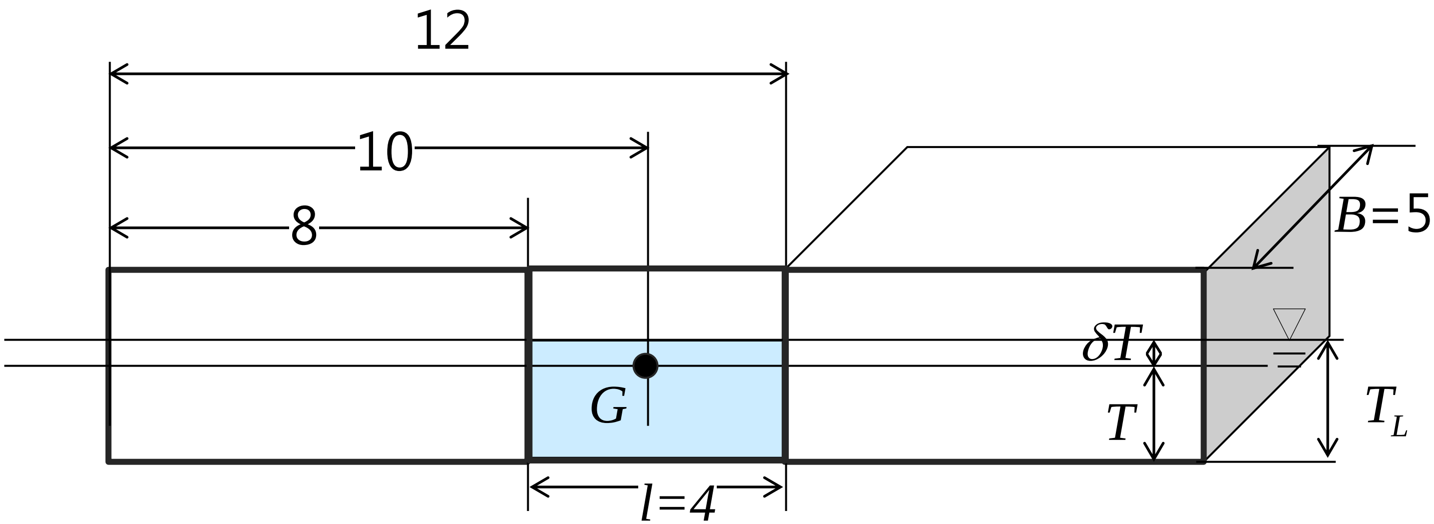


The draft after flooding: $T_A = T + \delta T$
$= 1.500 + 0.375 = 1.875\ m$

The volume of flooding water : $v = lBT_A = 4 \times 5 \times 1.875 = 37.5\ m^3$

The height of its centre of gravity: $kb = \dfrac{T_A}{2} = \dfrac{1.875}{2} = 0.938m$

The displacement volume of the flooded pontoon: $\nabla_A = LBT_A = 20 \times 5 \times 1.875 = 187.5\ m^3$

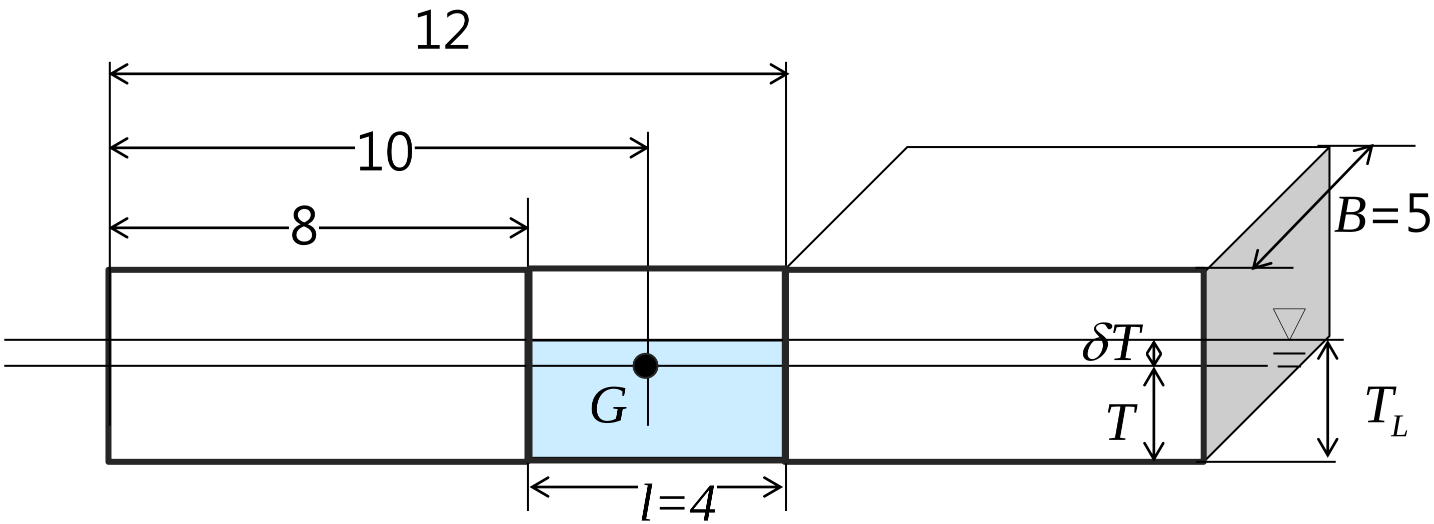


KG by the method of added weight:

	Volume	Centre of gravity	Moment
Initial	150.0	1.5	225.000
Added	37.5	0.938	35.156
Total	187.5	1.388	260.156

Moment of inertia of the waterplane area about the transverse axis through point G: $I_A = \dfrac{B^3 L}{12} = \dfrac{5^3 \times 20}{12} = 208.333\ m^4$

Metacentric radius: $BM_A = \dfrac{I_A}{\nabla_A} = \dfrac{208.333}{187.5} = 1.111\ m$

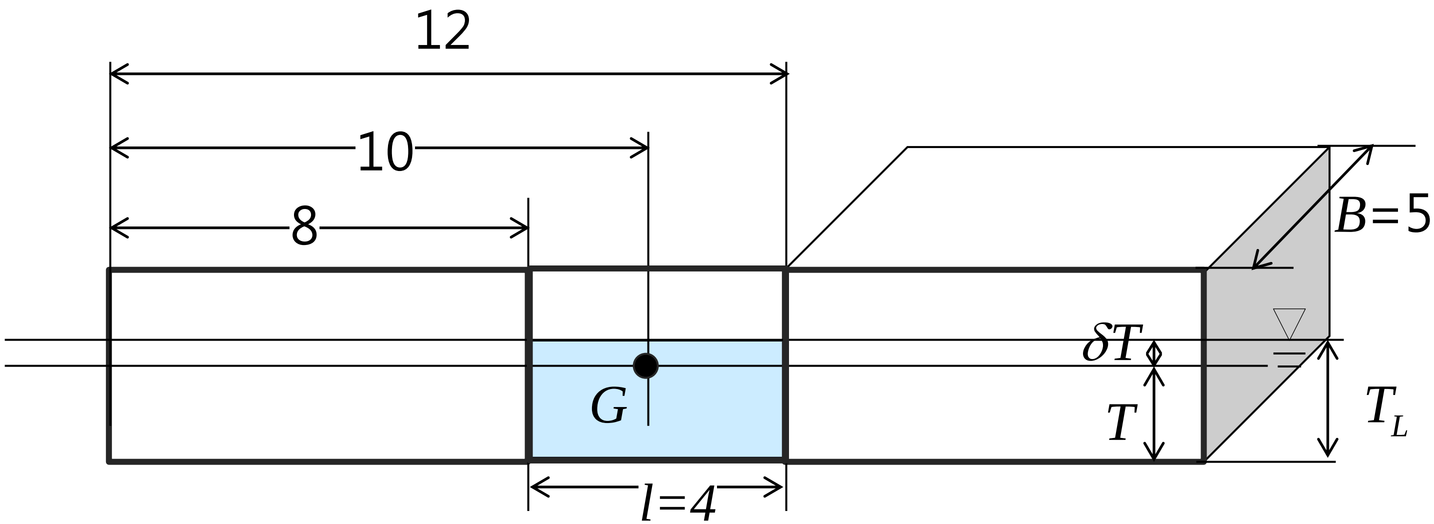


Free surface effect caused by the flooding water:

The moment of inertia of the free surface in the flooded compartment: $i = \frac{B^3 l}{12} = \frac{5^3 \times 4}{12} = 41.667\ m^4$

The lever arm of the free surface effect: $l_F = \frac{\rho i}{\rho \nabla_A} = \frac{41.667}{187.5} = 0.222\ m$

Free surface effect caused by the flooding water: $KB_A = \frac{T_A}{2} = \frac{1.875}{2} = 0.938m$



Metacentric height: $GM_A = KB_A + BM_A - KG_A - l_F$
$= 0.938 + 1.111 - 1.388 - 0.222 = 0.439\ m$

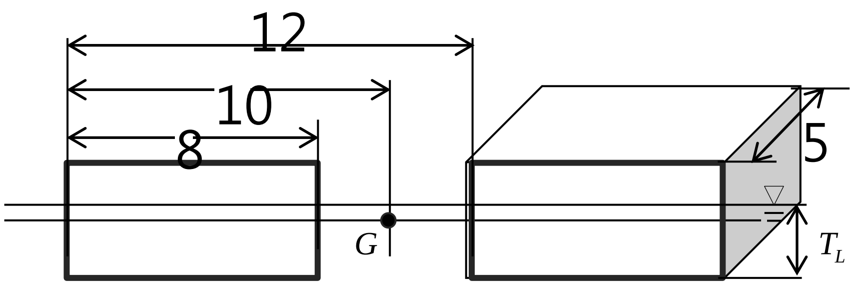
Mass displacement: $\Delta_A = \rho \nabla_A = 1.025 \times 187.5 = 192.188\ \ ton$

Righting moment for small angles of trim, in the added weight method:

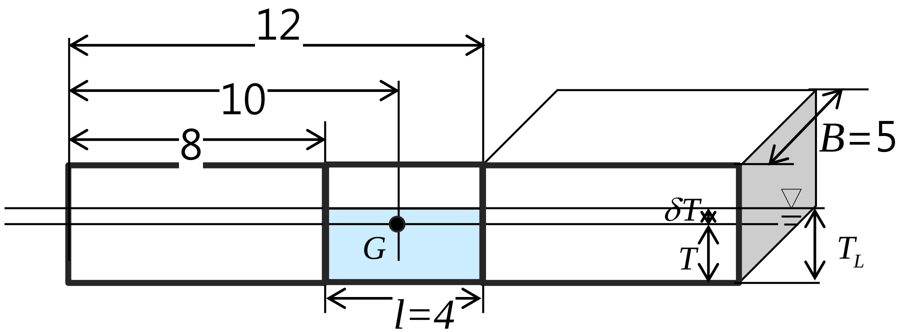
$$M_{RA} = \Delta_A GM_A \sin\theta = 192.188 \times 0.439 \sin\theta = 84.349 \sin\phi\ \ (t \cdot m)$$

Comparison of the results by two methods

Results by the lost buoyancy method



Results by the added weight method



	Intact condition	lost buoyancy method	added weight method
Draft, m	1.500	1.875	1.875
∇_L, m^3	150.000	150.000	187.500
Δ_L, ton	153.750	153.750	192.188
KB, m	0.750	0.938	0.938
BM, m	1.389	1.111	1.111
KG, m	1.500	1.500	1.388
GM, m	0.639	0.549	0.439
ΔGM, $t \cdot m$	98.229	84.349	84.349

12-3 Governing Equations of Computational Ship Stability in Flooded State

Governing Equations of Computational Ship Stability

When the ship is in intact state.

$$
\begin{bmatrix} \cancel{F_z^*} - F_z(z_{n0},\phi_0,\theta_0) \\ \cancel{M_T^*} - M_T(z_{n0},\phi_0,\theta_0) \\ \cancel{M_L^*} - M_L(z_{n0},\phi_0,\theta_0) \end{bmatrix} =
\begin{bmatrix}
-\rho g A_{WP} & -\rho g A_{WP}\,{}^{n}y_{F/E} & \rho g A_{WP}\,{}^{n}x_{F/E} \\
-\rho g A_{WP}\,{}^{n}y_{F/E} & \begin{matrix} -\rho g\left({}^{n}z_{B/E}\nabla + I_T\right) \\ -{}^{n}z_{G/E}\cdot F_G - {}^{n}z_{G_{ext}/E}\cdot F_{ext} \end{matrix} & \rho g I_P \\
\rho g A_{WP}\,{}^{n}x_{F/E} & \rho g I_P & \begin{matrix} -\rho g\left({}^{n}z_{B/E}\nabla + I_L\right) \\ -{}^{n}z_{G/E}\cdot F_G - {}^{n}z_{ext./E}\cdot F_{ext} \end{matrix}
\end{bmatrix}_{\substack{z_n=z_{n0}\\ \phi=\phi_0 \\ \theta=\theta_0}}
\begin{bmatrix} \delta z_n \\ \delta\phi \\ \delta\theta \end{bmatrix}
$$

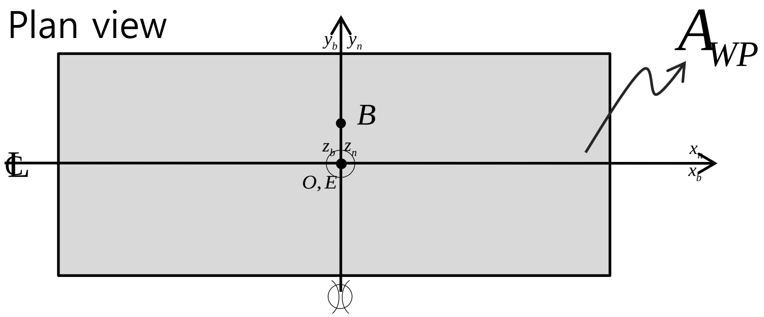
$= 0$

We want to find the static equilibrium position and orientation!

When the ship is in intact state, the waterplane area is as follows.

$$A_{WP}$$

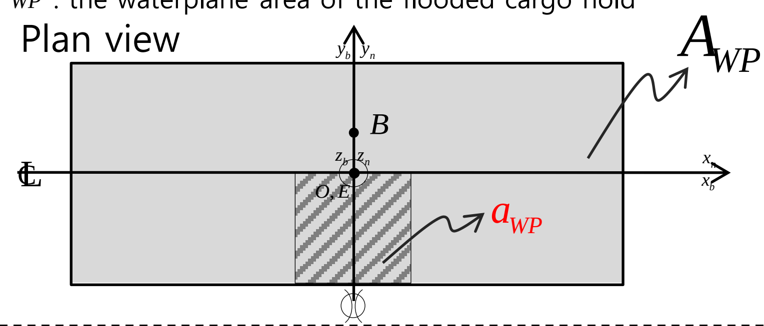
A_{WP} : the waterplane area of the intact ship



When the ship is flooded, the water plane area changes from A_{WP} to $A_{WP} - a_{WP}$.

A_{WP} : the waterplane area of the intact ship

a_{WP} : the waterplane area of the flooded cargo hold



In the same manner, the transverse inertia I_T, longitudinal inertia I_L and product of inertia of the waterplane area I_P change to $I_T - i_T$, $I_L - i_L$ and $I_P - i_P$ respectively.

i_T: The moment of inertia of the waterplane area of the flooded cargo hold about x_n axis

i_L: The moment of inertia of the waterplane area of the flooded cargo hold about y_n axis

i_P: The centrifugal moment of the waterplane area of the flooded cargo hold about x_n and y_n

Governing Equations of Computational Ship Stability in Intact State

When the ship is in intact state.

$$\begin{bmatrix} \cancel{F_z^*} - F_z(z_{n0},\phi_0,\theta_0) \\ \cancel{M_T^*} - M_T(z_{n0},\phi_0,\theta_0) \\ \cancel{M_L^*} - M_L(z_{n0},\phi_0,\theta_0) \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP} & -\rho g A_{WP}\,{}^{n}y_{F/E} & \rho g A_{WP}\,{}^{n}x_{F/E} \\ -\rho g A_{WP}\,{}^{n}y_{F/E} & -\rho g\left({}^{n}z_{B/E}\nabla + I_T\right) - {}^{n}z_{G/E}\cdot F_G - {}^{n}z_{G_{ext}/E}\cdot F_{ext} & \rho g I_P \\ \rho g A_{WP}\,{}^{n}x_{F/E} & \rho g I_P & -\rho g\left({}^{n}z_{B/E}\nabla + I_L\right) - {}^{n}z_{G/E}\cdot F_G - {}^{n}z_{ext./E}\cdot F_{ext} \end{bmatrix}_{\substack{z_n = z_{n0} \\ \phi=\phi_0 \\ \theta=\theta_0}} \begin{bmatrix} \delta z_n \\ \delta\phi \\ \delta\theta \end{bmatrix}$$

$= 0$

We want to find the static equilibrium position and orientation!

Intact State $\Rightarrow$ **After flooding**

$$A_{WP}, I_T, I_L, I_P \quad\Rightarrow\quad A_{WP} - a_{WP}, I_T - i_T, I_L - i_L, I_P - i_P$$

i_T: The moment of inertia of the waterplane area of the flooded cargo hold about x_n axis

i_L: The moment of inertia of the waterplane area of the flooded cargo hold about y_n axis

i_P: The centrifugal moment of the waterplane area of the flooded cargo hold about x_n and y_n

Governing Equations of Computational Ship Stability in Flooded State

When the ship is flooded.

$$
\begin{bmatrix} \cancel{F_z^*} - F_z(z_{n0},\phi_0,\theta_0) \\ \cancel{M_T^*} - M_T(z_{n0},\phi_0,\theta_0) \\ \cancel{M_L^*} - M_L(z_{n0},\phi_0,\theta_0) \end{bmatrix} =
\begin{bmatrix}
-\rho g A_{WP} + \rho g a_{WP} & -\rho g A_{WP}\,{}^n y_{F/E} + \rho g a_{WP}\,{}^n y_{f/E} & \rho g A_{WP}\,{}^n x_{F/E} - \rho g a_{WP}\,{}^n x_{f/E} \\
-\rho g A_{WP}\,{}^n y_{F/E} + \rho g a_{WP}\,{}^n y_{f/E} & \begin{array}{c} -\rho g\left({}^n z_{B/E}\nabla + I_T\right) \\ -{}^n z_{G/E}\cdot F_G - {}^n z_{G_{ext}/E}\cdot F_{ext} + \rho g i_T \end{array} & \rho g I_P - \rho g i_P \\
\rho g A_{WP}\,{}^n x_{F/E} - \rho g a_{WP}\,{}^n x_{f/E} & \rho g I_P - \rho g i_P & \begin{array}{c} -\rho g\left({}^n z_{B/E}\nabla + I_L\right) \\ -{}^n z_{G/E}\cdot F_G - {}^n z_{ext./E}\cdot F_{ext} + \rho g i_L \end{array}
\end{bmatrix}_{\substack{z_n = z_{n0} \\ \phi=\phi_0 \\ \theta=\theta_0}}
\begin{bmatrix} \delta z_n \\ \delta\phi \\ \delta\theta \end{bmatrix}
$$

$= 0$

We want to find the static equilibrium position and orientation!

F_G : the gravitational force exerted on the ship
M_T : the transverse moment of the ship about x_n axis through point E
M_L : the longitudinal moment of the ship about y_n axis through point E
A_{WP} : the waterplane area of the ship at current position
I_T : the transverse moment of inertia of the waterplane area of the ship about x_n axis through point E
I_L : the longitudinal moment of inertia of the waterplane area of the ship about y_n axis through point E
I_P : the centrifugal moment of the waterplane area of the ship about x_n and y_n axis through point E
F_B : the buoyant force exerted on the ship
F_{ext} : the external force exerted on the ship

x_F : the centroid of the waterplane area of the ship in x_n direction
y_F : the centroid of the waterplane area of the ship in y_n direction
z_B : the center of the displaced volume of the ship in z_n direction
z_G : the center of mass of the ship in z_n direction
δz_n : the change in draft
$\delta\phi$: the change in angle of heel
$\delta\theta$: the change in angle of trim

a_{WP} : the waterplane area of the flooded cargo hold
i_T : the transverse moment of inertia of the waterplane area of the flooded cargo hold about x_n axis through point E
i_L : the longitudinal moment of inertia of the waterplane area of the flooded cargo hold about y_n axis through point E
i_P : the centrifugal moment of the waterplane area of the flooded cargo hold about x_n and y_n axis through point E
x_f : the centroid of the waterplane area of the flooded cargo hold in x_n direction
y_f : the centroid of the waterplane area of the flooded cargo hold in y_n direction
z_{ext} : the center of the submerged volume of the flooded cargo hold in z_n direction

12-4 Coupled Immersion and Heel of a Box-Shaped Ship in Flooded State

Governing Equations of Computational Ship Stability in **Flooded State**

$$
\begin{bmatrix} \cancel{F_z} - F_z\left(z_n^{(k)},\phi^{(k)},\theta^{(k)}\right) \\ \cancel{M_T} - M_T\left(z_n^{(k)},\phi^{(k)},\theta^{(k)}\right) \\ \cancel{M_L} - M_L\left(z_n^{(k)},\phi^{(k)},\theta^{(k)}\right) \end{bmatrix} =
\begin{bmatrix}
\frac{\partial F_B}{\partial z_n}+\frac{\partial F_G}{\partial z_n}+\frac{\partial F_{ext}}{\partial z_n} & \frac{\partial F_B}{\partial \phi}+\frac{\partial F_G}{\partial \phi}+\frac{\partial F_{ext}}{\partial \phi} & \frac{\partial F_B}{\partial \theta}+\frac{\partial F_G}{\partial \theta}+\frac{\partial F_{ext}}{\partial \theta} \\
\boxed{-\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right)} & \boxed{-\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right)} & \boxed{\rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E}} \\
\frac{\partial M_{BT}}{\partial z_n}+\frac{\partial M_{GT}}{\partial z_n}+\frac{\partial M_{extT}}{\partial z_n} & \frac{\partial M_{BT}}{\partial \phi}+\frac{\partial M_{GT}}{\partial \phi}+\frac{\partial M_{extT}}{\partial \phi} & \frac{\partial M_{BT}}{\partial \theta}+\frac{\partial M_{GT}}{\partial \theta}+\frac{\partial M_{extT}}{\partial \theta} \\
\boxed{-\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right)} & \boxed{\begin{gathered} -\rho g\left({}^n z_{B^{(k)}/E}\nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E}\cdot F_G \\ - {}^n z_{G_{ext}^{(k)}/E}\cdot F_{ext}^{(k)} - \left(-\mu_F \cdot \rho g i_T^{(k)}\right) \end{gathered}} & \boxed{\rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)}} \\
\frac{\partial M_{BL}}{\partial z_n}+\frac{\partial M_{GL}}{\partial z_n}+\frac{\partial M_{extL}}{\partial z_n} & \frac{\partial M_{BL}}{\partial \phi}+\frac{\partial M_{GL}}{\partial \phi}+\frac{\partial M_{extL}}{\partial \phi} & \frac{\partial M_{BL}}{\partial \theta}+\frac{\partial M_{GL}}{\partial \theta}+\frac{\partial M_{extL}}{\partial \theta} \\
\boxed{\rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E}} & \boxed{\rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)}} & \boxed{\begin{gathered} -\rho g\left({}^n z_{B^{(k)}/E}\nabla^{(k)} + I_L^{(k)}\right) - {}^n z_{G^{(k)}/E}\cdot F_G \\ - {}^n z_{G_{ext}^{(k)}/E}\cdot F_{ext}^{(k)} - \left(-\mu_F \cdot \rho g i_L^{(k)}\right) \end{gathered}}
\end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi=\phi^{(k)} \\ \theta=\theta^{(k)}}}
\begin{bmatrix} \delta z_n^{(k)} \\ \delta\phi^{(k)} \\ \delta\theta^{(k)} \end{bmatrix}
$$

$= 0$

We want to find the static equilibrium position and orientation!

F_G: gravitational force exerted on a ship

M_T: transverse moment of a ship about x_n axis

M_L: longitudinal moment of a ship about y_n axis

$A_{WP}^{(k)}$: waterplane area of a ship at k[th] step

$I_T^{(k)}$: transverse moment of inertia of the waterplane area of a ship about x_n axis at k[th] step

$I_L^{(k)}$: longitudinal moment of inertia of the waterplane area of a ship about y_n axis at k[th] step

$I_P^{(k)}$: centrifugal moment of the waterplane area of a ship about x_n and y_n axis at k[th] step

F_B: buoyant force exerted on a ship

F_{ext}: external force exerted on a ship

${}^n x_{F^{(k)}/E}$: x_n coordinate of centroid of the waterplane area of a ship

${}^n y_{F^{(k)}/E}$: y_n coordinate of centroid of the waterplane area of a ship

${}^n z_{B^{(k)}/E}$: z_n coordinate of center of the displaced volume of a ship

${}^n z_{G^{(k)}/E}$: z_n coordinate of center of mass of the ship

$\delta z^{(k)}$: change in the draft at k[th] step

$\delta\phi^{(k)}$: change in the angle of heel at k[th] step

$\delta\theta^{(k)}$: change in the angle of trim at k[th] step

μ_V: permeability of a compartment

μ_F: surface permeability of a compartment

$a_{WP}^{(k)}$: waterplane area of a flooded compartment at k[th] step

$i_T^{(k)}$: transverse moment of inertia of the waterplane area of a flooded compartment about x_n axis at k[th] step

$i_L^{(k)}$: longitudinal moment of inertia of the waterplane area of a flooded compartment about y_n axis at k[th] step

$i_P^{(k)}$: centrifugal moment of the waterplane area of a flooded compartment about x_n and y_n axis at k[th] step

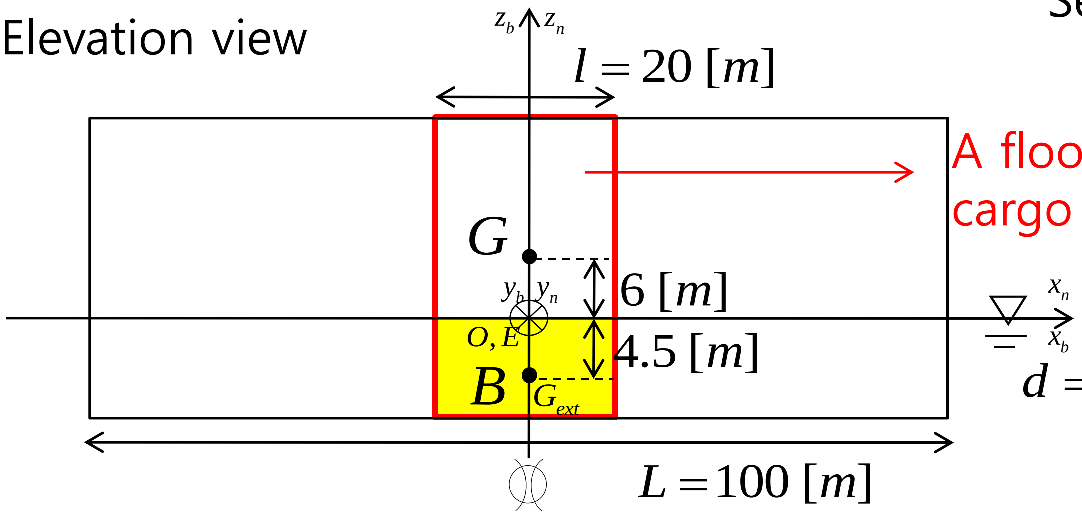
${}^n x_{f^{(k)}/E}$: x_n coordinate of centroid of the waterplane area of a flooded compartment at k[th] step

${}^n y_{f^{(k)}/E}$: y_n coordinate of centroid of the waterplane area of a flooded compartment at k[th] step

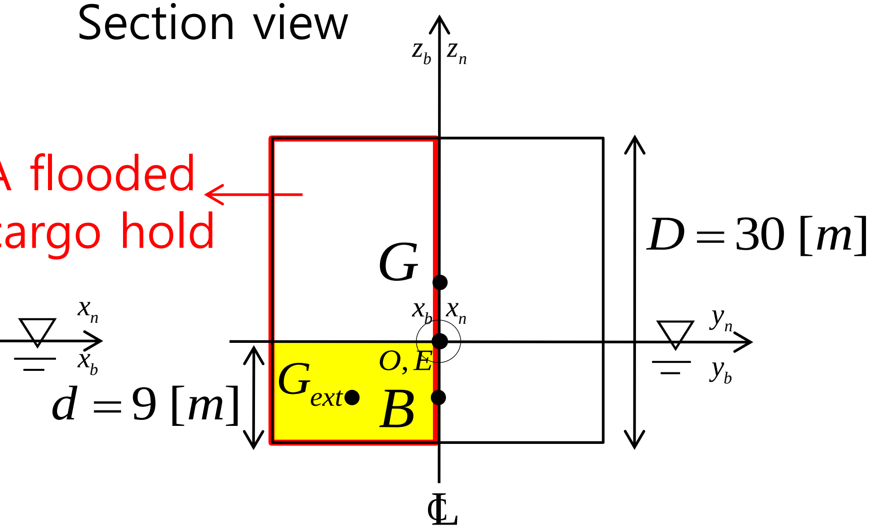
${}^n z_{G_{ext}^{(k)}/E}$: z_n coordinate of center of the submerged volume of a flooded compartment at k[th] step

Example of Coupled Immersion and Heel of a Box Shaped Ship in Flooded State - Problem Definition

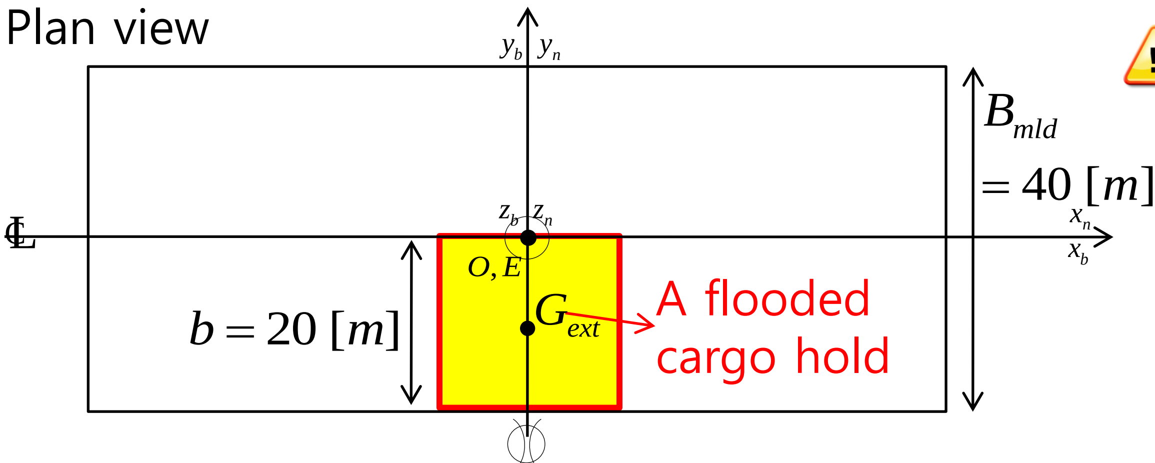
Elevation view



Section view



Plan view



In this case, **the external force** is given as a weight of the water in the flooded cargo hold which is not

$L = 100\ [m]$	${}^{n}\mathbf{r}_{G/E} = [0\ \ 0\ \ 6]^T\ [m]$
$B_{mld} = 40\ [m]$	${}^{n}\mathbf{r}_{B/E} = [0\ \ 0\ \ -4.5]^T\ [m]$
$D = 30\ [m]$	$l = 20\ [m]$
$d = 9\ [m]$	$b = 20\ [m]$

$$F_{G,z} = -3.6 \times 10^5\ [kN]$$

$$\rho g = 10\ [Mg / m^2 s^2]$$

n-frame : waterplane fixed reference frame (Inertial reference frame), ($x_n y_n z_n$ axis)

b-frame : body-fixed reference frame, ($x_b y_b z_b$ axis)

F_G : the gravitational force exerted on the ship, that is weight of the ship

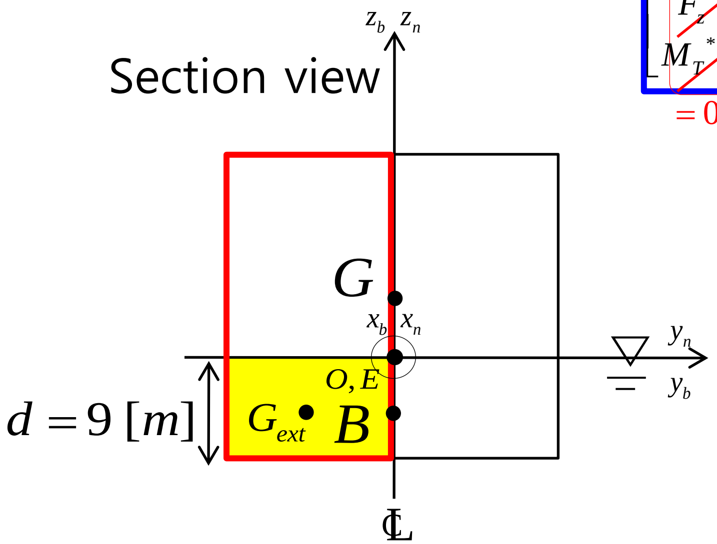
F_B : the buoyant force exerted on the ship (initial state)

F_{ext} : the external force exerted on the ship

1. Calculation of Forces and Moments at k=0 step

$$\begin{bmatrix} \cancel{F_z^{*}} - F_z^{(k)} \\ \cancel{M_T^{*}} - M_T^{(k)} \end{bmatrix} = \left. \begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) \\ -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) & -\rho g \left({}^{n}z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^{n}z_{G^{(k)}/E} \cdot F_{G,z} - {}^{n}z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\rho g i_T^{(k)}\right) \end{bmatrix} \right|_{\substack{z_n = z_n^{(0)} \\ \phi = \phi^{(0)} \\ \theta = \theta^{(0)}}} \begin{bmatrix} \delta z_n^{(k)} \\ \delta \phi^{(k)} \end{bmatrix}$$

$= 0$



$L = 100\ [m]$	${}^{n}\mathbf{r}_{G/E} = [0 \quad 0 \quad 6]^T\ [m]$
$B_{mld} = 40\ [m]$	${}^{n}\mathbf{r}_{B/E} = [0 \quad 0 \quad -4.5]^T\ [m]$
$D = 30\ [m]$	${}^{n}\mathbf{r}_{G_{ext}/E} = [0 \quad -10 \quad -4.5]^T\ [m]$
$d = 9\ [m]$	$l = 20\ [m] \quad b = 20\ [m]$

$F_{G,z} = -3.6 \times 10^5\ [kN]$

$\rho g = 10\ [Mg / m^2 s^2]$

$F_{ext,z} = -3.6 \times 10^4$ (Lost Buoyancy)

$$\begin{aligned} F_z &= F_{B,z} + F_{G,z} + F_{ext,z} \\ &= \rho g L B_{mld} d + F_{G,z} + F_{ext,z} \\ &= \rho g L B_{mld} d + F_G - \rho g l b d \\ &= 10 \cdot 100 \cdot 40 \cdot 9 - 3.6 \times 10^5 - 10 \cdot 20 \cdot 20 \cdot 9 \\ &= \boxed{3.6 \times 10^5 - 3.6 \times 10^5} - 3.6 \times 10^4 \\ &= \underline{-3.6 \times 10^4\ [kN]} \end{aligned}$$

Because the ship is in initial static equilibrium, the buoyant force is equal and opposite to the gravitational force.

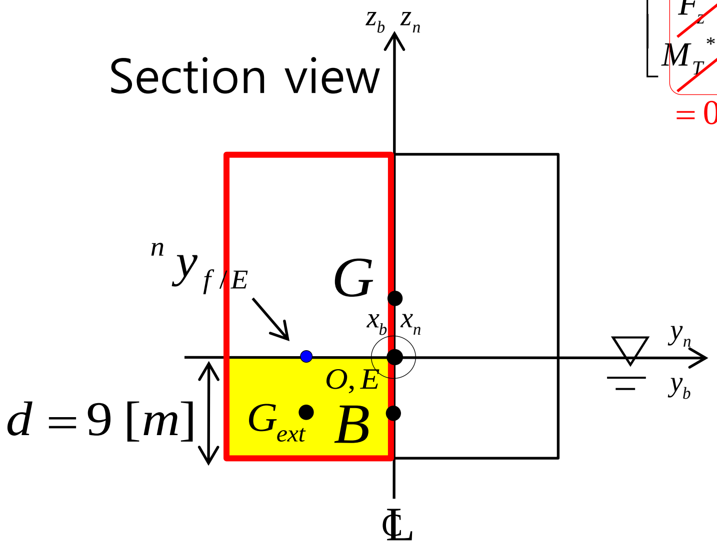
$$\begin{aligned} M_T &= M_{BT} + M_{GT} + M_{extT} \\ &= {}^{n}y_{B/E} \cdot F_{B,z} + {}^{n}y_{G/E} \cdot F_{G,z} + {}^{n}y_{G_{ext}/E} \cdot F_{ext,z} \\ &= \underline{0} \cdot (3.6 \times 10^5) + \underline{0} \cdot (-3.6 \times 10^5) + (-10) \cdot (-3.6 \times 10^4) \end{aligned}$$

The initial center of buoyancy and the initial center of mass lie on the center line of the ship.

$$= \underline{3.6 \times 10^5\ [kN \cdot m]}$$

2. Calculation of the Values of the Waterplane at k=0 step

Section view



$$\begin{bmatrix} \cancel{F_z^* - F_z^{(k)}} \\ \cancel{M_T^* - M_T^{(k)}} \end{bmatrix}_{=0} = \left[\begin{array}{c:c} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) \\ \hdashline -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} \\ -\left(-\rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -{}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\rho g i_T^{(k)}\right) \end{array}\right]_{\substack{z_n = z_n^{(0)} \\ \phi = \phi^{(0)} \\ \theta = \theta^{(0)}}} \begin{bmatrix} \delta z_n^{(k)} \\ \delta \phi^{(k)} \end{bmatrix}$$

$$-\rho g A_{WP} = -\rho g L B_{mld} = -10 \cdot 100 \cdot 40$$
$$= -4.0 \times 10^4\ [kN/m]$$

$$\rho g a_{WP} = \rho g l b = 10 \cdot 20 \cdot 20$$
$$= 4.0 \times 10^3\ [kN/m]$$

$$-\rho g A_{WP} + \rho g a_{WP} = -4.0 \times 10^4 + 4.0 \times 10^3$$
$$= \underline{-3.6 \times 10^4\ [kN/m]}$$

$$-\rho g A_{WP}\ {}^n y_{F/E} = -10 \cdot (4.0 \times 10^4) \cdot 0 = 0\ [kN]$$

$$\rho g a_{WP}\ {}^n y_{f/E} = (4.0 \times 10^3) \cdot (-10)$$
$$= -4.0 \times 10^4\ [kN]$$

$$-\rho g A_{WP}\ {}^n y_{F/E} + \rho g a_{WP}\ {}^n y_{f/E} = 0 - 4.0 \times 10^4$$
$$= \underline{-4.0 \times 10^4\ [kN]}$$

$L = 100\ [m]$	${}^n\mathbf{r}_{G/E} = [0 \quad 0 \quad 6]^T\ [m]$
$B_{mld} = 40\ [m]$	${}^n\mathbf{r}_{B/E} = [0 \quad 0 \quad -4.5]^T\ [m]$
$D = 30\ [m]$	${}^n\mathbf{r}_{G_{ext}/E} = [0 \quad -10 \quad -4.5]^T\ [m]$
$d = 9\ [m]$	$l = 20\ [m] \quad b = 20\ [m]$

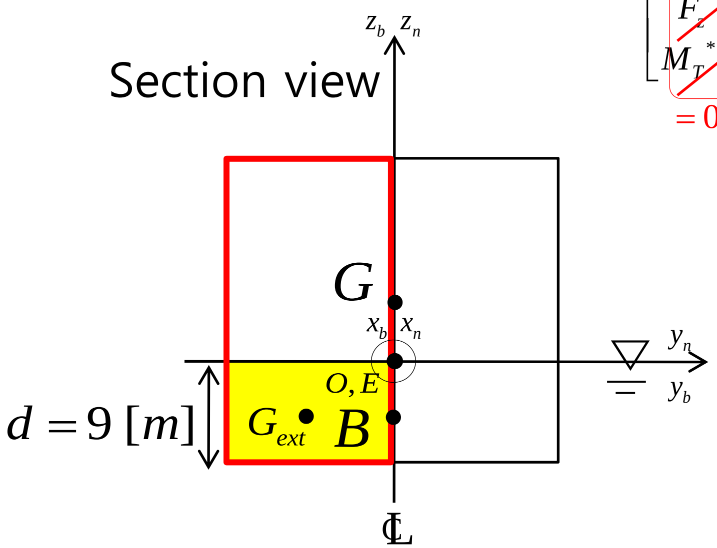
$F_{G,z} = -3.6 \times 10^5\ [kN]$

$F_{B,z} = 3.6 \times 10^5\ [kN]$

$F_{ext,z} = -3.6 \times 10^4\ [kN]$: (Lost Buoyancy)

$\rho g = 10\ [Mg/m^2 s^2]$

$$\underset{=0}{\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \end{bmatrix}} = \boxed{\left.\begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\rho g i_T^{(k)}\right) \end{bmatrix}\right|_{\substack{z_n = z_n^{(0)} \\ \phi = \phi^{(0)} \\ \theta = \theta^{(0)}}}} \begin{bmatrix} \delta z_n^{(k)} \\ \delta \phi^{(k)} \end{bmatrix}$$



$L = 100\ [m]$	${}^n\mathbf{r}_{G/E} = \begin{bmatrix} 0 & 0 & 6 \end{bmatrix}^T\ [m]$
$B_{mld} = 40\ [m]$	${}^n\mathbf{r}_{B/E} = \begin{bmatrix} 0 & 0 & -4.5 \end{bmatrix}^T\ [m]$
$D = 30\ [m]$	${}^n\mathbf{r}_{G_{ext}/E} = \begin{bmatrix} 0 & -10 & -4.5 \end{bmatrix}^T\ [m]$
$d = 9\ [m]$	$l = 20\ [m]$ $b = 20\ [m]$

$F_{G,z} = -3.6\times10^5\ [kN]$

$F_{B,z} = 3.6\times10^5\ [kN]$

$F_{ext,z} = -3.6\times10^4\ [kN]$

$\rho g = 10\ [Mg/m^2s^2]$

$$-\rho g\left({}^n z_{B/E}\nabla + I_T\right) = -\rho g\left(z_B L B_{mld} d + \frac{L B_{mld}^3}{12}\right)$$

$$= -10\left((-4.5)\cdot 100\cdot 40\cdot 9 + \frac{100\cdot 40^3}{12}\right)$$

$$= -3.71\times10^6\ [kN\cdot m]$$

$$-z_{G/E}\cdot F_{G,z} = -6\cdot(-3.6\times10^5) = 2.16\times10^6\ [kN\cdot m]$$

$$-{}^n z_{G_{ext}/E}\cdot F_{ext,z} = -(-4.5)(-3.6\times10^4)$$

$$= -1.62\times10^5\ [kN\cdot m]$$

$$\rho g i_T = \rho g\frac{lb^3}{12} + \rho g\left(\frac{b}{2}\right)^2 lb = \rho g\frac{lb^3}{3}$$

$$= 10\frac{20\cdot 20^3}{3} = 5.33\times10^5$$

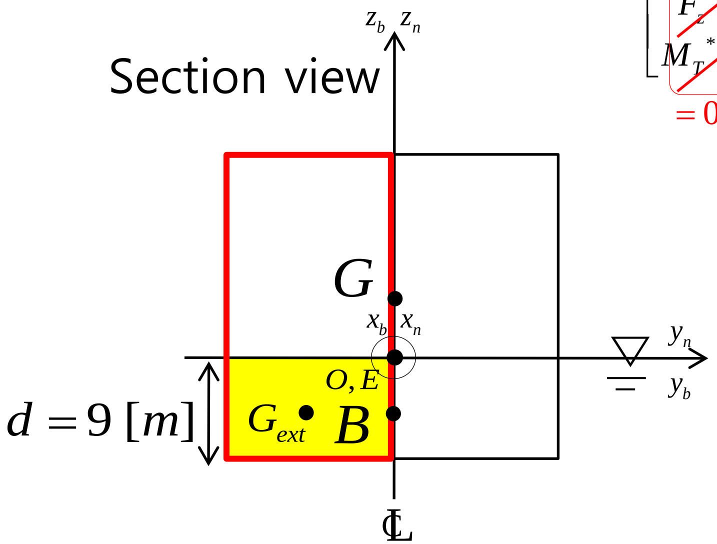
$$-\rho g\left({}^n z_{B/E}\nabla + I_T\right) - {}^n z_{G/E}\cdot F_{G,z} - {}^n z_{G_{ext}/E}\cdot F_{ext,z} + \rho g i_T$$

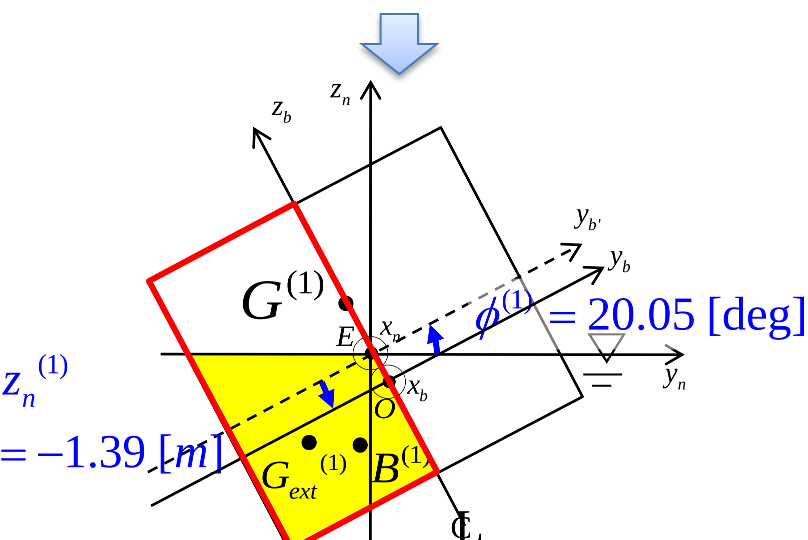
$$= -3.71\times10^6 + 2.16\times10^6 - 1.62\times10^5 + 5.33\times10^5$$

$$= \underline{-1.18\times10^6\ [kN\cdot m]}$$

3. Calculation of Immersion and Heel at k=0 step

$$\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \end{bmatrix}_{=0} = \left[\begin{array}{c:c} -\rho g A_{WP}^{(k)} - \left(-\mu_E \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{\cdots} - \left(-\rho g a_{WP}^{(k)} \cdot {}^n y_{\cdots}\right) \\ \hdashline -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} \\ -\left(-\rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -{}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\rho g i_T^{(k)}\right) \end{array}\right]_{\substack{z_n = z_n^{(0)} \\ \phi = \phi^{(0)} \\ \theta = \theta^{(0)}}} \boxed{\begin{bmatrix} \delta z_n^{(k)} \\ \delta \phi^{(k)} \end{bmatrix}}$$





$$\begin{bmatrix} 3.6\times10^4 \\ -3.6\times10^5 \end{bmatrix} = \begin{bmatrix} -3.6\times10^4 & -4.0\times10^4 \\ -4.0\times10^4 & -1.18\times10^6 \end{bmatrix} \begin{bmatrix} \delta z_n \\ \delta\phi \end{bmatrix}$$

$$\boxed{\begin{bmatrix} \delta z_n \\ \delta\phi \end{bmatrix}} = \begin{bmatrix} -3.6\times10^4 & -4.0\times10^4 \\ -4.0\times10^4 & -1.18\times10^6 \end{bmatrix}^{-1} \begin{bmatrix} 3.6\times10^4 \\ -3.6\times10^5 \end{bmatrix}$$

$$= \begin{bmatrix} -1.39\ [m] \\ 0.35\ [rad] \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} -1.39\ [m] \\ 20.05\ [deg] \end{bmatrix}}$$

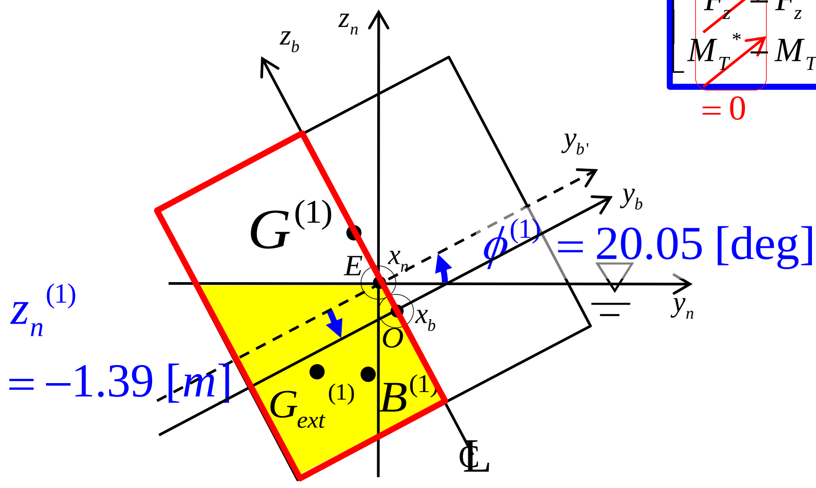
$$z_n^{(1)} = z_n^{(0)} + \delta z_n = 0 + (-1.39) = -1.39\ [m]$$

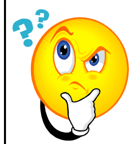
$$\phi^{(1)} = \phi^{(0)} + \delta\phi = 0 + (20.05) = 20.05\ [deg]$$

4. Check for the Ship to be in Static Equilibrium at k=0 step

$$\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y \ldots - \left(-\rho g a_{WP}^{(k)} \cdot {}^n y \ldots\right) \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\rho g i_T^{(k)}\right) \end{bmatrix}_{\substack{z_n = z_n^{(0)} \\ \phi = \phi^{(0)} \\ \theta = \theta^{(0)}}} \begin{bmatrix} \delta z_n^{(k)} \\ \delta \phi^{(k)} \end{bmatrix}$$

$= 0$





Is the ship in static equilibrium?

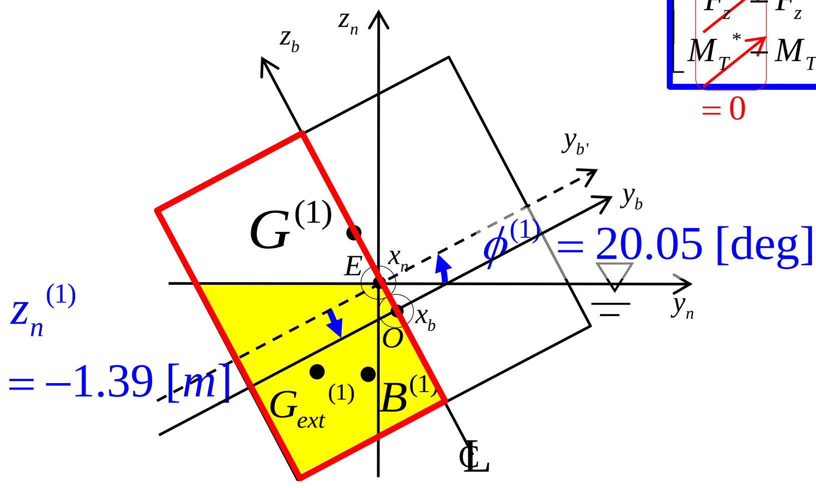
Let us check for the ship to be in static equilibrium!

$L = 100\ [m]$	${}^n\mathbf{r}_{G/E} = [0 \quad 0 \quad 6]^T\ [m]$
$B_{mld} = 40\ [m]$	${}^n\mathbf{r}_{B/E} = [0 \quad 0 \quad -4.5]^T\ [m]$
$D = 30\ [m]$	${}^n\mathbf{r}_{G_{ext}/E} = [0 \quad -10 \quad -4.5]^T\ [m]$
$d = 9\ [m]$	$l = 20\ [m] \quad b = 20\ [m]$

$$F_{G,z} = -3.6 \times 10^5\ [kN]$$
$$F_{B,z} = 3.6 \times 10^5\ [kN]$$
$$F_{ext,z} = -3.6 \times 10^4\ [kN]$$
$$\rho g = 10\ [Mg/m^2 s^2]$$

$$\begin{bmatrix} \cancel{F_z^{*}} - F_z^{(k)} \\ \cancel{M_T^{*}} - M_T^{(k)} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\rho g i_T^{(k)}\right) \end{bmatrix}_{\substack{z_n = z_n^{(0)} \\ \phi = \phi^{(0)} \\ \theta = \theta^{(0)}}} \begin{bmatrix} \delta z_n^{(k)} \\ \delta \phi^{(k)} \end{bmatrix}$$

$= 0$



For the convenience of the calculation, we rotate the ship and reference frames with an angle of $-\boldsymbol{\phi}$ in clock wise direction , and calculate the center of buoyancy w.r.t. the body-fixed frame.

$L = 100$ [m]

$B_{mld} = 40$ [m]

$D = 30$ [m]

$d = 9$ [m]

${}^n\mathbf{r}_{G/E} = [0 \quad 0 \quad 6]^T$ [m]

${}^n\mathbf{r}_{B/E} = [0 \quad 0 \quad -4.5]^T$ [m]

${}^n\mathbf{r}_{G_{ext}/E} = [0 \quad -10 \quad -4.5]^T$ [m]

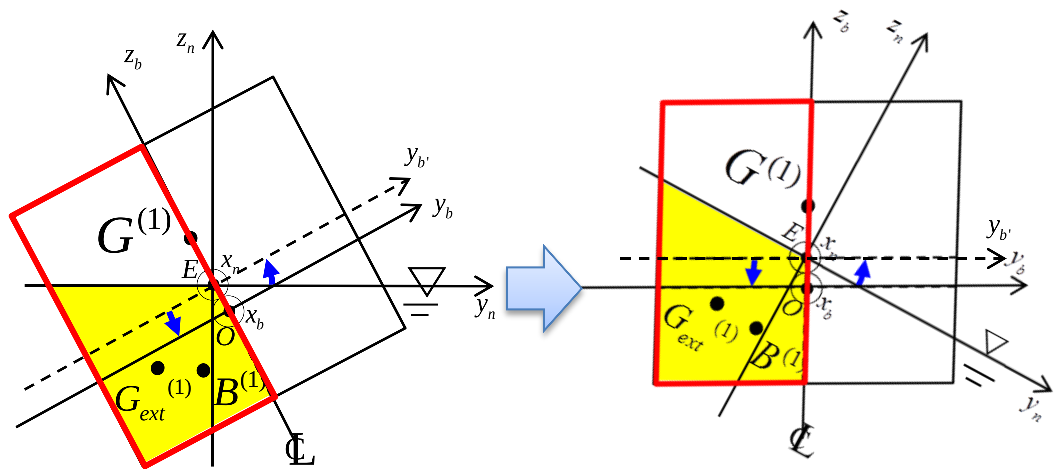
$l = 20$ [m] $\quad b = 20$ [m]

$F_{G,z} = -3.6 \times 10^5$ [kN]

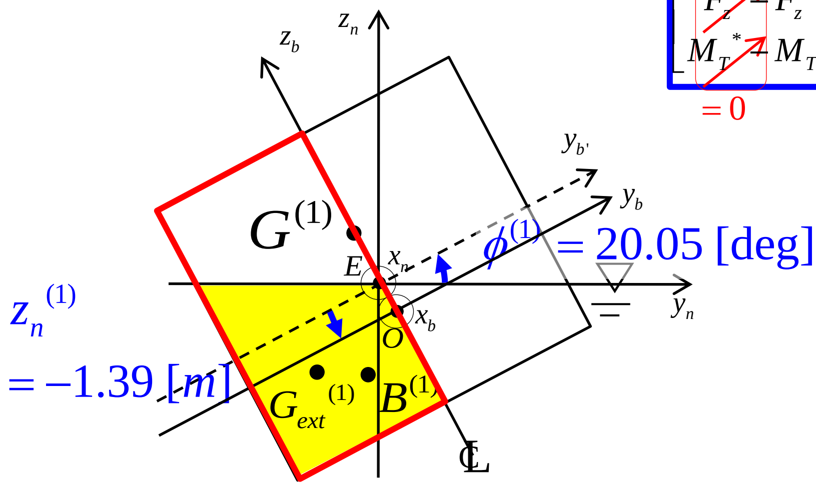
$F_{B,z} = 3.6 \times 10^5$ [kN]

$F_{ext,z} = -3.6 \times 10^4$ [kN]

$\rho g = 10\ [Mg / m^2 s^2]$



$$\boxed{\begin{bmatrix} \cancel{F_z^{*}} - F_z^{(k)} \\ \cancel{M_T^{*}} - M_T^{(k)} \end{bmatrix}}_{=0} = \left[\begin{array}{c:c} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) \\ \hdashline -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} & -\rho g \left({}^{n}z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^{n}z_{G^{(k)}/E} \cdot F_{G,z} \\ -\left(-\rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) & -{}^{n}z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\rho g i_T^{(k)}\right) \end{array}\right]_{\substack{z_n = z_n^{(0)} \\ \phi = \phi^{(0)} \\ \theta = \theta^{(0)}}} \begin{bmatrix} \delta z_n^{(k)} \\ \delta \phi^{(k)} \end{bmatrix}$$



Force equilibrium :

$$F_z^{(1)} = F_{B,z}^{(1)} + F_{G,z}^{(1)} + F_{ext,z}^{(1)}$$

$$F_{B,z}^{(1)} = \rho g \underline{V} = \rho g L \underline{A_{Section}}$$: Volume and area are independent of the reference frame

$$A_{Section} = A_1 + A_2 + A_3$$

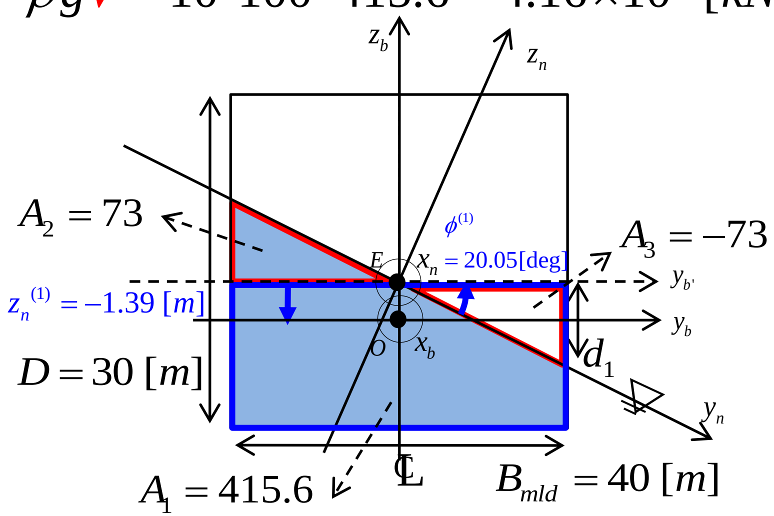
$$d_1 = \frac{B_{mld}}{2} \tan\left|\phi^{(1)}\right| = \frac{40}{2} \tan 20.05^\circ = 7.30\ [m]$$

$$A_{Section} = 415.6 + 73 + (-73) = 415.6\ [m^2]$$

$$F_{B,z}^{(1)} = \rho g V = 10 \cdot 100 \cdot 415.6 = 4.16 \times 10^5\ [kN]$$

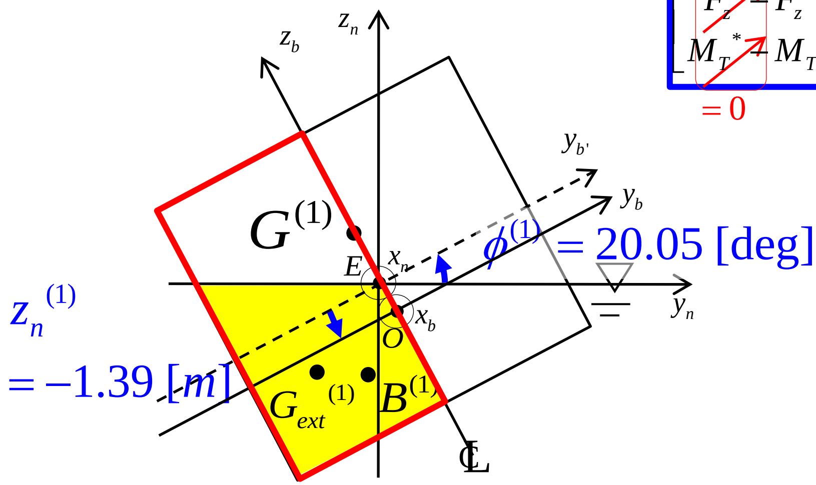
$L = 100\ [m]$	${}^{n}\mathbf{r}_{G/E} = [0 \quad 0 \quad 6]^T\ [m]$
$B_{mld} = 40\ [m]$	${}^{n}\mathbf{r}_{B/E} = [0 \quad 0 \quad -4.5]^T\ [m]$
$D = 30\ [m]$	${}^{n}\mathbf{r}_{G_{ext}/E} = [0 \quad -10 \quad -4.5]^T\ [m]$
$d = 9\ [m]$	$l = 20\ [m] \quad b = 20\ [m]$

$$F_{G,z} = -3.6 \times 10^5\ [kN]$$
$$F_{B,z} = 3.6 \times 10^5\ [kN]$$
$$F_{ext,z} = -3.6 \times 10^4\ [kN]$$
$$\rho g = 10\ [Mg/m^2 s^2]$$



$$\begin{bmatrix} \cancel{F_z^{*}} - F_z^{(k)} \\ \cancel{M_T^{*}} - M_T^{(k)} \end{bmatrix} \quad (=0)$$

$$= \left[\begin{array}{c:c} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) \\ \hdashline -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} & -\rho g\left({}^{n}z_{B^{(k)}/E}\nabla^{(k)} + I_T^{(k)}\right) - {}^{n}z_{G^{(k)}/E} \cdot F_{G,z} \\ -\left(-\rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) & -{}^{n}z_{G_{ext}^{(k)}/E} \cdot F_{ext,z} - \left(-\rho g i_T^{(k)}\right) \end{array}\right]_{\substack{z_n = z_n^{(0)} \\ \phi = \phi^{(0)} \\ \theta = \theta^{(0)}}} \begin{bmatrix} \delta z_n^{(k)} \\ \delta\phi^{(k)} \end{bmatrix}$$



Force equilibrium :

$$F_z^{(1)} = F_{B,z}^{(1)} + F_{G,z}^{(1)} + F_{ext,z}^{(1)}$$

$$F_{G,z}^{(1)} = F_{G,z} = -3.6\times10^5\ [kN]$$

$$F_{ext,z}^{(1)} = -\rho g v = -\rho g l a_{Section}$$ **(Lost Buoyancy)**

$$a_{Section} = A_1 + A_2$$

$$= 207.8 + 73 = 280.8\ [m^2]$$

$$F_{ext,z}^{(1)} = -\rho g v = -10\cdot20\cdot280.8 = -5.62\times10^4\ [kN]$$

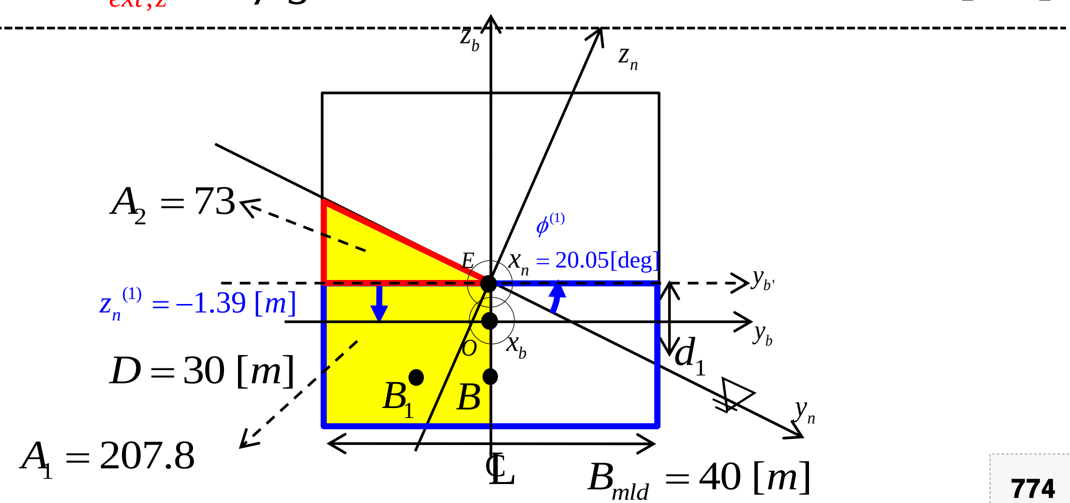
$L = 100\ [m]$	${}^{n}\mathbf{r}_{G/E} = [0\quad 0\quad 6]^T\ [m]$
$B_{mld} = 40\ [m]$	${}^{n}\mathbf{r}_{B/E} = [0\quad 0\quad -4.5]^T\ [m]$
$D = 30\ [m]$	${}^{n}\mathbf{r}_{G_{ext}/E} = [0\quad -10\quad -4.5]^T\ [m]$
$d = 9\ [m]$	$l = 20\ [m]\quad b = 20\ [m]$

$$F_{G,z} = -3.6\times10^5\ [kN]$$

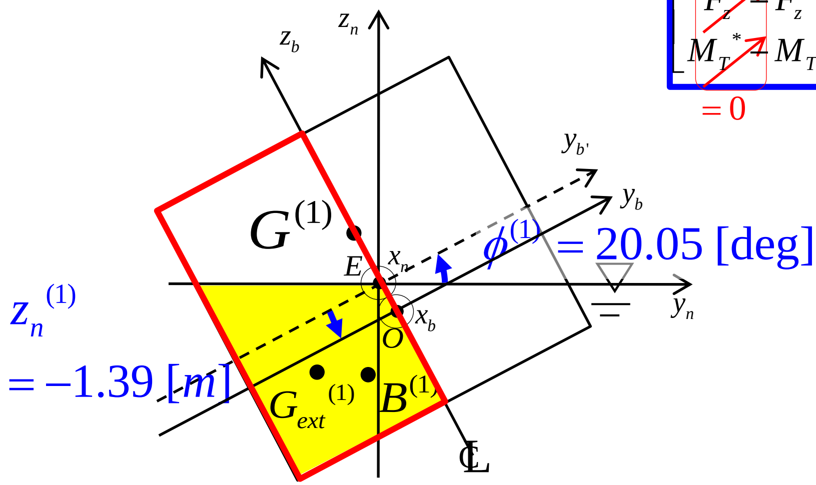
$$F_{B,z}^{(1)} = 4.16\times10^5\ [kN]$$

$$F_{ext,z} = -3.6\times10^4\ [kN]$$

$$\rho g = 10\ [Mg/m^2s^2]$$



$$\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \end{bmatrix}_{=0} = \left[\begin{array}{c:c} \text{[redacted]} & \\ \hdashline -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g\left({}^n z_{B^{(k)}/E}\nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} \\ -\left(-\rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -{}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z} - \left(-\rho g i_T^{(k)}\right) \end{array}\right]_{\substack{z_n = z_n^{(0)} \\ \phi = \phi^{(0)} \\ \theta = \theta^{(0)}}} \begin{bmatrix} \delta z_n^{(k)} \\ \delta \phi^{(k)} \end{bmatrix}$$



$L = 100\ [m]$ | ${}^n\mathbf{r}_{G/E} = [0\quad 0\quad 6]^T\ [m]$

$B_{mld} = 40\ [m]$ | ${}^n\mathbf{r}_{B/E} = [0\quad 0\quad -4.5]^T\ [m]$

$D = 30\ [m]$ | ${}^n\mathbf{r}_{G_{ext}/E} = [0\quad -10\quad -4.5]^T\ [m]$

$d = 9\ [m]$ | $l = 20\ [m]\quad b = 20\ [m]$

$F_{G,z} = -3.6\times 10^5\ [kN]$

$F_{B,z}^{(1)} = 4.16\times 10^5\ [kN]$

$F_{ext,z}^{(1)} = -5.62\times 10^4\ [kN]$

$\rho g = 10\ [Mg/m^2 s^2]$

Force equilibrium :

$$F_z^{(1)} = F_{B,z}^{(1)} + F_{G,z}^{(1)} + F_{ext,z}^{(1)}$$

$$F_{B,z}^{(1)} = 4.16\times 10^5\ [kN]$$

$$F_{G,z}^{(1)} = -3.6\times 10^5\ [kN]$$

$$F_{ext,z}^{(1)} = -5.62\times 10^4\ [kN]$$

$$F_z^{(1)} = 4.16\times 10^5 - 3.6\times 10^5 - 5.62\times 10^4$$

$$= \boxed{-2.0\times 10^2\ [kN]}$$

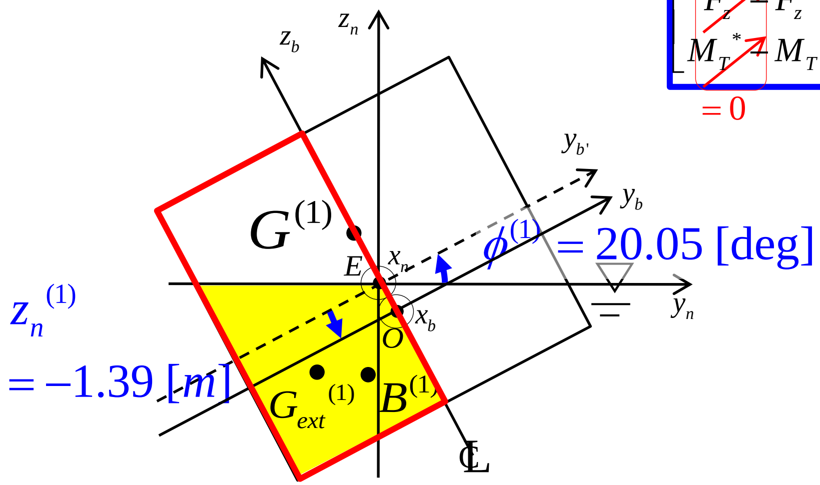
Tolerance $> e$

where, e(epsilon) : an arbitrarily small positive quantity

The static equilibrium of force is not satisfied!

We have to iterate!

$$\underset{=0}{\boxed{\begin{bmatrix} \cancel{F_z^{*}} - F_z^{(k)} \\ \cancel{M_T^{*}} - M_T^{(k)} \end{bmatrix}}} = \left[\begin{array}{c:c} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) \\ \hdashline -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} & -\rho g\left({}^{n}z_{B^{(k)}/E}\nabla^{(k)} + I_T^{(k)}\right) - {}^{n}z_{G^{(k)}/E} \cdot F_{G,z} \\ -\left(-\rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) & -{}^{n}z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\rho g i_T^{(k)}\right) \end{array}\right]_{\substack{z_n = z_n^{(0)} \\ \phi = \phi^{(0)} \\ \theta = \theta^{(0)}}} \begin{bmatrix} \delta z_n^{(k)} \\ \delta \phi^{(k)} \end{bmatrix}$$



Moment equilibrium :

$$M_T^{(1)} = M_{BT}^{(1)} + M_{GT}^{(1)} + M_{extT}^{(1)}$$

$$M_{BT}^{(1)} = {}^{n}y_{B^{(1)}/E} \cdot F_{B,z}^{(1)}$$

$${}^{b'}y_{B^{(1)}/E} = \frac{{}^{b'}y_{c_1/E}A_1 + {}^{b'}y_{c_2/E}A_2 + {}^{b'}y_{c_3/E}A_3}{A_1 + A_2 + A_3}$$

$$= \frac{0\cdot(415.6) + (-13.33)\cdot(73) + (13.33)\cdot(-73)}{415.6 + 73 - 73}$$

$$= -4.68\ [m]$$

$L = 100\ [m]$ | ${}^{n}\mathbf{r}_{G/E} = [0 \quad 0 \quad 6]^T\ [m]$

$B_{mld} = 40\ [m]$ | ${}^{n}\mathbf{r}_{B/E} = [0 \quad 0 \quad -4.5]^T\ [m]$

$D = 30\ [m]$ | ${}^{n}\mathbf{r}_{G_{ext}/E} = [0 \quad -10 \quad -4.5]^T\ [m]$

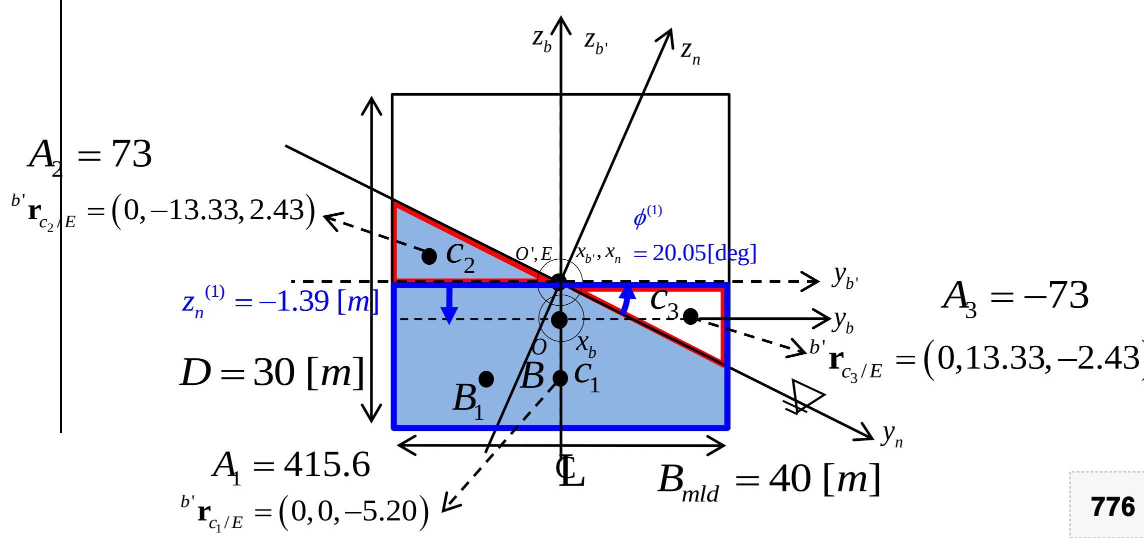
$d = 9\ [m]$ | $l = 20\ [m] \quad b = 20\ [m]$

$$F_{G,z} = -3.6\times 10^5\ [kN]$$

$$F_{B,z}^{(1)} = 4.16\times 10^5\ [kN]$$

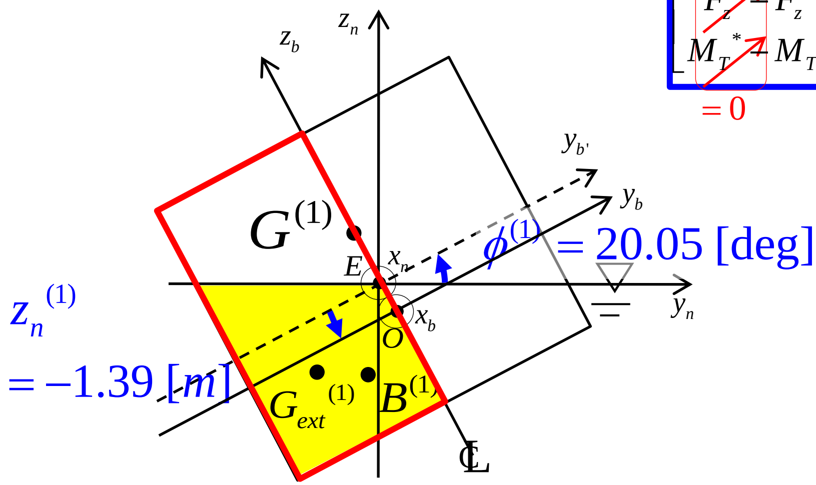
$$F_{ext,z}^{(1)} = -5.62\times 10^4\ [kN]$$

$$\rho g = 10\ [Mg/m^2 s^2]$$



$$\boxed{\begin{bmatrix} \cancel{F_z^{*} - F_z^{(k)}} \\ \cancel{M_T^{*} - M_T^{(k)}} \end{bmatrix}} = \left[\begin{array}{c:c} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) \\ \hdashline -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} & -\rho g\left({}^{n}z_{B^{(k)}/E}\nabla^{(k)} + I_T^{(k)}\right) - {}^{n}z_{G^{(k)}/E} \cdot F_{G,z} \\ -\left(-\rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) & -{}^{n}z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\rho g i_T^{(k)}\right) \end{array}\right]_{\substack{z_n = z_n^{(0)} \\ \phi = \phi^{(0)} \\ \theta = \theta^{(0)}}} \begin{bmatrix} \delta z_n^{(k)} \\ \delta \phi^{(k)} \end{bmatrix}$$

$= 0$



Moment equilibrium :

$$M_T^{(1)} = M_{BT}^{(1)} + M_{GT}^{(1)} + M_{extT}^{(1)}$$

$${}^{b'}y_{B^{(1)}/E} = -4.68\ [m]$$

$$M_{BT}^{(1)} = {}^{n}y_{B^{(1)}/E} \cdot F_{B,z}^{(1)}$$

$${}^{b'}z_{B^{(1)}/E} = \frac{{}^{b'}z_{c_1/E}A_1 + {}^{b'}z_{c_2/E}A_2 + {}^{b'}z_{c_3/E}A_3}{A_1 + A_2 + A_3}$$

$$= \frac{(-5.20)\cdot 415.6 + 2.43\cdot 73 + (-2.43)\cdot(-73)}{415.6 - 73 + 73}$$

$$= -4.35\ [m]$$

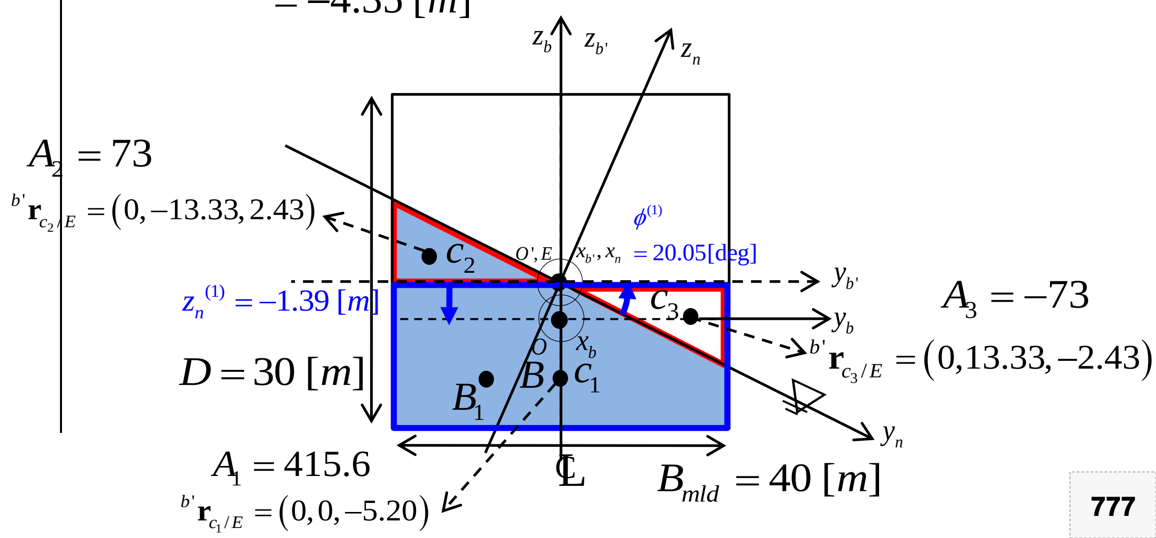
$L = 100\ [m]$	${}^{n}\mathbf{r}_{G/E} = [0 \quad 0 \quad 6]^T\ [m]$
$B_{mld} = 40\ [m]$	${}^{n}\mathbf{r}_{B/E} = [0 \quad 0 \quad -4.5]^T\ [m]$
$D = 30\ [m]$	${}^{n}\mathbf{r}_{G_{ext}/E} = [0 \quad -10 \quad -4.5]^T\ [m]$
$d = 9\ [m]$	$l = 20\ [m] \quad b = 20\ [m]$

$$F_{G,z} = -3.6\times 10^5\ [kN]$$

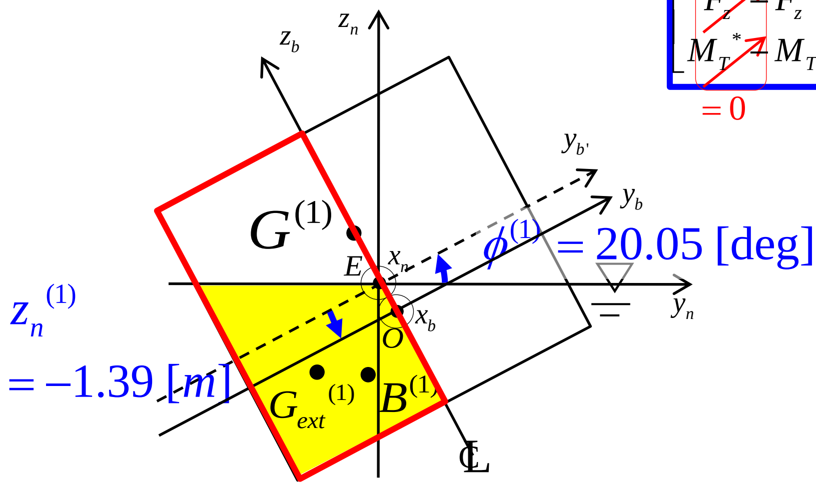
$$F_{B,z}^{(1)} = 4.16\times 10^5\ [kN]$$

$$F_{ext,z}^{(1)} = -5.62\times 10^4\ [kN]$$

$$\rho g = 10\ [Mg/m^2 s^2]$$



$$\boxed{\begin{bmatrix} \cancel{F_z^{*}} - F_z^{(k)} \\ \cancel{M_T^{*}} - M_T^{(k)} \end{bmatrix}}_{=0} = \left[\begin{array}{c:c} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) \\ \hdashline -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} & -\rho g\left({}^{n}z_{B^{(k)}/E}\nabla^{(k)} + I_T^{(k)}\right) - {}^{n}z_{G^{(k)}/E} \cdot F_{G,z} \\ -\left(-\rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) & -{}^{n}z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\rho g i_T^{(k)}\right) \end{array}\right]_{\substack{z_n = z_n^{(0)} \\ \phi = \phi^{(0)} \\ \theta = \theta^{(0)}}} \begin{bmatrix} \delta z_n^{(k)} \\ \delta \phi^{(k)} \end{bmatrix}$$



Moment equilibrium :

$$M_T^{(1)} = M_{BT}^{(1)} + M_{GT}^{(1)} + M_{extT}^{(1)}$$

$$M_{BT}^{(1)} = \underline{{}^{n}y_{B^{(1)}/E}} \cdot F_{B,z}^{(1)}$$

In this case, for convenience of calculating the center of displaced volume $B^{(1)}$ of the ship, we use b'-frame. The origin O' of b'-frame coincides with the origin E of n-frame. And the orientation of b'-frame is the same as that of b-frame.

So, to obtain the center of buoyancy with respect to n-frame, ${}^{n}r_{B^{(1)}/E}$, we have to perform the rotational transformation.

$L = 100$ [m] $\quad {}^{n}\mathbf{r}_{G/E} = [0 \quad 0 \quad 6]^T$ [m]

$B_{mld} = 40$ [m] $\quad {}^{n}\mathbf{r}_{B/E} = [0 \quad 0 \quad -4.5]^T$ [m]

$D = 30$ [m] $\quad {}^{n}\mathbf{r}_{G_{ext}/E} = [0 \quad -10 \quad -4.5]^T$ [m]

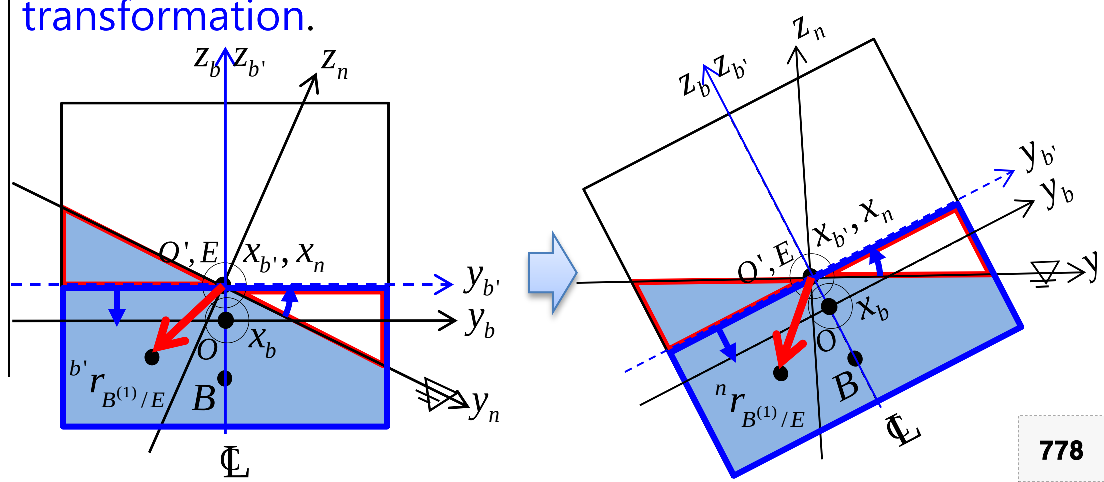
$d = 9$ [m] $\quad l = 20$ [m] $\quad b = 20$ [m]

$$F_{G,z} = -3.6 \times 10^5 \ [kN]$$

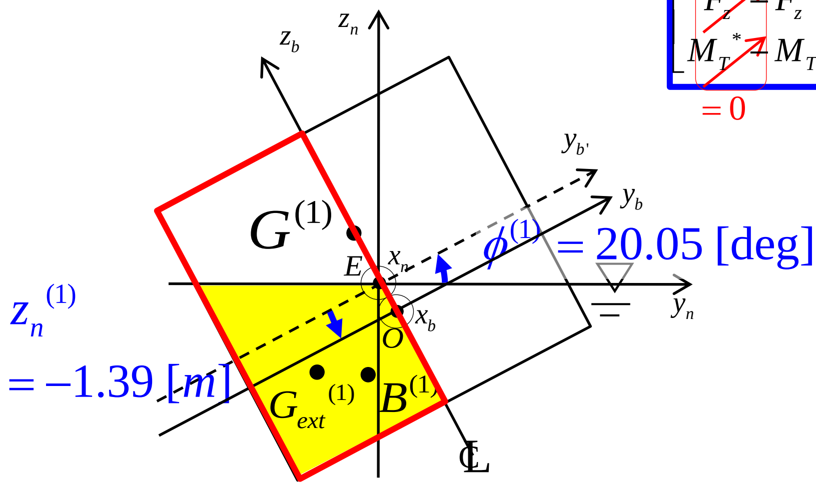
$$F_{B,z}^{(1)} = 4.16 \times 10^5 \ [kN]$$

$$F_{ext,z}^{(1)} = -5.62 \times 10^4 \ [kN]$$

$$\rho g = 10 \ [Mg/m^2 s^2]$$



$$\begin{bmatrix} \cancel{F_z^{*}} - F_z^{(k)} \\ \cancel{M_T^{*}} - M_T^{(k)} \end{bmatrix}_{=0} = \left[\begin{array}{c:c} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) \\ \hdashline -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} & -\rho g\left({}^{n}z_{B^{(k)}/E}\nabla^{(k)} + I_T^{(k)}\right) - {}^{n}z_{G^{(k)}/E} \cdot F_{G,z} \\ -\left(-\rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) & -{}^{n}z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\rho g i_T^{(k)}\right) \end{array}\right]_{\substack{z_n = z_n^{(0)} \\ \phi = \phi^{(0)} \\ \theta = \theta^{(0)}}} \begin{bmatrix} \delta z_n^{(k)} \\ \delta\phi^{(k)} \end{bmatrix}$$



Moment equilibrium :

$$M_T^{(1)} = M_{BT}^{(1)} + M_{GT}^{(1)} + M_{extT}^{(1)}$$

$$M_{BT}^{(1)} = {}^{n}y_{B^{(1)}/E} \cdot F_{B,z}^{(1)}$$

$${}^{b'}y_{B^{(1)}/E} = -4.68\ [m]$$

$${}^{b'}z_{B^{(1)}/E} = -4.35\ [m]$$

$${}^{n}\mathbf{r}_{B^{(1)}/E} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi^{(1)} & -\sin\phi^{(1)} \\ 0 & \sin\phi^{(1)} & \cos\phi^{(1)} \end{bmatrix} {}^{b'}\mathbf{r}_{B^{(1)}/E}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(20.05) & -\sin(20.05) \\ 0 & \sin(20.05) & \cos(20.05) \end{bmatrix} \begin{bmatrix} 0 \\ -4.68 \\ -4.35 \end{bmatrix}$$

$L = 100\ [m]$ | ${}^{n}\mathbf{r}_{G/E} = [0 \quad 0 \quad 6]^T\ [m]$

$B_{mld} = 40\ [m]$ | ${}^{n}\mathbf{r}_{B/E} = [0 \quad 0 \quad -4.5]^T\ [m]$

$D = 30\ [m]$ | ${}^{n}\mathbf{r}_{G_{ext}/E} = [0 \quad -10 \quad -4.5]^T\ [m]$

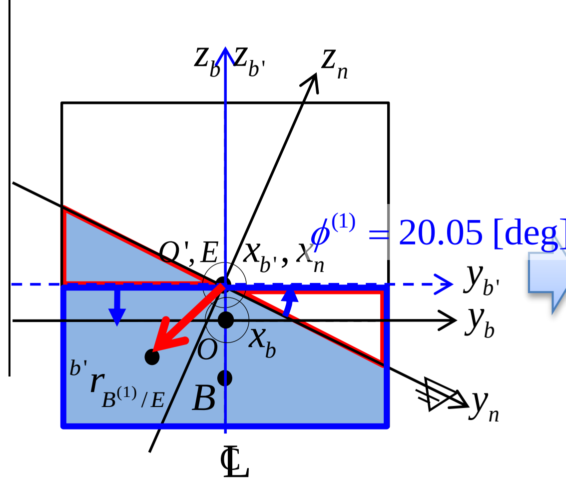
$d = 9\ [m]$ | $l = 20\ [m] \quad b = 20\ [m]$

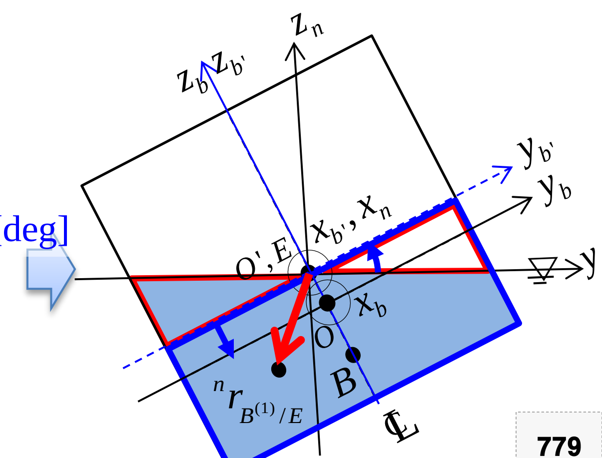
$$F_{G,z} = -3.6 \times 10^5\ [kN]$$

$$F_{B,z}^{(1)} = 4.16 \times 10^5\ [kN]$$

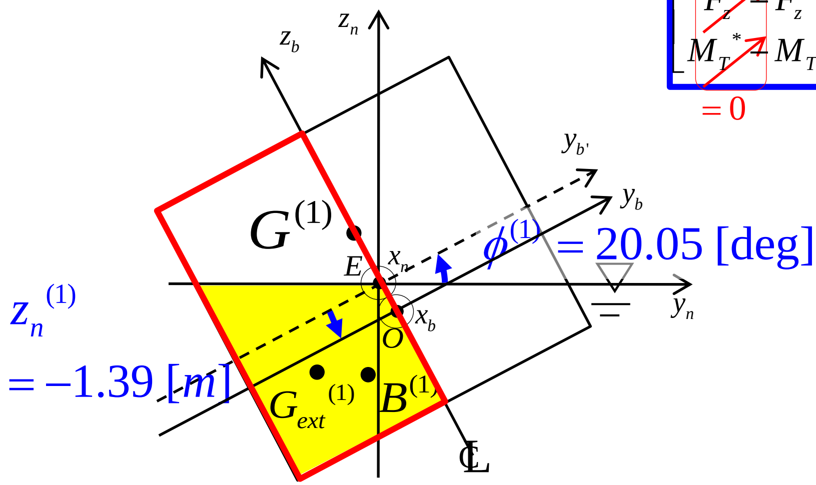
$$F_{ext,z}^{(1)} = -5.62 \times 10^4\ [kN]$$

$$\rho g = 10\ [Mg/m^2s^2]$$





$$\boxed{\begin{bmatrix} \cancel{F_z^{*} - F_z^{(k)}} \\ \cancel{M_T^{*} - M_T^{(k)}} \end{bmatrix}}_{=0} = \left[\begin{array}{c:c} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) \\ \hdashline -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} & -\rho g\left({}^{n}z_{B^{(k)}/E}\nabla^{(k)} + I_T^{(k)}\right) - {}^{n}z_{G^{(k)}/E} \cdot F_{G,z} \\ -\left(-\rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) & -{}^{n}z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\rho g i_T^{(k)}\right) \end{array}\right]_{\substack{z_n = z_n^{(0)} \\ \phi = \phi^{(0)} \\ \theta = \theta^{(0)}}} \begin{bmatrix} \delta z_n^{(k)} \\ \delta \phi^{(k)} \end{bmatrix}$$



Moment equilibrium :

$$M_T^{(1)} = M_{BT}^{(1)} + M_{GT}^{(1)} + M_{extT}^{(1)}$$

$$M_{BT}^{(1)} = {}^{n}y_{B^{(1)}/E} \cdot F_{B,z}^{(1)}$$

$${}^{b'}y_{B^{(1)}/E} = -4.68\ [m]$$

$${}^{b'}z_{B^{(1)}/E} = -4.35\ [m]$$

$${}^{n}\mathbf{r}_{B^{(1)}/E} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi^{(1)} & -\sin\phi^{(1)} \\ 0 & \sin\phi^{(1)} & \cos\phi^{(1)} \end{bmatrix} {}^{b'}\mathbf{r}_{B^{(1)}/E}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(20.05) & -\sin(20.05) \\ 0 & \sin(20.05) & \cos(20.05) \end{bmatrix} \begin{bmatrix} 0 \\ -4.68 \\ -4.35 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -2.91 \\ -5.69 \end{bmatrix}$$

$${}^{n}y_{B^{(1)}/E} = -2.91\ [m]$$

$$M_{BT}^{(1)} = (-2.91)\cdot(4.16\times10^{5})$$

$$= -1.21\times10^{6}$$

$L = 100\ [m]$	${}^{n}\mathbf{r}_{G/E} = [0 \quad 0 \quad 6]^T\ [m]$
$B_{mld} = 40\ [m]$	${}^{n}\mathbf{r}_{B^{(1)}/E} = [0 \quad -2.91 \quad -5.69]^T\ [m]$
$D = 30\ [m]$	${}^{n}\mathbf{r}_{G_{ext}/E} = [0 \quad -10 \quad -4.5]^T\ [m]$
$d = 9\ [m]$	$l = 20\ [m]$ $\quad b = 20\ [m]$

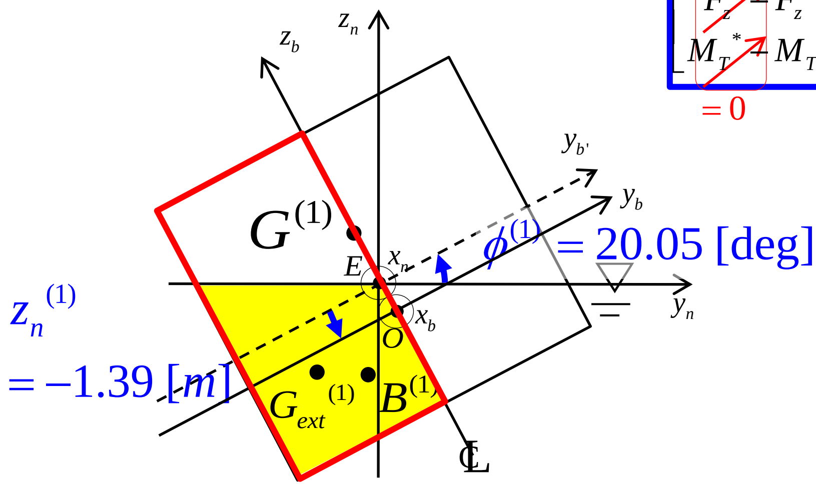
$$F_{G,z} = -3.6\times10^{5}\ [kN]$$

$$F_{B,z}^{(1)} = 4.16\times10^{5}\ [kN]$$

$$F_{ext,z}^{(1)} = -5.62\times10^{4}\ [kN]$$

$$\rho g = 10\ [Mg/m^2s^2]$$

$$\underbrace{\begin{bmatrix} \cancel{F_z^{*}-F_z^{(k)}} \\ \cancel{M_T^{*}-M_T^{(k)}} \end{bmatrix}}_{=0} = \left[\begin{array}{c:c} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) \\ \hdashline -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} & -\rho g\left({}^{n}z_{B^{(k)}/E}\nabla^{(k)} + I_T^{(k)}\right) - {}^{n}z_{G^{(k)}/E} \cdot F_{G,z} \\ -\left(-\rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) & -{}^{n}z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\rho g i_T^{(k)}\right) \end{array}\right]_{\substack{z_n=z_n^{(0)} \\ \phi=\phi^{(0)} \\ \theta=\theta^{(0)}}} \begin{bmatrix} \delta z_n^{(k)} \\ \delta\phi^{(k)} \end{bmatrix}$$



$L = 100\ [m]$	${}^{n}\mathbf{r}_{G/E} = [0 \quad 0 \quad 6]^T\ [m]$
$B_{mld} = 40\ [m]$	${}^{n}\mathbf{r}_{B^{(1)}/E} = [0 \quad -2.91 \quad -5.69]^T\ [m]$
$D = 30\ [m]$	${}^{n}\mathbf{r}_{G_{ext}/E} = [0 \quad -10 \quad -4.5]^T\ [m]$
$d = 9\ [m]$	$l = 20\ [m] \quad b = 20\ [m]$

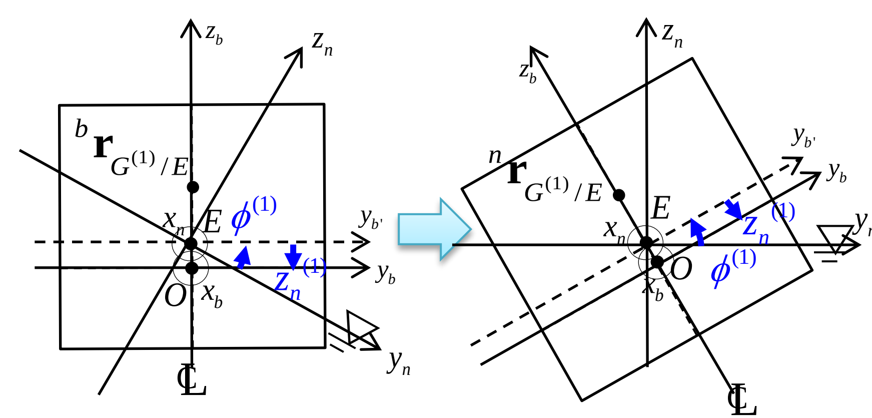
$$F_{G,z} = -3.6 \times 10^5\ [kN]$$
$$F_{B,z}^{(1)} = 4.16 \times 10^5\ [kN]$$
$$F_{ext,z}^{(1)} = -5.62 \times 10^4\ [kN]$$
$$\rho g = 10\ [Mg/m^2 s^2]$$

Moment equilibrium :

$$M_T^{(1)} = M_{BT}^{(1)} + M_{GT}^{(1)} + M_{extT}^{(1)} \qquad M_{BT}^{(1)} = -1.21 \times 10^6$$

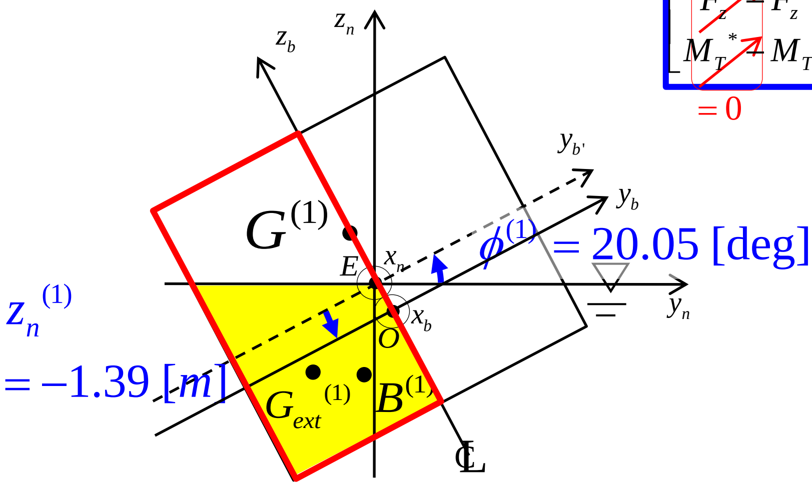
$$M_{GT}^{(1)} = {}^{n}y_{G^{(1)}/E} \cdot F_{G,z}^{(1)}$$

The center of mass, ${}^{b}\mathbf{r}_{G^{(1)}/E}$, with respect to the body fixed frame is identical with respect to the floating position. But the center of mass, ${}^{n}\mathbf{r}_{G^{(1)}/E}$, with respect to the waterplane-fixed frame changes with respect to the rotation. The change in the center of mass, ${}^{n}\mathbf{r}_{G^{(1)}/E}$, with respect to the waterplane-fixed frame causes an additional heeling moment arm.



$$\begin{bmatrix} \cancel{F_z^{*}} - F_z^{(k)} \\ \cancel{M_T^{*}} - M_T^{(k)} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) \\ -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) & -\rho g\left({}^{n}z_{B^{(k)}/E}\nabla^{(k)} + I_T^{(k)}\right) - {}^{n}z_{G^{(k)}/E} \cdot F_{G,z} - {}^{n}z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\rho g i_T^{(k)}\right) \end{bmatrix}_{\substack{z_n = z_n^{(0)} \\ \phi = \phi^{(0)} \\ \theta = \theta^{(0)}}} \begin{bmatrix} \delta z_n^{(k)} \\ \delta \phi^{(k)} \end{bmatrix}$$

$= 0$



$L = 100\ [m]$	${}^{n}\mathbf{r}_{G^{(1)}/E} = [0 \quad -1.58 \quad 4.33]^T\ [m]$
$B_{mld} = 40\ [m]$	${}^{n}\mathbf{r}_{B^{(1)}/E} = [0 \quad -2.91 \quad -5.69]^T\ [m]$
$D = 30\ [m]$	${}^{n}\mathbf{r}_{G_{ext}/E} = [0 \quad -10 \quad -4.5]^T\ [m]$
$d = 9\ [m]$	$l = 20\ [m] \quad b = 20\ [m]$

$F_{G,z} = -3.6 \times 10^5\ [kN]$

$F_{B,z}^{(1)} = 4.16 \times 10^5\ [kN]$

$F_{ext,z}^{(1)} = -5.62 \times 10^4\ [kN]$

$\rho g = 10\ [Mg/m^2 s^2]$

Moment equilibrium :

$$M_T^{(1)} = M_{BT}^{(1)} + M_{GT}^{(1)} + M_{extT}^{(1)} \qquad M_{BT}^{(1)} = -1.21 \times 10^6$$

$$M_{GT}^{(1)} = {}^{n}y_{G^{(1)}/E} \cdot F_{G,z}^{(1)}$$

2. Rotation with heel

1. Translation with immersion

$$ {}^{n}\mathbf{r}_{G^{(1)}/E} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi^{(1)} & -\sin\phi^{(1)} \\ 0 & \sin\phi^{(1)} & \cos\phi^{(1)} \end{bmatrix} {}^{b'}\mathbf{r}_{G^{(1)}/E}, \quad {}^{b'}\mathbf{r}_{G^{(1)}/E} = {}^{b}\mathbf{r}_{G^{(1)}/E} + \begin{bmatrix} 0 \\ 0 \\ z_n^{(1)} \end{bmatrix}$$

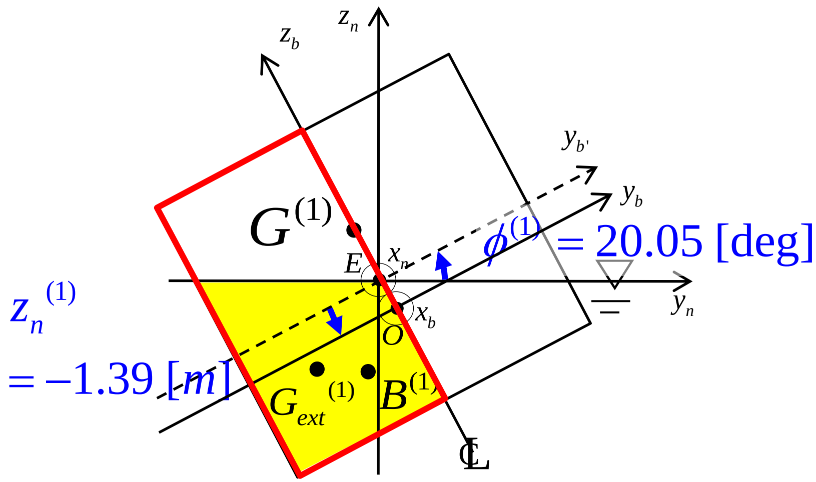
$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(20.05) & -\sin(20.05) \\ 0 & \sin(20.05) & \cos(20.05) \end{bmatrix} \left(\begin{bmatrix} 0 \\ 0 \\ 6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -1.39 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 0 \\ -1.58 \\ 4.33 \end{bmatrix}$$

$${}^{n}y_{G^{(1)}/E} = -1.58\ [m]$$

$$M_{GT}^{(1)} = (-1.58) \cdot (-3.6 \times 10^5)$$

$$= 5.69 \times 10^5$$



$L = 100\ [m]$ | ${}^{n}\mathbf{r}_{G^{(1)}/E} = [0 \quad -1.58 \quad 4.33]^T\ [m]$

$B_{mld} = 40\ [m]$ | ${}^{n}\mathbf{r}_{B^{(1)}/E} = [0 \quad -2.91 \quad -5.69]^T\ [m]$

$D = 30\ [m]$ | ${}^{n}\mathbf{r}_{G_{ext}/E} = [0 \quad -10 \quad -4.5]^T\ [m]$

$d = 9\ [m]$ | $l = 20\ [m] \quad b = 20\ [m]$

$$F_{G,z} = -3.6 \times 10^5\ [kN]$$

$$F_{B,z}{}^{(1)} = 4.16 \times 10^5\ [kN]$$

$$F_{ext,z}{}^{(1)} = -5.62 \times 10^4\ [kN]$$

$$\rho g = 10\ [Mg/m^2 s^2]$$

Moment equilibrium :

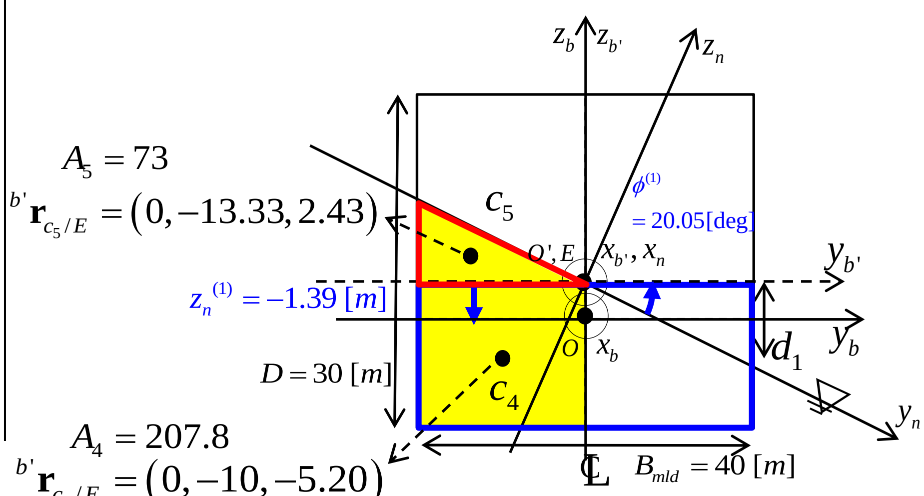
$$M_T^{(1)} = M_{BT}^{(1)} + M_{GT}^{(1)} + M_{extT}^{(1)}$$

$$M_{BT}^{(1)} = -1.21 \times 10^6$$

$$M_{GT}^{(1)} = 5.69 \times 10^5$$

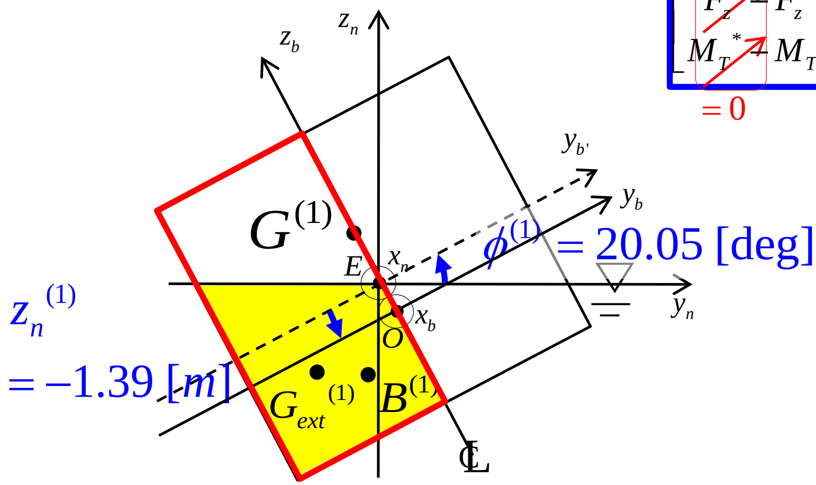
$$M_{extT}^{(1)} = {}^{n}y_{G_{ext}{}^{(1)}/E} \cdot F_{ext,z}^{(1)}$$

$$\begin{aligned} {}^{b'}y_{G_{ext}{}^{(1)}/E} &= \frac{{}^{b'}y_{c_4/E} A_4 + {}^{b'}y_{c_5/E} A_5}{A_4 + A_5} \\ &= \frac{(-10)\cdot(207.8) + (-13.33)\cdot(73)}{207.8 + 73} \\ &= -10.87\ [m] \end{aligned}$$



$$\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} \\ -\left(-\rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -{}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\rho g i_T^{(k)}\right) \end{bmatrix}_{\substack{z_n = z_n^{(0)} \\ \phi = \phi^{(0)} \\ \theta = \theta^{(0)}}} \begin{bmatrix} \delta z_n^{(k)} \\ \delta \phi^{(k)} \end{bmatrix}$$

$= 0$



$L = 100$ [m]

$B_{mld} = 40$ [m]

$D = 30$ [m]

$d = 9$ [m]

${}^n\mathbf{r}_{G^{(1)}/E} = [0 \quad -1.58 \quad 4.33]^T$ [m]

${}^n\mathbf{r}_{B^{(1)}/E} = [0 \quad -2.91 \quad -5.69]^T$ [m]

${}^n\mathbf{r}_{G_{ext}/E} = [0 \quad -10 \quad -4.5]^T$ [m]

$l = 20$ [m] $\quad b = 20$ [m]

$F_{G,z} = -3.6 \times 10^5$ [kN]

$F_{B,z}^{(1)} = 4.16 \times 10^5$ [kN]

$F_{ext,z}^{(1)} = -5.62 \times 10^4$ [kN]

$\rho g = 10$ [Mg / m²s²]

Moment equilibrium :

$$M_T^{(1)} = M_{BT}^{(1)} + M_{GT}^{(1)} + M_{extT}^{(1)}$$

$$M_{extT}^{(1)} = {}^n y_{G_{ext}^{(1)}/E} \cdot F_{ext,z}^{(1)}$$

$$M_{BT}^{(1)} = -1.21 \times 10^6$$

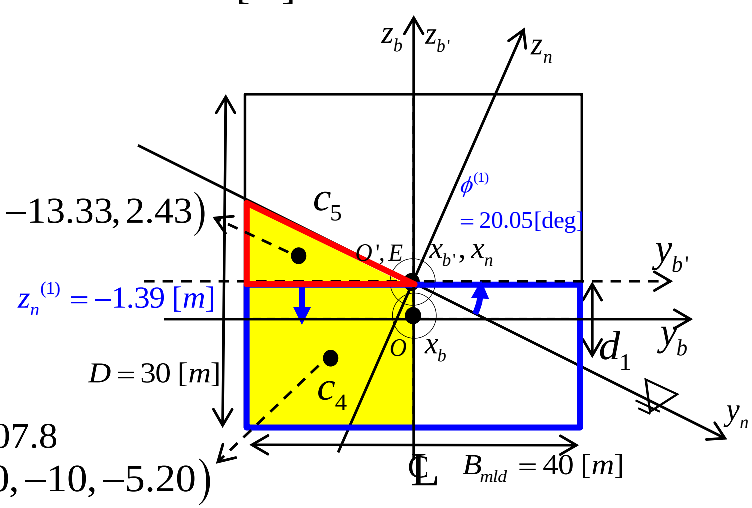
$$M_{GT}^{(1)} = 5.69 \times 10^5$$

$${}^{b'}y_{G_{ext}^{(1)}/E} = -10.87 \text{ [m]}$$

$${}^{b'}z_{G_{ext}^{(1)}/E} = \frac{{}^{b'}z_{c_4/E} A_4 + {}^{b'}z_{c_5/E} A_5}{A_4 + A_5} = \frac{(-5.20)\cdot 207.8 + 2.43 \cdot 73}{207.8 + 73} = -3.22 \text{ [m]}$$

$A_5 = 73$

${}^{b'}\mathbf{r}_{c_5/E} = (0, -13.33, 2.43)$

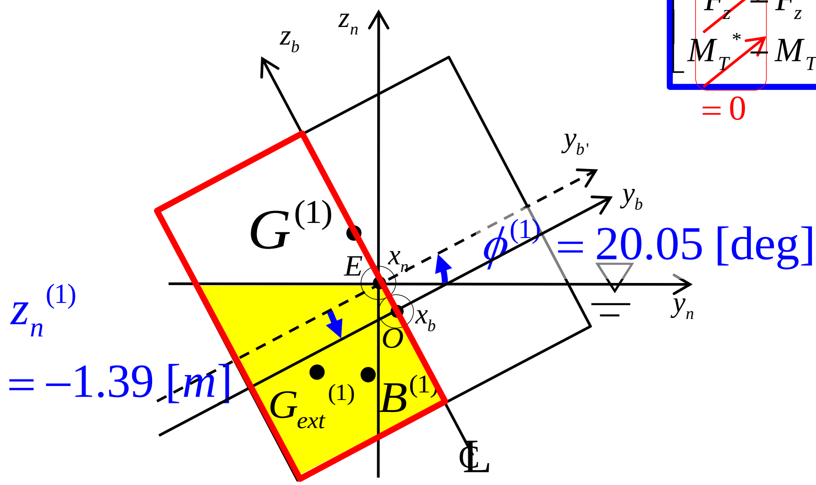


$A_4 = 207.8$

${}^{b'}\mathbf{r}_{c_4/E} = (0, -10, -5.20)$

$$\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \end{bmatrix} = \left[\begin{array}{c:c} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) \\ \hdashline -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} \\ -\left(-\rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -{}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\rho g i_T^{(k)}\right) \end{array}\right]_{\substack{z_n = z_n^{(0)} \\ \phi = \phi^{(0)} \\ \theta = \theta^{(0)}}} \begin{bmatrix} \delta z_n^{(k)} \\ \delta \phi^{(k)} \end{bmatrix}$$

$= 0$



Moment equilibrium :

$$M_T^{(1)} = M_{BT}^{(1)} + M_{GT}^{(1)} + M_{extT}^{(1)}$$

$$M_{BT}^{(1)} = -1.21 \times 10^6$$

$$M_{GT}^{(1)} = 5.69 \times 10^5$$

$$M_{extT}^{(1)} = \underline{{}^n y_{G_{ext}^{(1)}/E}} \cdot F_{ext,z}^{(1)}$$

In this case, for convenience of calculating the center of external force, $G_{ext}^{(1)}$ of the ship, we use b'-frame.

So, to obtain the center of external force with respect to n-frame, ${}^n r_{G_{ext}^{(1)}/E}$, we have to perform the rotational transformation.

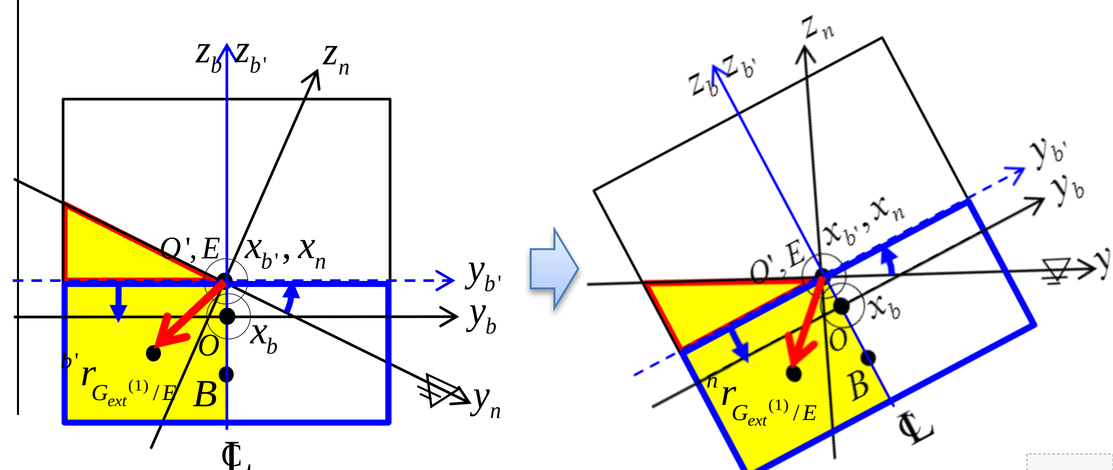
$L = 100\ [m]$	${}^n\mathbf{r}_{G^{(1)}/E} = \begin{bmatrix} 0 & -1.58 & 4.33 \end{bmatrix}^T [m]$
$B_{mld} = 40\ [m]$	${}^n\mathbf{r}_{B^{(1)}/E} = \begin{bmatrix} 0 & -2.91 & -5.69 \end{bmatrix}^T [m]$
$D = 30\ [m]$	${}^n\mathbf{r}_{G_{ext}/E} = \begin{bmatrix} 0 & -10 & -4.5 \end{bmatrix}^T [m]$
$d = 9\ [m]$	$l = 20\ [m] \quad b = 20\ [m]$

$$F_{G,z} = -3.6 \times 10^5\ [kN]$$

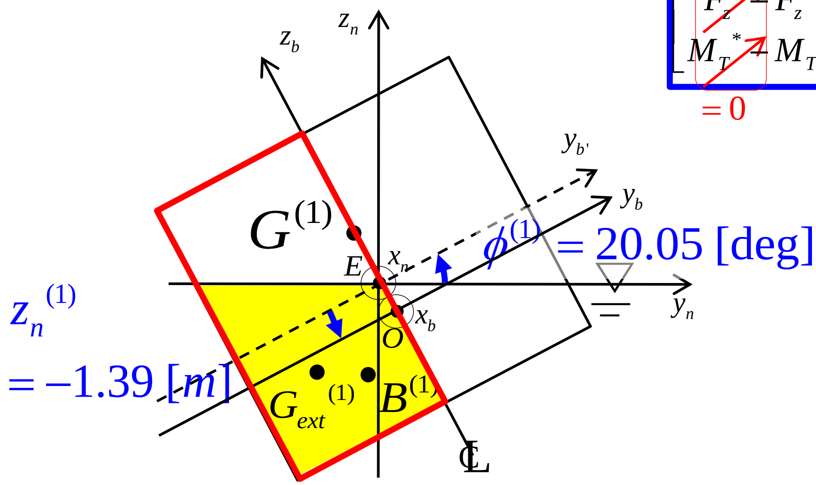
$$F_{B,z}^{(1)} = 4.16 \times 10^5\ [kN]$$

$$F_{ext,z}^{(1)} = -5.62 \times 10^4\ [kN]$$

$$\rho g = 10\ [Mg / m^2 s^2]$$



$$\boxed{\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \end{bmatrix}}_{=0} = \left[\begin{array}{c:c} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) \\ \hdashline -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} & -\rho g\left({}^{n}z_{B^{(k)}/E}\nabla^{(k)} + I_T^{(k)}\right) - {}^{n}z_{G^{(k)}/E} \cdot F_{G,z} \\ -\left(-\rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) & -{}^{n}z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\rho g i_T^{(k)}\right) \end{array}\right]_{\substack{z_n = z_n^{(0)} \\ \phi = \phi^{(0)} \\ \theta = \theta^{(0)}}} \begin{bmatrix} \delta z_n^{(k)} \\ \delta\phi^{(k)} \end{bmatrix}$$



$L = 100\ [m]$	${}^{n}\mathbf{r}_{G^{(1)}/E} = [0 \quad -1.58 \quad 4.33]^T\ [m]$
$B_{mld} = 40\ [m]$	${}^{n}\mathbf{r}_{B^{(1)}/E} = [0 \quad -2.91 \quad -5.69]^T\ [m]$
$D = 30\ [m]$	${}^{n}\mathbf{r}_{G_{ext}^{(1)}/E} = [0 \quad -9.11 \quad -6.75]^T\ [m]$
$d = 9\ [m]$	$l = 20\ [m] \quad b = 20\ [m]$

$$F_{G,z} = -3.6 \times 10^5\ [kN]$$
$$F_{B,z}^{(1)} = 4.16 \times 10^5\ [kN]$$
$$F_{ext,z}^{(1)} = -5.62 \times 10^4\ [kN]$$
$$\rho g = 10\ [Mg/m^2 s^2]$$

Moment equilibrium :

$$M_T^{(1)} = M_{BT}^{(1)} + M_{GT}^{(1)} + M_{extT}^{(1)}$$

$$M_{BT}^{(1)} = -1.21 \times 10^6$$
$$M_{GT}^{(1)} = 5.69 \times 10^5$$

$$M_{extT}^{(1)} = {}^{n}y_{G_{ext}^{(1)}/E} \cdot F_{ext,z}^{(1)}$$

$${}^{b'}y_{G_{ext}^{(1)}/E} = -10.87\ [m]$$
$${}^{b'}z_{G_{ext}^{(1)}/E} = -3.22\ [m]$$

$${}^{n}\mathbf{r}_{G_{ext}^{(1)}/E} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi^{(1)} & -\sin\phi^{(1)} \\ 0 & \sin\phi^{(1)} & \cos\phi^{(1)} \end{bmatrix} {}^{b'}\mathbf{r}_{G_{ext}^{(1)}/E}$$

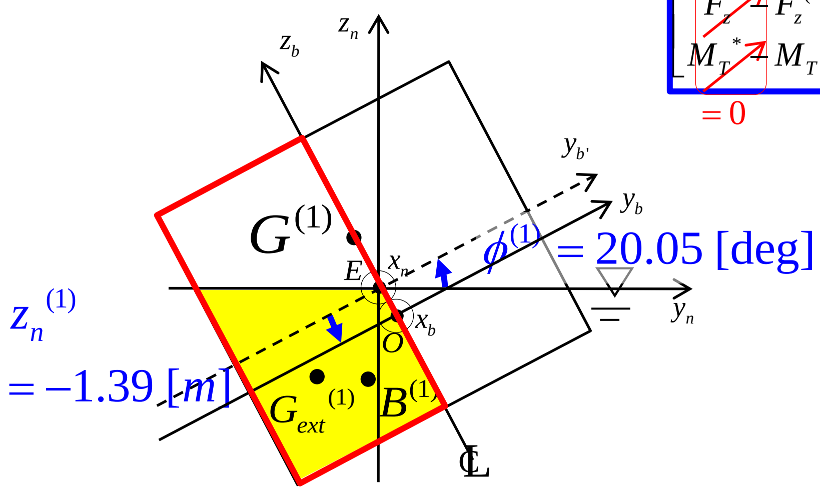
$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(20.05) & -\sin(20.05) \\ 0 & \sin(20.05) & \cos(20.05) \end{bmatrix} \begin{bmatrix} 0 \\ -10.87 \\ -3.22 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -9.11 \\ -6.75 \end{bmatrix}$$

$${}^{n}y_{G_{ext}^{(1)}/E} = -9.11\ [m]$$

$$M_{extT}^{(1)} = (-9.11) \cdot (-5.62 \times 10^4)$$
$$= 5.12 \times 10^5$$

$$\begin{bmatrix} \cancel{F_z^{*}} - F_z^{(k)} \\ \cancel{M_T^{*}} - M_T^{(k)} \end{bmatrix}_{=0} = \left[\begin{array}{c:c} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) \\ \hdashline -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} & -\rho g\left({}^{n}z_{B^{(k)}/E}\nabla^{(k)} + I_T^{(k)}\right) - {}^{n}z_{G^{(k)}/E} \cdot F_{G,z} \\ -\left(-\rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) & -{}^{n}z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\rho g i_T^{(k)}\right) \end{array}\right]_{\substack{z_n = z_n^{(0)} \\ \phi = \phi^{(0)} \\ \theta = \theta^{(0)}}} \begin{bmatrix} \delta z_n^{(k)} \\ \delta \phi^{(k)} \end{bmatrix}$$



$L = 100\ [m]$	${}^{n}\mathbf{r}_{G^{(1)}/E} = [0 \quad -1.58 \quad 4.33]^T\ [m]$
$B_{mld} = 40\ [m]$	${}^{n}\mathbf{r}_{B^{(1)}/E} = [0 \quad -2.91 \quad -5.69]^T\ [m]$
$D = 30\ [m]$	${}^{n}\mathbf{r}_{G_{ext}^{(1)}/E} = [0 \quad -9.11 \quad -6.75]^T\ [m]$
$d = 9\ [m]$	$l = 20\ [m] \quad b = 20\ [m]$

$F_{G,z} = -3.6\times10^5\ [kN]$

$F_{B,z}^{(1)} = 4.16\times10^5\ [kN]$

$F_{ext,z}^{(1)} = -5.62\times10^4\ [kN]$

$\rho g = 10\ [Mg/m^2s^2]$

Moment equilibrium :

$$M_T^{(1)} = M_{BT}^{(1)} + M_{GT}^{(1)} + M_{extT}^{(1)}$$

$M_{BT}^{(1)} = -1.21\times10^6$

$M_{GT}^{(1)} = 5.69\times10^5$

$M_{extT}^{(1)} = 5.12\times10^5$

$$M_{BT}^{(1)} = -1.21\times10^6\ [kN\cdot m]$$

$$M_{GT}^{(1)} = 5.69\times10^5\ [kN\cdot m]$$

$$M_{extT}^{(1)} = 5.12\times10^5\ [kN\cdot m]$$

$$M_T^{(1)} = -1.21\times10^6 + 5.69\times10^5 + 5.12\times10^5$$

$$= \boxed{-1.29\times10^5\ [kN\cdot m]}$$

$$\left|-1.29\times10^5\right| \overset{\text{Tolerance}}{>} e$$

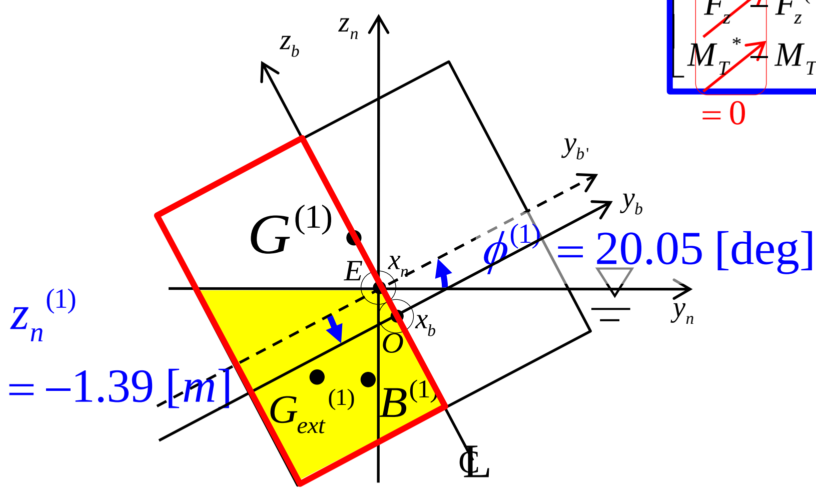
where, e(epsilon) : an arbitrarily small positive quantity

The static equilibrium of moment is not satisfied!

We have to iterate!

1. Calculation of Forces and Moments at k=1 step

$$\begin{bmatrix} \cancel{F_z^{*}} - F_z^{(k)} \\ \cancel{M_T^{*}} - M_T^{(k)} \end{bmatrix}_{=0} = \left[\begin{array}{c:c} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) \\ \hdashline -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} & -\rho g\left({}^{n}z_{B^{(k)}/E}\nabla^{(k)} + I_T^{(k)}\right) - {}^{n}z_{G^{(k)}/E} \cdot F_{G,z} \\ -\left(-\rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) & -{}^{n}z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\rho g i_T^{(k)}\right) \end{array}\right]_{\substack{z_n = z_n^{(0)} \\ \phi = \phi^{(0)} \\ \theta = \theta^{(0)}}} \begin{bmatrix} \delta z_n^{(k)} \\ \delta \phi^{(k)} \end{bmatrix}$$



$$F_z^{(1)} = F_{B,z}^{(1)} + F_{G,z}^{(1)} + F_{ext,z}^{(1)}$$
$$= 4.16\times10^5 - 3.6\times10^5 - 5.62\times10^4$$
$$= \underline{-2.00\times10^2\ [kN]}$$

$$M_T^{(1)} = M_{BT}^{(1)} + M_{GT}^{(1)} + M_{extT}^{(1)}$$
$$= {}^{n}y_{B^{(1)}/E} \cdot F_{B,z}^{(1)} + {}^{n}y_{G^{(1)}/E} \cdot F_{G,z}^{(1)} + {}^{n}y_{G_{ext}^{(1)}/E} \cdot F_{ext,z}^{(1)}$$
$$= (-2.91)\cdot(4.16\times10^5) + (-1.58)\times(-3.6\times10^5)$$
$$+ (-9.11)\times(-5.62\times10^4)$$
$$= \underline{-1.29\times10^5\ [kN\cdot m]}$$

$L = 100$ [m]

$B_{mld} = 40$ [m]

$D = 30$ [m]

$d = 9$ [m]

$${}^{n}\mathbf{r}_{G^{(1)}/E} = [0\quad -1.58\quad 4.33]^T\ [m]$$
$${}^{n}\mathbf{r}_{B^{(1)}/E} = [0\quad -2.91\quad -5.69]^T\ [m]$$
$${}^{n}\mathbf{r}_{G_{ext}^{(1)}/E} = [0\quad -9.11\quad -6.75]^T\ [m]$$

$l = 20$ [m] $b = 20$ [m]

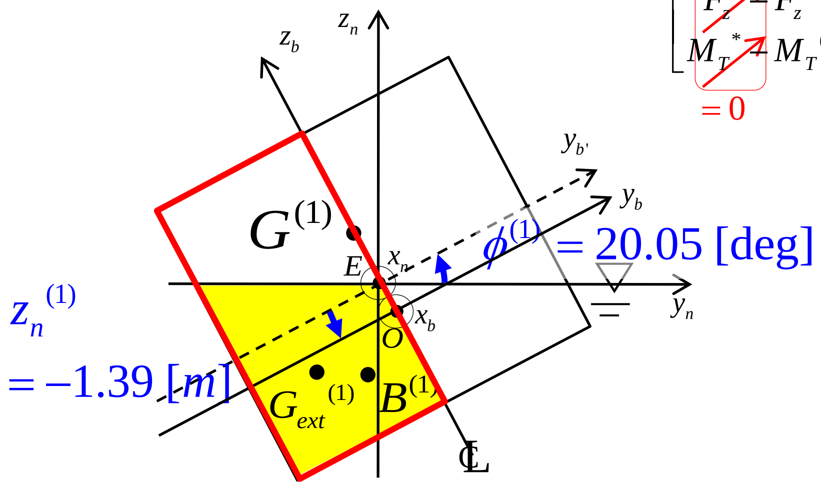
$$F_{G,z} = -3.6\times10^5\ [kN]$$
$$F_{B,z}^{(1)} = 4.16\times10^5\ [kN]$$
$$F_{ext,z}^{(1)} = -5.62\times10^4\ [kN]$$
$$\rho g = 10\ [Mg/m^2s^2]$$

※In previous state :

$$F_z = F_{B,z} + F_{G,z} + F_{ext,z}$$
$$= -3.6\times10^4\ [kN]$$
$$M_T = M_{BT} + M_{GT} + M_{extT}$$
$$= 3.6\times10^5\ [kN\cdot m]$$

2. Calculation of the Values of the Waterplane at k=1 step

$$\begin{bmatrix} \underbrace{F_z^{*}}_{=0} - F_z^{(k)} \\ M_T^{*} - M_T^{(k)} \end{bmatrix} = \left[\begin{array}{c|c} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) \\ \hline -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\rho g i_T^{(k)}\right) \end{array}\right]_{\substack{z_n = z_n^{(0)} \\ \phi = \phi^{(0)} \\ \theta = \theta^{(0)}}} \begin{bmatrix} \delta z_n^{(k)} \\ \delta \phi^{(k)} \end{bmatrix}$$



$L = 100\ [m]$	${}^n\mathbf{r}_{G^{(1)}/E} = [0 \quad -1.58 \quad 4.33]^T\ [m]$
$B_{mld} = 40\ [m]$	${}^n\mathbf{r}_{B^{(1)}/E} = [0 \quad -2.91 \quad -5.69]^T\ [m]$
$D = 30\ [m]$	${}^n\mathbf{r}_{G_{ext}^{(1)}/E} = [0 \quad -9.11 \quad -6.75]^T\ [m]$
$d = 9\ [m]$	$l = 20\ [m] \quad b = 20\ [m]$

$F_{G,z} = -3.6 \times 10^5\ [kN]$

$F_{B,z}^{(1)} = 4.16 \times 10^5\ [kN]$

$F_{ext,z}^{(1)} = -5.62 \times 10^4\ [kN]$

$\rho g = 10\ [Mg/m^2 s^2]$

$$-\rho g A_{WP} = -\rho g L B_{mld}^{(1)}, \quad B_{mld}^{(1)} = \frac{B_{mld}}{\cos|\phi^{(1)}|} = \frac{40}{\cos 20.05^\circ} = 42.58$$

$$= -10 \cdot 100 \cdot 42.58 \quad = -4.26 \times 10^4\ [kN/m]$$

$$\rho g a_{WP} = \rho g l b^{(1)}, \qquad b^{(1)} = \frac{b}{\cos|\phi^{(1)}|} = \frac{20}{\cos 20.05^\circ} = 21.29$$

$$= 10 \cdot 20 \cdot 21.29$$

$$-\rho g A_{WP} + \rho g a_{WP} = -4.26 \times 10^4 + 4.26 \times 10^3$$

$$= \underline{-3.83 \times 10^4\ [kN/m]}$$

$$-\rho g A_{WP}\, {}^n y_{F/E} = -(4.26 \times 10^4) \cdot 0 \quad = 0\ [kN]$$

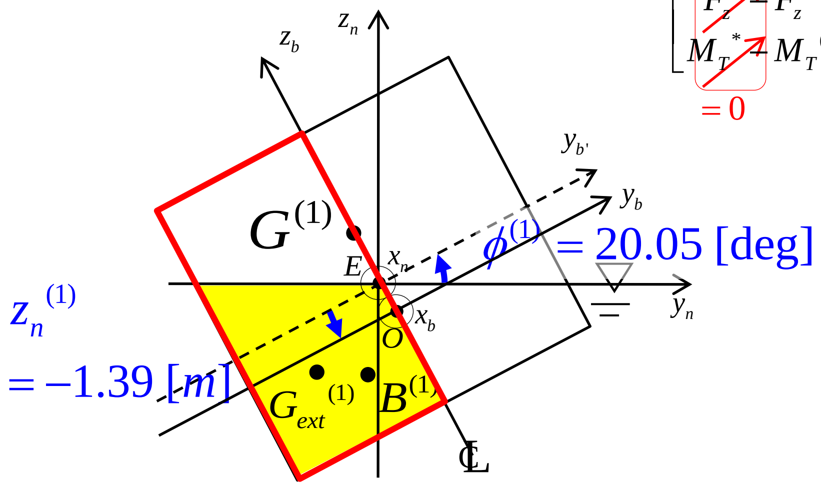
$$\rho g a_{WP}\, {}^n y_{f/E} = (4.26 \times 10^3) \cdot \left(-\frac{b^{(1)}}{2}\right)$$

$$= (4.26 \times 10^3) \cdot (-10.65) = -4.53 \times 10^4\ [kN]$$

$$-\rho g A_{WP}\, {}^n y_{F/E} + \rho g a_{WP}\, {}^n y_{f/E} = 0 - 3.88 \times 10^4$$

$$= \underline{-4.53 \times 10^4\ [kN]}$$

$$\begin{bmatrix} \cancel{F_z^* - F_z^{(k)}} \\ \cancel{M_T^* - M_T^{(k)}} \end{bmatrix}_{=0} = \boxed{\left[\begin{array}{c:c} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) \\ \hdashline -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} \\ -\left(-\rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\rho g i_T^{(k)}\right) \end{array}\right]_{\substack{z_n = z_n^{(0)} \\ \phi = \phi^{(0)} \\ \theta = \theta^{(0)}}}} \begin{bmatrix} \delta z_n^{(k)} \\ \delta \phi^{(k)} \end{bmatrix}$$



Summary of the results:

$$-\rho g A_{WP} = -4.26 \times 10^4 \ [kN/m]$$

$$\rho g a_{WP} = 4.26 \times 10^3 \ [kN/m]$$

$$-\rho g A_{WP} \, {}^n y_{F/E} = 0 \ [kN]$$

$$\rho g a_{WP} \, {}^n y_{f/E} = -4.53 \times 10^4 \ [kN]$$

※In previous state :

$$-\rho g A_{WP} = -4.0 \times 10^4 \ [kN/m]$$

$$\rho g a_{WP} = 4.0 \times 10^3 \ [kN/m]$$

$$-\rho g A_{WP} \, {}^n y_{F/E} = 0 \ [kN]$$

$$\rho g a_{WP} \, {}^n y_{f/E} = -4.0 \times 10^4 \ [kN]$$

$L = 100$ [m]	${}^n\mathbf{r}_{G^{(1)}/E} = [0 \quad -1.58 \quad 4.33]^T$ [m]
$B_{mld} = 40$ [m]	${}^n\mathbf{r}_{B^{(1)}/E} = [0 \quad -2.91 \quad -5.69]^T$ [m]
$D = 30$ [m]	${}^n\mathbf{r}_{G_{ext}^{(1)}/E} = [0 \quad -9.11 \quad -6.75]^T$ [m]
$d = 9$ [m]	$l = 20$ [m] $\quad b = 20$ [m]

$$F_{G,z} = -3.6 \times 10^5 \ [kN]$$

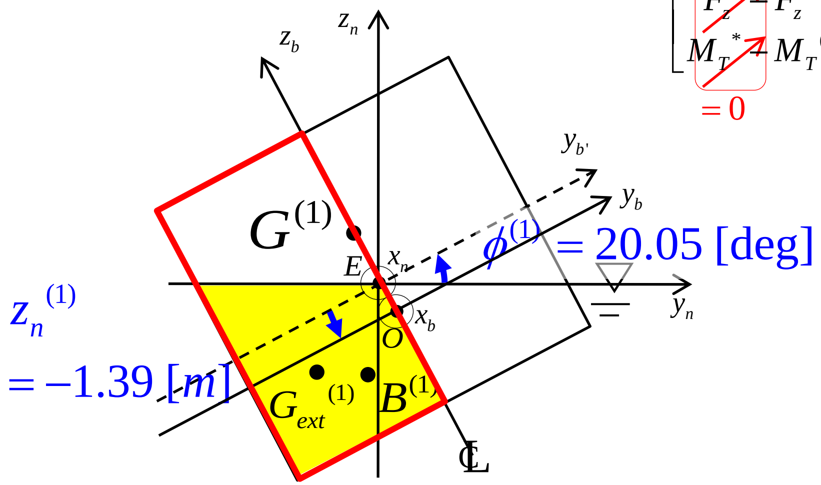
$$F_{B,z}^{(1)} = 4.16 \times 10^5 \ [kN]$$

$$F_{ext,z}^{(1)} = -5.62 \times 10^4 \ [kN]$$

$$\rho g = 10 \ [Mg/m^2 s^2]$$

$$\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \end{bmatrix} = \boxed{\begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) \\ -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) & -\rho g\left({}^{n}z_{B^{(k)}/E}\nabla^{(k)} + I_T^{(k)}\right) - {}^{n}z_{G^{(k)}/E} \cdot F_{G,z} - {}^{n}z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\rho g i_T^{(k)}\right) \end{bmatrix}_{\substack{z_n = z_n^{(0)} \\ \phi = \phi^{(0)} \\ \theta = \theta^{(0)}}}} \begin{bmatrix} \delta z_n^{(k)} \\ \delta \phi^{(k)} \end{bmatrix}$$

$= 0$



$$-\rho g\left({}^{n}z_{B/E}\nabla + I_T\right) = -{}^{n}z_{B/E}F_B^{(1)} - \rho g\frac{L(B_{mld}^{(1)})^3}{12}$$

$$= -(-5.69)\cdot 4.16\times10^5 - 10\frac{100\cdot 42.58^3}{12}$$

$$= -4.07\times10^6 \ [kN\cdot m]$$

$$-{}^{n}z_{G/E}\cdot F_{G,z} = -4.33\cdot(-3.6\times10^5)$$

$$= 1.56\times10^6 \ [kN\cdot m]$$

$$-{}^{n}z_{G_{ext}/E}\cdot F_{ext,z} = -(-6.75)(-5.62\times10^4)$$

$$= -3.79\times10^5 \ [kN\cdot m]$$

$$\rho g i_T = \rho g\frac{l\left(b^{(1)}\right)^3}{3} = 10\frac{20\cdot 21.29^3}{3} = 6.43\times10^5$$

$$-\rho g\left({}^{n}z_{B/E}\nabla + I_T\right) - {}^{n}z_{G/E}\cdot F_{G,z} - {}^{n}z_{G_{ext}/E}\cdot F_{ext,z} + \rho g i_T$$

$$= -4.07\times10^6 + 1.56\times10^6 - 3.79\times10^5 + 6.43\times10^5$$

$$= \underline{-2.25\times10^6 \ [kN\cdot m]}$$

$L = 100\ [m]$	${}^{n}\mathbf{r}_{G^{(1)}/E} = [0 \quad -1.58 \quad 4.33]^T\ [m]$
$B_{mld} = 40\ [m]$	${}^{n}\mathbf{r}_{B^{(1)}/E} = [0 \quad -2.91 \quad -5.69]^T\ [m]$
$D = 30\ [m]$	${}^{n}\mathbf{r}_{G_{ext}^{(1)}/E} = [0 \quad -9.11 \quad -6.75]^T\ [m]$
$d = 9\ [m]$	$l = 20\ [m] \quad b = 20\ [m]$

$$F_{G,z} = -3.6\times10^5\ [kN]$$

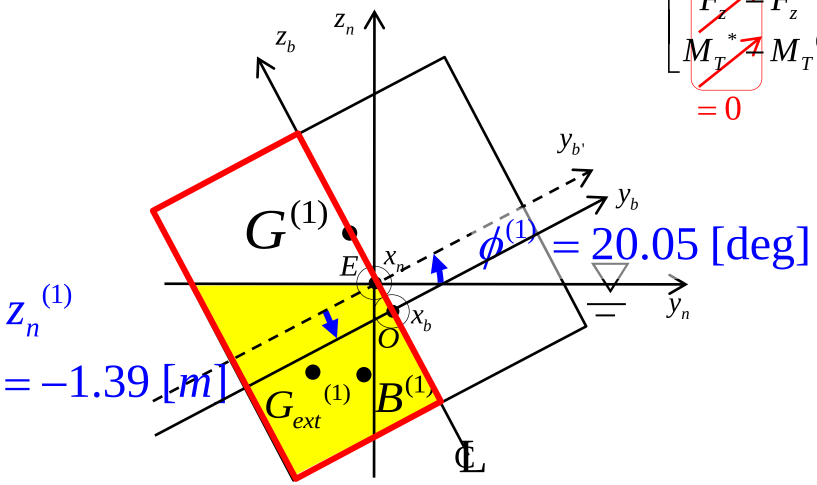
$$F_{B,z}^{(1)} = 4.16\times10^5\ [kN]$$

$$F_{ext,z}^{(1)} = -5.62\times10^4\ [kN]$$

$$\rho g = 10\ [Mg/m^2s^2]$$

$$\begin{pmatrix} B_{mld}^{(1)} = 42.58 \\ b^{(1)} = 21.29 \end{pmatrix}$$

$$
\begin{bmatrix} \cancel{F_z^{*}-F_z^{(k)}} \\ \cancel{M_T^{*}-M_T^{(k)}} \end{bmatrix}_{=0}
=
\left[\begin{array}{c:c}
-\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) \\ \hdashline
-\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} & -\rho g\left({}^{n}z_{B^{(k)}/E}\nabla^{(k)} + I_T^{(k)}\right) - {}^{n}z_{G^{(k)}/E}\cdot F_{G,z} \\
-\left(-\rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) & -{}^{n}z_{G_{ext}^{(k)}/E}\cdot F_{ext,z}^{(k)} - \left(-\rho g i_T^{(k)}\right)
\end{array}\right]_{\substack{z_n=z_n^{(0)} \\ \phi=\phi^{(0)} \\ \theta=\theta^{(0)}}}
\begin{bmatrix} \delta z_n^{(k)} \\ \delta\phi^{(k)} \end{bmatrix}
$$



$L = 100\ [m]$

$B_{mld} = 40\ [m]$

$D = 30\ [m]$

$d = 9\ [m]$

${}^{n}\mathbf{r}_{G^{(1)}/E} = [0 \quad -1.58 \quad 4.33]^T\ [m]$

${}^{n}\mathbf{r}_{B^{(1)}/E} = [0 \quad -2.91 \quad -5.69]^T\ [m]$

${}^{n}\mathbf{r}_{G_{ext}^{(1)}/E} = [0 \quad -9.11 \quad -6.75]^T\ [m]$

$l = 20\ [m] \quad b = 20\ [m]$

$F_{G,z} = -3.6\times10^5\ [kN]$

$F_{B,z}^{(1)} = 4.16\times10^5\ [kN]$

$F_{ext,z}^{(1)} = -5.62\times10^4\ [kN]$

$\rho g = 10\ [Mg/m^2s^2]$

Summary of the results:

$$-\rho g\left({}^{n}z_{B/E}\nabla + I_T\right) = -4.07\times10^6\ [kN\cdot m]$$

$$-{}^{n}z_{G/E}\cdot F_{G,z} = 1.56\times10^6\ [kN\cdot m]$$

$$-{}^{n}z_{G_{ext}/E}\cdot F_{ext,z} = -3.79\times10^5\ [kN\cdot m]$$

$$\rho g i_T = 6.43\times10^5\ [kN\cdot m]$$

※In previous state :

$$-\rho g\left({}^{n}z_{B/E}\nabla + I_T\right) = -3.71\times10^6\ [kN\cdot m]$$

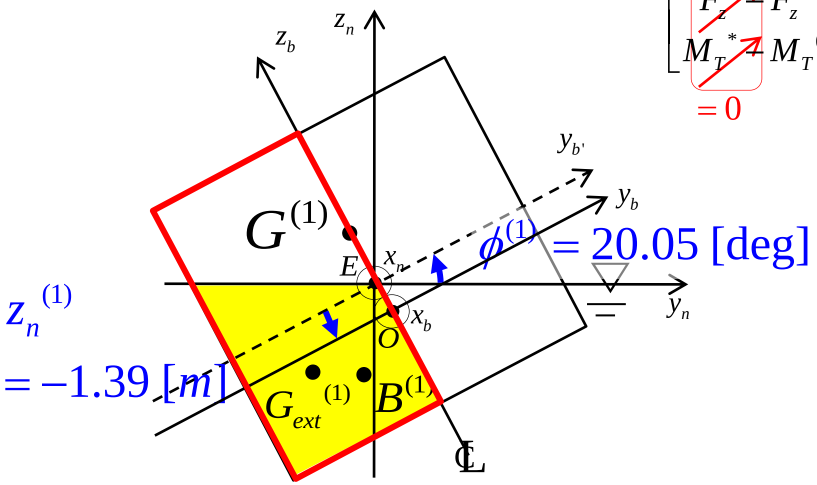
$$-{}^{n}z_{G/E}\cdot F_{G,z} = 2.16\times10^6\ [kN\cdot m]$$

$$-{}^{n}z_{G_{ext}/E}\cdot F_{ext,z} = -1.62\times10^5\ [kN\cdot m]$$

$$\rho g i_T = 5.33\times10^5\ [kN\cdot m]$$

3. Calculation of Immersion and Heel at k=1 step

$$\underbrace{\begin{bmatrix} \cancel{F_z^{*}} - F_z^{(k)} \\ \cancel{M_T^{*}} - M_T^{(k)} \end{bmatrix}}_{=0} = \begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\rho g i_T^{(k)}\right) \end{bmatrix}_{\substack{z_n = z_n^{(0)} \\ \phi = \phi^{(0)} \\ \theta = \theta^{(0)}}} \boxed{\begin{bmatrix} \delta z_n^{(k)} \\ \delta \phi^{(k)} \end{bmatrix}}$$

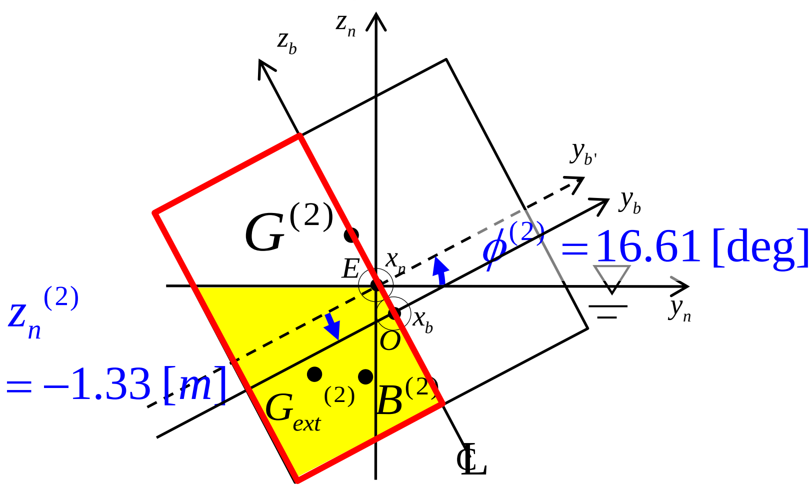


$$\begin{bmatrix} 2.00\times10^{2} \\ 1.29\times10^{5} \end{bmatrix} = \begin{bmatrix} -3.83\times10^{4} & -4.53\times10^{4} \\ -4.53\times10^{4} & -2.25\times10^{6} \end{bmatrix} \begin{bmatrix} \delta z_n \\ \delta\phi \end{bmatrix}$$

$$\begin{bmatrix} \delta z_n \\ \delta\phi \end{bmatrix} = \begin{bmatrix} -3.83\times10^{4} & -4.53\times10^{4} \\ -4.53\times10^{4} & -2.25\times10^{6} \end{bmatrix}^{-1} \begin{bmatrix} 2.00\times10^{2} \\ 1.29\times10^{5} \end{bmatrix}$$

$$= \begin{bmatrix} 0.06\ [m] \\ -0.06\ [rad] \end{bmatrix}$$

$$= \begin{bmatrix} 0.06\ [m] \\ -3.44\ [\text{deg}] \end{bmatrix}$$

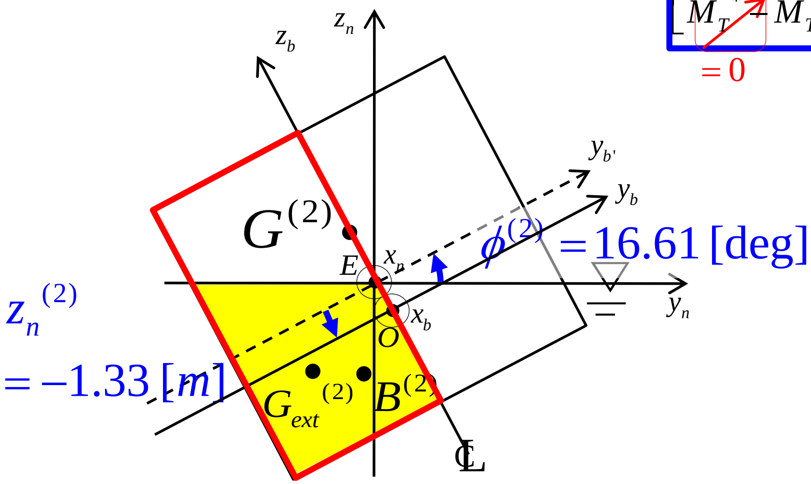


$$z_n^{(2)} = z_n^{(1)} + \delta z_n = -1.39 + (0.06) = -1.33\ [m]$$

$$\phi^{(2)} = \phi^{(1)} + \delta\phi = 20.05 - 3.44 = 16.61\ [\text{deg}]$$

4. Check for the Ship to be in Static Equilibrium at k=1 step

$$\boxed{\begin{bmatrix} \cancel{F_z^{*} - F_z^{(k)}} \\ \cancel{M_T^{*} - M_T^{(k)}} \end{bmatrix}}_{=0} = \begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g\left({}^n z_{B^{(k)}/E}\nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E}\cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E}\cdot F_{ext,z}^{(k)} - \left(-\rho g i_T^{(k)}\right) \end{bmatrix}_{\substack{z_n = z_n^{(0)} \\ \phi=\phi^{(0)} \\ \theta=\theta^{(0)}}} \begin{bmatrix} \delta z_n^{(k)} \\ \delta\phi^{(k)} \end{bmatrix}$$



$L = 100\ [m]$	${}^n\mathbf{r}_{G^{(1)}/E} = \begin{bmatrix}0 & -1.58 & 4.33\end{bmatrix}^T\ [m]$
$B_{mld} = 40\ [m]$	${}^n\mathbf{r}_{B^{(1)}/E} = \begin{bmatrix}0 & -2.91 & -5.69\end{bmatrix}^T\ [m]$
$D = 30\ [m]$	${}^n\mathbf{r}_{G_{ext}^{(1)}/E} = \begin{bmatrix}0 & -9.11 & -6.75\end{bmatrix}^T\ [m]$
$d = 9\ [m]$	$l = 20\ [m] \quad b = 20\ [m]$

$$F_{G,z} = -3.6\times 10^5\ [kN]$$
$$F_{B,z}^{(1)} = 4.16\times 10^5\ [kN]$$
$$F_{ext,z}^{(1)} = -5.62\times 10^4\ [kN]$$
$$\rho g = 10\ [Mg/m^2s^2]$$

Is the ship in static equilibrium?

Force equilibrium :

$$F_z^{(2)} = F_{B,z}^{(2)} + F_{G,z}^{(2)} + F_{ext,z}^{(2)}$$

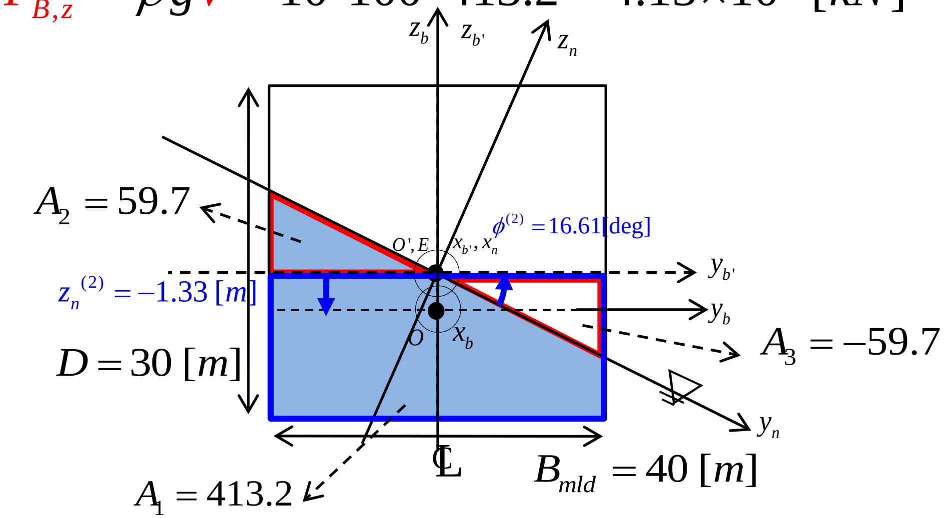
$$F_{B,z}^{(2)} = \rho g \underline{V} = \rho g L \underline{A_{Section}}$$: Volume and area are independent of the reference frame

$$A_{Section} = A_1 + A_2 + A_3$$

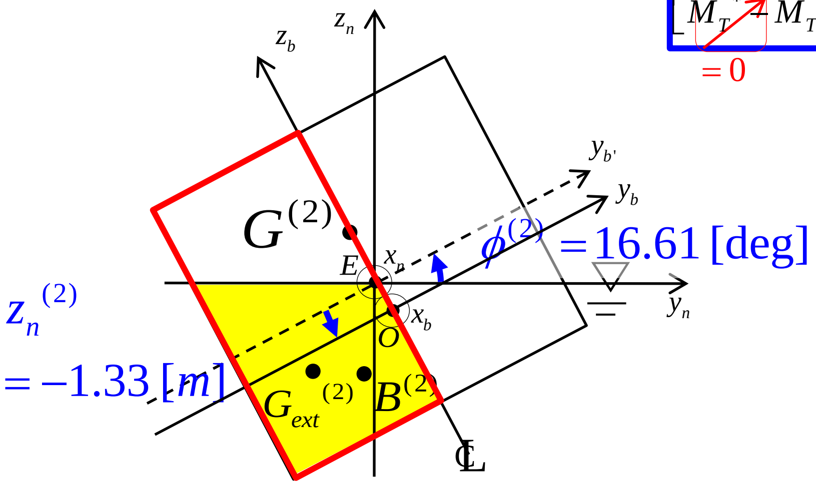
$$d_1 = \frac{B_{mld}}{2}\tan\left|\phi^{(2)}\right| = \frac{40}{2}\tan 16.61^\circ = 5.97\ [m]$$

$$A_{Section} = 413.2 + 59.7 + (-59.7) = 413.2\ [m^2]$$

$$F_{B,z}^{(2)} = \rho g V = 10\cdot 100\cdot 413.2 = 4.13\times 10^5\ [kN]$$



$$\begin{bmatrix} \cancel{F_z^{*}} - F_z^{(k)} \\ \cancel{M_T^{*}} - M_T^{(k)} \end{bmatrix}_{=0} = \left[\begin{array}{c:c} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) \\ \hdashline -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} & -\rho g\left({}^{n}z_{B^{(k)}/E}\nabla^{(k)} + I_T^{(k)}\right) - {}^{n}z_{G^{(k)}/E} \cdot F_{G,z} \\ -\left(-\rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) & -{}^{n}z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\rho g i_T^{(k)}\right) \end{array}\right]_{\substack{z_n = z_n^{(0)} \\ \phi = \phi^{(0)} \\ \theta = \theta^{(0)}}} \begin{bmatrix} \delta z_n^{(k)} \\ \delta \phi^{(k)} \end{bmatrix}$$



Is the ship in static equilibrium?

Force equilibrium :

$$F_z^{(2)} = F_{B,z}^{(2)} + F_{G,z}^{(2)} + F_{ext,z}^{(2)}$$

$$F_G^{(2)} = F_G = -3.6 \times 10^5 \ [kN]$$

$$F_{ext}^{(2)} = -\rho g v = -\rho g l a_{Section}$$

$$a_{Section} = A_1 + A_2$$

$$= 206.6 + 59.7 = 266.3 \ [m^2]$$

$$F_{ext}^{(2)} = -\rho g v = -10 \cdot 20 \cdot 266.3 = -5.33 \times 10^4 \ [kN]$$

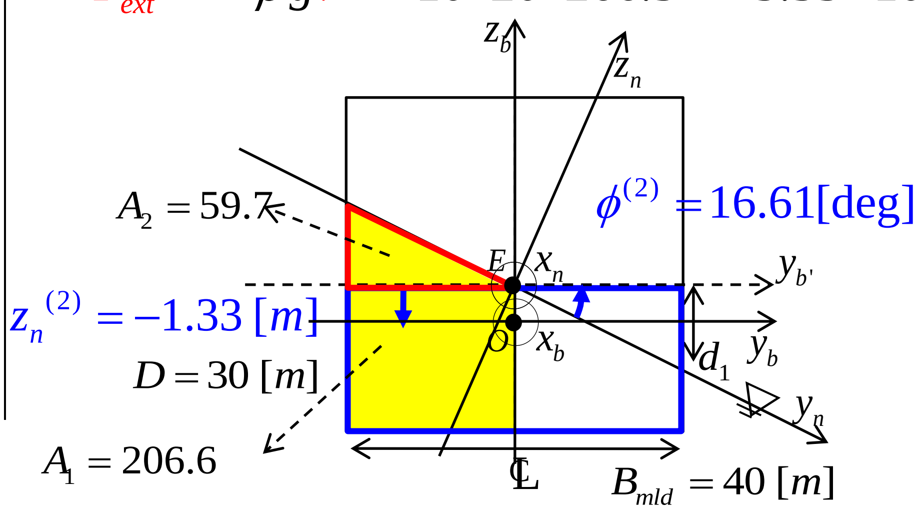
$L = 100\ [m]$	${}^{n}\mathbf{r}_{G^{(1)}/E} = [0 \quad -1.58 \quad 4.33]^T\ [m]$
$B_{mld} = 40\ [m]$	${}^{n}\mathbf{r}_{B^{(1)}/E} = [0 \quad -2.91 \quad -5.69]^T\ [m]$
$D = 30\ [m]$	${}^{n}\mathbf{r}_{G_{ext}^{(1)}/E} = [0 \quad -9.11 \quad -6.75]^T\ [m]$
$d = 9\ [m]$	$l = 20\ [m] \quad b = 20\ [m]$

$$F_{G,z} = -3.6 \times 10^5\ [kN]$$

$$F_{B,z}^{(2)} = 4.14 \times 10^5\ [kN]$$

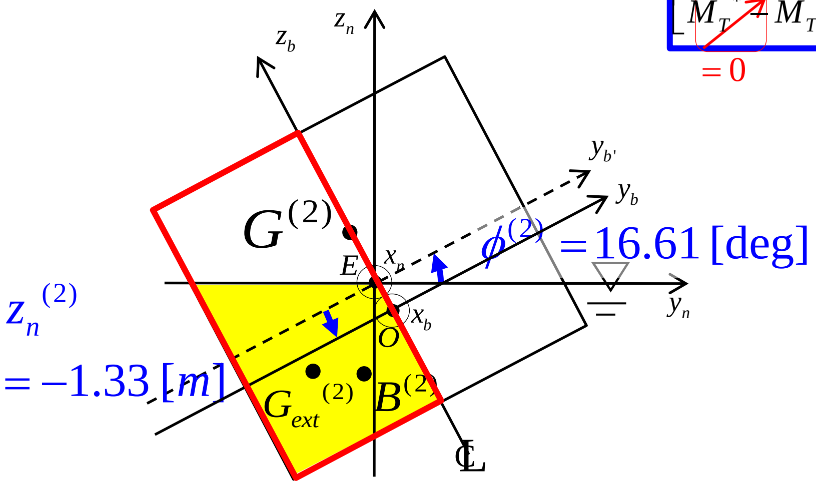
$$F_{ext,z}^{(1)} = -5.62 \times 10^4\ [kN]$$

$$\rho g = 10\ [Mg/m^2 s^2]$$



$$\begin{bmatrix} F_z^* - F_z^{(k)} \\ M_T^* - M_T^{(k)} \end{bmatrix} = \left[\begin{array}{c:c} -\rho g A_{WP}^{(k)} - \left(-\mu_E \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}} - \left(-\rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}}\right) \\ \hdashline -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} \\ -\left(-\rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -{}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\rho g i_T^{(k)}\right) \end{array}\right]_{\substack{z_n = z_n^{(0)} \\ \phi = \phi^{(0)} \\ \theta = \theta^{(0)}}} \begin{bmatrix} \delta z_n^{(k)} \\ \delta \phi^{(k)} \end{bmatrix}$$

$= 0$



$L = 100\ [m]$; ${}^n\mathbf{r}_{G^{(1)}/E} = [0 \quad -1.58 \quad 4.33]^T\ [m]$

$B_{mld} = 40\ [m]$; ${}^n\mathbf{r}_{B^{(1)}/E} = [0 \quad -2.91 \quad -5.69]^T\ [m]$

$D = 30\ [m]$; ${}^n\mathbf{r}_{G_{ext}^{(1)}/E} = [0 \quad -9.11 \quad -6.75]^T\ [m]$

$d = 9\ [m]$; $l = 20\ [m] \quad b = 20\ [m]$

$F_{G,z} = -3.6 \times 10^5\ [kN]$

$F_{B,z}^{(2)} = 4.14 \times 10^5\ [kN]$

$F_{ext,z}^{(2)} = -5.62 \times 10^4\ [kN]$

$\rho g = 10\ [Mg/m^2 s^2]$

Is the ship in static equilibrium?

Force equilibrium :

$$F_z^{(2)} = F_{B,z}^{(2)} + F_{G,z}^{(2)} + F_{ext,z}^{(2)}$$

$$F_{B,z}^{(2)} = 4.13 \times 10^5\ [kN]$$

$$F_{G,z}^{(2)} = -3.6 \times 10^5\ [kN]$$

$$F_{ext,z}^{(2)} = -5.33 \times 10^4\ [kN]$$

$$F_z^{(2)} = 4.13 \times 10^5 - 3.6 \times 10^5 - 5.33 \times 10^4$$

$$\boxed{= -3.0 \times 10^2\ [kN]} > e$$

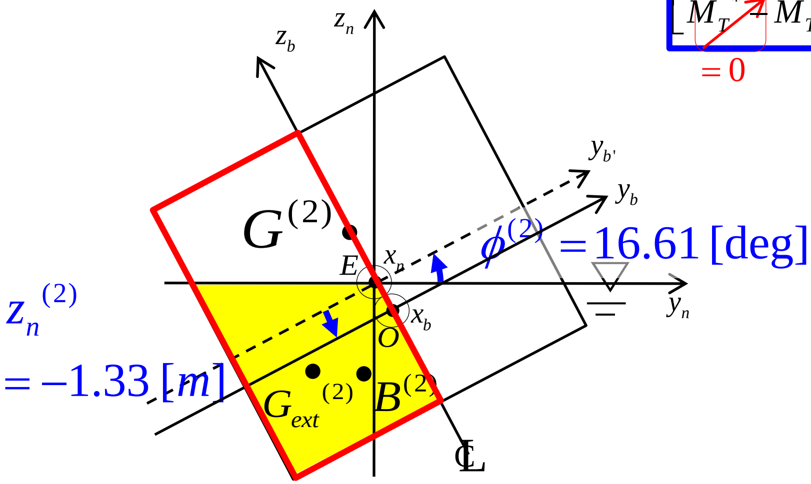
Tolerance

where, e(epsilon) : an arbitrarily small positive quantity

The static equilibrium of force is not satisfied!

We have to iterate!

$$\boxed{\begin{bmatrix} \cancel{F_z^* - F_z^{(k)}} \\ \cancel{M_T^* - M_T^{(k)}} \end{bmatrix}}_{=0} = \left[\begin{array}{c:c} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) \\ \hdashline -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} & -\rho g\left({}^{n}z_{B^{(k)}/E}\nabla^{(k)} + I_T^{(k)}\right) - {}^{n}z_{G^{(k)}/E} \cdot F_{G,z} \\ -\left(-\rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) & -{}^{n}z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\rho g i_T^{(k)}\right) \end{array}\right]_{\substack{z_n = z_n^{(0)} \\ \phi = \phi^{(0)} \\ \theta = \theta^{(0)}}} \begin{bmatrix} \delta z_n^{(k)} \\ \delta \phi^{(k)} \end{bmatrix}$$



Is the ship in static equilibrium?

Moment equilibrium :

$$M_T^{(2)} = M_{BT}^{(2)} + M_{GT}^{(2)} + M_{extT}^{(2)}$$

$$M_{BT}^{(2)} = {}^{n}y_{B^{(2)}/E} \cdot F_{B,z}^{(2)}$$

$$^{b'}y_{B^{(2)}/E} = \frac{^{b'}y_{c_1/E}A_1 + {}^{b'}y_{c_2/E}A_2 + {}^{b'}y_{c_3/E}A_3}{A_1 + A_2 + A_3}$$

$$= \frac{0 \cdot 413.2 + (-13.33) \cdot (59.7) + (13.33) \cdot (-59.7)}{413.2 + 59.7 - 59.7}$$

$$= -3.85\ [m]$$

$L = 100\ [m]$

$B_{mld} = 40\ [m]$

$D = 30\ [m]$

$d = 9\ [m]$

${}^{n}\mathbf{r}_{G^{(1)}/E} = [0 \quad -1.58 \quad 4.33]^T\ [m]$

${}^{n}\mathbf{r}_{B^{(1)}/E} = [0 \quad -2.91 \quad -5.69]^T\ [m]$

${}^{n}\mathbf{r}_{G_{ext}^{(1)}/E} = [0 \quad -9.11 \quad -6.75]^T\ [m]$

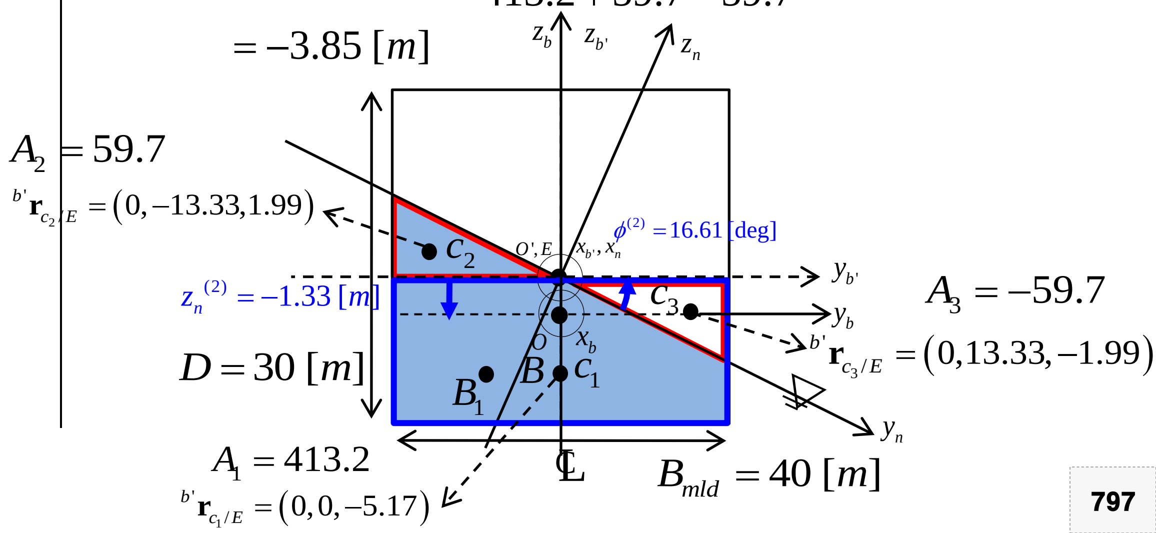
$l = 20\ [m] \quad b = 20\ [m]$

$F_{G,z} = -3.6 \times 10^5\ [kN]$

$F_{B,z}^{(2)} = 4.14 \times 10^5\ [kN]$

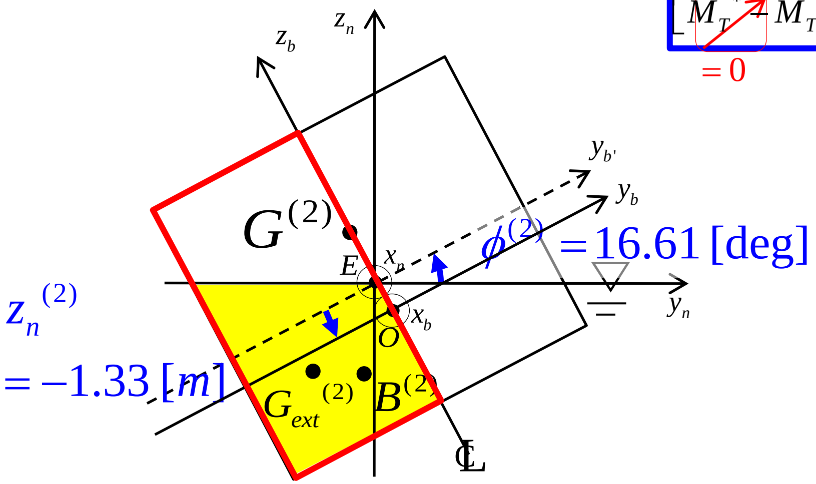
$F_{ext,z}^{(2)} = -5.62 \times 10^4\ [kN]$

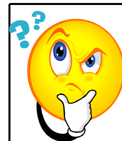
$\rho g = 10\ [Mg/m^2 s^2]$



$$\boxed{\begin{bmatrix} F_z^{*} - F_z^{(k)} \\ M_T^{*} - M_T^{(k)} \end{bmatrix}} = \begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) \\ -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) & -\rho g\left({}^{n}z_{B^{(k)}/E}\nabla^{(k)} + I_T^{(k)}\right) - {}^{n}z_{G^{(k)}/E} \cdot F_{G,z} - {}^{n}z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\rho g i_T^{(k)}\right) \end{bmatrix}_{\substack{z_n = z_n^{(0)} \\ \phi = \phi^{(0)} \\ \theta = \theta^{(0)}}} \begin{bmatrix} \delta z_n^{(k)} \\ \delta\phi^{(k)} \end{bmatrix}$$

$= 0$





Is the ship in static equilibrium?

Moment equilibrium :

$${}^{b'}y_{B^{(2)}/E} = -3.85\ [m]$$

$$M_T^{(2)} = M_{BT}^{(2)} + M_{GT}^{(2)} + M_{extT}^{(2)}$$

$$M_{BT}^{(2)} = {}^{n}y_{B^{(2)}/E} \cdot F_{B,z}^{(2)}$$

$${}^{b'}z_{B^{(2)}/E} = \frac{{}^{b'}z_{c_1/E}A_1 + {}^{b'}z_{c_2/E}A_2 + {}^{b'}z_{c_3/E}A_3}{A_1 + A_2 + A_3}$$

$$= \frac{(-5.17)\cdot 413.2 + 1.99 \cdot 59.7 + (-1.99)\cdot(-59.7)}{413.2 + 59.7 - 59.7}$$

$$= -4.59\ [m]$$

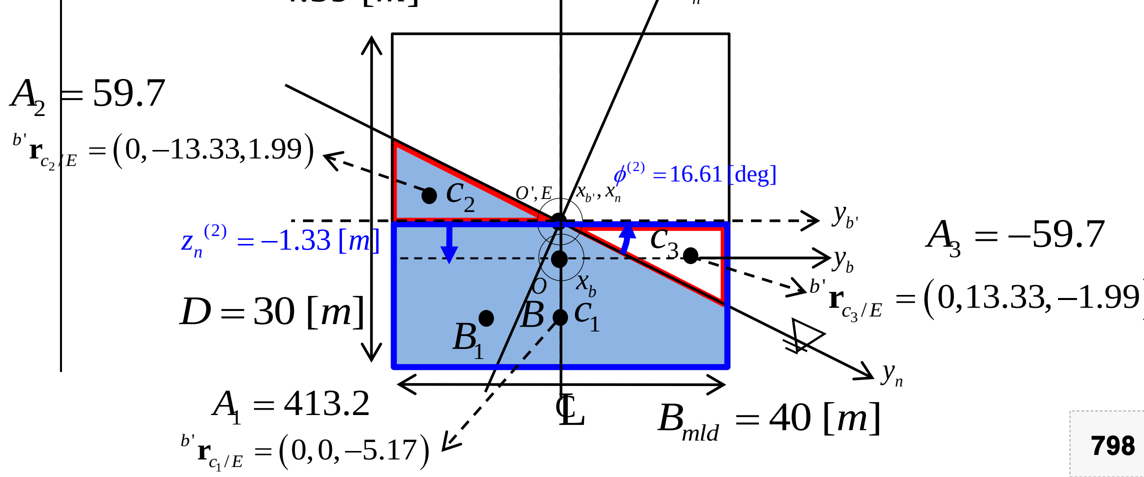
$L = 100\ [m]$	${}^{n}\mathbf{r}_{G^{(1)}/E} = [0 \quad -1.58 \quad 4.33]^T\ [m]$
$B_{mld} = 40\ [m]$	${}^{n}\mathbf{r}_{B^{(1)}/E} = [0 \quad -2.91 \quad -5.69]^T\ [m]$
$D = 30\ [m]$	${}^{n}\mathbf{r}_{G_{ext}^{(1)}/E} = [0 \quad -9.11 \quad -6.75]^T\ [m]$
$d = 9\ [m]$	$l = 20\ [m] \quad b = 20\ [m]$

$$F_{G,z} = -3.6 \times 10^5\ [kN]$$

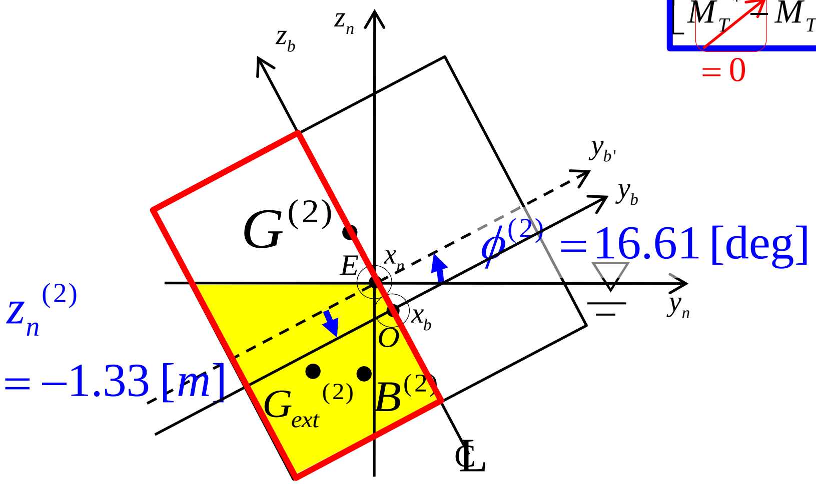
$$F_{B,z}^{(2)} = 4.14 \times 10^5\ [kN]$$

$$F_{ext,z}^{(2)} = -5.62 \times 10^4\ [kN]$$

$$\rho g = 10\ [Mg/m^2 s^2]$$



$$\begin{bmatrix} \underbrace{\cancel{F_z^{*}}}_{=0} - F_z^{(k)} \\ \cancel{M_T^{*}} - M_T^{(k)} \end{bmatrix} = \left[\begin{array}{c:c} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) \\ \hdashline -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} & -\rho g \left({}^{n}z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^{n}z_{G^{(k)}/E} \cdot F_{G,z} \\ -\left(-\rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) & -{}^{n}z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\rho g i_T^{(k)}\right) \end{array}\right]_{\substack{z_n = z_n^{(0)} \\ \phi = \phi^{(0)} \\ \theta = \theta^{(0)}}} \begin{bmatrix} \delta z_n^{(k)} \\ \delta \phi^{(k)} \end{bmatrix}$$



$L = 100\ [m]$	${}^{n}\mathbf{r}_{G^{(1)}/E} = \begin{bmatrix} 0 & -1.58 & 4.33 \end{bmatrix}^T\ [m]$
$B_{mld} = 40\ [m]$	${}^{n}\mathbf{r}_{B^{(1)}/E} = \begin{bmatrix} 0 & -2.91 & -5.69 \end{bmatrix}^T\ [m]$
$D = 30\ [m]$	${}^{n}\mathbf{r}_{G_{ext}^{(1)}/E} = \begin{bmatrix} 0 & -9.11 & -6.75 \end{bmatrix}^T\ [m]$
$d = 9\ [m]$	$l = 20\ [m]$ $\quad b = 20\ [m]$

$F_{G,z} = -3.6 \times 10^5\ [kN]$

$F_{B,z}^{(2)} = 4.14 \times 10^5\ [kN]$

$F_{ext,z}^{(2)} = -5.62 \times 10^4\ [kN]$

$\rho g = 10\ [Mg/m^2 s^2]$

Is the ship in static equilibrium?

Moment equilibrium :

$${}^{b'}y_{B^{(2)}/E} = -3.85\ [m]$$

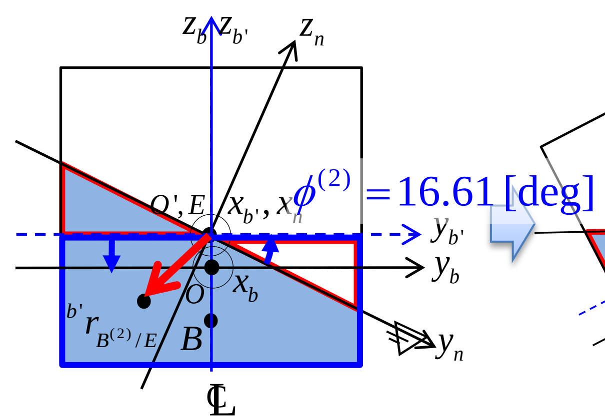
$${}^{b'}z_{B^{(2)}/E} = -4.59\ [m]$$

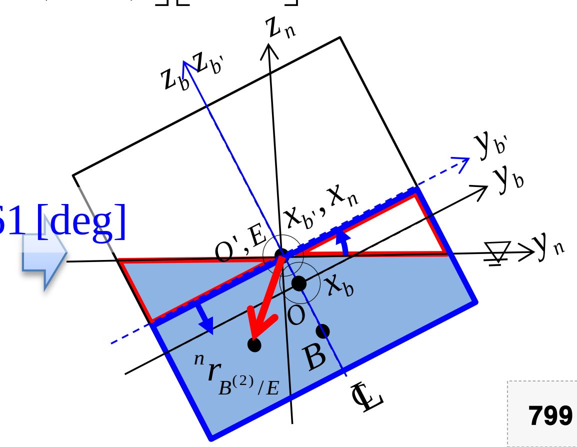
$$M_T^{(2)} = M_{BT}^{(2)} + M_{GT}^{(2)} + M_{extT}^{(2)}$$

$$M_{BT}^{(2)} = {}^{n}y_{B^{(2)}/E} \cdot F_{B,z}^{(2)}$$

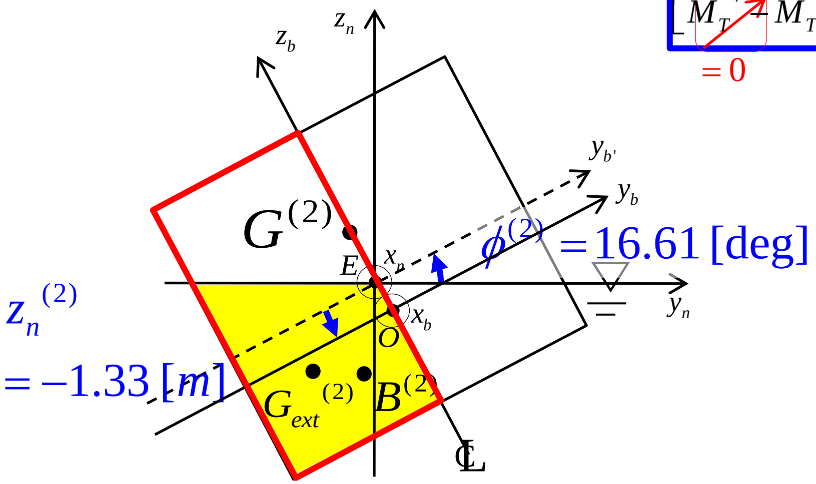
$${}^{n}\mathbf{r}_{B^{(2)}/E} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi^{(2)} & -\sin\phi^{(2)} \\ 0 & \sin\phi^{(2)} & \cos\phi^{(2)} \end{bmatrix} {}^{b'}\mathbf{r}_{B^{(2)}/E}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(16.61) & -\sin(16.61) \\ 0 & \sin(16.61) & \cos(16.61) \end{bmatrix} \begin{bmatrix} 0 \\ -3.85 \\ -4.59 \end{bmatrix}$$





$$\boxed{\begin{bmatrix} \cancel{F_z^{*}} - F_z^{(k)} \\ \cancel{M_T^{*}} - M_T^{(k)} \end{bmatrix}}_{=0} = \begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) \\ -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) & -\rho g\left({}^{n}z_{B^{(k)}/E}\nabla^{(k)} + I_T^{(k)}\right) - {}^{n}z_{G^{(k)}/E} \cdot F_{G,z} - {}^{n}z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\rho g i_T^{(k)}\right) \end{bmatrix}_{\substack{z_n = z_n^{(0)} \\ \phi = \phi^{(0)} \\ \theta = \theta^{(0)}}} \begin{bmatrix} \delta z_n^{(k)} \\ \delta \phi^{(k)} \end{bmatrix}$$



Is the ship in static equilibrium?

Moment equilibrium :

$${}^{b'}y_{B^{(2)}/E} = -3.85\ [m]$$

$${}^{b'}z_{B^{(2)}/E} = -4.59\ [m]$$

$$M_T^{(2)} = M_{BT}^{(2)} + M_{GT}^{(2)} + M_{extT}^{(2)}$$

$$M_{BT}^{(2)} = {}^{n}y_{B^{(2)}/E} \cdot F_{B,z}^{(2)}$$

$${}^{n}\mathbf{r}_{B^{(2)}/E} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi^{(2)} & -\sin\phi^{(2)} \\ 0 & \sin\phi^{(2)} & \cos\phi^{(2)} \end{bmatrix} {}^{b'}\mathbf{r}_{B^{(2)}/E}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(16.61) & -\sin(16.61) \\ 0 & \sin(16.61) & \cos(16.61) \end{bmatrix} \begin{bmatrix} 0 \\ -3.85 \\ -4.59 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -2.37 \\ -5.50 \end{bmatrix}$$

$${}^{n}y_{B^{(2)}/E} = -2.37\ [m]$$

$$M_{BT}^{(2)} = (-2.37)\cdot(4.14\times10^5)$$

$$= -9.80\times10^5$$

$L = 100\ [m]$	${}^{n}\mathbf{r}_{G^{(1)}/E} = [0\quad -1.58\quad 4.33]^T\ [m]$
$B_{mld} = 40\ [m]$	${}^{n}\mathbf{r}_{B^{(2)}/E} = [0\quad -2.37\quad -5.50]^T\ [m]$
$D = 30\ [m]$	${}^{n}\mathbf{r}_{G_{ext}^{(1)}/E} = [0\quad -9.11\quad -6.75]^T\ [m]$
$d = 9\ [m]$	$l = 20\ [m]$ $b = 20\ [m]$

$$F_{G,z} = -3.6\times10^5\ [kN]$$

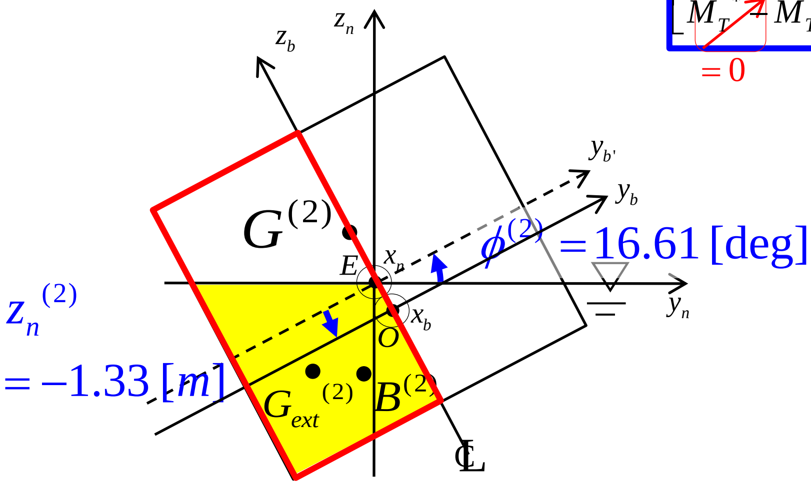
$$F_{B,z}^{(2)} = 4.14\times10^5\ [kN]$$

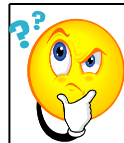
$$F_{ext,z}^{(2)} = -5.62\times10^4\ [kN]$$

$$\rho g = 10\ [Mg/m^2s^2]$$

$$\begin{bmatrix} F_z^* - F_z^{(k)} \\ M_T^* - M_T^{(k)} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\rho g i_T^{(k)}\right) \end{bmatrix}_{\substack{z_n = z_n^{(0)} \\ \phi = \phi^{(0)} \\ \theta = \theta^{(0)}}} \begin{bmatrix} \delta z_n^{(k)} \\ \delta \phi^{(k)} \end{bmatrix}$$

$= 0$





Is the ship in static equilibrium?

Moment equilibrium : $M_{BT}^{(2)} = -9.80 \times 10^5$

$$M_T^{(2)} = M_{BT}^{(2)} + M_{GT}^{(2)} + M_{extT}^{(2)}$$

$$M_{GT}^{(2)} = {}^n y_{G^{(2)}/E} \cdot F_{G,z}$$

2. Rotation with heel

1. Translation with immersion

$${}^n \mathbf{r}_{G^{(2)}/E} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi^{(2)} & -\sin\phi^{(2)} \\ 0 & \sin\phi^{(2)} & \cos\phi^{(2)} \end{bmatrix} {}^{b'}\mathbf{r}_{G^{(2)}/E}, \quad {}^{b'}\mathbf{r}_{G^{(2)}/E} = {}^b \mathbf{r}_{G^{(2)}/E} + \begin{bmatrix} 0 \\ 0 \\ z_n^{(2)} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(16.61) & -\sin(16.61) \\ 0 & \sin(16.61) & \cos(16.61) \end{bmatrix} \left(\begin{bmatrix} 0 \\ 0 \\ 6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -1.33 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 0 \\ -1.33 \\ 4.47 \end{bmatrix}$$

$${}^n y_{G^{(2)}/E} = -1.33 \, [m]$$

$$M_{GT}^{(2)} = (-1.33) \cdot (-3.6 \times 10^5)$$

$$= 4.79 \times 10^5$$

$L = 100 \, [m]$ | ${}^n \mathbf{r}_{G^{(2)}/E} = [0 \quad -1.33 \quad 4.47]^T \, [m]$

$B_{mld} = 40 \, [m]$ | ${}^n \mathbf{r}_{B^{(2)}/E} = [0 \quad -2.37 \quad -5.50]^T \, [m]$

$D = 30 \, [m]$ | ${}^n \mathbf{r}_{G_{ext}^{(1)}/E} = [0 \quad -9.11 \quad -6.75]^T \, [m]$

$d = 9 \, [m]$ | $l = 20 \, [m] \quad b = 20 \, [m]$

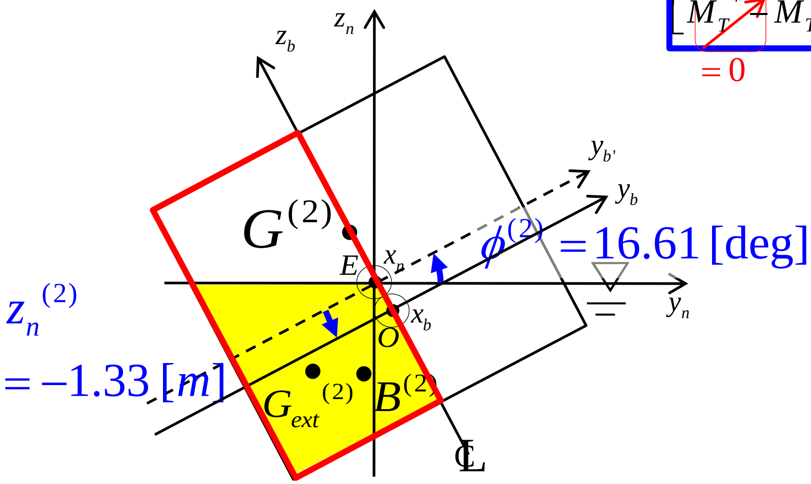
$F_{G,z} = -3.6 \times 10^5 \, [kN]$

$F_{B,z}^{(2)} = 4.14 \times 10^5 \, [kN]$

$F_{ext,z}^{(2)} = -5.62 \times 10^4 \, [kN]$

$\rho g = 10 \, [Mg / m^2 s^2]$

$$\begin{bmatrix} \cancel{F_z^* - F_z^{(k)}} \\ \cancel{M_T^* - M_T^{(k)}} \end{bmatrix}_{=0} = \left[\begin{array}{c|c} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) \\ \hline -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} \\ -\left(-\rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -{}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\rho g i_T^{(k)}\right) \end{array}\right]_{\substack{z_n = z_n^{(0)} \\ \phi = \phi^{(0)} \\ \theta = \theta^{(0)}}} \begin{bmatrix} \delta z_n^{(k)} \\ \delta \phi^{(k)} \end{bmatrix}$$



Is the ship in static equilibrium?

Moment equilibrium :

$M_{BT}^{(2)} = -9.80 \times 10^5$

$M_{GT}^{(2)} = 4.79 \times 10^5$

$$M_T^{(2)} = M_{BT}^{(2)} + M_{GT}^{(2)} + M_{extT}^{(2)}$$

$$M_{extT}^{(2)} = {}^n y_{G_{ext}^{(2)}/E} \cdot F_{ext,z}^{(2)}$$

$$\begin{aligned} {}^{b'}y_{G_{ext}^{(2)}/E} &= \frac{{}^{b'}y_{c_1'/E} A_1 + {}^{b'}y_{c_2'/E} A_2}{A_1 + A_2} \\ &= \frac{(-10)\cdot(206.6) + (-13.33)\cdot(59.7)}{206.6 + 59.7} \\ &= -10.75\ [m] \end{aligned}$$

$L = 100\ [m]$

$B_{mld} = 40\ [m]$

$D = 30\ [m]$

$d = 9\ [m]$

${}^n\mathbf{r}_{G^{(2)}/E} = [0\quad -1.33\quad 4.47]^T\ [m]$

${}^n\mathbf{r}_{B^{(2)}/E} = [0\quad -2.37\quad -5.50]^T\ [m]$

${}^n\mathbf{r}_{G_{ext}^{(1)}/E} = [0\quad -9.11\quad -6.75]^T\ [m]$

$l = 20\ [m]\quad b = 20\ [m]$

$F_{G,z} = -3.6 \times 10^5\ [kN]$

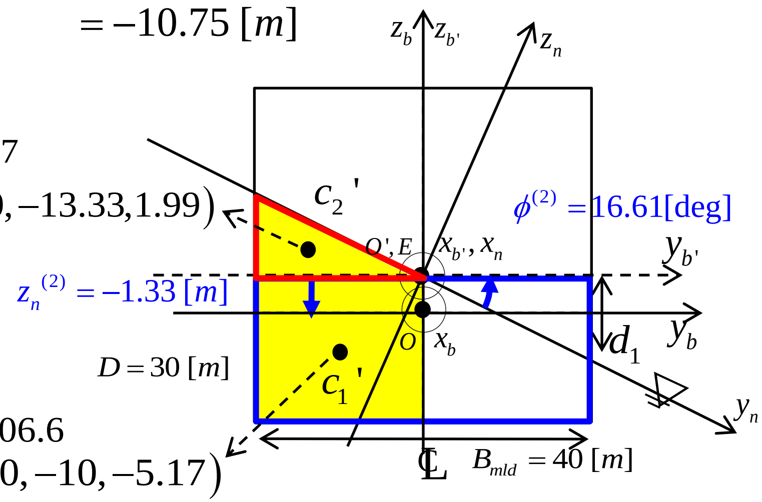
$F_{B,z}^{(2)} = 4.14 \times 10^5\ [kN]$

$F_{ext,z}^{(2)} = -5.62 \times 10^4\ [kN]$

$\rho g = 10\ [Mg/m^2s^2]$

$A_2 = 59.7$

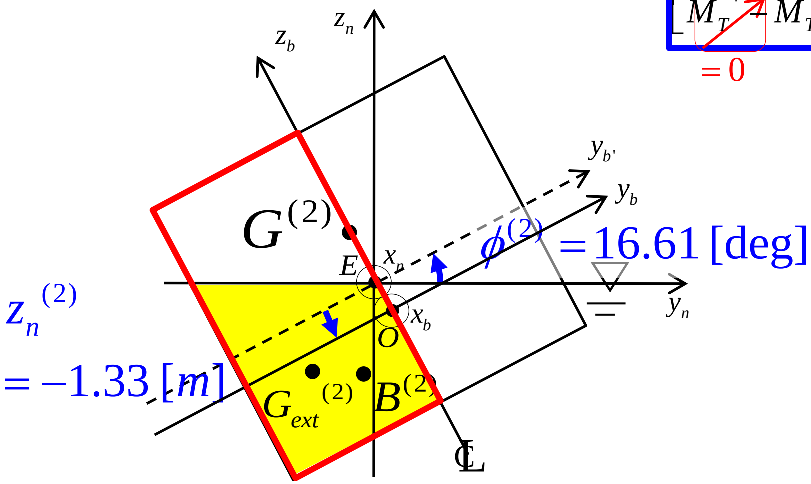
${}^{b'}\mathbf{r}_{c_2'/E} = (0, -13.33, 1.99)$



$A_1 = 206.6$

${}^{b'}\mathbf{r}_{c_1'/E} = (0, -10, -5.17)$

$$\boxed{\begin{matrix} \cancel{F_z{}^{*} - F_z{}^{(k)}} \\ \cancel{M_T{}^{*} - M_T{}^{(k)}} \end{matrix}}_{=0} = \left[\begin{array}{c:c} -\rho g A_{WP}{}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}{}^{(k)}\right) & -\rho g A_{WP}{}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} - \left(-\rho g a_{WP}{}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) \\ \hdashline -\rho g A_{WP}{}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} & -\rho g\left({}^{n}z_{B^{(k)}/E}\nabla^{(k)} + I_T{}^{(k)}\right) - {}^{n}z_{G^{(k)}/E} \cdot F_{G,z} \\ -\left(-\rho g a_{WP}{}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) & -{}^{n}z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} - \left(-\rho g i_T{}^{(k)}\right) \end{array}\right]_{\substack{z_n = z_n{}^{(0)} \\ \phi = \phi^{(0)} \\ \theta = \theta^{(0)}}} \begin{bmatrix} \delta z_n{}^{(k)} \\ \delta \phi^{(k)} \end{bmatrix}$$



$L = 100\ [m]$	${}^{n}\mathbf{r}_{G^{(2)}/E} = [0 \quad -1.33 \quad 4.47]^T\ [m]$
$B_{mld} = 40\ [m]$	${}^{n}\mathbf{r}_{B^{(2)}/E} = [0 \quad -2.37 \quad -5.50]^T\ [m]$
$D = 30\ [m]$	${}^{n}\mathbf{r}_{G_{ext}{}^{(1)}/E} = [0 \quad -9.11 \quad -6.75]^T\ [m]$
$d = 9\ [m]$	$l = 20\ [m] \quad b = 20\ [m]$

$$F_{G,z} = -3.6 \times 10^5\ [kN]$$

$$F_{B,z}{}^{(2)} = 4.14 \times 10^5\ [kN]$$

$$F_{ext,z}{}^{(2)} = -5.62 \times 10^4\ [kN]$$

$$\rho g = 10\ [Mg/m^2 s^2]$$

Is the ship in static equilibrium?

Moment equilibrium :

$$M_{BT}{}^{(2)} = -9.80 \times 10^5$$

$$M_{GT}{}^{(2)} = 4.79 \times 10^5$$

$$M_T{}^{(2)} = M_{BT}{}^{(2)} + M_{GT}{}^{(2)} + M_{extT}{}^{(2)}$$

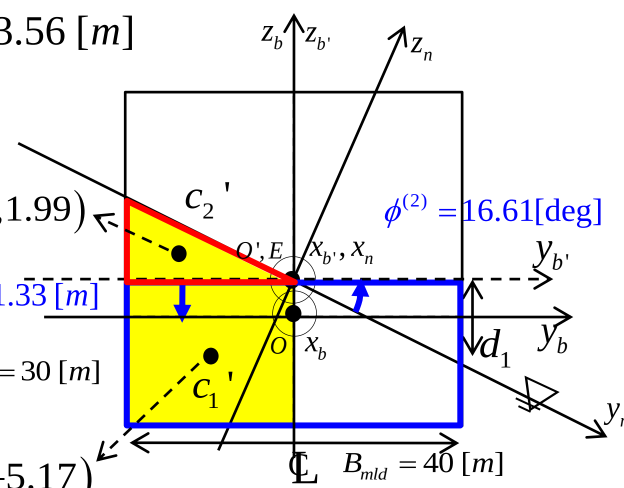
$$M_{extT}{}^{(2)} = {}^{n}y_{G_{ext}{}^{(2)}/E} \cdot F_{ext,z}{}^{(2)}$$

$${}^{b'}y_{G_{ext}{}^{(2)}/E} = -10.75\ [m]$$

$${}^{b'}z_{G_{ext}{}^{(2)}/E} = \frac{{}^{b'}z_{c_1'/E}A_1 + {}^{b'}z_{c_2'/E}A_2}{A_1 + A_2} = \frac{(-5.17)\cdot 206.6 + 1.99 \cdot 59.7}{206.6 + 59.7} = -3.56\ [m]$$

$A_2 = 59.7$

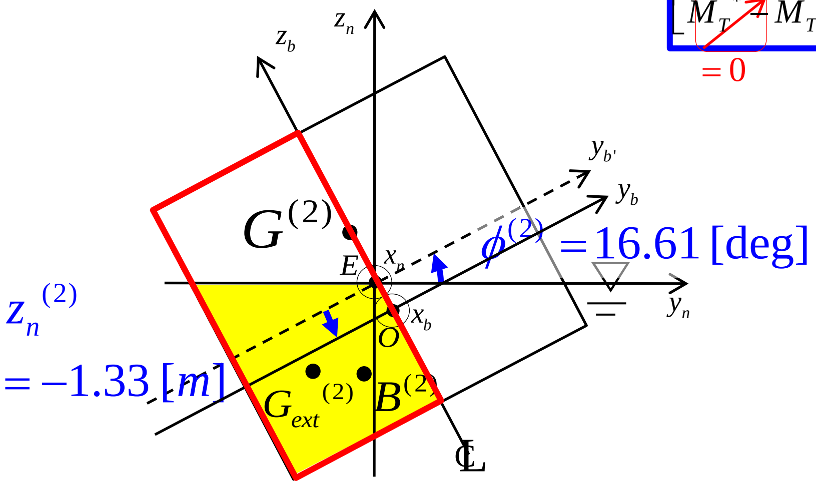
${}^{b'}\mathbf{r}_{c_2'/E} = (0, -13.33, 1.99)$



$A_1 = 206.6$

${}^{b'}\mathbf{r}_{c_1'/E} = (0, -10, -5.17)$

$$\underbrace{\begin{bmatrix} \cancel{F_z^{*}} - F_z^{(k)} \\ \cancel{M_T^{*}} - M_T^{(k)} \end{bmatrix}}_{=0} = \left.\begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) \\ -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) & -\rho g\left({}^{n}z_{B^{(k)}/E}\nabla^{(k)} + I_T^{(k)}\right) - {}^{n}z_{G^{(k)}/E} \cdot F_{G,z} - {}^{n}z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\rho g i_T^{(k)}\right) \end{bmatrix}\right|_{\substack{z_n = z_n^{(0)} \\ \phi = \phi^{(0)} \\ \theta = \theta^{(0)}}} \begin{bmatrix} \delta z_n^{(k)} \\ \delta \phi^{(k)} \end{bmatrix}$$



Is the ship in static equilibrium?

Moment equilibrium :

$M_{BT}^{(2)} = -9.80\times10^{5}$

$M_{GT}^{(2)} = 4.79\times10^{5}$

$${}^{b'}y_{G_{ext}^{(2)}/E} = -10.75\ [m]$$

$${}^{b'}z_{G_{ext}^{(2)}/E} = -3.56\ [m]$$

$$M_T^{(2)} = M_{BT}^{(2)} + M_{GT}^{(2)} + M_{extT}^{(2)}$$

$$M_{extT}^{(2)} = {}^{n}y_{G_{ext}^{(2)}/E} \cdot F_{ext,z}^{(2)}$$

$${}^{n}\mathbf{r}_{G_{ext}^{(2)}/E} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi^{(2)} & -\sin\phi^{(2)} \\ 0 & \sin\phi^{(2)} & \cos\phi^{(2)} \end{bmatrix} {}^{b'}\mathbf{r}_{G_{ext}^{(2)}/E}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(16.61) & -\sin(16.61) \\ 0 & \sin(16.61) & \cos(16.61) \end{bmatrix} \begin{bmatrix} 0 \\ -10.75 \\ -3.56 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -9.28 \\ -6.49 \end{bmatrix}$$

$${}^{n}y_{G_{ext}^{(2)}/E} = -9.28\ [m]$$

$$M_{extT}^{(2)} = (-9.28)\cdot(-5.33\times10^{4})$$

$$= 4.95\times10^{5}$$

$L = 100\ [m]$	${}^{n}\mathbf{r}_{G^{(2)}/E} = [0\quad -1.33\quad 4.47]^T\ [m]$
$B_{mld} = 40\ [m]$	${}^{n}\mathbf{r}_{B^{(2)}/E} = [0\quad -2.37\quad -5.50]^T\ [m]$
$D = 30\ [m]$	${}^{n}\mathbf{r}_{G_{ext}^{(2)}/E} = [0\quad -9.28\quad -6.49]^T\ [m]$
$d = 9\ [m]$	$l = 20\ [m]$ $b = 20\ [m]$

$F_{G,z} = -3.6\times10^{5}\ [kN]$

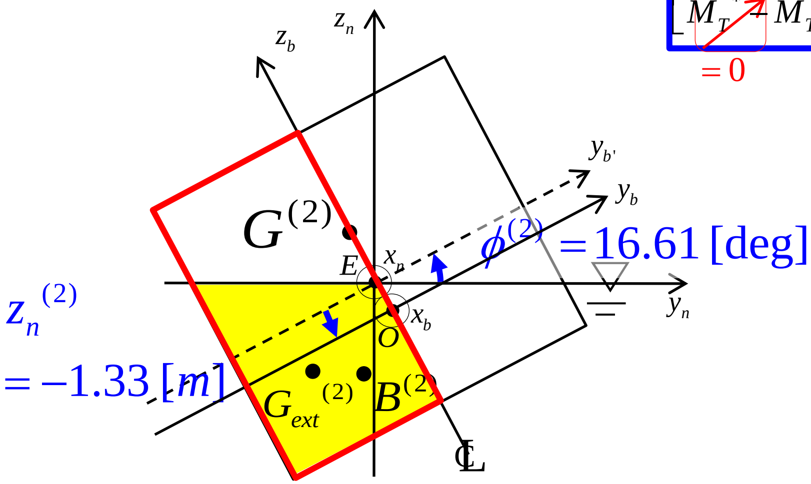
$F_{B,z}^{(2)} = 4.14\times10^{5}\ [kN]$

$F_{ext,z}^{(2)} = -5.62\times10^{4}\ [kN]$

$\rho g = 10\ [Mg/m^2 s^2]$

$$\boxed{\begin{bmatrix} \cancel{F_z^{*}} - F_z^{(k)} \\ \cancel{M_T^{*}} - M_T^{(k)} \end{bmatrix}} \quad \cancel{F_z^{*}},\ \cancel{M_T^{*}} = 0$$

$$= \left[\begin{array}{c:c} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} - \left(-\rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) \\ \hdashline -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} & -\rho g\left({}^{n}z_{B^{(k)}/E}\nabla^{(k)} + I_T^{(k)}\right) - {}^{n}z_{G^{(k)}/E} \cdot F_{G,z} \\ -\left(-\rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) & -{}^{n}z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\rho g i_T^{(k)}\right) \end{array}\right]_{\substack{z_n = z_n^{(0)} \\ \phi = \phi^{(0)} \\ \theta = \theta^{(0)}}} \begin{bmatrix} \delta z_n^{(k)} \\ \delta \phi^{(k)} \end{bmatrix}$$



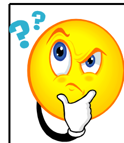
$L = 100\ [m]$	${}^{n}\mathbf{r}_{G^{(2)}/E} = [0\quad -1.33\quad 4.47]^T\ [m]$
$B_{mld} = 40\ [m]$	${}^{n}\mathbf{r}_{B^{(2)}/E} = [0\quad -2.37\quad -5.50]^T\ [m]$
$D = 30\ [m]$	${}^{n}\mathbf{r}_{G_{ext}^{(2)}/E} = [0\quad -9.28\quad -6.49]^T\ [m]$
$d = 9\ [m]$	$l = 20\ [m]\quad b = 20\ [m]$

$F_{G,z} = -3.6\times10^5\ [kN]$

$F_{B,z}^{(2)} = 4.14\times10^5\ [kN]$

$F_{ext,z}^{(2)} = -5.62\times10^4\ [kN]$

$\rho g = 10\ [Mg/m^2s^2]$



Is the ship in static equilibrium?

Moment equilibrium :

$M_{BT}^{(2)} = -9.80\times10^5$

$M_{GT}^{(2)} = 4.79\times10^5$

$M_{extT}^{(2)} = 4.95\times10^5$

$$M_T^{(2)} = M_{BT}^{(2)} + M_{GT}^{(2)} + M_{extT}^{(2)}$$

$$M_{BT}^{(2)} = -9.80\times10^5\ [kN\cdot m]$$
$$M_{GT}^{(2)} = 4.79\times10^5\ [kN\cdot m]$$
$$M_{extT}^{(2)} = 4.95\times10^5\ [kN\cdot m]$$

$$M_T^{(1)} = -9.80\times10^5 + 4.79\times10^5 + 4.95\times10^5$$

$$\boxed{= 6.00\times10^3\ [kN\cdot m]} > e$$

Tolerance

where, e(epsilon) : an arbitrarily small positive quantity

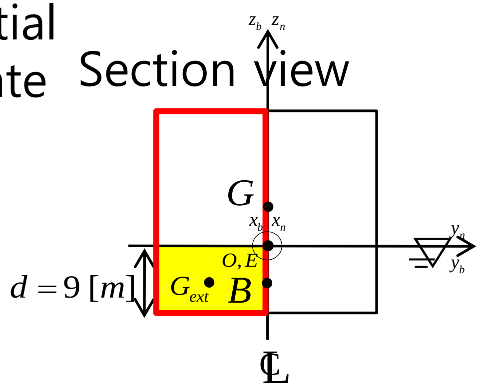
The static equilibrium of moment is not satisfied!

We have to iterate!

Example of Coupled Immersion and Heel of a Box Shaped Ship in Flooded State
- Summary

Initial State

Section View

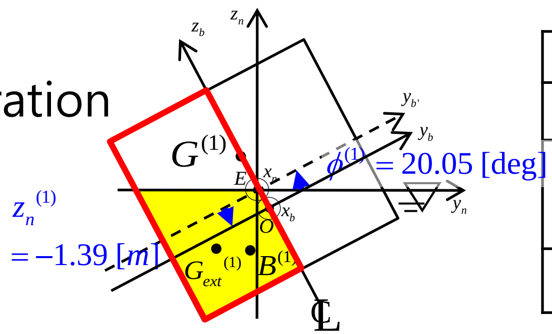


Force(KN)			Transverse Moment Arm(m)			Transverse Moment(KN·m)		
$F_{B,z}$	$F_{G,z}$	$F_{ext,z}$	${}^{n}y_{B/E}$	${}^{n}y_{G/E}$	${}^{n}y_{Gext/E}$	M_{BT}	M_{GT}	M_{extT}
3.60 x10^5	-3.60 x10^5	-3.60 x10^4	0	0	-10	0	0	3.60 X10^5
F = - 3.60x10^4						M_T = 3.60x10^5		

$\delta T = -1.39\,[m]$

$\delta\phi = 20.05\,[\text{deg}]$

1st Iteration

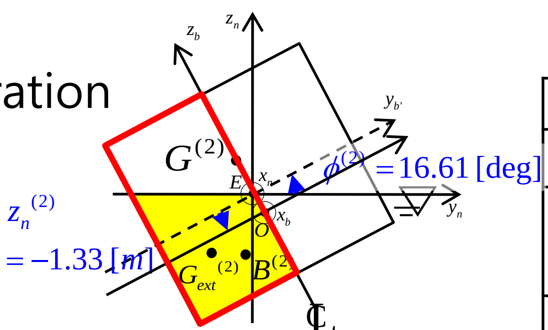


Force(KN)			Transverse Moment Arm(m)			Transverse Moment(KN·m)		
$F_{B,z}$	$F_{G,z}$	$F_{ext,z}$	${}^{n}y_{B/E}$	${}^{n}y_{G/E}$	${}^{n}y_{Gext/E}$	M_{BT}	M_{GT}	M_{extT}
4.16 x10^5	-3.60 x10^5	-5.62 x10^4	-2.91	-1.58	-10.6	-1.21 X10^6	5.69 X10^5	5.12 X10^5
F = - 2.00x10^2						M_T =- 1.29x10^5		

$\delta T = 0.06[m]$

$\delta\phi = -3.44\,[\text{deg}]$

2nd Iteration



Force(KN)			Transverse Moment Arm(m)			Transverse Moment(KN·m)		
$F_{B,z}$	$F_{G,z}$	$F_{ext,z}$	${}^{n}y_{B/E}$	${}^{n}y_{G/E}$	${}^{n}y_{Gext/E}$	M_{BT}	M_{GT}	M_{extT}
4.14 x10^5	-3.60 X10^5	-5.33 X10^4	-2.37	-1.32	-9.28	-9.80 X10^5	4.79 X10^5	4.95 X10^5
F = -3.00x10^1						M_T = 6.00x10^3		

12-5 Coupled Immersion, Heel, and Trim of a Box-Shaped Ship in Flooded State

Governing Equations of Computational Ship Stability in Flooded State

$$
\begin{bmatrix} \cancel{F_z} - F_z\left(z_n^{(k)},\phi^{(k)},\theta^{(k)}\right) \\ \cancel{M_T} - M_T\left(z_n^{(k)},\phi^{(k)},\theta^{(k)}\right) \\ \cancel{M_L} - M_L\left(z_n^{(k)},\phi^{(k)},\theta^{(k)}\right) \end{bmatrix}
=
\left.\begin{bmatrix}
\begin{gathered}\frac{\partial F_B}{\partial z_n}+\frac{\partial F_G}{\partial z_n}+\frac{\partial F_{ext}}{\partial z_n} \\ \boxed{-\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right)}\end{gathered} &
\begin{gathered}\frac{\partial F_B}{\partial \phi}+\frac{\partial F_G}{\partial \phi}+\frac{\partial F_{ext}}{\partial \phi} \\ \boxed{-\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right)}\end{gathered} &
\begin{gathered}\frac{\partial F_B}{\partial \theta}+\frac{\partial F_G}{\partial \theta}+\frac{\partial F_{ext}}{\partial \theta} \\ \boxed{\rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E}}\end{gathered} \\
\begin{gathered}\frac{\partial M_{BT}}{\partial z_n}+\frac{\partial M_{GT}}{\partial z_n}+\frac{\partial M_{extT}}{\partial z_n} \\ \boxed{-\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right)}\end{gathered} &
\begin{gathered}\frac{\partial M_{BT}}{\partial \phi}+\frac{\partial M_{GT}}{\partial \phi}+\frac{\partial M_{extT}}{\partial \phi} \\ \boxed{\begin{gathered}-\rho g\left({}^n z_{B^{(k)}/E}\nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_G \\ - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext}^{(k)} - \left(-\mu_F \cdot \rho g i_T^{(k)}\right)\end{gathered}}\end{gathered} &
\begin{gathered}\frac{\partial M_{BT}}{\partial \theta}+\frac{\partial M_{GT}}{\partial \theta}+\frac{\partial M_{extT}}{\partial \theta} \\ \boxed{\rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)}}\end{gathered} \\
\begin{gathered}\frac{\partial M_{BL}}{\partial z_n}+\frac{\partial M_{GL}}{\partial z_n}+\frac{\partial M_{extL}}{\partial z_n} \\ \boxed{\rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E}}\end{gathered} &
\begin{gathered}\frac{\partial M_{BL}}{\partial \phi}+\frac{\partial M_{GL}}{\partial \phi}+\frac{\partial M_{extL}}{\partial \phi} \\ \boxed{\rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)}}\end{gathered} &
\begin{gathered}\frac{\partial M_{BL}}{\partial \theta}+\frac{\partial M_{GL}}{\partial \theta}+\frac{\partial M_{extL}}{\partial \theta} \\ \boxed{\begin{gathered}-\rho g\left({}^n z_{B^{(k)}/E}\nabla^{(k)} + I_L^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_G \\ - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext}^{(k)} - \left(-\mu_F \cdot \rho g i_L^{(k)}\right)\end{gathered}}\end{gathered}
\end{bmatrix}\right|_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}}
\begin{bmatrix} \delta z_n^{(k)} \\ \delta\phi^{(k)} \\ \delta\theta^{(k)} \end{bmatrix}
$$

$= 0$

We want to find the static equilibrium position and orientation!

F_G: gravitational force exerted on a ship

M_T: transverse moment of a ship about x_n axis

M_L: longitudinal moment of a ship about y_n axis

$A_{WP}^{(k)}$: waterplane area of a ship at k^{th} step

$I_T^{(k)}$: transverse moment of inertia of the waterplane area of a ship about x_n axis at k^{th} step

$I_L^{(k)}$: longitudinal moment of inertia of the waterplane area of a ship about y_n axis at k^{th} step

$I_P^{(k)}$: centrifugal moment of the waterplane area of a ship about x_n and y_n axis at k^{th} step

F_B: buoyant force exerted on a ship

F_{ext}: external force exerted on a ship

${}^n x_{F^{(k)}/E}$: x_n coordinate of centroid of the waterplane area of a ship

${}^n y_{F^{(k)}/E}$: y_n coordinate of centroid of the waterplane area of a ship

${}^n z_{B^{(k)}/E}$: z_n coordinate of center of the displaced volume of a ship

${}^n z_{G^{(k)}/E}$: z_n coordinate of center of mass of the ship

$\delta z^{(k)}$: change in the draft at k^{th} step

$\delta\phi^{(k)}$: change in the angle of heel at k^{th} step

$\delta\theta^{(k)}$: change in the angle of trim at k^{th} step

μ_V: permeability of a compartment

μ_F: surface permeability of a compartment

$a_{WP}^{(k)}$: waterplane area of a flooded compartment at k^{th} step

$i_T^{(k)}$: transverse moment of inertia of the waterplane area of a flooded compartment about x_n axis at k^{th} step

$i_L^{(k)}$: longitudinal moment of inertia of the waterplane area of a flooded compartment about y_n axis at k^{th} step

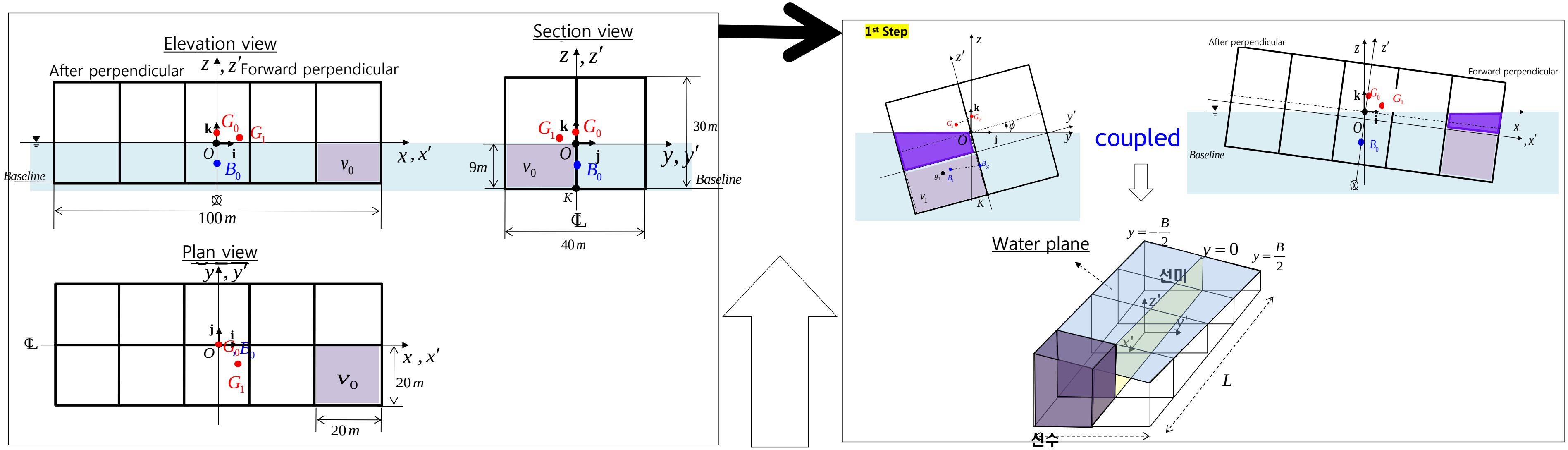
$i_P^{(k)}$: centrifugal moment of the waterplane area of a flooded compartment about x_n and y_n axis at k^{th} step

${}^n x_{f^{(k)}/E}$: x_n coordinate of centroid of the waterplane area of a flooded compartment at k^{th} step

${}^n y_{f^{(k)}/E}$: y_n coordinate of centroid of the waterplane area of a flooded compartment at k^{th} step

${}^n z_{G_{ext}^{(k)}/E}$: z_n coordinate of center of the submerged volume of a flooded compartment at k^{th} step

Calculation of Coupled Immersion, Trim, and Heel a Box-shaped Ship When a Cargo Hold Part is Flooded

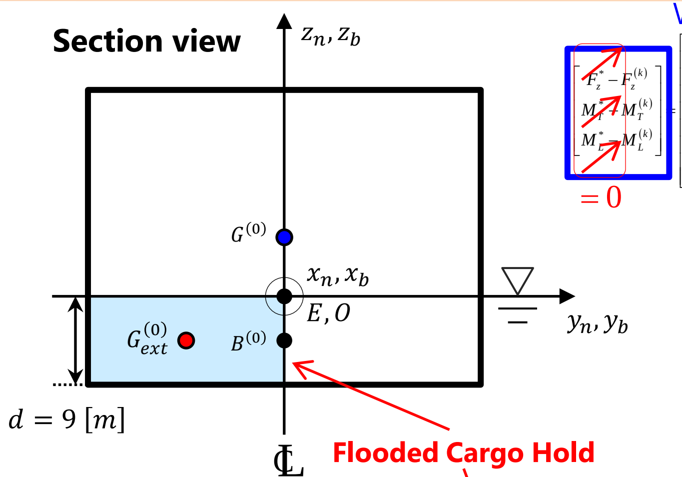


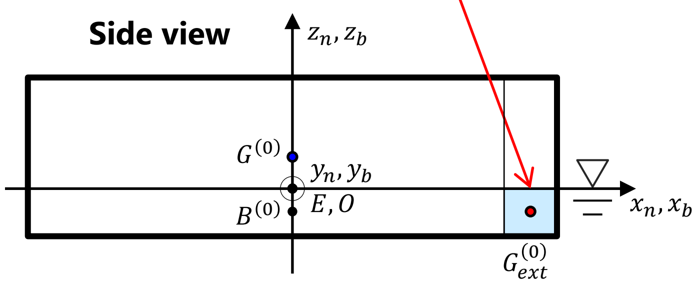
$$\begin{bmatrix} F_z^* - F_z^{(k)} \\ M_T^* - M_T^{(k)} \\ M_L^* - M_L^{(k)} \end{bmatrix} = \begin{bmatrix} \begin{array}{c} -\rho g A_{WP}^{(k)} \\ -\left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) \end{array} & \begin{array}{c} -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} \\ -\left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) \end{array} & \begin{array}{c} \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} \end{array} \\ \begin{array}{c} -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} \\ -\left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f/E}\right) \end{array} & \begin{array}{c} -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} \\ -{}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_T^{(k)}\right) \end{array} & \begin{array}{c} \rho g I_P^{(k)} \\ -\mu_F \cdot \rho g i_P^{(k)} \end{array} \\ \begin{array}{c} \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} \\ -\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} \end{array} & \begin{array}{c} \rho g I_P^{(k)} \\ -\mu_F \cdot \rho g i_P^{(k)} \end{array} & \begin{array}{c} -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} \\ -{}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_L^{(k)}\right) \end{array} \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

given known Find

Because the change of draft and heeling angle are coupled, the equations have to be solved simultaneously.

1. Calculation of Forces and Moments at k=0 step





We use the values in current floating position!

$$\boxed{\begin{bmatrix} \cancel{F_z^* - F_z^{(k)}} \\ \cancel{M_T^* - M_T^{(k)}} \\ \cancel{M_L^* - M_L^{(k)}} \end{bmatrix}} = \begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_T^{(k)}\right) & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_L^{(k)}\right) \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$

$$F_z^{(0)} = F_{B,z}^{(0)} + F_{G,z}^{(0)} + F_{ext,z}^{(0)}$$
$$= \rho g \nabla^{(0)} + F_{G,z} + \left(-\rho g v^{(0)}\right)$$
$$= 10 \cdot (3.6 \times 10^4) + (-3.6 \times 10^5) + 10 \cdot (-1.8 \times 10^3)$$
$$= -1.8 \times 10^4\ [kN]$$

$$\nabla^{(0)} = L \cdot B_{mld} \cdot d = 100 \cdot 40 \cdot 9 = 3.6 \times 10^4\ [m^3]$$
$$v^{(0)} = l \cdot b \cdot d = 10 \cdot 20 \cdot 9 = 1.8 \times 10^3\ [m^3]$$

$$M_T^{(0)} = M_{BT}^{(0)} + M_{GT}^{(0)} + M_{extT}^{(0)}$$
$$= {}^n y_{B^{(0)}/E} \cdot F_{B,z}^{(0)} + {}^n y_{G^{(0)}/E} \cdot F_{G,z} + {}^n y_{G_{ext}^{(0)}/E} \cdot F_{ext,z}$$
$$= 0 \cdot (3.6 \times 10^5) + 0 \cdot (-3.6 \times 10^5) + (-10) \cdot (-1.8 \times 10^4)$$
$$= 1.8 \times 10^5\ [kN \cdot m]$$

$$M_L^{(0)} = M_{BL}^{(0)} + M_{GL}^{(0)} + M_{extL}^{(0)}$$
$$= \left(-{}^n x_{B^{(0)}/E} \cdot F_{B,z}^{(0)}\right) + \left(-{}^n x_{G^{(0)}/E} \cdot F_{G,z}\right) + \left(-{}^n x_{G_{ext}^{(0)}/E} \cdot F_{ext,z}\right)$$
$$= [-0 \cdot (3.6 \times 10^5)] + [-0 \cdot (-3.6 \times 10^5)] + [-45 \cdot (-1.8 \times 10^4)]$$
$$= 8.1 \times 10^5\ [kN \cdot m]$$

$L = 100\ [m]$	$l = 10\ [m]$	${}^n\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$
$B_{mld} = 40\ [m]$	$b = 20\ [m]$	${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [45 \quad -10 \quad -4.5]^T\ [m]$
$D = 30\ [m]$	$\mu_F = 1.0$	${}^n\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T\ [m]$
$d = 9\ [m]$	$\mu_V = 1.0$	${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2s^2]$		${}^n\mathbf{r}_{f^{(0)}/E} = [45 \quad -10 \quad 0]^T\ [m]$

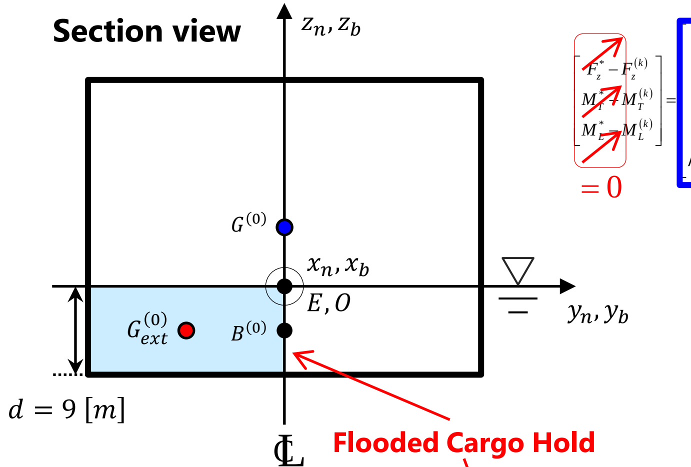
$\nabla^{(0)} = 3.6 \times 10^4\ [m^3]$ $\quad v^{(0)} = 1.8 \times 10^3\ [m^3]$

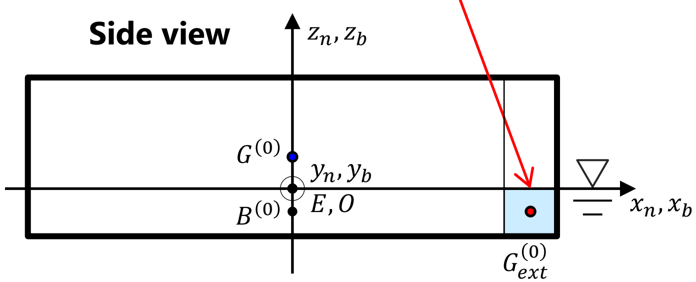
$F_{G,z} = -3.6 \times 10^5\ [kN]$

$F_{G_{ext},z}^{(0)} = -1.8 \times 10^4\ [kN]$

$F_{B,z}^{(0)} = 3.6 \times 10^5\ [kN]$

2. Calculation of the Values of the Waterplane at k=0 step





$$\underbrace{\begin{bmatrix} F_z^* - F_z^{(k)} \\ M_T^* - M_T^{(k)} \\ M_L^* - M_L^{(k)} \end{bmatrix}}_{=0} = \boxed{\begin{bmatrix} -\rho g A_{WP}{}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}{}^{(k)}\right) & -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^n x_{f^{(k)}/E} \\ -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T{}^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} - \left(-\mu_F \cdot \rho g i_T{}^{(k)}\right) & \rho g I_P{}^{(k)} - \mu_F \cdot \rho g i_P{}^{(k)} \\ \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^n x_{f^{(k)}/E} & \rho g I_P{}^{(k)} - \mu_F \cdot \rho g i_P{}^{(k)} & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L{}^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} - \left(-\mu_F \cdot \rho g i_L{}^{(k)}\right) \end{bmatrix}} \begin{bmatrix} \delta z^{(k)} \\ \delta\phi^{(k)} \\ \delta\theta^{(k)} \end{bmatrix}$$

$L = 100\ [m]$	$l = 10\ [m]$	${}^n\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$
$B_{mld} = 40\ [m]$	$b = 20\ [m]$	${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [45 \quad -10 \quad -4.5]^T\ [m]$
$D = 30\ [m]$	$\mu_F = 1.0$	${}^n\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T\ [m]$
$d = 9\ [m]$	$\mu_V = 1.0$	${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2 s^2]$		${}^n\mathbf{r}_{f^{(0)}/E} = [45 \quad -10 \quad 0]^T\ [m]$

$\nabla^{(0)} = 3.6 \times 10^4\ [m^3]$	$v^{(0)} = 1.8 \times 10^3\ [m^3]$
$F_{G,z} = -3.6 \times 10^5\ [kN]$	$F_z^{(0)} = -1.8 \times 10^4\ [kN]$
$F_{G_{ext},z}^{(0)} = -1.8 \times 10^4\ [kN]$	$M_T^{(0)} = 1.8 \times 10^5\ [kN]$
$F_{B,z}^{(0)} = 3.6 \times 10^5\ [kN]$	$M_L^{(0)} = 8.1 \times 10^5\ [kN]$

$$-\rho g A_{WP}^{(0)} = -\rho g(L \cdot B_{mld}) = -10 \cdot (100 \cdot 40) = -4.0 \times 10^4\ [kN/m]$$

$$-\mu_F \cdot \rho g a_{WP}^{(0)} = -\mu_F \cdot \rho g(l \cdot b) = -1.0 \cdot 10 \cdot (10 \cdot 20) = -2.0 \times 10^3\ [kN/m]$$

$$-\rho g A_{WP}^{(0)} - \left(-\mu_F \cdot \rho g a_{WP}^{(0)}\right) = -4.0 \times 10^4 - (-2.0 \times 10^3) = -3.8 \times 10^4\ [kN/m]$$

$$-\rho g A_{WP}^{(0)} \cdot {}^n y_{F^{(0)}/E} = (-4.0 \times 10^4) \cdot 0 = 0\ [kN]$$

$$-\mu_F \cdot \rho g a_{WP}^{(0)} \cdot {}^n y_{f^{(0)}/E} = (-2.0 \times 10^3) \cdot (-10) = 2.0 \times 10^4\ [kN]$$

$$-\rho g A_{WP}^{(0)} \cdot {}^n y_{F^{(0)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(0)} \cdot {}^n y_{f^{(0)}/E}\right)$$

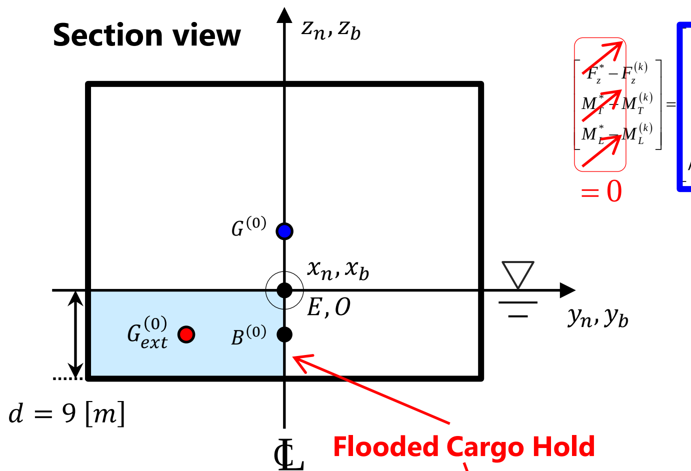
$$= 0 - (2.0 \times 10^4) = -2.0 \times 10^4\ [kN]$$

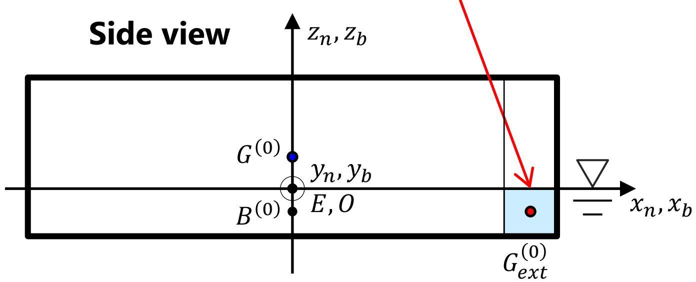
$$\rho g A_{WP}^{(0)} \cdot {}^n x_{F^{(0)}/E} = (-4.0 \times 10^4) \cdot 0 = 0\ [kN]$$

$$\mu_F \cdot \rho g a_{WP}^{(0)} \cdot {}^n x_{f^{(0)}/E} = (2.0 \times 10^3) \cdot 45 = 9.0 \times 10^4\ [kN]$$

$$\rho g A_{WP} \cdot {}^n x_{F^{(0)}/E} - \mu_F \cdot \rho g a_{WP} \cdot {}^n x_{f^{(0)}/E}$$

$$= 0 - (9.0 \times 10^4) = -9.0 \times 10^4\ [kN]$$





$$\begin{bmatrix} F_z^* - F_z^{(k)} \\ M_T^* - M_T^{(k)} \\ M_L^* - M_L^{(k)} \end{bmatrix} = \boxed{\begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_T^{(k)}\right) & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_L^{(k)}\right) \end{bmatrix}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$

$$I_T^{(0)} = \frac{1}{12} L \cdot B_{mld}^3 = \frac{1}{12} \cdot 100 \cdot 40^3 = 5.33 \times 10^5\ [m^4]$$

$$I_L^{(0)} = \frac{1}{12} L^3 \cdot B_{mld} = \frac{1}{12} \cdot 100^3 \cdot 40 = 3.33 \times 10^6\ [m^4]$$

$$I_P^{(0)} = 0\ [m^4]$$

$$i_T^{(0)} = \frac{1}{12} l \cdot b^3 + a_{WP}^{(0)} \cdot \left({}^n y_{f^{(0)}/E}\right)^2 = \frac{1}{12} \cdot 10 \cdot 20^3 + (2.0 \times 10^2) \times (-10)^2$$

$$= 2.67 \times 10^4\ [m^4]$$

$$i_L^{(0)} = \frac{1}{12} l^3 \cdot b + a_{WP}^{(0)} \cdot \left({}^n x_{f^{(0)}/E}\right)^2 = \frac{1}{12} \cdot 10^3 \cdot 20 + (2.0 \times 10^2) \times 45^2$$

$$= 4.07 \times 10^5\ [m^4]$$

$$i_P^{(0)} = -9.0 \times 10^4\ [m^4]$$

$$-\rho g \left({}^n z_{B^{(0)}/E} \nabla^{(0)} + I_T^{(0)}\right) - {}^n z_{G^{(0)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(0)}/E} \cdot F_{ext,z}^{(0)} - \left(-u_F \cdot \rho g i_T^{(0)}\right)$$

$$= -10 \cdot [-4.5 \cdot (3.6 \times 10^4) + (5.33 \times 10^5)] - 6 \cdot (-3.6 \times 10^5)$$

$$-(-4.5) \cdot (-1.8 \times 10^4) - \left(-1.0 \cdot 10 \cdot (2.67 \times 10^4)\right)$$

$$= -1.37 \times 10^6\ [kN \cdot m]$$

$$\rho g I_P^{(0)} - \mu_F \cdot \rho g i_P^{(0)} = 10 \cdot 0 - 1.0 \cdot 10 \cdot (-9.0 \times 10^4) = 9.0 \times 10^5\ [kN \cdot m]$$

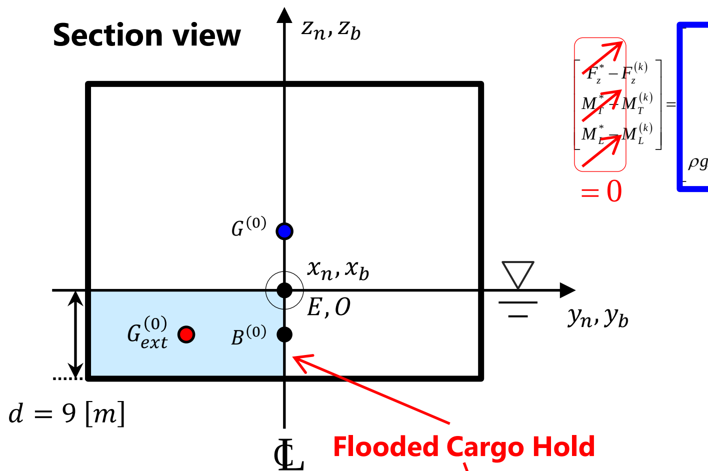
$L = 100\ [m]$
$B_{mld} = 40\ [m]$
$D = 30\ [m]$
$d = 9\ [m]$

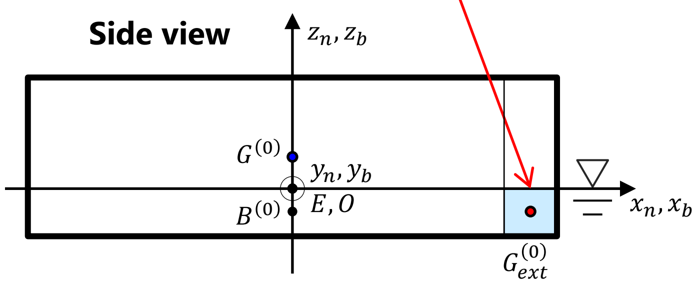
$l = 10\ [m]$
$b = 20\ [m]$
$\mu_F = 1.0$
$\mu_V = 1.0$

$\rho g = 10\ [Mg/m^2 s^2]$

${}^n\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$
${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [45 \quad -10 \quad -4.5]^T\ [m]$
${}^n\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T\ [m]$
${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
${}^n\mathbf{r}_{f^{(0)}/E} = [45 \quad -10 \quad 0]^T\ [m]$

$\nabla^{(0)} = 3.6 \times 10^4\ [m^3]$
$v^{(0)} = 1.8 \times 10^3\ [m^3]$
$F_{G,z} = -3.6 \times 10^5\ [kN]$
$F_z^{(0)} = -1.8 \times 10^4\ [kN]$
$F_{G_{ext},z}^{(0)} = -1.8 \times 10^4\ [kN]$
$M_T^{(0)} = 1.8 \times 10^5\ [kN]$
$F_{B,z}^{(0)} = 3.6 \times 10^5\ [kN]$
$M_L^{(0)} = 8.1 \times 10^5\ [kN]$





$$\begin{bmatrix} \cancel{F_z^* - F_z^{(k)}} \\ \cancel{M_T^* - M_T^{(k)}} \\ \cancel{M_L^* - M_L^{(k)}} \end{bmatrix} = \boxed{\begin{bmatrix} -\rho g A_{WP}{}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}{}^{(k)}\right) & -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^n x_{f^{(k)}/E} \\ -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T{}^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} - \left(-\mu_F \cdot \rho g i_T{}^{(k)}\right) & \rho g I_P{}^{(k)} - \mu_F \cdot \rho g i_P{}^{(k)} \\ \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^n x_{f^{(k)}/E} & \rho g I_P{}^{(k)} - \mu_F \cdot \rho g i_P{}^{(k)} & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L{}^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} - \left(-\mu_F \cdot \rho g i_L{}^{(k)}\right) \end{bmatrix}}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$

$L = 100\ [m]$ $B_{mld} = 40\ [m]$ $D = 30\ [m]$ $d = 9\ [m]$	$l = 10\ [m]$ $b = 20\ [m]$ $\mu_F = 1.0$ $\mu_V = 1.0$	${}^n\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$ ${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [45 \quad -10 \quad -4.5]^T\ [m]$ ${}^n\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T\ [m]$ ${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$ ${}^n\mathbf{r}_{f^{(0)}/E} = [45 \quad -10 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2s^2]$		

$\nabla^{(0)} = 3.6 \times 10^4\ [m^3]$	$v^{(0)} = 1.8 \times 10^3\ [m^3]$
$F_{G,z} = -3.6 \times 10^5\ [kN]$	$F_z^{(0)} = -1.8 \times 10^4\ [kN]$
$F_{G_{ext},z}^{(0)} = -1.8 \times 10^4\ [kN]$	$M_T^{(0)} = 1.8 \times 10^5\ [kN]$
$F_{B,z}^{(0)} = 3.6 \times 10^5\ [kN]$	$M_L^{(0)} = 8.1 \times 10^5\ [kN]$

$$I_T^{(0)} = \frac{1}{12} L \cdot B_{mld}^3 = \frac{1}{12} \cdot 100 \cdot 40^3 = 5.33 \times 10^5\ [m^4]$$

$$I_L^{(0)} = \frac{1}{12} L^3 \cdot B_{mld} = \frac{1}{12} \cdot 100^3 \cdot 40 = 3.33 \times 10^6\ [m^4]$$

$$I_P^{(0)} = 0\ [m^4]$$

$$i_T^{(0)} = \frac{1}{12} l \cdot b^3 + a_{WP}^{(0)} \cdot \left({}^n y_{f^{(0)}/E}\right)^2 = \frac{1}{12} \cdot 10 \cdot 20^3 + (2.0 \times 10^2) \times (-10)^2$$

$$= 2.67 \times 10^4\ [m^4]$$

$$i_L^{(0)} = \frac{1}{12} l^3 \cdot b + a_{WP}^{(0)} \cdot \left({}^n x_{f^{(0)}/E}\right)^2 = \frac{1}{12} \cdot 10^3 \cdot 20 + (2.0 \times 10^2) \times 45^2$$

$$= 4.07 \times 10^5\ [m^4]$$

$$i_P^{(0)} = -9.0 \times 10^4\ [m^4]$$

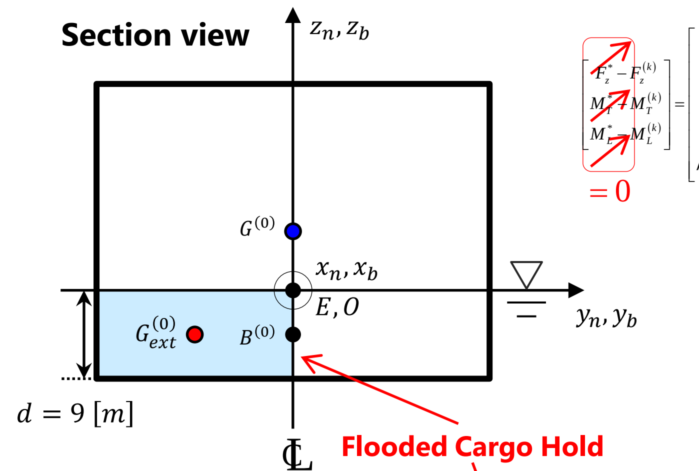
$$-\rho g \left({}^n z_{B^{(0)}/E} \nabla^{(0)} + I_L^{(0)}\right) - {}^n z_{G^{(0)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(0)}/E} \cdot F_{ext,z}^{(0)} - \left(-u_F \cdot \rho g i_L^{(0)}\right)$$

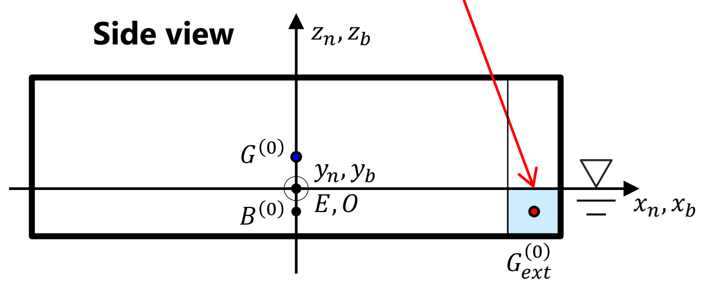
$$= -10 \cdot [-4.5 \cdot (3.6 \times 10^4) + (3.33 \times 10^6)] - 6 \cdot (-3.6 \times 10^5)$$

$$-(-4.5) \cdot (-1.8 \times 10^4) - \left(-1.0 \cdot 10 \cdot (4.07 \times 10^5)\right)$$

$$= -2.56 \times 10^7\ [kN \cdot m]$$

3. Calculation of Immersion, Trim, and Heel at k=0 step





$L = 100\ [m]$ $B_{mld} = 40\ [m]$ $D = 30\ [m]$ $d = 9\ [m]$	$l = 10\ [m]$ $b = 20\ [m]$ $\mu_F = 1.0$ $\mu_V = 1.0$	${}^n\mathbf{r}_{G^{(0)}/E} = [0\quad 0\quad 6]^T\ [m]$ ${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [45\quad -10\quad -4.5]^T\ [m]$ ${}^n\mathbf{r}_{B^{(0)}/E} = [0\quad 0\quad -4.5]^T\ [m]$ ${}^n\mathbf{r}_{F^{(0)}/E} = [0\quad 0\quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2s^2]$		${}^n\mathbf{r}_{f^{(0)}/E} = [45\quad -10\quad 0]^T\ [m]$

$\nabla^{(0)} = 3.6 \times 10^4\ [m^3]$	$v^{(0)} = 1.8 \times 10^3\ [m^3]$
$F_{G,z} = -3.6 \times 10^5\ [kN]$	$F_z^{(0)} = -1.8 \times 10^4\ [kN]$
$F_{G_{ext},z}^{(0)} = -1.8 \times 10^4\ [kN]$	$M_T^{(0)} = 1.8 \times 10^5\ [kN]$
$F_{B,z}^{(0)} = 3.6 \times 10^5\ [kN]$	$M_L^{(0)} = 8.1 \times 10^5\ [kN]$

$$\begin{bmatrix} \cancel{F_z^* - F_z^{(k)}} \\ \cancel{M_T^* - M_T^{(k)}} \\ \cancel{M_L^* - M_L^{(k)}} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_T^{(k)}\right) & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_L^{(k)}\right) \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \boxed{\begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}}$$

$= 0$

$$\begin{bmatrix} 1.8\times10^4 \\ -1.8\times10^5 \\ -8.1\times10^6 \end{bmatrix} = \begin{bmatrix} -3.8\times10^4 & -2.0\times10^4 & -9.0\times10^5 \\ -2.0\times10^4 & -1.37\times10^6 & 9.0\times10^5 \\ -9.0\times10^5 & 9.0\times10^5 & -2.56\times10^7 \end{bmatrix} \begin{bmatrix} \delta z_n^{(0)} \\ \delta\phi^{(0)} \\ \delta\theta^{(0)} \end{bmatrix}$$

$$\boxed{\begin{bmatrix} \delta z_n^{(0)} \\ \delta\phi^{(0)} \\ \delta\theta^{(0)} \end{bmatrix}} = \begin{bmatrix} -3.8\times10^4 & -2.0\times10^4 & -9.0\times10^5 \\ -2.0\times10^4 & -1.37\times10^6 & 9.0\times10^5 \\ -9.0\times10^5 & 9.0\times10^5 & -2.56\times10^7 \end{bmatrix}^{-1} \begin{bmatrix} 1.8\times10^4 \\ -1.8\times10^5 \\ -8.1\times10^6 \end{bmatrix}$$

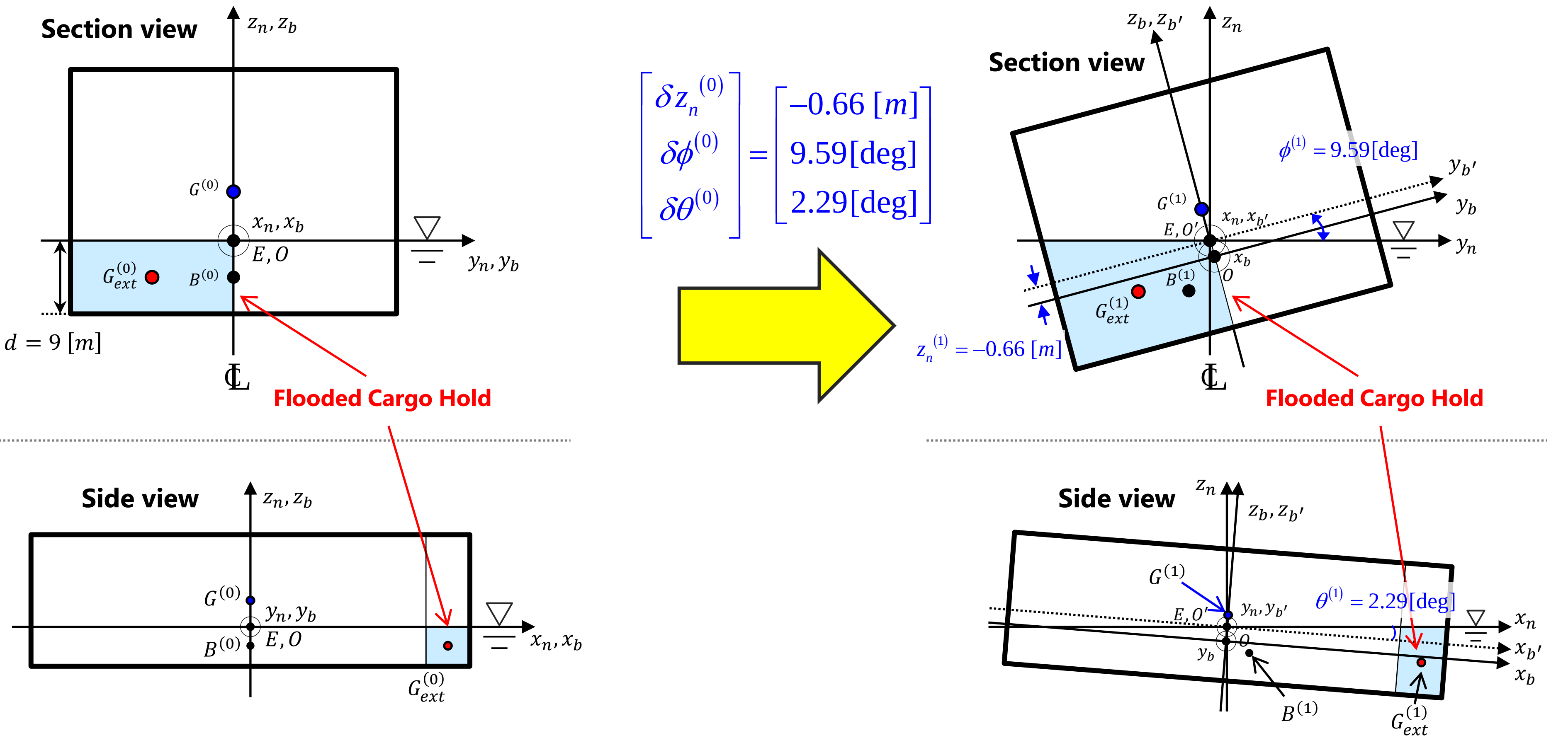
$$= \begin{bmatrix} -0.6563\ [m] \\ 0.1675[rad] \\ 0.0399[rad] \end{bmatrix} = \boxed{\begin{bmatrix} -0.6563\ [m] \\ 9.5945[\deg] \\ 2.2853[\deg] \end{bmatrix}}$$

$$z_n^{(1)} = z_n^{(0)} + \delta z_n^{(0)} = 0 + (-0.6563) = -0.66\ [m]$$

$$\phi^{(1)} = \phi^{(0)} + \delta\phi = 0 + (9.5945) = 9.59\ [\deg]$$

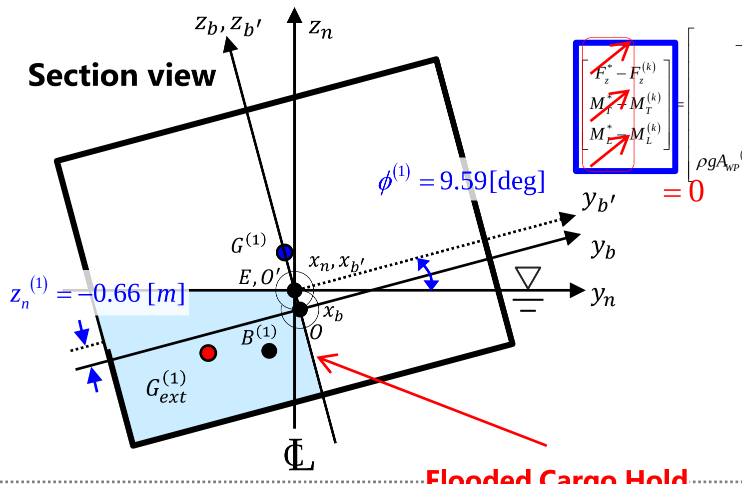
$$\theta^{(1)} = \theta^{(0)} + \delta\theta = 0 + (2.2853) = 2.29\ [\deg]$$

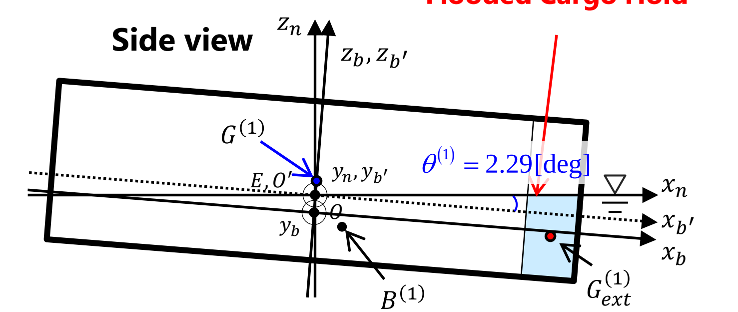
회전 변환 정의: Trim, Heel 순서로 한다



b′-frame: b-frame을 n-frame의 원점 E으로 translation한 coordinate system

4. Check for the Ship to be in Static Equilibrium at k=0 step





$$\begin{bmatrix} F_z^* - F_z^{(k)} \\ M_T^* - M_T^{(k)} \\ M_L^* - M_L^{(k)} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_T^{(k)}\right) & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_L^{(k)}\right) \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$

Is the ship in static equilibrium?

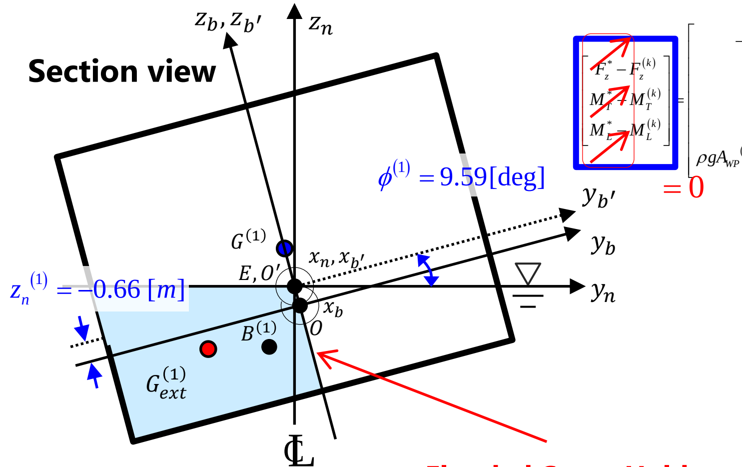
Let us check for the ship to be in static equilibrium!

$L = 100\ [m]$	$l = 10\ [m]$	${}^n\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$
$B_{mld} = 40\ [m]$	$b = 20\ [m]$	${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [45 \quad -10 \quad -4.5]^T\ [m]$
$D = 30\ [m]$	$\mu_F = 1.0$	${}^n\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T\ [m]$
$d = 9\ [m]$	$\mu_V = 1.0$	${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2 s^2]$		${}^n\mathbf{r}_{f^{(0)}/E} = [45 \quad -10 \quad 0]^T\ [m]$

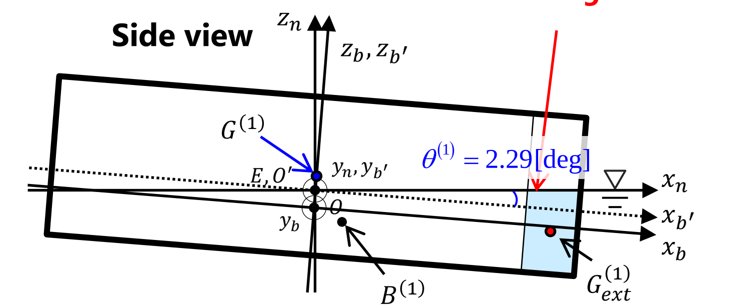
$\nabla^{(0)} = 3.6 \times 10^4\ [m^3]$	$v^{(0)} = 1.8 \times 10^3\ [m^3]$
$F_{G,z} = -3.6 \times 10^5\ [kN]$	$F_z^{(0)} = -1.8 \times 10^4\ [kN]$
$F_{G_{ext},z}^{(0)} = -1.8 \times 10^4\ [kN]$	$M_T^{(0)} = 1.8 \times 10^5\ [kN]$
$F_{B,z}^{(0)} = 3.6 \times 10^5\ [kN]$	$M_L^{(0)} = 8.1 \times 10^5\ [kN]$

$$\begin{bmatrix} F_z^* - F_z^{(k)} \\ M_T^* - M_T^{(k)} \\ M_L^* - M_L^{(k)} \end{bmatrix} = \left.\begin{bmatrix} -\rho g A_{WP}{}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}{}^{(k)}\right) & -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^n x_{f^{(k)}/E} \\ -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T{}^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} - \left(-\mu_F \cdot \rho g i_T{}^{(k)}\right) & \rho g I_P{}^{(k)} - \mu_F \cdot \rho g i_P{}^{(k)} \\ \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^n x_{f^{(k)}/E} & \rho g I_P{}^{(k)} - \mu_F \cdot \rho g i_P{}^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L{}^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} - \left(-\mu_F \cdot \rho g i_L{}^{(k)}\right) \end{bmatrix}\right|_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$



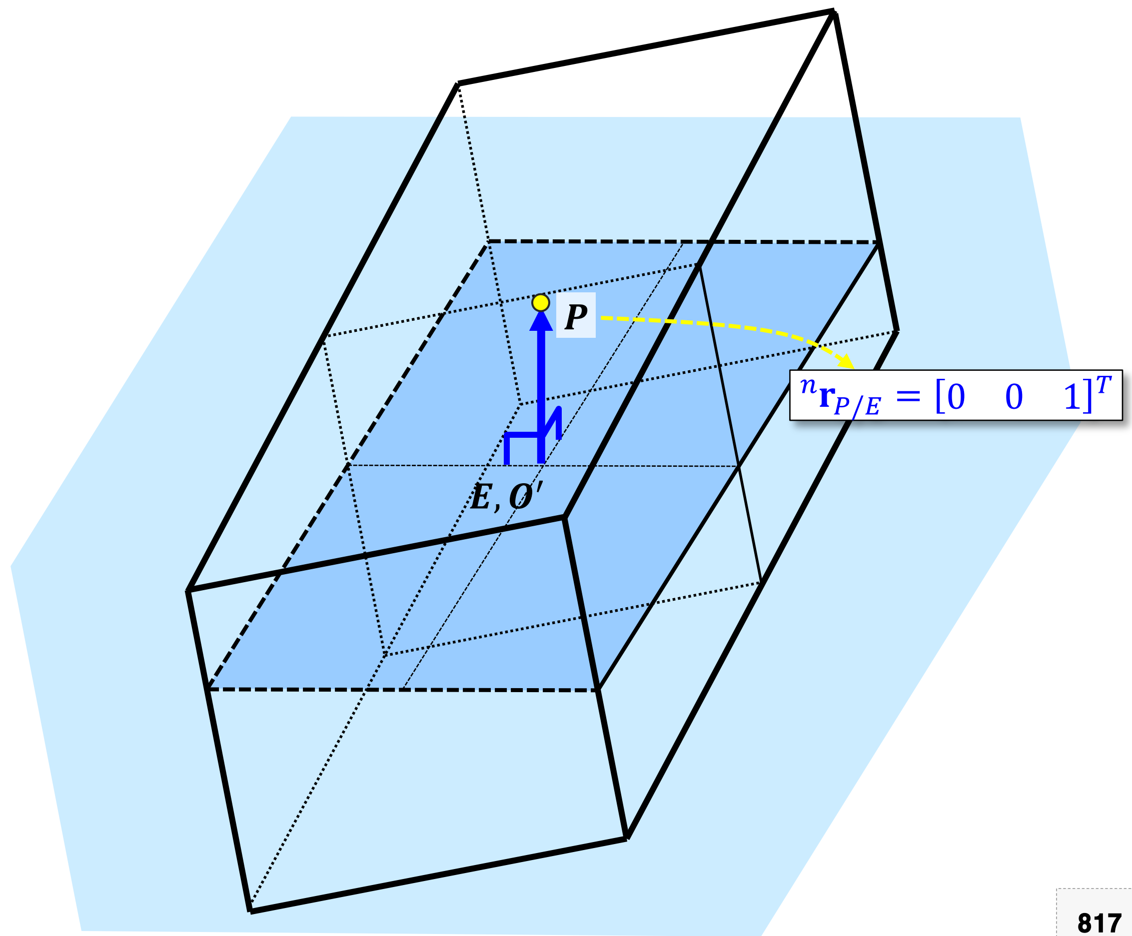
Flooded Cargo Hold



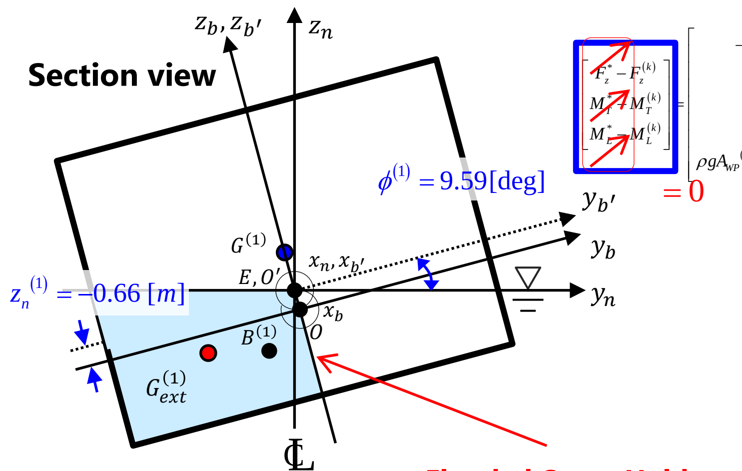
$L = 100\ [m]$	$l = 10\ [m]$	${}^n\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$
$B_{mld} = 40\ [m]$	$b = 20\ [m]$	${}^n\mathbf{r}_{G^{(0)}_{ext}/E} = [45 \quad -10 \quad -4.5]^T\ [m]$
$D = 30\ [m]$	$\mu_F = 1.0$	${}^n\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T\ [m]$
$d = 9\ [m]$	$\mu_V = 1.0$	${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2 s^2]$		${}^n\mathbf{r}_{f^{(0)}/E} = [45 \quad -10 \quad 0]^T\ [m]$

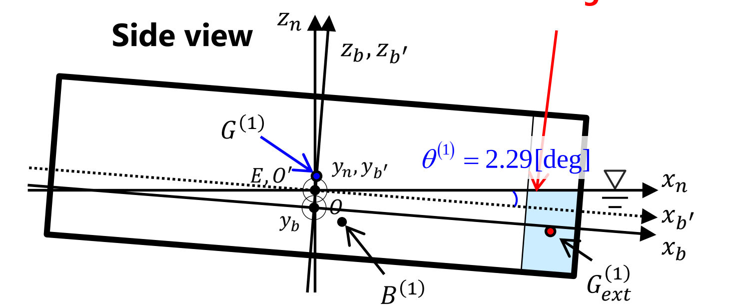
$\nabla^{(0)} = 3.6 \times 10^4\ [m^3]$	$v^{(0)} = 1.8 \times 10^3\ [m^3]$
$F_{G,z} = -3.6 \times 10^5\ [kN]$	$F_z^{(0)} = -1.8 \times 10^4\ [kN]$
$F_{G_{ext},z}^{(0)} = -1.8 \times 10^4\ [kN]$	$M_T^{(0)} = 1.8 \times 10^5\ [kN]$
$F_{B,z}^{(0)} = 3.6 \times 10^5\ [kN]$	$M_L^{(0)} = 8.1 \times 10^5\ [kN]$

Normal vector of waterplane
decomposed in the n-frame: ${}^n\mathbf{r}_{P/E}$



$$\underbrace{\begin{bmatrix} F_z^* - F_z^{(k)} \\ M_T^* - M_T^{(k)} \\ M_L^* - M_L^{(k)} \end{bmatrix}}_{=0} = \begin{bmatrix} -\rho g A_{WP}{}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}{}^{(k)}\right) & -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^n x_{f^{(k)}/E} \\ -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T{}^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} - \left(-\mu_F \cdot \rho g i_T{}^{(k)}\right) & \rho g I_P{}^{(k)} - \mu_F \cdot \rho g i_P{}^{(k)} \\ \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^n x_{f^{(k)}/E} & \rho g I_P{}^{(k)} - \mu_F \cdot \rho g i_P{}^{(k)} & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L{}^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} - \left(-\mu_F \cdot \rho g i_L{}^{(k)}\right) \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$





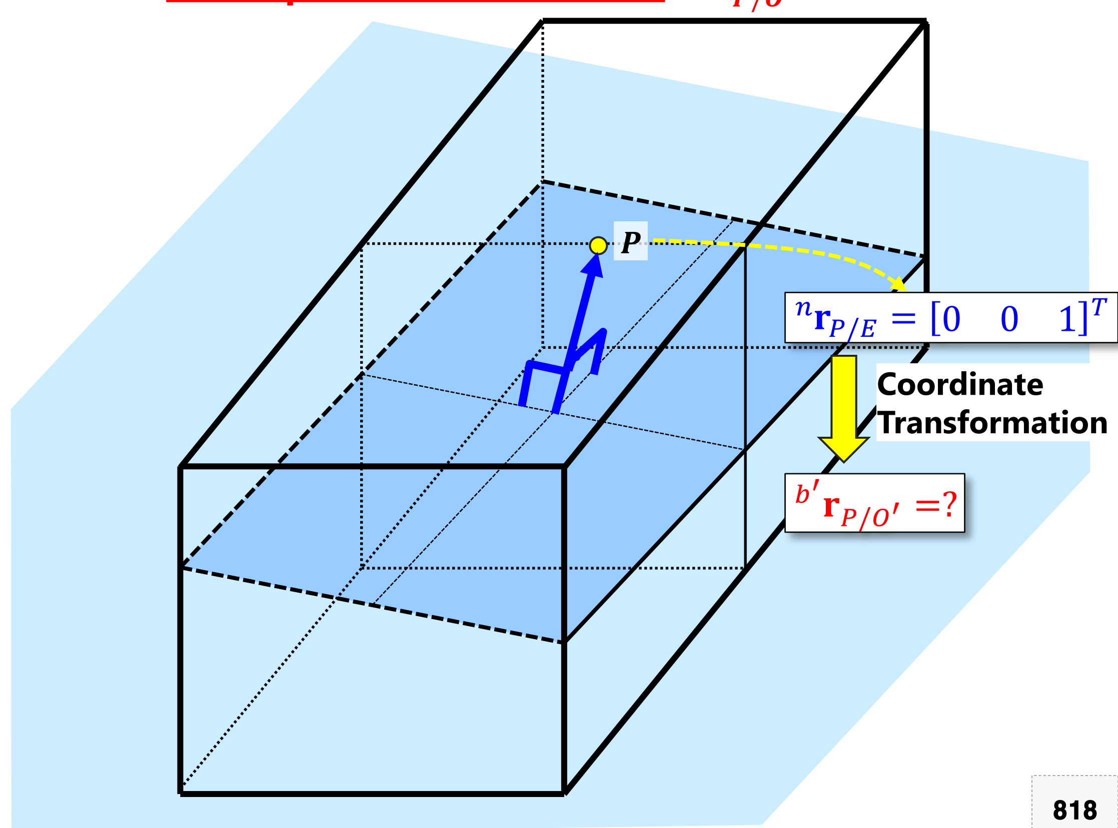
$L = 100\ [m]$	$l = 10\ [m]$	${}^n\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$
$B_{mld} = 40\ [m]$	$b = 20\ [m]$	${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [45 \quad -10 \quad -4.5]^T\ [m]$
$D = 30\ [m]$	$\mu_F = 1.0$	${}^n\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T\ [m]$
$d = 9\ [m]$	$\mu_V = 1.0$	${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2s^2]$		${}^n\mathbf{r}_{f^{(0)}/E} = [45 \quad -10 \quad 0]^T\ [m]$

$\nabla^{(0)} = 3.6 \times 10^4\ [m^3]$	$v^{(0)} = 1.8 \times 10^3\ [m^3]$
$F_{G,z} = -3.6 \times 10^5\ [kN]$	$F_z^{(0)} = -1.8 \times 10^4\ [kN]$
$F_{G_{ext},z}^{(0)} = -1.8 \times 10^4\ [kN]$	$M_T^{(0)} = 1.8 \times 10^5\ [kN]$
$F_{B,z}^{(0)} = 3.6 \times 10^5\ [kN]$	$M_L^{(0)} = 8.1 \times 10^5\ [kN]$

Normal vector of waterplane
decomposed in the n-frame: ${}^n\mathbf{r}_{P/E}$

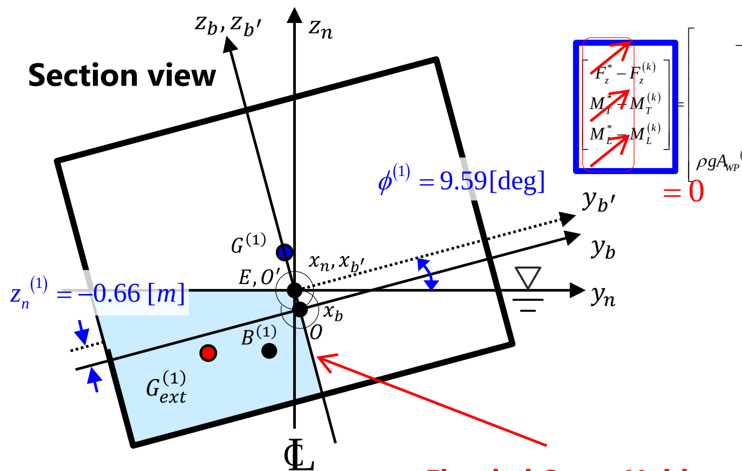
Coordinate Transformation

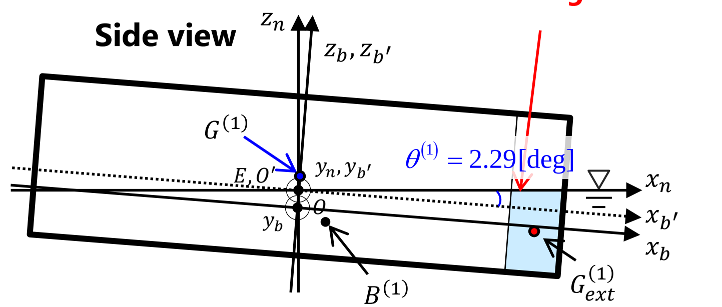
Normal vector of waterplane
decomposed in the b'-frame: ${}^{b'}\mathbf{r}_{P/O'}$



$$
\boxed{\begin{bmatrix} F_z^* - F_z^{(k)} \\ M_T^* - M_T^{(k)} \\ M_L^* - M_L^{(k)} \end{bmatrix}} = \begin{bmatrix} -\rho g A_{WP}{}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}{}^{(k)}\right) & -\rho g A_{WP}{}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) & \rho g A_{WP}{}^{(k)} \cdot {}^{n}x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^{n}x_{f^{(k)}/E} \\ -\rho g A_{WP}{}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) & -\rho g\left({}^{n}z_{B^{(k)}/E}\nabla^{(k)} + I_T{}^{(k)}\right) - {}^{n}z_{G^{(k)}/E} \cdot F_{G,z} - {}^{n}z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} - \left(-\mu_F \cdot \rho g i_T{}^{(k)}\right) & \rho g I_P{}^{(k)} - \mu_F \cdot \rho g i_P{}^{(k)} \\ \rho g A_{WP}{}^{(k)} \cdot {}^{n}x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^{n}x_{f^{(k)}/E} & \rho g I_P{}^{(k)} - \mu_F \cdot \rho g i_P{}^{(k)} & -\rho g\left({}^{n}z_{B^{(k)}/E}\nabla^{(k)} + I_L{}^{(k)}\right) - {}^{n}z_{G^{(k)}/E} \cdot F_{G,z} - {}^{n}z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} - \left(-\mu_F \cdot \rho g i_L{}^{(k)}\right) \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta\phi^{(k)} \\ \delta\theta^{(k)} \end{bmatrix}
$$

$= 0$

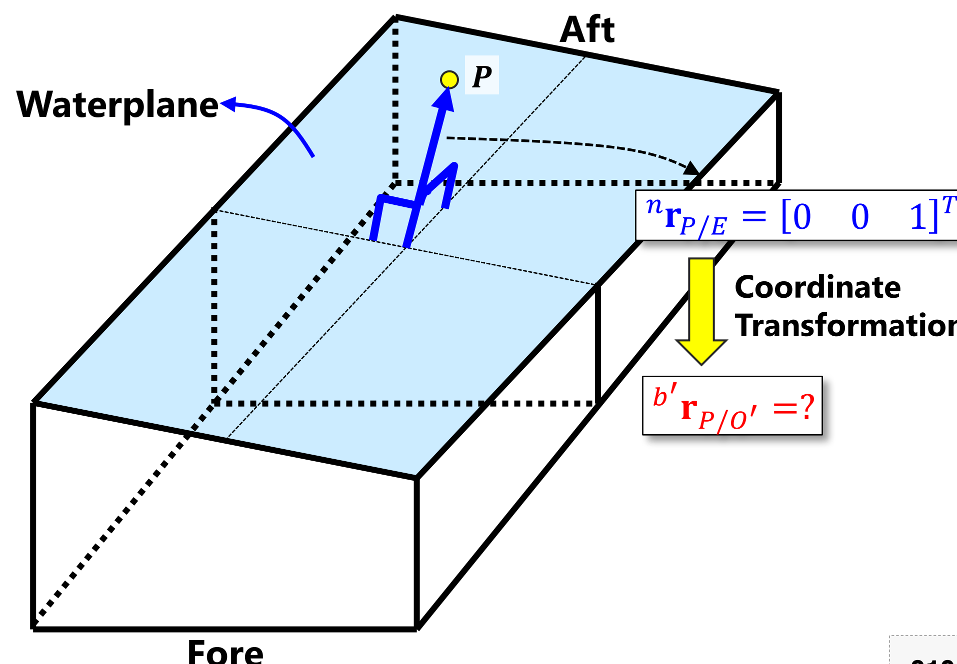




Normal vector of waterplane

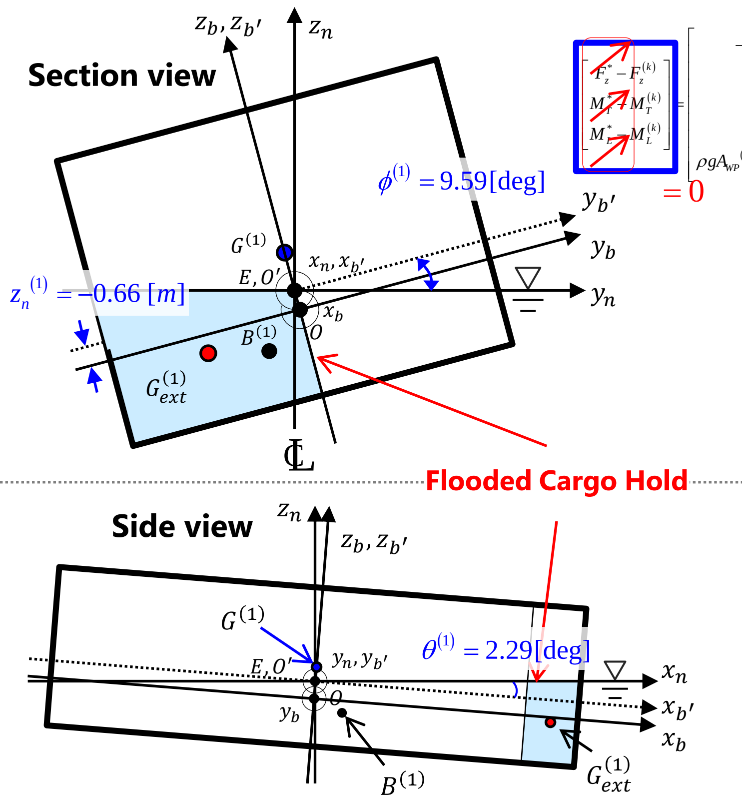
decomposed in the b′-frame: ${}^{b'}\mathbf{r}_{P/O'}$

$$
\underbrace{{}^{n}\mathbf{r}_{P/E}}_{\text{Given}} = \begin{bmatrix} \cos\theta^{(1)} & 0 & \sin\theta^{(1)} \\ 0 & 1 & 0 \\ -\sin\theta^{(1)} & 0 & \cos\theta^{(1)} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi^{(1)} & -\sin\phi^{(1)} \\ 0 & \sin\phi^{(1)} & \cos\phi^{(1)} \end{bmatrix} \underbrace{{}^{b'}\mathbf{r}_{P/O'}}_{\text{Find}}
$$



$L = 100\ [m]$	$l = 10\ [m]$	${}^{n}\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$
$B_{mld} = 40\ [m]$	$b = 20\ [m]$	${}^{n}\mathbf{r}_{G_{ext}^{(0)}/E} = [45 \quad -10 \quad -4.5]^T\ [m]$
$D = 30\ [m]$	$\mu_F = 1.0$	${}^{n}\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T\ [m]$
$d = 9\ [m]$	$\mu_V = 1.0$	${}^{n}\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2s^2]$		${}^{n}\mathbf{r}_{f^{(0)}/E} = [45 \quad -10 \quad 0]^T\ [m]$

$\nabla^{(0)} = 3.6 \times 10^4\ [m^3]$	$v^{(0)} = 1.8 \times 10^3\ [m^3]$
$F_{G,z} = -3.6 \times 10^5\ [kN]$	$F_z^{(0)} = -1.8 \times 10^4\ [kN]$
$F_{G_{ext},z}^{(0)} = -1.8 \times 10^4\ [kN]$	$M_T^{(0)} = 1.8 \times 10^5\ [kN]$
$F_{B,z}^{(0)} = 3.6 \times 10^5\ [kN]$	$M_L^{(0)} = 8.1 \times 10^5\ [kN]$



$$\boxed{\begin{bmatrix} F_z^* - F_z^{(k)} \\ M_T^* - M_T^{(k)} \\ M_L^* - M_L^{(k)} \end{bmatrix}} = \begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_T^{(k)}\right) & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_L^{(k)}\right) \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$

$L = 100$ [m]	$l = 10$ [m]	${}^n\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T$ [m]
$B_{mld} = 40$ [m]	$b = 20$ [m]	${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [45 \quad -10 \quad -4.5]^T$ [m]
$D = 30$ [m]	$\mu_F = 1.0$	${}^n\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T$ [m]
$d = 9$ [m]	$\mu_V = 1.0$	${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T$ [m]
$\rho g = 10$ [Mg/m^2s^2]		${}^n\mathbf{r}_{f^{(0)}/E} = [45 \quad -10 \quad 0]^T$ [m]

$\nabla^{(0)} = 3.6 \times 10^4$ [m^3]	$v^{(0)} = 1.8 \times 10^3$ [m^3]
$F_{G,z} = -3.6 \times 10^5$ [kN]	$F_z^{(0)} = -1.8 \times 10^4$ [kN]
$F_{G_{ext},z}^{(0)} = -1.8 \times 10^4$ [kN]	$M_T^{(0)} = 1.8 \times 10^5$ [kN]
$F_{B,z}^{(0)} = 3.6 \times 10^5$ [kN]	$M_L^{(0)} = 8.1 \times 10^5$ [kN]

Normal vector of waterplane

decomposed in the b′-frame: ${}^{b'}\mathbf{r}_{P/O'}$

$${}^n\mathbf{r}_{P/E} = \begin{bmatrix} \cos\theta^{(1)} & 0 & \sin\theta^{(1)} \\ 0 & 1 & 0 \\ -\sin\theta^{(1)} & 0 & \cos\theta^{(1)} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi^{(1)} & -\sin\phi^{(1)} \\ 0 & \sin\phi^{(1)} & \cos\phi^{(1)} \end{bmatrix} {}^{b'}\mathbf{r}_{P/O'}$$

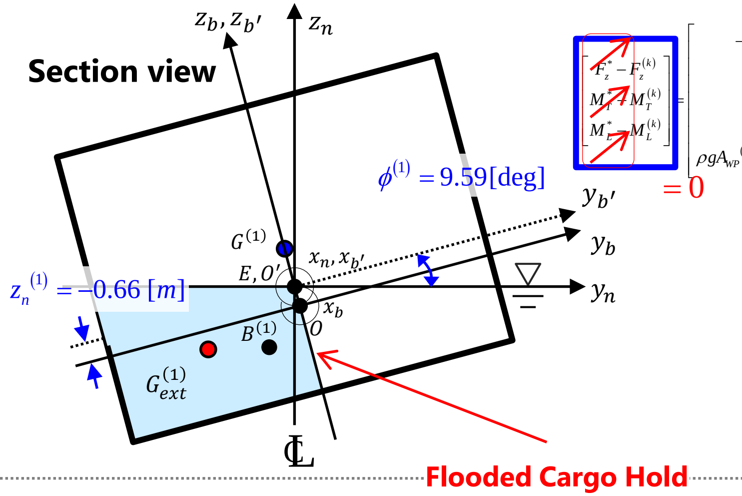
$${}^{b'}\mathbf{r}_{P/O'} = \left(\begin{bmatrix} \cos\theta^{(1)} & 0 & \sin\theta^{(1)} \\ 0 & 1 & 0 \\ -\sin\theta^{(1)} & 0 & \cos\theta^{(1)} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi^{(1)} & -\sin\phi^{(1)} \\ 0 & \sin\phi^{(1)} & \cos\phi^{(1)} \end{bmatrix} \right)^{-1} {}^n\mathbf{r}_{P/E}$$

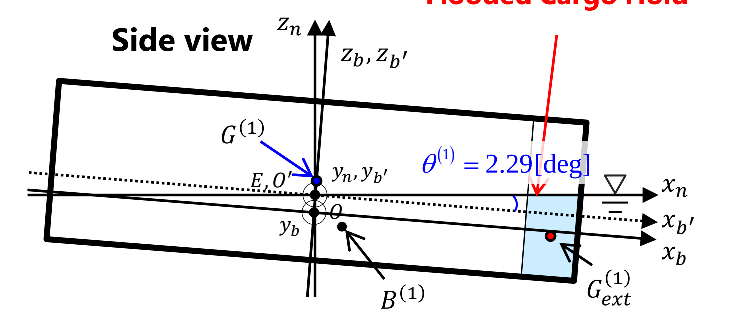
$$= \left(\begin{bmatrix} \cos 2.29 & 0 & \sin 2.29 \\ 0 & 1 & 0 \\ -\sin 2.29 & 0 & \cos 2.29 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 9.59 & -\sin 9.59 \\ 0 & \sin 9.59 & \cos 9.59 \end{bmatrix} \right)^{-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -0.0399 \\ 0.1665 \\ 0.9852 \end{bmatrix}$$

$$\begin{bmatrix} F_z^* - F_z^{(k)} \\ M_T^* - M_T^{(k)} \\ M_L^* - M_L^{(k)} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP}{}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}{}^{(k)}\right) & -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^n x_{f^{(k)}/E} \\ \begin{gathered} -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} \\ -\left(-\mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) \end{gathered} & \begin{gathered} -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T{}^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} \\ - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} - \left(-\mu_F \cdot \rho g i_T{}^{(k)}\right) \end{gathered} & \rho g I_P{}^{(k)} - \mu_F \cdot \rho g i_P{}^{(k)} \\ \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^n x_{f^{(k)}/E} & \rho g I_P{}^{(k)} - \mu_F \cdot \rho g i_P{}^{(k)} & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L{}^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} - \left(-\mu_F \cdot \rho g i_L{}^{(k)}\right) \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta\phi^{(k)} \\ \delta\theta^{(k)} \end{bmatrix}$$

$= 0$





$L = 100\ [m]$ $B_{mld} = 40\ [m]$ $D = 30\ [m]$ $d = 9\ [m]$	$l = 10\ [m]$ $b = 20\ [m]$ $\mu_F = 1.0$ $\mu_V = 1.0$	${}^n\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$ ${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [45 \quad -10 \quad -4.5]^T\ [m]$ ${}^n\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T\ [m]$ ${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2s^2]$		${}^n\mathbf{r}_{f^{(0)}/E} = [45 \quad -10 \quad 0]^T\ [m]$

$\nabla^{(0)} = 3.6 \times 10^4\ [m^3]$	$v^{(0)} = 1.8 \times 10^3\ [m^3]$
$F_{G,z} = -3.6 \times 10^5\ [kN]$	$F_z^{(0)} = -1.8 \times 10^4\ [kN]$
$F_{G_{ext},z}^{(0)} = -1.8 \times 10^4\ [kN]$	$M_T^{(0)} = 1.8 \times 10^5\ [kN]$
$F_{B,z}^{(0)} = 3.6 \times 10^5\ [kN]$	$M_L^{(0)} = 8.1 \times 10^5\ [kN]$

Normal vector of waterplane decomposed in the b′-frame:

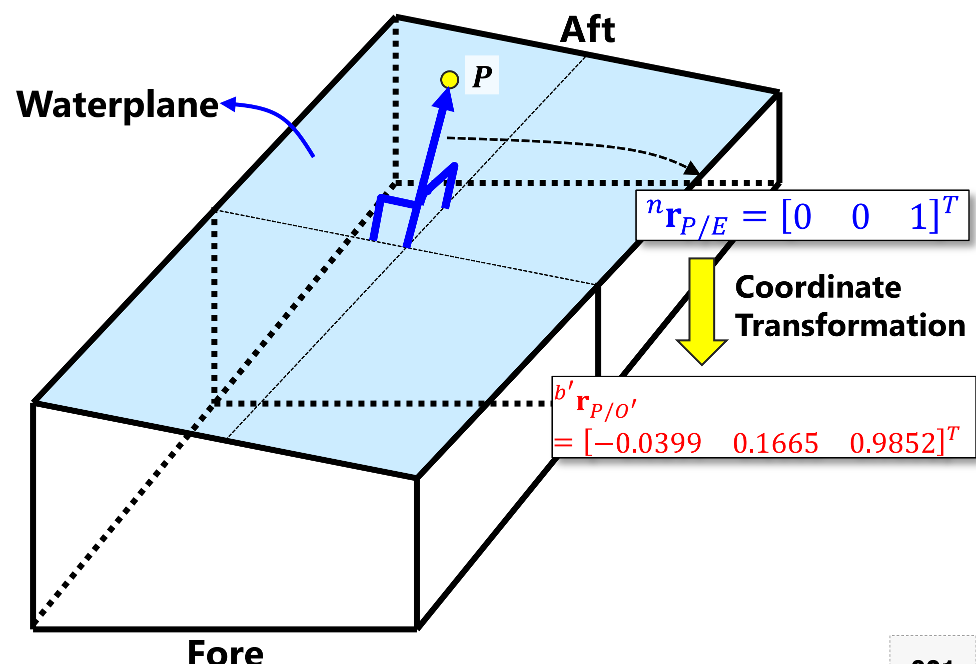
$${}^{b'}\mathbf{r}_{P/O'} = [-0.0399 \quad 0.1665 \quad 0.9852]^T$$

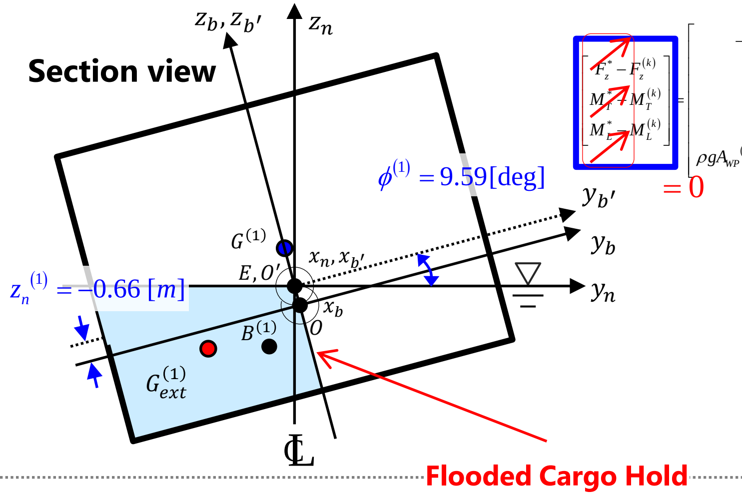
Equation of waterplane: ${}^{b'}\mathbf{r}_{P/O'} \cdot \left({}^{b'}\mathbf{r} - {}^{b'}\mathbf{r}_{E/O'}\right) = 0$

$$-0.0399x + 0.1665y + 0.9852z = 0$$

(x, y, z are defined in the b′-frame)

$$z = (0.0399x - 0.1665y)/0.9852$$





$$\boxed{\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \\ \cancel{M_L^*} - M_L^{(k)} \end{bmatrix}} = \begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_T^{(k)}\right) & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_L^{(k)}\right) \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$

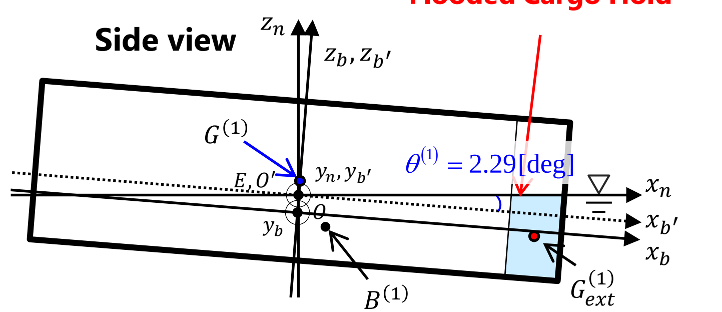
Equation of waterplane: $z = (0.0399x - 0.1665y)/0.9852$

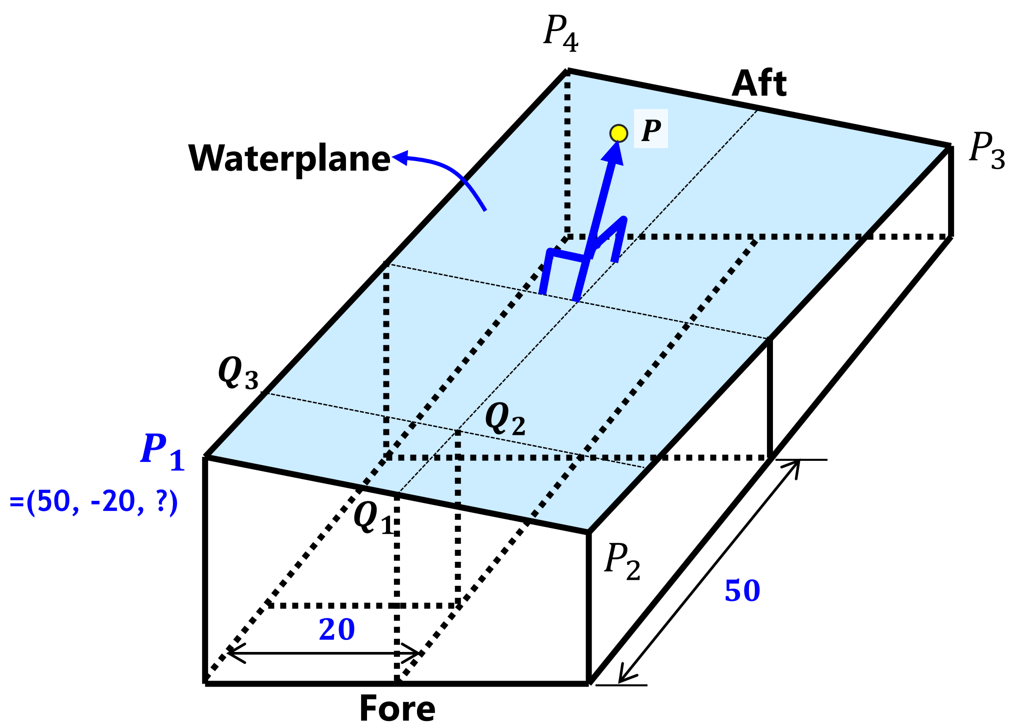
$${}^{b'}\mathbf{r}_{P_1/O'}:\ {}^{b'}x_{P_1/O'} = \mathbf{50},\ {}^{b'}y_{P_1/O'} = \mathbf{-20}$$

$${}^{b'}z_{P_1/O'} = \left(0.0399 \cdot {}^{b'}x_{P_1/O'} - 0.1665 \cdot {}^{b'}y_{P_1/O'}\right)/0.9852$$

$$= (0.0399 \cdot 50 - 0.1665 \cdot (-20)\ )/0.9852$$

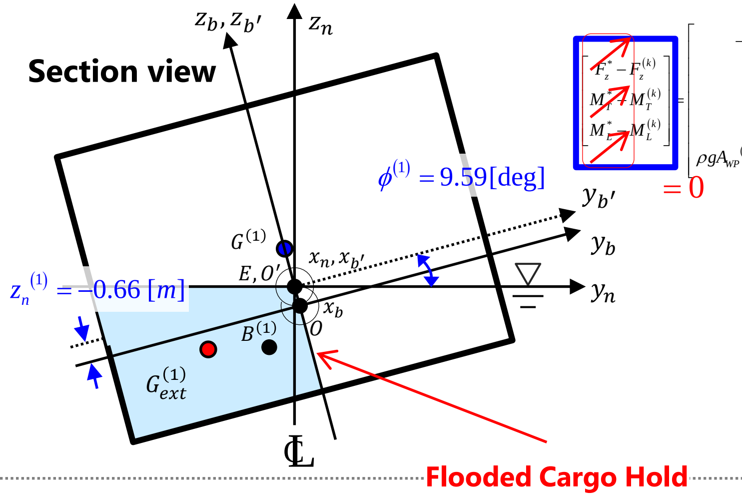
$$= \mathbf{5.4044}$$





$L = 100\ [m]$	$l = 10\ [m]$	${}^n\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$
$B_{mld} = 40\ [m]$	$b = 20\ [m]$	${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [45 \quad -10 \quad -4.5]^T\ [m]$
$D = 30\ [m]$	$\mu_F = 1.0$	${}^n\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T\ [m]$
$d = 9\ [m]$	$\mu_V = 1.0$	${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2 s^2]$		${}^n\mathbf{r}_{f^{(0)}/E} = [45 \quad -10 \quad 0]^T\ [m]$

$\nabla^{(0)} = 3.6 \times 10^4\ [m^3]$	$v^{(0)} = 1.8 \times 10^3\ [m^3]$
$F_{G,z} = -3.6 \times 10^5\ [kN]$	$F_z^{(0)} = -1.8 \times 10^4\ [kN]$
$F_{G_{ext},z}^{(0)} = -1.8 \times 10^4\ [kN]$	$M_T^{(0)} = 1.8 \times 10^5\ [kN]$
$F_{B,z}^{(0)} = 3.6 \times 10^5\ [kN]$	$M_L^{(0)} = 8.1 \times 10^5\ [kN]$



$$\boxed{\begin{bmatrix} F_z^* - F_z^{(k)} \\ M_T^* - M_T^{(k)} \\ M_L^* - M_L^{(k)} \end{bmatrix}} = \begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_T^{(k)}\right) & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_L^{(k)}\right) \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta\phi^{(k)} \\ \delta\theta^{(k)} \end{bmatrix}$$

$= 0$

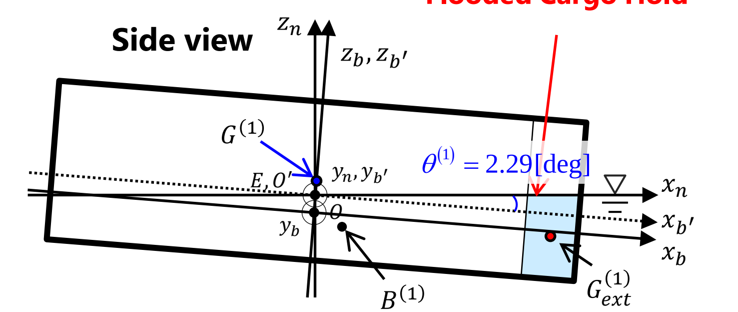
Equation of waterplane: $z = (0.0399x - 0.1665y)/0.9852$

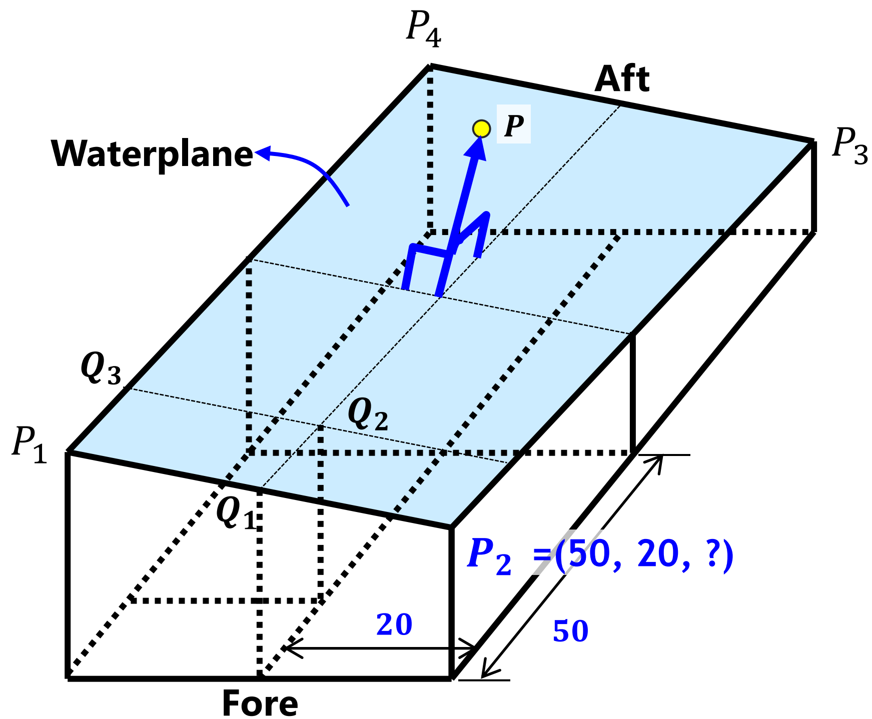
$${}^{b'}\mathbf{r}_{P_2/O'}: \ {}^{b'}x_{P_2/O'} = \mathbf{50}, \ {}^{b'}y_{P_2/O'} = \mathbf{20}$$

$${}^{b'}z_{P_2/O'} = \left(0.0399 \cdot {}^{b'}x_{P_2/O'} - 0.1665 \cdot {}^{b'}y_{P_2/O'}\right)/0.9852$$

$$= (0.0399 \cdot 50 - 0.1665 \cdot 20\,)/0.9852$$

$$= \mathbf{-1.3572}$$



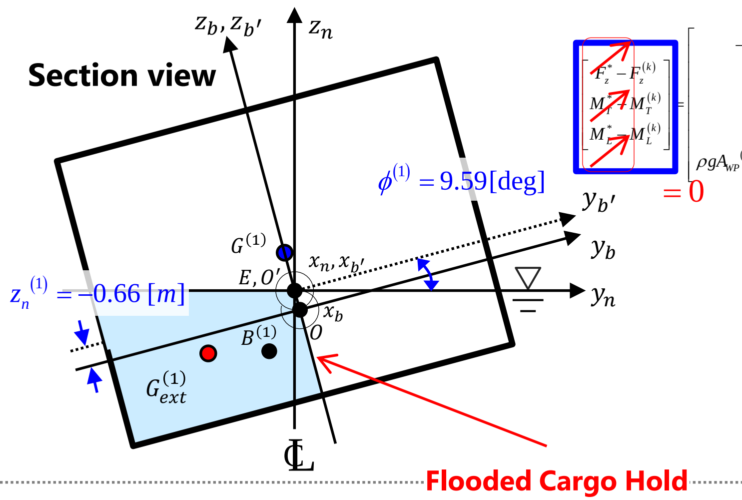


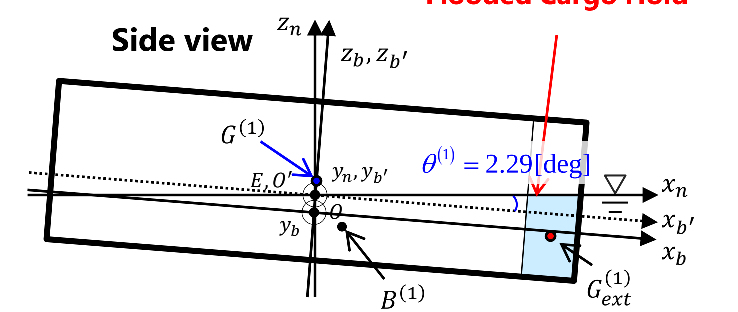
$L = 100\ [m]$	$l = 10\ [m]$	${}^n\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$
$B_{mld} = 40\ [m]$	$b = 20\ [m]$	${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [45 \quad -10 \quad -4.5]^T\ [m]$
$D = 30\ [m]$	$\mu_F = 1.0$	${}^n\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T\ [m]$
$d = 9\ [m]$	$\mu_V = 1.0$	${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2s^2]$		${}^n\mathbf{r}_{f^{(0)}/E} = [45 \quad -10 \quad 0]^T\ [m]$

$\nabla^{(0)} = 3.6 \times 10^4\ [m^3]$	$v^{(0)} = 1.8 \times 10^3\ [m^3]$
$F_{G,z} = -3.6 \times 10^5\ [kN]$	$F_z^{(0)} = -1.8 \times 10^4\ [kN]$
$F_{G_{ext},z}^{(0)} = -1.8 \times 10^4\ [kN]$	$M_T^{(0)} = 1.8 \times 10^5\ [kN]$
$F_{B,z}^{(0)} = 3.6 \times 10^5\ [kN]$	$M_L^{(0)} = 8.1 \times 10^5\ [kN]$

$$\boxed{\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \\ \cancel{M_L^*} - M_L^{(k)} \end{bmatrix}} = \begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) & \rho g A_{WP}^{(k)} \cdot {}^{n}x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^{n}x_{f^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) & -\rho g\left({}^{n}z_{B^{(k)}/E}\nabla^{(k)} + I_T^{(k)}\right) - {}^{n}z_{G^{(k)}/E} \cdot F_{G,z} - {}^{n}z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_T^{(k)}\right) & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^{n}x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^{n}x_{f^{(k)}/E} & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} & -\rho g\left({}^{n}z_{B^{(k)}/E}\nabla^{(k)} + I_L^{(k)}\right) - {}^{n}z_{G^{(k)}/E} \cdot F_{G,z} - {}^{n}z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_L^{(k)}\right) \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta\phi^{(k)} \\ \delta\theta^{(k)} \end{bmatrix}$$

$= 0$





$L = 100\ [m]$	$l = 10\ [m]$	${}^{n}\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$
$B_{mld} = 40\ [m]$	$b = 20\ [m]$	${}^{n}\mathbf{r}_{G_{ext}^{(0)}/E} = [45 \quad -10 \quad -4.5]^T\ [m]$
$D = 30\ [m]$	$\mu_F = 1.0$	${}^{n}\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T\ [m]$
$d = 9\ [m]$	$\mu_V = 1.0$	${}^{n}\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2 s^2]$		${}^{n}\mathbf{r}_{f^{(0)}/E} = [45 \quad -10 \quad 0]^T\ [m]$

$\nabla^{(0)} = 3.6 \times 10^4\ [m^3]$	$v^{(0)} = 1.8 \times 10^3\ [m^3]$
$F_{G,z} = -3.6 \times 10^5\ [kN]$	$F_z^{(0)} = -1.8 \times 10^4\ [kN]$
$F_{G_{ext},z}^{(0)} = -1.8 \times 10^4\ [kN]$	$M_T^{(0)} = 1.8 \times 10^5\ [kN]$
$F_{B,z}^{(0)} = 3.6 \times 10^5\ [kN]$	$M_L^{(0)} = 8.1 \times 10^5\ [kN]$

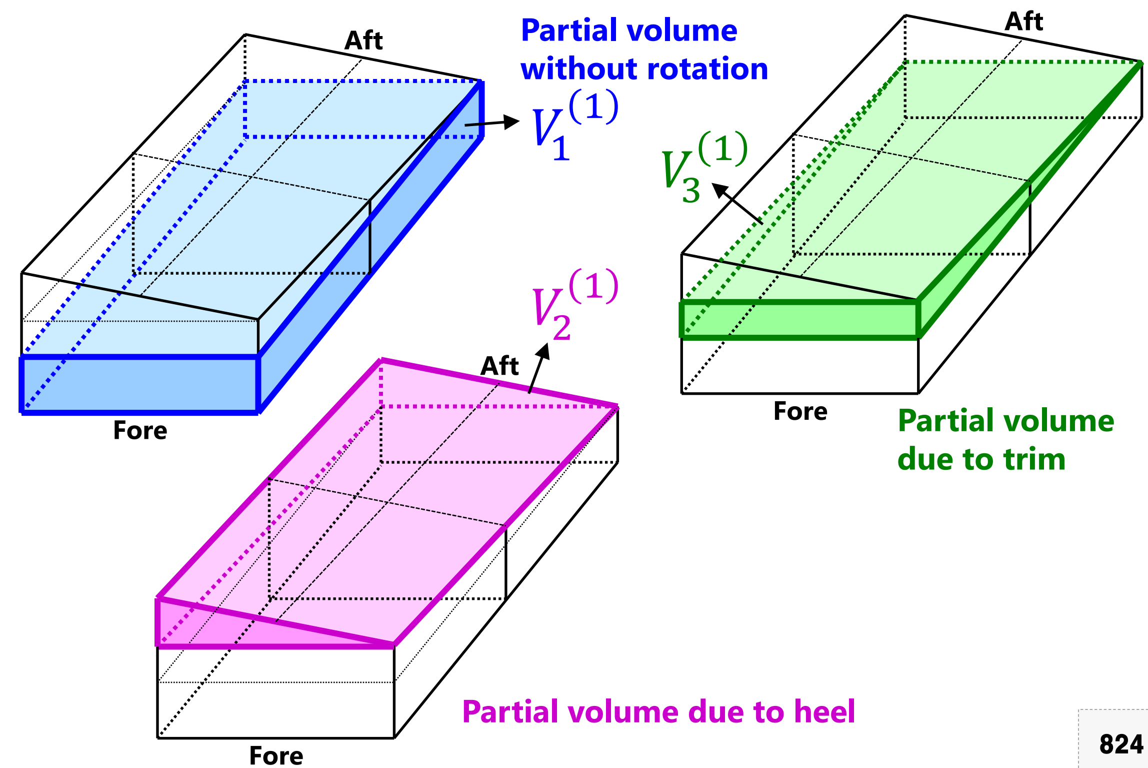
Force equilibrium:

$$F_z^{(1)} = F_{B,z}^{(1)} + F_{G,z}^{(1)} + F_{ext,z}^{(1)}$$

$$F_{B,z}^{(1)} = \rho g \nabla^{(1)}$$

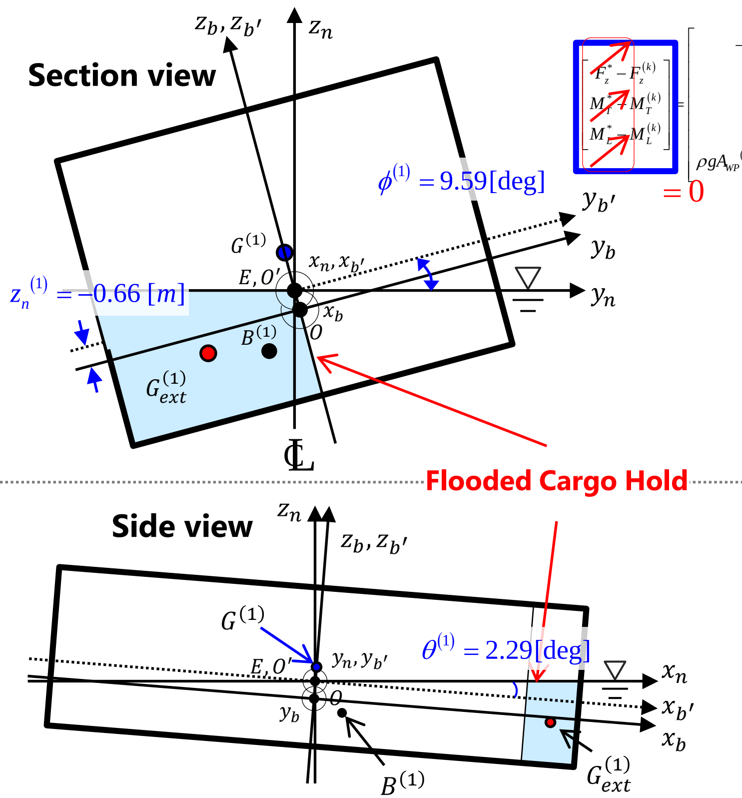
$$\nabla^{(1)} = V_1^{(1)} + V_2^{(1)} + V_3^{(1)}$$

$${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 5.4044]^T\ [m]$$
$${}^{b'}\mathbf{r}_{P_2/O'} = [50 \quad 20 \quad -1.3571]^T\ [m]$$
$${}^{b'}\mathbf{r}_{P_3/O'} = [-50 \quad 20 \quad -5.4044]^T\ [m]$$
$${}^{b'}\mathbf{r}_{P_4/O'} = [-50 \quad -20 \quad 1.3571]^T\ [m]$$



$$\boxed{\begin{bmatrix} F_z^* - F_z^{(k)} \\ M_T^* - M_T^{(k)} \\ M_L^* - M_L^{(k)} \end{bmatrix}} = \begin{bmatrix} -\rho g A_{WP}{}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}{}^{(k)}\right) & -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^n x_{f^{(k)}/E} \\ -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T{}^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} - \left(-\mu_F \cdot \rho g i_T{}^{(k)}\right) & \rho g I_P{}^{(k)} - \mu_F \cdot \rho g i_P{}^{(k)} \\ \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^n x_{f^{(k)}/E} & \rho g I_P{}^{(k)} - \mu_F \cdot \rho g i_P{}^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L{}^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} - \left(-\mu_F \cdot \rho g i_L{}^{(k)}\right) \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$



$L = 100\ [m]$ $\quad l = 10\ [m]$ $\quad {}^n\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$

$B_{mld} = 40\ [m]$ $\quad b = 20\ [m]$ $\quad {}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [45 \quad -10 \quad -4.5]^T\ [m]$

$D = 30\ [m]$ $\quad \mu_F = 1.0$ $\quad {}^n\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T\ [m]$

$d = 9\ [m]$ $\quad \mu_V = 1.0$ $\quad {}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$

$\rho g = 10\ [Mg/m^2s^2]$ $\quad {}^n\mathbf{r}_{f^{(0)}/E} = [45 \quad -10 \quad 0]^T\ [m]$

$\nabla^{(0)} = 3.6 \times 10^4\ [m^3]$ $\quad v^{(0)} = 1.8 \times 10^3\ [m^3]$

$F_{G,z} = -3.6 \times 10^5\ [kN]$ $\quad F_z^{(0)} = -1.8 \times 10^4\ [kN]$

$F_{G_{ext},z}^{(0)} = -1.8 \times 10^4\ [kN]$ $\quad M_T^{(0)} = 1.8 \times 10^5\ [kN]$

$F_{B,z}^{(0)} = 3.6 \times 10^5\ [kN]$ $\quad M_L^{(0)} = 8.1 \times 10^5\ [kN]$

Force equilibrium:

$$F_z^{(1)} = F_{B,z}^{(1)} + F_{G,z}^{(1)} + F_{ext,z}^{(1)}$$

$$F_{B,z}^{(1)} = \rho g \nabla^{(1)}$$

$$\nabla^{(1)} = V_1^{(1)} + V_2^{(1)} + V_3^{(1)}$$

$$V_1^{(1)} = L \cdot B_{mld} \cdot \overline{P_3 A_3} = 40 \cdot 100 \cdot \left(-5.4044 - (-9.9563)\right)$$

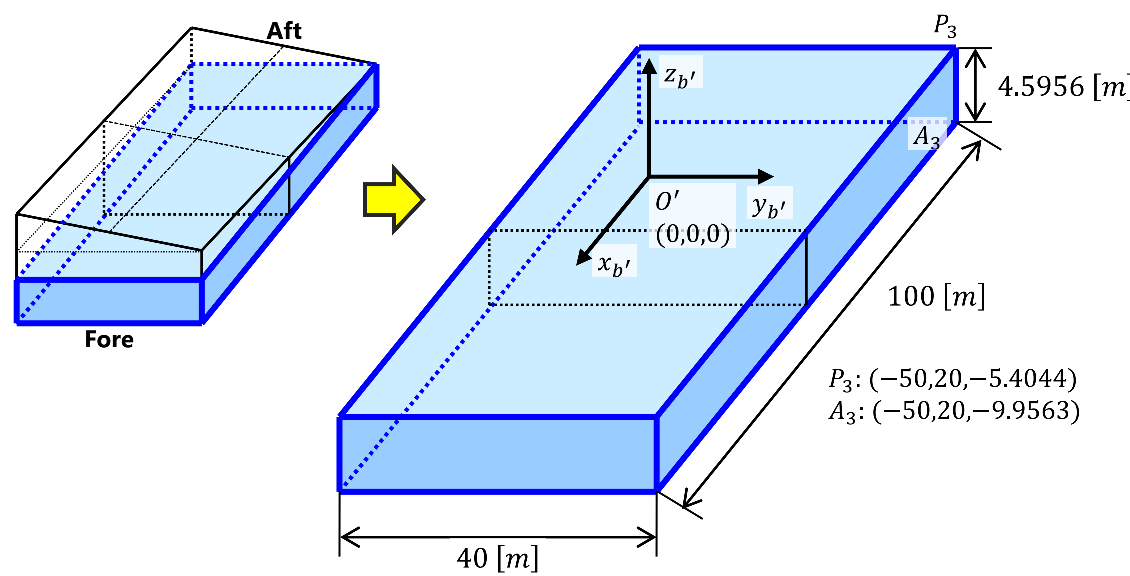
$$= 1.8208 \times 10^4\ [m^3]$$

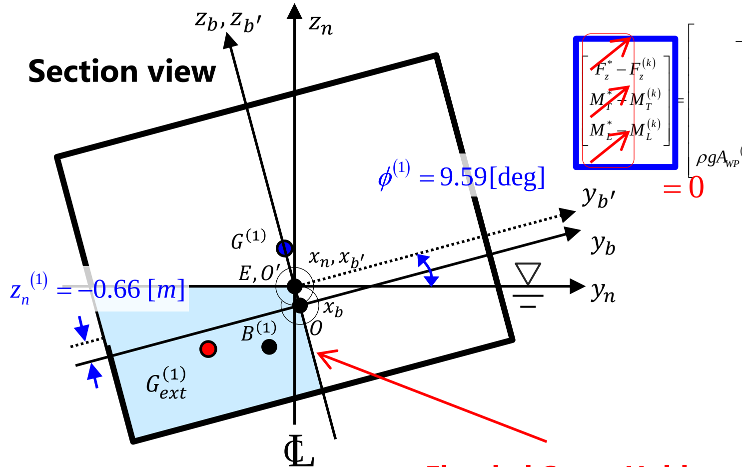
$${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 5.4044]^T\ [m]$$

$${}^{b'}\mathbf{r}_{P_2/O'} = [50 \quad 20 \quad -1.3571]^T\ [m]$$

$${}^{b'}\mathbf{r}_{P_3/O'} = [-50 \quad 20 \quad -5.4044]^T\ [m]$$

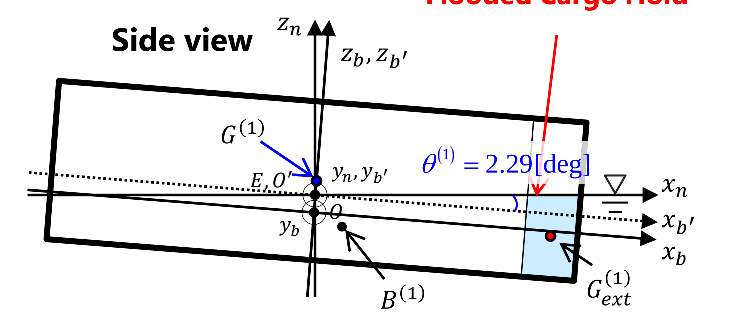
$${}^{b'}\mathbf{r}_{P_4/O'} = [-50 \quad -20 \quad 1.3571]^T\ [m]$$





$$\begin{bmatrix} \cancel{F_z^* - F_z^{(k)}} \\ \cancel{M_T^* - M_T^{(k)}} \\ \cancel{M_L^* - M_L^{(k)}} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_T^{(k)}\right) & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_L^{(k)}\right) \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta\phi^{(k)} \\ \delta\theta^{(k)} \end{bmatrix}$$

$= 0$



$L = 100\ [m]$
$B_{mld} = 40\ [m]$
$D = 30\ [m]$
$d = 9\ [m]$

$l = 10\ [m]$
$b = 20\ [m]$
$\mu_F = 1.0$
$\mu_V = 1.0$

$\rho g = 10\ [Mg/m^2 s^2]$

${}^n\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$
${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [45 \quad -10 \quad -4.5]^T\ [m]$
${}^n\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T\ [m]$
${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
${}^n\mathbf{r}_{f^{(0)}/E} = [45 \quad -10 \quad 0]^T\ [m]$

$\nabla^{(0)} = 3.6 \times 10^4\ [m^3]$
$F_{G,z} = -3.6 \times 10^5\ [kN]$
$F_{G_{ext},z}^{(0)} = -1.8 \times 10^4\ [kN]$
$F_{B,z}^{(0)} = 3.6 \times 10^5\ [kN]$

$v^{(0)} = 1.8 \times 10^3\ [m^3]$
$F_z^{(0)} = -1.8 \times 10^4\ [kN]$
$M_T^{(0)} = 1.8 \times 10^5\ [kN]$
$M_L^{(0)} = 8.1 \times 10^5\ [kN]$

Force equilibrium:

$${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 5.4044]^T\ [m]$$
$${}^{b'}\mathbf{r}_{P_2/O'} = [50 \quad 20 \quad -1.3571]^T\ [m]$$
$${}^{b'}\mathbf{r}_{P_3/O'} = [-50 \quad 20 \quad -5.4044]^T\ [m]$$
$${}^{b'}\mathbf{r}_{P_4/O'} = [-50 \quad -20 \quad 1.3571]^T\ [m]$$
$$V_1 = 1.8208 \times 10^4\ [m^3]$$

$$F_z^{(1)} = F_{B,z}^{(1)} + F_{G,z}^{(1)} + F_{ext,z}^{(1)}$$

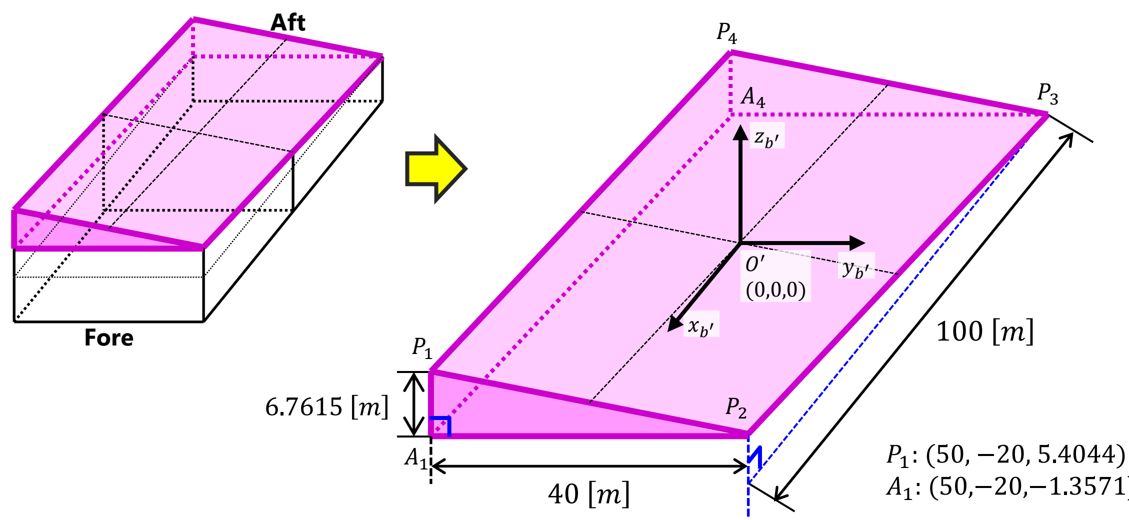
$$F_{B,z}^{(1)} = \rho g \nabla^{(1)}$$

$$\nabla^{(1)} = V_1^{(1)} + V_2^{(1)} + V_3^{(1)}$$

$$V_2^{(1)} = Triangular\ Area \times Length = \left(\frac{1}{2} \cdot B_{mld} \cdot \overline{P_1 A_1}\right) \cdot L$$

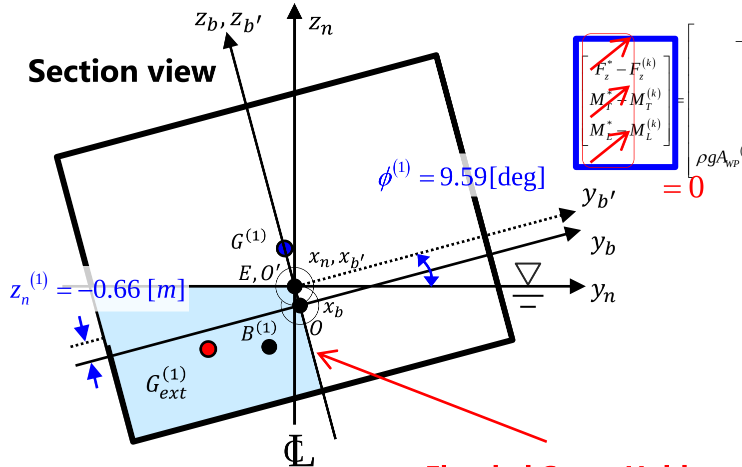
$$= [0.5 \cdot 40 \cdot \{5.4044 - (-1.3571)\}] \cdot 100$$

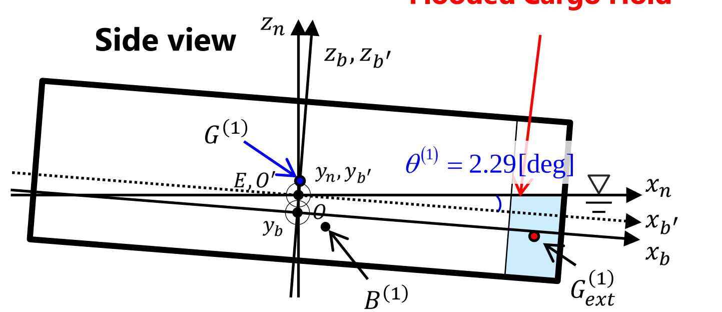
$$= 1.3523 \times 10^4\ [m^3]$$



$$
\begin{bmatrix} F_z^* - F_z^{(k)} \\ M_T^* - M_T^{(k)} \\ M_L^* - M_L^{(k)} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_T^{(k)}\right) & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_L^{(k)}\right) \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}
$$

$= 0$





$L = 100\ [m]$	$l = 10\ [m]$	${}^n\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$
$B_{mld} = 40\ [m]$	$b = 20\ [m]$	${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [45 \quad -10 \quad -4.5]^T\ [m]$
$D = 30\ [m]$	$\mu_F = 1.0$	${}^n\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T\ [m]$
$d = 9\ [m]$	$\mu_V = 1.0$	${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2 s^2]$		${}^n\mathbf{r}_{f^{(0)}/E} = [45 \quad -10 \quad 0]^T\ [m]$

$\nabla^{(0)} = 3.6 \times 10^4\ [m^3]$	$v^{(0)} = 1.8 \times 10^3\ [m^3]$
$F_{G,z} = -3.6 \times 10^5\ [kN]$	$F_z^{(0)} = -1.8 \times 10^4\ [kN]$
$F_{G_{ext},z}^{(0)} = -1.8 \times 10^4\ [kN]$	$M_T^{(0)} = 1.8 \times 10^5\ [kN]$
$F_{B,z}^{(0)} = 3.6 \times 10^5\ [kN]$	$M_L^{(0)} = 8.1 \times 10^5\ [kN]$

Force equilibrium:

$$F_z^{(1)} = F_{B,z}^{(1)} + F_{G,z}^{(1)} + F_{ext,z}^{(1)}$$

$$F_{B,z}^{(1)} = \rho g \nabla^{(1)}$$

$$\nabla^{(1)} = V_1^{(1)} + V_2^{(1)} + V_3^{(1)}$$

$$V_3^{(3)} = Triangular\ Area \times Height = \left(\frac{1}{2} \cdot L \cdot \overline{P_2 A_2}\right) \cdot B_{mld}$$

$$= [0.5 \cdot 100 \cdot \{-1.3571 - (-5.4044)\}] \cdot 40$$

$$= 8.0945 \times 10^4\ [m^3]$$

$${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 5.4044]^T\ [m]$$

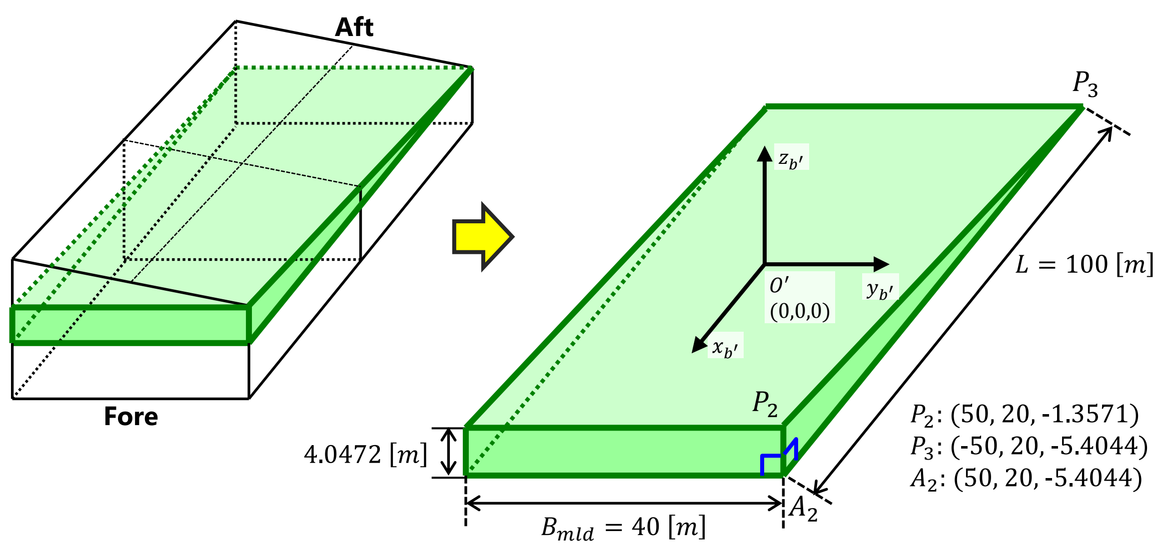
$${}^{b'}\mathbf{r}_{P_2/O'} = [50 \quad 20 \quad -1.3571]^T\ [m]$$

$${}^{b'}\mathbf{r}_{P_3/O'} = [-50 \quad 20 \quad -5.4044]^T\ [m]$$

$${}^{b'}\mathbf{r}_{P_4/O'} = [-50 \quad -20 \quad 1.3571]^T\ [m]$$

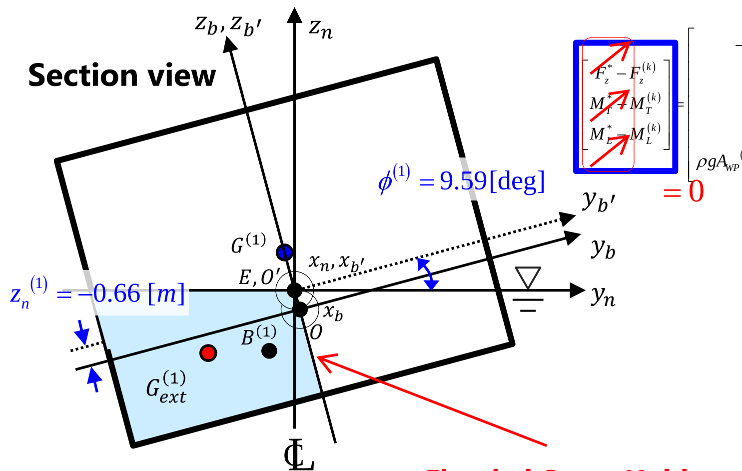
$$V_1 = 1.8208 \times 10^4\ [m^3]$$

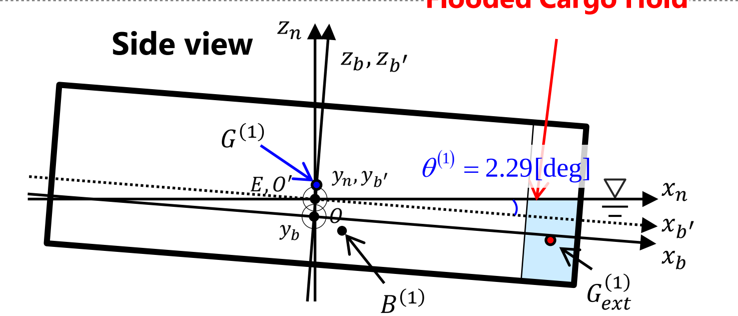
$$V_2 = 1.3523 \times 10^4\ [m^3]$$



$$
\boxed{\begin{bmatrix} F_z^* - F_z^{(k)} \\ M_T^* - M_T^{(k)} \\ M_L^* - M_L^{(k)} \end{bmatrix}} = \begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_T^{(k)}\right) & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_L^{(k)}\right) \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}
$$

$= 0$





$L = 100\ [m]$	$l = 10\ [m]$	${}^n\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$
$B_{mld} = 40\ [m]$	$b = 20\ [m]$	${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [45 \quad -10 \quad -4.5]^T\ [m]$
$D = 30\ [m]$	$\mu_F = 1.0$	${}^n\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T\ [m]$
$d = 9\ [m]$	$\mu_V = 1.0$	${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2s^2]$		${}^n\mathbf{r}_{f^{(0)}/E} = [45 \quad -10 \quad 0]^T\ [m]$

$\nabla^{(0)} = 3.6 \times 10^4\ [m^3]$	$v^{(0)} = 1.8 \times 10^3\ [m^3]$
$F_{G,z} = -3.6 \times 10^5\ [kN]$	$F_z^{(0)} = -1.8 \times 10^4\ [kN]$
$F_{G_{ext},z}^{(0)} = -1.8 \times 10^4\ [kN]$	$M_T^{(0)} = 1.8 \times 10^5\ [kN]$
$F_{B,z}^{(0)} = 3.6 \times 10^5\ [kN]$	$M_L^{(0)} = 8.1 \times 10^5\ [kN]$

Force equilibrium:

$${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 5.4044]^T\ [m]$$
$${}^{b'}\mathbf{r}_{P_2/O'} = [50 \quad 20 \quad -1.3571]^T\ [m]$$
$${}^{b'}\mathbf{r}_{P_3/O'} = [-50 \quad 20 \quad -5.4044]^T\ [m]$$
$${}^{b'}\mathbf{r}_{P_4/O'} = [-50 \quad -20 \quad 1.3571]^T\ [m]$$

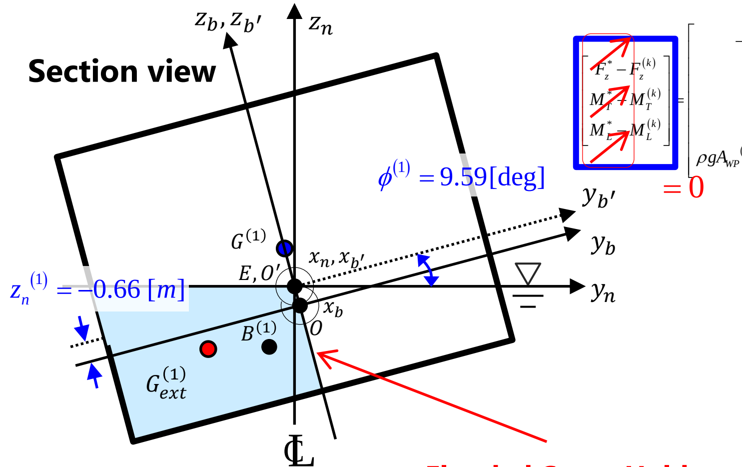
$$V_1 = 1.8208 \times 10^4\ [m^3]$$
$$V_2 = 1.3523 \times 10^4\ [m^3]$$
$$V_3 = 8.0945 \times 10^4\ [m^3]$$

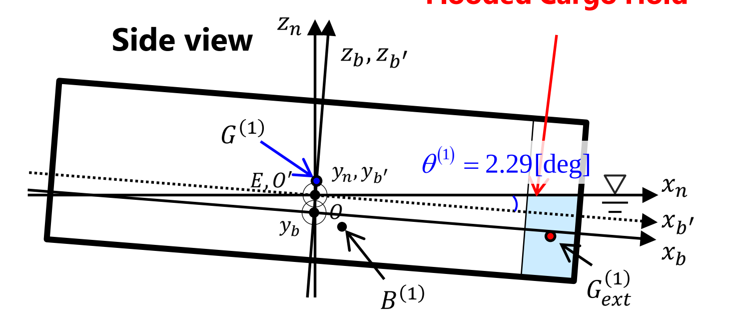
$$F_z^{(1)} = F_{B,z}^{(1)} + F_{G,z}^{(1)} + F_{ext,z}^{(1)}$$

$$F_{B,z}^{(1)} = \rho g \nabla^{(1)}$$

$$\nabla^{(1)} = V_1^{(1)} + V_2^{(1)} + V_3^{(1)}$$
$$= (1.8208 \times 10^4) + (\quad)1.3523 \times 10^4$$
$$+ (8.0945 \times 10^4)$$
$$= 3.9285 \times 10^4\ [m^3]$$

$$F_{B,z}^{(1)} = \rho g \nabla^{(1)} = 10 \cdot (4.0 \times 10^4) = 3.9285 \times 10^5\ [kN]$$





$$\begin{bmatrix} F_z^* - F_z^{(k)} \\ M_T^* - M_T^{(k)} \\ M_L^* - M_L^{(k)} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_T^{(k)}\right) & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_L^{(k)}\right) \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta\phi^{(k)} \\ \delta\theta^{(k)} \end{bmatrix}$$

$= 0$

Force equilibrium:

$$F_z^{(1)} = F_{B,z}^{(1)} + F_{G,z}^{(1)} + F_{ext,z}^{(1)}$$

$$F_{ext,z}^{(1)} = -\rho g v^{(1)}$$

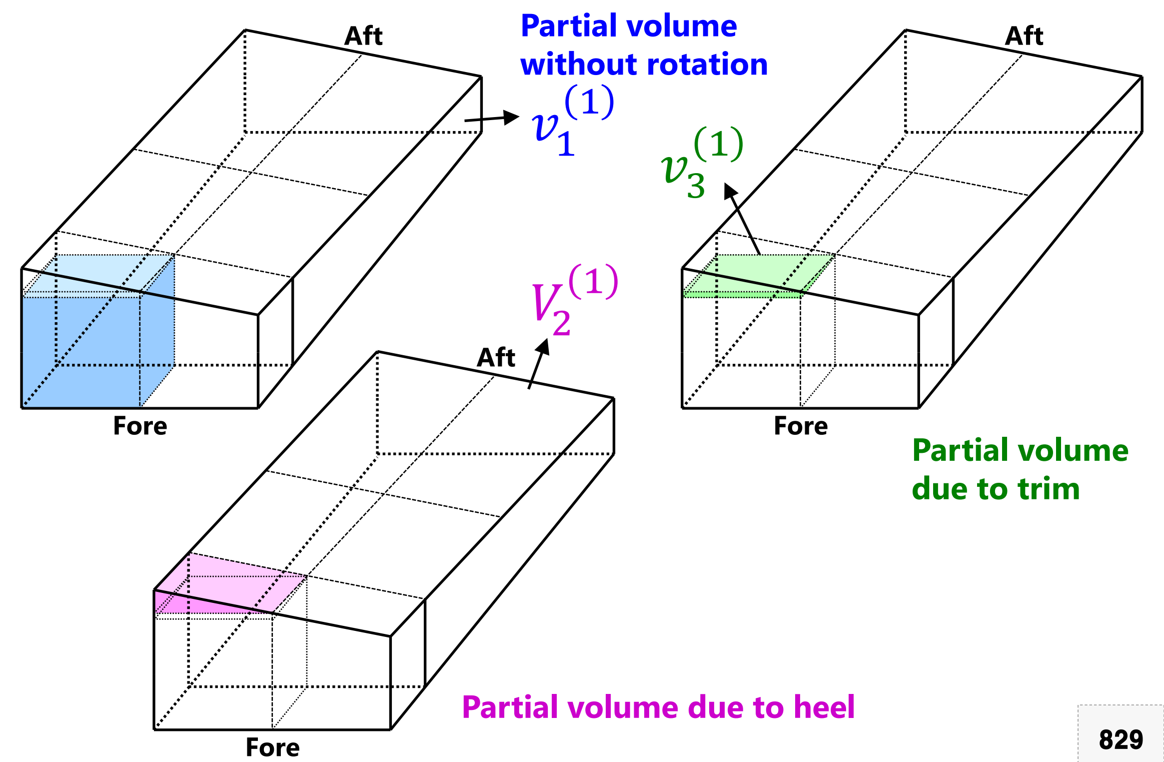
$$v^{(1)} = v_1^{(1)} + v_2^{(1)} + v_3^{(1)}$$

$${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 5.4044]^T \ [m]$$

$${}^{b'}\mathbf{r}_{Q_1/O'} = [50 \quad 0 \quad 2.0236]^T \ [m]$$

$${}^{b'}\mathbf{r}_{Q_2/O'} = [40 \quad 0 \quad 1.6189]^T \ [m]$$

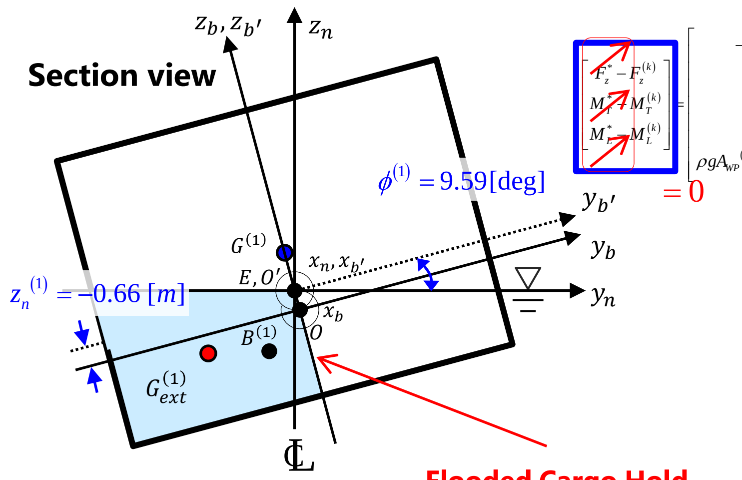
$${}^{b'}\mathbf{r}_{Q_3/O'} = [40 \quad -20 \quad 4.9997]^T \ [m]$$

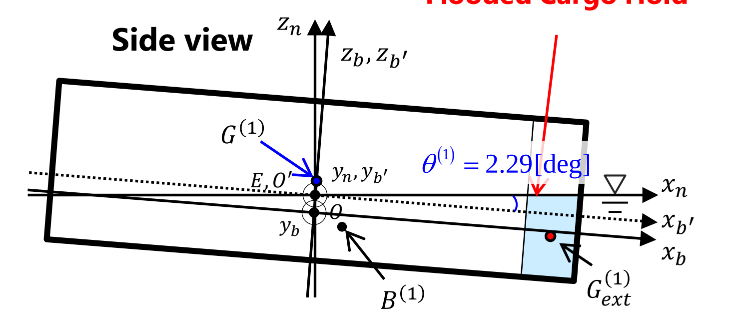


$L = 100\ [m]$	$l = 10\ [m]$	${}^n\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$
$B_{mld} = 40\ [m]$	$b = 20\ [m]$	${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [45 \quad -10 \quad -4.5]^T\ [m]$
$D = 30\ [m]$	$\mu_F = 1.0$	${}^n\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T\ [m]$
$d = 9\ [m]$	$\mu_V = 1.0$	${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2 s^2]$		${}^n\mathbf{r}_{f^{(0)}/E} = [45 \quad -10 \quad 0]^T\ [m]$

$\nabla^{(1)} = 3.9285 \times 10^4\ [m^3]$	$v^{(0)} = 1.8 \times 10^3\ [m^3]$
$F_{G,z} = -3.6 \times 10^5\ [kN]$	$F_z^{(0)} = -1.8 \times 10^4\ [kN]$
$F_{G_{ext},z}^{(0)} = -1.8 \times 10^4\ [kN]$	$M_T^{(0)} = 1.8 \times 10^5\ [kN]$
$F_{B,z}^{(1)} = 3.9285 \times 10^5\ [kN]$	$M_L^{(0)} = 8.1 \times 10^5\ [kN]$

$$\underbrace{\boxed{\begin{bmatrix} F_z^* - F_z^{(k)} \\ M_T^* - M_T^{(k)} \\ M_L^* - M_L^{(k)} \end{bmatrix}}}_{=0} = \begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_T^{(k)}\right) & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_L^{(k)}\right) \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$





$L = 100\ [m]$	$l = 10\ [m]$	${}^n\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$
$B_{mld} = 40\ [m]$	$b = 20\ [m]$	${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [45 \quad -10 \quad -4.5]^T\ [m]$
$D = 30\ [m]$	$\mu_F = 1.0$	${}^n\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T\ [m]$
$d = 9\ [m]$	$\mu_V = 1.0$	${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2 s^2]$		${}^n\mathbf{r}_{f^{(0)}/E} = [45 \quad -10 \quad 0]^T\ [m]$

$\nabla^{(1)} = 3.9285 \times 10^4\ [m^3]$	$v^{(0)} = 1.8 \times 10^3\ [m^3]$
$F_{G,z} = -3.6 \times 10^5\ [kN]$	$F_z^{(0)} = -1.8 \times 10^4\ [kN]$
$F_{G_{ext},z}^{(0)} = -1.8 \times 10^4\ [kN]$	$M_T^{(0)} = 1.8 \times 10^5\ [kN]$
$F_{B,z}^{(1)} = 3.9285 \times 10^5\ [kN]$	$M_L^{(0)} = 8.1 \times 10^5\ [kN]$

Force equilibrium:

$$F_z^{(1)} = F_{B,z}^{(1)} + F_{G,z}^{(1)} + F_{ext,z}^{(1)}$$

$$F_{ext,z}^{(1)} = -\rho g v^{(1)}$$

$$v^{(1)} = v_1^{(1)} + v_2^{(1)} + v_3^{(1)}$$

$$v_1^{(1)} = l \cdot b \cdot \overline{Q_2 R_2} = 40 \cdot 100 \cdot \left(1.6189 - (-9.9563)\right)$$

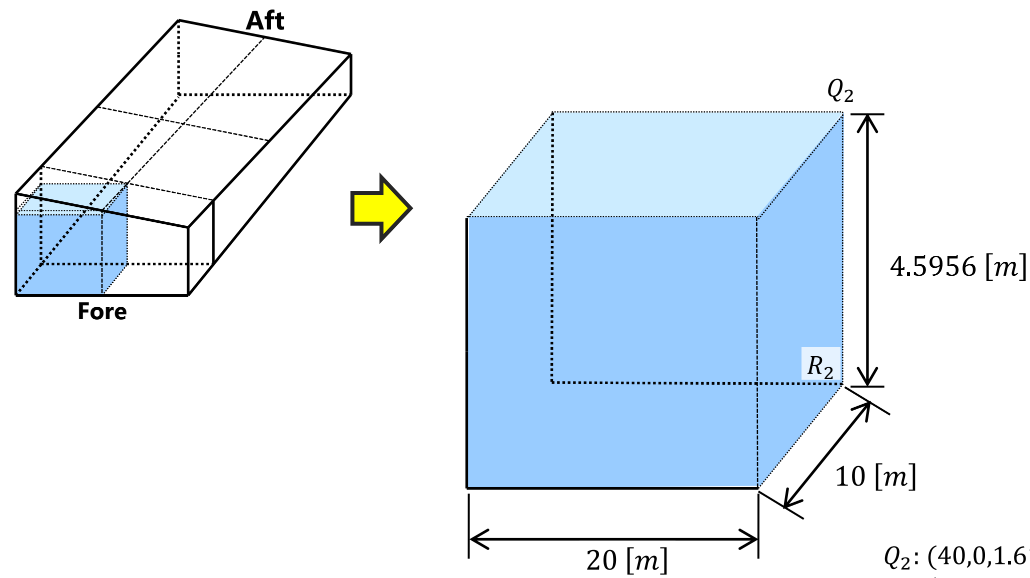
$$= 2.3150 \times 10^3\ [m^3]$$

$${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 5.4044]^T\ [m]$$

$${}^{b'}\mathbf{r}_{Q_1/O'} = [50 \quad 0 \quad 2.0236]^T\ [m]$$

$${}^{b'}\mathbf{r}_{Q_2/O'} = [40 \quad 0 \quad 1.6189]^T\ [m]$$

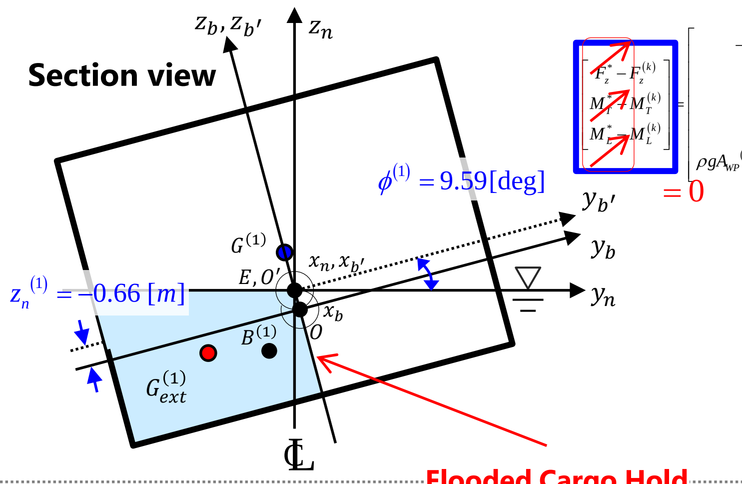
$${}^{b'}\mathbf{r}_{Q_3/O'} = [40 \quad -20 \quad 4.9997]^T\ [m]$$

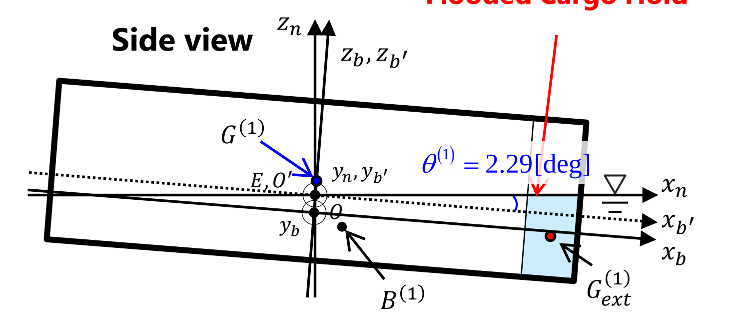


Q_2: (40,0,1.6189)
R_2: (40,0,−9.9563)

$$\boxed{\begin{bmatrix} \cancel{F_z^* - F_z^{(k)}} \\ \cancel{M_T^* - M_T^{(k)}} \\ \cancel{M_L^* - M_L^{(k)}} \end{bmatrix}} = \begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_T^{(k)}\right) & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_L^{(k)}\right) \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$





$L = 100\ [m]$
$B_{mld} = 40\ [m]$
$D = 30\ [m]$
$d = 9\ [m]$

$l = 10\ [m]$
$b = 20\ [m]$
$\mu_F = 1.0$
$\mu_V = 1.0$

$\rho g = 10\ [Mg/m^2 s^2]$

${}^n\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$
${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [45 \quad -10 \quad -4.5]^T\ [m]$
${}^n\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T\ [m]$
${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
${}^n\mathbf{r}_{f^{(0)}/E} = [45 \quad -10 \quad 0]^T\ [m]$

$\nabla^{(1)} = 3.9285 \times 10^4\ [m^3]$
$F_{G,z} = -3.6 \times 10^5\ [kN]$
$F_{G_{ext},z}^{(0)} = -1.8 \times 10^4\ [kN]$
$F_{B,z}^{(1)} = 3.9285 \times 10^5\ [kN]$

$v^{(0)} = 1.8 \times 10^3\ [m^3]$
$F_z^{(0)} = -1.8 \times 10^4\ [kN]$
$M_T^{(0)} = 1.8 \times 10^5\ [kN]$
$M_L^{(0)} = 8.1 \times 10^5\ [kN]$

Force equilibrium:

$$F_z^{(1)} = F_{B,z}^{(1)} + F_{G,z}^{(1)} + F_{ext,z}^{(1)}$$

$$F_{ext,z}^{(1)} = -\rho g v^{(1)}$$

$$v^{(1)} = v_1^{(1)} + v_2^{(1)} + v_3^{(1)}$$

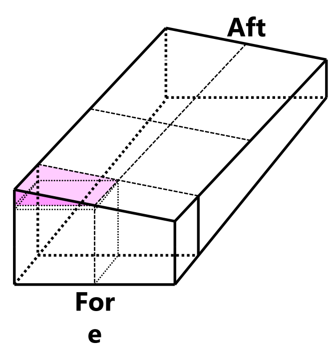
$$v_2^{(1)} = Triangular\ Area \times Length = \left(\frac{1}{2} \cdot b \cdot \overline{P_1 R_1}\right) \cdot l$$

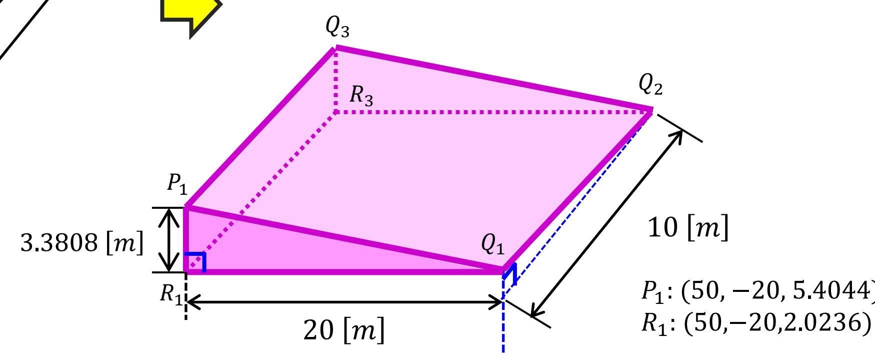
$$= [0.5 \cdot 20 \cdot \{5.4044 - (2.0236)\}] \cdot 10$$

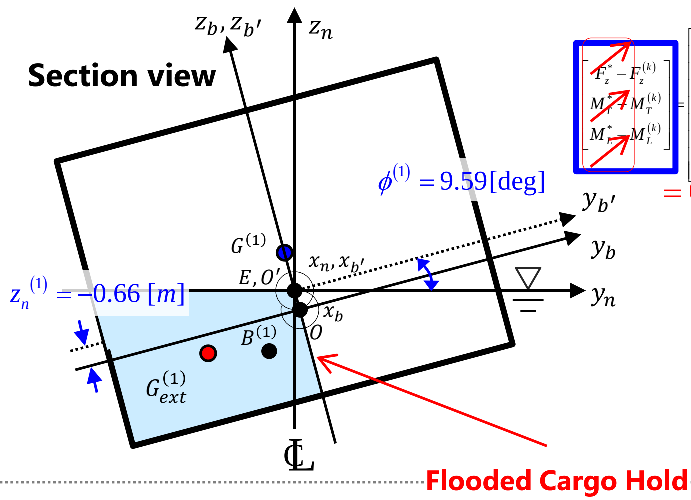
$$= 3.3808 \times 10^2\ [m^3]$$

${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 5.4044]^T\ [m]$
${}^{b'}\mathbf{r}_{Q_1/O'} = [50 \quad 0 \quad 2.0236]^T\ [m]$
${}^{b'}\mathbf{r}_{Q_2/O'} = [40 \quad 0 \quad 1.6189]^T\ [m]$
${}^{b'}\mathbf{r}_{Q_3/O'} = [40 \quad -20 \quad 4.9997]^T\ [m]$

$v_1 = 2.3150 \times 10^3\ [m^3]$

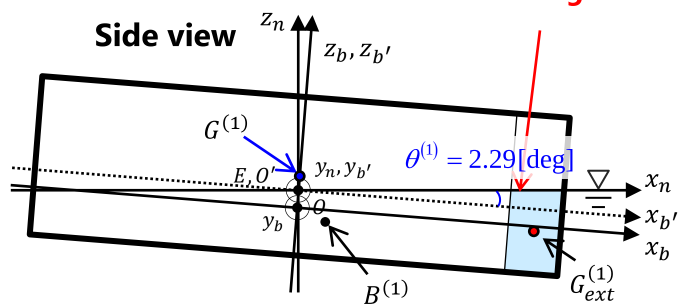






$$\boxed{\begin{bmatrix} F_z^* - F_z^{(k)} \\ M_T^* - M_T^{(k)} \\ M_L^* - M_L^{(k)} \end{bmatrix}} = \begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_T^{(k)}\right) & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_L^{(k)}\right) \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta\phi^{(k)} \\ \delta\theta^{(k)} \end{bmatrix}$$

$= 0$



$L = 100\ [m]$	$l = 10\ [m]$	${}^n\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$
$B_{mld} = 40\ [m]$	$b = 20\ [m]$	${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [45 \quad -10 \quad -4.5]^T\ [m]$
$D = 30\ [m]$	$\mu_F = 1.0$	${}^n\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T\ [m]$
$d = 9\ [m]$	$\mu_V = 1.0$	${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2s^2]$		${}^n\mathbf{r}_{f^{(0)}/E} = [45 \quad -10 \quad 0]^T\ [m]$

$\nabla^{(1)} = 3.9285 \times 10^4\ [m^3]$	$v^{(0)} = 1.8 \times 10^3\ [m^3]$
$F_{G,z} = -3.6 \times 10^5\ [kN]$	$F_z^{(0)} = -1.8 \times 10^4\ [kN]$
$F_{G_{ext},z}^{(0)} = -1.8 \times 10^4\ [kN]$	$M_T^{(0)} = 1.8 \times 10^5\ [kN]$
$F_{B,z}^{(1)} = 3.9285 \times 10^5\ [kN]$	$M_L^{(0)} = 8.1 \times 10^5\ [kN]$

Force equilibrium:

$${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 5.4044]^T\ [m]$$
$${}^{b'}\mathbf{r}_{Q_1/O'} = [50 \quad 0 \quad 2.0236]^T\ [m]$$
$${}^{b'}\mathbf{r}_{Q_2/O'} = [40 \quad 0 \quad 1.6189]^T\ [m]$$
$${}^{b'}\mathbf{r}_{Q_3/O'} = [40 \quad -20 \quad 4.9997]^T\ [m]$$

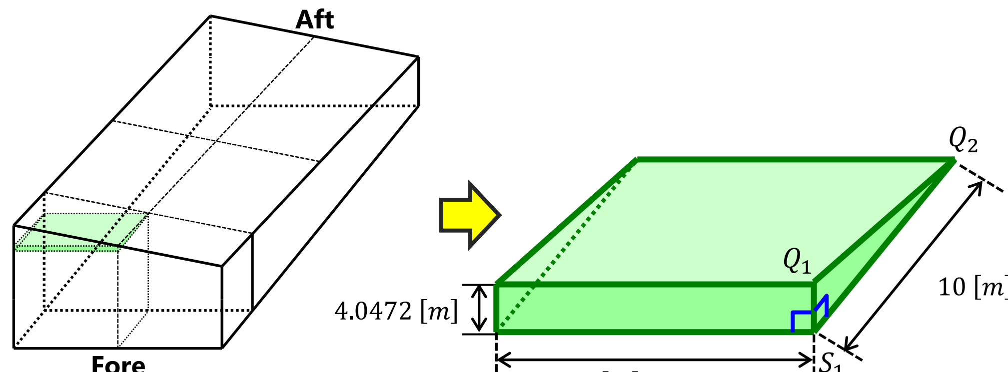
$$F_z^{(1)} = F_{B,z}^{(1)} + F_{G,z}^{(1)} + F_{ext,z}^{(1)}$$

$$F_{ext,z}^{(1)} = -\rho g v^{(1)}$$

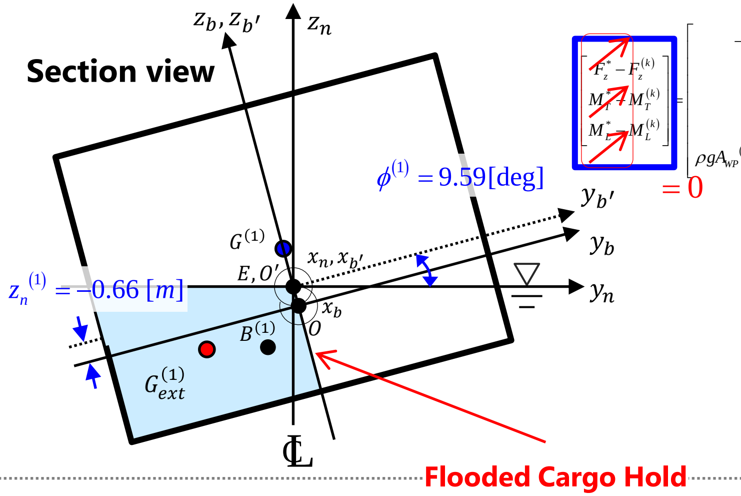
$$v^{(1)} = v_1^{(1)} + v_2^{(1)} + v_3^{(1)}$$

$$v_1 = 2.3150 \times 10^3\ [m^3]$$
$$v_2 = 3.3808 \times 10^2\ [m^3]$$

$$v_3^{(3)} = Triangular\ Area \times Height = \left(\frac{1}{2} \cdot l \cdot \overline{Q_1 S_1}\right) \cdot b$$
$$= [0.5 \cdot 10 \cdot \{2.0236 - 1.6189\}] \cdot 20$$
$$= 4.0472 \times 10^1\ [m^3]$$

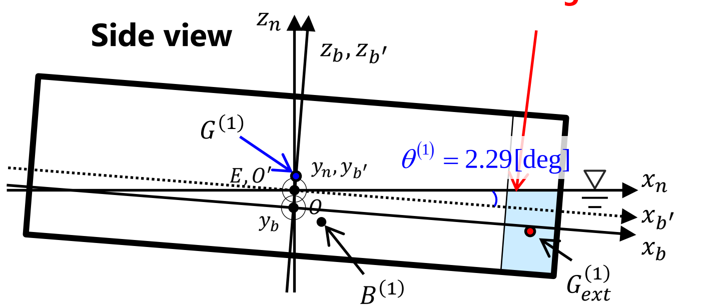


Q_1: (50, 0, 2.0236)
S_1: (50, 0, 1.6189)



$$\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \\ \cancel{M_L^*} - M_L^{(k)} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP}{}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}{}^{(k)}\right) & -\rho g A_{WP}{}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) & \rho g A_{WP}{}^{(k)} \cdot {}^{n}x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^{n}x_{f^{(k)}/E} \\ -\rho g A_{WP}{}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) & -\rho g\left({}^{n}z_{B^{(k)}/E}\nabla^{(k)} + I_T{}^{(k)}\right) - {}^{n}z_{G^{(k)}/E} \cdot F_{G,z} - {}^{n}z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} - \left(-\mu_F \cdot \rho g i_T{}^{(k)}\right) & \rho g I_P{}^{(k)} - \mu_F \cdot \rho g i_P{}^{(k)} \\ \rho g A_{WP}{}^{(k)} \cdot {}^{n}x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^{n}x_{f^{(k)}/E} & \rho g I_P{}^{(k)} - \mu_F \cdot \rho g i_P{}^{(k)} & -\rho g\left({}^{n}z_{B^{(k)}/E}\nabla^{(k)} + I_L{}^{(k)}\right) - {}^{n}z_{G^{(k)}/E} \cdot F_{G,z} - {}^{n}z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} - \left(-\mu_F \cdot \rho g i_L{}^{(k)}\right) \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$



Force equilibrium:

$$F_z^{(1)} = F_{B,z}^{(1)} + F_{G,z}^{(1)} + F_{ext,z}^{(1)}$$

$$F_{ext,z}^{(1)} = -\rho g v^{(1)}$$

$$v^{(1)} = v_1^{(1)} + v_2^{(1)} + v_3^{(1)}$$

$$= (2.3150 \times 10^3) + (3.3808 \times 10^2)$$

$$+(4.0472 \times 10^1)$$

$$= 2.6936 \times 10^3\ [m^3]$$

$$F_{ext,z}^{(1)} = -\rho g v^{(1)} = 10 \cdot (26936 \times 10^3) = -2.6936 \times 10^4\ [kN]$$

$${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 5.4044]^T\ [m]$$

$${}^{b'}\mathbf{r}_{Q_1/O'} = [50 \quad 0 \quad 2.0236]^T\ [m]$$

$${}^{b'}\mathbf{r}_{Q_2/O'} = [40 \quad 0 \quad 1.6189]^T\ [m]$$

$${}^{b'}\mathbf{r}_{Q_3/O'} = [40 \quad -20 \quad 4.9997]^T\ [m]$$

$$v_1 = 2.3238 \times 10^3\ [m^3]$$

$$v_2 = 3.3808 \times 10^2\ [m^3]$$

$$V_3 = 4.0472 \times 10^1\ [m^3]$$

$$F_z^{(1)} = F_{B,z}^{(1)} + F_{G,z}^{(1)} + F_{ext,z}^{(1)}$$

$$= (3.9285 \times 10^5) + (-3.6 \times 10^5) + (-2.6936 \times 10^4)$$

$$= \boxed{1.1316 \times 10^4\ [kN]} \overset{\text{Tolerance}}{<} e$$

where, e(epsilon) : an arbitrarily small positive quantity

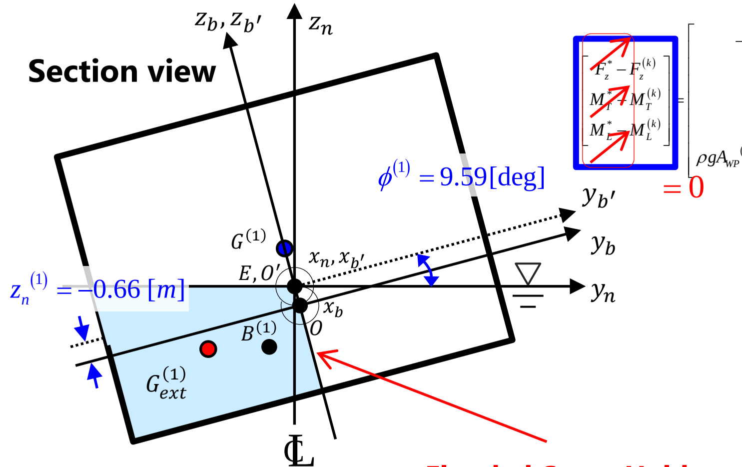
$L = 100\ [m]$	$l = 10\ [m]$	${}^{n}\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$
$B_{mld} = 40\ [m]$	$b = 20\ [m]$	${}^{n}\mathbf{r}_{G_{ext}^{(0)}/E} = [45 \quad -10 \quad -4.5]^T\ [m]$
$D = 30\ [m]$	$\mu_F = 1.0$	${}^{n}\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T\ [m]$
$d = 9\ [m]$	$\mu_V = 1.0$	${}^{n}\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2 s^2]$		${}^{n}\mathbf{r}_{f^{(0)}/E} = [45 \quad -10 \quad 0]^T\ [m]$

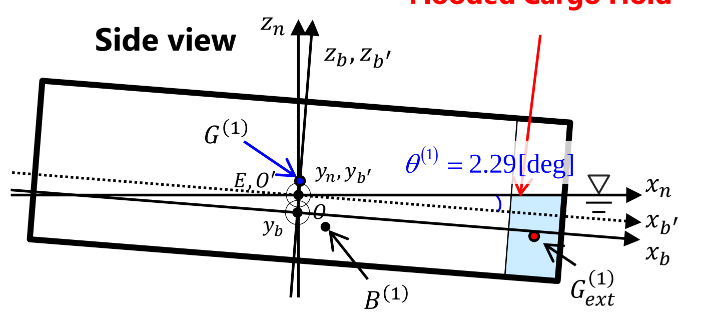
$\nabla^{(1)} = 3.9285 \times 10^4\ [m^3]$	$v^{(0)} = 1.8 \times 10^3\ [m^3]$
$F_{G,z} = -3.6 \times 10^5\ [kN]$	$F_z^{(0)} = -1.8 \times 10^4\ [kN]$
$F_{G_{ext},z}^{(0)} = -1.8 \times 10^4\ [kN]$	$M_T^{(0)} = 1.8 \times 10^5\ [kN]$
$F_{B,z}^{(1)} = 3.9285 \times 10^5\ [kN]$	$M_L^{(0)} = 8.1 \times 10^5\ [kN]$

The static equilibrium of force is satisfied!

$$\begin{bmatrix} F_z^* - F_z^{(k)} \\ M_T^* - M_T^{(k)} \\ M_L^* - M_L^{(k)} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_T^{(k)}\right) & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_L^{(k)}\right) \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$





$L = 100\ [m]$	$l = 10\ [m]$	${}^n\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$
$B_{mld} = 40\ [m]$	$b = 20\ [m]$	${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [45 \quad -10 \quad -4.5]^T\ [m]$
$D = 30\ [m]$	$\mu_F = 1.0$	${}^n\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T\ [m]$
$d = 9\ [m]$	$\mu_V = 1.0$	${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2 s^2]$		${}^n\mathbf{r}_{f^{(0)}/E} = [45 \quad -10 \quad 0]^T\ [m]$

$\nabla^{(1)} = 3.9285 \times 10^4\ [m^3]$	$v^{(1)} = 2.6936 \times 10^3\ [m^3]$
$F_{G,z} = -3.6 \times 10^5\ [kN]$	$F_z^{(1)} = 1.1316 \times 10^4\ [kN]$
$F_{G_{ext},z}^{(1)} = -2.6936 \times 10^4\ [kN]$	$M_T^{(0)} = 1.8 \times 10^5\ [kN]$
$F_{B,z}^{(1)} = 3.9285 \times 10^5\ [kN]$	$M_L^{(0)} = 8.1 \times 10^5\ [kN]$

Moment equilibrium:

First, we calculate center of volume for each volume V_1, V_2, and V_3.

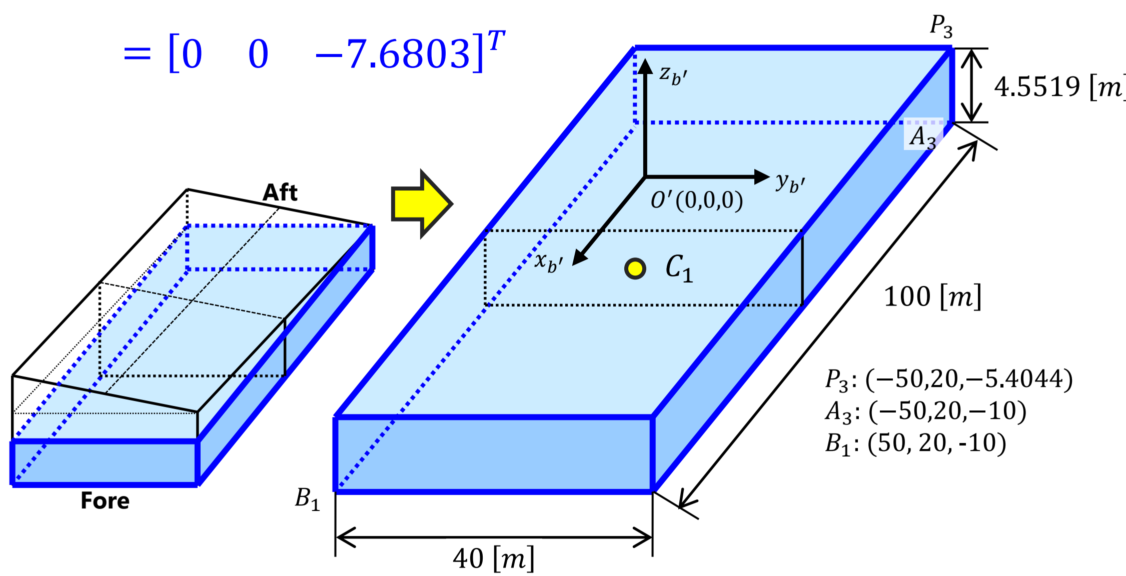
$${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 5.4044]^T\ [m]$$
$${}^{b'}\mathbf{r}_{P_2/O'} = [50 \quad 20 \quad -1.3571]^T\ [m]$$
$${}^{b'}\mathbf{r}_{P_3/O'} = [-50 \quad 20 \quad -5.4044]^T\ [m]$$
$${}^{b'}\mathbf{r}_{P_4/O'} = [-50 \quad -20 \quad 1.3571]^T\ [m]$$

Volume V_1: Box

C_1: located at centerline, midship and $T_0/2$

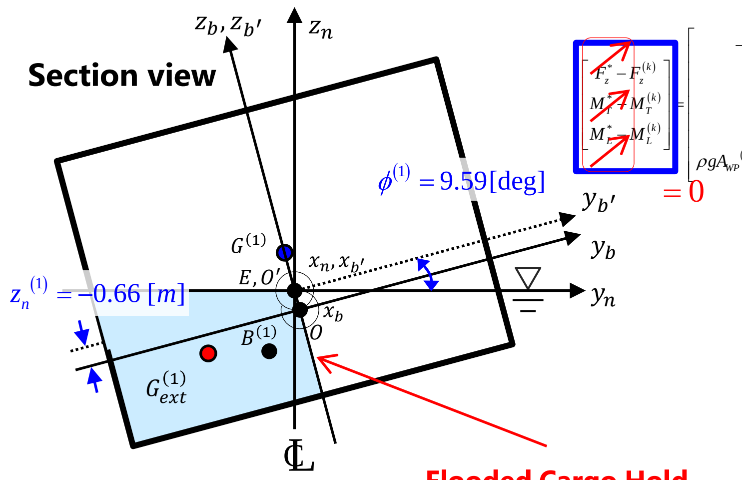
$${}^{b'}\mathbf{r}_{C_1/O'} = ({}^{b'}\mathbf{r}_{P_3/O'} + {}^{b'}\mathbf{r}_{B_1/O'})/2 = [\underbrace{0}_{\text{midship}} \quad \underbrace{0}_{\text{centerline}} \quad (-9.9563 + (-5.4044))/2]^T$$

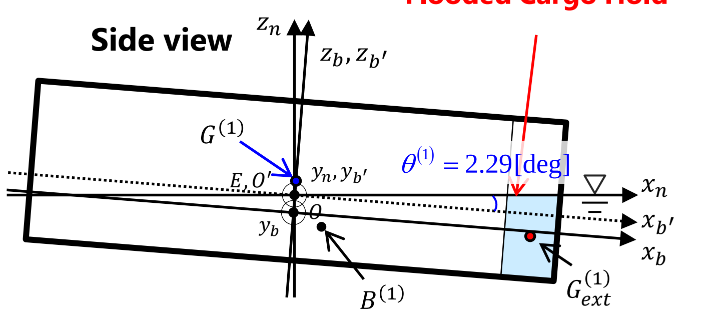
$$= [0 \quad 0 \quad -7.6803]^T$$



$$\boxed{\begin{bmatrix} \cancel{F_z^* - F_z^{(k)}} \\ \cancel{M_T^* - M_T^{(k)}} \\ \cancel{M_L^* - M_L^{(k)}} \end{bmatrix}} = \begin{bmatrix} -\rho g A_{WP}{}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}{}^{(k)}\right) & -\rho g A_{WP}{}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) & \rho g A_{WP}{}^{(k)} \cdot {}^{n}x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^{n}x_{f^{(k)}/E} \\ -\rho g A_{WP}{}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) & -\rho g \left({}^{n}z_{B^{(k)}/E} \nabla^{(k)} + I_T{}^{(k)}\right) - {}^{n}z_{G^{(k)}/E} \cdot F_{G,z} - {}^{n}z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} - \left(-\mu_F \cdot \rho g i_T{}^{(k)}\right) & \rho g I_P{}^{(k)} - \mu_F \cdot \rho g i_P{}^{(k)} \\ \rho g A_{WP}{}^{(k)} \cdot {}^{n}x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^{n}x_{f^{(k)}/E} & \rho g I_P{}^{(k)} - \mu_F \cdot \rho g i_P{}^{(k)} & -\rho g \left({}^{n}z_{B^{(k)}/E} \nabla^{(k)} + I_L{}^{(k)}\right) - {}^{n}z_{G^{(k)}/E} \cdot F_{G,z} - {}^{n}z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} - \left(-\mu_F \cdot \rho g i_L{}^{(k)}\right) \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$





$L = 100\ [m]$	$l = 10\ [m]$	${}^{n}\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$
$B_{mld} = 40\ [m]$	$b = 20\ [m]$	${}^{n}\mathbf{r}_{G_{ext}^{(0)}/E} = [45 \quad -10 \quad -4.5]^T\ [m]$
$D = 30\ [m]$	$\mu_F = 1.0$	${}^{n}\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T\ [m]$
$d = 9\ [m]$	$\mu_V = 1.0$	${}^{n}\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2s^2]$		${}^{n}\mathbf{r}_{f^{(0)}/E} = [45 \quad -10 \quad 0]^T\ [m]$

$\nabla^{(1)} = 3.9285 \times 10^4\ [m^3]$	$v^{(1)} = 2.6936 \times 10^3\ [m^3]$
$F_{G,z} = -3.6 \times 10^5\ [kN]$	$F_z^{(1)} = 1.1316 \times 10^4\ [kN]$
$F_{G_{ext},z}^{(1)} = -2.6936 \times 10^4\ [kN]$	$M_T^{(0)} = 1.8 \times 10^5\ [kN]$
$F_{B,z}^{(1)} = 3.9285 \times 10^5\ [kN]$	$M_L^{(0)} = 8.1 \times 10^5\ [kN]$

Moment equilibrium:

First, we calculate center of volume for each volume V_1, V_2, and V_3.

Volume V_2: Trigonal prism

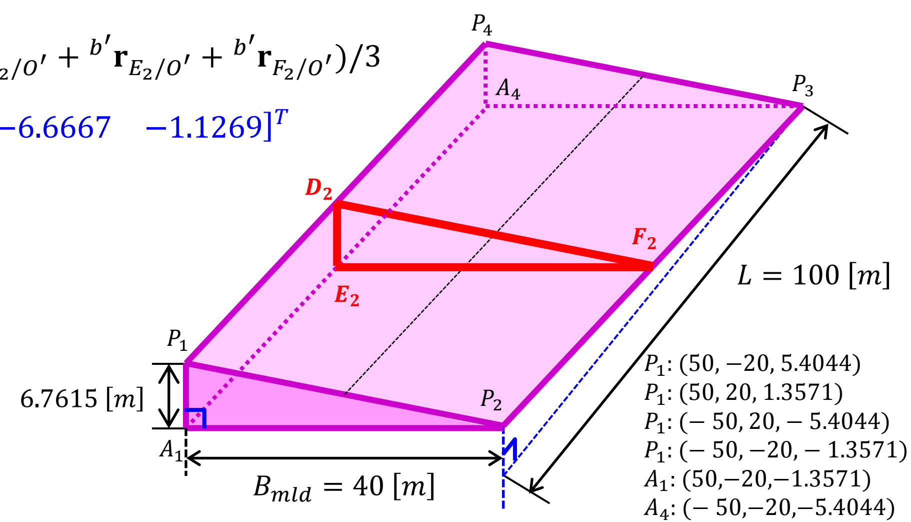
$${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 5.4044]^T\ [m]$$
$${}^{b'}\mathbf{r}_{P_2/O'} = [50 \quad 20 \quad -1.3571]^T\ [m]$$
$${}^{b'}\mathbf{r}_{P_3/O'} = [-50 \quad 20 \quad -5.4044]^T\ [m]$$
$${}^{b'}\mathbf{r}_{P_4/O'} = [-50 \quad -20 \quad 1.3571]^T\ [m]$$
$${}^{b'}\mathbf{r}_{C_1/O'} = [0 \quad 0 \quad -7.6803]^T\ [m]$$

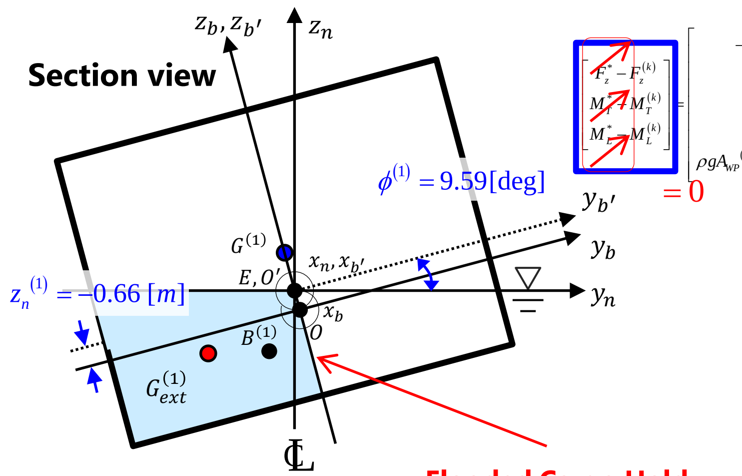
Center of volume:

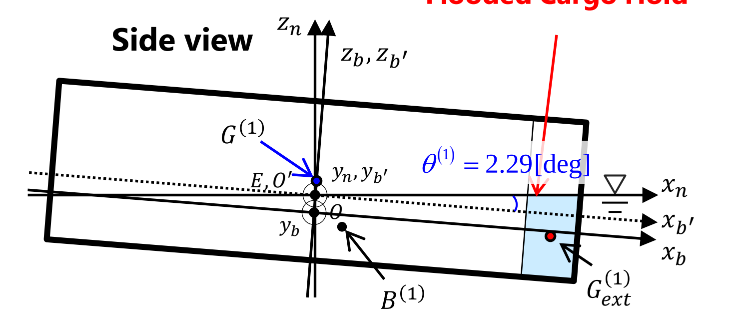
= Center of area of triangle $D_2E_2F_2$

$${}^{b'}\mathbf{r}_{D_2/O'} = ({}^{b'}\mathbf{r}_{P_1/O'} + {}^{b'}\mathbf{r}_{P_4/O'})/2 = (0,\ -20,\ 3{,}3808)$$
$${}^{b'}\mathbf{r}_{E_2/O'} = ({}^{b'}\mathbf{r}_{A_1/O'} + {}^{b'}\mathbf{r}_{A_4/O'})/2 = (0,\ -20,\ -3{,}3808)$$
$${}^{b'}\mathbf{r}_{F_2/O'} = ({}^{b'}\mathbf{r}_{P_2/O'} + {}^{b'}\mathbf{r}_{P_3/O'})/2 = (0,\ 20,\ -3{,}3808)$$

$${}^{b'}\mathbf{r}_{C_2/O'} = ({}^{b'}\mathbf{r}_{D_2/O'} + {}^{b'}\mathbf{r}_{E_2/O'} + {}^{b'}\mathbf{r}_{F_2/O'})/3$$
$$= [0 \quad -6.6667 \quad -1.1269]^T$$







$L = 100\ [m]$	$l = 10\ [m]$	${}^n\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$
$B_{mld} = 40\ [m]$	$b = 20\ [m]$	${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [45 \quad -10 \quad -4.5]^T\ [m]$
$D = 30\ [m]$	$\mu_F = 1.0$	${}^n\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T\ [m]$
$d = 9\ [m]$	$\mu_V = 1.0$	${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2s^2]$		${}^n\mathbf{r}_{f^{(0)}/E} = [45 \quad -10 \quad 0]^T\ [m]$

$\nabla^{(1)} = 3.9285 \times 10^4\ [m^3]$	$v^{(1)} = 2.6936 \times 10^3\ [m^3]$
$F_{G,z} = -3.6 \times 10^5\ [kN]$	$F_z^{(1)} = 1.1316 \times 10^4\ [kN]$
$F_{G_{ext},z}^{(1)} = -2.6936 \times 10^4\ [kN]$	$M_T^{(0)} = 1.8 \times 10^5\ [kN]$
$F_{B,z}^{(1)} = 3.9285 \times 10^5\ [kN]$	$M_L^{(0)} = 8.1 \times 10^5\ [kN]$

$$\underbrace{\boxed{\begin{bmatrix} F_z^* - F_z^{(k)} \\ M_T^* - M_T^{(k)} \\ M_L^* - M_L^{(k)} \end{bmatrix}}}_{=0} = \begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_T^{(k)}\right) & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_L^{(k)}\right) \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

Moment equilibrium:

First, we calculate center of volume for each volume V_1, V_2, and V_3.

$${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 5.4044]^T\ [m]$$
$${}^{b'}\mathbf{r}_{P_2/O'} = [50 \quad 20 \quad -1.3571]^T\ [m]$$
$${}^{b'}\mathbf{r}_{P_3/O'} = [-50 \quad 20 \quad -5.4044]^T\ [m]$$
$${}^{b'}\mathbf{r}_{P_4/O'} = [-50 \quad -20 \quad 1.3571]^T\ [m]$$
$${}^{b'}\mathbf{r}_{C_1/O'} = [0 \quad 0 \quad -7.6803]^T\ [m]$$
$${}^{b'}\mathbf{r}_{C_2/O'} = [0 \quad -6.6667 \quad -1.1269]^T$$

Volume V_3: Trigonal prism

Center of volume

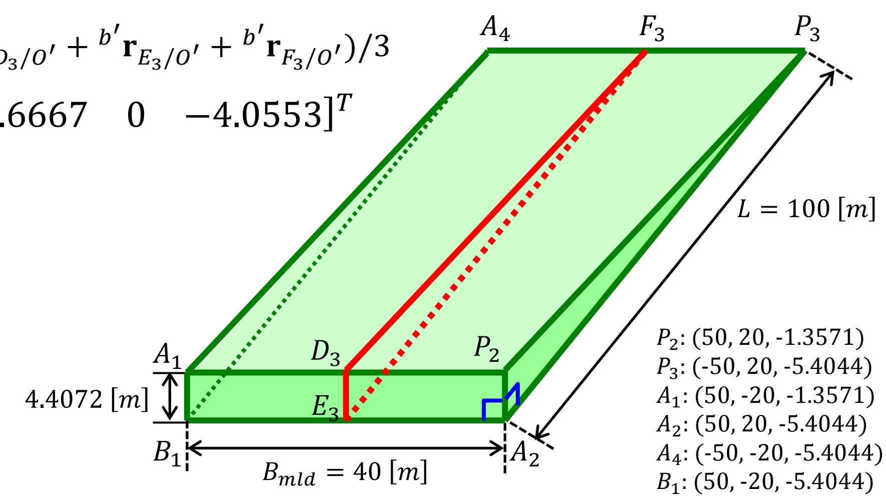
= Center of area of triangle $D_3E_3F_3$

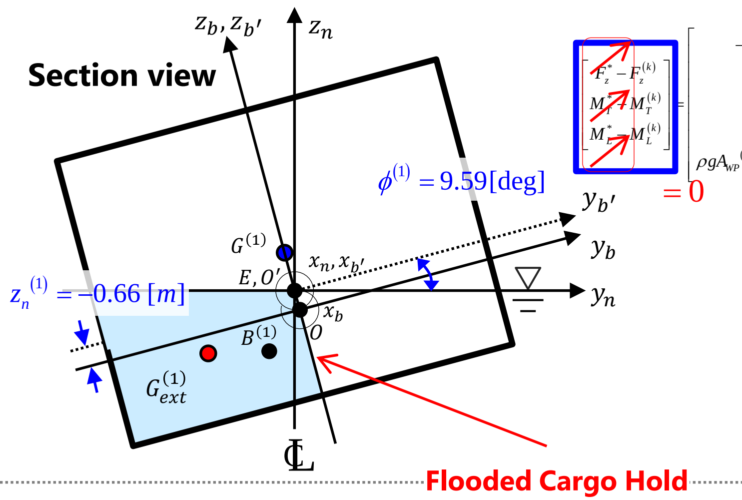
$${}^{b'}\mathbf{r}_{D_3/O'} = ({}^{b'}\mathbf{r}_{A_1/O'} + {}^{b'}\mathbf{r}_{P_2/O'})/2 = (50,\ 0,\ -1.3571)$$

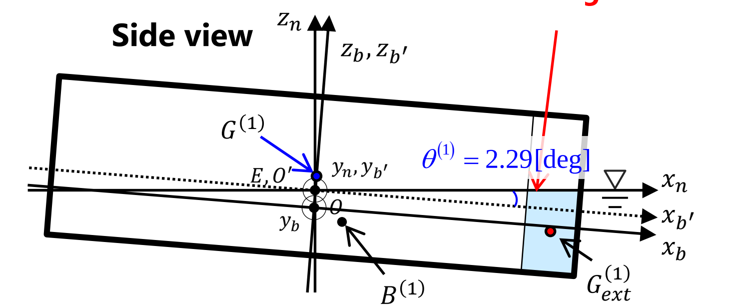
$${}^{b'}\mathbf{r}_{E_3/O'} = ({}^{b'}\mathbf{r}_{B_1/O'} + {}^{b'}\mathbf{r}_{A_2/O'})/2 = (50,\ 0,\ -5.4044)$$

$${}^{b'}\mathbf{r}_{F_3/O'} = ({}^{b'}\mathbf{r}_{P_3/O'} + {}^{b'}\mathbf{r}_{P_4/O'})/2 = (-50,\ 0,\ -5.4044)$$

$${}^{b'}\mathbf{r}_{C_3/O'} = ({}^{b'}\mathbf{r}_{D_3/O'} + {}^{b'}\mathbf{r}_{E_3/O'} + {}^{b'}\mathbf{r}_{F_3/O'})/3$$
$$= [16.6667 \quad 0 \quad -4.0553]^T$$







$$\begin{bmatrix} F_z^* - F_z^{(k)} \\ M_T^* - M_T^{(k)} \\ M_L^* - M_L^{(k)} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_T^{(k)}\right) & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_L^{(k)}\right) \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$

$L = 100\ [m]$	$l = 10\ [m]$	${}^n\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$
$B_{mld} = 40\ [m]$	$b = 20\ [m]$	${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [45 \quad -10 \quad -4.5]^T\ [m]$
$D = 30\ [m]$	$\mu_F = 1.0$	${}^n\mathbf{r}_{B^{(0)}/E} = [0 \quad 0 \quad -4.5]^T\ [m]$
$d = 9\ [m]$	$\mu_V = 1.0$	${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2 s^2]$		${}^n\mathbf{r}_{f^{(0)}/E} = [45 \quad -10 \quad 0]^T\ [m]$

$\nabla^{(1)} = 3.9285 \times 10^4\ [m^3]$	$v^{(1)} = 2.6936 \times 10^3\ [m^3]$
$F_{G,z} = -3.6 \times 10^5\ [kN]$	$F_z^{(1)} = 1.1316 \times 10^4\ [kN]$
$F_{G_{ext},z}^{(1)} = -2.6936 \times 10^4\ [kN]$	$M_T^{(0)} = 1.8 \times 10^5\ [kN]$
$F_{B,z}^{(1)} = 3.9285 \times 10^5\ [kN]$	$M_L^{(0)} = 8.1 \times 10^5\ [kN]$

Moment equilibrium:

$${}^{b'}\mathbf{r}_{C_1/O'} = [0 \quad 0 \quad -7.6803]^T\ [m]$$
$${}^{b'}\mathbf{r}_{C_2/O'} = [0 \quad -6.6667 \quad -1.1269]^T\ [m]$$
$${}^{b'}\mathbf{r}_{C_3/O'E} = [16.6667 \quad 0 \quad -4.0553]^T\ [m]$$

Center of Buoyancy:

$V_1 = 1.8208 \times 10^4\ [m^3]$

$V_2 = 1.3523 \times 10^4\ [m^3]$

$V_3 = 8.0945 \times 10^4\ [m^3]$

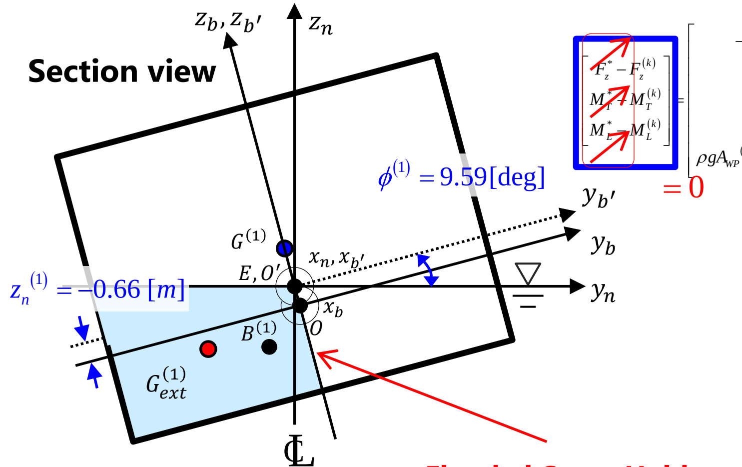
$${}^{b'}\mathbf{r}_{B^{(1)}/O'} = \frac{{}^{b'}\mathbf{r}_{C_1/O'} \cdot V_1 + {}^{b'}\mathbf{r}_{C_2/O'} \cdot V_2 + {}^{b'}\mathbf{r}_{C_3/O'} \cdot V_3}{V_1 + V_2 + V_3}$$

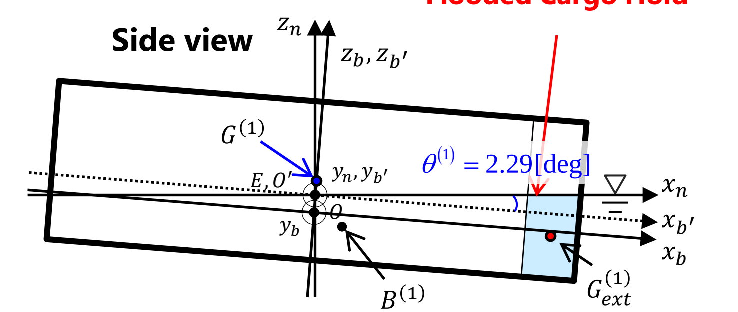
$$= [3.3875 \quad -2.2637 \quad -4.7183]^T\ [m]$$

In this case, for convenience of calculating the center of displaced volume ${}^{b'}\mathbf{r}_{B^{(1)}/O'}$ of the ship, we use b′-frame. The origin O′ of b′-frame coincides with the origin E of n-frame. And the orientation of b′-frame is the same as that of b-frame. So, to obtain the center of buoyancy with respect to n-frame, ${}^{n}\mathbf{r}_{B^{(1)}/E}$, we have to perform the rotational transformation.

$${}^{n}\mathbf{r}_{B^{(1)}/E} = \begin{bmatrix} \cos\theta^{(1)} & 0 & \sin\theta^{(1)} \\ 0 & 1 & 0 \\ -\sin\theta^{(1)} & 0 & \cos\theta^{(1)} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi^{(1)} & -\sin\phi^{(1)} \\ 0 & \sin\phi^{(1)} & \cos\phi^{(1)} \end{bmatrix} {}^{b'}\mathbf{r}_{B^{(1)}/O'}$$

$$= \begin{bmatrix} \cos 2.29 & 0 & \sin 2.29 \\ 0 & 1 & 0 \\ -\sin 2.29 & 0 & \cos 2.29 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 9.59 & -\sin 9.59 \\ 0 & \sin 9.59 & \cos 9.59 \end{bmatrix} \begin{bmatrix} 3.3875 \\ -2.2637 \\ -4.7183 \end{bmatrix} = \boxed{\begin{bmatrix} 3.1843 \\ -1.4457 \\ -5.1606 \end{bmatrix}}$$





$$\begin{bmatrix} F_z^* - F_z^{(k)} \\ M_T^* - M_T^{(k)} \\ M_L^* - M_L^{(k)} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_T^{(k)}\right) & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_L^{(k)}\right) \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$

Moment equilibrium:

Center of Gravity:

The center of mass, ${}^b\mathbf{r}_{G^{(1)}/O}$, with respect to the body fixed frame is identical with respect to the floating position. But the center of mass, ${}^n\mathbf{r}_{G^{(1)}/E}$, with respect to the waterplane-fixed frame changes with respect to the rotation. The change in the center of mass, ${}^n\mathbf{r}_{G^{(1)}/E}$, with respect to the waterplane-fixed frame causes an additional heeling moment arm.

$${}^n\mathbf{r}_{G^{(1)}/E} = \begin{bmatrix} \cos\theta^{(1)} & 0 & \sin\theta^{(1)} \\ 0 & 1 & 0 \\ -\sin\theta^{(1)} & 0 & \cos\theta^{(1)} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi^{(1)} & -\sin\phi^{(1)} \\ 0 & \sin\phi^{(1)} & \cos\phi^{(1)} \end{bmatrix} {}^{b'}\mathbf{r}_{G^{(1)}/O'}, \quad {}^{b'}\mathbf{r}_{G^{(1)}/O'} = {}^b\mathbf{r}_{G^{(1)}/O} + \begin{bmatrix} 0 \\ 0 \\ z_n^{(1)} \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2.29 & 0 & \sin 2.29 \\ 0 & 1 & 0 \\ -\sin 2.29 & 0 & \cos 2.29 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 9.59 & -\sin 9.59 \\ 0 & \sin 9.59 & \cos 9.59 \end{bmatrix} \left(\begin{bmatrix} 0 \\ 0 \\ 6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -0.6563 \end{bmatrix} \right)$$

$$= \boxed{\begin{bmatrix} 0.1708 \\ -0.7240 \\ 4.2795 \end{bmatrix}}$$

$L = 100\ [m]$	$l = 10\ [m]$	${}^n\mathbf{r}_{G^{(0)}/E} = [0 \quad 0 \quad 6]^T\ [m]$
$B_{mld} = 40\ [m]$	$b = 20\ [m]$	${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [45 \quad -10 \quad -4.5]^T\ [m]$
$D = 30\ [m]$	$\mu_F = 1.0$	${}^n\mathbf{r}_{B^{(1)}/E} = [3.18 \quad -1.45 \quad -5.16]^T\ [m]$
$d = 9\ [m]$	$\mu_V = 1.0$	${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2 s^2]$		${}^n\mathbf{r}_{f^{(0)}/E} = [45 \quad -10 \quad 0]^T\ [m]$

$\nabla^{(1)} = 3.9285 \times 10^4\ [m^3]$	$v^{(1)} = 2.6936 \times 10^3\ [m^3]$
$F_{G,z} = -3.6 \times 10^5\ [kN]$	$F_z^{(1)} = 1.1316 \times 10^4\ [kN]$
$F_{G_{ext},z}^{(1)} = -2.6936 \times 10^4\ [kN]$	$M_T^{(0)} = 1.8 \times 10^5\ [kN]$
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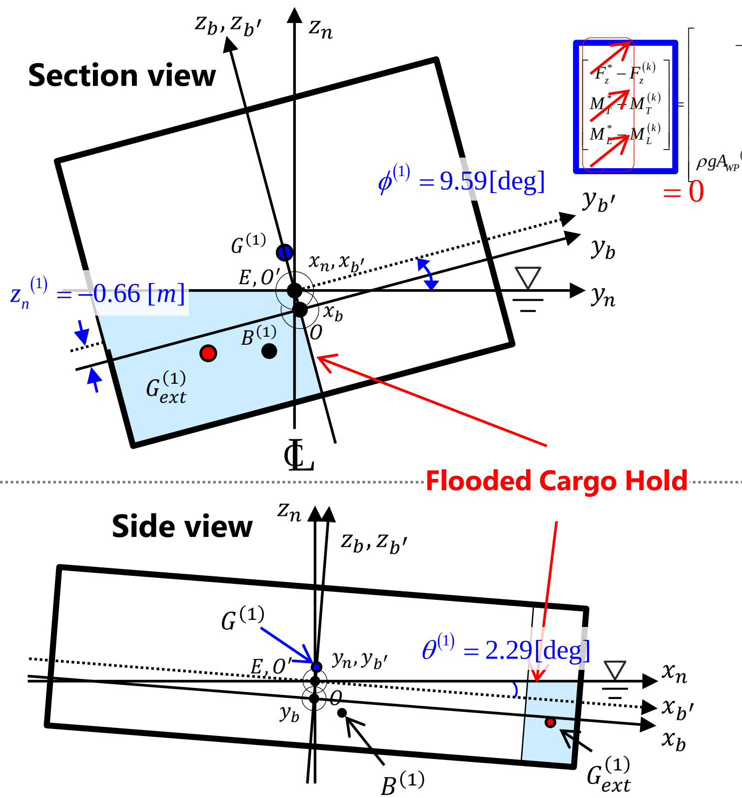

$$\begin{bmatrix} F_z^* - F_z^{(k)} \\ M_T^* - M_T^{(k)} \\ M_L^* - M_L^{(k)} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_T^{(k)}\right) & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_L^{(k)}\right) \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$

$L = 100$ [m]	$l = 10$ [m]	${}^n\mathbf{r}_{G^{(1)}/E} = [0.17 \quad -0.72 \quad 4.28]^T$ [m]
$B_{mld} = 40$ [m]	$b = 20$ [m]	${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [45 \quad -10 \quad -4.5]^T$ [m]
$D = 30$ [m]	$\mu_F = 1.0$	${}^n\mathbf{r}_{B^{(1)}/E} = [3.18 \quad -1.45 \quad -5.16]^T$ [m]
$d = 9$ [m]	$\mu_V = 1.0$	${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T$ [m]
$\rho g = 10$ [Mg/m^2s^2]		${}^n\mathbf{r}_{f^{(0)}/E} = [45 \quad -10 \quad 0]^T$ [m]

$\nabla^{(1)} = 3.9285 \times 10^4$ [m^3]	$v^{(1)} = 2.6936 \times 10^3$ [m^3]
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$F_{B,z}^{(1)} = 3.9285 \times 10^5$ [kN]	$M_L^{(0)} = 8.1 \times 10^5$ [kN]

Moment equilibrium:

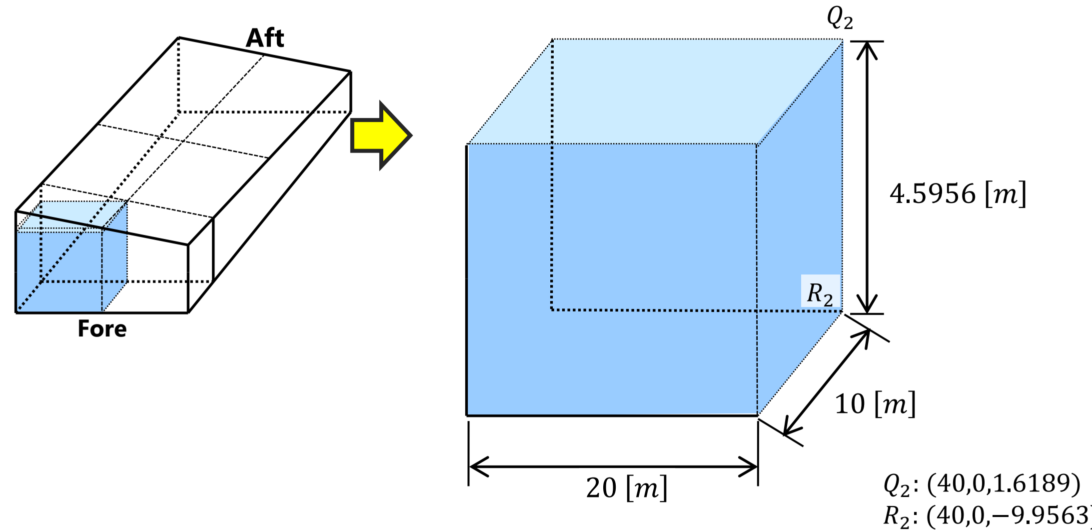
$${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 5.4044]^T \text{ [}m\text{]}$$
$${}^{b'}\mathbf{r}_{Q_1/O'} = [50 \quad 0 \quad 2.0236]^T \text{ [}m\text{]}$$
$${}^{b'}\mathbf{r}_{Q_2/O'} = [40 \quad 0 \quad 1.6189]^T \text{ [}m\text{]}$$
$${}^{b'}\mathbf{r}_{Q_3/O'} = [40 \quad -20 \quad 4.9997]^T \text{ [}m\text{]}$$

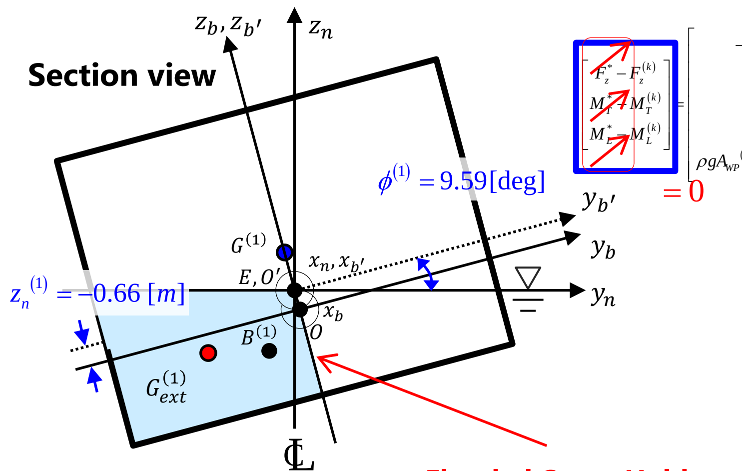
Volume v_1: Box

C_1: located at centerline, midship and $T_0/2$

$${}^{b'}\mathbf{r}_{H_1/O'} = \left({}^{b'}\mathbf{r}_{Q_2/O'} + {}^{b'}\mathbf{r}_{R_2/O'}\right)/2 = [\underbrace{0}_{\text{midship}} \quad \underbrace{0}_{\text{centerline}} \quad (-9.9563 + (1.6189))/2]^T$$

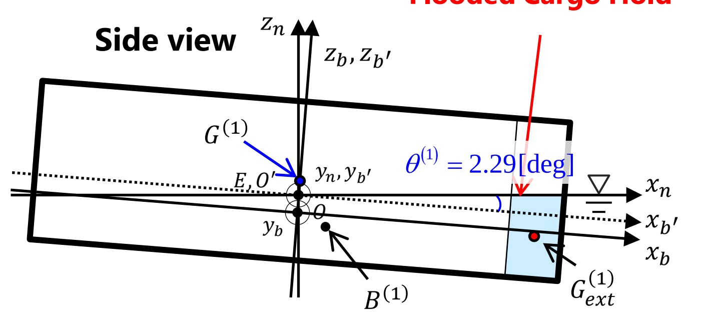
$$= [0 \quad 0 \quad -4.1687]^T$$





$$\begin{bmatrix} F_z^* - F_z^{(k)} \\ M_T^* - M_T^{(k)} \\ M_L^* - M_L^{(k)} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP}{}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}{}^{(k)}\right) & -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^n x_{f^{(k)}/E} \\ -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T{}^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} - \left(-\mu_F \cdot \rho g i_T{}^{(k)}\right) & \rho g I_P{}^{(k)} - \mu_F \cdot \rho g i_P{}^{(k)} \\ \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^n x_{f^{(k)}/E} & \rho g I_P{}^{(k)} - \mu_F \cdot \rho g i_P{}^{(k)} & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L{}^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} - \left(-\mu_F \cdot \rho g i_L{}^{(k)}\right) \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta\phi^{(k)} \\ \delta\theta^{(k)} \end{bmatrix}$$

$= 0$



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$B_{mld} = 40\ [m]$	$b = 20\ [m]$	${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [45 \quad -10 \quad -4.5]^T\ [m]$
$D = 30\ [m]$	$\mu_F = 1.0$	${}^n\mathbf{r}_{B^{(1)}/E} = [3.18 \quad -1.45 \quad -5.16]^T\ [m]$
$d = 9\ [m]$	$\mu_V = 1.0$	${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2 s^2]$		${}^n\mathbf{r}_{f^{(0)}/E} = [45 \quad -10 \quad 0]^T\ [m]$

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Moment equilibrium:

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$${}^{b'}\mathbf{r}_{Q_1/O'} = [50 \quad 0 \quad 2.0236]^T\ [m]$$
$${}^{b'}\mathbf{r}_{Q_2/O'} = [40 \quad 0 \quad 1.6189]^T\ [m]$$
$${}^{b'}\mathbf{r}_{Q_3/O'} = [40 \quad -20 \quad 4.9997]^T\ [m]$$

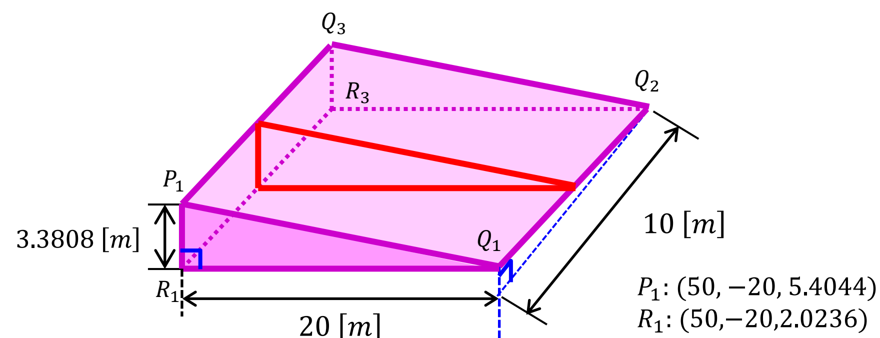
Volume V_2: Trigonal prism

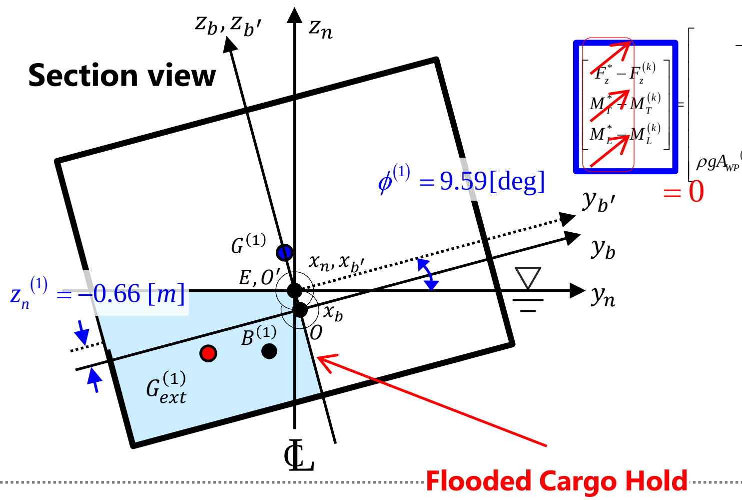
Center of volume:

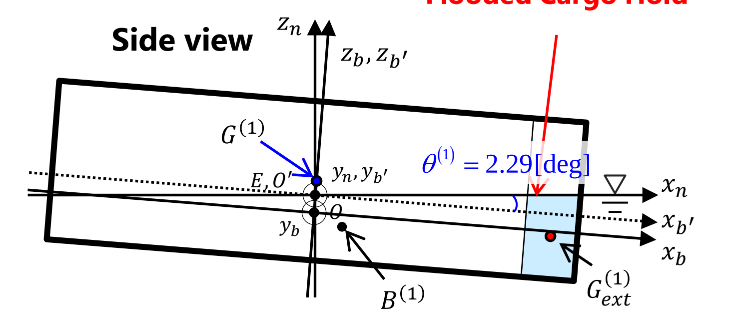
= Center of area of triangle $D_2E_2F_2$

$${}^{b'}\mathbf{r}_{D_2/O'} = ({}^{b'}\mathbf{r}_{P_1/O'} + {}^{b'}\mathbf{r}_{Q_3/O'})/2 = (45,\ -20,\ 3.5116)$$
$${}^{b'}\mathbf{r}_{E_2/O'} = ({}^{b'}\mathbf{r}_{R_1/O'} + {}^{b'}\mathbf{r}_{R_3/O'})/2 = (45,\ -20,\ 1.8213)$$
$${}^{b'}\mathbf{r}_{F_2/O'} = ({}^{b'}\mathbf{r}_{Q_1/O'} + {}^{b'}\mathbf{r}_{Q_2/O'})/2 = (45,\ -10,\ 1.8213)$$

$${}^{b'}\mathbf{r}_{C_2/O'} = ({}^{b'}\mathbf{r}_{D_2/O'} + {}^{b'}\mathbf{r}_{E_2/O'} + {}^{b'}\mathbf{r}_{F_2/O'})/3$$
$$= [45 \quad -13.3333 \quad 2.9482]^T$$







$$\boxed{\begin{bmatrix} F_z^* - F_z^{(k)} \\ M_T^* - M_T^{(k)} \\ M_L^* - M_L^{(k)} \end{bmatrix}} = \begin{bmatrix} -\rho g A_{WP}{}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}{}^{(k)}\right) & -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^n x_{f^{(k)}/E} \\ -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T{}^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} - \left(-\mu_F \cdot \rho g i_T{}^{(k)}\right) & \rho g I_P{}^{(k)} - \mu_F \cdot \rho g i_P{}^{(k)} \\ \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^n x_{f^{(k)}/E} & \rho g I_P{}^{(k)} - \mu_F \cdot \rho g i_P{}^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L{}^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} - \left(-\mu_F \cdot \rho g i_L{}^{(k)}\right) \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$

$L = 100\ [m]$	$l = 10\ [m]$	${}^n\mathbf{r}_{G^{(1)}/E} = [0.17 \quad -0.72 \quad 4.28]^T\ [m]$
$B_{mld} = 40\ [m]$	$b = 20\ [m]$	${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [45 \quad -10 \quad -4.5]^T\ [m]$
$D = 30\ [m]$	$\mu_F = 1.0$	${}^n\mathbf{r}_{B^{(1)}/E} = [3.18 \quad -1.45 \quad -5.16]^T\ [m]$
$d = 9\ [m]$	$\mu_V = 1.0$	${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2s^2]$		${}^n\mathbf{r}_{f^{(0)}/E} = [45 \quad -10 \quad 0]^T\ [m]$

$\nabla^{(1)} = 3.9285 \times 10^4\ [m^3]$	$v^{(1)} = 2.6936 \times 10^3\ [m^3]$
$F_{G,z} = -3.6 \times 10^5\ [kN]$	$F_z^{(1)} = 1.1316 \times 10^4\ [kN]$
$F_{G_{ext},z}^{(1)} = -2.6936 \times 10^4\ [kN]$	$M_T^{(0)} = 1.8 \times 10^5\ [kN]$
$F_{B,z}^{(1)} = 3.9285 \times 10^5\ [kN]$	$M_L^{(0)} = 8.1 \times 10^5\ [kN]$

Moment equilibrium:

Volume V_3: Trigonal prism

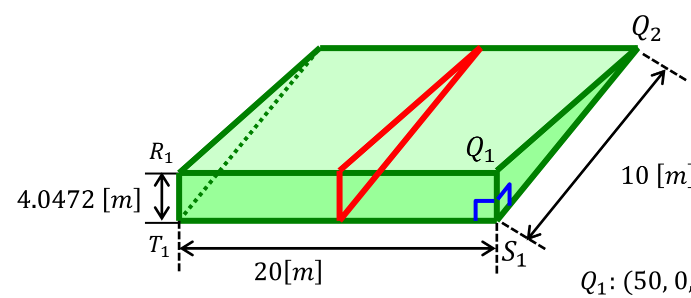
$${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 5.4044]^T\ [m]$$
$${}^{b'}\mathbf{r}_{Q_1/O'} = [50 \quad 0 \quad 2.0236]^T\ [m]$$
$${}^{b'}\mathbf{r}_{Q_2/O'} = [40 \quad 0 \quad 1.6189]^T\ [m]$$
$${}^{b'}\mathbf{r}_{Q_3/O'} = [40 \quad -20 \quad 4.9997]^T\ [m]$$

Center of volume

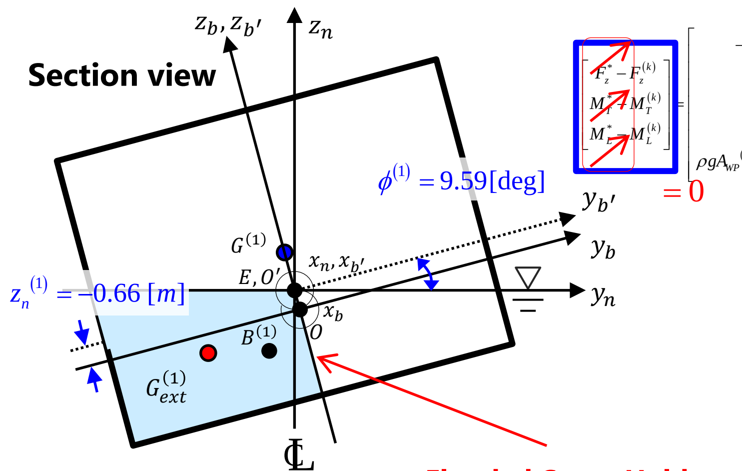
= Center of area of triangle $D_3E_3F_3$

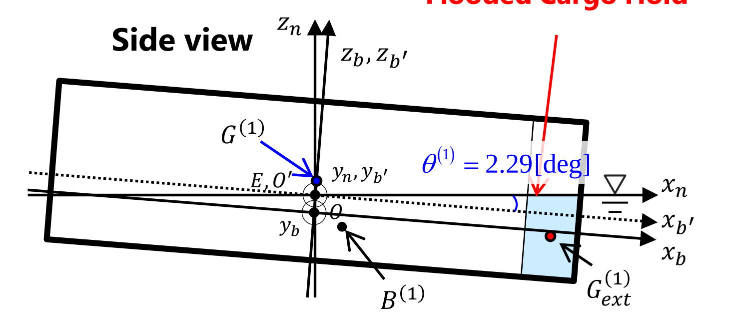
$${}^{b'}\mathbf{r}_{D_3/O'} = ({}^{b'}\mathbf{r}_{R_1/O'} + {}^{b'}\mathbf{r}_{Q_1/O'})/2 = (50,\ 10,\ 2.0236)$$
$${}^{b'}\mathbf{r}_{E_3/O'} = ({}^{b'}\mathbf{r}_{T_1/O'} + {}^{b'}\mathbf{r}_{S_1/O'})/2 = (50,\ 10,\ 1.6189)$$
$${}^{b'}\mathbf{r}_{F_3/O'} = ({}^{b'}\mathbf{r}_{Q_2/O'} + {}^{b'}\mathbf{r}_{Q_3/O'})/2 = (40,\ 10,\ 1.6189)$$

$${}^{b'}\mathbf{r}_{H_3/O'} = ({}^{b'}\mathbf{r}_{D_3/O'} + {}^{b'}\mathbf{r}_{E_3/O'} + {}^{b'}\mathbf{r}_{F_3/O'})/3$$
$$= [46.6667 \quad 0 \quad 1.7538]^T$$



Q_1: (50, 0, 2.0236)
S_1: (50, 0, 1.6189)





$$\begin{bmatrix} F_z^* - F_z^{(k)} \\ M_T^* - M_T^{(k)} \\ M_L^* - M_L^{(k)} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_T^{(k)}\right) & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_L^{(k)}\right) \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$

$L = 100\ [m]$	$l = 10\ [m]$	${}^n\mathbf{r}_{G^{(1)}/E} = [0.17 \quad -0.72 \quad 4.28]^T\ [m]$
$B_{mld} = 40\ [m]$	$b = 20\ [m]$	${}^n\mathbf{r}_{G_{ext}^{(0)}/E} = [45 \quad -10 \quad -4.5]^T\ [m]$
$D = 30\ [m]$	$\mu_F = 1.0$	${}^n\mathbf{r}_{B^{(1)}/E} = [3.18 \quad -1.45 \quad -5.16]^T\ [m]$
$d = 9\ [m]$	$\mu_V = 1.0$	${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2s^2]$		${}^n\mathbf{r}_{f^{(0)}/E} = [45 \quad -10 \quad 0]^T\ [m]$

$\nabla^{(1)} = 3.9285 \times 10^4\ [m^3]$	$v^{(1)} = 2.6936 \times 10^3\ [m^3]$
$F_{G,z} = -3.6 \times 10^5\ [kN]$	$F_z^{(1)} = 1.1316 \times 10^4\ [kN]$
$F_{G_{ext},z}^{(1)} = -2.6936 \times 10^4\ [kN]$	$M_T^{(0)} = 1.8 \times 10^5\ [kN]$
$F_{B,z}^{(1)} = 3.9285 \times 10^5\ [kN]$	$M_L^{(0)} = 8.1 \times 10^5\ [kN]$

Moment equilibrium:

Center of Gravity (Cargo):

$$ {}^{b'}\mathbf{r}_{G_{ext}^{(1)}/O'} = \frac{{}^{b'}\mathbf{r}_{H_1/O'} \cdot v_1 + {}^{b'}\mathbf{r}_{H_2/O'} \cdot v_2 + {}^{b'}\mathbf{r}_{H_3/O'} \cdot v_3}{v_1 + v_2 + v_3}$$

$$= [45.0256 \quad -10.4279 \quad -3.0356]^T\ [m]$$

In this case, for convenience of calculating the center of displaced volume ${}^{b'}\mathbf{r}_{G_{ext}^{(1)}/O'}$ of the ship, we use b′-frame. The origin O′ of b′-frame coincides with the origin E of n-frame. And the orientation of b′-frame is the same as that of b-frame. So, to obtain the center of buoyancy with respect to n-frame, ${}^{n}\mathbf{r}_{G_{ext}^{(1)}/E}$, we have to perform the rotational transformation.

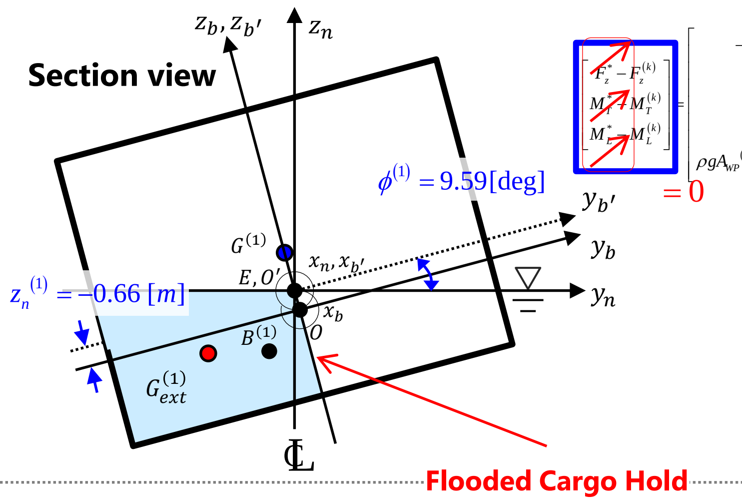
$${}^{n}\mathbf{r}_{G_{ext}^{(1)}/E} = \begin{bmatrix} \cos\theta^{(1)} & 0 & \sin\theta^{(1)} \\ 0 & 1 & 0 \\ -\sin\theta^{(1)} & 0 & \cos\theta^{(1)} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi^{(1)} & -\sin\phi^{(1)} \\ 0 & \sin\phi^{(1)} & \cos\phi^{(1)} \end{bmatrix} {}^{b'}\mathbf{r}_{G_{ext}^{(1)}/O'}$$

$$= \begin{bmatrix} \cos 2.29 & 0 & \sin 2.29 \\ 0 & 1 & 0 \\ -\sin 2.29 & 0 & \cos 2.29 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 9.59 & -\sin 9.59 \\ 0 & \sin 9.59 & \cos 9.59 \end{bmatrix} \begin{bmatrix} 45.0256 \\ -10.4279 \\ -3.0356 \end{bmatrix} = \boxed{\begin{bmatrix} 44.8011 \\ -9.7761 \\ -6.5228 \end{bmatrix}}$$

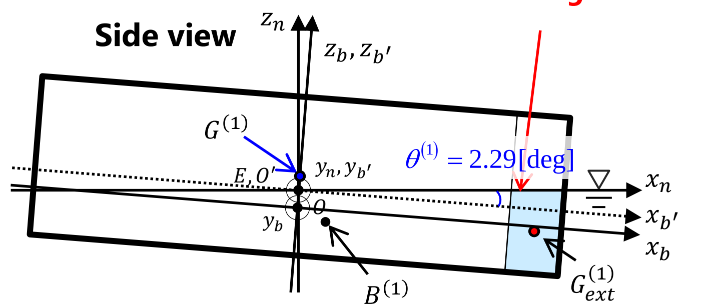
$$\boxed{\begin{bmatrix} F_z^* - F_z^{(k)} \\ M_T^* - M_T^{(k)} \\ M_L^* - M_L^{(k)} \end{bmatrix}} = \begin{bmatrix} -\rho g A_{WP}{}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}{}^{(k)}\right) & -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^n x_{f^{(k)}/E} \\ -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T{}^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} - \left(-\mu_F \cdot \rho g i_T{}^{(k)}\right) & \rho g I_P{}^{(k)} - \mu_F \cdot \rho g i_P{}^{(k)} \\ \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^n x_{f^{(k)}/E} & \rho g I_P{}^{(k)} - \mu_F \cdot \rho g i_P{}^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L{}^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} - \left(-\mu_F \cdot \rho g i_L{}^{(k)}\right) \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$

Section view



Side view



$L = 100\ [m]$	$l = 10\ [m]$	${}^n\mathbf{r}_{G^{(1)}/E} = [0.17\quad -0.72\quad 4.28]^T\ [m]$
$B_{mld} = 40\ [m]$	$b = 20\ [m]$	${}^n\mathbf{r}_{G_{ext}^{(1)}/E} = [44.80\quad -9.78\quad -6.52]^T\ [m]$
$D = 30\ [m]$	$\mu_F = 1.0$	${}^n\mathbf{r}_{B^{(1)}/E} = [3.18\quad -1.45\quad -5.16]^T\ [m]$
$d = 9\ [m]$	$\mu_V = 1.0$	${}^n\mathbf{r}_{F^{(0)}/E} = [0\quad 0\quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2 s^2]$		${}^n\mathbf{r}_{f^{(0)}/E} = [45\quad -10\quad 0]^T\ [m]$

$\nabla^{(1)} = 3.9285 \times 10^4\ [m^3]$	$v^{(1)} = 2.6936 \times 10^3\ [m^3]$
$F_{G,z} = -3.6 \times 10^5\ [kN]$	$F_z^{(1)} = 1.1316 \times 10^4\ [kN]$
$F_{G_{ext},z}^{(1)} = -2.6936 \times 10^4\ [kN]$	$M_T^{(0)} = 1.8 \times 10^5\ [kN]$
$F_{B,z}^{(1)} = 3.9285 \times 10^5\ [kN]$	$M_L^{(0)} = 8.1 \times 10^5\ [kN]$

Moment equilibrium:

$$M_T^{(1)} = M_{BT}^{(1)} + M_{GT}^{(1)} + M_{extT}^{(1)}$$

$$= {}^n y_{B^{(1)}/E} \cdot F_{B,z}^{(1)} + {}^n y_{G^{(1)}/E} \cdot F_{G,z} + {}^n y_{G_{ext}^{(1)}/E} \cdot F_{ext,z}$$

$$= -1.45 \cdot (3.93 \times 10^5) + (-0.72) \cdot (-3.6 \times 10^5) + (-9.78) \cdot (-2.69 \times 10^4)$$

$$= -1.7252 \times 10^4\ [kN \cdot m]$$

Tolerance

$\rightarrow |-1.7252 \times 10^4| > e$

where, e(epsilon) : an arbitrarily small positive quantity

$$M_L^{(1)} = M_{BL}^{(1)} + M_{GL}^{(1)} + M_{extL}^{(1)}$$

$$= \left(-{}^n x_{B^{(1)}/E} \cdot F_{B,z}^{(1)}\right) + \left(-{}^n x_{G^{(1)}/E} \cdot F_{G,z}\right) + \left(-{}^n x_{G_{ext}^{(1)}/E} \cdot F_{ext,z}\right)$$

$$= [3.18 \cdot (3.93 \times 10^5)] + [-0.17 \cdot (-3.6 \times 10^5)] + [-44.80 \cdot (-2.69 \times 10^4)]$$

$$= -1.7258 \times 10^4\ [kN \cdot m]$$

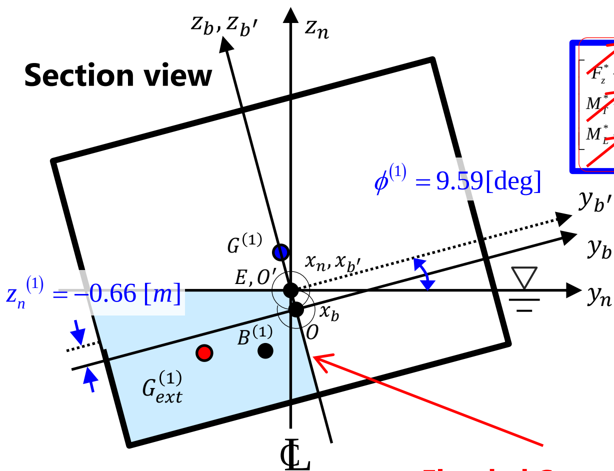
Tolerance

$\rightarrow |-1.7258 \times 10^4| > e$

where, e(epsilon) : an arbitrarily small positive quantity

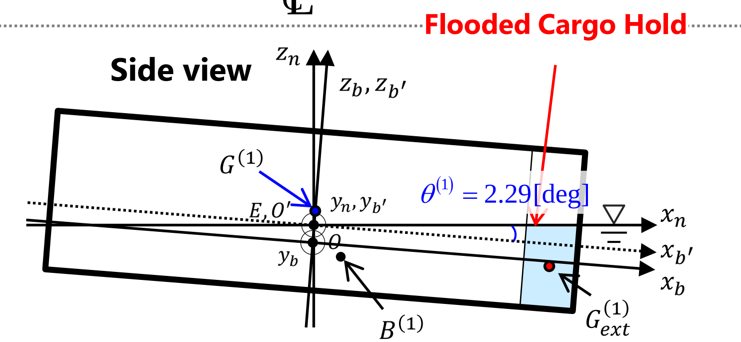
The static equilibrium of moment is not satisfied!
We have to iterate!

1. Calculation of Forces and Moments at k=1 step



$$
\begin{bmatrix} \cancel{F_z^* - F_z^{(k)}} \\ \cancel{M_T^* - M_T^{(k)}} \\ \cancel{M_L^* - M_L^{(k)}} \end{bmatrix} = \left[\begin{array}{ccc} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_T^{(k)}\right) & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_L^{(k)}\right) \end{array}\right]_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}
$$

$= 0$



$L = 100\ [m]$	$l = 10\ [m]$	${}^n\mathbf{r}_{G^{(1)}/E} = [0.17 \quad -0.72 \quad 4.28]^T\ [m]$
$B_{mld} = 40\ [m]$	$b = 20\ [m]$	${}^n\mathbf{r}_{G_{ext}^{(1)}/E} = [44.80 \quad -9.78 \quad -6.52]^T\ [m]$
$D = 30\ [m]$	$\mu_F = 1.0$	${}^n\mathbf{r}_{B^{(1)}/E} = [3.18 \quad -1.45 \quad -5.16]^T\ [m]$
$d = 9\ [m]$	$\mu_V = 1.0$	${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2 s^2]$		${}^n\mathbf{r}_{f^{(0)}/E} = [45 \quad -10 \quad 0]^T\ [m]$

$\nabla^{(1)} = 3.9285 \times 10^4\ [m^3]$	$v^{(1)} = 2.6936 \times 10^3\ [m^3]$
$F_{G,z} = -3.6 \times 10^5\ [kN]$	$F_z^{(1)} = 1.1316 \times 10^4\ [kN]$
$F_{G_{ext},z}^{(1)} = -2.6936 \times 10^4\ [kN]$	$M_T^{(1)} = -1.73 \times 10^4\ [kN]$
$F_{B,z}^{(1)} = 3.9285 \times 10^5\ [kN]$	$M_L^{(1)} = -1.73 \times 10^4\ [kN]$

Force:

$$F_z^{(1)} = F_{B,z}^{(1)} + F_{G,z}^{(1)} + F_{ext,z}^{(1)}$$

$$= 1.13 \times 10^4\ [kN]$$

Moment:

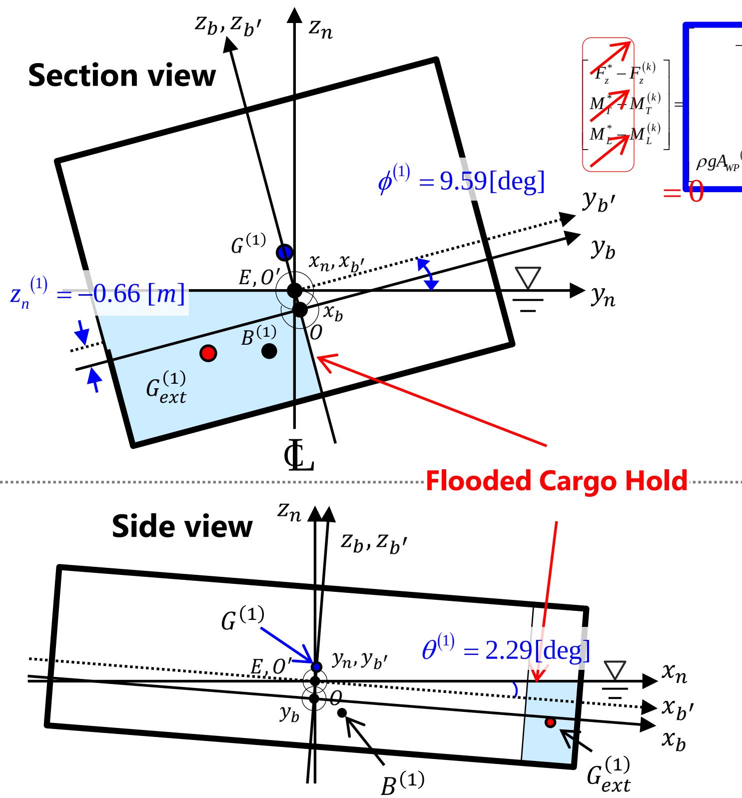
$$M_T^{(1)} = M_{BT}^{(1)} + M_{GT}^{(1)} + M_{extT}^{(1)}$$

$$= -1.73 \times 10^4\ [kN \cdot m]$$

$$M_L^{(1)} = M_{BL}^{(1)} + M_{GL}^{(1)} + M_{extL}^{(1)}$$

$$= -1.73 \times 10^4\ [kN \cdot m]$$

2. Calculation of the Values of the Waterplane at k=1 step



$$\begin{bmatrix} \cancel{F_z^* - F_z^{(k)}} \\ \cancel{M_T^* - M_T^{(k)}} \\ \cancel{M_L^* - M_L^{(k)}} \end{bmatrix} = \boxed{\begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_T^{(k)}\right) & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_L^{(k)}\right) \end{bmatrix}}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$

$L = 100\ [m]$	$l = 10\ [m]$	${}^n\mathbf{r}_{G^{(1)}/E} = [0.17 \quad -0.72 \quad 4.28]^T\ [m]$
$B_{mld} = 40\ [m]$	$b = 20\ [m]$	${}^n\mathbf{r}_{G_{ext}^{(1)}/E} = [44.80 \quad -9.78 \quad -6.52]^T\ [m]$
$D = 30\ [m]$	$\mu_F = 1.0$	${}^n\mathbf{r}_{B^{(1)}/E} = [3.18 \quad -1.45 \quad -5.16]^T\ [m]$
$d = 9\ [m]$	$\mu_V = 1.0$	${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2s^2]$		${}^n\mathbf{r}_{f^{(0)}/E} = [45 \quad -10 \quad 0]^T\ [m]$

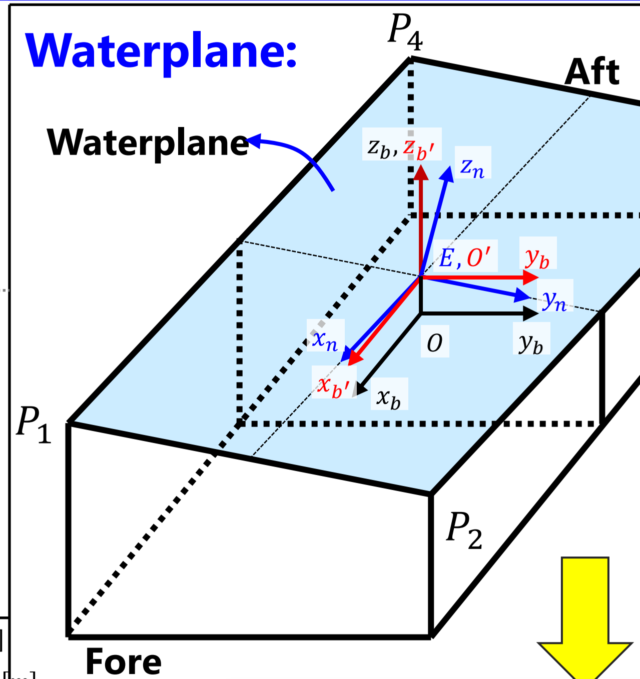
$\nabla^{(1)} = 3.9285 \times 10^4\ [m^3]$ $\quad v^{(1)} = 2.6936 \times 10^3\ [m^3]$

$F_{G,z} = -3.6 \times 10^5\ [kN]$ $\quad F_z^{(1)} = 1.1316 \times 10^4\ [kN]$

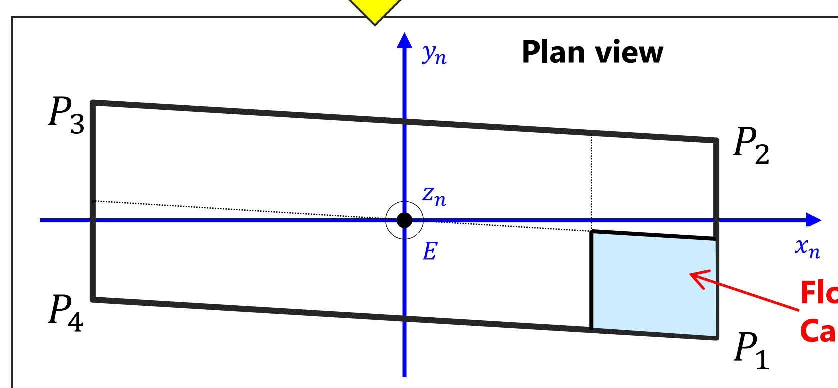
$F_{G_{ext},z}^{(1)} = -2.6936 \times 10^4\ [kN]$ $\quad M_T^{(1)} = -1.73 \times 10^4\ [kN]$

$F_{B,z}^{(1)} = 3.9285 \times 10^5\ [kN]$ $\quad M_L^{(1)} = -1.73 \times 10^4\ [kN]$

Waterplane:

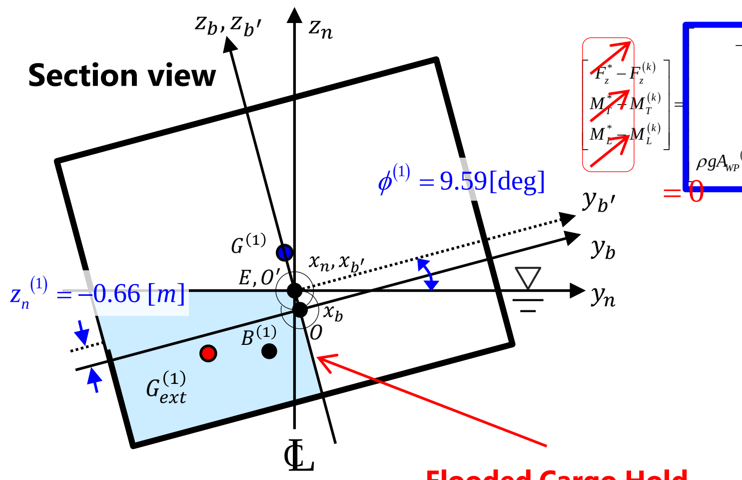


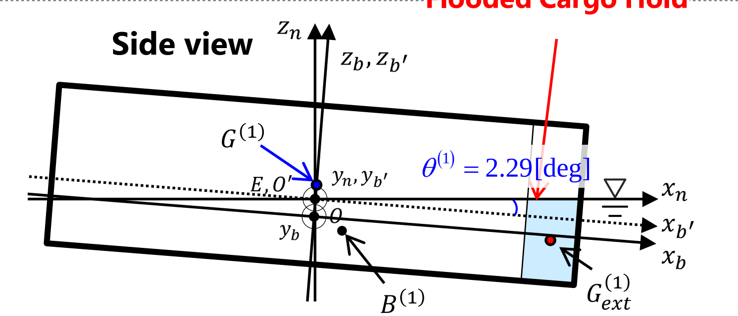
$${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 5.4044]^T\ [m]$$
$${}^{b'}\mathbf{r}_{P_2/O'} = [50 \quad 20 \quad -1.3571]^T\ [m]$$
$${}^{b'}\mathbf{r}_{P_3/O'} = [-50 \quad 20 \quad -5.4044]^T\ [m]$$
$${}^{b'}\mathbf{r}_{P_4/O'} = [-50 \quad -20 \quad 1.3571]^T\ [m]$$



$$\begin{bmatrix} \cancel{F_z^*} - F_z^{(k)} \\ \cancel{M_T^*} - M_T^{(k)} \\ \cancel{M_L^*} - M_L^{(k)} \end{bmatrix} = \boxed{\begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) & \rho g A_{WP}^{(k)} \cdot {}^{n}x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^{n}x_{f^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^{n}y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^{n}y_{f^{(k)}/E}\right) & -\rho g\left({}^{n}z_{B^{(k)}/E}\nabla^{(k)} + I_T^{(k)}\right) - {}^{n}z_{G^{(k)}/E} \cdot F_{G,z} - {}^{n}z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_T^{(k)}\right) & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^{n}x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^{n}x_{f^{(k)}/E} & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} & -\rho g\left({}^{n}z_{B^{(k)}/E}\nabla^{(k)} + I_L^{(k)}\right) - {}^{n}z_{G^{(k)}/E} \cdot F_{G,z} - {}^{n}z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_L^{(k)}\right) \end{bmatrix}}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$





$L = 100$ [m]	$l = 10$ [m]	${}^{n}\mathbf{r}_{G^{(1)}/E} = [0.17 \quad -0.72 \quad 4.28]^T$ [m]
$B_{mld} = 40$ [m]	$b = 20$ [m]	${}^{n}\mathbf{r}_{G_{ext}^{(1)}/E} = [44.80 \quad -9.78 \quad -6.52]^T$ [m]
$D = 30$ [m]	$\mu_F = 1.0$	${}^{n}\mathbf{r}_{B^{(1)}/E} = [3.18 \quad -1.45 \quad -5.16]^T$ [m]
$d = 9$ [m]	$\mu_V = 1.0$	${}^{n}\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T$ [m]
$\rho g = 10$ [Mg/m^2s^2]		${}^{n}\mathbf{r}_{f^{(0)}/E} = [45 \quad -10 \quad 0]^T$ [m]

$\nabla^{(1)} = 3.9285 \times 10^4$ [m^3]	$v^{(1)} = 2.6936 \times 10^3$ [m^3]
$F_{G,z} = -3.6 \times 10^5$ [kN]	$F_z^{(1)} = 1.1316 \times 10^4$ [kN]
$F_{G_{ext},z}^{(1)} = -2.6936 \times 10^4$ [kN]	$M_T^{(1)} = -1.73 \times 10^4$ [kN]
$F_{B,z}^{(1)} = 3.9285 \times 10^5$ [kN]	$M_L^{(1)} = -1.73 \times 10^4$ [kN]

Waterplane: length of each side

$${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 5.4044]^T \ [m]$$

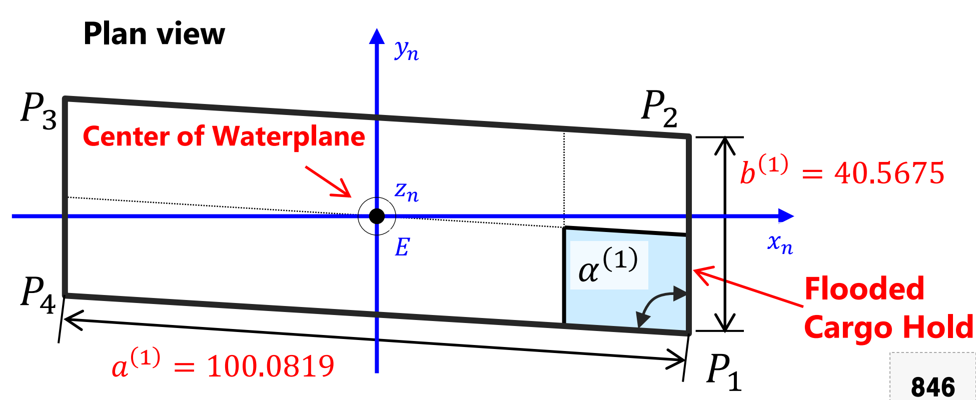
$${}^{b'}\mathbf{r}_{P_2/O'} = [50 \quad 20 \quad -1.3571]^T \ [m]$$

$${}^{b'}\mathbf{r}_{P_3/O'} = [-50 \quad 20 \quad -5.4044]^T \ [m]$$

$${}^{b'}\mathbf{r}_{P_4/O'} = [-50 \quad -20 \quad 1.3571]^T \ [m]$$

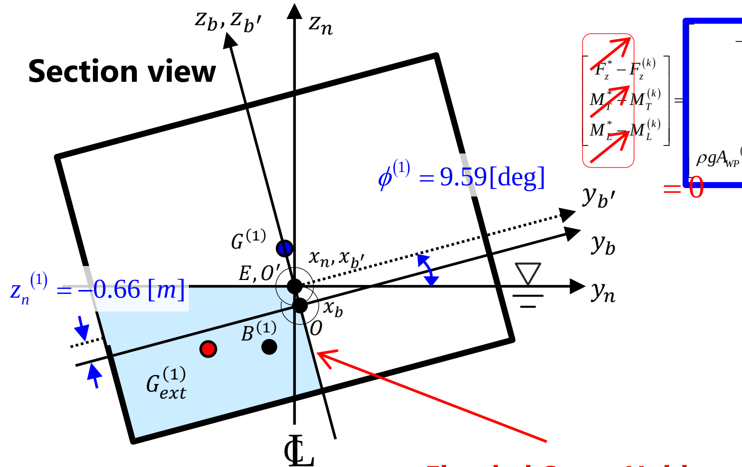
$$\overrightarrow{P_1P_2} = [50 \quad 20 \quad -1.3571]^T - [50 \quad -20 \quad 5.4044]^T$$
$$= [0 \quad 40 \quad -6.7615]^T$$
$$\left|\overrightarrow{P_1P_2}\right| = 40.5675 \ \left(= a^{(1)}\right)$$

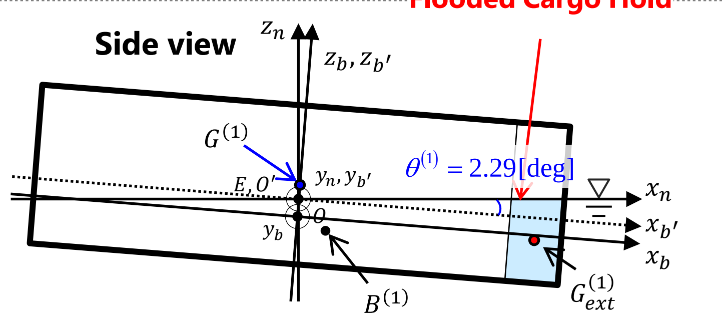
$$\overrightarrow{P_1P_4} = [-50 \quad -20 \quad 1.3571]^T - [50 \quad -20 \quad 5.4044]^T$$
$$= [-100 \quad 0 \quad -4.0472]^T$$
$$\left|\overrightarrow{P_1P_4}\right| = 100.0819 \ \left(= b^{(1)}\right)$$



$$\begin{bmatrix} \cancel{F_z^* - F_z^{(k)}} \\ \cancel{M_T^* - M_T^{(k)}} \\ \cancel{M_L^* - M_L^{(k)}} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_T^{(k)}\right) & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_L^{(k)}\right) \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$





$L = 100\,[m]$	$l = 10\,[m]$	${}^n\mathbf{r}_{G^{(1)}/E} = [0.17 \quad -0.72 \quad 4.28]^T\,[m]$
$B_{mld} = 40\,[m]$	$b = 20\,[m]$	${}^n\mathbf{r}_{G_{ext}^{(1)}/E} = [44.80 \quad -9.78 \quad -6.52]^T\,[m]$
$D = 30\,[m]$	$\mu_F = 1.0$	${}^n\mathbf{r}_{B^{(1)}/E} = [3.18 \quad -1.45 \quad -5.16]^T\,[m]$
$d = 9\,[m]$	$\mu_V = 1.0$	${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\,[m]$
$\rho g = 10\,[Mg/m^2 s^2]$		${}^n\mathbf{r}_{f^{(0)}/E} = [45 \quad -10 \quad 0]^T\,[m]$

$\nabla^{(1)} = 3.9285 \times 10^4\,[m^3]$	$v^{(1)} = 2.6936 \times 10^3\,[m^3]$
$F_{G,z} = -3.6 \times 10^5\,[kN]$	$F_z^{(1)} = 1.1316 \times 10^4\,[kN]$
$F_{G_{ext},z}^{(1)} = -2.6936 \times 10^4\,[kN]$	$M_T^{(1)} = -1.73 \times 10^4\,[kN]$
$F_{B,z}^{(1)} = 3.9285 \times 10^5\,[kN]$	$M_L^{(1)} = -1.73 \times 10^4\,[kN]$

Waterplane: angle $\alpha^{(1)}$

$$\alpha^{(1)} = \cos^{-1} \frac{\overrightarrow{P_1P_2} \cdot \overrightarrow{P_1P_4}}{\left|\overrightarrow{P_1P_2}\right| \cdot \left|\overrightarrow{P_1P_4}\right|}$$

$$= \cos^{-1}\left[\frac{(0 \cdot (-100)) + 40 \cdot 0 + (-6.7615) \cdot (-4.0472)}{40.5675 \cdot 100.0819}\right]$$

$$= 89.6138\,[deg]$$

${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 5.4044]^T\,[m]$

${}^{b'}\mathbf{r}_{P_2/O'} = [50 \quad 20 \quad -1.3571]^T\,[m]$

${}^{b'}\mathbf{r}_{P_3/O'} = [-50 \quad 20 \quad -5.4044]^T\,[m]$

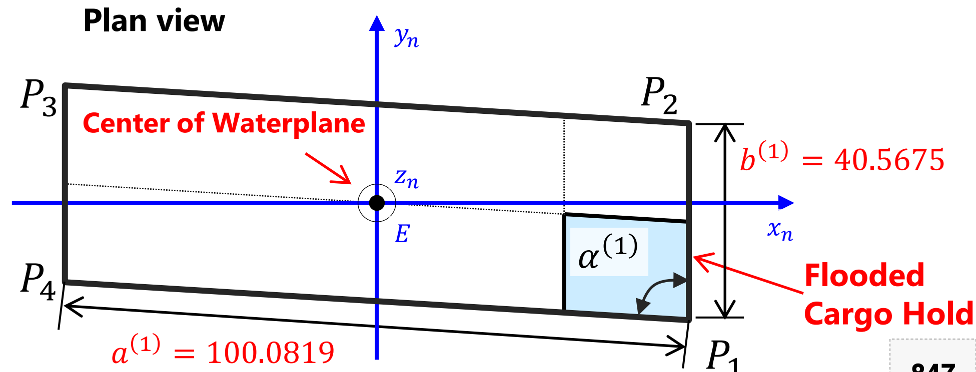
${}^{b'}\mathbf{r}_{P_4/O'} = [-50 \quad -20 \quad 1.3571]^T\,[m]$

$\overrightarrow{P_1P_2} = [0 \quad 40 \quad -6.7615]^T$

$\left|\overrightarrow{P_1P_2}\right| = 40.5675 \; (= a^{(1)})$

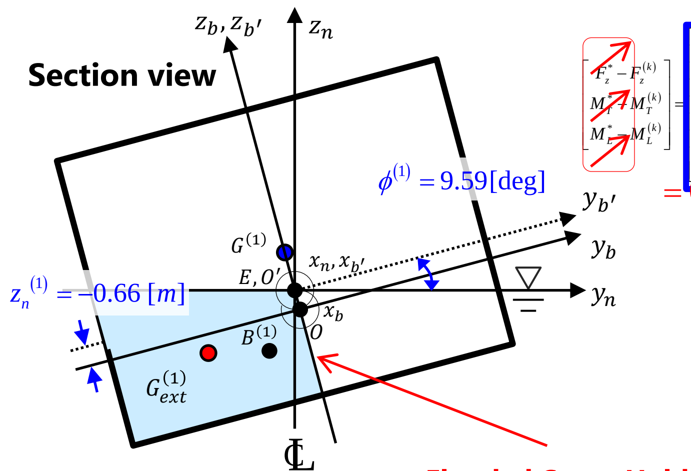
$\overrightarrow{P_1P_4} = [-100 \quad 0 \quad -4.0472]^T$

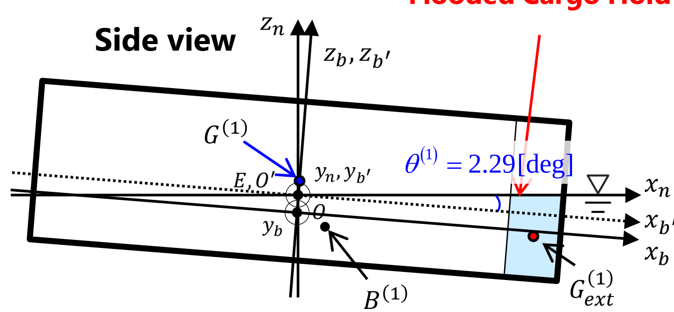
$\left|\overrightarrow{P_1P_4}\right| = 100.0819 \; (= b^{(1)})$



$$\begin{bmatrix} \cancel{F_z^* - F_z^{(k)}} \\ \cancel{M_T^* - M_T^{(k)}} \\ \cancel{M_L^* - M_L^{(k)}} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_T^{(k)}\right) & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_L^{(k)}\right) \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$





$L = 100\ [m]$	$l = 10\ [m]$	${}^n\mathbf{r}_{G^{(1)}/E} = [0.17 \quad -0.72 \quad 4.28]^T\ [m]$
$B_{mld} = 40\ [m]$	$b = 20\ [m]$	${}^n\mathbf{r}_{G_{ext}^{(1)}/E} = [44.80 \quad -9.78 \quad -6.52]^T\ [m]$
$D = 30\ [m]$	$\mu_F = 1.0$	${}^n\mathbf{r}_{B^{(1)}/E} = [3.18 \quad -1.45 \quad -5.16]^T\ [m]$
$d = 9\ [m]$	$\mu_V = 1.0$	${}^n\mathbf{r}_{F^{(0)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2s^2]$		${}^n\mathbf{r}_{f^{(0)}/E} = [45 \quad -10 \quad 0]^T\ [m]$

$\nabla^{(1)} = 3.9285 \times 10^4\ [m^3]$	$v^{(1)} = 2.6936 \times 10^3\ [m^3]$
$F_{G,z} = -3.6 \times 10^5\ [kN]$	$F_z^{(1)} = 1.1316 \times 10^4\ [kN]$
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$F_{B,z}^{(1)} = 3.9285 \times 10^5\ [kN]$	$M_L^{(1)} = -1.73 \times 10^4\ [kN]$

Waterplane Area: A_{WP}

$$A_{WP}^{(1)} = a^{(1)} \cdot b^{(1)} \cdot \sin \alpha^{(1)}$$

$$= 100.0819 \cdot 40.5675 \cdot \sin 89.6138$$

$$= 4.0600 \times 10^3\ [m^3]$$

$${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 5.4044]^T\ [m]$$

$${}^{b'}\mathbf{r}_{P_2/O'} = [50 \quad 20 \quad -1.3571]^T\ [m]$$

$${}^{b'}\mathbf{r}_{P_3/O'} = [-50 \quad 20 \quad -5.4044]^T\ [m]$$

$${}^{b'}\mathbf{r}_{P_4/O'} = [-50 \quad -20 \quad 1.3571]^T\ [m]$$

$$\overrightarrow{P_1P_2} = [0 \quad 40 \quad -6.7615]^T$$

$$\left|\overrightarrow{P_1P_2}\right| = 40.5675 \ (= a^{(1)})$$

$$\overrightarrow{P_1P_4} = [-100 \quad 0 \quad -4.0472]^T$$

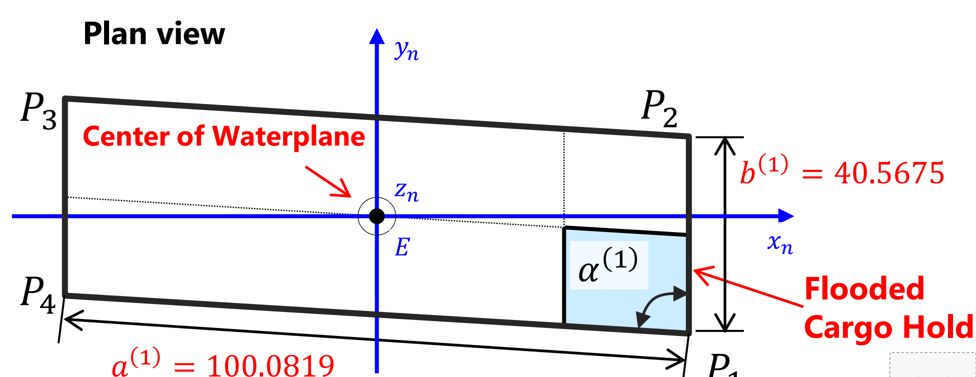
$$\left|\overrightarrow{P_1P_4}\right| = 100.0819 \ (= b^{(1)})$$

$$\alpha^{(1)} = 89.6138\ [deg]$$

Center of waterplane:

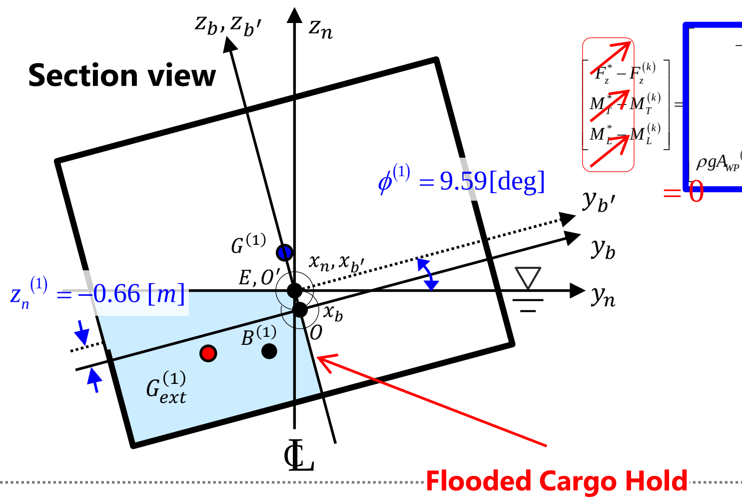
(equal to center of area)

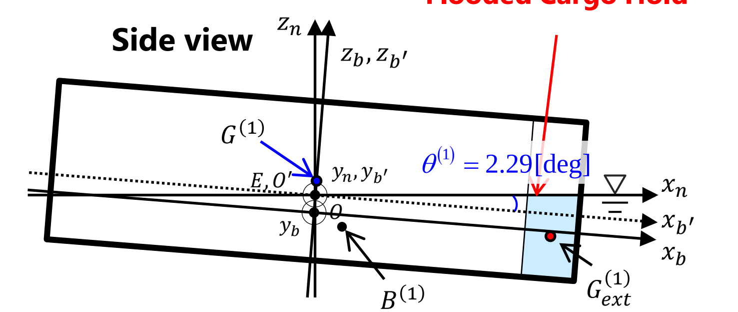
$${}^n\mathbf{r}_{F^{(1)}/E} = (P_1 + P_3)/2 = [0 \quad 0 \quad 0]^T\ [m]$$



$$
\cancel{\begin{bmatrix} F_z^* - F_z^{(k)} \\ M_T^* - M_T^{(k)} \\ M_L^* - M_L^{(k)} \end{bmatrix}} = \boxed{\begin{bmatrix}
-\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} \\
-\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_T^{(k)}\right) & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} \\
\rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_L^{(k)}\right)
\end{bmatrix}}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta\phi^{(k)} \\ \delta\theta^{(k)} \end{bmatrix}
$$

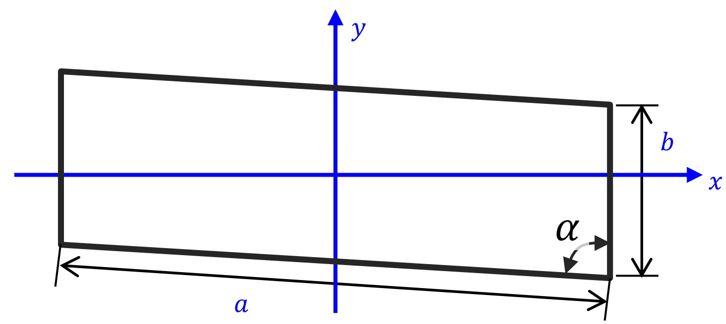
$= 0$





Second moment of area:

Second moment of area of parallelogram:



$$I_{xx} = \frac{1}{12} ab \sin\theta \left(b^2 + a^2 \cos^2 \alpha\right)$$

$$I_{yy} = \frac{1}{12} a^3 b \sin^3 \alpha$$

$$I_{zz} = 0$$

${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 5.4044]^T\ [m]$

${}^{b'}\mathbf{r}_{P_2/O'} = [50 \quad 20 \quad -1.3571]^T\ [m]$

${}^{b'}\mathbf{r}_{P_3/O'} = [-50 \quad 20 \quad -5.4044]^T\ [m]$

${}^{b'}\mathbf{r}_{P_4/O'} = [-50 \quad -20 \quad 1.3571]^T\ [m]$

$\overrightarrow{P_1P_2} = [0 \quad 40 \quad -6.7615]^T$

$\left|\overrightarrow{P_1P_2}\right| = 40.5675\ (= a^{(1)})$

$\overrightarrow{P_1P_4} = [-100 \quad 0 \quad -4.0472]^T$

$\left|\overrightarrow{P_1P_4}\right| = 100.0819\ (= b^{(1)})$

$\alpha^{(1)} = 89.6138\ [deg]$

$L = 100\ [m]$ $B_{mld} = 40\ [m]$ $D = 30\ [m]$ $d = 9\ [m]$	$l = 10\ [m]$ $b = 20\ [m]$ $\mu_F = 1.0$ $\mu_V = 1.0$	${}^n\mathbf{r}_{G^{(1)}/E} = [0.17 \quad -0.72 \quad 4.28]^T\ [m]$ ${}^n\mathbf{r}_{G_{ext}^{(1)}/E} = [44.80 \quad -9.78 \quad -6.52]^T\ [m]$ ${}^n\mathbf{r}_{B^{(1)}/E} = [3.18 \quad -1.45 \quad -5.16]^T\ [m]$ ${}^n\mathbf{r}_{F^{(1)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2s^2]$		${}^n\mathbf{r}_{f^{(0)}/E} = [45 \quad -10 \quad 0]^T\ [m]$

$\nabla^{(1)} = 3.9285 \times 10^4\ [m^3]$ $\quad v^{(1)} = 2.6936 \times 10^3\ [m^3]$

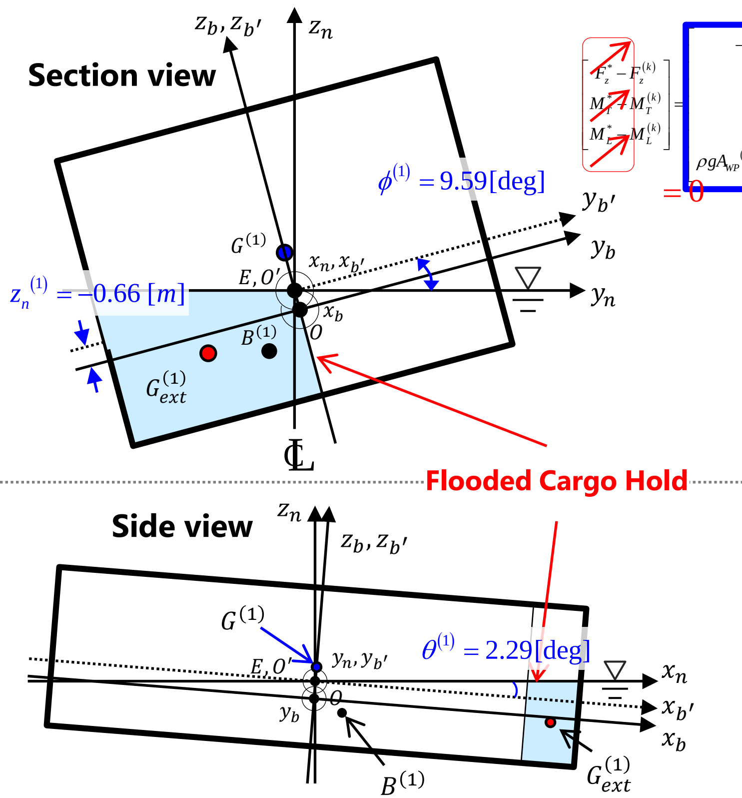
$F_{G,z} = -3.6 \times 10^5\ [kN]$ $\quad F_z^{(1)} = 1.1316 \times 10^4\ [kN]$

$F_{G_{ext},z}^{(1)} = -2.6936 \times 10^4\ [kN]$ $\quad M_T^{(1)} = -1.73 \times 10^4\ [kN]$

$F_{B,z}^{(1)} = 3.9285 \times 10^5\ [kN]$ $\quad M_L^{(1)} = -1.73 \times 10^4\ [kN]$

$$\begin{bmatrix} F_z^* - F_z^{(k)} \\ M_T^* - M_T^{(k)} \\ M_L^* - M_L^{(k)} \end{bmatrix} = \boxed{\begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_T^{(k)}\right) & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_L^{(k)}\right) \end{bmatrix}}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$



Waterplane: second moment of area

$$I_T = I_{x_n x_n}$$

$$= \frac{1}{12} a^{(1)} b^{(1)} \sin \alpha^{(1)} \left[b^{(1)^2} + a^{(1)^2} \cos^2 \alpha^{(1)} \right]$$

$$= \frac{1}{12} \cdot 100.0819 \cdot 40.5675 \cdot \sin 89.6138$$

$$\times [40.5675^2 + 100.0819^2 \cos^2 89.6138]$$

$$= 5.5695 \times 10^5\ [m^4]$$

$${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 5.4044]^T\ [m]$$

$${}^{b'}\mathbf{r}_{P_2/O'} = [50 \quad 20 \quad -1.3571]^T\ [m]$$

$${}^{b'}\mathbf{r}_{P_3/O'} = [-50 \quad 20 \quad -5.4044]^T\ [m]$$

$${}^{b'}\mathbf{r}_{P_4/O'} = [-50 \quad -20 \quad 1.3571]^T\ [m]$$

$$\overrightarrow{P_1P_2} = [0 \quad 40 \quad -6.7615]^T$$

$$|\overrightarrow{P_1P_2}| = 40.5675 \ (= a^{(1)})$$

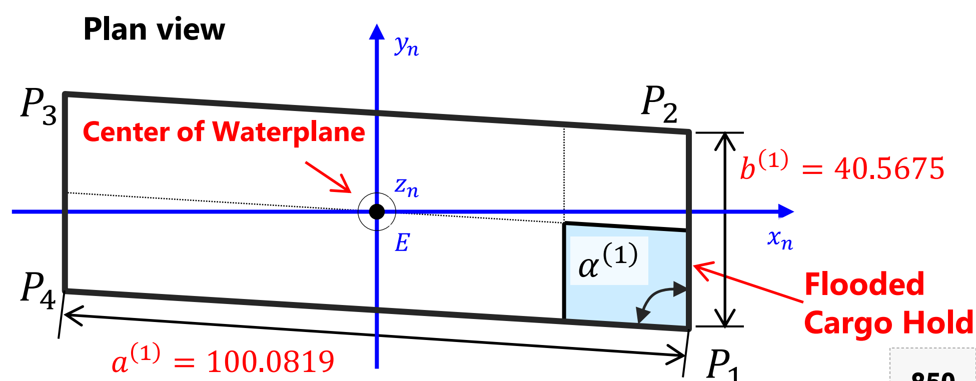
$$\overrightarrow{P_1P_4} = [-100 \quad 0 \quad -4.0472]^T$$

$$|\overrightarrow{P_1P_4}| = 100.0819 \ (= b^{(1)})$$

$$\alpha^{(1)} = 89.6138\ [deg]$$

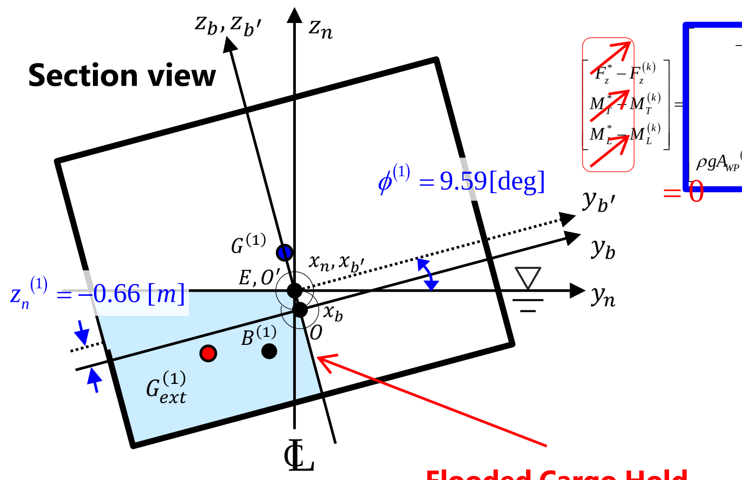
$L = 100\ [m]$ $B_{mld} = 40\ [m]$ $D = 30\ [m]$ $d = 9\ [m]$	$l = 10\ [m]$ $b = 20\ [m]$ $\mu_F = 1.0$ $\mu_V = 1.0$	${}^n\mathbf{r}_{G^{(1)}/E} = [0.17 \quad -0.72 \quad 4.28]^T\ [m]$ ${}^n\mathbf{r}_{G_{ext}^{(1)}/E} = [44.80 \quad -9.78 \quad -6.52]^T\ [m]$ ${}^n\mathbf{r}_{B^{(1)}/E} = [3.18 \quad -1.45 \quad -5.16]^T\ [m]$ ${}^n\mathbf{r}_{F^{(1)}/E} = [0 \quad 0 \quad 0]^T\ [m]$ ${}^n\mathbf{r}_{f^{(0)}/E} = [45 \quad -10 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2 s^2]$		

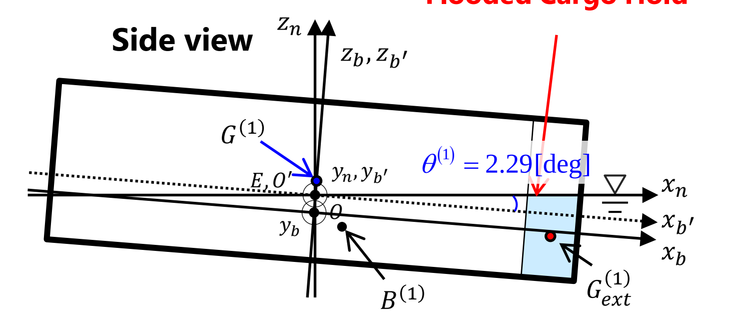
$\nabla^{(1)} = 3.9285 \times 10^4\ [m^3]$	$v^{(1)} = 2.6936 \times 10^3\ [m^3]$
$F_{G,z} = -3.6 \times 10^5\ [kN]$	$F_z^{(1)} = 1.1316 \times 10^4\ [kN]$
$F_{G_{ext},z}^{(1)} = -2.6936 \times 10^4\ [kN]$	$M_T^{(1)} = -1.73 \times 10^4\ [kN]$
$F_{B,z}^{(1)} = 3.9285 \times 10^5\ [kN]$	$M_L^{(1)} = -1.73 \times 10^4\ [kN]$



$$
\begin{bmatrix} \cancel{F_z^* - F_z^{(k)}} \\ \cancel{M_T^* - M_T^{(k)}} \\ \cancel{M_L^* - M_L^{(k)}} \end{bmatrix} = \boxed{\begin{bmatrix}
-\rho g A_{WP}{}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}{}^{(k)}\right) & -\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & \rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^n x_{f^{(k)}/E} \\
-\rho g A_{WP}{}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T{}^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} - \left(-\mu_F \cdot \rho g i_T{}^{(k)}\right) & \rho g I_P{}^{(k)} - \mu_F \cdot \rho g i_P{}^{(k)} \\
\rho g A_{WP}{}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}{}^{(k)} \cdot {}^n x_{f^{(k)}/E} & \rho g I_P{}^{(k)} - \mu_F \cdot \rho g i_P{}^{(k)} & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L{}^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}{}^{(k)}/E} \cdot F_{ext,z}{}^{(k)} - \left(-\mu_F \cdot \rho g i_L{}^{(k)}\right)
\end{bmatrix}}_{z_n=z_n^{(k)},\ \phi=\phi^{(k)},\ \theta=\theta^{(k)}} \begin{bmatrix} \delta z^{(k)} \\ \delta\phi^{(k)} \\ \delta\theta^{(k)} \end{bmatrix}
$$

$= 0$





$L = 100\ [m]$	$l = 10\ [m]$	${}^n\mathbf{r}_{G^{(1)}/E} = [0.17 \quad -0.72 \quad 4.28]^T\ [m]$
$B_{mld} = 40\ [m]$	$b = 20\ [m]$	${}^n\mathbf{r}_{G_{ext}^{(1)}/E} = [44.80 \quad -9.78 \quad -6.52]^T\ [m]$
$D = 30\ [m]$	$\mu_F = 1.0$	${}^n\mathbf{r}_{B^{(1)}/E} = [3.18 \quad -1.45 \quad -5.16]^T\ [m]$
$d = 9\ [m]$	$\mu_V = 1.0$	${}^n\mathbf{r}_{F^{(1)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2s^2]$		${}^n\mathbf{r}_{f^{(0)}/E} = [45 \quad -10 \quad 0]^T\ [m]$

$\nabla^{(1)} = 3.9285 \times 10^4\ [m^3]$	$v^{(1)} = 2.6936 \times 10^3\ [m^3]$
$F_{G,z} = -3.6 \times 10^5\ [kN]$	$F_z^{(1)} = 1.1316 \times 10^4\ [kN]$
$F_{G_{ext},z}^{(1)} = -2.6936 \times 10^4\ [kN]$	$M_T^{(1)} = -1.73 \times 10^4\ [kN]$
$F_{B,z}^{(1)} = 3.9285 \times 10^5\ [kN]$	$M_L^{(1)} = -1.73 \times 10^4\ [kN]$

Waterplane: second moment of area

$$I_L = I_{y_n y_n}$$

$$= \frac{1}{12} a^{(1)^3} b^{(1)} \sin^3 \alpha^{(1)}$$

$$= \frac{1}{12} \cdot 100.0819^3 \cdot 40.5675 \cdot \sin^3 89.6138$$

$$= 3.3578 \times 10^6\ [m^4]$$

$$I_P = I_{z_n z_n} = 0\ [m^4]$$

$${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 5.4044]^T\ [m]$$

$${}^{b'}\mathbf{r}_{P_2/O'} = [50 \quad 20 \quad -1.3571]^T\ [m]$$

$${}^{b'}\mathbf{r}_{P_3/O'} = [-50 \quad 20 \quad -5.4044]^T\ [m]$$

$${}^{b'}\mathbf{r}_{P_4/O'} = [-50 \quad -20 \quad 1.3571]^T\ [m]$$

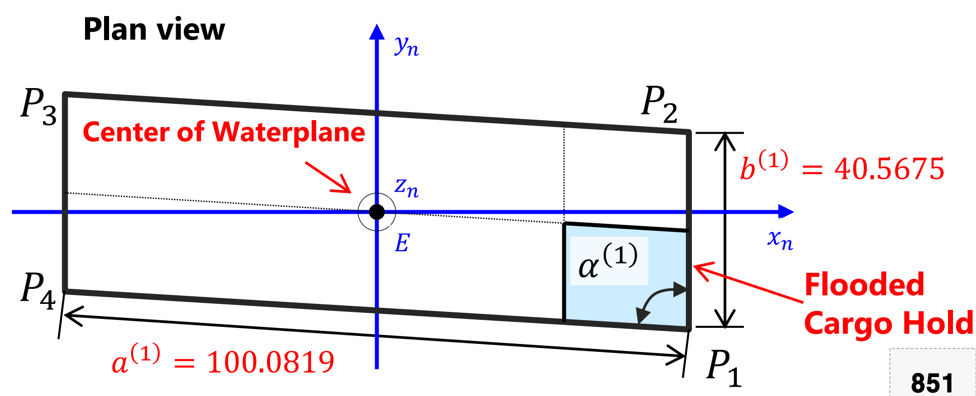
$$\overrightarrow{P_1P_2} = [0 \quad 40 \quad -6.7615]^T$$

$$|\overrightarrow{P_1P_2}| = 40.5675 \ (= l^{(1)})$$

$$\overrightarrow{P_1P_4} = [-100 \quad 0 \quad -4.0472]^T$$

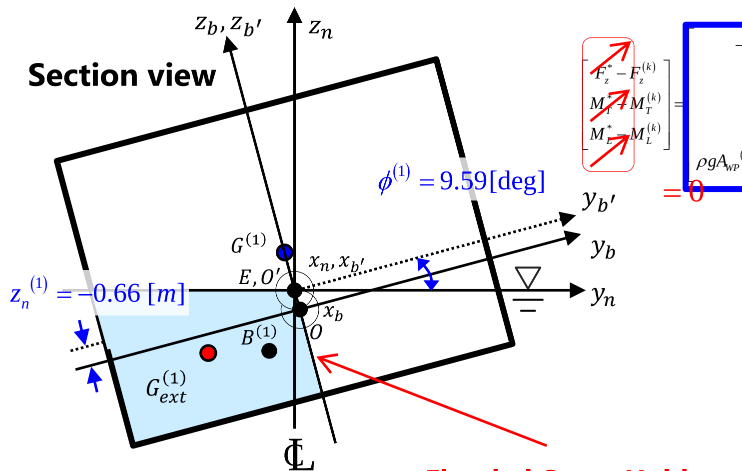
$$|\overrightarrow{P_1P_4}| = 100.0819 \ (= b^{(1)})$$

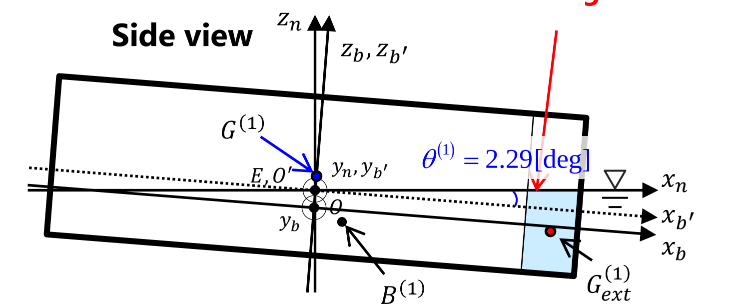
$$\alpha^{(1)} = 89.6138\ [deg]$$



$$\begin{bmatrix} \cancel{F_z^* - F_z^{(k)}} \\ \cancel{M_T^* - M_T^{(k)}} \\ \cancel{M_L^* - M_L^{(k)}} \end{bmatrix} = \boxed{\begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_T^{(k)}\right) & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_L^{(k)}\right) \end{bmatrix}}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$





$L = 100\ [m]$	$l = 10\ [m]$	${}^n\mathbf{r}_{G^{(1)}/E} = [0.17 \quad -0.72 \quad 4.28]^T\ [m]$
$B_{mld} = 40\ [m]$	$b = 20\ [m]$	${}^n\mathbf{r}_{G_{ext}^{(1)}/E} = [44.80 \quad -9.78 \quad -6.52]^T\ [m]$
$D = 30\ [m]$	$\mu_F = 1.0$	${}^n\mathbf{r}_{B^{(1)}/E} = [3.18 \quad -1.45 \quad -5.16]^T\ [m]$
$d = 9\ [m]$	$\mu_V = 1.0$	${}^n\mathbf{r}_{F^{(1)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2 s^2]$		${}^n\mathbf{r}_{f^{(0)}/E} = [45 \quad -10 \quad 0]^T\ [m]$

$\nabla^{(1)} = 3.9285 \times 10^4\ [m^3]$	$v^{(1)} = 2.6936 \times 10^3\ [m^3]$
$F_{G,z} = -3.6 \times 10^5\ [kN]$	$F_z^{(1)} = 1.1316 \times 10^4\ [kN]$
$F_{G_{ext},z}^{(1)} = -2.6936 \times 10^4\ [kN]$	$M_T^{(1)} = -1.73 \times 10^4\ [kN]$
$F_{B,z}^{(1)} = 3.9285 \times 10^5\ [kN]$	$M_L^{(1)} = -1.73 \times 10^4\ [kN]$

Flooded region:

Length of each side

$$l^{(1)} = \frac{1}{10} a^{(1)} = \frac{1}{10} \cdot 100.0819 = 10.0082$$

$$m^{(1)} = \frac{1}{2} b^{(1)} = \frac{1}{2} \cdot 40.5675 = 20.2873$$

${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 5.4044]^T\ [m]$

${}^{b'}\mathbf{r}_{P_2/O'} = [50 \quad 20 \quad -1.3571]^T\ [m]$

${}^{b'}\mathbf{r}_{P_3/O'} = [-50 \quad 20 \quad -5.4044]^T\ [m]$

${}^{b'}\mathbf{r}_{P_4/O'} = [-50 \quad -20 \quad 1.3571]^T\ [m]$

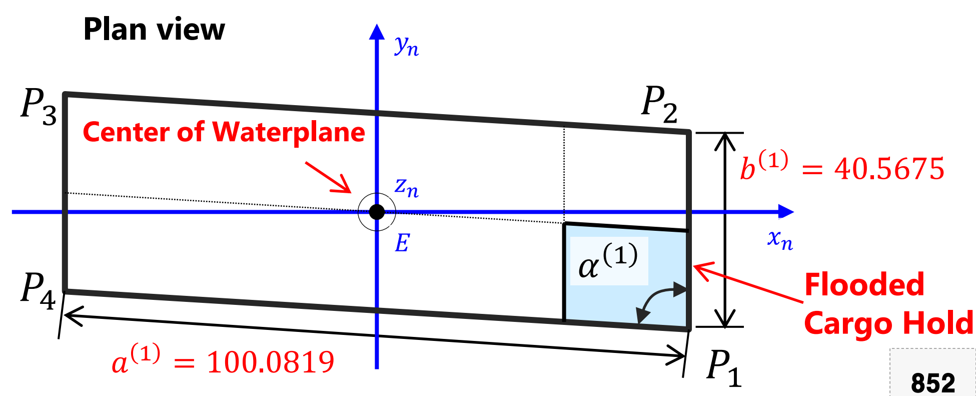
$\overrightarrow{P_1P_2} = [0 \quad 40 \quad -6.7615]^T$

$|\overrightarrow{P_1P_2}| = 40.5675 \ (= L^{(1)})$

$\overrightarrow{P_1P_4} = [-100 \quad 0 \quad -4.0472]^T$

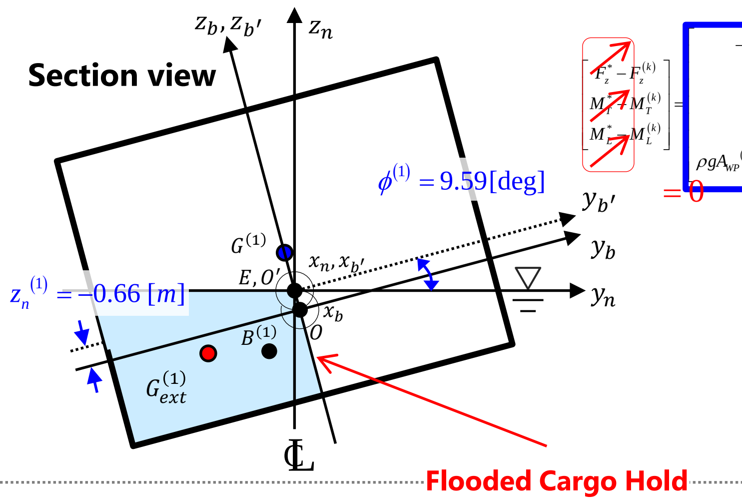
$|\overrightarrow{P_1P_4}| = 100.0819 \ (= B_{mld}^{(1)})$

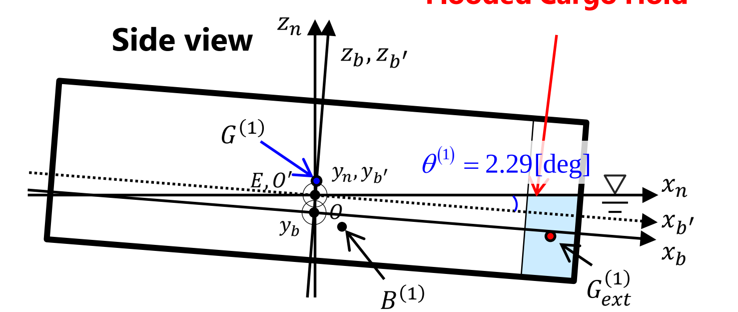
$\alpha^{(1)} = 89.6138\ [deg]$



$$\begin{bmatrix} F_z^* - F_z^{(k)} \\ M_T^* - M_T^{(k)} \\ M_L^* - M_L^{(k)} \end{bmatrix} = \boxed{\begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_T^{(k)}\right) & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_L^{(k)}\right) \end{bmatrix}}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$





$L = 100\ [m]$ $B_{mld} = 40\ [m]$ $D = 30\ [m]$ $d = 9\ [m]$	$l = 10\ [m]$ $b = 20\ [m]$ $\mu_F = 1.0$ $\mu_V = 1.0$	${}^n\mathbf{r}_{G^{(1)}/E} = [0.17 \quad -0.72 \quad 4.28]^T\ [m]$ ${}^n\mathbf{r}_{G_{ext}^{(1)}/E} = [44.80 \quad -9.78 \quad -6.52]^T\ [m]$ ${}^n\mathbf{r}_{B^{(1)}/E} = [3.18 \quad -1.45 \quad -5.16]^T\ [m]$ ${}^n\mathbf{r}_{F^{(1)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2s^2]$		${}^n\mathbf{r}_{f^{(0)}/E} = [45 \quad -10 \quad 0]^T\ [m]$

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$F_{G,z} = -3.6 \times 10^5\ [kN]$

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$F_{G_{ext},z}^{(1)} = -2.6936 \times 10^4\ [kN]$

$M_T^{(1)} = -1.73 \times 10^4\ [kN]$

$F_{B,z}^{(1)} = 3.9285 \times 10^5\ [kN]$

$M_L^{(1)} = -1.73 \times 10^4\ [kN]$

Flooded Region: a_{WP}

$$a_{WP}^{(1)} = l^{(1)} \cdot m^{(1)} \cdot \sin \alpha^{(1)}$$

$$= 10.0082 \cdot 20.2837 \cdot \sin 89.6138$$

$$= 2.03 \times 10^2\ [m^3]$$

${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 5.4044]^T\ [m]$

${}^{b'}\mathbf{r}_{P_2/O'} = [50 \quad 20 \quad -1.3571]^T\ [m]$

${}^{b'}\mathbf{r}_{P_3/O'} = [-50 \quad 20 \quad -5.4044]^T\ [m]$

${}^{b'}\mathbf{r}_{P_4/O'} = [-50 \quad -20 \quad 1.3571]^T\ [m]$

$\overrightarrow{P_1P_2} = [0 \quad 40 \quad -6.7615]^T$

$|\overrightarrow{P_1P_2}| = 40.5675\ (= a^{(1)})$

$\overrightarrow{P_1P_4} = [-100 \quad 0 \quad -4.0472]^T$

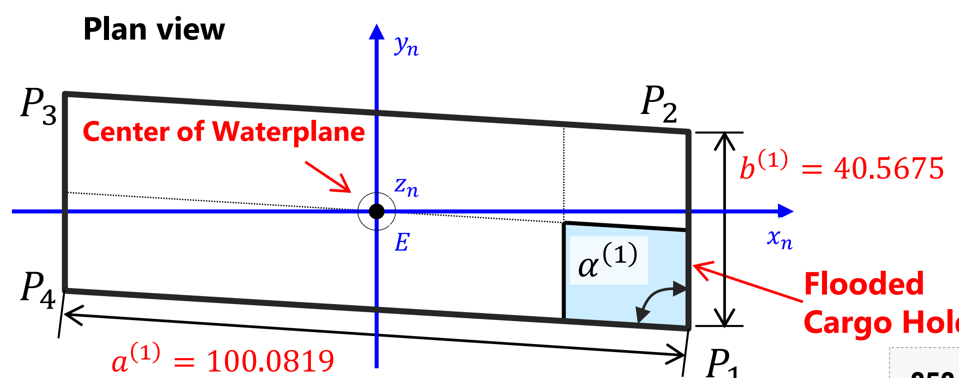
$|\overrightarrow{P_1P_4}| = 100.0819\ (= b^{(1)})$

$\alpha^{(1)} = 89.6138\ [deg]$

Center of waterplane:

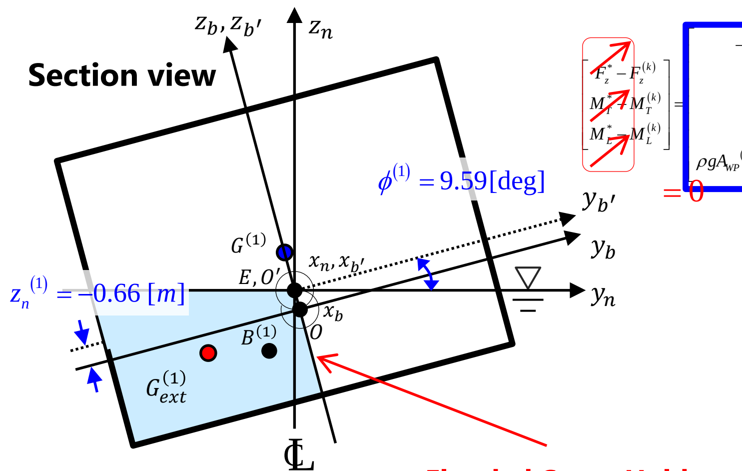
(equal to center of area)

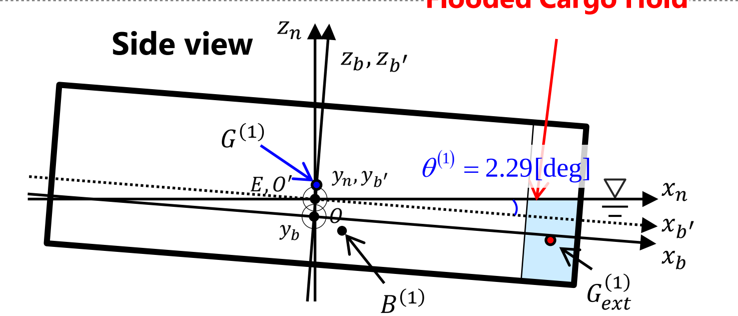
$${}^n\mathbf{r}_{f^{(1)}/E} = (P_1 + Q_2)/2 = [45.0358 \quad -10.4454 \quad 0]^T\ [m]$$



$$\begin{bmatrix} F_z^* - F_z^{(k)} \\ M_T^* - M_T^{(k)} \\ M_L^* - M_L^{(k)} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_T^{(k)}\right) & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_L^{(k)}\right) \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$= 0$





$L = 100\ [m]$	$l = 10\ [m]$	${}^n\mathbf{r}_{G^{(1)}/E} = [0.17 \quad -0.72 \quad 4.28]^T\ [m]$
$B_{mld} = 40\ [m]$	$b = 20\ [m]$	${}^n\mathbf{r}_{G_{ext}^{(1)}/E} = [44.80 \quad -9.78 \quad -6.52]^T\ [m]$
$D = 30\ [m]$	$\mu_F = 1.0$	${}^n\mathbf{r}_{B^{(1)}/E} = [3.18 \quad -1.45 \quad -5.16]^T\ [m]$
$d = 9\ [m]$	$\mu_V = 1.0$	${}^n\mathbf{r}_{F^{(1)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2s^2]$		${}^n\mathbf{r}_{f^{(1)}/E} = [45.03 \quad -10.45 \quad 0]^T\ [m]$

$\nabla^{(1)} = 3.9285 \times 10^4\ [m^3]$	$v^{(1)} = 2.6936 \times 10^3\ [m^3]$
$F_{G,z} = -3.6 \times 10^5\ [kN]$	$F_z^{(1)} = 1.1316 \times 10^4\ [kN]$
$F_{G_{ext},z}^{(1)} = -2.6936 \times 10^4\ [kN]$	$M_T^{(1)} = -1.73 \times 10^4\ [kN]$
$F_{B,z}^{(1)} = 3.9285 \times 10^5\ [kN]$	$M_L^{(1)} = -1.73 \times 10^4\ [kN]$

Flooded Region: second moment of area

$$i_T = i_{x_n x_n}$$

$$= \frac{1}{12} l^{(1)} m^{(1)} \sin \alpha^{(1)} \left[m^{(1)^2} + l^{(1)^2} \cos^2 \alpha^{(1)} \right]$$

$$+ A \cdot d_y^2$$

$$= \frac{1}{12} \cdot 10.0082 \cdot 20.2837 \cdot \sin 89.6138$$

$$\times [20.2837^2 + 10.0082^2 \cos^2 89.6138]$$

$$+ 2.03 \times 10^2 \cdot 10.4454^2$$

$$= 2.9109 \times 10^4\ [m^4]$$

${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 5.4044]^T\ [m]$

${}^{b'}\mathbf{r}_{P_2/O'} = [50 \quad 20 \quad -1.3571]^T\ [m]$

${}^{b'}\mathbf{r}_{P_3/O'} = [-50 \quad 20 \quad -5.4044]^T\ [m]$

${}^{b'}\mathbf{r}_{P_4/O'} = [-50 \quad -20 \quad 1.3571]^T\ [m]$

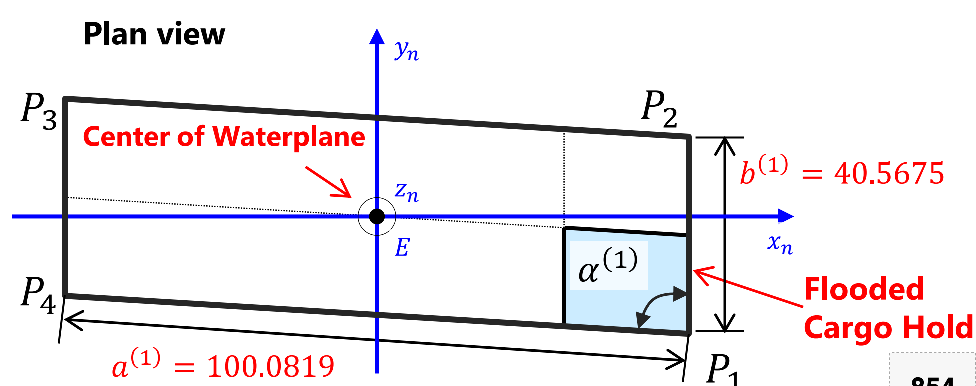
$\overrightarrow{P_1P_2} = [0 \quad 40 \quad -6.7615]^T$

$|\overrightarrow{P_1P_2}| = 40.5675\ (= a^{(1)})$

$\overrightarrow{P_1P_4} = [-100 \quad 0 \quad -4.0472]^T$

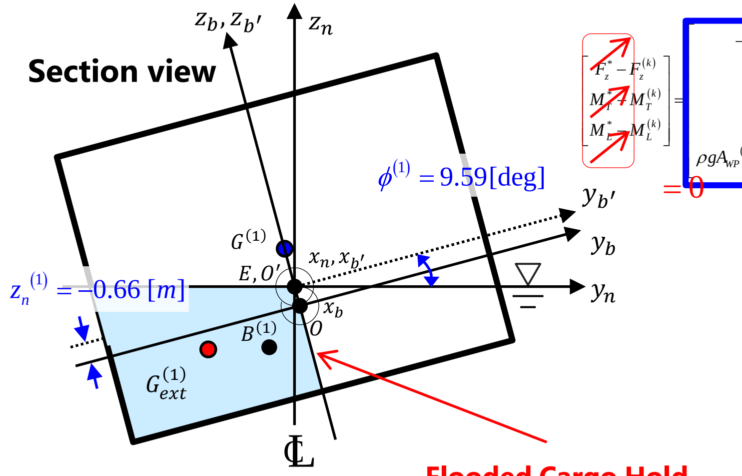
$|\overrightarrow{P_1P_4}| = 100.0819\ (= b^{(1)})$

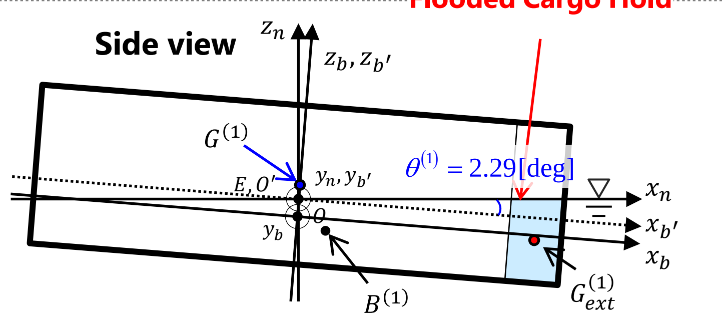
$\alpha^{(1)} = 89.6138\ [deg]$



$$\begin{bmatrix} F_z^* - F_z^{(k)} \\ M_T^* - M_T^{(k)} \\ M_L^* - M_L^{(k)} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_T^{(k)}\right) & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_L^{(k)}\right) \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta \phi^{(k)} \\ \delta \theta^{(k)} \end{bmatrix}$$

$$= 0$$





$L = 100\ [m]$	$l = 10\ [m]$	${}^n\mathbf{r}_{G^{(1)}/E} = [0.17 \quad -0.72 \quad 4.28]^T\ [m]$
$B_{mld} = 40\ [m]$	$b = 20\ [m]$	${}^n\mathbf{r}_{G_{ext}^{(1)}/E} = [44.80 \quad -9.78 \quad -6.52]^T\ [m]$
$D = 30\ [m]$	$\mu_F = 1.0$	${}^n\mathbf{r}_{B^{(1)}/E} = [3.18 \quad -1.45 \quad -5.16]^T\ [m]$
$d = 9\ [m]$	$\mu_V = 1.0$	${}^n\mathbf{r}_{F^{(1)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2s^2]$		${}^n\mathbf{r}_{f^{(1)}/E} = [45.03 \quad -10.45 \quad 0]^T\ [m]$

$\nabla^{(1)} = 3.9285 \times 10^4\ [m^3]$	$v^{(1)} = 2.6936 \times 10^3\ [m^3]$
$F_{G,z} = -3.6 \times 10^5\ [kN]$	$F_z^{(1)} = 1.1316 \times 10^4\ [kN]$
$F_{G_{ext},z}^{(1)} = -2.6936 \times 10^4\ [kN]$	$M_T^{(1)} = -1.73 \times 10^4\ [kN]$
$F_{B,z}^{(1)} = 3.9285 \times 10^5\ [kN]$	$M_L^{(1)} = -1.73 \times 10^4\ [kN]$

Flooded Region: second moment of area

$$i_L = i_{y_n y_n}$$

$$= \frac{1}{12} {a^{(1)}}^3 b^{(1)} \sin^3 \alpha^{(1)} + A \cdot d_x^2$$

$$= \frac{1}{12} \cdot 100.0819^3 \cdot 40.5675 \cdot \sin^3 89.6138$$

$$+ 2.02 \times 10^2 \cdot 45.0385^2$$

$$= 4.1342 \times 10^5\ [m^4]$$

$$i_P = i_{z_n z_n} = -9.55 \times 10^4\ [m^4]$$

${}^{b'}\mathbf{r}_{P_1/O'} = [50 \quad -20 \quad 5.4044]^T\ [m]$

${}^{b'}\mathbf{r}_{P_2/O'} = [50 \quad 20 \quad -1.3571]^T\ [m]$

${}^{b'}\mathbf{r}_{P_3/O'} = [-50 \quad 20 \quad -5.4044]^T\ [m]$

${}^{b'}\mathbf{r}_{P_4/O'} = [-50 \quad -20 \quad 1.3571]^T\ [m]$

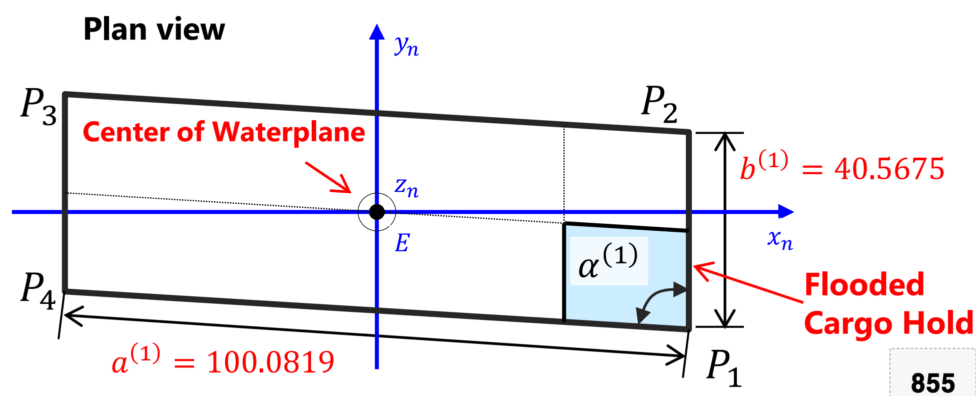
$\overrightarrow{P_1P_2} = [0 \quad 40 \quad -6.7615]^T$

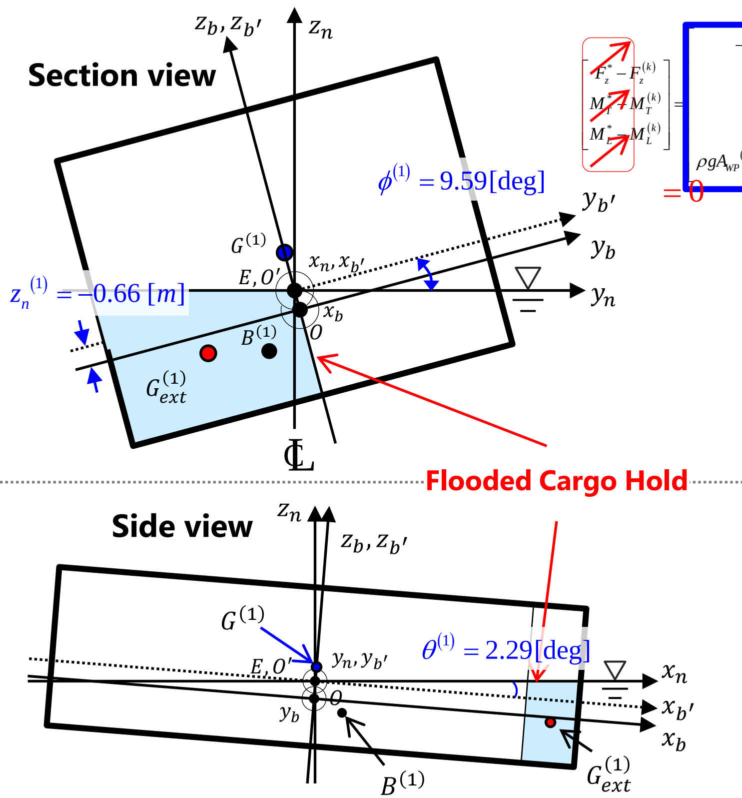
$|\overrightarrow{P_1P_2}| = 40.5675 \ (= l^{(1)})$

$\overrightarrow{P_1P_4} = [-100 \quad 0 \quad -4.0472]^T$

$|\overrightarrow{P_1P_4}| = 100.0819 \ (= b^{(1)})$

$\alpha^{(1)} = 89.6138\ [deg]$





$$\begin{bmatrix} \cancel{F_z^* - F_z^{(k)}} \\ \cancel{M_T^* - M_T^{(k)}} \\ \cancel{M_L^* - M_L^{(k)}} \end{bmatrix} = \boxed{\begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_T^{(k)}\right) & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} & -\rho g\left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_L^{(k)}\right) \end{bmatrix}}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \begin{bmatrix} \delta z^{(k)} \\ \delta\phi^{(k)} \\ \delta\theta^{(k)} \end{bmatrix}$$

$= 0$

$$-\rho g A_{WP}^{(1)} = -10 \cdot (4.06 \times 10^3) = -4.06 \times 10^4\ [kN/m]$$

$$-\mu_F \cdot \rho g a_{WP}^{(1)} = -1.0 \cdot 10 \cdot (2.02 \times 10^2) = -2.02 \times 10^3\ [kN/m]$$

$$-\rho g A_{WP}^{(1)} - \left(-\mu_F \cdot \rho g a_{WP}^{(0)}\right) = -4.06 \times 10^4 - (-2.02 \times 10^3) = -3.86 \times 10^4\ [kN/m]$$

$$-\rho g A_{WP}^{(1)} \cdot {}^n y_{F^{(1)}/E} = (-4.02 \times 10^4) \cdot 0 = 0\ [kN]$$

$$-\mu_F \cdot \rho g a_{WP}^{(1)} \cdot {}^n y_{f^{(1)}/E} = (-2.02 \times 10^3) \cdot (-10.45) = 2.11 \times 10^4\ [kN]$$

$$-\rho g A_{WP}^{(1)} \cdot {}^n y_{F^{(1)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(1)} \cdot {}^n y_{f^{(1)}/E}\right)$$
$$= 0 - (2.11 \times 10^4) = -2.11 \times 10^4\ [kN]$$

$$\rho g A_{WP}^{(1)} \cdot {}^n x_{F^{(1)}/E} = (-4.06 \times 10^4) \cdot 0 = 0\ [kN]$$

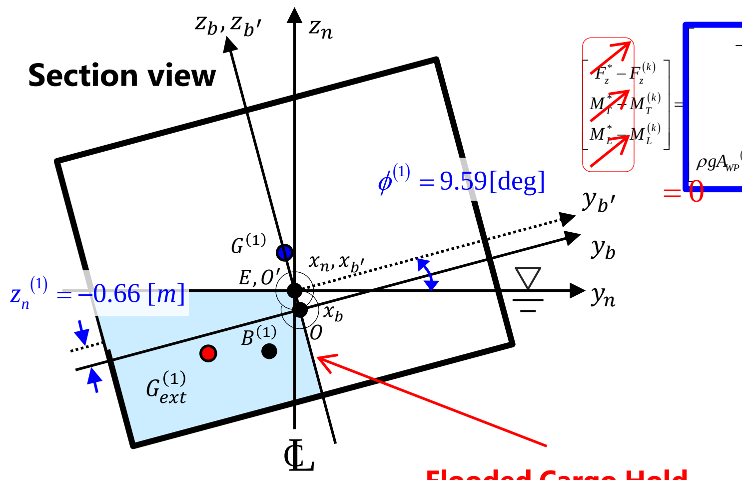
$$\mu_F \cdot \rho g a_{WP}^{(1)} \cdot {}^n x_{f^{(1)}/E} = (2.02 \times 10^3) \cdot 45.03 = 9.10 \times 10^4\ [kN]$$

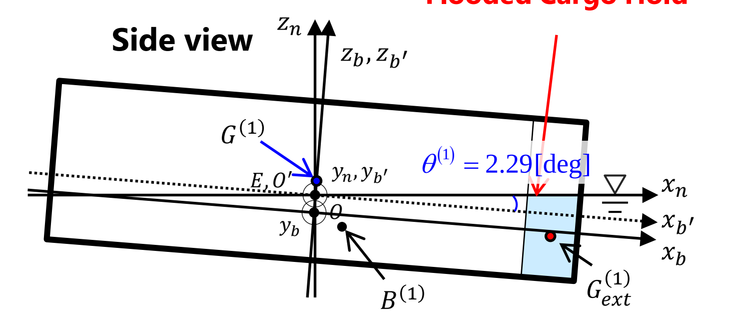
$$\rho g A_{WP}^{(1)} \cdot {}^n x_{F^{(1)}/E} - \mu_F \cdot \rho g a_{WP}^{(1)} \cdot {}^n x_{f^{(1)}/E}$$
$$= 0 - (9.10 \times 10^4) = -9.10 \times 10^4\ [kN]$$

$L = 100\ [m]$ $B_{mld} = 40\ [m]$ $D = 30\ [m]$ $d = 9\ [m]$	$l = 10\ [m]$ $b = 20\ [m]$ $\mu_F = 1.0$ $\mu_V = 1.0$	${}^n\mathbf{r}_{G^{(1)}/E} = [0.17\ \ -0.72\ \ 4.28]^T\ [m]$ ${}^n\mathbf{r}_{G_{ext}^{(1)}/E} = [44.80\ \ -9.78\ \ -6.52]^T\ [m]$ ${}^n\mathbf{r}_{B^{(1)}/E} = [3.18\ \ -1.45\ \ -5.16]^T\ [m]$ ${}^n\mathbf{r}_{F^{(1)}/E} = [0\ \ 0\ \ 0]^T\ [m]$ ${}^n\mathbf{r}_{f^{(1)}/E} = [45.03\ \ -10.45\ \ 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2s^2]$		
$\nabla^{(1)} = 3.9285 \times 10^4\ [m^3]$		$v^{(1)} = 2.6936 \times 10^3\ [m^3]$
$F_{G,z} = -3.6 \times 10^5\ [kN]$		$F_z^{(1)} = 1.1316 \times 10^4\ [kN]$
$F_{G_{ext},z}^{(1)} = -2.6936 \times 10^4\ [kN]$		$M_T^{(1)} = -1.73 \times 10^4\ [kN]$
$F_{B,z}^{(1)} = 3.9285 \times 10^5\ [kN]$		$M_L^{(1)} = -1.73 \times 10^4\ [kN]$

$$
\begin{bmatrix} F_z^* - F_z^{(k)} \\ M_T^* - M_T^{(k)} \\ M_L^* - M_L^{(k)} \end{bmatrix} = \boxed{\begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_T^{(k)}\right) & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_L^{(k)}\right) \end{bmatrix}}_{z_n = z_n^{(k)},\ \phi = \phi^{(k)},\ \theta = \theta^{(k)}} \begin{bmatrix} \delta z^{(k)} \\ \delta\phi^{(k)} \\ \delta\theta^{(k)} \end{bmatrix}
$$

$= 0$





$$
-\rho g \left({}^n z_{B^{(1)}/E} \nabla^{(1)} + I_T^{(1)}\right) - {}^n z_{G^{(1)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(1)}/E} \cdot F_{ext,z}^{(1)} - \left(-u_F \cdot \rho g i_T^{(1)}\right)
$$
$$
= -10 \cdot [-5.16 \cdot (3.93 \times 10^4) + (5.57 \times 10^5)] - 4.28 \cdot (-3.6 \times 10^5)
$$
$$
-(-6.52) \cdot (-2.69 \times 10^4) - \left(-1.0 \cdot 10 \cdot (2.91 \times 10^4)\right)
$$
$$
= -1.62 \times 10^6\ [kN \cdot m]
$$

$$
\rho g I_P^{(1)} - \mu_F \cdot \rho g i_P^{(1)} = 10 \cdot 0 - 1.0 \cdot 10 \cdot (-9.0 \times 10^4) = 9.0 \times 10^5\ [kN \cdot m]
$$

$$
-\rho g \left({}^n z_{B^{(1)}/E} \nabla^{(1)} + I_L^{(1)}\right) - {}^n z_{G^{(1)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(1)}/E} \cdot F_{ext,z}^{(1)} - \left(-u_F \cdot \rho g i_L^{(1)}\right)
$$
$$
= -10 \cdot [-5.16 \cdot (3.93 \times 10^4) + (1.18 \times 10^6)] - 4.28 \cdot (-3.6 \times 10^5)
$$
$$
-(-6.52) \cdot (-2.69 \times 10^4) - \left(-1.0 \cdot 10 \cdot (4.13 \times 10^5)\right)
$$
$$
= -2.61 \times 10^7\ [kN \cdot m]
$$

$L = 100\ [m]$ $B_{mld} = 40\ [m]$ $D = 30\ [m]$ $d = 9\ [m]$	$l = 10\ [m]$ $b = 20\ [m]$ $\mu_F = 1.0$ $\mu_V = 1.0$	${}^n\mathbf{r}_{G^{(1)}/E} = [0.17 \quad -0.72 \quad 4.28]^T\ [m]$ ${}^n\mathbf{r}_{G_{ext}^{(1)}/E} = [44.80 \quad -9.78 \quad -6.52]^T\ [m]$ ${}^n\mathbf{r}_{B^{(1)}/E} = [3.18 \quad -1.45 \quad -5.16]^T\ [m]$ ${}^n\mathbf{r}_{F^{(1)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
$\rho g = 10\ [Mg/m^2s^2]$		${}^n\mathbf{r}_{f^{(1)}/E} = [45.03 \quad -10.45 \quad 0]^T\ [m]$

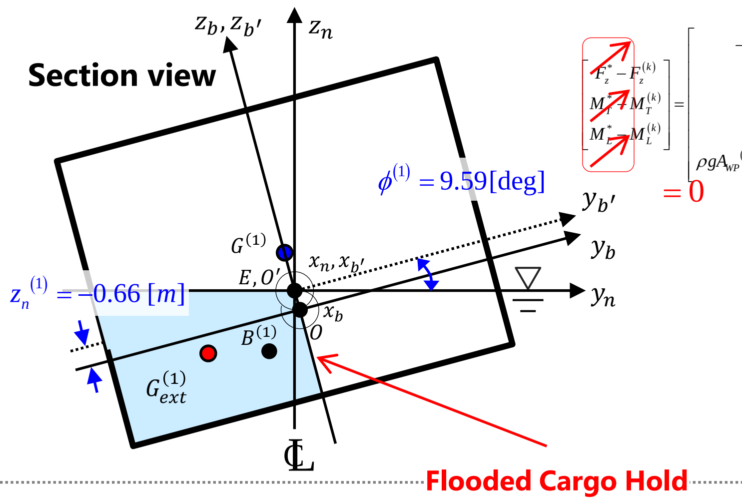
$\nabla^{(1)} = 3.9285 \times 10^4\ [m^3]$ $\quad v^{(1)} = 2.6936 \times 10^3\ [m^3]$

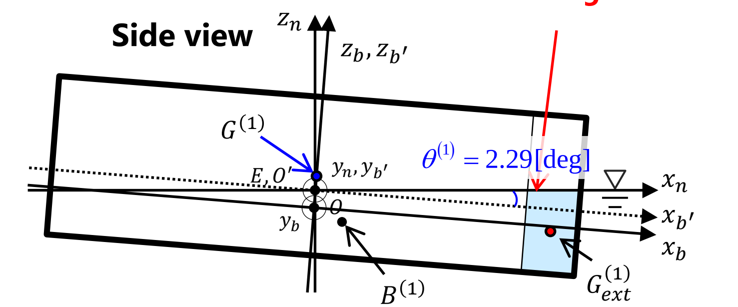
$F_{G,z} = -3.6 \times 10^5\ [kN]$ $\quad F_z^{(1)} = 1.1316 \times 10^4\ [kN]$

$F_{G_{ext},z}^{(1)} = -2.6936 \times 10^4\ [kN]$ $\quad M_T^{(1)} = -1.73 \times 10^4\ [kN]$

$F_{B,z}^{(1)} = 3.9285 \times 10^5\ [kN]$ $\quad M_L^{(1)} = -1.73 \times 10^4\ [kN]$

3. Calculation of Immersion, Trim, and Heel at k=1 step





$L = 100\ [m]$
$B_{mld} = 40\ [m]$
$D = 30\ [m]$
$d = 9\ [m]$

$l = 10\ [m]$
$b = 20\ [m]$
$\mu_F = 1.0$
$\mu_V = 1.0$

$\rho g = 10\ [Mg/m^2 s^2]$

${}^n\mathbf{r}_{G^{(1)}/E} = [0.17 \quad -0.72 \quad 4.28]^T\ [m]$
${}^n\mathbf{r}_{G_{ext}^{(1)}/E} = [44.80 \quad -9.78 \quad -6.52]^T\ [m]$
${}^n\mathbf{r}_{B^{(1)}/E} = [3.18 \quad -1.45 \quad -5.16]^T\ [m]$
${}^n\mathbf{r}_{F^{(1)}/E} = [0 \quad 0 \quad 0]^T\ [m]$
${}^n\mathbf{r}_{f^{(1)}/E} = [45.03 \quad -10.45 \quad 0]^T\ [m]$

$\nabla^{(1)} = 3.9285 \times 10^4\ [m^3]$ $\quad v^{(1)} = 2.6936 \times 10^3\ [m^3]$

$F_{G,z} = -3.6 \times 10^5\ [kN]$ $\quad F_z^{(1)} = 1.1316 \times 10^4\ [kN]$

$F_{G_{ext},z}^{(1)} = -2.6936 \times 10^4\ [kN]$ $\quad M_T^{(1)} = -1.73 \times 10^4\ [kN]$

$F_{B,z}^{(1)} = 3.9285 \times 10^5\ [kN]$ $\quad M_L^{(1)} = -1.73 \times 10^4\ [kN]$

$$\begin{bmatrix} F_z^* - F_z^{(k)} \\ M_T^* - M_T^{(k)} \\ M_L^* - M_L^{(k)} \end{bmatrix} = \begin{bmatrix} -\rho g A_{WP}^{(k)} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)}\right) & -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} \\ -\rho g A_{WP}^{(k)} \cdot {}^n y_{F^{(k)}/E} - \left(-\mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n y_{f^{(k)}/E}\right) & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_T^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_T^{(k)}\right) & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} \\ \rho g A_{WP}^{(k)} \cdot {}^n x_{F^{(k)}/E} - \mu_F \cdot \rho g a_{WP}^{(k)} \cdot {}^n x_{f^{(k)}/E} & \rho g I_P^{(k)} - \mu_F \cdot \rho g i_P^{(k)} & -\rho g \left({}^n z_{B^{(k)}/E} \nabla^{(k)} + I_L^{(k)}\right) - {}^n z_{G^{(k)}/E} \cdot F_{G,z} - {}^n z_{G_{ext}^{(k)}/E} \cdot F_{ext,z}^{(k)} - \left(-\mu_F \cdot \rho g i_L^{(k)}\right) \end{bmatrix}_{\substack{z_n = z_n^{(k)} \\ \phi = \phi^{(k)} \\ \theta = \theta^{(k)}}} \boxed{\begin{bmatrix} \delta z^{(k)} \\ \delta\phi^{(k)} \\ \delta\theta^{(k)} \end{bmatrix}}$$

$$= 0$$

$$\begin{bmatrix} 1.13\times10^4 \\ -1.73\times10^5 \\ -1.73\times10^6 \end{bmatrix} = \begin{bmatrix} -3.86\times10^4 & -2.12\times10^4 & -9.14\times10^4 \\ -2.12\times10^4 & -1.62\times10^6 & 7.27\times10^5 \\ -9.14\times10^4 & 7.27\times10^5 & -2.61\times10^7 \end{bmatrix} \begin{bmatrix} \delta z_n^{(1)} \\ \delta\phi^{(1)} \\ \delta\theta^{(1)} \end{bmatrix}$$

$$\boxed{\begin{bmatrix} \delta z_n^{(1)} \\ \delta\phi^{(1)} \\ \delta\theta^{(1)} \end{bmatrix}} = \begin{bmatrix} -3.86\times10^4 & -2.12\times10^4 & -9.14\times10^4 \\ -2.12\times10^4 & -1.62\times10^6 & 7.27\times10^5 \\ -9.14\times10^4 & 7.27\times10^5 & -2.61\times10^7 \end{bmatrix}^{-1} \begin{bmatrix} 1.13\times10^4 \\ -1.73\times10^5 \\ -1.73\times10^6 \end{bmatrix}$$

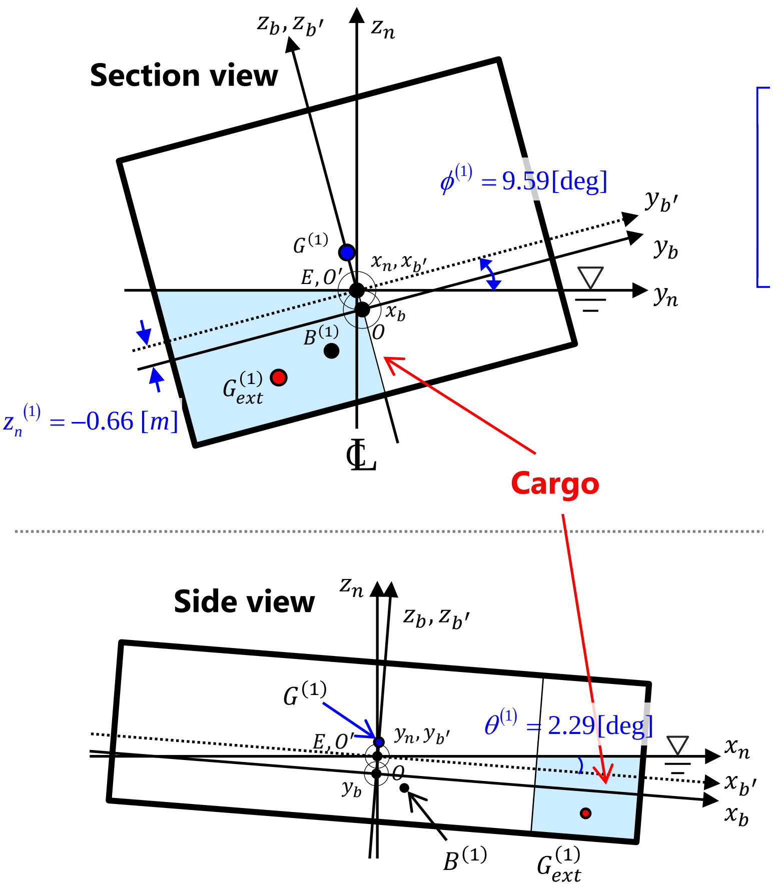
$$= \begin{bmatrix} 0.0063\ [m] \\ -0.0112[rad] \\ -0.0010[rad] \end{bmatrix} = \boxed{\begin{bmatrix} 0.0063\ [m] \\ -0.6422[\deg] \\ -0.0571[\deg] \end{bmatrix}}$$

$$z_n^{(2)} = z_n^{(1)} + \delta z_n^{(1)} = -0.66 + 0.0063 = -0.67\ [m]$$

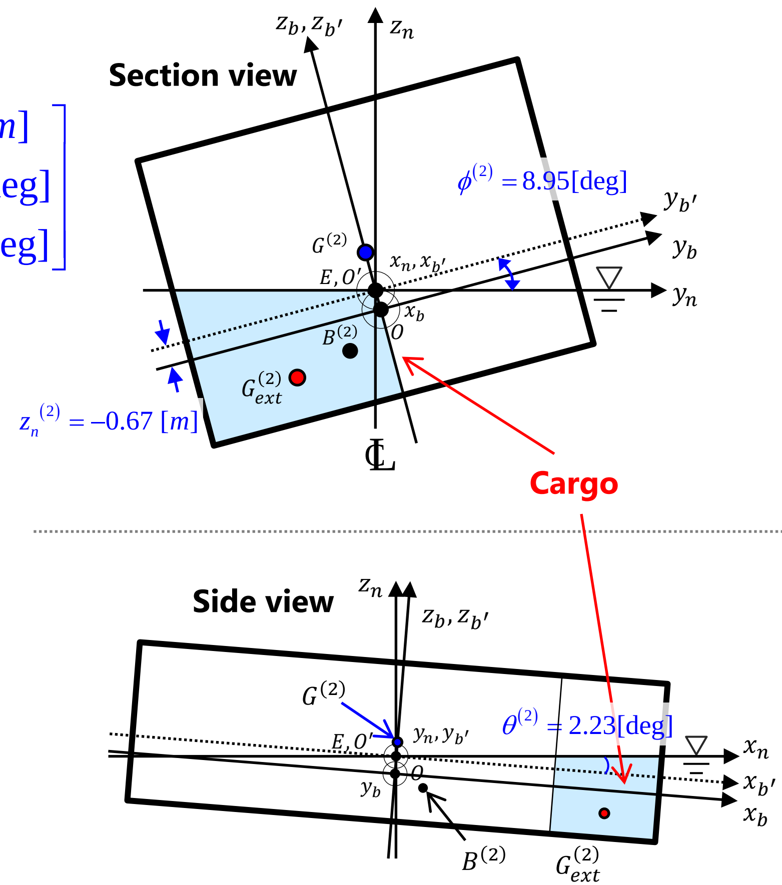
$$\phi^{(2)} = \phi^{(1)} + \delta\phi^{(1)} = 9.59 + (-0.6422) = 8.95\ [\deg]$$

$$\theta^{(2)} = \theta^{(1)} + \delta\theta^{(1)} = 2.29 + (-0.0571) = 2.23\ [\deg]$$

3. Calculation of Immersion, Trim, and Heel at k=1 step



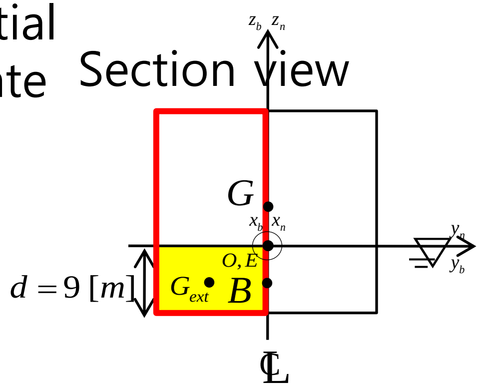
$$\begin{bmatrix} \delta z_n^{(1)} \\ \delta\phi^{(1)} \\ \delta\theta^{(1)} \end{bmatrix} = \begin{bmatrix} 0.0063\ [m] \\ -0.6422[deg] \\ -0.0571[deg] \end{bmatrix}$$



Summary of the coupled Immersion, Heel, and Trim of a Box-Shaped Ship in Flooded State(1/2)

Initial State

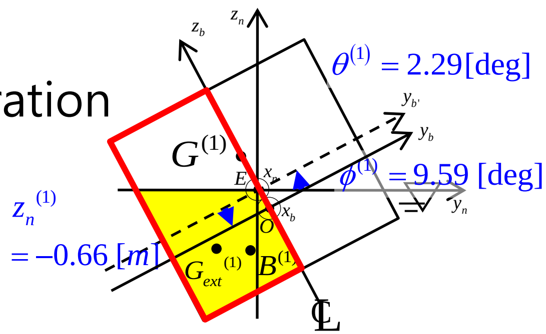
Section view



Force(KN)	Transverse Moment(KN·m)	Longitudinal Moment(KN·m)
$F = -1.800\text{x}10^4$	$M_T = 1.800\text{x}10^5$	$M_L = 8.100\text{x}10^5$

$$\begin{bmatrix} \delta z_n^{(0)} \\ \delta\phi^{(0)} \\ \delta\theta^{(0)} \end{bmatrix} = \begin{bmatrix} -0.6563\ [m] \\ 9.5945\ [\text{deg}] \\ 2.2853\ [\text{deg}] \end{bmatrix}$$

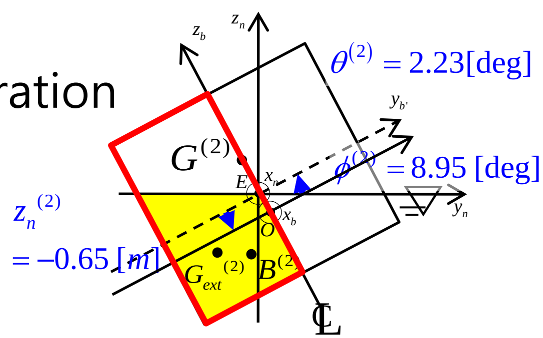
1st Iteration



Force(KN)	Transverse Moment(KN·m)	Longitudinal Moment(KN·m)
$F = -8.4515\text{x}10^1$	$M_T = -1.7252\text{x}10^4$	$M_L = -1.7258\text{x}10^4$

$$\begin{bmatrix} \delta z_n^{(1)} \\ \delta\phi^{(1)} \\ \delta\theta^{(1)} \end{bmatrix} = \begin{bmatrix} 0.0063\ [m] \\ -0.6423\ [\text{deg}] \\ -0.0571\ [\text{deg}] \end{bmatrix}$$

2nd Iteration

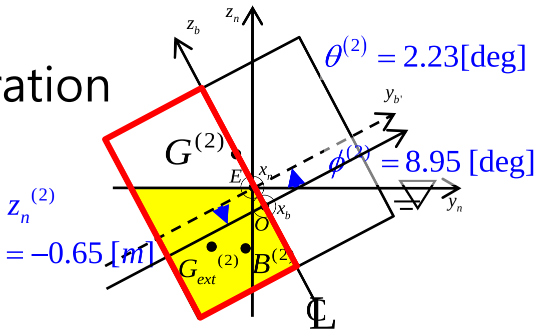


Force(KN)	Transverse Moment(KN·m)	Longitudinal Moment(KN·m)
$F = 2.5808\text{x}10^0$	$M_T = -1.7484\text{x}10^2$	$M_L = -1.0379\text{x}10^2$

$$\begin{bmatrix} \delta z_n^{(2)} \\ \delta\phi^{(2)} \\ \delta\theta^{(2)} \end{bmatrix} = \begin{bmatrix} 0.0001\ [m] \\ -0.0066\ [\text{deg}] \\ -0.0004\ [\text{deg}] \end{bmatrix}$$

Summary of the coupled Immersion, Heel, and Trim of a Box-Shaped Ship in Flooded State(2/2)

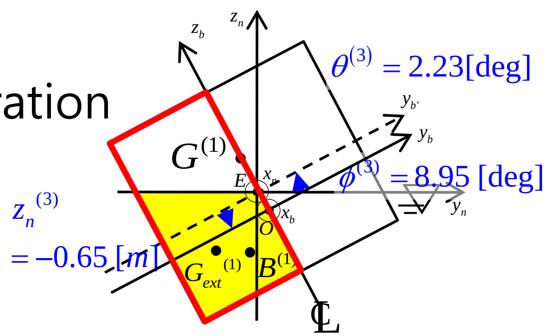
2nd Iteration



Force(KN)	Transverse Moment(KN·m)	Longitudinal Moment(KN·m)
F = 2.5808×10^{0}	M_T = -1.7484×10^{2}	M_L = -1.0379×10^{2}

$$\begin{bmatrix} \delta z_n^{(2)} \\ \delta\phi^{(2)} \\ \delta\theta^{(2)} \end{bmatrix} = \begin{bmatrix} 0.0001\,[m] \\ \text{-}0.0066\ [\text{deg}] \\ \text{-}0.0004\ [\text{deg}] \end{bmatrix}$$

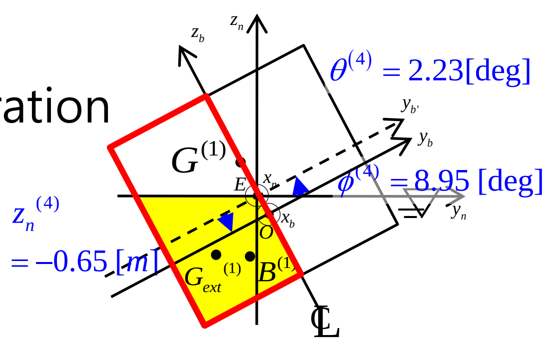
3rd Iteration



Force(KN)	Transverse Moment(KN·m)	Longitudinal Moment(KN·m)
F = 7.2185×10^{-2}	M_T = -1.1485×10^{-1}	M_L = 2.2971×10^{-1}

$$\begin{bmatrix} \delta z_n^{(3)} \\ \delta\phi^{(3)} \\ \delta\theta^{(3)} \end{bmatrix} = \begin{bmatrix} 0.0\ [m] \\ 0.0\ [\text{deg}] \\ 0.0\ [\text{deg}] \end{bmatrix}$$

4th Iteration



Force(KN)	Transverse Moment(KN·m)	Longitudinal Moment(KN·m)
F = 9.5798×10^{-4}	M_T = 4.0717×10^{-4}	M_L = 2.3295×10^{-3}

Chapter 13. Probabilistic Damage Stability(Subdivision and Damage Stability:SDS)

13-1 Concept of Probabilistic Damage Stability

Required Documents

In general, the document which contains the following list is submitted to shipowner and classification society, and get approval from them 9 months before steel cutting.

- Principle particulars
- General arrangement
- Midship section plan
- Lines plan
- Hydrostatic table
- Bonjean table
- Tank capacity table
- Light weight summary
- Allowable Minimum GM Curve
- Trim & stability calculation
- Damage stability calculation(Subdivision and damage stability)
- Freeboard Calculation
- Visibility Check
- Equipment number calculation

……………………

Main subject of Chapter 13

Probabilistic Method : Subdivision & Damage Stability

Probabilistic Method

The probability of damage "p_i" that a compartment or group of compartments may be flooded at the level of the deepest subdivision draught (scantling draft).

The probability of survivability "s_i" after flooding in a given damage condition.

The attained subdivision index "A" is the summation of the probability of all damage cases.

$$A = p_1 \times s_1 + p_2 \times s_2 + p_3 \times s_3 + \cdots p_i \times s_i$$
$$= \sum p_i \times s_i$$

The required subdivision index "R" is requirement of a minimum value of index "A" for a particular ship.

$$R = 1 - \frac{128}{L_s + 152}$$

where: "L_S" is called subdivision length and related with the ship's length

$$\boxed{A \geq R}$$

Attained index "A"

$$A = \sum p_i \times s_i$$

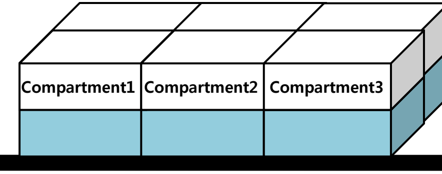
p_i : accounts for the **probability** that only the compartment or group of compartments under consideration may **be flooded**, disregarding any horizontal subdivision.

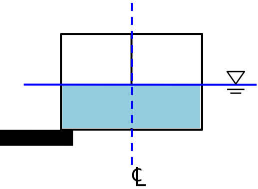
$$p_i = p_i(x1, x2, b, j, n, k)$$

s_i : accounts for the **probability** **of survival after flooding** the compartment or group of compartments under consideration, and includes the effect of any horizontal subdivision

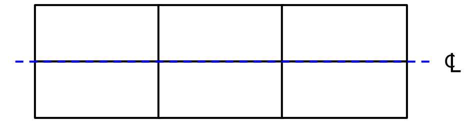
$$s_i = s_i(\theta e, \theta v, GZ_{\max}, Range, Flooding\ stage)$$

Probability of Damage : Subdivision





$$A = \sum p_i \times s_i$$



What is the factor "p_i"?

: **Probability of damage** **that a compartment or group of compartments** **may be flooded** **at the level of the** **deepest subdivision draught "ds",** **that is, scantling draft.**

: Related to the Generation of "Damage Case"

→Dependent on the **geometry of the ship**.
(Watertight arrangement and main dimensions of the ship)

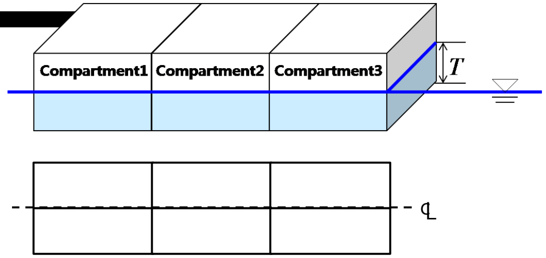
$$p_i = p \cdot r$$

p : The probability of damage in the longitudinal subdivision.
r : The probability of damage in the transverse subdivision.

13-2 Probability of Damage in Longitudinal Subdivision (p)

$p_i = p \cdot r$

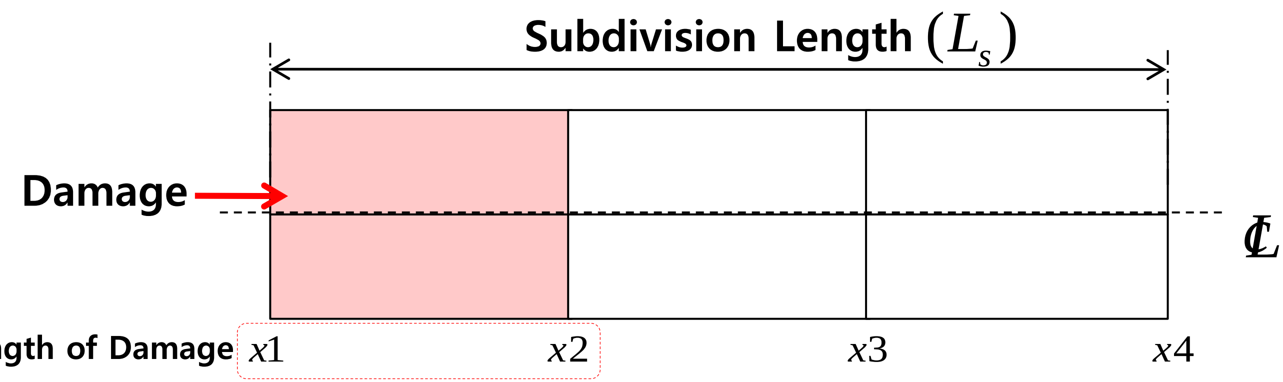
Probability related to the longitudinal subdivision



What is the factor "p"?

: Probability of damage in the longitudinal subdivision "p"

$$p = p(x1, x2, L_s)$$



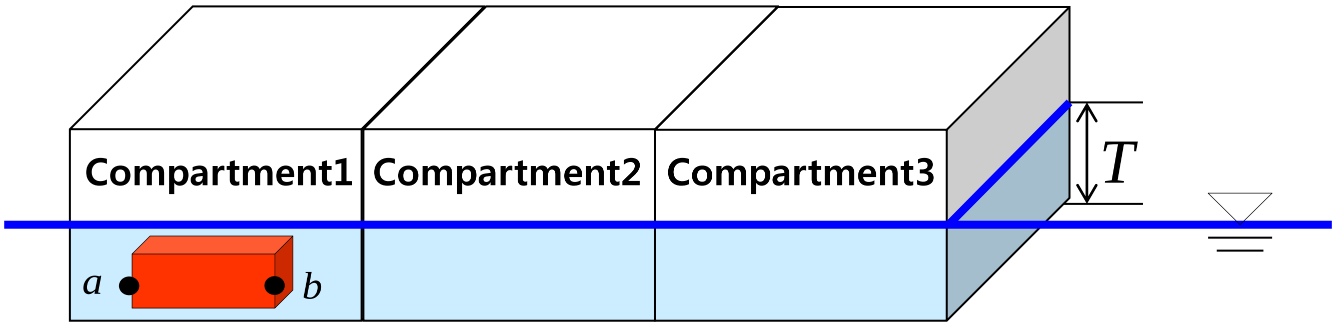
: the **factor "p"** is dependent on **the length of damage ($x2$ - $x1$)** and the **subdivision length "L_s"** of a ship.

- Example) Box-Shaped Ship – Damage Generator

$p_i = p \cdot r$

$p = p(x1, x2, L_s)$

How can you obtain the value of "p" for a box-shaped ship?



✓Assume that the dimensions of the compartments are same.

✓The ship is damaged by the "**Damage generator**" defined by **damage extent in horizontal, transverse and vertical direction.**

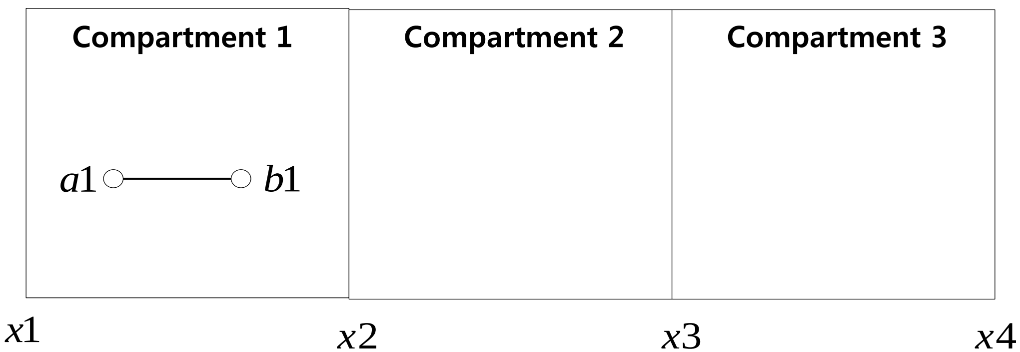
✓ Define that the **each end point of the "Damage generator"** is "a" and "b".

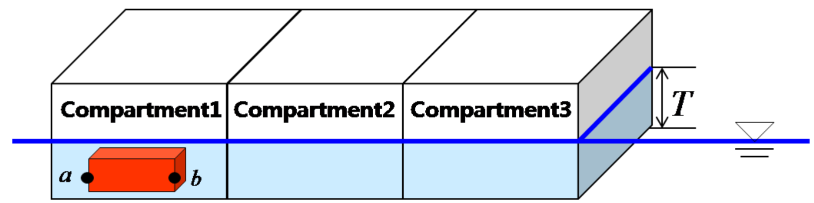
Damage Length

$$p_i = p \cdot r$$

$$p = p(x1, x2, L_s)$$

What is the "Damage length" (length of the damage)?





For example, when one compartment is damaged, the end points become *"a1" and "b1"*.

✓What we consider in this part is "Damage Length". **Each end of the damage length is "*x1*" (left) and "*x2*" (right)** and we can calculate the probability of damage by this **length (*x2 –x1*)**.

*The damage length is represented by the non-dimensional damage length in the SOLAS regulation:

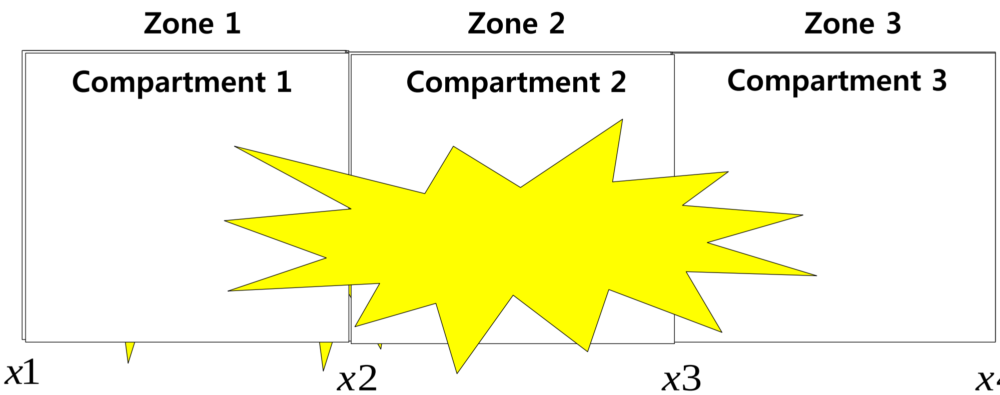
$$\text{Non-dimensional damage length "}J\text{"} = \frac{(x2 - x1)}{L_s}$$

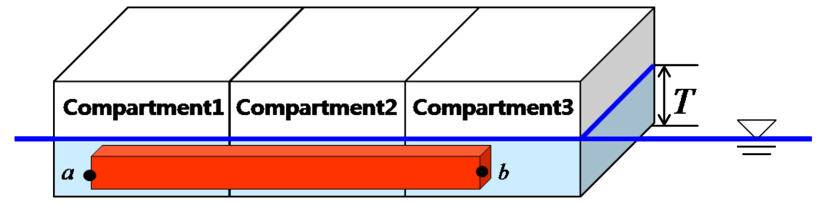
Damage Zone

$$p_i = p \cdot r$$

$$p = p(x1, x2, L_s)$$

What is the "Damage zone"?





$x4$: terminal of the zones

Damage zone is a **longitudinal interval of the ship** within the subdivision length.

In general, the zones are placed in accordance with the watertight arrangement. However, **the zones can be placed in accordance with the virtual subdivision**.

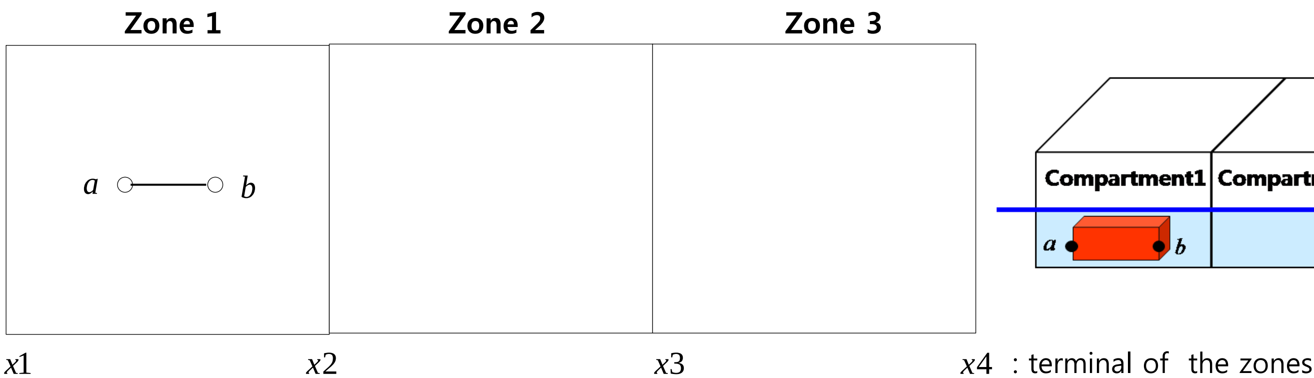
For this example, we place the zones in accordance with the compartments (the watertight arrangement).

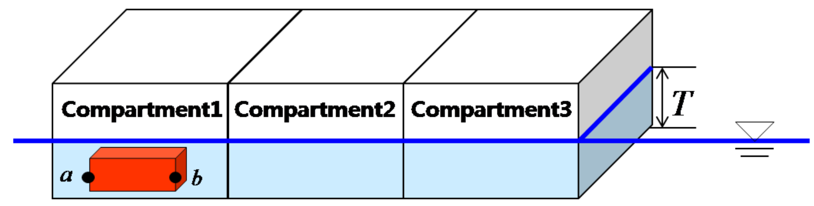
One Zone Damage Case

$$p_i = p \cdot r$$

$$p = p(x1, x2, L_s)$$

How can you obtain the value of "p" when one zone is damaged?





Example) What is the probability that zone1 is damaged?

Probability that "a" is located in zone1 $\times$ **Probability that "b" is located in zone1**

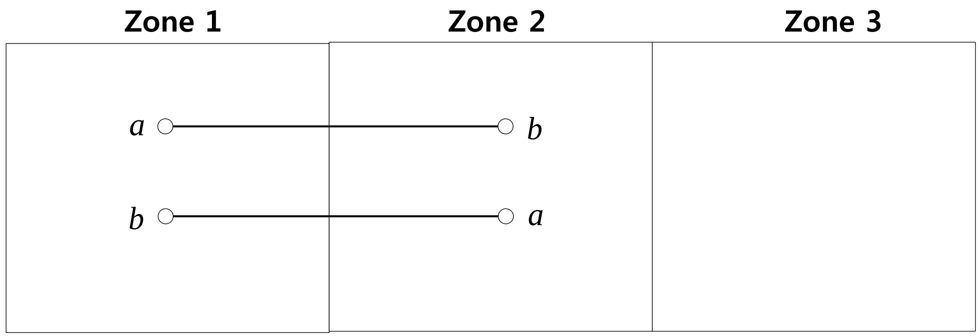
$$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$$

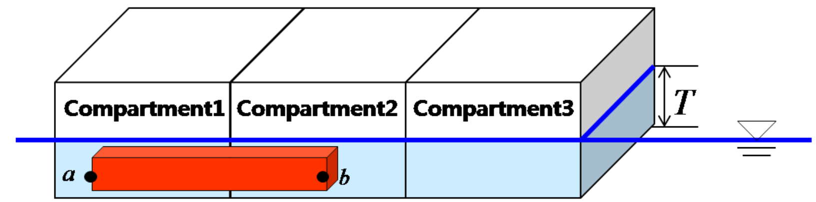
Two Zones Damage Case

$$p_i = p \cdot r$$

$$p = p(x1, x2, L_s)$$

How can you obtain the value of "p" when two adjacent zones are damaged?





$x1$ $x2$ $x3$ $x4$: terminal of the zones

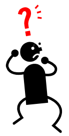
Example) What is the probability that zone 1 and zone 2 are damaged simultaneously?

Probability that "a" is located in zone1	×	**Probability that "b" is located in zone2**	+	**Probability that "a" is located in zone2**	×	**Probability that "b" is located in zone1**

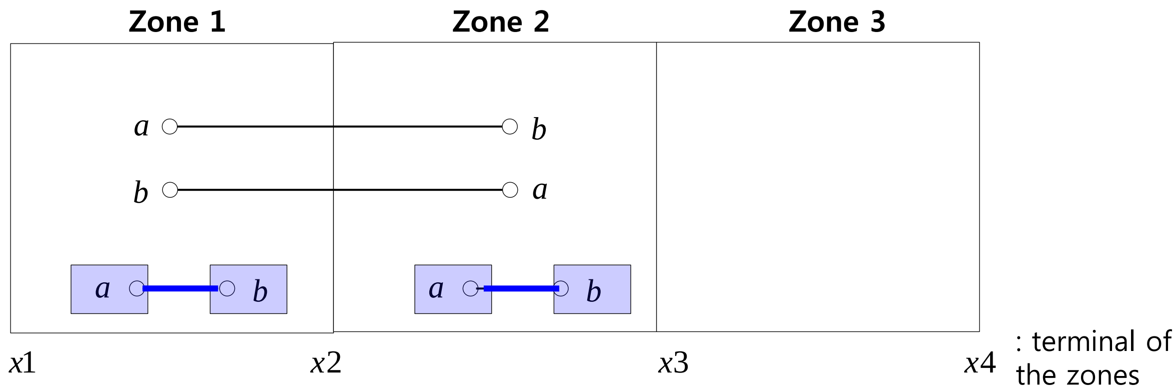
$$\frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{1}{3} = \frac{2}{9}$$

$$p_i = p \cdot r$$

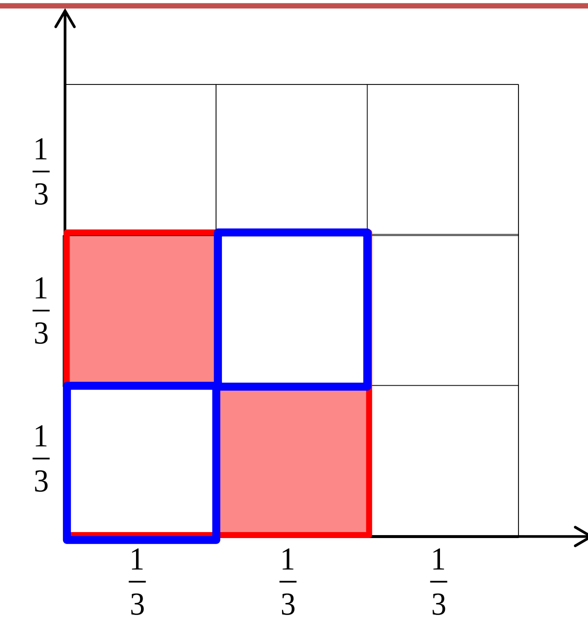
$$p = p(x1, x2, L_s)$$



How can you obtain the value of "p" that two adjacent zones are damaged by different representation method?



Example) What is the probability that zone 1 and zone 2 are damaged simultaneously?

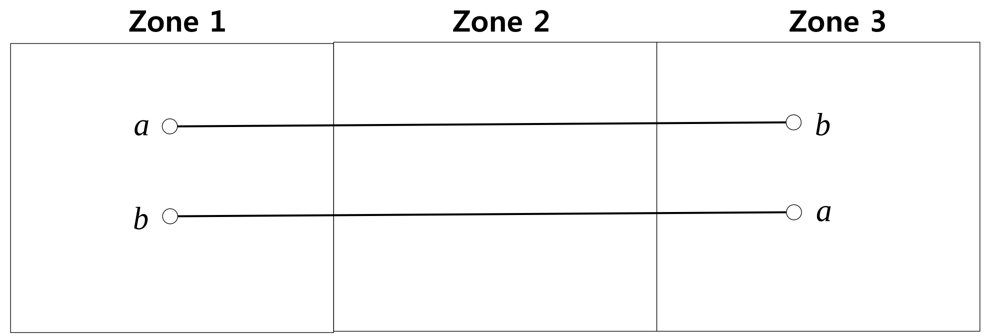


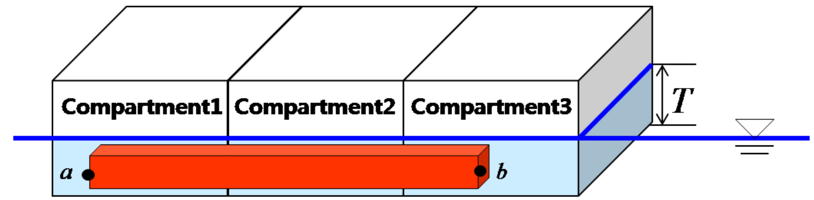
In the figure, the red area means the probability that zone 1 and zone 2 are damaged simultaneously.

Probability that "a" is located in zone1 or zone 2 $\times$ **Probability that "b" is located in zone1 or zone 2** $-$ **Probability that "a" is located in zone1** $\times$ **Probability that "b" is located in zone1** $-$ **Probability that "a" is located in zone 2** $\times$ **Probability that "b" is located in zone 2**

$$\frac{2}{3} \times \frac{2}{3} - \frac{1}{3} \times \frac{1}{3} - \frac{1}{3} \times \frac{1}{3} = \frac{2}{9}$$

How can you obtain the value of "p" when three zones are damaged?





$x1$ $x2$ $x3$ $x4$: terminal of the zones

Example) What is the probability that zone 1, zone 2 and zone 3 are damaged simultaneously?

Probability that "a" is located in zone1 $\times$ **Probability that "b" is located in zone3** $+$ **Probability that "a" is located in zone3** $\times$ **Probability that "b" is located in zone1**

$$\frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{1}{3} = \frac{2}{9}$$

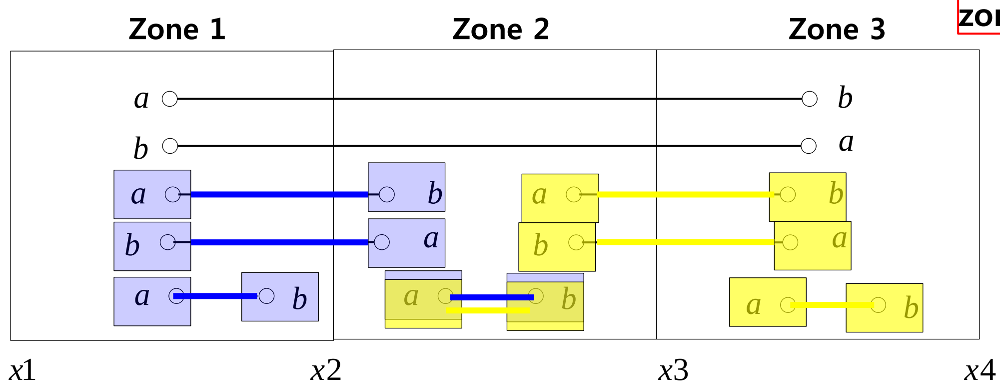
Three Zones Damage Case

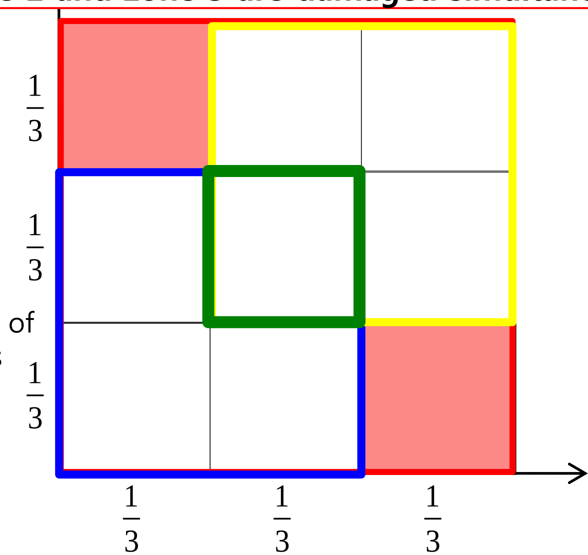
$$p_i = p \cdot r$$

$$p = p(x1, x2, L_s)$$

How can you obtain the value of "p" by different representation method when three zones are damaged?

In the figure, the red area means the probability that zone 1, zone 2 and zone 3 are damaged simultaneously.





Example) What is the probability that zone 1, zone 2 and zone 3 are damaged simultaneously?

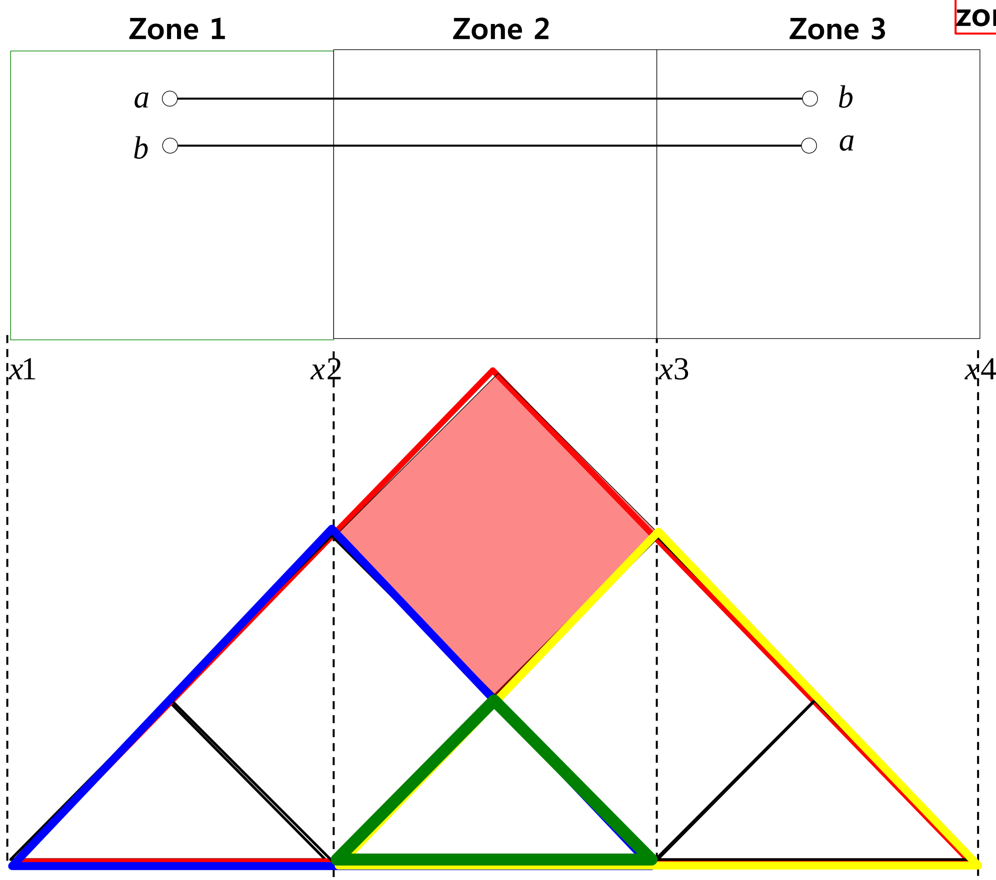
					Representation in terms of "p"
	Probability that "a" is located in zone1 or zone 2 or zone 3	$\times$	**Probability that "b" is located in zone1 or zone 2 or zone 3**	$\frac{3}{3} \times \frac{3}{3}$	$p(x_1, x_4)$
$-$	**Probability that "a" is located in zone1 or zone 2**	$\times$	**Probability that "b" is located in zone1 or zone 2**	$\frac{2}{3} \times \frac{2}{3}$	$p(x_1, x_3)$
$-$	**Probability that "a" is located in zone 2 or zone 3**	$\times$	**Probability that "b" is located in zone 2 or zone 3**	$\frac{2}{3} \times \frac{2}{3}$	$p(x_2, x_4)$
$+$	**Probability that "a" is located in zone 2**	$\times$	**Probability that "b" is located in zone 2**	$\frac{1}{3} \times \frac{1}{3}$	$p(x_2, x_3)$

$$= \frac{2}{9}$$

$$p_i = p \cdot r$$

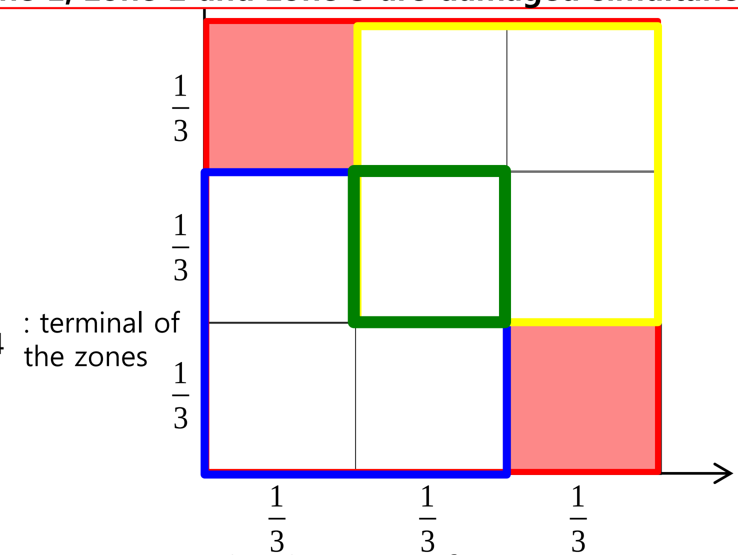
$$p = p(x1, x2, L_s)$$

How can you obtain the value of "p" by different representation method when three zones are damaged?



In the figure, the red area means the probability that zone 1, zone 2 and zone 3 are damaged simultaneously.

In the figure, the red area means the probability that zone 1, zone 2 and zone 3 are damaged simultaneously.



Representation in terms of "p"

$\frac{3}{3} \times \frac{3}{3} \Rightarrow p(x_1, x_4)$

$\frac{2}{3} \times \frac{2}{3} \Rightarrow p(x_1, x_3)$

$\frac{2}{3} \times \frac{2}{3} \Rightarrow p(x_2, x_4)$

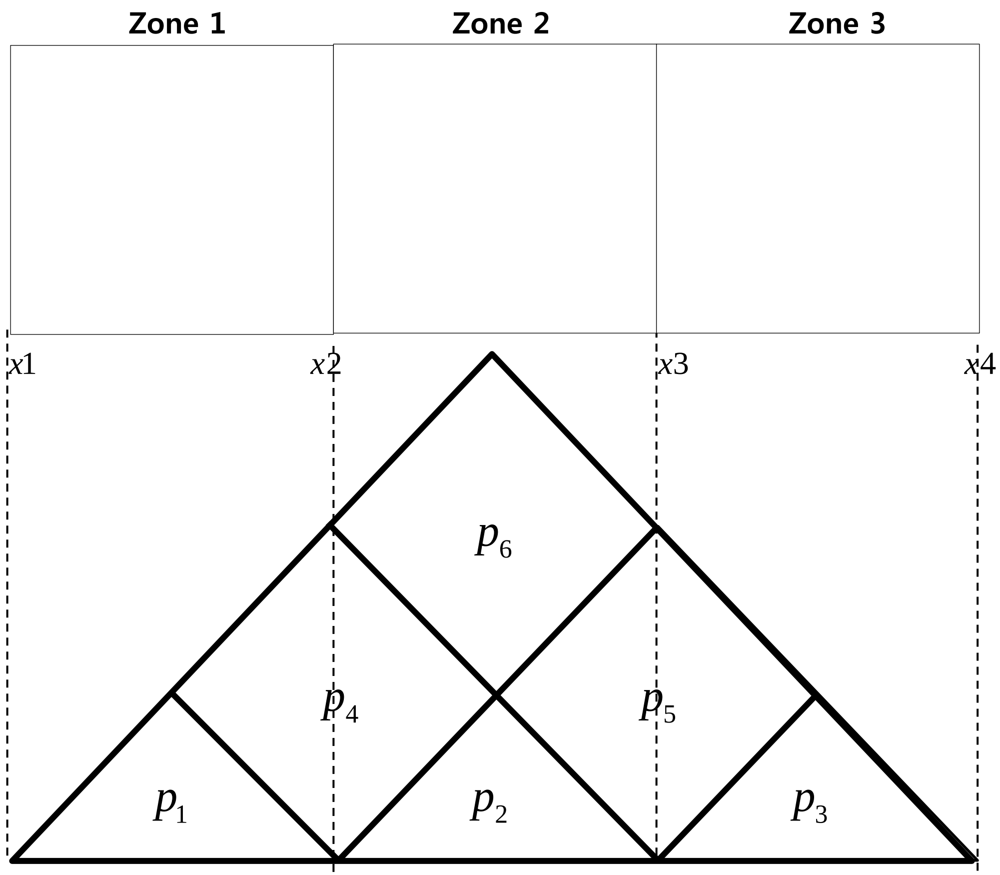
$\frac{1}{3} \times \frac{1}{3} \Rightarrow p(x_2, x_3)$

$\boxed{= \frac{2}{9}}$

Example) Box-Shaped Ship – Total Damage Cases

$$p_i = p \cdot r$$

$$p = p(x1, x2, L_s)$$



One zone damage case

$$p_1 = p(x_1, x_2)$$
$$p_2 = p(x_2, x_3)$$
$$p_3 = p(x_3, x_4)$$

Two zones damage case

$$p_4 = p(x_1, x_3) - p(x_1, x_2) - p(x_2, x_3)$$
$$p_5 = p(x_2, x_4) - p(x_2, x_3) - p(x_3, x_4)$$

Three zones damage case

$$p_6 = p(x_1, x_4) - p(x_1, x_3) - p(x_2, x_4) + p(x_2, x_3)$$

*Assume that the factor "r" is constant(r=1)

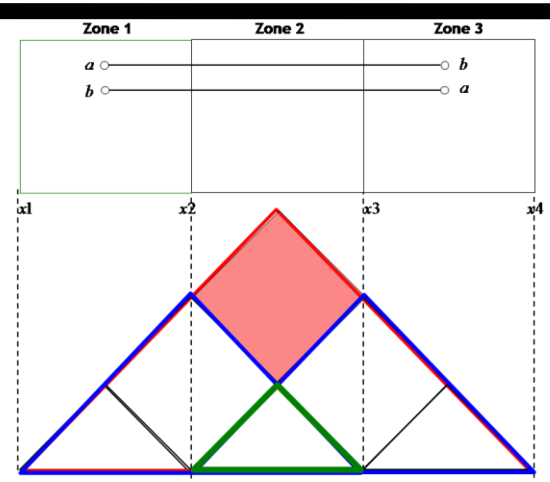
After the calculation of the factor "p_i" in each damage case, we can calculate "s_i" of "that damage case" in a given draft such as "*dp*", "*ds*", "*dl*".

$$A = \sum p_i \cdot s_i$$

$p(x_i, x_j)$: **This function gives the probability of all cases when the compartments between i[th] subdivision line and j[th] subdivision line can be damaged.**

Probability of Damage in Longitudinal Subdivision (p)

- Recurrence Formula for Three or More Adjacent Zones Damage Case

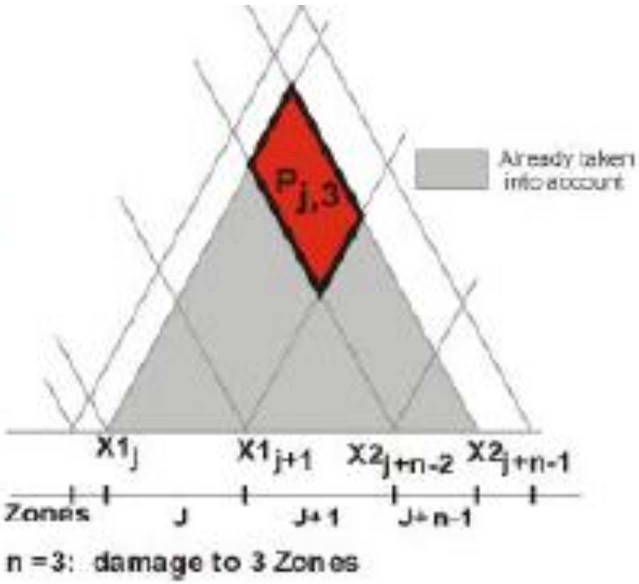


Three zones damage case

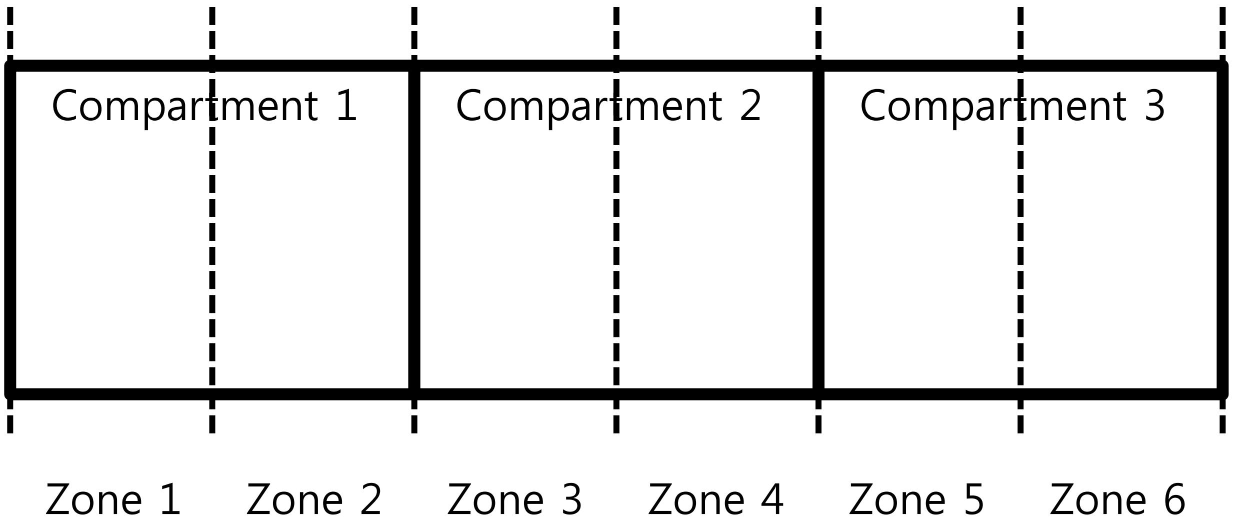
$$p_6 = p(x_1, x_4) - p(x_1, x_3) - p(x_2, x_4) + p(x_2, x_3)$$

Three or more adjacent zones, pure transverse subdivision:

$$p_{j,n} = p(x1_j, x2_{j+n-1}) - p(x1_j, x2_{j+n-2}) - p(x1_{j+1}, x2_{j+n-1}) + p(x1_{j+1}, x2_{j+n-2})$$



"Virtual Subdivision"
- Compartment vs Zone



Compartment – an onboard space within watertight boundaries.

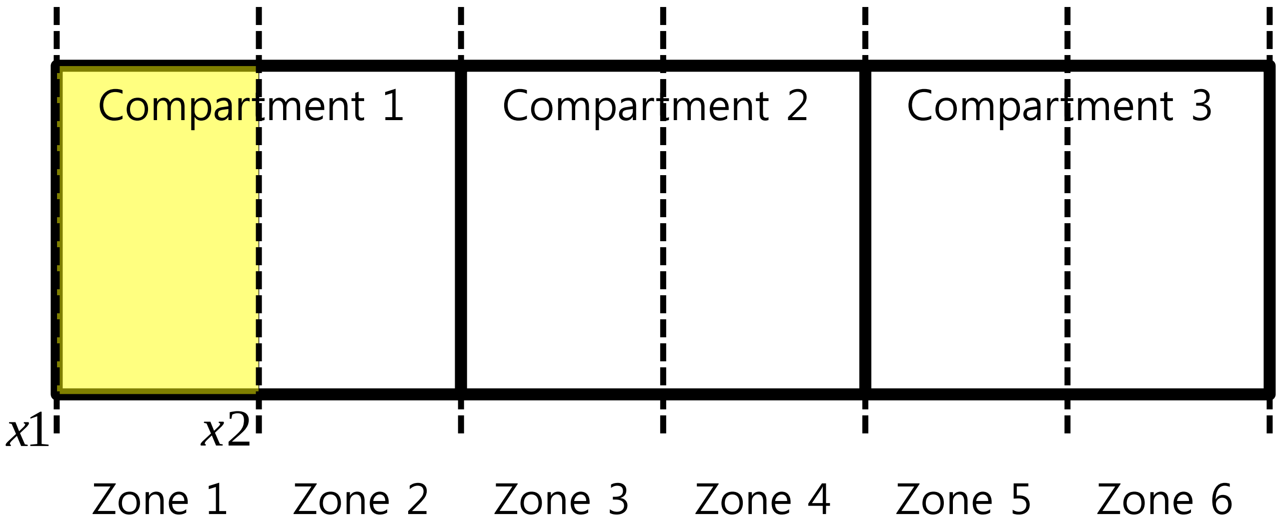
: **Actual subdivision** of the ship.

Zone – a longitudinal interval of the ship within the subdivision length.

: **Conceptual subdivision** for calculation of the probability of damage "p_i".

"Virtual Subdivision"
- One zone damage case vs Multi zone damage case

Compartment – an onboard space within watertight boundaries.
Zone – a longitudinal interval of the ship within the subdivision length.



Only one zone is damaged, this case is called "one zone damage case".
Two adjacent zones are damaged, this case is called "two zone damage case"

Example) One zone damage case : (Zone 1), (Zone 2), …
Two zone damage case : (Zone 1, Zone 2), (Zone 2, Zone 3), …

And, the length of damage in this case can be expressed by x1 and x2.

$x1$ = the distance from the aft terminal to the aft end of the zone in question.
$x2$ = the distance from the aft terminal to the forward end of the zone in question.

$x1$ and $x2$ represent the terminals of the compartment or group of compartments.

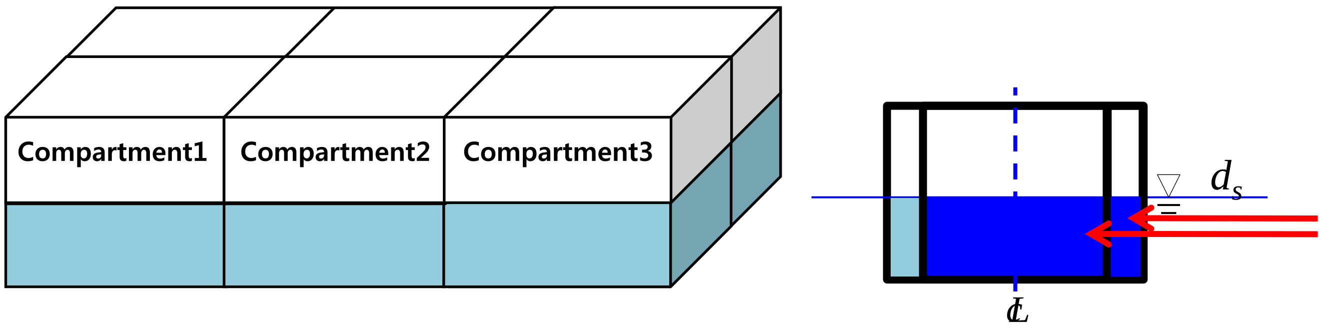
13-3 Probability of Damage in Transverse Subdivision (r)

$p_i = p \cdot r$

Probability related to the transverse subdivision

Is there only longitudinal subdivision to consider "p_i"?

d_s : Deepest subdivision draft



No!

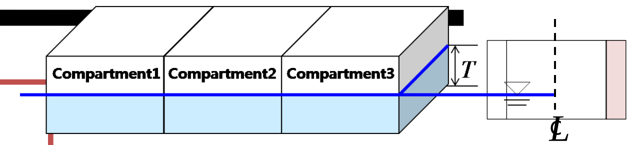
We have to consider the probability related to the **transverse subdivision and penetration**.

The probability of damage in transverse subdivision and penetration is represented by the **factor "r"**.

The factor "r" is determined **after deciding the longitudinal damage case**.

Probability of Damage in Transverse Subdivision (r)

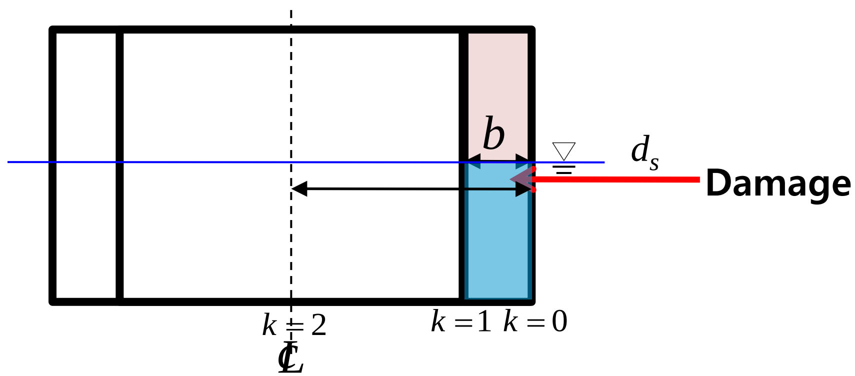
$$p_i = p \cdot r$$



What is the factor "r"?

: Probability of damage in the transverse subdivision "r"

$$r = r(x1, x2, L_s, b, k)$$

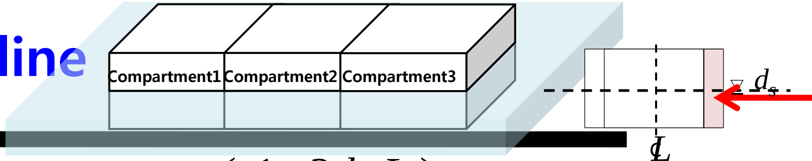


: the **factor "r"** is dependent on the **penetration depth "b"** and **the number of a particular longitudinal bulkhead "k".**

<u>**"k" is counted from shell towards the centerline**</u>.

<u>**"b" is measured at deepest subdivision draught "d_s"**</u>

Range of the factor “b”: toward the centerline



$p_i = p \cdot r \quad r = r(x1, x2, b, L_s)$

b : penetration depth
k : the number of a particular longitudinal bulkhead

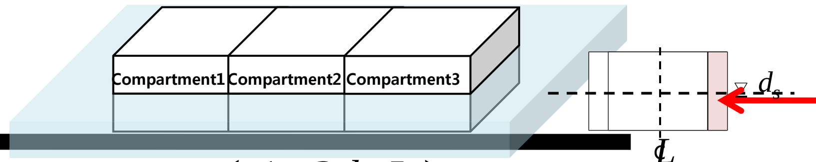
What is the factor “r“ when this factor “b” is zero?
And what is the factor “r“ when this factor “b” is $B/2$?

Where “B” is the maximum breadth of the ship at the deepest subdivision draught “d_s”.

The value of “r” is equal to 0, if the penetration depth is 0.

The value of “r” is equal to 1, if the penetration depth is $B/2$.

“b" is not being taken greater than $B/2$.
The transverse penetration is calculated **only considering one side of the ship**.

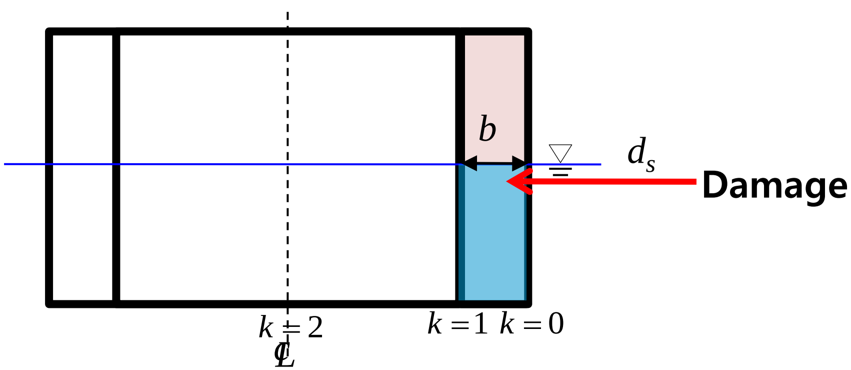


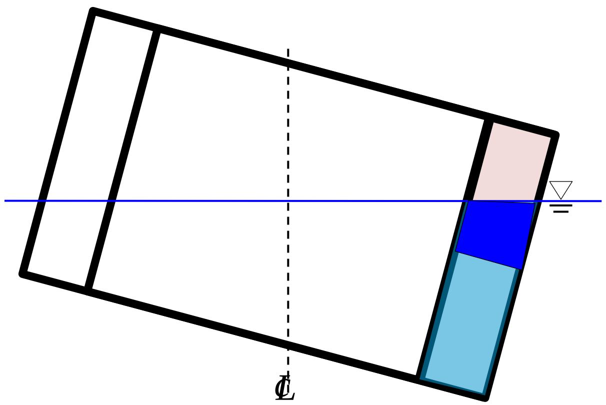
$p_i = p \cdot r \quad r = r(x1, x2, b, L_s)$

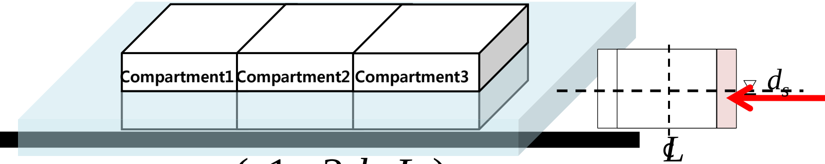
b : penetration depth
k : the number of a particular longitudinal bulkhead

Why the factor "b" is only considered to extend to $B/2$?

When the first compartment is damaged.







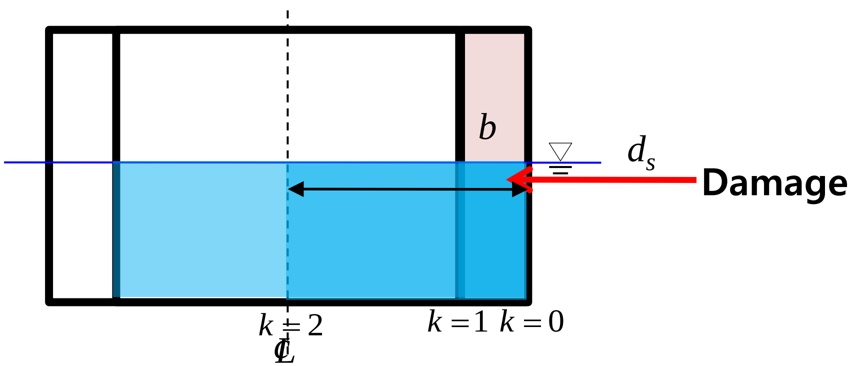
$p_i = p \cdot r \quad r = r(x1, x2, b, L_s)$

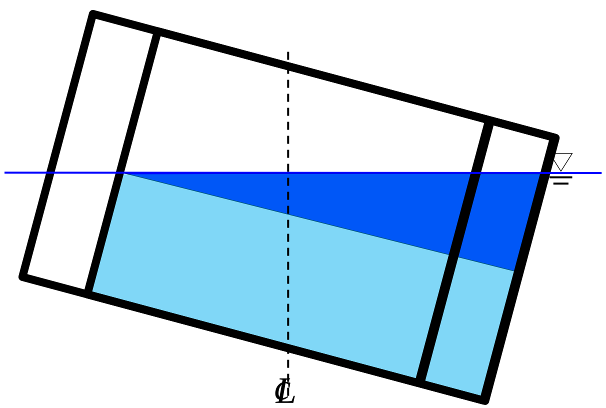
b : penetration depth
k : the number of a particular longitudinal bulkhead

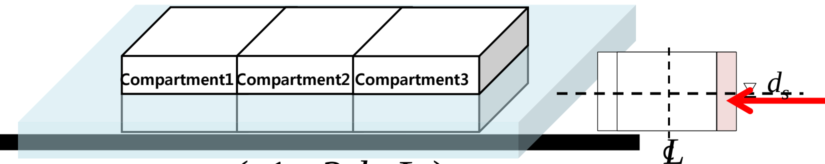
Why the factor "b" is only considered to extend to $B/2$?

When the second compartment is damaged.

It is the most severe damage case because the factor "b" is considered to extent to B/2







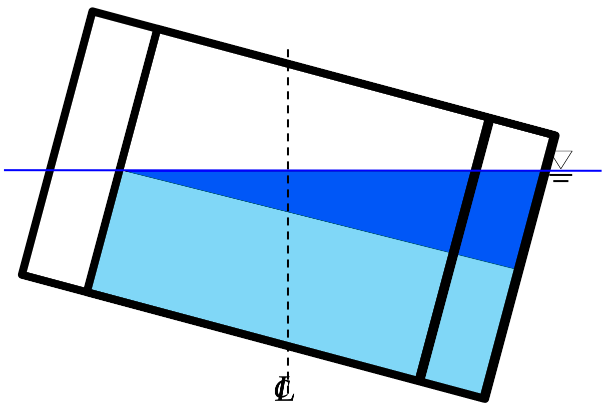
$p_i = p \cdot r \quad r = r(x1, x2, b, L_s)$

b : **penetration depth**

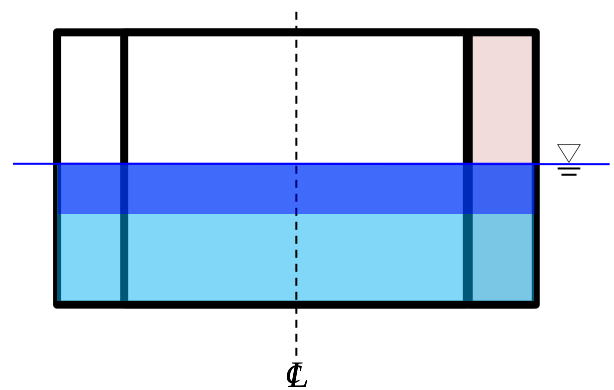
k : **the number of a particular longitudinal bulkhead**

Why the factor "b" is only considered to extend to $B/2$?

It is the most severe damage case because the factor "b" is considered to extent to B/2



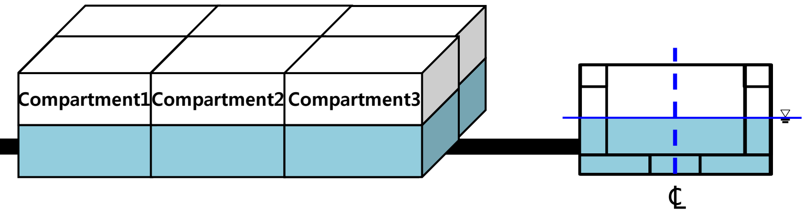
What if the factor "b" is considered to extent to B?



Because the result calculated for one side of the ship causes **more severe result** than for both side of the ship, the factor "b" is only considered to extend to B/2.

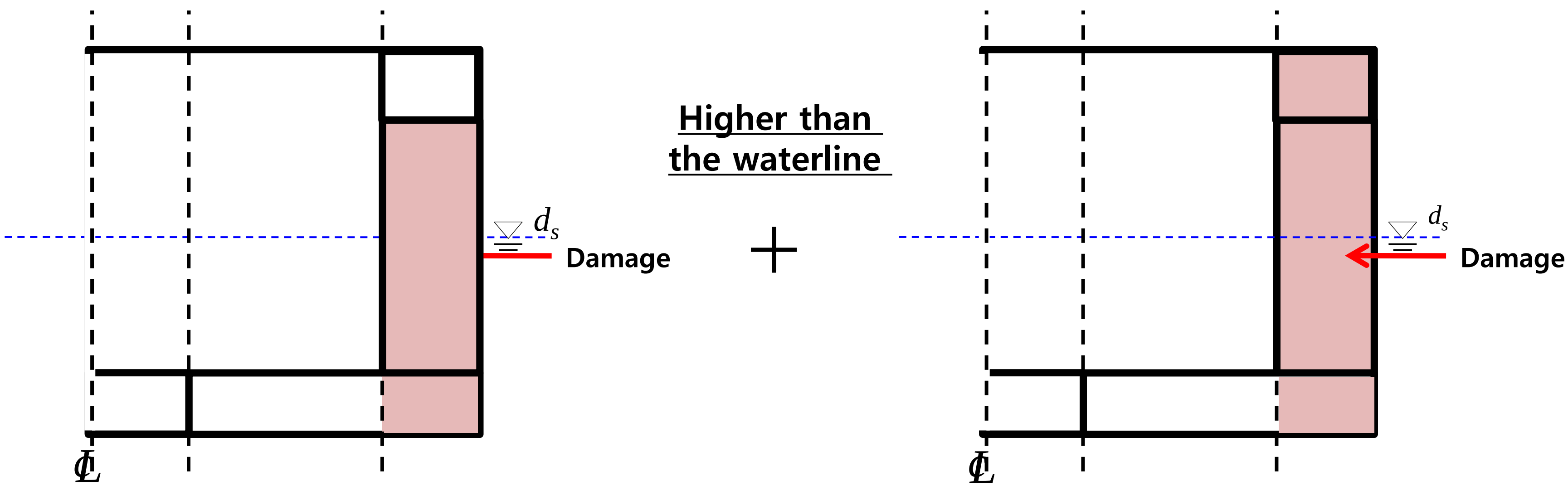
13-4 Probability of Damage
- Vertical Extent

Probability of Damage
- Vertical Extent

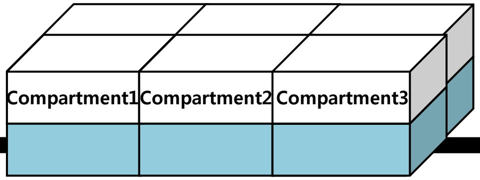


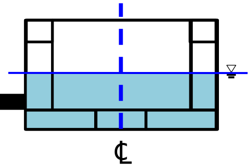
The assumed vertical extent of damage is to extend from the baseline upwards to any watertight horizontal subdivision **above the waterline** or **higher**. That is, higher horizontal subdivision is also to be assumed.

Example) $k=1$



Vertical Extent : "Lesser Extent"



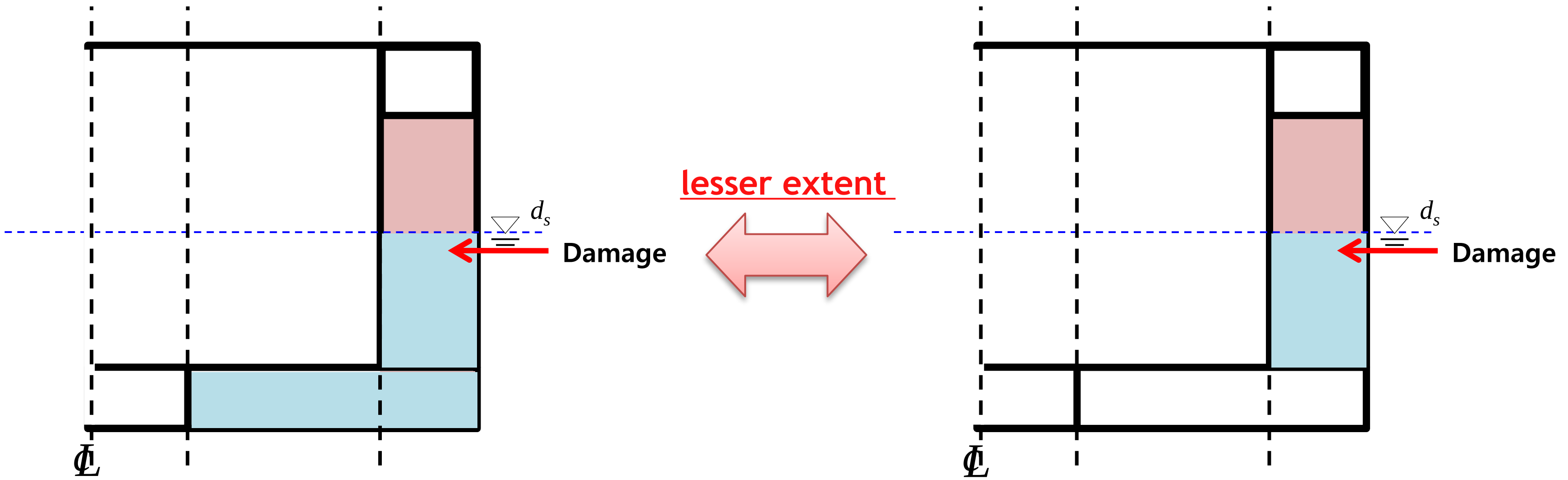


The flooding always extends to baseline?

No.

If a lesser extent of damage will give a more severe result, such extent is to be assumed.

Example) $k=1$



13-5 Example of Probability Damage Stability of a Simplified 7,000 TEU Container Carrier

Example) 7,000 TEU Container Carrier
- One(1) Zone Damage – Z8

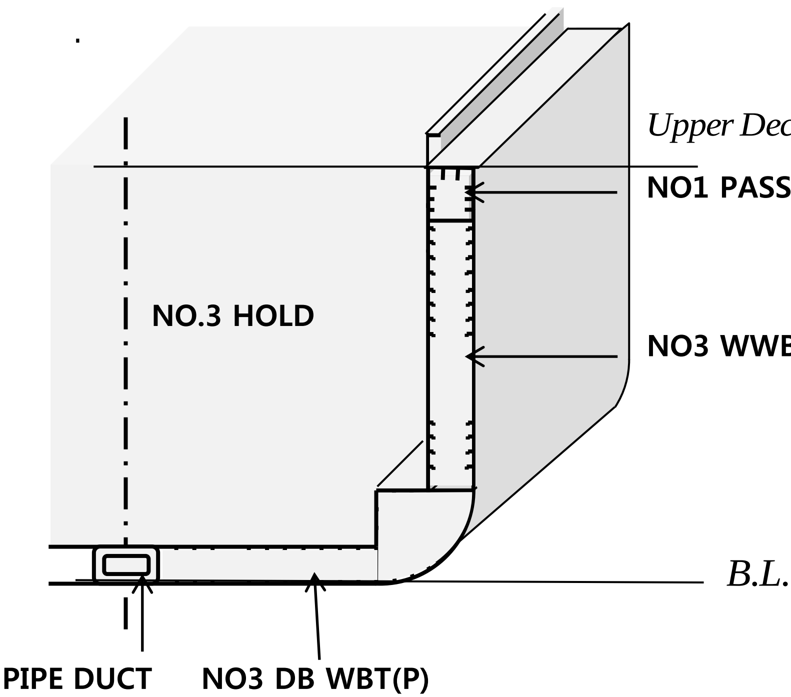
$r = r(x1, x2, b, L_s)$

b : penetration depth
k : the number of a particular longitudinal bulkhead

How can you obtain the values of r?

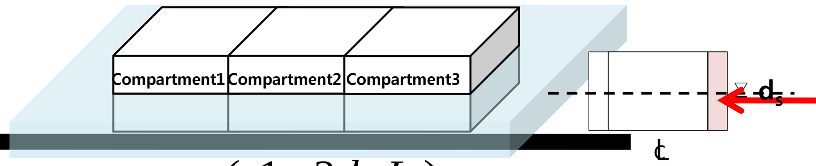
7,000 TEU Container Carrier

Example) One(1) Zone Damage – Z8



Extend the concept learned from the examples of box shaped barge.

Case 1) Three longitudinal Bulkheads (two wing tanks + two cargo holds)



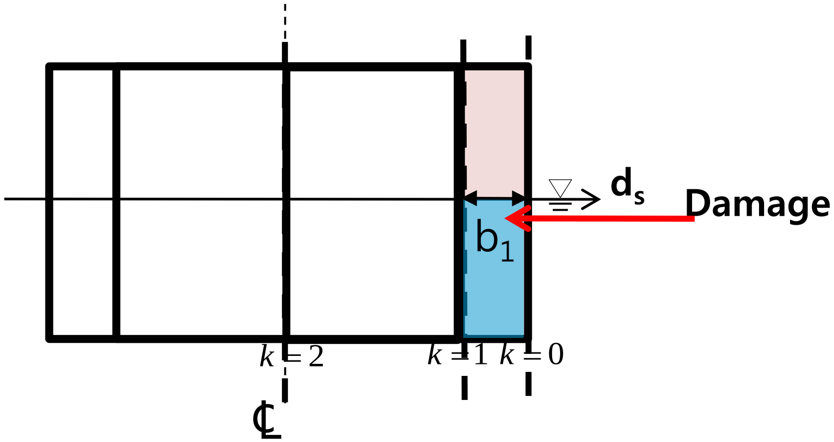
Assume that we calculate the value of r in the port side.

$$r = r(x1, x2, b, L_s)$$

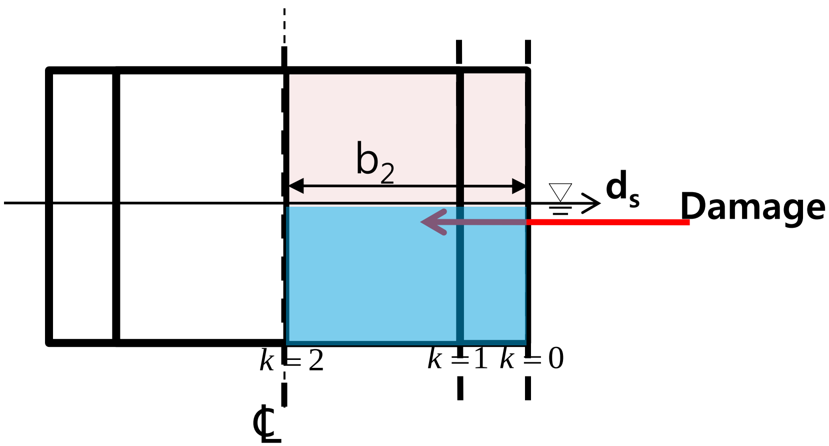
* b is measured at **deepest subdivision draught**

b : penetration depth
k : the number of a particular longitudinal bulkhead

How can you obtain the value of r for a box shaped ship?

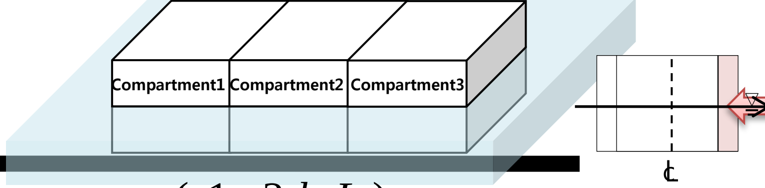


K=1 : b=b_1
(wing tank(P))



K=2 : b=b_2=B/2
(wing tank(P)+cargo hold(P))

Case 2) Two longitudinal Bulkheads (two wing tanks + one cargo holds)



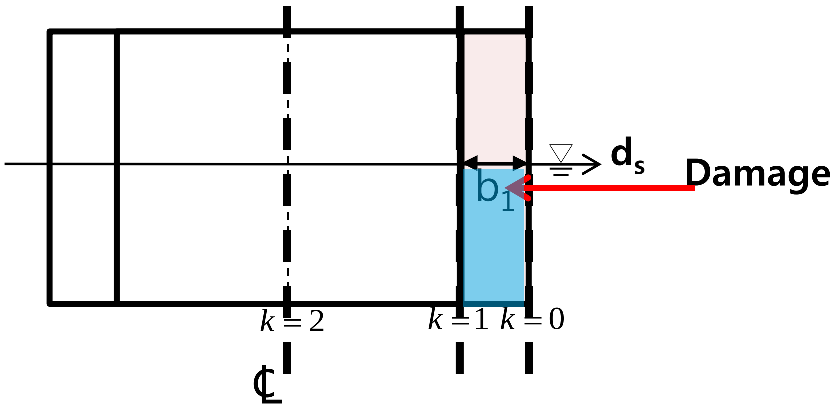
Assume that we calculate the value of r in the port side.

$r = r(x1, x2, b, L_s)$

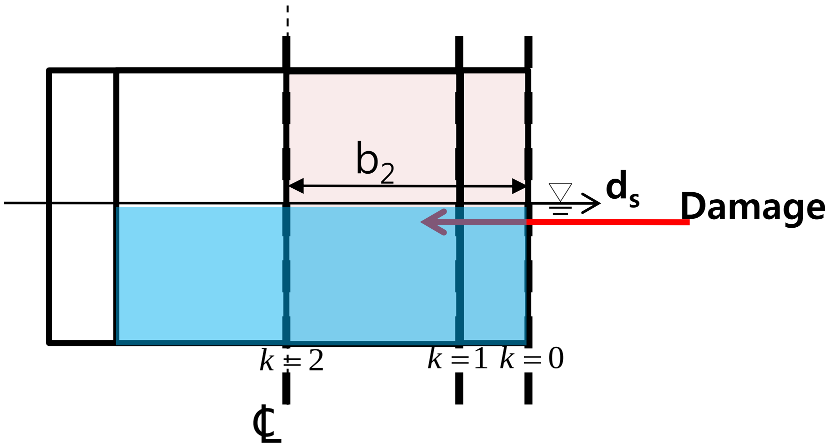
* b is measured at **deepest subdivision draught**

b : penetration depth
k : the number of a particular longitudinal bulkhead

How can you obtain the value of r for a box shaped ship?



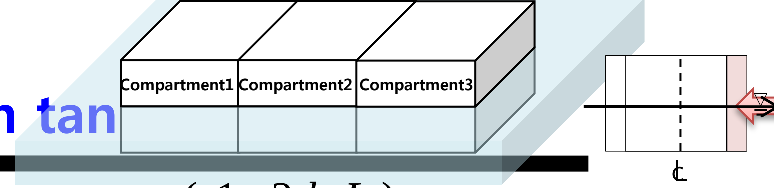
K=1 : b=b_1
(wing tank(P))



K=2 : b=b_2=B/2
(wing tank(P)+cargo hold)

Case 3) Two longitudinal Bulkheads

(two wing tanks + one cargo hold + two double bottom tan



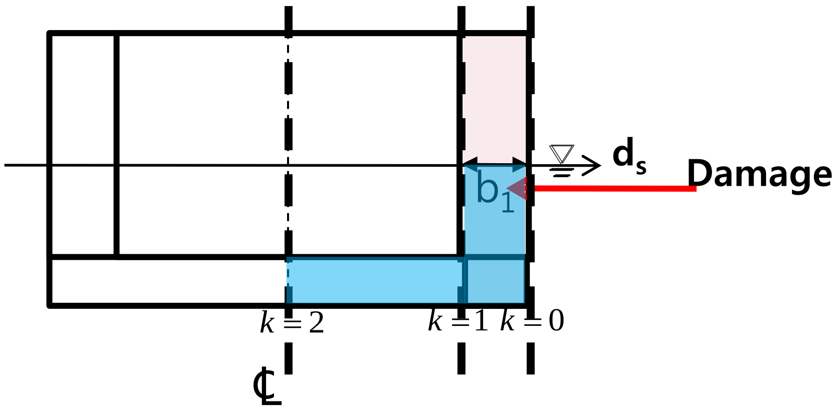
Assume that we calculate the value of r in the port side.

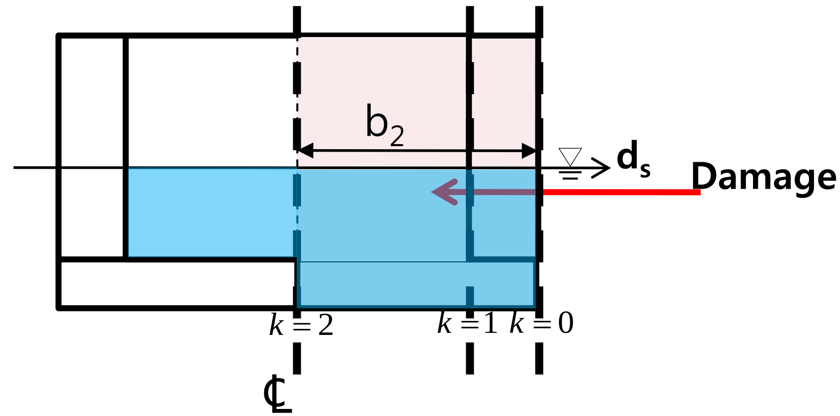
* b is measured at **deepest subdivision draught**

$$r = r(x1, x2, b, L_s)$$

b : penetration depth
k : the number of a particular longitudinal bulkhead

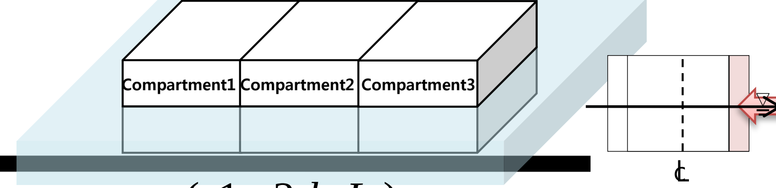
How can you obtain the value of r for a box shaped ship?





K=1 : $b=b_1$
(wing tank(P)+double bottom tank(P))

K=2 : $b=b_2=B/2$
(wing tank(P)+double bottom tank(P)+cargo hold)



Assume that we calculate the value of r in the port side.

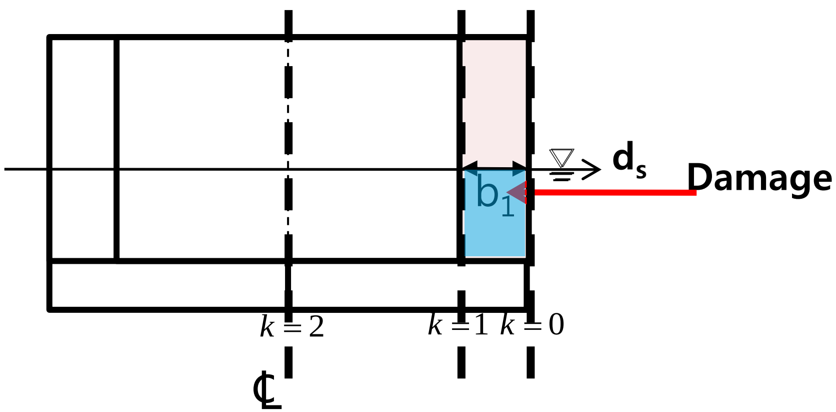
* b is measured at **deepest subdivision draught**

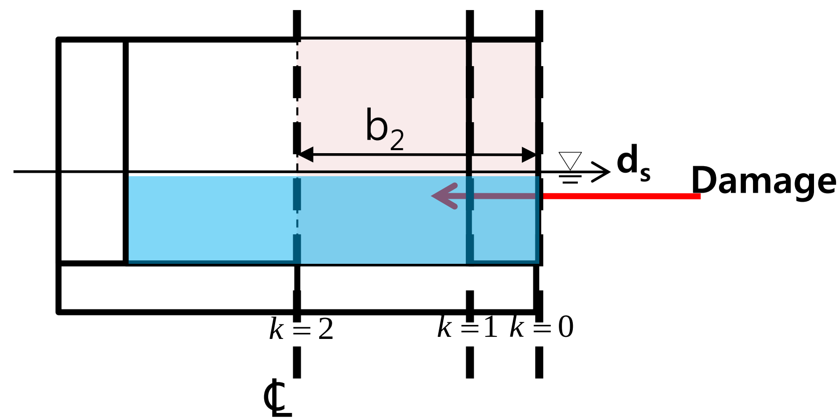
$$r = r(x1, x2, b, L_s)$$

b : penetration depth
k : the number of a particular longitudinal bulkhead

How can you obtain the value of r for a box shaped ship?

*Lesser extent damage cases



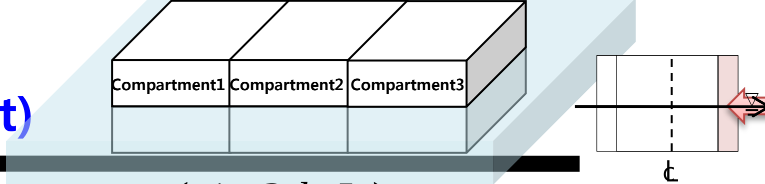


K=1 : $b=b_1$
(wing tank(P))

K=2 : $b=b_2=B/2$
(wing tank(P)+cargo hold)

Case 4) Two longitudinal Bulkheads

(two wing tanks + one cargo hold+ two double bottom tanks+ pipe duct)



Assume that we calculate the value of r in the port side.

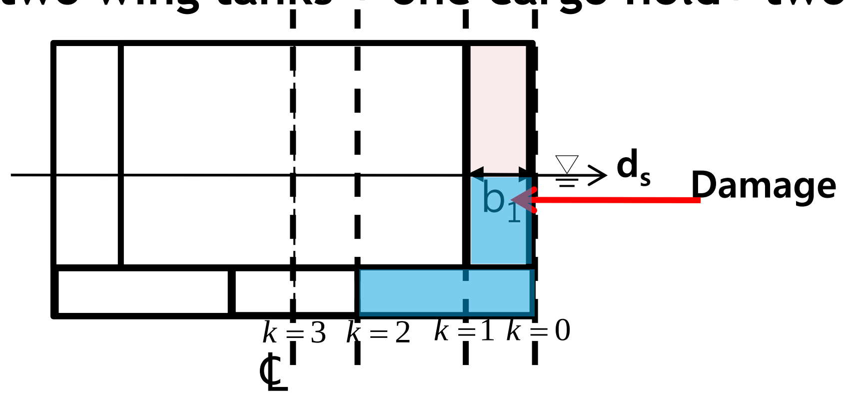
* b is measured at **deepest subdivision draught**

$$r = r(x1, x2, b, L_s)$$

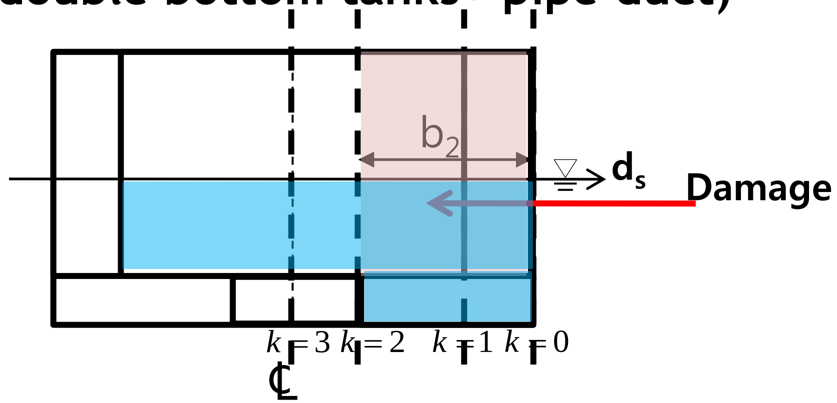
b : penetration depth
k : the number of a particular longitudinal bulkhead

How can you obtain the value of r for a box shaped ship?

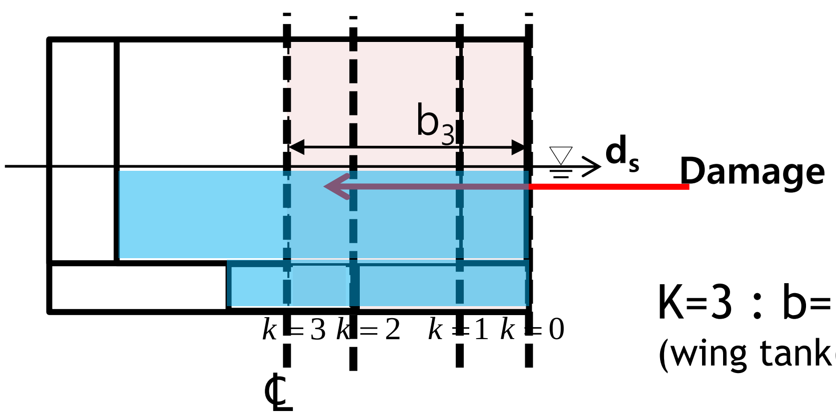
Case : two longitudinal bulkhead
(two wing tanks + one cargo hold+ two double bottom tanks+ pipe duct)



K=1 : $b=b_1$
(wing tank(P)+double bottom tank(P))

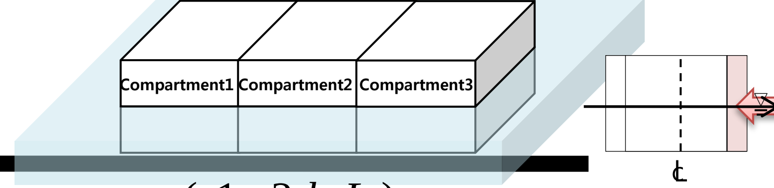


K=2 : $b=b_2$
(wing tank(P)+double bottom tank(P)+cargo hold)



If the upper part of a longitudinal bulkhead is below the deepest subdivision loadline the vertical plane used for determination of b is assumed to <u>extend upwards to the deepest subdivision waterline.</u>

K=3 : $b=b_3=B/2$
(wing tank(P) + double bottom tank(P) + cargo hold + pipe duct)



Assume that we calculate the value of r in the port side.

$r = r(x1, x2, b, L_s)$

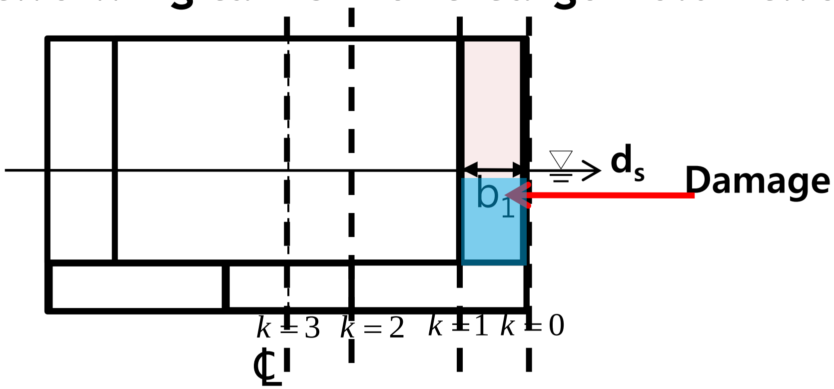
* b is measured at **deepest subdivision draught**

b : penetration depth
k : the number of a particular longitudinal bulkhead

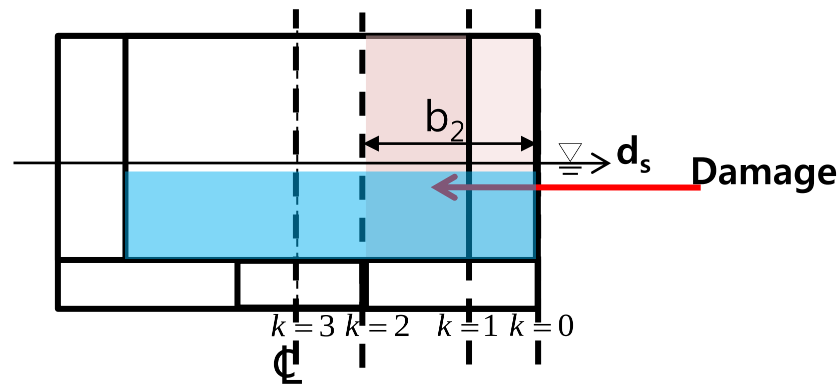
How can you obtain the value of r for a box shaped ship?

***Lesser extent damage cases**

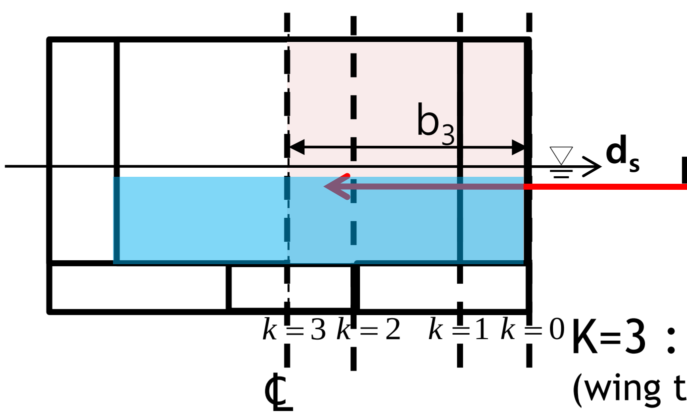
Case : two longitudinal bulkhead
(two wing tanks + one cargo hold+ two double bottom tanks+ pipe duct)



K=1 : b=b_1
(wing tank(P))



K=2 : b=b_2
(wing tank(P) +cargo hold)

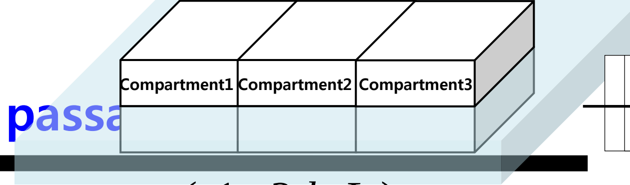


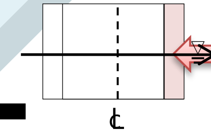
K=3 : b=b_3=B/2
(wing tank(P) + cargo hold)

In the flooding calculations carried out according to the regulations, only one breach of the hull and only one free surface need to be assumed. The assumed vertical extent of damage is to extend from the baseline upwards to any watertight horizontal subdivision above the waterline or higher. However, if a lesser extent of damage will give a more severe result, such extent is to be assumed.

Case 4) Two longitudinal Bulkheads

(two wing tanks + one cargo hold+ two double bottom tanks+ pipe duct+passa





Assume that we calculate the value of r in the port side.

* b is measured at **deepest subdivision draught**

$$r = r(x1, x2, b, L_s)$$

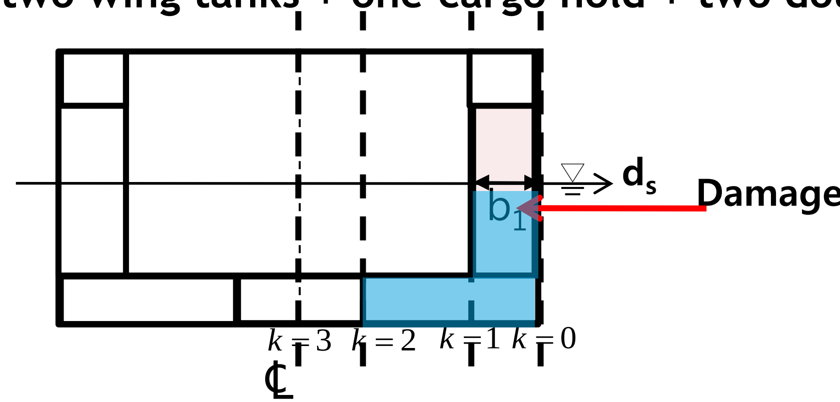
b : penetration depth

k : the number of a particular longitudinal bulkhead

How can you obtain the value of r for a box shaped ship?

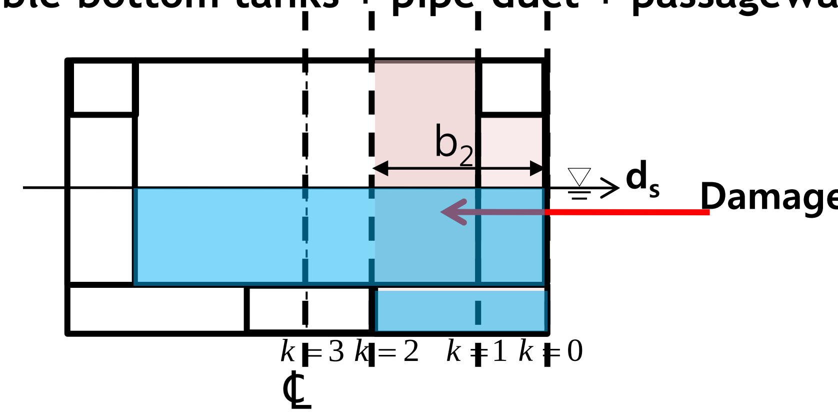
Case : two longitudinal bulkhead

(two wing tanks + one cargo hold + two double bottom tanks + pipe duct + passageway)



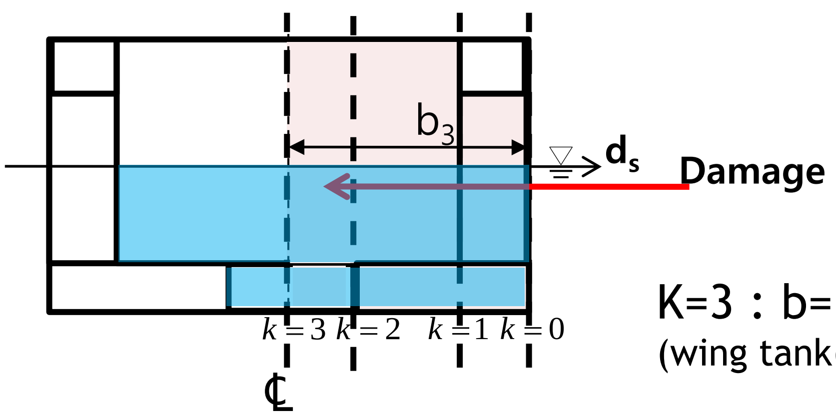
K=1 : b=b_1

(wing tank(P)+double bottom tank(P))



K=2 : b=b_2

(wing tank(P)+double bottom tank(P)+cargo hold)

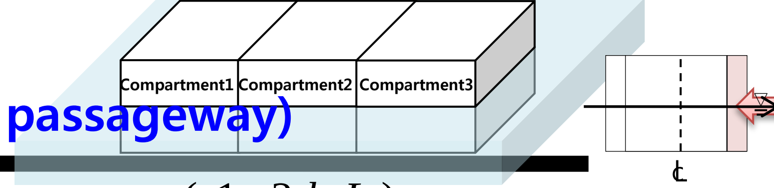


K=3 : b=b_3=B/2

(wing tank(P) + double bottom tank(P) + cargo hold + pipe duct)

Case 4) Two longitudinal Bulkheads

(two wing tanks + one cargo hold+ two double bottom tanks+ pipe duct+passageway)



Assume that we calculate the value of r in the port side.

* b is measured at **deepest subdivision draught**

$r = r(x1, x2, b, L_s)$

b : penetration depth

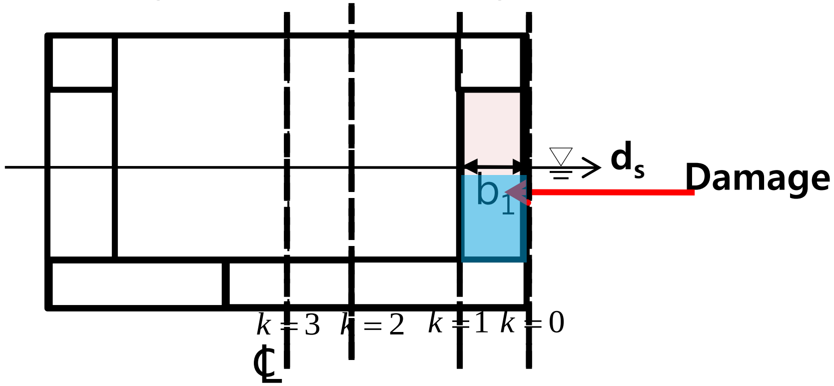
k : the number of a particular longitudinal bulkhead

How can you obtain the value of r for a box shaped ship?

***Lesser extent damage cases**

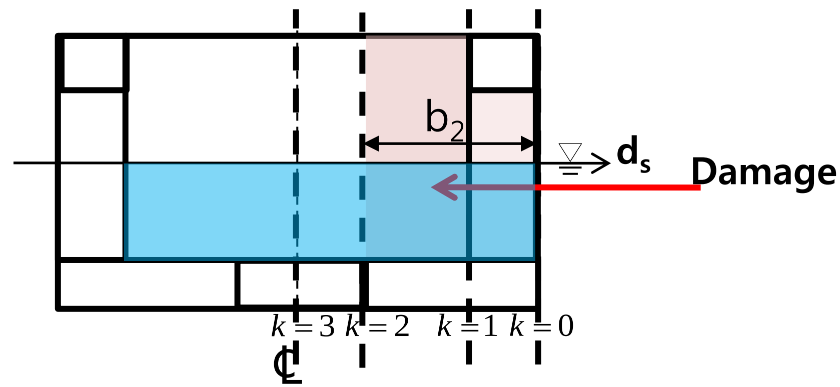
Case : two longitudinal bulkhead

(two wing tanks + one cargo hold + two double bottom tanks + pipe duct + passageway)



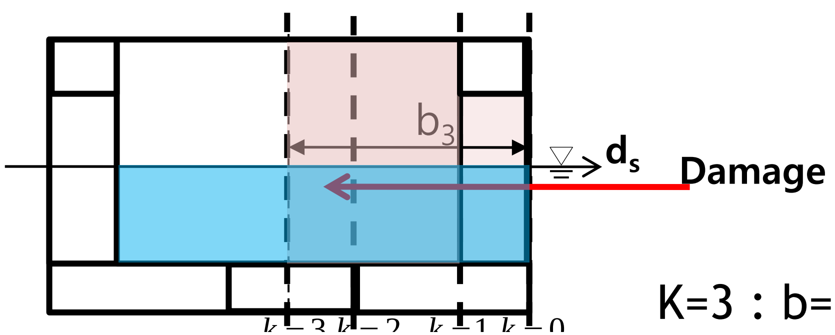
K=1 : b=b_1

(wing tank(P))



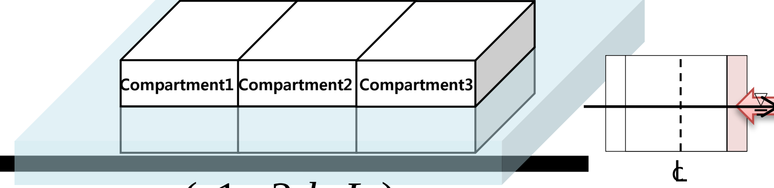
K=2 : b=b_2

(wing tank(P)+cargo hold)



K=3 : b=b_3=B/2

(wing tank(P) + cargo hold)



Assume that we calculate the value of r in the port side.

* b is measured at **deepest subdivision draught**

$$r = r(x1, x2, b, L_s)$$

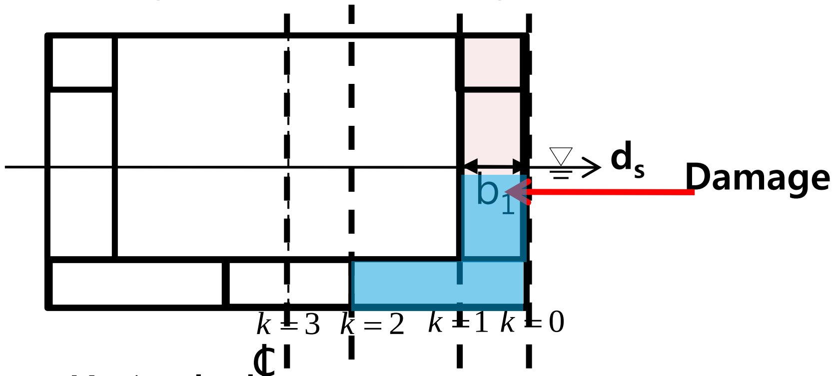
b : penetration depth
k : the number of a particular longitudinal bulkhead

How can you obtain the value of r for a box shaped ship?

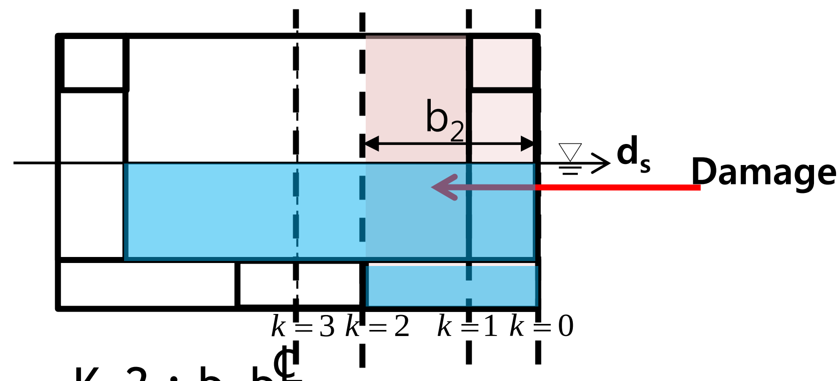
*** Higher horizontal subdivision**

Case : two longitudinal bulkhead

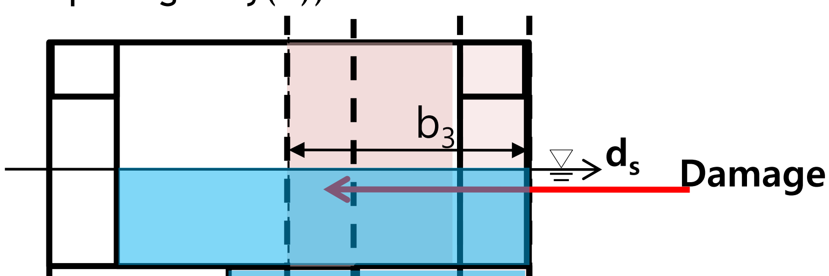
(two wing tanks + one cargo hold + two double bottom tanks + pipe duct + passageway)



K=1 : $b = b_1$
(wing tank(P)+double bottom tank(P)+ passageway(P))



K=2 : $b = b_2$
(wing tank(P)+double bottom tank(P)+cargo hold+ passageway(P))

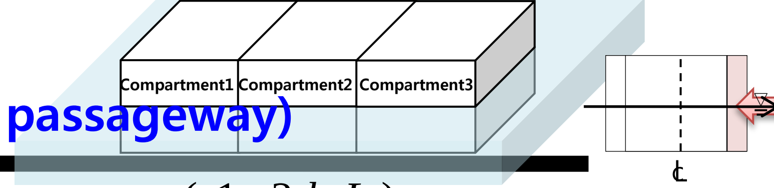


K=3 : $b = b_3 = B/2$
(wing tank(P) + double bottom tank(P) + cargo hold + pipe duct+ passageway(P))

In the flooding calculations carried out according to the regulations, only one breach of the hull and only one free surface need to be assumed. The assumed vertical extent of damage is to extend from the baseline upwards to any watertight horizontal subdivision above the waterline or higher. However, if a lesser extent of damage will give a more severe result, such extent is to be assumed.

Case 4) Two longitudinal Bulkheads

(two wing tanks + one cargo hold+ two double bottom tanks+ pipe duct+passageway)



Assume that we calculate the value of r in the port side.

$r = r(x1, x2, b, L_s)$

* b is measured at **deepest subdivision draught**

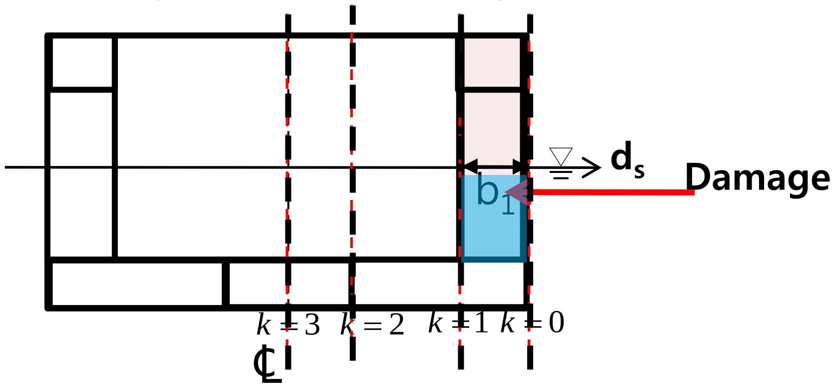
b : penetration depth
k : the number of a particular longitudinal bulkhead

How can you obtain the value of r for a box shaped ship?

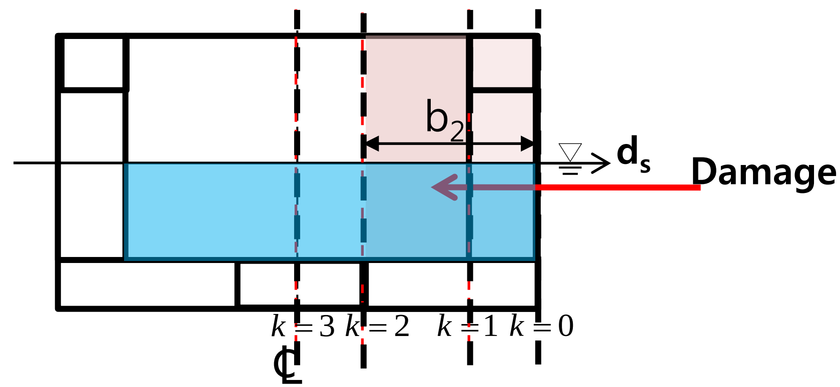
*** Higher horizontal subdivision**

***Lesser extent damage cases**

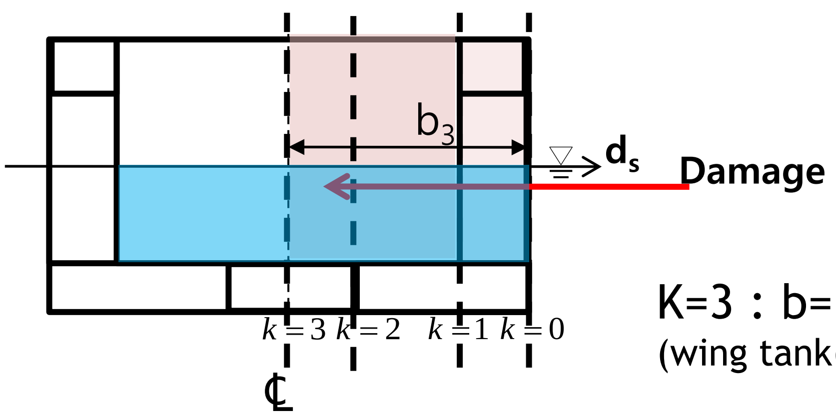
Case : two longitudinal
(two wing tanks + one cargo hold + two double bottom tanks + pipe duct + passageway)



K=1 : b=b_1
(wing tank(P)+ passageway(P))

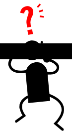


K=2 : b=b_2
(wing tank(P) +cargo hold+ passageway(P))



K=3 : b=b_3=B/2
(wing tank(P) cargo hold + pipe duct+ passageway(P))

Permeability



When the ship is flooding, how to calculate the actual amount of flooding water?

The compartment of the ship already contains cargo, machinery, liquids, accommodations, or any other equipment or material.

To consider this characteristic, the concept of permeability is introduced.

Permeability (μ) of a space is the proportion of the immersed volume of that space which can be occupied by water.

For the purpose of the subdivision and damage stability calculations of the regulations, the permeability of each general compartment or part of a compartment shall be as follows:

Spaces	Permeability
Appropriated to stores	0.60
Occupied by accommodation	0.95
Occupied by machinery	0.85
Void spaces	0.95
Intended for liquids	0 or 0.95*

For the purpose of the subdivision and damage stability calculations of the regulations, the permeability of each cargo compartment or part of a compartment shall be as follows:

Spaces	Permeability at draught *ds*	Permeability at draught *dp*	Permeability at draught *dl*
Dry cargo spaces	0.70	0.80	0.95
Container spaces	0.70	0.80	0.95
Ro-ro spaces	0.90	0.90	0.95
Cargo liquids	0.70	0.80	0.95

Attained Subdivision Index "A" : Check attained index "A"

Producing an index A requires calculation of various damage scenarios defined by the extent of damage and the initial loading conditions of the ship before damage.

Three loading conditions are to be considered and the result weighted as follows:

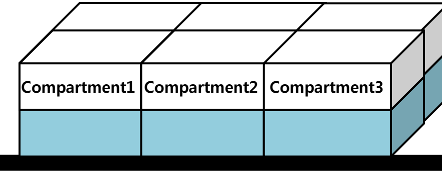
$$
\begin{array}{lll}
As, Ap, Al \geq 0.5R & & \text{: for cargo ships} \\
 \geq 0.9R & & \text{: for passenger ships} \\
A \geq R & \text{Where } A = 0.4A_s + 0.4A_p + 0.2A_l &
\end{array}
$$

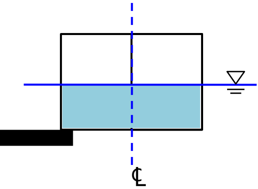
Where the indices s, p and l represent the three loading conditions and the factor to be multiplied to the index indicates **how the index A from each loading condition is weighted**.

We can assume that the meaning of the weight factors 0.4, 0.4, 0.2.
In the ship's lifecycle, the lightship condition is rarely exist.
Normally, the loading condition is performed between the scantling draft and design draft. Thus, the weight factor considers this cruising condition.

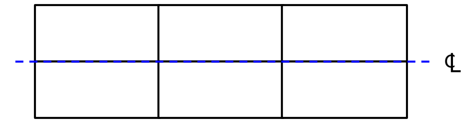
13-6 Probability of Survivability

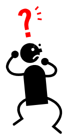
Probability of Survivability





$$A = \sum p_i \times s_i$$





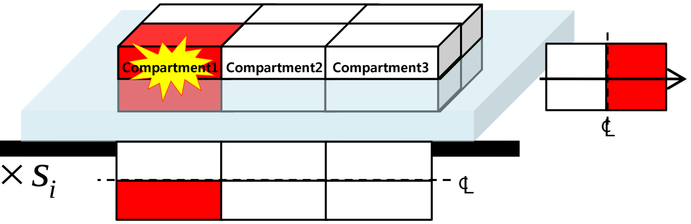
What is the factor "s_i"?

: The factor "s_i" is the probability of survivability after flooding in a <u>given damage condition</u>.

: Calculation the probability of survivability in a given "Damage Case"

→ Dependent on the "initial draft (ds, dp, dl)"

Probability of Survivability : Factor "s_i"

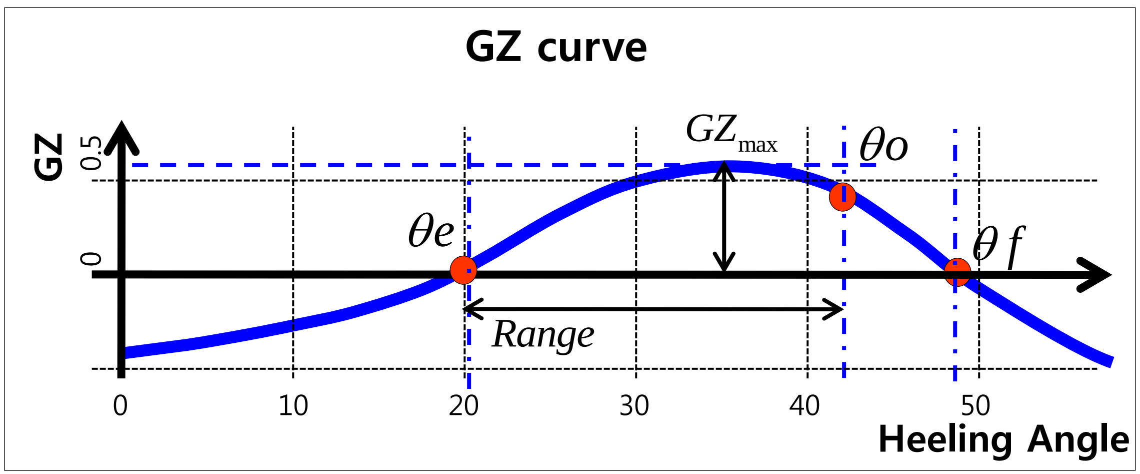


$$A = \sum p_i \times s_i$$

What is related to the factor "s_i"?

$$s_i = s_i(\theta e, \theta v, GZ_{\max}, Range, Flooding\ stage)$$ **(For cargo ships)**

: the **factor "s"** is to be calculated according to the range of GZ curve and GZ max.



θe : Equilibrium heel angle.
θv : $\theta v = \text{minimum}(\theta f, \theta o)$
(in this case, θv equals to θo)
GZ_{max} : Maximum value of GZ.
$Range$: Range of positive righting arm.
$Flooding\ stage$: Discrete step during the flooding process.

θf : angle of flooding (righting arm becomes negative)
θo : angle at which an **"opening"** incapable of being closed weathertight becomes submerged.

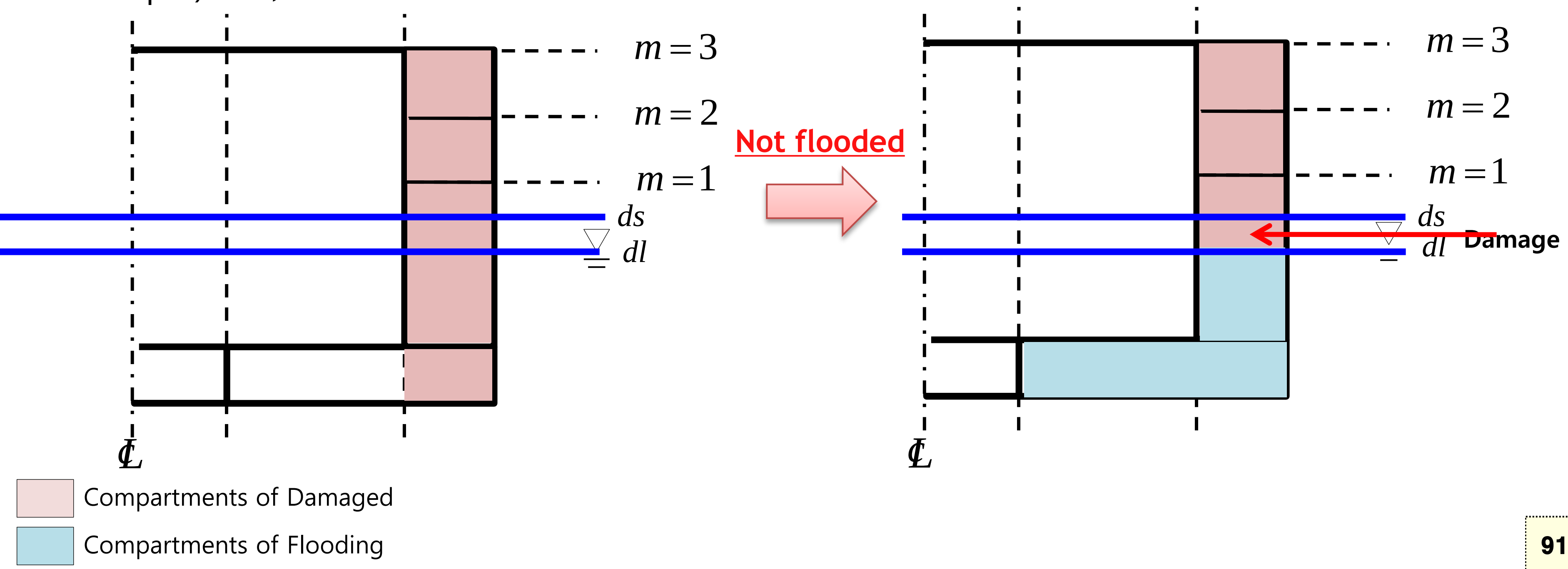
Consideration of Horizontal Subdivision in Flooding Stage - Factor "v_m"

When the horizontal watertight boundaries above the waterline are considered, the "s_i" value is obtained **by multiplying the reduction factor "v_m"**.

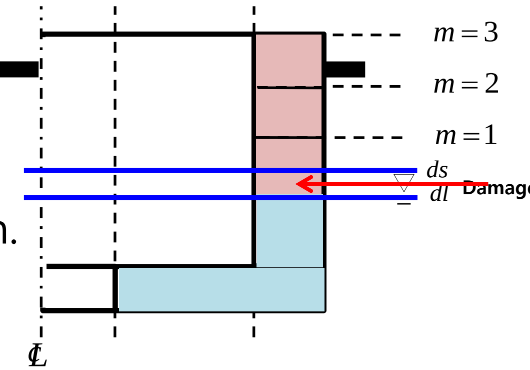
"v_m" represents **the probability that the spaces above the horizontal subdivision will not be flooded**.

Where "m" represents each **horizontal boundary** counted upwards from the waterline **under consideration**.

Example) $k=1$, $m=3$ $d=dl$

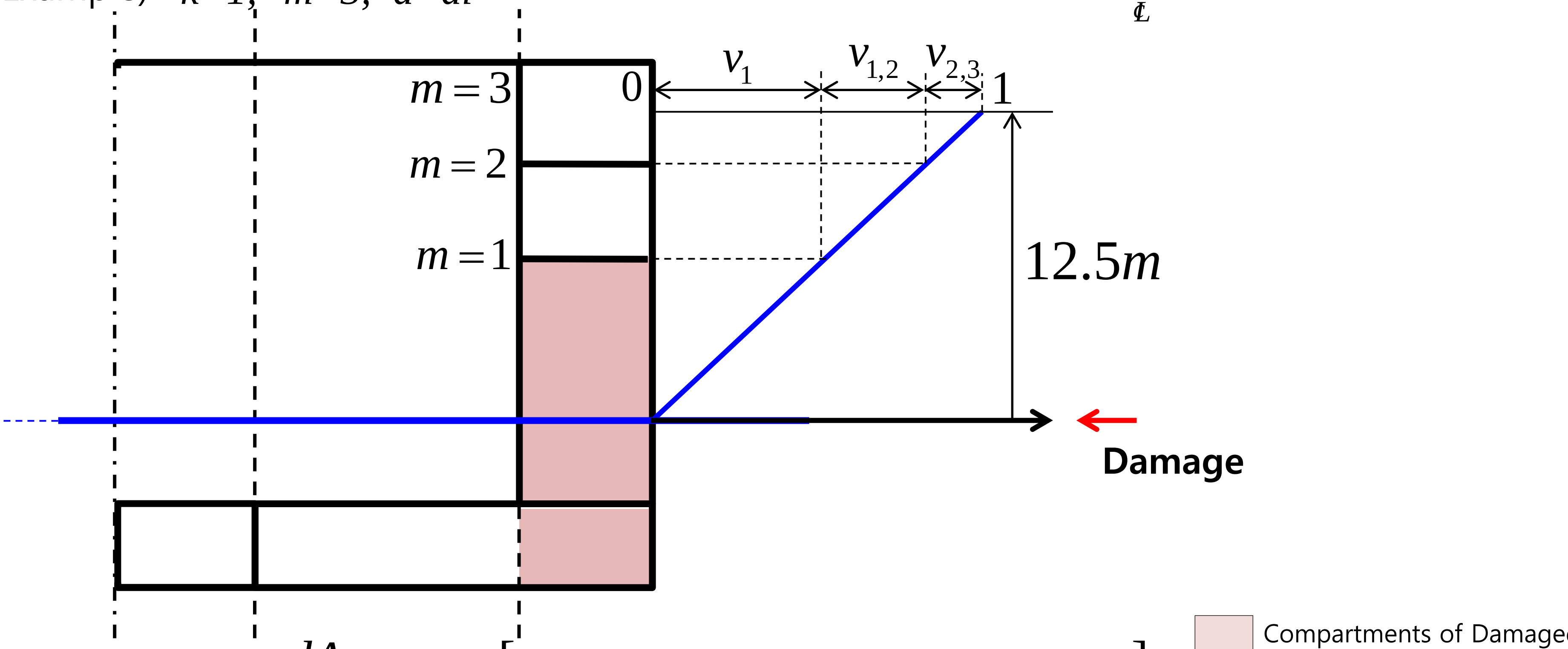


Consideration of Horizontal Subdivision in Flooding Stage
- Factor "v_m" : Stage 1) Damage (Initial condition)



After determination of the longitudinal and transverse damage case, i.e., **p and v is determined**, $v_{m-1,m}$ and $s_{min,m}$ is calculated. $s_{min,m}$ is survivability when the compartment is flooded up to deck number m.

Example) $k=1$, $m=3$, $d=dl$

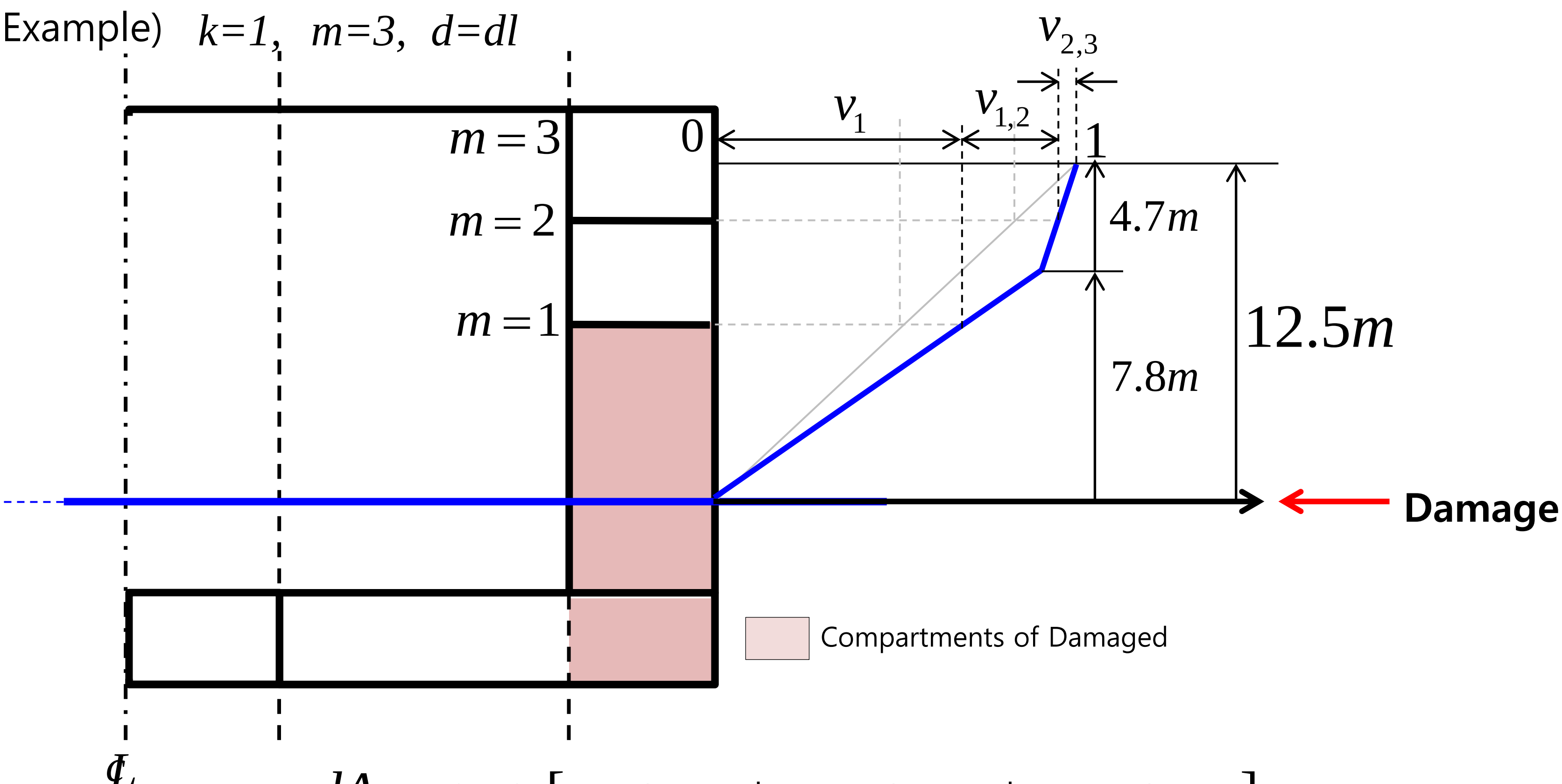


$$dA = p \cdot r \cdot [v_1 \cdot s_{\min,1} + v_{1,2} \cdot s_{\min,2} + v_{2,3} \cdot s_{\min,3}]$$

v_1은 m=1번까지 물에 잠길 확률, $v_{1,2}$는 m=2번까지 물에 잠길 확률, $v_{2,3}$은 m=3번 까지 물에 잠길 확률을 의미한다. 각각의 확률은 1) damage된 부분으로부터 12.5m까지의 길이를 1로 normalize한 뒤, 2) 이전 번호의 수평선으로부터 해당 수평선까지의 높이 비로 결정한다.
여기서 주의할 것은 이미 길이, 폭 방향의 damage case는 이미 결정된 후 위의 계산을 진행한다는 것이다.

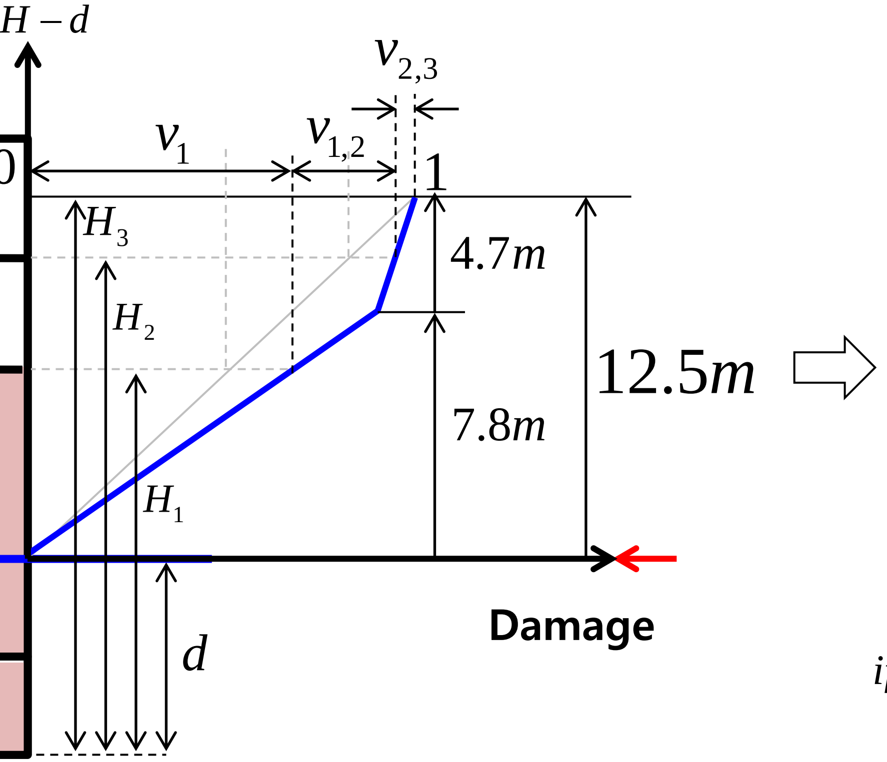
However, the horizontal subdivision line located lower can be flooded easier than that located higher. Therefore, the interpolation line between zero and one is modified as shown in following figure.

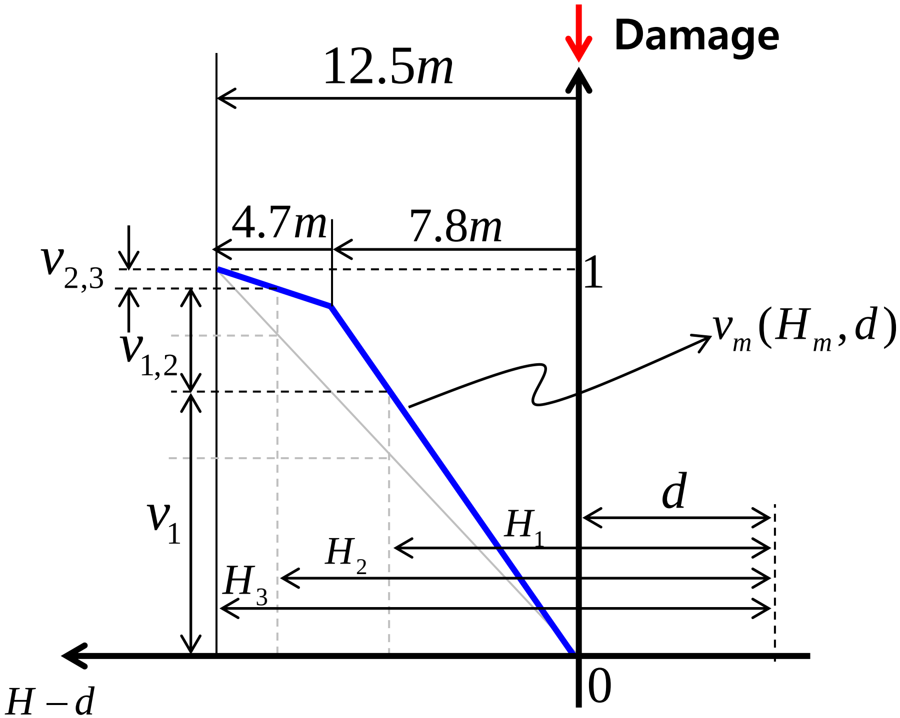
Example) $k=1$, $m=3$, $d=dl$



$$dA = p \cdot r \cdot [v_1 \cdot s_{\min,1} + v_{1,2} \cdot s_{\min,2} + v_{2,3} \cdot s_{\min,3}]$$

수선면과 가까운 수평 line이 잠길 확률이 수선면과 먼 곳의 수평 line이 잠길 확률보다 높으므로, 0과 1사이의 보간 line을 위의 그림에서 보는 바와 같이 수정하였음.





$$if\ \ 0 \le (H-d) < 7.8 \quad then\ v_m(H_m, d) = 0.8\frac{(H_m - d)}{7.8}$$

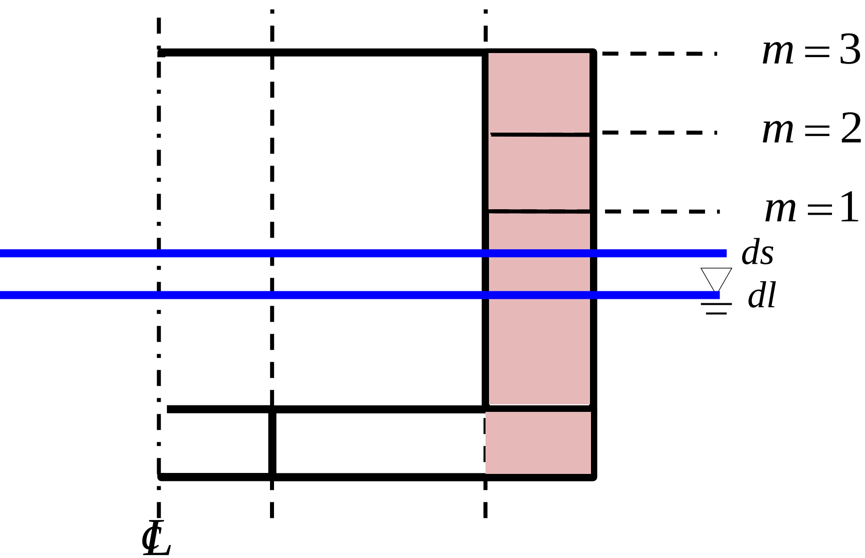
$$if\ \ 7.8 < (H-d) \quad then\ v_m(H_m, d) = 0.8 + 0.2\frac{(H_m - d) - 7.8}{4.7}$$

therefore

$$v_1 = v_1(H_1, d), \quad v_{1,2} = v_2(H_2, d) - v_1(H_1, d)$$

$$v_{2,3} = v_3(H_3, d) - v_2(H_2, d)$$

Example) $k=1, \quad m=3 \quad d=dl$



$$dA = p \cdot r \cdot [v_1 \cdot s_{\min,1} + v_{1,2} \cdot s_{\min,2} + v_{2,3} \cdot s_{\min,3}]$$

,where

$$v_1 = v_1(H_1, d), \quad v_{1,2} = v_2(H_2, d) - v_1(H_1, d)$$
$$v_{2,3} = v_3(H_3, d) - v_2(H_2, d)$$

$$if \;\; 0 \le (H-d) < 7.8 \quad then \; v_m(H_m, d) = 0.8\frac{(H_m - d)}{7.8}$$
$$if \;\; 7.8 < (H-d) \quad then \; v_m(H_m, d) = 0.8 + 0.2\frac{(H_m - d) - 7.8}{4.7}$$

The factor "v_m" is dependent if the **geometry of the watertight arrangement (decks) "H_m"** of the ship and **the draught of the initial loading condition (**$d : ds, dp, dl$**)**.

$$v_m = v(H_m, d) - v(H_{m-1}, d)$$
$$dA = p_i \cdot [v_1 \cdot s_{\min 1} + (v_2 - v_1) \cdot s_{\min 2} + \cdots + (1 - v_{m-1}) \cdot s_{\min m}]$$

Where $A = \sum dA$**.** The maximum possible vertical extent of damage is $d+12.5m$. Then the factor "H_m" equals 1.

Attained Subdivision Index "A" : Calculation of Factor "s_i"
- Factor "v_m" : Stage 2) Flooding up to $m=1$

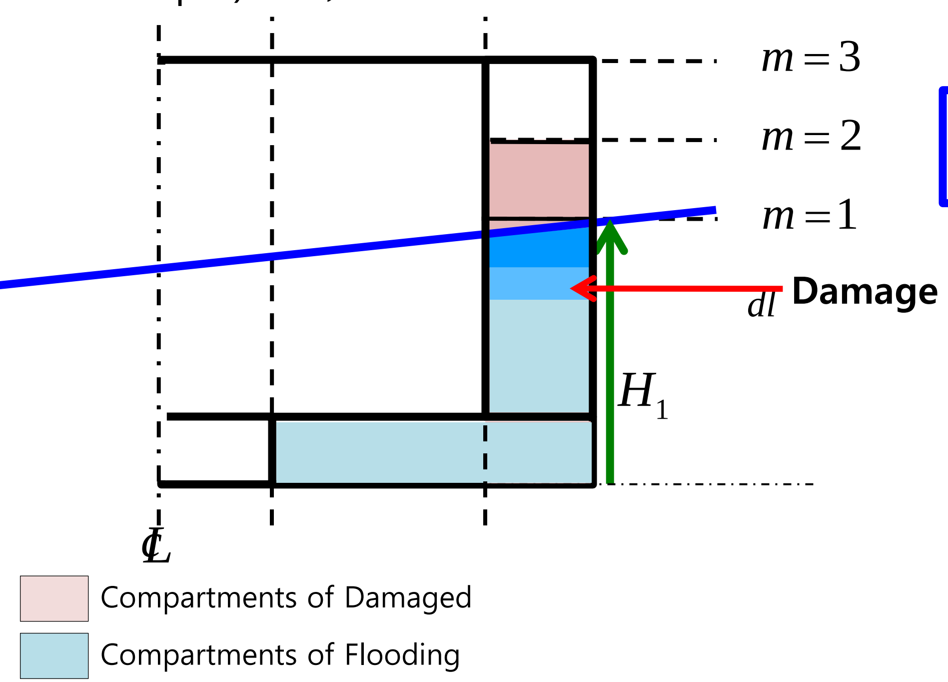
The factor "v_m" is dependent on the **geometry of the watertight arrangement (decks) "H_m"** of the ship and **the draught of the initial loading condition (**d : ds, dp, dl**)**.

$$v_m = v(H_m, d) - v(H_{m-1}, d)$$

$$dA = p_i \cdot [v_1 \cdot s_{\min 1} + (v_2 - v_1) \cdot s_{\min 2} + \cdots + (1 - v_{m-1}) \cdot s_{\min m}]$$

Where $A = \sum dA$**.** The maximum possible vertical extent of damage is $d+12.5m$. Then the factor "H_m" equals 1.

Example) $k=1$, $m=1$ $d=dl$



Stage 2) Flooding up to $m=1$

$$\boxed{s_i = v_1 \cdot s_{\min 1}}$$

Attained Subdivision Index "A" : Calculation of Factor "s_i"
- Factor "v_m" : Stage 3) Flooding up to $m=2$

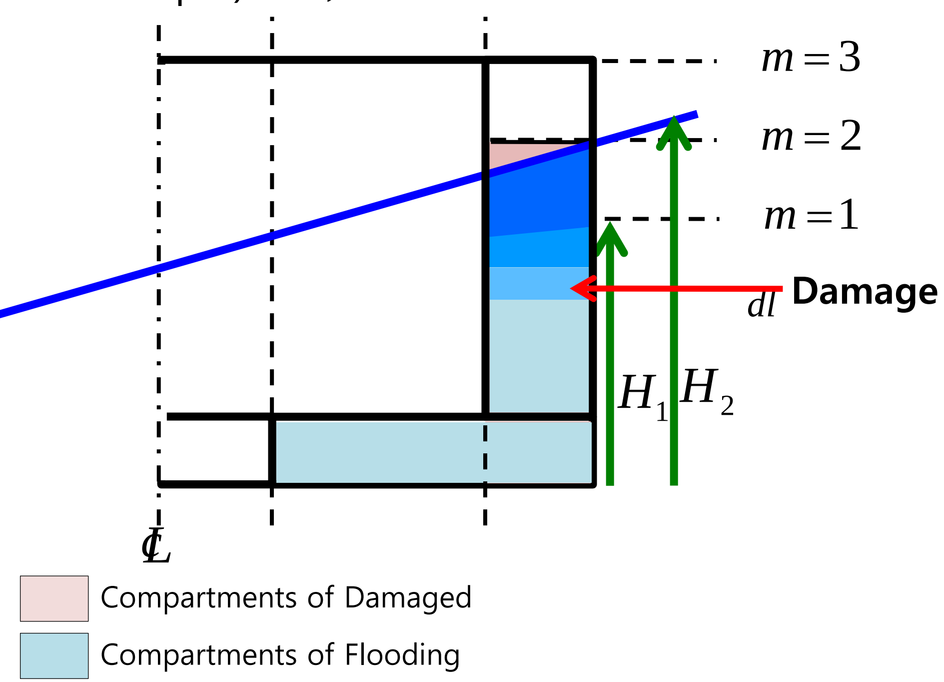
The factor "v_m" is dependent on the **geometry of the watertight arrangement (decks) "H_m"** of the ship and **the draught of the initial loading condition** (*d : ds, dp, dl*).

$$v_m = v(H_m, d) - v(H_{m-1}, d)$$

$$dA = p_i \cdot [v_1 \cdot s_{\min 1} + (v_2 - v_1) \cdot s_{\min 2} + \cdots + (1 - v_{m-1}) \cdot s_{\min m}]$$

Where $A = \sum dA$. The maximum possible vertical extent of damage is $d+12.5m$. Then the factor "H_m" equals 1.

Example) $k=1$, $m=2$ $d=dl$



Stage 3) Flooding up to $m=2$

$$s_i = v_1 \cdot s_{\min 1} + \boxed{(v_2 - v_1) \cdot s_{\min 2}}$$

Attained Subdivision Index "A" : Calculation of Factor "s_i" - Factor "v_m" : Stage 4) Flooding up to $m=3$

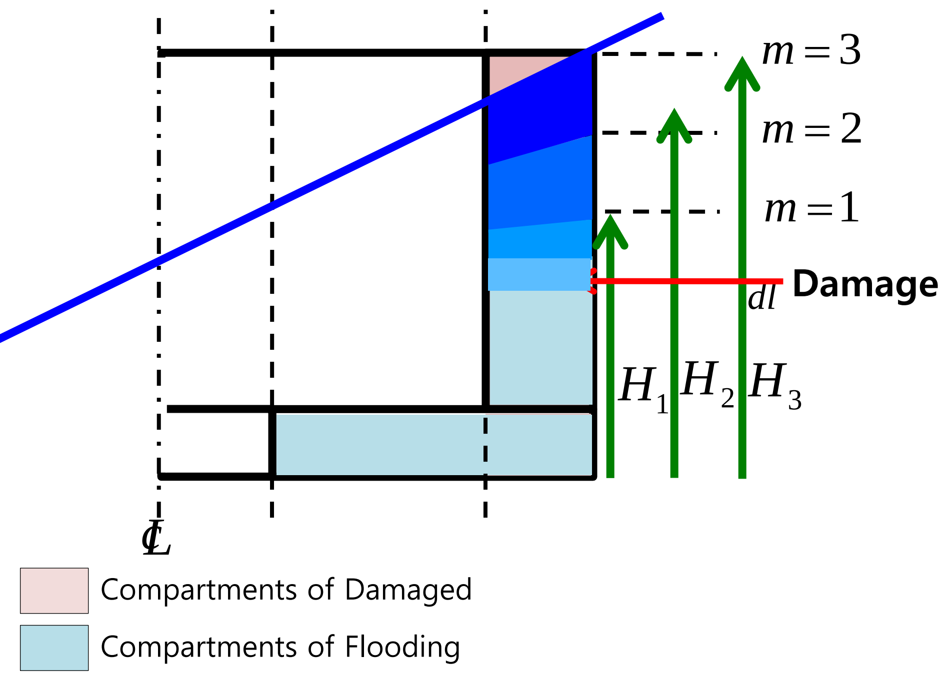
The factor "v_m" is dependent on the **geometry of the watertight arrangement (decks) "H_m"** of the ship and **the draught of the initial loading condition (**d : ds, dp, dl**)**.

$$v_m = v(H_m, d) - v(H_{m-1}, d)$$

$$dA = p_i \cdot [v_1 \cdot s_{\min 1} + (v_2 - v_1) \cdot s_{\min 2} + \cdots + (1 - v_{m-1}) \cdot s_{\min m}]$$

Where $A = \sum dA$. The maximum possible vertical extent of damage is $d+12.5m$. Then the factor "H_m" equals 1.

Example) $k=1$, $m=3$ $d=dl$



Stage 4) Flooding up to $m=3$

$$s_i = v_1 \cdot s_{\min 1} + (v_2 - v_1) \cdot s_{\min 2} + \boxed{(1 - v_3) \cdot s_{\min 3}}$$

Three loading conditions are to be considered and the result weighted as follows:

$$As, Ap, Al \geq 0.5R \quad \text{: for cargo ships}$$

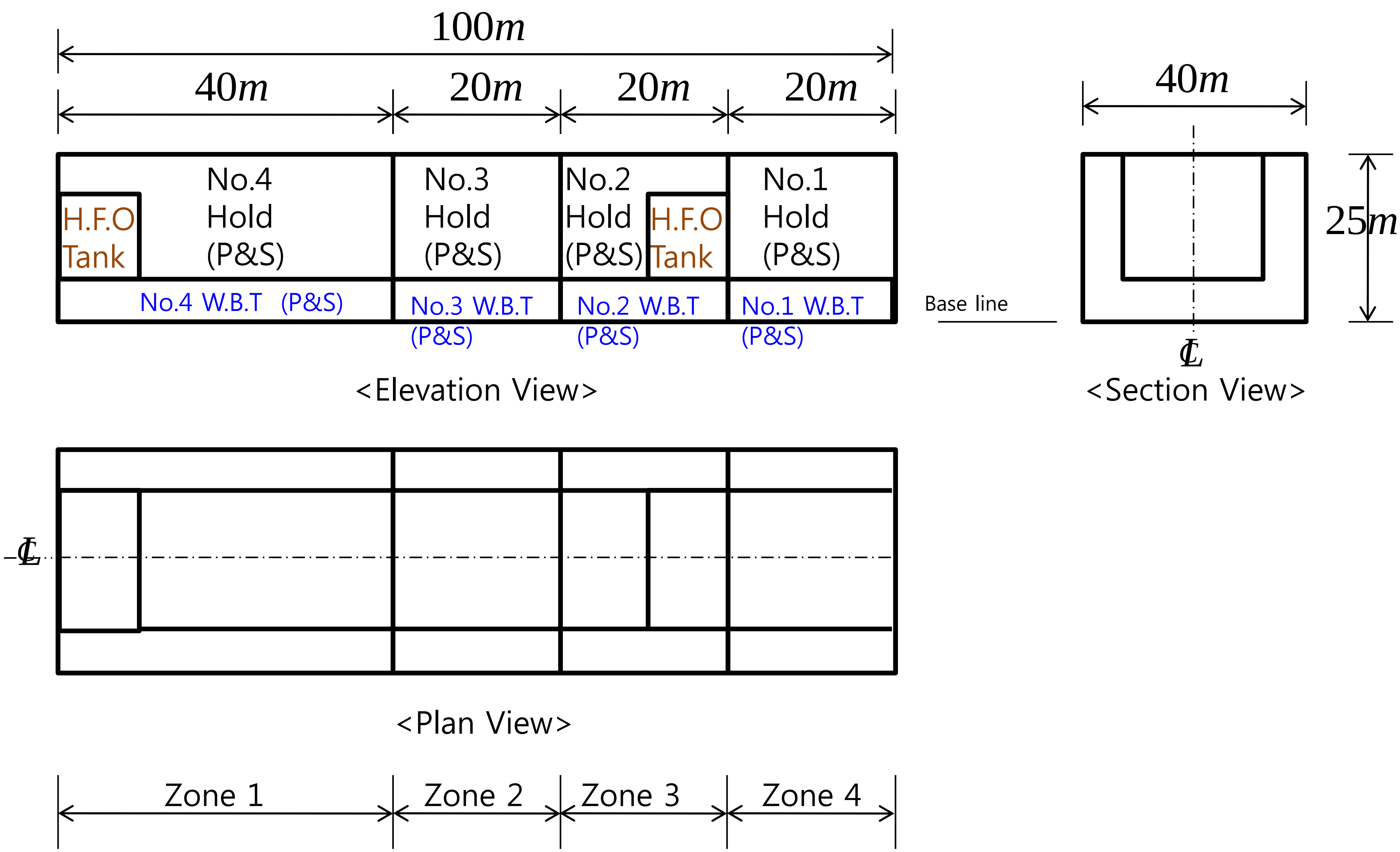
$$\geq 0.9R \quad \text{: for passenger ships}$$

$$\boxed{A \geq R} \qquad \text{Where } A = 0.4A_s + 0.4A_p + 0.2A_l$$

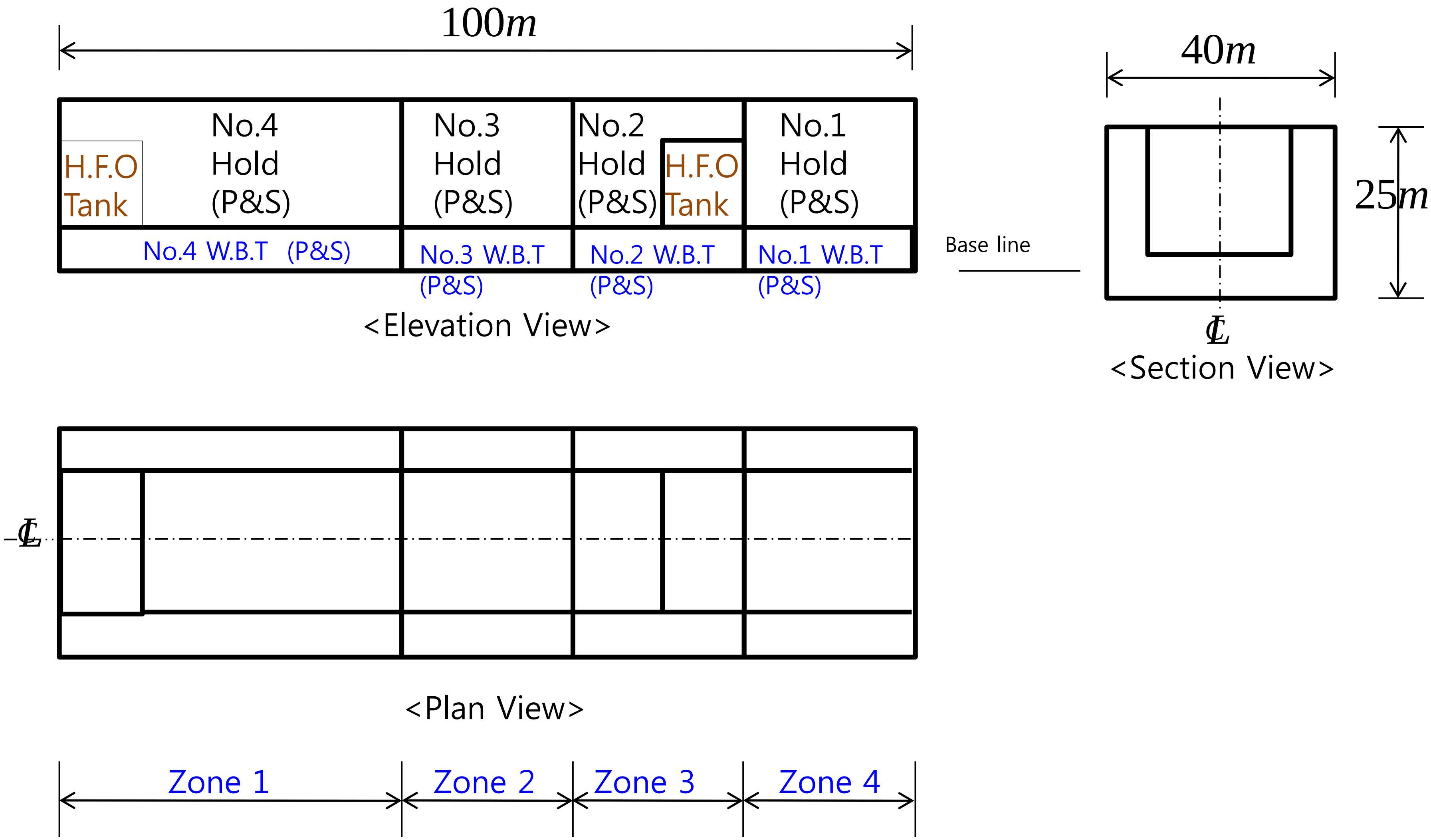
Where the indices "s", "p" and "l" represent three loading conditions and the factor to be multiplied to the index indicates **how the index "A" from each loading condition is weighted**.

13-7 Example of Subdivision & Damage Stability Calculation of a Box-Shaped Ship

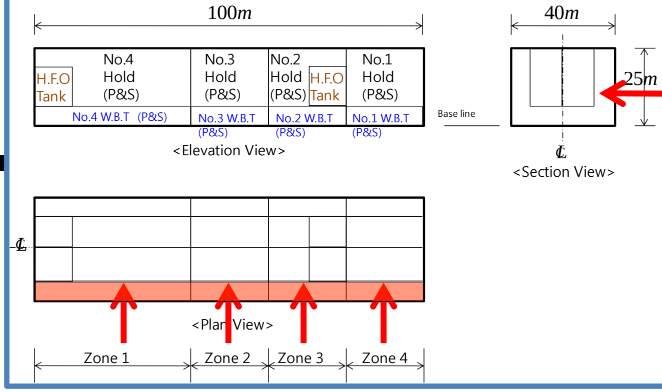
Example of Calculation of Attained Index A for a Box-Shaped Ship



Supposition of Subdivision Zone



Calculation of Probability of Damage(p_i)



$$pi = p(x1, x2, Ls) \times r(x1, x2, Ls, b)$$

Calculation Condition

: Scantling Draft (18.0 m), b=4.0

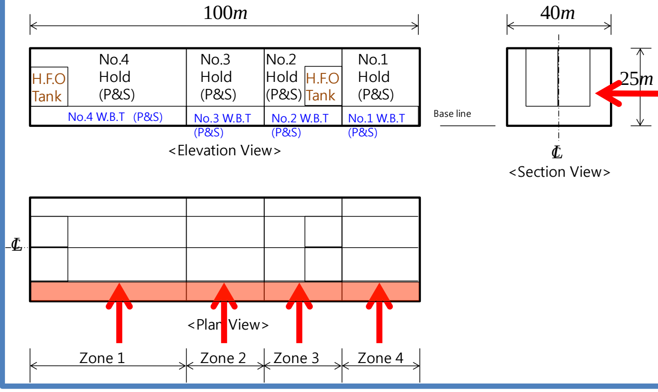
DAMAGES	x1	x2	Damage Length	J	p	r	pi
< 1 zone damage >				Cause	Effect		Effect
1.1.1	0	40	40	0.4	0.4[illegible]21	0.42119	0.1[illegible]25
2.1.1	40	60	20	0.2	0.1[illegible]73	0.36117	0.0[illegible]16
3.1.1	60	80	20	0.2	0.1[illegible]73	0.36117	0.0[illegible]16
3.1.1	80	100	20	0.2	0.1[illegible]7	0.58293	0.1[illegible]21
				Bigger	Bigger		Bigger
< 2 zone damage >							
1-2.1.1	0	60	60	0.6	0.6[illegible]21	0.37975	0.2[illegible]45
2-3.1.1	40	80	40	0.4	0.4[illegible]42	0.34515	0.1[illegible]97
3-4.1.1	60	100	40	0.4	0.4[illegible]1	0.42119	0.1[illegible]5
				Bigger	Bigger		Bigger
< 3 zone damage >							
1-3.1.1	0	80	80	0.8	0.8[illegible]21	0.35892	0.2[illegible]55
2-4.1.1	40	100	60	0.6	0.6[illegible]42	0.34563	0.2[illegible]9
				Bigger	Bigger		Bigger

J : Non-dimensional damage length $J = \frac{|x2 - x1|}{Ls}$

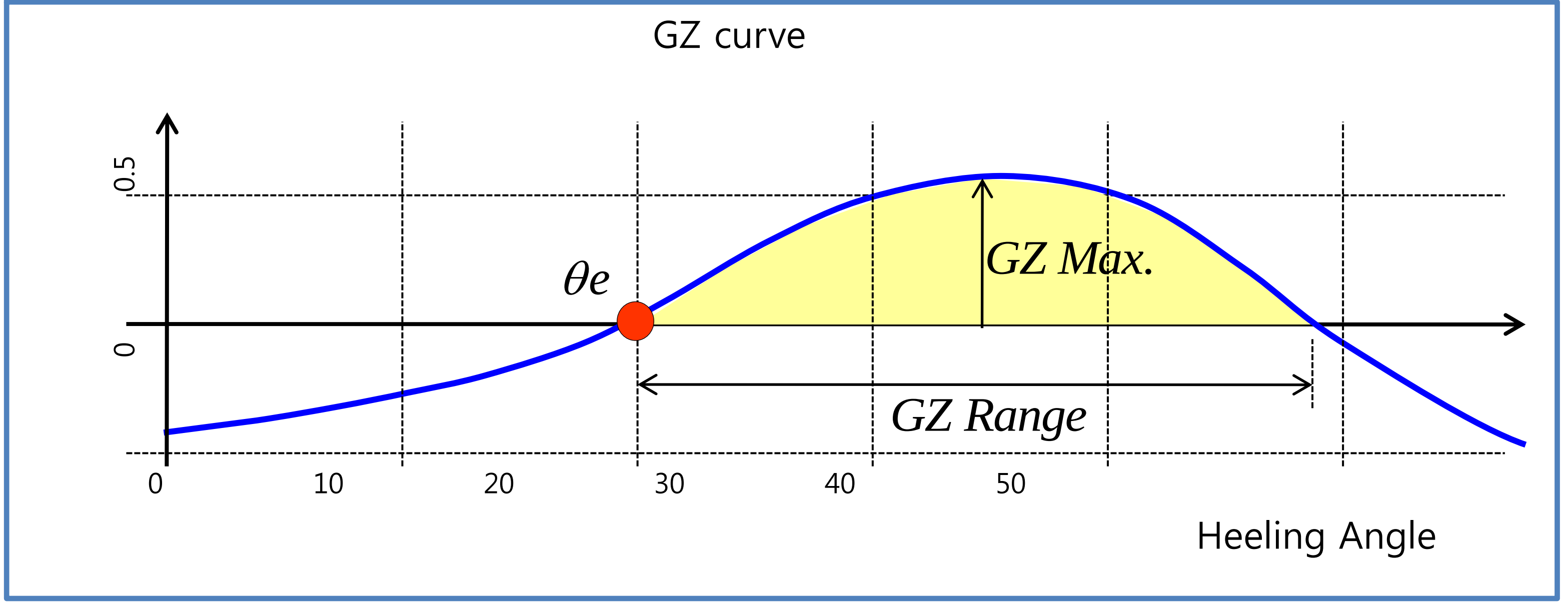
b : Mean transverse distance

※ Each results are obtained using manual calculation.

Calculation of Probability of Survivability(p_i)

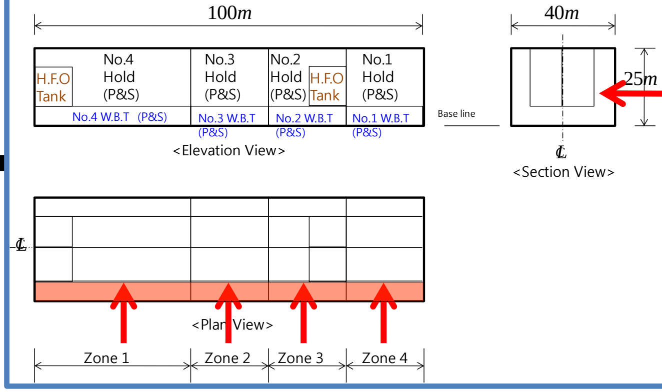


Typical GZ curve in damage condition



θ_e : the equilibrium heel angle in any stage of flooding, in degrees
GZmax : the maximum positive righting lever, in meters
Range : the range of positive righting levers, in degrees, measured from the angle θ_e

$$si = si(\theta_e, \theta_v, GZmax, Range)$$



Calculation Condition

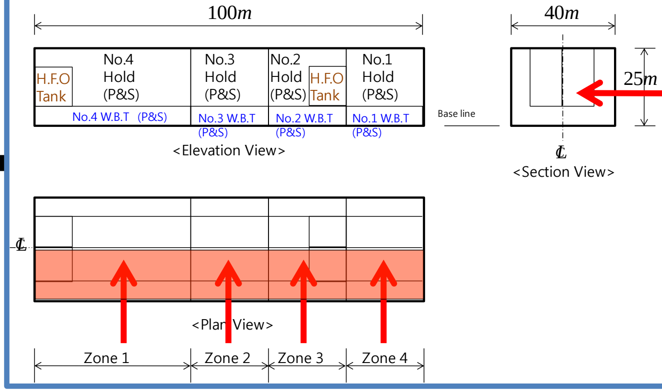
: Scantling Draft (18.0 m), b=4.0

DAMAGES	x1	x2	J	θ_e	Max_GZ	GZ Range	Si	pi	A
< 1 zone damage >			Cause	Effect	Effect	Effect	Effect		
1.1.1	0	40	0.4	[illegible]	[illegible]	[illegible]	[illegible]	0.17025	0.02666
2.1.1	40	60	0.2	[illegible]	[illegible]	[illegible]	[illegible]	0.05516	0.00864
3.1.1	60	80	0.2	[illegible]	[illegible]	[illegible]	[illegible]	0.05516	0.00864
3.1.1	80	100	0.2	[illegible]	[illegible]	[illegible]	[illegible]	0.10281	0.01610
< 2 zone damage >			Bigger	Bigger	Smaller	Smaller	Smaller		
1-2.1.1	0	60	0.6	[illegible]	[illegible]	[illegible]	[illegible]	0.22945	0.03195
2-3.1.1	40	80	0.4	[illegible]	[illegible]	[illegible]	[illegible]	0.14097	0.02207
3-4.1.1	60	100	0.4	[illegible]	[illegible]	[illegible]	[illegible]	0.17025	0.02666
< 3 zone damage >			Bigger	Bigger	Smaller	Smaller	Smaller		
1-3.1.1	0	80	0.8	[illegible]	[illegible]	[illegible]	[illegible]	0.28865	0.00000
2-4.1.1	40	100	0.6	[illegible]	[illegible]	[illegible]	[illegible]	0.21029	0.02928
			Bigger	Bigger	Smaller	Smaller	Smaller		

θ_e : Non-dimensional damage length

※ θ_e, GZ, GZ range are obtained using computer ship calculation software, "Ez-compart".

Calculation of Probability of Damage(p_i)



$$pi = p(x1, x2, Ls) \times r(x1, x2, Ls, b)$$

Calculation Condition

: Scantling Draft (18.0 m), b=20.0 **Cause** **Bigger**

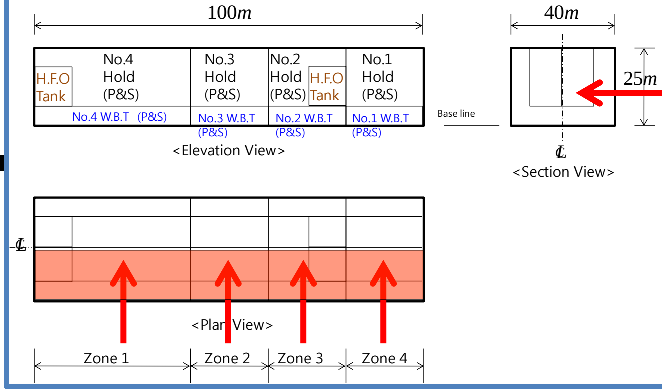
DAMAGES	x1	x2	Damage Length	J	p	r	pi
< 1 zone damage >					**Effect**	**Effect**	**Effect**
1.2.1	0	40	40	0.4000	0.4[illegible]1	1.0[illegible]0	0.4[illegible]21
2.2.1	40	60	20	0.2000	0.15[illegible]73	1.00[illegible]00	0.1[illegible]73
3.2.1	60	80	20	0.2000	0.15[illegible]73	1.00[illegible]00	0.1[illegible]73
3.2.1	80	100	20	0.2000	0.17[illegible]37	1.00[illegible]00	0.1[illegible]37
< 2 zone damage >					**Bigger**	**Bigger**	**Bigger**
1-2.2.1	0	60	60	0.6000	0.60[illegible]21	1.00[illegible]00	0.6[illegible]21
2-3.2.1	40	80	40	0.4000	0.40[illegible]42	1.00[illegible]00	0.4[illegible]42
3-4.2.1	60	100	40	0.4000	0.40[illegible]21	1.00[illegible]00	0.4[illegible]21
< 3 zone damage >							
1-3.2.1	0	80	80	0.8000	0.80[illegible]21	1.00[illegible]00	0.8[illegible]21
2-4.2.1	40	100	60	0.6000	0.60[illegible]42	1.00[illegible]00	0.60[illegible]42

J : Non-dimensional damage length $J = \frac{|x2 - x1|}{Ls}$

b : Mean transverse distance

※ Each results are obtained using manual calculation.

Calculation of Probability of Survivability(p_i)



$$si = si(\theta_e, \theta_v, GZmax, Range)$$

Calculation Condition

: Scantling Draft (18.0 m), b=20.0 **Cause** **Bigger**

※ θ_e, GZ, GZ range are obtained using computer ship calculation software, "Ez-compart".

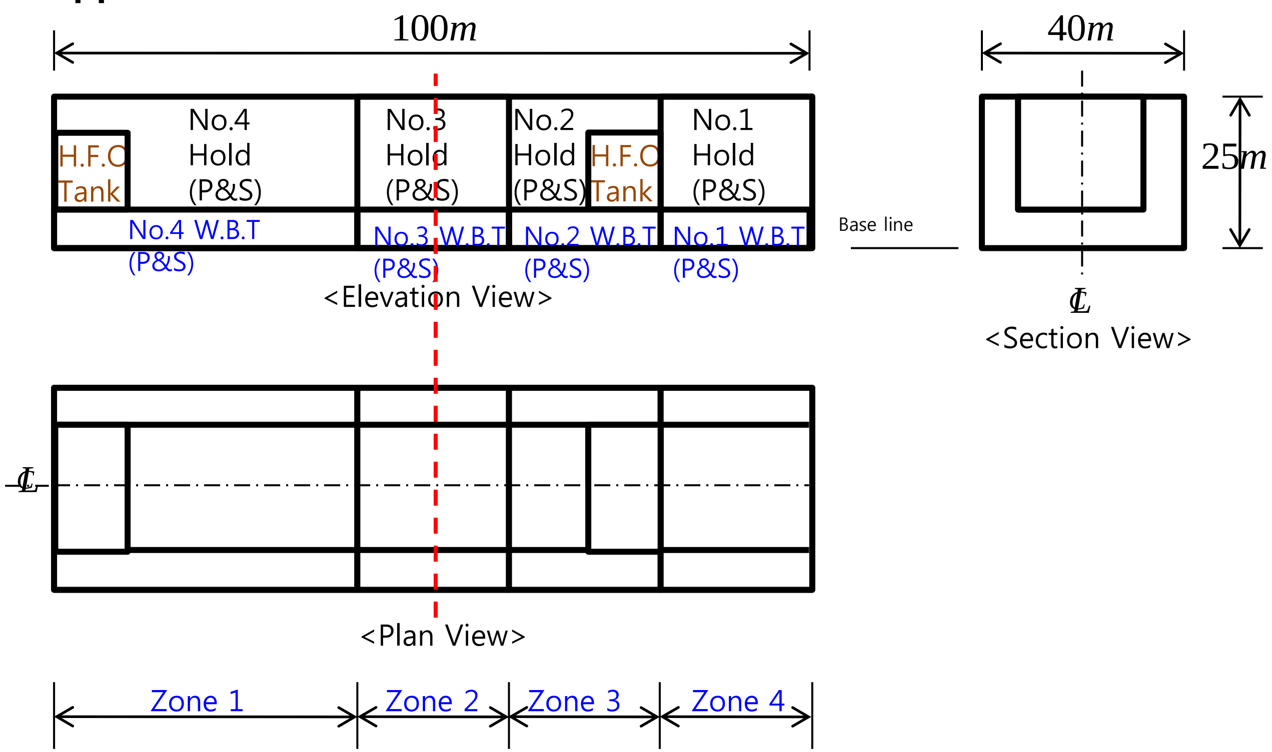
DAMAGES	x1	x2	J	θe	Max_GZ	Range	Si	pi	A
< 1 zone damage >				**Effect**	**Effect**	**Effect**	**Effect**		
1.2.1	0	40	0.4000	[illegible]	[illegible]	[illegible]	[illegible]	0.40421	~~0.00000~~
2.2.1	40	60	0.2000	[illegible]	[illegible]	[illegible]	[illegible]	0.15273	0.02392
3.2.1	60	80	0.2000	[illegible]	[illegible]	[illegible]	[illegible]	0.15273	0.02392
3.2.1	80	100	0.2000	[illegible]	[illegible]	[illegible]	[illegible]	0.17637	~~0.02099~~
< 2 zone damage >				**Smaller**	**Smaller**	**Smaller**	**Smaller**		
1-2.2.1	0	60	0.6000	[illegible]	[illegible]	[illegible]	[illegible]	0.60421	~~0.00000~~
2-3.2.1	40	80	0.4000	[illegible]	[illegible]	[illegible]	[illegible]	0.40842	~~0.00000~~
3-4.2.1	60	100	0.4000	[illegible]	[illegible]	[illegible]	[illegible]	0.40421	~~0.00000~~
< 3 zone damage >									
1-3.2.1	0	80	0.8000	[illegible]	[illegible]	[illegible]	[illegible]	0.80421	~~0.00000~~
2-4.2.1	40	100	0.6000	[illegible]	[illegible]	[illegible]	[illegible]	0.60842	~~0.00000~~

Attained index(A) is zero in most case, because too large areas are damaged.
We can expect that calculating '4 zone damage' cases are meaningless.

Effect of the Virtual Subdivision Bulkhead in Zone 2

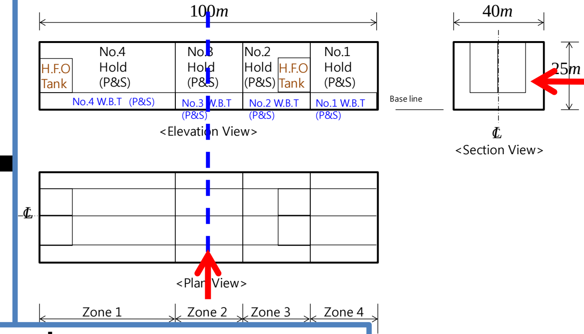
Effect of the Virtual Subdivision Bulkhead in Zone 2

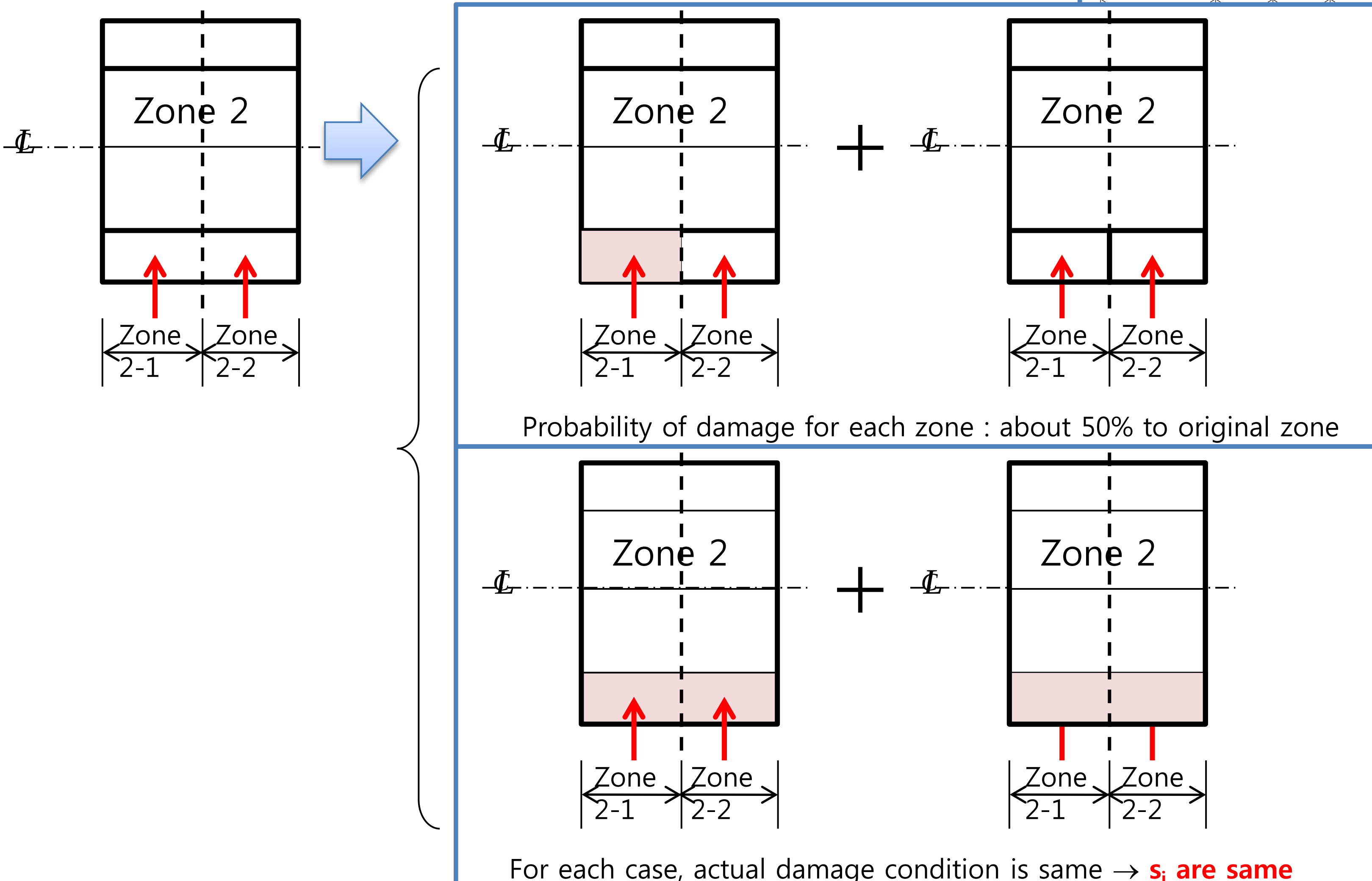
Supposition of Subdivision Zone



What could it happen, if there is virtual subdivision bulkhead in zone 2??

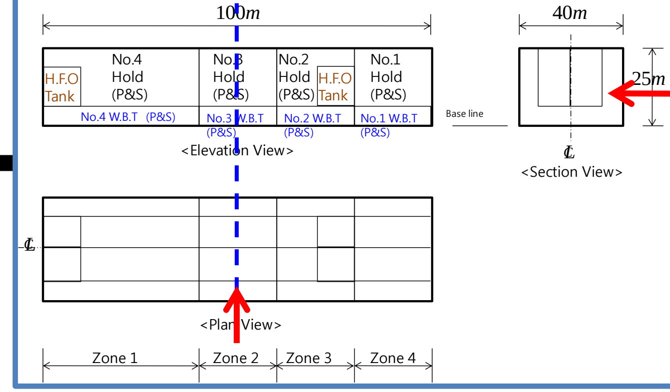
What could it happen, if there is virtual subdivision bulkhead in zone 2??





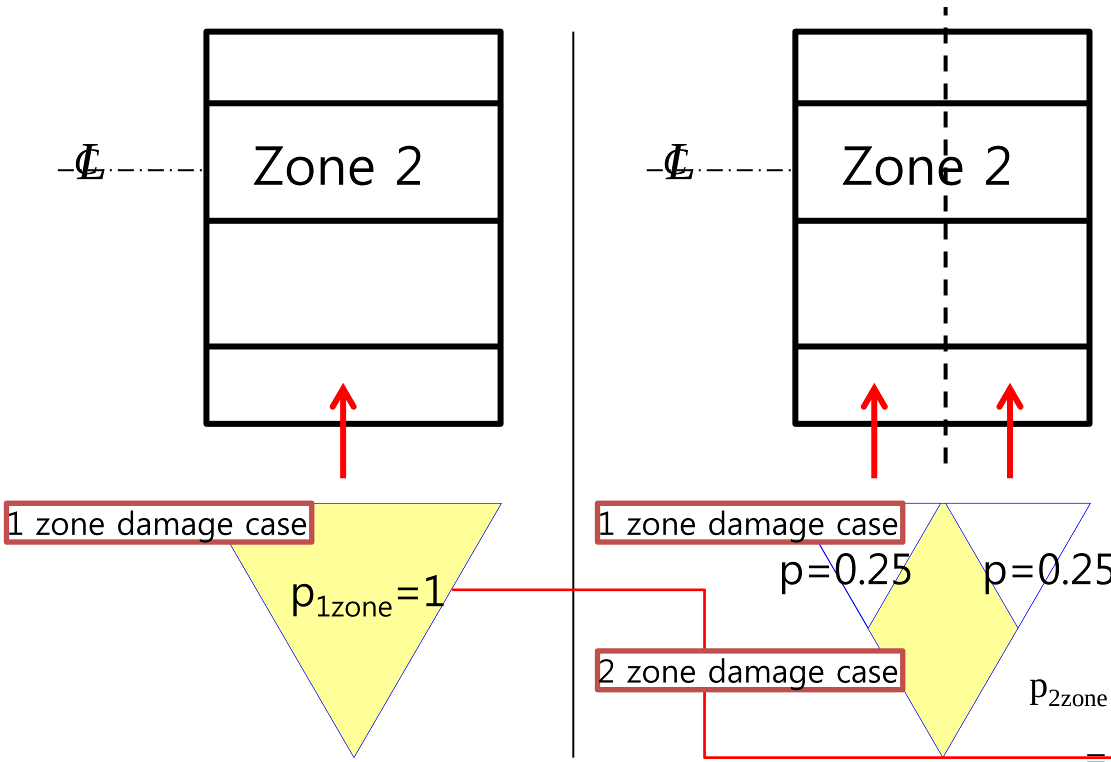
Probability of damage for each zone : about 50% to original zone

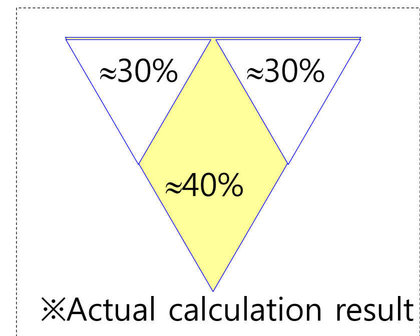
For each case, actual damage condition is same → **s_i are same**



What could it happen, if there is virtual subdivision bulkhead in zone 2??

Assumption : Original Probability of damage
in "1 zone damage" of zone 2 is '1'



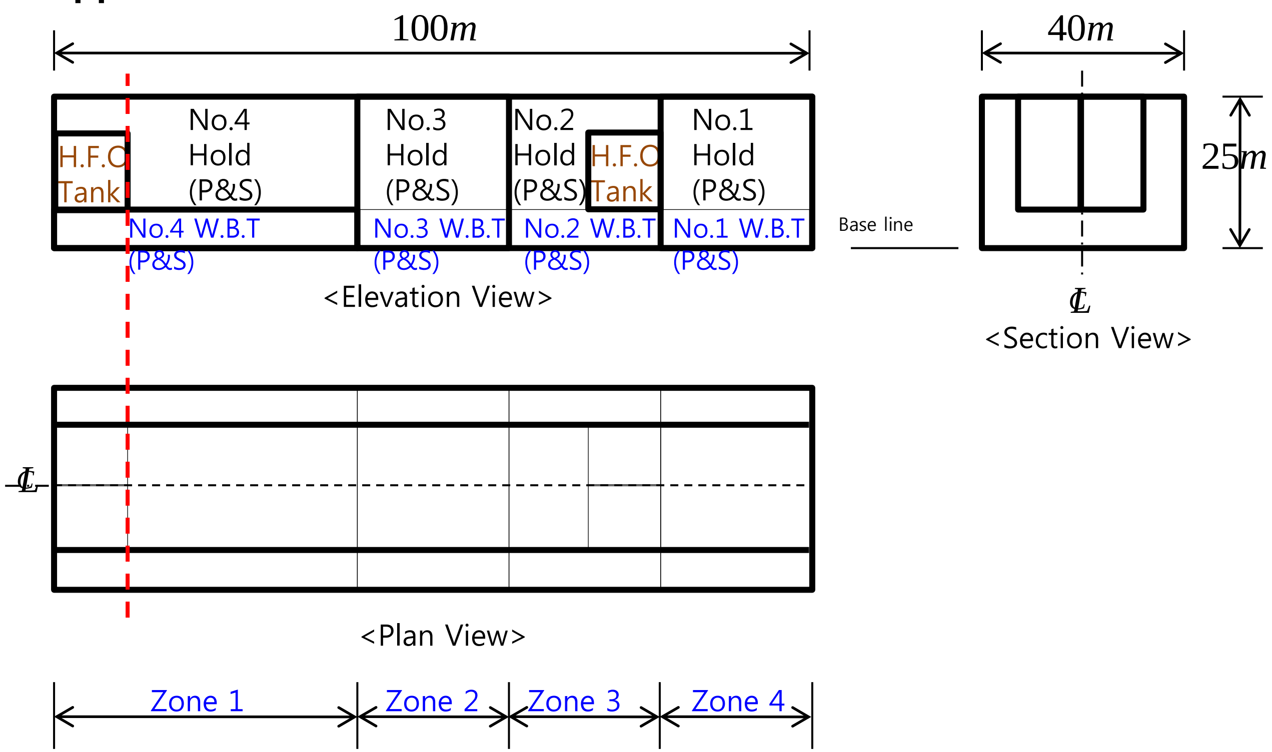


$$p_{2zone} = p(x1_j,x2_{j+1}) - p(x1_j,x2_j) - p(x1_{j+1},x2_{j+1})$$
$$= \boxed{1} - 0.25 - 0.25$$
$$= 0.5$$

Total probability of damage before and after dividing zone by virtual bulkhead are **same** :

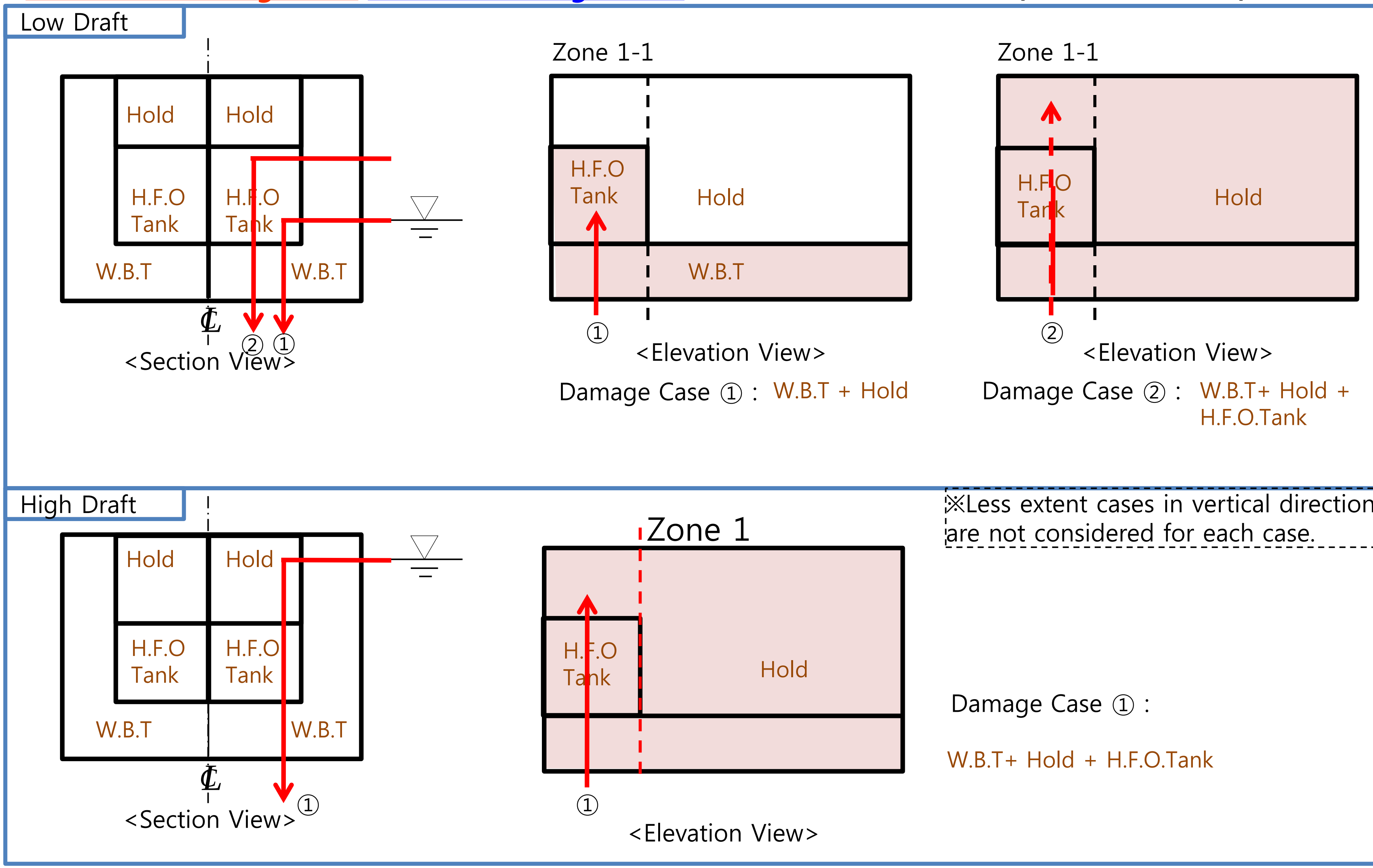
Effect of the Virtual Subdivision Bulkhead in Zone 1

Supposition of Subdivision Zone



What could it happen, if there is virtual subdivision bulkhead in zone 1??

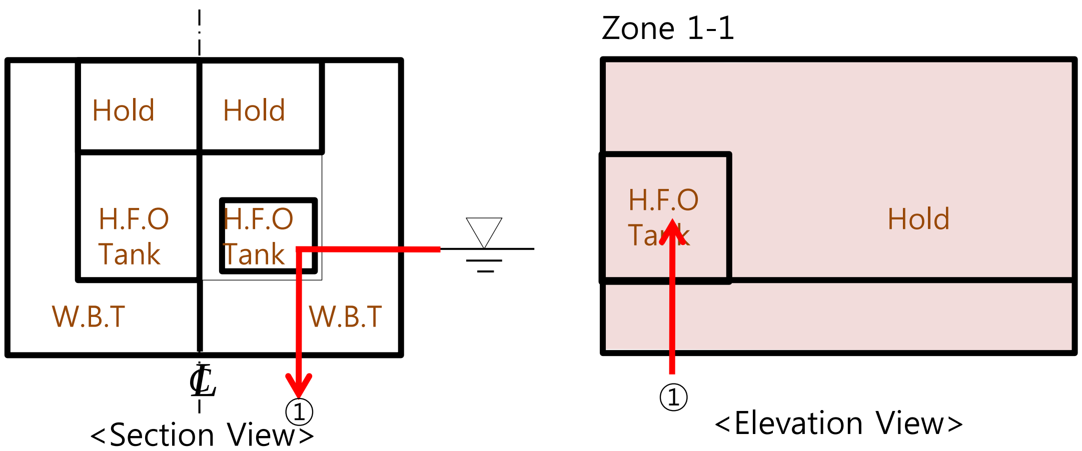
For different scantling draft, different damage cases occurs , (b : 20 m same)



Low draft case is considered for description of the effect of the virtual subdivision bulkhead.

No virtual subdivision bulkhead

, (b : 20 m same)



Damage Case ① : W.B.T+ Hold + H.F.O.Tank

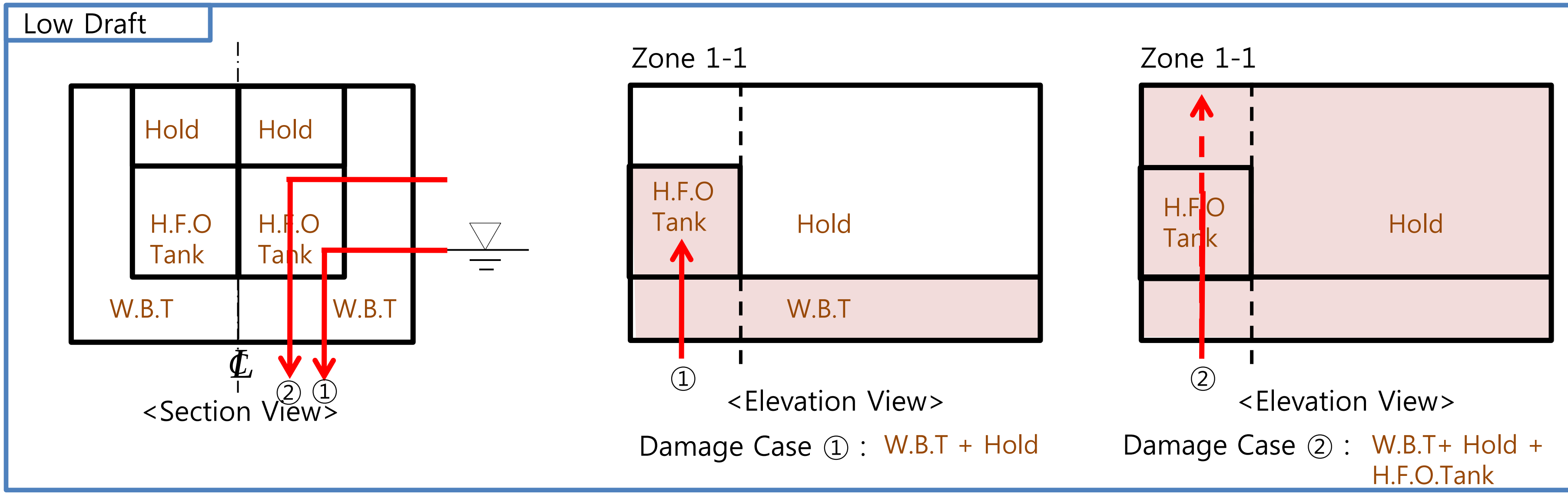
At previous calculation,

$$A_i = p_i \times s_i$$
$$= 0.60842 \times 0 \quad \text{(Because GZ, Range are 0)}$$
$$= 0$$

※ θ_e, GZ, GZ range are obtained using computer ship calculation software, "Ez-compart".

With virtual subdivision bulkhead

, (b : 20 m same)



※ θ_e, GZ, GZ range are obtained using computer ship calculation software, "Ez-compart".

Damage case ①

$$A_i = p_i \times s_i$$

$$= 0.01795 \times 0.9306 \quad = 0.01635$$

(Because GZ, Range are 0)

Damage case ②

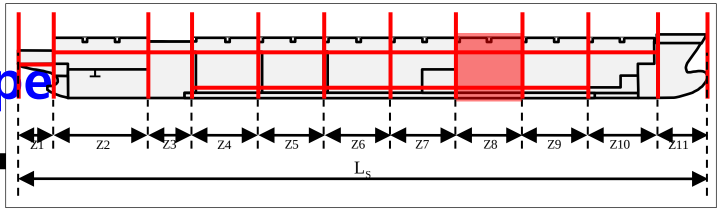
$$A_i = p_i \times s_i$$

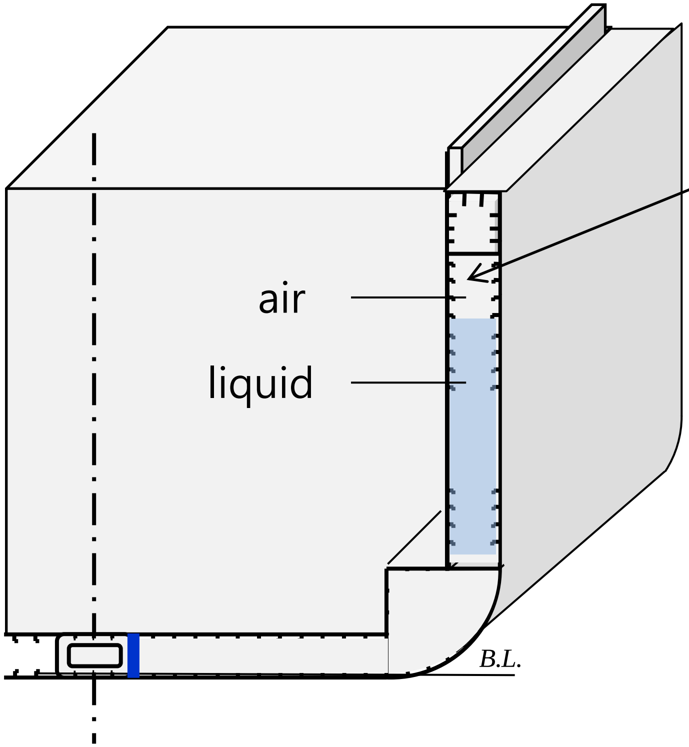
$$= 0.05516 \times 0 \quad = 0$$

In damage case ①, we obtain attained index which is greater than 0.
So, dividing zone by virtual bulkhead is meaningful if attained index which is greater than 0 is obtained.

13-8 Opening & Air Escape Pipe

Information of a Opening and a Air Escape Pipe





Where is the air moved,
When the liquid are filled in tank of a ship?

There must be a route for escape!

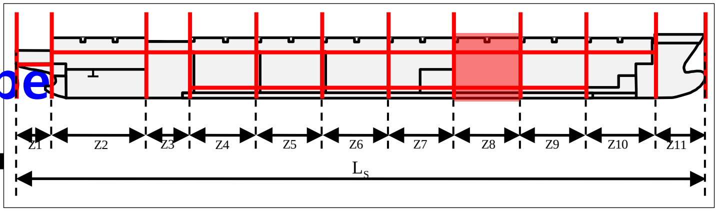
Air (escape) pipe !

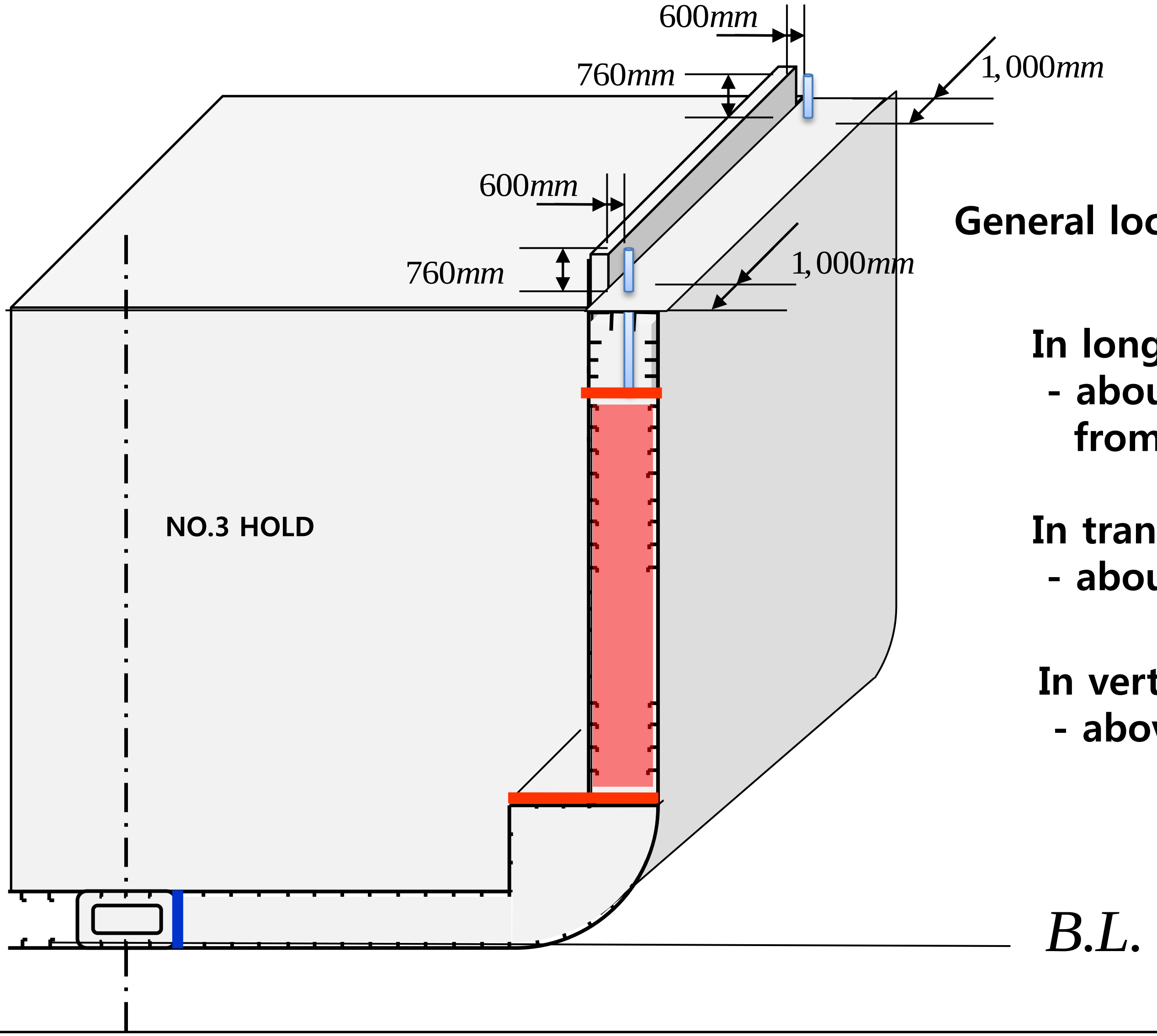
Air pipe : Pipes which are provided for all tanks to **prevent air being trapped** under pressure **in tank** when it is filled, or a vacuum being created when it is emptied. 1)

Air escape pipes of sufficient size and number should be led from the highest point of the tank, having regard to the various conditions of trim and heel of the ship.2)

1) Eyres,D.J. , Ship Construction, Elsevier, 2007, 6th edition
2) Walton.T, Know your own ship, Charles griffin and company, 1927

Information of a Opening and a Air Escape Pipe





General location of opening

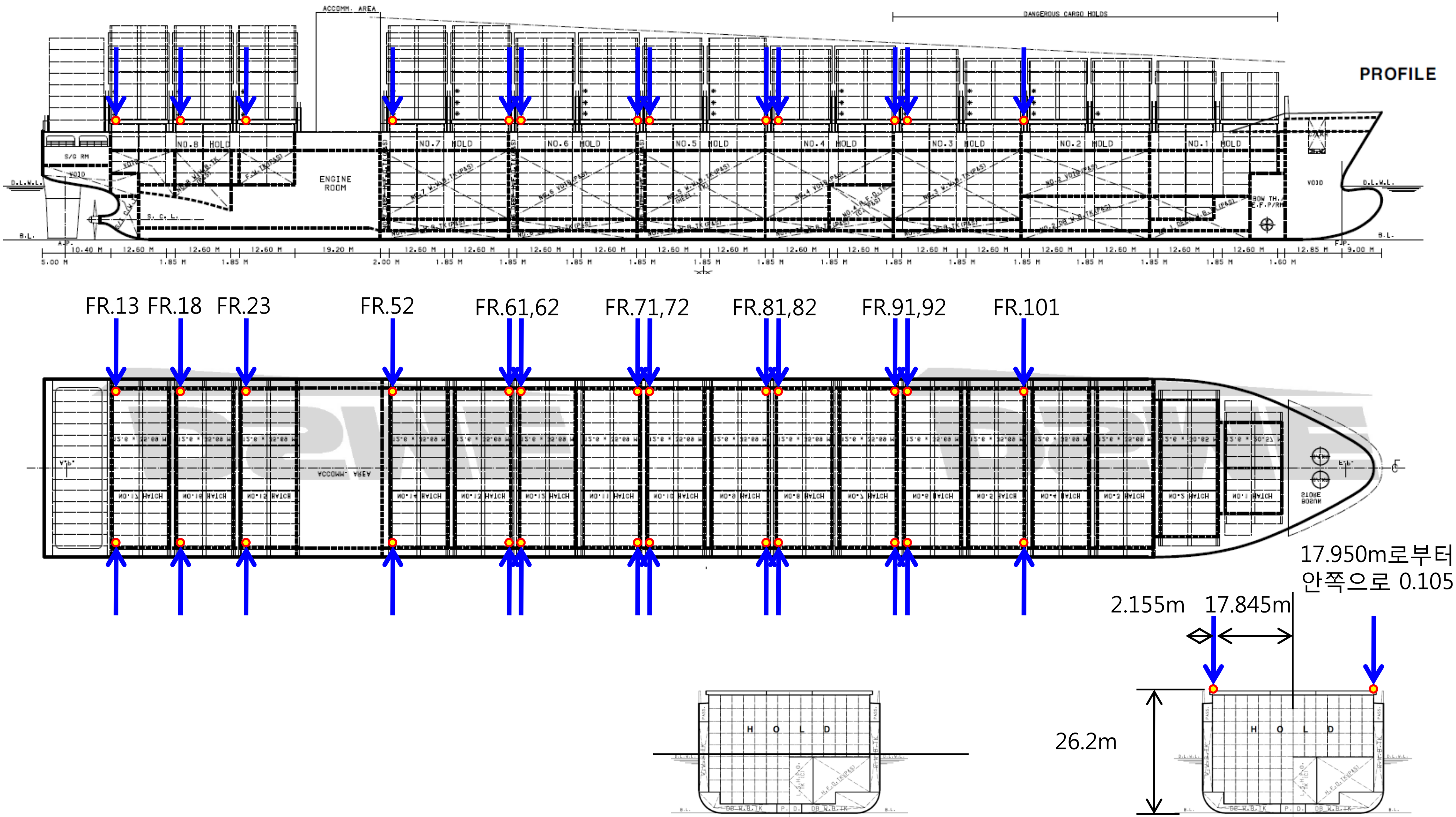
In longitudinal direction
- about 1,000 mm off from transverse bulkhead

In transverse direction
- about 600 mm off

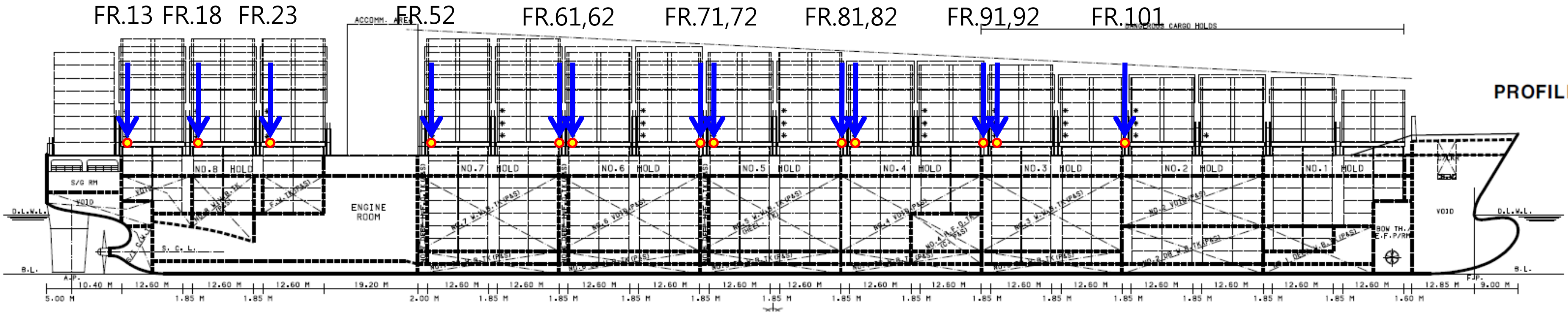
In vertical direction
- above 760 mm deck

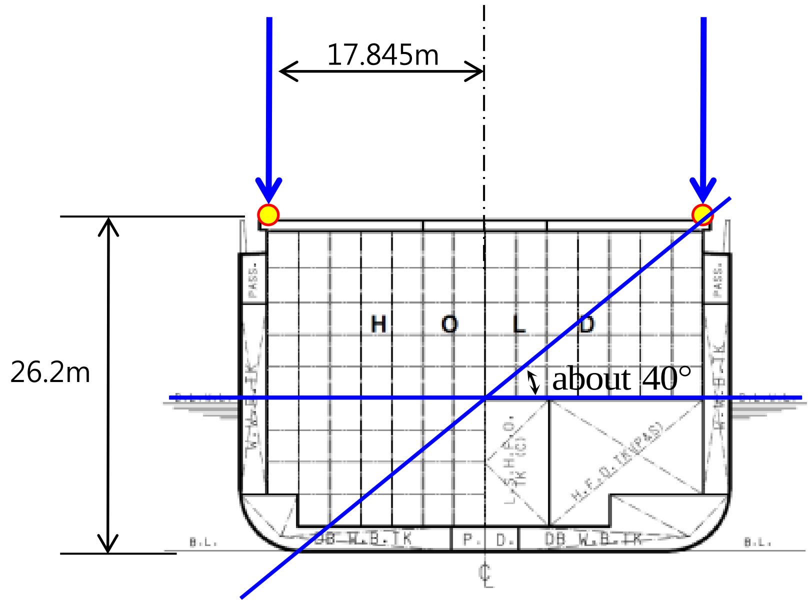
Example of Openings & Air Pipes of 7,000TEU Container Ship

Opening Location for Cargo Hold : Unprotected opening

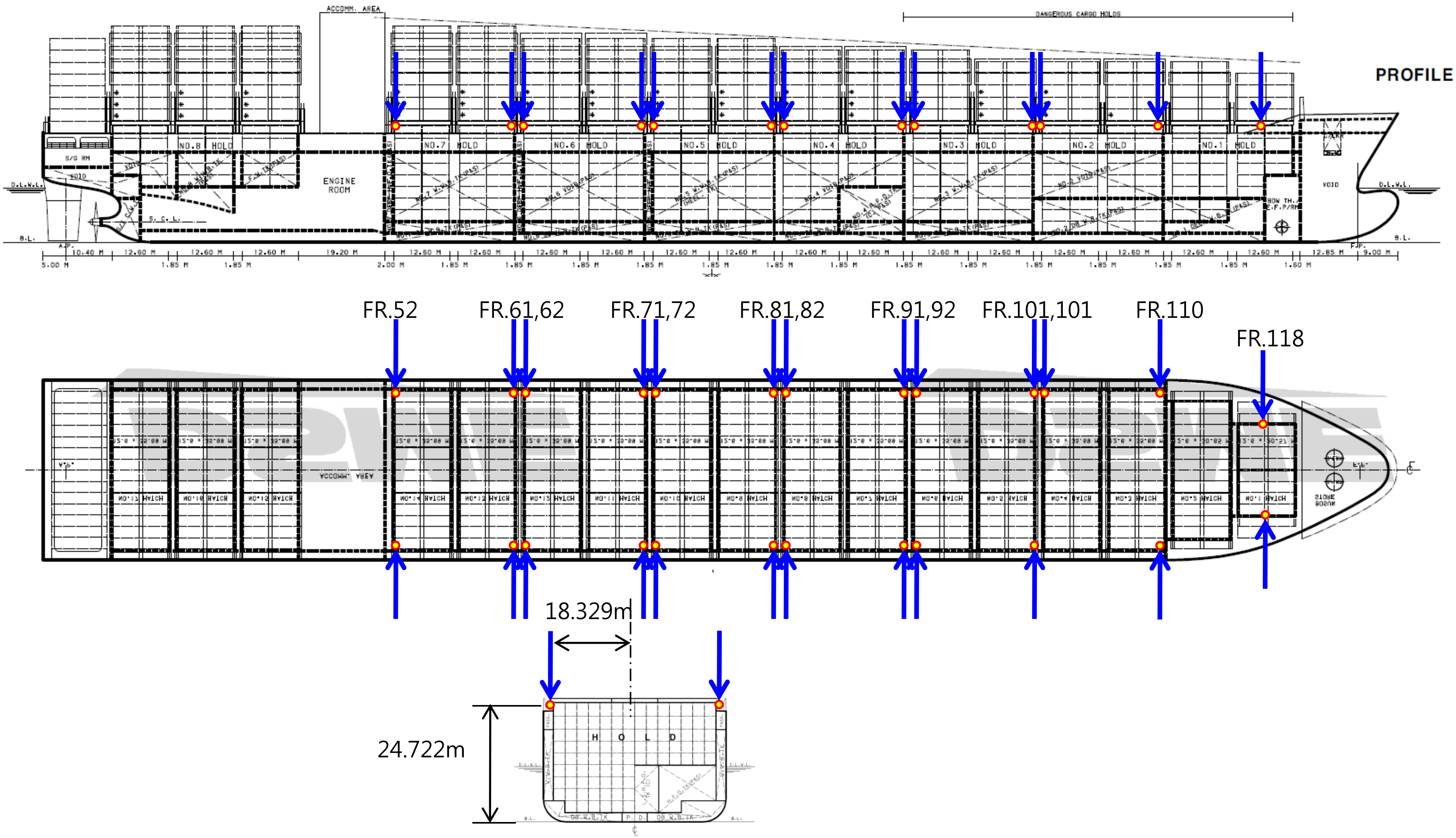


Opening Location for Cargo Hold : Unprotected opening

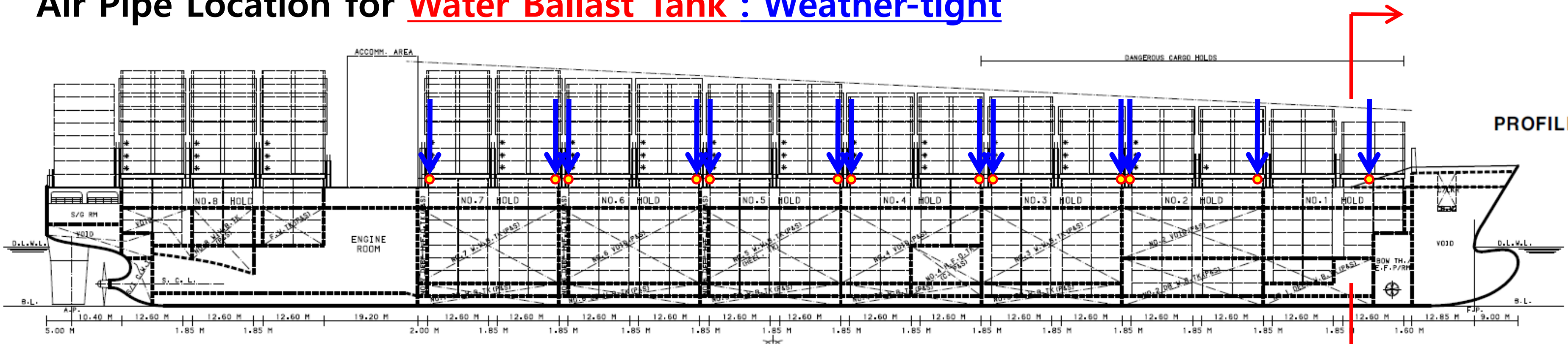


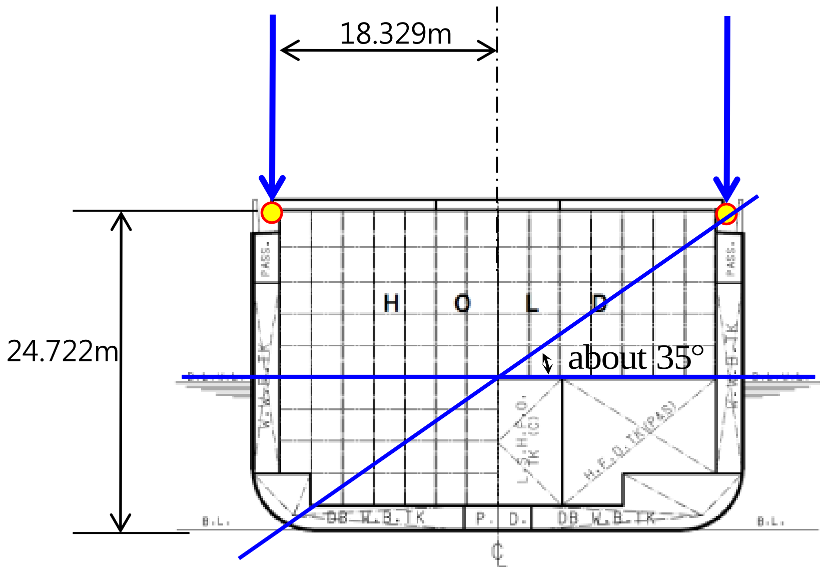


Air Pipe Location for Water Ballast Tank : Weather-tight

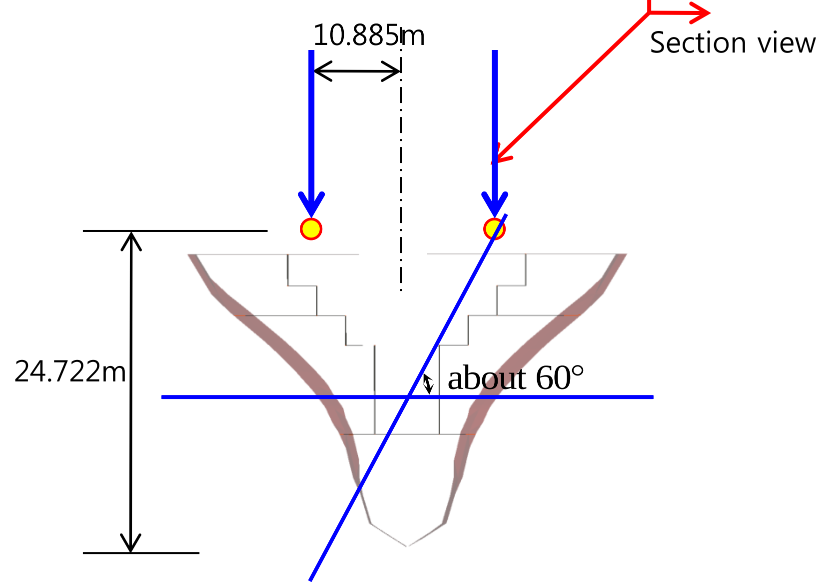


Air Pipe Location for Water Ballast Tank : Weather-tight





<Typical section view>

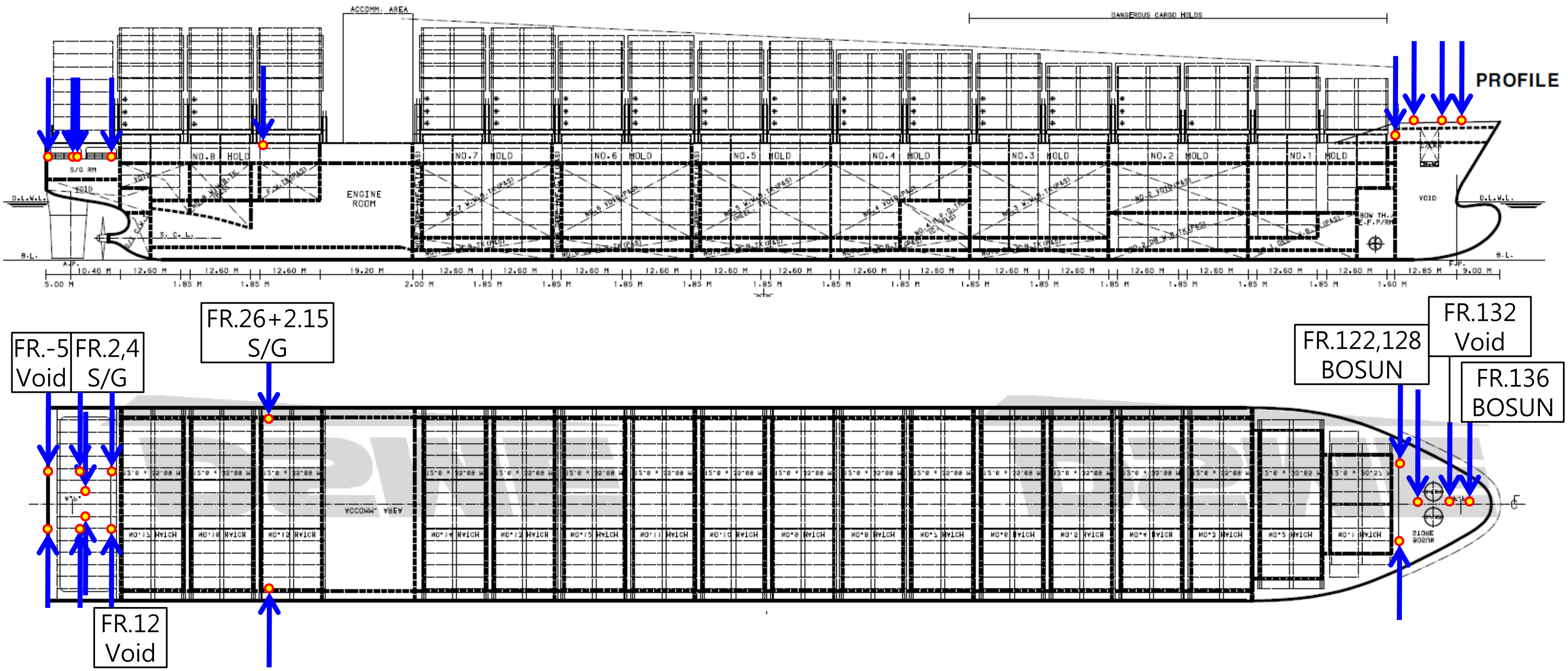


<Section view of forward area>

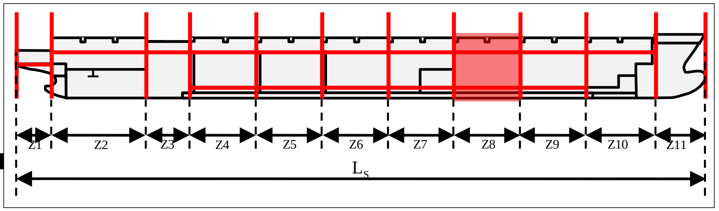
Which is dangerous?

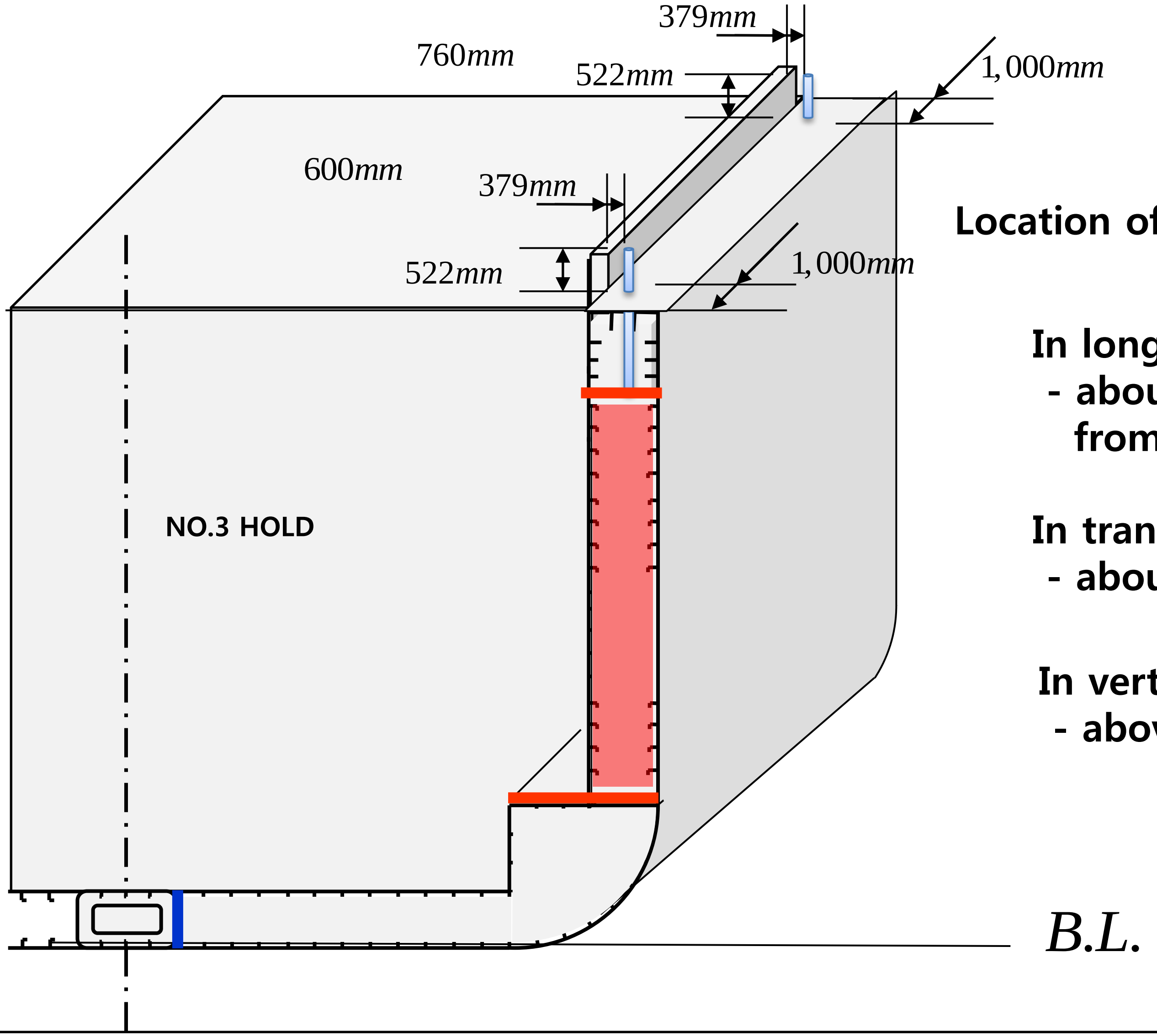
Air pipe located outboard is more easily exposed to flooding

Opening Location for other compartments



Opening and a Air Escape Pipe





Location of opening

In longitudinal direction
- about 1,000 mm off from transverse bulkhead

In transverse direction
- about 600 mm off

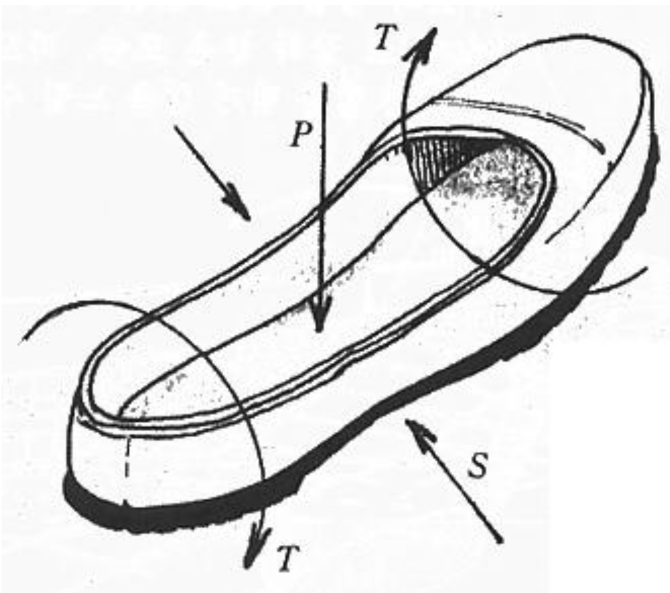
In vertical direction
- above 760 mm deck

Chapter 14.
Structural Arrangement of a Ship

1. Structure Plans
2. Structural Arrangement of a VLCC
3. Structural Arrangement of a Container Carrier
4. Structural Arrangement of a Bulk Carrier

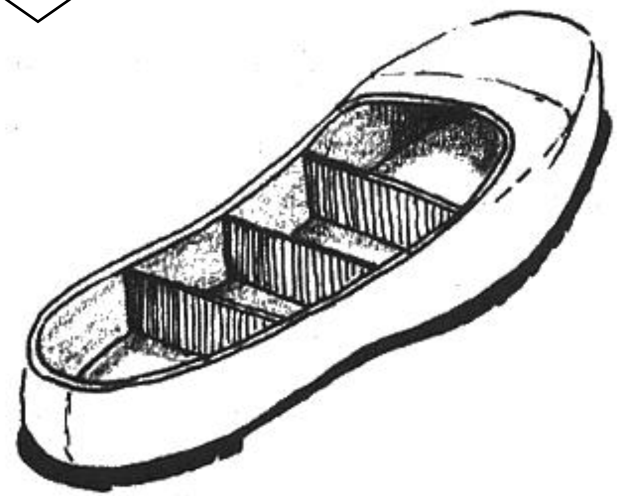
14-1. Ship Structure

Ship Structure



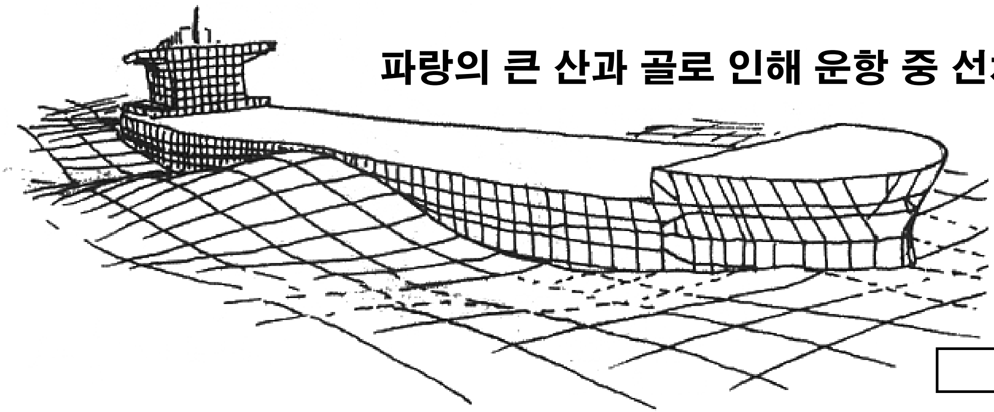
어떠한 힘이나 굽힘 모멘트에도
쉽게 변형이 일어남

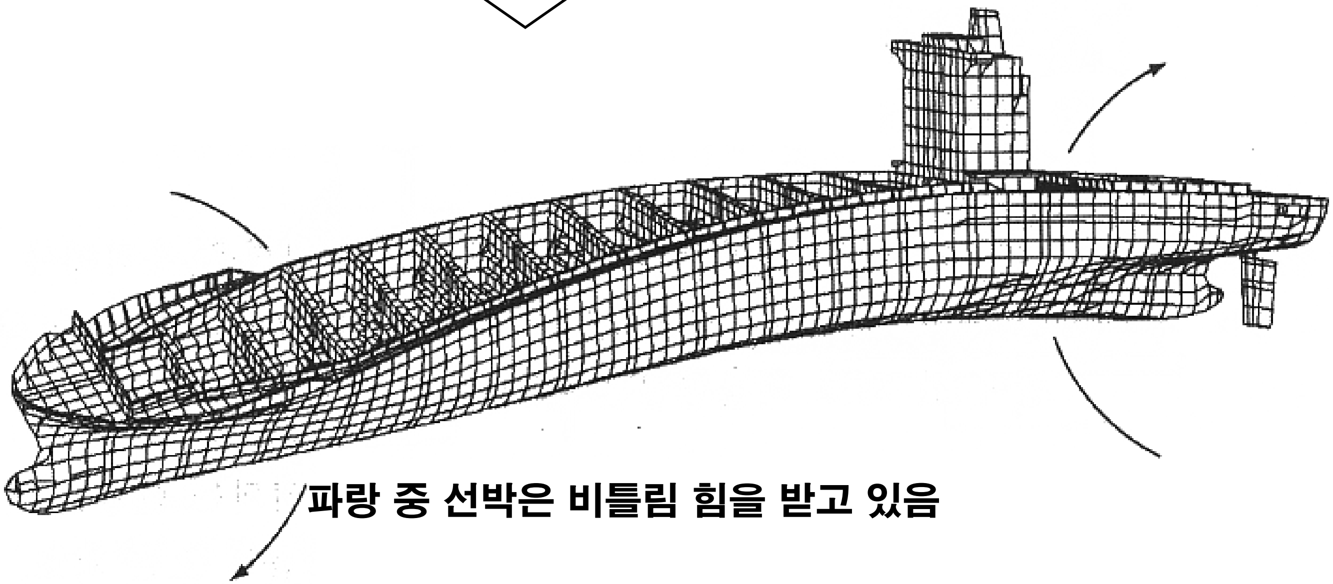
어떠한 힘이나 굽힘 모멘트에도
훨씬 강하게 대응할 수 있음



Ship in Waves

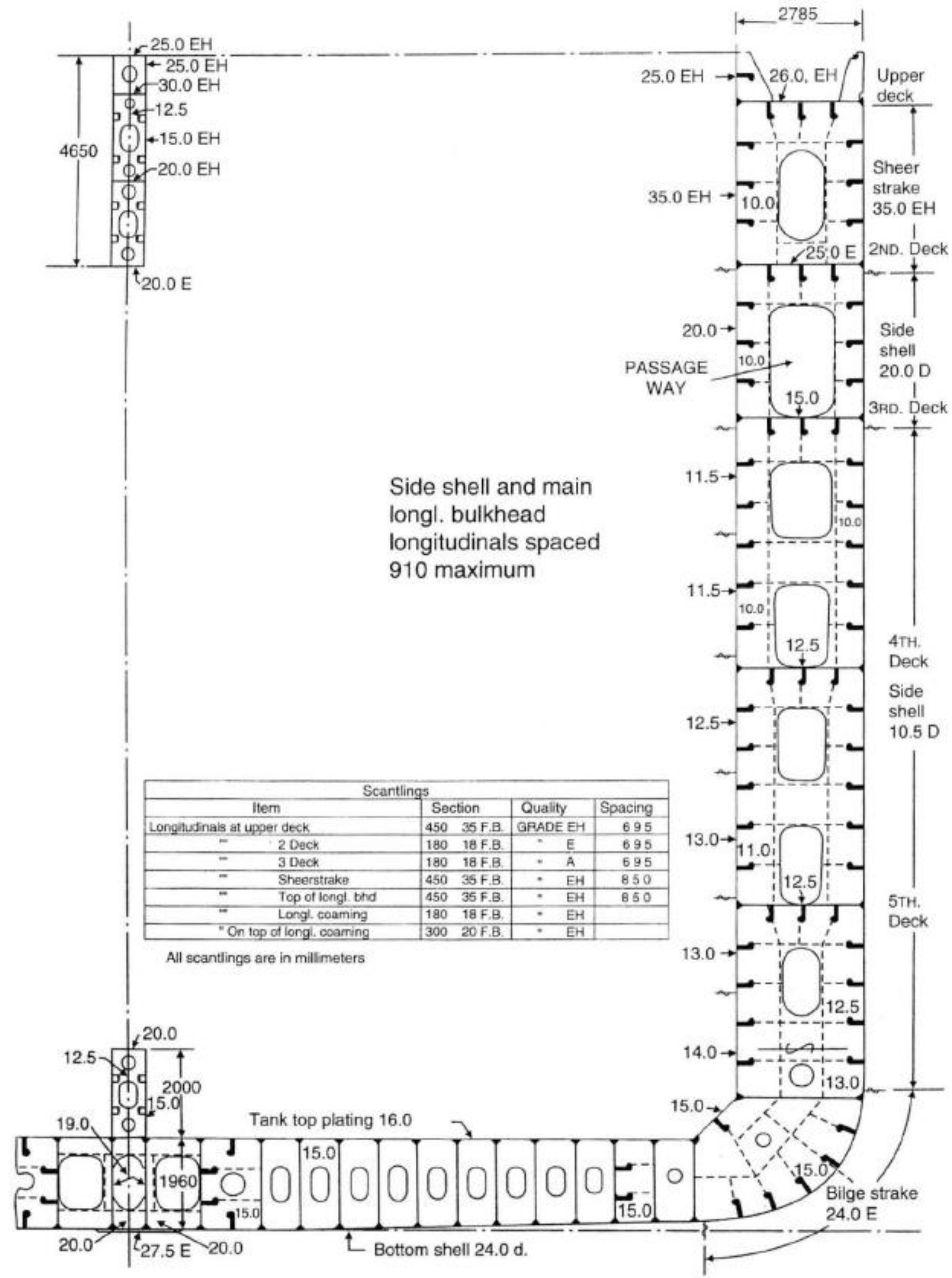
파랑의 큰 산과 골로 인해 운항 중 선체는 큰 힘을 받게 됨





파랑 중 선박은 비틀림 힘을 받고 있음

Web Frame of a Container Carrier

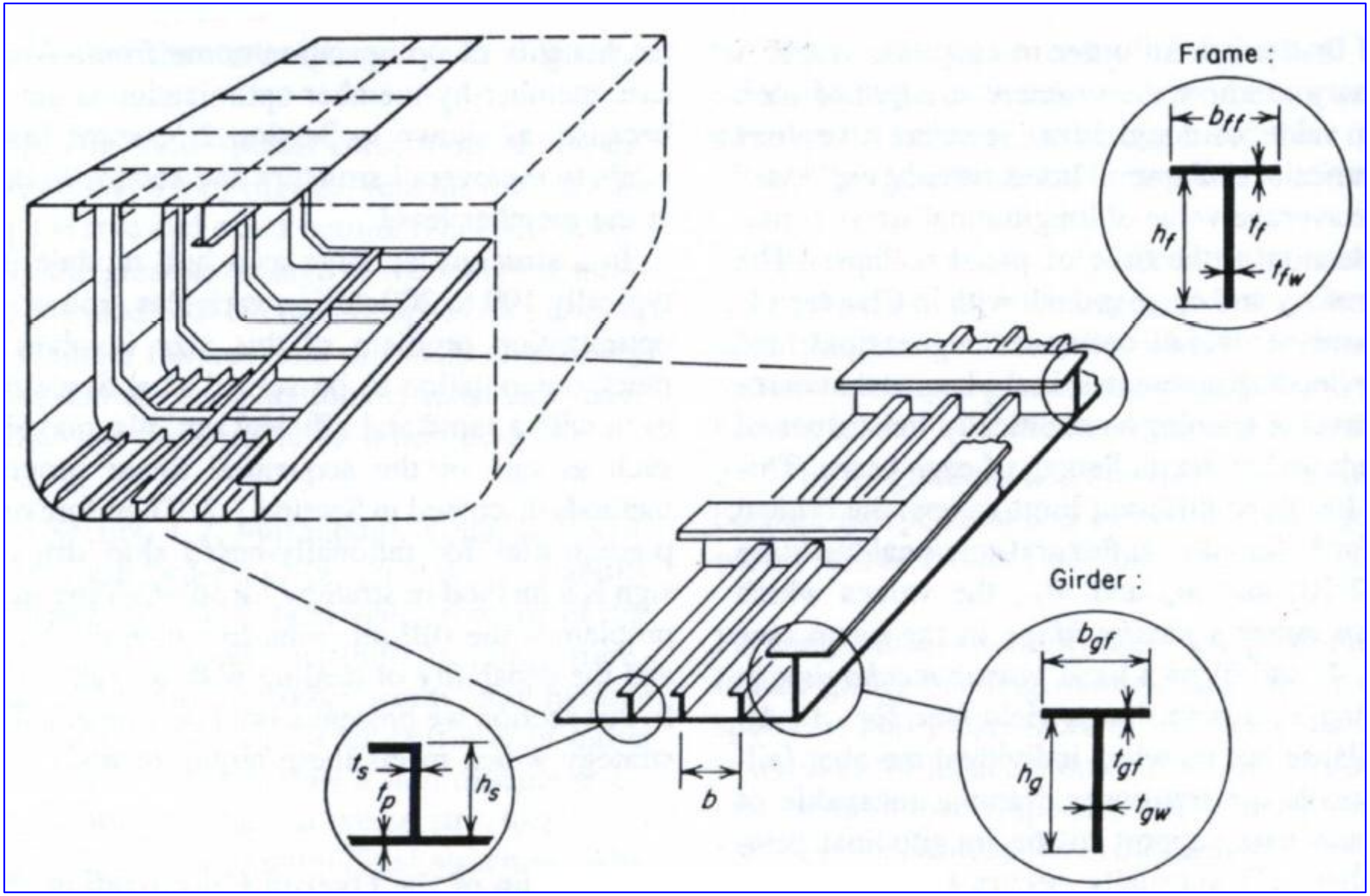


Scantlings			
Item	Section	Quality	Spacing
Longitudinals at upper deck	450 35 F.B.	GRADE EH	6 9 5
" 2 Deck	180 18 F.B.	" E	6 9 5
" 3 Deck	180 18 F.B.	" A	6 9 5
" Sheerstrake	450 35 F.B.	" EH	8 5 0
" Top of longl. bhd	450 35 F.B.	" EH	8 5 0
" Longl. coaming	180 18 F.B.	" EH	
" On top of longl. coaming	300 20 F.B.	" EH	

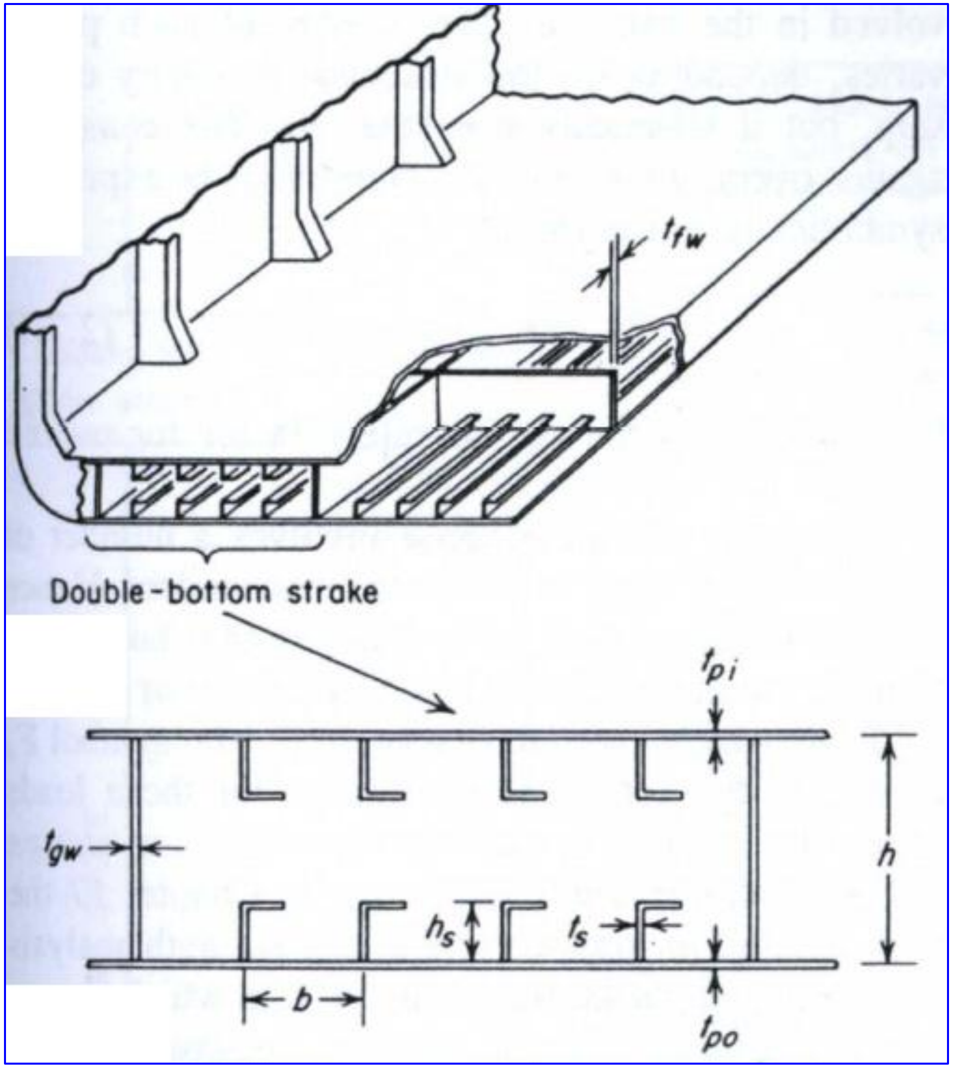
All scantlings are in millimeters

MOLLAND, F. A., THE MARITIME ENGINEERING REFERENCE BOOK, ELSEVIER, 2008

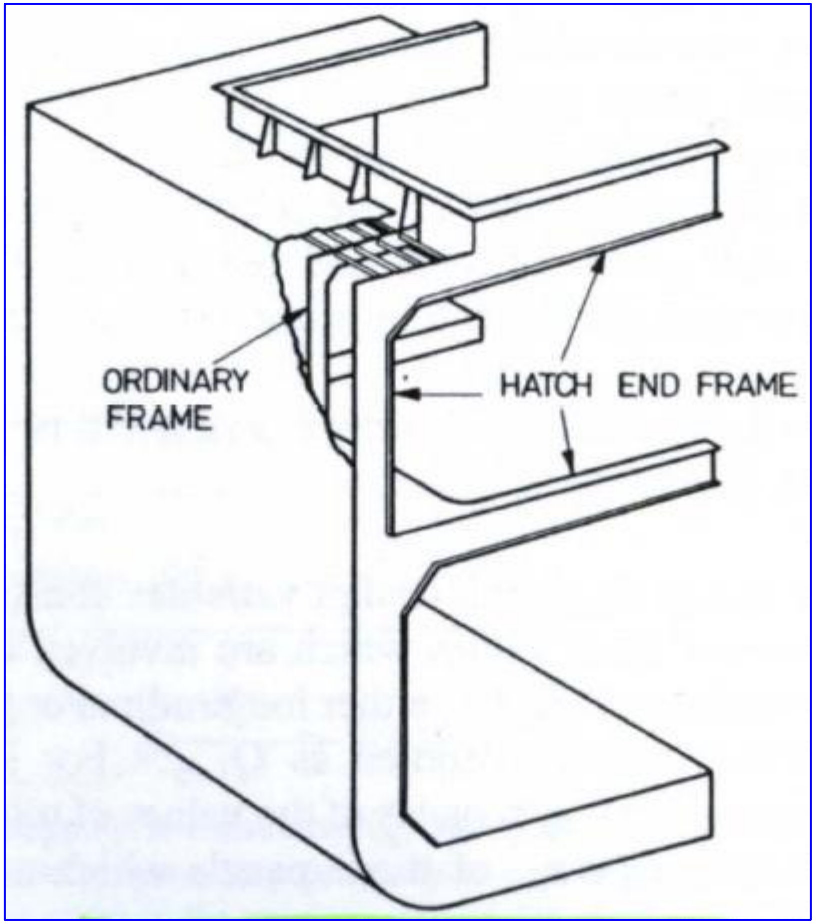
Bottom Structure



Double Bottom Structure, Frame & Hatch Coaming

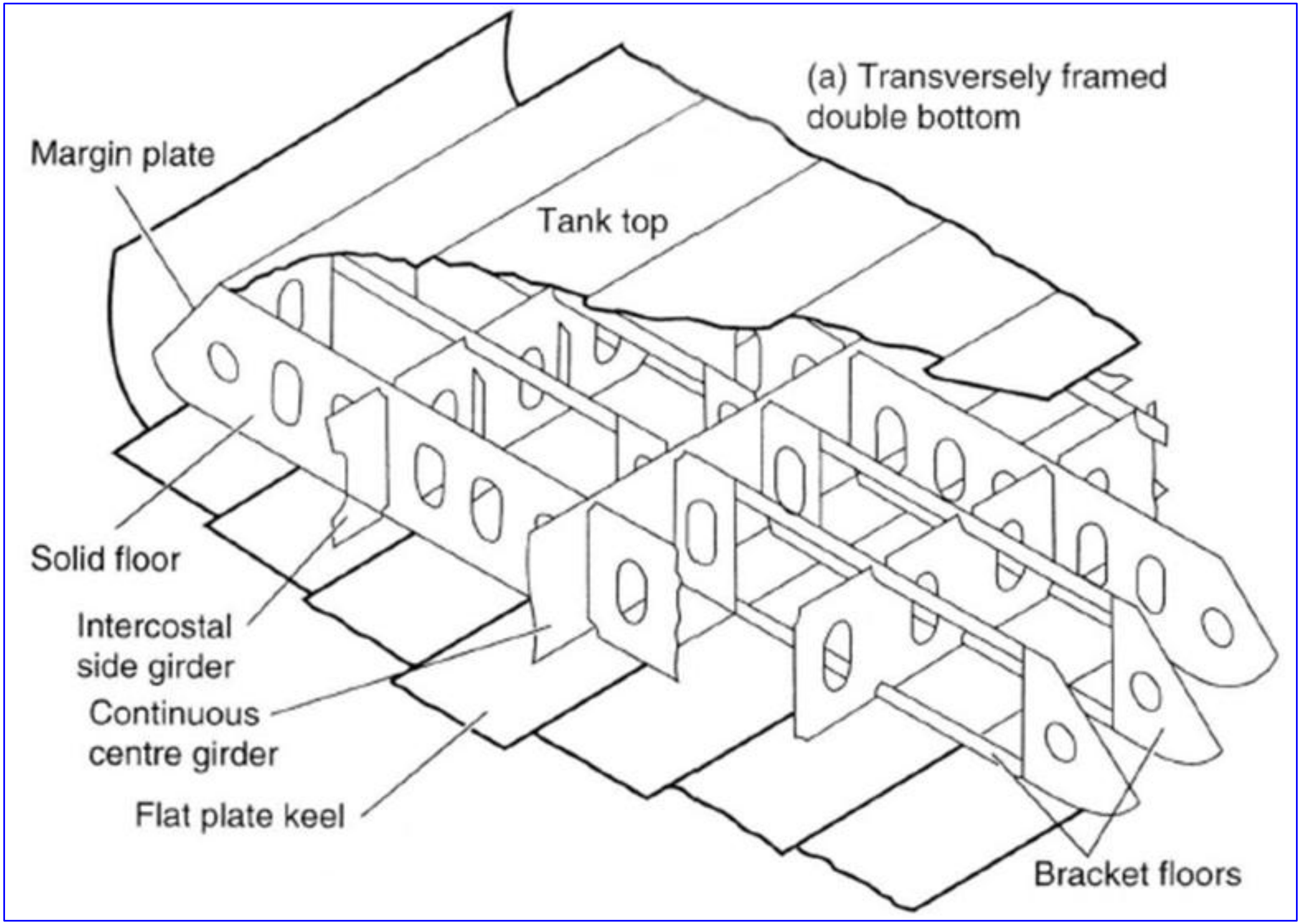


Double Bottom Structure

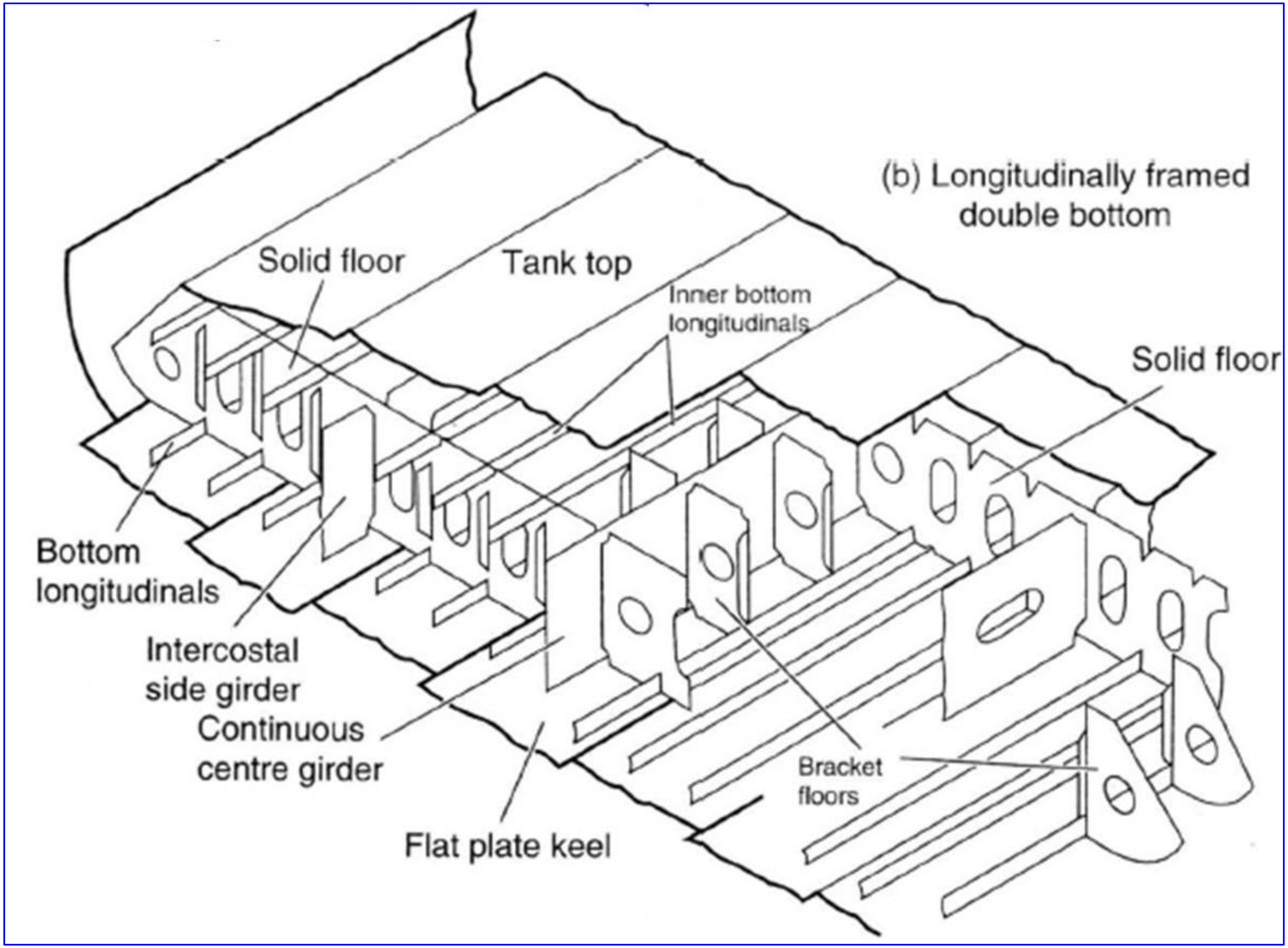


Frame & Hatch Coaming

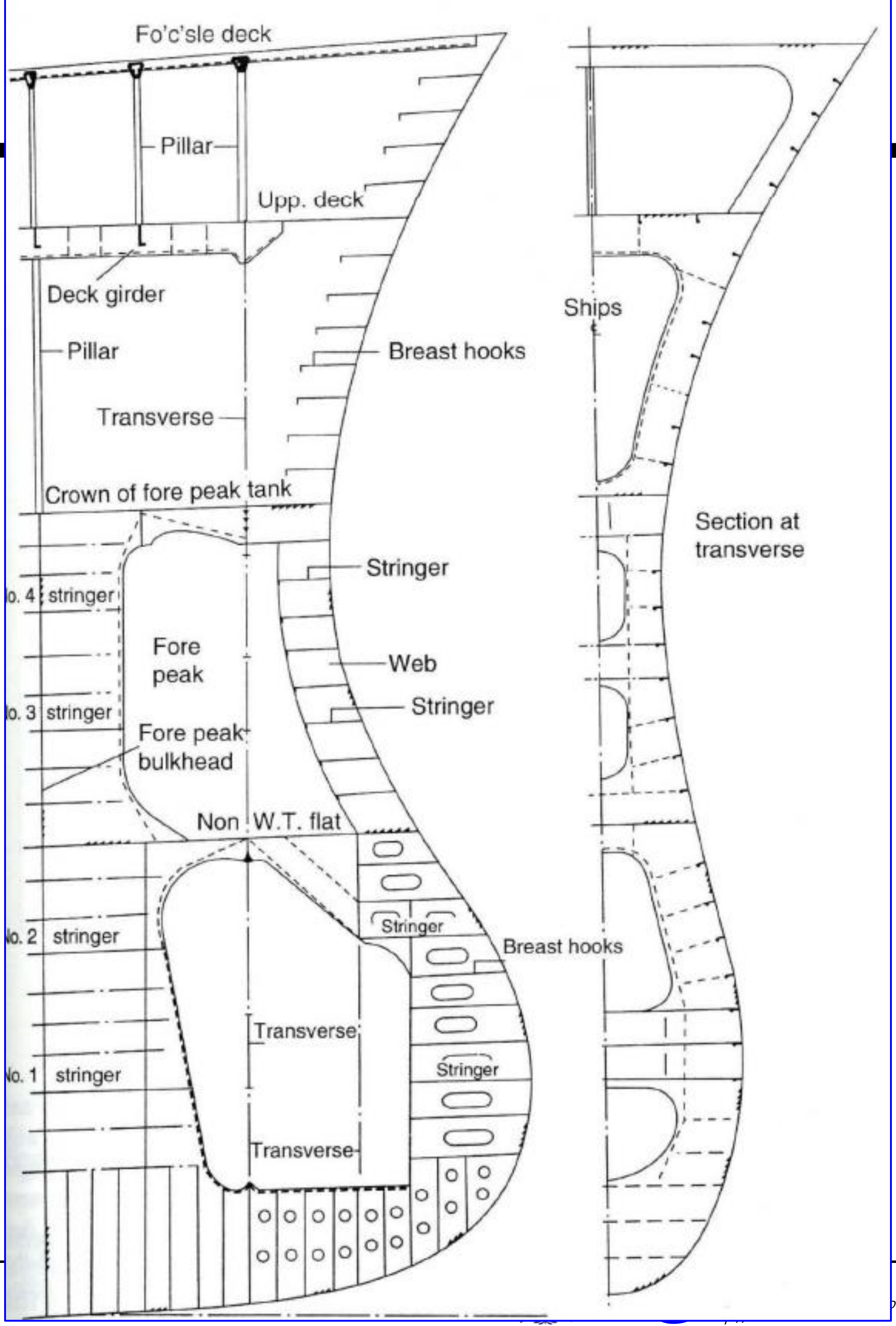
Transversely Framed Double Bottom



Longitudinally Framed Double Bottom



Construction Profile of Stem Part



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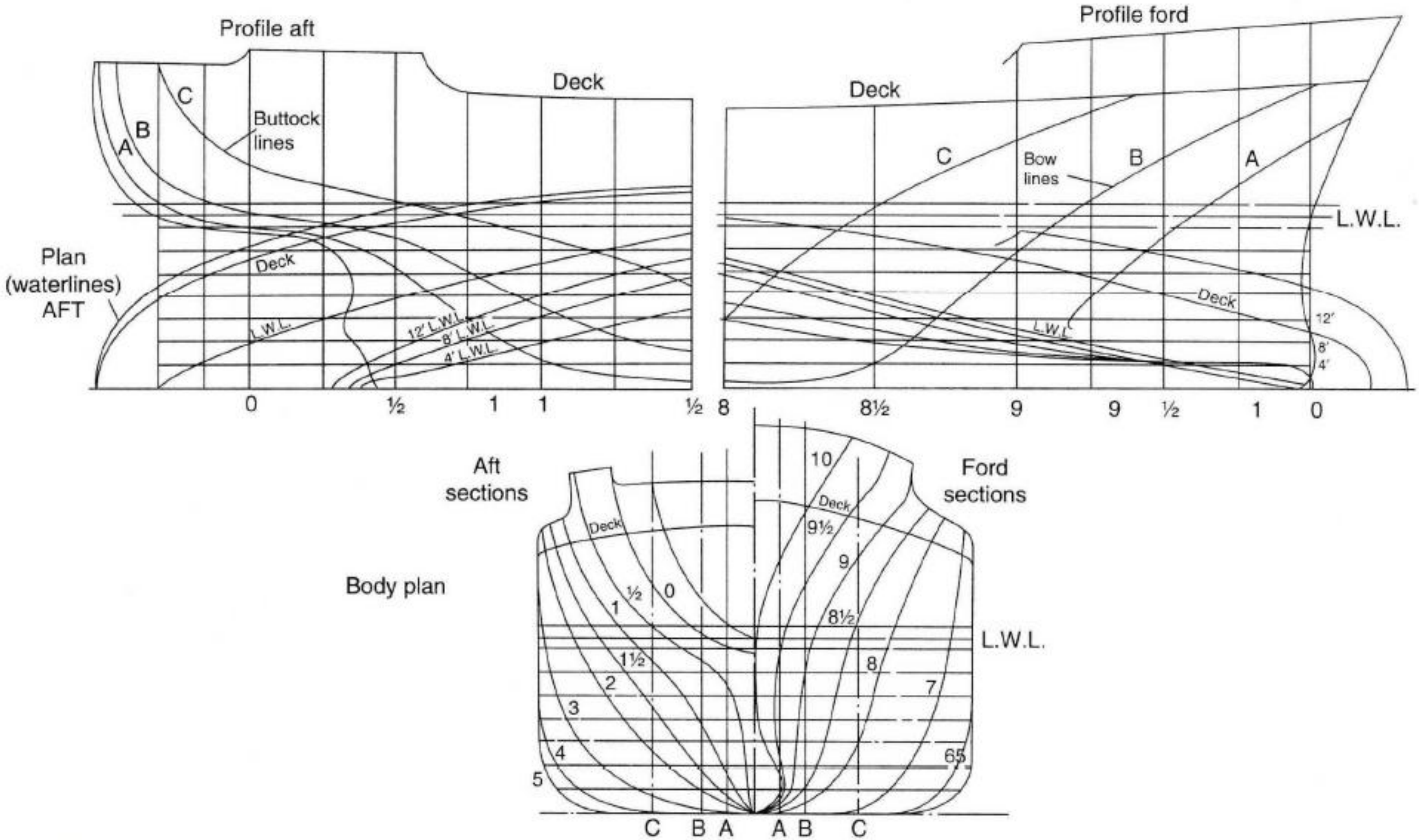
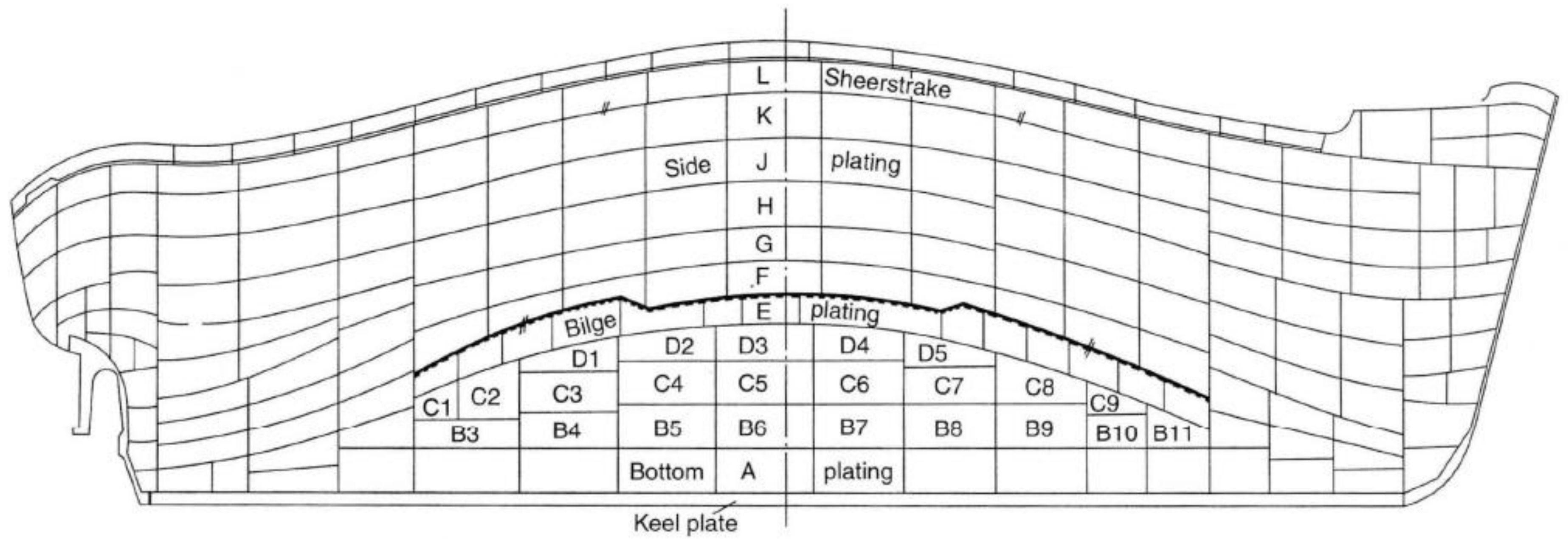
Lines Plans



Profile aft
Deck
C
B
A
Buttock lines
Plan (waterlines) AFT
Deck
L.W.L.
12' L.W.L.
8' L.W.L.
4' L.W.L.
0
½
1
1
½
Profile ford
Deck
C
Bow lines
B
A
L.W.L.
Deck
L.W.L.
12'
8'
4'
8
8½
9
9
½
1
0
Aft sections
Ford sections
Body plan
Deck
10
Deck
9½
9
8½
½
0
1
1½
2
3
4
5
L.W.L.
8
7
65
C
B
A
A
B
C

Shell Expansions



Framing, stringers, decks and openings in side shell are also shown on the shell expansion but have been omitted for clarity

14-2. Structural Arrangement of a VLCC (Very Large Crude oil Carrier)

- DWT : Deadweight

VLCC(Very Large Crude Oil Carrier)

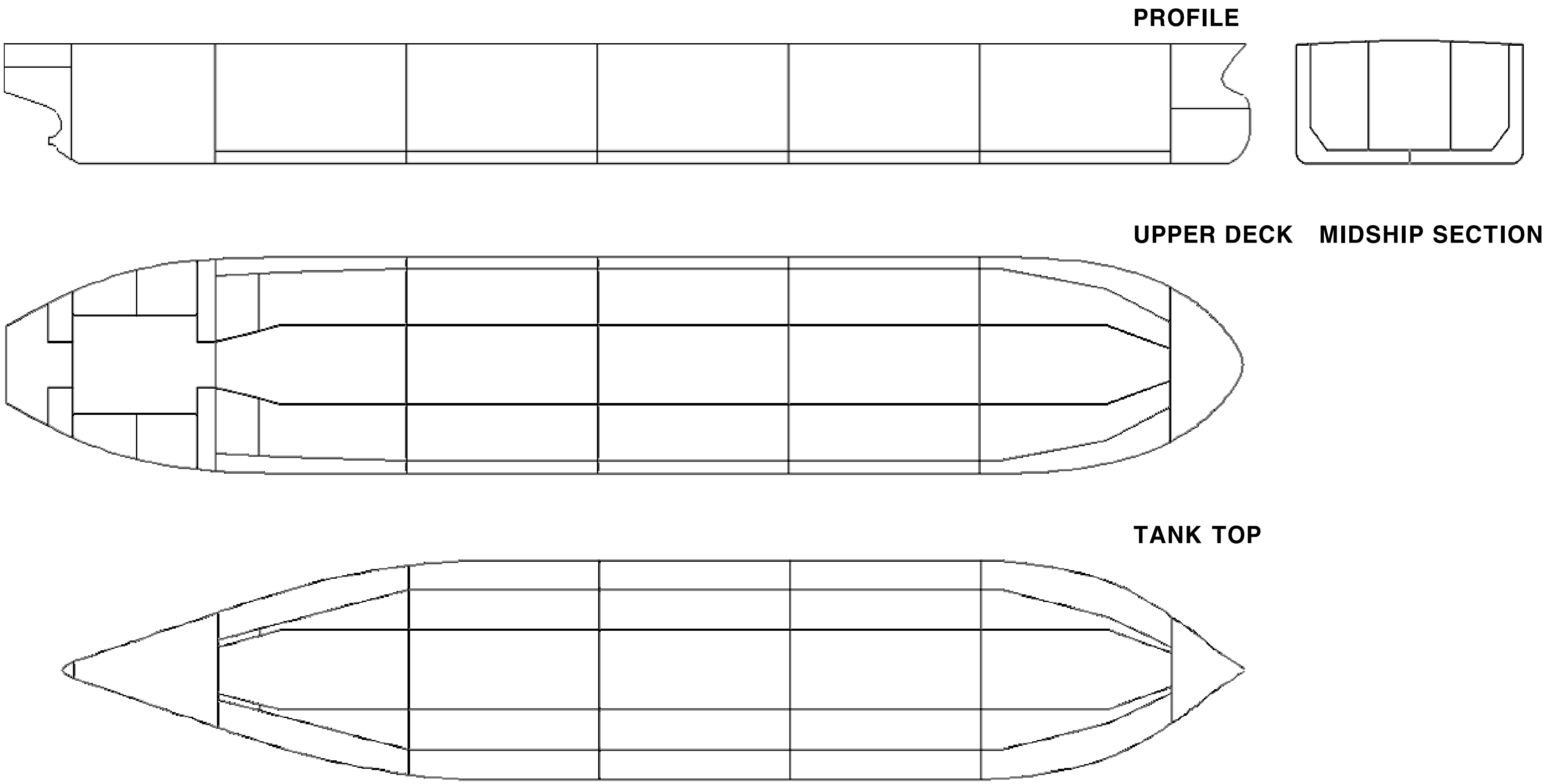
442,000 ton DWT
ULCC(Ultra Large Crude Oil Carrier)

LxBxDxTxCb= 320m x 60m x 30m x 20 x 0.81

Ballast draft : 14m

300,000 ton DWT VLCC

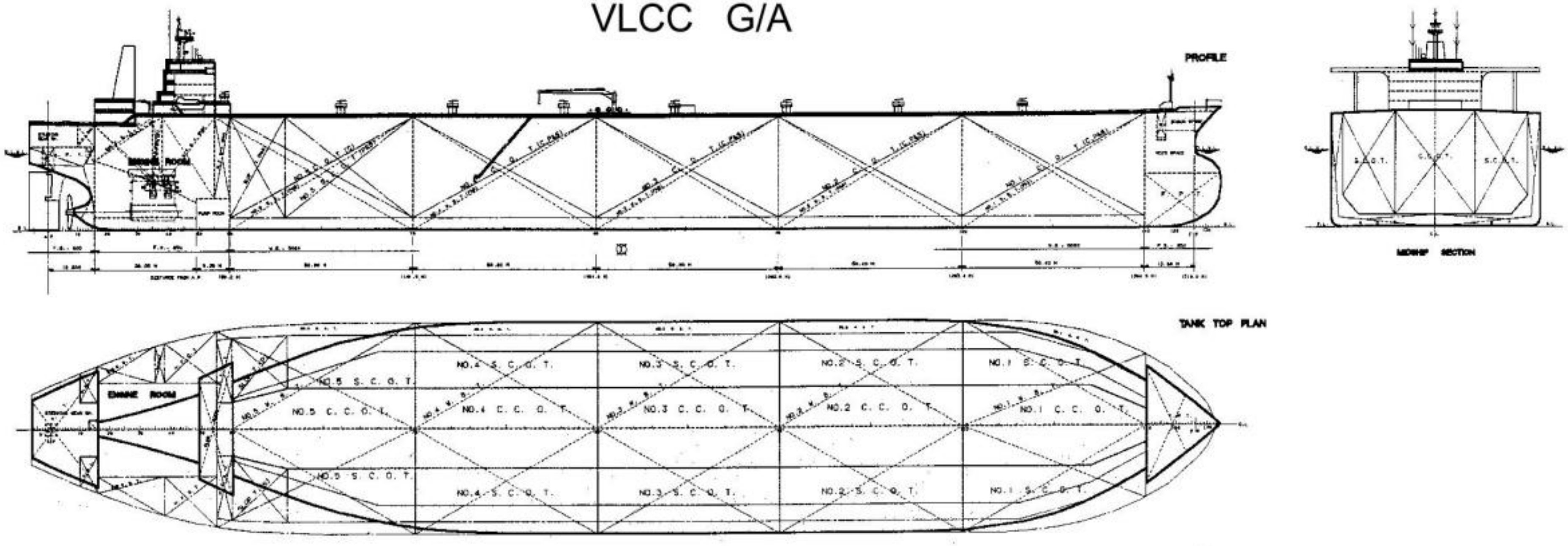
Example of compartment arrangement of a Tanker



General Arrangement(G/A) of 308,000 ton DWT VLCC

General Arrangement(G/A) of a VLCC

VLCC G/A



MAIN PARTICULARS

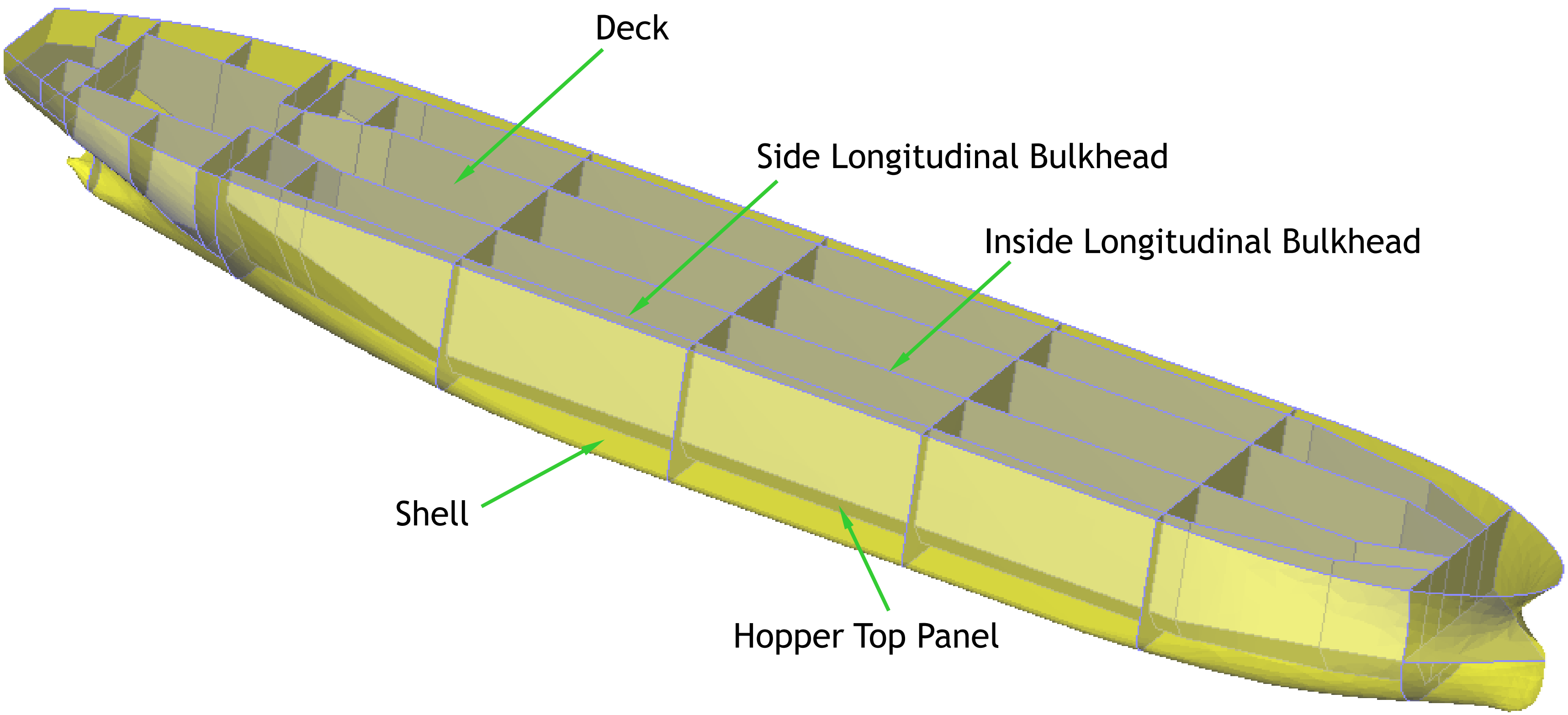
- -Length over all: apprx. 330.50m
- -Length betw. Perpendicular : 318.00m
- -Bredth(moulded) : 58.00m
- -Depth(moulded) : 31.25m

- -Draught(Designed, moulded) : 21.40m
- -Draught(Scantling, moulded) : 22.60m
- -Deadweight at Td : apprx. 288,000MT
- at Ts : apprx. 308,500MT

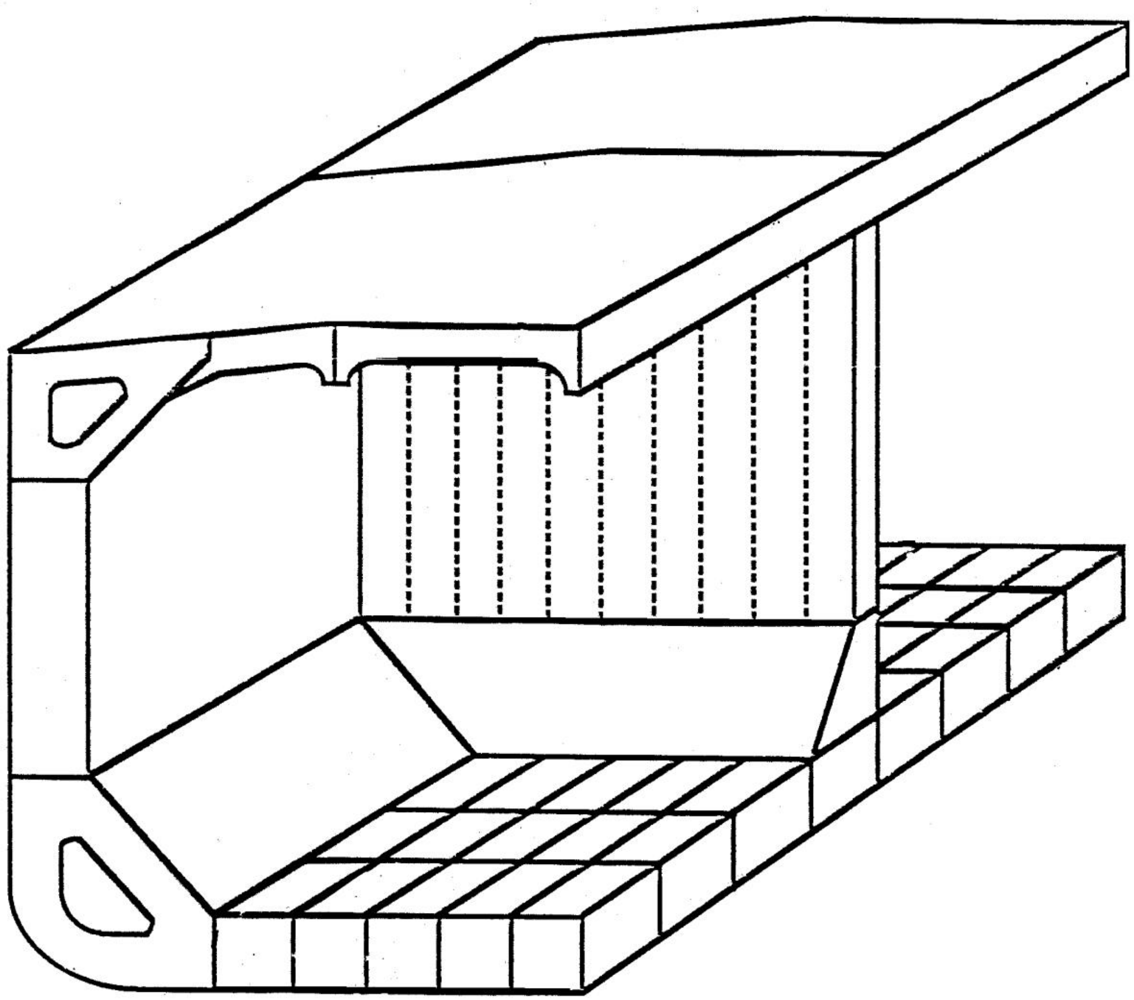
- -Crusing Range : apprx. 30,000NM
- -Service Speed : apprx. 15.3knots
- (Designed draught, 90% MCR, 15% Sea margin)
- -Class : DNV or ABS or LR equivalent
- -Gross Tonnage : apprx. 160,480 tons
- -Complements : 30 persons + 6suez crews

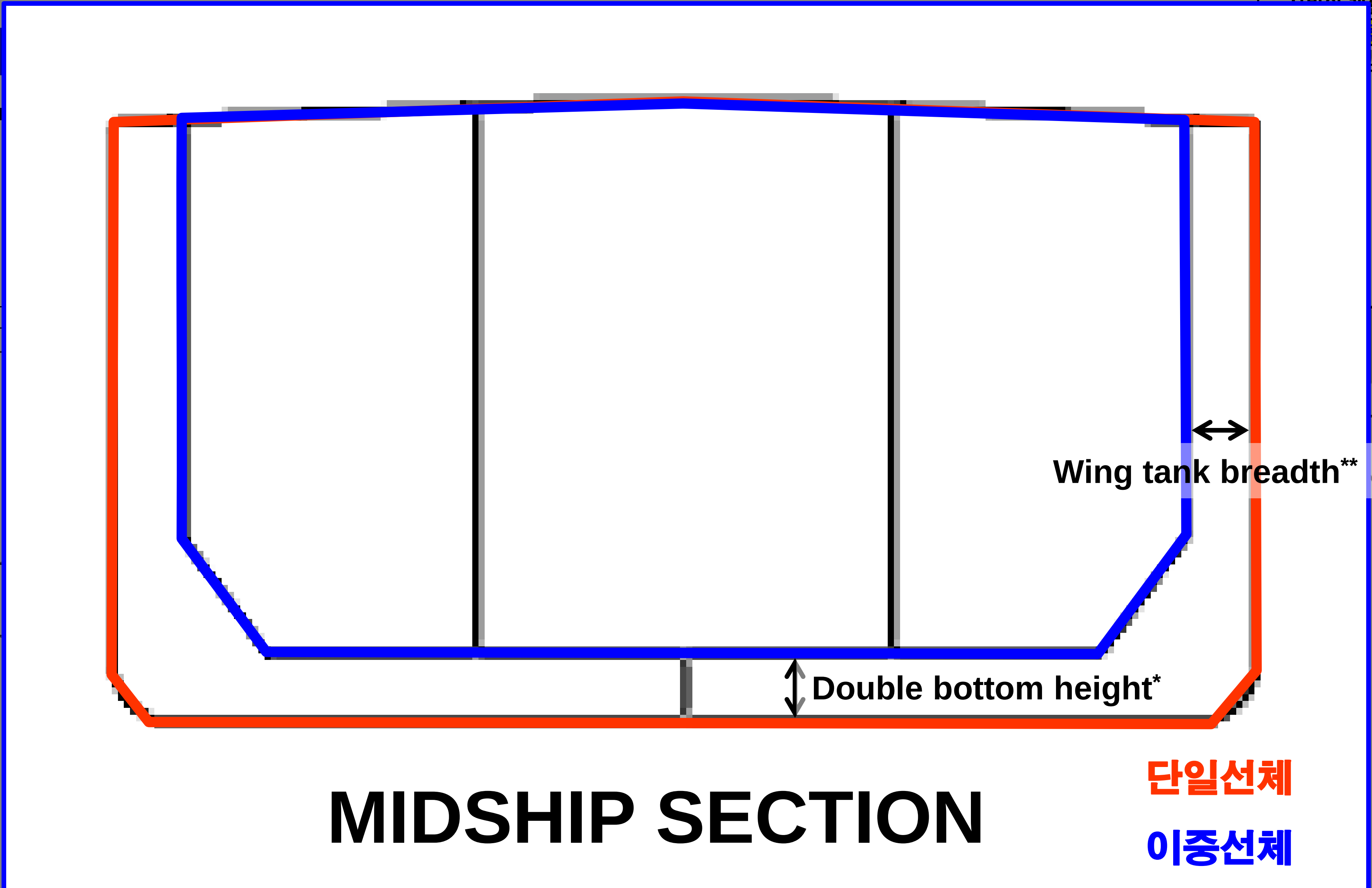
- DWT : Deadweight

3D Structure Model of a VLCC



Cargo hold part arrangement of a tanker



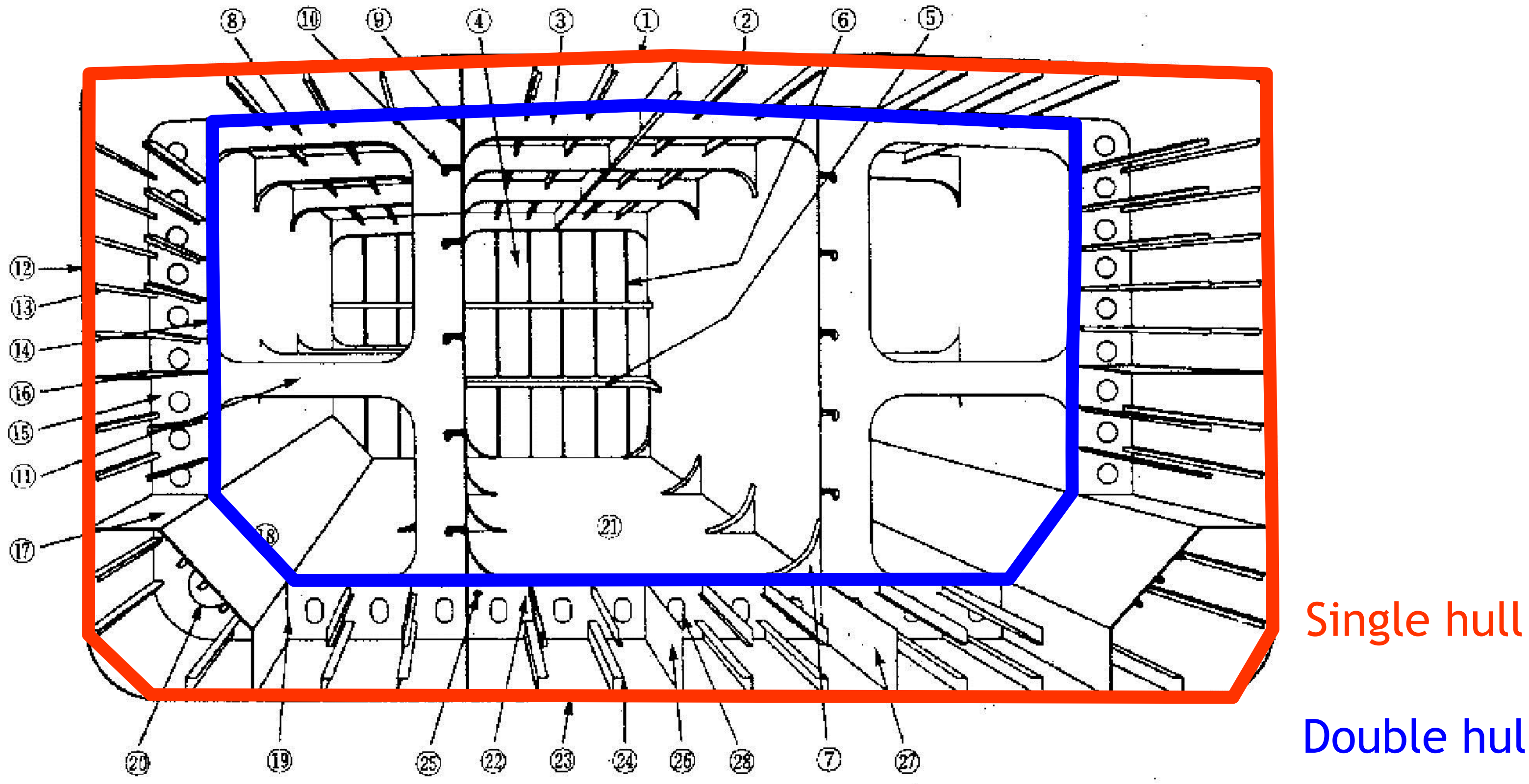


이중 선체 요건(MARPOL 73/78 Reg. 13F)

Double bottom height* : h이상 ; h=B/15 (m) 또는 2.0m 중 작은 값, 단 최소 1.0m ; 보통 3m

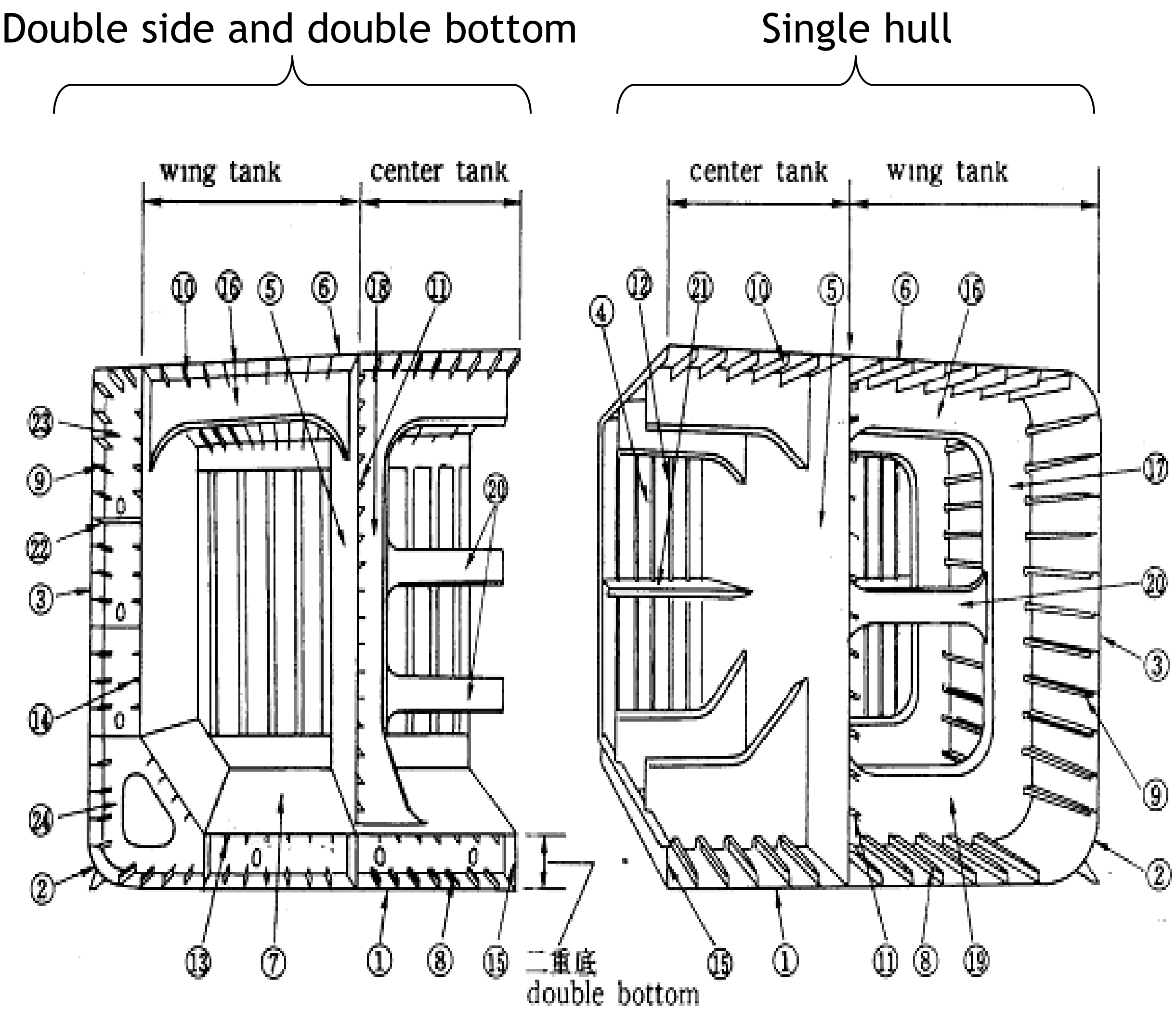
Wing tank breadth** : w이상 ; w=0.5+DWT/20,000 (m) 또는 2.0m 중 작은 값. 단 최소 1.0m ; 보통 3m

Naming of Cargo Hold Structure Members of a Tanker



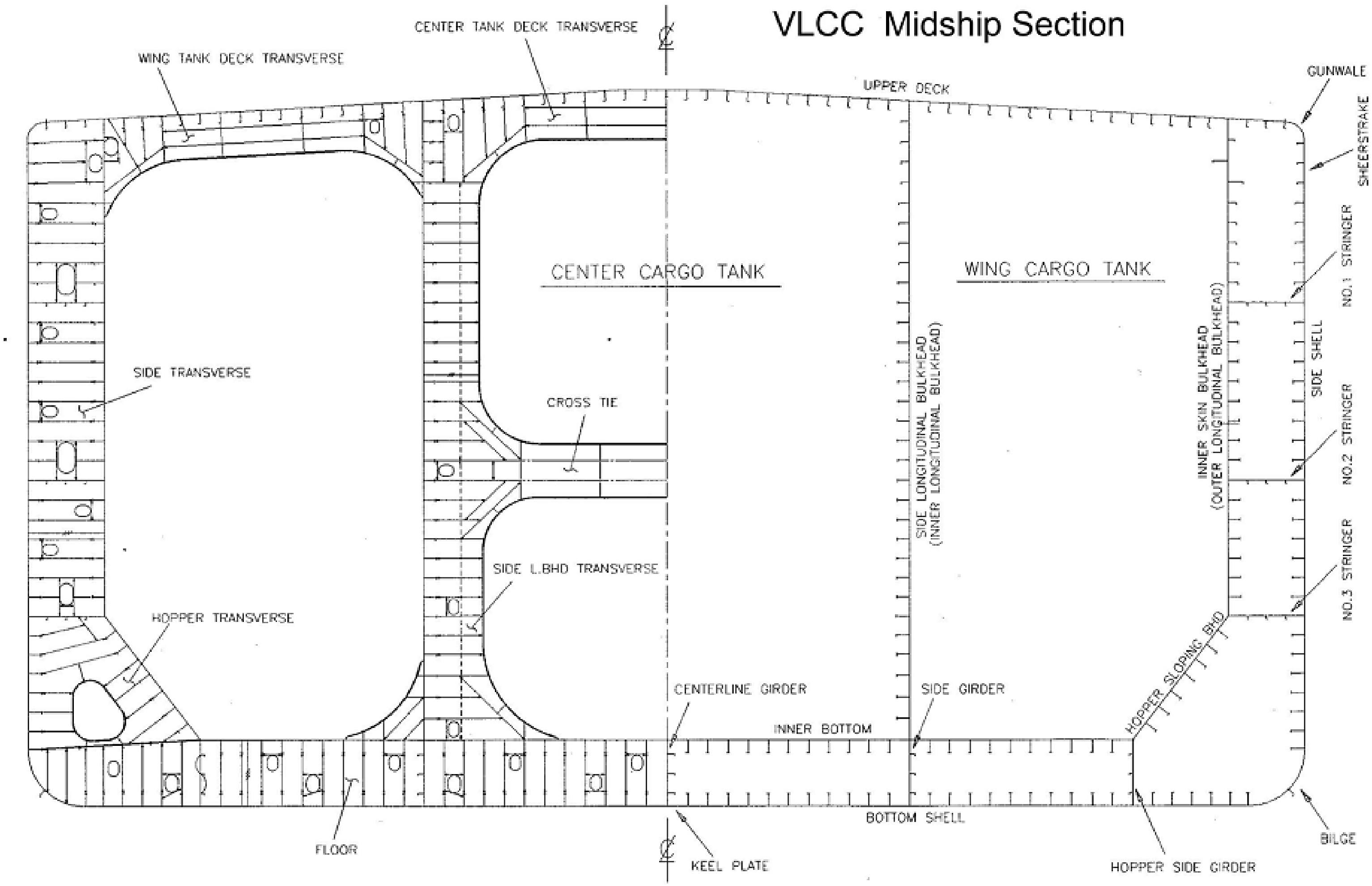
1. upper deck plate
2. deck longitudinal
3. transverse web
4. transverse bulkhead
5. stringer
6. transverse bulkhead stiffener
7. bracket
8. transverse ring
9. longitudinal bulkhead
10. longitudinal stiffener on L BHD
11. cross tie
12. side shell plate
13. side longitudinal
14. inner hull longitudinal bulkhead
15. wing tank transverse
16. horizontal girder
17. hopper horizontal girder
18. hopper plate
19. hopper side girder
20. hopper hole
21. inner bottom plate
22. Inner bottom longitudinal
23. bottom plate
24. bottom longitudinal
25. double bottom floor
26. center girder
27. side girder
28. manhole

Naming of Cargo Hold Structure Members of a Tanker

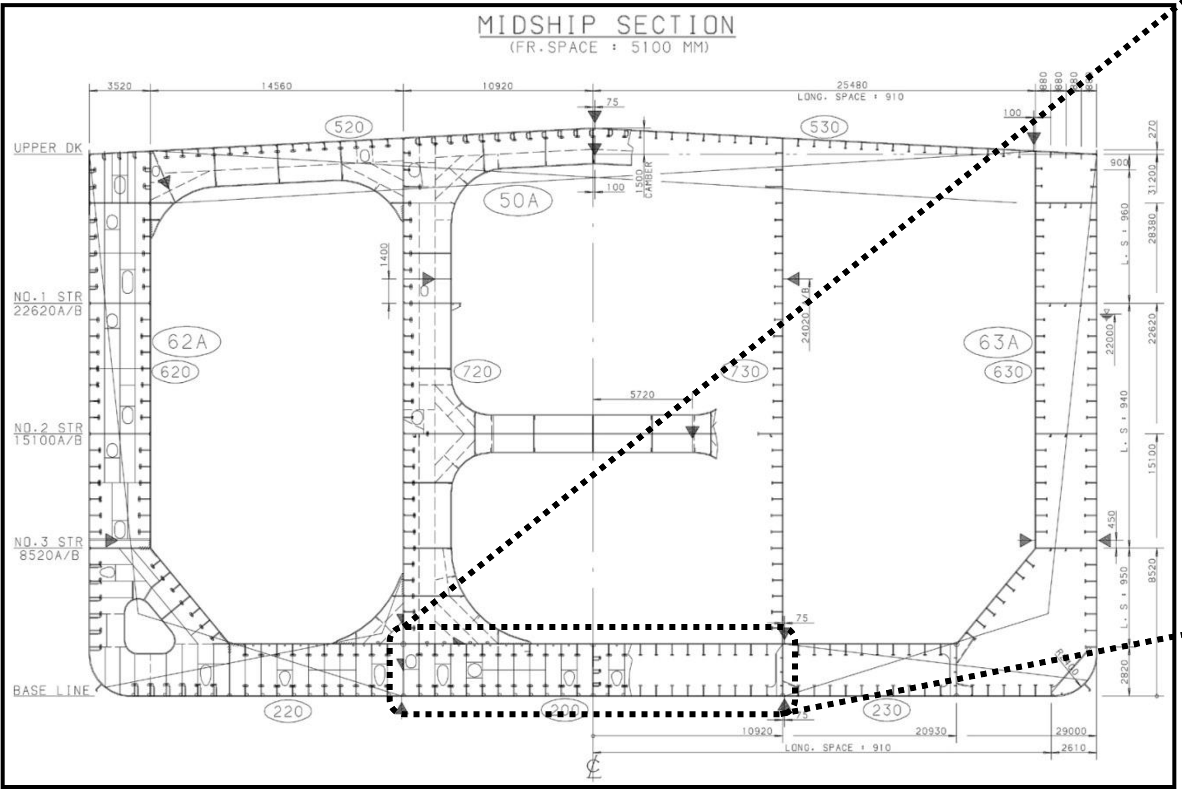


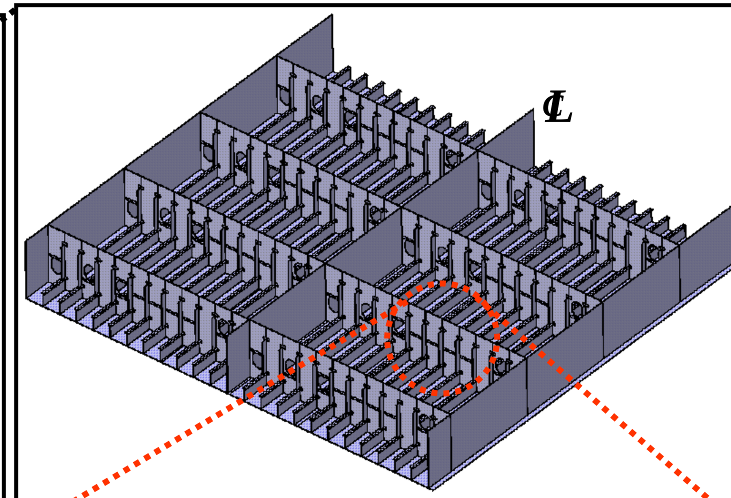
1) bottom plate
2) bilge strake
3) side shell plate
4) transverse bulkhead plate
5) longitudinal bulkhead plate
6) upper deck plate
7) inner bottom plate
8) bottom longitudinal
9) side longitudinal
10) deck longitudinal
11) longitudinal stiffener on L BHD
12) transverse bulkhead stiffener
13) inner bottom longitudinal
14) longitudinal stiffener on Inner hull
15) bottom center girder
16) deck transverse
17) side transverse
18) vertical web
19) bottom transverse
20) strut
21) horizontal girder
22) side stringer
23) side transverse
24) bilge transverse

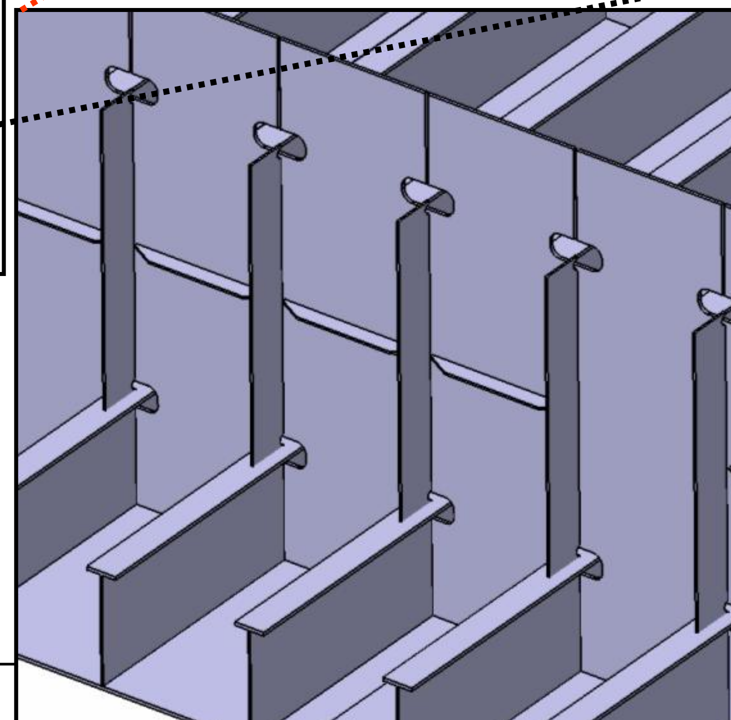
Midship section plan(중앙단면도) of a VLCC



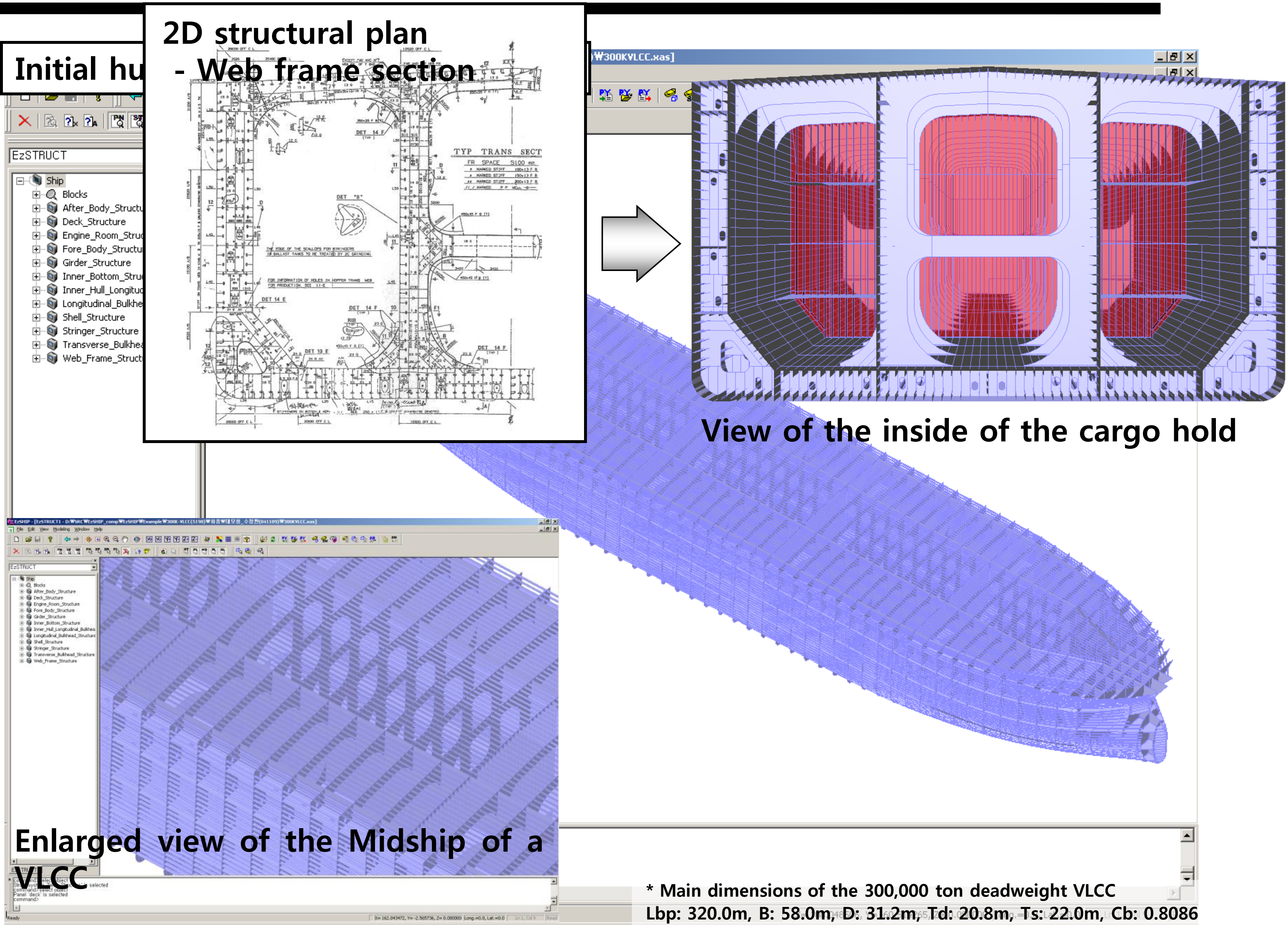
Structure Model of Midship Section of a VLCC







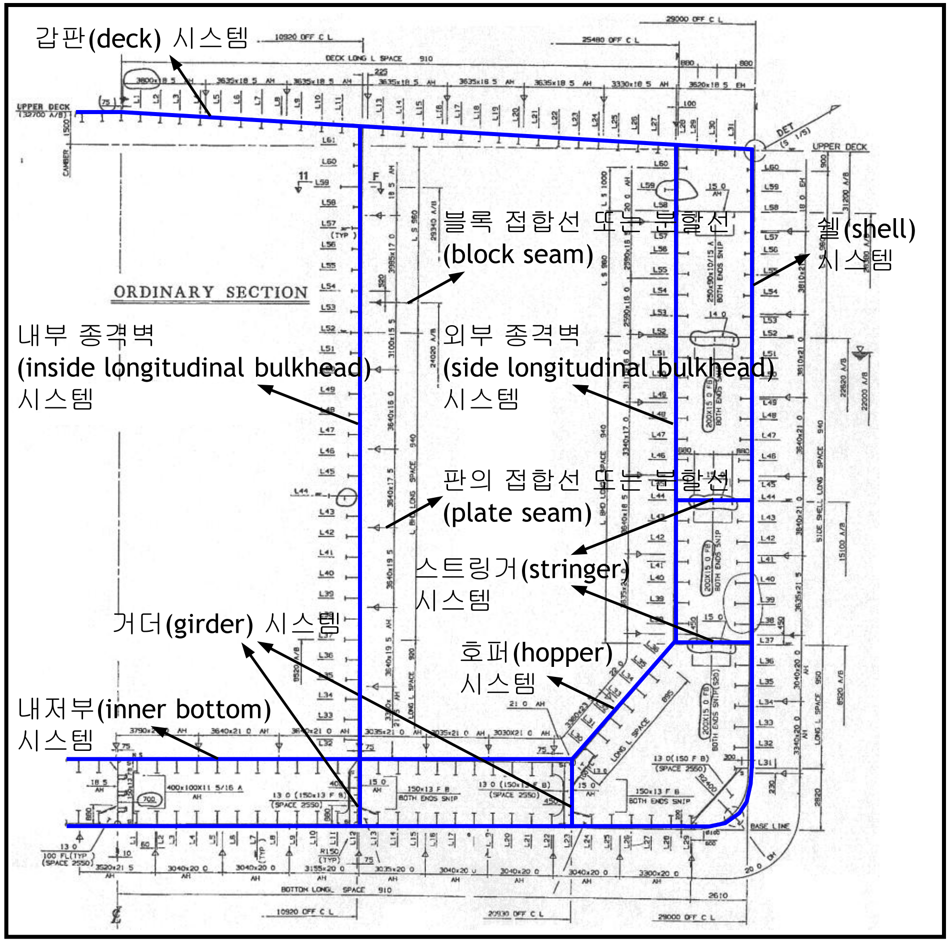
3D Structure Model of a 300,000 ton deadweight VLCC



* Main dimensions of the 300,000 ton deadweight VLCC

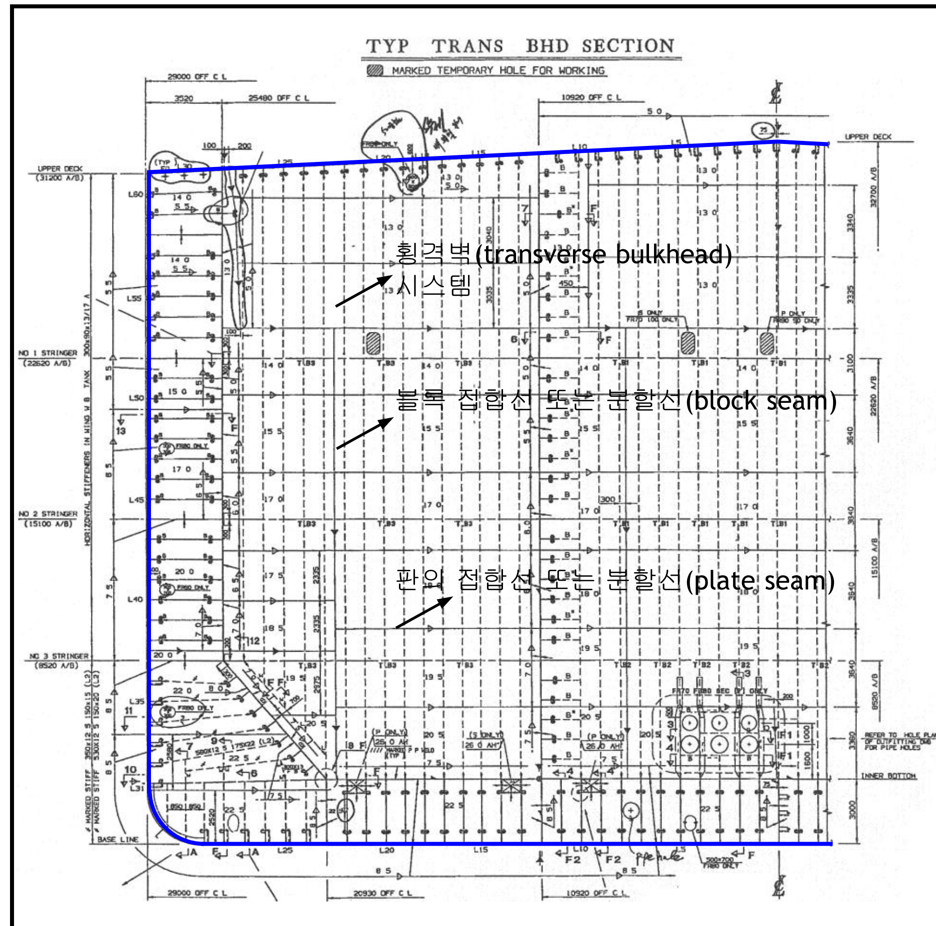
Lbp: 320.0m, B: 58.0m, D: 31.2m, Td: 20.8m, Ts: 22.0m, Cb: 0.8086

Midship section plan(중앙단면도) of a VLCC

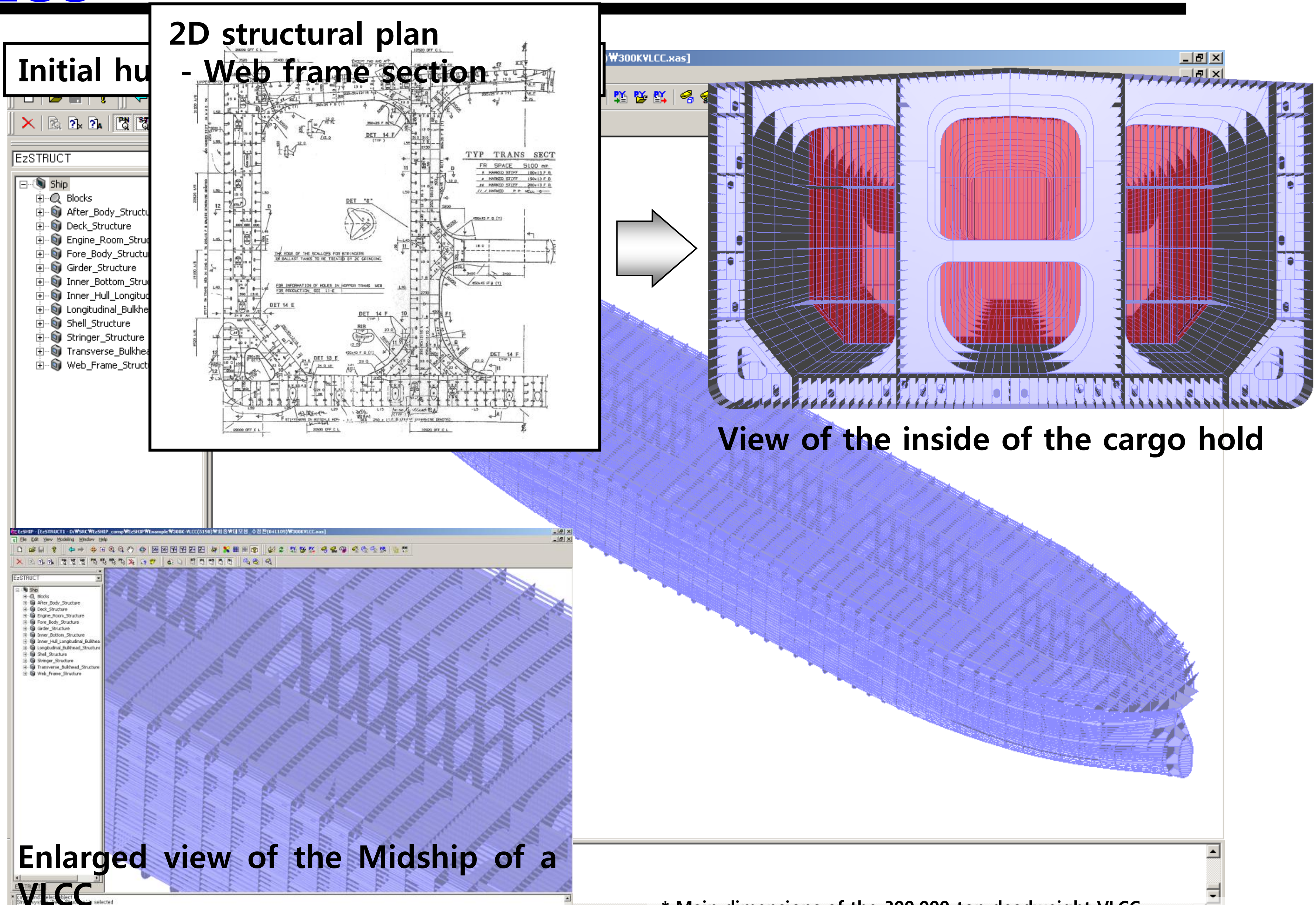


Midship section plan

Typical transverse bulkhead plan



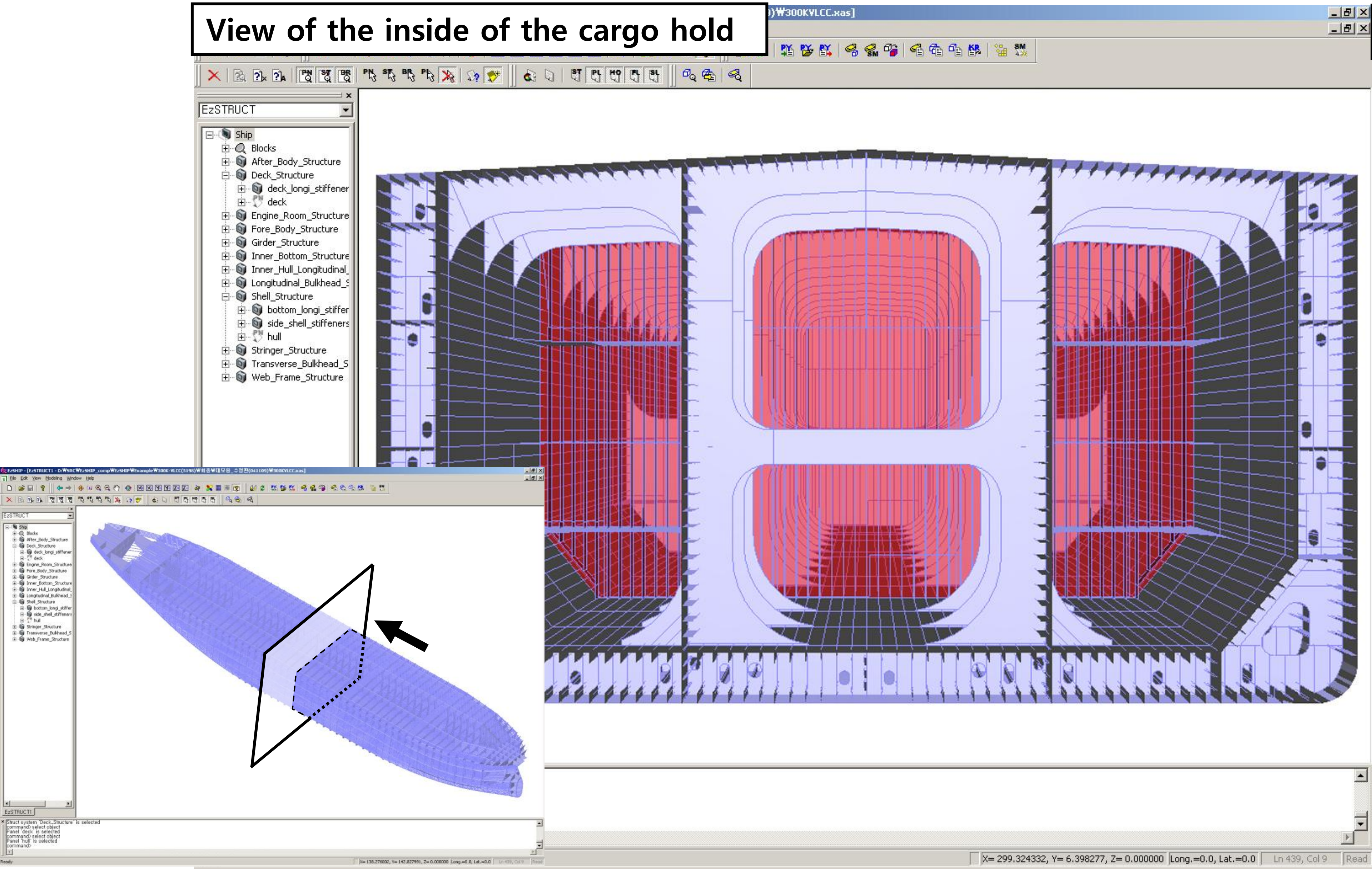
3D Structure Model of a 300,000 ton deadweight VLCC



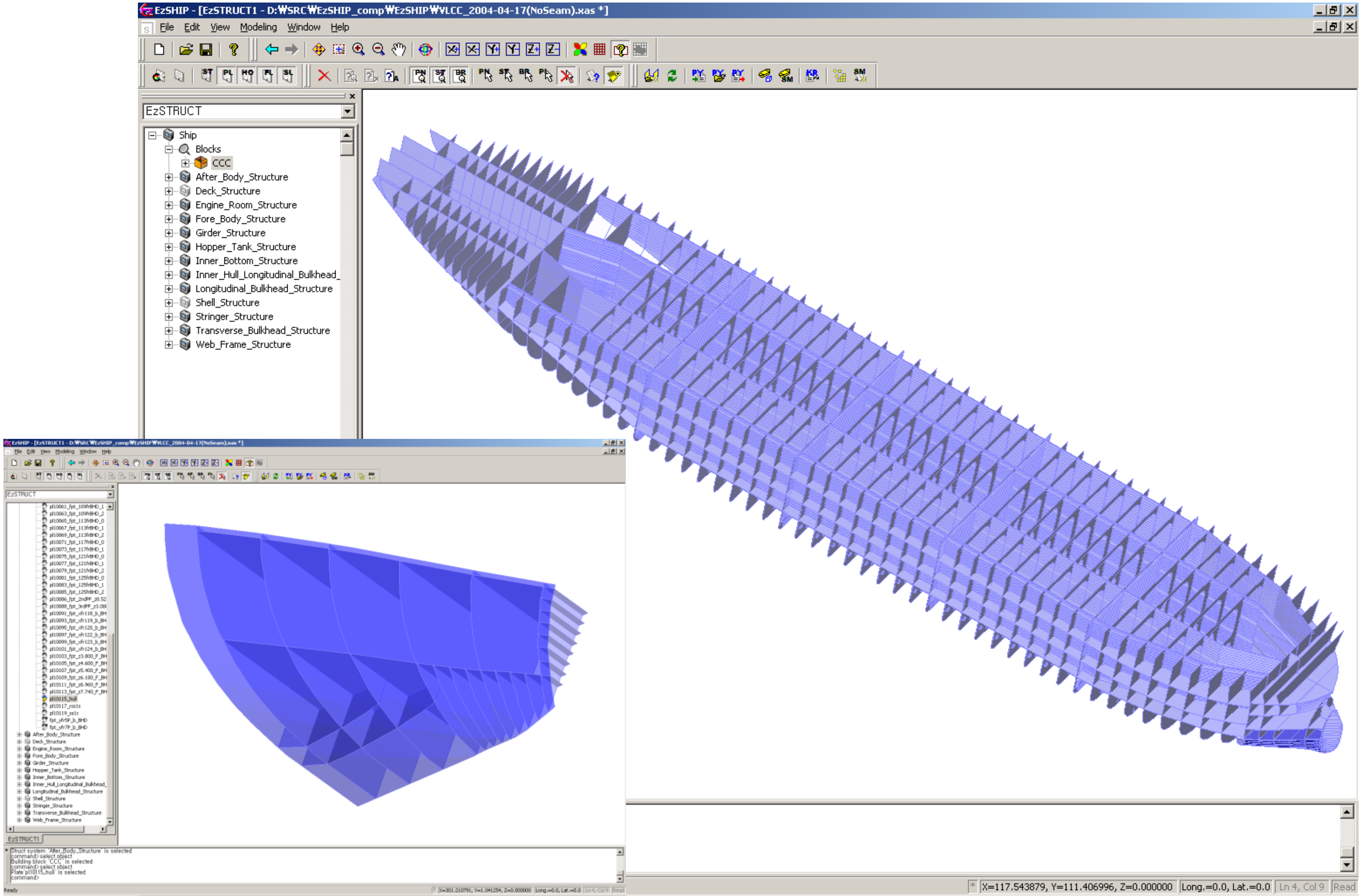
* Main dimensions of the 300,000 ton deadweight VLCC

Lbp: 320.0m, B: 58.0m, D: 31.2m, Td: 20.8m, Ts: 22.0m, Cb: 0.8086

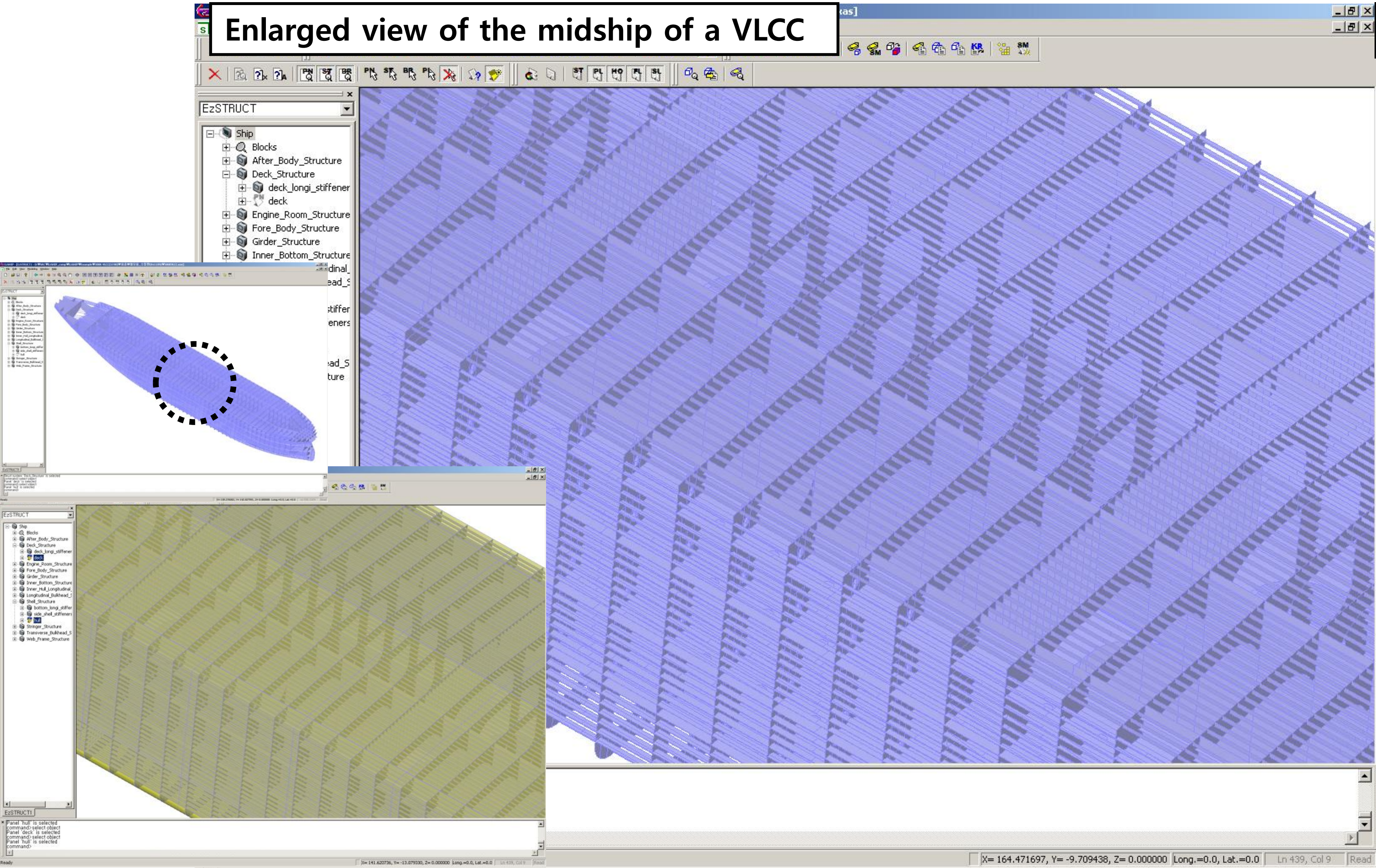
3D Structure Model of a 320,000 ton DWT VLCC :
- Cargo Hold



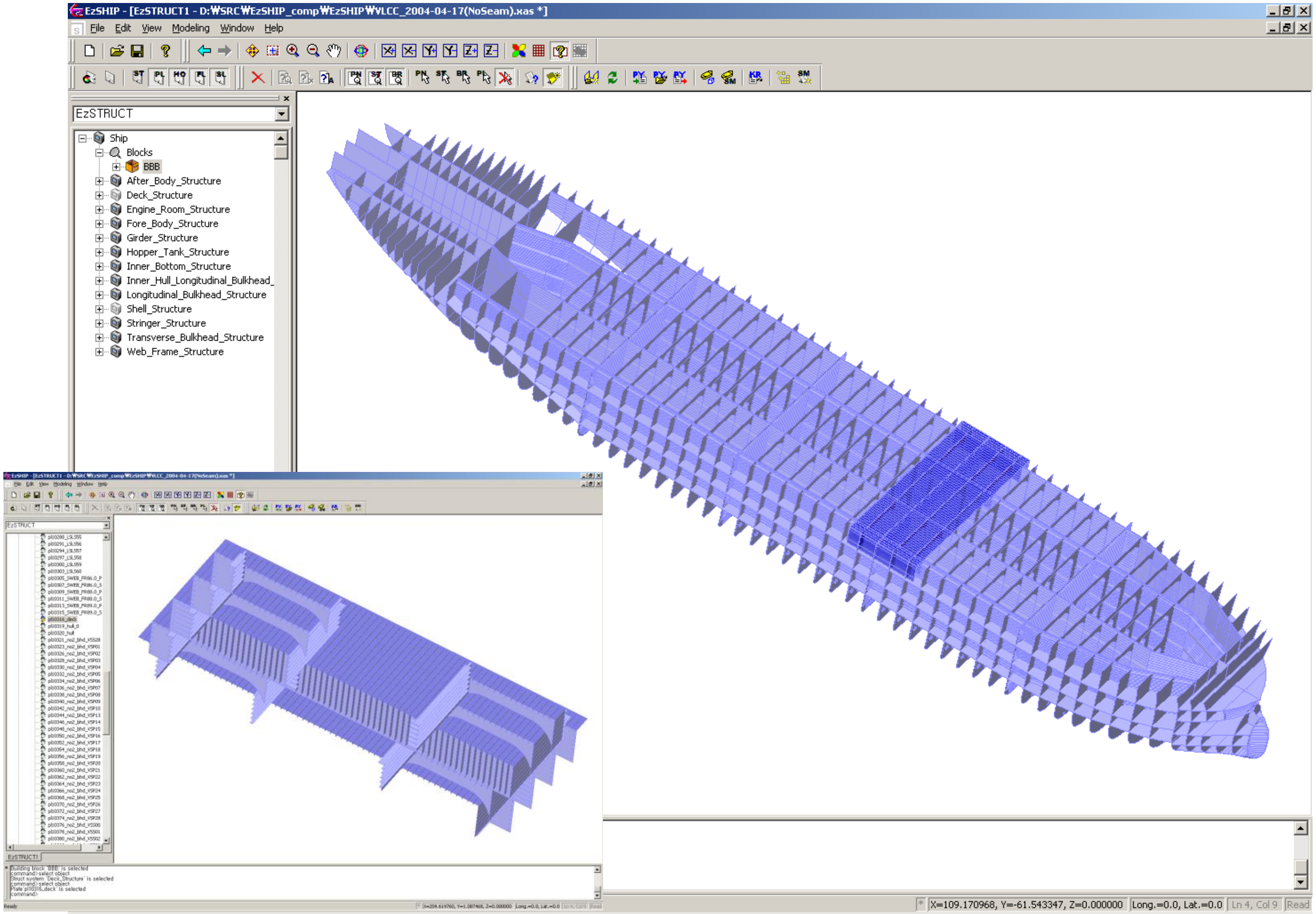
3D Structure Model of a 320,000 ton DWT VLCC :



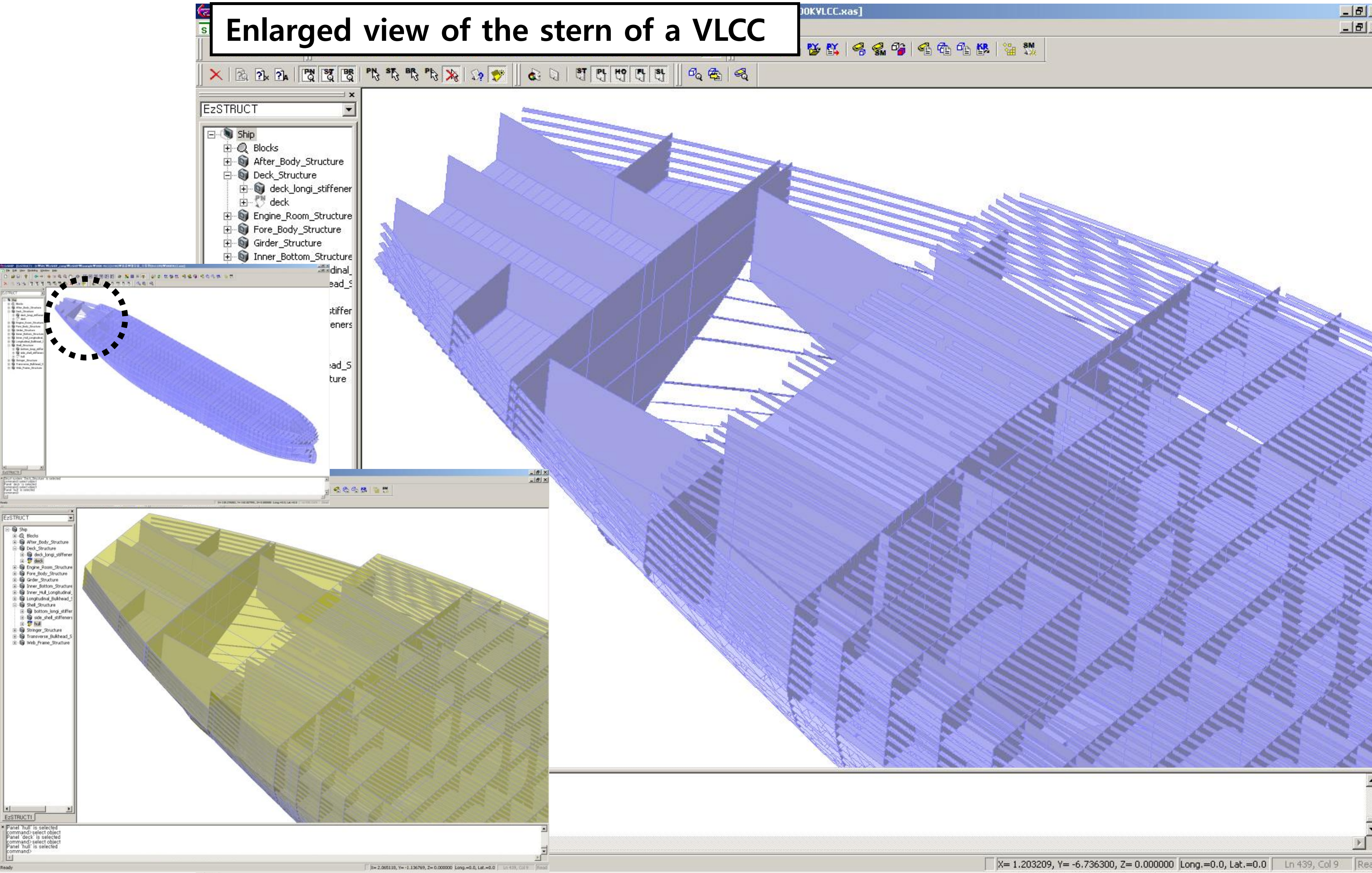
3D Structure Model of a 320,000 ton DWT VLCC : - Midship



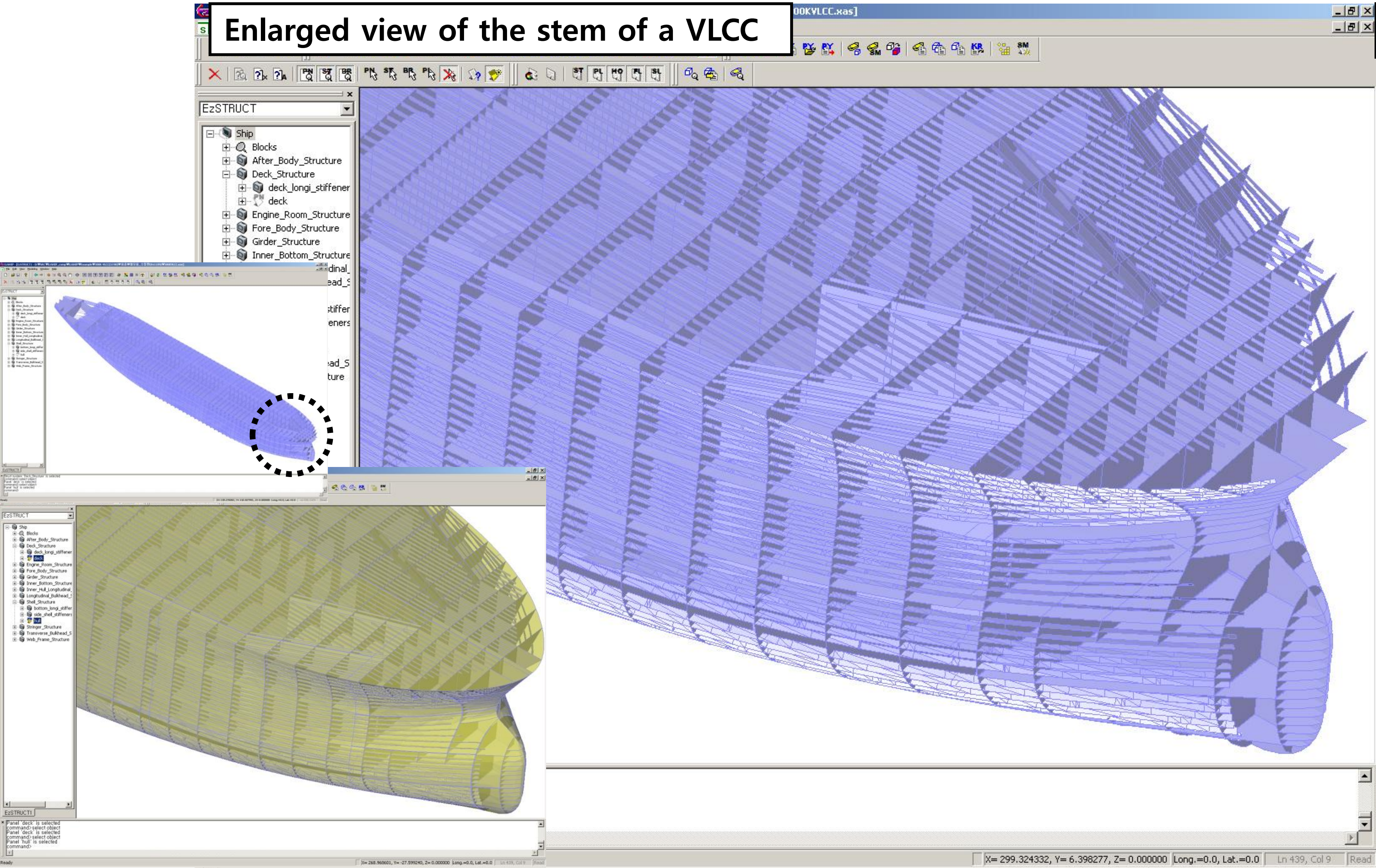
3D Structure Model of a 320,000 ton DWT VLCC :
- Deck Structure



3D Structure Model of a 320,000 ton DWT VLCC :
- Stern

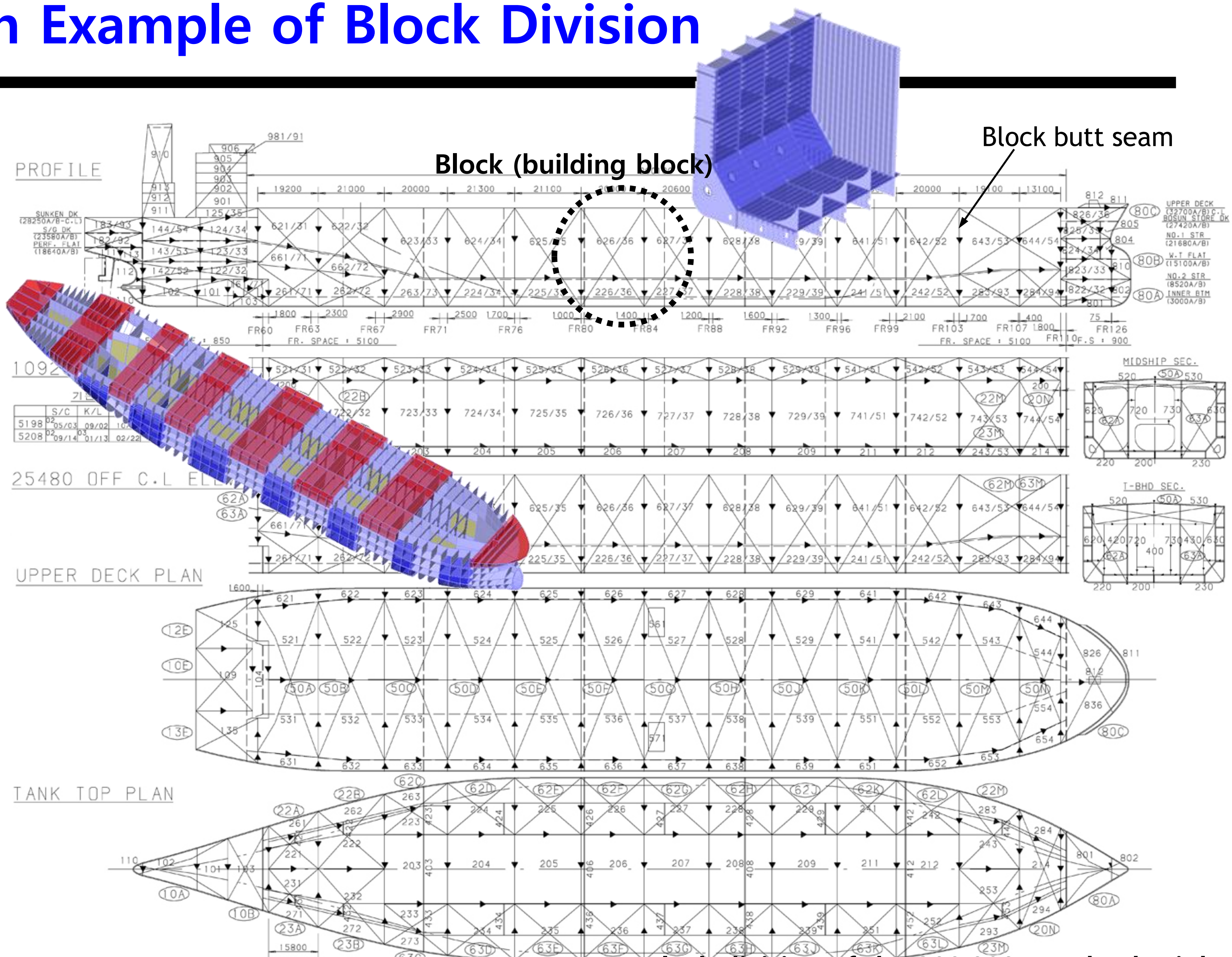


3D Structure Model of a 320,000 ton DWT VLCC : - Stem



14-3. Block Division of a VLCC (Very Large Crude oil Carrier)

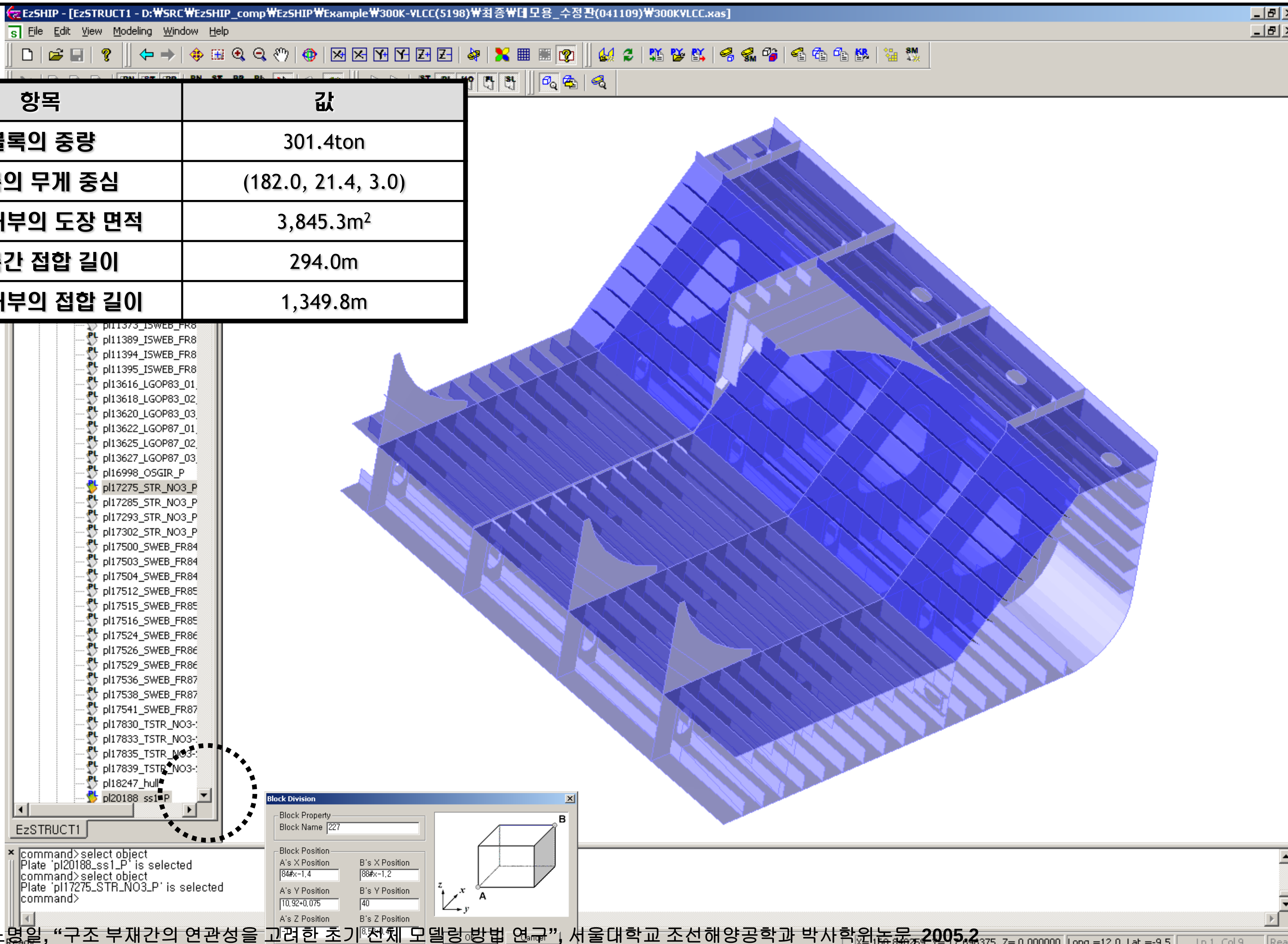
An Example of Block Division



Block division of the 300,000 ton deadweight VLCC

Production Information of a Cargo Hold Part
[화물창 중앙부 선체 블록에 대한 초기 공정 및 일정 계획용 물량 정보 생성 예]

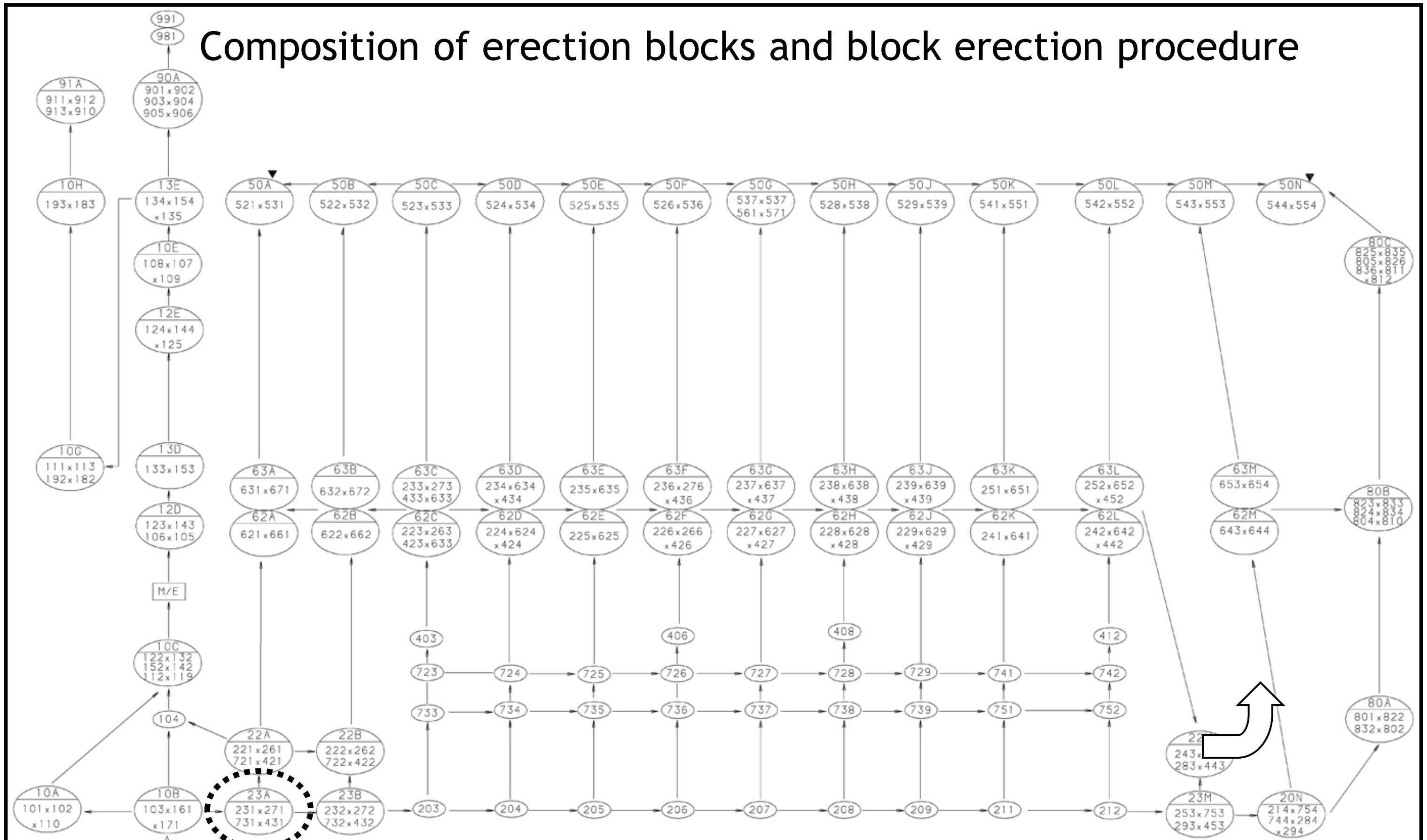
항목	값
블록의 중량	301.4ton
블록의 무게 중심	(182.0, 21.4, 3.0)
블록 내부의 도장 면적	3,845.3m^2
블록간 접합 길이	294.0m
블록 내부의 접합 길이	1,349.8m

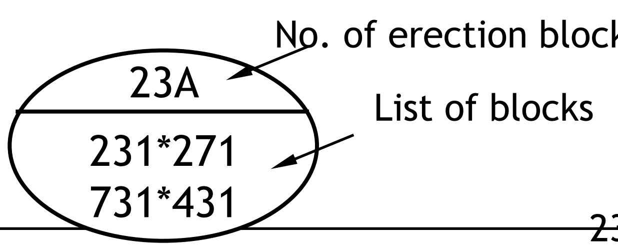


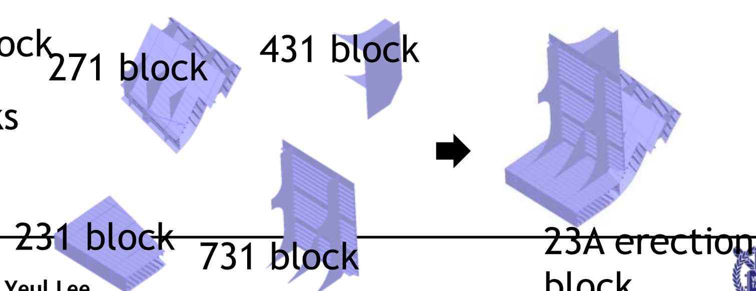
* 자료 출처: 노명일, "구조 부재간의 연관성을 고려한 초기 선체 모델링 방법 연구", 서울대학교 조선해양공학과 박사학위논문, 2005.2

Procedure of Block Erection

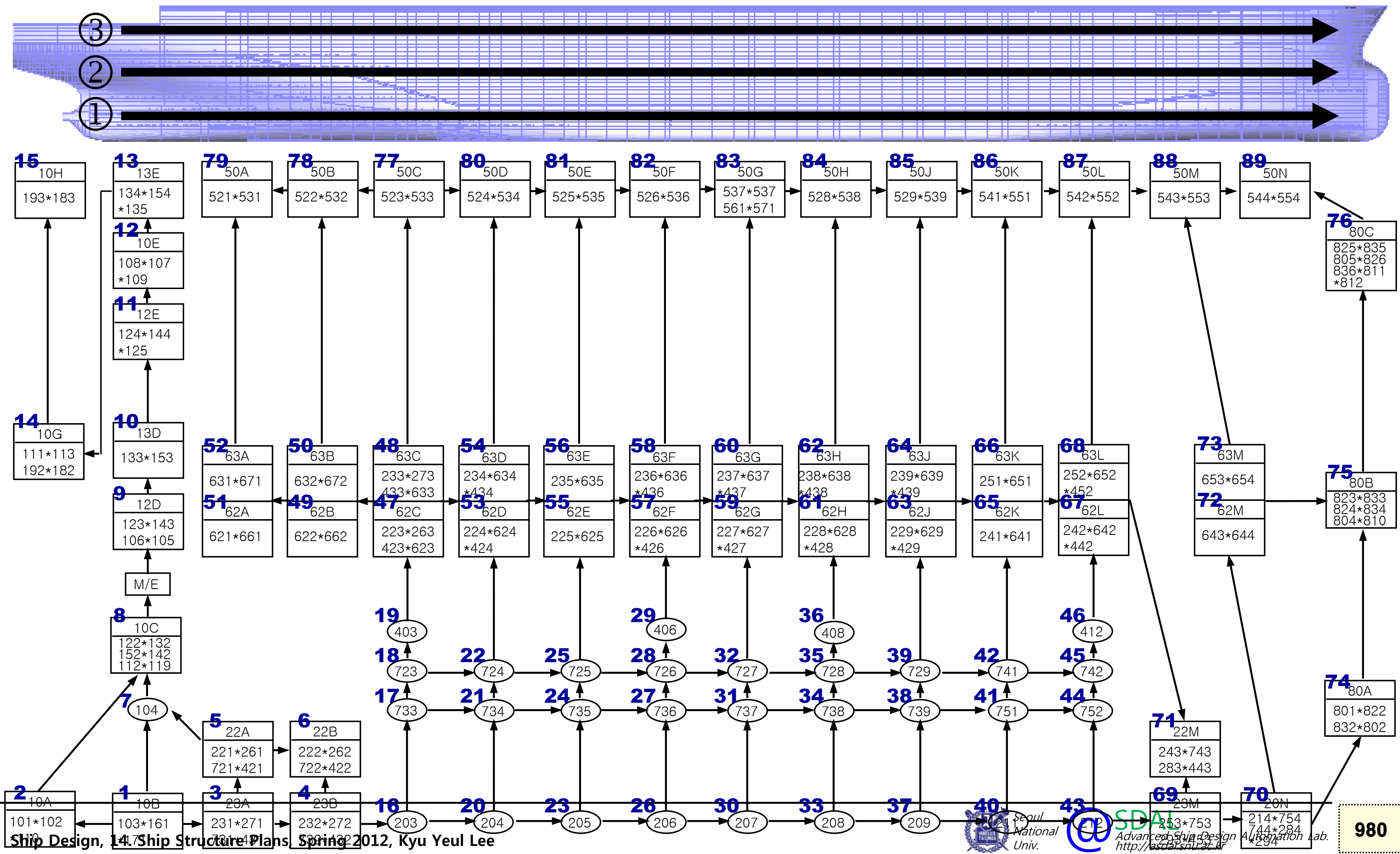
Composition of erection blocks and block erection procedure





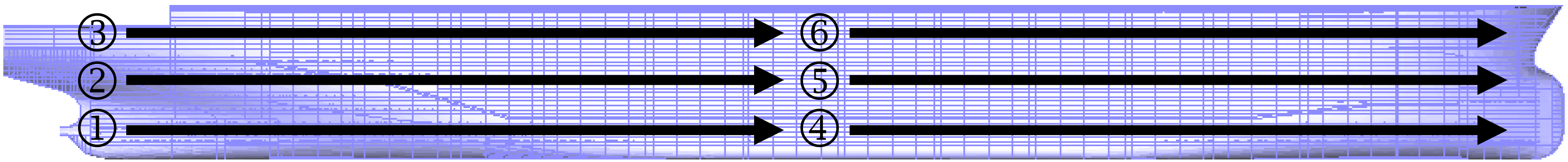


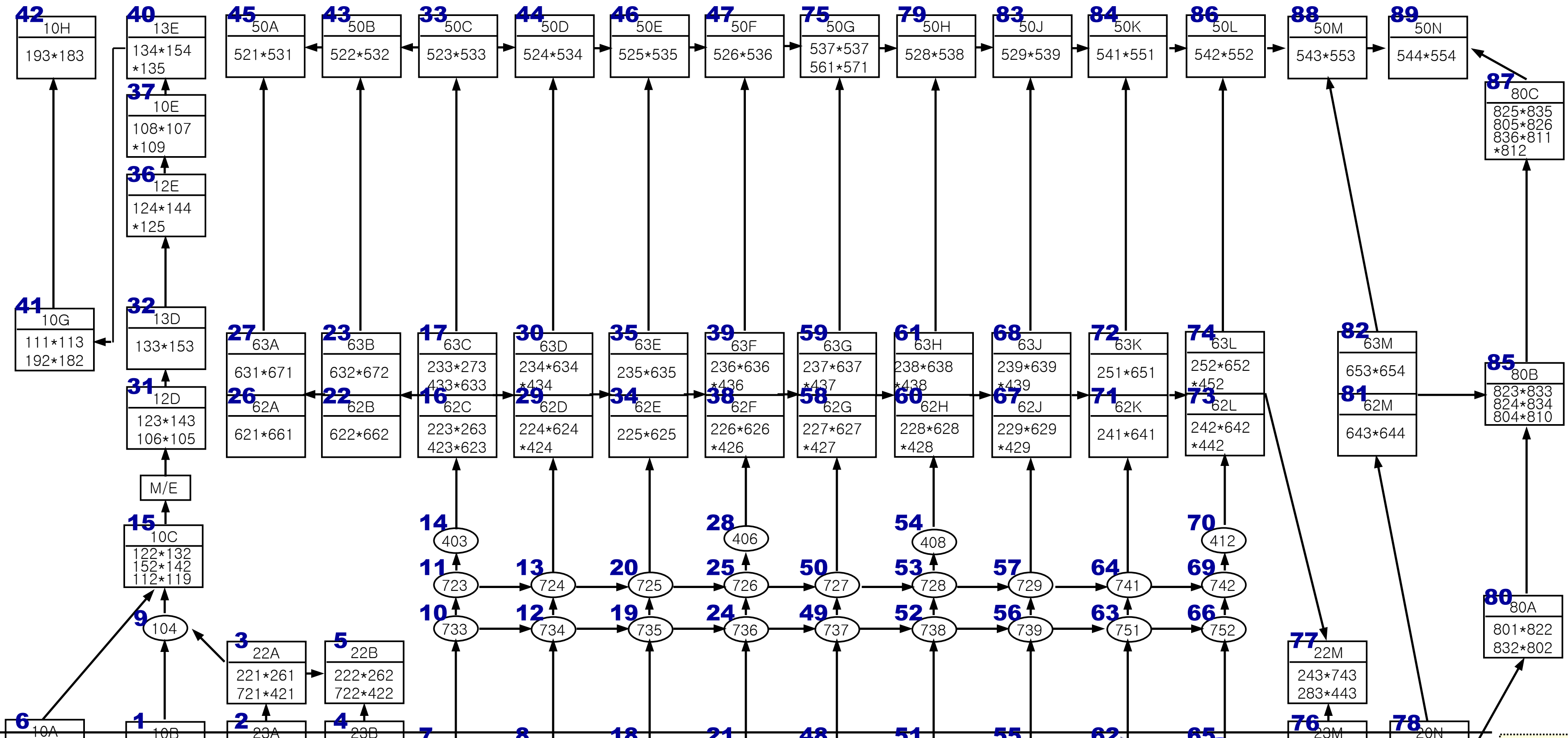
Scenario of a Block Erection Procedure - 1



Scenario of a Block Erection Procedure - 2

Block Erection considering a method
of construction of tandem block



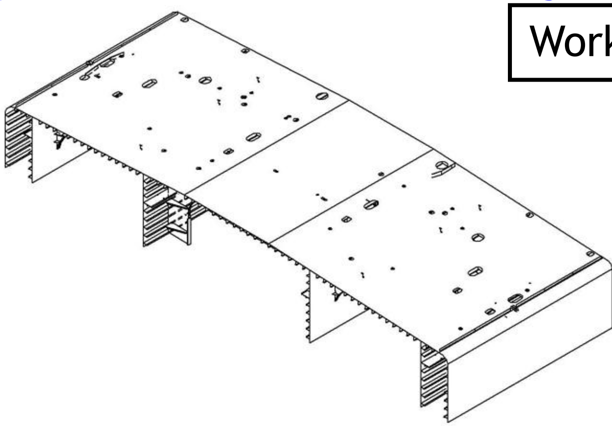


14-4. Assembly Procedure of the Double Bottom of a VLCC

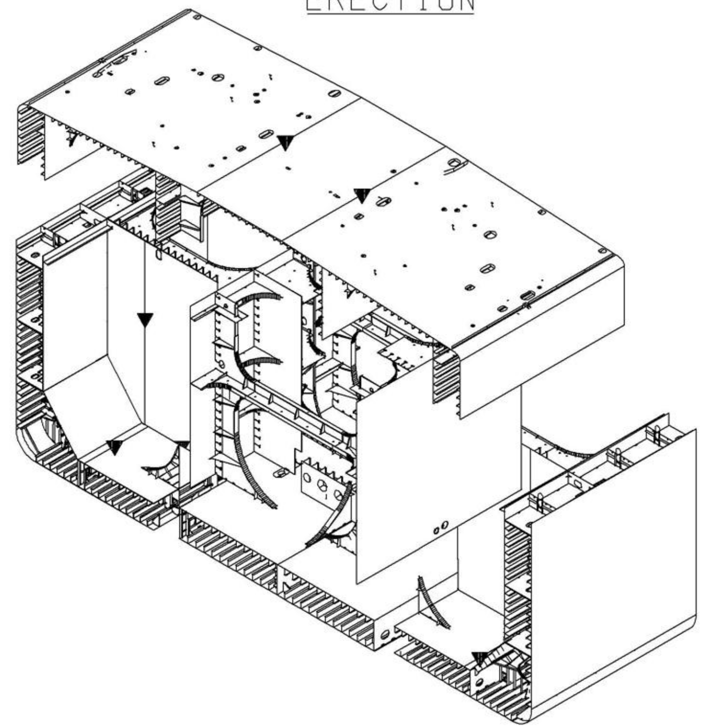
Example of an Assembly Procedure

Work Sequence Diagram

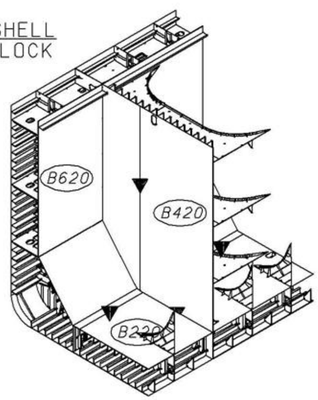
UPPER DECK
P.E BLOCK



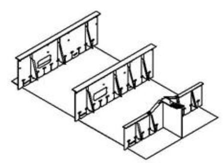
ERECTION



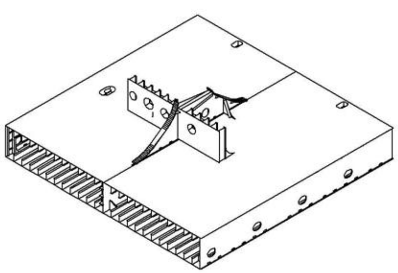
SIDE SHELL
P.E BLOCK



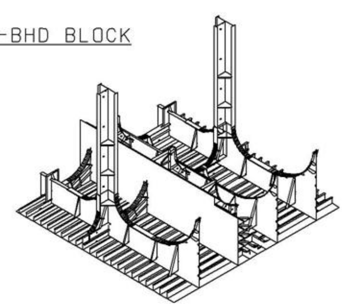
T-BHD BLOCK



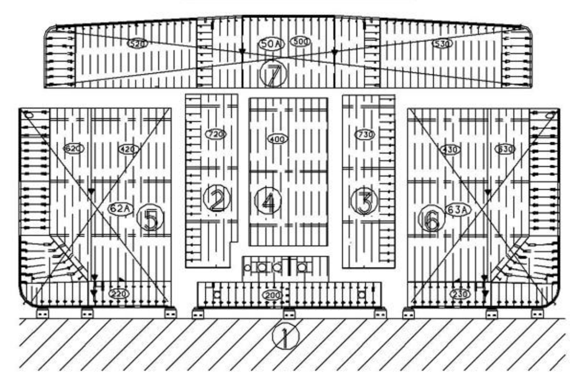
CENTER BTM BLOCK



L-BHD BLOCK

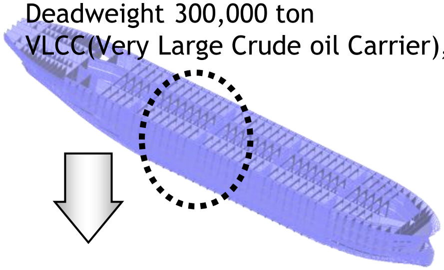


ERECTION SEQUENCE

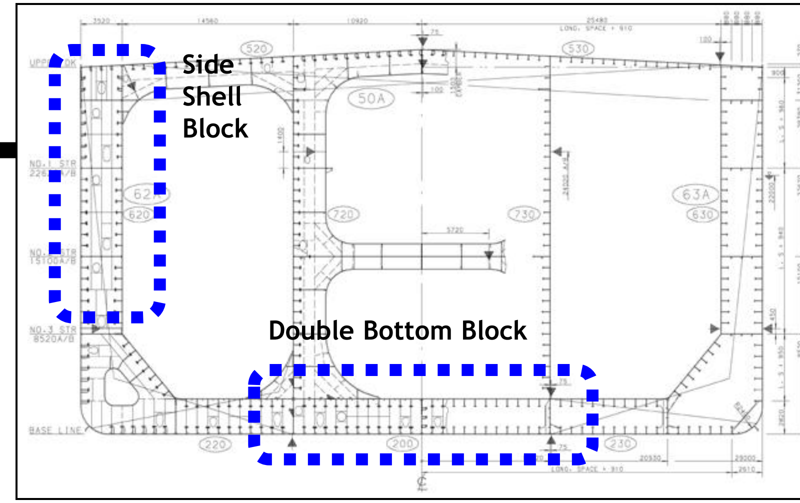


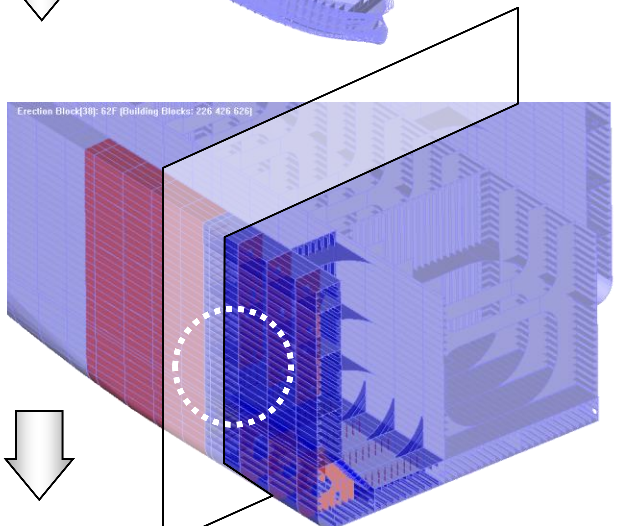
Double Bottom Structure of a VLCC

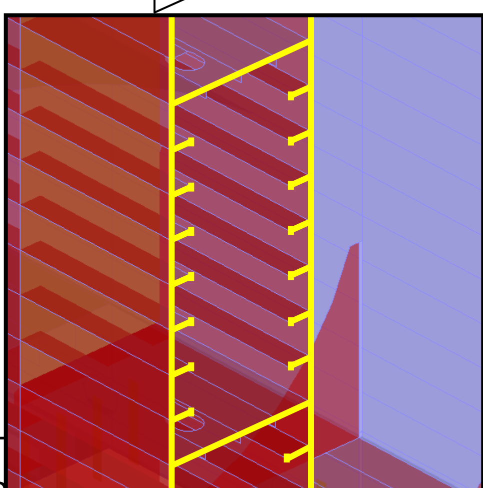
Deadweight 300,000 ton
VLCC(Very Large Crude oil Carrier), L=320m, B= 60m, D= 30m, T=20m

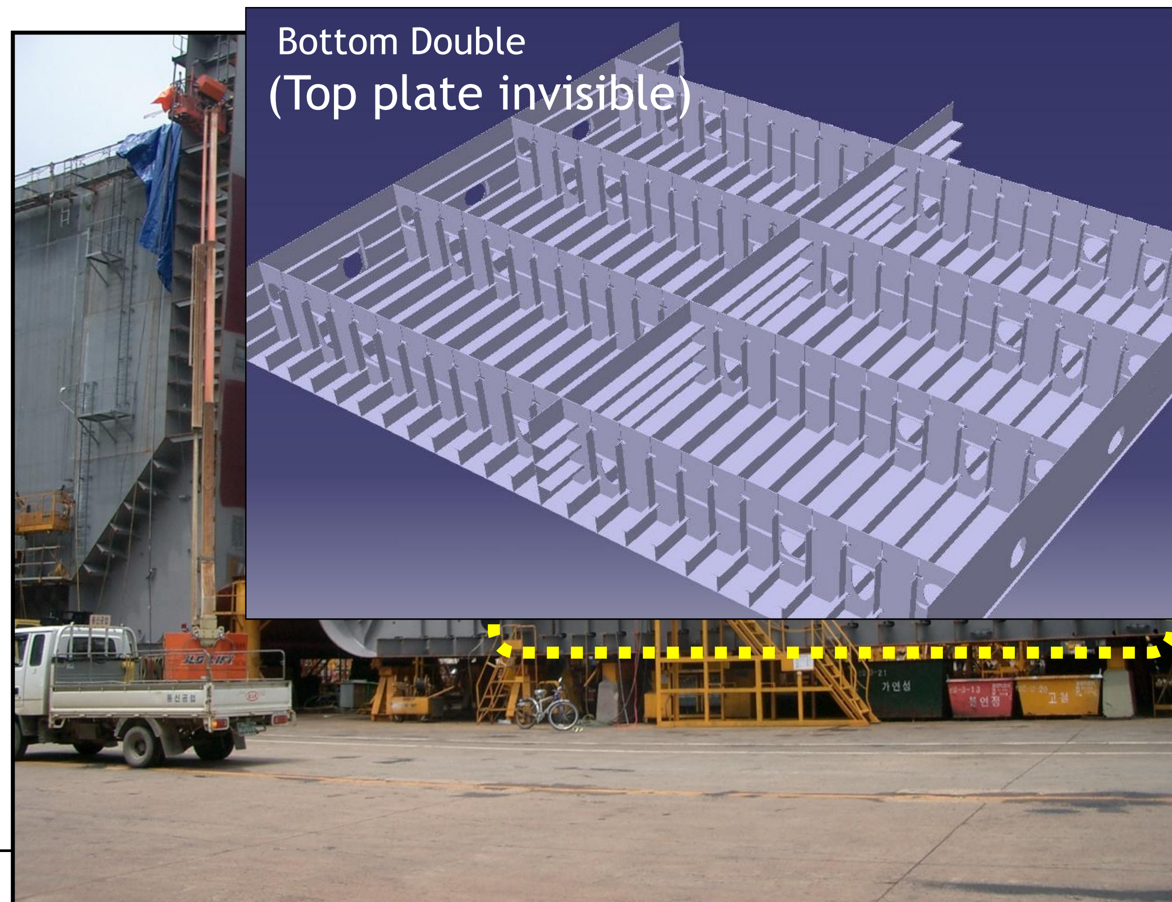


Section

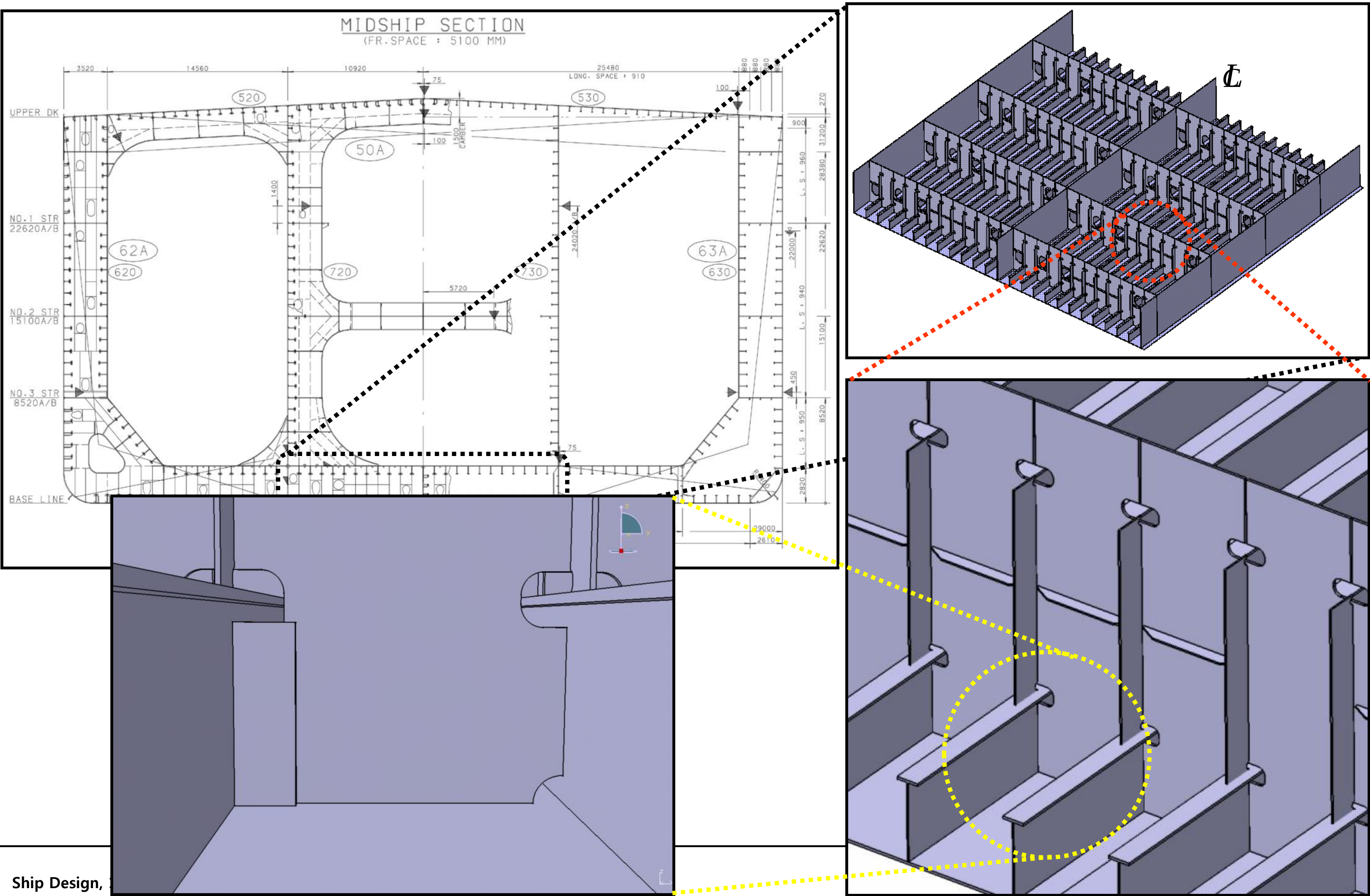




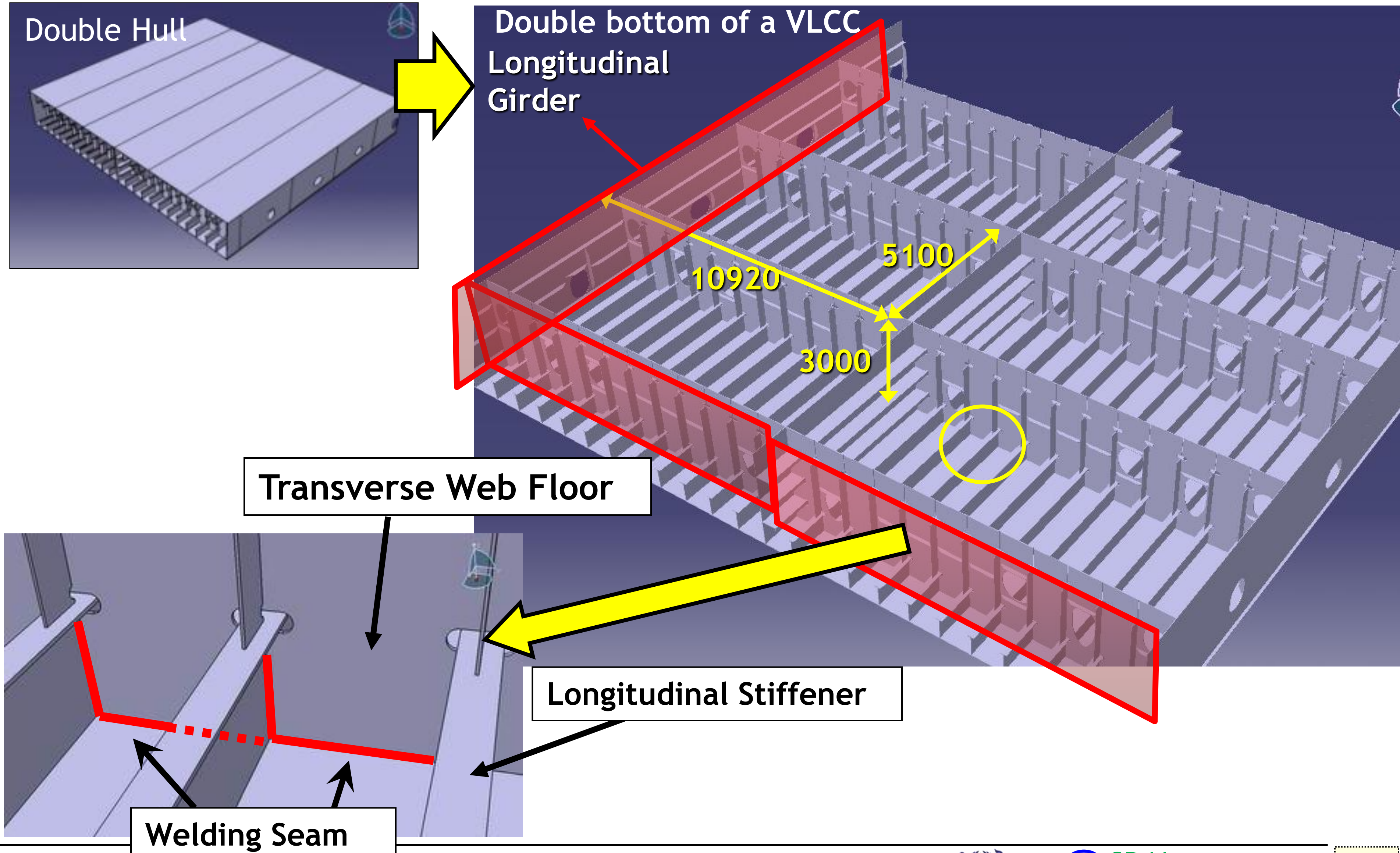




Double Bottom Structure of a VLCC

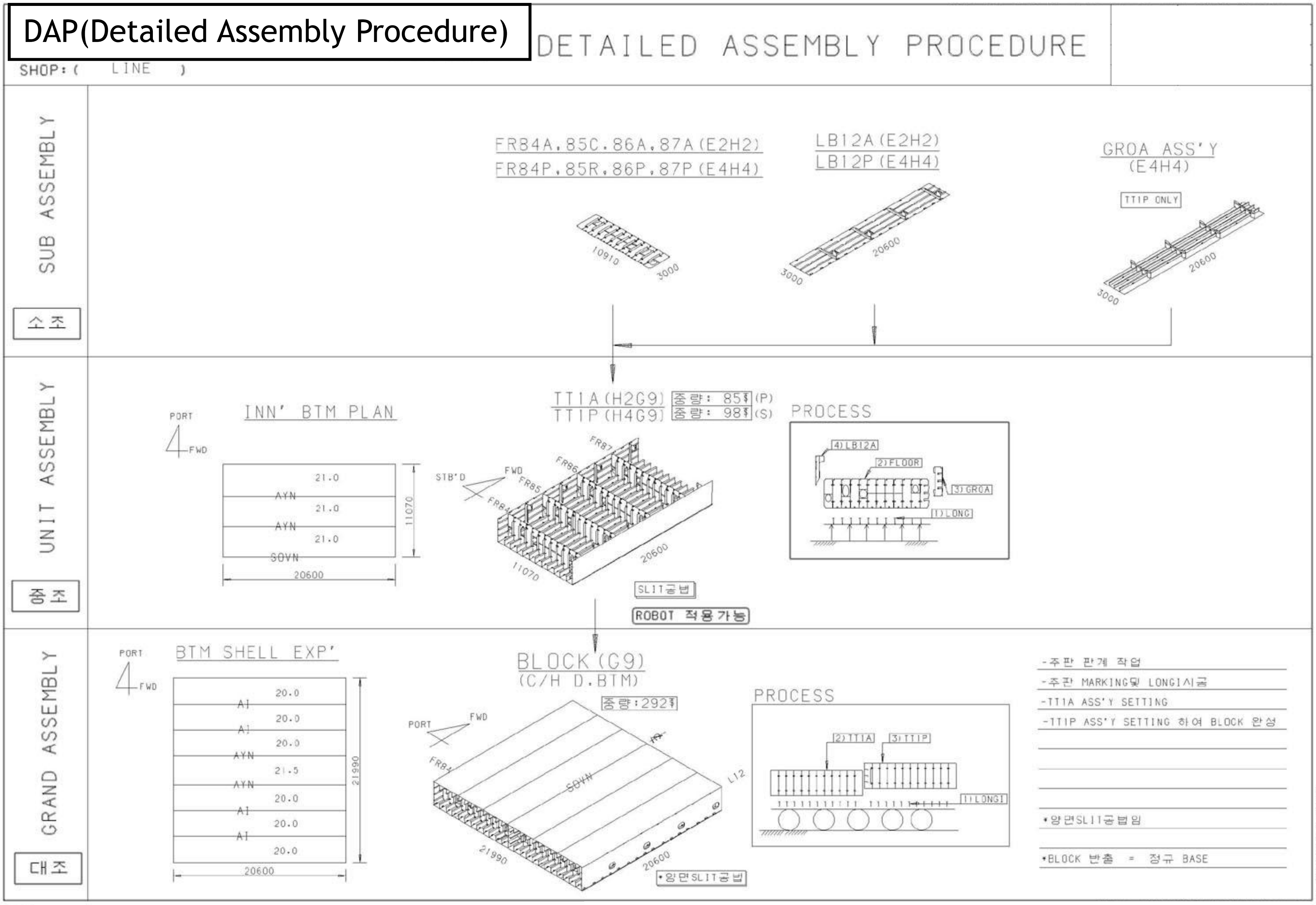


Double Bottom Structure of a VLCC

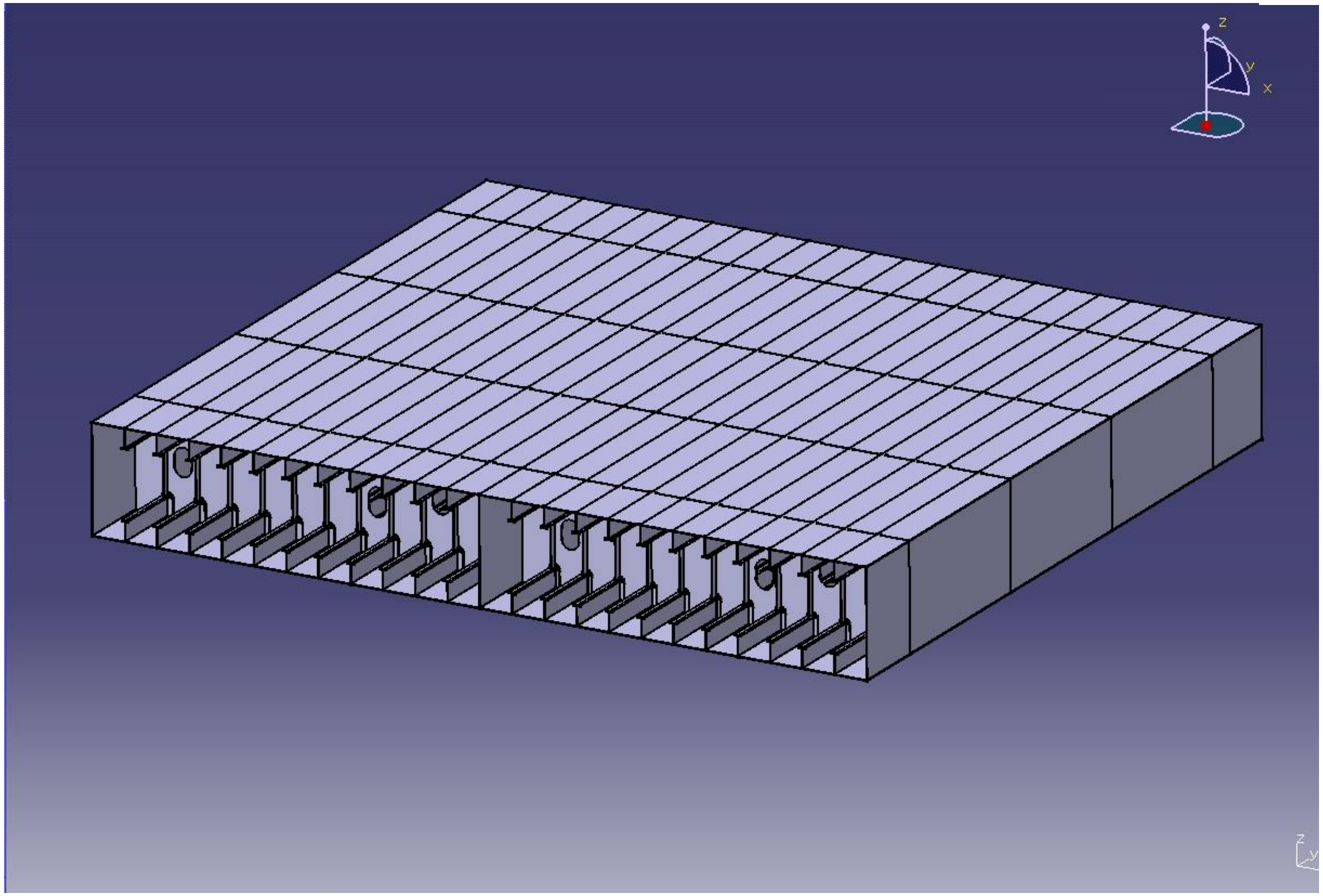


DAP (Detailed Assembly Procedure) of Double Bottom

DAP(Detailed Assembly Procedure)



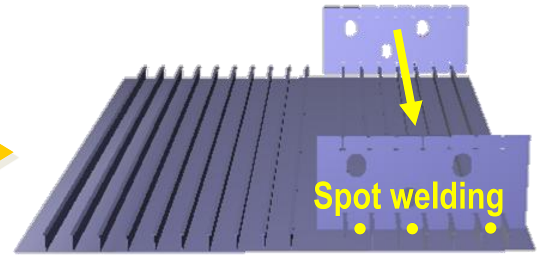
Assembly Procedure of Double Bottom



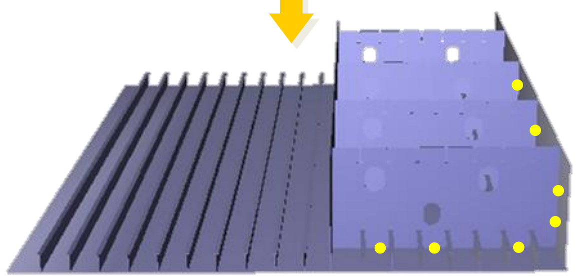
Assembly Procedure of a Double Bottom
- Double Side Slit Method of Construction (1)

Slit Method

Inner Bottom Shell

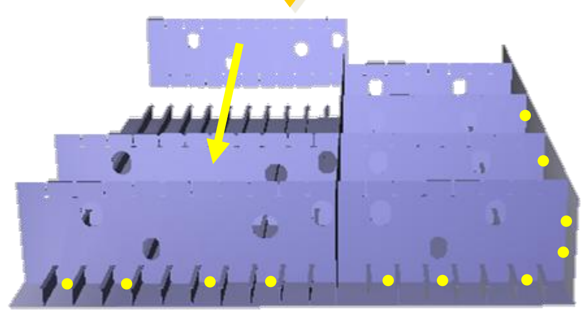


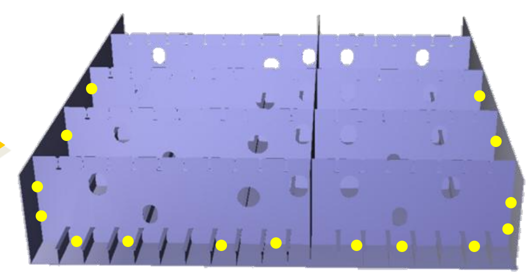
Assemble the web floor by using the slit



Spot welding after girder assembly

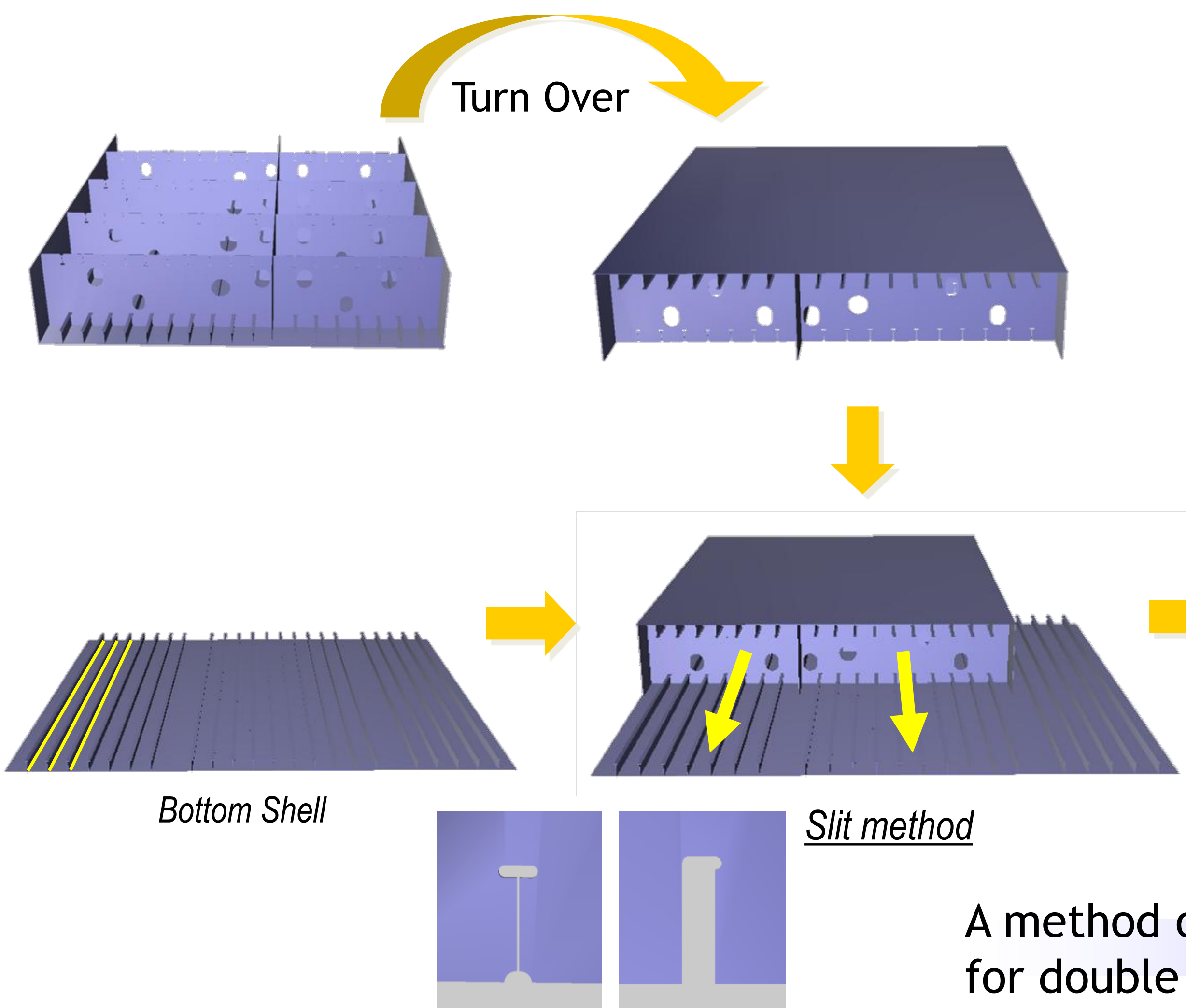
Assemble the web floor by using the slit





Inner Bottom Block

Assembly Procedure of a Double Bottom
- Double Side Slit Method of Construction (2)

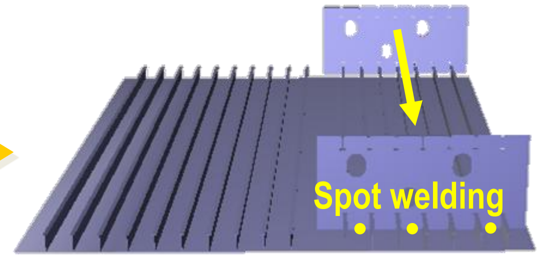


A method of construction of slit for double side

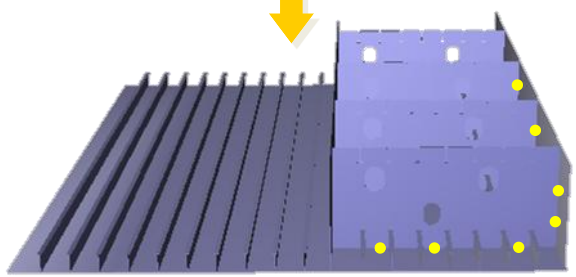
Assembly Procedure of a Double Bottom
- One Side Slit Method of Construction + Open Type Method of Construction (1)

Slit Method

Inner Bottom Shell

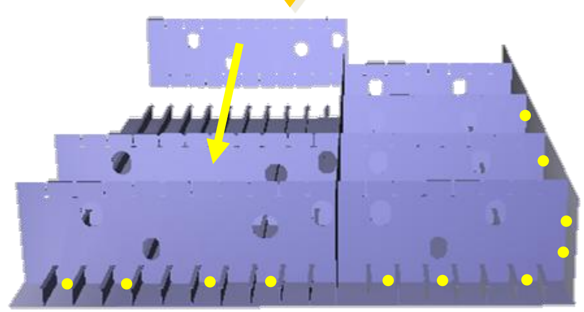


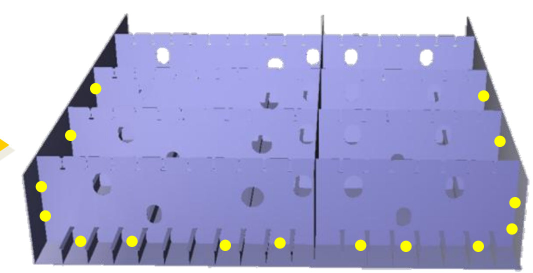
Assemble the web floor by using the slit



Spot welding after girder assembly

Assemble the web floor by using the slit

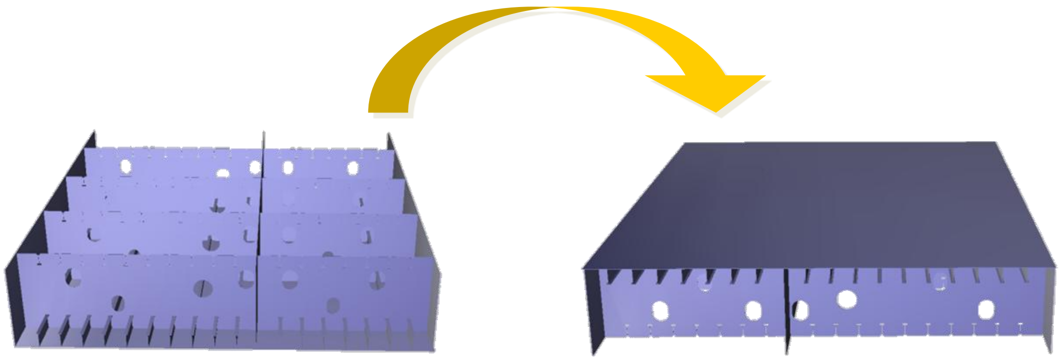


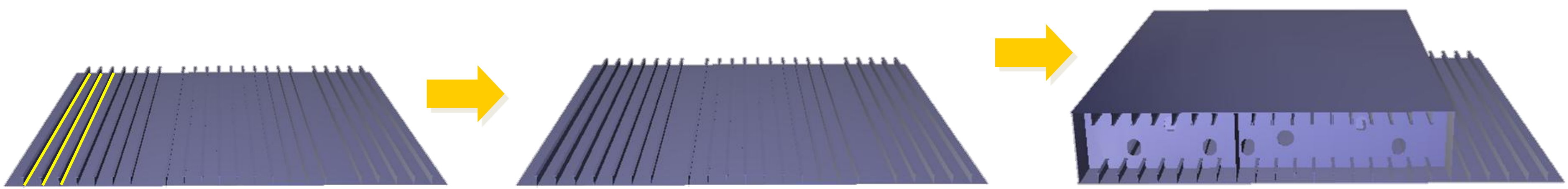


InnerBottom Block

A method of construction of slit for one side

Assembly Procedure of a Double Bottom
- One Side Slit Method of Construction + Open Type Method of Construction (2)





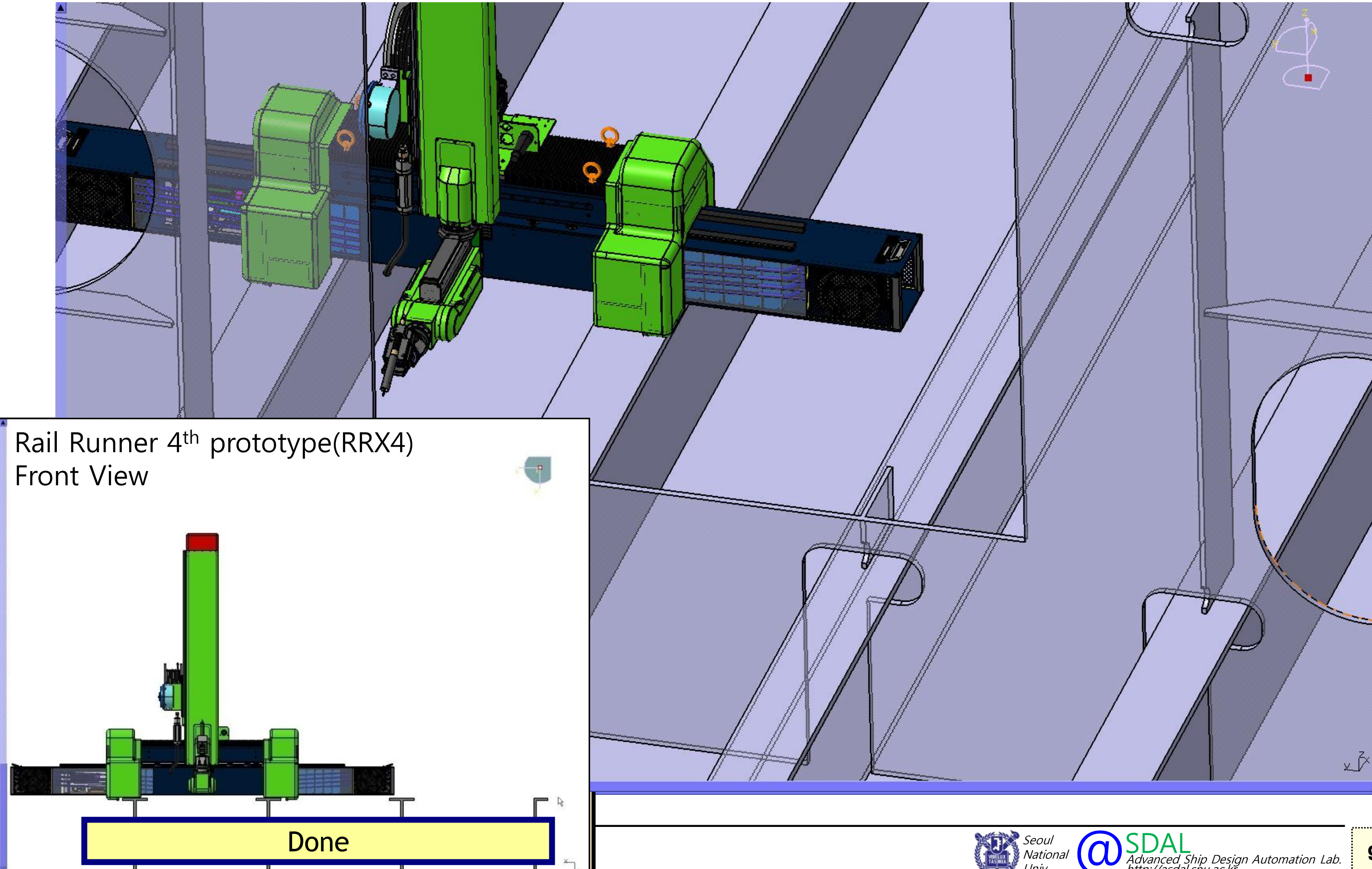
Bottom Shell

Open type method

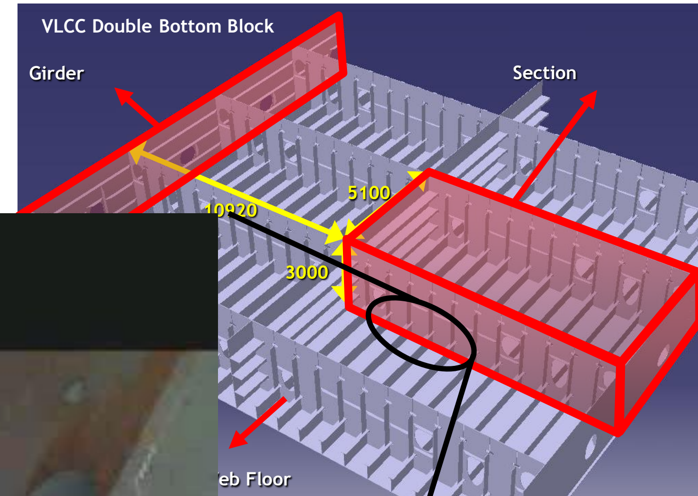
Double Bottom Block

A method of construction
of open type

Moving Welding Robot for Double Bottom Structure



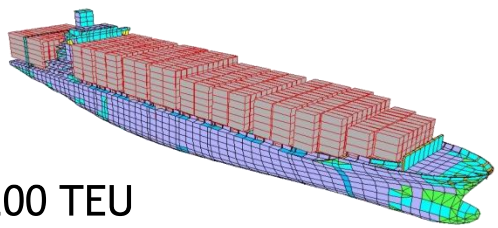
Moving Welding Robot for Double Bottom Structure



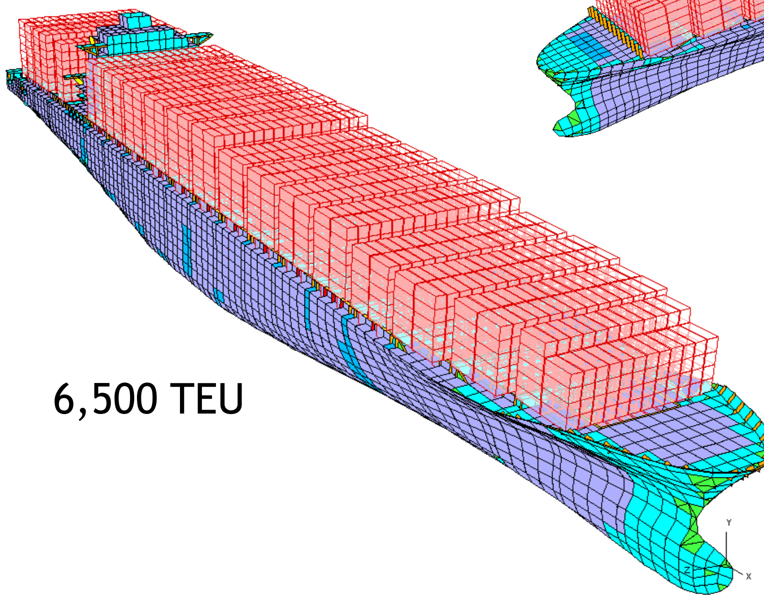
14-5 Structure Plans of Container Carrier

Container Carrier

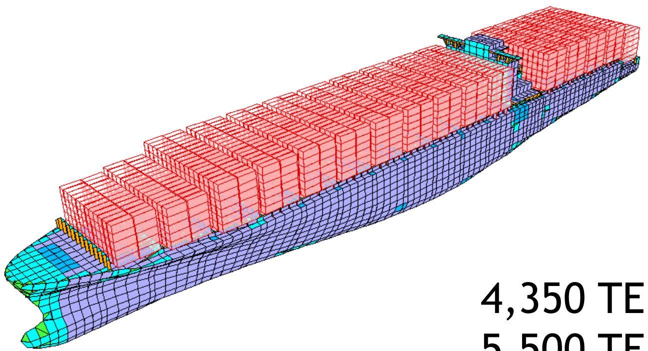
Container Carrier



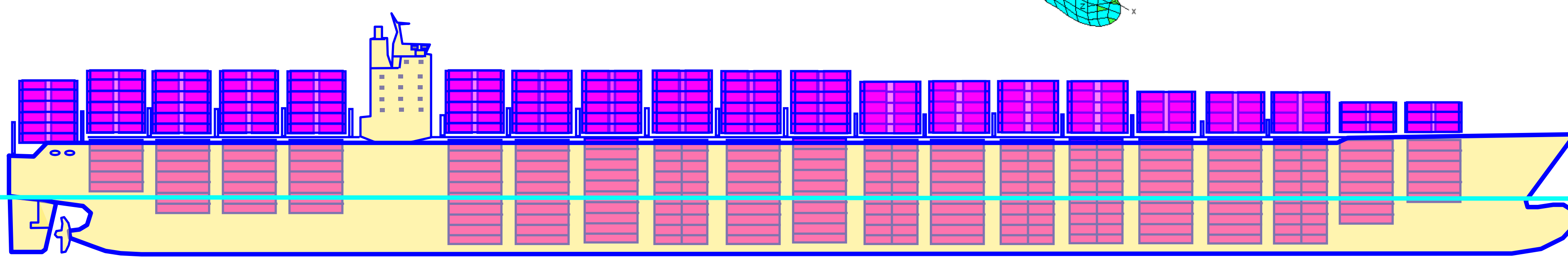
2,200 TEU



6,500 TEU



4,350 TEU
5,500 TEU



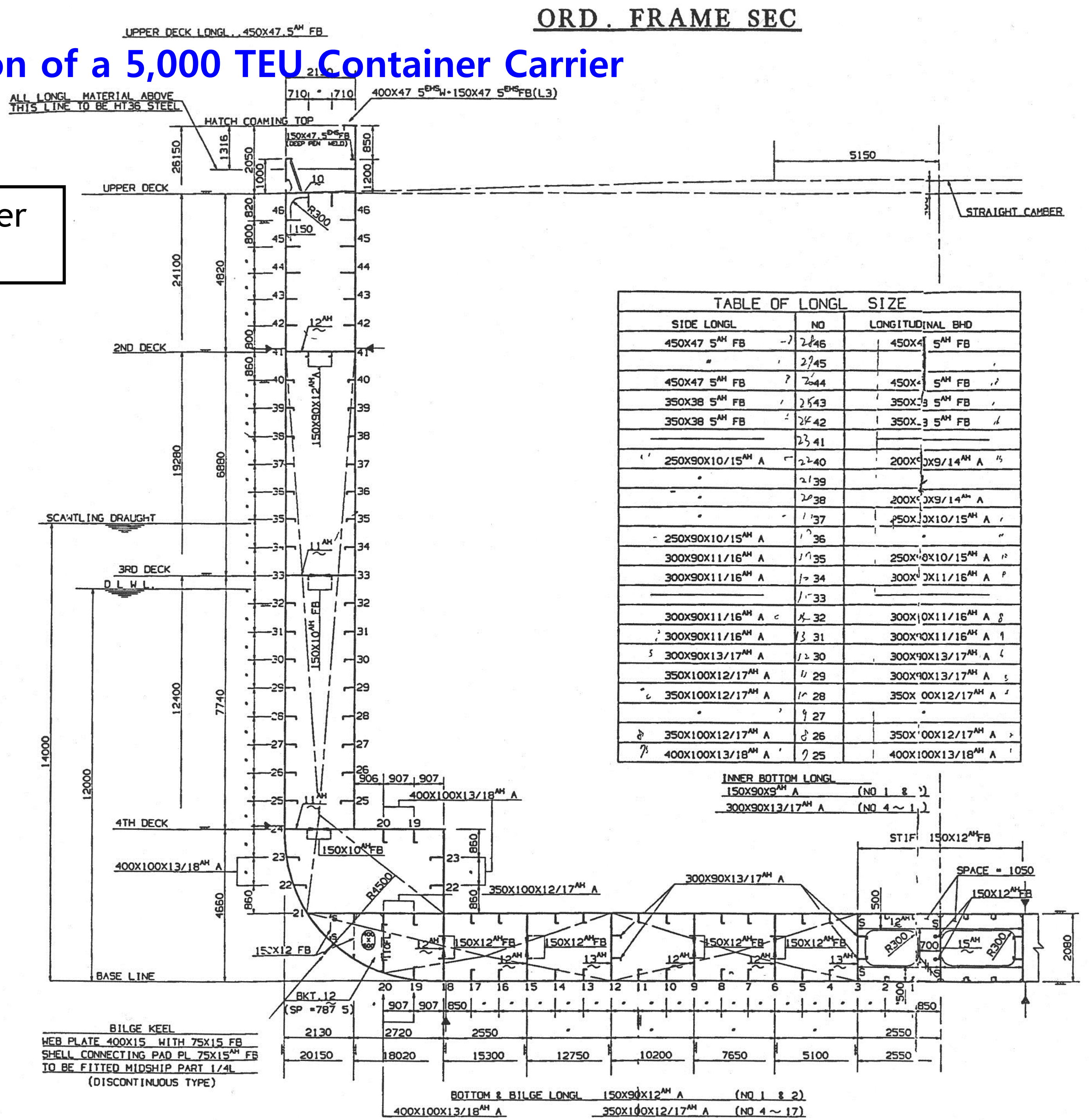
9,100 TEU

Structure Plans of a 5,000 TEU Container Carrier

Ordinary Frame Section of a 5,000 TEU Container Carrier

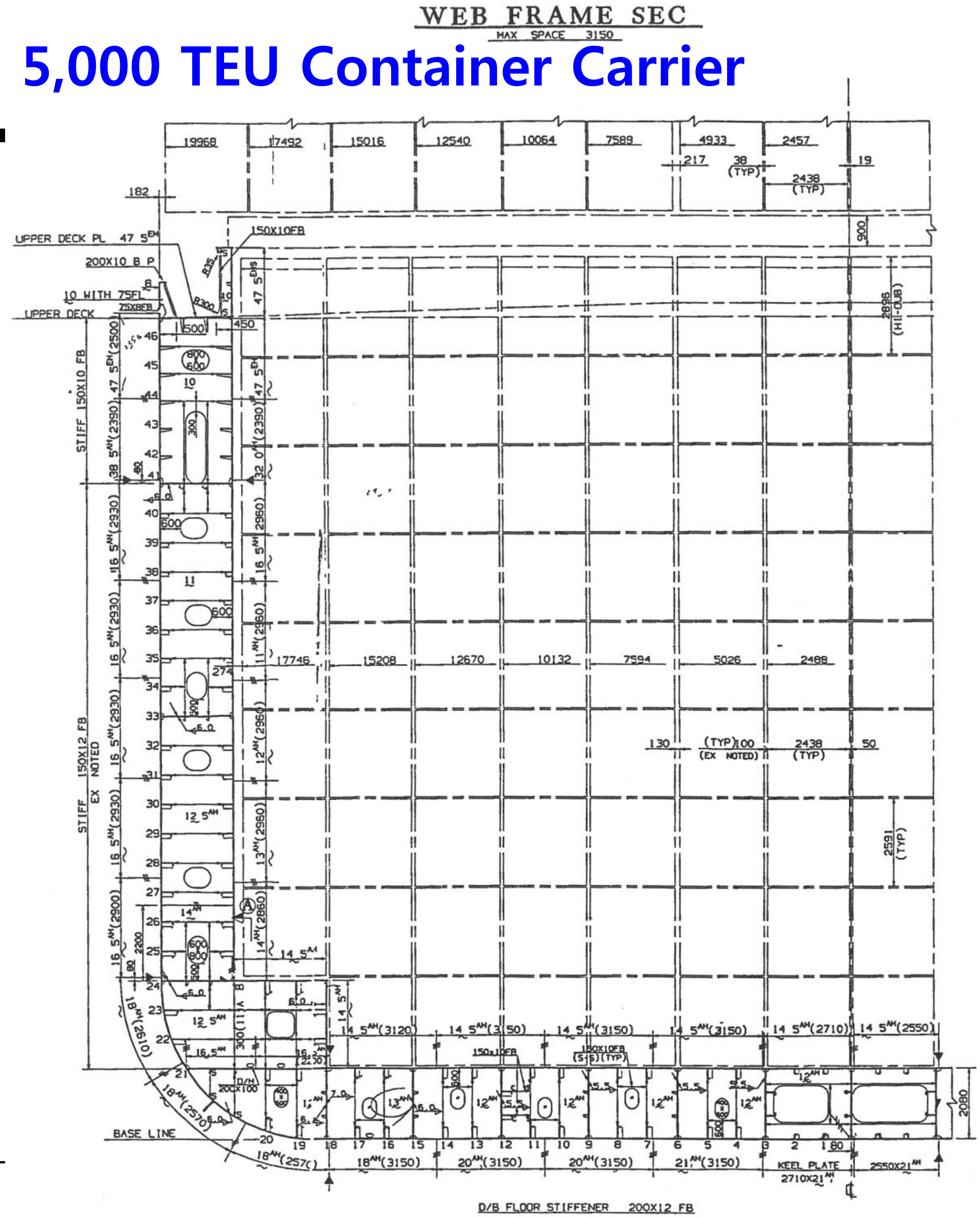
5,000TEU Container Carrier
- Ordinary Frame Section

Principal Dimensions	Values
LOA	279.0 m
LBP	265.0 m
Breadth MLD	40.3 m
Depth MLD	24.1 m
Draft Design	12.0 m
Draft Scant.	14.0 m



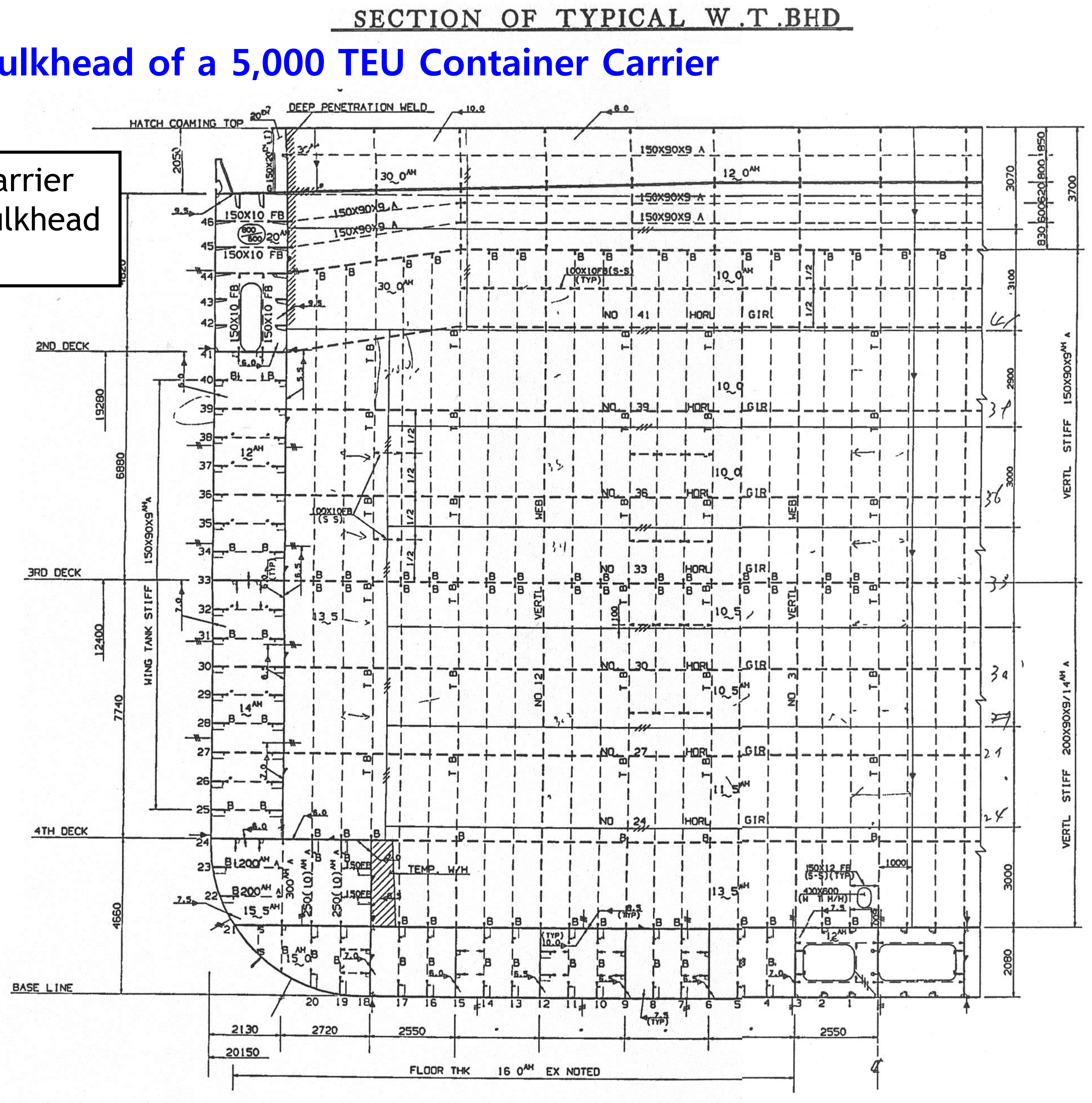
Web Frame Section of a 5,000 TEU Container Carrier

5,000TEU Container Carrier
- Web Frame Section



Typical Transverse Bulkhead of a 5,000 TEU Container Carrier

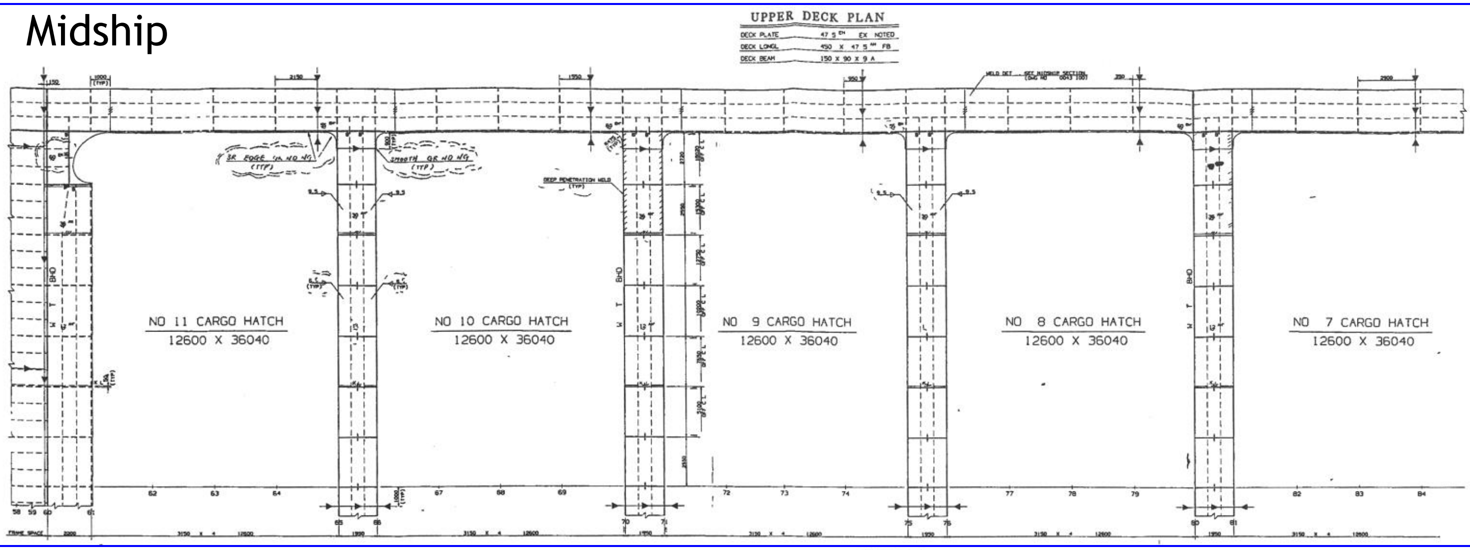
5,000TEU Container Carrier
- Typical Transverse Bulkhead



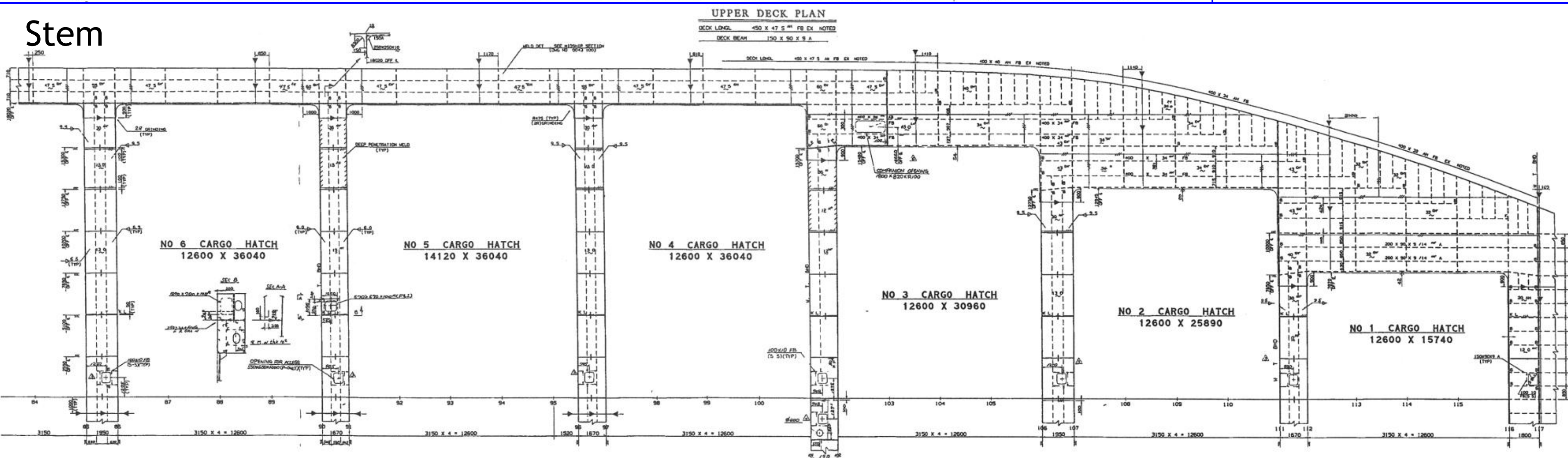
Upper Deck Plan of a 5,000 TEU Container Carrier

5,000TEU Container Carrier - Upper Deck Plan

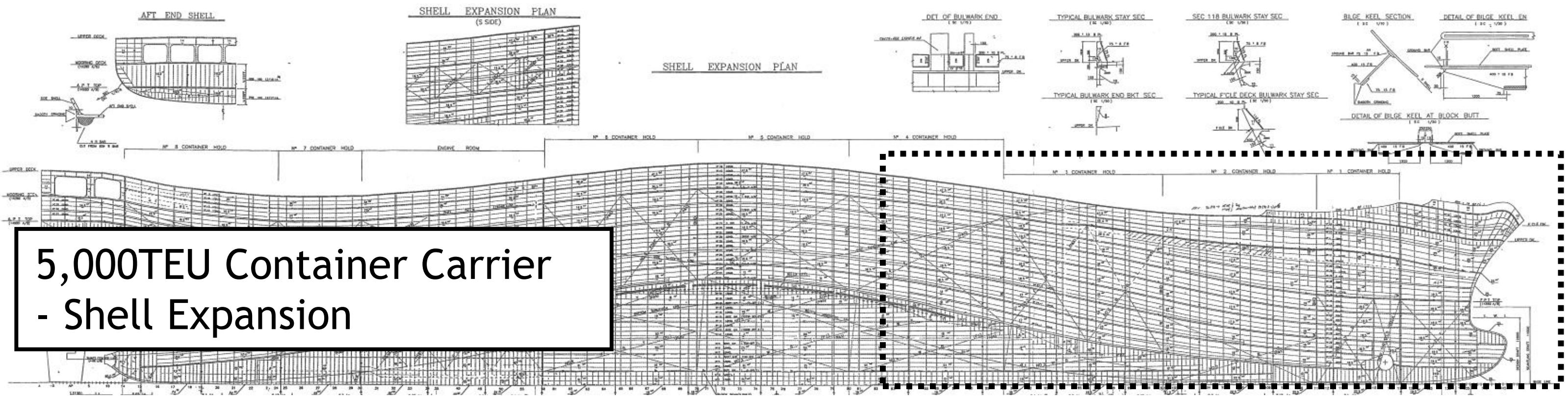
Midship



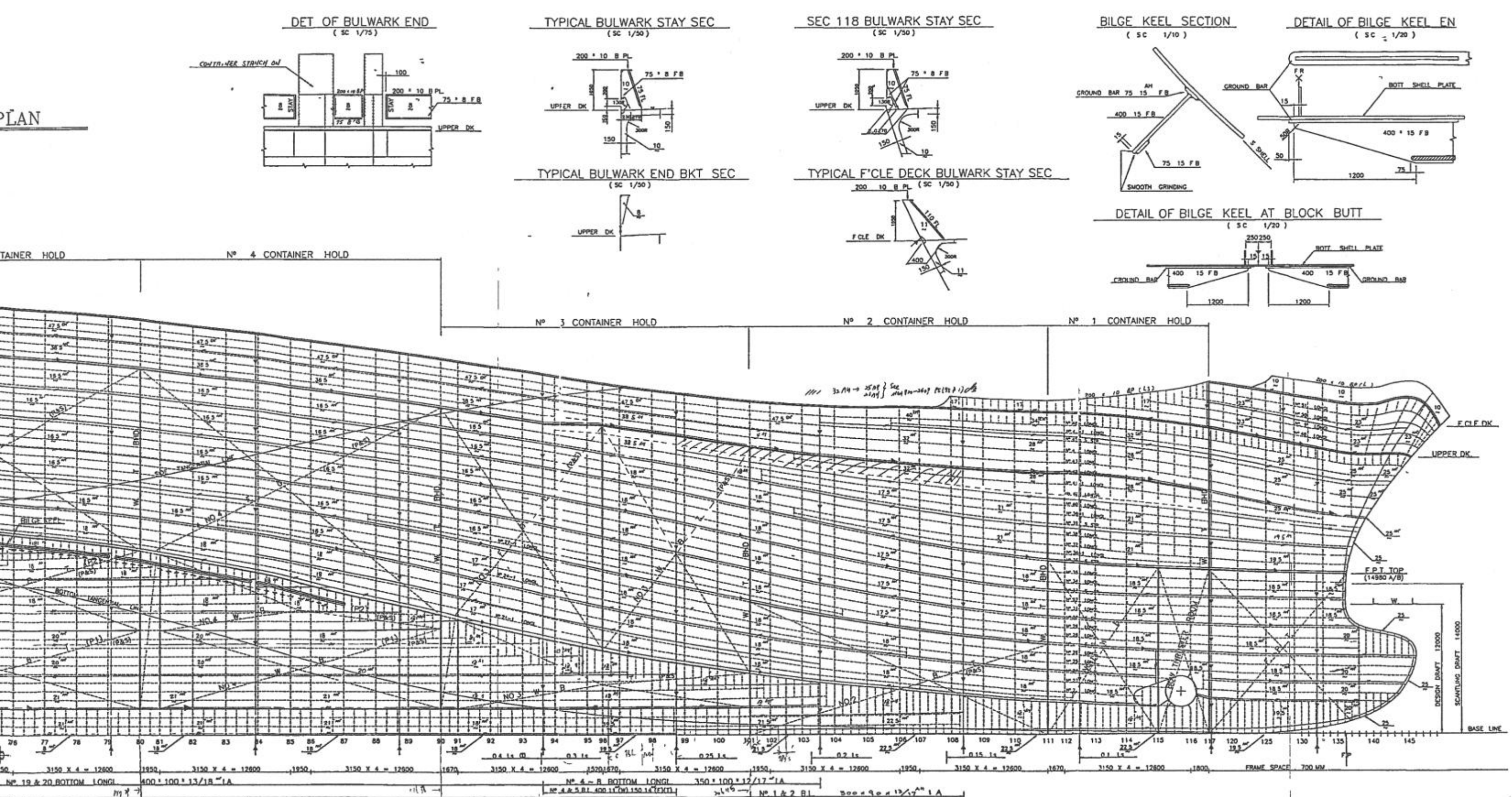
Stem



Shell Expansion of a 5,000 TEU Container Carrier

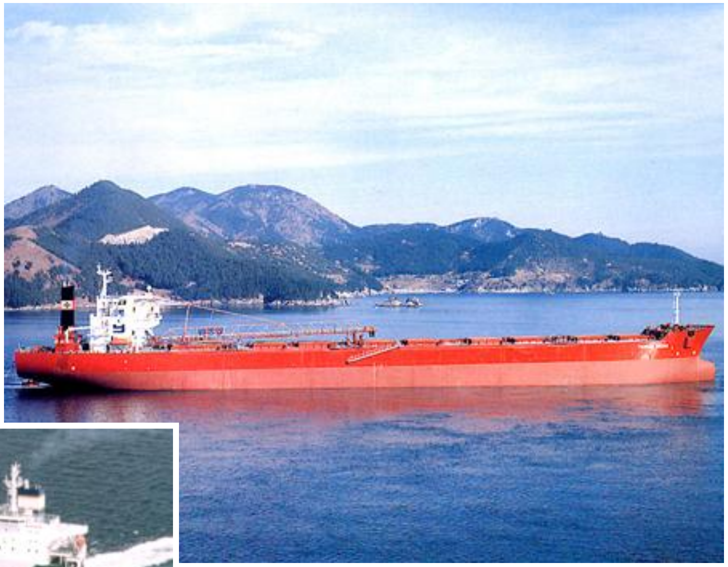


5,000TEU Container Carrier
- Shell Expansion

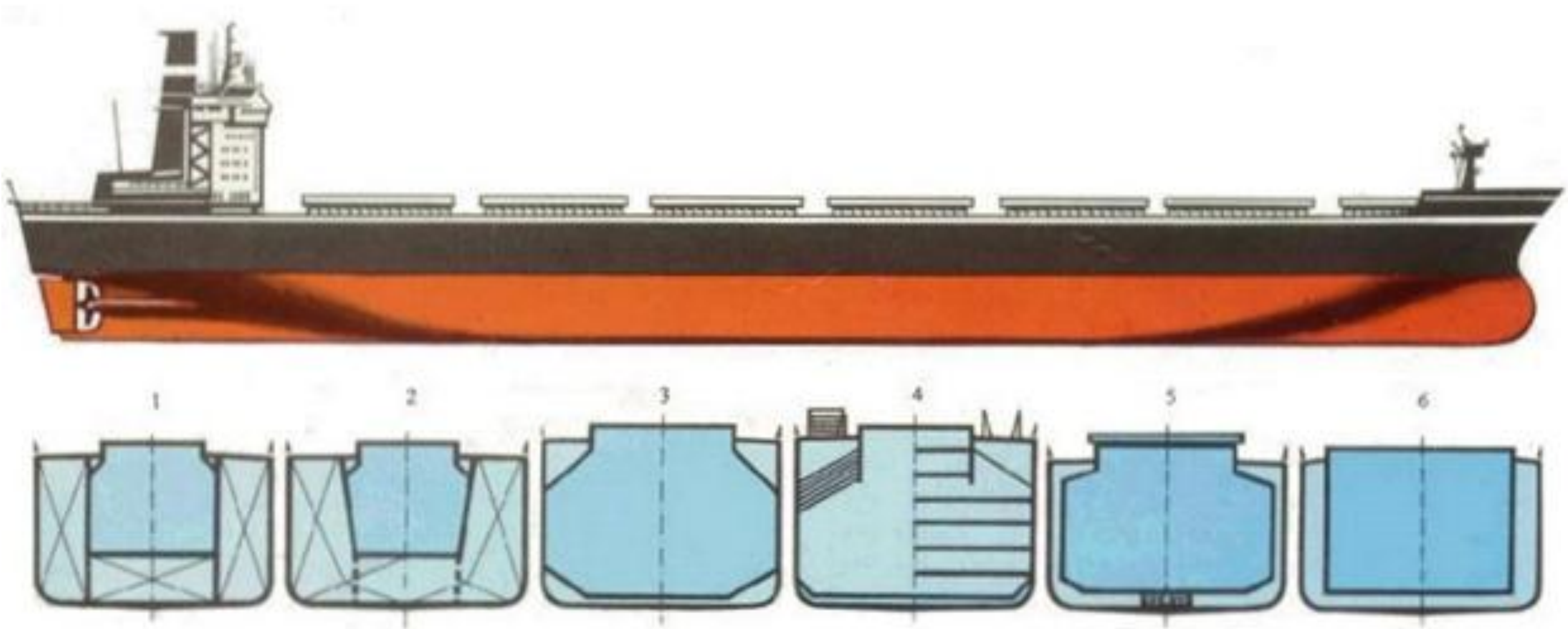


14-6 Structure Plans of Bulk Carrier

Bulk Carrier

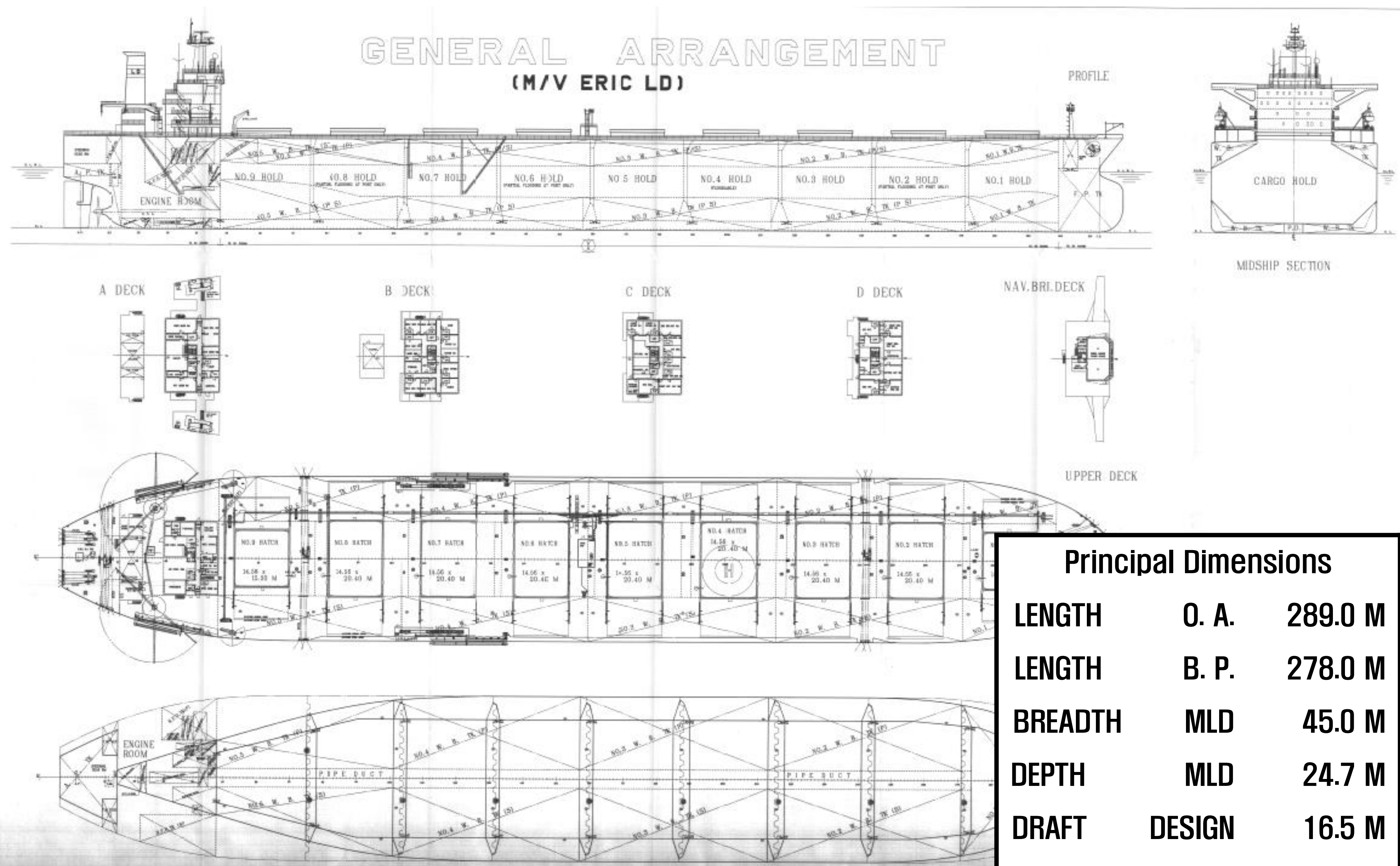


Kinds of midship section of bulk carrier



1,2 : Ore carrier, 3,4 : Bulk carrier
5 : Double hull bulk carrier
6 : Open bulk carrier

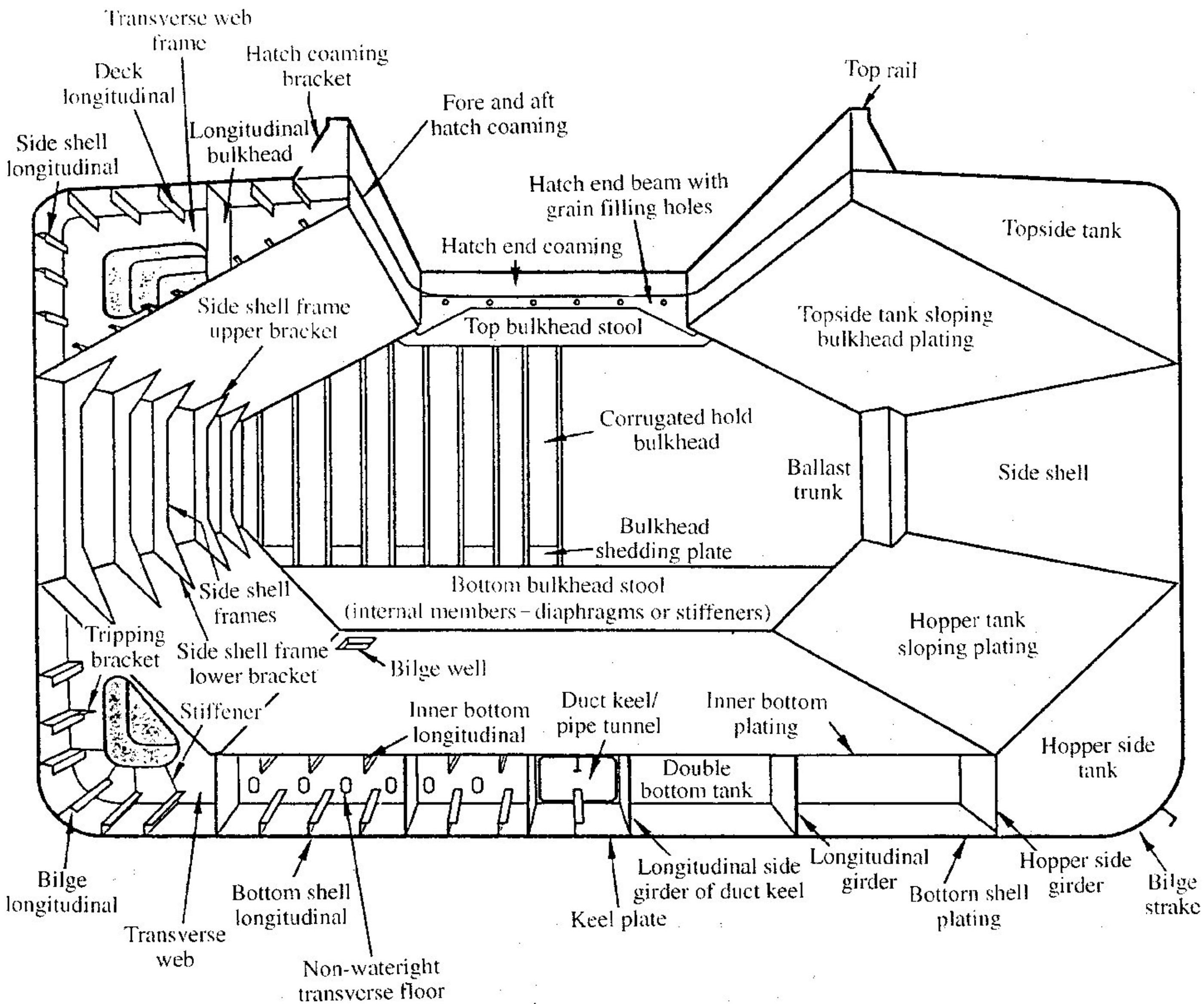
General Arrangement of a 170,000 ton DWT Bulk Carrier

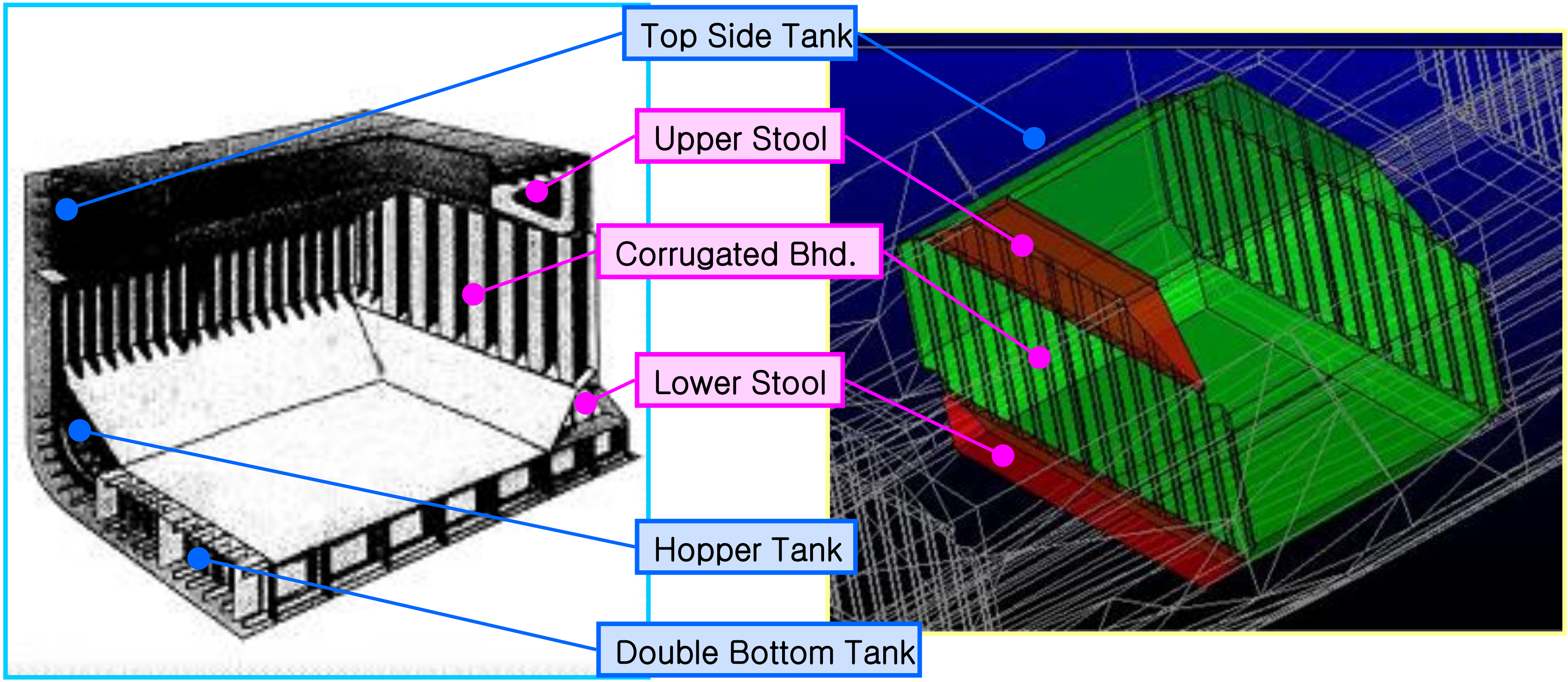


Principal Dimensions		
LENGTH	O. A.	289.0 M
LENGTH	B. P.	278.0 M
BREADTH	MLD	45.0 M
DEPTH	MLD	24.7 M
DRAFT	DESIGN	16.5 M
	SCANT.	17.5 M

3D STRUCTURE MODEL OF A BULK CARRIER

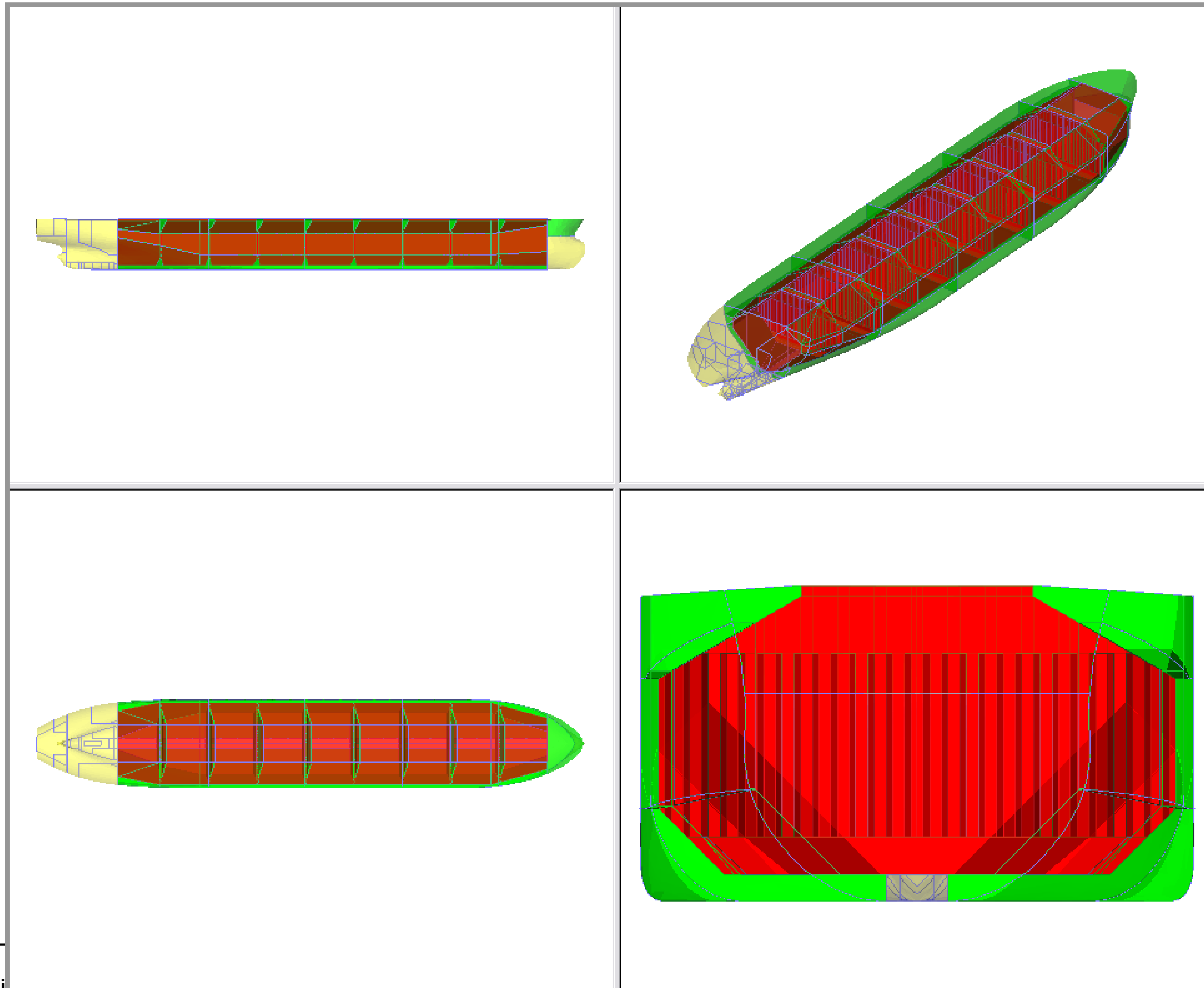
Naming of Structure Members of a Bulk Carrier



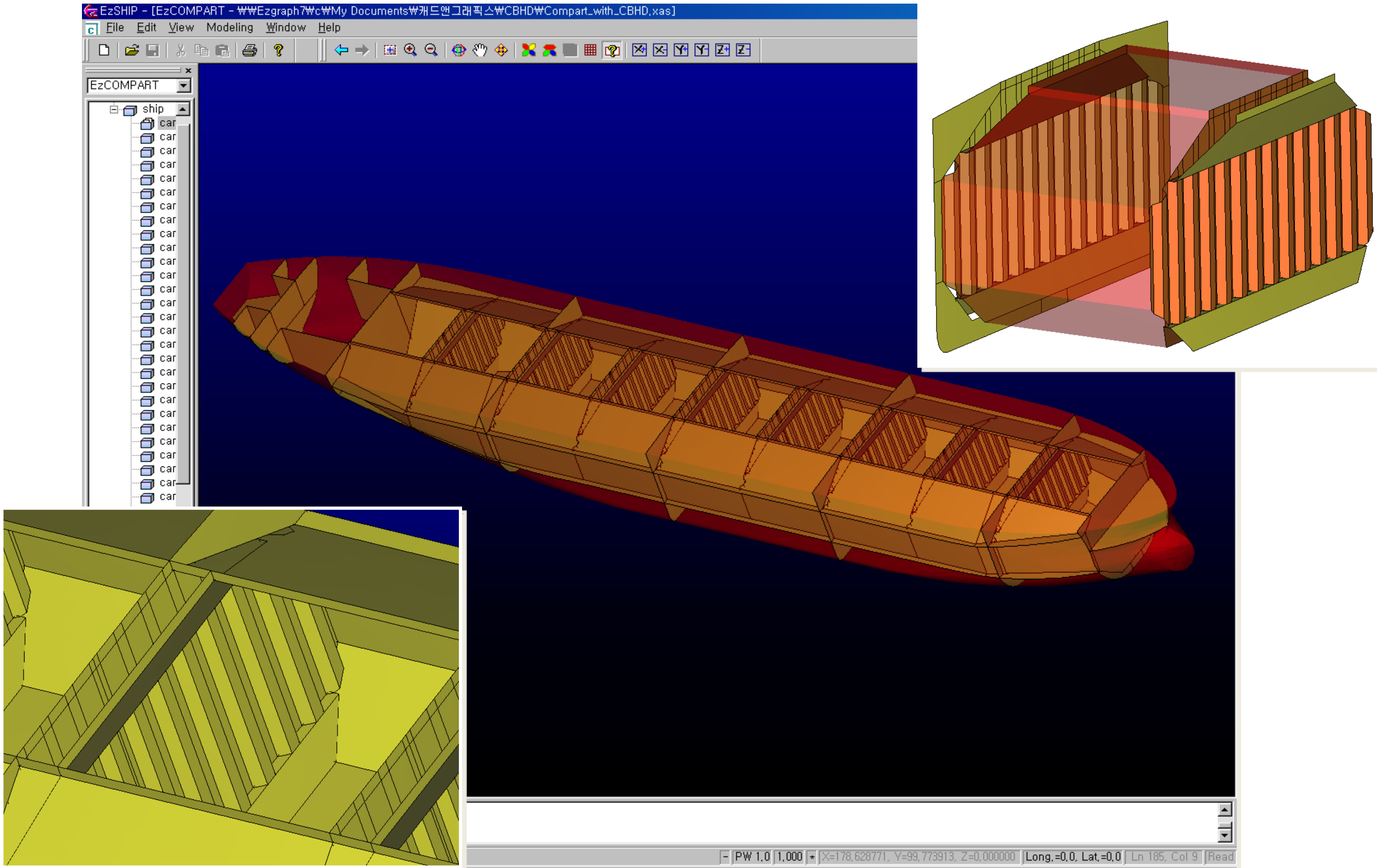


Top Side Tank
Upper Stool
Corrugated Bhd.
Lower Stool
Hopper Tank
Double Bottom Tank

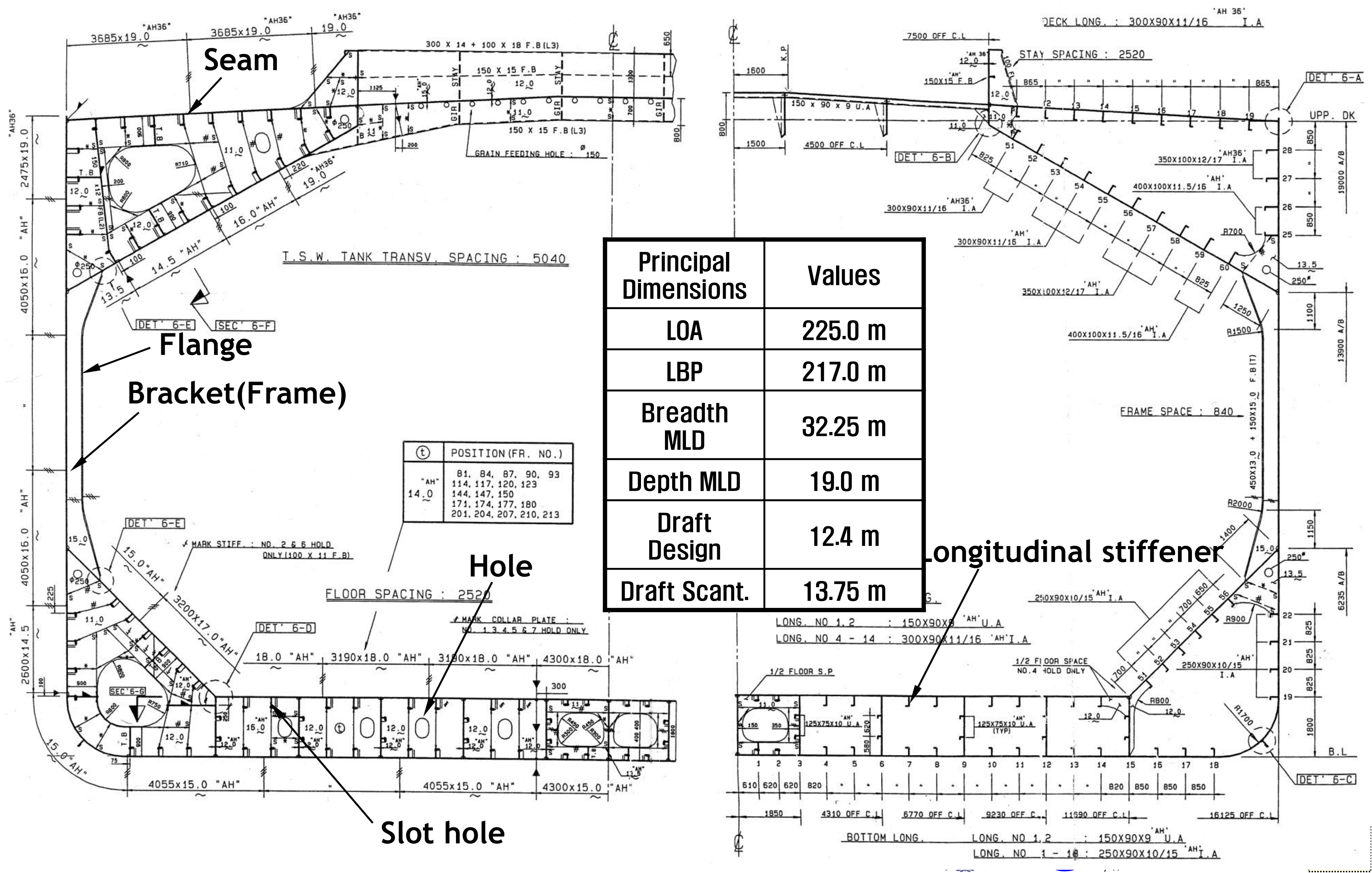
3D Structure Model of a 170,000 ton DWT Bulk Carrier



3D Structure Model of a 170,000 ton DWT Bulk Carrier

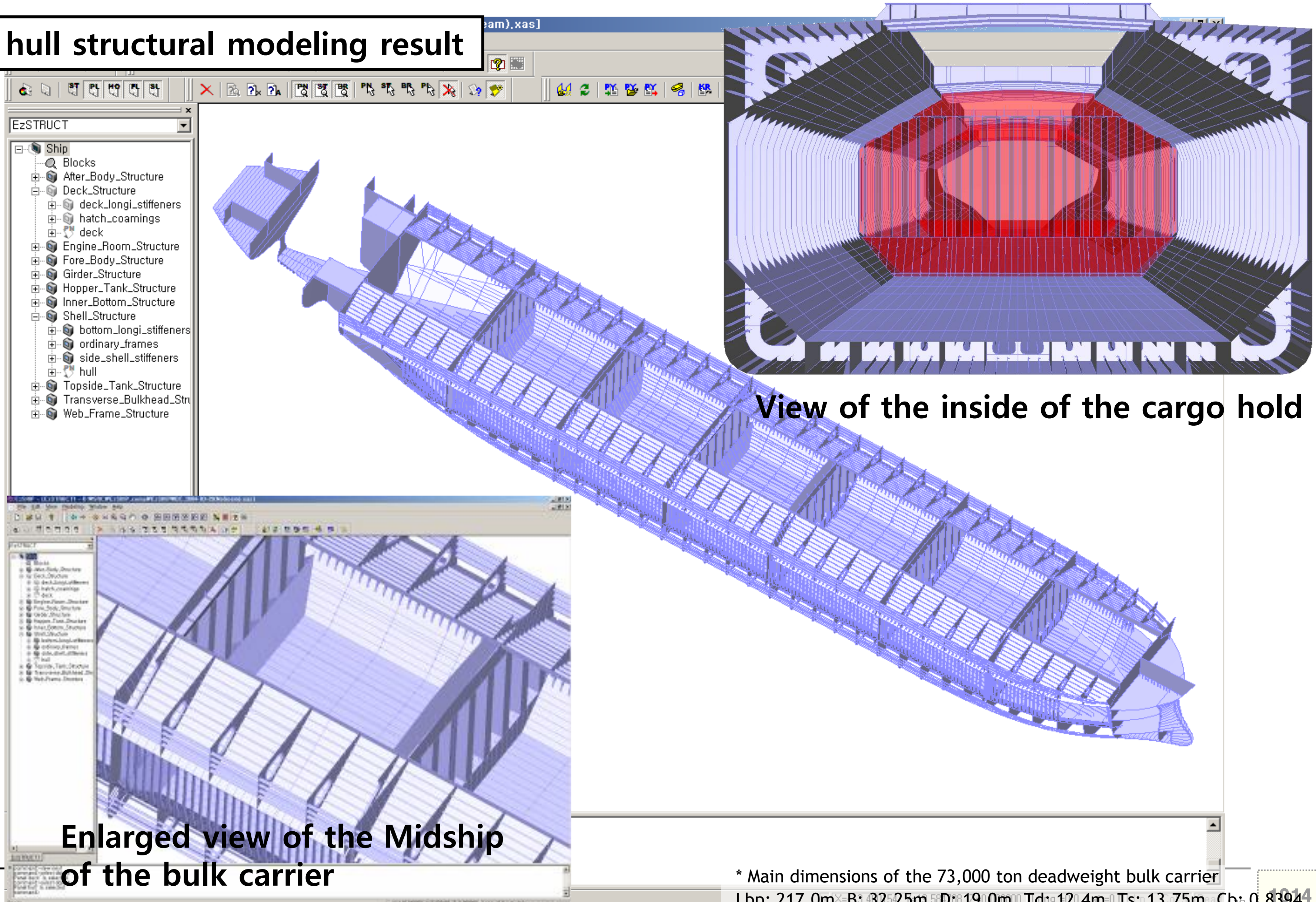


Structure Plan of a 73,000 ton DWT Bulk Carrier
: Midship Section



Principal Dimensions	Values
LOA	225.0 m
LBP	217.0 m
Breadth MLD	32.25 m
Depth MLD	19.0 m
Draft Design	12.4 m
Draft Scant.	13.75 m

3D Structure Model of a 73,000 ton DWT Bulk Carrier : Cargo Hold



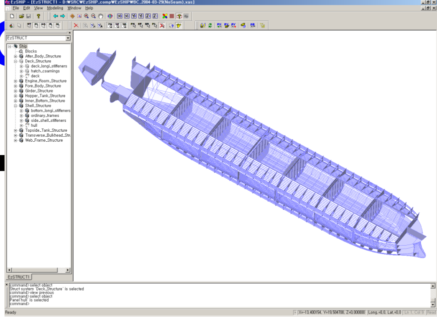
hull structural modeling result

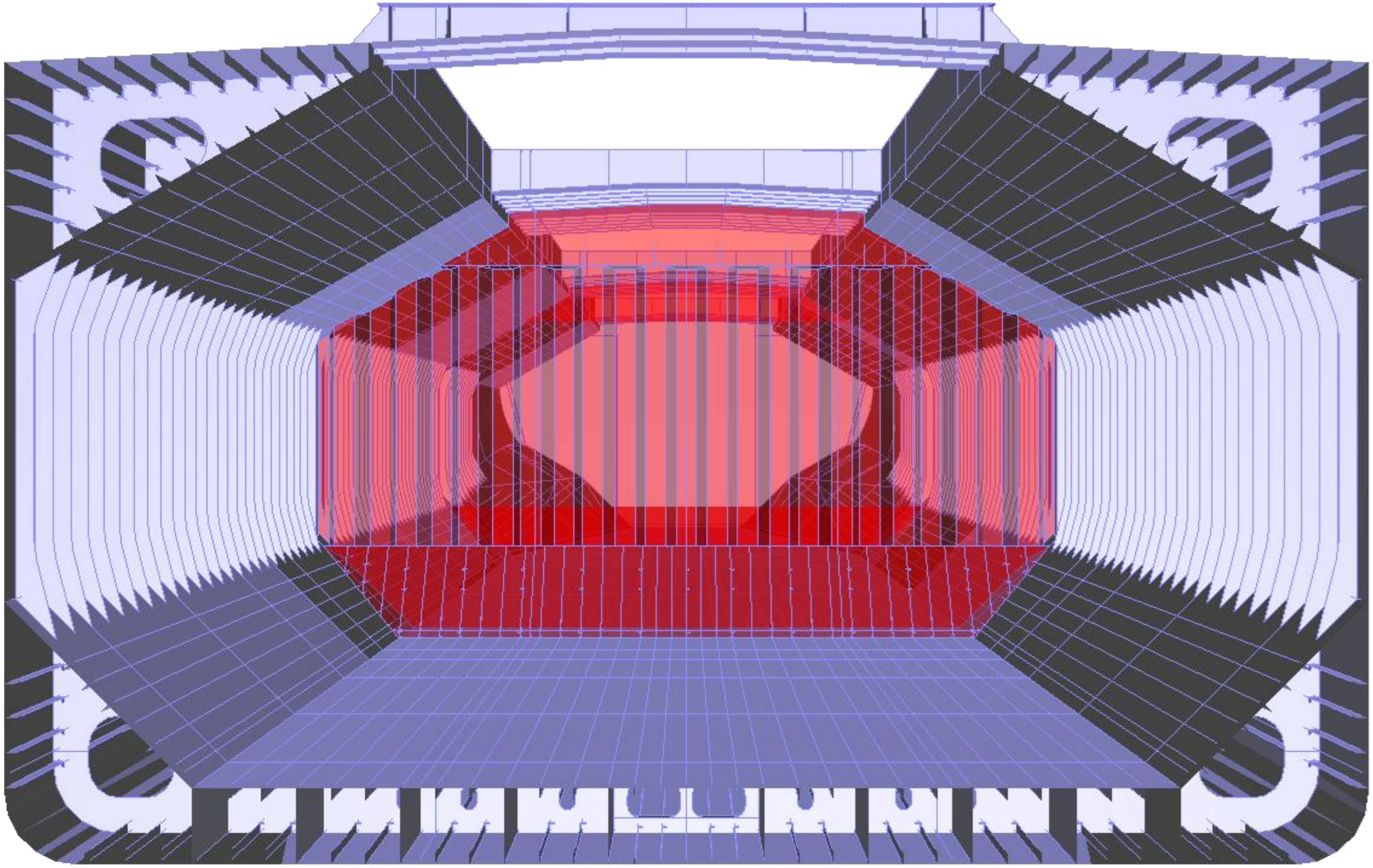
View of the inside of the cargo hold

Enlarged view of the Midship of the bulk carrier

* Main dimensions of the 73,000 ton deadweight bulk carrier
Lbp: 217.0m, B: 32.25m, D: 19.0m, Td: 12.4m, Ts: 13.75m, Cb: 0.8394

3D Structure Model of a 73,000 ton DWT Bulk C
: Cargo Hold

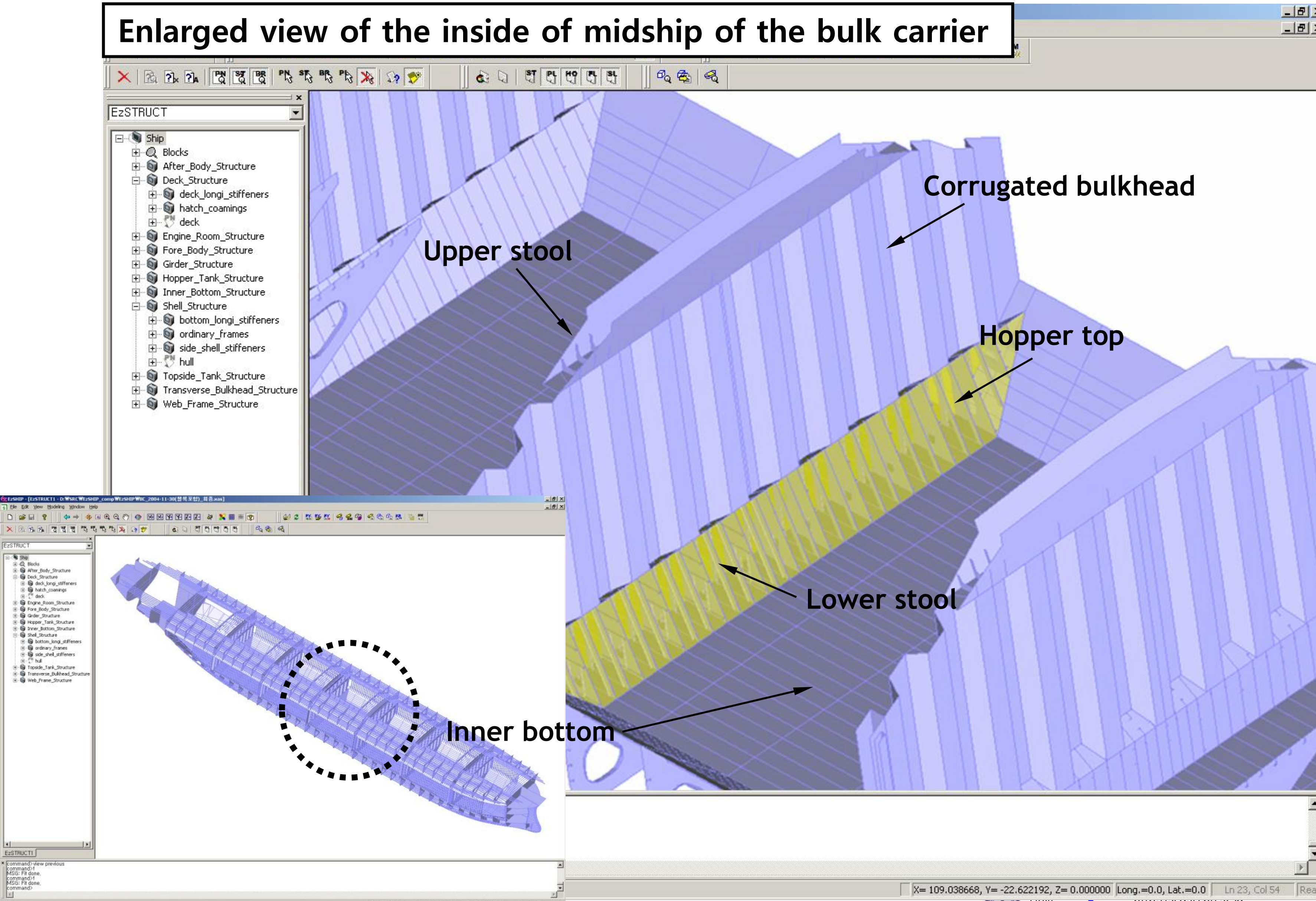




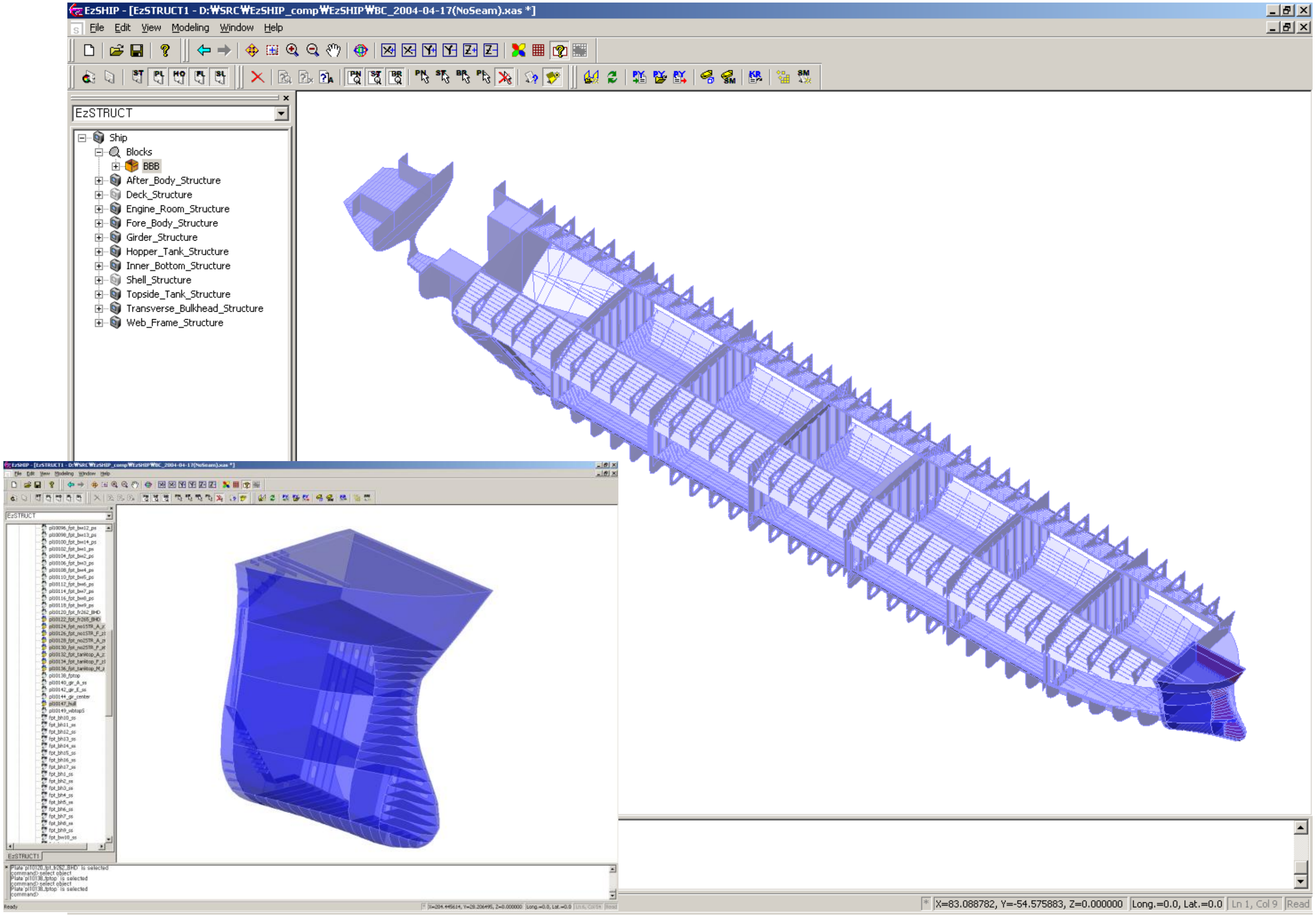
View of the inside of the cargo hold

* Main dimensions of the 73,000 ton deadweight bulk carrier
Lbp: 217.0m, B: 32.25m, D: 19.0m, Td: 12.4m, Ts: 13.75m, Cb: 0.8394

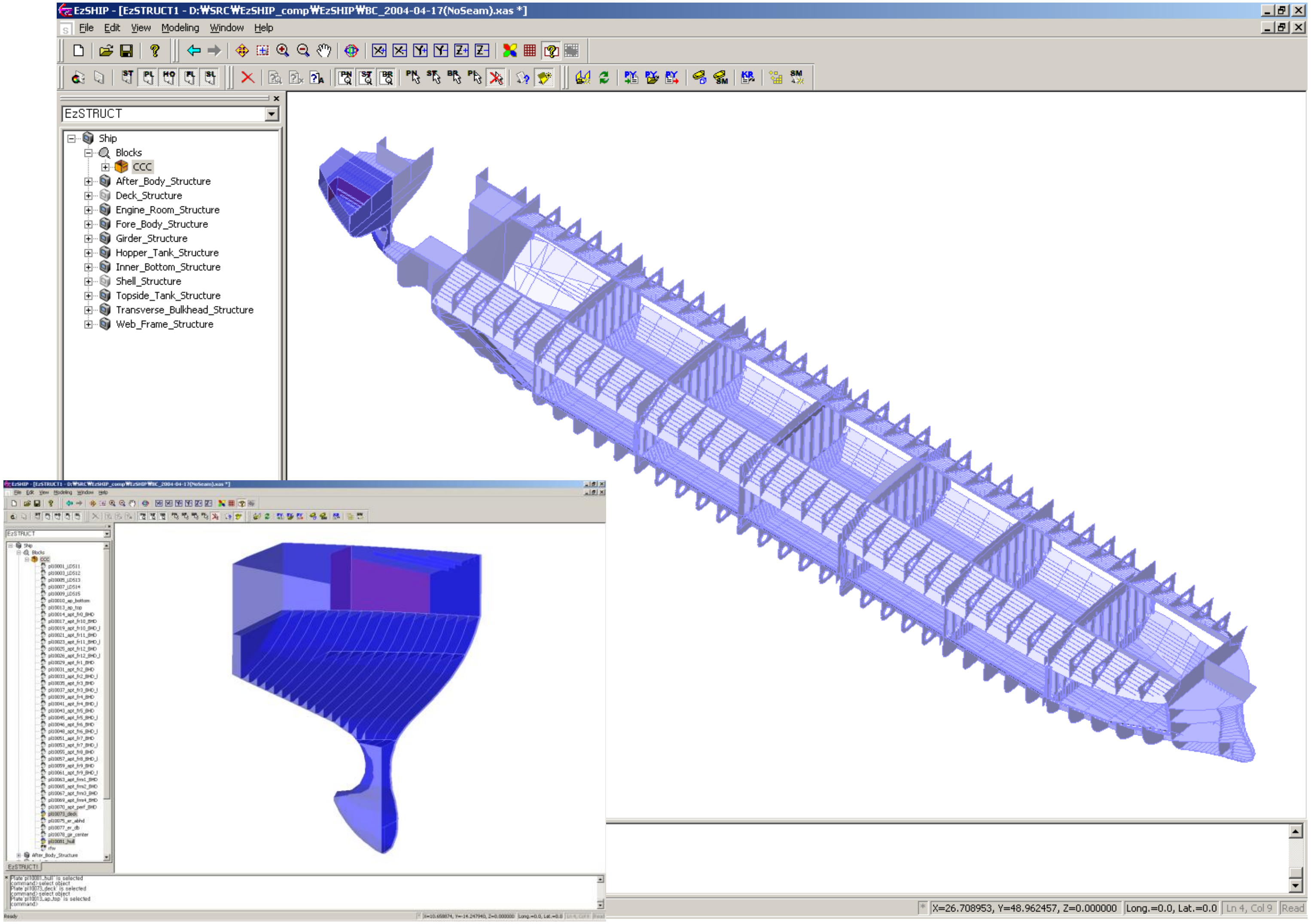
3D Structure Model of a 73,000 ton DWT Bulk Carrier
: Cargo Hold



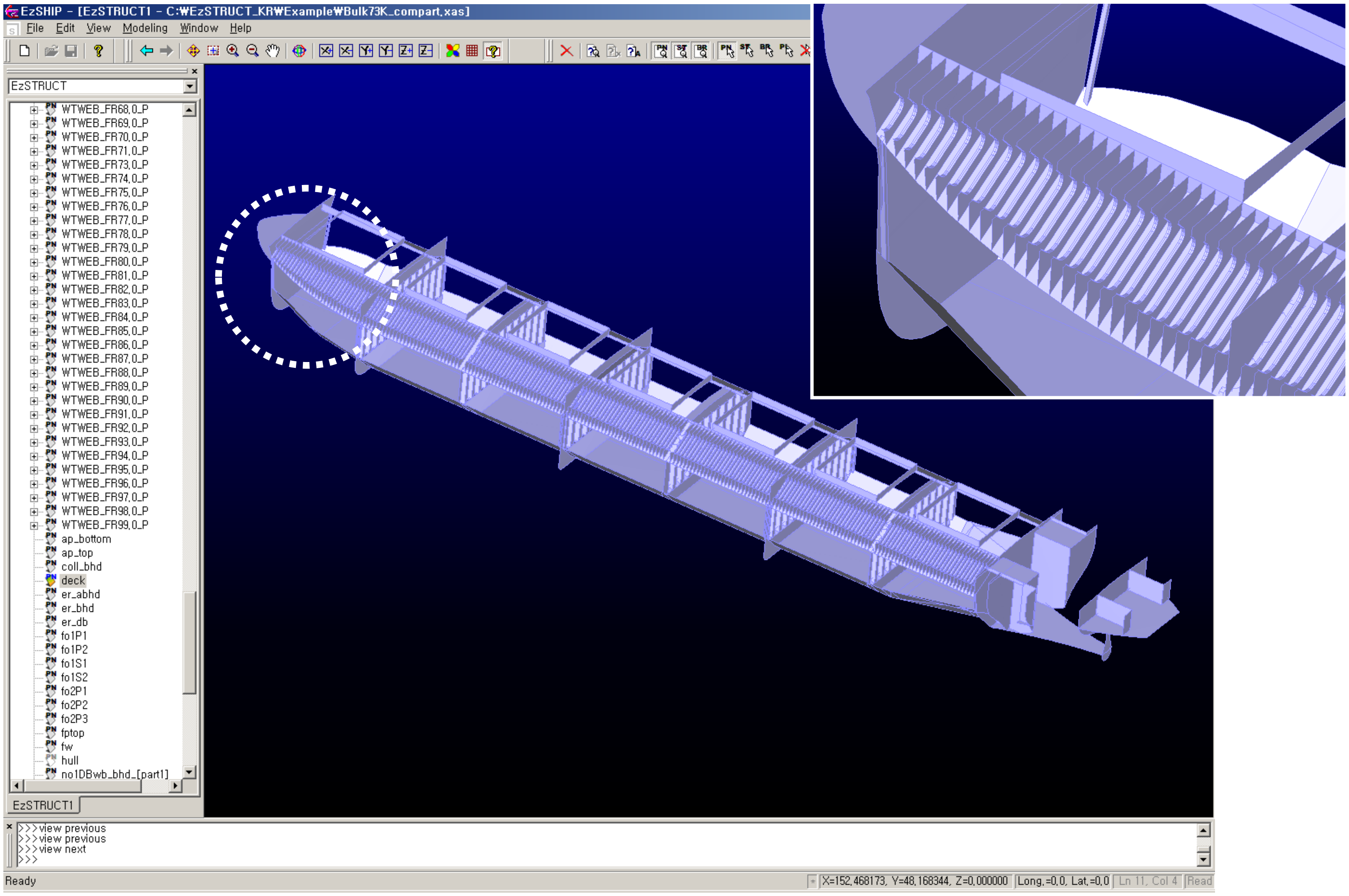
3D Structure Model of a 73,000 ton DWT Bulk Carrier : Stem



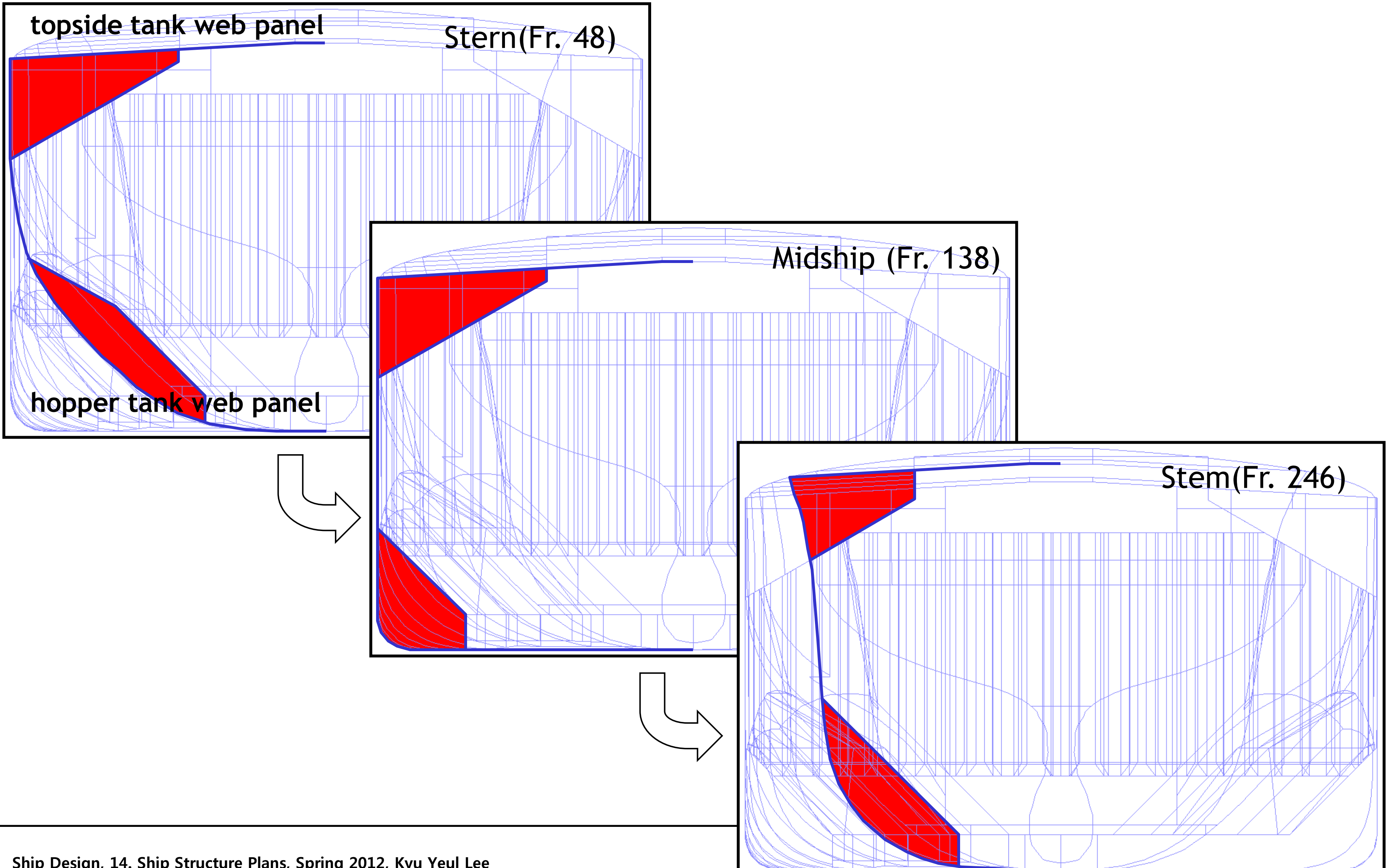
3D Structure Model of a 73,000 ton DWT Bulk Carrier
: Stem



3D Structure Model of a 73,000 ton DWT Bulk Carrier
: Topside Tank



3D Structure Model of a 73,000 ton DWT Bulk Carrier
: Transverse Structural Member



Chapter 15. Ship Structure Design

15-1. Global Hull Girder Strength (Longitudinal Strength)

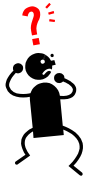
15-2. Local Strength

15-3. Midship Section Structure Design of a 3,700 TEU Container Carrier

15-1. Global Hull Girder Strength (Longitudinal Strength)

Interest of "Ship Structure Design"

- **Ship Structure Design**



what is designer's major interest?

- **Safety :**

Won't 'it' fail under the load?

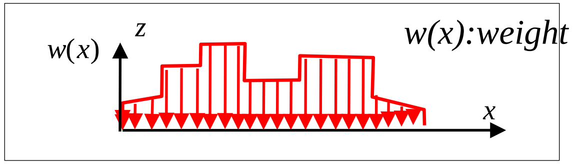
a ship } **global**

a stiffener
a plate } **local**

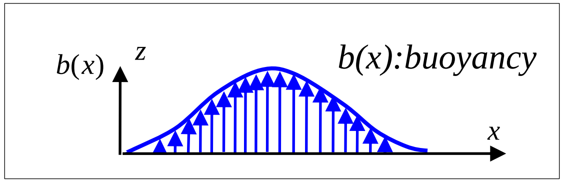
Let's consider the safety of the ship <u>from the point of global strength</u> first.

Global Hull Girder Strength (Longitudinal Strength)
- Dominant forces acting on a ship

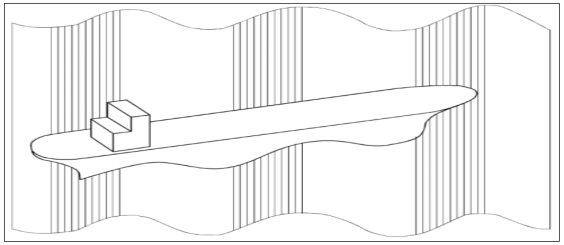
What are dominant forces acting on a ship in view of the longi. strength?



weight of light ship, weight of cargo and consumables



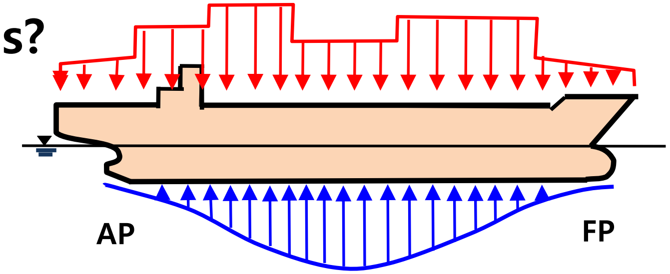
hydrostatic force(buoyancy) on the submerged hull



hydrodynamic force induced by the wave

What is the direction of the dominant forces?

The forces act in vertical ~~lateral~~ direction along the ship's length



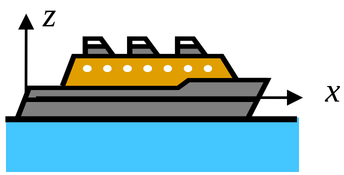
Longitudinal Strength:

Longitudinal Strength: Overall strength of ship's hull which resists the bending moment, shear force and torsional moment acting on a hull girder.

- **Longitudinal strength loads:**

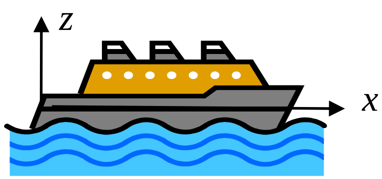
  : Load concerning the overall strength of the ship's hull, such as the bending moment, shear force and torsional moment acting on a hull girder

  - **Static longitudinal loads :**



Loads are caused by differences between weight and buoyancy in longitudinal direction in the still water condition

  - **Hydrodynamic longitudinal loads :**



: loads are induced by waves

[1)] Okumoto,Y., Design of Ship Hull Structures, Springers, 2009, P.17

Idealization of the ship hull girder structure

How can we idealize a ship as structural member?

- **Structural member according to the types of loads**

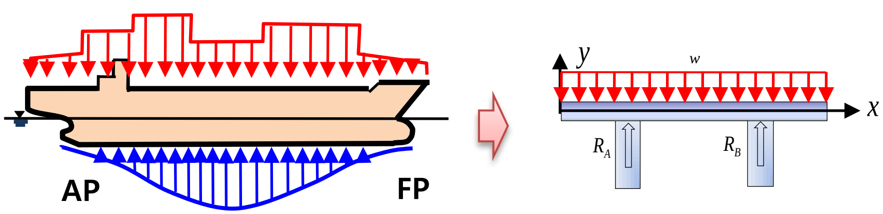
① Axially loaded bar : structural member which supports forces directed along the axis of the bar

② Bar in torsion : structural member which supports torques (or couples) having their moment about the longitudinal axis

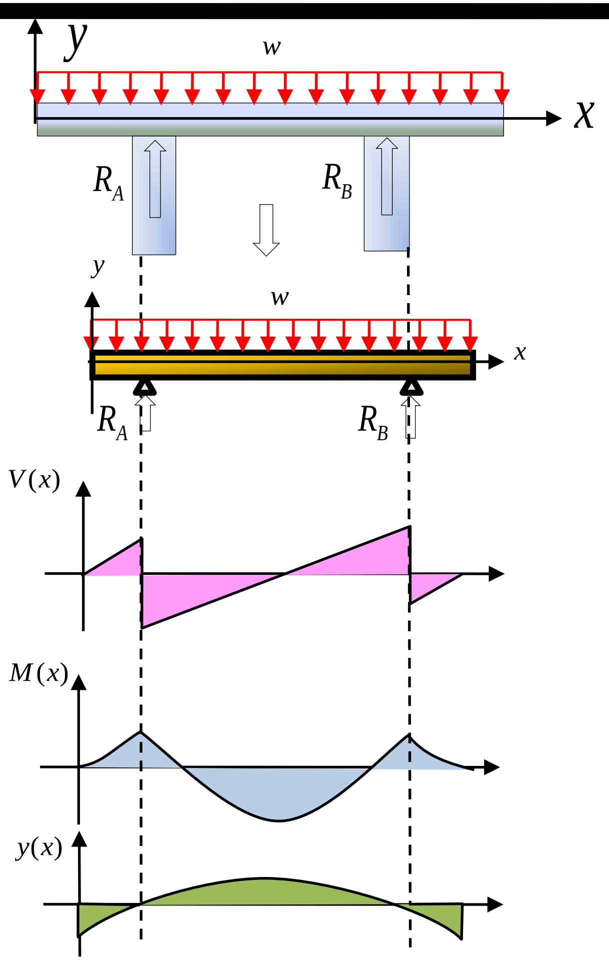
③ **Beam** : structural members subjected to **lateral loads**, that is, forces or moments perpendicular to the axis of the bar

Since a ship has a slender shape and subject to lateral loads, it will behave like a beam from the point view of structural member.

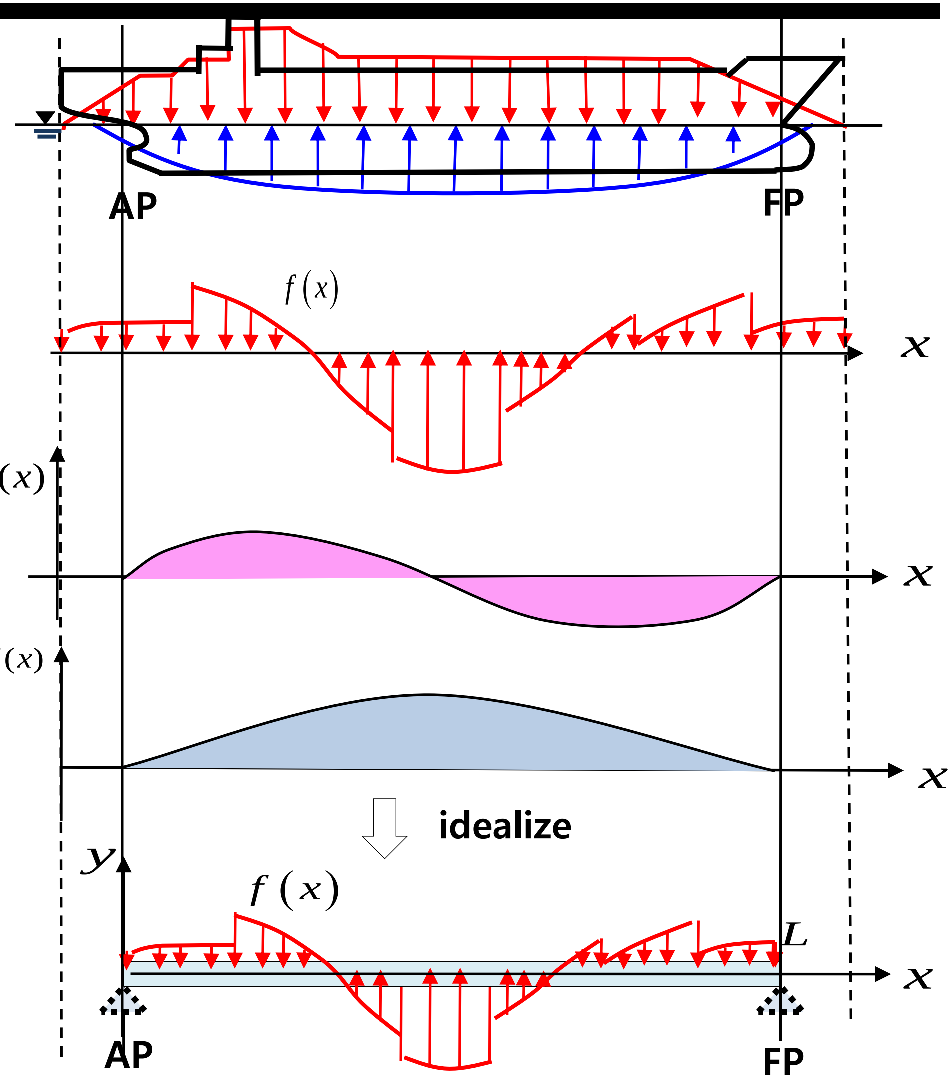
Ship is regarded as a beam.



Applying **Beam theory** to a ship

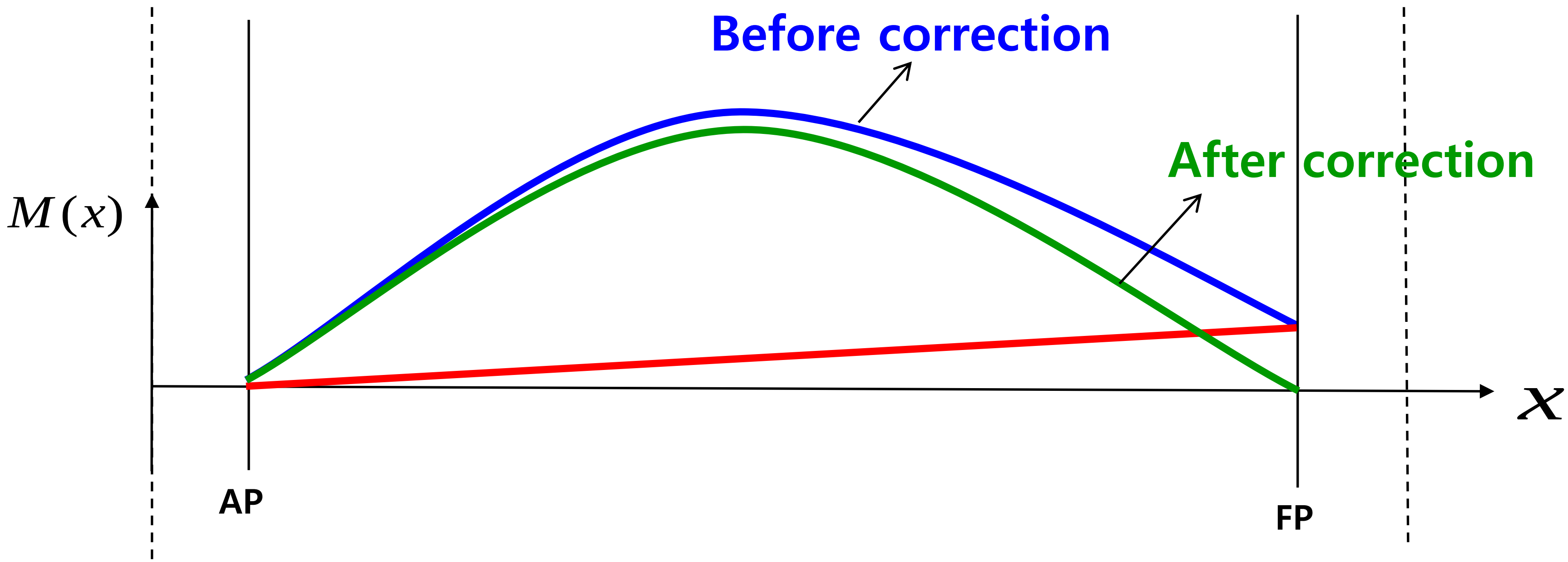


If there are no supports at the ends, deflection and slope of the beam occur.

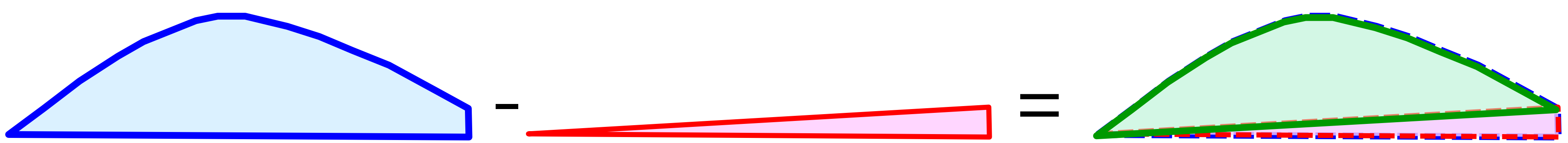


Actually, there are no supports at the ends of the ship.
And the deflection and slope could occur due to inequality of the buoyancy and the weight of a ship. For this problem, we assume that there are simple supports at the A.P and the F.P .

Correction of a bending moment curve



What if the bending moment is not zero at FP?



Actual Stress ≤ Allowable Stress
- Bending Stress and Allowable Bending Stress

• The actual bending stress($\sigma_{act.}$) shall not be greater than the allowable bending stress(σ_l)

M_S : Largest SWBM among all loading conditions and class rule

M_W : calculated by class rule or direct calculation

$$\boxed{\sigma_{act.} = \frac{|M_S + M_W|}{Z} 10^3 \ \ (kg / cm^2)}$$

$$\boxed{\sigma_{act.} \le \sigma_l}$$

(DnV Pt 3 Ch1 Sec. 5 C303)

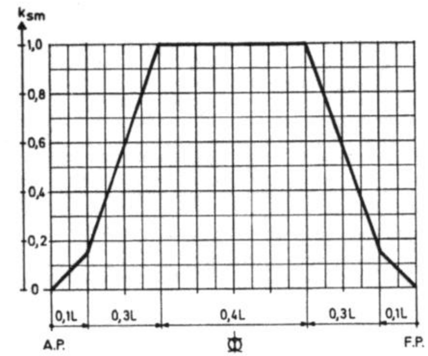


Fig. 2
Stillwater bending moment

$$\sigma_l = \sigma_{allow} = 175 f_1 N / mm^2$$ within 0.4 L amidship

$$= 125 f_1 N / mm^2$$ within 0.1 L from A.P. or F.P.

(DnV Pt 3 Ch1 Sec. 5 C303) - 2011

303 The section modulus requirements about the transverse neutral axis based on cargo and ballast conditions are given by:

$$Z_O = \frac{|M_S + M_W|}{\sigma_l} 10^3 \quad (\text{cm}^3)$$

σ_l = 175 f_1 N/mm^2 within 0.4 L amidship
= 125 f_1 N/mm^2 within 0.1 L from A.P. or F.P.

Between specified positions σ_l shall be varied linearly.

304 The midship section modulus about the vertical neutral axis (centre line) is normally not to be less than:

$$Z_{OH} = \frac{5}{f_1} L^{9/4} (T + 0.3B)\, C_B \quad (cm^3)$$

The above requirement may be disregarded provided the combined effects of vertical and horizontal bending stresses at bilge and deck corners are proved to be within 195 f_1 N/mm^2.

The combined effect may be taken as:

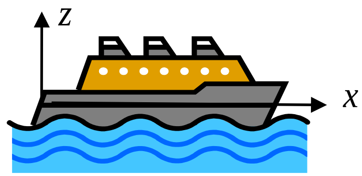
$$\sigma_s + \sqrt{\sigma_w^{\ 2} + \sigma_{wh}^{\ 2}}$$

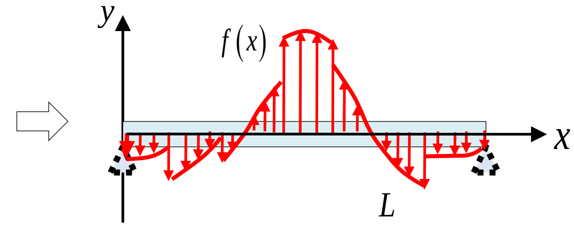
σ_s = stress due to M_S
σ_w = stress due to M_W
σ_{wh} = stress due to M_{WH}, the horizontal wave bending moment as given in B205.

Criteria of Structure Design

- **Ship Structure Design**

a ship





- Actual bending stress(σ) shall not be greater than the allowable bending stress(σ_l)

$$\boxed{\sigma \le \sigma_l}$$

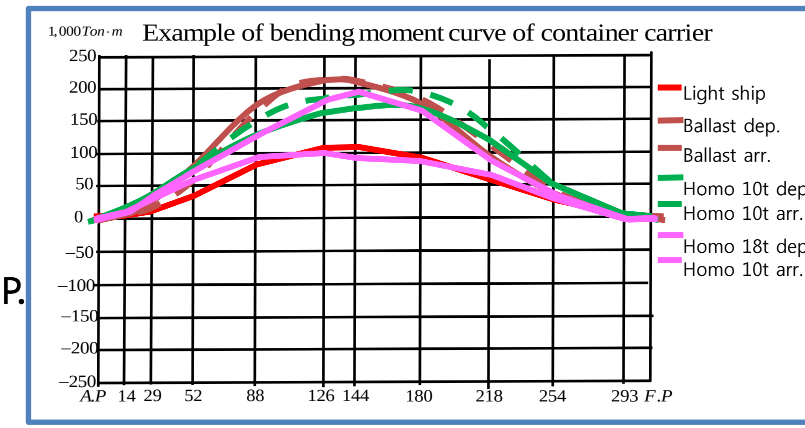
$$, \sigma = \frac{M}{I_{N.A}/y} = \frac{|M_S + M_W|}{I_{N.A}/y}$$

σ_l **: allowable stress**

For instance, allowable bending stresses by DNV rule are given as follows :

$\sigma_l = 175 f_1 \ [N/mm^2]$ within 0.4 L amidship

$= 125 f_1 \ [N/mm^2]$ within 0.1 L from A.P. or F.P.



- Actual bending moments at aft and forward area are smaller than that at the midship

What is, then, the f_1?

$$\boxed{\sigma \leq \sigma_l}$$

$$\sigma = \frac{M}{I_{N.A}/y} = \frac{|M_S + M_W|}{I_{N.A}/y}$$

1) Ms
2) Mw
3) I N.A
4) Allowable Stress

Ms : Still water Bending Moment

$$\sigma = \frac{M}{I_{N.A}/y} = \frac{|M_S + M_W|}{I_{N.A}/y}$$

, M_S = Still water bending moment
M_W = Vertical wave bending moment

Hydrostatic loads along ship's length

Caused by the weight & the buoyancy"

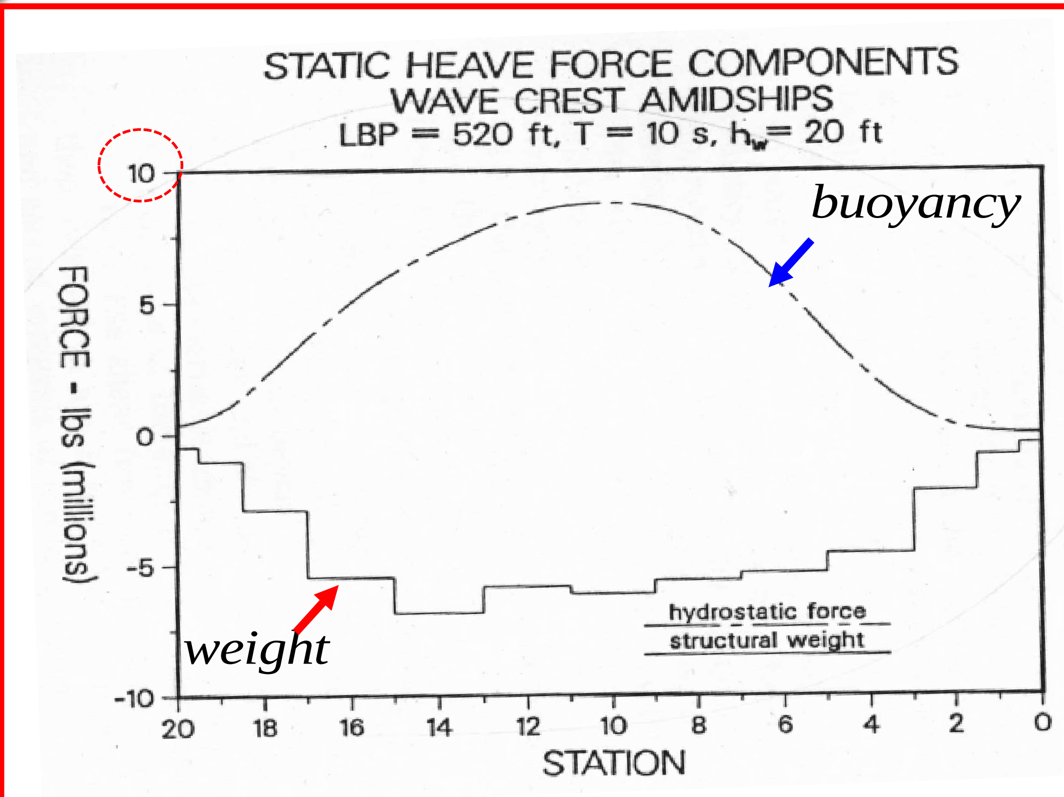
$f_S(x)$: distributed loads in longitudinal direction in still water

$$V_S(x) = \int_0^x f_S(x)dx$$

$V_S(x)$: still water shear force

$$M_S(x) = \int_0^x V_S(x)dx$$

$M_S(x)$: still water bending moment



15-2. Still water shear force, Qs & Still water bending moment, Ms

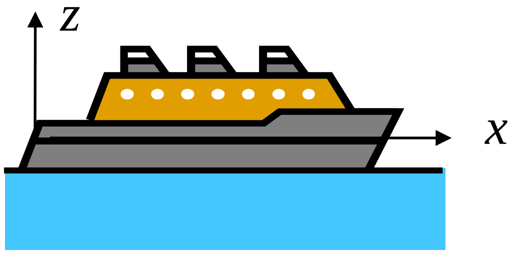
Distributed Loads in longitudinal direction

$$f(x) = f_S(x) + f_W(x)$$

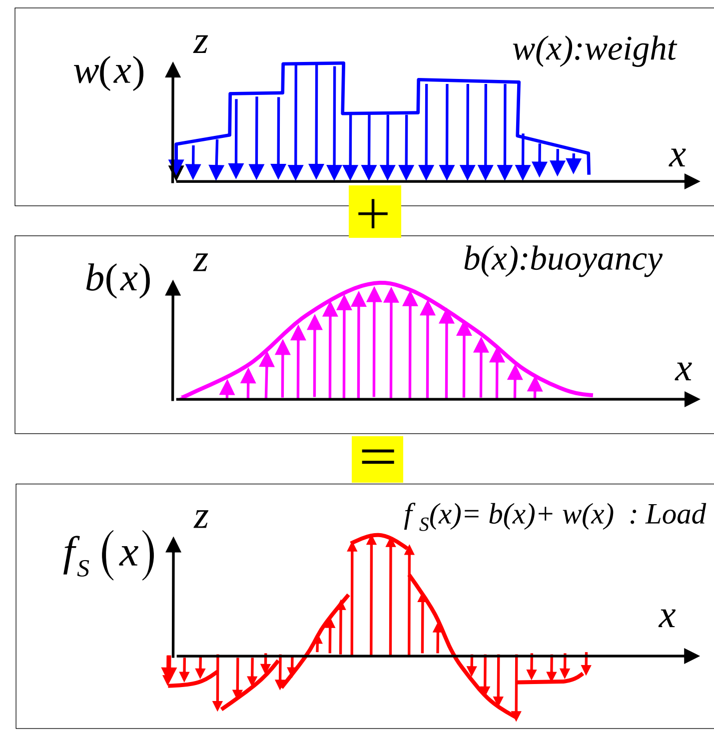
f(x) : distributed loads in longitudinal direction

$f_S(x)$: **Static longitudinal loads** in longitudinal direction
$f_W(x)$: **Hydrodynamic longitudinal loads** induced by wave

In still water



$$f_S(x) = b(x) + w(x)$$



b(x) : distributed buoyancy in longitudinal direction
w(x) = LWT(x) + DWT(x)
- *w(x)* : weight distribution in longitudinal direction
- *LWT(x)* : lightweight distribution
- *DWT(x)* : deadweight distribution

Distributed Loads in Still Water

Load curve, $f_S(x)$ → Actual Still water Shear Force, $V_S(x)$ → Actual Still water Bending Moment, $M_S(x)$

Weight, $w(x)$
Buoyancy, $b(x)$

$$V_S(x) = \int^{x} f_S(x)dx \qquad M_S(x) = \int_0^{x} V_S(x)dx$$

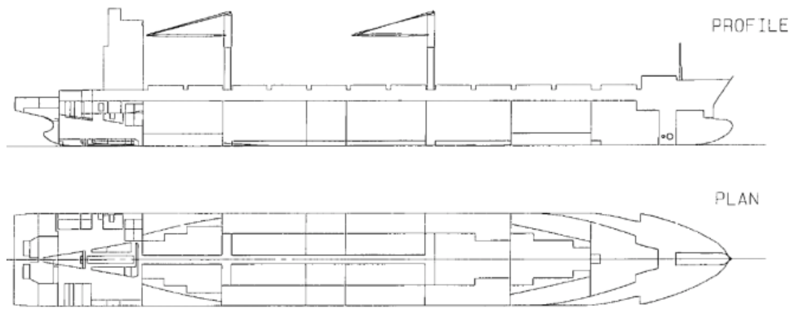
✓ Example of A 3,700 TEU Container Ship in Homogeneous 10t Scantling Condition

- Principal dimensions & Plans

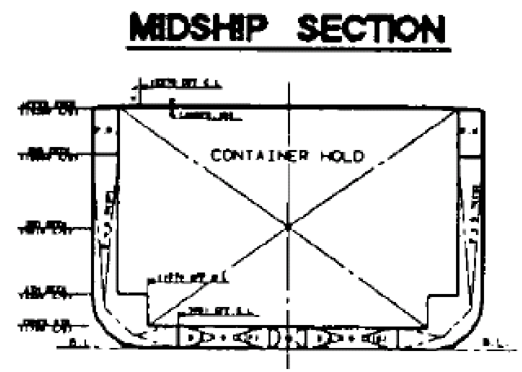
  - principal dimension

LENGTH O. A.	257. 368 M
LENGTH B. P.	245. 240 M
BREADTH MOULDED	32. 20 M
DEPTH MOULDED	19. 30 M
DESIGNED DRAUGHT MOULDED	10. 10 M
SCANTLING DRAUGHT MOULDED	12. 50 M

  - profile & plan



  - midship section



- Loading Condition (Sailing state) in homogeneous 10t scantling condition

SAILING STATE

DRAUGHT F.P	=	12.260 M	K.M.T	=	14.889 M
DRAUGHT MIDSHIP	=	12.457 M	KG (SOLID)	=	13.586 M
DRAUGHT A.P	=	12.654 M	GM (SOLID)	=	1.303 M
TRIM BY STERN	=	.394 M	FREE SURF. CORR. (GGo)	=	.059 M
PROPELLER I/D	=	160.3 %	GoM (FLUID)	=	1.244 M
DISPLACEMENT	=	66813.6 T	KGo ACTUAL (FLUID)	=	13.645 M
DRAUGHT AT LCF	=	12.483 M	TRIM (DIS*A)/(MTC*100)	=	.394 M
LCB FROM A.P	=	115.677 M	FREE SURF. MOM.	=	3921 T-M
LCG FROM A.P	=	115.045 M	M.T.C.	=	1072.0 T-M
TRIM LEVER : A	=	.632 M	LCF FROM A.P	=	106.275 M

DEGREE =	.0	5.0	10.0	15.0	20.0	30.0	40.0	50.0	60.0	75.0
KN =	.000	1.296	2.591	3.882	5.168	7.614	9.592	10.930	11.697	11.959
KGo*SINθ =	.000	1.189	2.369	3.532	4.667	6.823	8.771	10.453	11.817	13.180
GZ =	.000	.107	.222	.350	.501	.791	.821	.477	-.120	-1.221

- Frame space : 800mm

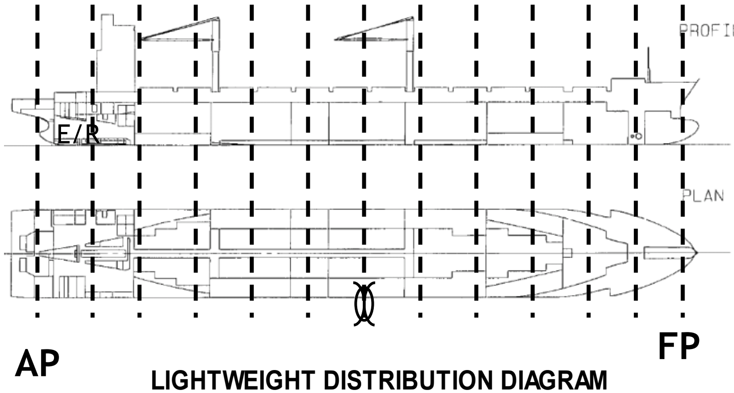
Distributed Loads in Still Water
- Lightweight (Example of 3,700TEU Container carrier)

Load curve, $f_S(x)$ → Actual Still water Shear Force, $V_S(x)$ → Actual Still water Bending Moment, $M_S(x)$

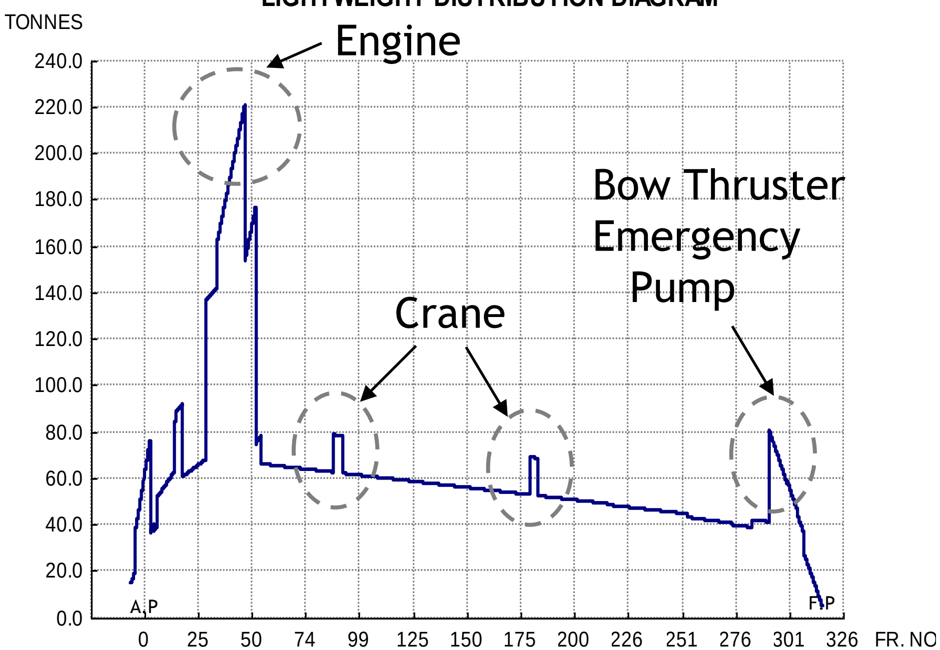
Weight, $w(x)$
Buoyancy, $b(x)$

$$V_S(x) = \int_0^x f_S(x)dx$$

$$M_S(x) = \int_0^x V_S(x)dx$$



LIGHTWEIGHT DISTRIBUTION DIAGRAM



L I G H T   W E I G H T   S U M M A R Y

Hull No. : 1329. 3,700 TEU CONTAINER VESSEL

NO	AFT END	FORE END	WEIGHT	L.C.G	MOMENT
1	-5.000	14.350	616.00	7.000	4312.0
2	14.350	43.400	1387.10	31.400	43554.9
3	43.400	232.320	7591.50	128.620	976418.7
4	232.320	252.240	732.30	239.280	175224.7
5	27.200	41.600	476.40	35.800	17055.1
6	.000	245.240	30.00	122.620	3678.6
7	43.400	232.320	340.00	134.200	45628.0
8	-3.600	232.320	119.00	114.400	13613.6
9	-3.400	2.400	151.90	.000	.0
10	.000	252.240	224.00	120.000	26880.0
11	202.240	232.320	137.90	217.000	29924.3
12	43.400	202.240	1053.00	121.700	128150.1
13	143.280	146.680	55.00	144.980	7973.9
14	70.480	73.880	55.00	72.180	3969.9
15	14.350	232.320	115.90	114.360	13254.3
16	-3.600	232.320	128.00	114.360	14638.1
17	232.320	245.240	118.30	238.600	28226.4
18	36.000	170.000	3.00	81.000	243.0
19	-5.000	4.000	50.00	-.500	-25.0
20	29.000	41.600	15.50	37.100	575.0
21	-3.500	4.000	19.20	.000	.0
22	4.000	11.200	34.30	7.600	260.7
23	41.600	173.900	62.50	105.760	6610.0
24	226.160	232.320	20.40	229.240	4676.5
25	239.000	243.000	5.40	241.000	1301.4
26	11.200	232.320	39.20	121.700	4770.6
27	11.200	232.320	191.30	121.700	23281.2
28	27.200	41.600	214.50	36.000	7722.0
29	23.230	37.600	979.00	30.400	29761.6
30	11.200	41.600	289.50	22.000	6369.0
31	5.000	23.230	111.30	11.200	1246.6
32	12.000	41.600	150.70	28.000	4219.6
33	11.200	41.600	158.60	28.000	4440.8
34	11.200	41.600	95.90	28.000	2685.2
35	11.200	218.480	165.00	114.240	18849.6
36	27.200	41.600	8.50	36.000	306.0
37	11.200	41.600	43.00	30.000	1290.0
38	27.200	41.600	4.30	36.000	154.8
39	27.200	41.600	5.70	36.000	205.2
LIGHT SHIP TOTAL =			15998.10	103.228	1651446.5

Distributed Loads in Still Water

- Deadweight (Example of 3,700TEU Container carrier)

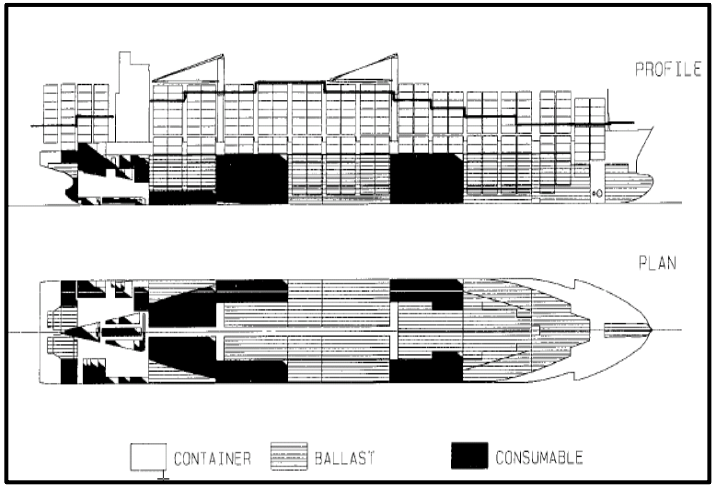
Load curve, $f_S(x)$ → Actual Still water Shear Force, $V_S(x)$ → Actual Still water Bending Moment, $M_S(x)$

Weight, $w(x)$
Buoyancy, $b(x)$

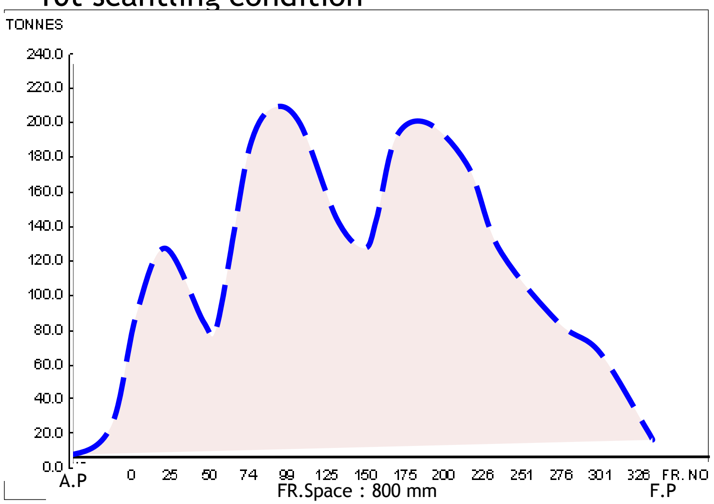
$$V_S(x) = \int^{x} f_S(x)dx$$

$$M_S(x) = \int_0^{x} V_S(x)dx$$

- Loading Plan in homogenous 10t scantling condition



- Deadweight distribution curve in homogenous 10t scantling condition



Deadweight distribution in longitudinal direction in homogenous 10t scantling condition

SHI — NO.6L HOMO.10T SCANTLING DEPARTURE (2.918 TEU) — 69

COMPARTMENT	LOCATION FR.NO.	FILL. RATIO (%)	S.G OR UNIT WEIGHT	WEIGHT (MT)	L.C.G FROM A.P.	LONGI. MOMENT (T-M)	V.C.G ABOVE B.L	VERT. MOMENT (T-M)	FREE MOMENT (T-M)
NO.1 HOLD(71)	254.0-292.0		10.00	710.0	216.728	153877	15.653	11114	0
NO.2 HOLD(204)	218.0-254.0		10.00	2040.0	187.243	381976	12.449	25396	0
NO.3 HOLD(282)	180.0-218.0		10.00	2820.0	159.759	450520	11.347	31999	0
NO.4 HOLD(300)	144.0-180.0		10.00	3000.0	129.840	389520	10.946	32838	0
NO.5 HOLD(150)	126.0-144.0		10.00	1500.0	108.480	162720	10.946	16419	0
NO.6 HOLD(298)	88.0-126.0		10.00	2980.0	87.188	259820	11.000	32780	0
NO.7 HOLD(260)	52.0- 88.0		10.00	2600.0	57.325	149045	11.920	30992	0
TOTAL CONTAINER LOADED HOLD				15650.0		1947478		181538	0
NO.1 HATCH(22)	254.0-292.0		10.00	220.0	211.760	46587	22.944	5048	0
NO.2 HATCH(81)	218.0-254.0		10.00	810.0	186.451	151025	23.882	19344	0
NO.3 HATCH(182)	180.0-218.0		10.00	1820.0	159.103	289567	26.314	47891	0
NO.4 HATCH(260)	144.0-180.0		10.00	2600.0	129.840	337584	28.186	73284	0
NO.5 HATCH(156)	126.0-144.0		10.00	1560.0	108.480	169229	29.497	46015	0
NO.6 HATCH(286)	88.0-126.0		10.00	2860.0	87.767	251014	28.901	82657	0
NO.7 HATCH(234)	52.0- 88.0		10.00	2340.0	57.631	134857	27.604	64593	0
AFT DECK(132)	-4.4- 30.0		10.00	1320.0	11.551	15247	22.631	29873	0
TOTAL CONTAINER LOADED DECK				13530.0		1395110		368705	0
F.P.TK(C)	292.0-316.9	100.0	1.0250	535.9	240.444	128854	5.980	3205	0
NO.1 W.W.B.TK(P)	254.0-284.0	100.0	1.0250	996.3	212.092	211307	8.003	7973	0
NO.1 W.W.B.TK(S)	254.0-284.0	100.0	1.0250	996.3	212.092	211307	8.003	7973	0
NO.2 D.B.W.B.TK(P)	218.0-254.0	100.0	1.0250	541.3	186.645	101031	2.136	1156	0
NO.2 D.B.W.B.TK(S)	218.0-254.0	100.0	1.0250	541.3	186.645	101031	2.136	1156	0
NO.2 W.W.B.TK(P)	218.0-254.0	100.0	1.0250	989.3	187.893	185883	9.662	9559	0
NO.2 W.W.B.TK(S)	218.0-254.0	100.0	1.0250	989.3	187.893	185883	9.662	9559	0
NO.3 D.B.W.B.TK(P)	184.0-218.0	100.0	1.0250	363.2	159.025	57758	.852	309	0
NO.3 D.B.W.B.TK(S)	184.0-218.0	100.0	1.0250	363.2	159.025	57758	.852	309	0
NO.4 D.B.W.B.TK(P)	144.0-180.0	100.0	1.0250	371.5	129.040	47938	.850	316	0
NO.4 D.B.W.B.TK(S)	144.0-180.0	100.0	1.0250	371.5	129.040	47938	.850	316	0
NO.4 W.W.B.TK(P)	144.0-180.0	100.0	1.0250	1229.1	128.858	158379	6.435	7909	0
NO.4 W.W.B.TK(S)	144.0-180.0	100.0	1.0250	1229.1	128.858	158379	6.435	7909	0
NO.5 D.B.W.B.TK(P)	126.0-144.0	100.0	1.0250	185.7	107.680	19996	.850	158	0
NO.5 D.B.W.B.TK(S)	126.0-144.0	100.0	1.0250	185.7	107.680	19996	.850	158	0
NO.5 W.W.B.TK(P)	126.0-144.0	100.0	1.0250	621.0	107.718	66893	6.391	3969	0
NO.5 W.W.B.TK(S)	126.0-144.0	100.0	1.0250	621.0	107.718	66893	6.391	3969	0
NO.6 D.B.W.B.TK(P)	92.0-126.0	100.0	1.0250	345.3	87.269	30134	.861	297	0
NO.6 D.B.W.B.TK(S)	92.0-126.0	100.0	1.0250	345.3	87.269	30134	.861	297	0
NO.7 W.W.B.TK(P)	52.0- 88.0	100.0	1.0250	929.2	54.797	50917	9.175	8526	0
NO.7 W.W.B.TK(S)	52.0- 88.0	100.0	1.0250	929.2	54.797	50917	9.175	8526	0
A.P.TK(C)	-2.0- 14.0	100.0	1.0250	466.6	6.018	2808	11.899	5552	0
TOTAL BALLAST WATER				14146.3		1952134		89101	0
F.W.TK(P)	5.0- 14.0	100.0	1.0000	172.9	7.326	1267	15.113	2613	275
F.W.TK(S)	5.0- 14.0	100.0	1.0000	189.8	7.634	1449	15.111	2868	295
TOTAL FRESH WATER				362.7		2716		5481	570
NO.1 H.F.O.TK(P)	180.0-218.0	98.0	.9900	1202.4	159.059	191253	6.778	8150	22
NO.1 H.F.O.TK(S)	180.0-218.0	98.0	.9900	1202.4	159.059	191253	6.778	8150	22
NO.2 H.F.O.TK(P)	88.0-126.0	98.0	.9900	1107.4	85.697	94901	6.944	7690	22
NO.2 H.F.O.TK(S)	88.0-126.0	98.0	.9900	1107.4	85.697	94901	6.944	7690	22
NO.3 H.F.O.TK(P)	52.4- 88.0	98.0	.9900	576.2	57.390	33068	2.313	1333	1114
NO.3 H.F.O.TK(S)	52.4- 88.0	98.0	.9900	576.2	57.390	33068	2.313	1333	1114
H.F.O SERV.TK(P)	44.0- 52.0	90.0	.9900	52.8	38.192	2017	12.999	686	19
NO.1 HFO SETT.TK(P)	48.0- 52.0	90.0	.9900	109.2	40.011	4369	10.529	1150	20
NO.2 HFO SETT.TK(P)	44.0- 48.0	90.0	.9900	104.4	36.814	3843	10.484	1095	20
TOTAL FUEL OIL				6038.4		648673		37277	2375
D.O.STOR.TK(P)	14.0- 29.0	80.0	.8600	246.5	16.748	4128	14.146	3487	107
D.O.SERV.TK(P)	24.0- 29.0	80.0	.8600	38.6	21.200	818	12.988	501	10
TOTAL DIESEL OIL				285.1		4946		3988	117
MAIN L.O.SUMP TK(C)	27.0- 40.0	90.0	.9000	41.0	29.894	1226	1.134	46	5
MAIN L.O.SETT.TK(S)	36.0- 42.0	75.0	.9000	28.1	31.214	877	12.931	363	3
MAIN L.O.STOR.TK(S)	42.0- 52.0	75.0	.9000	47.8	37.609	1798	12.887	616	5
NO1 CYL.O.STOR.TK(S)	25.0- 29.0	75.0	.9000	87.9	21.623	1901	12.177	1070	118
NO2 CYL.O.STOR.TK(S)	21.0- 25.0	75.0	.9000	81.7	18.429	1506	12.394	1013	118
G.E L.O.SETT.TK(S)	17.0- 19.0	75.0	.9000	36.6	14.409	527	12.092	465	57
G.E L.O.STOR.TK(S)	19.0- 21.0	75.0	.9000	38.3	16.008	613	12.570	481	50
TOTAL LUBRICATING OIL				361.4		8448		4054	364
SEWAGE HOLDING TK(P)	32.0- 34.0		1.0000	2.5	26.405	66	12.546	31	1
BILGE HOLDING TK(C)	14.0- 25.0		1.0000	31.3	16.661	521	.938	29	75
S.T L.O. DRAIN TK(C)	24.0- 25.0		.9000	1.3	19.600	25	1.313	2	1
RESIDUE TK(S)	29.0- 44.0		1.0000	7.5	31.111	233	1.160	9	10
DIRTY OIL TK(S)	29.0- 36.0		1.0000	20.3	26.093	530	12.389	251	4
L.O.SLUDGE TK(P)	37.0- 39.0		.9000	2.0	30.436	61	10.195	20	2
HFO SLUDGE TK(P)	34.0- 43.0		.9900	27.1	31.417	851	9.490	257	60
C.F.W.DRAIN TK(S)	44.0- 47.0		1.0000	4.7	36.439	171	1.231	6	6
HFO.LO.LEAK O.TK(S)	29.0- 36.0		1.0000	3.7	26.536	98	1.493	6	1
C.W.TK(C)	7.3- 14.0		1.0000	35.5	9.480	337	3.554	126	6
F.O. OVERFLOW TK(C)	36.0- 50.0		.9900	22.7	36.871	837	1.052	24	325
STUFF BOX.L.O.TK(P)	25.0- 26.0		.9000	2.0	20.403	41	.889	2	2
STUFF BOX.L.O.TK(S)	25.0- 26.0		.9000	2.0	20.403	41	.889	2	2
TOTAL MISCELL.				162.6		3812		765	495
STORE & PROV.	34.0-251.4		1.0000	67.0	95.400	6392	17.500	1173	0
TOTAL STORE & PROVISION				67.0		6392		1173	0
CREW EFFECT	34.0- 51.0		1.0000	7.0	34.400	241	29.900	209	0
DWT CONSTANT	-3.7-282.6		1.0000	205.0	122.620	25137	20.850	4274	0
TOTAL DEADWEIGHT CONSTANT				212.0		25378		4483	0
TOTAL DEADWEIGHT				50815.5		6035087		696565	3921
LIGHT SHIP				15998.1	103.228	1651452	13.200	211175	0
TOTAL WEIGHT				66813.6	115.045	7686539	13.586	907740	3921

Distributed Loads in Still Water
- Buoyancy curve
(Example of 3,700TEU Container carrier)

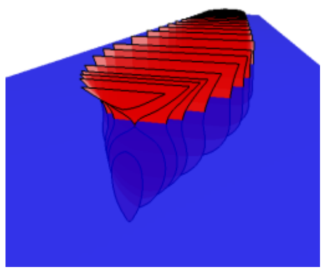
Load curve, $f_S(x)$ → Actual Still water Shear Force, $V_S(x)$ → Actual Still water Bending Moment, $M_S(x)$

Weight, $w(x)$

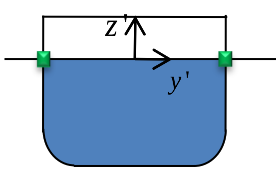
Buoyancy, $b(x)$

$$V_S(x) = \int^x f_S(x)dx \qquad M_S(x) = \int_0^x V_S(x)dx$$

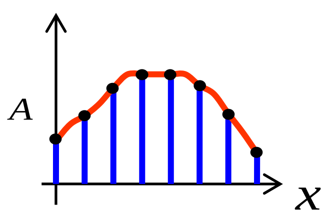
✓ Calculation of buoyancy



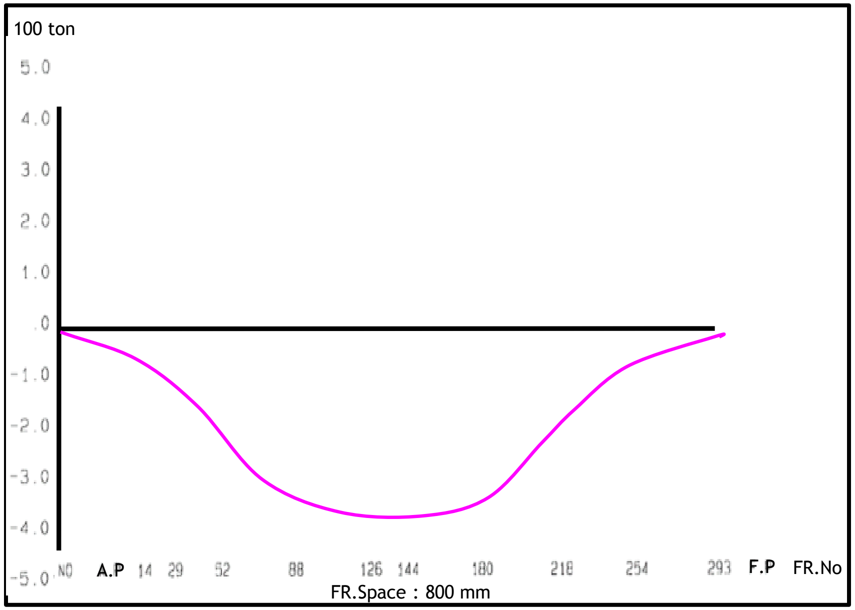
(1) Calculation of sectional area below waterline



(2) Integration of sectional area over the ship's length



✓ Buoyancy Curve in Homogeneous 10ton Scantling Condition



DRAUGHT F.P	= 12.260 M	K.M.T	= 14.889 M
DRAUGHT MIDSHIP	= 12.457 M	KG (SOLID)	= 13.586 M
DRAUGHT A.P	= 12.654 M	GM (SOLID)	= 1.303 M
TRIM BY STERN	= .394 M	FREE SURF. CORR.(GGo)	= .059 M
PROPELLER I/D	= 160.3 %	GoM (FLUID)	= 1.244 M
DISPLACEMENT	= 66813.6 T	KGo ACTUAL (FLUID)	= 13.645 M
DRAUGHT AT LCF	= 12.483 M	TRIM (DIS*A)/(MTC*100)	= .394 M
LCB FROM A.P	= 115.677 M	FREE SURF. MOM.	= 3921 T-M
LCG FROM A.P	= 115.045 M	M.T.C.	= 1072.0 T-M
TRIM LEVER : A	= .632 M	LCF FROM A.P	= 106.275 M

Distributed Loads in Still Water

- Example of 3,700TEU Container carrier

Load curve, $f_S(x)$ → Actual Still water Shear Force, $V_S(x)$ → Actual Still water Bending Moment, $M_S(x)$

Weight, $w(x)$
Buoyancy, $b(x)$

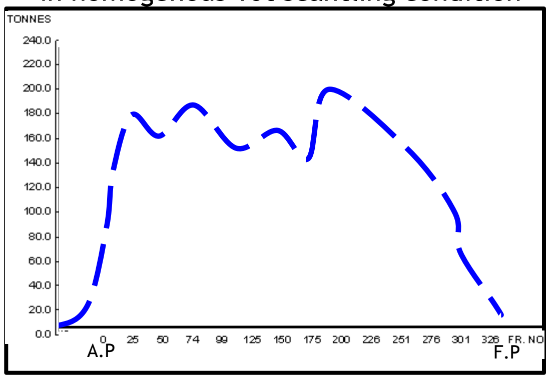
$$V_S(x) = \int_0^x f_S(x)dx \qquad M_S(x) = \int_0^x V_S(x)dx$$

Load curve

$f_S(x)$: Loads curve = Weight + Buoyancy

Weight curve =Lightweight + Deadweight

in homogenous 10t scantling condition



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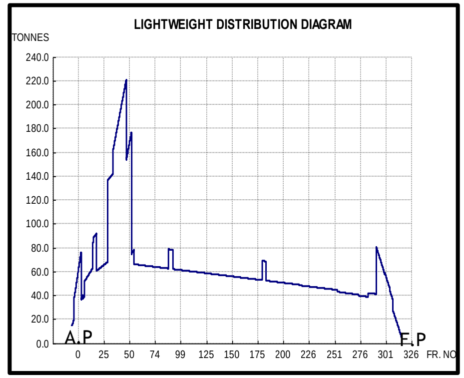
Lightweight Distribution Curve



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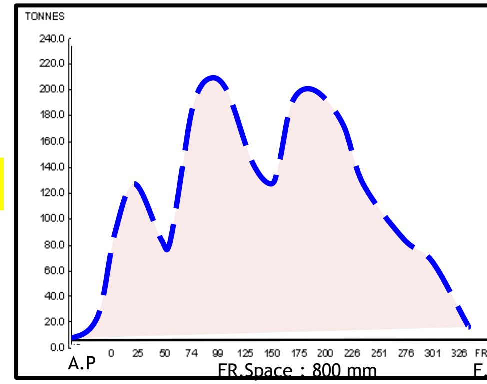
Deadweight Distribution Curve

in homogenous 10t scantling condition



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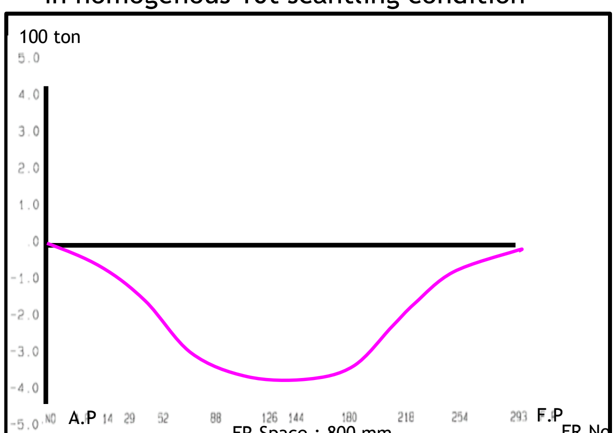
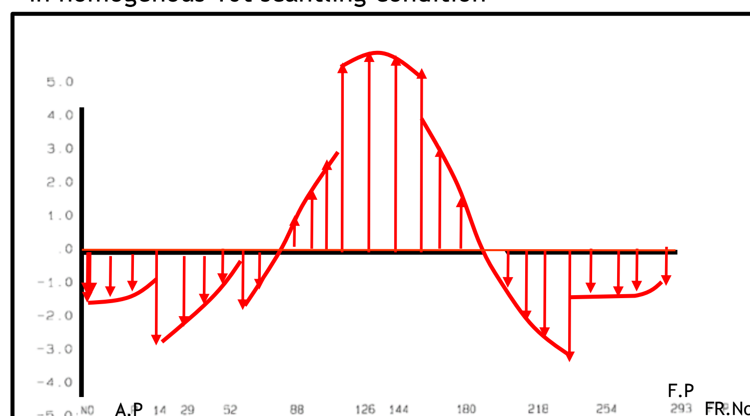
Bouyancy Curve

in homogenous 10t scantling condition



Load curve = Weight $w(x)$ + Buoyancy $b(x)$

in homogenous 10t scantling condition



Actual Still Water shear Force

Actual still water shear force & Actual still water bending moment

Load curve, $f_S(x)$ → Actual Still water Shear Force, $V_S(x)$ → Actual Still water Bending Moment, $M_S(x)$

Weight, $w(x)$
Buoyancy, $b(x)$

$V_S(x) = \int_0^x f_S(x)dx$

$M_S(x) = \int_0^x V_S(x)dx$

(Example of 3,700TEU Container carrier)

Load curve

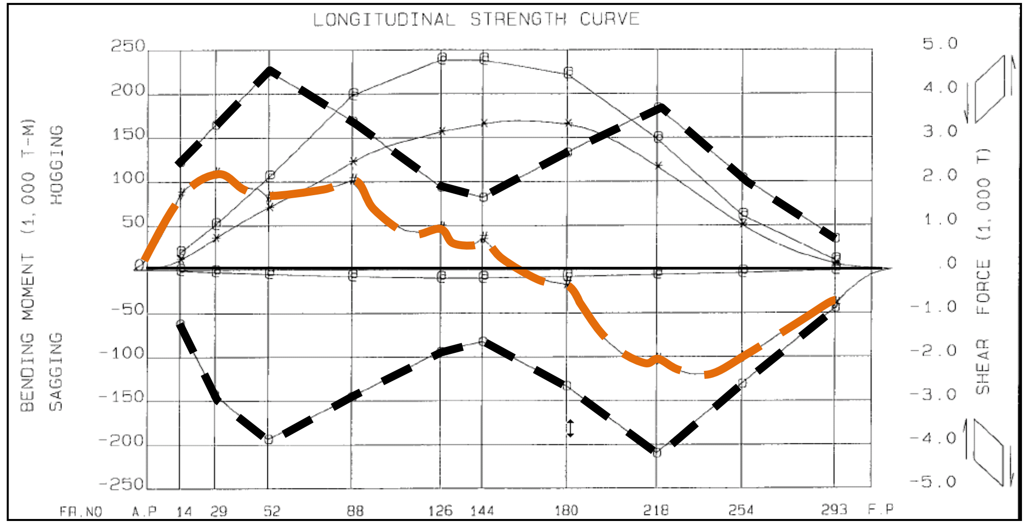
$$f_S(x) = b(x) + w(x)$$

Actual still water Shear force

$$V_S(x) = \int_0^x f_S(x)dx$$

Actual Still water shear force

Design Still water shear force

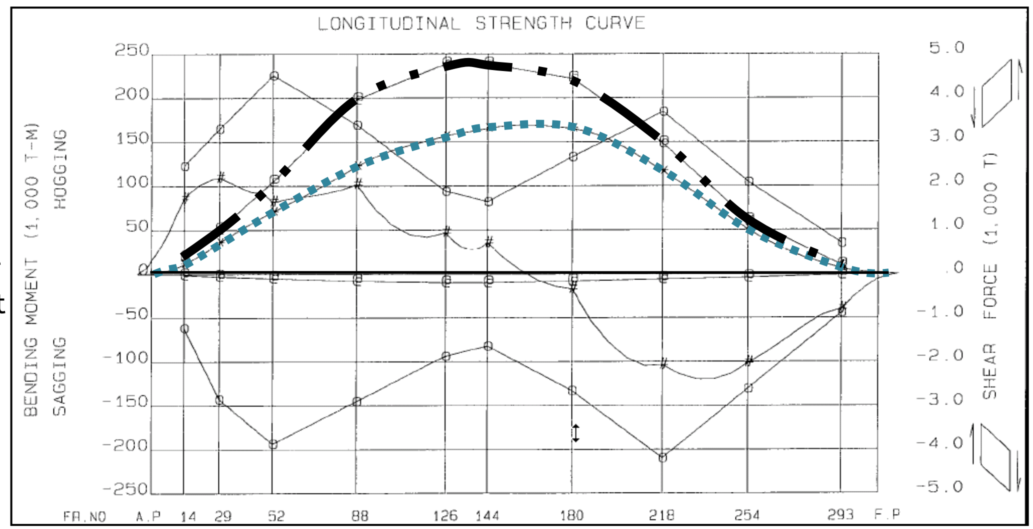


Actual still water Bending moment

$$M_S(x) = \int_0^x V_S(x)dx$$

Actual still water bending Moment

Design still water bending moment



Rule Still Water Bending Moment by the Classification Rule

Recently, actual still water bending moment based on the load conditions is used for still water bending moment, because the rule still water bending moment is only for the tanker.

- The design still water bending moments amidships are not to be taken less than

(DnV Pt 3 Ch1 Sec. 5 A105)

$$\boxed{M_S = M_{SO}(kNm)}$$

$$M_{SO} = \underline{-0.065} C_{WU} L^2 B (C_B + 0.7) \quad \text{(kNm) in sagging}$$

$$= C_{WU} L^2 B (0.1225 - 0.015 C_B) \quad \text{(kNm) in hogging}$$

C_{WU}:wave coefficient, for unrestricted service.

The still water bending moment shall not be less than the large of : the largest actual still water bending moment based on the load conditions and the rule still water bending moment .

(DnV Pt 3 Ch1 Sec. 5 A106) 2011

106 The design stillwater bending moments amidships (sagging and hogging) are normally not to be taken less than:

$$M_S = M_{SO} \quad (\text{kNm})$$

M_{SO} = $-0.065\, C_{WU}\, L^2\, B\, (C_B + 0.7)$ (kNm) in sagging

= $C_{WU}\, L^2\, B\, (0.1225 - 0.015\, C_B)$ (kNm) in hogging

C_{WU} = C_W for unrestricted service.

Larger values of M_{SO} based on cargo and ballast conditions shall be applied when relevant, see 102.

For ships with arrangement giving small possibilities for variation of the distribution of cargo and ballast, M_{SO} may be dispensed with as design basis.

107 When required in connection with stress analysis or buckling control, the stillwater bending moments at arbitrary positions along the length of the ship are normally not to be taken less than:

$$M_S = k_{sm} M_{SO} \quad (\text{kNm})$$

M_{SO} = as given in 106
k_{sm} = 1.0 within 0.4 L amidships
= 0.15 at 0.1 L from A.P. or F.P.

= 0.0 at A.P. and F.P.

Between specified positions k_{sm} shall be varied linearly.

Values of k_{sm} may also be obtained from Fig.3.

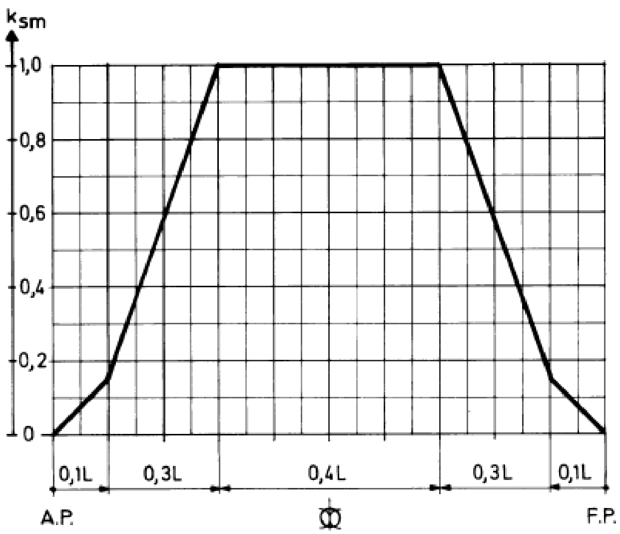


Fig. 3
Stillwater bending moment

The extent of the constant design bending moments amidships may be adjusted after special consideration.

Rule Still Water Shear Force by the Classification Rule

• The design values of still water shear forces along the length of the ship are normally not to be taken less than :

(Dnv Pt 3 Ch1 Sec. 5 B107)

$$\boxed{Q_S = k_{sq} Q_{SO} (kN)}$$

$$Q_{SO} = 5\frac{M_{SO}}{L}(kN)$$

k_{sq} = 0 at A.P. and F.P.
= 1.0 between 0.15 L and 0.3 L from A.P.
= 0.8 between 0.4 L and 0.6 L from A.P.
= 1.0 between 0.7 L and 0.85 L from A.P.

$M_{SO} = -0.065 C_{WU} L^2 B (C_B + 0.7)$ (kNm) in sagging
$= C_{WU} L^2 B (0.1225 - 0.015 C_B)$ (kNm) in hogging

C_{WU}:wave coefficient, for unrestricted service.

The still water shear force shall not be less than the large of : the largest actual still water shear forces based on load conditions and the rule still water shear force .

(DnV Pt 3 Ch1 Sec. 5 B108) 2011

108 The design values of stillwater shear forces along the length of the ship are normally not to be taken less than:

$$Q_S = k_{sq}\, Q_{SO} \quad \text{(kN)}$$

$$Q_{SO} = 5\frac{M_{SO}}{L} \quad \text{(kN)}$$

M_{SO} = design stillwater bending moments (sagging or hogging) given in 106.

Larger values of Q_S based on load conditions ($Q_S = Q_{SL}$) shall be applied when relevant, see 102. For ships with arrangement giving small possibilities for variation in the distribution of cargo and ballast, Q_{SO} may be dispensed with as design basis

k_{sq} = 0 at A.P. and F.P.
= 1.0 between 0.15 L and 0.3 L from A.P.
= 0.8 between 0.4 L and 0.6 L from A.P.
= 1.0 between 0.7 L and 0.85 L from A.P.

Between specified positions k_{sq} shall be varied linearly.

Sign convention to be applied:

— when sagging condition positive in forebody, negative in afterbody
— when hogging condition negative in forebody, positive in afterbody.

15-3. Wave Shear Force, Qw & Wave Bending Moment, Mw

Mw: Vertical Wave Bending Moment

$\boxed{\sigma \le \sigma_l}$ $\quad , \sigma = \dfrac{M}{I_{N.A}/y} = \dfrac{|M_S + M_W|}{I_{N.A}/y}$, M_S = Still water bending moment; M_W = Vertical wave bending moment

Hydrodynamic loads induced by waves along ship's length

$f_W(x)$: distributed loads induced by waves

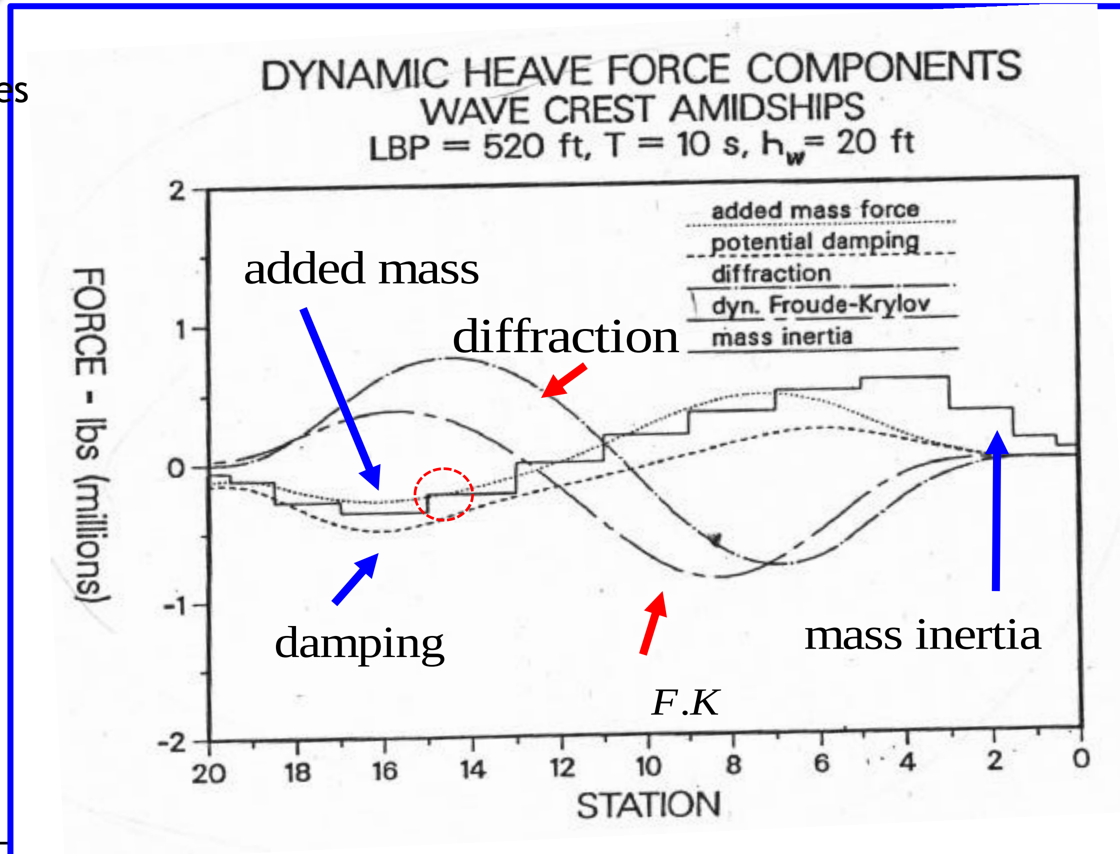
= Froude-Krylov force + diffraction force
+ added mass force + damping force
+ mass inertia force

$$V_W(x) = \int_0^x f_W(x)dx$$

$V_W(x)$: vertical wave shear force

$$M_W(x) = \int_0^x V_W(x)dx$$

$M_W(x)$: vertical wave bending moment



Comparison of Ship Heave Motion and Mass-Spring-Damping system

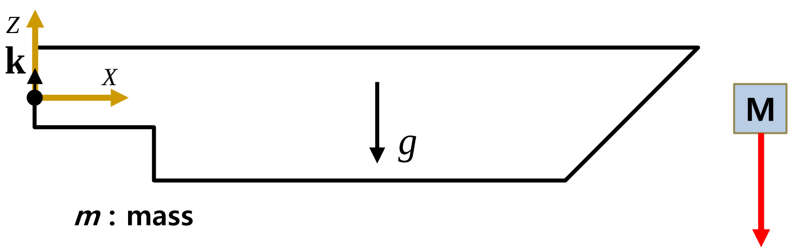
Equation of Heave Motion of a Ship

Nonlinearity of the nature → Nonlinear Mathematical Model → Numerical Method

↓ **Linearization**

Linear Mathematical Model → Analytic Solution

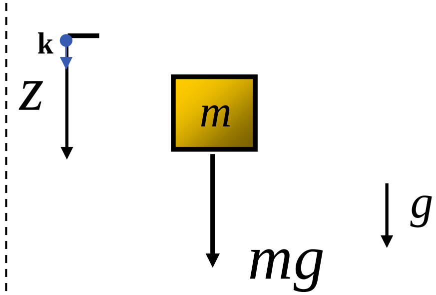
Ex) Heave Motion of a Ship – step 1



$$
\begin{aligned}
m\ddot{\mathbf{z}} &= \mathbf{F} \\
&= \mathbf{F}_{gravity} \\
&= -mg\mathbf{k}
\end{aligned}
$$

✓ Mass-Spring-Damper system

①



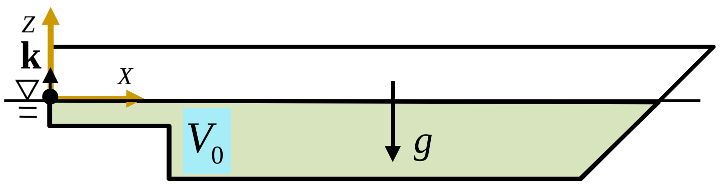
By Newton's 2nd law,

$$
\begin{aligned}
m\mathbf{z}'' &= \mathbf{F} \\
&= mg\mathbf{k}
\end{aligned}
$$

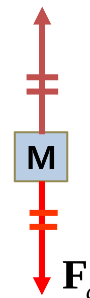
Ex) Heave Motion of a Ship – step 2

$$\mathbf{F}_{static} = \iint_{S_B} P_{static}\mathbf{n}dS = \rho g V_0 \mathbf{k}$$

✓ Archimedes' Principle $\mathbf{F}_{static} = \rho g V_0 \mathbf{k}$



m : mass
ρ : density of sea water
V_0: submerged volume
S_B : submerged surface area



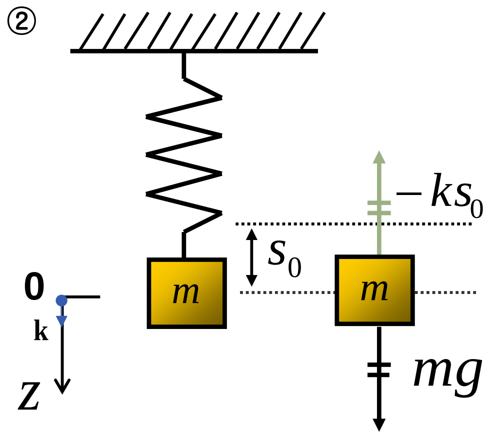
$\mathbf{F}_{gravity}$

$= -mg\mathbf{k}$

$$m\ddot{\mathbf{z}} = \mathbf{F}$$
$$= \mathbf{F}_{gravity} + \mathbf{F}_{static}$$
$$= -mg\mathbf{k} + \rho g V_0 \mathbf{k}$$
$$= 0 \ (\because \ddot{z} = 0)$$: static equilibrium

✓ Mass-Spring-Damper system

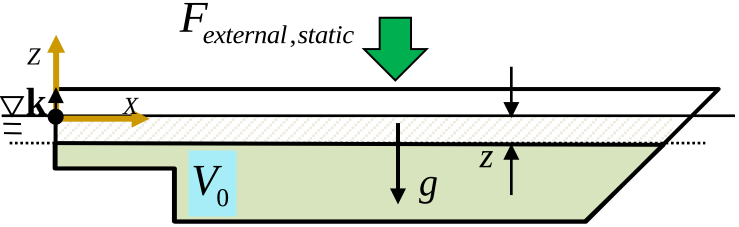
②



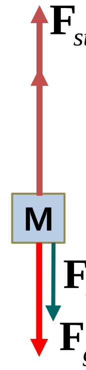
$$m\mathbf{z}'' = \mathbf{F}$$
$$= mg\mathbf{k} - ks_0\mathbf{k}$$
$$= 0 \ (\because z'' = 0)$$

: static equilibrium

Ex) Heave Motion of a Ship – step 3



m : mass ρ : density of sea water
V_0: submerged volume
S_B : submerged surface area
Awp : waterplane area



$$\mathbf{F}_{static} = \iint_{S_B} P_{static}\mathbf{n}dS = \rho g V_0 \mathbf{k} + \mathbf{F}_{addtional\,bouyancy}$$

additional buoyancy caused by additional displacement z

$$\mathbf{F}_{gravity} = -mg\mathbf{k}$$

if, z is small

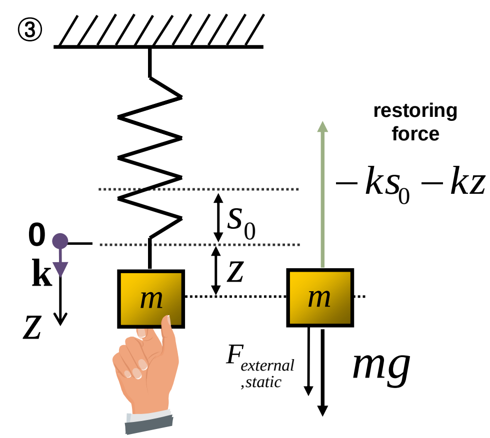
$$\mathbf{F}_{addtional\,bouyancy} = -\rho g A_{WP}\mathbf{z} = -k\mathbf{z}, \quad k = \rho g A_{WP}$$

$$m\ddot{\mathbf{z}} = \mathbf{F}$$
$$= \mathbf{F}_{gravity} + \mathbf{F}_{static} + \mathbf{F}_{external,static}$$
$$= -mg\mathbf{k} + \rho g V_0 \mathbf{k} - \rho g A_{wp}\mathbf{z} + \mathbf{F}_{external,static}$$
$$= -\rho g A_{wp}\mathbf{z} + \mathbf{F}_{external,static}$$
$$= -k\mathbf{z} + \mathbf{F}_{external,static}$$
$$= 0 \ (\because \ddot{z} = 0)$$

✓ Archimedes' Principle $\mathbf{F}_{static} = \rho g V_0 \mathbf{k}$

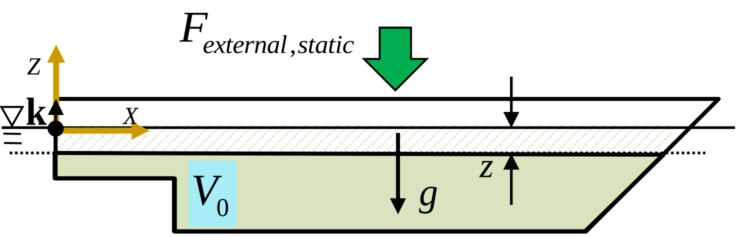
✓ **Mass-Spring-Damper system**

③

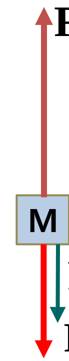


$$m\mathbf{z}'' = \mathbf{F}$$
$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} + \mathbf{F}_{external,static}$$
$$= -kz\mathbf{k} + \mathbf{F}_{external,static}$$
$$= 0 \quad (\because z'' = 0)$$

Ex) Heave Motion of a Ship – step 4



m : mass
ρ : density of sea water
V_0: submerged volume
S_B : submerged surface area
Awp : waterplane area



$$\mathbf{F}_{static} = \iint_{S_B} P_{static}\mathbf{n}dS$$
$$= \rho g V_0 \mathbf{k} + \mathbf{F}_{addtional\,bouyancy}$$

additional buoyancy caused by additional displacement z

$$\mathbf{F}_{external,static}$$
$$\mathbf{F}_{gravity} = -mg\mathbf{k}$$

if, z is small

$$\mathbf{F}_{addtional\,bouyancy} = -\rho g A_{WP}\mathbf{z} = -k\mathbf{z} \quad , k = \rho g A_{WP}$$

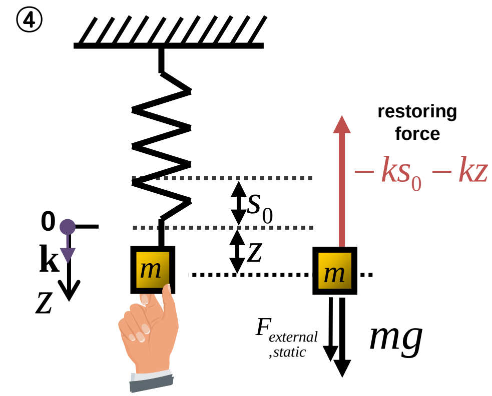
$$m\ddot{\mathbf{z}} = \mathbf{F}$$
$$= \mathbf{F}_{gravity} + \mathbf{F}_{static} + \mathbf{F}_{external,static}$$
$$= -mg\mathbf{k} + \rho g V_0 \mathbf{k} - \rho g A_{wp}\mathbf{z} + \mathbf{F}_{external,static}$$
$$= -\rho g A_{wp}\mathbf{z} + \mathbf{F}_{external,static}$$
$$= -k\mathbf{z} + \mathbf{F}_{external,static} \quad , k = \rho g A_{WP}$$
$$= 0 \; (\because \ddot{z} = 0)$$

Linearized Restoring Force

✓ Archimedes' Principle $\mathbf{F}_{static} = \rho g V_0 \mathbf{k}$

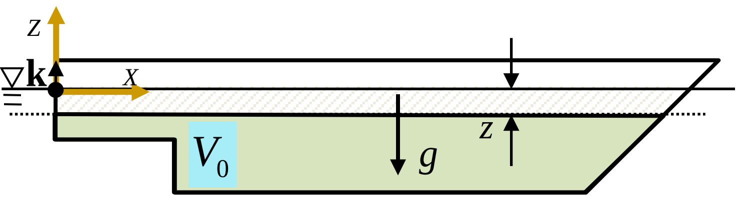
✓ Mass-Spring-Damper system

④

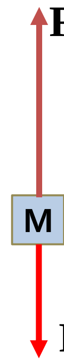


$$m\mathbf{z}'' = \mathbf{F}$$
$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} + \mathbf{F}_{external,static}$$
$$= -kz\mathbf{k} + \mathbf{F}_{external,static}$$

$$m\mathbf{z}'' + k\mathbf{z} = 0$$ Oscillation by the restoring force



m : mass ρ : density of sea water
V_0: submerged volume
S_B : submerged surface area
Awp : waterplane area



$$\mathbf{F}_{static} = \iint_{S_B} P_{static}\mathbf{n}dS = \rho g V_0 \mathbf{k} - \rho g A_{WP}\mathbf{z} = \rho g V_0 \mathbf{k} - k\mathbf{z}$$

$$\mathbf{F}_{gravity} = -mg\mathbf{k}$$

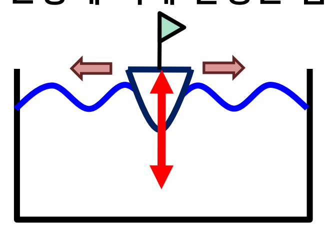
✓ Archimedes' Principle $\mathbf{F}_{static} = \rho g V_0 \mathbf{k}$

$$m\ddot{\mathbf{z}} = \mathbf{F}$$
$$= \mathbf{F}_{gravity} + \mathbf{F}_{static}$$
$$= -mg\mathbf{k} + \rho g V_0 \mathbf{k} - \rho g A_{wp}\mathbf{z}$$
$$= -\rho g A_{wp}\mathbf{z}$$
$$= -k\mathbf{z}$$

Ship will oscillate forever?

Energy is dissipated by radiation wave

정수 중 선박의 강제 운동에 의해 발생한 힘

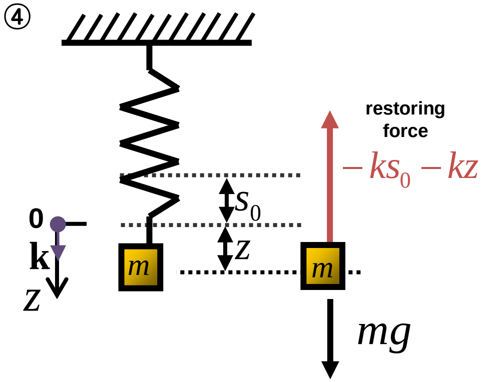


Radiation Force

$$\mathbf{F}_{radiation} = \iint_{S_B} P_{radiation}\mathbf{n}dS$$

✓ Mass-Spring-Damper system

④

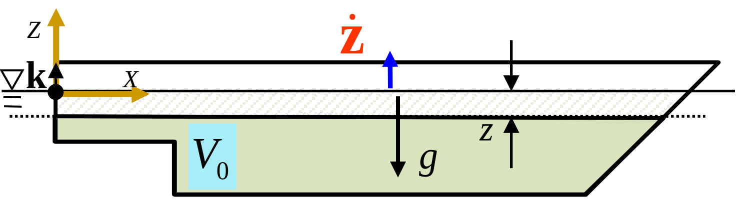


$$m\mathbf{z}'' = \mathbf{F}$$
$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k}$$
$$= -kz\mathbf{k}$$

$$m\mathbf{z}'' + k\mathbf{z} = 0$$

Oscillation by the restoring force

Ex) Heave Motion of a Ship – step 5



m : mass ***ρ*** : density of sea water
V_0: submerged volume
S_B : submerged surface area
Awp : waterplane area

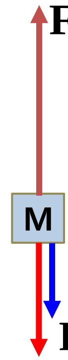
$$m\ddot{\mathbf{z}} = \mathbf{F}$$

$$= \mathbf{F}_{gravity} + \mathbf{F}_{static} + \mathbf{F}_{radiation}$$

$$= -mg\mathbf{k} + \rho g V_0 \mathbf{k} - \rho g A_{wp}\mathbf{z} - c\dot{\mathbf{z}}$$

$$= -\rho g A_{wp}\mathbf{z} - c\dot{\mathbf{z}}$$

$$= -k\mathbf{z} - c\dot{\mathbf{z}}$$



$$\mathbf{F}_{static} = \iint_{S_B} P_{static}\mathbf{n}dS$$

$$= \rho g V_0 \mathbf{k} - \rho g A_{WP}\mathbf{z}$$

$$= \rho g V_0 \mathbf{k} - k\mathbf{z}$$

$$\mathbf{F}_{radiation} = -c\dot{\mathbf{z}}$$

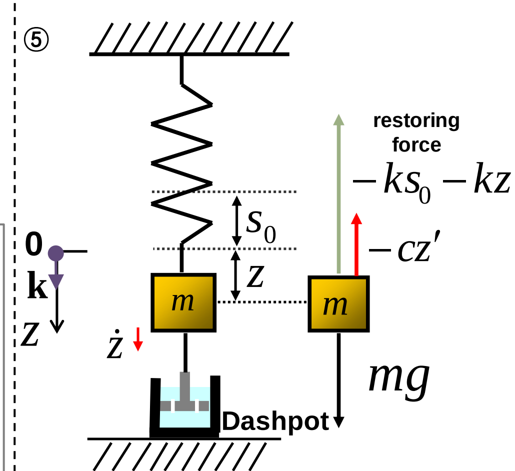
$$\mathbf{F}_{gravity}$$

$$= -mg\mathbf{k}$$

✓ Archimedes' Principle $\mathbf{F}_{static} = \rho g V_0 \mathbf{k}$

✓ Mass-Spring-Damper system

⑤

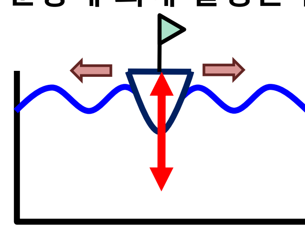


$$m\mathbf{z}'' = \mathbf{F}$$

$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} - cz'\mathbf{k}$$

$$= -kz\mathbf{k} - cz'\mathbf{k}$$

정수 중 선박의 강제
운동에 의해 발생한 힘



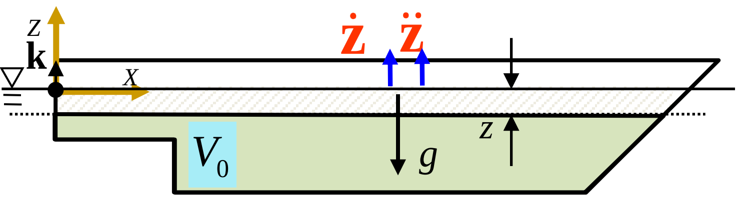
Radiation Force

$$\mathbf{F}_{radiation} = \iint_{S_B} P_{radiation}\mathbf{n}dS$$

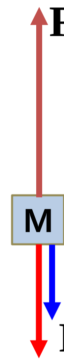
$$= -c\dot{\mathbf{z}}$$

opposite to velocity

c : damping coefficient



m : mass ρ : density of sea water
V_0: submerged volume
S_B : submerged surface area
Awp : waterplane area



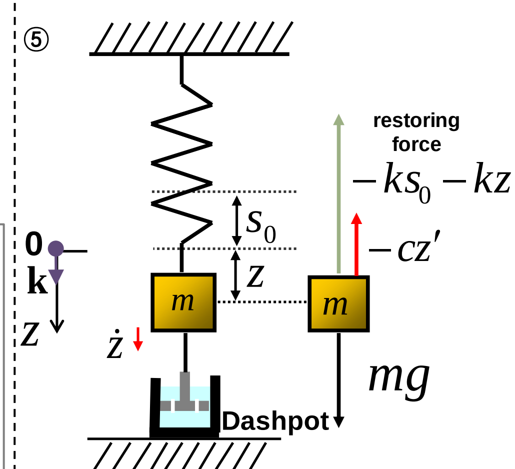
$$\mathbf{F}_{static} = \iint_{S_B} P_{static}\mathbf{n}dS$$
$$= \rho g V_0 \mathbf{k} - \rho g A_{WP}\mathbf{z}$$
$$= \rho g V_0 \mathbf{k} - k\mathbf{z}$$

$$\mathbf{F}_{radiation} = -c\dot{\mathbf{z}} - m_a\ddot{\mathbf{z}}$$

$$\mathbf{F}_{gravity} = -mg\mathbf{k}$$

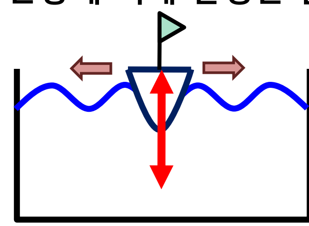
✓ Archimedes' Principle $\mathbf{F}_{static} = \rho g V_0 \mathbf{k}$

✓ Mass-Spring-Damper system



$$m\ddot{\mathbf{z}} = \mathbf{F}$$
$$= \mathbf{F}_{gravity} + \mathbf{F}_{static} + \mathbf{F}_{radiation}$$
$$= -mg\mathbf{k} + \rho g V_0 \mathbf{k} - \rho g A_{wp}\mathbf{z} - c\dot{\mathbf{z}} - m_a\ddot{\mathbf{z}}$$
$$= -\rho g A_{wp}\mathbf{z} - c\dot{\mathbf{z}} - m_a\ddot{\mathbf{z}}$$
$$= -k\mathbf{z} - c\dot{\mathbf{z}} - m_a\ddot{\mathbf{z}}$$

정수 중 선박의 강제 운동에 의해 발생한 힘



Radiation Force

$$\mathbf{F}_{radiation} = \iint_{S_B} P_{radiation}\mathbf{n}dS$$
$$= -c\dot{\mathbf{z}} - m_a\ddot{\mathbf{z}}$$

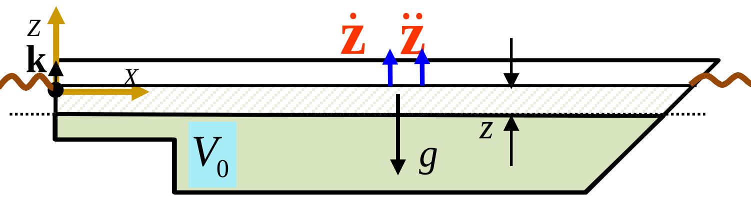
opposite to velocity

opposite to acceleration

c : damping coefficient
m_a : added mass

$$m\mathbf{z}'' = \mathbf{F}$$
$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} - cz'\mathbf{k}$$
$$= -kz\mathbf{k} - cz'\mathbf{k}$$

Ex) Heave Motion of a Ship – step 6



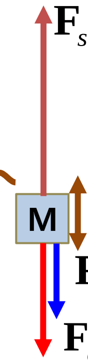
m : mass

ρ : density of sea water

V_0: submerged volume

S_B : submerged surface area

Awp : waterplane area



$$\mathbf{F}_{static} = \iint_{S_B} P_{static}\mathbf{n}dS = \rho g V_0 \mathbf{k} - \rho g A_{WP}\mathbf{z} = \rho g V_0 \mathbf{k} - k\mathbf{z}$$

$$\mathbf{F}_{exciting}$$

$$\mathbf{F}_{radiation} = -c\dot{\mathbf{z}} - m_a\ddot{\mathbf{z}}$$

$$\mathbf{F}_{gravity} = -mg\mathbf{k}$$

✓ Archimedes' Principle $\mathbf{F}_{static} = \rho g V_0 \mathbf{k}$

$$m\ddot{\mathbf{z}} = \mathbf{F}$$

$$= \mathbf{F}_{gravity} + \mathbf{F}_{static} + \mathbf{F}_{radiation} + \mathbf{F}_{exciting}$$

$$= -mg\mathbf{k} + \rho g V_0 \mathbf{k} - \rho g A_{wp}\mathbf{z} - c\dot{\mathbf{z}} - m_a\ddot{\mathbf{z}} + \mathbf{F}_{exciting}$$

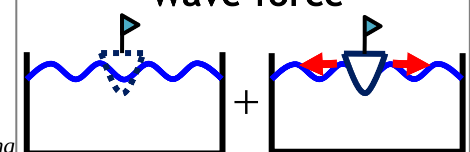
$$= -\rho g A_{wp}\mathbf{z} - c\dot{\mathbf{z}} - m_a\ddot{\mathbf{z}} + \mathbf{F}_{exciting}$$

$$= -k\mathbf{z} - c\dot{\mathbf{z}} - m_a\ddot{\mathbf{z}} + \mathbf{F}_{exciting}$$

c : damping coefficient

m_a : added mass

Wave force

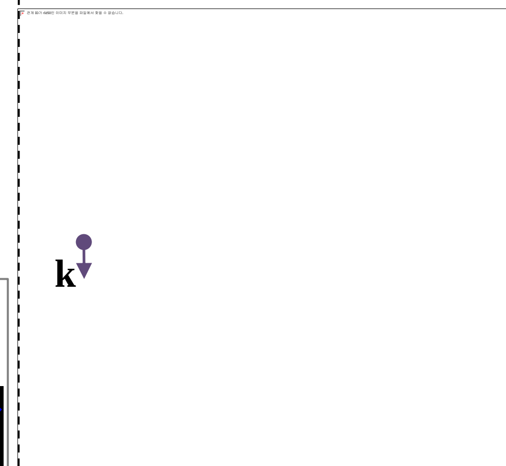


Froude-Kriloff Force

Diffraction Force

$$\mathbf{F}_{wave\ exciting} = \iint_{S_B} P_{wave\ exciting}\mathbf{n}dS \quad (= \mathbf{F}_{exciting})$$

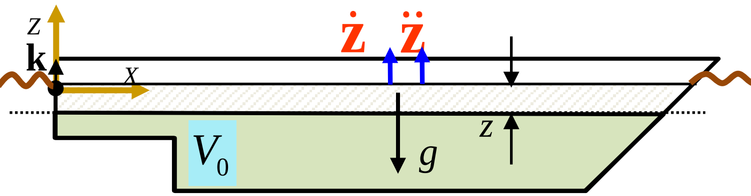
✓ **Mass-Spring-Damper system**



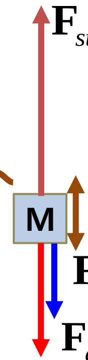
$$m\mathbf{z}'' = \mathbf{F}$$

$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} - cz'\mathbf{k} + \mathbf{F}_0 \cos\omega t$$

$$= -kz\mathbf{k} - cz'\mathbf{k} + \mathbf{F}_0 \cos\omega t$$



m : mass
ρ : density of sea water
V_0: submerged volume
S_B : submerged surface area
Awp : waterplane area



$$\mathbf{F}_{static} = \iint_{S_B} P_{static} \mathbf{n} dS$$
$$= \rho g V_0 \mathbf{k} - \rho g A_{WP} \mathbf{z}$$
$$= \rho g V_0 \mathbf{k} - k\mathbf{z}$$

$\mathbf{F}_{exciting}$

$$\mathbf{F}_{radiation} = -c\dot{\mathbf{z}} - m_a \ddot{\mathbf{z}}$$

$$\mathbf{F}_{gravity} = -mg\mathbf{k}$$

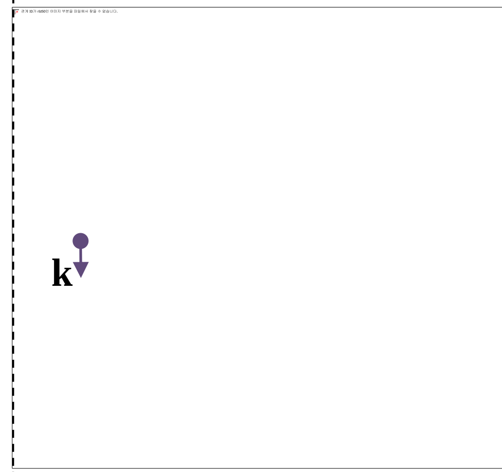
$$m\ddot{\mathbf{z}} = \mathbf{F}$$
$$= \mathbf{F}_{gravity} + \mathbf{F}_{static} + \mathbf{F}_{radiation} + \mathbf{F}_{exciting}$$
$$= -mg\mathbf{k} + \rho g V_0 \mathbf{k} - \rho g A_{wp} \mathbf{z} - c\dot{\mathbf{z}} - m_a \ddot{\mathbf{z}} + \mathbf{F}_{exciting}$$
$$= -\rho g A_{wp} \mathbf{z} - c\dot{\mathbf{z}} - m_a \ddot{\mathbf{z}} + \mathbf{F}_{exciting}$$
$$= -k\mathbf{z} - c\dot{\mathbf{z}} - m_a \ddot{\mathbf{z}} + \mathbf{F}_{exciting}$$

$$\boxed{(m + m_a)\ddot{\mathbf{z}} + c\dot{\mathbf{z}} + k\mathbf{z} = \mathbf{F}_{exciting}}$$

c : damping coefficient
m_a : added mass

✓ Archimedes' Principle $\mathbf{F}_{static} = \rho g V_0 \mathbf{k}$

✓ **Mass-Spring-Damper system**

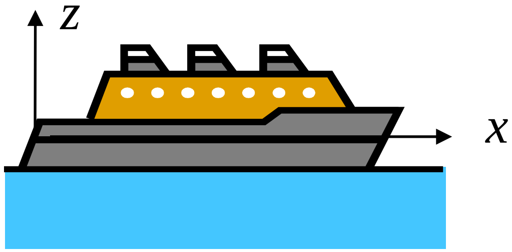


$$m\mathbf{z}'' = \mathbf{F}$$
$$= mg\mathbf{k} - ks_0\mathbf{k} - kz\mathbf{k} - cz'\mathbf{k} + \mathbf{F}_0 \cos \omega t$$
$$= -kz\mathbf{k} - cz'\mathbf{k} + \mathbf{F}_0 \cos \omega t$$

$$\boxed{m\mathbf{z}'' + c\mathbf{z}' + k\mathbf{z} = \mathbf{F}_0 \cos \omega t}$$

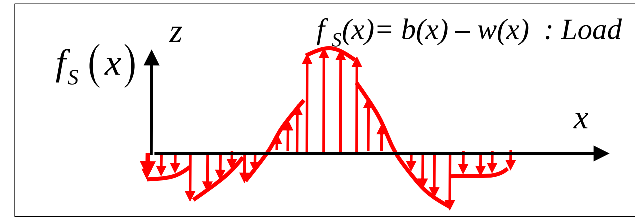
Dynamic Longitudinal Loads

In still water



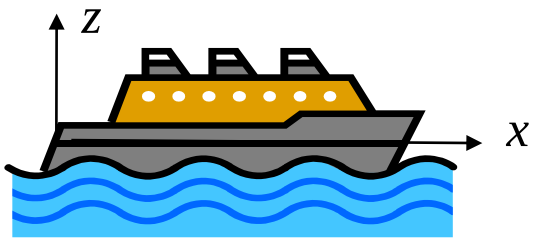
$$f_S(x) = b(x) + w(x)$$

$f(x)$: longitudinal strength loads
$f_S(x)$: static longitudinal loads
$f_W(x)$: dynamic longitudinal loads



+

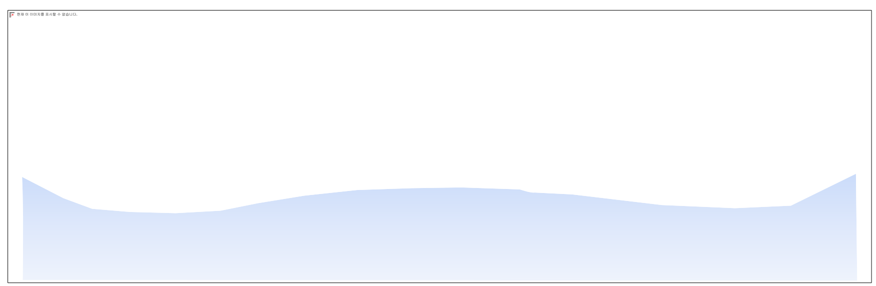
In wave



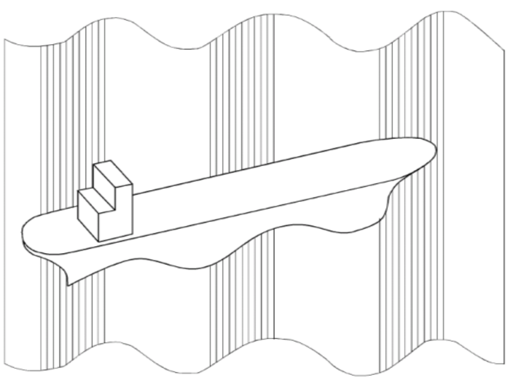
✓ Dynamic longitudinal loads

: Loads are induced by waves

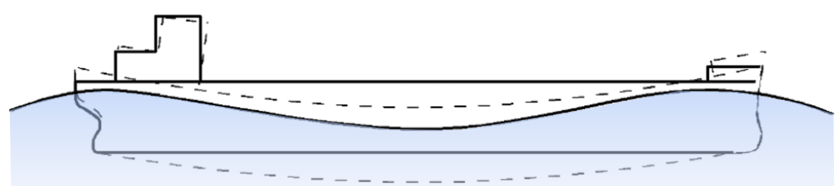
Vertical bending due to waves



- Ship in oblique waves

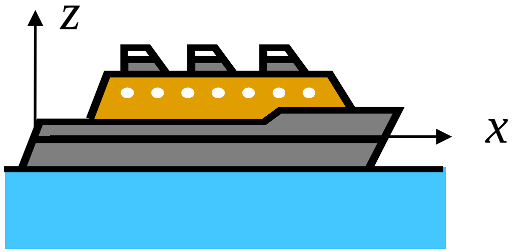


Sagging



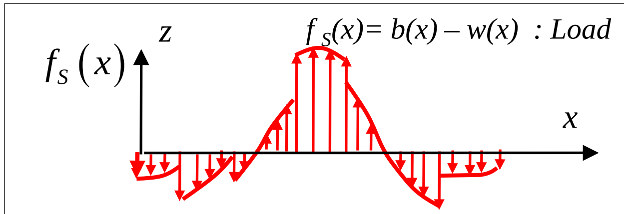
Dynamic Longitudinal Loads
: Direct Calculation of Dynamic Longitudinal Loads

In still water



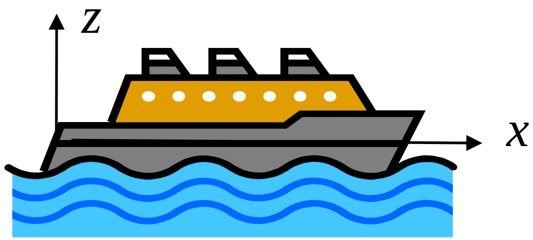
$$f_S(x) = b(x) + w(x)$$

$f(x)$: longitudinal strength loads
$f_S(x)$: static longitudinal loads
$f_W(x)$: dynamic longitudinal loads



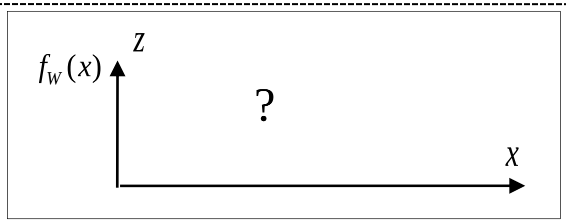
+

In wave



✓ Dynamic longitudinal loads

- Loads are induced by waves

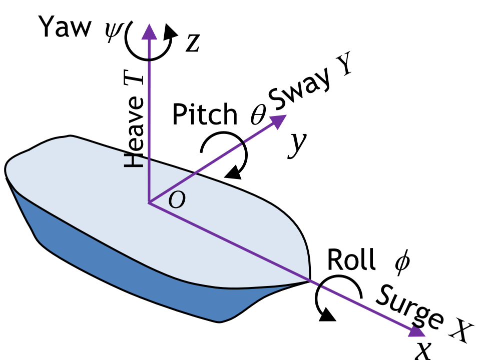


✓ Direct Calculation of dynamic longitudinal loads

- from 6DOF motion of ship

$$\mathbf{x} = [X, Y, T, \phi, \theta, \psi]^T$$

✓ ref.> 6 DOF motion of ship



$$f(x) = f_S(x) \quad \boxed{+ f_W(x)}$$

$$= b(x) + w(x) + f_D(x) + f_{F.K}(x) + f_R(x) + f_{inertia}(x)$$

additional loads in wave

Where,

$$f_R(x) = -a(x)\boxed{\ddot{\mathbf{x}}} - b(x)\boxed{\dot{\mathbf{x}}}$$

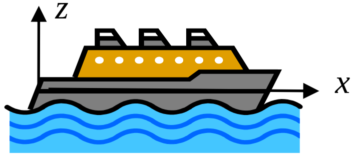
$f_D(x)$: Diffraction force at x
$f_R(x)$: Radiation force at x
$f_{F.K}(x)$: Froude-Krylov force at x

In order to calculate loads in waves,
First we have to determine $\ddot{\xi}_3$, $\dot{\xi}_3$

How to determine $\ddot{\xi}_3$, $\dot{\xi}_3$

Direct Calculation of Dynamic Longitudinal Loads

In wave



✓ Direct Calculation of dynamic longitudinal loads

Load induced by wave

$$f_W(x) = f_D(x) + f_{F.K}(x) + f_R(x) + f_{inertia}(x)$$

Where, $f_R(x) = -a(x)\,\ddot{\mathbf{x}} - b(x)\,\dot{\mathbf{x}}$

Actual vertical wave shear force

$$Q_W(x) = \int_0^x f_W(x)dx$$

Actual vertical wave bending moment

$$M_W(x) = \int_0^x Q_W(x)dx$$

Rule values of Vertical Wave Bending Moments

✓ Direct Calculation of dynamic longitudinal loads

- Loads are induced by waves

Actual vertical wave Shear force

$$Q_W(x) = \int_0^x f_W(x)dx$$

↓

Actual vertical wave bending moment

$$M_W(x) = \int_0^x Q_W(x)dx$$

Recently, rule values of vertical wave moments are used, because of the uncertainty of the direct calculation values of vertical wave bending moments

- The rule vertical wave bending moments amidships are given by :

$$\boxed{M_w = M_{WO} \quad (kNm)}$$

(DnV Pt 3 Ch1 Sec. 5 B201)

$$M_{WO} = \underline{-0.11}\alpha C_W L^2 B(C_B + 0.7) \quad \text{(kNm) in sagging}$$

$$= 0.19\alpha C_W L^2 BC_B \quad \text{(kNm) in hogging}$$

L	C_W
$L \le 100$	$0.0792 \cdot L$
$100 < L < 300$	$10.75 - [(300 - L)/100]^{3/2}$
$300 \le L \le 350$	10.75
$L > 350$	$10.75 - [(L - 350)/150]^{3/2}$

$\alpha = 1.0$ for seagoing condition

$= 0.5$ for harbour and sheltered water conditions (enclosed fjords, lakes, rivers)

C_W : wave coefficient

C_B : block coefficient, not be taken less than 0.6

Direct calculation values of vertical wave bending moments can be used for vertical wave bending moment instead of the rule values of vertical wave moments, if the value of the direct calculation is smaller than that of the rule value.

Rule values of Vertical Wave Shear Forces

✓ Direct Calculation of dynamic longitudinal loads

- Loads are induced by waves

Load induced by wave

$$f_W(x) = f_D(x) + f_{F.K}(x) + f_R(x) + f_{inertia}(x)$$

Where, $f_R(x) = -a(x)\,\ddot{\mathbf{x}} - b(x)\,\dot{\mathbf{x}}$

↓

Actual vertical wave Shear force

$$Q_W(x) = \int_0^x f_W(x)dx$$

- The rule values of vertical wave shear forces along the length of the ship
- are given by :

(DnV Pt 3 Ch1 Sec. 5 B203)

Positive shear force : $$\boxed{Q_{WP} = 0.3\beta k_{wqp} C_W LB(C_B + 0.7)}$$

Negative shear force : $$\boxed{Q_{WN} = -0.3\beta k_{wqn} C_W LB(C_B + 0.7)}$$

β : coefficient according to operating condition

k_{wqp} , k_{wqn} : coefficients according to location in lengthwise

C_W : wave coefficient

Direct calculation values of vertical wave shear forces can be used for vertical wave shear force instead of the rule values of vertical shear forces, if the value of the direct calculation is smaller than that of the rule value.

Example: Rule values of still water bending moments, Ms and vertical wave bending moment, Mw

Calculate L_S, $C_{B,SCANT}$ and vertical wave bending moment at amidships (0.5L) of a ship in hogging condition for sea going condition

$$\text{Dimension}: L_{OA} = 332.0m,\ L_{BP} = 317.2m,\ L_{EXT} = 322.85m,\ B = 43.2m,\ T_s = 14.5m$$

$$\nabla\left(Displacement\,(Ton)\,at\,T_s\right) = 140{,}960\,Ton$$

$$\boxed{M_S = M_{SO}(kNm)}$$

$$M_{SO} = -0.065C_{WU}L^2B(C_B + 0.7),\ \text{(in sagging)}$$

$$= C_{WU}L^2B(0.1225 - 0.015C_B),\ \text{(in hogging)}$$

$$\boxed{M_w = M_{WO}\quad (kNm)}$$

$$M_{WO} = -0.11\alpha C_W L^2 B(C_B + 0.7),\ \text{(in sagging)}$$

$$= 0.19\alpha C_W L^2 B C_B,\ \text{(in hogging)}$$

(Sol.) $L_s = 0.97 \times L_{EXT} = 0.97 \times 322.85$

$$C_{B,SCANT} = \nabla / \left(1.025 \times L_s \times B \times T_s\right) = \frac{140{,}906}{1.025 \times 313.17 \times 43.2 \times 14.5} = 0.701$$

$\alpha = 1.0$, for sea going condition,

$C_W = 10.75$, if $300 \le \mathrm{L} \le 350$ (wave Coefficient)

$k_{wm} = 1.0$ between 0.4L and 0.65 L from A.P=0.0 and F.P

$$M_{WO} = 0.19 \times \alpha \times C_W \times L^2 \times B \times C_{B,SCANT}\ (kNm)$$

$$= 0.19 \times 1.0 \times 10.75 \times 313.17^2 \times 43.2 \times 0.701 = 6{,}066{,}303\ (kNm)$$

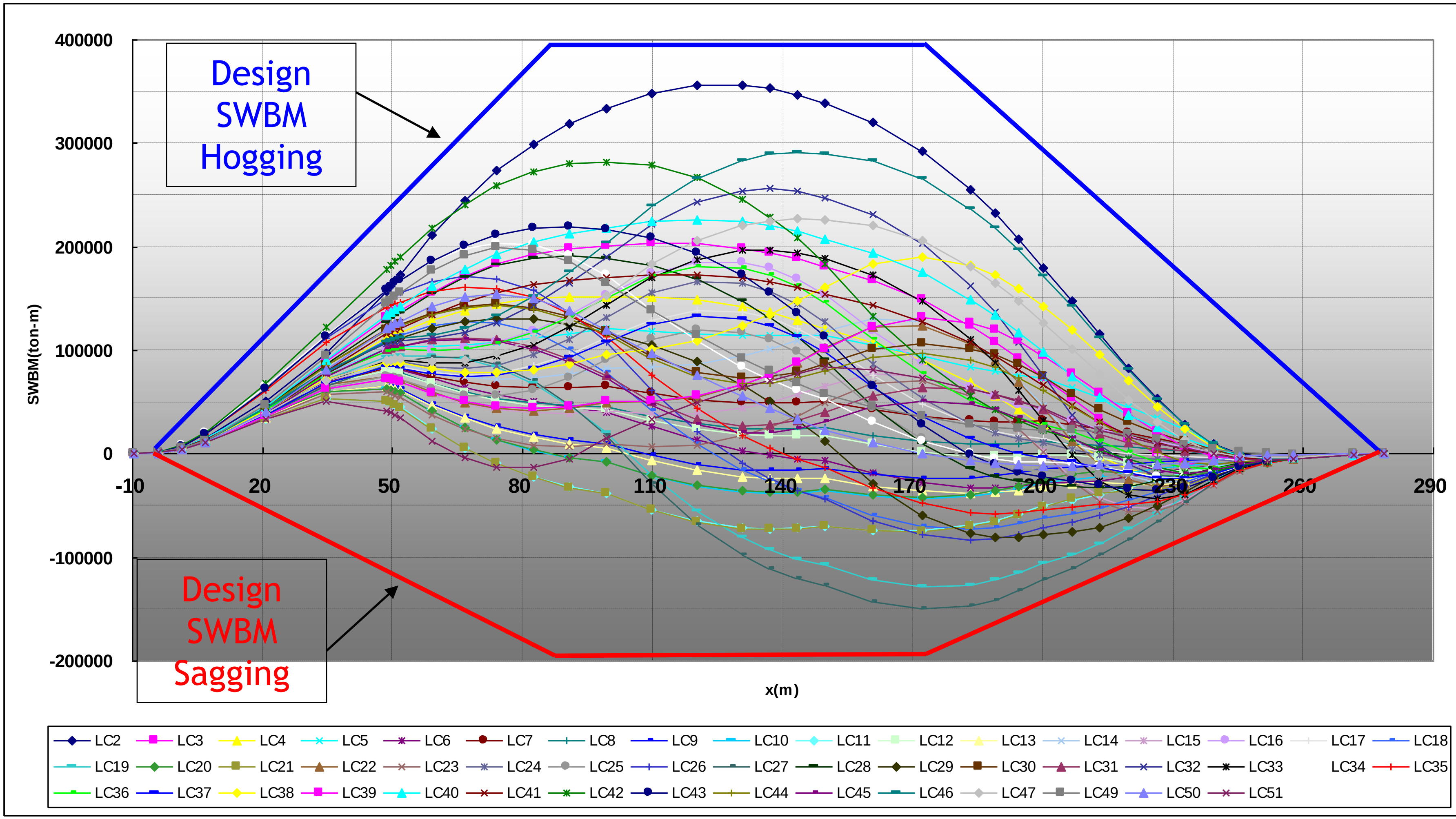
At 0.5L, $k_{wm} = 1.0$

$M_W = 1.0 \times M_{WO}$

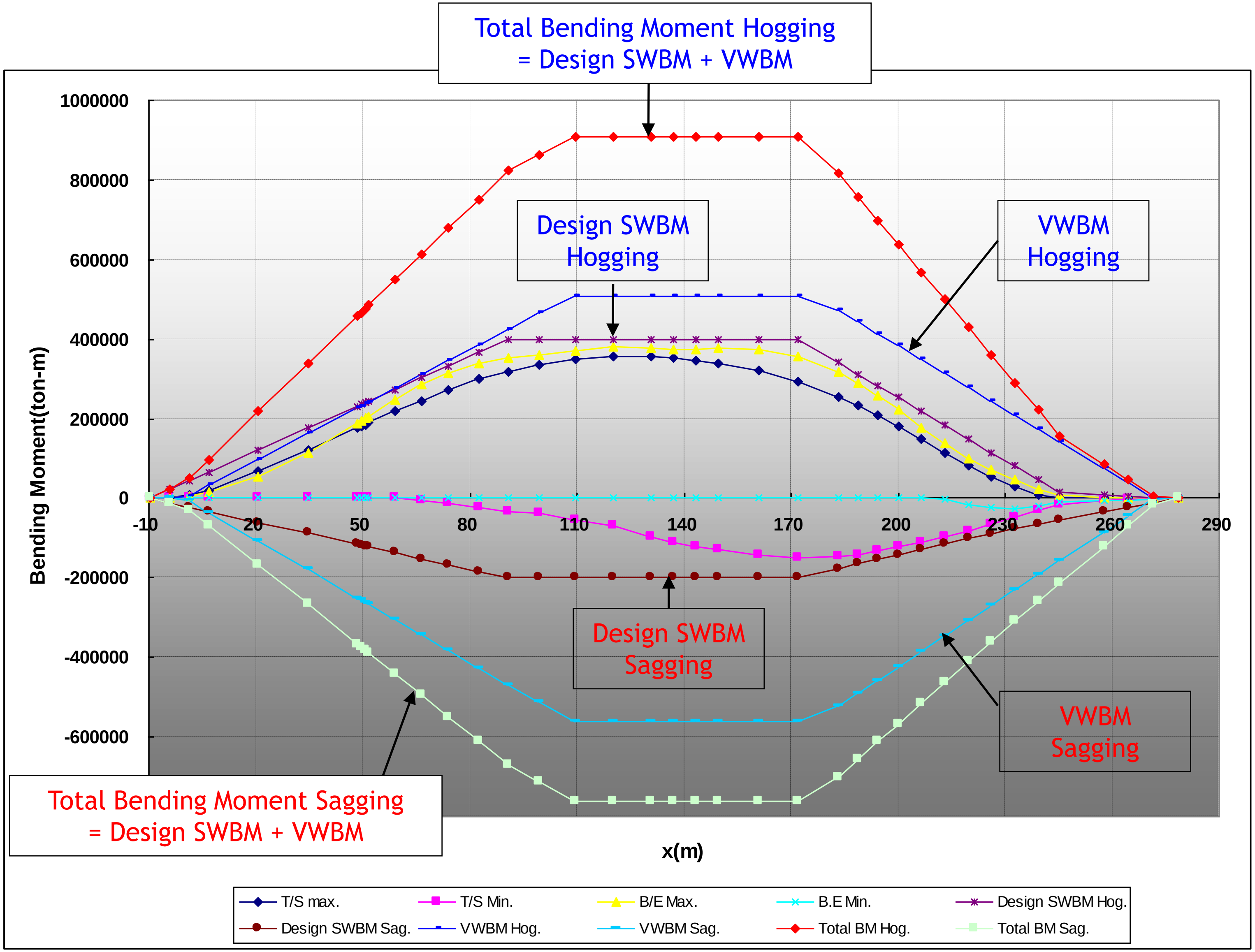
So, $M_W = 1.0 \times M_{WO} = 6{,}066{,}303\ (kNm)$

[1] 대우조선해양, 선박구조설계, 5-2 Load on Hull structure, 예제4, 2005.

Still Water Bending Moment Curve (T&S Booklet)



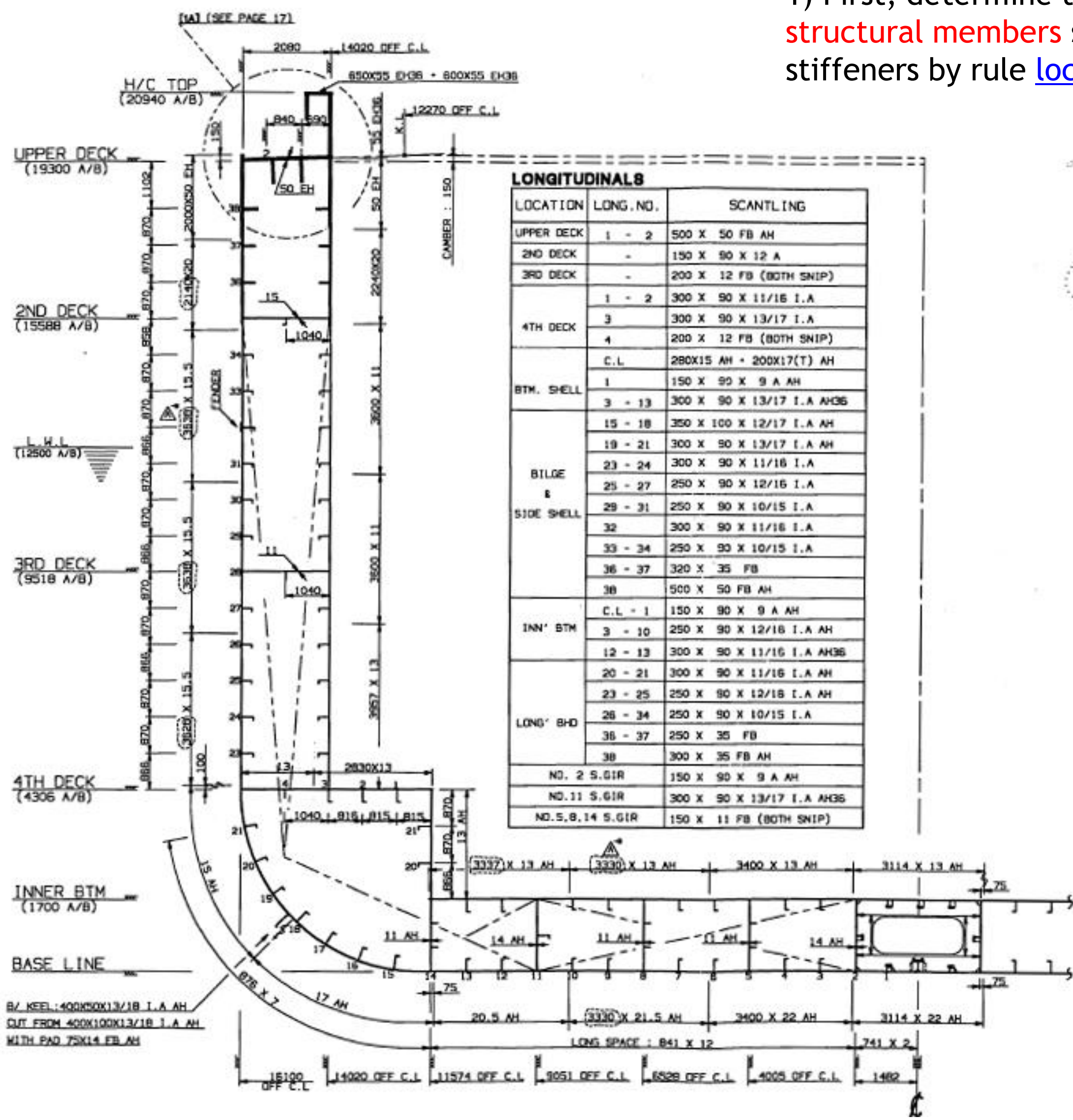
Total Bending Moment Curve



15-4 Calculation of Section Modulus

Example of Midship section of a 3,700 TEU Containership

1) First, determine the dimensions of the longitudinal structural members such as longitudinal plates and longitudinal stiffeners by rule local scantling



LONGITUDINALS

LOCATION	LONG. NO.	SCANTLING
UPPER DECK	1 - 2	500 X 50 FB AH
2ND DECK	-	150 X 90 X 12 A
3RD DECK	-	200 X 12 FB (BOTH SNIP)
4TH DECK	1 - 2	300 X 90 X 11/16 I.A
	3	300 X 90 X 13/17 I.A
	4	200 X 12 FB (BOTH SNIP)
BTM. SHELL	C.L	280X15 AH + 200X17(T) AH
	1	150 X 90 X 9 A AH
	3 - 13	300 X 90 X 13/17 I.A AH36
BILGE & SIDE SHELL	15 - 18	350 X 100 X 12/17 I.A AH
	19 - 21	300 X 90 X 13/17 I.A AH
	23 - 24	300 X 90 X 11/16 I.A
	25 - 27	250 X 90 X 12/16 I.A
	29 - 31	250 X 90 X 10/15 I.A
	32	300 X 90 X 11/16 I.A
	33 - 34	250 X 90 X 10/15 I.A
	36 - 37	320 X 35 FB
	38	500 X 50 FB AH
INN' BTM	C.L - 1	150 X 90 X 9 A AH
	3 - 10	250 X 90 X 12/16 I.A AH
	12 - 13	300 X 90 X 11/16 I.A AH36
LONG' BHD	20 - 21	300 X 90 X 11/16 I.A AH
	23 - 25	250 X 90 X 12/16 I.A AH
	26 - 34	250 X 90 X 10/15 I.A
	36 - 37	250 X 35 FB
	38	300 X 35 FB AH
NO. 2 S.GIR		150 X 90 X 9 A AH
NO.11 S.GIR		300 X 90 X 13/17 I.A AH36
NO.5,8,14 S.GIR		150 X 11 FB (BOTH SNIP)

Vertical location of Neutral Axis about baseline

•2) Second, calculate the moment of sectional area about the baseline.

$$\sum h_i A_i$$

h_i : vertical center of structural member

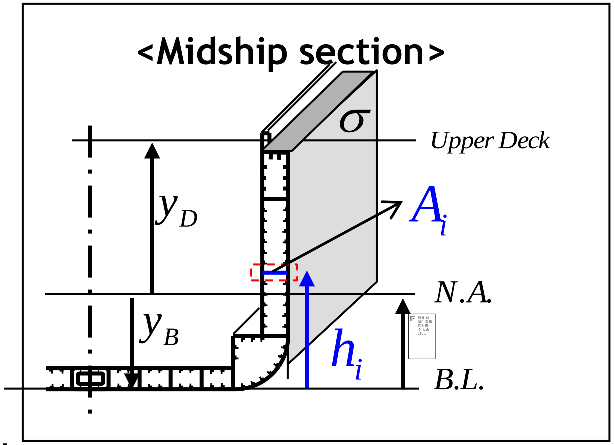
A_i : area of structural member

• 3) Vertical location of neutral axis from baseline($\bar{h}$) is, then, calculated (by)dividing the moment of area by the total sectional area

$$\bar{h} = \frac{\sum h_i A_i}{A}$$

$\bar{h}$: vertical location of neutral axis

A: total area



•By definition, neutral axis pass through the centroid of the cross section.

Moment of Inertia of the sectional area about N.A

- The midship section moment of Inertia about baseline($I_{B.L}$)

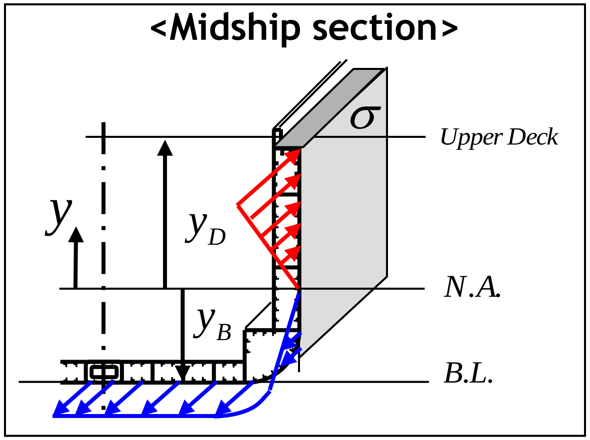
$$I_{B.L} = I_{N.A.} + A\,\bar{h}^2$$

- then calculate the midship section moment of inertia about neutral axis ($I_{N.A}$) using $I_{B.L}$.

$$I_{N.A.} = I_{B.L} - A\,\bar{h}^2$$

$$\boxed{\sigma \le \sigma_l}\,,\ \sigma = \frac{M}{I_{N.A}/y} = \frac{M}{Z}$$

Calculate section modulus and actual stress at deck and bottom



σ : bending stress

M_T : Total bending moment

A : Total Area

$I_{N.A.}$: Inertia moment of the midship section area about neutral axis (N.A.)

B.L : Base Line

$$\boxed{\sigma \le \sigma_l}\,,\ \sigma = \frac{M}{I_{N.A}/y} = \frac{M}{Z}$$

Section modulus

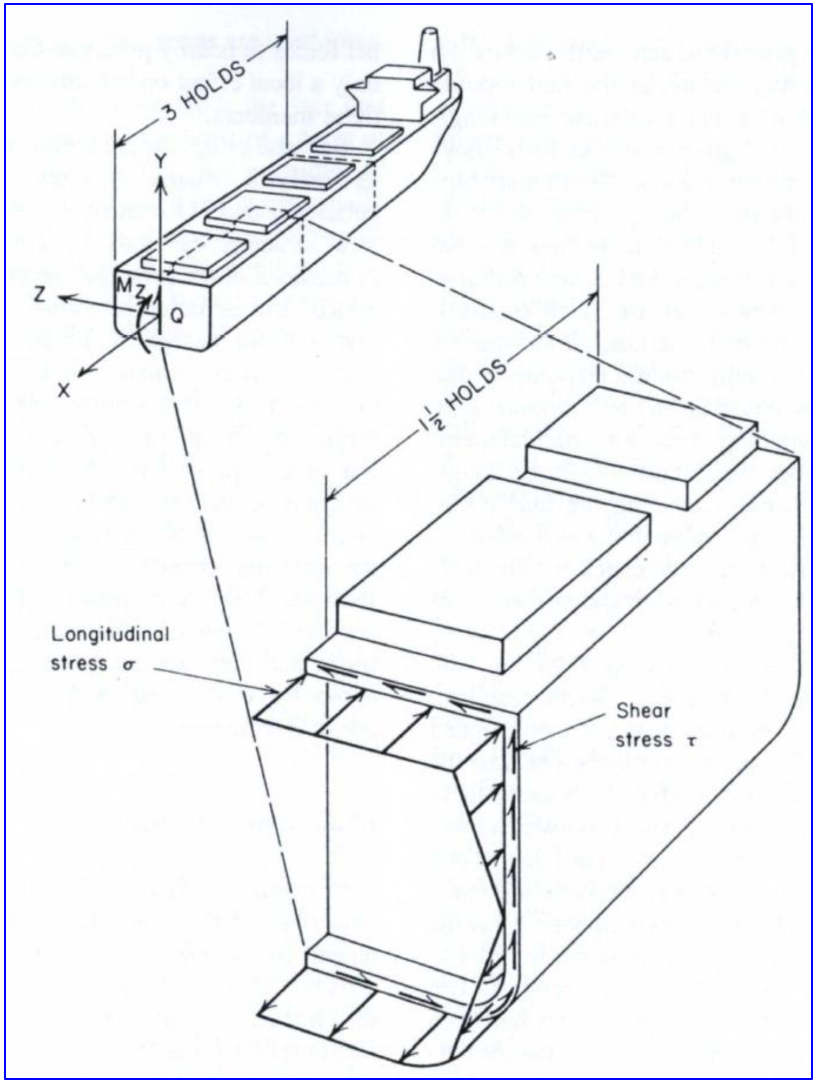
$$\frac{I_{N.A.}}{y_D} = Z_D, \qquad \frac{I_{N.A.}}{y_B} = Z_B$$

Calculation of Actual Stress at Deck and Bottom

$$\sigma_{Deck} = \frac{M}{I_{N.A}/y_D} = \frac{M}{Z_D}$$

$$\sigma_{Bottom} = \frac{M}{I_{N.A}/y_B} = \frac{M}{Z_B}$$

Global Hull Girder Strength (Longitudinal Strength)
- Definition of the longitudinal Strength members



Application of hull girder load effects

※ Example of Requirement for Longitudinal Structural Member

DNV Rules for Classification of Ships
Part 3 Chapter 1 HULL STRUCTUREALDESIGN SHIPS WITH LENGTH 100 METERS AND ABOVE
Sec. 5 Longitudinal Strength
C 300 Section modulus
301 The requirements given in 302 and 303 will normally be satisfied when calculated for the midship section only, provided the following rules for tapering are complied with:

a) **Scantlings of all continuous longitudinal strength members shall be maintained within 0.4 L amidships**. I

b) **Scantlings outside 0.4 L amidships are gradually reduced** to the local requirements at the ends, and the same material strength group is applied over the full length of the ship.

The section modulus at other positions along the length of the ship may have to be specially considered for ships with small block coefficient, high speed and large flare in the forebody or when considered necessary due to structural arrangement, see A106.

(DnV Pt 3 Ch1 Sec. 5 C300) 2011

C 300 Section modulus

301 The requirements given in 302 and 303 will normally be satisfied when calculated for the midship section only, provided the following rules for tapering are complied with:

a) Scantlings of all continuous longitudinal strength members shall be maintained within 0.4 L amidships. In special cases, based on consideration of type of ship, hull form and loading conditions, the scantlings may be gradually reduced towards the ends of the 0.4 L amidship part, bearing in mind the desire not to inhibit the vessel's loading flexibility.

b) Scantlings outside 0.4 L amidships are gradually reduced to the local requirements at the ends, and the same material strength group is applied over the full length of the ship.

The section modulus at other positions along the length of the ship may have to be specially considered for ships with small block coefficient, high speed and large flare in the forebody or when considered necessary due

to structural arrangement, see A106.

In particular this applies to ships of length L > 120 m and speed V > 17 knots.

Min. Required Midship Section Modulus and Inertia Moment by DnV Rule

DnV Rules, Jan. 2004,Pt.3 Ch.1 Sec.5

- The Midship section modulus about the transverse neutral axis shall not be less than :

(Pt 3 Ch1 Sec. 5 C302)

$$Z_O = \frac{C_{WO}}{f_1} L^2 B(C_B + 0.7)$$

C_{WO} : wave coefficient

L	C_{WO}
$L < 300$	$10.75 - [(300 - L)/100]^{3/2}$
$300 \le L \le 350$	10.75
$L > 350$	$10.75 - [(L - 350)/150]^{3/2}$

C_B is in this case not to be taken less than 0.60.

- The midship section moment of inertia about the transverse neutral axis shall not
- be less than :

(Pt 3 Ch1 Sec. 5 C400)

$$I_{ship} = 3 C_W L^3 B(C_B + 0.7) \quad (cm^4)$$

1)DnV Rules, Jan. 2004,Pt.3 Ch.1 Sec.5

(DnV Pt 3 Ch1 Sec. 5 C302) 2011

302 The midship section modulus about the transverse neutral axis shall not be less than:

$$Z_O = \frac{C_{WO}}{f_1} L^2 B (C_B + 0.7) \quad (cm^3)$$

C_{WO} = 10.75 – [(300 – L) /100] $^{3/2}$ for L < 300
= 10.75 for 300 ≤ L ≤ 350
= 10.75 – [(L – 350) /150] $^{3/2}$ for L > 350

Values of C_{WO} are also given in Table C1.

C_B is in this case not to be taken less than 0.60.

Table C1 Values for C_{WO}

L	C_{WO}	L	C_{WO}	L	C_{WO}
		160	9.09	260	10.50
		170	9.27	280	10.66
		180	9.44	300	10.75
		190	9.60	350	10.75
100	7.92	200	9.75	370	10.70
110	8.14	210	9.90	390	10.61
120	8.34	220	10.03	410	10.50
130	8.53	230	10.16	440	10.29
140	8.73	240	10.29	470	10.03
150	8.91	250	10.40	500	9.75

For ships with restricted service, C_{WO} may be reduced as follows:

— service area notation **R0**: No reduction
— service area notation **R1**: 5%
— service area notation **R2**: 10%
— service area notation **R3**: 15%
— service area notation **R4**: 20%
— service area notation **RE**: 25%.

(DnV Pt 3 Ch1 Sec. 5 C401) 2011

C 400 Moment of inertia

401 The midship section moment of inertia about the transverse neutral axis shall not be less than:

$$I = 3\, C_W\, L^3\, B\, (C_B + 0.7) \quad (cm^4)$$

Material Factor (f_1)

$$\boxed{\sigma \le \sigma_l}, \sigma = \frac{M}{I_{N.A}/y} = \frac{M}{Z} \qquad \sigma_l = 175 \, f_1 \; N/mm^2$$

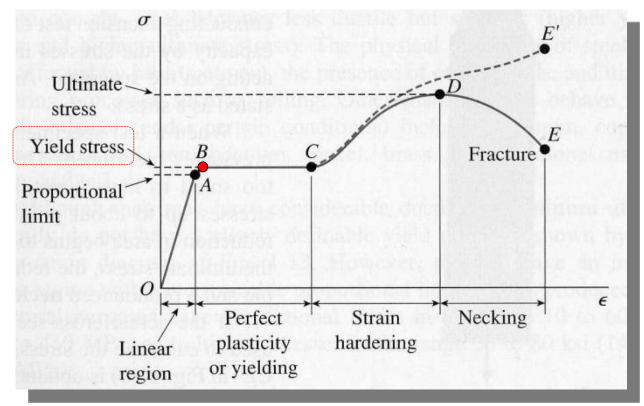
within 0.4 L amidship

- The material factor (f_1) included in the various formulae for scantlings and in expressions giving allowable stresses.[1)]

Material Designation	Yield Stress (N/mm²)	$f_1 = \dfrac{\sigma}{\sigma_{NV-NS}}$	Material Factor (f_1)
NV-NS	235	235/235 = 1.00	1.00
NV-27	265	265/235 = 1.13	1.08
NV-32	315	315/235 = 1.34	1.28
NV-36	355	355/235 = 1.51	1.39
NV-40	390	390/235 = 1.65	1.43

*NV-NS : Normal Strength Steel(Mild Steel)

*NV-XX : High tensile Steel



*Yield Stress(σ_y) [N/mm²] or [MPa]: The magnitude of the load required to cause yielding in the beam.[2)]

*High tensile steel

A type of alloy steel that provides better mechanical properties or greater resistance to corrosion than carbon steel. They have a carbon content between 0.05-0.25% to retain formability and weldability and include up to 2.0% manganese and other elements are added for strengthening purposes..

*A: 'A' grade 'Normal strength Steel'

AH: 'A' grade 'High tensile steel'

1) DnV Rules, Jan. 2004,Pt.3 Ch.1 Sec.2

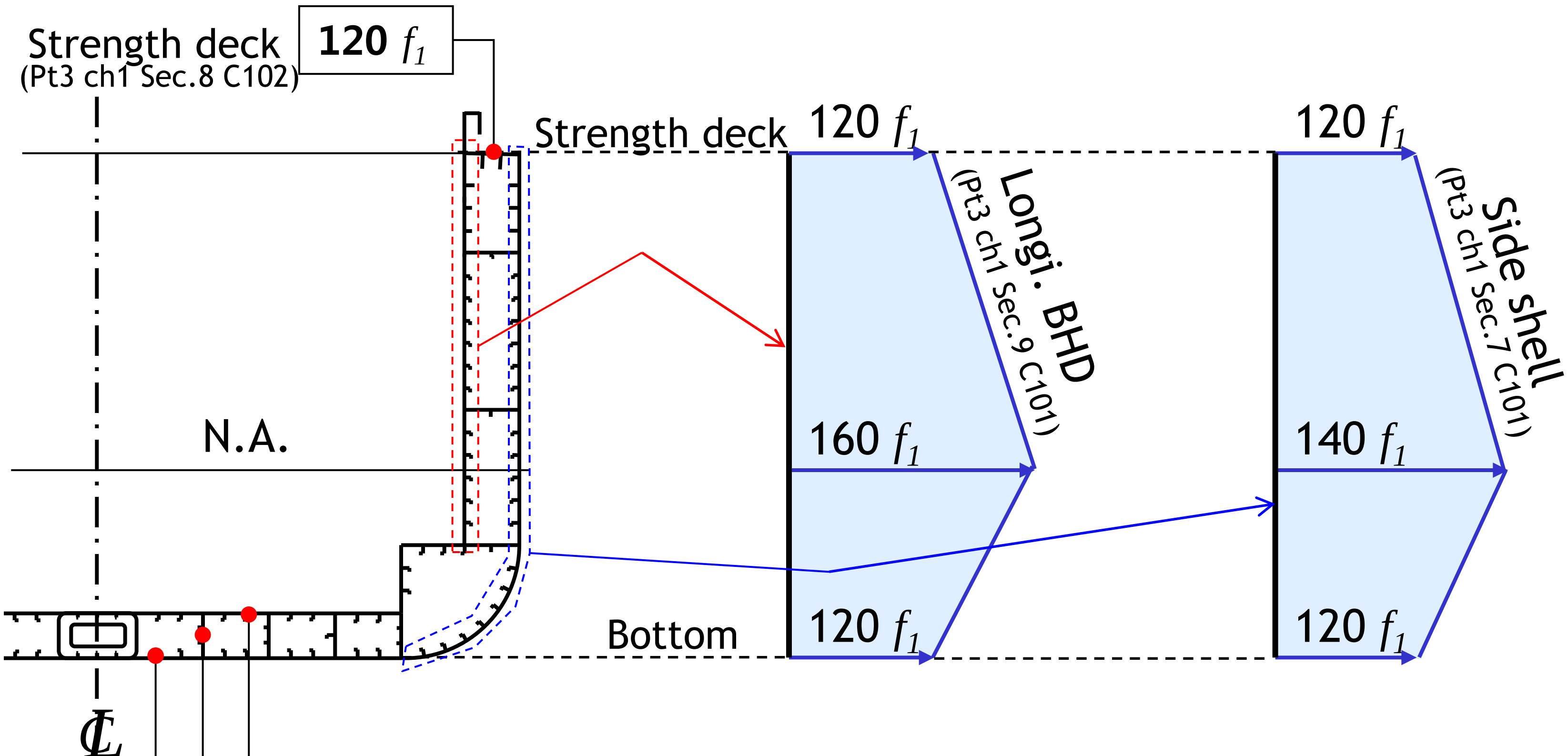
2) James M. Gere, Mechanics of Materials 7th Edition, Thomson, Chap.1, pp.15~26

15-5 Allowable stresses (σ_l)

Allowable stresses (σ_l)

DnV Rules, Jan. 2004,Pt.3 Ch.1 Sec.6,7,8,9

for Bottom Plating, Deck Plating, Bulkhead Plating, Side Plating



140 f_1 Inner Bottom Plating (Pt3 ch1 Sec.6 C401)

130 f_1 Longi. Girder Plating (Pt3 ch1 Sec.6 C501)

120 f_1 Bottom Plating (Pt3 ch1 Sec.6 C302)

✓ Allowable stress at the neutral axis(N.A.) is **largest**. And the allowable stress decreases proportionally from the neutral axis(N.A) to the deck and bottom of the section.
Because actual bending stress is **smallest** at the neutral axis(N.A.), and increases proportionally from N.A to the deck and bottom of the section.

f_1 : material factor

(DnV Pt 3 Ch1 Sec. 6 C302) 2011

302 The thickness requirement corresponding to lateral pressure is given by:

$$t = \frac{15.8 k_a s \sqrt{p}}{\sqrt{\sigma}} + t_k \quad \text{(mm)}$$

p = p_1 to p_3 (when relevant) in Table B1
σ = 175 f_1 – 120 f_{2b}, maximum 120 f_1 when transverse frames, within 0.4 L
= 120 f_1 when longitudinals, within 0.4 L
= 160 f_1 within 0.1 L from the perpendiculars.

Between specified regions the σ-value may be varied linearly.

f_{2b} = stress factor as given in A 200

(DnV Pt 3 Ch1 Sec. 6 C401) 2011

C 400 Inner bottom plating

401 The thickness requirement corresponding to lateral pressure is given by:

$$t = \frac{15.8 k_a s \sqrt{p}}{\sqrt{\sigma}} + t_k \quad \text{(mm)}$$

p = p_4 to p_{15} (whichever is relevant) as given in Table B1

σ = 200 f_1 – 110 f_{2b}, maximum 140 f_1 when transverse frames, within 0.4 L

= 140 f_1 when longitudinals, within 0.4 L

= 160 f_1 within 0.1 L from the perpendiculars.

Between specified regions the σ-value may be varied linearly.

f_{2b} = stress factor as given in A200.

(DnV Pt 3 Ch1 Sec. 6 C501) 2011

501 The thickness requirement of floors and longitudinal girders forming boundaries of double bottom tanks is given by:

$$t = \frac{15.8\, k_a s \sqrt{p}}{\sqrt{\sigma}} + t_k \quad \text{(mm)}$$

p = p_{13} to p_{15} (when relevant) as given in Table B1
p = p_1 for sea chest boundaries (including top and partial bulkheads)
σ = allowable stress, for longitudinal girders within 0.4 L given by:

Transversely stiffened	*Longitudinally stiffened*
$190\, f_1 - 120\, f_{2b}$, maximum $130\, f_1$	$130\, f_1$

σ = $160\, f_1$ within 0.1 L from the perpendiculars and for floors in general
= $120\, f_1$ for sea chest boundaries (including top and partial bulkheads)
f_{2b} = stress factor as given in A200.
Between specified regions of longitudinal girders the σ-value may be varied linearly.

(DnV Pt 3 Ch1 Sec. 7 C101) 2011

101 The thickness requirement corresponding to lateral pressure is given by:

$$t = \frac{15.8 k_a s \sqrt{p}}{\sqrt{\sigma}} + t_k \quad \text{(mm)}$$

p = $p_1 - p_8$, whichever is relevant, as given in Table B1

σ = 140 f_1 for longitudinally stiffened side plating at neutral axis, within 0.4 L amidship

= 120 f_1 for transversely stiffened side plating at neutral axis, within 0.4 L amidship.

Above and below the neutral axis the σ-values shall be reduced linearly to the values for the deck and bottom plating, assuming the same stiffening direction and material factor f_1 as for the plating considered

= 160 f_1 within 0.05 L from F.P. and 0.1 L from A.P.

Between specified regions the σ-value may be varied linearly.

(DnV Pt 3 Ch1 Sec. 8 C102) 2011

102 The thickness requirement corresponding to lateral pressure is given by:

$$t = \frac{15.8 k_a s \sqrt{p}}{\sqrt{\sigma}} + t_k \quad \text{(mm)}$$

p = $p_1 - p_{13}$, whichever is relevant, as given in Table B1
σ = allowable stress within 0.4 L, given by:

Transversely stiffened	*Longitudinally stiffened*
$175 f_1 - 120 f_{2d}$, maximum $120 f_1$	$120 f_1$

σ = $160 f_1$ within 0.1 L from the perpendiculars and within line of large deck openings.

Between specified regions the σ-value may be varied linearly.

f_{2D} = stress factor as given in A 200.

(DnV Pt 3 Ch1 Sec. 9 C101) 2011

C 100 Bulkhead plating

101 The thickness requirement corresponding to lateral pressure is given by:

$$t = \frac{15.8 k_a s \sqrt{p}}{\sqrt{\sigma}} + t_k \quad \text{(mm)}$$

p = $p_1 - p_9$, whichever is relevant, as given in Table B1

σ = 160 f_1 for longitudinally stiffened longitudinal bulkhead plating at neutral axis irrespective of ship length

= 140 f_1 for transversely stiffened longitudinal bulkhead plating at neutral axis within 0.4 L amidships, may however be taken as 160 f_1 when p_6 or p_7 are used.

Above and below the neutral axis the σ-values shall be reduced linearly to the values for the deck and bottom plating, assuming the same stiffening direction and material factor as for the plating considered

= 160 f_1 for longitudinal bulkheads outside 0.05 L from F.P. and 0.1 L from A.P. and for transverse bulkheads in general

= 220 f_1 for watertight bulkheads except the collision bulkhead, when p_1 is applied.

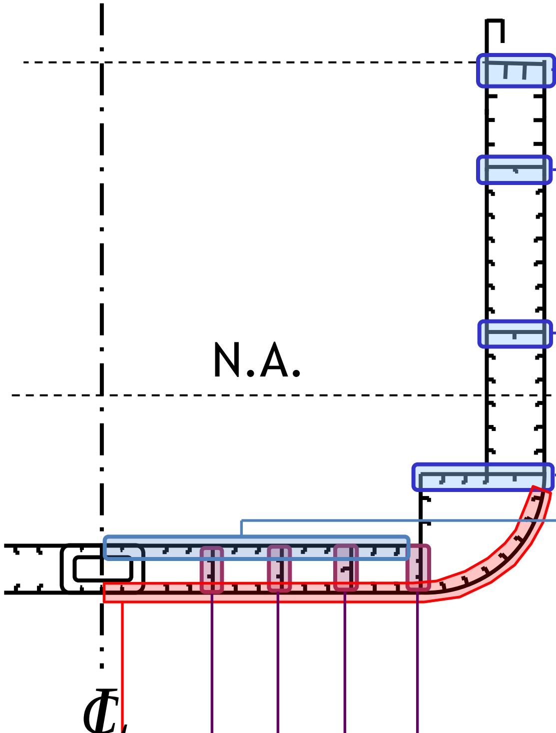
Between specified regions the σ-value may be varied linearly.

In corrugated bulkheads formed by welded plate strips, the thickness in flange and web plates may be differing. The thickness requirement then is given by the following modified formula:

$$t = \sqrt{\frac{500 s^2 p}{\sigma} - t_n^2} + t_k \quad \text{(mm)}$$

t_n = thickness in mm of neighbouring plate (flange or web), not to be taken greater than t.

Allowable Stresses (σ_l) for Longitudinal Stiffeners



Decks (Pt3 ch1 Sec.8 C301)

$$\boxed{225 f_1 - \underline{130 f_{2d}}, \max 160 f_1}$$: Strength deck

$\rightarrow \sigma_L$

$$\boxed{225 f_1 - \underline{130 f_{2d} \frac{z_n - z_a}{z_n}}, \max 160 f_1}$$: Continuous decks below strength deck

$\rightarrow \sigma_L$

Inner Bottom (Pt3 ch1 Sec.6 C801)

$$\boxed{225 f_1 - \underline{100 f_{2b}} - 0.7\sigma_{db}, \max 160 f_1}$$

$\rightarrow \sigma_L$

Longi. Stiffeners on double bottom girders (Pt3 ch1 Sec.6 C901)

$$\boxed{225 f_1 - \underline{110 f_{2b}}, \max 160 f_1}$$

$\rightarrow \sigma_L$

Double Bottom (Pt3 ch1 Sec.6 C701)

$$\boxed{225 f_1 - \underline{130 f_{2b}} - 0.7\sigma_{db}}$$

σ_L

σ_{db} = mean double bottom stress at plate flanges, normally not to be taken less than
- = 20 f1 for cargo holds in general cargo vessel
- = 50 f1 for holds for ballast
- = 85 f1 b/B for tanks for liquid cargo

Zn : Vertical distance in m from the baseline or deckline to the neutral axis of the hull girder, whichever is relevant

Z_a : vertical distance in m from the baseline or deckline to the point in question below or above the neutral axis, respectively

$$f_{2b,2d} = \frac{5.7(M_S + M_W)}{Z_{b,d}}$$

M_S : the largest design SWBM (kN·m)

M_W: rule VWBM in (kN·m)

$Z_{b,d}$:midship section modulus (cm^3) at bottom or deck as built

(f2b: Pt3 ch1 Sec.6 A201)

302 The section modulus requirement is given by:

$$Z = \frac{1000 l^2 s p w_k}{m \sigma} \quad (cm^3)$$

p = p_1 to p_8 whichever is relevant, as given in Pt.3 Ch.1 Sec.7 Table B1
w_k = 1.05 when calculating sectional modulus for midspan and upper end
= 1.15 when calculating sectional modulus for lower end
σ = 130 f_1 for internal loads p_3 to p_8
σ = 150 f_1 for external loads p_1, p_2 and p_{min} given above
m = 18 in general
m = 12 at upper end (including bracket) in combination with internal loads, p_3 to p_8
m = 9 at lower end (including bracket) and for upper end in combination with external loads p_1, p_2 and p_{min}.

For main frames situated next to plane transverse bulkheads, e.g. at the ends of the cargo region, the section modulus of the mid portion of the frame is generally to exceed the section modulus of the adjacent frame by a factor $3h_a/h$ where:

h_a = web height of adjacent frame
h = web height of considered frame.

The increased section modulus of the main frame adjacent to plane transverse bulkheads need not be fitted if other equivalent means are applied to limit the deflection of these frames.

(DnV Pt 3 Ch1 Sec. 8 C301) 2011

301 The section modulus requirement is given by:

$$Z = \frac{83\, l^2\, s\, p\, w_k}{\sigma} \quad (\mathrm{cm}^3), \quad \text{minimum } 15\ \mathrm{cm}^3$$

p = $p_1 - p_{13}$, whichever is relevant, as given in Table B1.

σ = allowable stress, within 0.4 L midship given in Table C1

= 160 f_1 for continuous decks within 0.1 L from the perpendiculars and for other deck longitudinals in general.

Between specified regions the σ-value shall be varied linearly.

For longitudinals $\sigma = 160\ f_1$ may be used in any case in combination with heeled condition pressures p_9 and sloshing load pressures, p_{11} and p_{12}.

For definition of other parameters used in the formula, see A200.

(DnV Pt 3 Ch1 Sec. 6 C701) 2011

701 The section modulus requirement is given by:

$$Z = \frac{83\, l^2\, s\, p\, w_k}{\sigma} \quad (\mathrm{cm}^3)$$

p = p_1 to p_3 (when relevant) as given in Table B1
σ = allowable stress (maximum 160 f_1) given by:

— within 0.4 L:

Single bottom	*Double bottom*
$225\, f_1 - 130\, f_{2b}$	$225\, f_1 - 130\, f_{2b} - 0.7\, \sigma_{db}$

For bilge longitudinals the allowable stress σ shall be taken as $225\, f_1 - 130\, f_2\, (z_n - z_a)/z_n$, where z_n, z_a are taken as defined in Sec.7 A201.

— within 0.1 L from perpendiculars: $\sigma = 160\, f_1$

Between specified regions the σ- value may be varied linearly.

σ_{db} = mean double bottom stress at plate flanges, normally not to be taken less than:
= $20\, f_1$ for cargo holds in general cargo vessels
= $50\, f_1$ for holds for ballast
= $85\, f_1\, b/B$ for tanks for liquid cargo
f_{2b} = stress factor as given in A200
b = breadth of tank at double bottom.

Longitudinals connected to vertical girders on transverse bulkheads shall be checked by a direct stress analysis, see Sec.12 C.

(DnV Pt 3 Ch1 Sec. 6 C801) 2011

801 The section modulus requirement is given by:

$$Z = \frac{83\, l^2 s\, p\, w_k}{\sigma} \quad (\text{cm}^3)$$

p = p_4 to p_{15} (whichever is relevant) as given in Table B1
σ = $225\, f_1 - 100\, f_{2B} - 0.7\, \sigma_{db}$ within 0.4 L (maximum $160\, f_1$)
= $160\, f_1$ within 0.1 L from the perpendiculars.

Between specified regions the σ-value may be varied linearly.

σ_{db} = mean double bottom stress at plate flanges, normally not to be taken less than:
= $20\, f_1$ for cargo holds in general cargo vessels
= $50\, f_1$ for holds for ballast
= $85\, f_1\, b/B$ for tanks for liquid cargo
f_{2b} = stress factor as given in A200
b = breadth of tank at double bottom.

(DnV Pt 3 Ch1 Sec. 6 C901) 2011

901 The section modulus requirement of stiffeners on floors and longitudinal girders forming boundary of double bottom tanks is given by:

$$Z = \frac{100\, l^2\, s\, p\, w_k}{\sigma} \quad (\mathrm{cm}^3)$$

p = p_{13} to p_{15} as given in Table B1

p = p_1 for sea chest boundaries (including top and partial bulkheads)

σ = $225\, f_1 - 110\, f_{2b}$, maximum $160\, f_1$ for longitudinal stiffeners within 0.4 L

= $160\, f_1$ for longitudinal stiffeners within 0.1 L from perpendiculars and for transverse and vertical stiffeners in general.

= $120\, f_1$ for sea chest boundaries (including top and partial bulkheads).

Between specified regions of longitudinal stiffeners the σ-value may be varied linearly.

f_{2b} = stress factor as given in A200.

Derivation of coefficient 5.7 of Stress factor, f_2 (f_{2b}, f_{2d})

Rule still water bending moment

$$M_{SO} = -0.065 C_{WU} L^2 B(C_B + 0.7)$$

Rule vertical wave bending moment

$$M_{WO} = -0.11 \alpha C_W L^2 B(C_B + 0.7)$$

$$f_2 = \frac{Z_O}{Z_A}$$

Z_O : In case of mild steel(f1 = 1.0), rule midship section modulus

Pt.3 Sh.1 Sec5 C302

$$Z_O = \frac{C_{WO}}{f_1} L^2 B(C_b + 0.7) \qquad (cm^3)$$

Z_A : Actual midship section modulus (cm³).

f2 is the ratio of the rule midship section modulus (ZO) to the actual midship section modulus(ZA)

$$f_2 = \frac{C_{WO} L^2 B(C_b + 0.7)}{Z_A} = \boxed{\frac{1}{0.175}} \frac{\boxed{0.175} \cdot C_{WO} L^2 B(C_b + 0.7)}{Z_A}$$

$$= \boxed{5.7} \times \frac{\boxed{0.065} \cdot C_{WO} L^2 B(C_b + 0.7) + \boxed{0.11} \cdot C_{WO} L^2 B(C_b + 0.7)}{Z_A}$$

$$\boxed{\therefore\ f_2 = 5.7 \times \frac{(M_S + M_W)}{Z_A}}$$

M_S : Normally to be taken as the largest design still water bending moment in kNm. Ms shall not be taken less than 0.5 Mso. When actual design moment is not known, Ms may be taken equal to Mso.

M_W : Rule wave bending moment in kNm. Hogging or sagging moment to be chosen in relation to the applied still water moment

(DnV Pt 3 Ch1 Sec. 5 C302) 2011

302 The thickness requirement corresponding to lateral pressure is given by:

$$t = \frac{15.8 k_a s \sqrt{p}}{\sqrt{\sigma}} + t_k \quad \text{(mm)}$$

p = p_1 to p_3 (when relevant) in Table B1
σ = 175 f_1 – 120 f_{2b}, maximum 120 f_1 when transverse frames, within 0.4 L
= 120 f_1 when longitudinals, within 0.4 L
= 160 f_1 within 0.1 L from the perpendiculars.

Between specified regions the σ-value may be varied linearly.

f_{2b} = stress factor as given in A 200

Design Procedure of Structures
- Stress factor

1) DnV Rules, Jan. 2004,Pt.3 Ch.1 Sec.6 c800

Why the iteration is needed for the calculation of local scantling?

Example) Inner bottom Longitudinals[1)]

▪Minimum Longi . stiffener section modulus

$$\boxed{Z = \frac{83 l^2 s p w_k}{\sigma} \quad (cm^3)}$$

l : stiffener span in m
s : stiffener spacing in m
p : design loads
w_k : section modulus corrosion factor in tanks, Sec.3 C1004

σ_{db} = mean double bottom stress at plate flanges, normally not to be taken less than
= 20 f1 for cargo holds in general cargo vessel
= 50 f1 for holds for ballast
= 85 f1 b/B for tanks for liquid cargo

Where, $\sigma = 225 f_1 - 100 f_{2b} - 0.7\sigma_{db}$

f_1 : material factor as defined in DnV Rules Pt.3 Ch.1 Sec.2

f_{2b} : stress factor

$$f_{2b,2d} = \frac{5.7(M_S + M_W)}{Z_{b,d}}$$

required midship section modulus (cm³) at bottom or deck

actual midship section modulus (cm³) at bottom or deck as built

M_S : the largest design SWBM[2)] (kN·m)

M_W : VWBM by class rule of direct calculation in (kN·m)

2) Largest SWBM among all loading conditions and class rule

The actual midship section modulus at bottom or deck is needed.

However, the section modulus can be calculated after the scantlings of the members are determined.

→ **Assumption!**

Therefore, actual section modulus is calculated to be equal to the assumed section modulus by the iteration.

Example of Midship Scantling

- Midship Scantling for 4,100 TEU Container Ship

< Calculation of Design Bending Moment (Hogging)>

- Still water bending moment
  - Larger value shall be used for still water bending moment between the largest actual still water bending moment based on load conditions and design still water bending moment by rule

✓**Design still water bending moment by rule** [1)]

$$C_{WU} = C_W$$
$$= 10.75 - [(300 - L)/100]^{3/2}$$
$$= 10.75 - [(300 - 247.64)/100]^{3/2}$$
$$= 10.37$$

$$M_{SO} = C_{WU}\, L^2\, B(0.1225 - 0.015\, C_B)$$
$$= 10.37 \times 247.64^2 \times 32.2$$
$$\times (0.1225 - 0.015 \times 0.6581)$$
$$\boxed{= 2{,}364{,}171.77 \quad (kNm)}$$

$$M_S = k_{sm} M_{SO}$$
$$= 1.0 \times 2{,}364{,}171.77$$
$$\boxed{= 2{,}364{,}171.77 \quad (kNm)}$$

✓**Largest actual still water bending moment based on the load conditions**

- At ballast arrival condition

$$M_S = \boxed{2{,}227{,}221\ (kNm)}$$

$$\therefore M_S = 2{,}364{,}171.77 \quad (kNm)$$

- Wave bending moment[2)]

$$M_{WO} = 0.19\,\alpha\, C_{WU}\, L^2\, B\, C_B$$
$$= 0.19 \times 1.0 \times 10.37 \times 247.64^2 \times 32.2 \times 0.6581$$
$$= 2{,}560{,}481.90 \quad (kNm)$$

$$M_w = k_{wm} M_{WO}$$
$$= 1.0 \times 2{,}560{,}481.90$$
$$= 2{,}560{,}481.90 \quad (kNm)$$

$$M = M_S + M_W$$
$$= 2{,}364{,}171.77 + 2{,}560{,}481.90$$
$$= 4{,}924{,}653.67\ (kNm)$$

< Calculation of Design Bending Moment (Sagging)>

Design bending moment at sagging condition is calculated in the same way.

$$M_S = -1{,}807{,}679.05 \quad (kNm)$$
$$M_W = -3{,}059{,}149.16\ (kNm)$$

$$M = M_S + M_W$$
$$= -1{,}807{,}679.05 - 3{,}059{,}149.16 = -4{,}866{,}828.21\ (kNm)$$

B. Still Water and Wave Induced Hull Girder Bending Moments and Shear Forces

B 100 Stillwater conditions

101 The design stillwater bending moments, M_S, and stillwater shear forces, Q_S, shall be calculated along the ship length for design cargo and ballast loading conditions as specified in 102.

For these calculations, downward loads are assumed to be taken as positive values, and shall be integrated in the forward direction from the aft end of L. The sign conventions of Q_S and M_S are as shown in Fig.1.

(IACS UR S11.2.1.1 Rev.5)

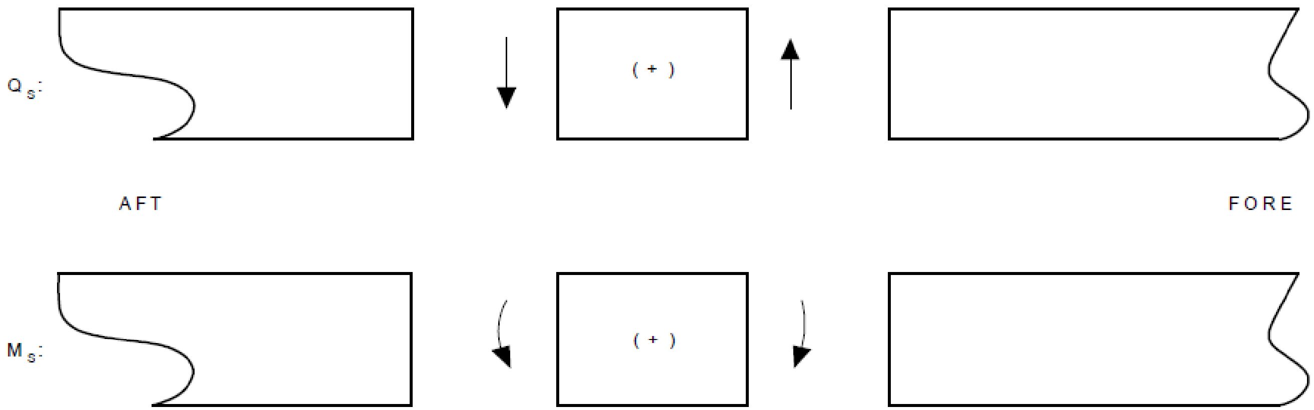


Fig. 1
Sign Conventions of Q_S and M_S

102 In general, the following design cargo and ballast loading conditions, based on amount of bunker, fresh water and stores at departure and arrival, shall be considered for the M_S and Q_S calculations. Where the amount and disposition of consumables at any intermediate stage of the voyage are considered more severe, calculations for such intermediate conditions shall be submitted in addition to those for departure and arrival conditions. Also, where any ballasting and or deballasting is intended during voyage, calculations of the

(DnV Pt 3 Ch1 Sec. 5 B201) 2011

B 200 Wave load conditions

201 The rule vertical wave bending moments amidships are given by:

$$M_W = M_{WO} \quad \text{(kNm)}$$

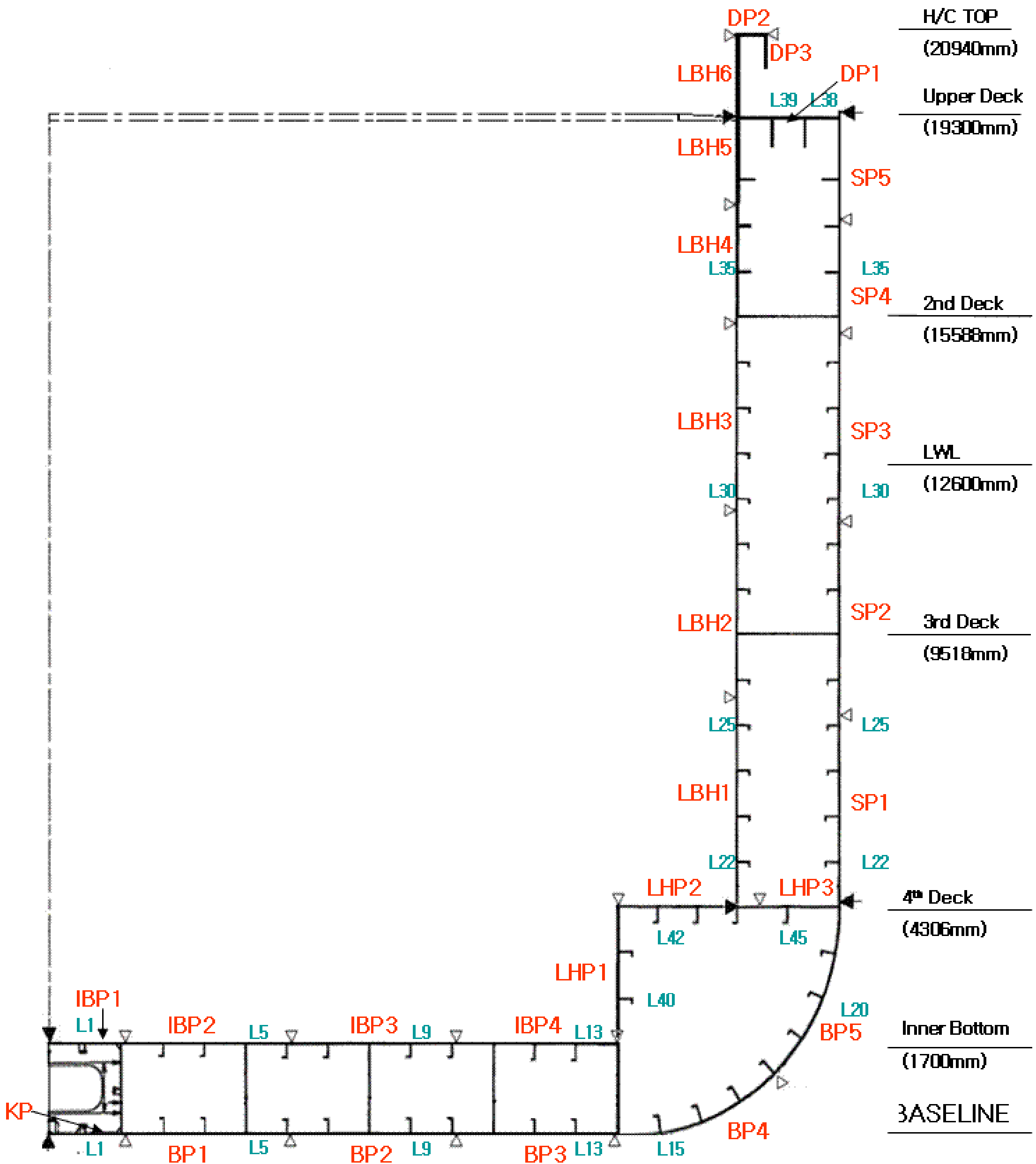
M_{WO} = $-0.11\ \alpha C_W L^2 B (C_B + 0.7)$ (kNm) in sagging
= $0.19\ \alpha C_W L^2 B C_B$ (kNm) in hogging

α = 1.0 for seagoing conditions
= 0.5 for harbour and sheltered water conditions (enclosed fjords, lakes, rivers).

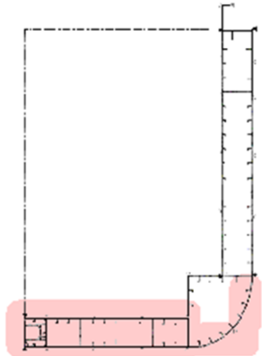
C_B is not be taken less than 0.6.

Example of Midship Scantling

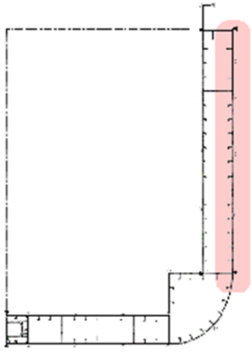
- Midship Scantling for 4,100 TEU Container Ship



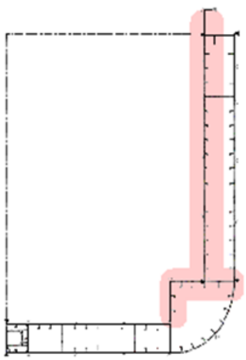
In this example, we consider the midship section is composed of bottom structure, side structure, bulkhead structure, and deck structure.



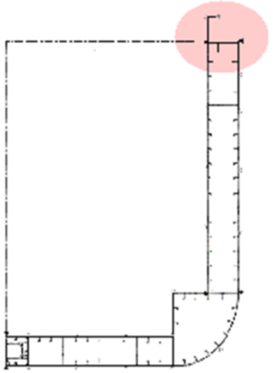
Bottom Structure



Side Structure



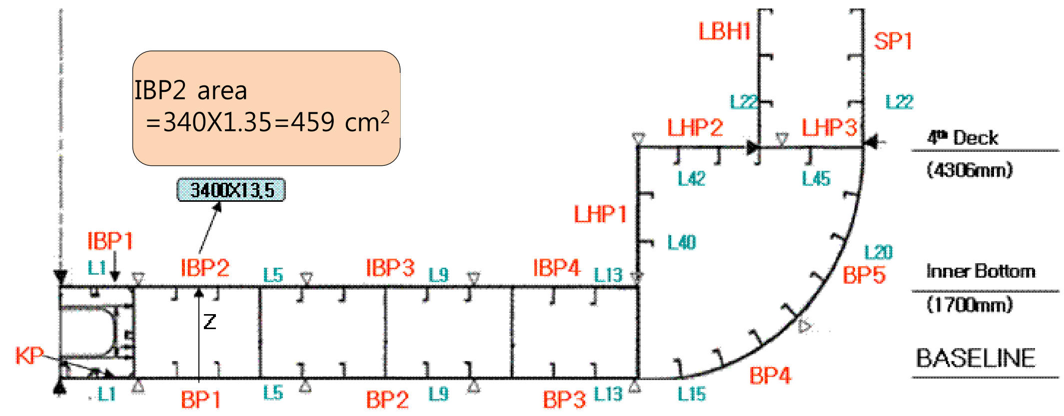
Bulkhead Structure



Deck Structure

Example of Midship Scantling
- Midship Scantling for 4,100 TEU Container Ship

Plate at Bottom Structure



<Bottom Structure Plate>								
	Name	Width (cm)	Thickness (cm)	Area (cm^2)	Vertical center of IBP (cm)	1st moment of IBP area about base line (cm^3)	2nd moment of IBP area about base line (cm^4)	Moment of inertia of IBP area(Ix) (cm^4)
Plate	KP	155.70	2.05	319	0.0	0	0.000E+00	1.118E+02
	BP1	340.00	1.60	544	0.0	0	0.000E+00	1.161E+02
	BP2	333.00	1.60	533	0.0	0	0.000E+00	1.137E+02
	BP3	326.20	1.60	522	0.0	0	0.000E+00	1.113E+02
	BP4	350.40	2.15	753	34.3	25,825	8.853E+05	2.902E+02
	BP5	350.40	2.15	753	27.8	20,943	5.822E+05	2.902E+02
	IBP1	155.70	1.35	210	170.0	35,733	6.075E+06	3.192E+01
	IBP2	340.00	1.35	459	170.0	78,030	1.327E+07	6.971E+01
	IBP3	333.00	1.35	450	170.0	76,424	1.299E+07	6.828E+01
	IBP4	326.20	1.35	440	170.0	74,863	1.273E+07	6.688E+01
girder	Name	Width (cm)	Thickness (cm)	Area (cm^2)	Vertical center of IBP (cm)	1st moment of IBP area about base line (cm^3)	2nd moment of IBP area about base line (cm^4)	Moment of inertia of IBP area(Ix) (cm^4)
	L0	1.10	170.00	187	85.0	15,895	1.351E+06	4.504E+05
	L2	1.40	170.00	238	85.0	20,230	1.720E+06	5.732E+05
	L5	1.25	170.00	213	85.0	18,063	1.535E+06	5.118E+05
	L8	1.25	170.00	213	85.0	18,063	1.535E+06	5.118E+05
	L11	1.25	170.00	213	85.0	18,063	1.535E+06	5.118E+05
	L14	1.25	170.00	213	85.0	18,063	1.535E+06	5.118E+05

IBP(inner bottom plate) area(A)
= width of IBP X thickness of IBP

Ex) Area of IBP2 $=340\times13.5=459\ cm^2$

1st moment of IBP area about base line
= Area of IBP(A) X Vertical center of IBP(b)

Ex) 1st moment of Area of IBP2

$$=459\times170=78,030\ cm^3$$

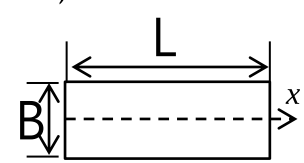
2nd moment of IBP area about base line
= Area of IBP(A) X Vertical center of IBP(b)²

Ex) 2nd moment of Area of IBP2

$$=459\times170^2=1.327e^{07}cm^4$$

Moment of inertia of IBP area(I_x)

Ex) Moment of inertia of IBP2 area

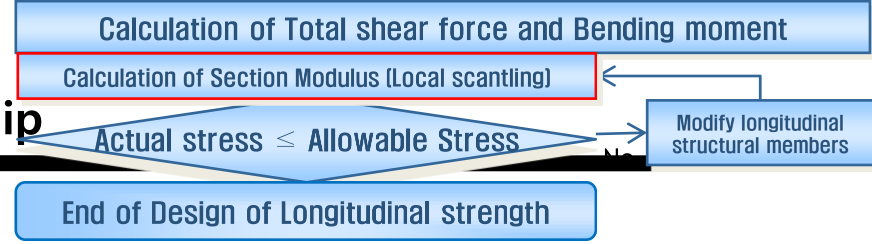


$$=\frac{BL^3}{12}\ (??)=\frac{340\times1.35^3}{12}=6.971e^{01}cm$$

Moment of inertia of IBP area about baseline($I_{x'}$) is obtained by using the parallel-axis theorem.

$$I_{x'}=I_x+b^2A$$

Example of Midship Scantling
- Midship Scantling for 4,100 TEU Container Ship



Stiffener at Bottom Structure

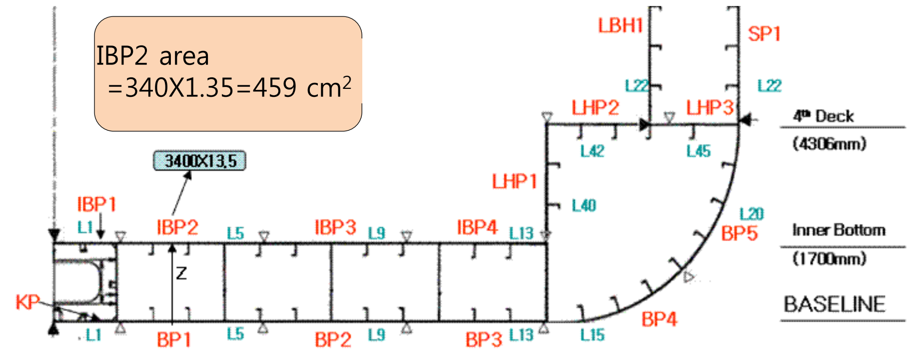
<Bottom Structure Stiffener Web Plate>

	Name	Width (cm)	Thickness (cm)	Area (cm^2)	Vertical center of IBP (cm)	1st moment of IBP area about base line (cm^3)	2nd moment of IBP area about base line (cm^4)	Moment of inertia of IBP area(Ix) (cm^4)
Bottom Longi. Web	L1	1.20	45.00	54	22.5	1,215	2.734E+04	9.113E+03
	L3	1.20	55.00	66	27.5	1,815	4.991E+04	1.664E+04
	L4	1.20	55.00	66	27.5	1,815	4.991E+04	1.664E+04
	L6	1.20	55.00	66	27.5	1,815	4.991E+04	1.664E+04
	L7	1.20	55.00	66	27.5	1,815	4.991E+04	1.664E+04
	L9	1.20	55.00	66	27.5	1,815	4.991E+04	1.664E+04
	L10	1.20	55.00	66	27.5	1,815	4.991E+04	1.664E+04
	L12	1.20	55.00	66	27.5	1,815	4.991E+04	1.664E+04
	L13	1.20	55.00	66	27.5	1,815	4.991E+04	1.664E+04
InnerBTM Longi.Web	Inner Bottom	In case of Inner Bottom 1.20	35.00	378	152.5	57,645	8.791E+06	3.859E+04
	Name	Width (cm)	Thickness (cm)	Area (cm^2)	Vertical center of IBP (cm)	1st moment of IBP area about base line (cm^3)	2nd moment of IBP area about base line (cm^4)	Moment of inertia of IBP area(Ix) (cm^4)
Bilge ~~Longi.~~ Web	L15	1.20	45.00	54	362.5	19,575	7.096E+06	9.113E+03
	L16	1.20	45.00	54	278.0	19,183	4.173E+06	9.113E+03
	L17	1.20	45.00	54	200.1	13,810	2.162E+06	9.113E+03
	L18	1.20	40.00	48	131.9	8,310	8.351E+05	6.400E+03
	L19	1.20	40.00	48	75.9	4,782	2.765E+05	6.400E+03
	L20	1.20	35.00	42	34.3	1,954	4.941E+04	4.288E+03
	L21	1.20	35.00	42	8.7	493	3.179E+03	4.288E+03

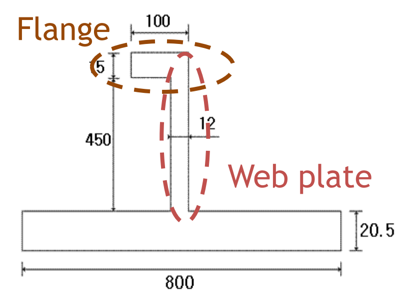
<Bottom Structure Stiffener Flange>

	Name	Width (cm)	Thickness (cm)	Area (cm^2)	Vertical center of IBP (cm)	1st moment of IBP area about base line (cm^3)	2nd moment of IBP area about base line (cm^4)	Moment of inertia of IBP area(Ix) (cm^4)
Bottom Longi. Flange	L1	10.00	1.50	15	45.0	675	3.038E+04	2.813E+00
	L3	50.00	1.50	75	55.0	4,125	2.269E+05	1.406E+01
	L4	50.00	1.50	75	55.0	4,125	2.269E+05	1.406E+01
	L6	50.00	1.50	75	55.0	4,125	2.269E+05	1.406E+01
	L7	50.00	1.50	75	55.0	4,125	2.269E+05	1.406E+01
	L9	50.00	1.50	75	55.0	4,125	2.269E+05	1.406E+01
	L10	10.00	1.50	15	55.0	825	4.538E+04	2.813E+00
	L12	10.00	1.50	15	55.0	825	4.538E+04	2.813E+00
	L13	10.00	1.50	15	55.0	825	4.538E+04	2.813E+00
InnerBTM Longi.F	Inner Bottom	In case of Inner Bottom 10.00	1.50	135	166.5	22,478	3.743E+06	2.531E+01
	Name	Width (cm)	Thickness (cm)	Area (cm^2)	Vertical center of IBP (cm)	1st moment of IBP area about base line (cm^3)	2nd moment of IBP area about base line (cm^4)	Moment of inertia of IBP area(Ix) (cm^4)
Bilge Longi. Flange	L15	1.20	45.00	69	362.5	25,013	9.067E+06	9.113E+03
	L16	1.20	45.00	69	278.0	19,183	5.333E+06	9.113E+03
	L17	1.20	45.00	69	200.1	13,810	2.764E+06	9.113E+03
	L18	1.20	40.00	63	131.9	8,310	1.096E+06	6.400E+03
	L19	1.20	40.00	63	75.9	4,782	3.629E+05	6.400E+03
	L20	1.20	35.00	57	34.3	1,954	6.699E+04	4.288E+03
	L21	1.20	35.00	57	8.7	493	4.268E+03	4.288E+03
Toal				7,519		590,637	87,554,496	3,350,227

IBP2 area
=340X1.35=459 cm^2



For convenience of calculation of moment of inertia of the stiffener area about base line, we consider that the stiffener is composed of flange and web plate.



Neutral axis of bottom ~~deck~~ structure
= total 1st moment of area about baseline/total area

$$= \frac{590{,}637}{7519} = 78.55\ cm$$

Example of Midship Scantling

- Midship Scantling for 4,100 TEU Container Ship

Calculation of moment of inertia of sectional area from neutral axis

Area, neutral axis, 1st moment, 2nd moment and moment of inertia about baseline of side structure , bulkhead structure, deck structure are calculated in the same way and the results are as follows:

Structure	Area	Neutral axis	1st moment of area about baseline	2nd moment of area about baseline	Moment of inertia of area
Bottom	7,519	79	5.906E+05	8.755E+08	2.627E+07
Side	3,135	1,158	3.630E+06	4.203E+09	1.261E+08
Bulkhead	5,273	1,250	6.592E+06	8.242E+09	2.472E+08
Deck	2,200	2,130	5.015E+06	1.208E+10	3.624E+08
Total	18,127		1.583E+07	2.540E+10	7.620E+08

Vertical location of neutral axis of midship section from baseline($\bar{h}$)is calculated by using the above table.

$$\bar{h} = \text{total 1st moment of area about baseline/total area}$$

$$= \frac{1.583e^{07}}{18{,}127} = 873.2 \;\; cm$$

Moment of inertia of area about neutral axis of midship section:

$$I_{Base,Total} = I_{N.A.,Total} + \bar{h}^2 \sum A_i$$

(Parallel-axis theorem.)

$$I_{N.A.,Total} = I_{Base,Total} - \bar{h}^2 \sum A_i$$

$$= \sum\left(I_{Local,i} + A_i h_i^2\right) - \bar{h}^2 \sum A_i \qquad I_{Base,Total} = \sum\left(I_{Local,i} + A_i h_i^2\right)$$

$$= \boxed{\sum I_{Local,i}} + \boxed{\sum A_i h_i^2} - \bar{h}^2 \sum A_i$$

$$= \left(7.620e^{08} + 2.540e^{10}\right) - 873.2^2 \times 18{,}127 = 1.234e^{10} \;\; \left(cm^4\right)$$

$I_{N.A.}$: moment of inertia *of* midship section *area about* neutral axis(cm^3)

I_{Base} : moment of inertia *of* midship section *area about* baseline(cm^3)

h_i : vertical center of structural member(cm)

A_i : area of structural member(cm)

Example of Midship Scantling
- Midship Scantling for 4,100 TEU Container Ship

$$f_{2b,2d} = \frac{5.7(M_S + M_W)}{Z_{b,d}}$$

① Assumed section modulus.

- Bottom stress factor of the basis ship

$$Z_B = 2.595e^7 \ cm^3 \quad , f_{2b} = 1.030$$

- Deck stress factor of the basis ship

$$Z_D = 2.345e^7 \ cm^3 \quad , f_{2d} = 1.140$$

② Actual section modulus

- Bottom section modulus

$$Z_B = 2 \times I / y_B$$
$$= 2 \times 1.234e^{10} / 873.2$$
$$= 2.826e^7 \ (cm^3)$$

(y_B: Vertical distance from N.A to bottom=873.2cm)

Because the section modulus at bottom is larger than that of the basis ship, the stress factor should be decreased.

- Bottom Stress Factor

$$f_{2b} = \frac{5.7(M_S + M_W)}{f_1 \times Z_B}$$
$$= \frac{5.7 \times 4,924,653}{1.0 \times 2.826e^7} = 0.993$$

- Deck section modulus

$$Z_D = 2 \times I / y_D$$
$$= 2 \times 1.234e^{10} / 1,226.8$$
$$= 2.012e^7 \ (cm^3)$$

(y_D:Vertical distance from N.A to deck=2094-873.2=1,226.8cm)

Because the section modulus at deck is smaller than that of the basis ship, the stress factor should be increased. However, if HT-36 is used ,then the stress factor will be:

- Deck Stress Factor

$$f_{2d} = \frac{5.7(M_S + M_W)}{f_1 \times Z_D}$$
$$= \frac{5.7 \times 4,924,653.67}{1.39 \times 2.012e^7} = 1.004$$

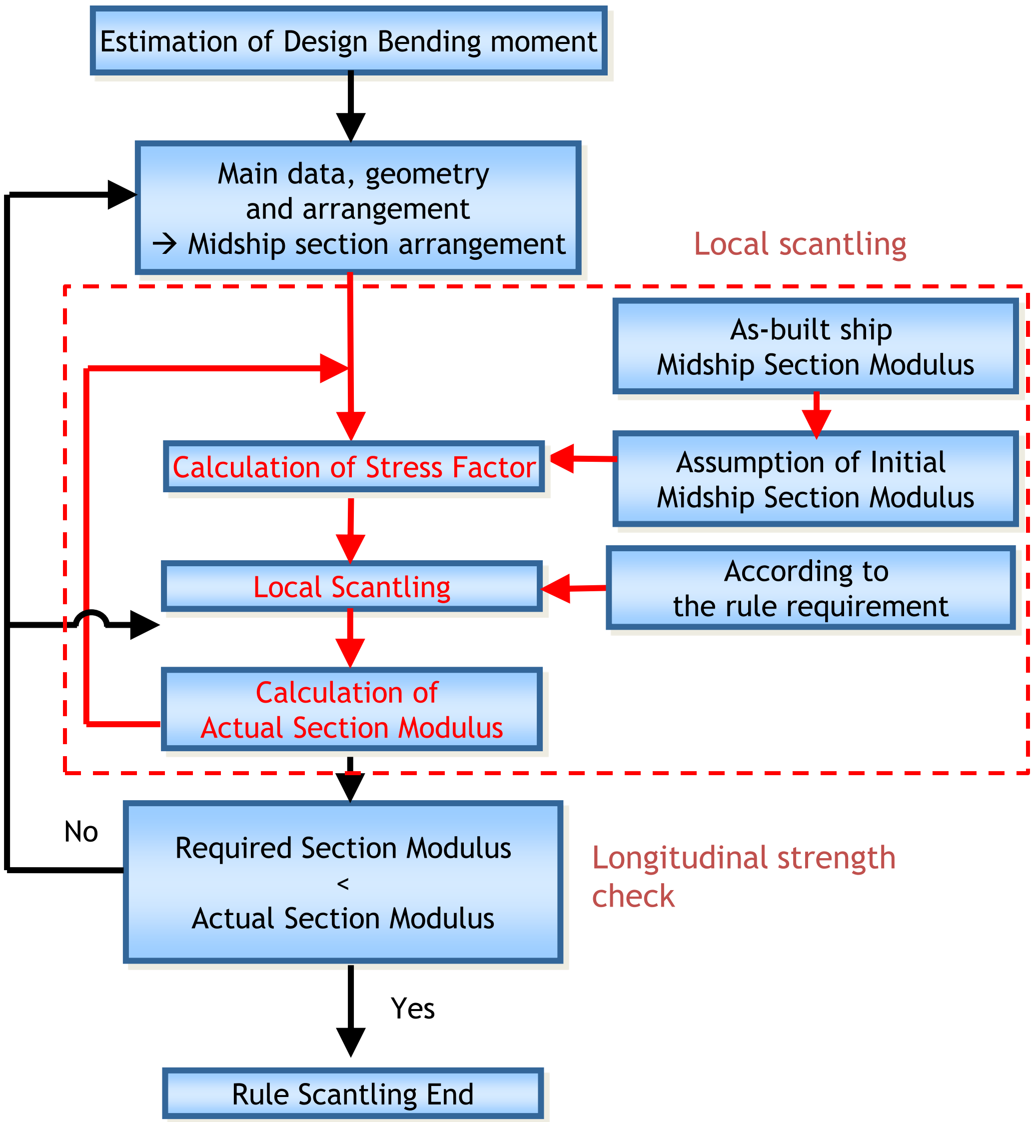
③ Because the stress factor (f_{2b}) is decreased, the allowable stress is increased.

$$\sigma = 225 f_1 - 100 f_{2b} - 0.7\sigma_{db}$$

④ Because the allowable stress is increased, the required section modulus is decreased. So, we can reduce the size of the structure member.

$$\boxed{Z = \frac{83 l^2 s p w_k}{\sigma} \ (cm^3)}$$

Example of Midship Scantling
- Midship Scantling for 4,100 TEU Container Ship



The local scantling is determined assuming that initial midship section modulus of the design ship is equal to that of the basis ship.

The midship section modulus of the basis ship:

$$Z_B = 2.595e^{07}\ cm^3 \qquad f_{2b} = 1.030$$

$$Z_D = 2.345e^{07}\ cm^3 \qquad f_{2d} = 1.140$$

The <u>actual bending stress</u>($\sigma_{act.}$) shall not be greater than the <u>allowable bending stress</u>(σ_l).

Therefore, we have to repeat the calculation.

Summary of Longitudinal Strength

Calculation of Hull Girder total shear force & bending moment

Still water shear forces Q_S
Still water bending moments M_S

(Q_S , M_S) based on the loading conditions

1. Weight curve $W(x)$
2. Buoyancy curve $B(x)$
3. Load curve $f_S(x) = W(x) + B(x)$
4. Shear force curve $Q_s = \int f_S \, dx$
5. Bending moment curve $M_S = \int Q_s \, dx$

(Q_S , M_S) Min. rule requirements

Larger value shall be used for the still water bending moment between the largest actual still water bending moment based on load conditions and design still water bending moment by rule

+

Wave Shear force Q_W
Wave Bending moment M_W

Direct Calculation (Q_W , M_W)

1. Wave Load curve
$$f_W(x) = f_D(x) + f_{F.K}(x) + f_R(x)$$
2. Vertical Wave Shear force curve
$$Q_W = \int f_W dx$$
3. Vertical Wave Bending moment curve
$$M_W = \int Q_W dx$$

Class rule (Q_W , M_W)

Direct calculation values can be used for wave shear force and wave bending moment.

Calculation of Section Modulus (Local Scantling)

Actual Bending Stress $\leq$ Allowable Bending Stress

No → Modify longitudinal structural members → Calculation of Section Modulus (Local Scantling)

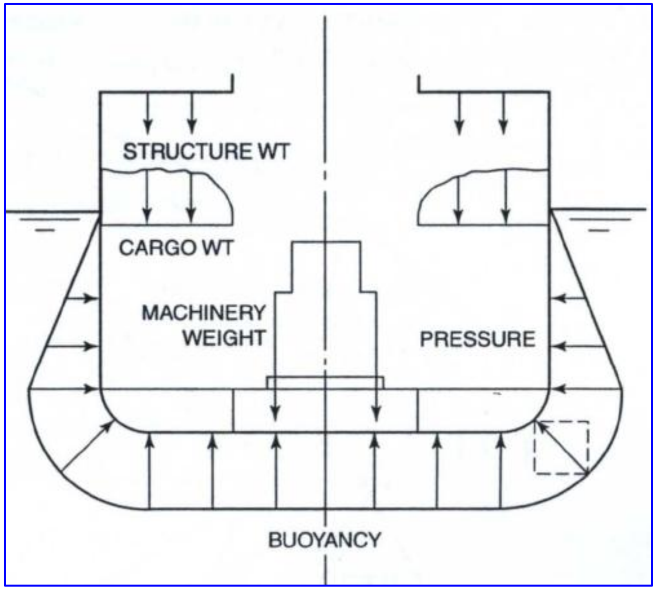
Yes → End of Design of Longitudinal strength

15-6 Local Scantling

1) Scantling of Stiffeners
2) Scantling of Plates
3) Sectional Properties of Steel Sections

Local Scantling

Ship structure members are designed to endure the loads acting on the ship structure such as hydrostatic and hydrodynamic loads.



2)

For instance, the structural member is subjected to :

Hydrostatic pressure due to surrounding water.

Internal loading due to self weight and cargo weight.

Inertia force of cargo or ballast due to ship motion.

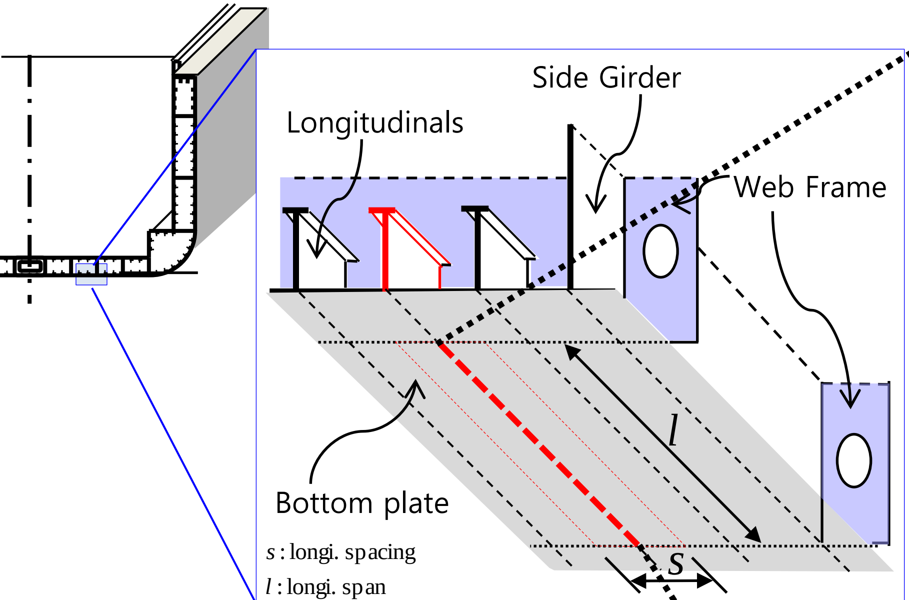
Okumoto, Y., Takeda, Y., Mano, M., Design of Ship Hull Structures, Springer, 2009, pp17-32.

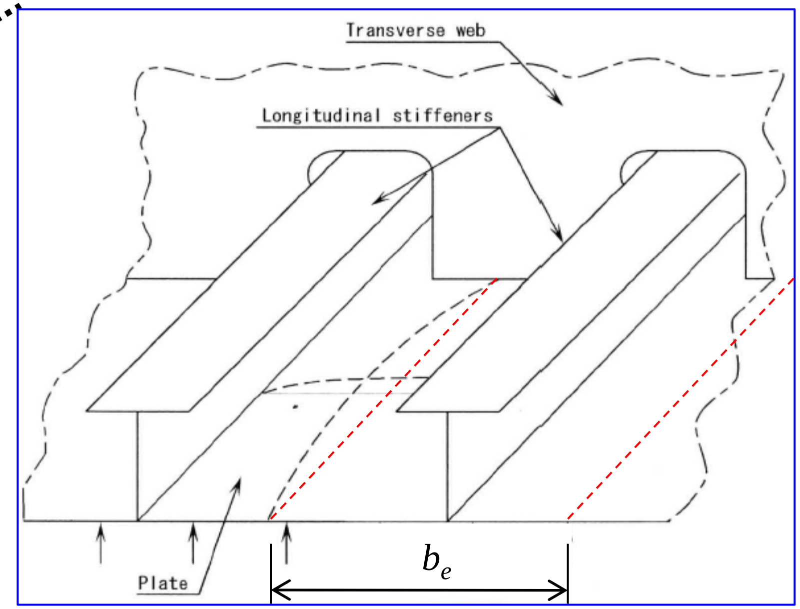
[2] Mansour, A., liu, d., the principles of naval architecture series – strength of ships and ocean structures, the society of naval architects and marine engineers, 2008

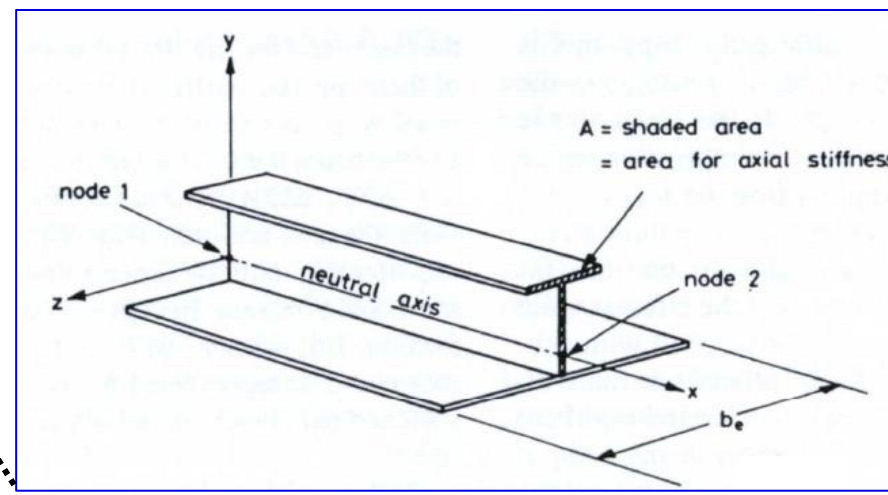
1) SCANTLING OF STIFFENERS

Scantling of Stiffeners

p : "pressure" on the load point for the stiffener



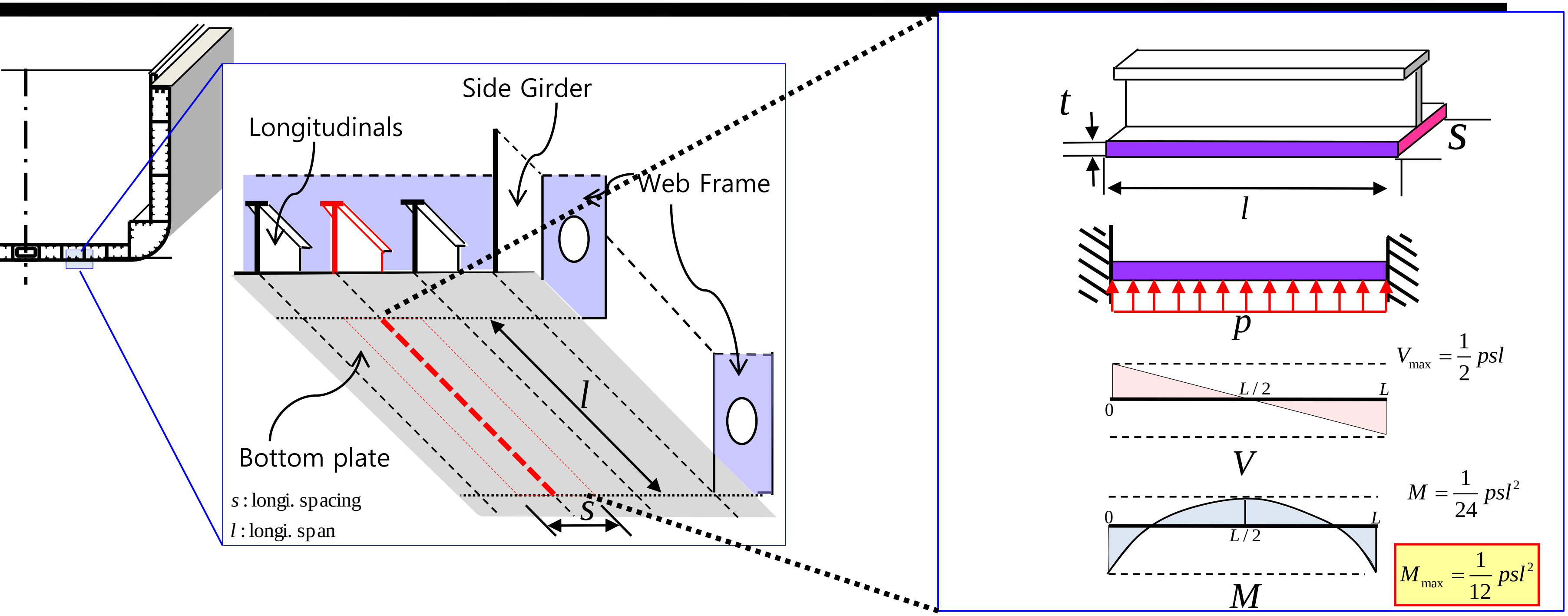




b_e : effective breadth

•Okumomto, Y., et al., design of ship hull structures – a practical guide for engineers, springer, 2009

p : "pressure" on the load point for the stiffener



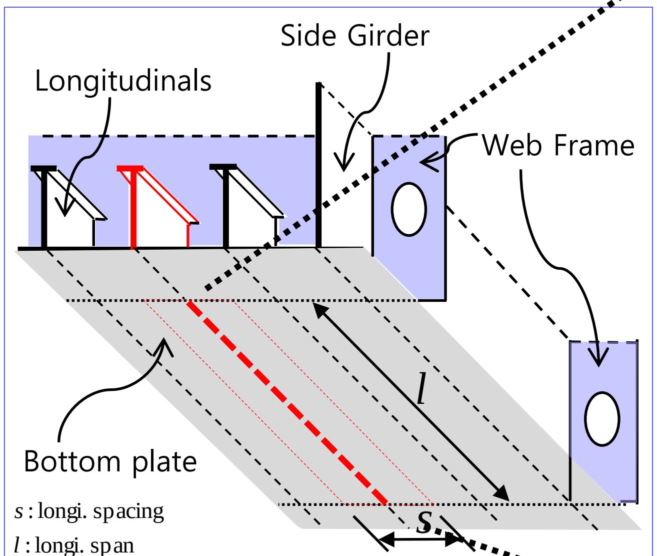
Assumption 1. Cut off the **stiffener and attached plate with effective breadth**. Sectional properties of stiffener are calculated including attached plate.

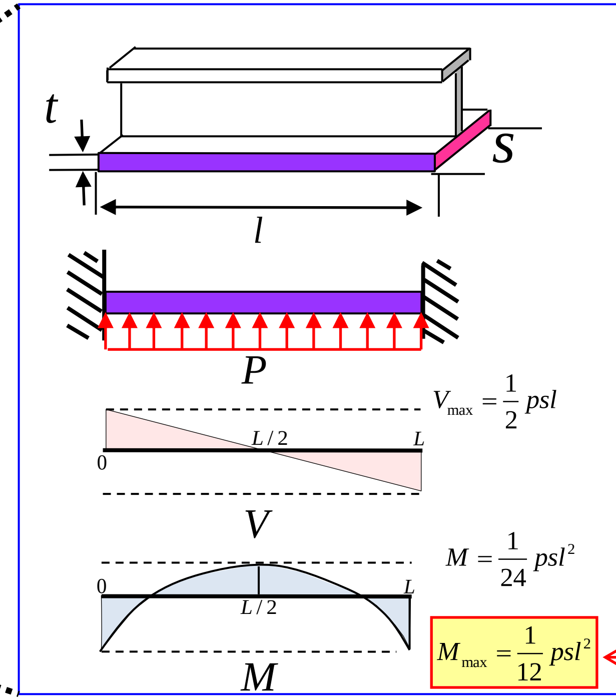
Assumption 2. Consider the stiffener and attached plate as a **"fixed-end beam"** supported by the **web frames**.

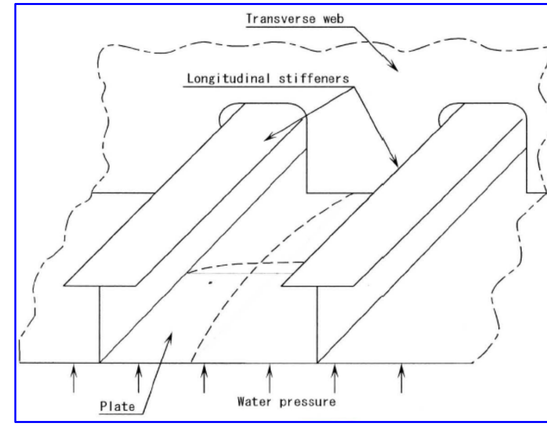
Assumption 3. Consider the **lateral load** of the beam as a **uniformly distributed load.** (Assume the **"pressure"** on the load point as an intensity of uniformly distributed load.)

Assumption 4. The design of stiffener is based on the **elastic design** (When the load is removed, the material returns to its original dimensions)

p : “pressure” on the load point for the stiffener







Relation between $p\,s$ and w

p : pressure
(load per unit area)

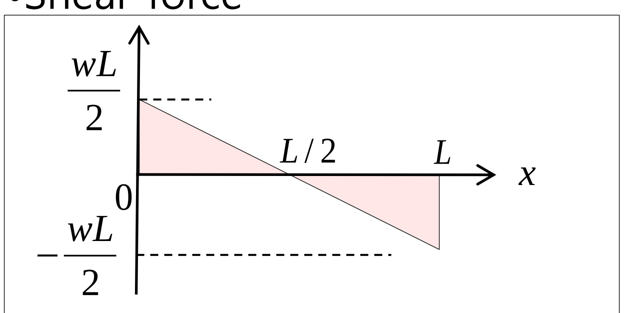
$p\,s$: distributed load
(load per unit length)

$\parallel$

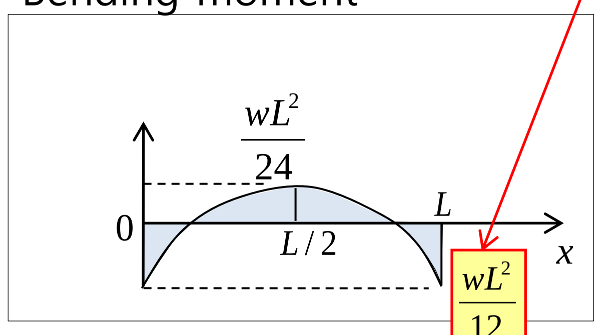
w : distributed load
(load per unit length)

$\boxed{\frac{wL^2}{12}} \xrightarrow[L=l]{w=p\cdot s} \boxed{\frac{psl^2}{12}}$ **Same!**

•Shear force



•Bending moment



Derivation of the formula for the scantling of the Stiffener

Flexure formula

$$\sigma = \frac{M}{I / y} = \frac{M}{Z}$$

Given : Bending Moment "M"

Find : Required Section Modulus "Z"

$$\boxed{\sigma \le \sigma_l}$$

$$\sigma = \sigma_l$$

Substituting into the formula:

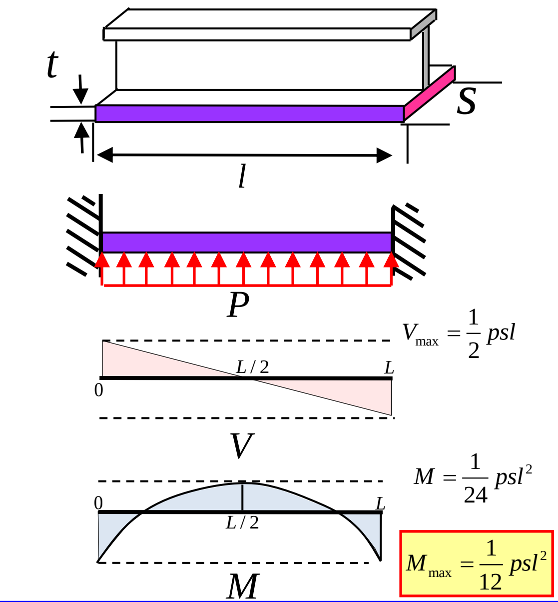
$$Z_{req.} = \frac{M}{\sigma_l}$$

Substituting into the formula:

Elastic moment(M)

$$M = \frac{p \cdot s \cdot l^2}{12}$$

$$Z_{req.} = \frac{p \cdot s \cdot l^2}{12\sigma_l}$$



For example, the allowable stress of bottom longitudinal stiffener is given by:

$$\sigma_l = 225 f_1 - 100 f_{2b} - 0.7\sigma_{db}$$

f_1 : material factor $\quad$ f_{2b} : stress factor

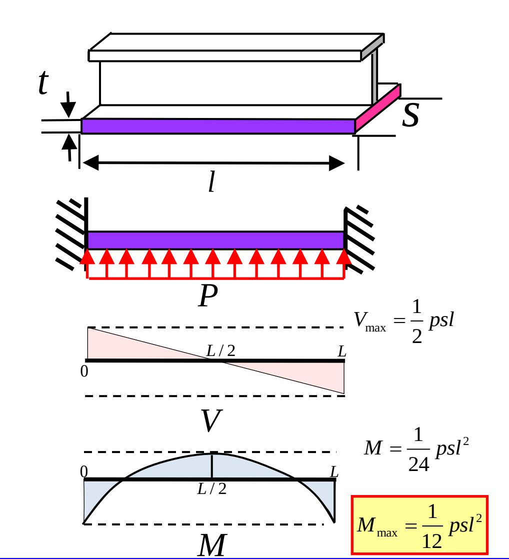
σ_{db} : mean double bottom stress

Flexure formula

$$\sigma = \frac{M}{I/y} = \frac{M}{Z}$$

Given : Moment "M"

Find : Required Section Modulus "Z"



$$Z_{req.} = \frac{p \cdot s \cdot l^2}{12\sigma_l}$$

Considering different units: $p(kN/m^2),\ s(m),\ l(m),\ \sigma(N/mm^2)$

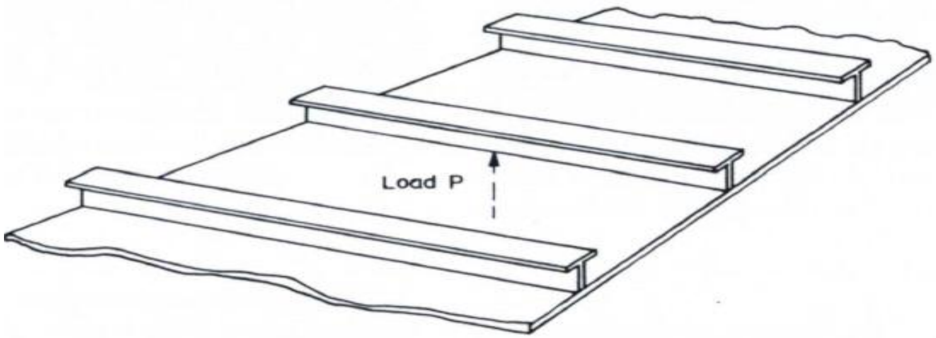
$$Z_{req.} = \frac{p \cdot s \cdot l^2}{12\sigma_l} = \frac{1}{12}\frac{p}{\sigma_l}\left(\frac{1000/1000^2[\cancel{N/mm^2}]}{1[\cancel{N/mm^2}]}\right)\frac{s \cdot l^2}{1}\left(\frac{100[cm]\cdot 100^2[cm^2]}{1}\right)$$

$$= \frac{\boxed{83}\, p \cdot s \cdot l^2}{\sigma_l}\,[cm^3]$$

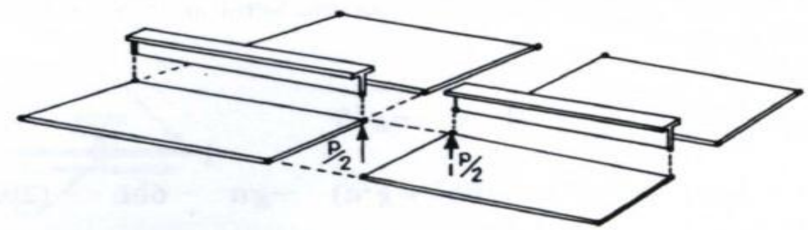
$$Z_{req.} = \frac{83 l^2 \cdot s \cdot p \cdot w_k}{\sigma_l} \quad \left(cm^3\right)$$

w_k : section modulus corrosion factor in tanks.

2) SCANTLING OF PLATES



•(0) BEAMS ATIACHED TO PLATING

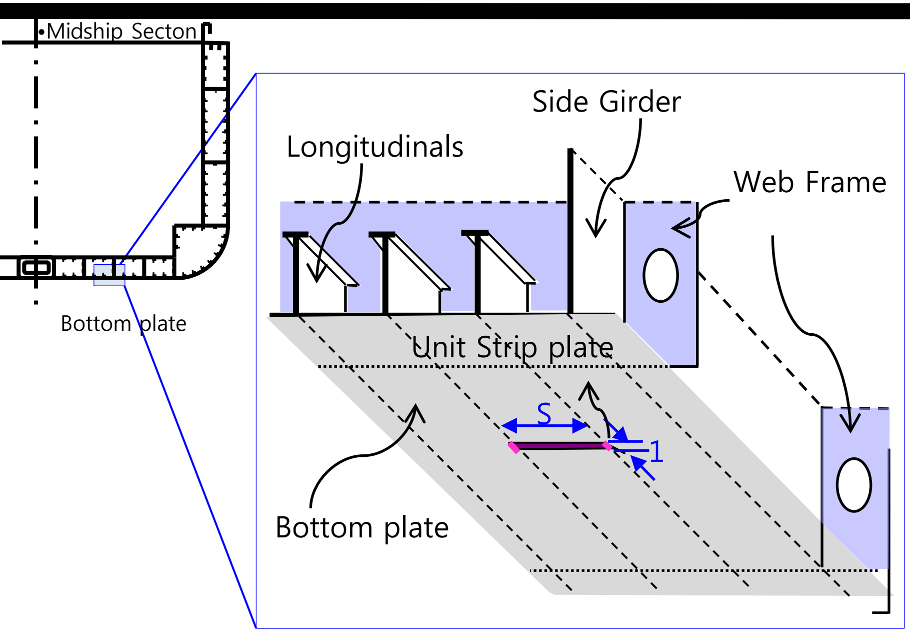


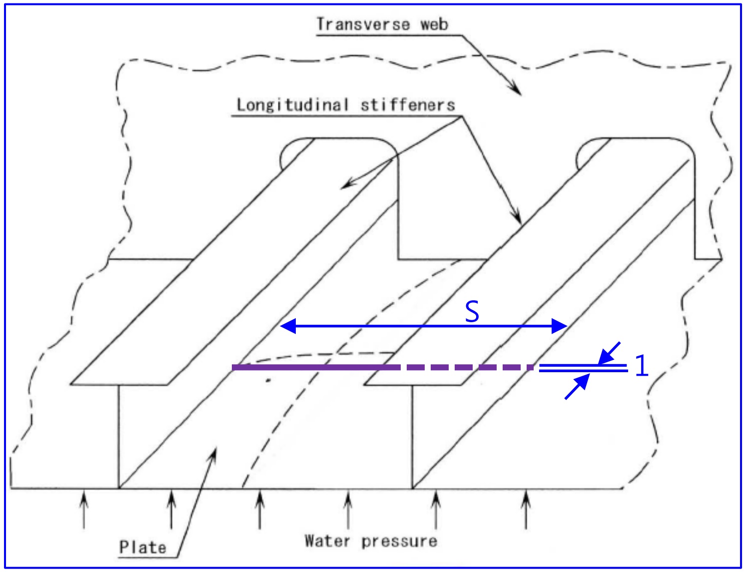
•(b) STRUCTURAL MODEL USING ECCENTRIC BEAM ELEMENT

Use of eccentric beam element:

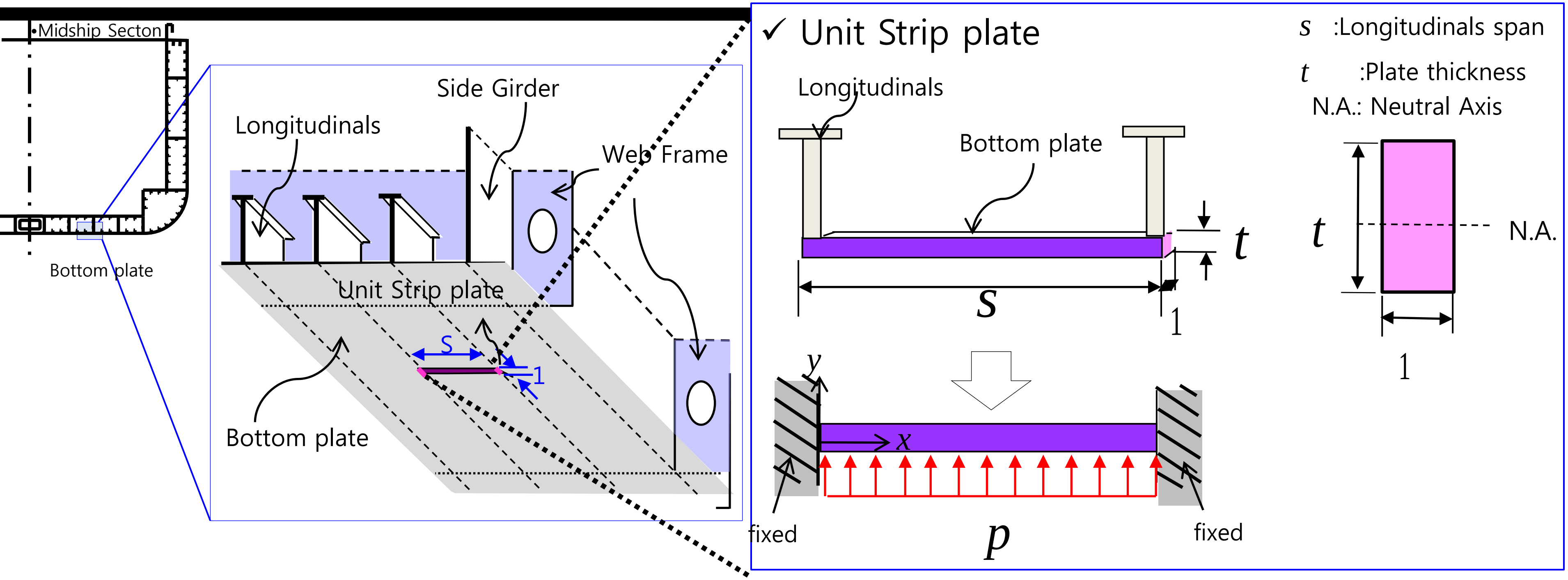
Scantling of Plates

p : "pressure" on the load point for the stiffener





p : "pressure" on the load point for the stiffener



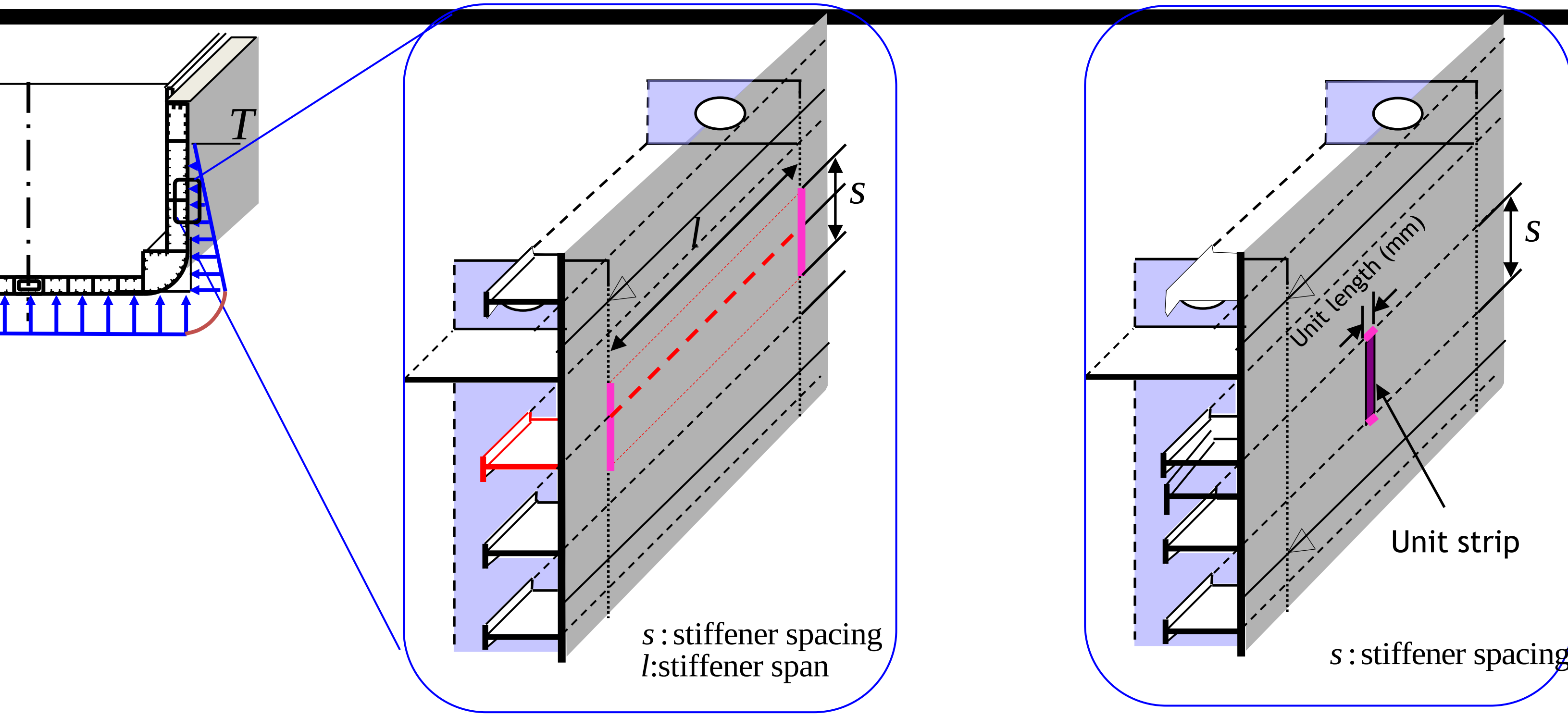
Assumption 1. Cut off the **unit strip plate** supported by the **longitudinals** or **girder.** And consider the unit strip plate as a **"fixed-end beam"** which has a span 's' , thickness 't'.

Assumption 2. Consider the lateral load of the beam as a uniformly distributed load. (Assume the pressure on the load point as an intensity of uniformly distributed load.)

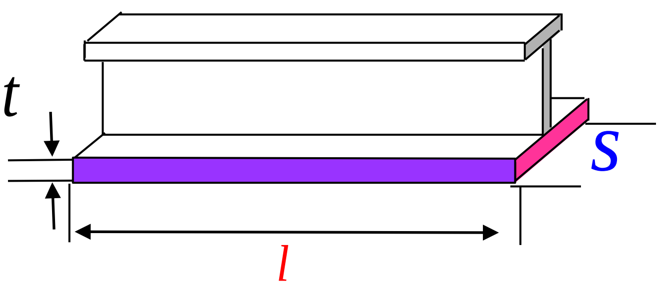
Assumption 3. The design of plates is based on the **plastic design**.

Scantling of Plates
- Comparison between Stiffener and Plate

p : “pressure” on the load point for the stiffener



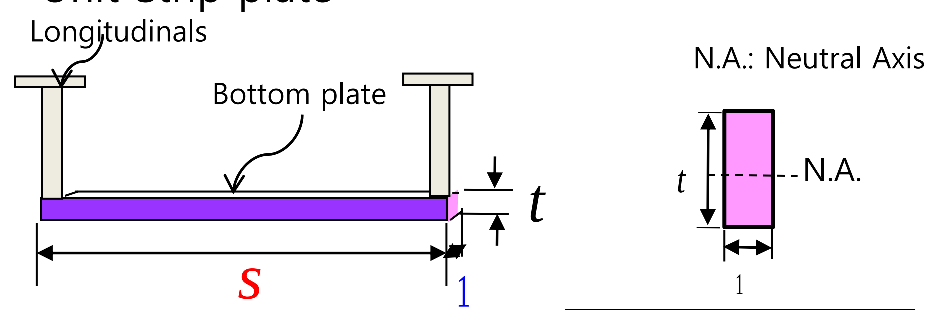
✓Longitudinal Stiffener attached to the Plate



l : Stiffener span
s : Stiffener spacing

$$M = \frac{1}{12} p \cdot s \cdot l^2$$

✓ Unit Strip plate



S : Stiffener spacing
1 : Unit length of strip

$$M_P = \frac{1}{16} p \cdot 1 \cdot s^2$$

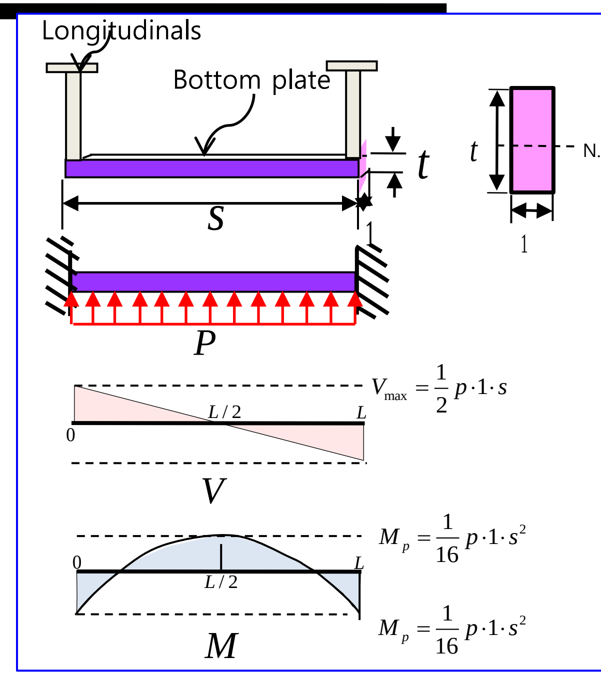
Derivation of the thickness requirement of the plates

Flexure formula

$$\sigma = \frac{M}{I/y} = \frac{M}{Z}$$

Given : Moment "M"

Find : Thickness requirement "t"



Substituting formula:

$$\boxed{\sigma \le \sigma_l}$$

$$\sigma = \sigma_l$$

$$Z_{req.} = \frac{M}{\sigma_l}$$

Substituting formula:

Plastic moment(M_p)

$$M_p = \frac{p \cdot 1 \cdot s^2}{16}$$

$$Z_{req.} = \frac{p \cdot 1 \cdot s^2}{16 \cdot \sigma_l}$$

Substituting formula:

Plastic section modulus(Z_p)

$$Z_p = \frac{1 \cdot t^2}{4} = \frac{t^2}{4}$$

$$\frac{t_{req.}^{\;2}}{4} = \frac{p \cdot 1 \cdot s^2}{16 \cdot \sigma_l} \Rightarrow t_{req.} = \frac{s\sqrt{p}}{2\sqrt{\sigma}}$$

For example, the allowable stress of bottom plating is given by:

$$\sigma_l = 120 f_1$$

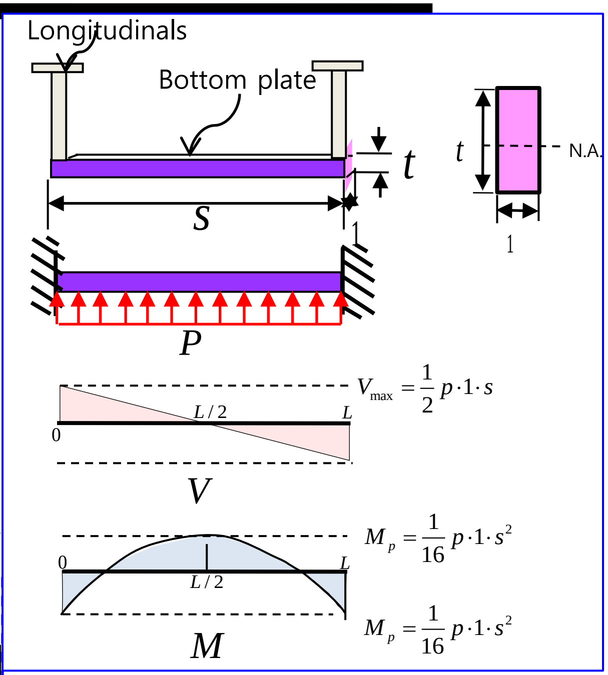
Where, f_1 **: material factor**

Flexure formula

$$\sigma = \frac{M}{I/y} = \frac{M}{Z}$$

Given : Moment "M"

Find : Thickness Requirement "t"



$$t_{req.} = \frac{s\sqrt{p}}{2\sqrt{\sigma_l}}$$

Considering different units: $t(mm),\ s(m),\ p(kN/m^2),\ \sigma(N/mm^2)$

$$t_{req.} = \frac{s\sqrt{p}}{2\sqrt{\sigma_l}} \cdot \frac{1000[mm] \cdot \sqrt{1000/1000^2\ [N/mm^2]}}{\sqrt{1\,[N/mm^2]}}\ [mm]$$

$$= \frac{\boxed{15.8}\, s\sqrt{p}}{\sqrt{\sigma_l}}$$

$$t_{req.} = \frac{15.8 k_a s\sqrt{p}}{\sqrt{\sigma_l}} + t_k \ (mm)$$

k_a = correction factor for aspect ratio of plate field

t_k = corrosion addition

Comparison of the elastic and plastic design of the plate

Flexure formula

$$\sigma = \frac{M}{I/y} = \frac{M}{Z}$$

Plastic Design

Substituting formula:

Plastic moment (M_p)

$$M_p = \frac{p \cdot 1 \cdot s^2}{16}$$

Plastic section modulus (Z_p)

$$Z_p = \frac{1 \cdot t^2}{4} = \frac{t^2}{4}$$

$$\sigma = \frac{M_p}{Z_p} = \frac{ps^2}{4t^2} \quad , t = \frac{s\sqrt{p}}{2\sqrt{\sigma}}$$

$$\text{assumption}: \sigma = \sigma_l$$

$$t_{req.} = \frac{15.8 k_a s \sqrt{p}}{\sqrt{\sigma_l}} + t_k \ (mm)$$

Elastic Design

Substituting formula:

Elastic moment (M)

$$M = \frac{p \cdot 1 \cdot s^2}{12}$$

Elastic section modulus (Z)

$$Z = \frac{1 \cdot t^2}{6} = \frac{t^2}{6}$$

$$\sigma = \frac{M}{Z} = \frac{ps^2}{2t^2} \quad , t = \frac{s\sqrt{p}}{\sqrt{2}\sqrt{\sigma}}$$

$$\text{assumption}: \sigma = \sigma_l$$

$$t_{req.} = \frac{22.4 k_a s \sqrt{p}}{\sqrt{\sigma_l}} + t_k \ (mm)$$

k_a = correction factor for aspect ratio of plate field

t_k = corrosion addition

Comparison of the Elastic and Plastic Design:
Example) Thickness requirements

Plastic moment(M_p)

$$M_p = \frac{p \cdot 1 \cdot s^2}{16}$$

Plastic section modulus(Z_p)

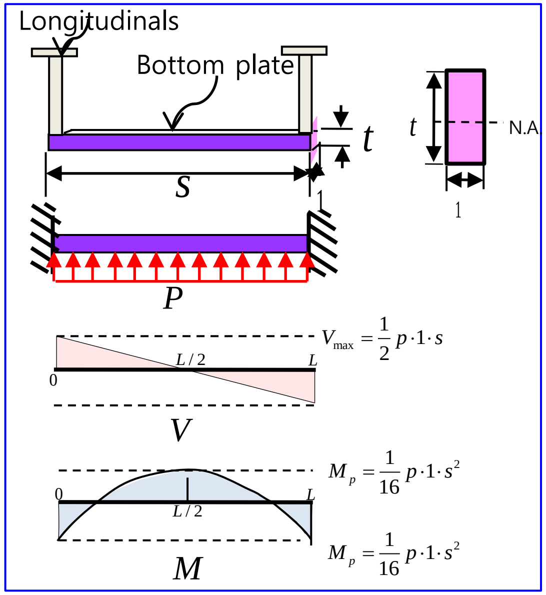
$$Z_p = \frac{1 \cdot t^2}{4} = \frac{t^2}{4}$$

Cf) Elastic moment(M)

$$M = \frac{p \cdot 1 \cdot s^2}{12}$$

Elastic section modulus(Z)

$$Z = \frac{1 \cdot t^2}{6} = \frac{t^2}{6}$$



① Ex: A mild steel plate carries the uniform pressure of 100 kN/m^2 on a span length of 800mm.
Compare the thickness requirement depending on the plastic design and elastic design.

$$t_{req.\,plastic} = \frac{15.8 k_a s \sqrt{p}}{\sqrt{\sigma_l}} = \frac{15.8 \times 1 \times 0.8 \times \sqrt{100}}{\sqrt{235}} = \boxed{8.24\ (mm)}$$

$$t_{req.\,elastic} = \frac{22.4 k_a s \sqrt{p}}{\sqrt{\sigma_l}} = \frac{22.4 \times 1 \times 0.8 \times \sqrt{100}}{\sqrt{235}} = \boxed{11.69\ (mm)}$$

The thickness requirement of the plate of plastic design is smaller than that of the elastic design at the same pressure and on the same span.

k_a = correction factor for aspect ratio of plate field

Example) Design Pressure

Plastic moment(M_p)

$$M_p = \frac{p \cdot 1 \cdot s^2}{16}$$

Plastic section modulus(Z_p)

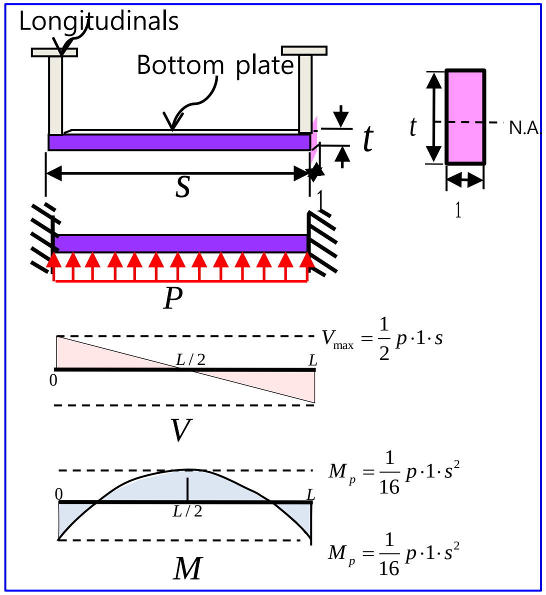
$$Z_p = \frac{1 \cdot t^2}{4} = \frac{t^2}{4}$$

Cf) Elastic moment(M)

$$M = \frac{p \cdot 1 \cdot s^2}{12}$$

Elastic section modulus(Z)

$$Z = \frac{1 \cdot t^2}{6} = \frac{t^2}{6}$$



② Ex: A mild steel plate has a thickness of 10 mm on a span length of 800 mm.
Compare the design pressure that the maximum stresses of the plate reaches the yield stress depending on the plastic design and elastic design.

$$p_{plastic} = \frac{t^2 \sigma_l}{15.8^2 s^2} = \frac{10^2 \times 235}{15.8^2 \, 0.8^2} = \boxed{147 \left(kN/m^2\right)}$$

$$p_{elastic} = \frac{t^2 \sigma_l}{22.4^2 s^2} = \frac{10^2 \times 235}{22.4^2 \, 0.8^2} = \boxed{73 \left(kN/m^2\right)}$$

The design pressure of plastic design that reaches the yield stress, is higher than that of the elastic design on the same span with the same thickness.

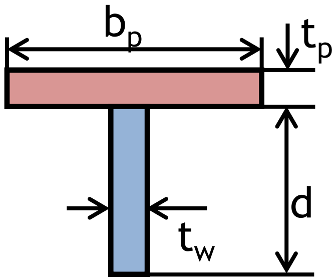
3) Sectional Properties of Steel Sections for Shipbuilding

Sectional Properties of Steel Sections for Shipbuilding [1)]

[1)] "조선설계편람", 제 4판 (일본어), 일본관서조선협회, 1996

<Sectional properties including attached plate >

(Base plate dimension : $b_p x t_p$ = 420 x 8)



b_p = breadth of plate (mm)
t_p = thickness of plate (mm)

A = Area including plate(cm^2)
Z = Section modulus including plate (cm^3)
I = Moment of inertia of area including plate(cm^4)

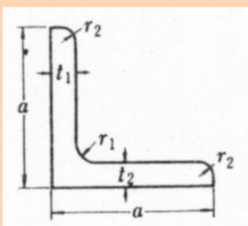
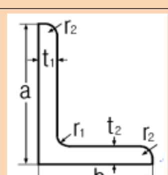
d \ t_w		6	9	11	12.7	14
50	A	3.00	4.5	5.50	6.35	7.00
	Z	6.05	8.81	10.6	12.1	13.3
	I	31.2	44.5	53.0	59.7	75.2
65	A	3.90	5.85	7.15	8.26	9.10
	Z	9.55	14.0	16.8	19.3	21.1
	I	62.3	88.8	105	119	129
75	A	4.50	6.75	8.25	9.53	10.5
	Z	12.3	18.1	21.8	25.0	27.3
	I	91.4	130	154	174	189
90	A	5.40	8.10	9.90	11.4	12.6
	Z	17.2	25.3	30.5	34.8	38.0
	I	150	214	252	284	307
100	A	6.00	9.00	11.0	12.7	14.0
	Z	20.9	30.6	37.0	42.2	46.1
	I	200	284	335	376	407
125	A	7.50	11.3	13.8	15.9	17.5
	Z	31.7	46.4	55.8	63.6	69.5
	I	370	521	612	685	738
150	A	9.00	13.5	16.5	19.1	21.0
	Z	44.7	65.2	78.3	89.1	97.2
	I	614	856	1000	1120	1200

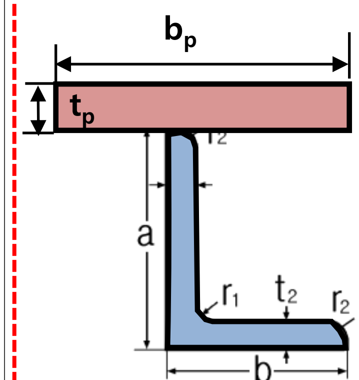
d \ t_w		16	19	22	25.4	28	32	35	38
200	A	32.0	38.0	44.03	50.8	56.0	64.0	70.0	76.0
	Z	215	259	05	359	401	469	521	576
	I	3900	4730	5600	6640	7460	8790	9830	10900
250	A	40.0	47.5	55.0	63.5	70.0	80.0	87.5	95.0
	Z	325	390	458	536	597	694	769	845
	I	7120	8600	10100	11900	13400	15600	17400	19200
300	A	48.0	57.0	66.0	76.2	84.0	96.0	105.0	114.0
	Z	455	546	639	746	829	961	1060	1160
	I	11700	14000	16500	19300	21600	25100	27800	30700
350	A	56.0	66.5	77.0	88.9	98.0	112.0	122.5	133.0
	Z	606	726	847	988	1100	1270	1400	1530
	I	17700	21200	24800	29100	32400	37600	41600	45700
400	A	64.0	76.0	88.0	101.6	112.0	128.0	140.0	152.0
	Z	776	928	1080	1260	1400	1610	1780	1940
	I	25300	30300	35400	41400	46000	53300	58900	64600
450	A	72.0	85.5	99.0	114.3	126.0	144.0	157.5	171.0
	Z	965	1150	1340	1560	1730	2000	2200	2400
	I	34700	41500	48500	56500	62800	72600	80100	87700
500	A	80.0	95.0	110.0	127.0	140.0	160.0	175.0	190.0
	Z	1170	1400	1630	18907	2100	2420	2660	2900
	I	46000	55000	64200	4700	82900	95700	10500	11500

$A = 42*0.8 + 15*1.4 = 21\ [cm^2]$
$Z_{Top} = 349.6\ [cm^3]$
$Z_{Bottom} = 97.2\ [cm^3]$

< Sectional Properties of Steel Sections including attached plate>

- Use the standard dimension of plate depending on “a” ($b_p \times t_p$) => ($a \le 75$: 420×8, $75 < a < 150$: 610×10, $150 \le a$: 610×15)

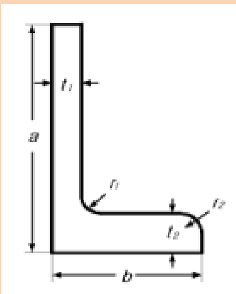
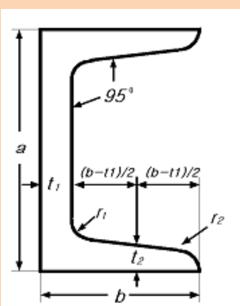
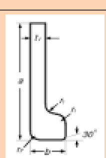
Symbol	a	b	t_1	t_2	r_1	r_2	A	I (Including plate)	Z (Including plate)
Unit	mm						cm^2	cm^4	cm^3
Equal Angle	ㄴ								
	50		6		6.5	4.5	5.64		
	65		6		8.5	4	7.53		
	65		8		8.5	6	9.76		
	75		6		8.5	4	8.73		
	75		9		8.5	6	12.69	90.1	18.7
	75		12		8.5	6	16.56	191	31.9
	90		10		10	7	17.00	229	39.7
	90		13		10	7	21.71	284	42.5
	100		10		10	7	19.00	369	58.2
	100	‘‘	13	‘‘	10	7	24.31	433	71.6
	130		9		12	6	11.74	767	96.0
	130		12		12	8.5	19.76	905	117
	130		15		12	8.5	36.75	1030	119
	150		12		14	7	34.77	1220	147
	150		15		14	10	42.74		
	150		19		14	10	53.38		
	200		20		17	12	76.00		
	200		25		17	12	93.75		
	200		29		17	12	107.6		
Unequal angle	ㄴ								
	100	75	7		10	5	11.87	674	72.5
	100	75	10		10	7	16.50	860	96.2
	125	75	7	‘‘	10	5	13.62	110	97.2
	125	75	10		10	7	19.00	1420	130
	150	90	9		12	6	20.94	2490	181
	150	90	12		12	8.5	27.36	3060	230

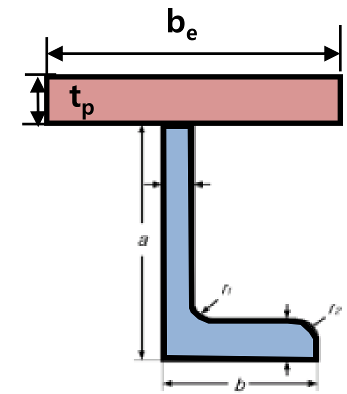


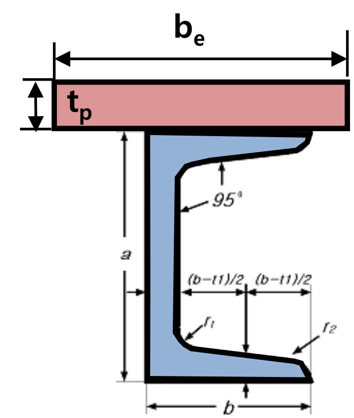
< Sectional Properties of Steel Sections including attached plate>

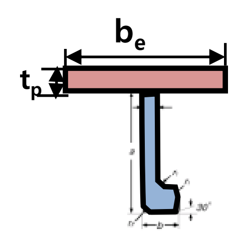
- Use the standard dimension of plate depending on "a" ($b_p \times t_p$) => (a≤75 : 420×8,
- 75<a<150 : 610×10, 150≤a : 610×15)

1) "조선설계편람", 제 4판 (일본어), 일본관서조선협회, 1996

	Dimension						Area	Including plate	
Symbol	a	b	t_1	t_2	r_1	r_2	A	I	Z
Unit	mm						cm^2	cm^4	cm^3
Unequal Angle									
	200	90	9	14	14	7.0	29.66	5870	340
	250	90	10	15	17	8.5	37.47	10300	494
	250	90	12	16	17	8.5	42.95	11000	540
	300	90	11	16	19	9.5	46.22	16400	681
	300	90	13	17	19	9.5	52.67	17600	743
	400	100	11.5	16	24	12	61.09	34200	1120
	400	100	13	18	24	12	68.59	36700	1230
	450	125	11.5	18	24	12	73.11	51200	1570
	450	150	11.5	15	24	12	73.45	51700	1590
	500	150	11.5	18	24	12	83.6	70400	2020
	550	150	12	21	24	12	95.91	93300	2520
	600	150	12.5	23	24	12	107.6	118000	3000
Channels									
	150	75	6.5	10	10	5	23.71	2160	154
	200	90	8	13.5	14	7	38.65	5650	322
	250	90	9	13	14	7	44.07	9420	439
	250	90	11	14.5	17	8.5	51.17	10500	499
	300	90	9	13	14	7	48.57	14300	567
	300	90	10	15.5	19	9.5	55.74	16000	646
	300	90	12	16	19	9.5	61.90	16900	693
	380	100	10.5	16	18	9	69.39	29900	989
	380	100	13	20	24	12	85.71	34900	1190
Bulb flats									
	180	32.5	9.5	–	7	2	21.06	2860	172
	200	36.5	10	–	8	2	25.23	4160	231
	230	41	11	–	9	2	31.98	6610	330
	250	45	12	–	10	2	38.13	8960	424



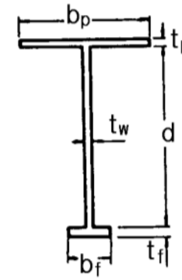




<판을 포함한 조립형강재의 단면계수

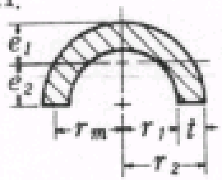
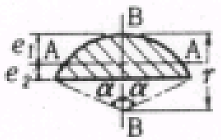
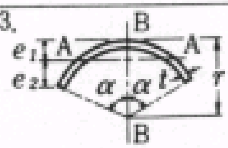
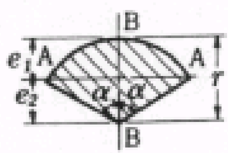
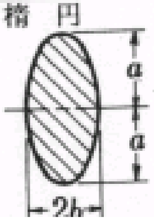
($b_p x t_p$ = 610 x 15)

$d \times t_w$ \ $b_f \times t_f$		100 ×16	125 ×16	150 ×16	150 ×19	150 ×22	150 ×25·4	180 ×25·4	200 ×25·4	200 ×28	200 ×32	230 ×32	230 ×35	230 ×38
	A_f	16·0	20·0	24·0	28·5	33·0	38·1	45·6	50·8	56·0	64·0	73·6	80·5	87·4
300 × 11·5	A	50·5	54·5	58·5	63·0	67·5	72·6							
	Z	775	890	1000	1130	1250	1390							
	I	19400	21600	23800	26300	28700	31300							
350 × 11·5	A	56·3	60·3	64·3	68·8	73·3	78·4							
	Z	955	1090	·1220	1360	1500	1660							
	I	27100	30100	32900	36100	39300	42700							
400 × 11·5	A	62·0	66·0	70·0	74·5	79·0	84·1							
	Z	1150	1300	1450	1610	1770	1950							
	I	36500	40200	43800	47900	51800	56100							
450 × 11·5	A	67·8	71·8	75·8	80·3	84·8	89·9							
	Z	1350	1520	1690	1870	2050	2250							
	I	47600	52200	56500	61500	66300	71500							
500 × 11·5	A	73·5	77·5	81·5	86·0	90·5	95·6							
	Z	1570	1760	1940	2140	2340	2560							
	I	60400	65900	71200	77100	82900	89200							
550 × 12	A	82·0	86·0	90·0	94·5	99·0	104·1							
	Z	1840	2040	2240	2460	2680	2920							
	I	76300	82700	88900	95800	103000	110000							
600 × 12·7	A	92·2	96·2	100·2	104·7	109·2	114·3	121·9	127·0	132·2	140·2	149·8	156·7	163·6
	Z	2150	2370	2590	2820	3050	3310	3720	3990	4260	4660	5170	5510	5850
	I	95300	103000	110000	118000	125000	134000	145000	152000	160000	171000	182000	190000	198000
650 × 12·7	A	98·6	102·6	106·6	111·1	115·6	120·7	128·3	133·4	138·6	146·6	156·2	163·1	170·0
	Z	2430	2660	2890	3140	3390	3670	4110	4410	4690	5130	5670	6050	6420
	I	115000	123000	131000	141000	149000	159000	172000	180000	189000	202000	215000	224000	234000
700 × 12·7	A	104·9	108·9	112·9	117·4	121·9	127·0	134·6	139·7	144·9	152·9	162·5	169·4	176·3
	Z	2720	2960	3210	3480	3750	4050	4520	4830	5140	5610	6190	6590	6990
	I	137000	146000	156000	166000	176000	187000	202000	211000	221000	236000	251000	262000	272000
700 × 16	A	128·0	132·0	136·0	140·5	145·0	150·1	157·7	162·8	168·0	176·0	185·6	192·5	199·4
	Z	3070	3310	3560	3820	4070	4370	4830	5130	5430	5890	6460	6850	7230
	I	150000	159000	168000	178000	187000	198000	212000	222000	231000	245000	260000	271000	282000
800 × 12·7	A	117·6	121·6	125·6	130·1	134·6	139·7	147·3	152·4	157·6	165·6	175·2	182·1	189·0
	Z	3330	3610	3890	4200	4500	4840	5370	5720	6080	6600	7260	7720	8170
	I	188000	200000	211000	224000	237000	251000	270000	282000	294000	313000	332000	346000	360000
800 × 16	A	144·0	148·0	152·0	156·5	161·0	166·1	173·7	178·8	184·0	192·0	201·6	208·5	215·4
	Z	3780	4060	4330	4630	4920	5250	5770	6110	6450	6960	7610	8050	8490
	I	207000	218000	229000	242000	354000	267000	285000	297000	309000	327000	346000	360000	374000
900 × 14	A	142·0	146·0	150·0	154·5	159·0	164·1	171·7	176·8	182·0	190·0	199·6	206·5	213·4
	Z	4220	4530	4840	5170	5500	5880	6460	6860	7240	7820	8550	9050	9550
	I	259000	274000	287000	303000	318000	335000	358000	373000	388000	410000	435000	452000	469000
900 × 18	A	178·0	182·0	186·0	190·5	195·0	200·1	207·7	212·8	218·0	226·0	235·6	242·5	249·4
	Z	4880	5190	5490	5810	6130	6490	7060	7440	7810	8370	9090	9570	10100
	I	290000	304000	317000	332000	346000	362000	384000	399000	413000	435000	459000	476000	493000
1000 × 16	A	176·0	180·0	184·0	188·5	193·0	198·1	205·7	210·8	216·0	224·0	233·6	240·5	247·4
	Z	5390	5730	6070	6420	6780	7180	7820	8250	8660	9290	10100	10600	11200
	I	355000	372000	388000	407000	424000	444000	471000	489000	507000	534000	563000	584000	605000
1000 × 19	A	206·0	210·0	214·0	218·5	223·0	228·1	235·7	240·8	246·0	254·0	263·6	270·5	277·4
	Z	5990	6320	6650	7000	7340	7740	8360	8780	9180	9800	10600	11100	11600
	I	386000	402000	418000	435000	452000	472000	498000	515000	533000	559000	588000	609000	630000



단위 mm

A_f = 면재의 단면적(cm^2)
A = 판을 포함한 단면적(cm^2)
Z = 판을 포함한 단면계수(cm^3)
I = 판을 포함한 단면 2차 모멘트(cm^4)

断 面 形	A	I	Z, e	Z_P
11. 	$\frac{1}{2}\pi(r_2^2-r_1^2)$ t/r_mが小さいとき $A_{rm}=\pi r_m t$	$\left(\frac{\pi}{8}-\frac{8}{9\pi}\right)(r_2^4-r_1^4)$ $-\frac{8r_2^2r_1^2(r_2-r_1)}{9\pi(r_2+r_1)}$ $I_{rm}=\left(\frac{\pi}{2}-\frac{4}{\pi}\right)r_m^3t$ $\fallingdotseq 0{\cdot}2976\,r_m^3t$	$e_1=r_2-e_2$ $e_2=\frac{4(r_2^2+r_2r_1+r_1^2)}{3\pi(r_2+r_1)}$ $e_{2rm}=\frac{2}{\pi}r_m\fallingdotseq 0{\cdot}6366\,r_m$	$2[2(r_2^3\sin^3\theta_2$ $-r_1^3\sin^3\theta_1)$ $-(r_2^3-r_1^3)]/3$ ここに, $r_1\cos\theta_1=r_2\cos\theta_2$
12. 	$\frac{1}{2}r^2(2\alpha-\sin 2\alpha)$	$I_A=r^4\left[\frac{1}{16}(4\alpha-\sin 4\alpha)-\frac{8\sin^6\alpha}{9(2\alpha-\sin 2\alpha)}\right]$ $I_B=\frac{r^4}{12}\left[3\alpha-2\sin 2\alpha+\frac{1}{4}\sin 4\alpha\right]$	$e_1=r\left(1-\frac{4\sin^3\alpha}{6\alpha-3\sin 2\alpha}\right)$ $e_2=r\left(\frac{4\sin^3\alpha}{6\alpha-3\sin 2\alpha}-\cos\alpha\right)$	$\frac{2}{3}r^3(2\sin^3\alpha_0-\sin^3\alpha)$ ここに, $\frac{2\alpha-\sin 2\alpha}{2\alpha_0-\sin 2\alpha_0}=4$
13. 	$2\alpha rt$	$I_A=r^3t(\alpha+\sin\alpha\cos\alpha$ $-2\frac{\sin^2\alpha}{\alpha})$ $I_B=r^3t(\alpha-\sin\alpha\cos\alpha)$	$e_1=r\left(1-\frac{\sin\alpha}{\alpha}\right)$ $e_2=r\left(\frac{\sin\alpha}{\alpha}-\cos\alpha\right)$	$2rt(r-t/2)$ $\times\left(2\sin\frac{\alpha}{2}-\sin\alpha\right)$
14. 	αr^2	$I_A=\frac{1}{4}r^4(\alpha+\sin\alpha\cos\alpha$ $-\frac{16\sin^2\alpha}{9\alpha})$ $I_B=\frac{1}{4}r^4(\alpha-\sin\alpha\cos\alpha)$	$e_1=r\left(1-\frac{2\sin\alpha}{3\alpha}\right)$ $e_2=r\frac{2\sin\alpha}{3\alpha}$	$\alpha>0{\cdot}996$, $(2\alpha'-\sin 2\alpha'=\alpha)$ $2r^3(2\sin\alpha'-\sin\alpha)/3$ $\alpha<0{\cdot}996$ $\frac{2r^3}{3}\left[\sin\alpha-\sqrt{\frac{\alpha^3}{2\tan\alpha}}\right]$
15. 楕 円 	πab	$\frac{\pi}{4}a^3b\fallingdotseq 0{\cdot}7854\,a^3b$	$\frac{\pi}{4}a^2b\fallingdotseq 0{\cdot}7854\,a^2b$	$\frac{4}{3}a^2b$

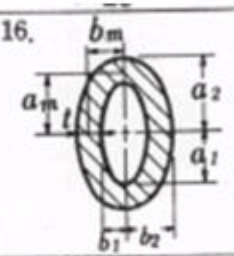
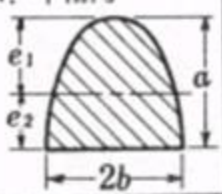
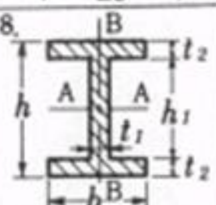
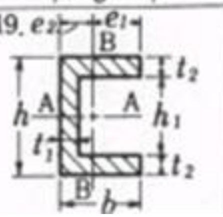
断 面 形	A	I	Z, e	Z_r
16. 	$\pi(a_2b_2-a_1b_1)$ t/a_m, t/b_m が小さいとき $A_m=\pi(a_m+b_m)t$	$\frac{\pi}{4}(a_2^3b_2-a_1^3b_1)$ $I_m=\frac{\pi}{4}a_m^2(a_m+3b_m)t$	$\frac{\pi}{4}\,\frac{a_2^3b_2-a_1^3b_1}{a_2}$ $Z_m=\frac{\pi}{4}a_m(a_m+3b_m)t$	$\frac{4}{3}(a_2^2b_2-a_1^2b_1)$
17. 半楕円 	$\frac{1}{2}\pi ab$	$\left(\frac{\pi}{8}-\frac{8}{9\pi}\right)a^3b$ $\fallingdotseq 0{\cdot}1098\,a^3b$	$e_1=\left(1-\frac{4}{3\pi}\right)a\fallingdotseq 0{\cdot}5756a$ $Z_1\fallingdotseq 0{\cdot}1908\,a^2b$ $e_2=\frac{4r}{3\pi}\fallingdotseq 0{\cdot}4244\,a$ $Z_2\fallingdotseq 0{\cdot}2587\,a^2b$	$\fallingdotseq 0{\cdot}35362\,a^2b$
18. 	$2bt_2+h_1t_1$	$I_A=\frac{bh^3-(b-t_1)h_1^3}{12}$ $I_B=\frac{2b^3t_2+h_1t_1^3}{12}$	$Z_A=\frac{bh^3-(b-t_1)h_1^3}{6h}$ $Z_B=\frac{2b^3t_2+h_1t_1^3}{6b}$	$\frac{h_1^2t_1}{4}+\frac{bt_2}{2}(h+h_1)$
19. 	$2bt_2+h_1t_1$	$I_A=\frac{bh^3-(b-t_1)h_1^3}{12}$ $I_B=\frac{2b^3t_2+h_1t_1^3}{3}-Ae_2^2$	$e_1=b-e_2$ $e_2=\frac{2b^2t_2+h_1t_1^2}{4bt_2+2h_1t_1}$	18. と同じ

表 27　各種断面の断面積 A，断面 2 次モーメント I，断面係数 Z 及び塑性断面係数 Z_p

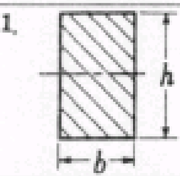
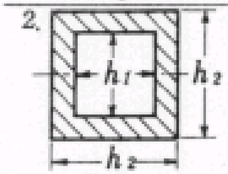
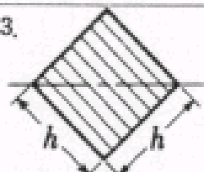
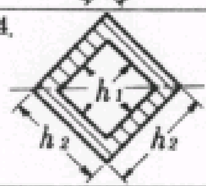
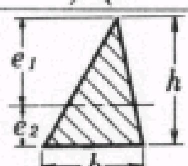
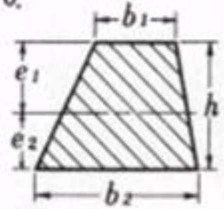
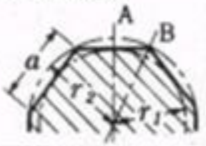
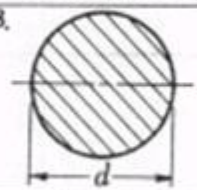
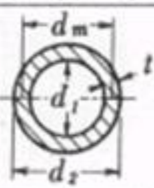
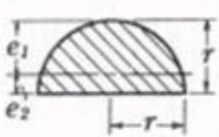
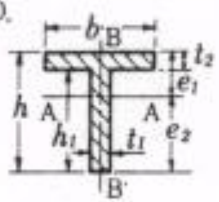
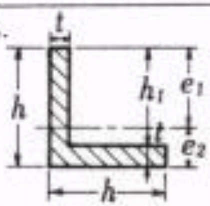
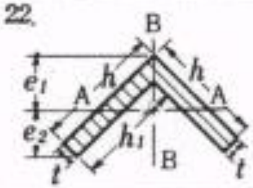
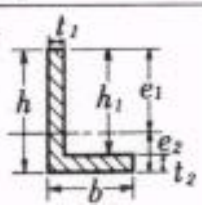
断　面　形	A	I	Z，e	Z_p
1. 	bh	$\frac{1}{12}bh^3$	$\frac{1}{6}bh^2$	$\frac{1}{4}bh^2$
2. 	$h_2^2-h_1^2$	$\frac{1}{12}(h_2^4-h_1^4)$	$\frac{1}{6}\ \frac{h_2^4-h_1^4}{h_2}$	$\frac{1}{4}(h_2^3-h_1^3)$
3. 	h^2	$\frac{1}{12}h^4$	$\frac{\sqrt{2}}{12}h^3$	$\frac{\sqrt{2}}{6}h^3$
4. 	$h_2^2-h_1^2$	$\frac{1}{12}(h_2^4-h_1^4)$	$\frac{\sqrt{2}}{12}\ \frac{h_2^4-h_1^4}{h_2}$	$\frac{\sqrt{2}}{6}(h_2^3-h_1^3)$
5. 	$\frac{1}{2}bh$	$\frac{1}{36}bh^3$	$e_1=\frac{2}{3}h,\ Z_1=\frac{bh^2}{24}$ $e_2=\frac{1}{3}h,\ Z_2=\frac{bh^2}{12}$	$\frac{2-\sqrt{2}}{6}bh^2$

表 27　各種断面の断面積 A，断面 2 次モーメント I，断面係数 Z 及び塑性断面係数 Z_p

断　面　形	A	I	Z，e	Z_p
6. 	$\frac{1}{2}(b_1+b_2)h$	$\frac{h^3(b_1^2+4b_1b_2+b_2^2)}{36(b_1+b_2)}$	$e_1=\frac{h(b_1+2b_2)}{3(b_1+b_2)}$ $Z_1=\frac{h^2(b_1^2+4b_1b_2+b_2^2)}{12(b_1+2b_2)}$ $e_2=\frac{h(2b_1+b_2)}{3(b_1+b_2)}$ $Z_2=\frac{h^2(b_1^2+4b_1b_2+b_2^2)}{12(2b_1+b_2)}$	$\frac{Ah}{3}\frac{(b_1b_2+b_2b_3+b_3b_1)}{(b_1+b_3)(b_2+b_3)}$ ここに， $b_3^2=(b_1^2+b_2^2)/2$
7. 正 n 角形 	$\frac{1}{2}nar_1$	$\frac{A}{24}(6r_2^2-a^2)$ $=\frac{A}{48}(12r_1^2+a^2)$	$Z_A=\frac{A}{48r_1}(12r_1^2+a^2)$ $Z_B=\frac{A}{24r_2}(6r_2^2-a^2)$	n：偶数，$Z_{PA}=\frac{a^2r_1}{6}$ $+\frac{2}{3}ar_1^2\sum_{k=1}^{\frac{n}{2}-1}\sin\frac{2k\pi}{n}$
8. 	$\frac{1}{4}\pi d^2$	$\frac{1}{64}\pi d^4$	$\frac{1}{32}\pi d^3$	$\frac{1}{6}d^3$
9. 	$\frac{1}{4}\pi(d_2^2-d_1^2)$ t/d_mが小さいとき $A_{dm}=\pi d_m t$	$\frac{1}{64}\pi(d_2^4-d_1^4)$ $I_{dm}=\frac{1}{8}\pi d_m^3 t$	$\frac{\pi}{32}\ \frac{d_2^4-d_1^4}{d_2}$ $Z_{dm}=\frac{1}{4}\pi d_m^2 t$	$\frac{1}{6}(d_2^3-d_1^3)$
10. 	$\frac{1}{2}\pi r^2$	$\left(\frac{\pi}{8}-\frac{8}{9\pi}\right)r^4$ $\fallingdotseq 0\cdot1098\,r^4$	$e_1=\left(1-\frac{4}{3\pi}\right)r\fallingdotseq 0\cdot5756r$ $Z_1\fallingdotseq 0\cdot1908\,r^3$ $e_2=\frac{4r}{3\pi}\fallingdotseq 0\cdot4244\,r$ $Z_2\fallingdotseq 0\cdot2587\,r^3$	$\fallingdotseq 0\cdot37982\,r^3$

断　面　形	A	I	$Z,\ e$	Z_r
20. 	$bt_2+h_1t_1$	$I_A=\frac{h^3t_1+(b-t_1)t_2^3}{3}-Ae_1^2$ $I_B=\frac{b^3t_2+h_1t_1^3}{12}$	$e_1=\frac{h^2t_1+(b-t_1)t_2^2}{2(bt_2+h_1t_1)}$ $e_2=h-e_1$	$t_2\leqq h_1t_1/b$ のとき $\frac{bt_2}{2}\left(h-\frac{t_2}{t_1}b\right)+\frac{h_1t_1}{4}\left[h_1+\left(\frac{t_2}{t_1}\right)^2\times\left(\frac{b}{h_1}\right)b\right]$ $t_2>h_1t_1/b$ のとき $\frac{bt_2^2}{4}\left[1-\left(\frac{h_1t_1}{bt_2}\right)^2\right]+\frac{h\,h_1t_1}{2}$
21. 	$(h+h_1)t$	$\frac{t}{3}(h^3+h_1t^2)-Ae_2^2$	$e_1=h-e_2$ $e_2=\frac{h^2+h_1t}{2(h+h_1)}$	$\frac{t}{4}[(h-t)^2+h^2]$
22. 	$(h+h_1)t$	$I_A=\frac{(h+t)^4}{24}-\frac{h_1^4+2t^4}{24}-Ae_2^2$ $I_B=\frac{1}{12}(h^4-h_1^4)$	$e_1=\frac{h^2+h_1t}{\sqrt{2}(h+h_1)}$ $e_2=\frac{h^2}{\sqrt{2}(h+h_1)}$	$\frac{t}{\sqrt{2}}[h(h-t)+t^2]$
23. 	$bt_2+h_1t_1$	$\frac{h^3t_1+(b-t_1)t_2^3}{3}-Ae_2^2$	$e_1=h-e_2$ $e_2=\frac{h^2t_1+(b-t_1)t_2^2}{2(bt_2+h_1t_1)}$	20. と同じ

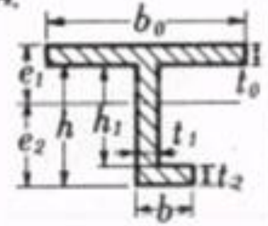
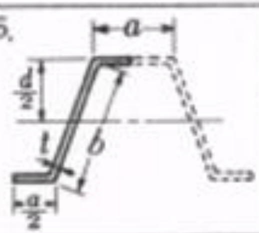
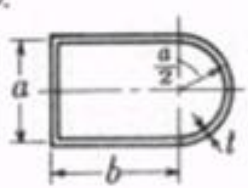
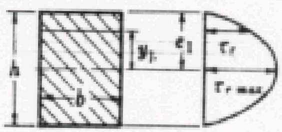
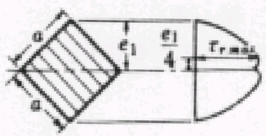
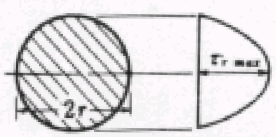
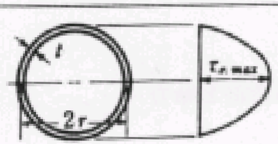
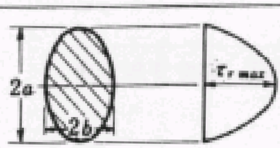
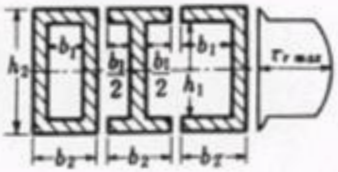
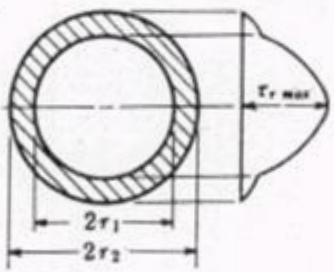
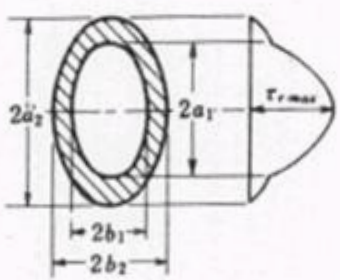
断　面　形	A	I	$Z,\ e$	Z_r
24. 	$b_0t_0+bt_2+h_1t_1$	$I=\dfrac{b_0t_0^3}{3}+\dfrac{bh^3}{3}-\dfrac{(b-t_1)h_1^3}{3}-A(e_1-t_0)^2$ $e_1=t_0+\dfrac{bh^2-(b-t_1)h_1^2-b_0t_0^2}{2A}$ $e_2=h-\dfrac{bh^2-(b-t_1)h_1^2-b_0t_0^2}{2A}$		$t_0\leqq(bt_2+h_1t_1)/b_0$のとき $\dfrac{b_0t_0}{2}(h_1+t_0)+\dfrac{bt_2h}{2}$ $+\dfrac{h_1^2t_1}{4}-\dfrac{1}{4t_1}$ $\times(bt_2-b_0t_0)^2$ $t_0>(bt_2+h_1t_1)/b_0$のとき $\dfrac{b_0t_0^2}{4}-\dfrac{1}{4b_0}(bt_2+h_1t_1)^2$ $+\dfrac{(h_1+t_0)(h_1t_1+bt_2)}{2}$ $+\dfrac{bt_2h}{2}$
25. 	$t(a+b)$	$\dfrac{td^2}{12}(3a+b)$	$\dfrac{td}{6}(3a+b)$	$\dfrac{adt}{2}+\dfrac{bdt}{4}$
26. 	$at\left(1+\dfrac{\pi}{2}\right)+2bt$ $\fallingdotseq 2\cdot5708\,at+2bt$	$\dfrac{a^3t}{12}\left(1+\dfrac{3}{4}\pi\right)+\dfrac{1}{2}a^2bt$ $\fallingdotseq 0\cdot2797a^3t+0\cdot5a^2bt$	$\dfrac{a^2t}{6}\left(1+\dfrac{3}{4}\pi\right)+abt$ $\fallingdotseq 0\cdot5594a^2t+abt$	$\dfrac{3}{4}a^2t+abt+\dfrac{t^3}{6}$

表 28　各種断面の剪断応力分布

断面形及び剪断応力分布	$\tau_r=\dfrac{F}{z_1 I}\displaystyle\int_{y_1}^{e_1} zy\,dy$	$\tau_{r\,max}=\dfrac{\alpha F}{A}$
1. 	$\dfrac{3}{2}\cdot\dfrac{F}{bh}\left\{1-\left(\dfrac{2y_1}{h}\right)^2\right\}$	$\dfrac{3}{2}\cdot\dfrac{F}{bh}=\dfrac{3}{2}\cdot\dfrac{F}{A}$
2. 	$\sqrt{2}\,\dfrac{F}{a^2}\left\{1+\sqrt{2}\,\dfrac{y_1}{a}-4\left(\dfrac{y_1}{a}\right)^2\right\}$	$\dfrac{9}{8}\sqrt{2}\,\dfrac{F}{a^2}=1{\cdot}591\dfrac{F}{A}$
3. 	$\dfrac{4}{3}\cdot\dfrac{F}{\pi r^2}\left\{1-\left(\dfrac{y_1}{r}\right)^2\right\}$	$\dfrac{4}{3}\cdot\dfrac{F}{\pi r^2}=\dfrac{4}{3}\cdot\dfrac{F}{A}$
4. 	$\dfrac{F}{\pi rt}\left\{1-\left(\dfrac{y_1}{r}\right)^2\right\}$	$\dfrac{F}{\pi rt}=2\dfrac{F}{A}$
5. 	$\dfrac{4}{3}\cdot\dfrac{F}{\pi ab}\left\{1-\left(\dfrac{y_1}{a}\right)^2\right\}$	$\dfrac{4}{3}\cdot\dfrac{F}{\pi ab}=\dfrac{4}{3}\cdot\dfrac{F}{A}$

断面形及び剪断応力分布	$\tau_r = \dfrac{F}{z_1 I}\displaystyle\int_{y_1}^{e_1} zy\,dy$	$\tau_{r\,max} = \dfrac{\alpha F}{A}$
6. 	$\dfrac{h_2}{2} \geqq y_1 \geqq \dfrac{h_1}{2}$: $\dfrac{3F}{2(b_2h_2^3-b_1h_1^3)}(h_2^2-4y_1^2)$ $\dfrac{h_1}{2} \geqq y_1 \geqq 0$: $\dfrac{3F}{2(b_2h_2^3-b_1h_1^3)}\left(\dfrac{b_2h_2^2-b_1h_1^2}{b_2-b_1}-4y_1^2\right)$	$\dfrac{3(b_2h_2^2-b_1h_1^2)F}{2(b_2h_2^3-b_1h_1^3)(b_2-b_1)}$ $=\dfrac{3(b_2h_2^2-b_1h_1^2)(b_2h_2-b_1h_1)}{2(b_2h_2^3-b_1h_1^3)(b_2-b_1)}\cdot\dfrac{F}{A}$
7. 	$r_2 \geqq y_1 \geqq r_1$: $\dfrac{4F}{3\pi(r_2^4-r_1^4)}(r_2^2-y_1^2)$ $r_1 \geqq y_1 \geqq 0$: $\dfrac{4F}{3\pi(r_2^4-r_1^4)}\{r_2^2+r_1^2-2y_1^2+\sqrt{(r_2^2-y_1^2)(r_1^2-y_1^2)}\}$	$\dfrac{4(r_2^2+r_2r_1+r_1^2)F}{3\pi(r_2^4-r_1^4)}$ $=\dfrac{4(r_2^2+r_2r_1+r_1^2)}{3(r_2^2+r_1^2)}\cdot\dfrac{F}{A}$
8. 	$a_2 \geqq y_1 \geqq a_1$: $\dfrac{4F}{3\pi(a_2^3b_2-a_1^3b_1)}(a_2^2-y_1^2)$ $a_1 \geqq y_1 \geqq 0$: $\dfrac{4F}{3\pi(a_2^3b_2-a_1^3b_1)}$ $\times\dfrac{\frac{b_2}{a_2}(a_2^2-y_1^2)^{\frac{3}{2}}-\frac{b_1}{a_1}(a_1^2-y_1^2)^{\frac{3}{2}}}{\frac{b_2}{a_2}(a_2^2-y_1^2)^{\frac{1}{2}}-\frac{b_1}{a_1}(a_1^2-y_1^2)^{\frac{1}{2}}}$	$\dfrac{4(a_2^2b_2-a_1^2b_1)F}{3\pi(a_2^3b_2-a_1^3b_1)(b_2-b_1)}$ $=\dfrac{4(a_2^2b_2-a_1^2b_1)(a_2b_2-a_1b_1)}{3(a_2^3b_2-a_1^3b_1)(b_2-b_1)}\cdot\dfrac{F}{A}$

Equation of Deflection Curve of Beam

15-7 Deflection of Beam

Relation between the deformation and the distributed load

$$\boxed{\frac{d^2y}{dx^2} = \frac{M}{EI}} \rightarrow \boxed{EI\frac{d^2y}{dx^2} = M}$$

Differentiation with respect to *x*

$$\boxed{EI\frac{d^3y}{dx^3} = V}$$

Differentiation with respect to *x*

$$\boxed{EI\frac{d^4y}{dx^4} = -f(x)}$$

"Deflection Curve of a Beam"

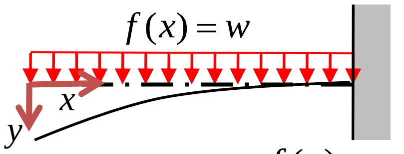
$$\frac{M}{EI} = \frac{d^2y}{dx^2}$$

$$\frac{dM}{dx} = V(x)$$

$$\frac{dV}{dx} = -f(x)$$

Examples of Deflection, Shear Forces & Bending Moments of a beam

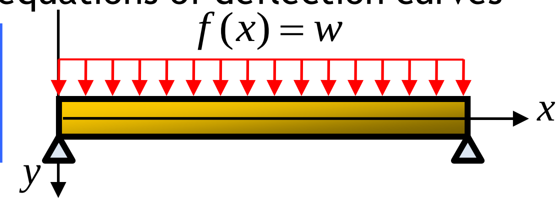
Deflection, Shear Forces & Bending Moments of beam
Example 1 – Cantilever beam



distributed load $f(x)=w$
(y-coordinate modified)

recall, differential equations of deflection curves*

$$\boxed{EI\frac{d^4y}{dx^4}=f(x)}$$



Boundary Condition

①Shear force P at x=0 → $EIy'''(0)=0$

②No bending moment at x=0 → $EIy''(0)=0$

③No slop x=L → $y'(L)=0$

④No displacement e at x=L → $y(L)=0$

$$EI\frac{d^4y(x)}{dx^4}=w$$

Integrate four times

$$y'''(x)=\frac{w}{EI}x+c_1$$

$$y''(x)=\frac{w}{2EI}x^2+c_1x+c_2$$

$$y'(x)=\frac{w}{6EI}x^3+\frac{1}{2}c_1x^2+c_2x+c_3$$

$$y(x)=\frac{w}{24EI}x^4+\frac{1}{6}c_1x^3+\frac{1}{2}c_2x^2+c_3x+c_4$$

by the boundary condition $(E\neq 0,\ I\neq 0)$

① $\left.\begin{aligned}EIy'''(0)&=c_1\\0&=c_1\end{aligned}\right\}\Rightarrow\ \therefore c_1=0$

② $\left.\begin{aligned}EIy''(0)&=c_2\\0&=c_2\end{aligned}\right\}\Rightarrow\ \therefore c_2=0$

③ $\left.\begin{aligned}y'(L)&=\frac{w}{6EI}L^3+c_3\\0&=\frac{w}{6EI}L^3+c_3\end{aligned}\right\}\Rightarrow\ \therefore c_3=-\frac{wL^3}{6EI}$

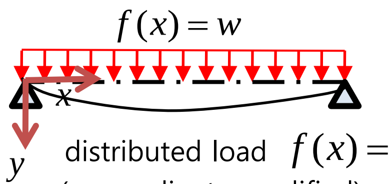
④ $\left.\begin{aligned}y(L)&=\frac{wL^4}{24EI}-\frac{wL^4}{6EI}+c_4\\0&=\frac{wL^4}{24EI}-\frac{wL^4}{6EI}+c_4\end{aligned}\right\}\Rightarrow\ \therefore c_4=\frac{wL^4}{8EI}$

$$y(x)=\frac{w}{24EI}x^4-\frac{wL^3}{6EI}x+\frac{wL^4}{8EI}=\frac{wl^4}{8EI}\left(1-\frac{4x}{3l}+\frac{x^4}{3l^4}\right)$$

$$M(x)=EIy''=\frac{w}{2}x^2$$

$$V(x)=EIy'''=wx$$

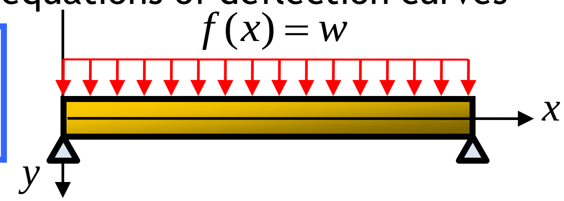
Deflection, Shear Forces & Bending Moments of beam
Example 2 – Simply supported beam



distributed load $f(x)=w$
(y-coordinate modified)

recall, differential equations of deflection curves*

$$\boxed{EI\frac{d^4y}{dx^4}=f(x)}$$



Boundary Condition

①No displacement at x=0 → $y(0)=0$

②No displacement at x=L → $y(L)=0$

③No bending moment x=0 → $EIy''(0)=0$

④No bending moment x=L → $EIy''(L)=0$

$$EI\frac{d^4y(x)}{dx^4}=w$$

Integrate four times

$$y'''(x)=\frac{w}{EI}x+c_1$$

$$y''(x)=\frac{w}{2EI}x^2+c_1x+c_2$$

$$y'(x)=\frac{w}{6EI}x^3+\frac{1}{2}c_1x^2+c_2x+c_3$$

$$y(x)=\frac{w}{24EI}x^4+\frac{1}{6}c_1x^3+\frac{1}{2}c_2x^2+c_3x+c_4$$

by the boundary condition *(E≠0, I≠0)*

$$③\ \left.\begin{aligned}EIy''(0)&=c_2\\0&=c_2\end{aligned}\right\}\Rightarrow\ \therefore c_2=0$$

$$①\ \left.\begin{aligned}y(0)&=c_4\\0&=c_4\end{aligned}\right\}\Rightarrow\ \therefore c_4=0$$

$$④\ \left.\begin{aligned}EIy''(L)&=\frac{wL^2}{2EI}+c_1L\\0&=\frac{wL^2}{2EI}+c_1L\end{aligned}\right\}\Rightarrow\ \therefore c_1=-\frac{wL}{2EI}$$

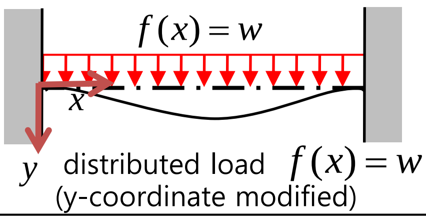
$$②\ \left.\begin{aligned}y(L)&=\frac{L^4}{24EI}+\frac{wL^4}{12EI}+c_3L\\0&=\frac{L^4}{24EI}+\frac{wL^4}{12EI}+c_3L\end{aligned}\right\}\Rightarrow\ \therefore c_3=\frac{wL^3}{24EI}$$

$$y(x)=\frac{w}{24EI}x^4-\frac{wL}{12EI}x^3+\frac{wL^3}{24EI}=\frac{wL^3x}{24EI}\left(1-\frac{2x^2}{l^2}+\frac{x^3}{l^3}\right)$$

$$M(x)=EIy''=\frac{w}{2}x^2-\frac{wL}{2}x$$

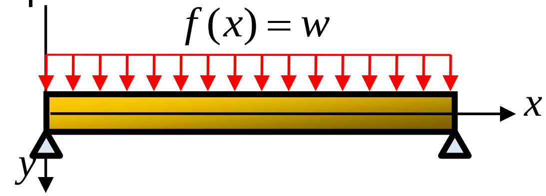
$$V(x)=EIy'''=wx-\frac{wL}{2}$$

Deflection, Shear Forces & Bending Moments of beam
Example 3 – Both end embeded beam



• recall, differential equations of deflection curves*

$$\boxed{EI\frac{d^4y}{dx^4} = f(x)}$$



Boundary Condition		
①No displacement at x=0	→	$y(0) = 0$
②No displacement at x=L	→	$y(L) = 0$
③No slope at x=0	→	$y'(0) = 0$
④No slope at x=0	→	$y'(L) = 0$

$$EI\frac{d^4y(x)}{dx^4} = w$$

Integrate four times

$$y'''(x) = \frac{w}{EI}x + c_1$$

$$y''(x) = \frac{w}{2EI}x^2 + c_1x + c_2$$

$$y'(x) = \frac{w}{6EI}x^3 + \frac{1}{2}c_1x^2 + c_2x + c_3$$

$$y(x) = \frac{w}{24EI}x^4 + \frac{1}{6}c_1x^3 + \frac{1}{2}c_2x^2 + c_3x + c_4$$

by the boundary condition *(E≠0, I≠0)*

$$① \left.\begin{aligned} y(0) &= c_4 \\ 0 &= c_4 \end{aligned}\right\} \Rightarrow \therefore c_4 = 0$$

$$③ \left.\begin{aligned} y'(0) &= c_3 \\ 0 &= c_3 \end{aligned}\right\} \Rightarrow \therefore c_3 = 0$$

$$② \; y(L) = \frac{1}{2}c_2L^2 + \frac{1}{6}c_1L^3 + \frac{1}{24EI}L^4$$

$$0 = \frac{1}{2}c_2L^2 + \frac{1}{6}c_1L^3 + \frac{1}{24EI}L^4$$

$$④ \; y'(L) = c_2L + \frac{1}{2}c_1L^2 + \frac{1}{6EI}L^3$$

$$0 = c_2L + \frac{1}{2}c_1L^2 + \frac{1}{6EI}L^3$$

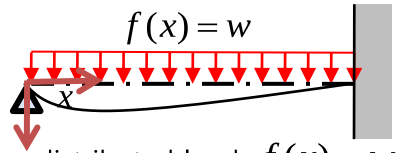
$$\rightarrow \quad c_2 = \frac{wL^2}{12EI}, \quad c_1 = -\frac{wL}{2EI}$$

$$EIy(x) = \frac{w}{24}x^4 - \frac{wL}{12}x^3 + \frac{wL^2}{24}x^2 = \frac{wL^2x^2}{24EI}\left(1 - \frac{x}{L}\right)^2$$

$$M(x) = EIy'' = \frac{wl^2}{12}\left(1 - \frac{6x}{l} + \frac{6x^2}{l^2}\right)$$

$$V(x) = EIy''' = -\frac{wl}{2}\left(1 - \frac{wx}{l}\right)$$

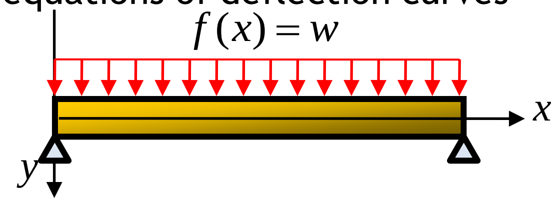
Deflection, Shear Forces & Bending Moments of beam
Example 4 – Simply supported and embeded beam



distributed load $f(x)=w$
(y-coordinate modified)

•recall, differential equations of deflection curves*

$$\boxed{EI\frac{d^4y}{dx^4}=f(x)}$$



Boundary Condition

①No displacement at x=0 → $y(0)=0$

②No bending moment at x=0 → $EIy''(0)=0$

③No displacement at x=L → $y(L)=0$

④No slope at x=L → $y'(L)=0$

$$EI\frac{d^4y(x)}{dx^4}=w$$

Integrate four times

$$y'''(x)=\frac{w}{EI}x+c_1$$

$$y''(x)=\frac{w}{2EI}x^2+c_1x+c_2$$

$$y'(x)=\frac{w}{6EI}x^3+\frac{1}{2}c_1x^2+c_2x+c_3$$

$$y(x)=\frac{w}{24EI}x^4+\frac{1}{6}c_1x^3+\frac{1}{2}c_2x^2+c_3x+c_4$$

by the boundary condition *(E≠0, I≠0)*

$$① \left.\begin{aligned} y(0)&=c_4\\ 0&=c_4\end{aligned}\right\} \Rightarrow \therefore c_4=0$$

$$② \left.\begin{aligned} y''(0)&=c_2\\ 0&=c_2\end{aligned}\right\} \Rightarrow \therefore c_3=0$$

$$③\ y'(L)=\frac{wL^3}{6EI}+\frac{L^2}{2}c_1+c_3$$

$$0=\frac{wL^3}{6EI}+\frac{L^2}{2}c_1+c_3 \Rightarrow \boxed{c_3}=-\frac{wL^3}{6EI}-\frac{L^2}{2}c_1$$

$$④\ y(L)=\frac{wL^4}{24EI}+\frac{L^3}{6}c_1+Lc_3$$

$$0=\frac{wL^4}{24EI}+\frac{L^3}{6}c_1+L\boxed{c_3}$$

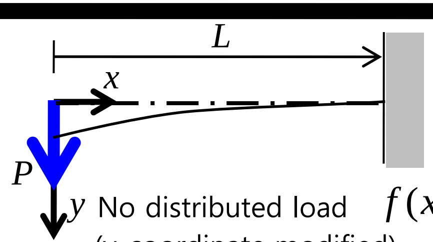
$$\rightarrow \quad c_1=-\frac{3}{8}\frac{wL}{EI}, \qquad c_3=-\frac{wL^3}{48EI}$$

$$EIy(x)=\frac{w}{24}x^4-\frac{wL}{16}x^3+\frac{wL^3}{48}x=\frac{wL^3x}{48EI}\left(1-\frac{3x^2}{L^2}+\frac{2x^3}{l^3}\right)$$

$$M(x)=EIy''=\frac{w}{2}x^2-\frac{3wL}{8}x$$

$$V(x)=EIy'''=wx-\frac{3wL}{8}$$

Deflection, Shear Forces & Bending Moments of beam
Example 5 - Cantilever

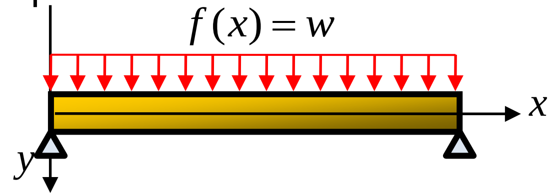


No distributed load $f(x)=0$

(y-coordinate modified)

• recall, differential equations of deflection curves*

$$\boxed{EI\frac{d^4y}{dx^4}=f(x)}$$



Boundary Condition

①No bending moment at x=0 → $EIy''(0)=0$

②Shear force P at x=0 → $EIy'''(0)=P$

③No displacement at x=0 → $y(L)=0$

④No slope at x=0 → $y'(L)=0,$

$$EI\frac{d^4y(x)}{dx^4}=0$$

• Integrate four times

$$y'''(x)=\frac{1}{EI}c_1$$

$$y''(x)=\frac{1}{EI}c_1x+c_2$$

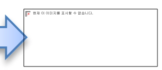
$$y'(x)=\frac{1}{2EI}c_1x^2+c_2x+c_3$$

$$y(x)=\frac{1}{6EI}c_1x^3+\frac{1}{2}c_2x^2+c_3x+c_4$$

by the boundary condition *(E≠0, I≠0)*

$$①\left.\begin{aligned}y''(0)&=c_2\\0&=c_2\end{aligned}\right\}\ \therefore c_2=0$$

$$②\left.\begin{aligned}EIy'''(0)&=c_1\\P&=c_1\end{aligned}\right\}$$



$$④\left.\begin{aligned}y'(L)&=\frac{PL^2}{2EI}+c_3\\0&=\frac{PL^2}{2EI}+c_3\end{aligned}\right\}\ c_3=-\frac{PL^2}{2EI}$$

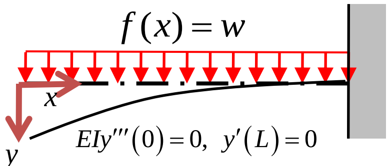
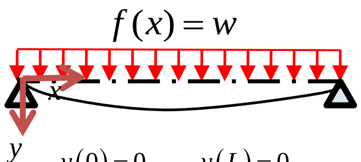
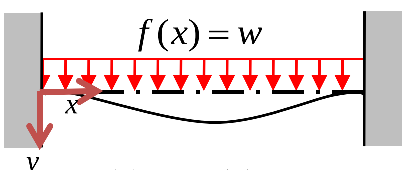
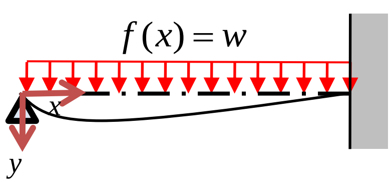
$$③\ c_4-\frac{PL^3}{2EI}+\frac{PL^3}{6}=0\ \ c_4=\frac{PL^3}{3}$$

$$\therefore y(x)=\frac{1}{6EI}Px^3-\frac{1}{2EI}PL^2x-\frac{1}{3EI}PL^3$$

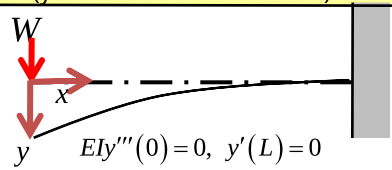
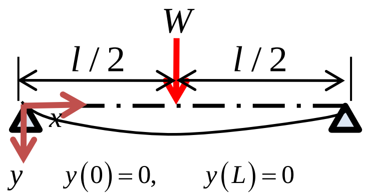
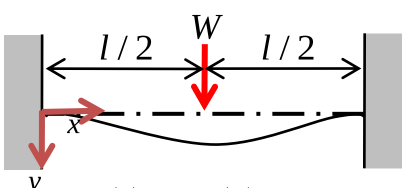
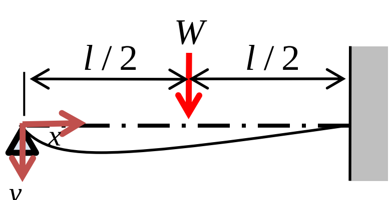
$$M(x)=EIy''=wx$$

$$V(x)=EIy'''=w$$

Table of Deflection, Shear Forces & Bending Moments of beam

	(y-coordinate modified)	Shear force V Reaction force R	Moment M	Deflection y, angle of rotation θ
1	 $EIy'''(0)=0,\ y'(L)=0$ $EIy''(0)=0,\ y(L)=0$	$V=wx$ $R_2=wl$	$M=\frac{wx^2}{2}$ $x=l: M_{\max}=\frac{wl^2}{2}$	$y=\frac{wl^4}{8EI}\left(1-\frac{4x}{3l}+\frac{x^4}{3l^4}\right)$ $x=0;\ y_{\max}=\frac{wl^4}{8EI}$ $\quad x=0;\ \theta=-\frac{wl^3}{6EI}$
2	 $y(0)=0,\quad y(L)=0$ $EIy''(0)=0,\ EIy''(L)=0$	$V=-\frac{wl}{2}\left(1-\frac{wx}{l}\right)$ $R_1=R_2=\frac{wl}{2}$	$M=-\frac{wlx}{2}\left(1-\frac{x}{l}\right)$ $x=\frac{l}{2}; M_{\max}=-\frac{wl^2}{8}$	$y=\frac{wl^3x}{24EI}\left(1-\frac{2x^2}{l^2}+\frac{x^3}{l^3}\right),\quad x=\frac{l}{2}: y_{\max}=\frac{5wl^4}{384EI}$ $x=0:\theta=-\frac{wl^3}{24EI};\quad x=l:\theta=-\frac{wl^3}{24EI}$
3	 $y(0)=0,\ y(L)=0$ $y'(0)=0,\ y'(L)=0$	$V=-\frac{wl}{2}\left(1-\frac{wx}{l}\right)$ $R_1=R_2=\frac{wl}{2}$	$M_{\max}=M_{x=0}=M_{x=l}=\frac{wl^2}{12}$ $M_{x=1/2}=-\frac{wl^2}{24}$	$y=\frac{wl^2x^2}{24EI}\left(1-\frac{x}{l}\right)^2$ $x=\frac{1}{2};\ y_{\max}=\frac{wl^4}{384EI}$
4	 $y(0)=0,\ EIy''(0)=0$ $y(L)=0,\ y'(L)=0$	$V=-\frac{3wl}{8}\left(1-\frac{8x}{3l}\right)$ $R_1=\frac{3wl}{8},\ R_2=\frac{wl5}{8}$	$M=-\frac{3wlx}{8}\left(1-\frac{4x}{3l}\right)$ $x=l: M_{\max}=\frac{wl^2}{8}$ $x=l: M_{\max}=\frac{wl^2}{8}$	$y=\frac{wl^3x}{48EI}\left(1-\frac{3x^2}{l^2}+\frac{2x^3}{l^3}\right)$ $y_{\max}=\frac{\left(39+55\sqrt{33}\right)wl^4}{16^4}\simeq\frac{wl^4}{184.6EI}$ $x=0:\theta=\frac{wl^3}{48EI}$

1) 대우조선해양, 선박구조설계, 6-8 보의 도표, 2005.

	(y-coordinate modified)	Shear force V Reaction force R	Moment M	Deflection y, angle of rotation θ
5	 $EIy'''(0)=0,\ y'(L)=0$ $EIy''(0)=0,\ y(L)=0$	$V=W$ $R_2=W$	$M=Wx$ $x=l: M_{\max}=Wl$	$y=\frac{Wl^3}{3EI}\left(1-\frac{3x}{2l}+\frac{x^3}{2l^3}\right)$ $x=0;\ y_{\max}=\frac{wl^3}{3EI}$ $\quad x=0;\ \theta=-\frac{wl^2}{2EI}$
6	 $y(0)=0,\quad y(L)=0$ $EIy''(0)=0,\ EIy''(L)=0$	$0<x<\frac{1}{2}, V=-\frac{W}{2}$ $\frac{1}{2}<x<l, V=\frac{W}{2}$ $R_1=R_2=\frac{W}{2}$	$0<x<\frac{1}{2}, M=-\frac{Wx}{2}$ $\frac{1}{2}<x<l, M=-\frac{W(l-x)}{2}$ $x=\frac{1}{2}: M_{\max}=-\frac{Wl}{4}$	$0<x<\frac{1}{2}: y_1=\frac{Wl^2x}{16EI}\left(1-\frac{4x^2}{3l^2}\right)$ $\frac{1}{2}<x<l: y_2=\frac{Wl^3}{48EI}\left(1-\frac{9x}{l}+\frac{12x^2}{l^2}-\frac{4x^3}{l^3}\right)$ $x=\frac{1}{2}: y_{\max}=\frac{Wl^3}{48EI}$ $\quad ,x=0,l: \theta=-\frac{wl^2}{16EI}$
7	 $y(0)=0,\ y(L)=0$ $y'(0)=0,\ y'(L)=0$	$0<x<\frac{1}{2}, V=-\frac{W}{2}$ $\frac{1}{2}<x<l, V=\frac{W}{2}$ $R_1=R_2=\frac{W}{2}$	$0<x<\frac{1}{2}, M=\frac{Wl}{8}\left(1-\frac{4x}{l}\right)$ $\frac{1}{2}<x<l, M=-\frac{3Wl}{8}\left(1-\frac{4x}{3l}\right)$ $M_{\substack{x=0\\x=l}}=\frac{Wl}{8};\ M_{x=l/2}=-\frac{Wl}{8}$	$0<x<\frac{1}{2}: y_1=\frac{Wl^2x}{16EI}\left(1-\frac{4x^2}{3l^2}\right)$ $\frac{1}{2}<x<l: y_2=\frac{Wl^3}{48EI}\left(1-\frac{9x}{l}+\frac{12x^2}{l^2}-\frac{4x^3}{l^3}\right)$ $x=\frac{1}{2}: y_{\max}=\frac{Wl^3}{192EI}$
8	 $y(0)=0,\ EIy''(0)=0$ $y(L)=0,\ y'(L)=0$	$0<x<\frac{1}{2}, V=-\frac{5W}{16}$ $\frac{1}{2}<x<l, V=\frac{11W}{16}$ $R_1=\frac{5W}{16},\ R_2=\frac{11W}{16}$	$0<x<\frac{1}{2}, M=-\frac{5Wl}{16}$ $\frac{1}{2}<x<l, M=-\frac{Wl}{2}\left(1-\frac{11x}{8l}\right)$ $M_{x=\frac{l}{2}}=\frac{5Wl}{32};\ M_{\substack{\max\\x=l}}=-\frac{3Wl}{16}$	$0<x<\frac{1}{2}: y_1=\frac{Wl^2x}{32EI}\left(1-\frac{5x^2}{3l^2}\right)$ $\frac{1}{2}<x<l: y_2=\frac{Wl^3}{48EI}\left(1-\frac{15x}{2l}+\frac{12x^2}{l^2}-\frac{11x^3}{2l^3}\right)$ $x=\frac{1}{\sqrt{5}}: y_{\max}=\frac{Wl^3}{48\sqrt{5}EI}$

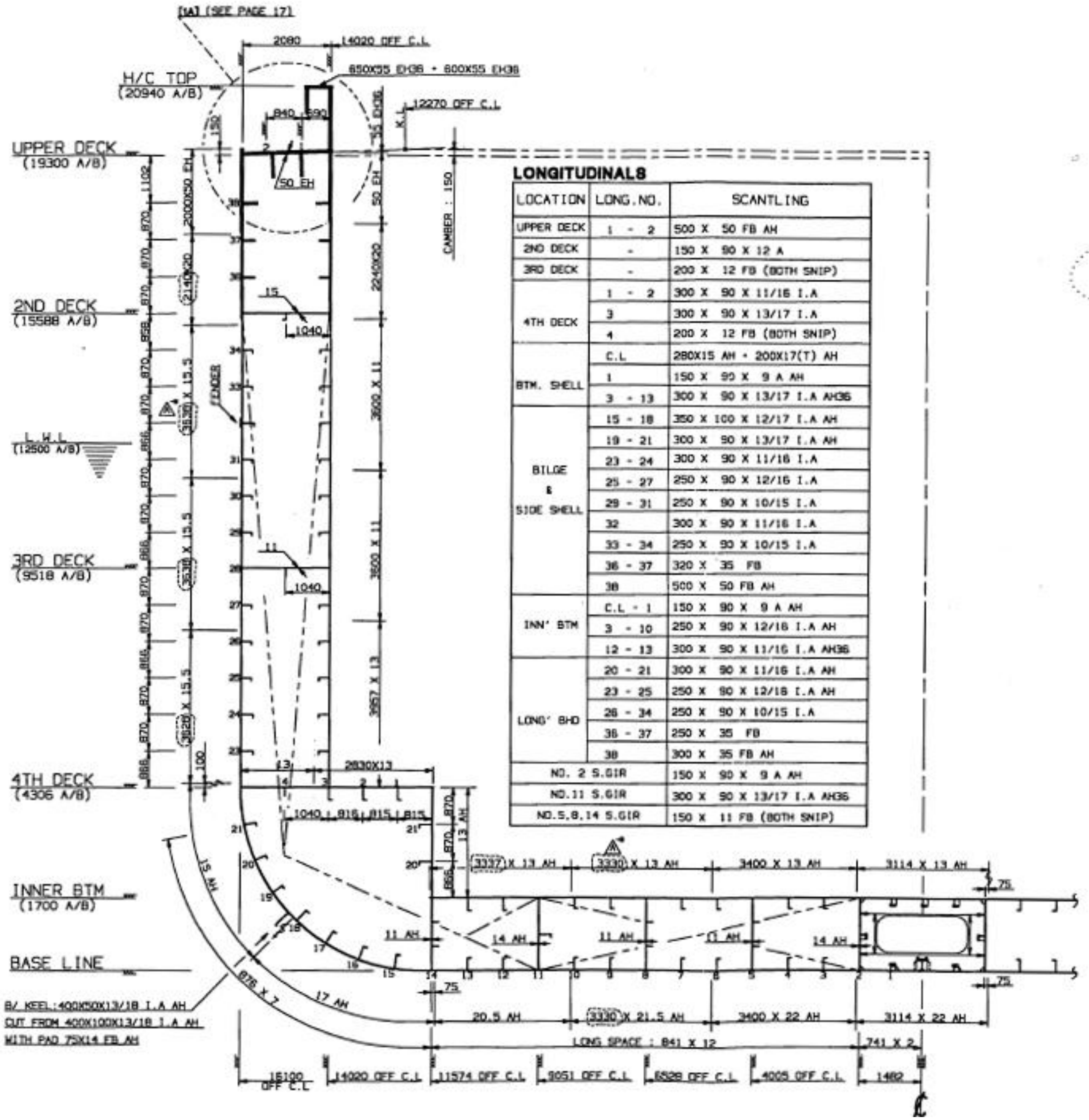
1) 대우조선해양, 선박구조설계, 6-8 보의 도표, 2005.

16. Midship Section Structure Design of a 3,700 TEU Container

16.1 General & Materials

1) Midship section for 3,700 TEU Containership
2) Stress transmission
3) Principal dimensions
4) Criteria for the selection of plate thickness,
   Grouping of longitudinal stiffener
5) Material Factors

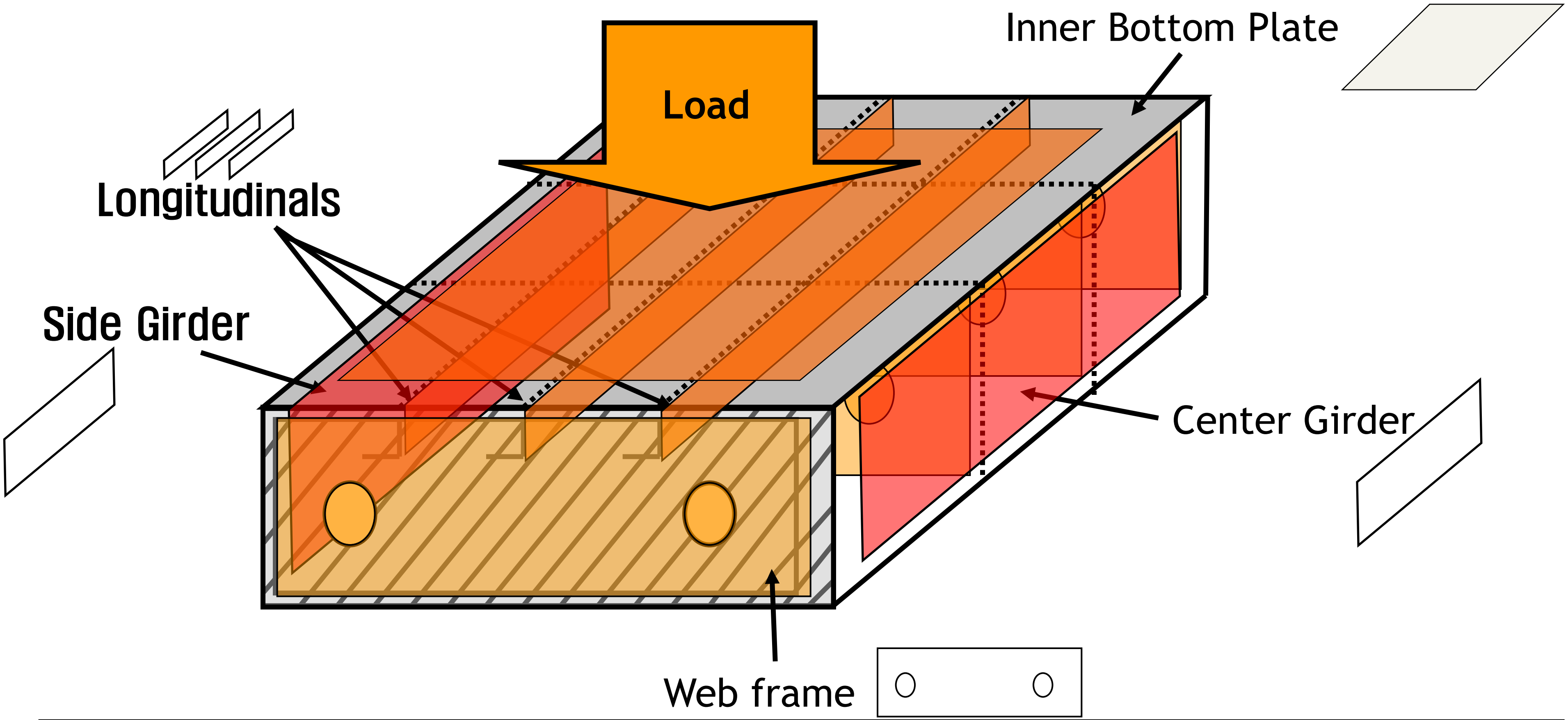
1) Midship section for 3,700 TEU Containership

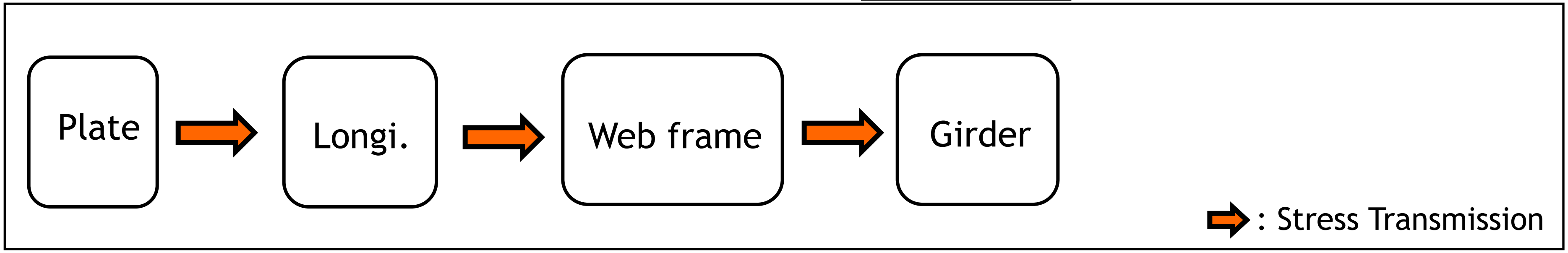


LONGITUDINALS

LOCATION	LONG.NO.	SCANTLING
UPPER DECK	1 - 2	500 X 50 FB AH
2ND DECK	-	150 X 90 X 12 A
3RD DECK	-	200 X 12 FB (BOTH SNIP)
4TH DECK	1 - 2	300 X 90 X 11/16 I.A
	3	300 X 90 X 13/17 I.A
	4	200 X 12 FB (BOTH SNIP)
BTM. SHELL	C.L	280X15 AH + 200X17(T) AH
	1	150 X 90 X 9 A AH
	3 - 13	300 X 90 X 13/17 I.A AH36
BILGE & SIDE SHELL	15 - 18	350 X 100 X 12/17 I.A AH
	19 - 21	300 X 90 X 13/17 I.A AH
	23 - 24	300 X 90 X 11/16 I.A
	25 - 27	250 X 90 X 12/16 I.A
	29 - 31	250 X 90 X 10/15 I.A
	32	300 X 90 X 11/16 I.A
	33 - 34	250 X 90 X 10/15 I.A
	36 - 37	320 X 35 FB
	38	500 X 50 FB AH
INN' BTM	C.L - 1	150 X 90 X 9 A AH
	3 - 10	250 X 90 X 12/16 I.A AH
	12 - 13	300 X 90 X 11/16 I.A AH36
LONG' BHD	20 - 21	300 X 90 X 11/16 I.A AH
	23 - 25	250 X 90 X 12/16 I.A AH
	26 - 34	250 X 90 X 10/15 I.A
	36 - 37	250 X 35 FB
	38	300 X 35 FB AH
NO. 2 S.GIR		150 X 90 X 9 A AH
NO.11 S.GIR		300 X 90 X 13/17 I.A AH36
NO.5,8,14 S.GIR		150 X 11 FB (BOTH SNIP)

2) Stress transmission





3) Principal dimensions

The following principal dimensions are used in accordance with DnV rule.

1) Rule length (L)

•Rule Scantling시 사용하는 선박의 길이

: length of a ship used for rule scantling procedure

$$\boxed{0.96 \cdot LWL < L < 0.97 \cdot LWL}$$

- Distance on the summer load waterline (LWL) from the fore side of the stem to the axis of the rudder stock
- Not to be taken less than 96%, and need not be taken greater than 97%, of the extreme length on the summer load waterline (L_{WL})
- Starting point of rule length : F.P

ex)

LBP	LWL	0.96·LWL	0.97 ·LWL	L
250	261	250.56	253.17	250.56
250	258	247.68	250.26	250.00
250	255	244.80	247.35	247.35

2) Breadth

- Greatest moulded breadth in [m], measured at the summer load waterline

B. Definitions

B 100 Symbols

101 The following symbols are used:

L = length of the ship in m defined as the distance on the summer load waterline from the fore side of the stem to the axis of the rudder stock.

L shall not be taken less than 96%, and need not to be taken greater than 97%, of the extreme length on the summer load waterline. For ships with unusual stern and bow arrangement, the length L will be especially considered.

F.P. = the forward perpendicular is the perpendicular at the intersection of the summer load waterline with the fore side of the stem. For ships with unusual bow arrangements the position of the F.P. will be especially considered.

A.P. = the after perpendicular is the perpendicular at the after end of the length L.

L_F = length of the ship as defined in the International Convention of Load Lines:

The length shall be taken as 96 per cent of the total length on a waterline at 85 per cent of the least moulded depth measured from the top of the keel, or as the length from the fore side of the stem to the axis of the rudder stock on that waterline, if that be greater. In ships designed with a rake of keel the waterline on which this length is measured shall be parallel to the designed waterline.

B = greatest moulded breadth in m, measured at the summer waterline.

D = moulded depth defined as the vertical distance in m from baseline to moulded deckline at the uppermost continuous deck measured amidships.

D_F = least moulded depth taken as the vertical distance in m from the top of the keel to the top of the freeboard deck beam at side.

In ships having rounded gunwales, the moulded depth shall be measured to the point of intersection of the moulded lines of the deck and side shell plating, the lines extending as though the gunwale was of angular design.

Where the freeboard deck is stepped and the raised part of the deck extends over the point at which the moulded depth shall be determined, the moulded depth shall be measured to a line of reference extending from the lower part of the deck along a line parallel with the raised part.

T = mean moulded summer draught in m.

Δ = moulded displacement in t in salt water (density 1.025 t/m^3) on draught T.

C_B = block coefficient,

$$= \frac{\Delta}{1.025\, L\, B\, T}$$

For barge rigidly connected to a push-tug C_B shall be calculated for the combination barge/ push-tug.

C_{BF} = block coefficient as defined in the International Convention of Load Lines:

$$= \frac{\nabla}{L_F\; B\; T_F}$$

∇ = volume of the moulded displacement, excluding bossings, taken at the moulded draught T_F.

T_F = 85% of the least moulded depth.

V = maximum service speed in knots, defined as the greatest speed which the ship is designed to maintain in service at her deepest seagoing draught.

g_0 = standard acceleration of gravity
= 9.81 m/s^2.

f_1 = material factor depending on material strength group. See Sec.2.

t_k = corrosion addition as given in Sec.2 D200 and D300, as relevant.

x = axis in the ship's longitudinal direction.

y = axis in the ship's athwartships direction.

z = axis in the ship's vertical direction.

E = modulus of elasticity of the material
= $2.06 \cdot 10^5$ N/mm^2 for steel
= $0.69 \cdot 10^5$ N/mm^2 for aluminium alloy.

C_W = wave load coefficient given in Sec.4 B200.

Amidships = the middle of the length L.

3) Principal dimensions

3) Depth (D)

: moulded depth defined as the vertical distance in [m] from baseline to moulded deckline at the uppermost continuous deck measured amidships

4) Draught (T)

: mean moulded summer draught in [m]

5) Brock coefficient (C_B)

- To be calculated based on the rule length

$$C_B = \frac{\Delta}{1.025 \cdot L \cdot B \cdot T}$$,(Δ : moulded displacement in salt water on draught T)

4) Criteria for the selection of plate thickness, Grouping of longitudinal stiffener

1) Criteria for the selection of plate thickness

→When selecting plate thickness, use the provided plate thickness.

1) 0.5 mm interval
2) Above 0.25 mm : 0.5 mm
3) Below 0.25 mm : 0.00 mm

ex) 15.75 mm → 16.0 mm
15.74 mm → 15.5 mm

2) Grouping of longitudinal stiffener

•For the efficiency of productivity, each member is arranged by grouping longitudinal stiffeners.
The grouping members should satisfy the following rule.
Average value but not to be taken less than 90% of the largest individual requirement. (DNV)

ex) The longitudinal stiffeners have design thickness of 100, 90, 80, 70, 60 mm. The average thickness is given by 80 mm × 5. However, the average value is less than 100mm × 90% = 90mm of the largest individual requirement, 100mm.

Therefore,the average value should be taken 90mm × 5.

5) Material Factors

[1] DnV Rules, Jan. 2004,Pt.3 Ch.1 Sec.2

[2] James M. Gere, Mechanics of Materials 7th Edition, Thomson, Chap.1, pp.15~26

- The material factor f_1 is included in the various formulae for scantlings and in expressions giving allowable stresses.[1]

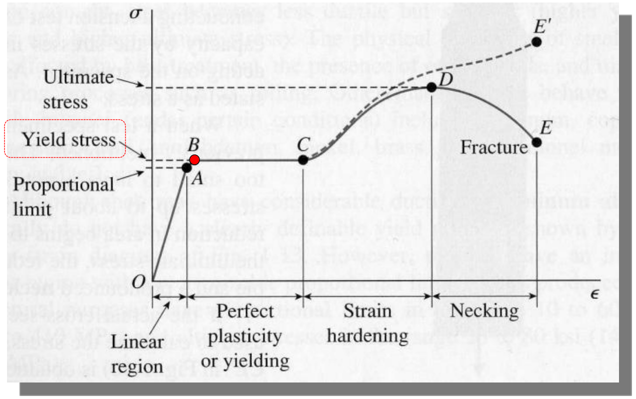
Material Designation	Yield Stress (N/mm²)	$\frac{\sigma}{\sigma_{NV-NS}}$	Material Factor
NV-NS	235	235/235 = 1.00	1.00
NV-27	265	265/235 = 1.13	1.08
NV-32	315	315/235 = 1.34	1.28
NV-36	355	355/235 = 1.51	1.39
NV-40	390	390/235 = 1.65	1.43

*NV-NS : Normal Strength Steel(Mild Steel)

*NV-XX : High tensile Steel

*High tensile steel

A type of alloy steel that provides better mechanical properties or greater resistance to corrosion than carbon steel. They have a carbon content between 0.05-0.25% to retain formability and weldability and include up to 2.0% manganese and other elements are added for strengthening purposes..



*Yield Stress(σ_y) [N/mm²] or [MPa]: The magnitude of the load required to cause yielding in the beam.[2]

*A: 'A' grade 'Normal strength Steel'

AH: 'A' grade 'High tensile steel'

Material Classes

[1]DnV Rules, Jan. 2004,Pt.3 Ch.1 Sec.2

- In order to distinguish between the material grade requirements for different hull parts, various material classes are applied as defined in Table.[1]

Thickness (mm)	Class				
	I	II	III	IV	V
$t \leq 15$	A/AH	A/AH	A/AH	A/AH	D/DH
$15 < t \leq 20$	A/AH	A/AH	A/AH	B/AH	E/DH
$20 < t \leq 25$	A/AH	A/AH	B/AH	D/DH	E/EH
$25 < t \leq 30$	A/AH	A/AH	D/DH	E/DH	E/EH
$30 < t \leq 35$	A/AH	B/AH	D/DH	E/EH	E/EH
$35 < t \leq 40$	A/AH	B/AH	D/DH	E/EH	E/EH
$40 < t \leq 50$	B/AH	D/DH	E/EH	E/EH	E/EH

Price for Steel Grade

08.05.30

Grade	$/ton
A	$1,340
AH36	$1,384
…	…
E	$1,425
EH36	$1,502

✓ Mechanical properties and chemical composition are different according to Steel Grade.[2]

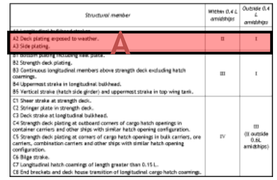
✓ High grade steels are developed by modifying alloy elements, de-oxidation methods and normalizing heat treatments.

*A: 'A' grade 'Normal strength Steel'
AH: 'A' grade 'High tensile steel'

➔ Advantage : strength regarding brittleness and toughness in the cold.
Disadvantage: expensive.

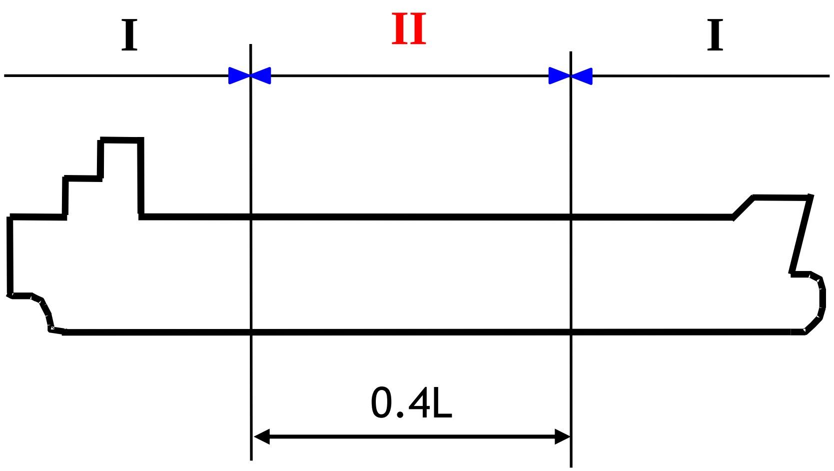
*brittleness : material property that has little tendency to deform before fracture. This fracture absorbs relatively little energy, even in materials of high strength.

Material Classes
- Typical Example (1)



Structural member	0.4 L	Outside 0.4 L
A1 Longitudinal bulkhead strakes. A2 Deck plating exposed to weather. A3 Side plating.	II	I

DnV Rules, Jan. 2004,Pt.3 Ch.1 Sec.2 Table B2



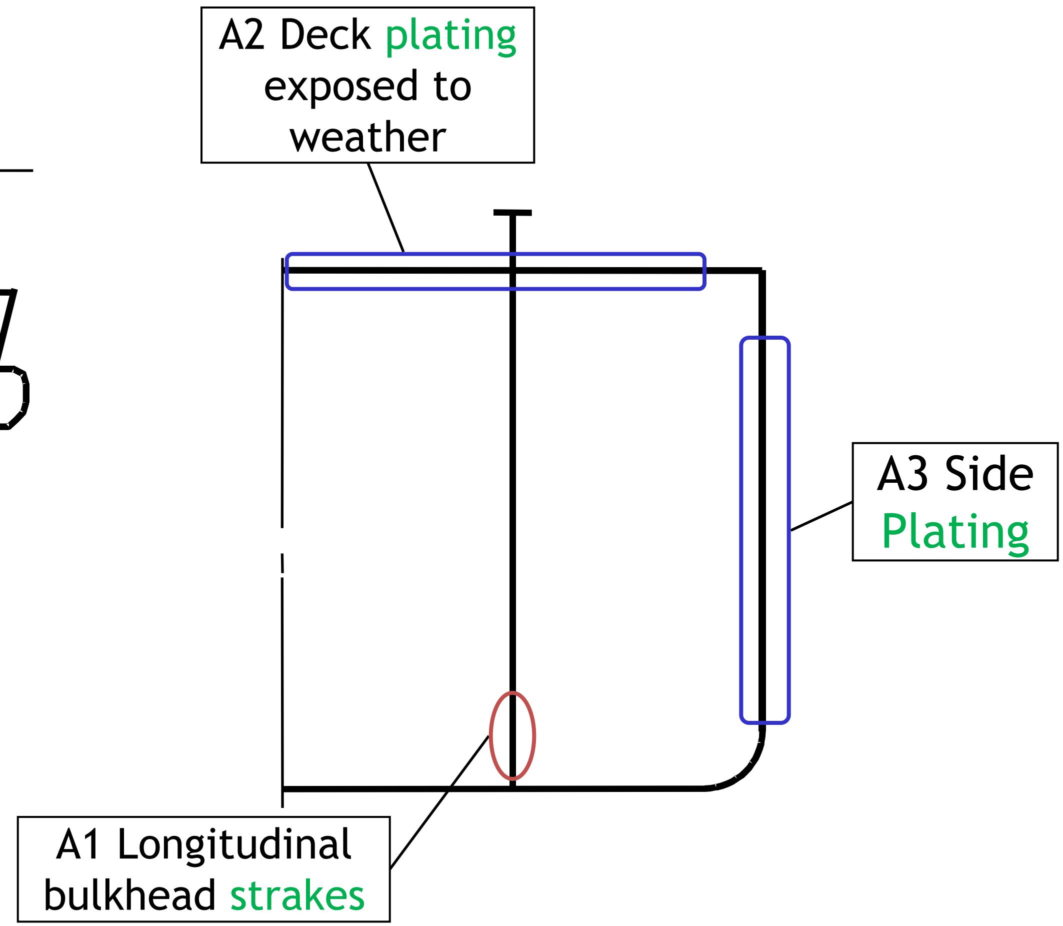


Table B2 Material Classes and Grades for ships in general

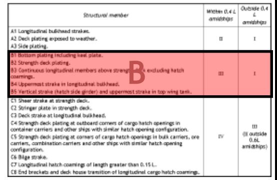
Structural member category		*Material class/grade*
SECONDARY:		
A1.	Longitudinal bulkhead strakes, other than that belonging to the Primary category	— Class II within 0.4L amidships — Grade A/AH outside 0.4L amidships
A2.	Deck plating exposed to weather, other than that belonging to the Primary or Special category	
A3.	Side plating	
PRIMARY:		
B1.	Bottom plating, including keel plate	— Class III within 0.4L amidships — Grade A/AH outside 0.4L amidships
B2.	Strength deck plating, excluding that belonging to the Special category	
B3.	Continuous longitudinal members above strength deck, excluding hatch coamings	
B4.	Uppermost strake in longitudinal bulkhead	
B5.	Vertical strake (hatch side girder) and uppermost sloped strake in top wing tank	
SPECIAL:		
C1.	Sheer strake at strength deck *)	— Class IV within 0.4L amidships — Class III outside 0.4L amidships — Class II outside 0.6L amidships
C2.	Stringer plate in strength deck *)	
C3.	Deck strake at longitudinal bulkhead, excluding deck plating in way of inner-skin bulkhead of double-hull ships *)	
C4.	Strength deck plating at outboard corners of cargo hatch openings in container carriers and other ships with similar hatch opening configurations	— Class IV within 0.4L amidships — Class III outside 0.4L amidships — Class II outside 0.6L amidships — Min. Class IV within the cargo region
C5.	Strength deck plating at corners of cargo hatch openings in bulk carriers, ore carriers combination carriers and other ships with similar hatch opening configurations	— Class IV within 0.6L amidships — Class III within rest of cargo region

Table B2 Material Classes and Grades for ships in general (Continued)

Structural member category		*Material class/grade*
C6.	Bilge strake in ships with double bottom over the full breadth and length less than 150 m *)	— Class III within 0.6L amidships — Class II outside 0.6L amidships
C7.	Bilge strake in other ships *)	— Class IV within 0.4L amidships — Class III outside 0.4L amidships — Class II outside 0.6L amidships
C8.	Longitudinal hatch coamings of length greater than 0.15L	— Class IV within 0.4L amidships — Class III outside 0.4L amidships — Class II outside 0.6L amidships — Not to be less than Grade D/DH
C9.	End brackets and deck house transition of longitudinal cargo hatch coamings	

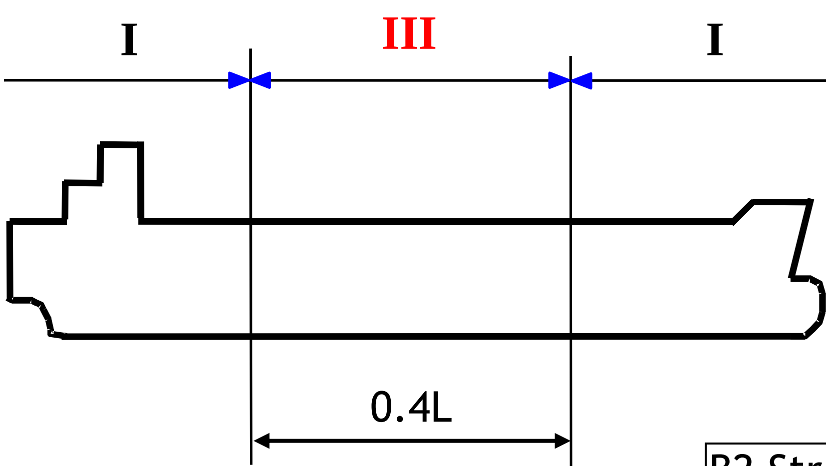
*) Single strakes required to be of Class IV within 0.4L amidships are to have breadths not less than 800 + 5L (mm), need not be greater than 1800 (mm), unless limited by the geometry of the ship's design.

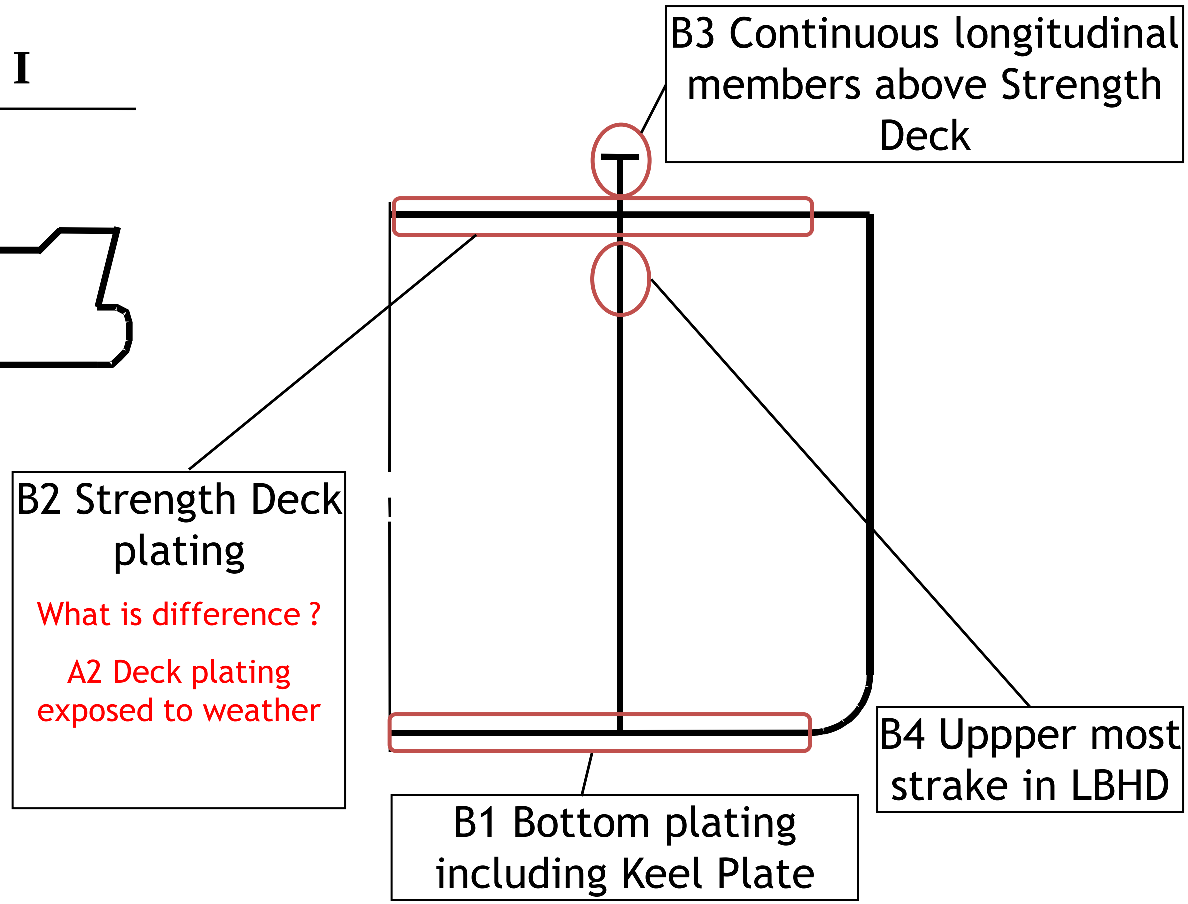
Material Classes
- Typical Example (2)



Structural member	0.4 L	Outside 0.4 L
B1 Bottom plating including keel plate. B2 Strength deck plating. B3 Continuous longitudinal members above strength deck B4 Uppermost strake in longitudinal bulkhead.	III	I

DnV Rules, Jan. 2004,Pt.3 Ch.1 Sec.2 Table B2

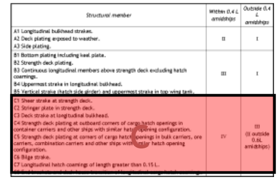




Strength Deck:

General defined as the uppermost continuous deck

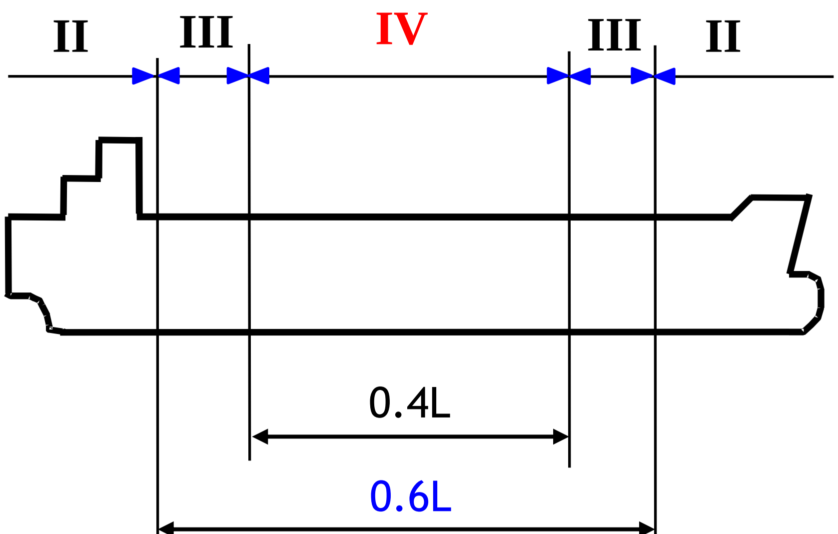
Material Classes
- Typical Example (3)



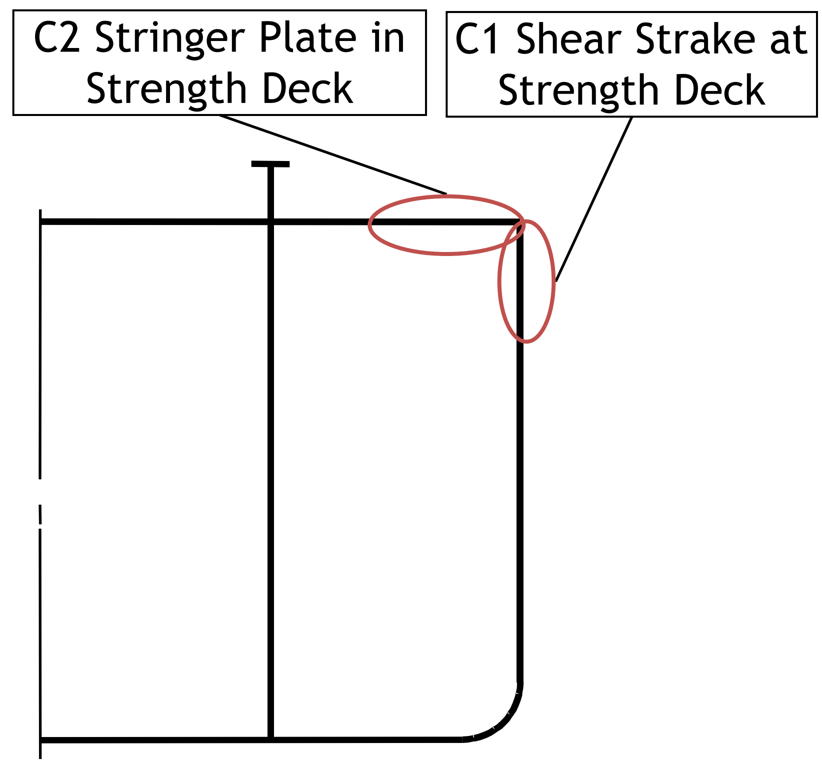
Structural member	0.4 L	Outside 0.4 L
C1 Sheer strake at strength deck. C2 Stringer plate in strength deck	IV	III (II outside 0.6L amidships) ?

DnV Rules, Jan. 2004,Pt.3 Ch.1 Sec.2 Table B2

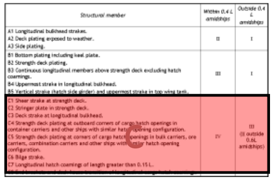
E/EH ← **When L> 250 m**



In the case of container carrier,
0.6L is replaced to cargo region
by 0.4 L(?).

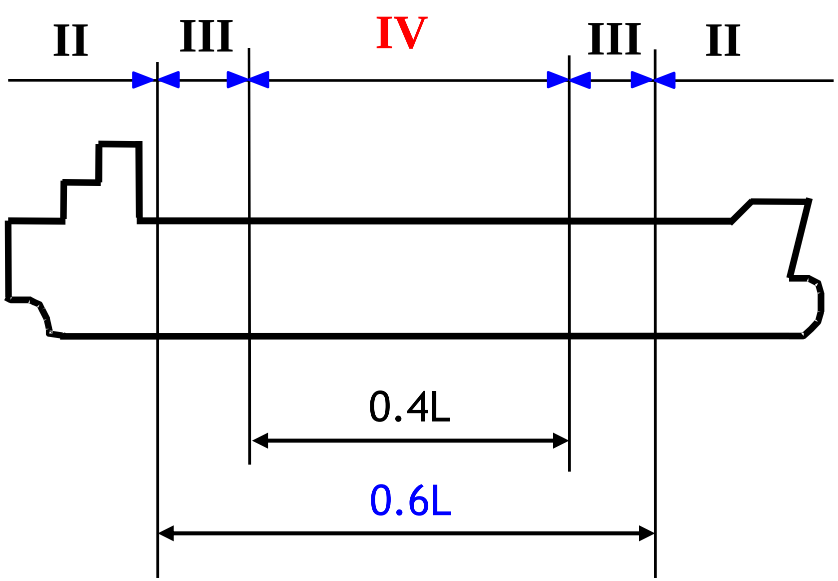


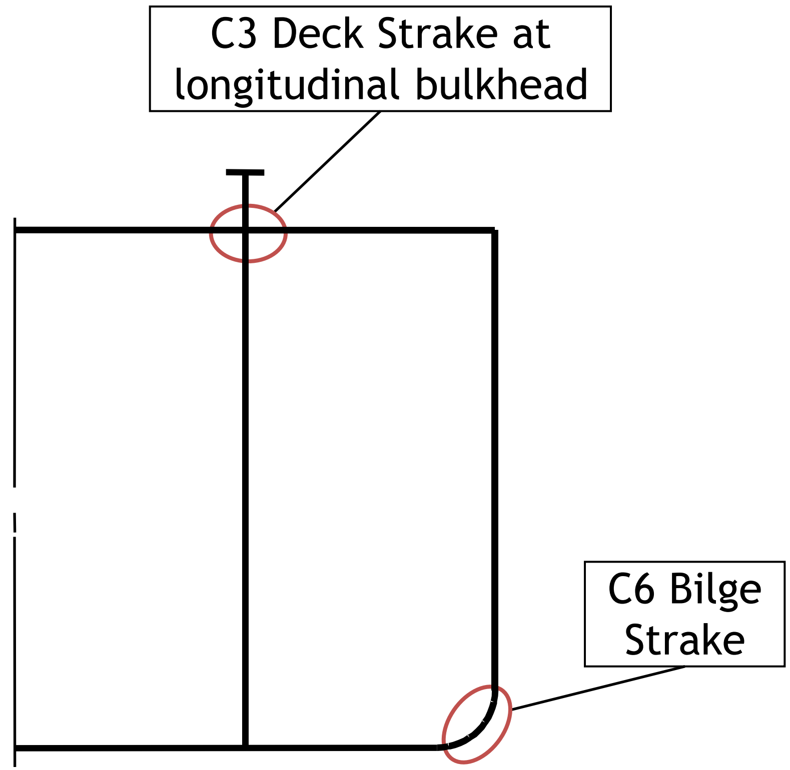
Material Classes
- Typical Example (4)



Structural member	0.4 L	Outside 0.4 L
C3 Deck strake at longitudinal bulkhead. C6 Bilge strake.	IV	III (II outside 0.6L amidships) ?

DnV Rules, Jan. 2004,Pt.3 Ch.1 Sec.2 Table B2





Corrosion Addition (tk , tc)

- In tanks for cargo oil and or water ballast the scantlings of the steel structures shall be increased by corrosion additions.[1)]

→Reduction of the plate thickness by corrosion is well known. Corrosion margin of plate is considered by the classification societies. [2)]

(Refer to : DnV Rules, Jan. 2004,Pt.3 Ch.1 Sec.2 Table D1, D2)

ex) Bottom plate thickness[1)]

$$t_1 = \frac{15.8 k_a s \sqrt{p}}{\sqrt{\sigma}} + t_k \; (mm)$$

Corrosion Addition

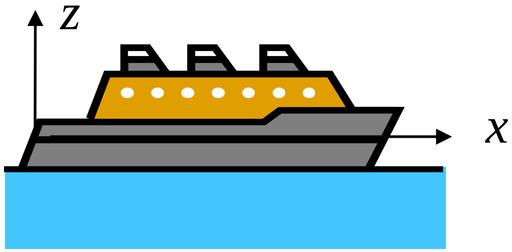
•Corrosion: deterioration of useful properties in a material due to reactions with its environment. Weakening of steel due to oxidation of the iron atoms is a well-known example of electrochemical corrosion.

16-2 Design Load

1) Ship motion and acceleration
2) Combined Acceleration
3) Design Probability Level
4) Load point
5) Pressure & Force
    a) Sea Pressure
    b) Liquid Tank Pressure

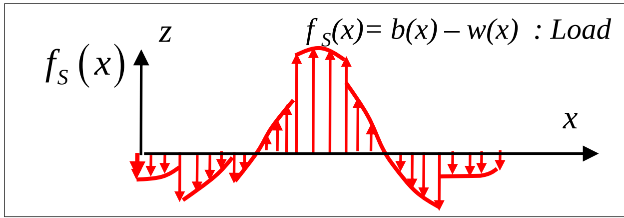
(Review) Loads in wave

In still water



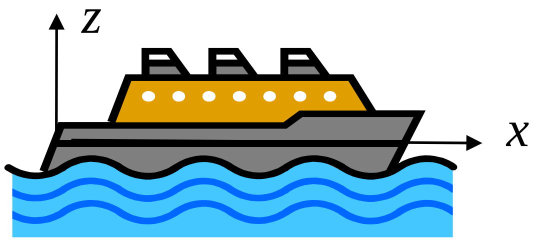
$$f_S(x) = b(x) - w(x)$$

$f(x)$: Distributed load in longitudinal direction
$f_S(x)$: Distributed load in longitudinal direction in still water
$f_W(x)$: Distributed load in longitudinal direction in wave



$+$

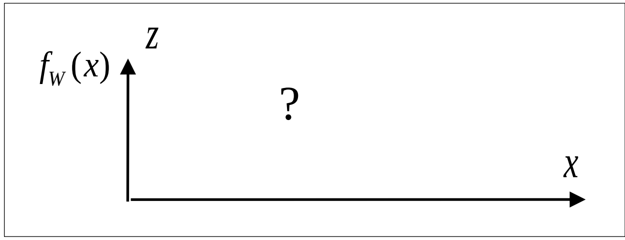
In wave



✓ Loads in wave

- from 6DOF motion of ship

$$\mathbf{x} = [\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6]^T$$



- for example, consider heave motion ξ_3

$$f(x) = f_S(x) + f_W(x)$$

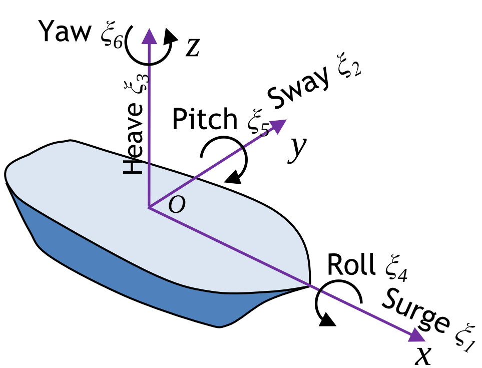
$$= b(x) - w(x) - m(x)\boxed{\ddot{\xi}_3} + f_D(x) + f_{F.K}(x) + f_R(x)$$

additional loads in wave

Where,

$$f_R(x) = -a_{33}(x)\boxed{\ddot{\xi}_3} - b_{33}(x)\boxed{\dot{\xi}_3}$$

✓ ref.> 6 DOF motion of ship



$f_D(x)$: Diffraction force in a unit length
$f_R(x)$: Radiation force in a unit length
$f_{F.K}(x)$: Froude-Krylov force in a unit length

In order to calculate loads in wave,
we have to know $\ddot{\xi}_3$, $\dot{\xi}_3$

How to know $\ddot{\xi}_3$, $\dot{\xi}_3$?

(Review) 6DOF Equation of motion of ship

How to know $\ddot{\mathbf{x}}, \dot{\mathbf{x}}$?

By solving equations of motion, we get the velocities and accelerations.

✓ Pressure acting on hull

Linearized Bernoulli Eq. $$P_{Fluid} = -\rho g z - \rho \frac{\partial \Phi}{\partial t} = -\rho g z - \rho \left(\frac{\partial \Phi_I}{\partial t} + \frac{\partial \Phi_D}{\partial t} + \frac{\partial \Phi_R}{\partial t} \right)$$

✓ Fluid force acting on hull

$$\mathbf{F}_{Fluid} = \iint_{S_B} P\mathbf{n}dS = -\iint_{S_B} \rho g z \,\mathbf{n} dS - \rho \iint_{S_B} \left(\frac{\partial \Phi_I}{\partial t} + \frac{\partial \Phi_D}{\partial t} + \frac{\partial \Phi_R}{\partial t} \right) dS$$
$$= \mathbf{F}_{Static} + \mathbf{F}_{F.K} + \mathbf{F}_D + \mathbf{F}_R$$

✓ 6 D.O.F equations of motion of a ship in waves

Newton's 2nd Law

Pressure force acting as surface force on hull

$$\mathbf{M}\ddot{\mathbf{x}} = \sum \mathbf{F} = \mathbf{F}_{Body} + \mathbf{F}_{Surface}$$
$$= \mathbf{F}_{Gravity} + \mathbf{F}_{Fluid} + \mathbf{F}_{External}$$

External force excluding wave exciting force (ex. control force..)

Body force | Surface force

$$\mathbf{M}\ddot{\mathbf{x}} = \underbrace{\mathbf{F}_{Gravity} + \mathbf{F}_{Static}}_{\mathbf{F}_{Restoring}} + \underbrace{\mathbf{F}_{F.K} + \mathbf{F}_D}_{\mathbf{F}_{Wave\ exciting}} + \mathbf{F}_R + \mathbf{F}_{External,\,dynamic} + \mathbf{F}_{External,\,static}$$

$$\mathbf{F}_R = -\mathbf{A}\ddot{\mathbf{x}} - \mathbf{B}\dot{\mathbf{x}}$$

added mass | Damping Coefficient

$$\mathbf{M}\ddot{\mathbf{x}} = (\mathbf{F}_{Gravity} + \mathbf{F}_{Static}) + \mathbf{F}_{Wave\ exciting} - \mathbf{A}\ddot{\mathbf{x}} - \mathbf{B}\dot{\mathbf{x}} + \mathbf{F}_{External,\,dynamic} + \mathbf{F}_{External,\,static}$$

Linearization , $(F_{Restoring} = (\mathbf{F}_{Gravity} + \mathbf{F}_{Static}) \approx -\mathbf{C}\mathbf{x})$

$$(\mathbf{M} + \mathbf{A})\ddot{\mathbf{x}} + \mathbf{B}\dot{\mathbf{x}} + \mathbf{C}\mathbf{x} = \mathbf{F}_{Wave\ exciting} + \mathbf{F}_{External,\,dynamic} + \mathbf{F}_{External,\,static}$$

By solving equations of motion, we get the velocities and accelerations!!

$F_{F.K}$: Froude-Krylov force
F_D : Diffraction force
F_R : Radiation force

Φ_I : Incident wave velocity potential

Φ_D : Diffraction wave velocity potential

Φ_R : Radiation wave velocity potential

$\mathbf{M}_A$: 6×6 added mass matrix
$\mathbf{B}$: 6×6 damping coeff. matrix
$\mathbf{C}$: 6×6 restoring coeff. matrix

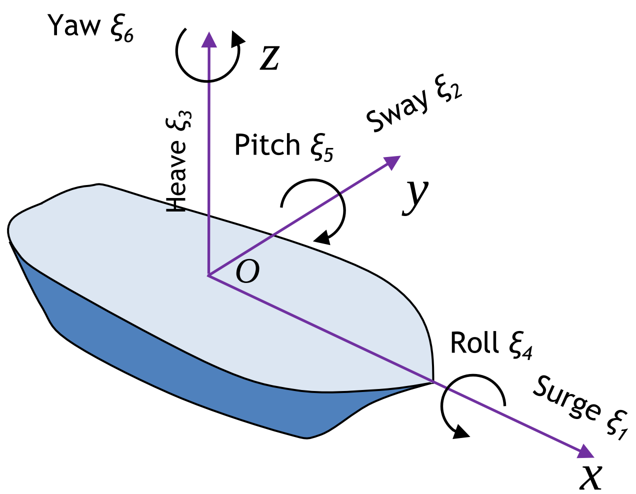
1) Ship Motion and Acceleration - Empirical Formula of DnV Rule

Common Acceleration Parameter	$a_0 = \frac{3C_w}{L} + C_v C_{v1}$
Surge Acceleration	$a_x = 0.2 g_0 a_0 \sqrt{C_b}$
Combined Sway/Yaw Acceleration	$a_y = 0.3 g_0 a_0$
Heave Acceleration	$a_z = 0.7 g_0 \frac{a_0}{\sqrt{C_b}}$
Tangential Roll Acceleration	$a_r = \phi \left(\frac{2\pi}{T_r}\right)^2 R_r$
Tangential Pitch Acceleration	$a_p = \theta \left(\frac{2\pi}{T_p}\right)^2 R_p$

g_0 : standard acceleration of gravity

=9.81m/s^2

✓ ref.> 6 DOF motion of ship



• A common Acceleration Parameter, a_0

$$\boxed{a_0 = \frac{3C_W}{L} + C_V C_{V1}}$$

$C_V = \frac{\sqrt{L}}{50}$, maximum 0.2

$C_{V1} = \frac{V}{\sqrt{L}}$, minimum 0.8

C_W = Wave coefficient

L	C_W
$L \le 100$	$0.0792 \cdot L$
$100 < L < 300$	$10.75 - [(300-L)/100]^{3/2}$
$300 \le L \le 350$	10.75
$L > 350$	$10.75 - [(L-350)/150]^{3/2}$

B 400 Roll motion and acceleration

B 300 Surge, sway /yaw and heave accelerations

301 The surge acceleration is given by:

$$a_x = 0.2\, g_0\, a_0 \sqrt{C_B} \qquad (m/s^2)$$

302 The combined sway/yaw acceleration is given by:

$$a_y = 0.3\, g_0\, a_0 \quad (m/s^2)$$

303 The heave acceleration is given by:

$$a_z = 0.7\, g_0 \frac{a_0}{\sqrt{C_B}} \qquad (m/s^2)$$

B 400 Roll motion and acceleration

401 The roll angle (single amplitude) is given by:

$$\phi = \frac{50c}{B + 75} \qquad (rad)$$

c = $(1.25 - 0.025\, T_R)\, k$
k = 1.2 for ships without bilge keel
= 1.0 for ships with bilge keel
= 0.8 for ships with active roll damping facilities
T_R = as defined in 402, not to be taken greater than 30.

402 The period of roll is generally given by:

$$T_R = \frac{2k_r}{\sqrt{GM}} \qquad (s)$$

k_r = roll radius of gyration in m
GM = metacentric height in m.

B 400 Roll motion and acceleration, B 500 Pitch motion and acceleration

The values of k_r and GM to be used shall give the minimum realistic value of T_R for the load considered.

In case k_r and GM have not been calculated for such condition, the following approximate design values may be used:

k_r = 0.39 B for ships with even transverse distribution of mass
= 0.35 B for tankers in ballast
= 0.25 B for ships loaded with ore between longitudinal bulkheads

GM = 0.07 B in general
= 0.12 B for tankers and bulk carriers.
= 0.05 B for container ship with B < 32.2 m
= 0.08 B for container ship with B > 40.0 m
with interpolation for B in between.

403 The tangential roll acceleration (gravity component not included) is generally given by:

$$a_r = \phi \left(\frac{2\pi}{T_R}\right)^2 R_R \qquad (\mathrm{m/s^2})$$

R_R = distance in m from the centre of mass to the axis of rotation.

The roll axis of rotation may be taken at a height z m above the baseline.

z = the smaller of $\left[\frac{D}{4} + \frac{T}{2}\right]$ and $\left[\frac{D}{2}\right]$

404 The radial roll acceleration may normally be neglected.

B 500 Pitch motion and acceleration

501 The pitch angle is given by:

$$\theta = 0.25 \frac{a_0}{C_B} \qquad (\mathrm{rad})$$

502 The period of pitch may normally be taken as:

$$T_P = 1.8 \sqrt{\frac{L}{g_0}} \qquad (\mathrm{s})$$

503 The tangential pitch acceleration (gravity component not included) is generally given by:

$$a_p = \theta \left[\frac{2\pi}{T_P}\right]^2 R_P \quad (m/s^2)$$

T_P = period of pitch
R_P = distance in m from the centre of mass to the axis of rotation.

The pitch axis of rotation may be taken at the cross- section 0.45 L from A.P. z meters above the baseline.

z = as given in 403.

With T_P as indicated in 502 the pitch acceleration is given by:

$$a_p = 120 \ \theta \frac{R_P}{L} \quad (m/s^2)$$

504 The radial pitch acceleration may normally be neglected.

B 600 Combined vertical acceleration

601 Normally the combined vertical acceleration (acceleration of gravity not included) may be approximated by:

$$a_v = \frac{k_v g_0 a_0}{C_B} \quad (m/s^2)$$

k_v = 1.3 aft of A.P.
= 0.7 between 0.3 L and 0.6 L from A.P.
= 1.5 forward of F.P.

Between mentioned regions k_v shall be varied linearly, see Fig.3.

1) Ship Motions and Accelerations
- Roll angle & Roll Period

✓ Roll angle

$$\phi = \frac{50c}{B + 75} \quad \text{(rad)}$$

c = $(1.25 - 0.025\, T_R)\, k$
k = 1.2 for ships without bilge keel
= 1.0 for ships with bilge keel
= 0.8 for ships with active roll damping facilities
T_R = as defined in 402, not to be taken greater than 30.

✓ Roll Period

$$T_R = \frac{2k_r}{\sqrt{GM}} \quad \text{(s)}$$

k_r= 0.39B for ships with even transverse distribution of mass
=0.35B for tankers in ballast
=0.25B for ships loaded with ore between longitudinal bulkheads
GM= 0.07B in general
= 0.12B for tankers and bulk carriers

✓ Pitch angle

$$\theta = 0.25 \frac{a_0}{C_B} \quad \text{(rad)}$$

$$a_0 = \frac{3C_W}{L} + C_V C_{V1}$$

✓ Pitch Period

$$T_P = 1.8 \sqrt{\frac{L}{g_0}} \quad \text{(s)}$$

g_0 : standard acceleration of gravity
=9.81m/s^2

2) Combined Acceleration
- Combined Vertical Acceleration, a_v

1) DnV Rules, Jan. 2004,Pt.3 Ch.1 Sec.4 B602
2) DnV Rules, Jan. 2004,Pt.3 Ch.1 Sec.4 B401
3) DnV Rules, Jan. 2004,Pt.3 Ch.1 Sec.4 B402
4) DnV Rules, Jan. 2004,Pt.3 Ch.1 Sec.4 B303

✓ The acceleration along the ship's vertical axis considering combined effect of heave, pitch & roll motion[1)]

$$\boxed{a_v = \frac{k_v g_o a_o}{C_b}}$$

K_v = Acceleration distribution factor along the length of vessel

= 0.7 between 0.3L and 0.6L from A.P.

a_0 = Common Acceleration Parameter

$$a_v = \max\left\{\sqrt{a_z^{\,2} + a_{rz}^{\,2}},\ \sqrt{a_z^{\,2} + a_{pz}^{\,2}}\right\}$$

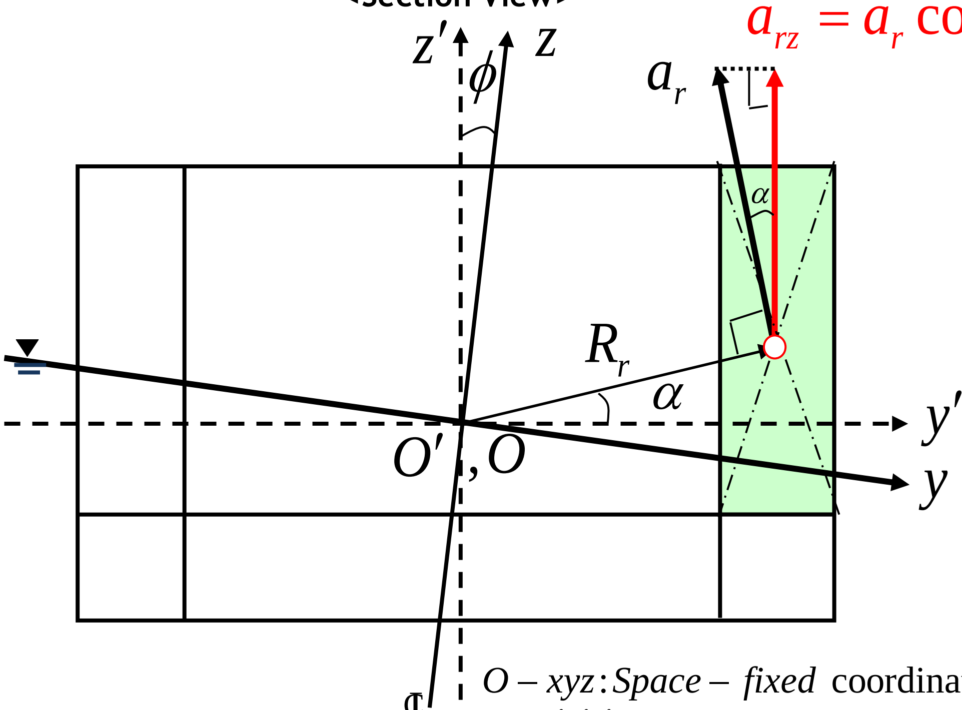
Heave Acceleration[4)] (a_z)

Vertical component of tangential roll acceleration (a_{rz})

Vertical component of tangential pitch acceleration (a_{pz})

<Section View>

$$a_{rz} = a_r \cos\alpha$$



$O - xyz$: Space − fixed coordinate system

$O - x'y'z'$: body fixed coordinate system

$$\phi = \phi^A \cos\left(\frac{2\pi}{T_R} t\right)$$

$$\ddot{\phi} = \phi^A \left(\frac{2\pi}{T_R}\right)^2 \cdot \cos\left(\frac{2\pi}{T_R} t\right)$$

$$a_r = \ddot{\phi} \cdot R_r$$

$$= \phi^A \left(\frac{2\pi}{T_r}\right)^2 R_r$$

a_r : tangential roll acceleration

R_r : distance in m from the center of the mass to the axis of rotation

α : angle of center of mass about the body fixed coordinate system

ϕ : roll angle

ϕ^A : roll angle amplitude[2)]

T_R : period of roll[3)]

g_0 : standard acceleration of gravity

B 400 Roll motion and acceleration

401 The roll angle (single amplitude) is given by:

$$\phi = \frac{50c}{B + 75} \quad \text{(rad)}$$

c = $(1.25 - 0.025\ T_R)\ k$
k = 1.2 for ships without bilge keel
= 1.0 for ships with bilge keel
= 0.8 for ships with active roll damping facilities
T_R = as defined in 402, not to be taken greater than 30.

402 The period of roll is generally given by:

$$T_R = \frac{2k_r}{\sqrt{GM}} \quad \text{(s)}$$

k_r = roll radius of gyration in m
GM = metacentric height in m.

The values of k_r and GM to be used shall give the minimum realistic value of T_R for the load considered.

In case k_r and GM have not been calculated for such condition, the following approximate design values may be used:

k_r = 0.39 B for ships with even transverse distribution of mass
= 0.35 B for tankers in ballast
= 0.25 B for ships loaded with ore between longitudinal bulkheads
GM = 0.07 B in general
= 0.12 B for tankers and bulk carriers.
= 0.05 B for container ship with B < 32.2 m
= 0.08 B for container ship with B > 40.0 m
with interpolation for B in between.

303 The heave acceleration is given by:

$$a_z = 0.7\ g_0 \frac{a_0}{\sqrt{C_B}} \quad (\text{m/s}^2)$$

2) Combined Acceleration
- Combined Vertical Acceleration, a_v

1) DnV Rules, Jan. 2004,Pt.3 Ch.1 Sec.4 B602
2) DnV Rules, Jan. 2004,Pt.3 Ch.1 Sec.4 B401
3) DnV Rules, Jan. 2004,Pt.3 Ch.1 Sec.4 B402
4) DnV Rules, Jan. 2004,Pt.3 Ch.1 Sec.4 B303

✓ The acceleration along the ship's vertical axis considering combined effect of heave, pitch & roll motion[1)]

$$\boxed{a_v = \frac{k_v g_o a_o}{C_b}}$$

K_v = Acceleration distribution factor along the length of vessel
= 0.7 between 0.3L and 0.6L from A.P.
a_0 = Common Acceleration Parameter

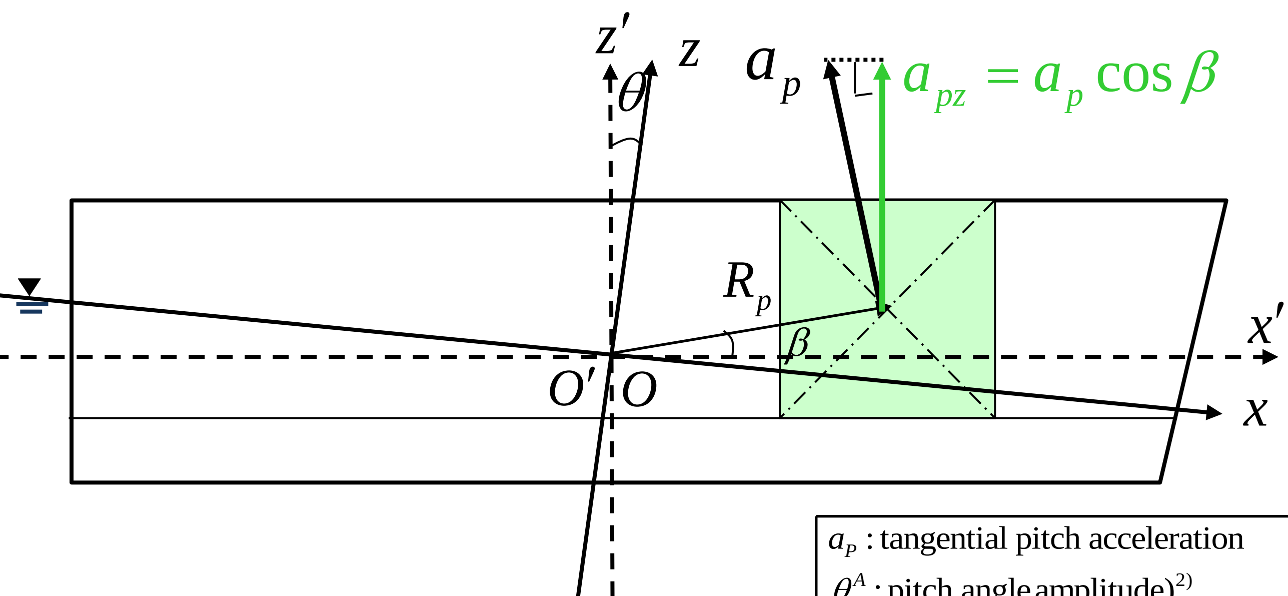
$$\boxed{a_v = \max\left\{\sqrt{a_z^{\ 2} + a_{rz}^{\ 2}}, \sqrt{a_z^{\ 2} + a_{pz}^{\ 2}}\right\}}$$

Heave Acceleration[4)]

Vertical component of tangential roll acceleration

Vertical component of tangential pitch acceleration

<Elevation View>



$$\theta = \theta^A \cos(\frac{2\pi}{T_P} t)$$

$$\ddot{\theta} = \theta^A \left(\frac{2\pi}{T_P}\right)^2 \cdot \cos(\frac{2\pi}{T_P} t)$$

$$a_P = \ddot{\theta} \cdot R_r$$

$$= \theta^A \left(\frac{2\pi}{T_P}\right)^2 R_r$$

$O - xyz : Space - fixed$ coordinate system
$O - x'y'z'$: body fixed coordinate system

a_P : tangential pitch acceleration
θ^A : pitch angle amplitude)[2)]
θ : pitch angle
T_P : period of pitch[3)]

R_P : distance in m from the center of the mass to the axis of rotation
β : angle of center of mass about the body fixed coordinate system

2) Combined Acceleration
- Combined Transverse Acceleration, a_t

1) DnV Rules, Jan. 2004,Pt.3 Ch.1 Sec.4 B700

✓ The acceleration along the ship's transverse axis considering combined effect of sway, yaw & roll motion[1)]

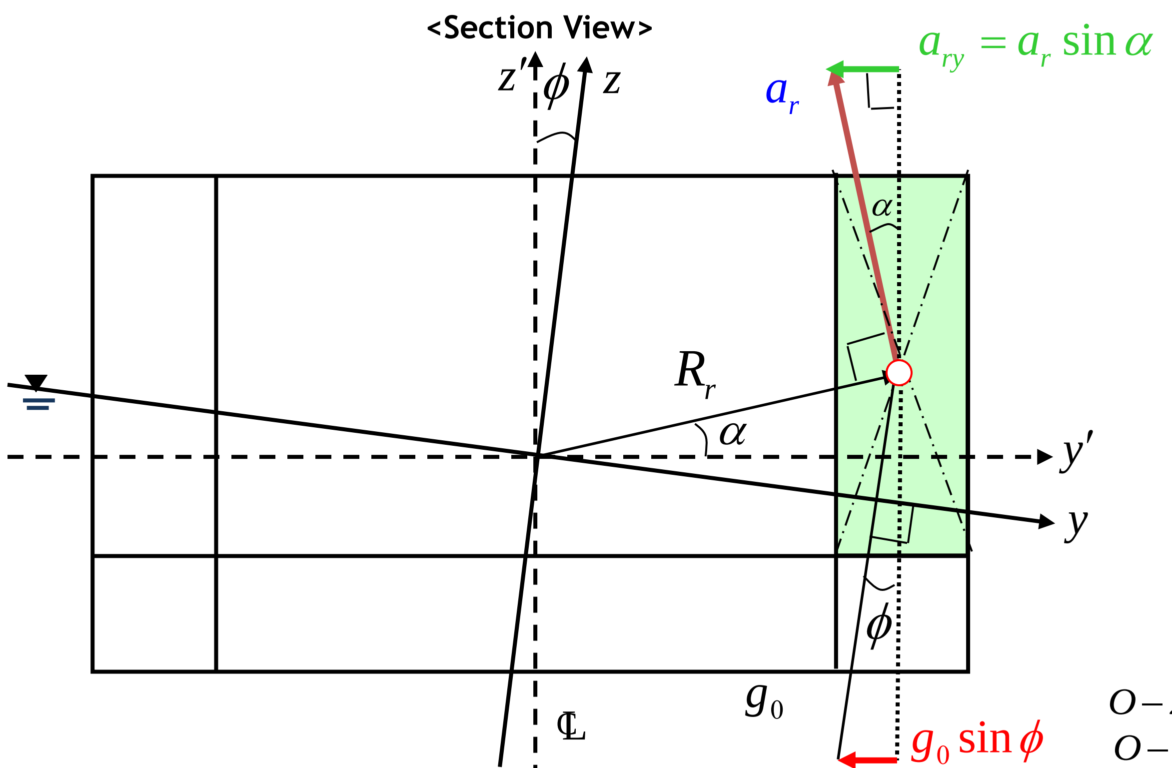
$$a_t = \sqrt{a_y{}^2 + (g_o \sin\phi + a_{ry})^2}$$

Combined Sway & yaw acceleration

$a_y = 0.3 g_0 a_0$

Transverse component of acceleration of gravity by roll angle

Transverse component of the tangential roll acceleration



$, a_r = \phi^A \left(\frac{2\pi}{T_r}\right)^2 R_r$

a_r : tangential roll acceleration

R_r : distance in m from the center of the mass to the axis of rotation

ϕ : roll angle

ϕ^A : roll angle amplitude

T_R : period of roll[3)]

g_0 : standard acceleration of gravity =9.81m/s^2

$O-xyz$: Global coordinate system

$O-x'y'z'$: body fixed coordinate system

Pt.3 Ch.1 Sec.4 B701 2011

701 Acceleration along the ship's transverse axis is given as the combined effect of sway/yaw and roll calculated as indicated in 100, i.e.:

$$a_t = \sqrt{a_y^{\;2} + (g_0 \sin\phi + a_{ry})^2} \qquad (\mathrm{m/s^2})$$

a_{ry} = transverse component of the roll acceleration given in 403.

Note that a_{ry} is equal to a_r using the vertical projection of R_R.

2) Combined Acceleration
- Combined Longitudinal Acceleration, a_l

1) DnV Rules, Jan. 2004,Pt.3 Ch.1 Sec.4 B800

✓ The acceleration along the ship's longitudinal axis considering combined effect of surge & pitch motion[1)]

$$a_l = \sqrt{a_x{}^2 + (g_o \sin\theta + a_{px})^2}$$

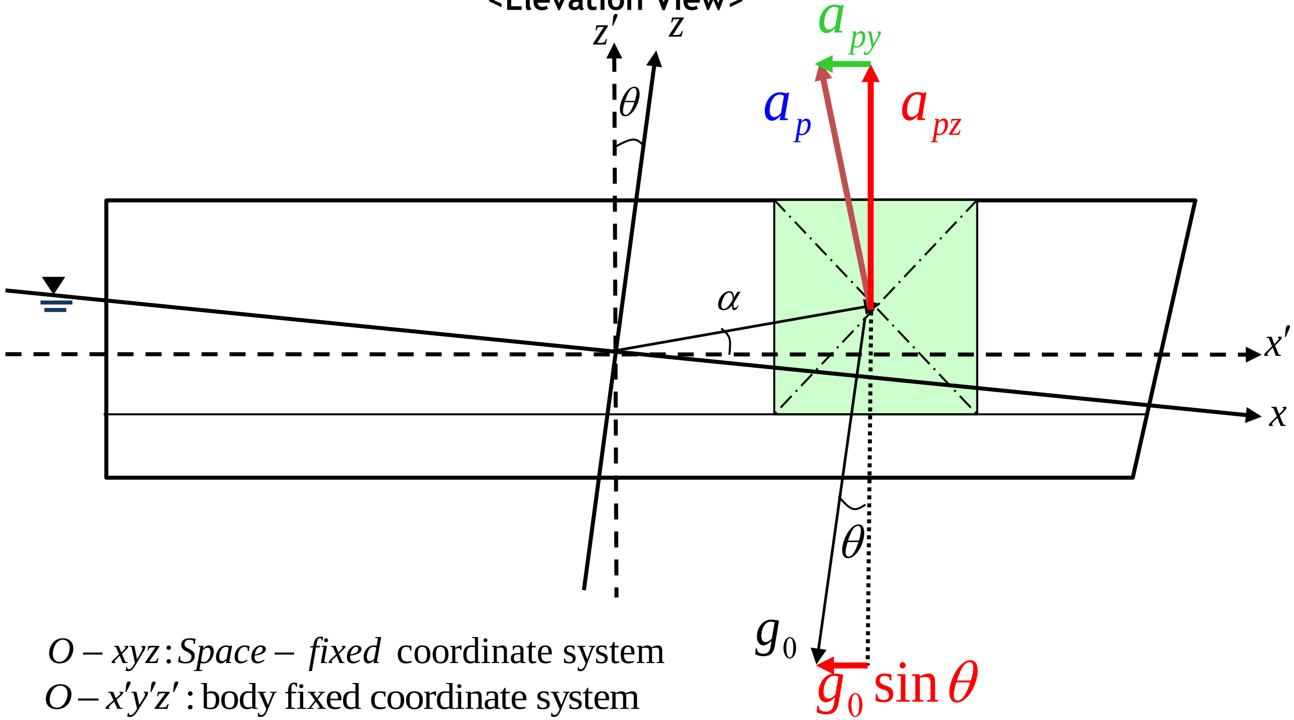
Combined Sway & yaw acceleration

$$a_x = 0.2 g_0 a_0 \sqrt{C_b}$$

Longitudinal component of gravitational acceleration by Pitch angle

Longitudinal component of the Pitch acceleration

<Elevation View>



$O - xyz$: *Space – fixed* coordinate system

$O - x'y'z'$: body fixed coordinate system

$$a_P = \theta^A \left(\frac{2\pi}{T_P}\right)^2 R_r$$

a_P : tangential pitch acceleration

R_P : distance in m from the center of the mass to the axis of rotation

θ : pitch angle

θ^A : pitch angle amplitude[2)]

T_P : period of pitch[3)]

g_0 : standard acceleration of gravity $=9.81 \mathrm{m/s^2}$

2) Combined Acceleration
- Example) Vertical Acceleration

(Example) Calculate the vertical acceleration of a given ship at 0.5L (amidships) by DNV Rule

[Dimension] L_s=315.79 m, V=15.5 knots, C_B=0.832

$$\boxed{a_v = \frac{k_v g_o a_o}{C_b}}$$

K_v = Acceleration distribution factor along the length of vessel

= 0.7 between 0.3L and 0.6L from A.P.

a_0 = Common Acceleration Parameter

g_0 = Standard acceleration of gravity (=9.81m/sec^2)

(Sol.) $$a_v = (k_v\, g_0\, a_0) / C_B = (0.7 \times 9.81 \times 0.277)/0.832$$

$$= 2.286 \,(m/\sec^2)$$

where, $k_v = 0.7$ at mid ship

$$a_0 = 3C_W / L + C_v\, C_{v1} = 3 \times 10.75/315.79 + 0.2 \times 0.872 = 0.277$$

$$C_v = L^{0.5}/50 = 315.79^{0.5}/50 = 0.355 \text{ or Max. } 0.2$$

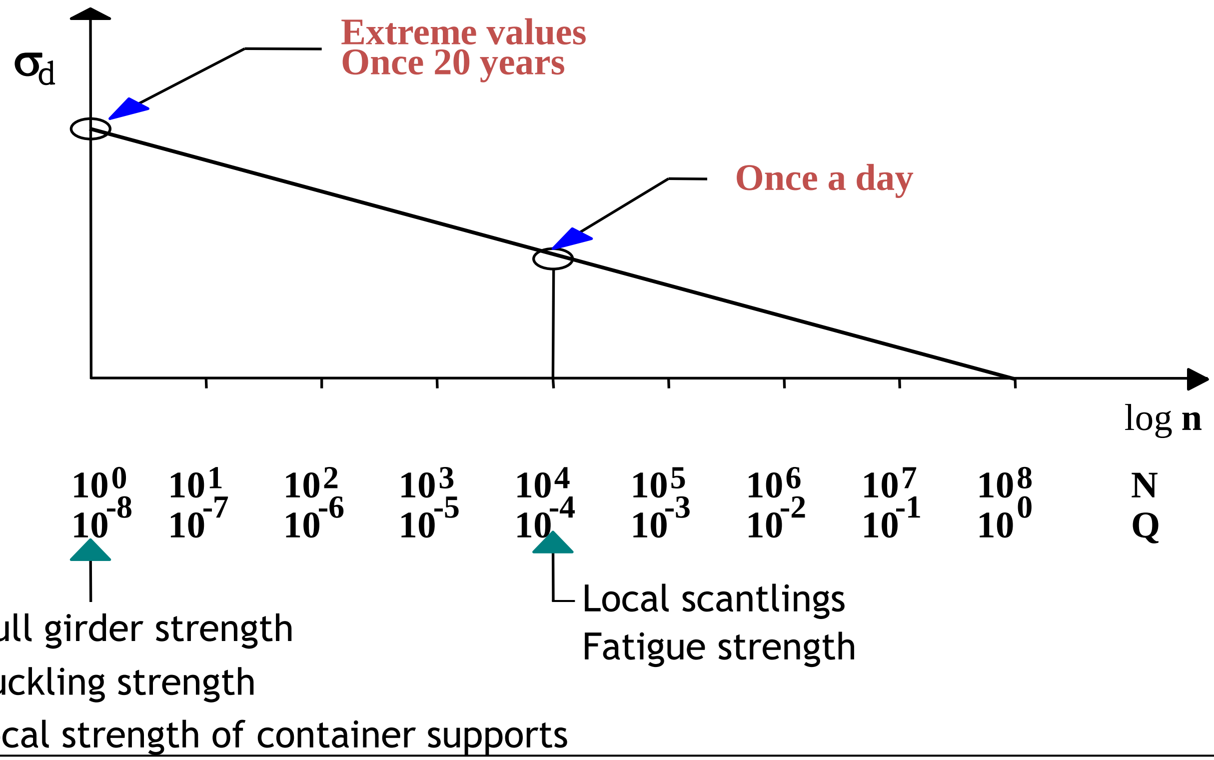
$$= 0.2$$

$$C_{v1} = V/L^{0.5} = 15.5/315.79^{0.5} = 0.872 \text{ or Min. } 0.8$$

$$= 0.872$$

3) Design Probability Level

◆Probability Level[1)]



◆Design probability Level[2)]

✓ Number of waves that the ship experiences during the ship's life (for 20 years) : about 10^8

→ The ship is design to endure the extreme wave (10^{-8} probability) which the ship encounter once for 20 years

(Extreme condition), (Ship motion, Acceleration is given as extreme value)

✓ In case of design pressure, use the reduced value of 10^{-4} (Reduction Value = 0.5 × Extreme value)

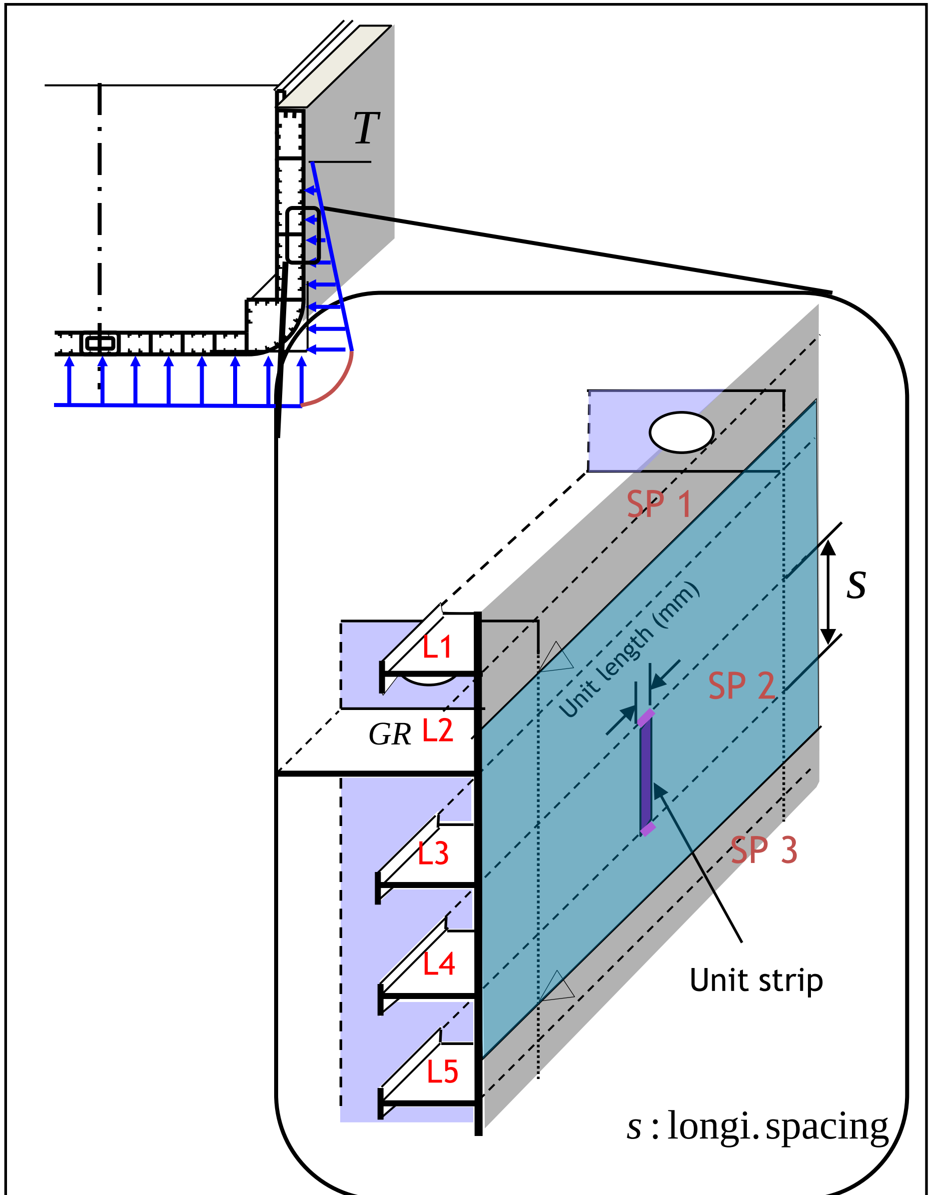
예) Liquid Tank Pressure: Pressure, P_1 , considering Vertical Acceleration

$$p_1 = \rho\left(g_o + 0.5 a_v\right) h_s$$

4) Load point
- Horizontally stiffened plate

DnV Rules, Jan. 2004,Pt.3 Ch.1 Sec.4 A 202

$$p_2 = \rho\, g_o \left[\, 0.67(h_s + \phi\, b) - 0.12\sqrt{H\, b_t\, \phi}\, \right]$$



✓ **The pressure at the load point** is considered as **uniform load** of unit strip

✓ Definition of Load point

1. General

   : Midpoint of stiffened plate field

2. Seam & butt (In case two plates are welded)

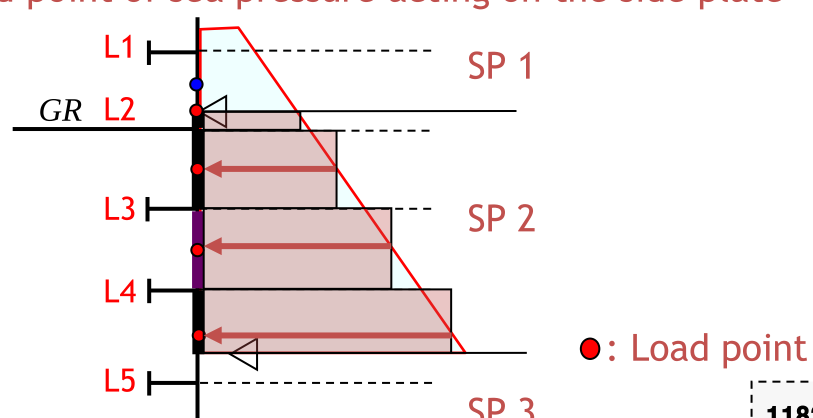
   1) When considered plate includes the midpoint of stiffened plate field

      : Midpoint of stiffened plate field

   2) When considered plate does not include the midpoint of stiffened plate field

      : Nearest seam or butt line from midpoint

✓ Load point of sea pressure acting on the side plate



Pt.3 Ch.1 Sec.4 A201 ,202 2011

A 200 Definitions

201 Symbols:

p = design pressure in kN/m^2
ρ = density of liquid or stowage rate of dry cargo in t/m^3.

202 The load point for which the design pressure shall be calculated is defined for various strength members as follows:

a) For plates:
midpoint of horizontally stiffened plate field.
Half of the stiffener spacing above the lower support of vertically stiffened plate field, or at lower edge of plate when the thickness is changed within the plate field.

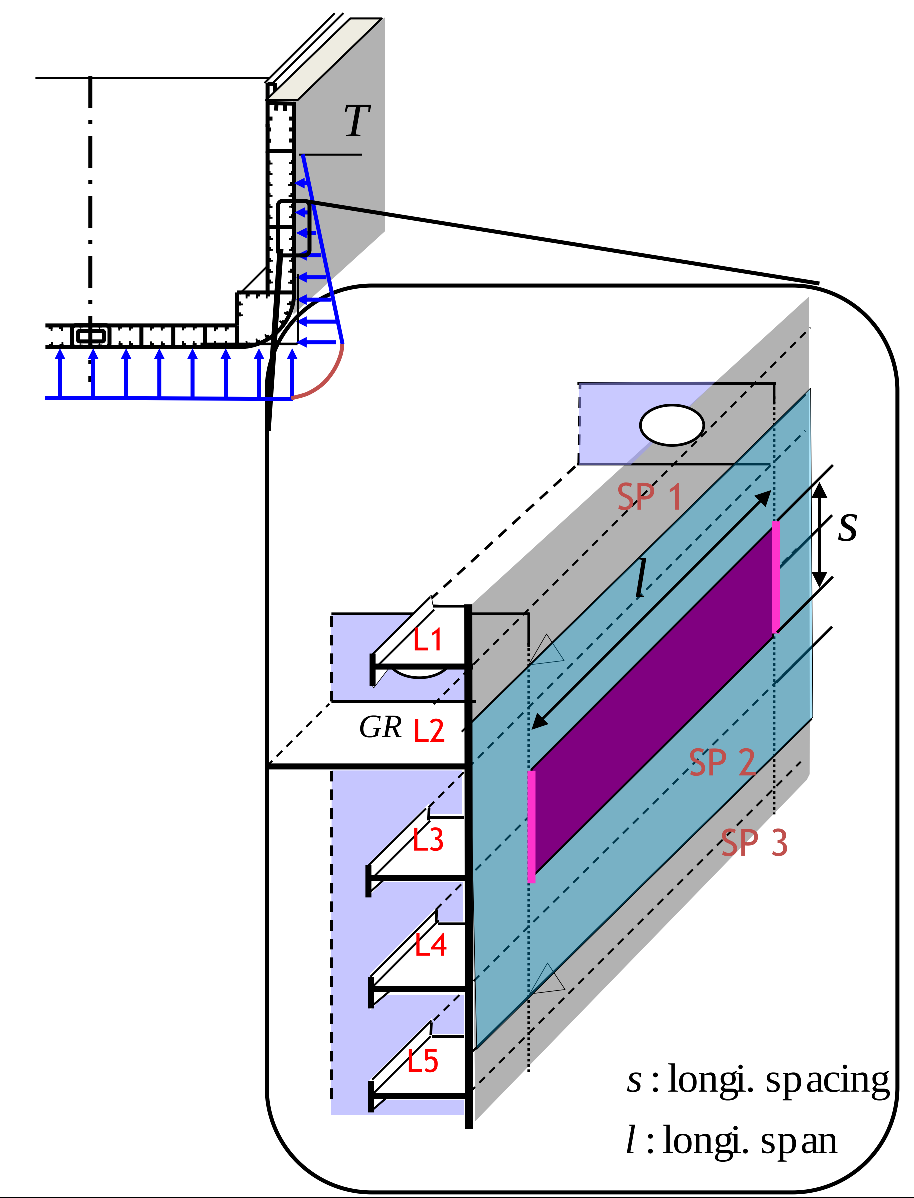
b) For stiffeners:
midpoint of span.
When the pressure is not varied linearly over the span the design pressure shall be taken as the greater of:

$$p_m \quad \text{and} \quad \frac{p_a + p_b}{2}$$

p_m, p_a and p_b are calculated pressure at the midpoint and at each end respectively.

c) For girders:
midpoint of load area.

4) Load point
- Longitudinal stiffeners



✓ **The pressure at the load point** is considered as **uniform load**

✓ Definition of load point

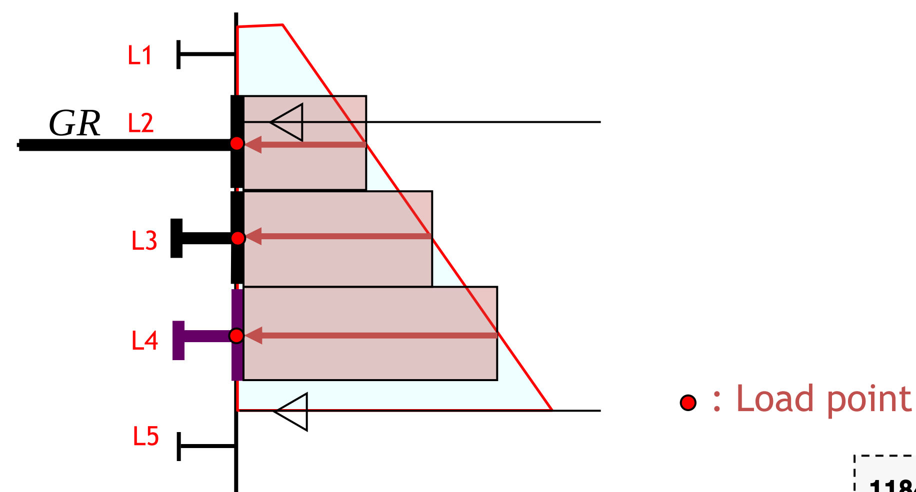
1. In vertical direction

: The point of intersection between a plate and a stiffener

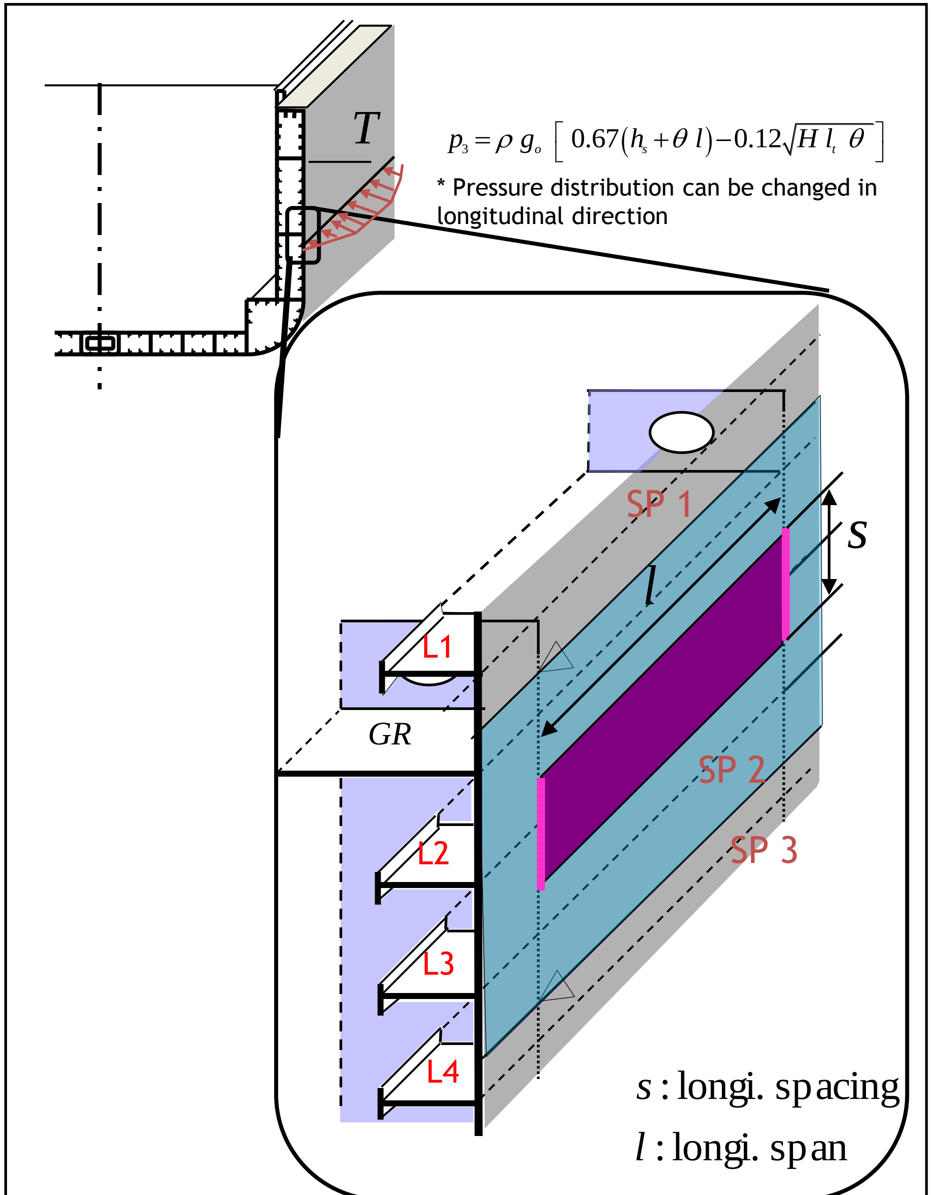
2. In longitudinal direction

: Midpoint of span

✓ Load point of sea pressure acting on the side plate
- In vertical direction



4) Load point
- Longitudinal stiffeners



✓ **The pressure at the load point** is considered as **uniform load**

✓ Definition of load point

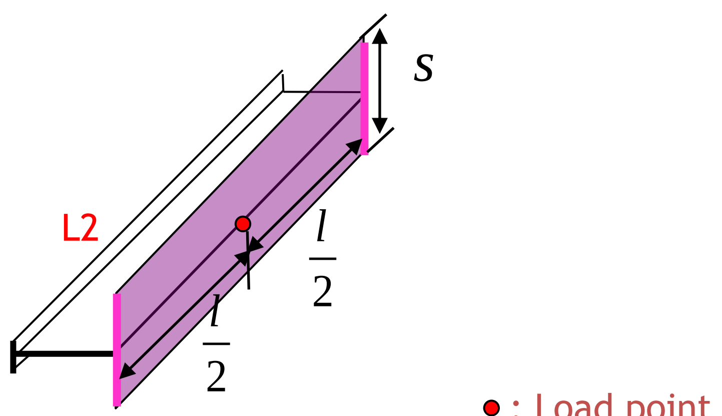
1. In vertical direction

   : The point of intersection between a plate and a stiffener

2. In longitudinal direction

   : Midpoint of span

✓ Load point of sea pressure acting on the side plate
- In longitudinal direction

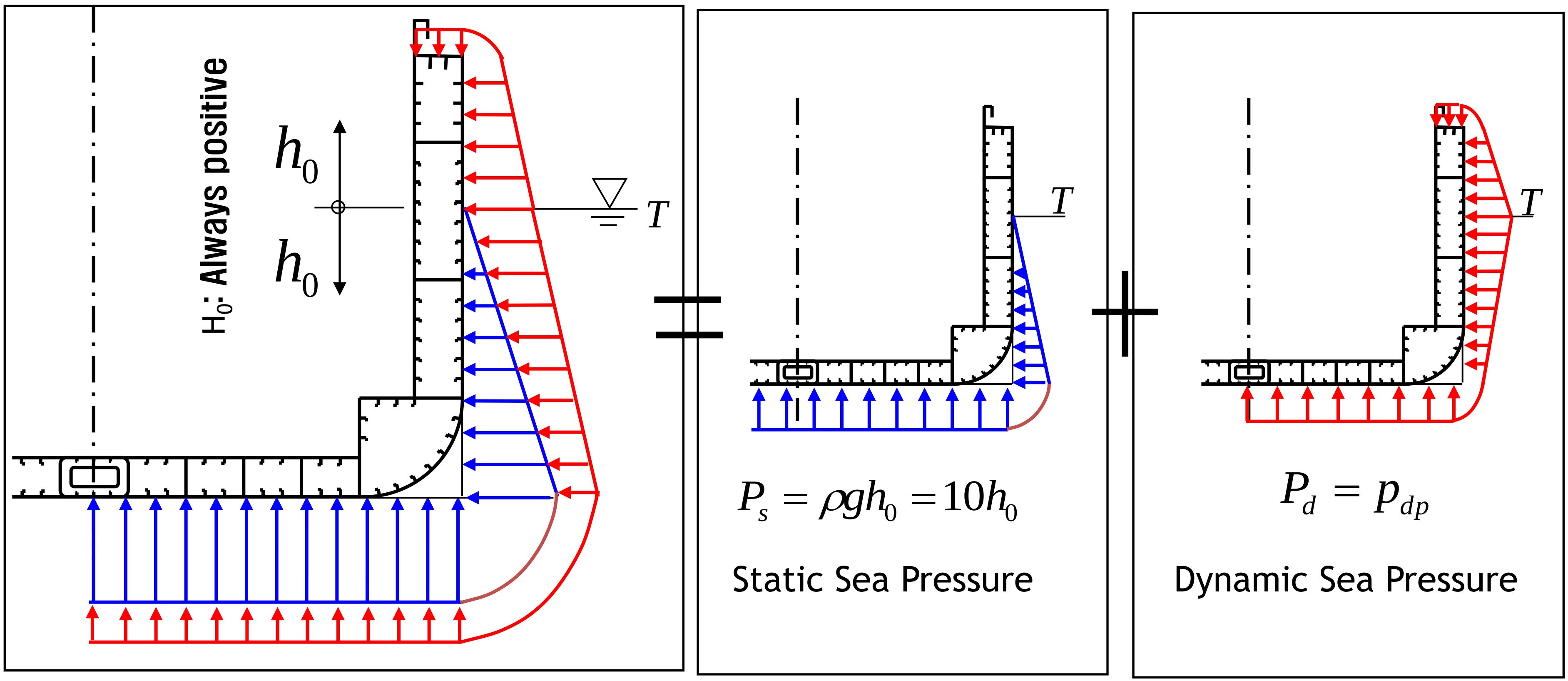


5) Pressure and Force

a) Sea Pressure

✓ sea pressures = static sea pressure + dynamic sea pressure

$$P = P_s + P_d$$



5) Pressure and Force
b) Liquid Tank Pressure (1)

1)DnV Rules, Jan. 2004,Pt.3 Ch.1 Sec.4 C 300

- The pressure in full tanks shall be taken as the greater of p_1 ~ p_5 [1)]

$p_1 = \rho\ (g_o + 0.5a_v)\, h_s$	P_1: Considering vertical acceleration
$p_2 = \rho\, g_o \left[\ 0.67(h_s + \phi\, b) - 0.12\sqrt{H\, b_t\ \phi}\ \right]$	P_2: Considering rolling motion
$p_3 = \rho\, g_o \left[\ 0.67(h_s + \theta\, l) - 0.12\sqrt{H\, l_t\ \theta}\ \right]$	P_3: Considering pitching motion
$p_4 = 0.67(\rho\, g_o h_p + \Delta P_{dyn})$	P_4: Considering overflow
$p_5 = \rho\, g_o h_s + p_o$	P_5: Considering tank test pressure

a_v : Vertical acceleration

ϕ : Roll angle

b : the largest athwartship distance in m form the load point to the tank corner at top of tank

$b_t\, \&\, l_t$: breadth and length in m of top of tank

ρ : density of liquid cargo

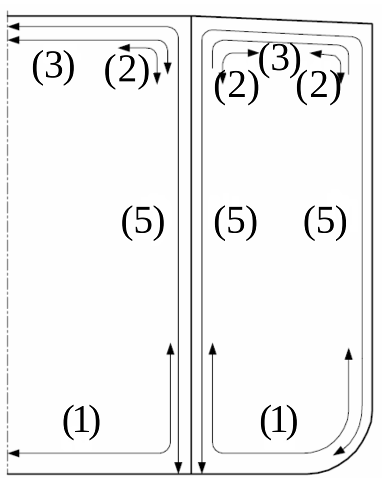
h_s : Vertical distance from the load point to tank top in tank

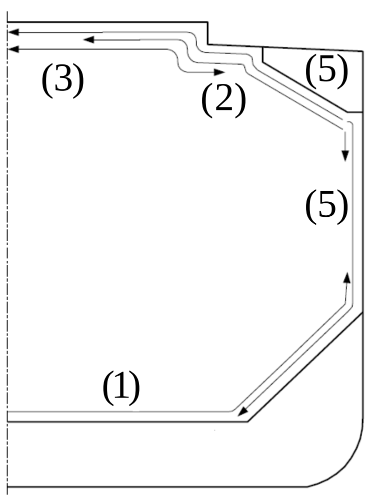
h_p : Vertical distance from the load point to the top of air pipe

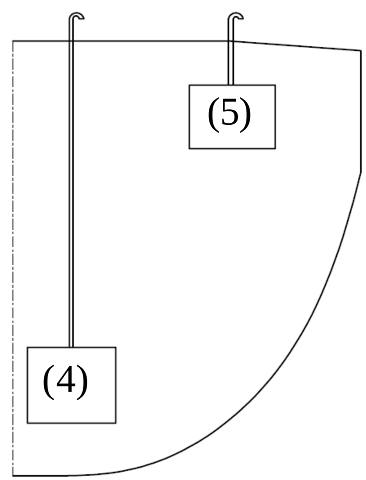
p_0 : 25 kN/m² general

Δp_{dyn} : calculated pressure drop

- Maximum pressure is different depending on locations







Pt.3 Ch.1 Sec.4 C302 2011

301 Tanks for crude oil or bunkers are normally to be designed for liquids of density equal to that of sea water, taken as $\rho = 1.025$ t/m^3 (i.e. $\rho\, g_0 \approx 10$). Tanks for heavier liquids may be approved after special consideration. Vessels designed for 100% filling of specified tanks with a heavier liquid will be given the notation **HL(ρ)**, indicating the highest cargo density applied as basis for approval. The density upon which the scantling of individual tanks are based, will be given in the appendix to the classification certificate.

302 The pressure in full tanks shall be taken as the greater of:

$$p = \rho\,(g_0 + 0.5\, a_v)\, h_s \quad (\text{kN/m}^2) \qquad [\,1\,]$$

$$p = \rho\, g_0\,[0.67(h_s + \phi b) - 0.12\sqrt{H b_t \phi}] \quad (\text{kN/m}^2) \quad [\,2\,]$$

$$p = \rho\, g_0\,[0.67(h_s + \theta l) - 0.12\sqrt{H l_t \theta}] \quad (\text{kN/m}^2) \quad [\,3\,]$$

$$p = 0.67\,(\rho\, g_0\, h_p + \Delta p_{dyn}) \quad (\text{kN/m}^2) \qquad [\,4\,]$$

$$p = \rho\, g_0\, h_s + p_0 \quad (\text{kN/m}^2) \qquad [\,5\,]$$

a_v = vertical acceleration as given in B600, taken in centre of gravity of tank.
ϕ = as given in B400
θ = as given in B500
H = height in m of the tank
ρ = density of ballast, bunkers or liquid cargo in t/m^3, normally not to be taken less than 1.025 t/m^3 (i.e. $\rho\, g_0 \approx 10$)
b = the largest athwartship distance in m from the load point to the tank corner at top of the tank which is situated most distant from the load point. For tank tops with stepped contour, the uppermost tank corner will normally be decisive

b_t = breadth in m of top of tank

l = the largest longitudinal distance in m from the load point to the tank corner at top of tank which is situated most distant from the load point. For tank tops with stepped contour, the uppermost tank corner will normally be decisive

l_t = length in m of top of tank

h_s = vertical distance in m from the load point to the top of tank, excluding smaller hatchways.

h_p = vertical distance in m from the load point to the top of air pipe

p_0 = 25 kN/m^2 in general

= 15 kN/m^2 in ballast holds in dry cargo vessels

= tank pressure valve opening pressure when exceeding the general value.

Δp_{dyn} = calculated pressure drop according to Pt.4 Ch.6 Sec.4 K201.

For calculation of girder structures the pressure [4] shall be increased by a factor 1.15.

The formulae normally giving the greatest pressure are indicated in Figs. 4 to 6 for various types.

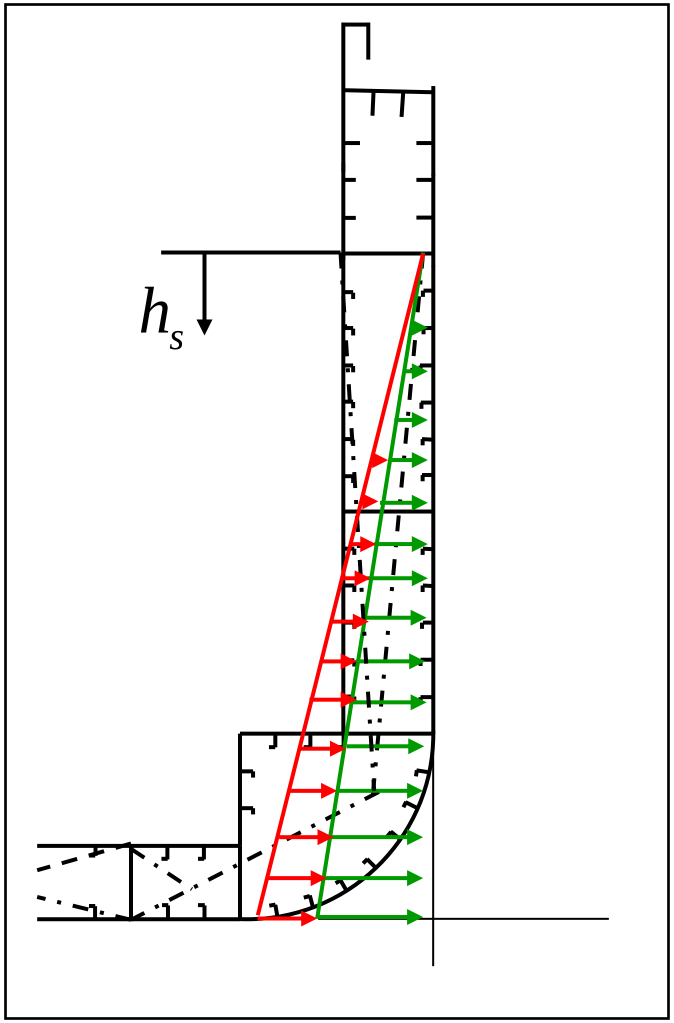
For sea pressure at minimum design draught which may be deduced from formulae above, see 202.

Formulae [2] and [3] are based on a 2% ullage in large tanks.

5) Pressure and Force
b) Liquid Tank Pressure (2)

$p_1 = \rho(g_o + 0.5a_v)h_s$	P_1: Considering vertical acceleration
$p_2 = \rho\, g_o\left[0.67(h_s + \phi\, b) - 0.12\sqrt{H\, b_t\, \phi}\right]$	P_2: Considering rolling motion
$p_3 = \rho\, g_o\left[0.67(h_s + \theta\, l) - 0.12\sqrt{H\, l_t\, \theta}\right]$	P_3: Considering pitching motion
$p_4 = 0.67(\rho\, g_o h_p + \Delta P_{dyn})$	P_4: Considering overflow
$p_5 = \rho\, g_o h_s + p_o$	P_5: Considering tank test pressure

✓Design Pressure, P_1 considering vertical acceleration **(General)**



$$P_1 = \underbrace{\rho g_0 h_s}_{\text{Static Pressure}} + \underbrace{0.5\rho a_v h_s}_{\text{Dynamic Pressure}}$$

Reduced value of 10^{-4} by probability level is used

(reduction value=0.5×extreme value)

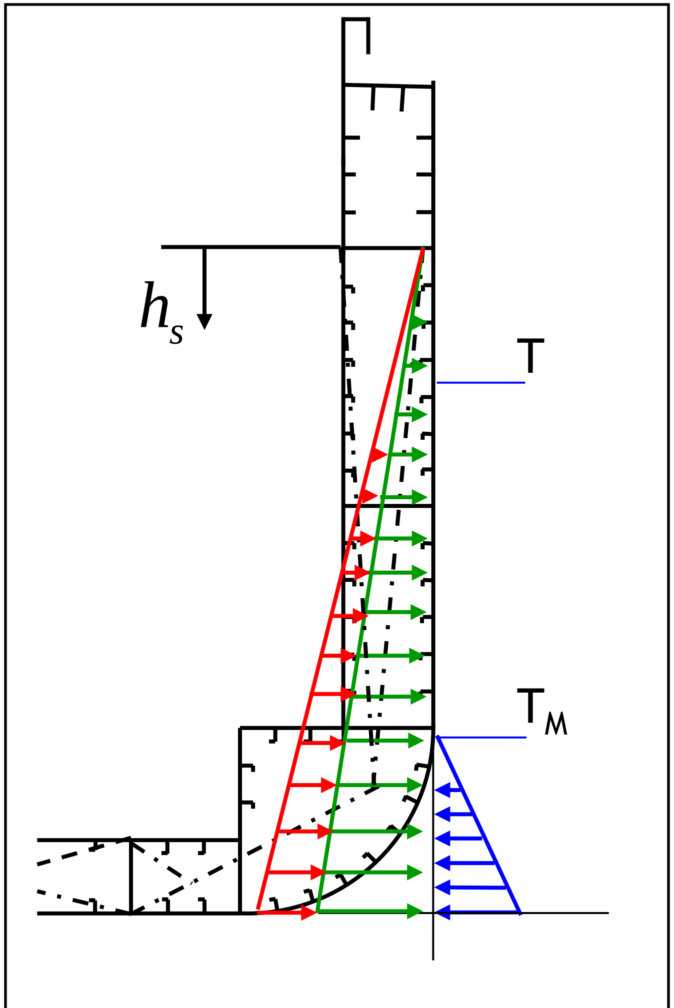
$$\boxed{p = \rho(g_0 + 0.5a_v)h_s}$$

a_v : Vertical acceleration

h_s : virtical distance in m from load point to top of tank

$p_1 = \rho\,(g_o + 0.5a_v)\,h_s$	P_1: Considering vertical acceleration
$p_2 = \rho\, g_o \left[0.67(h_s + \phi\, b) - 0.12\sqrt{H\, b_t\, \phi}\right]$	P_2: Considering rolling motion
$p_3 = \rho\, g_o \left[0.67(h_s + \theta\, l) - 0.12\sqrt{H\, l_t\, \theta}\right]$	P_3: Considering pitching motion
$p_4 = 0.67(\rho\, g_o h_p + \Delta P_{dyn})$	P_4: Considering overflow
$p_5 = \rho\, g_o h_s + p_o$	P_5: Considering tank test pressure

✓Design Pressure, P_1 considering vertical acceleration (In case of side shell)



In case of side shell, the effect of sea pressure is considered

$$P = \underbrace{\rho g_0 h_s}_{\text{Static Pressure}} + \underbrace{0.5\rho a_v h_s}_{\text{Dynamic Pressure}} \boxed{-10h_b}$$

sea pressure

When we consider the design pressure, the largest value shall be applied.
The liquid cargo pressure acting on the side shell is the highest when the sea pressure is the lowest, i.e. in case of minimum draft.

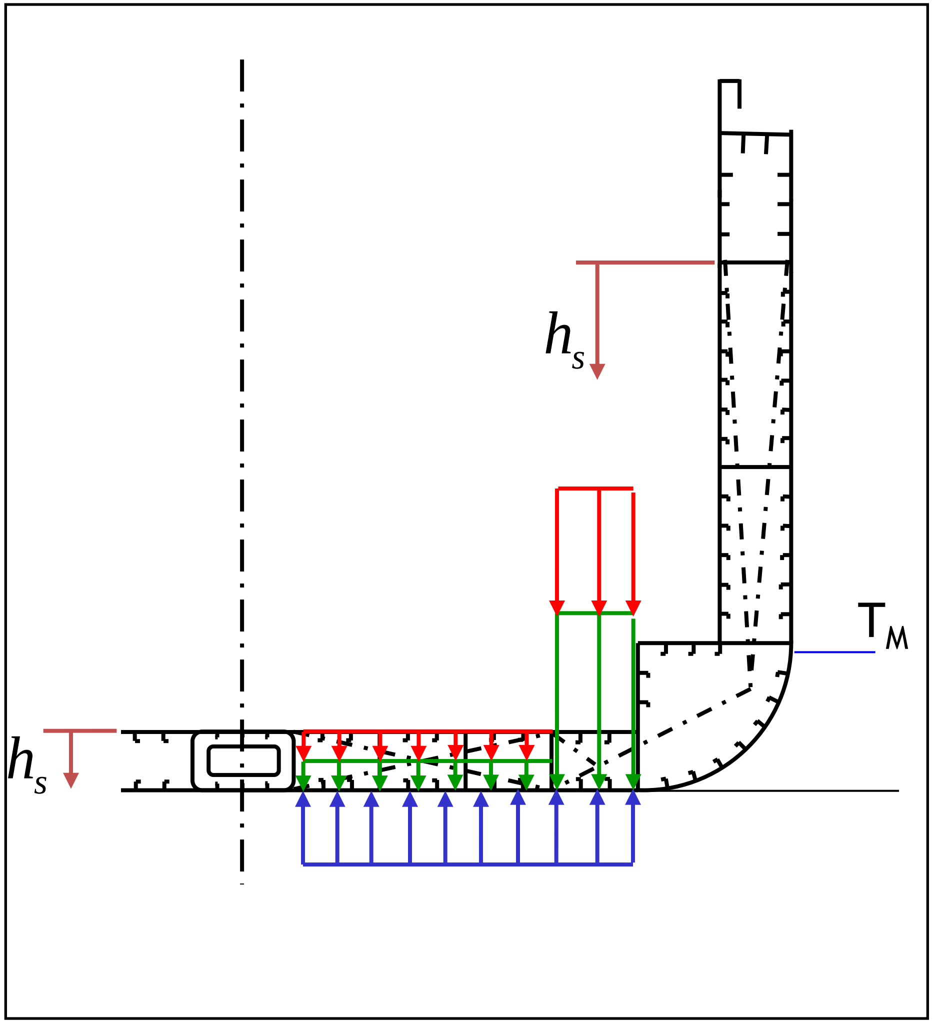
$$p = \rho(g_0 + 0.5a_v)h_s - 10h_b$$

h_b : vertical distance in m from load point to minimum design draft
= 2 + 0.02L for Tanker
= 0.35 T for Dry Cargo
(T : Rule Draft)

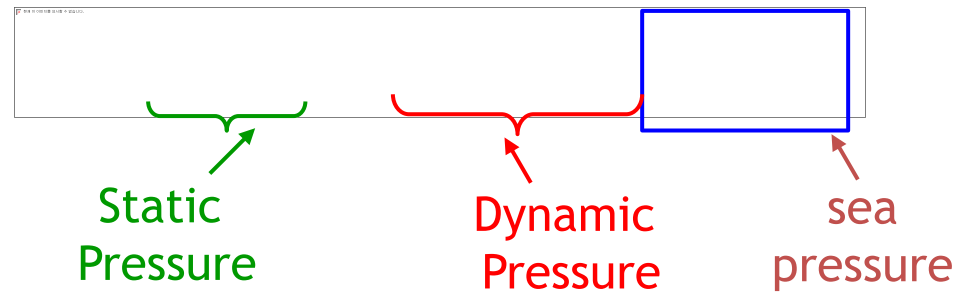
5) Pressure and Force
b) Liquid Tank Pressure (4)

$p_1 = \rho\,(g_o + 0.5a_v)\,h_s$	P_1: Considering vertical acceleration
$p_2 = \rho\, g_o \left[0.67(h_s + \phi\, b) - 0.12\sqrt{H\, b_t\, \phi}\right]$	P_2: Considering rolling motion
$p_3 = \rho\, g_o \left[0.67(h_s + \theta\, l) - 0.12\sqrt{H\, l_t\, \theta}\right]$	P_3: Considering pitching motion
$p_4 = 0.67(\rho\, g_o h_p + \Delta P_{dyn})$	P_4: Considering overflow
$p_5 = \rho\, g_o h_s + p_o$	P_5: Considering tank test pressure

✓Design Pressure, P_1 considering vertical acceleration **(In case of bottom shell)**



In case of bottom shell, the effect of sea pressure is considered



When we consider the design pressure, the largest value shall be applied.
The liquid cargo pressure acting on the bottom shell is the highest when the sea pressure is the lowest, i.e. in case of minimum draft.

$$p = \rho(g_0 + 0.5a_v)h_s - 10T_M$$

T_M : vertical distance in m from load point to minimum design draught

= 2 + 0.02L for Tanker
= 0.35 T for Dry Cargo
(T : Rule Draft)

5) Pressure and Force
- Example) Calculation of P_1 Pressure

(Example) When the tank is filled up , calculate the P_1 pressure of inner bottom and deck by using vertical acceleration (a_v=2.286 m/sec^2) and dimensions of tank which is given below.

[Dimension] Inner bottom height : 3.0 m, Deck height : 31.2m, ρ = 1.025 ton/m^3

$$\boxed{P_1 = \rho\left(g_0 + 0.5a_v\right)h_s}$$

ρ = density (ton/m^3)

a_v = Vertical acceleration

g_0 = Standard acceleration of gravity (=9.81m/sec^2)

h_s : virtical distance in m from load point to top of tank

(Sol.) $a_v = 2.286 \text{ m/sec}^2$

① Inner Bottom

$$h_s = 31.2 - 3.0 = 28.8 \ m$$

$$P_1 = \rho\left(g_0 + 0.5a_v\right)h_s$$
$$= 1.025\left(9.81 + 0.5 \times 2.286\right) \times 28.2$$
$$= 316.6 \, kN/m^2$$

② Deck

$$h_s = 31.2 - 31.2 = 0 \ m$$

$$P_1 = \rho\left(g_0 + 0.5a_v\right)h_s$$
$$= 1.025\left(9.81 + 0.5 \times 2.286\right) \times 0$$
$$= 0 \, kN/m^2$$

B 800 Combined longitudinal accelerations

801 Acceleration along the ship's longitudinal axis is given as the combined effect of surge and pitch calculated as indicated in 100, i.e.:

$$a_l = \sqrt{a_x^{\ 2} + (g_0 \sin\theta + a_{px})^2} \qquad (m/s^2)$$

a_{px} = longitudinal component of pitch acceleration given in 503.

Note that a_{px} is equal to a_p using the vertical projection of R_P.

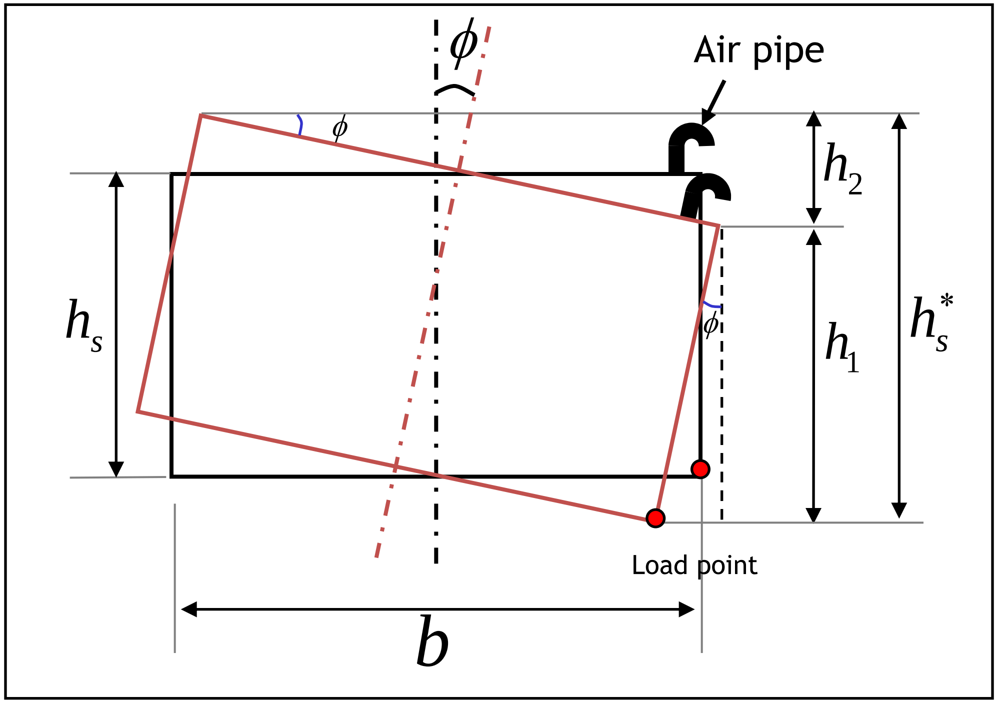
5) Pressure and Force
b) Liquid Tank Pressure (5)

$p_1 = \rho(g_o + 0.5a_v)h_s$	P_1: Considering vertical acceleration
$p_2 = \rho g_o\left[0.67(h_s + \phi b) - 0.12\sqrt{H b_t \phi}\right]$	P_2: Considering rolling motion
$p_3 = \rho g_o\left[0.67(h_s + \theta l) - 0.12\sqrt{H l_t \theta}\right]$	P_3: Considering pitching motion
$p_4 = 0.67(\rho g_o h_p + \Delta P_{dyn})$	P_4: Considering overflow
$p_5 = \rho g_o h_s + p_o$	P_5: Considering tank test pressure

DSME, 선박구조설계 5-3

DnV Rules, Jan. 2004,Pt.3 Ch.1 Sec.4

✓Design Pressure P_2 considering the rolling motion



When the ship is rolling, the higher static pressure is applied.

Assumption : $\phi << 1$

$$h_1 = h_s \cos\phi \approx h_s$$

$$h_2 = b\sin\phi \approx b\phi$$

$$\therefore h_s^* = h_1 + h_2$$

$$= (h_s + b\phi)$$

$$p_2 = \rho g_0[0.67(h_s + \phi b) - 0.12\sqrt{H\phi b_t}\,]$$

In case of rolling of a ship, two third (=0.67) of actual pressure is applied considering pressure drop by overflow.

The filling ratio of the most tank is about 98%.

That (about 2%) is considered.

H : height in m of the tank

b_t : breadth in m of top of tank

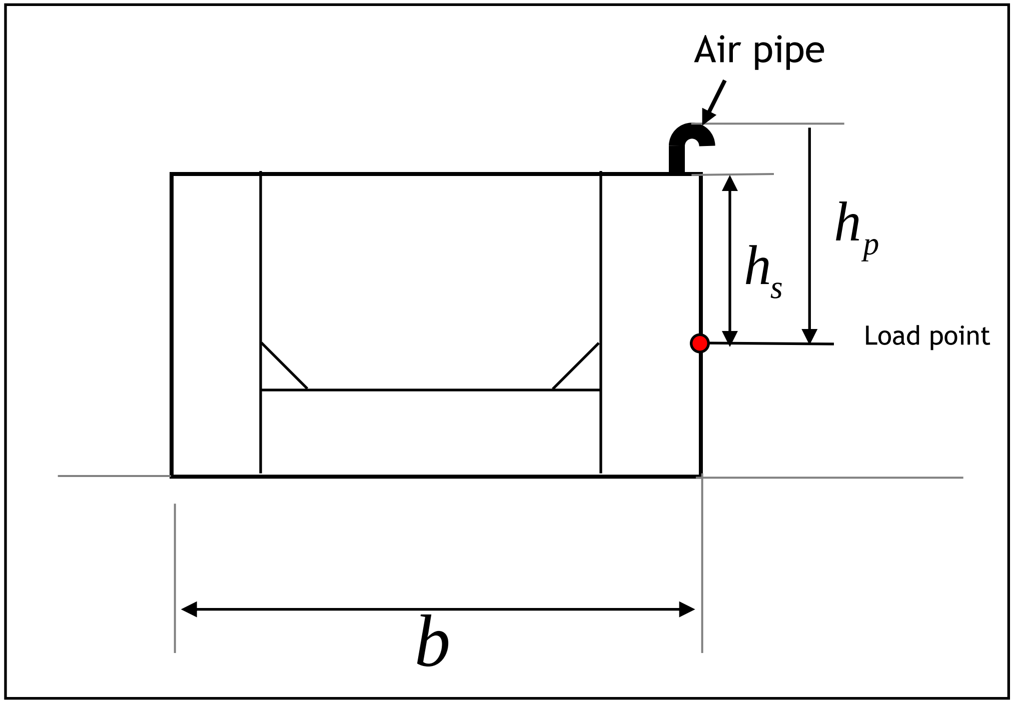
5) Pressure and Force
b) Liquid Tank Pressure (6)

$p_1 = \rho (g_o + 0.5 a_v) h_s$	P_1: Considering vertical acceleration
$p_2 = \rho g_o \left[0.67(h_s + \phi b) - 0.12\sqrt{H b_t \phi} \right]$	P_2: Considering rolling motion
$p_3 = \rho g_o \left[0.67(h_s + \theta l) - 0.12\sqrt{H l_t \theta} \right]$	P_3: Considering pitching motion
$p_4 = 0.67(\rho g_o h_p + \Delta P_{dyn})$	P_4: Considering overflow
$p_5 = \rho g_o h_s + p_o$	P_5: Considering tank test pressure

DSME, 선박구조설계 5-3

DnV Rules, Jan. 2004,Pt.3 Ch.1 Sec.4

✓Design Pressure P_4 considering the tank overflow



The liquid of tank is filled up to air pipe in case of tank overflow.
So h_p is used for calculating static pressure.

h_p = vertical distance in m from the load point to the top of air pipe

$$p = 0.67\left(\rho g_0 h_p + \Delta P_{dyn}\right)$$

Calculated Pressure drop

Generally, 25kN/m^2

In case of rolling of a ship, two third (=0.67) of actual pressure is applied considering pressure drop by overflow.

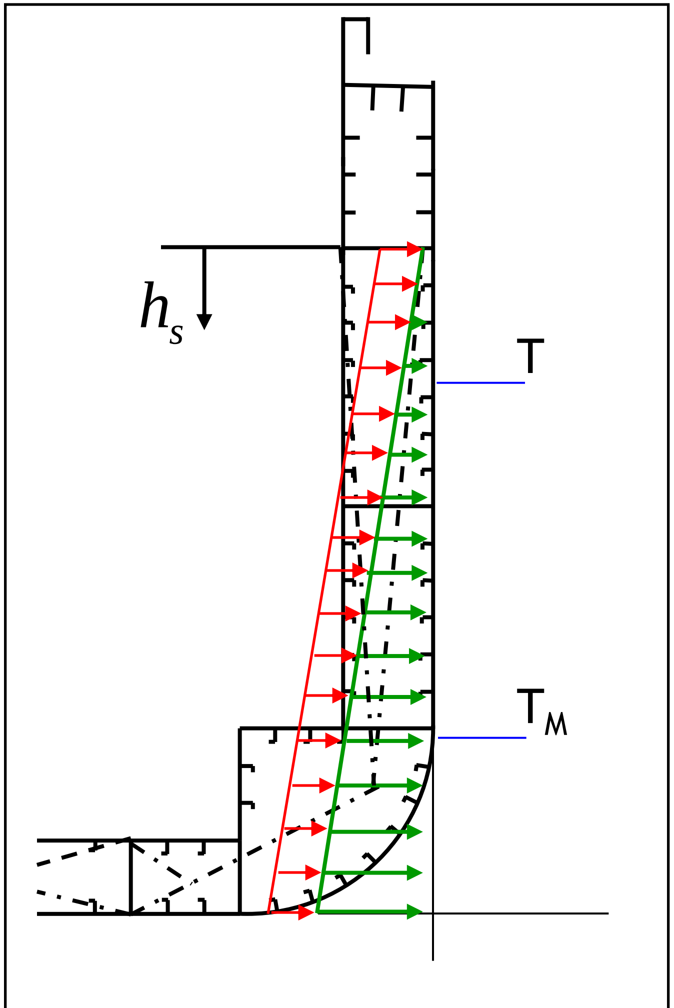
5) Pressure and Force
b) Liquid Tank Pressure (7)

$p_1 = \rho\left(g_o + 0.5a_v\right)h_s$	P_1: Considering vertical acceleration
$p_2 = \rho\, g_o\left[0.67(h_s + \phi\, b) - 0.12\sqrt{H\, b_t\, \phi}\right]$	P_2: Considering rolling motion
$p_3 = \rho\, g_o\left[0.67(h_s + \theta\, l) - 0.12\sqrt{H\, l_t\, \theta}\right]$	P_3: Considering pitching motion
$p_4 = 0.67\left(\rho\, g_o h_p + \Delta P_{dyn}\right)$	P_4: Considering overflow
$p_5 = \rho\, g_o h_s + p_o$	P_5: Considering tank test pressure

✓Design Pressure P_5 considering the tank test pressure

DSME, 선박구조설계 5-3

DnV Rules, Jan. 2004,Pt.3 Ch.1 Sec.4



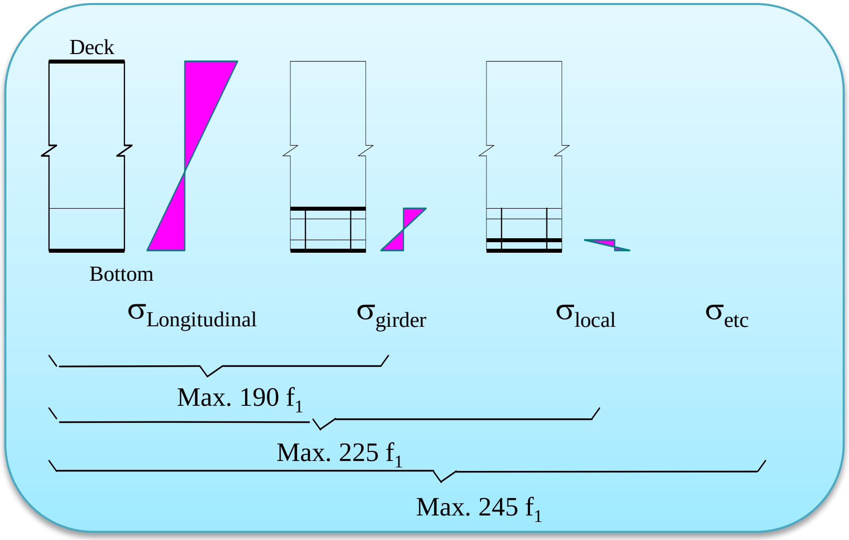
Over-pressure is applied in order to have the water head of 'tank height + 2.5 ' (m) in case of tank test for leakage.
(water head of over-pressure of tank test : 2.5m)

$$\boxed{p = \rho g_0 h_s + p_o}$$

$$p_o = \rho g_0 \times 2.5$$
$$= 10 \times 2.5$$
$$= 25kN/m^2$$

16-3. Local Strength & Allowable stress

Allowable stresses
- Allowable Stress for Local Strength



Relationship between Load and Stress

1) longitudinal Load induced by waves (Hogging or Sagging)

↓

$\sigma_{Longitudinal}$

2) Cargo Load

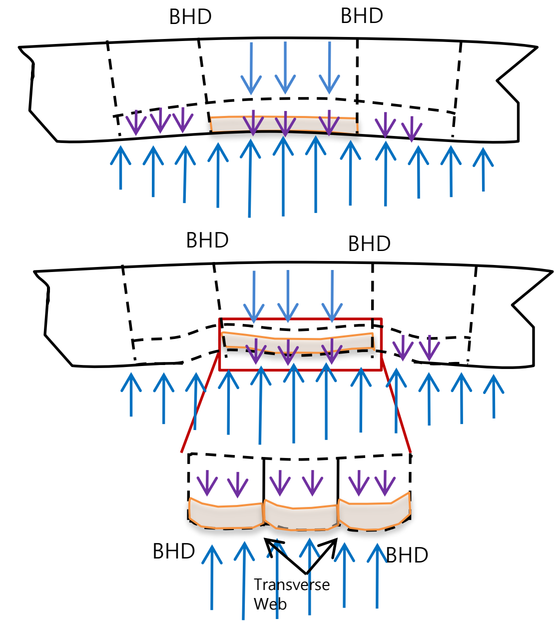
↓

σ_{girder}

3) Ballasting Load

↓

σ_{local}



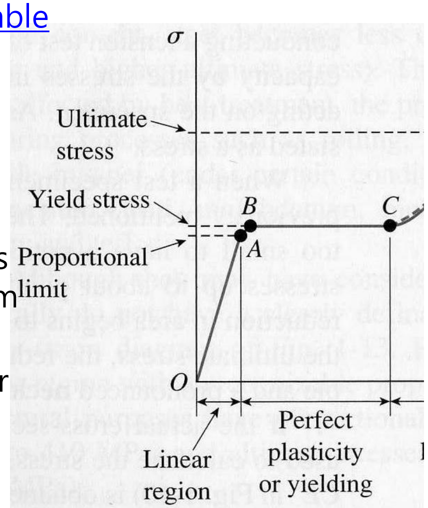
In the figure above, the meaning of the coefficients of the maximum allowable stresses is as follows:

245 f_1 : Maximum Yield Stress

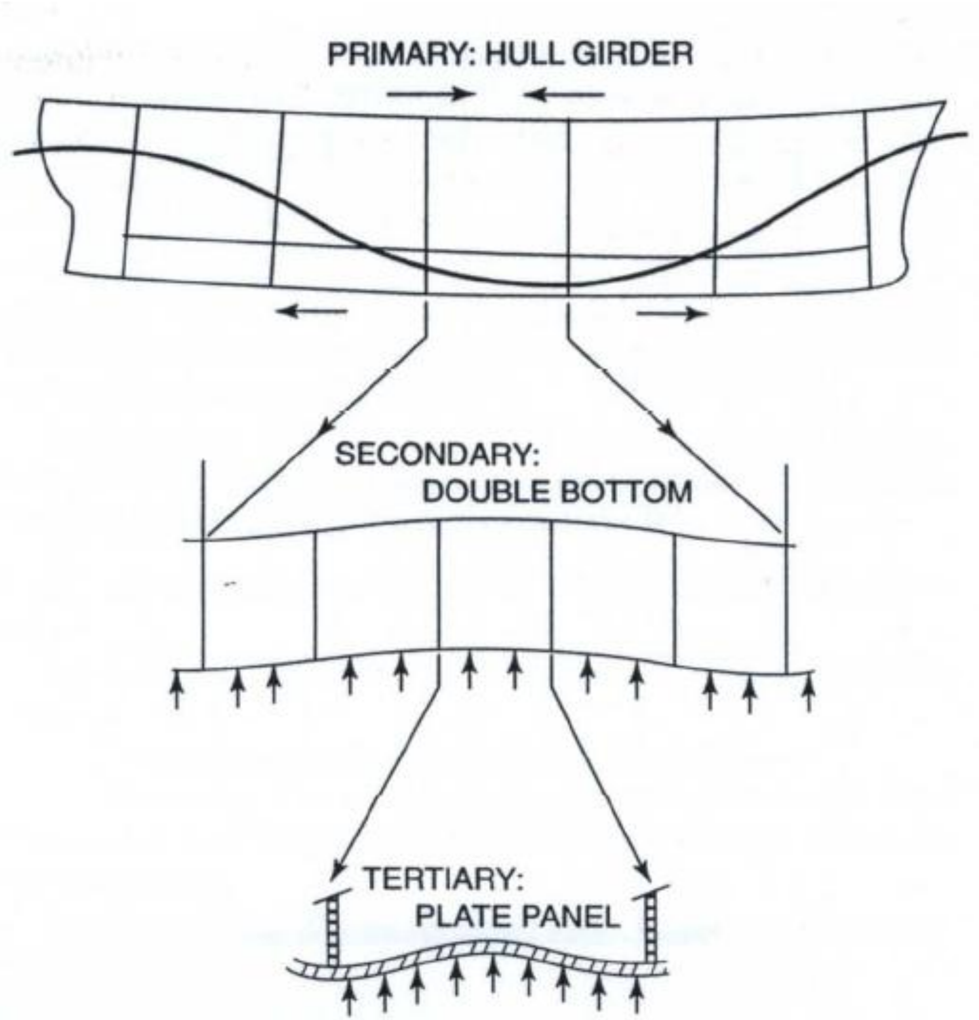
235 f_1 : Proportional Limit

→ 225 f_1 : The maximum allowable stress for the local strength uses the value less than the maximum yield stress.

In other words, 225 f_1 is used for the yield stress, except for the other effects.



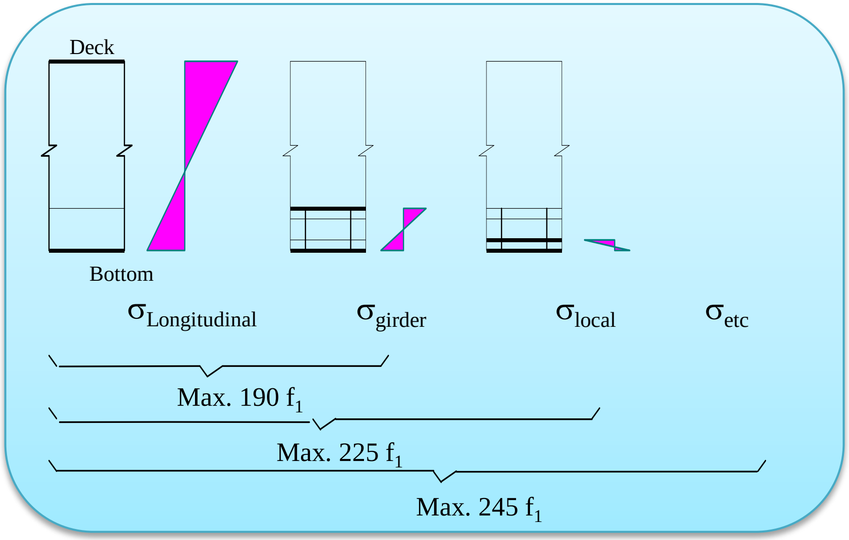
Local Strength & Allowable Stresses



•Primary, secondary, and tertiary structure

* Mansour, A., liu, d., the principles of naval architecture series – strength of ships and ocean structures, the society of naval architects and marine engineers, 2008 -

Allowable stresses
- Allowable Stress for Local Strength



Another interpretation of the figure

Example) Inner bottom Longitudinals[1)]

1) DnV Rules, Jan. 2004,Pt.3 Ch.1 Sec.6 c800

The section modulus requirement is given by:

$$Z = \frac{83 l^2 s p w_k}{\sigma} \quad (cm^3)$$

Where p is the local pressure on bottom structure.

The nominal allowable bending stress due to lateral pressure is used except for the longitudinal stress and the double bottom girder stress.

$$\sigma = 225 f_1 - 100 f_{2b} - 0.7\sigma_{db}$$

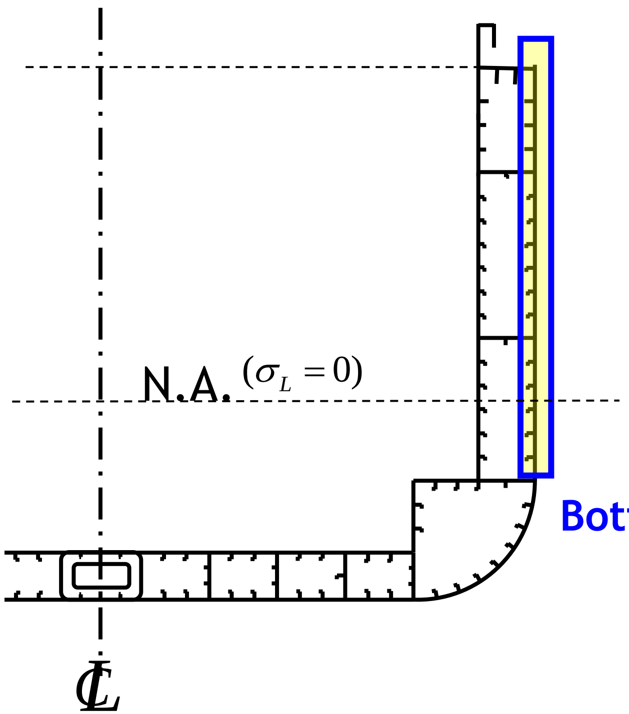
The longitudinal stress is given by the stress factor. And the double bottom stress is given by:

σ_{db} = mean double bottom stress at plate flanges, normally not to be taken less than
= 20 f1 for cargo holds in general cargo vessel
= 50 f1 for holds for ballast
= 85 f1 b/B for tanks for liquid cargo

Allowable Stresses
- Longitudinal Stiffeners (2)

-> check !!!

$$Z_{req.} = \frac{83 l^2 \cdot s \cdot p \cdot w_k}{\sigma_l} \quad (cm^3)$$



✓ $f_{2b,2d}$

$$f_{2b,2d} = \frac{5.7(M_S + M_W)}{Z_{b,d}}$$

M_S : the largest design SWBM (kN·m)
M_W: rule VWBM in (kN·m)
$Z_{b,d}$: midship section modulus (cm³) at bottom or deck as built

For example, 4,100TEU Container Carrier: $I = 2.343e^{10}\ cm^4$

Bottom: $y_B = 9.028e^2\ cm$

$$Z_B = 2.595e^{07}\ cm^3 \longrightarrow f_{2b} = \frac{5.7(M_S + M_W)}{Z_b} = 1.030$$

Deck: $y_B = 10.272e^2\ cm$

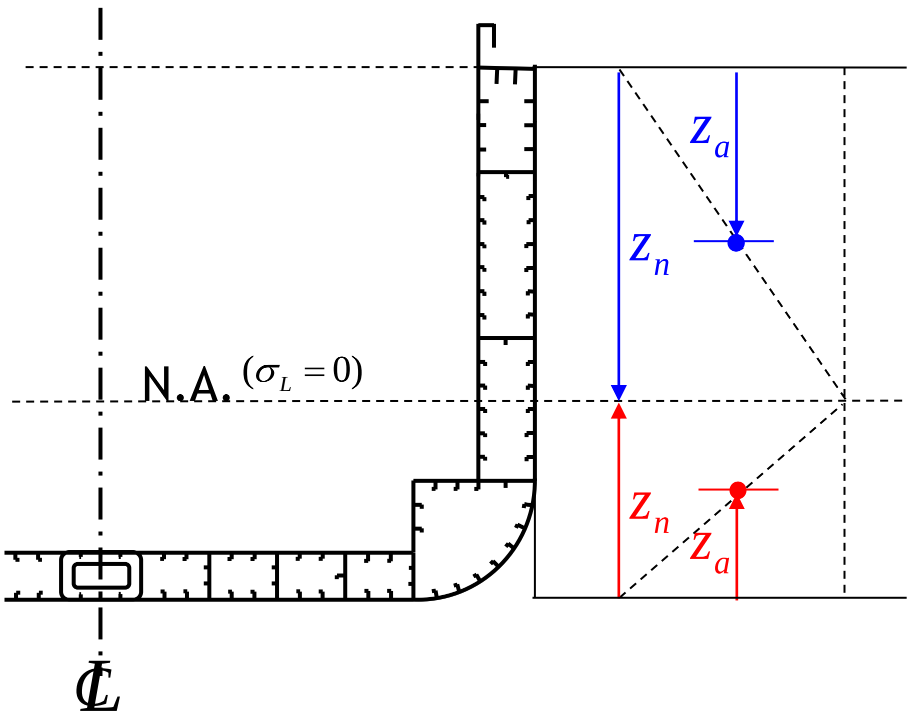
$$Z_D = 2.345e^{07}\ cm^3 \longrightarrow f_{2d} = \frac{5.7(M_S + M_W)}{Z_d} = 1.140$$

Section modulus of bottom is larger than that of deck,
So stress factor f_{2b} is smaller than f_{2d}

Allowable Stresses
- Longitudinal Stiffeners (2)

$$Z_{req.} = \frac{83 l^2 \cdot s \cdot p \cdot w_k}{\sigma_l} \quad (cm^3)$$

✓ Z_n, Z_a



Zn : Vertical distance in m from the baseline or deckline to the neutral axis of the hull girder, whichever is relevant

Z_a : vertical distance in m from the baseline or deckline to the point in question below or above the neutral axis, respectively

$$f_{2b,2d} = \frac{5.7(M_S + M_W)}{Z_{b,d}}$$

M_S : the largest design SWBM (kN·m)
M_W: rule VWBM in (kN·m)
$Z_{b,d}$: midship section modulus (cm^3) at bottom or deck as built

Allowable Stresses (σ_l) - Longitudinal Stiffeners (1)

DnV Rules, Jan. 2004,Pt.3 Ch.1 Sec.6,7,8,9
DSME, Dnv Rule 해설서, 1991.8

-> check !!!

$$Z_{req.} = \frac{83 l^2 \cdot s \cdot p \cdot w_k}{\sigma_l} \quad (cm^3)$$

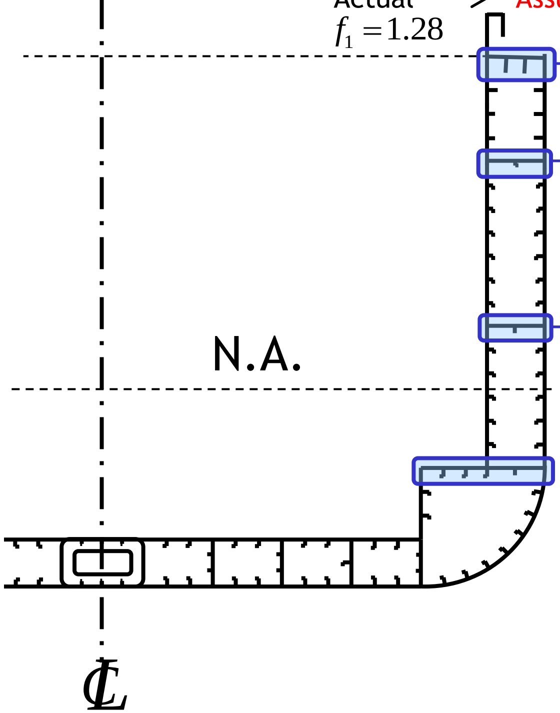
Decks (Pt3 ch1 Sec.8 C301)

$$\sigma_l = 225 f_1 - \underbrace{130 f_{2d} \frac{z_n - z_a}{z_n}}_{\to \sigma_L}$$

For example, 4,100TEU Container Carrier: $f_{2d} = 1.19$

Actual $\rightarrow$ Assumption :

Actual $f_1 = 1.28$



$f_1 = 1.0$ $\quad z_n = 10.272,\ z_a = 0.000 \rightarrow \frac{z_n - z_a}{z_n} = 1, \quad \rightarrow \sigma_l = 70.3\ [Mpa]$

Actual $f_1 = 1.0$ $\quad z_n = 10.272,\ z_a = 3.712 \rightarrow \frac{z_n - z_a}{z_n} = 0.639, \quad \rightarrow \sigma_l = 126.147\ [Mpa]$

Actual $f_1 = 1.0$ $\quad z_n = 10.272,\ z_a = 9.782 \rightarrow \frac{z_n - z_a}{z_n} = 0.048, \quad \rightarrow \sigma_l = \cancel{217.574}\ [Mpa]$

$= 160\ [Mpa]$

(Maximum : 160)

$$f_{2b,2d} = \frac{5.7(M_S + M_W)}{Z_{b,d}}$$

M_S : the largest design SWBM (kN·m)
M_W: rule VWBM in (kN·m)
$Z_{b,d}$:midship section modulus (cm^3) at bottom or deck as built

Zn : Vertical distance in m from the baseline or deckline to the neutral axis of the hull girder, whichever is relevant
Z_a : vertical distance in m from the baseline or deckline to the point in question below or above the neutral axis, respectively

301 The section modulus requirement is given by:

$$Z = \frac{83\, l^2\, s\, p\, w_k}{\sigma} \quad (cm^3), \quad \text{minimum } 15\ cm^3$$

p = $p_1 - p_{13}$, whichever is relevant, as given in Table B1.
σ = allowable stress, within 0.4 L midship given in Table C1
= 160 f_1 for continuous decks within 0.1 L from the perpendiculars and for other deck longitudinals in general.

Between specified regions the σ-value shall be varied linearly.

For longitudinals $\sigma = 160\ f_1$ may be used in any case in combination with heeled condition pressures p_9 and sloshing load pressures, p_{11} and p_{12}.

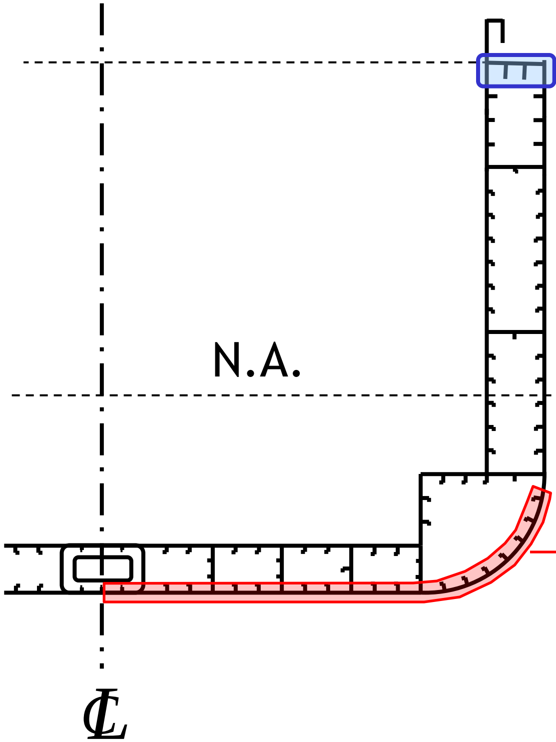
For definition of other parameters used in the formula, see A200.

Allowable Stresses (σ_l)
- Longitudinal Stiffeners (1)

DnV Rules, Jan. 2004,Pt.3 Ch.1 Sec.6,7,8,9
DSME, Dnv Rule 해설서, 1991.8

-> check !!!

$$Z_{req.} = \frac{83 l^2 \cdot s \cdot p \cdot w_k}{\sigma_l} \quad (cm^3)$$



Decks (Pt3 ch1 Sec.8 C301)

$$225 f_1 - 130 f_{2d} \frac{z_n - z_a}{z_n}$$

For example, 4,100TEU Container Carrier:

$$f_1 = 1.28 \quad f_{2d} = 1.140$$

$$z_n = 10.272, \; z_a = 0.000 \rightarrow \frac{z_n - z_a}{z_n} = 1, \rightarrow \boxed{\sigma_l = 139.8[Mpa]}$$

Double Bottom (Pt3 ch1 Sec.6 C701)

$$225 f_1 - 130 f_{2b} - 0.7\sigma_{db}$$

For example, 4,100TEU Container Carrier, Assumption :σ_{db} = 0

$$f_1 = 1.0 \quad f_{2b} = 1.030$$

$$z_n = 9.208, \; z_a = 0.000 \rightarrow \frac{z_n - z_a}{z_n} = 1, \rightarrow \boxed{\sigma_l = 154.1[Mpa]}$$

Allowable stresses at deck are **smaller** than those at deck, because the distance from N.A to bottom is shorter than deck.

$$f_{2b,2d} = \frac{5.7(M_S + M_W)}{Z_{b,d}}$$

M_S : the largest design SWBM (kN·m)
M_W: rule VWBM in (kN·m)
$Z_{b,d}$:midship section modulus (cm^3) at bottom or deck as built

Zn : Vertical distance in m from the baseline or deckline to the neutral axis of the hull girder, whichever is relevant
Z_a : vertical distance in m from the baseline or deckline to the point in question below or above the neutral axis, respectively
σ_{db} = mean double bottom stress at plate flanges, normally not to be taken less than
= 20 f1 for cargo holds in general cargo vessel
= 50 f1 for holds for ballast
= 85 f1 b/B for tanks for liquid cargo

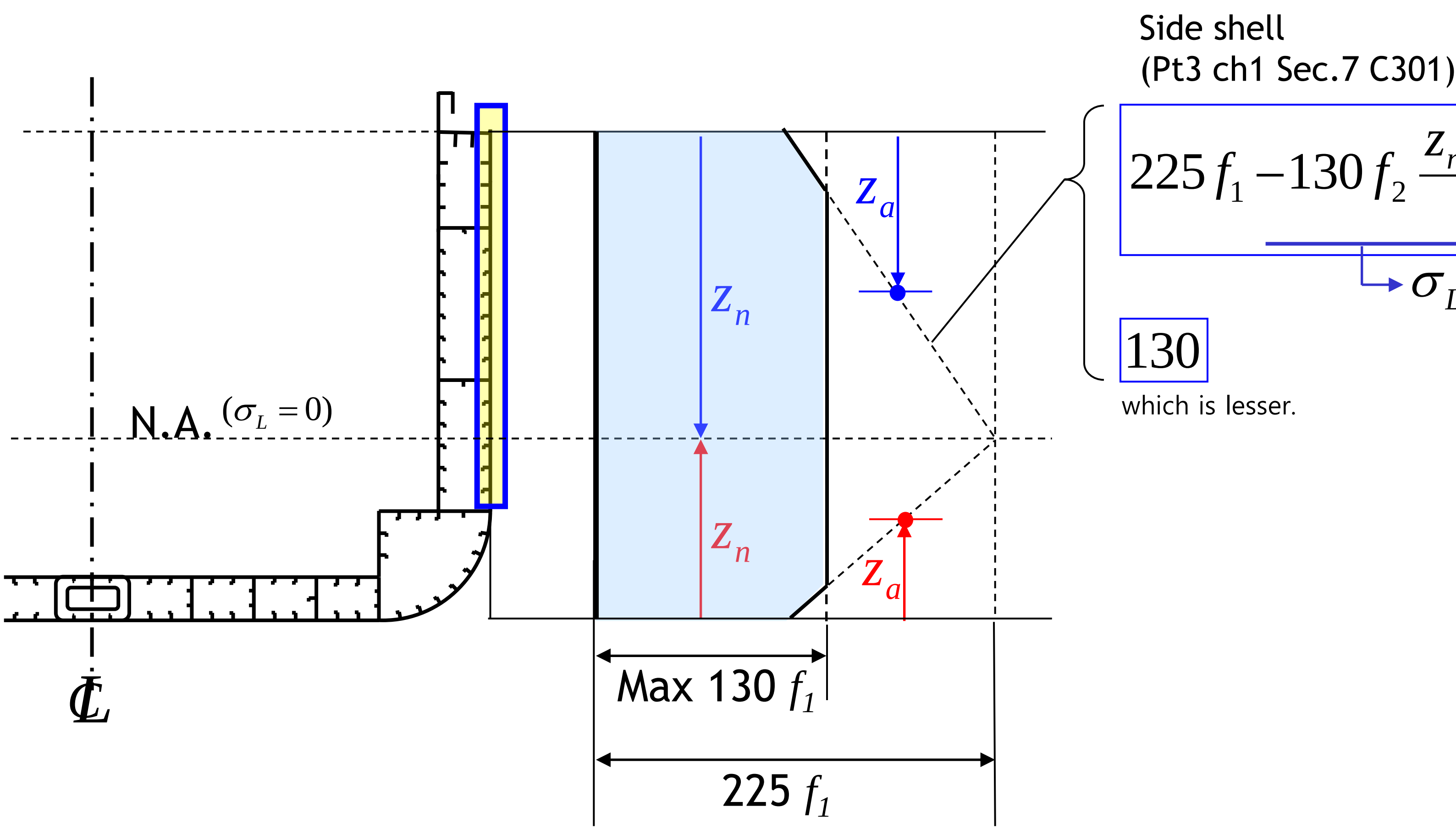
If the mean double bottom stress(σ_{db}) is considered as 20

$$225 f_1 - 130 f_{2b} - 0.7\sigma_{db}$$

$$225 \times 1.28 - 130 \times 1 - 0.7 \times 20 = \quad \rightarrow \boxed{\sigma_l = 140.1[Mpa]}$$

Allowable Stresses
- Longitudinal Stiffeners (2)

$$Z_{req.} = \frac{83l^2 \cdot s \cdot p \cdot w_k}{\sigma_l} \quad (cm^3)$$



$$f_{2b,2d} = \frac{5.7(M_S + M_W)}{Z_{b,d}}$$

M_S : the largest design SWBM (kN·m)
M_W: rule VWBM in (kN·m)
$Z_{b,d}$:midship section modulus (cm^3) at bottom or deck as built

Zn : Vertical distance in m from the baseline or deckline to the neutral axis of the hull girder, whichever is relevant
Z_a : vertical distance in m from the baseline or deckline to the point in question below or above the neutral axis, respectively

301 The section modulus requirement is given by:

$$Z = \frac{83\, l^2 s\, p\, w_k}{\sigma} \quad (\mathrm{cm}^3), \text{ minimum } 15 \text{ cm}^3$$

p = $p_1 - p_8$, whichever is relevant, as given in Table B1
σ = allowable stress (maximum 160 f_1) given by:

Within 0.4 L amidships:

$$\sigma = 225 f_1 - 130 f_2 \frac{z_n - z_a}{z_n}$$

= maximum 130 f_1 for longitudinals supported by side verticals in single deck constructions.

Within 0.1 L from perpendiculars:

$$\sigma = 160 f_1$$

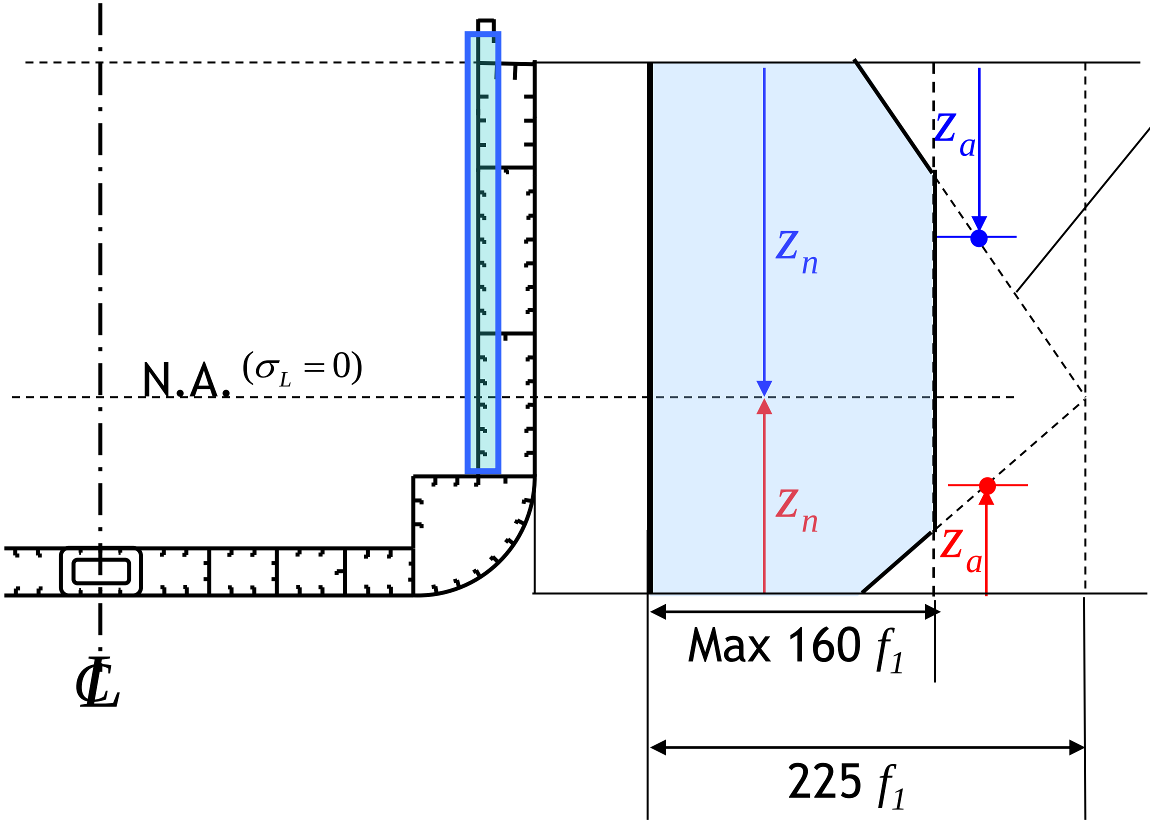
Between specified regions the σ-value may be varied linearly.

For longitudinals $\sigma = 160 f_1$ may be used in any case in combination with heeled condition pressures p_6 and p_8.

f_2 = stress factor f_{2b} as given in Sec.6 A200 below the neutral axis
= stress factor f_{2d} as given in Sec.8 A200 above the neutral axis.

$Z_{req.} = \frac{83 l^2 \cdot s \cdot p \cdot w_k}{\sigma_l} \quad (cm^3)$

Allowable Stresses - Longitudinal Stiffeners (3)



Longitudinal Bulkhead
(Pt3 ch1 Sec.9 C201)

$225 f_1 - 130 f_2 \frac{z_n - z_a}{z_n} \rightarrow \sigma_L$

$\boxed{160}$

which is lesser.

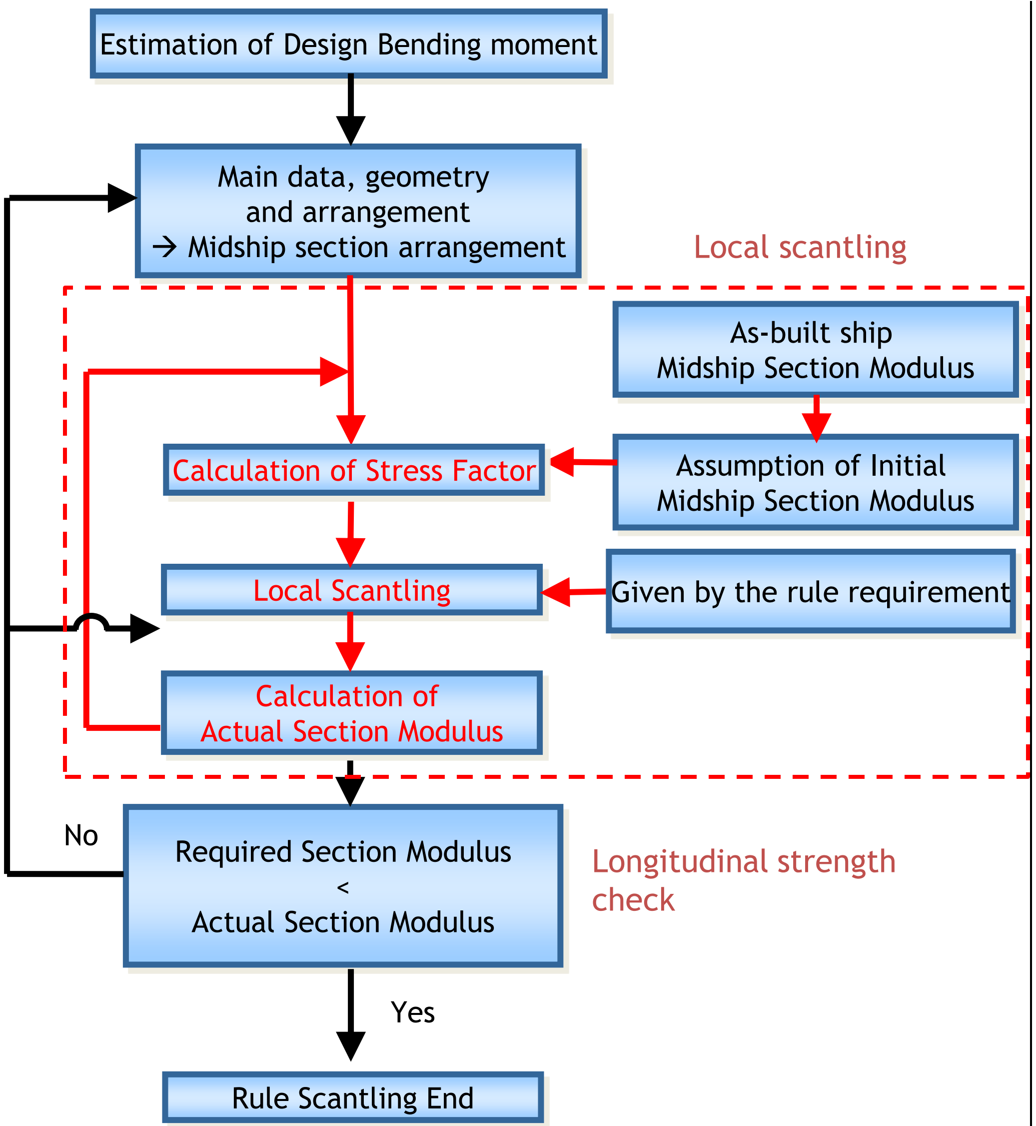
$$f_{2b,2d} = \frac{5.7(M_S + M_W)}{Z_{b,d}}$$

M_S : the largest design SWBM (kN·m)
M_W: rule VWBM in (kN·m)
$Z_{b,d}$:midship section modulus (cm^3) at bottom or deck as built

Zn : Vertical distance in m from the baseline or deckline to the neutral axis of the hull girder, whichever is relevant
Z_a : vertical distance in m from the baseline or deckline to the point in question below or above the neutral axis, respectively

16-4 Procedure of Local Scantling

Local Scantling
- Design Procedure of Structures



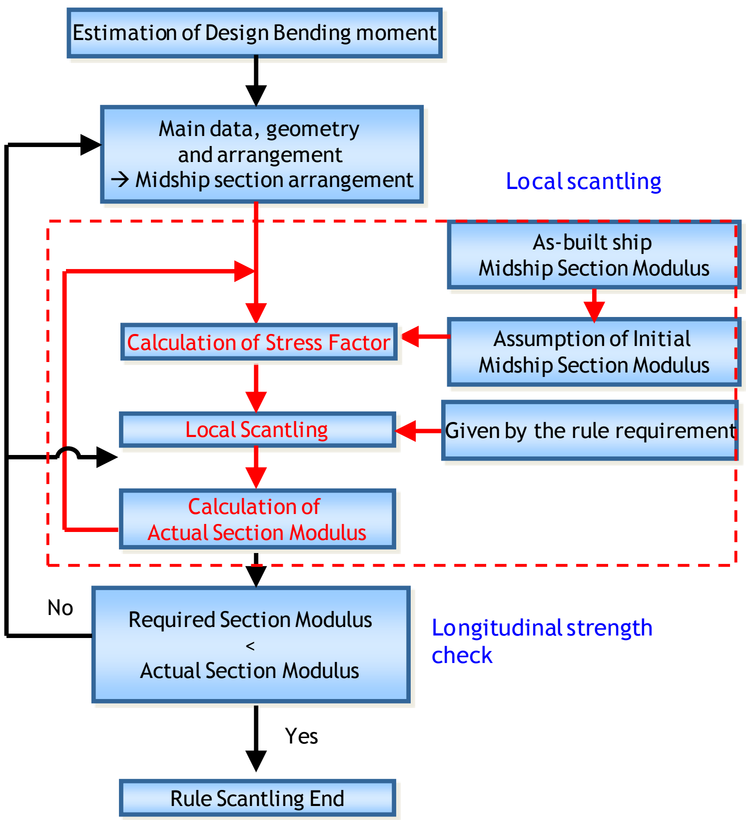
Ship structure design is carried out in accordance with the procedure shown in the figure.

Each member is adjusted to have enough local strength given by the rule of Classification Societies based on the mechanics of materials.

This is called the “local scantling”.

Design Procedure of Structures
- Stress factor

Why iteration is needed for the calculation of local scantling?



The actual midship section modulus at bottom or deck is needed.

However, the section modulus can be calculated after the scantlings of the members are determined.

→ Assumption!

Therefore, actual section modulus is calculated to be equal to the assumed section modulus by the iteration.

Design Procedure of Structures
- Stress factor

1) DnV Rules, Jan. 2004, Pt.3 Ch.1 Sec.6 c800

Why iteration is needed for the calculation of local scantling?

Example) Inner bottom Longitudinals[1)]

▪**Minimum Longi . stiffener section modulus**

$$Z = \frac{83 l^2 s p w_k}{\sigma} \quad (cm^3)$$

l : stiffener span in m
s : stiffener spacing in m
p : design loads
w_k : section modulus corrosion factor in tanks, Sec.3 C1004

σ_{db} = mean double bottom stress at plate flanges, normally not to be taken less than
= 20 f1 for cargo holds in general cargo vessel
= 50 f1 for holds for ballast
= 85 f1 b/B for tanks for liquid cargo

Where, $\sigma = 225 f_1 - 100 f_{2b} - 0.7\sigma_{db}$

f_1 : material factor as defined in DnV Rules Pt.3 Ch.1 Sec.2

f_{2b} **: stress factor**

$$f_{2b,2d} = \frac{5.7(M_S + M_W)}{Z_{b,d}}$$

required midship section modulus (cm³) at bottom or deck

actual midship section modulus (cm³) at bottom or deck as built

M_S : the largest design SWBM[2)] (kN·m)

M_W : VWBM by class rule of direct calculation in (kN·m)

2) Largest SWBM among all loading conditions and class rule

The actual midship section modulus at bottom or deck is needed.

However, the section modulus can be calculated after the scantlings of the members are determined.

→ **Assumption!**

Therefore, actual section modulus is calculated to be equal to the assumed section modulus by the iteration.

801 The section modulus requirement is given by:

$$Z = \frac{83\, l^2 s\, p\, w_k}{\sigma} \qquad (\mathrm{cm}^3)$$

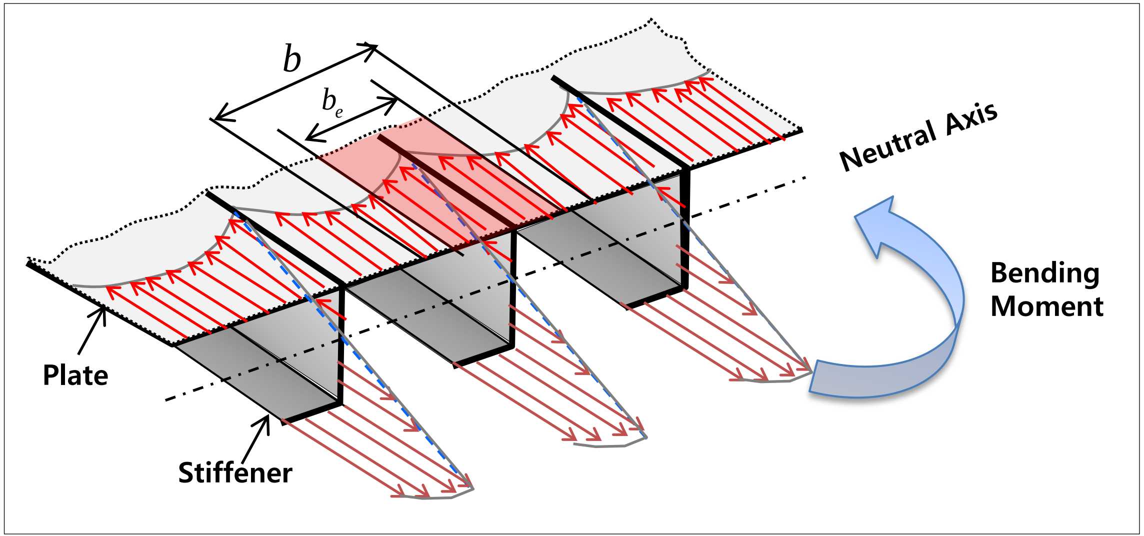
p = p_4 to p_{15} (whichever is relevant) as given in Table B1
σ = $225\, f_1 - 100\, f_{2B} - 0.7\, \sigma_{db}$ within 0.4 L (maximum $160\, f_1$)
= $160\, f_1$ within 0.1 L from the perpendiculars.

Between specified regions the σ-value may be varied linearly.

σ_{db} = mean double bottom stress at plate flanges, normally not to be taken less than:
= $20\, f_1$ for cargo holds in general cargo vessels
= $50\, f_1$ for holds for ballast
= $85\, f_1$ b/B for tanks for liquid cargo
f_{2b} = stress factor as given in A200
b = breadth of tank at double bottom.

Effective Breadth, Span Point

Scantling of Stiffeners
- Effective Breadth of Attached Plates



When the lateral pressure is imposed, the stress distribution in the plates and the stiffeners is complex as shown in the Figure.

The longitudinal stress in the attached plate will be a maximum at the connection line to the stiffener and become smaller gradually beyond this line.

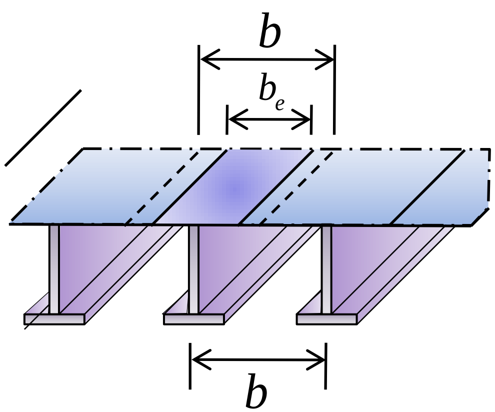
Considering the strength, **the stiffened panel will be assumed to be a collection of beams which include some parts of the attached plate. The breadth of this plate is called the "effective breadth".**

---

ex) DNV Rule : effective flange of girder [2)]

The effective plate flange area is defined as the cross sectional area of plating within the effective flange width. Continuous stiffeners within the effective range may be included. The effective flange width b_e is determined by the following formula:

$$\underbrace{b_e}_{\text{Effective Breadth}} = \underset{\text{reduction Factor}}{C} \bullet \underbrace{b}_{\text{Actual Breadth}} \ (m)$$



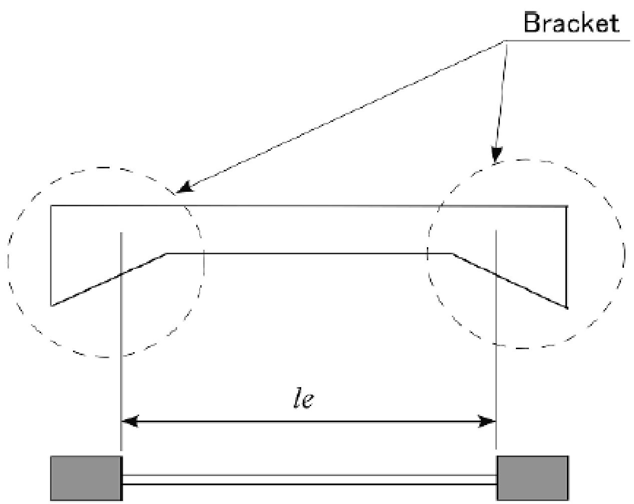
C = as given in Table for various numbers of evenly spaced point loads(r) on the span

b = sum of plate flange width on each side of girder, normally taken to half the distance from nearest girder or bulkhead

[1)] 대우조선해양, 선박구조설계, 3.4 선체구조 단면특성, 2005.

[2)] DNV rules for ships, Pt.3 Ch.1 Sec.3 C400

Scantling of Stiffeners
- Span Point of a Beams



(a) Definition of span point

Ship structure members are usually connected with brackets or other structures.

When we consider a member as a beam, **it is convenient to assume the member to be a uniform section beam, having an equivalent length between two span points,** and to assume the outside structures of the span points to be rigid bodies as illustrated in the figure.

The span point depends on structural details and loading conditions.

---

ex) DNV Rule : Definition of span for stiffeners and girders. [1)]

The effective span of a stiffener (l) or girder (S) depends on the design of the end connections in relation to adjacent structures. Unless otherwise stated the span points at each end of the member, between which the span is measured, shall be determined as shown in Fig.
It is assumed that brackets are effectively supported by the adjacent structure.

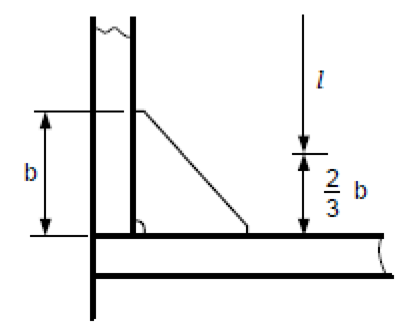


Figure: Example of Span point

[1)] DNV rules for ships, Pt.3 Ch.1 Sec.3 C100

16-5 PLASTIC DESIGN OF PLATE

References :

Hughes, O. F., Ship Structural Design, Wiley-Interscience, 1983.

-Ch.10 : Plastic Frame Analysis (pp508-540)

Okumoto, Y., Takeda, Y., Mono, M., Design of Ship Hull Structures, Springer, 2009.

-Part1.Sec3.6 : Plastic Strength (pp.60-69)

Gere J.M., Mechanics of Materials, 7th edition, Thomson, 2009.

-Sec.1.3 : Mechanical Properties of Materials (pp.15-24)

-Sec.1.4 : Elasticity, Plasticity, and Creep (pp.24-26)

-Sec.2.12 : Elastoplastic Analysis (pp.175-178)

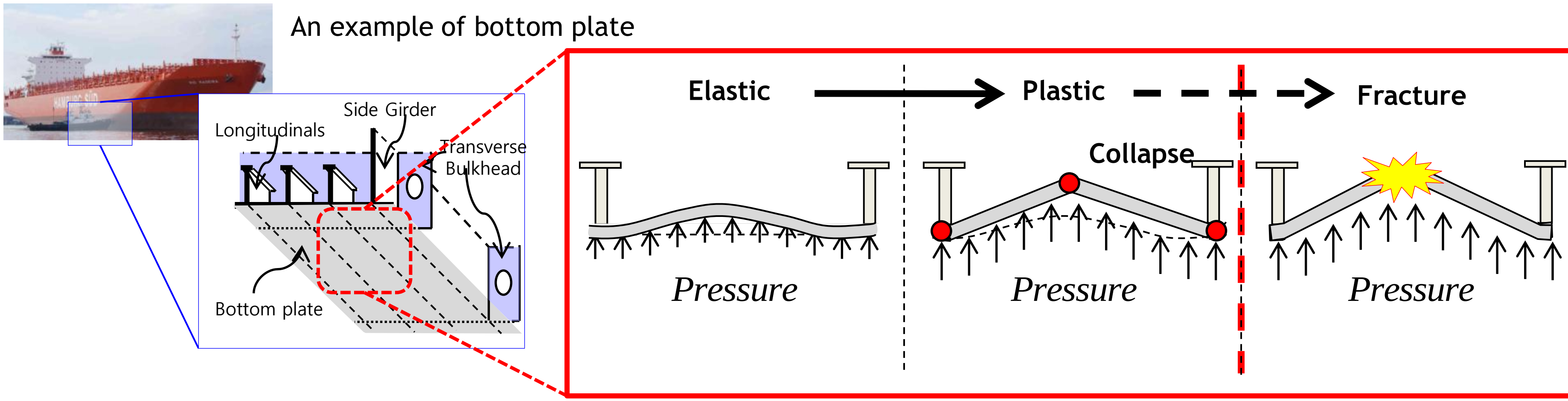
-Sec.6.10 : Elastoplastic Bending (pp.504-510)

Lewis, E. V., Principles of Naval Architecture, Vol1, 1988.

-Ch.4, Sec4 : Load Carrying Capability and Structural performance Criteria (pp.275-290)

Introduction to Plastic Design for the Plate Scantling

Why do we consider plastic design for the plate scantling?



Function of the plate	To keep the (sea) water outside the ship
	To preserve the cargo or fuel inside the ship

The plate can perform its own function before a fracture occurs.

Therefore, it is reasonable to consider a structure until a collapse occurs.

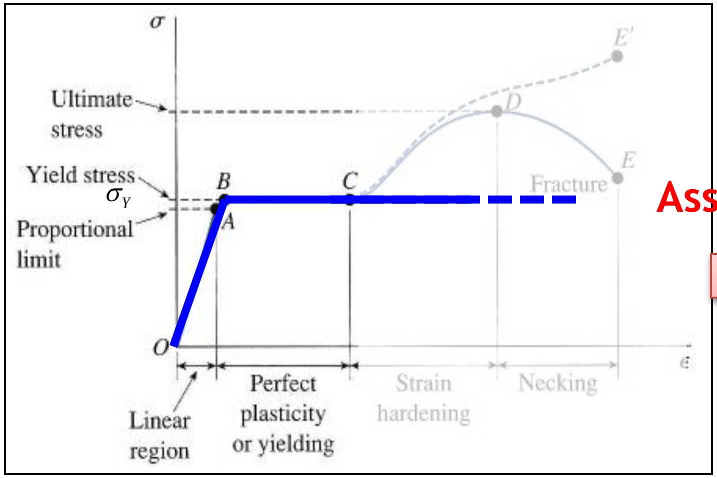
We consider plastic design for the plate scantling

Reasonable!

Plastic Design
- Assumption

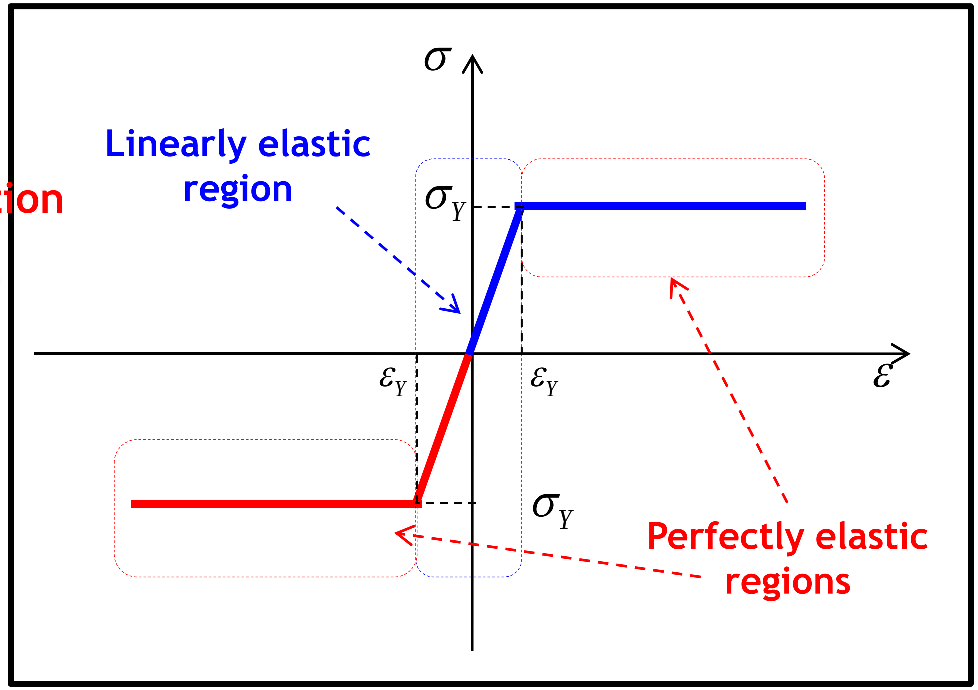
Assumption: Consider the material as "elastoplastic" material

Actual stress-strain curve



Assumption

Idealized stress-strain curve for an "elastoplastic" material



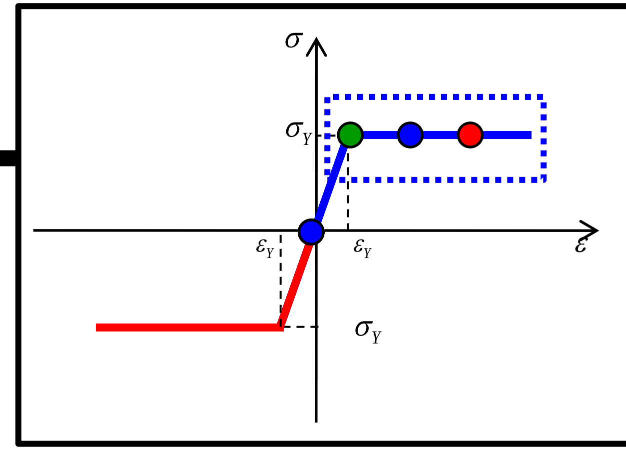
*We will assume that the material has **the same yield stress** σ_Y and **same yield strain** ε_Y in both **tension and compression.**

✓ **"Elastoplastic" material follows Hooke's law up to the yield stress σ_Y and then yield plastically under constant stress**

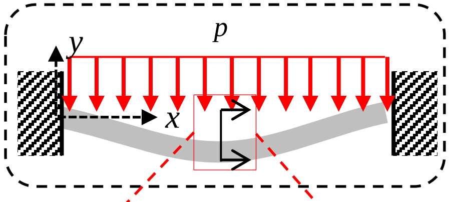
✓ **The maximum stress reaches yield stress σ_Y**

Gere J.M., Mechanics of Materials, 7th edition, Thomson, 2009, pp504-510.

Plastic Design
- Stress Distribution of Elastoplastic Material

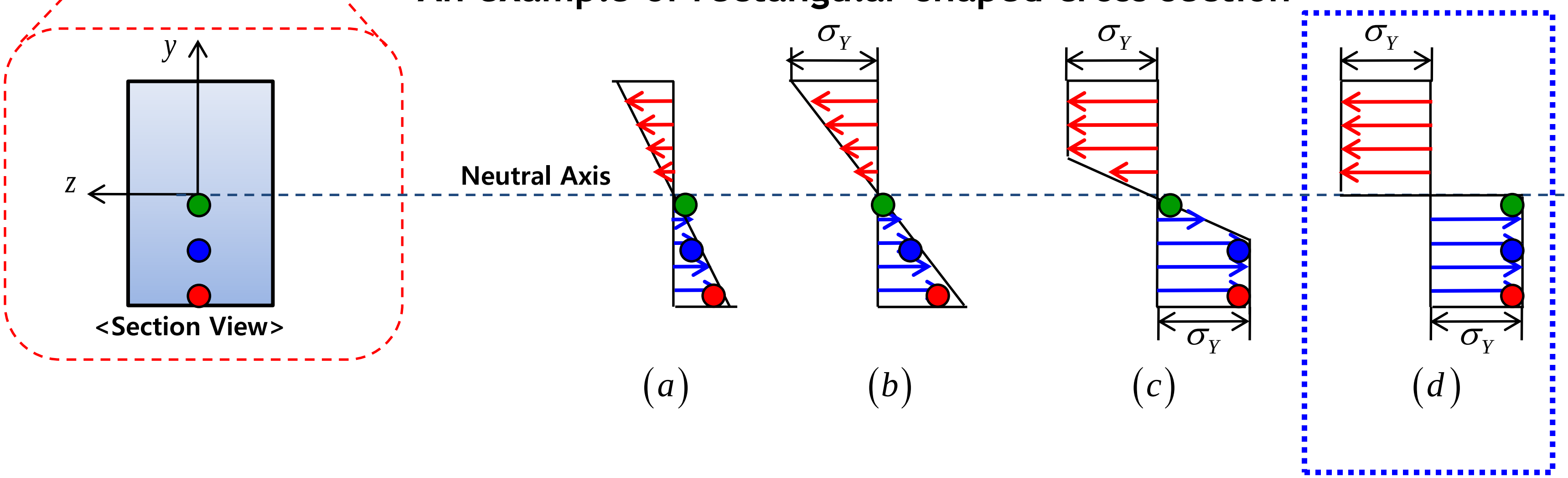


Let us consider a cross section of a beam.



p : intensity of uniformly distributed load

An example of rectangular-shaped cross section



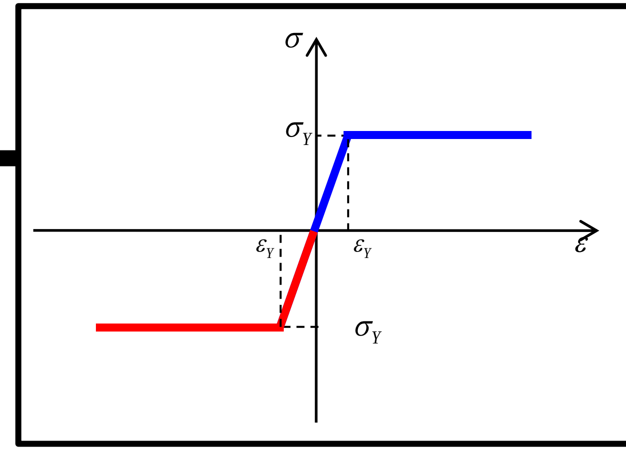
(a), (b): The stress in the section is **proportional** **to the distance from the neutral axis.**

(c): If the load is increased further, **the stress does not exceed** σ_Y and it **spreads inside the section** **until the section becomes fully plastic.**

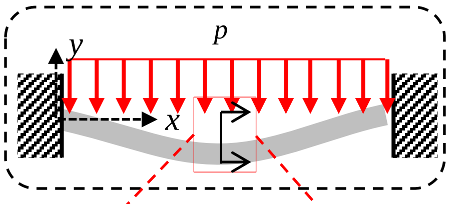
(d): The stress becomes yield stress **wholly in the section.**

Plastic Design
- Plastic Moment (M_P)

$$M = -\int_A \sigma y \, dA$$

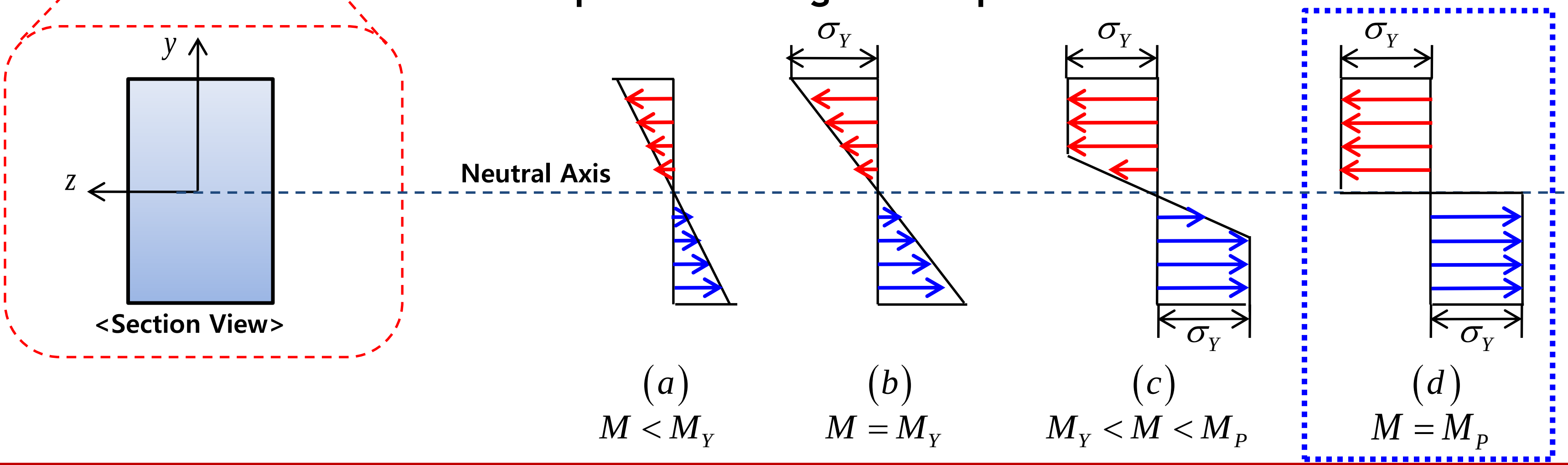


Let us consider a cross section of a beam.



p : intensity of uniformly distributed load

An example of rectangular-shaped cross section



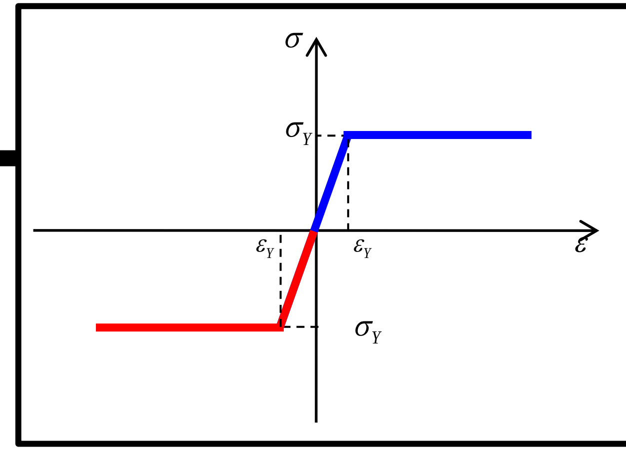
The bending moment corresponding to the condition (d) : **"Plastic Moment"** M_P

M_P : Moment when the stress becomes yield stress wholly in the section.

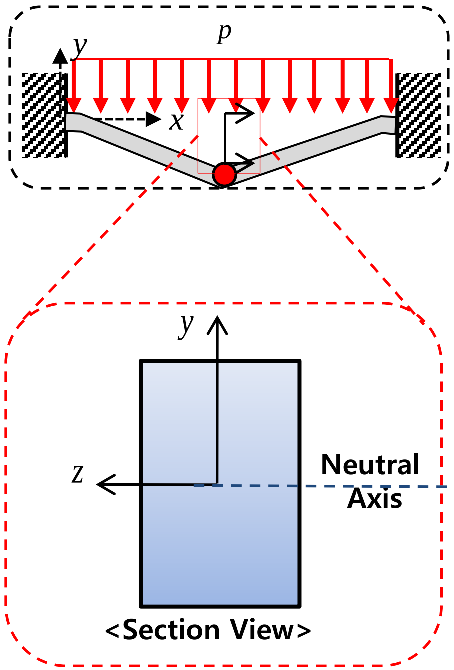
M_Y : Moment when the maximum stress in the section reaches yield stress

Plastic Design
- Plastic Hinge

$$M = -\int_A \sigma y \, dA$$

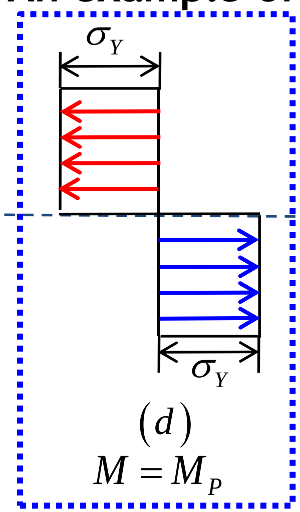


Let us consider a cross section of a beam.



p : intensity of uniformly distributed load

An example of rectangular-shaped cross section



If this section becomes this condition,
the beam can not absorb any further bending moment at this section.

It is as if a **"hinge" had been inserted** in the beam at this point.

Therefore, we refer to this condition as a **"plastic hinge"**.

Hughes, O. F., Ship Structural Design, Wiley-Interscience, 1983, p509.

Plastic Design

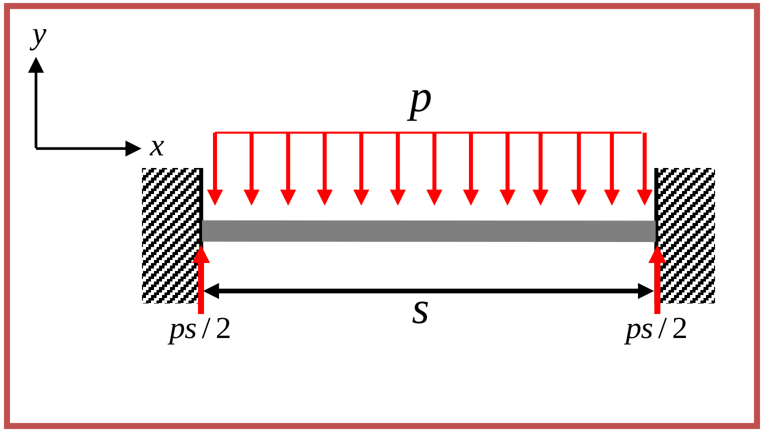
- Plastic Design at a Fixed-end Beam

$p_{Ult.}$: intensity of uniformly distributed load when the beam collapses.

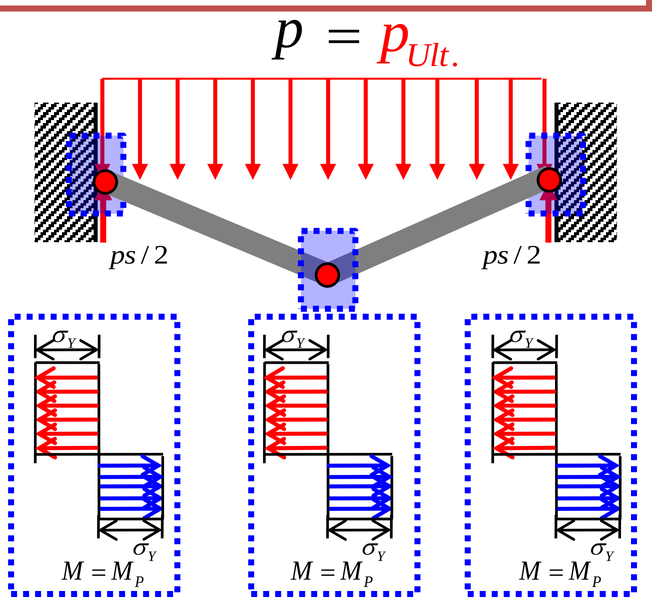
s :span of longitudinals

Let us take the case of a "fixed-end beam".

An example of fixed-end beam which carries a uniformly distributed load of intensity p.



"Plastic Design":
The beam is designed to sustain the load until the beam collapses and cannot sustain any further load.



In this case, the beam will collapses when the **three plastic hinges** are formed **at the ends and middle of the beam.**

The collapse or "ultimate" load : $p_{Ult.}$

Okumoto, Y., Takeda, Y., Mano, M., Design of Ship Hull Structures, Springer, 2009, pp60-69.

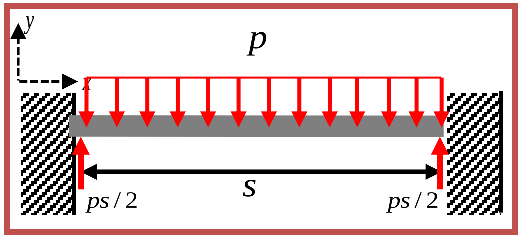
Plastic Design

- Plastic Design at a Fixed-end Beam : Comparison to Elastic Design

p_Y : intensity of uniformly distributed load when the maximum stress at any section of the beam reaches yield stress.

Let us take the case of a fixed-end beam

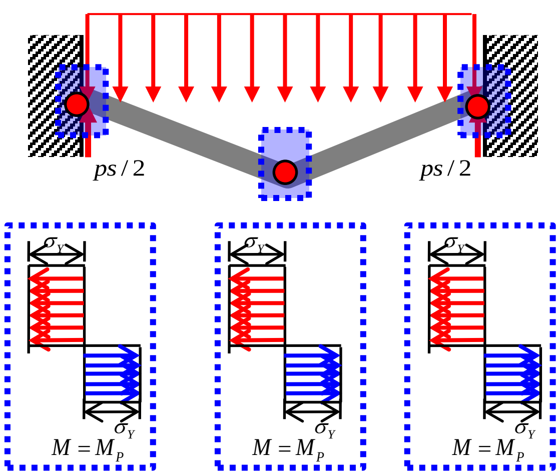
An example of fixed-end beam which carries a uniformly distributed load of intensity p.



Plastic Design

: The beam is designed to sustain the load until the beam collapses and cannot sustain any further load.

$$p = p_{Ult.}$$

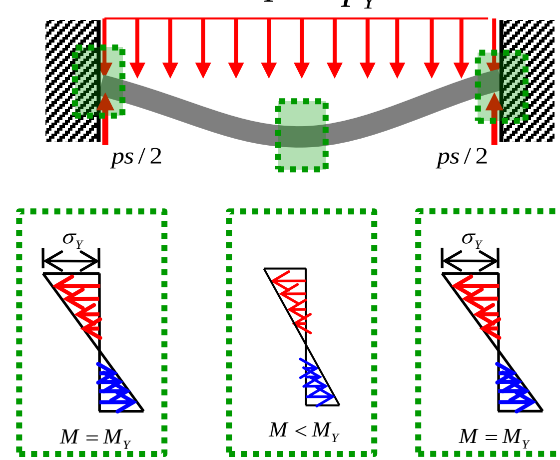


On the other hand,

Elastic Design

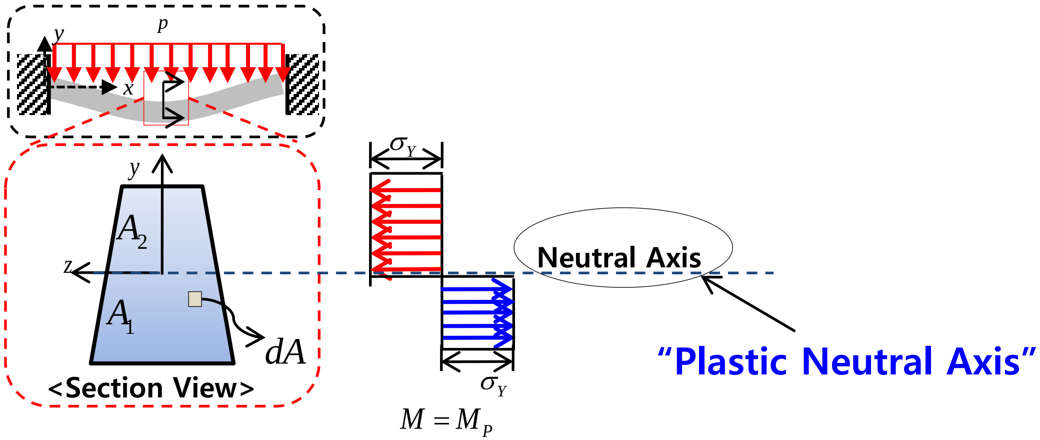
: The beam is designed to sustain the load until maximum stress at any section of the beam reaches yield stress σ_Y

$$p = p_Y$$



Plastic Design
- Plastic Neutral Axis (*P.N.A.*)

A certain section:



Plastic neutral axis divides the section into two equal areas.

Proof)

$\sum F_x = 0$ (Assumption: There is **no axial force** acting in the beam and we shall neglect the effect of shear.)

In the plastic design:

$$\sum F_x = \int_A \sigma_y dA$$

$$= \int_{A_1} \sigma_Y dA_1 + \int_{A_2} (-\sigma_Y) dA_2$$

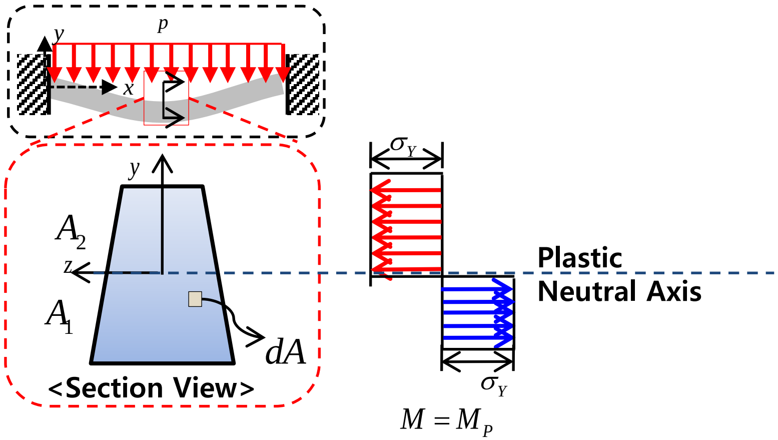
$$= \sigma_Y (A_1 - A_2)$$

$$\boxed{\therefore A_1 = A_2}$$

Okumoto, Y., Takeda, Y., Mano, M., Design of Ship Hull Structures, Springer, 2009, pp60-69.

Plastic Design
- Plastic Neutral Axis (*P.N.A.*) : Comparison to Elastic Neutral Axis

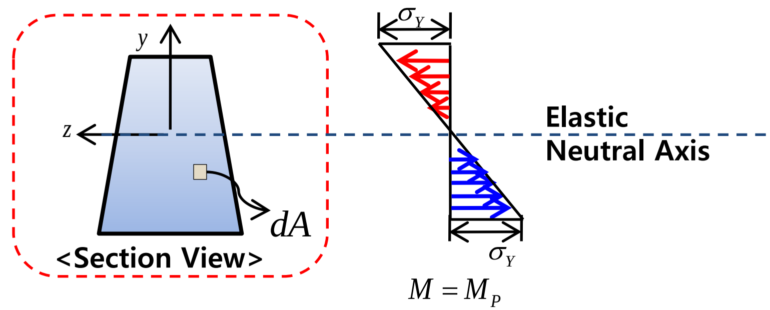
A certain section:



In the plastic design:

<u>Plastic neutral axis</u> <u>divides the section into two equal areas</u>.

$$\boxed{A_1 = A_2}$$



On the other hand,
In the elastic design:

Elastic neutral axis **passes through** <u>the centroid of the sectional area</u>

$$\boxed{\int_A y\,dA = 0}$$

Hence, if the cross section is **symmetric vertically**, the neutral axis in the plastic condition is the **same** as the elastic one, while it is **not the same** for **non-symmetric sections.**

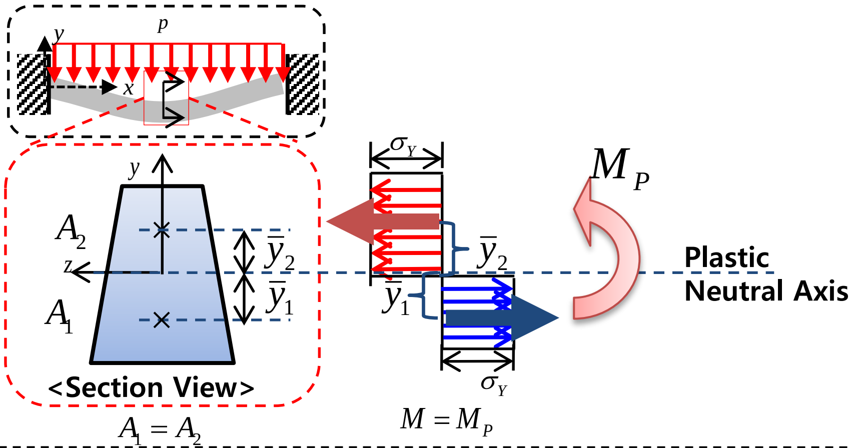
Okumoto, Y., Takeda, Y., Mano, M., Design of Ship Hull Structures, Springer, 2009, pp60-69.

Plastic Design
- Plastic Section Modulus

$$M = -\int_A \sigma_y y dA$$

A certain section:

$\bar{y}_1, \bar{y}_2$: vertical distance from the plastic neutral axis to the centroids of the two half area portions. The distance must be treated as positive.



Cf) Elastic Moment:

$$M_Y = \sigma_Y \frac{I}{y}$$

- ✓ Since the plastic moment M_P is the **resultant of the stresses** acting on the cross section when **the stress becomes yield stress wholly in the section**, it can be written as:

$$\begin{aligned} M_P &= -\int_{A_1} (\sigma_Y) y dA_1 - \int_{A_2} (-\sigma_Y) y dA_2 \\ &= -\sigma_Y(-\bar{y}_1)A_1 - (-\sigma_Y)\bar{y}_2 A_2 \\ &= \sigma_Y \cdot \bar{y}_1 (A_1 + A_2) \\ &= \sigma_Y \frac{A(\bar{y}_1 + \bar{y}_2)}{2} \end{aligned}$$

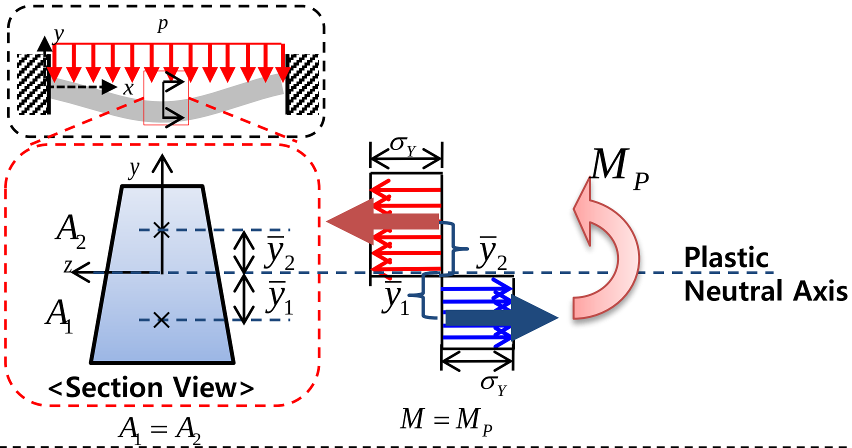
$$(A_1 + A_2 = A)$$

$$\boxed{\therefore M_P = \sigma_Y \frac{A(\bar{y}_1 + \bar{y}_2)}{2}}$$

Gere J.M., Mechanics of Materials, 7th edition, Thomson, 2009, pp504-510.

Plastic Design
- Plastic Section Modulus

A certain section:



Cf) Elastic Section Modulus:

$$M_Y = \sigma_Y \frac{I}{y}$$

We write it as:

$$M_Y = \sigma_Y Z \quad , \ where \ \ Z = \frac{I}{y}$$

Plastic Moment:

$$M_P = \sigma_Y \frac{A(\bar{y}_1 + \bar{y}_2)}{2}$$

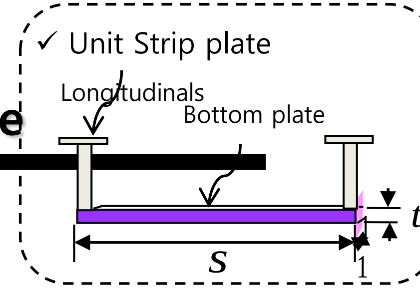
We write it as:

$$M_P = \sigma_Y Z_P \quad , \quad where \quad \boxed{Z_P = \frac{A(\bar{y}_1 + \bar{y}_2)}{2}}$$

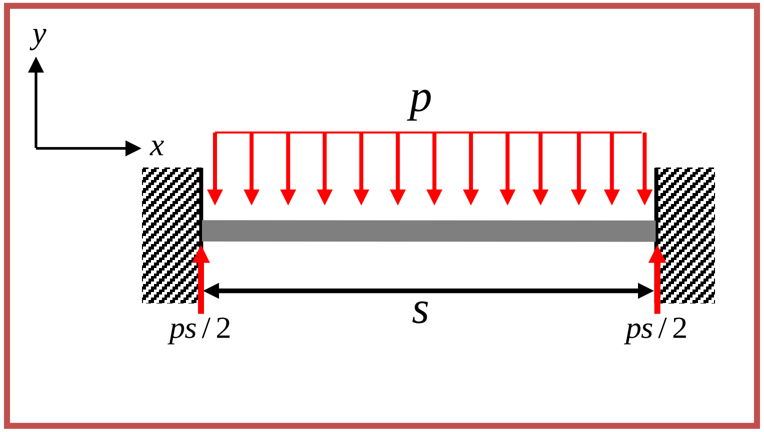
"Plastic Section Modulus"

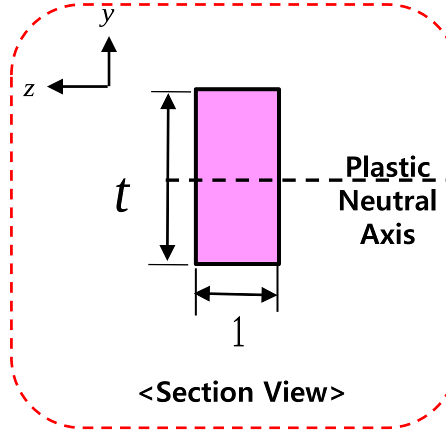
Plastic Design

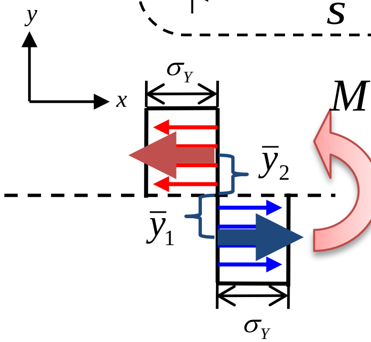
- Derivation of Plastic Section Modulus (Z_P) in a Unit Strip Plate



- **Fixed-end beam which carries a uniformly distributed load of intensity** p







Plastic Section Modulus:

$$Z_P = \frac{A(\bar{y}_1 + \bar{y}_2)}{2}$$

In the unit strip plate:

$$Z_P = \frac{1 \cdot t \cdot (t/4 + t/4)}{2}$$

$$= \frac{1 \cdot t^2}{4}$$

$$\boxed{\therefore Z_p = \frac{1 \cdot t^2}{4}}$$

Cf) Elastic Section Modulus:

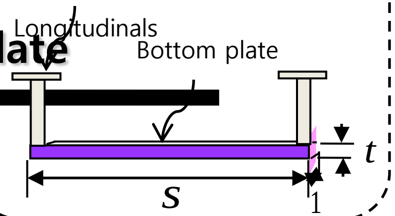
$$Z = \frac{I}{y}$$

$$= \frac{1 \cdot t^3}{12} / \frac{t}{2}$$

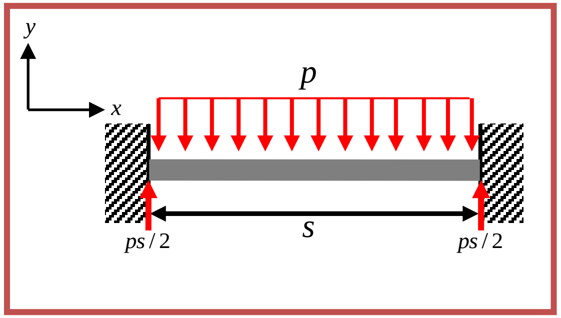
$$= \frac{1 \cdot t^2}{6}$$

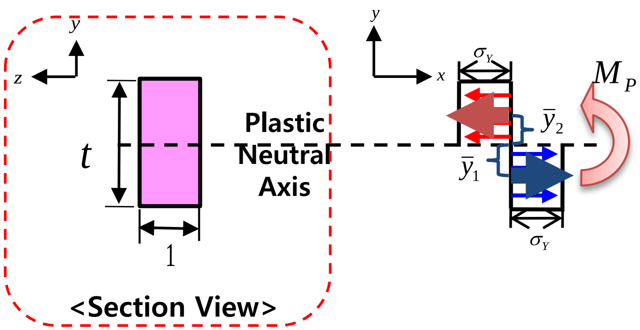
Plastic Design
- Comparison of the Plastic and Elastic Section Modulus in a Unit Strip Plate

✓ Unit Strip plate



▪ **Fixed-end beam which carries a uniformly distributed load of intensity** p





✓ Comparison of the plastic and elastic section modulus in a unit strip plate:

Plastic Section Modulus:

$$Z_p = \frac{1 \cdot t^2}{4} \quad , \textit{where} \;\; M_P = \sigma_Y Z_P$$

Elastic Section Modulus:

$$Z = \frac{1 \cdot t^2}{6} \quad , \textit{where} \;\; M_Y = \sigma_Y Z$$

$$\frac{Z_P}{Z} = \frac{Z_P \cdot \sigma_Y}{Z \cdot \sigma_Y} = \frac{M_P}{M_Y} \qquad = \frac{1 \cdot t^2}{4} / \frac{1 \cdot t^2}{6} = \frac{6}{4} = 1.5$$

The ratio of section modulus is equal to the ratio of bending moment at a given section

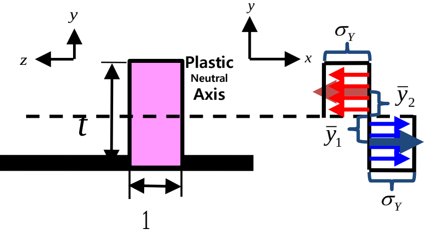
: Plastic section modulus is always larger than elastic section modulus.

: The **plastic moment exceeds** the initial **yield moment** at the given section of the beam.

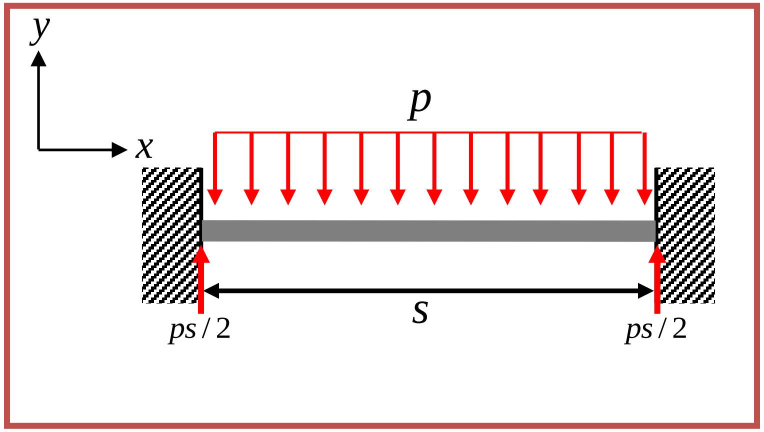
* For thin-walled, flange-and-web beams this ratio is generally in the range 1.1~1.2

Plastic Design

- Derivation of Plastic Moment (M_P) in Unit Strip Plate (1)



- Fixed-end beam which carries a uniformly distributed load of intensity p



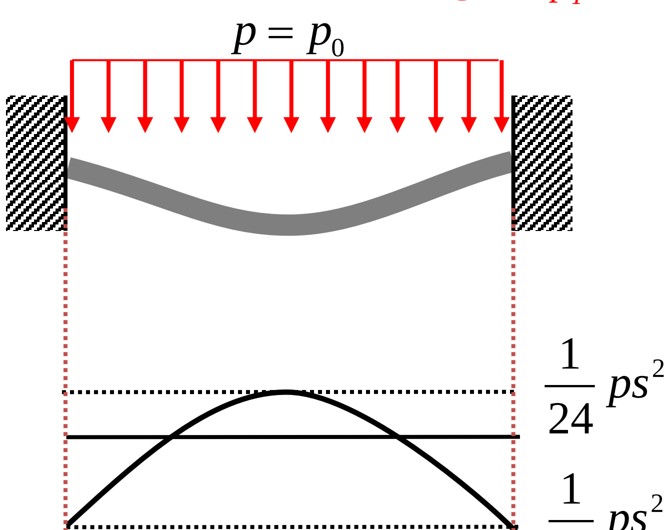
When a distributed load becomes larger, the condition of beam is changed as follows ;

1) Whole beam is in elastic region (p_0)

2) Plastic hinge occurs at both ends (p_{EH})

3) Plastic hinge occurs at midpoint (point of collapse)($p_{EH}+\Delta p$)

1) Whole beam is in elastic region (p_Y)



Bending moment diagram

When yield stress occurs at both ends: $p = p_Y$

Both ends: $M_E = M_{\max} = \boxed{\frac{1}{12} p_Y \cdot 1 \cdot s^2 = M_Y}$

: Maximum moment in elastic design

$$p_Y = \frac{12 M_Y}{s^2}$$

Center: $M_C = \frac{1}{24} p_{EH} \cdot 1 \cdot s^2$

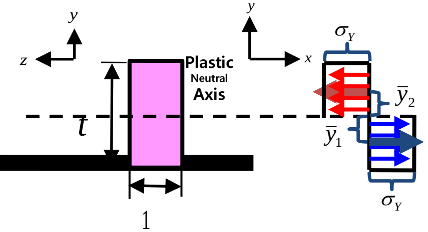
$$= \frac{1}{2} M_Y$$

Bending moment at the end of the beam: M_E

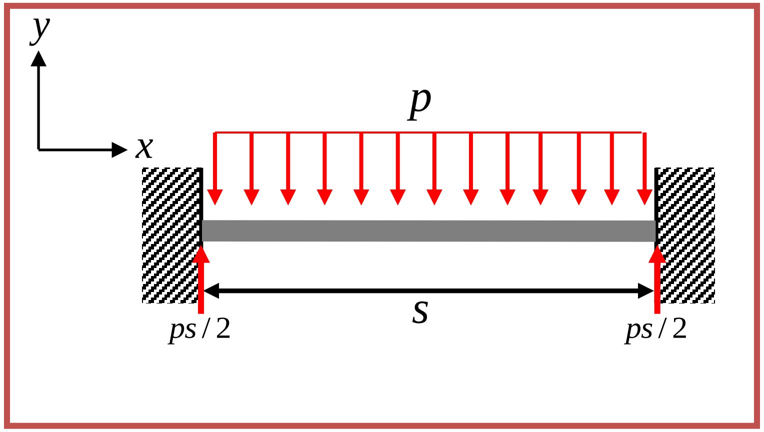
Bending moment at the center of the beam: M_C

Hughes, O. F., Ship Structural Design, Wiley-Interscience, 1983, pp513-515.

Plastic Design
- Derivation of Plastic Moment (M_P) in Unit Strip Plate (2)



- **Fixed-end beam which carries a uniformly distributed load of intensity** p

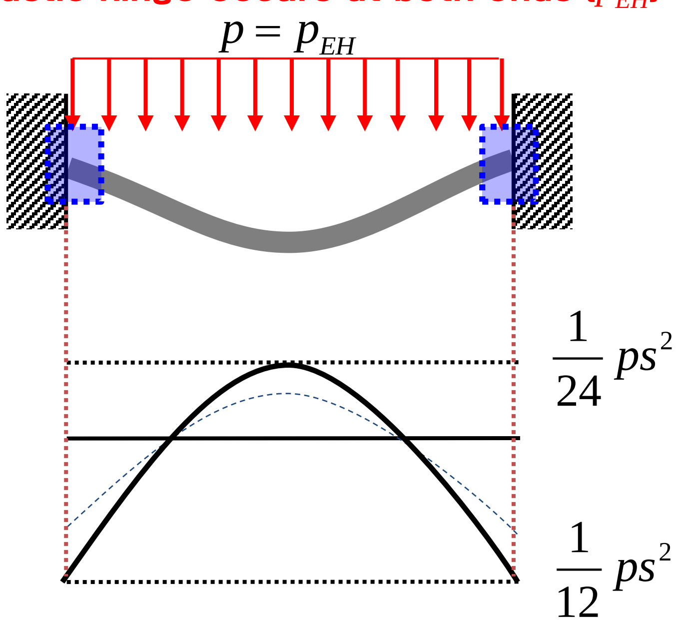


When a distributed load becomes larger, the condition of beam is changed as follows ;

1) Whole beam is in elastic region (p_0)
2) Plastic hinge occurs at both ends (p_{EH})
3) Plastic hinge occurs at midpoint
   (point of collapse)($p_{EH}+\Delta p$)

---

2) Plastic hinge occurs at both ends (p_{EH})



Bending moment diagram

When plastic hinge occurs at both ends: $p = p_{EH}$

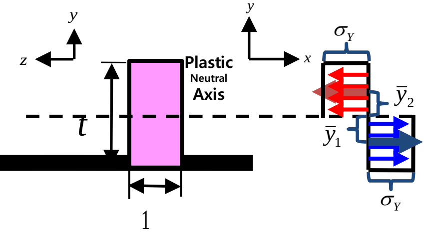
Both ends: $$M_E = M_{\max} = \frac{1}{12}\, p_{EH} \cdot 1 \cdot s^2$$

$$p_{EH} = \frac{12 M_P}{s^2}$$

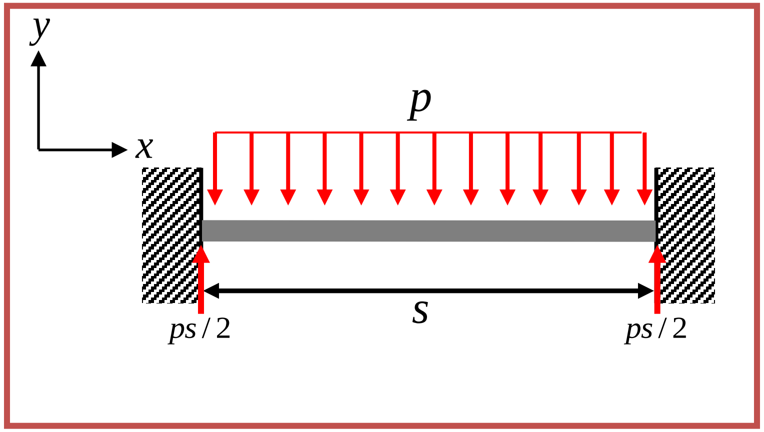
Center: $$M_C = \frac{1}{24}\, p_{EH} \cdot 1 \cdot s^2$$

$$= \frac{1}{2} M_P$$

Plastic Design
- Derivation of Plastic Moment (M_P) in Unit Strip Plate (3)



- **Fixed-end beam which carries a uniformly distributed load of intensity** p



When a distributed load becomes larger, the condition of beam is changed as follows ;

1) Whole beam is in elastic region (p_0)

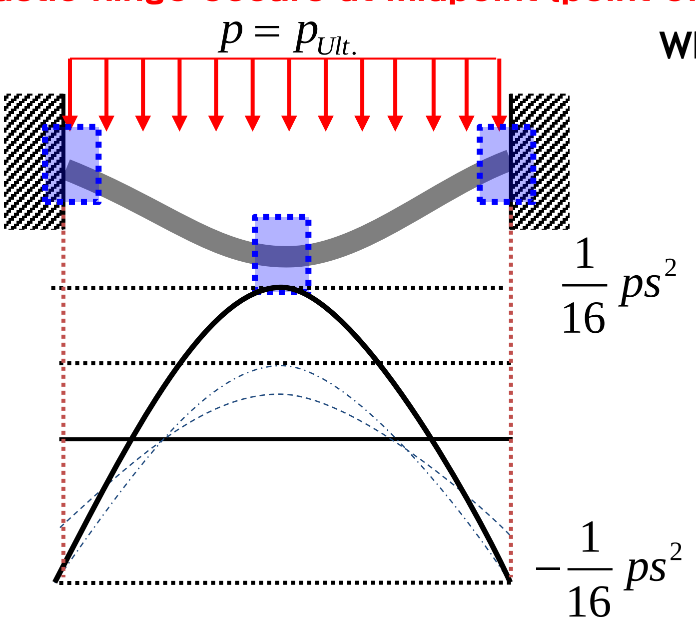
2) Plastic hinge occurs at both ends (p_{EH})

3) Plastic hinge occurs at midpoint (point of collapse)($p_{EH}+\Delta p$)

---

3) Plastic hinge occurs at midpoint (point of collapse)($p_{EH}+\Delta p$)

Bending moment diagram



When plastic hinge occurs at midpoint: $p = p_{EH} + \Delta p = p_{Ult.}$

Both ends: $M_E = M_P$: Can not larger than M_P

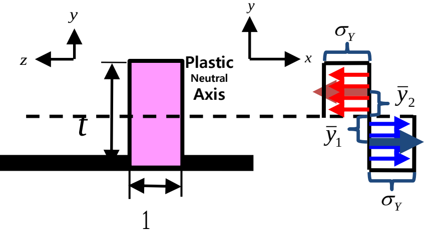
Center: $M_C = \frac{1}{24} p_{EH} \cdot 1 \cdot s^2 + \frac{1}{8} \Delta p \cdot 1 \cdot s^2 = \boxed{\frac{1}{16} p_{Ult.} \cdot 1 \cdot s^2}$

$= M_P$

$$\rightarrow M_E = M_C = M_P = \frac{1}{16} p_{Ult.} \cdot 1 \cdot s^2, \; p_{Ult.} = \frac{16 M_P}{s^2}$$

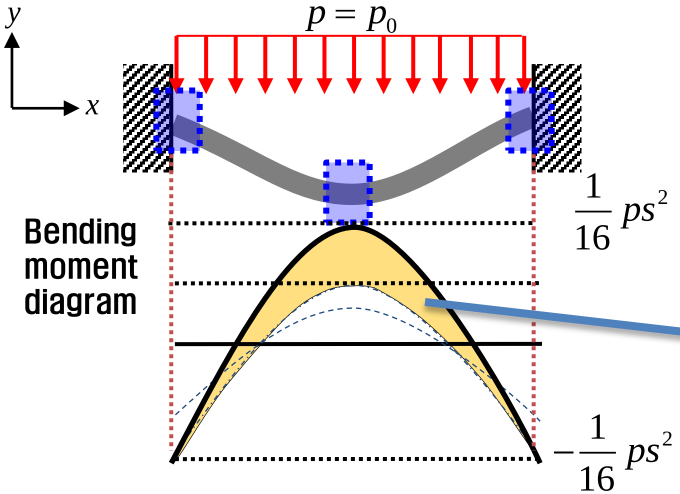
Hughes, O. F., Ship Structural Design, Wiley-Interscience, 1983, pp513-515.

Plastic Design
- Derivation of Plastic Moment (M_P) in Unit Strip Plate (3)



- **Derivation Detail of** M_P

3) Plastic hinge occurs at midpoint (point of collapse)($p_0+\Delta p'$)



When plastic hinge occurs at midpoint: $p = p_{Ult.}$

$$M_P = \boxed{\frac{1}{16}p_{Ult.}\cdot 1\cdot s^2}$$

The bending moment at the ends remains constant at M_P. Therefore, with respect to the additional load Δp, the beam behaves as if it were **simply supported at the ends**.

$p = p_{EH}$

Both ends: $M_P = \frac{1}{12}p_{EH}\cdot 1\cdot s^2$

Center: $M_C = \frac{1}{24}p_{EH}\cdot 1\cdot s^2$

$$= \frac{1}{2}M_P$$

$$p_{EH} = \frac{12M_P}{s^2}$$

$p = p_{EH} + \Delta p = p_{Ult.}$

Center: $M_C = \frac{1}{2}M_P + \frac{1}{8}\Delta p\cdot 1\cdot s^2 = M_P, \quad \Delta p = \frac{4M_P}{s^2}$

$$p_{Ult.} = p + \Delta p = \frac{12M_P}{s^2} + \frac{4M_P}{s^2} = \frac{16M_P}{s^2}$$

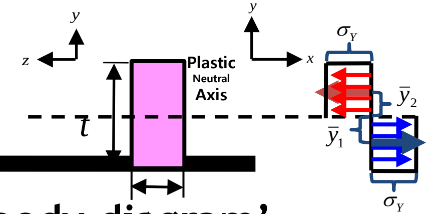
$$M_P = \frac{p_{Ult.}\cdot 1\cdot s^2}{16}$$

$$\boxed{\therefore M_P = \frac{p\cdot 1\cdot s^2}{16}}$$

Hughes, O. F., Ship Structural Design, Wiley-Interscience, 1983, pp513-515.

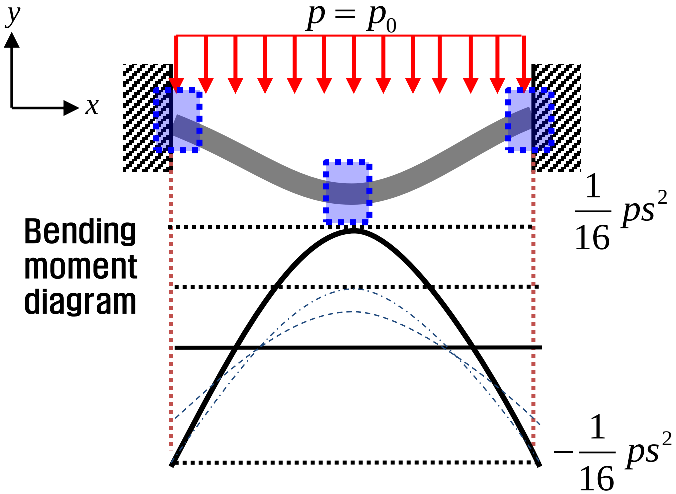
Plastic Design

- Derivation of Plastic Moment (M_P) in Unit Strip Plate (3)



- Derivation Detail of M_P : another derivation method by using 'free body diagram'

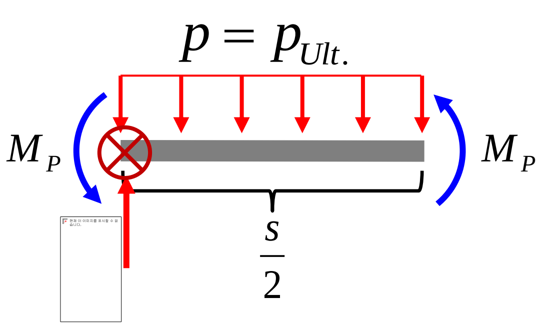
3) Plastic hinge occurs at midpoint (point of collapse)($p_0+\Delta p'$)



When plastic hinge occurs at midpoint: $p = p_{Ult.}$

$$M_P = \boxed{\frac{1}{16} p_{Ult.} \cdot 1 \cdot s^2}$$

If we draw 'free body diagram' from x=0 to x=s/2, then by using deformation sign convention the free body diagram becomes:



The moment equilibrium along the z axis through the left end point:

$$\sum M = I\ddot{\theta} = 0$$

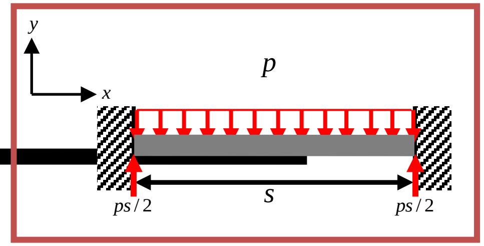
$$= M_P - \int_0^{s/2} p \cdot 1 \cdot x\,dx + M_P$$

$$2M_P = \int_0^{s/2} p \cdot 1 \cdot x\,dx = \frac{1}{2} px^2 \Big|_{x=0}^{x=s/2} = \frac{ps^2}{8}$$

$$\boxed{\therefore M_P = \frac{ps^2}{16}}$$

Plastic Design
- Comparison of Elastic and Plastic Design

- **Unit Strip Plate**



p : Intensity of uniformly distributed load

Elastic Design

Bending Moment: $M_Y = \frac{1}{12} p_Y \cdot 1 \cdot s^2$

Section Modulus: $Z = \frac{1 \cdot t^2}{6}$

$$\sigma = \frac{M_Y}{Z} = \frac{1}{12} p_Y \cdot 1 \cdot s^2 / \frac{1 \cdot t^2}{6} = \frac{p_Y s^2}{2t^2}$$

Plastic Design

Bending Moment: $M_P = \frac{1}{16} p_{Ult.} \cdot 1 \cdot s^2$

Section Modulus: $Z_P = \frac{1 \cdot t^2}{4}$

$$\sigma = \frac{M_P}{Z_P} = \frac{1}{16} p_{Ult.} \cdot 1 \cdot s^2 / \frac{1 \cdot t^2}{4} = \frac{p_{Ult.} s^2}{4t^2}$$

$\sigma = \sigma_Y$

$$p_Y = \frac{2t^2}{s^2}\sigma_Y < p_{Ult.} = \frac{4t^2}{s^2}\sigma_Y$$

: When the stress reaches the yield stress, the design pressure of plastic design is higher than that of the elastic design on the same span with the same thickness.

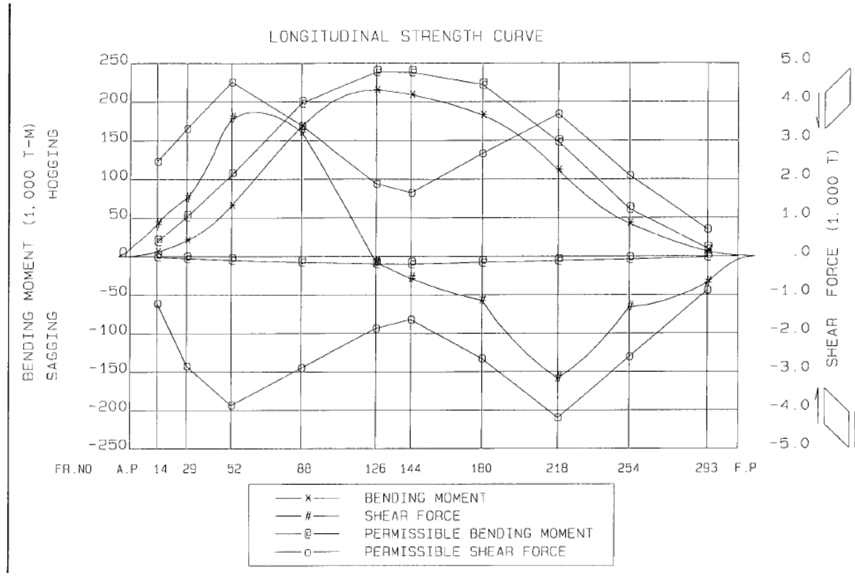
$\sigma = \sigma_Y$

$$t_{\substack{req. \\ elastic}} = \frac{s\sqrt{p}}{\sqrt{2\sigma_Y}} > t_{\substack{req. \\ plastic}} = \frac{s\sqrt{p}}{2\sqrt{\sigma_Y}}$$

: The thickness requirement of the plate of plastic design is smaller than that of the elastic design at the same pressure and on the same span.

16-6 Example of Midship Section Scantling of a 3,700TEU CONTAINER CARRIER

Example of 3,700TEU Container Ship
- Ballast Arrival Condition



**LONGITUDINAL STRESS VALUES ON THE BULKHEADS.

NO	FR.NO	DIST FROM A.P	ACT. SWSF & SWBM VALUES SHEAR FORCE (%)		ACT. SWSF & SWBM VALUES BENDING MOMENT (%)		ALLOWABLE SWSF & SWBM VALUES SHEAR FORCE (POSI.)	ALLOWABLE SHEAR FORCE (NEGA.)	ALLOWABLE BENDING MOMENT (HOG.)	ALLOWABLE BENDING MOMENT (SAG.)
1	14.0	11.200	836.8	(34.1)	5986	(31.4)	2457.0	-1223.0	19068	-1427
2	29.0	23.200	1501.3	(45.5)	20353	(39.8)	3303.0	-2855.0	51189	-3059
3	52.0	41.600	3563.7	(79.1)	65688	(62.2)	4507.0	-3874.0	105642	-5608
4	88.0	70.280	3213.3	(94.9)	168241	(84.8)	3385.0	-2906.0	198334	-8157
5	126.0	100.560	-157.9	(8.4)	214842	(90.2)	1876.0	-1876.0	238103	-10197
6	144.0	114.800	-576.1	(35.1)	208976	(87.8)	1641.0	-1641.0	238103	-10197
7	180.0	143.280	-1142 6	(42.9)	182393	(81.8)	2661.0	-2661.0	222909	-8667
8	218.0	173.560	-3168.8	(75 4)	111139	(74.8)	3681.0	-4201.0	148572	-6628
9	254.0	202.040	-1318.2	(50.5)	42122	(68.7)	2090.0	-2610.0	61284	-4690
10	293.0	233.120	-684 5	(77.6)	6009	(55.1)	698.0	-882.0	10911	-1529

MAXIMUM SHEAR FORCE = 3725.9 AT A DISTANCE OF = 53.072 (FR 66 + .272)

MAXIMUM BENDING MOMENT = $\boxed{214994}$ AT A DISTANCE OF = 99.212 (FR 124 + .272)

$M_S = 214,994 \quad (ton \cdot m)$

Example of 3,700TEU Container Ship
- Design Still Water Bending Moment

NOTES

1. DESIGN STILL WATER BENDING MOMENT IN SEAGOING CONDITION.
   HOGGING CONDITION : 238,000 TON-M (2,335,000 kN-M)

From ballast arrival condition,

$$M_S = 214{,}994 \quad (ton \cdot m)$$

2. MIN. LEG LENGTH OF FILLET WELDING 4.5 EXCEPT AS SHOWN.
3. BOTH SIDES ARE SYMMETRICAL UNLESS OTHERWISE SHOWN.
4. SECTIONS ARE SHOWN IN LOOKING FORWARD AND ELEVATIONS ARE SHOWN TO PORT.
5. THE DETAILS NOT SHOWN IN THIS DRAWING ARE REFERRED TO "STRUCTURAL DETAILS FOR HULL" (DWG. NO. SF091.20)

By calculating the section modulus and stress factor of the basis ship, we can assume the stress factor for the design ship.

Example of Midship Scantling

Outer Bottom & Bilge plate

Outer Bottom Longitudinals

Inner Bottom Plate

Inner Bottom Longitudinals

Side Shell Plate

Side Shell Longitudinals

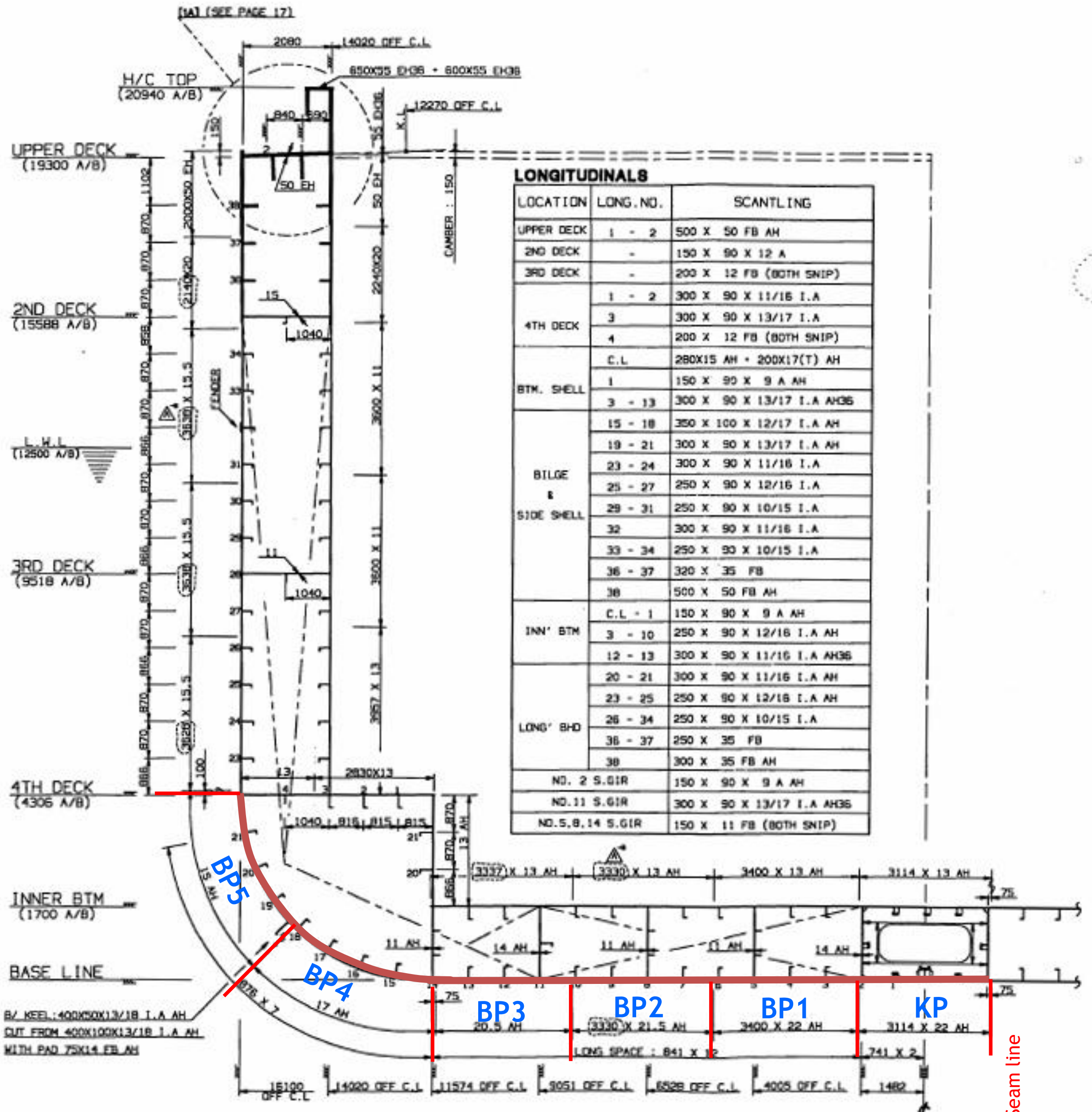
Deck Plate

Deck Longitudinals

Longitudinal Bulkhead Plate

Longitudinal Bulkhead Longitudinals

Outer Bottom & Bilge plate



LONGITUDINALS

LOCATION	LONG. NO.	SCANTLING
UPPER DECK	1 - 2	500 X 50 FB AH
2ND DECK	-	150 X 90 X 12 A
3RD DECK	-	200 X 12 FB (BOTH SNIP)
4TH DECK	1 - 2	300 X 90 X 11/16 I.A
	3	300 X 90 X 13/17 I.A
	4	200 X 12 FB (BOTH SNIP)
BTM. SHELL	C.L	280X15 AH + 200X17(T) AH
	1	150 X 90 X 9 A AH
	3 - 13	300 X 90 X 13/17 I.A AH36
BILGE & SIDE SHELL	15 - 18	350 X 100 X 12/17 I.A AH
	19 - 21	300 X 90 X 13/17 I.A AH
	23 - 24	300 X 90 X 11/16 I.A
	25 - 27	250 X 90 X 12/16 I.A
	29 - 31	250 X 90 X 10/15 I.A
	32	300 X 90 X 11/16 I.A
	33 - 34	250 X 90 X 10/15 I.A
	36 - 37	320 X 35 FB
	38	500 X 50 FB AH
INN' BTM	C.L - 1	150 X 90 X 9 A AH
	3 - 10	250 X 90 X 12/16 I.A AH
	12 - 13	300 X 90 X 11/16 I.A AH36
LONG' BHD	20 - 21	300 X 90 X 11/16 I.A AH
	23 - 25	250 X 90 X 12/16 I.A AH
	26 - 34	250 X 90 X 10/15 I.A
	36 - 37	250 X 35 FB
	38	300 X 35 FB AH
NO. 2 S.GIR		150 X 90 X 9 A AH
NO.11 S.GIR		300 X 90 X 13/17 I.A AH36
NO.5,8,14 S.GIR		150 X 11 FB (BOTH SNIP)

KP : Keel plate, BPn: n-th Outer Bottom plate

Main particulars of design ship	
LOA(m)	259.64
LBP(m)	247.64
L_scant(m)	245.11318
B(m)	32.2
D(m)	19.3
Td(m)	11
Ts(m)	12.6
Vs(knt)	24.5
C_b	0.6563

M_S : Largest SWBM among all loading conditions and class rule

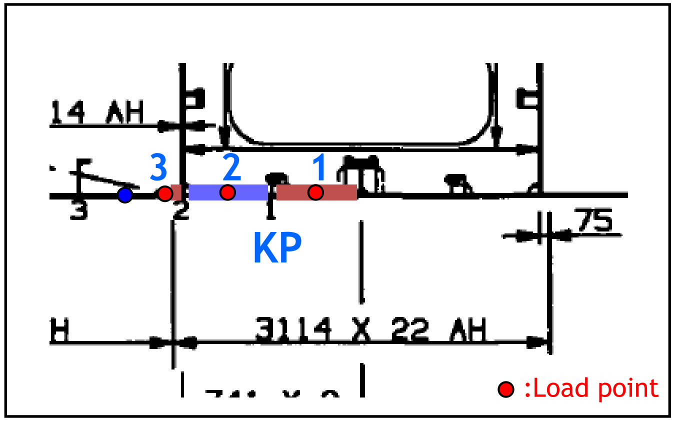
M_W : calculated by class rule or direct calculation

✓ **It is assume that the initial stress factor is equal to the stress factor of basis ship.**

$$Z_B = 2.595e^{07} \ cm^3 \qquad f_{2b} = 1.030$$

$$Z_D = 2.345e^{07} \ cm^3 \qquad f_{2d} = 1.140$$

Keel Plate (KP) (1)



✓ Keel plate is composed of the three unit strips.

✓ Load point of the unit strip :
1,2: Midpoint
3: Point nearest the midpoint

✓ Calculate the required thickness of each unit strip. And thickest value shall be used for thickness of the plate.

✓ The material of keel plate of basis ship(NV-32) is used for that of design ship. (f_1=1.28)

✓ Design Load

DnV Rules, Jan. 2004,Pt.3 Ch.1 Sec.6 Table B1

Structure	Load Type	$p \ (kN/m^2)$
Outer bottom	Sea pressure	$p_1 = 10T + p_{dp}$

: Design load acting on the keel plate is only the sea pressure.

① Design load acting on the unit strip 1 of keel plate, P1

p1	pdp	pl	ks		2	0.2L~0.7L from A.P. ks=2
			Cw		10.343	100 < L < 300, 10.75 - [(300-L)/100]^(3/2)
			kf	f	6.7	f= vertical distance from the waterline to the top of the ship's side at transverse section considered, maximum 0.8*Cw (m)
					6.7	
					28.33795639	$p_l = (k_s C_W + k_f)(0.8 + 0.15V/\sqrt{L})$
		y			8.05	horizontal distance in m from the ship's centre line to the load point, minimum B/4(m)=8.05
		z			0	vertical distance in m from the ship's baseline to the load point, maximum T(m)
					23.355	$p_{dp} = p_l + 135\frac{y}{B+75} - 1.2(T-z) \ (kN/m^2)$
					149.355	$p_1 = 10T + p_{dp}$

✓ The design loads of the unit strip2 and 3 are calculated in the same way.

Unit strip2 : p1 = 149.355(kN/m^2)

Unit strip3 : p1 = 149.355(kN/m^2)

Pt.3 Ch.1 Sec.6 Table B1 2011

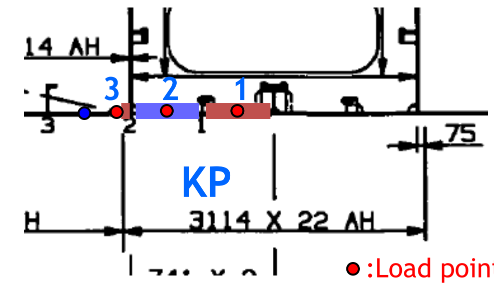
Table B1 Design loads

Structure	*Load type*	*p (kN/m²)*
Outer bottom	Sea pressure	$p_1 = 10\,T + p_{dp}$ (kN/m²) [1]
	Net pressure in way of cargo tank or deep tank	$p_2 = \rho\,(g_0 + 0.5\,a_v)\,h_s - 10\,T_M$ $p_3 = \rho\,g_0\,h_s + p_0 - 10\,T_M$
Inner bottom	Dry cargo in cargo holds	$p_4 = \rho\,(g_0 + 0.5\,a_v)\,H_C$
	Ballast in cargo holds	$p_5 = (10 + 0.5a_v)h_s$ $p_6 = 6.7(h_s + \phi b) - 1.2\sqrt{H\,\phi b_t}$ [2] $p_7 = 0.67(10h_p + \Delta p_{dyn})$ $p_8 = 10h_s + p_0$
	Liquid cargo in tank above	$p_9 = \rho\,(g_0 + 0.5a_v)\,h_s$ $p_{10} = \rho g_0[0.67(h_s + \phi b) - 0.12\sqrt{H\,\phi b_t}]$ [2] $p_{11} = 0.67(\rho g_0 h_p + \Delta p_{dyn})$ $p_{12} = \rho g_0 h_s + p_0$
Inner bottom, floors and girders	Pressure on tank boundaries in double bottom	$p_{13} = 0.67\,(10\,h_p + \Delta\,p_{dyn})$ $p_{14} = 10\,h_s + p_0$
	Minimum pressure	$p_{15} = 10\,T$

1) For ships with service restrictions the last term in p_1 may be reduced by the percentages given in Sec.4 B202.

2) p_6 and p_{10} to be used in tanks/holds with largest breadth > 0.4 B.

Keel Plate (KP) (2)



②

✓ Required Thickness	✓ Allowable stress for Bottom Plate
$t_1 = \dfrac{15.8 k_a s \sqrt{p}}{\sqrt{\sigma}} + t_k \ (mm)$	$\sigma = 120 f_1$

Required thickness of the unit strip 1 of the keel plate

T_1	p	149.355	Maximum Design Load
	ka	1.0	$k_a = (1.1 - 0.25 s / l)^2$, maximum 1.0 for s/l = 0.4 minimum 0.72 for s/l = 1-0
	s	0.741	stiffener spacing in m
	f1	1.28	Material factor = 1.28 for NV-32
	σ	153.6	$\sigma = 120 f_1$
	tk	1.5	Corrosion addition
		13.04	$t_1 = \dfrac{15.8 k_a s \sqrt{p}}{\sqrt{\sigma}} + t_k \ (mm)$

The required thickness of the unit strip2 and 3 are calculated in the same way.

Unit strip2 : t_1 = 13.04 (mm)

Unit strip3 : t_1 = 14.603(mm)

③

✓ Minimum Thickness

$$t_2 = 7.0 + \frac{0.05 L_1}{\sqrt{f_1}} + t_k \ (mm)$$

t_2	L1	245.11	Min (L, 300) (m)
	f1	1.28	Material factor = 1.28 for NV-32
	tk	1.5	Corrosion addition
		19.33	$t_2 = 7.0 + \dfrac{0.05 L_1}{\sqrt{f_1}} + t_k \ (mm)$

cf) Minimum Breadth

$$b = 800 + 5L (mm)$$

b	Rule	2025.566	
	Arr.	3154	Breadth of keel plate

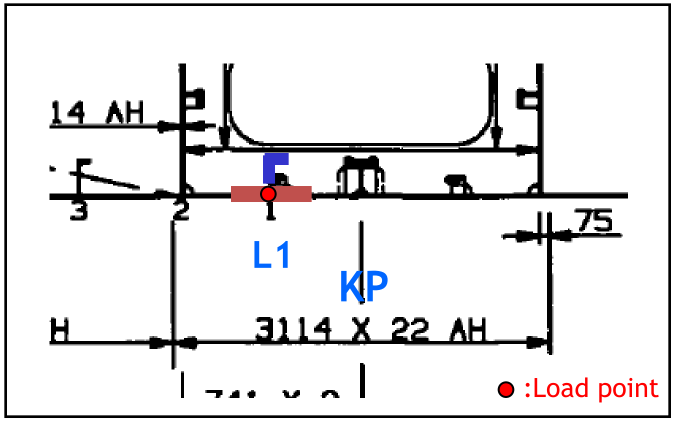
→ Rule is satisfied.

④	$t = \max(t_1, t_2) \ [mm]$
Unit strip 1	19.33
Unit strip 2	19.33
Unit Strip 3	19.33

⑤ The thickest value between the thickness of unit strips shall be used for thickness of keel plate.

$$t = 19.33 \approx 19.5 \ [mm]$$

Longitudinals at Keel Plate (L1)(1)



✓Load point: Midpoint

✓ The material of L1 of basis ship(NV-32) is used for that of design ship. (f_1=1.28)

✓ Design Load

DnV Rules, Jan. 2004,Pt.3 Ch.1 Sec.6 Table B1

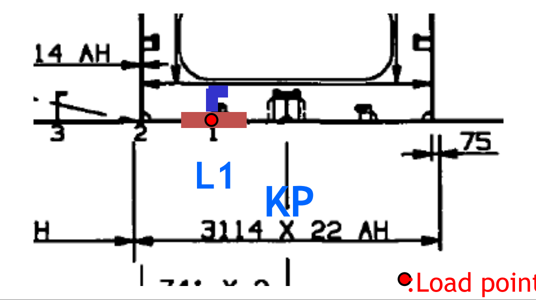
Structure	Load Type	$p\ (kN/m^2)$
Outer bottom	Sea pressure	$p_1 = 10T + p_{dp}$

: Design load acting on the keel plate is only the sea pressure.

① Design Load acting on the L1 (P1)

p_1	pdp	pl	ks		2	0.2L~0.7L from A.P. ks=2
			Cw		10.343	100 < L < 300, 10.75 - [(300-L)/100]^(3/2)
			kf	f	6.7	f = vertical distance from the waterline to the top of the ship's side at transverse section considered, maximum 0.8*Cw (m)
					6.7	
					28.33795639	$p_l = (k_s C_W + k_f)(0.8 + 0.15V/\sqrt{L})$
		y			8.05	horizontal distance in m from the ship's centre line to the load point, minimum B/4(m)=8.05
		z			0	vertical distance in m from the ship's baseline to the load point, maximum T(m)
					23.355	$p_{dp} = p_l + 135\frac{y}{B+75} - 1.2(T-z)\ (kN/m^2)$
					149.355	$p_1 = 10T + p_{dp}$

Longitudinals at Keel Plate (L1)(2)



②

✓ Required Section Modulus

$$Z = \frac{83 l^2 s p w_k}{\sigma} \ (cm^3)$$

✓ Allowable stress

$$\sigma = 225 f_1 - 130 f_{2b} - 0.7 \sigma_{db}$$

Z	le		2.96	Distance between web frame (3.16m) - 0.2 m(braket)
	s		0.741	stiffener spacing in m
	p		149.355	Maximum Design Load
	wk	tkw	1.0	Corrosion addition
		tkf	1.0	Corrosion addition
			1.15	$1 + 0.05(t_{kw} + t_{kf})$ for flanged section
	σ	f1	1.28	Material factor = 1.28 for NV-32
		f2b	1.04	It is obtained from the section modulus of the basis ship.
		σdb	25.6	$20f_1$ in general
			134.88	$\sigma = 225 f_1 - 130 f_{2b} - 0.7 \sigma_{db}$
			744.91	$Z = \frac{83 l^2 s p w_k}{\sigma} \ (cm^3)$

③

✓ Minimum Thickness of Web and Flange

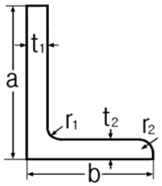
$$t_1 = 5.0 + \frac{k}{\sqrt{f_1}} + t_k \ (mm), \quad t_2 = \frac{h}{g} + t_k \ (mm)$$

t1	k	4.9022	$0.02\ L_1$
	f1	1.28	Material factor = 1.28 for NV-32
	tk	1.0	Corrosion addition
		10.83	$t_1 = 5.0 + \frac{k}{\sqrt{f_1}} + t_k \ (mm)$
t2	h	400	Profile height in m
	g	70	70 for flanged profile webs
	tk	1.0	Corrosion addition
		7.21	$t_2 = \frac{h}{g} + t_k \ (mm)$

$$t = \max(t_1, t_2) = t_1$$

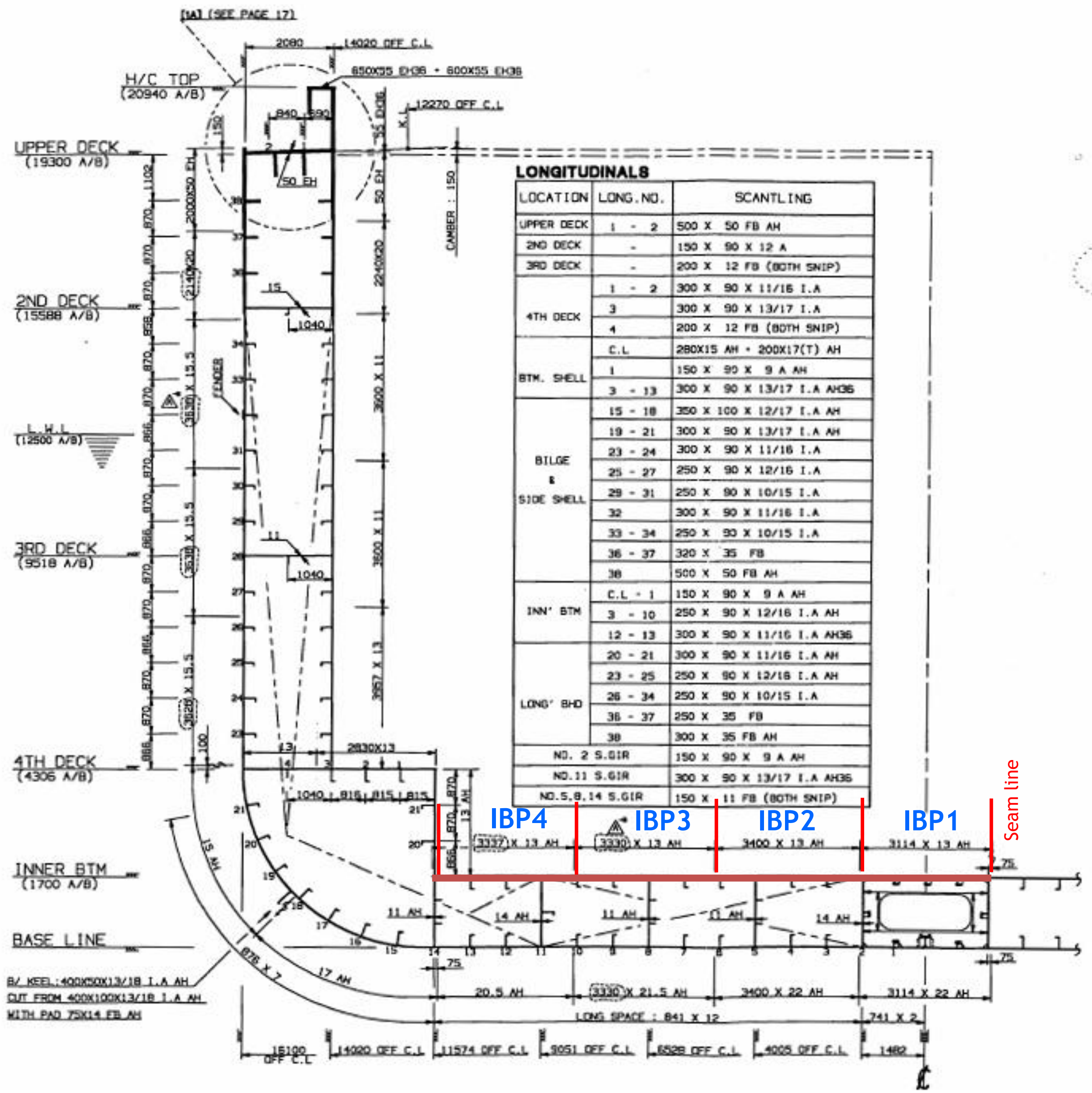
④ Select the longitudinal whose section modulus is larger than the required section modulus from the table.

"조선설계편람", 제 4판 (일본어), 일본관서조선협회, 1996



a	b	t_1	t_2	r_1	r_2	A	I	Z
mm						cm^2	cm^4	cm^3
400	100	11.5	16	24	12	61.09	34,200	1,120

Inner Bottom Plate



LONGITUDINALS

LOCATION	LONG.NO.	SCANTLING
UPPER DECK	1 - 2	500 X 50 FB AH
2ND DECK	-	150 X 90 X 12 A
3RD DECK	-	200 X 12 FB (BOTH SNIP)
4TH DECK	1 - 2	300 X 90 X 11/16 I.A
	3	300 X 90 X 13/17 I.A
	4	200 X 12 FB (BOTH SNIP)
BTM. SHELL	C.L	280X15 AH + 200X17(T) AH
	1	150 X 90 X 9 A AH
	3 - 13	300 X 90 X 13/17 I.A AH36
BILGE & SIDE SHELL	15 - 18	350 X 100 X 12/17 I.A AH
	19 - 21	300 X 90 X 13/17 I.A AH
	23 - 24	300 X 90 X 11/16 I.A
	25 - 27	250 X 90 X 12/16 I.A
	29 - 31	250 X 90 X 10/15 I.A
	32	300 X 90 X 11/16 I.A
	33 - 34	250 X 90 X 10/15 I.A
	36 - 37	320 X 35 FB
	38	500 X 50 FB AH
INN' BTM	C.L - 1	150 X 90 X 9 A AH
	3 - 10	250 X 90 X 12/16 I.A AH
	12 - 13	300 X 90 X 11/16 I.A AH36
LONG' BHD	20 - 21	300 X 90 X 11/16 I.A AH
	23 - 25	250 X 90 X 12/16 I.A AH
	26 - 34	250 X 90 X 10/15 I.A
	36 - 37	250 X 35 FB
	38	300 X 35 FB AH
NO. 2 S.GIR		150 X 90 X 9 A AH
NO.11 S.GIR		300 X 90 X 13/17 I.A AH36
NO.5,8,14 S.GIR		150 X 11 FB (BOTH SNIP)

Main particulars of design ship	
LOA(m)	259.64
LBP(m)	247.64
L_scant(m)	245.11318
B(m)	32.2
D(m)	19.3
Td(m)	11
Ts(m)	**12.6**
Vs(knt)	24.5
C_b	0.6563

M_S : Largest SWBM among all loading conditions and class rule

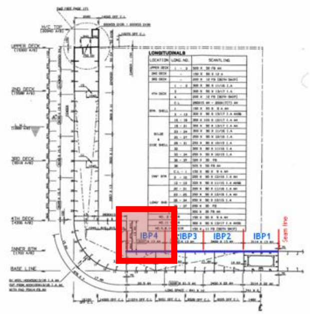
M_W : calculated by class rule or direct calculation

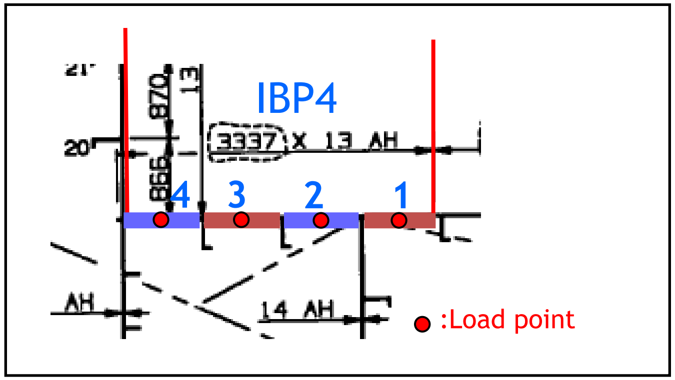
✓ **It is assume that the initial stress factor is equal to the stress factor of basis ship.**

$Z_B = 2.595e^{07}\ cm^3 \Rightarrow f_{2b} = 1.030$

$Z_D = 2.345e^{07}\ cm^3 \Rightarrow f_{2d} = 1.140$

Inner Bottom Plate (IBP4) (1)





✓ Inner bottom plate 4(IBP4) is composed of the four unit strips.

✓ Load point of the unit strip :
1, 2, 3, 4: Midpoint

✓ Calculate the required thickness of each unit strip. And thickest value shall be used for thickness of the plate.

✓ The material of inner bottom plate of basis ship(NV-32) is used for that of design ship. (f_1=1.28)

✓ Design Load

Structure	Load Type	
Inner bottom	Dry cargo in cargo holds	$p_4 = \rho(g_0 + 0.5a_V)H_C$
Inner Bottom, floors and girders	Pressure on tank boundary in double bottom	$p_{13} = 0.67(10h_p + \Delta p_{dyn})$ $p_{14} = 10h_s + p_0$
	Minimum pressure	$p_{15} = 10T$

① Design load acting on the unit strip 1 of IBP4 (P13)

✓ Dry cargo in cargo holds

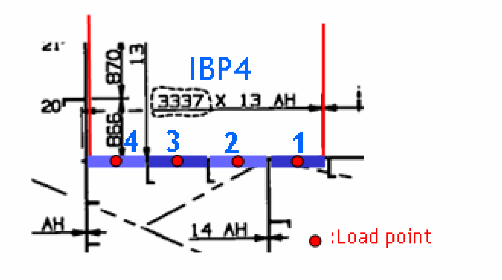
Container is considered as a light cargo, so load by container can be negligible during local scantling.
(Opinion by expert in structural design)

✓ Design load acting on the inner bottom plate considering the overflow of the cargo tank.

p_{13}	Δp_{dyn}	25	25 in general
	hp	14.648	vertical distance in m from the load point to the top of air pipe (Air pipe is located on 0.76 m above the second deck)
		114.89	$P_4 = 0.67(\rho g_0 h_p + \Delta P_{dyn})$

The design loads of the unit strip 2,3 and 4 are equal to that of the unit strip 1.

Inner Bottom Plate (IBP4) (2)



✓ Design load acting on the inner bottom plate considering the static pressure on the tank.

① Design load acting on the unit strip 1 of IBP4 (P14)

p_{14}	h_s	0	vertical distance in m from the load point to top of tank(= 0)
	p0	15	15 in ballast hold of dry cargo vessels
		15	$p_{14} = 10h_s + p_0$

Design loads acting on the unit strip 2, 3 and 4 are calculated in the same way.

Unit strip2 : p_{14} = 153.88(kN/m^2)
Unit strip3 : p_{14} = 153.88(kN/m^2)
Unit strip4 : p_{14} = 153.88(kN/m^2)

$h_s = 13.88m$
,(h_s of the unit strip 2, 3, 4 is different from that of the unit strip1.)

✓ Design load acting on the inner bottom plate considering the damaged condition.

p_{15}	**126**	$p_{15} = 10T$

The design loads of the unit strip 2,3 and 4 are equal to that of the unit strip 1.

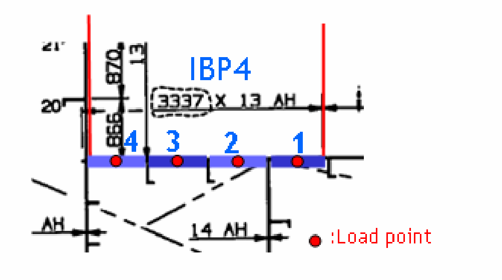
Largest value between p_{13}, p_{14} and p_{15} shall be used for pressure acting the unit strip.

$$p = \max(p_{13}, p_{14}, p_{15})$$

$$[kN/m^2]$$

Unit strip1 : p = p_{15} = 126
Unit strip2 : p = p_{14} =153.88
Unit strip3 : p = p_{14} =153.88
Unit strip4 : p = p_{14} = 153.88

Inner Bottom Plate (IBP4) (3)



②

✓ Required Thickness	✓ Allowable stress for Bottom Plate
$t_1 = \dfrac{15.8 k_a s \sqrt{p}}{\sqrt{\sigma}} + t_k \ (mm)$	$\sigma = 140 f_1$

Required thickness of the unit strip 1 of the inner bottom plate

t_1	p	126	Maximum Design Load
	ka	1.0	$k_a = (1.1 - 0.25 s/l)^2$, maximum 1.0 for s/l = 0.4 minimum 0.72 for s/l = 1-0
	s	0.841	stiffener spacing in m
	f1	1.28	Material factor = 1.28 for NV-32
	σ	179.2	$\sigma = 140 f_1$
	tk	1	Corrosion addition
		12.14	$t_1 = \dfrac{15.8 k_a s \sqrt{p}}{\sqrt{\sigma}} + t_k \ (mm)$

The required thicknesses of the unit strip2 and 3 are calculated in the same way.

Unit strip2 : t_1 = 13.31 (mm)

Unit strip3 : t_1 = 13.31(mm)

Unit strip4 : t_1 = 13.31(mm)

③ ✓ Minimum Thickness

$$t_2 = t_0 + \frac{0.03 L_1}{\sqrt{f_1}} + t_k \ (mm)$$

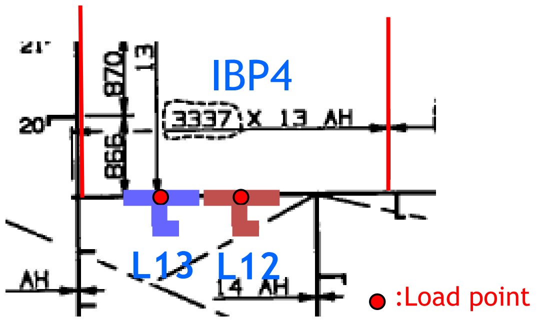
t_2	t_0	5.0	5.0 in general
	L1	245.11	Min (L, 300) (m)
	f1	1.28	Material factor = 1.28 for NV-32
	tk	1	Corrosion addition
		12.50	

④	$t = \max(t_1, t_2)\ [mm]$
Unit strip 1	12.50
Unit strip 2	13.31
Unit Strip 3	13.31

⑤ The thickest value between the thickness of unit strips shall be used for thickness of inner bottom plate.

$$t = 13.31 \approx 13.5\ [mm]$$

Longitudinals at Inner Bottom (L12)(1)



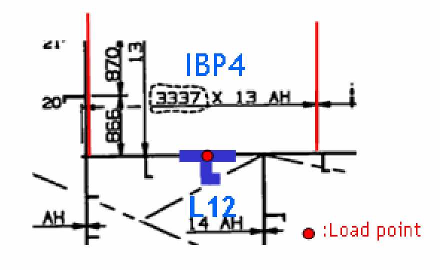
✓ Design Load

Structure	Load Type	
Inner bottom	Dry cargo in cargo holds	$p_4 = \rho(g_0 + 0.5a_V)H_C$
Inner Bottom, floors and girders	Pressure on tank boundary in double bottom	$p_{13} = 0.67(10h_p + \Delta p_{dyn})$ $p_{14} = 10h_s + p_0$
	Minimum pressure	$p_{15} = 10T$

✓Load point: Midpoint

✓ The materials of L12 and L13 of basis ship(NV-32) are used for those of design ship. (f_1=1.28)

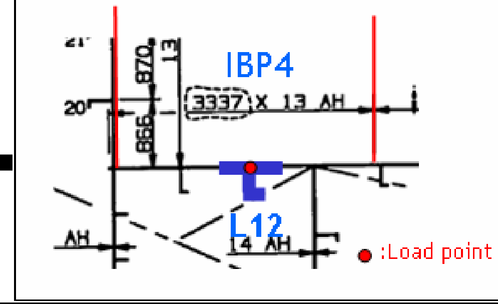
① Design Load acting on the L12, (P)



Design load acting on the longitudinals at inner bottom is equal to that on the inner bottom plate.

L14 : p = p_{14} =153.88

Longitudinals at Inner Bottom (L12)(2)



②

✓ Required Section Modulus

$$Z = \frac{83 l^2 s p w_k}{\sigma} \quad (cm^3)$$

✓ Allowable stress

$$\sigma = 225 f_1 - 100 f_{2b} - 0.7\sigma_{db}$$

Z	le		2.96	Distance between web frame (3.16m) - 0.2 m(braket)
	s		0.841	stiffener spacing in m
	p		153.88	Maximum Design Load
	wk	tkw	1.0	Corrosion addition
		tkf	1.0	Corrosion addition
			1.1	$1 + 0.05(t_{kw} + t_{kf})$ for flanged section
	σ	f1	1.28	Material factor = 1.28 for NV-32
		f2b	1.04	It is obtained from the section modulus of the basis ship.
		σdb	25.6	$20f_1$ in general
			166.08	$\sigma = 225 f_1 - 100 f_{2b} - 0.7\sigma_{db}$
	623.33			$Z = \frac{83 l^2 s p w_k}{\sigma} \; (cm^3)$

③

✓ Minimum Thickness of Web and Flange

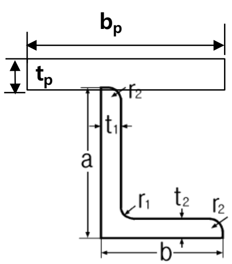
$$t_1 = 5.0 + \frac{k}{\sqrt{f_1}} + t_k \; (mm), \quad t_2 = \frac{h}{g} + t_k \; (mm)$$

t1	k	4.9022	0.02 L_1
	f1	1.28	Material factor = 1.28 for NV-32
	tk	1.0	Corrosion addition
	10.33		$t_1 = 5.0 + \frac{k}{\sqrt{f_1}} + t_k \; (mm)$
t2	h	300	Profile height in m
	g	70	70 for flanged profile webs
	tk	1.0	Corrosion addition
	5.29		$t_2 = \frac{h}{g} + t_k \; (mm)$

$$t = \max(t_1, t_2) = t_1$$

④ Select the longitudinal whose section modulus is larger than the required section modulus from the table.

"조선설계편람", 제 4판 (일본어), 일본관서조선협회, 1996

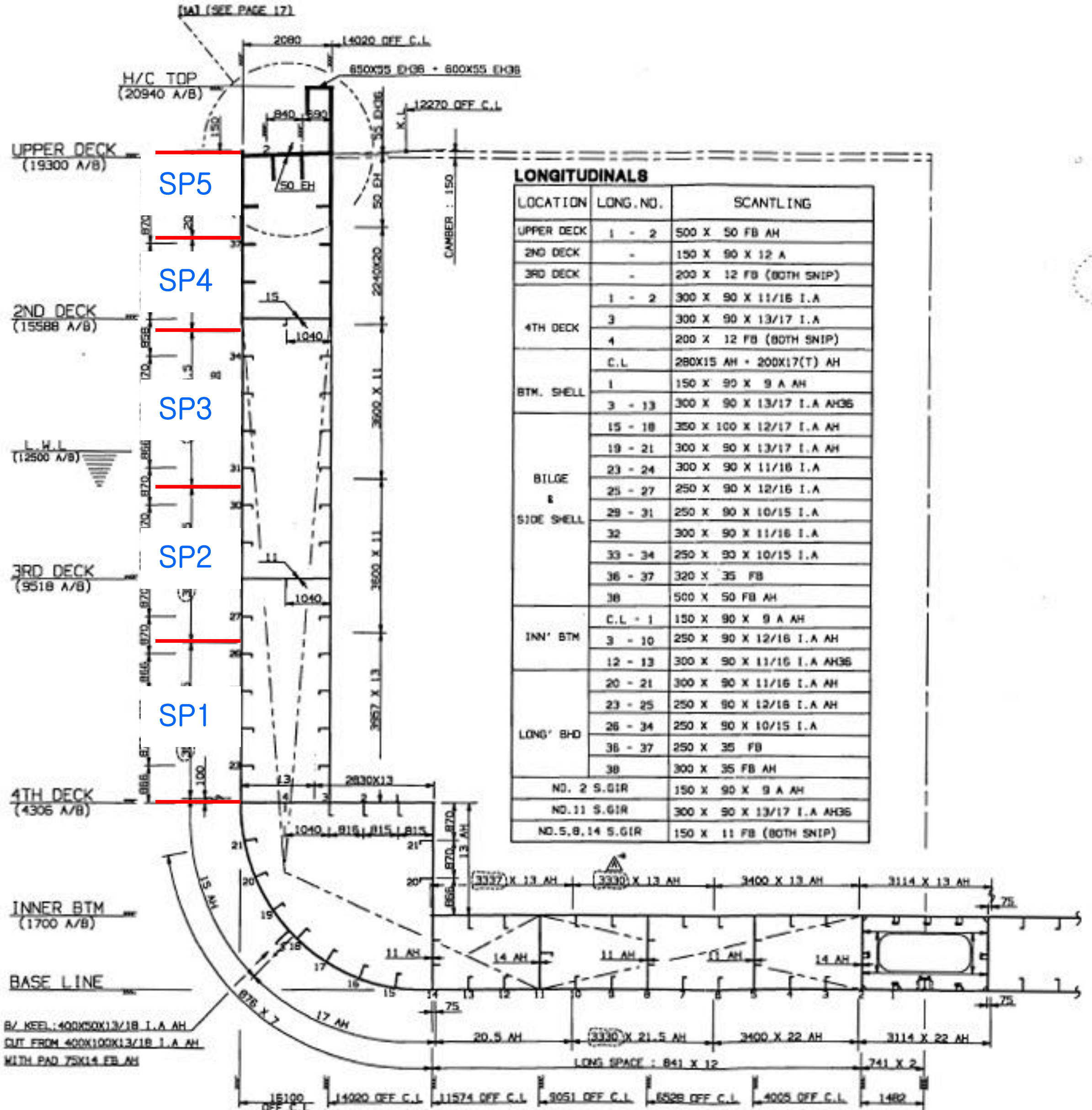


Section modulus whose longitudinal involves the plate.[1)]

a	b	t_1	t_2	r_1	r_2	A	I	Z
mm						cm^2	cm^4	cm^3
300	90	11	16	19	9.5	46.22	16,400	681

1) When the section modulus is calculated, standard breadth depending on a is used for effective breadth for simplicity. But effective breadth in accordance with rule should be used in actual calculation. **(b_p×t_p)** => (a≤75 : 420×8, 75<a<150 : 610×10, 150≤a : 610×15)

Side Shell Plate



LONGITUDINALS

LOCATION	LONG. NO.	SCANTLING
UPPER DECK	1 - 2	500 X 50 FB AH
2ND DECK	-	150 X 90 X 12 A
3RD DECK	-	200 X 12 FB (BOTH SNIP)
4TH DECK	1 - 2	300 X 90 X 11/16 I.A
	3	300 X 90 X 13/17 I.A
	4	200 X 12 FB (BOTH SNIP)
BTM. SHELL	C.L	280X15 AH + 200X17(T) AH
	1	150 X 90 X 9 A AH
	3 - 13	300 X 90 X 13/17 I.A AH36
BILGE & SIDE SHELL	15 - 18	350 X 100 X 12/17 I.A AH
	19 - 21	300 X 90 X 13/17 I.A AH
	23 - 24	300 X 90 X 11/16 I.A
	25 - 27	250 X 90 X 12/16 I.A
	29 - 31	250 X 90 X 10/15 I.A
	32	300 X 90 X 11/16 I.A
	33 - 34	250 X 90 X 10/15 I.A
	36 - 37	320 X 35 FB
	38	500 X 50 FB AH
INN' BTM	C.L - 1	150 X 90 X 9 A AH
	3 - 10	250 X 90 X 12/16 I.A AH
	12 - 13	300 X 90 X 11/16 I.A AH36
LONG' BHD	20 - 21	300 X 90 X 11/16 I.A AH
	23 - 25	250 X 90 X 12/16 I.A AH
	26 - 34	250 X 90 X 10/15 I.A
	36 - 37	250 X 35 FB
	38	300 X 35 FB AH
NO. 2 S.GIR		150 X 90 X 9 A AH
NO.11 S.GIR		300 X 90 X 13/17 I.A AH36
NO.5,8,14 S.GIR		150 X 11 FB (BOTH SNIP)

- SP : Side shell Plate

Main particulars of design ship	
LOA(m)	259.64
LBP(m)	247.64
L_scant(m)	245.11318
B(m)	32.2
D(m)	19.3
Td(m)	11
Ts(m)	**12.6**
Vs(knt)	24.5
C_b	0.6563

✓ M_S: Largest SWBM among all loading conditions and class rule

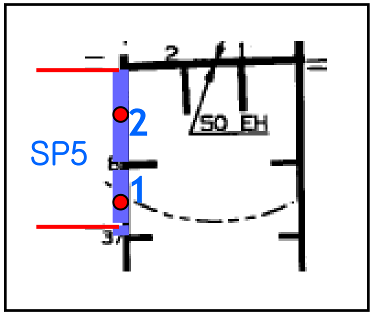
M_W : calculated by class rule or direct calculation

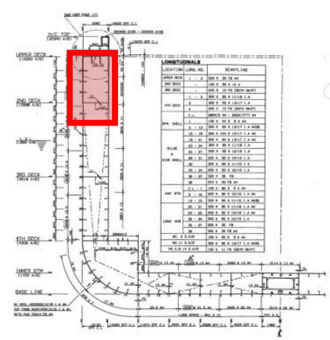
✓ **It is assume that the initial stress factor is equal to the stress factor of basis ship.**

$$Z_B = 2.595e^{07}\ cm^3 \quad \Rightarrow \quad f_{2b} = 1.030$$

$$Z_D = 2.345e^{07}\ cm^3 \quad \quad f_{2d} = 1.140$$

Side Shell Plate (1)
(Design Load & Load Point)





● :Load point

✓ Side shell plate(SP5) is composed of the two unit strips.

✓ Load point of the unit strip :
1, 2: Midpoint

✓ Calculate the required thickness of each unit strip. And thickest value shall be used for thickness of the plate(SP5).

✓ The material of SP5 of basis ship(NV-32) is used for that of design ship. (f_1=1.28)

✓ Because SP5 is side plate and shear strake at strength deck, required thickness of SP5 considering both required side plating and strength deck plating. (DnV Rules, Jan. 2004,Pt.3 Ch.1 Sec.7 C202)

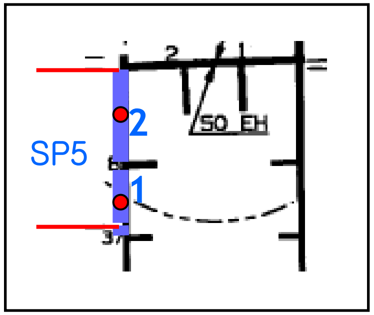
$$t = \frac{t_1 + t_2}{2} \quad (mm)$$

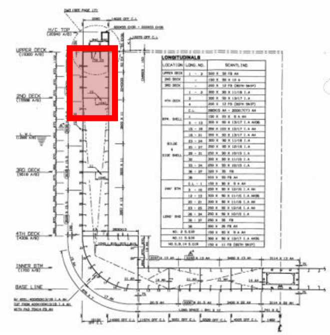
✓t1 : required side plating in mm

✓t2 : strength deck plating in mm

✓t2 shall not be taken less than t1.

Side Shell Plate (2) (SP5 - Side plating)





● :Load point

✓ Side plate(SP5) is composed of the two unit strips.

✓ Load point of the unit strip :
1, 2: Midpoint

✓ Calculate the required thickness of each unit strip. And thickest value shall be used for thickness of the plate(SP5).

✓ The material of SP5 of basis ship(NV-32) is used for that of design ship. (f_1=1.28)

DnV Rules, Jan. 2004,Pt.3 Ch.1 Sec.7 Table B1

Structure	Load Type	$p \ (kN/m^2)$
External	Sea pressure above summer load waterline	$p_2 = p_{dp} - (4+0.2k_s)h_0$

: Design load acting on the SP5 is only the sea pressure.

① Design load acting on the unit strip 1 of SP5, P2

p2	pdp	pl	ks		2	0.2L~0.7L from A.P. ks=2
			Cw		10.343	100 < L < 300, 10.75 - [(300-L)/100]^(3/2)
			kf	f	6.7	f= vertical distance from the waterline to the top of the ship's side at transverse section considered, maximum 0.8*Cw (m)
					6.7	
					28.33795639	$p_l = (k_s C_W + k_f)(0.8 + 0.15V/\sqrt{L})$
		y			16.1	horizontal distance in m from the ship's centre line to the load point, minimum B/4(m)=8.05
		z			12.6	vertical distance in m from the ship's baseline to the load point, maximum T(m))
					48.613	$p_{dp} = p_l + 135\frac{y}{B+75} - 1.2(T-z) \ (kN/m^2)$
	h0				5.163	vertical distance in m from the waterline considered to the load point
					25.896	$p_2 = p_{dp} - (4+0.2k_s)h_0$

✓ The design loads of the unit strip2 is calculated in the same way.
Unit strip2 : p2 = 21.558(kN/m^2)

Pt.3 Ch.1 Sec.7 Table B1 2011

Table B1 Design loads

Load type		*P (kN/m^2)*
External	Sea pressure below summer load waterline	$p_1 = 10\ h_0 + p_{dp}$ [1]
	Sea pressure above summer load waterline	$p_2 = (p_{dp} - (4 + 0.2\ k_s)\ h_0)$ [1] minimum $6.25 + 0.025\ L_1$
Internal	Ballast, bunker or liquid cargo in side tanks in general	$p_3 = \rho\ (g_0 + 0.5\ a_v)\ h_s - 10\ h_b$ $p_4 = \rho\ g_0\ h_s - 10\ h_b + p_o$ $p_5 = 0.67\ (\rho\ g_0\ h_p + \Delta\ p_{dyn}) - 10\ h_b$
	Above the ballast waterline at ballast, bunker or liquid cargo tanks with a breadth > 0.4 B	$p_6 = \rho g_0 [0.67(h_s + \phi b) - 0.12\sqrt{H\ \phi b_t}]$
	Above the ballast waterline and towards ends of tanks for ballast, bunker or liquid cargo with length > 0.15 L	$p_7 = \rho g_0 [0.67(h_s + \theta l) - 0.12\sqrt{H\ \theta\ l_t}]$
	In tanks with no restriction on their filling height [2]	$p_8 = \rho\left[3 - \frac{B}{100}\right] b_b$

1) For ships with service restrictions, p_2 and the last term in p_1 may be reduced by the percentages given in Sec.4 B202.

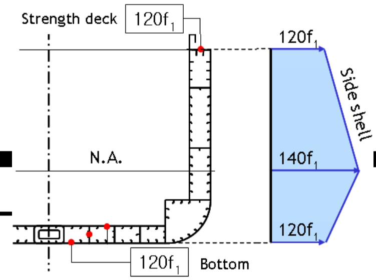
2) For tanks with free breadth $b_s > 0.56$ B the design pressure will be specially considered according to Sec.4 C305.

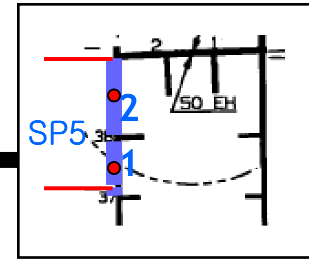
Pt.3 Ch.1 Sec.7 Table B1 2011

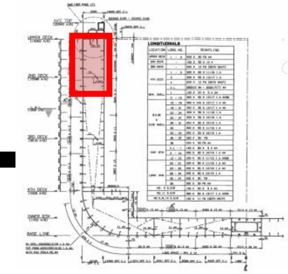
h_0 = vertical distance in m from the waterline at draught T to the load point
T = rule draught in m, see Sec.1 B
z = vertical distance from the baseline to the load point, maximum T (m)
p_{dp}, k_s = as given in Sec.4 C201
L_1 = ship length, need not be taken greater than 300 (m)
a_v = vertical acceleration as given in Sec.4 B600
h_s = vertical distance in m from load point to top of tank, excluding smaller hatchways.

h_p = vertical distance in m from the load point to the top of air pipe
h_b = vertical distance in m from the load point to the minimum design draught, which may normally be taken as 0.35 T for dry cargo vessels and 2 + 0.02 L for tankers. For load points above the ballast waterline $h_b = 0$
p_o = 25 in general
= 15 in ballast holds in dry cargo vessels
= tank pressure valve opening pressure when exceeding the general value
ρ = density of ballast, bunker or liquid cargo in t/m^3, normally not to be taken less than 1.025 t/m^3 (i.e. $\rho g_0 \approx 10$)
Δp_{dyn} = as given in Sec.4 C300
H = height in m of tank
b = the largest athwartship distance in m from the load point to the tank corner at the top of tank/ hold most distant from the load point, see Fig.2
b_t = breadth in m of top of tank/hold
l = the largest longitudinal distance in m from the load point to the tank corner at top of tank most distant from the load point
l_t = length in m of top of tank
ϕ = roll angle in radians as given in Sec.4 B400
θ = pitch angle in radians as given in Sec.4 B500
b_b = distance in m between tank sides or effective longitudinal wash bulkhead at the height at which the strength member is located.

Side Shell Plate (3)
(SP5 - Side plating)







●:Load point

②

✓ Required Thickness

$$t = \frac{15.8 k_a s \sqrt{p}}{\sqrt{\sigma}} + t_k \ (mm)$$

✓ Allowable stress for Side shell Plate

$\sigma = 140 f_1$ at N.A.

σshall be reduced linearly.

Required thickness of the unit strip 1 of the SP5

t_{1_1}	p	25.896	Maximum Design Load
	ka	1.0	$k_a = (1.1 - 0.25 s/l)^2$, maximum 1.0 for s/l = 0.4 minimum 0.72 for s/l = 1-0
	s	0.87	stiffener spacing in m
	f1	1.28	Material factor = 1.28 for NV-32
	sigma	157.431	N.A(140f1)~ deck(140f1), It shall be reduced linearly.
	tk	3	Corrosion addition
		8.575	$t_1 = \frac{15.8 k_a s \sqrt{p}}{\sqrt{\sigma}} + t_k \ (mm)$

The required thickness of the unit strip2 is calculated in the same way.

Unit strip2 : t_1 = 9.993 (mm)

③ ✓ Minimum Thickness

$$t = 5.0 + \frac{kL_1}{\sqrt{f_1}} + t_k \ (mm)$$

t_{1_2}	k	0.03	Min (L, 300) (m)
	L1	245.11	Min (L, 300) (m)
	f1	1.28	Material factor = 1.28 for NV-32
	tk	3	Corrosion addition
		14. 5	$t = 5.0 + \frac{kL_1}{\sqrt{f_1}} + t_k \ (mm)$

cf) Minimum Breadth

$$b = 800 + 5L (mm)$$

b	Rule	2025.566	
	Arr.	3154	Breadth of side shell plate

→ Rule is satisfied.

④	$t_1 = \max(t_{1\_1}, t_{1\_2}) \quad [mm]$
Unit strip 1	14.5
Unit strip 2	14.5

⑤ The thickest value between the thickness of unit strips shall be used for thickness of SP5.

$$t_1 = 14.5$$

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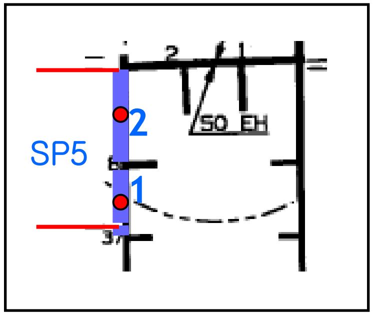
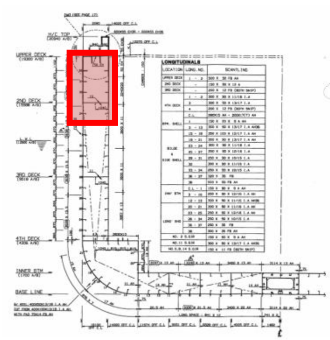
Side Shell Plate (4)
(SP5 – Shear strake strength deck plating)

DnV Rules, Jan. 2004,Pt.3 Ch.1 Sec.7 Table B1

Structure	Load Type	$p\ (kN/m^2)$
Weather deck	Sea pressure	$p_1 = a(p_{dp} - (4+0.2k_s)h_0)$

: Design load acting on the SP5 is only the sea pressure.





● :Load point

✓ Shear strake at strength deck(SP5) is composed of the two unit strips.

✓ Load point of the unit strip :
1, 2: Midpoint

✓ Calculate the required thickness of each unit strip. And thickest value shall be used for thickness of the plate(SP5).

✓ The material of SP5 of basis ship(NV-32) is used for that of design ship. (f_1=1.28)

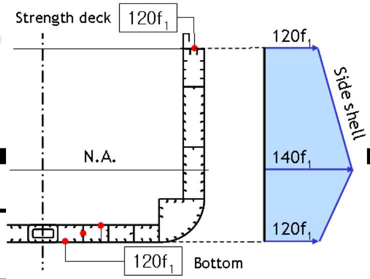
① Design load acting on the unit strip 1 of SP5, P1

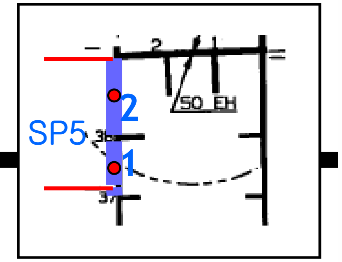
p1	pdp	pl	ks		2	0.2L~0.7L from A.P. ks=2
			Cw		10.343	100 < L < 300, 10.75 - [(300-L)/100]^(3/2)
			kf	f	6.7	f= vertical distance from the waterline to the top of the ship's side at transverse section considered, maximum 0.8*Cw (m)
					6.7	
					28.33795639	$p_l = (k_s C_W + k_f)(0.8 + 0.15V/\sqrt{L})$
		y			16.1	horizontal distance in m from the ship's centre line to the load point, minimum B/4(m)=8.05
		z			12.6	vertical distance in m from the ship's baseline to the load point, maximum T(m)
					48.613	$p_{dp} = p_l + 135\frac{y}{B+75} - 1.2(T-z)\ (kN/m^2)$
	a				0.8	1.0 for weather decks forward of 0.15L from FP, or forward of deckhouse front, whichever is the foremost position or 0.8 for weather decks elsewhere
	h0				6.7	vertical distance in m from the waterline considered to the load point
					15.743	$p_1 = a(p_{dp} - (4+0.2k_s)h_0)$

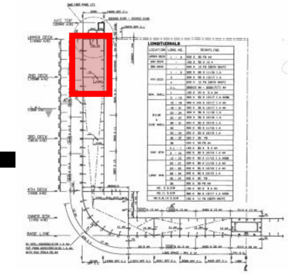
✓ The design loads of the unit strip2 is calculated in the same way.

Unit strip2 : p1 =15.743(kN/m²)

Side Shell Plate (5) (SP5 – Shear strake at strength deck)







●:Load point

②

✓ Required Thickness	✓ Allowable stress for Side shell Plate
$t = \dfrac{15.8 k_a s \sqrt{p}}{\sqrt{\sigma}} + t_k \ (mm)$	$\sigma = 140 f_1$ at N.A. σ shall be reduced linearly.

Required thickness of the unit strip 1 of the SP5

t_{2_1}	p	15.743	Maximum Design Load
	ka	1.0	$k_a = (1.1 - 0.25 s/l)^2$, maximum 1.0 for s/l = 0.4 minimum 0.72 for s/l = 1-0
	s	0.87	stiffener spacing in m
	f1	1.28	Material factor = 1.28 for NV-32
	sigma	153.6	120f1
	tk	3	Corrosion addition
	7.401		$t_1 = \dfrac{15.8 k_a s \sqrt{p}}{\sqrt{\sigma}} + t_k \ (mm)$

The required thickness of the unit strip2 is calculated in the same way.
Unit strip2 : t_{1_1} =8.574 (mm)

③ ✓ Minimum Thickness $t = t_0 + \dfrac{k L_1}{\sqrt{f_1}} + t_k \ (mm)$

t_{2_2}	t0	5.5	5.5 for unsheathed weather and cargo deck
	k	0.02	0.02 in vessels with single continuous deck
	L1	245.11	Min (L, 300) (m)
	f1	1.28	Material factor = 1.28 for NV-32
	tk	3	Corrosion addition
	12.883		$t = t_0 + \dfrac{k L_1}{\sqrt{f_1}} + t_k \ (mm)$

cf) Minimum Breadth

$$b = 800 + 5L (mm)$$

b	Rule	2025.566	
	Arr.	3154	Breadth of shear strake

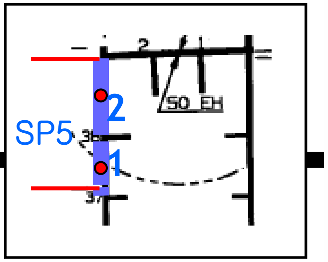
→ Rule is satisfied.

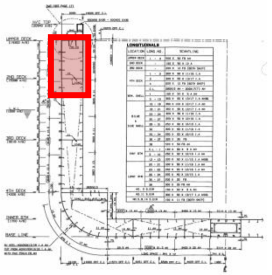
④	$t_2 = \max(t_{2\_1}, t_{2\_2}) \ [mm]$
Unit strip 1	12.883
Unit strip 2	12.883

⑤ The thickest value between the thickness of unit strips shall be used for thickness of SP5.

$$t_2 = 12.883 \approx 13.0$$

Side Shell Plate (6)
(SP5 – Shear strake at strength deck)





●:Load point

✓ Side shell plate(SP5)

$$t = \frac{t_1 + t_2}{2} \quad (mm)$$

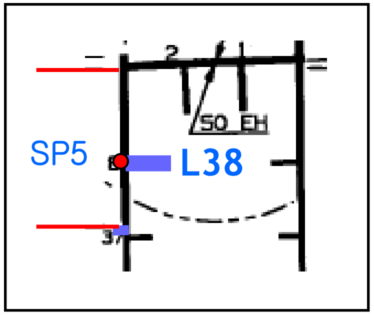
✓t1 : required side plating in mm $\quad t_1 = 14.5$

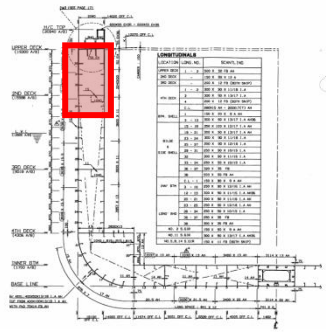
✓t2 : strength deck plating in mm $\quad t_2 = 13.0$

✓t2 shall not be taken less than t1. $\quad \therefore t_2 = 14.5$

$$\therefore t = \frac{t_1 + t_2}{2} = \frac{14.5 + 14.5}{2} = 14.5 \ (mm)$$

Longitudinals at Side Shell Plate (1) (L38 – Deck structure)





● :Load point

- ✓ Load point: Midpoint
- ✓ The material of L38 of basis ship(NV-32) is used for that of design ship. (f_1=1.28)
- ✓ L38 to be considered is the longitudinals located between the side structure and deck structure.

DnV Rules, Jan. 2004,Pt.3 Ch.1 Sec.7 Table B1

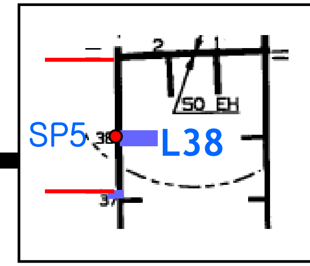
Structure	Load Type	$p\ (kN/m^2)$
External	Sea pressure above summer load waterline	$p_2 = p_{dp} - (4 + 0.2k_s)h_0$

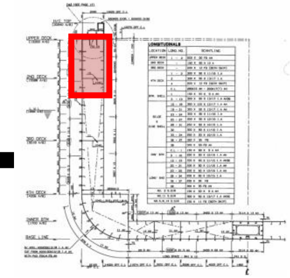
: Design load acting on the L_{38} is only the sea pressure.

① Design load acting on the L_{38} of the SP5 (P2)

p2	pdp	pl	ks		2	0.2L~0.7L from A.P. ks=2
			Cw		10.343	100 < L < 300, 10.75 - [(300-L)/100]^(3/2)
			kf	f	6.7	f= vertical distance from the waterline to the top of the ship's side at transverse section considered, maximum 0.8*Cw (m)
					6.7	
					28.33795639	$p_l = (k_s C_W + k_f)(0.8 + 0.15V/\sqrt{L})$
		y			16.1	horizontal distance in m from the ship's centre line to the load point, minimum B/4(m)=8.05
		z			12.6	vertical distance in m from the ship's baseline to the load point, maximum T(m))
					48.613	$p_{dp} = p_l + 135\dfrac{y}{B+75} - 1.2(T-z)\ (kN/m^2)$
	h0				5.598	vertical distance in m from the waterline considered to the load point
					23.982	$p_2 = p_{dp} - (4 + 0.2k_s)h_0$

Longitudinals at Side Shell Plate (2) (L38 – Deck structure)





●:Load point

②

✓ Required Section Modulus

$$Z = \frac{83 l^2 s p w_k}{\sigma} \quad (cm^3)$$

✓ Allowable stress

$$\sigma = 225 f_1 - 130 f_2 - \frac{z_n - z_a}{z_n}$$

Z_1	le		2.96	Distance between web frame (3.16m) - 0.2 m(braket)
	s		0.986	= (0.87+1.102)/2, stiffener spacing in m
	p		23.95194	Maximum Design Load
	wk	tkw	3	Corrosion addition
		tkf	3	Corrosion addition
			1.3	$1+0.05(t_{kw}+t_{kf})$ for flanged section
	σ	f1	1.28	Material factor = 1.28 for NV-32
		f2	1.19	It is obtained from the section modulus of the basis ship.
		zn	10.272	=19.3 - 9.028, vertical distance in m from the neutral axis to the deck
		za	1.102	vertical distance in m from the deck to the load point
			150.383	$\sigma = 225 f_1 - 130 f_2 - \frac{z_n - z_a}{z_n}$
			148.651	$Z = \frac{83 l^2 s p w_k}{\sigma} \quad (cm^3)$

③

✓ Minimum Thickness of Web and Flange

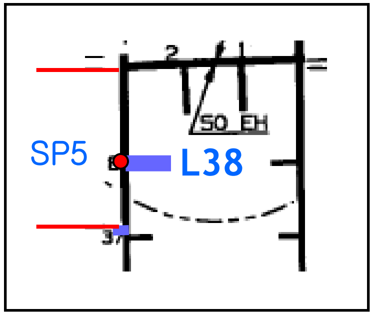
$$t_1 = 5.0 + \frac{k}{\sqrt{f_1}} + t_k \ (mm), \quad t_2 = \frac{h}{g} + t_k \ (mm)$$

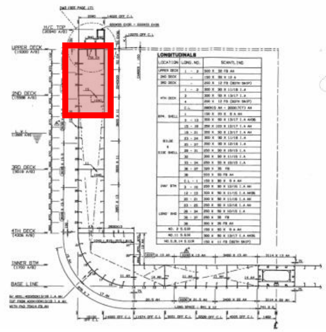
t1_1	k	4.9022	0.02 L_1
	f1	1.28	Material factor = 1.28 for NV-32
	tk	3	Corrosion addition
		12.33	$t_1 = 5.0 + \frac{k}{\sqrt{f_1}} + t_k \ (mm)$
t1_2	h	200	Profile height in m
	g	20	20 for plat bar profile
	tk	3	Corrosion addition
		13	$t_2 = \frac{h}{g} + t_k \ (mm)$

$$t = \max(t_1, t_2) = t_2$$

$$\therefore Z_1 = 148.651 \ cm^3, \quad t_1 = 13 \ mm$$

Longitudinals at Side Shell Plate (3) (L38 – Deck structure)





● :Load point

- ✓ Load point: Midpoint
- ✓ The material of L38 of basis ship(NV-32) is used for that of design ship. (f_1=1.28)
- ✓ L38 to be considered is the longitudinals located between the side structure and deck structure.

DnV Rules, Jan. 2004,Pt.3 Ch.1 Sec.7 Table B1

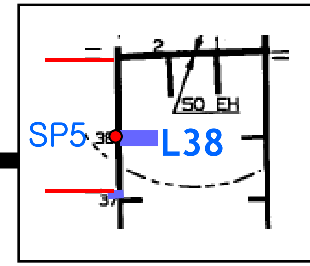
Structure	Load Type	$p\ (kN/m^2)$
Weather deck	Sea pressure	$p_1 = a(p_{dp} - (4+0.2k_s)h_0)$

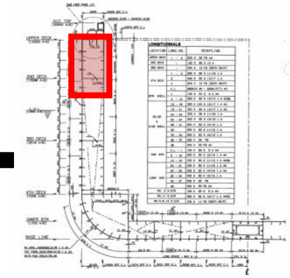
: Design load acting on the L_{38} is only the sea pressure.

① Design load acting on the L_{38} of the SP5 (P2)

p1	pdp	pl	ks		2	0.2L~0.7L from A.P. ks=2
			Cw		10.343	100 < L < 300, 10.75 - [(300-L)/100]^(3/2)
			kf	f	6.7	f= vertical distance from the waterline to the top of the ship's side at transverse section considered, maximum 0.8*Cw (m)
					6.7	
					28.33795639	$p_l = (k_s C_W + k_f)(0.8 + 0.15V/\sqrt{L})$
		y			16.1	horizontal distance in m from the ship's centre line to the load point, minimum B/4(m)=8.05
		z			12.6	vertical distance in m from the ship's baseline to the load point, maximum T(m))
					48.613	$p_{dp} = p_l + 135\frac{y}{B+75} - 1.2(T-z)\ (kN/m^2)$
	a				0.8	1.0 for weather decks forward of 0.15L from FP, or forward of deckhouse front, whichever is the foremost position or 0.8 for weather decks elsewhere
	h0				6.7	vertical distance in m from the waterline considered to the load point
					15.307	$p_1 = a(p_{dp} - (4+0.2k_s)h_0)$

Longitudinals at Side Shell Plate (4) (L38 – Deck structure)





●:Load point

②

✓ Required Section Modulus

$$Z = \frac{83 l^2 s p w_k}{\sigma} \quad (cm^3)$$

✓ Allowable stress

$$\sigma = 225 f_1 - 130 f_{2d} \frac{z_n - z_a}{z_n}$$

Z_2	le		2.96	Distance between web frame (3.16m) - 0.2 m(braket)
	s		0.986	= (0.87+1.102)/2, stiffener spacing in m
	p		15.307	Maximum Design Load
	wk	tkw	3	Corrosion addition
		tkf	3	Corrosion addition
			1.3	$1+0.05(t_{kw}+t_{kf})$ for flanged section
	σ	f1	1.28	Material factor = 1.28 for NV-32
		f2d	1.19	It is obtained from the section modulus of the basis ship.
		zn	10.272	=19.3 - 9.028, vertical distance in m from the neutral axis to the deck
		za	1.102	vertical distance in m from the deck to the load point
			150.383	$\sigma = 225 f_1 - 130 f_{2d} \frac{z_n - z_a}{z_n}$
	94.877			$Z = \frac{83 l^2 s p w_k}{\sigma} \ (cm^3)$

③

✓ Minimum Thickness of Web and Flange

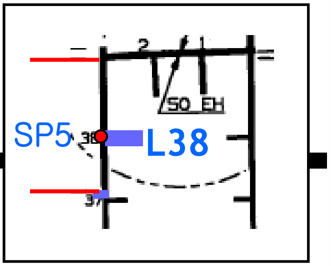
$$t_1 = 5.0 + \frac{k}{\sqrt{f_1}} + t_k \ (mm), \quad t_2 = \frac{h}{g} + t_k \ (mm)$$

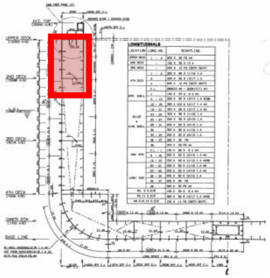
t2_1	k	4.9022	0.02 L_1
	f1	1.28	Material factor = 1.28 for NV-32
	tk	3	Corrosion addition
	12.33		$t_1 = 5.0 + \frac{k}{\sqrt{f_1}} + t_k \ (mm)$
t2_2	h	200	Profile height in m
	g	20	20 for plat bar profile
	tk	3	Corrosion addition
	13		$t_2 = \frac{h}{g} + t_k \ (mm)$

$$t = \max(t_1, t_2) = t_2$$

$$\therefore Z_2 = 94.877 \ cm^3, \quad t_2 = 13 \ mm$$

Longitudinals at Side Shell Plate (5)
(L38 – Side structure & deck structure)





●:Load point

- ✓ Side structure : $Z_1 = 148.651\ cm^3,\ t_1 = 13\ mm$
- ✓ Deck structure : $Z_2 = 94.877\ cm^3,\ t_2 = 13\ mm$
- ✓ Side structure & Deck structure

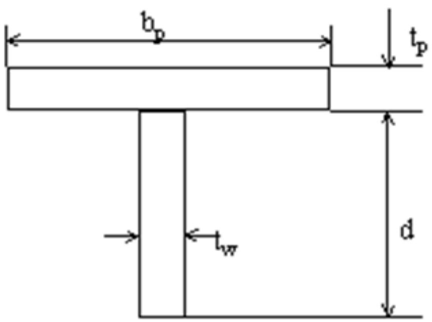
$$Z = \max(Z_1, Z_2) = Z_1 = 148.651\ cm^3$$
$$t = \max(t_1, t_2) = 13\ mm$$

"조선설계편람", 제 4판 (일본어), 일본관서조선협회, 1996

④ Select the longitudinal whose section modulus is larger than the required section modulus from the table.

Section modulus of flange whose longitudinal involves the plate.[1)]

d \ tw		6	9	11	12.7	14
150	A	9	13.5	16.5	19.1	21
	Z	44.7	65.2	78.3	89.1	97.2
	I	614	856	1000	1120	1200

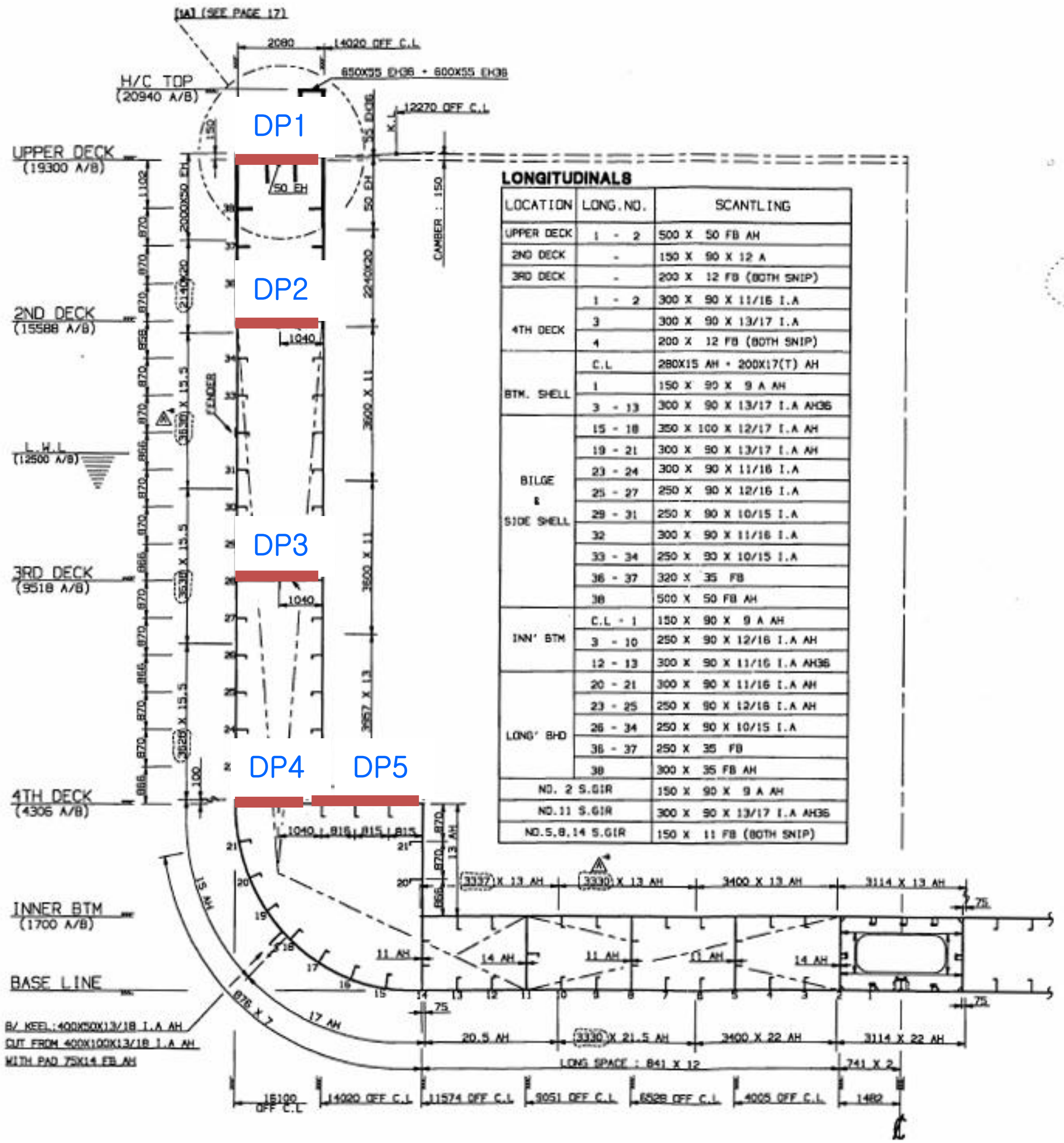


Section of web plate modulus whose longitudinal involves the plate.

d \ tw		16	19	22	25.4	28	32	35	38
200	A	32	38	44	50.8	56	64	70	76
	Z	215	259	305	359	401	469	521	576
	I	3900	4730	5600	6640	7460	8790	9830	10900

1) When the section modulus is calculated, standard breadth depending on a is used for effective breadth for simplicity. But effective breadth in accordance with rule should be used in actual calculation. ($b_p \times t_p$) => (a≤75 : 420×8, 75<a<150 : 610×10, 150≤a : 610×15)

Deck Plate



LOCATION	LONG. NO.	SCANTLING
UPPER DECK	1 - 2	500 X 50 FB AH
2ND DECK	-	150 X 90 X 12 A
3RD DECK	-	200 X 12 FB (BOTH SNIP)
4TH DECK	1 - 2	300 X 90 X 11/16 I.A
	3	300 X 90 X 13/17 I.A
	4	200 X 12 FB (BOTH SNIP)
BTM. SHELL	C.L	280X15 AH + 200X17(T) AH
	1	150 X 90 X 9 A AH
	3 - 13	300 X 90 X 13/17 I.A AH36
BILGE & SIDE SHELL	15 - 18	350 X 100 X 12/17 I.A AH
	19 - 21	300 X 90 X 13/17 I.A AH
	23 - 24	300 X 90 X 11/16 I.A
	25 - 27	250 X 90 X 12/16 I.A
	29 - 31	250 X 90 X 10/15 I.A
	32	300 X 90 X 11/16 I.A
	33 - 34	250 X 90 X 10/15 I.A
	36 - 37	320 X 35 FB
	38	500 X 50 FB AH
INN' BTM	C.L - 1	150 X 90 X 9 A AH
	3 - 10	250 X 90 X 12/16 I.A AH
	12 - 13	300 X 90 X 11/16 I.A AH36
LONG' BHD	20 - 21	300 X 90 X 11/16 I.A AH
	23 - 25	250 X 90 X 12/16 I.A AH
	26 - 34	250 X 90 X 10/15 I.A
	36 - 37	250 X 35 FB
	38	300 X 35 FB AH
NO. 2 S.GIR		150 X 90 X 9 A AH
NO.11 S.GIR		300 X 90 X 13/17 I.A AH36
NO.5,8,14 S.GIR		150 X 11 FB (BOTH SNIP)

Main particulars of design ship	
LOA(m)	259.64
LBP(m)	247.64
L_scant(m)	245.11318
B(m)	32.2
D(m)	19.3
Td(m)	11
Ts(m)	**12.6**
Vs(knt)	24.5
C_b	0.6563

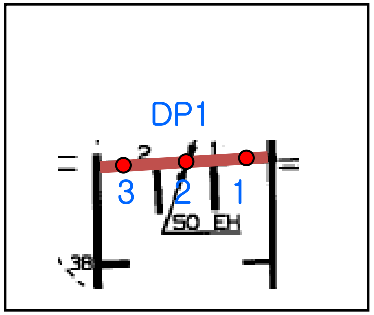
- ✓ M_S : Largest SWBM among all loading conditions and class rule
- M_W : calculated by class rule or direct calculation

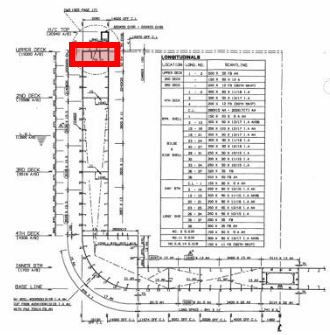
✓ **It is assume that the initial stress factor is equal to the stress factor of basis ship.**

$$Z_B = 2.595e^{07}\ cm^3 \qquad f_{2b} = 1.030$$

$$Z_D = 2.345e^{07}\ cm^3 \qquad f_{2d} = 1.140$$

Deck Plate (1)





● :Load point

✓ Deck plate(DP1) is composed of the three unit strips.

✓ Load point of the unit strip :
1, 2, 3: Midpoint

✓ Calculate the required thickness of each unit strip. And thickest value shall be used for thickness of the plate(DP1).

✓ The material of DP1 of basis ship(NV-32) is used for that of design ship. (f_1=1.28)

DnV Rules, Jan. 2004,Pt.3 Ch.1 Sec.7 Table B1

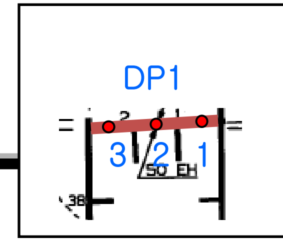
Structure	Load Type	$p\ (kN/m^2)$
Weather deck	Sea pressure	$p_1 = a(p_{dp} - (4+0.2k_s)h_0)$

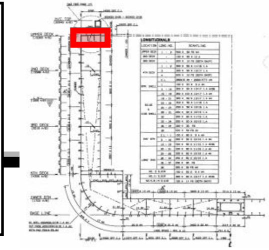
: Design load acting on the DP1 is only the sea pressure.

① Design load acting on the unit strip 3 of DP1, P1

p1	pdp	pl	ks	2	0.2L~0.7L from A.P. ks=2
			Cw	10.343	100 < L < 300, 10.75 - [(300-L)/100]^(3/2)
			kf	f 6.7	f= vertical distance from the waterline to the top of the ship's side at transverse section considered, maximum 0.8*Cw (m)
				6.7	
			28.33795639		$p_l = (k_s C_W + k_f)(0.8 + 0.15V/\sqrt{L})$
		y		15.825	horizontal distance in m from the ship's centre line to the load point, minimum B/4(m)=8.05
		z		12.6	vertical distance in m from the ship's baseline to the load point, maximum T(m))
		48.267			$p_{dp} = p_l + 135\frac{y}{B+75} - 1.2(T-z)\ (kN/m^2)$
	a	0.8			1.0 for weather decks forward of 0.15L from FP, or forward of deckhouse front, whichever is the foremost position or 0.8 for weather decks elsewhere
	h0	6.7			vertical distance in m from the waterline considered to the load point
	16.853				$p_1 = a(p_{dp} - (4+0.2k_s)h_0)$

Deck Plate (2)





● :Load point

②

✓ Required Thickness	✓ Allowable stress for Side shell Plate
$t = \dfrac{15.8 k_a s \sqrt{p}}{\sqrt{\sigma}} + t_k \ (mm)$	$\sigma = 140 f_1$ at N.A. σshall be reduced linearly.

Required thickness of the unit strip 3 of the DP1

t_1	p	16.853	Maximum Design Load
	ka	1.0	$k_a = (1.1 - 0.25 s/l)^2$, maximum 1.0 for s/l = 0.4 minimum 0.72 for s/l = 1-0
	s	0.765	= (0.69 + 0.84)/2, stiffener spacing in m
	f1	1.28	Material factor = 1.28 for NV-32
	sigma	153.6	120f1
	tk	3	Corrosion addition
		6.611	$t_1 = \dfrac{15.8 k_a s \sqrt{p}}{\sqrt{\sigma}} + t_k \ (mm)$

The required thicknesses of the unit strip 1 and 2 are calculated in the same way.

Unit strip1 : t_1 = 6.45 (mm)

Unit strip2 : t_1 = 6.535 (mm)

③ ✓ Minimum Thickness

$$t = t_0 + \frac{kL_1}{\sqrt{f_1}} + t_k \ (mm)$$

t_2	t0	5.5	5.5 for unsheathed weather and cargo deck
	k	0.02	0.02 in vessels with single continuous deck
	L1	245.11	Min (L, 300) (m)
	f1	1.28	Material factor = 1.28 for NV-32
	tk	3	Corrosion addition
		12.883	$t = t_0 + \dfrac{kL_1}{\sqrt{f_1}} + t_k \ (mm)$

cf) Minimum Breadth

$$b = 800 + 5L (mm)$$

b	Rule	2025.566	
	Arr.	3154	Breadth of side deck plate

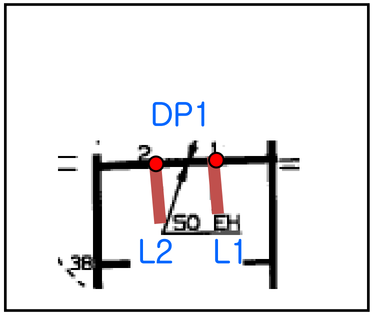
→ Rule is satisfied.

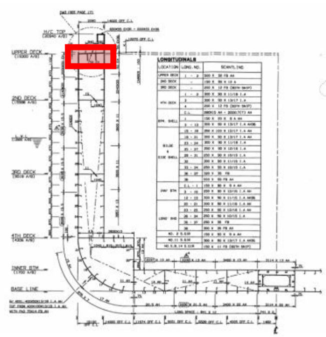
④	$t_2 = \max(t_{2\_1}, t_{2\_2}) \ [mm]$
Unit strip 1	12.883
Unit strip 2	12.883

⑤ The thickest value between the thickness of unit strips shall be used for thickness of DP1.

$$t_2 = 12.883 \approx 13.0$$

Longitudinals at Deck Plate (1)





● :Load point

✓Load point: Midpoint

✓ The materials of L_1, L_2 of basis ship(NV-32) are used for that of design ship. (f_1=1.28)

DnV Rules, Jan. 2004,Pt.3 Ch.1 Sec.7 Table B1

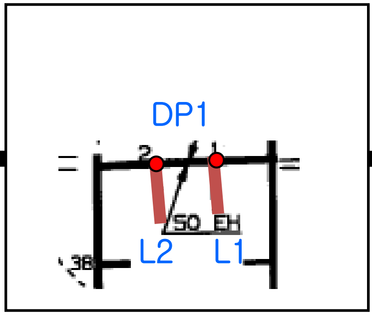
Structure	Load Type	$p\ (kN/m^2)$
Weather deck	Sea pressure	$p_1 = a(p_{dp} - (4+0.2k_s)h_0)$

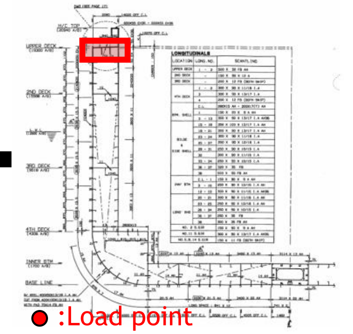
: Design load acting on the L1, L2 is only the sea pressure.

① Design load acting on the L1 and L2, P1

p1	pdp	pl	ks		2	0.2L~0.7L from A.P. ks=2
			Cw		10.343	100 < L < 300, 10.75 - [(300-L)/100]^(3/2)
			kf	f	6.7	f= vertical distance from the waterline to the top of the ship's side at transverse section considered, maximum 0.8*Cw (m)
					6.7	
					28.33795639	$p_l = (k_s C_W + k_f)(0.8 + 0.15V/\sqrt{L})$
		y			15.55	horizontal distance in m from the ship's centre line to the load point, minimum B/4(m)=8.05
		z			12.6	vertical distance in m from the ship's baseline to the load point, maximum T(m)
					47.921	$p_{dp} = p_l + 135\frac{y}{B+75} - 1.2(T-z)\ (kN/m^2)$
	a				0.8	1.0 for weather decks forward of 0.15L from FP, or forward of deckhouse front, whichever is the foremost position or 0.8 for weather decks elsewhere
	h0				6.7	vertical distance in m from the waterline considered to the load point
					16.576	$p_1 = a(p_{dp} - (4+0.2k_s)h_0)$

Longitudinals at Deck Plate (2)





② ✓ Required Section Modulus

$$Z = \frac{83 l^2 s p w_k}{\sigma} \quad (cm^3)$$

✓ Allowable stress

$$\sigma = 225 f_1 - 130 f_{2d} \frac{z_n - z_a}{z_n}$$

Z	le		2.96	Distance between web frame (3.16m) - 0.2 m(braket)
	s		0.695	= (0.550 + 0.840)/2, stiffener spacing in m
	p		16.576	Maximum Design Load
	wk	tkw	3	Corrosion addition
		tkf	3	Corrosion addition
			1.3	
	σ	f1	1.28	Material factor = 1.28 for NV-32
		f2d	1.19	It is obtained from the section modulus of the basis ship.
		zn	10.272	=19.3 - 9.028, vertical distance in m from the neutral axis to the deck
		za	0	vertical distance in m from the deck to the load point
			150.383	
	72.422			$1 + 0.05(t_{kw} + t_{kf})$ for flanged section

③ ✓ Minimum Thickness of Web and Flange

$$t_1 = 5.0 + \frac{k}{\sqrt{f_1}} + t_k \ (mm), \quad t_2 = \frac{h}{g} + t_k \ (mm)$$

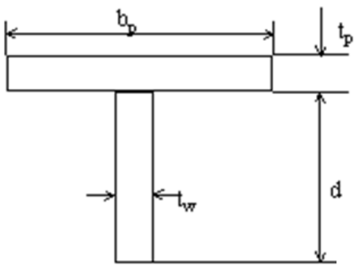
t1	k	4.9022	$0.02\, L_1$
	f1	1.28	Material factor = 1.28 for NV-32
	tk	3	Corrosion addition
	12.33		$t_1 = 5.0 + \frac{k}{\sqrt{f_1}} + t_k \ (mm)$
t2_2	h	150	Profile height in m
	g	20	20 for plat bar profile
	tk	3	Corrosion addition
	10.5		$t_2 = \frac{h}{g} + t_k \ (mm)$

$$t = \max(t_1, t_2) = t_1$$

④ Select the longitudinal whose section modulus is larger than the required section modulus from the table.

"조선설계편람", 제 4판 (일본어), 일본관서조선협회, 1996

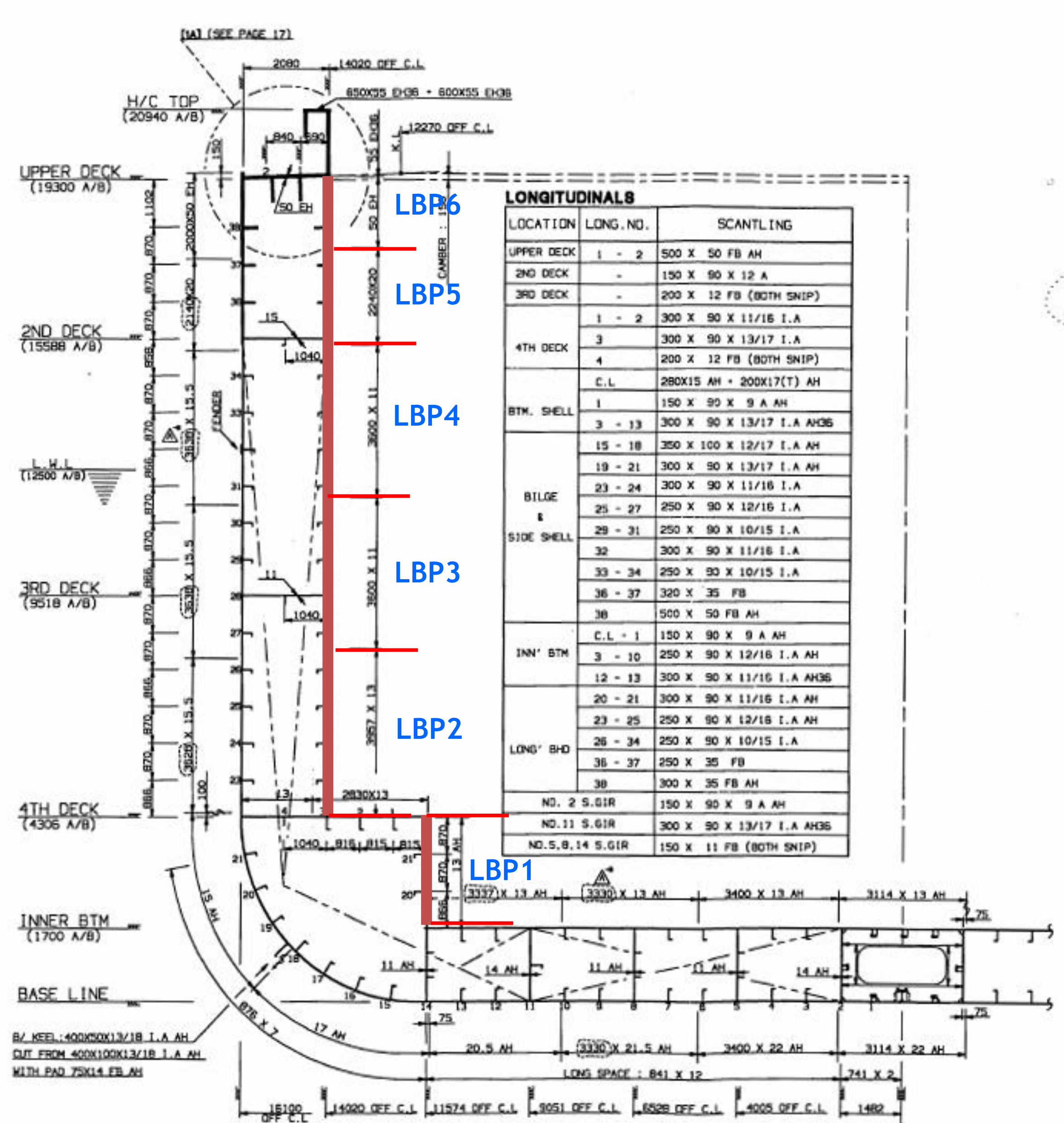
Section modulus of flange whose longitudinal involves the plate.[1)]



d \ tw		6	9	11	12.7	14
150	A	9	13.5	16.5	19.1	21
	Z	44.7	65.2	78.3	89.1	97.2
	I	614	856	1000	1120	1200

1) When the section modulus is calculated, standard breadth depending on a is used for effective breadth for simplicity. But effective breadth in accordance with rule should be used in actual calculation. ($b_p \times t_p$) => (a≤75 : 420×8, 75<a<150 : 610×10, 150≤a : 610×15)

Longitudinal Bulkhead Plate



LONGITUDINALS

LOCATION	LONG. NO.	SCANTLING
UPPER DECK	1 - 2	500 X 50 FB AH
2ND DECK	-	150 X 90 X 12 A
3RD DECK	-	200 X 12 FB (BOTH SNIP)
4TH DECK	1 - 2	300 X 90 X 11/16 I.A
	3	300 X 90 X 13/17 I.A
	4	200 X 12 FB (BOTH SNIP)
BTM. SHELL	C.L	280X15 AH + 200X17(T) AH
	1	150 X 90 X 9 A AH
	3 - 13	300 X 90 X 13/17 I.A AH36
BILGE & SIDE SHELL	15 - 18	350 X 100 X 12/17 I.A AH
	19 - 21	300 X 90 X 13/17 I.A AH
	23 - 24	300 X 90 X 11/16 I.A
	25 - 27	250 X 90 X 12/16 I.A
	29 - 31	250 X 90 X 10/15 I.A
	32	300 X 90 X 11/16 I.A
	33 - 34	250 X 90 X 10/15 I.A
	36 - 37	320 X 35 FB
	38	500 X 50 FB AH
INN' BTM	C.L - 1	150 X 90 X 9 A AH
	3 - 10	250 X 90 X 12/16 I.A AH
	12 - 13	300 X 90 X 11/16 I.A AH36
LONG' BHD	20 - 21	300 X 90 X 11/16 I.A AH
	23 - 25	250 X 90 X 12/16 I.A AH
	26 - 34	250 X 90 X 10/15 I.A
	36 - 37	250 X 35 FB
	38	300 X 35 FB AH
NO. 2 S.GIR		150 X 90 X 9 A AH
NO.11 S.GIR		300 X 90 X 13/17 I.A AH36
NO.5,8,14 S.GIR		150 X 11 FB (BOTH SNIP)

LBP : Longitudinal Bulkhead Plate

Main particulars of design ship	
LOA(m)	259.64
LBP(m)	247.64
L_scant(m)	245.11318
B(m)	32.2
D(m)	19.3
Td(m)	11
Ts(m)	**12.6**
Vs(knt)	24.5
C_b	0.6563

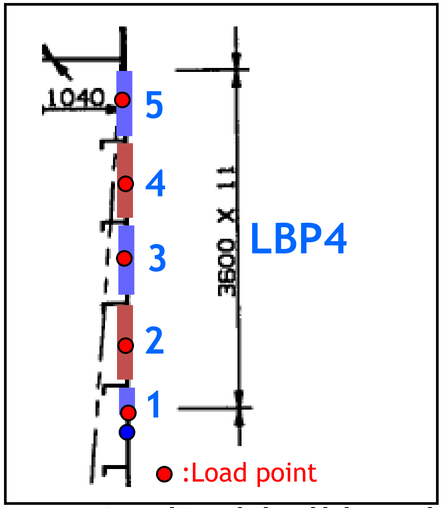
- ✓ M_S : Largest SWBM among all loading conditions and class rule
- M_W : calculated by class rule or direct calculation

✓ **It is assume that the initial stress factor is equal to the stress factor of basis ship.**

$$Z_B = 2.595e^{07}\ cm^3 \qquad f_{2b} = 1.030$$

$$Z_D = 2.345e^{07}\ cm^3 \qquad f_{2d} = 1.140$$

Longitudinal Bulkhead Plate (LBP4) (1)



✓ Longitudinal bulkhead plate(LBP4) is composed of the five unit strips.

✓ Load point of the unit strip :

1: Point nearest the midpoint

2, 3, 4, 5 : Midpoint

✓ Calculate the required thickness of each unit strip. And thickest value shall be used for thickness of the plate(LBP4).

✓ The material of LBP4 of basis ship(NV-NS) is used for that of design ship. (f_1=1.00)

✓ Design Load

DnV Rules, Jan. 2004,Pt.3 Ch.1 Sec.6 Table B1

Structure	Load Type	$p\ (kN/m^2)$
Watertight bulkheads	Sea pressure when flooded or general dry cargo minimum	$p_1 = 10h_b$
Tank bulkheads in general		$P_3 = \rho(g_0 + 0.5a_V)\cdot h_s$ $P_4 = 0.67(\rho g_0 h_p + \Delta P_{dyn})$ $P_5 = \rho g_0 h_s + p_0$

①Design load acting on the unit strip 1 of LBP4, P1

Watertight decks submerged in damaged condition

p_1	h_b	3.725	vertical distance in metres from the load point to the deepest equilibrium waterline in damaged condition obtained from applicable damage stability calculations. The deepest equilibrium waterline in damaged condition should be indicated on the drawing of the deck in question.
	37.25		$p_1 = 10h_b$

The design loads of the unit strip2, 3, 4, and 5 are calculated in the same way.

Unit strip2 : p1 = 30.31(kN/m^{2})

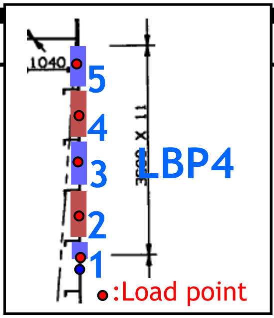
Unit strip3 : p1 = 21.63(kN/m^{2})

Unit strip4 : p1 = 12.93(kN/m^{2})

Unit strip5 : p1 = 4.29(kN/m^{2})

Longitudinal Bulkhead Plate (LBP4) (2)

① Design load acting on the unit strip 1 of LBP4, P1



Considering vertical acceleration (P_3)

p_3	av	a0	Cw	10.34	100 < L < 300, 10.75 - [(300-L)/100]^(3/2)
			Cv	0.2	$C_V = \sqrt{L}/50$, max 0.2
			Cv1	1.56	$C_{V1} = V/\sqrt{L}$, max 0.8
				0.4396	$a_0 = 3C_W \cdot / L + C_V C_{V1}$
		kv		0.7	0.7 between 0.3L and 0.6L from A.P.
				4.599	$a_v = k_v g_0 a_0 / C_B$
	hb			3.725	vertical distance in metres from the load point to the deepest equilibrium waterline in damaged condition obtained from applicable damage stability calculations. The deepest equilibrium waterline in damaged condition should be indicated on the drawing of the bulkhead in question.
				46.24	$p_3 = \rho(g_0 + 0.5a_v)h_s - 10h_b$

The design loads of the unit strip2, 3, 4, and 5 are calculated in the same way.

Unit strip2 : p2 = 30.31(kN/m^{2})

Unit strip3 : p2 = 21.63(kN/m^{2})

Unit strip4 : p2 = 12.93(kN/m^{2})

Unit strip5 : p2 = 4.29(kN/m^{2})

Considering the tank overflow(P_4)

p_4	Δpdyn	25	25 in general
	hp	4.485	vertical distance in m from the load point to the top of air pipe
		46.97	$P_4 = 0.67(\rho g_0 h_p + \Delta P_{dyn})$

The design loads of the unit strip2, 3, 4, and 5 are calculated in the same way.

Unit strip2 : p4 = 42.29(kN/m^{2})

Unit strip3 : p4 = 36.44(kN/m^{2})

Unit strip4 : p4 = 30.58(kN/m^{2})

Unit strip5 : p4 = 24.76(kN/m^{2})

Considering tank test pressure(P_5)

p_5	P_0	25	25 in general
	Hs	3.725	vertical distance in m from the load point to the top of tank or hatchway excluding smaller hatchways
		52.46	$P_5 = \rho g_0 h_s + p_0$

The design loads of the unit strip2, 3, 4, and 5 are calculated in the same way.

Unit strip2 : p5 = 45.48(kN/m^{2})

Unit strip3 : p5 = 36.75(kN/m^{2})

Unit strip4 : p5 = 28.00(kN/m^{2})

Unit strip5 : p5 = 19.31(kN/m^{2})

Largest value between p_1, p_3, p_4 and p_5 shall be used for pressure acting the unit strip.

$$p = \max(p_1, p_3, p_4, p_5)$$

$$[kN/m^2]$$

Unit strip1 : p = p_5 = 52.46

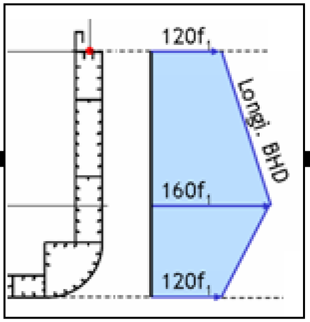
Unit strip2 : p = p_5 =45.48

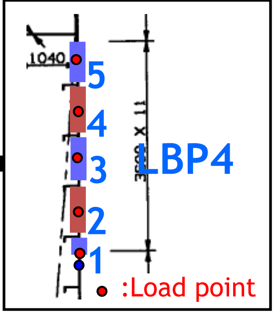
Unit strip3 : p = p_5 =36.74

Unit strip4 : p = p_4 = 30.58

Unit strip5 : p = p_4 24.76

Longitudinal Bulkhead Plate (LBP4) (2)





②

✓ Required Thickness	✓ Allowable stress for Bottom Plate
$t_1 = \dfrac{15.8 k_a s \sqrt{p}}{\sqrt{\sigma}} + t_k \ (mm)$	

Required thickness of the unit strip 1 of the LBP4

t_1	p	52.46	Maximum Design Load
	ka	1.0	$k_a = (1.1 - 0.25 s/l)^2$, maximum 1.0 for s/l = 0.4 minimum 0.72 for s/l = 1-0
	s	0.87	stiffener spacing in m
	f1	1	Material factor = 1.00 for NV-NS
	σ	134.48	
	tk	3	Corrosion addition
		11.59	$t_1 = \dfrac{15.8 k_a s \sqrt{p}}{\sqrt{\sigma}} + t_k \ (mm)$

The required thickness of the unit strip2, 3, 4 and 5 are calculated in the same way.

Unit strip2 : t_1 = 10.99 (mm)

Unit strip3 : t_1 = 10.26 (mm)

Unit strip4 : t_1 = 9.67 (mm)

Unit strip5 : t_1 = 8.96 (mm)

③

✓ Minimum Thickness

$$t_2 = 5.0 + \frac{k \cdot L_1}{\sqrt{f_1}} + t_k \ (mm)$$

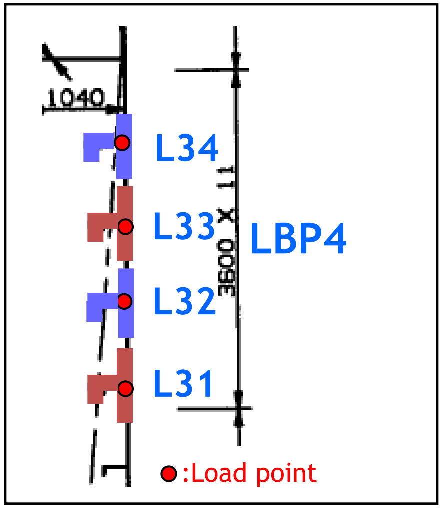
t_2	L1	245.11	Min (L, 300) (m)
	f1	1.00	Material factor = 1.00 for NV-NS
	Tk	3	Corrosion addition
	k	0.01	0.01 for other bulkheads
		8	$t_2 = 5.0 + \dfrac{k \cdot L_1}{\sqrt{f_1}} + t_k \ (mm)$

④	$t = \max(t_1, t_2)$ [mm]
Unit strip 1	11.59
Unit strip 2	10.99
Unit Strip 3	10.26
Unit Strip 4	9.67
Unit Strip 5	8.96

⑤ The thickest value between the thickness of unit strips shall be used for thickness of LBP4.

$$t = 11.59 \approx 11.5 \ [mm]$$

Longitudinals at Longitudinal Bulkhead Plate (LBP4) (1)



✓Load point: Midpoint

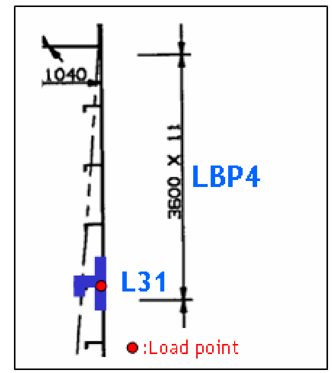
✓ The material of LBP4 of basis ship(NV-NS) is used for that of design ship. (f_1=1.00)

✓ Design Load

DnV Rules, Jan. 2004,Pt.3 Ch.1 Sec.6 Table B1

Structure	Load Type	$p\ (kN/m^2)$
Watertight bulkheads	Sea pressure when flooded or general dry cargo minimum	$p_1 = 10h_b$
Tank bulkheads in general		$P_3 = \rho(g_0 + 0.5a_V)\cdot h_s$ $P_4 = 0.67(\rho g_0 h_p + \Delta P_{dyn})$ $P_5 = \rho g_0 h_s + p_0$

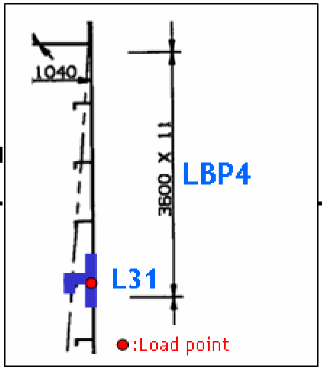
① Design load acting on the L31(P1)



Considering the sea pressure at damaged condition (P1)

p_1	h_b	3.464	vertical distance in metres from the load point to the deepest equilibrium waterline in damaged condition obtained from applicable damage stability calculations. The deepest equilibrium waterline in damaged condition should be indicated on the drawing of the deck in question.
	34.64		$p_1 = 10h_b$

Longitudinal Bulkhead Plate (LBP4) (2)



① Design load acting on the L31(P1)

Considering vertical acceleration (P_3)

p3	av	a0	Cw	10.34	100 < L < 300, 10.75 - [(300-L)/100]^(3/2)
			Cv	0.2	$C_V = \sqrt{L}/50$, max 0.2
			Cv1	1.56	$C_{V1} = V/\sqrt{L}$, max 0.8
				0.4396	$a_0 = 3C_W \cdot / L + C_V C_{V1}$
		kv		0.7	0.7 between 0.3L and 0.6L from A.P.
				4.599	$a_v = k_v g_0 a_0 / C_B$
	hb			3.464	vertical distance in metres from the load point to the deepest equilibrium waterline in damaged condition obtained from applicable damage stability calculations. The deepest equilibrium waterline in damaged condition should be indicated on the drawing of the bulkhead in question.
				43.00	$p_3 = \rho(g_0 + 0.5a_v)h_s - 10h_b$

Considering the tank overflow(P_4)

p4	Δpdyn	25	25 in general
	hp	4.224	vertical distance in m from the load point to the top of air pipe
		45.21	$P_4 = 0.67(\rho g_0 h_p + \Delta P_{dyn})$

Considering tank test pressure(P_5)

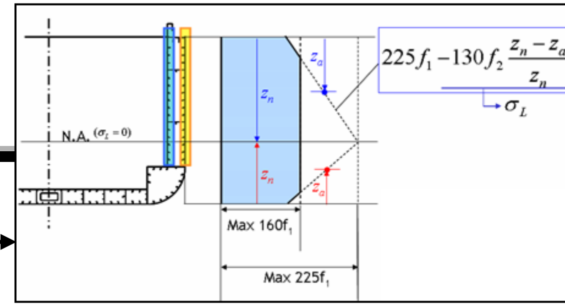
p5	P_0	25	25 in general
	Hs	3.464	vertical distance in m from the load point to the top of tank or hatchway excluding smaller hatchways
		49.83	$P_5 = \rho g_0 h_s + p_0$

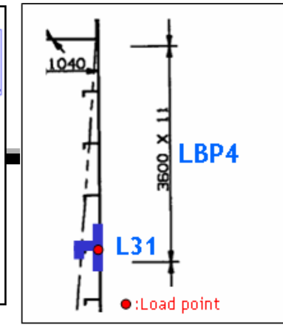
Largest value between p_1, p_3, p_4 and p_5 shall be used for pressure acting the unit strip.

$$p = \max(p_1, p_3, p_4, p_5)\ [kN/m^2]$$

$p = p_5 = 49.83$

Longitudinal Bulkhead Plate (LBP4) (3)





②

✓ Required Section Modulus

$$Z=\frac{83l^2spw_k}{\sigma}\ (cm^3)$$

✓ Allowable stress

$$\sigma=225f_1-130f_2\frac{z_n-z_a}{z_n}$$

Z	le		2.96	Distance between web frame (3.16m) - 0.2 m(braket)
	s		0.87	stiffener spacing in m
	p		49.83	Maximum Design Load
	wk	tkw	1.5	Corrosion addition
		tkf	1.5	Corrosion addition
			1.15	$1+0.05(t_{kw}+t_{kf})$ for flanged section
	σ		160	
	226.08			$Z=\dfrac{83l^2spw_k}{\sigma}\ (cm^3)$

③

✓ Minimum Thickness of Web and Flange

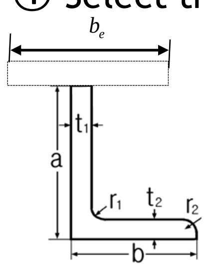
$$t_1=5.0+\frac{k}{\sqrt{f_1}}+t_k\ (mm),\quad t_2=\frac{h}{g}+t_k\ (mm)$$

t_1	k	4.9022	0.01L$_1$
	f1	1.00	Material factor = 1.00 for NV-NS
	tk	1.5	Corrosion addition
	8.95		$t_1=5.0+\dfrac{k}{\sqrt{f_1}}+t_k\ (mm)$
t_2	h	340	Profile height in m
	g	70	70 for flanged profile webs
	tk	1.5	Corrosion addition
	4.36		$t_2=\dfrac{h}{g}+t_k\ (mm)$

$$t=\max(t_1,t_2)=t_1$$

④ Select the longitudinal whose section modulus is larger than the required section modulus from the table.

“조선설계편람”, 제 4판 (일본어), 일본관서조선협회, 1996



a	b	t_1	t_2	r_1	r_2	A	I	Z
mm						cm^2	cm^4	cm^3
200	90	9	14	14	7	29.66	5,870	340

Section modulus whose longitudinal involves the plate.[1)]

1) When the section modulus is calculated, standard breadth depending on a is used for effective breadth for simplicity. But effective breadth in accordance with rule should be used in actual calculation. ($b_p\times t_p$) => (a≤75 : 420×8, 75<a<150 : 610×10, 150≤a : 610×15)

16-7. Buckling

1) Column Buckling
2) Buckling Strength of Stiffener
3) Buckling Strength of Plate
4) Buckling Strength by DNV Rule
5) Buckling Strength of Stiffener by DNV Rule
6) Buckling Strength of Plate by DNV Rule

Buckling

- **Definition: The phenomenon where lateral deflection may arise in the athwart direction* against the axial working load**

*선측(船側)에서 선측으로 선체를 가로지르는

- **This section covers buckling control for plate and longitudinal stiffener.**

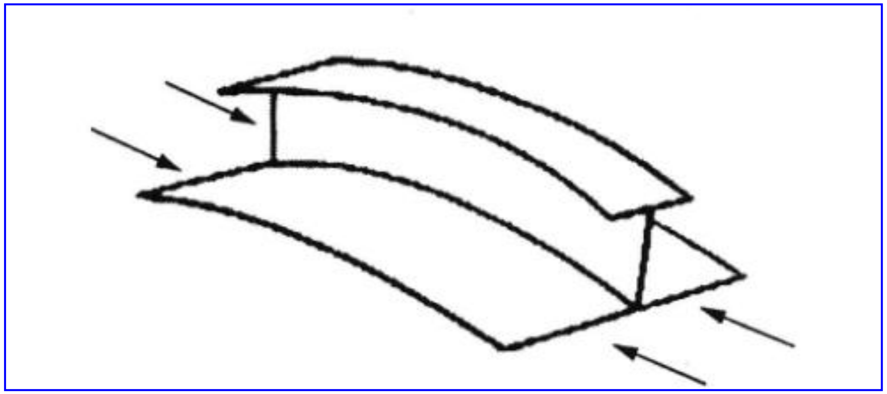


Figure 1. Flexural buckling of stiffeners plus plating

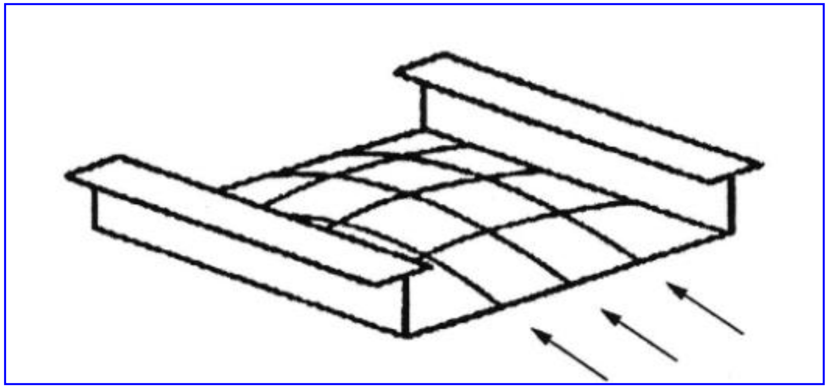
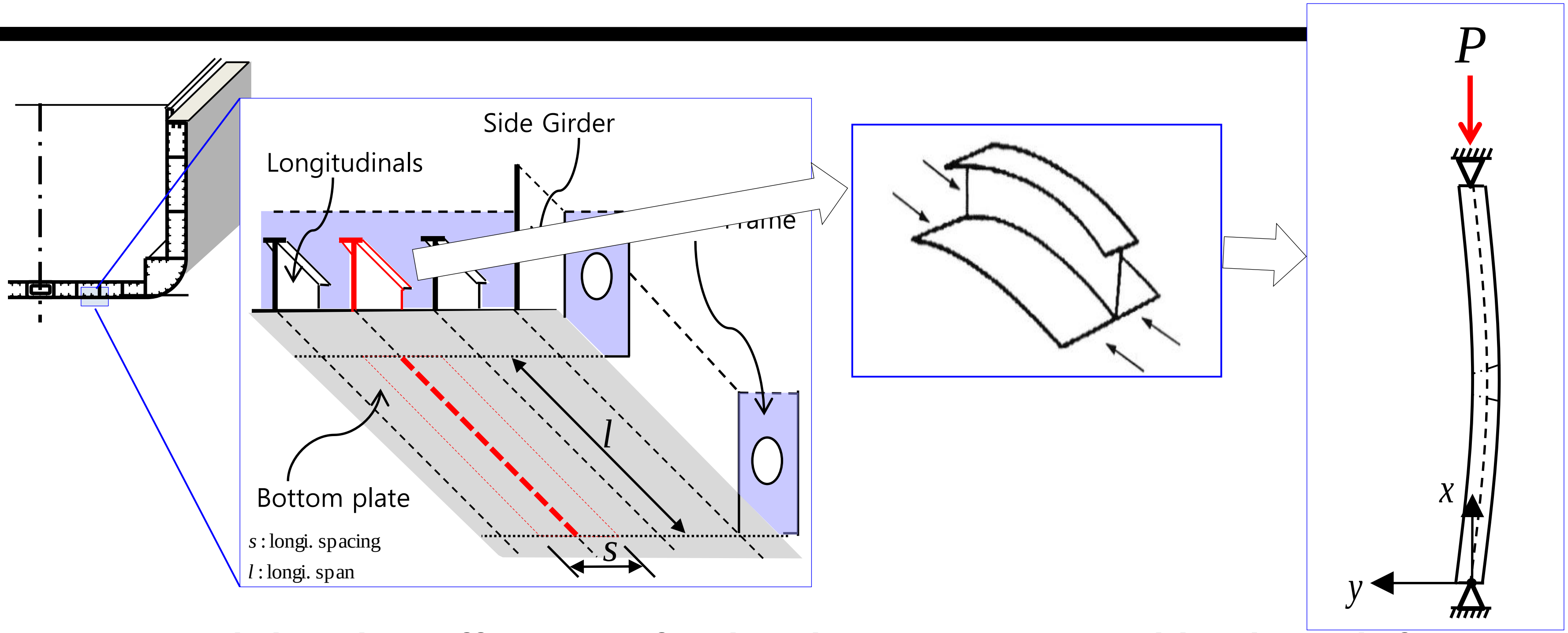


Figure 2. Plate alone buckles between stiffeners

Mansour, A., Liu, d., the principles of naval architecture series – strength of ships and ocean structures, the society of naval architects and marine engineers, 2008

2) Buckling Strength of Stiffener



It is assumed that the <u>stiffener is a fixed-end column supported by the web frames.</u>

Hull girder bending moment is acting on the cross section of the ship as moment from the point view of global deformation. And it is acting on the each stiffener as axial load from the point view of local deformation.

what is our interest?

- **Safety :**

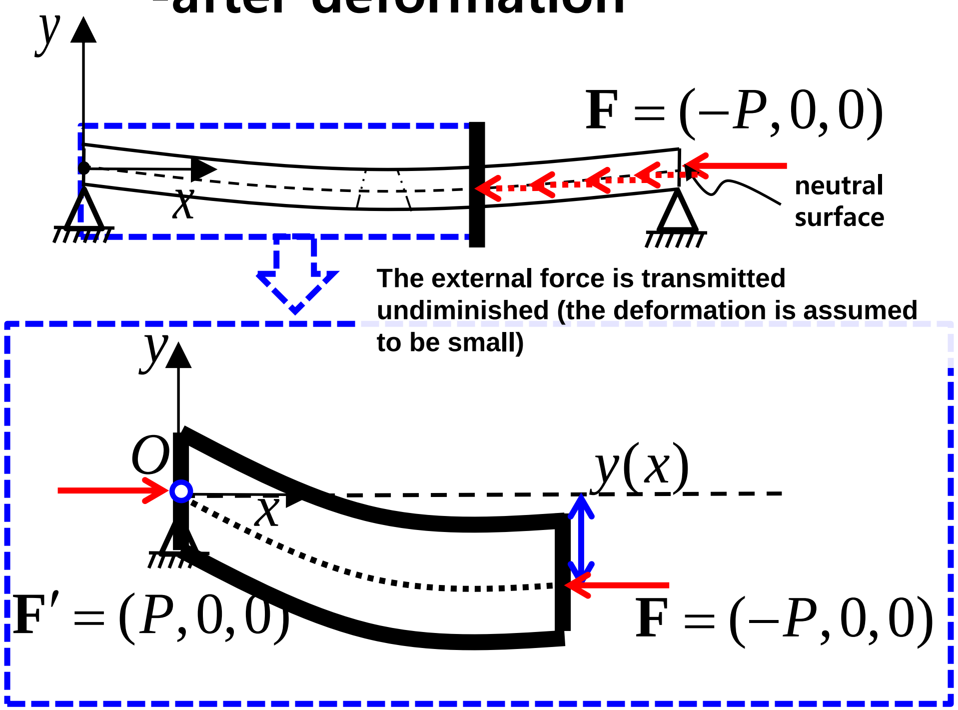
Won't it fail under the load?

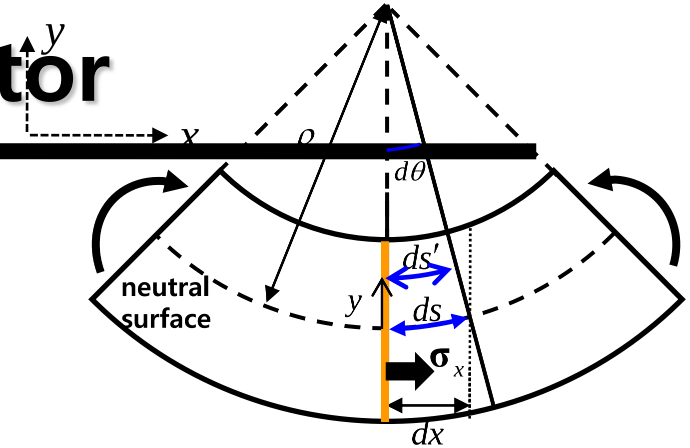
- The <u>actual compressive stress</u>(σ_a) shall not be greate than the <u>critical buckling stress</u>(σ_{cr})

$$\boxed{\sigma_a \le \sigma_{cr}} \quad , \text{ where } \sigma_a = \frac{M}{I_{N.A}/y} = \frac{M}{Z} \quad , Z = Z(y)$$

Deflection of Beam with Vector

-after deformation





- Differential Equation of the deflection curve of a beam: $M = EI\dfrac{d^2y}{dx^2}$

Equilibrium of forces:

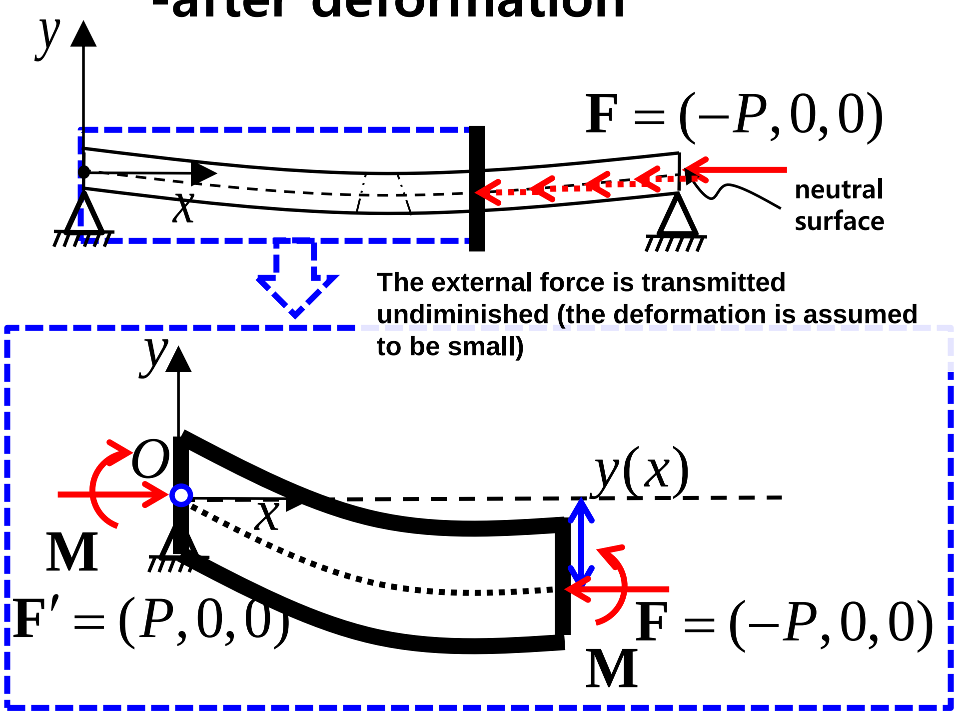
$$\mathbf{F}' + \mathbf{F} = \mathbf{0}$$

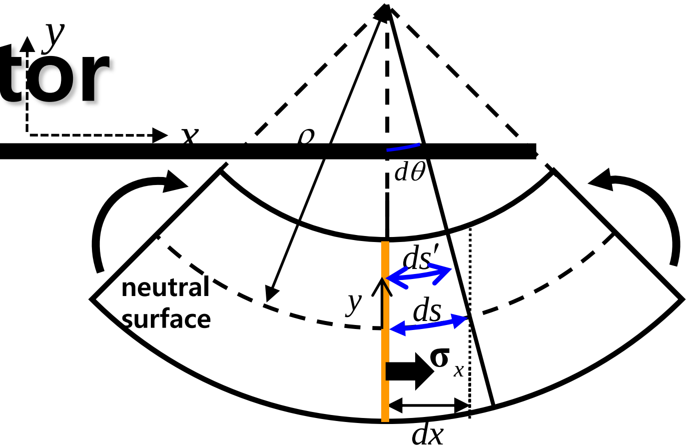
$$(\cancel{P}, 0, 0) + (-\cancel{P}, 0, 0) = \mathbf{0}$$

F′: reaction force

Deflection of Beam with Vector

-after deformation





- Differential Equation of the deflection curve of a beam: $M = EI\dfrac{d^2y}{dx^2}$

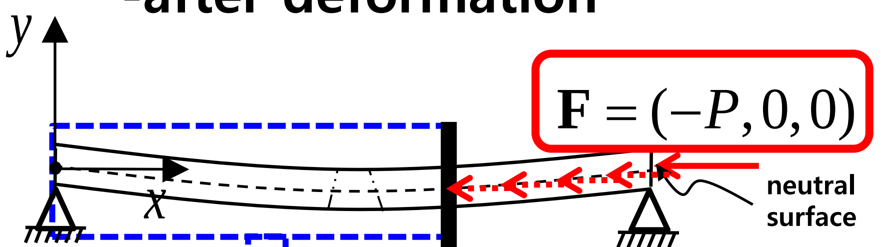
Equilibrium of forces: $\mathbf{F}'+\mathbf{F}=\mathbf{0}$

Equilibrium of moments about point O:

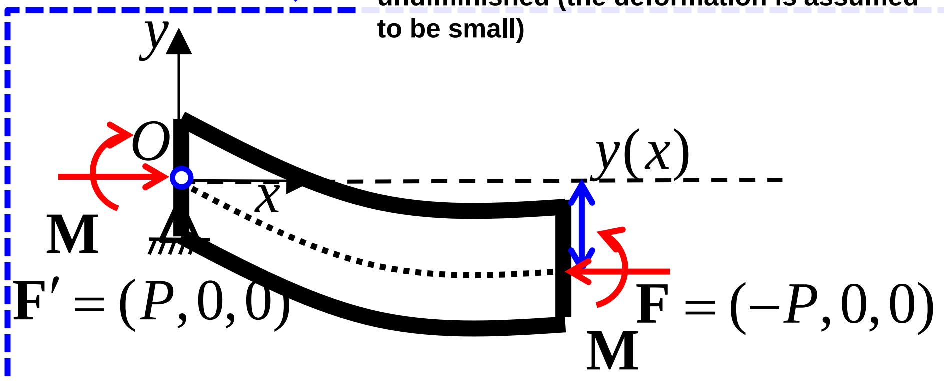
F′: reaction force

Deflection of Beam with Vector

-after deformation



The external force is transmitted undiminished (the deformation is assumed to be small)

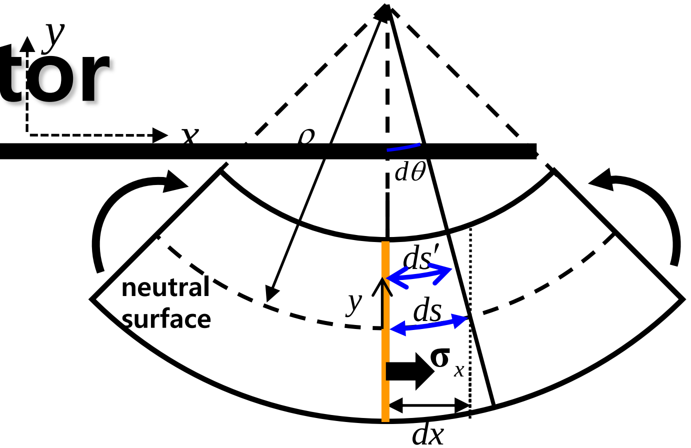


Equilibrium of forces: $\mathbf{F}'+\mathbf{F}=\mathbf{0}$

Equilibrium of moments about z-axis:

$$\mathbf{i}\cdot\cancel{M_x}\ +\mathbf{i}(y\cdot\cancel{F_z}-\cancel{z}\cdot\cancel{F_y})$$
$$+\mathbf{j}\cdot\cancel{M_y}\ +\mathbf{j}(-x\cdot\cancel{F_z}+\cancel{z}\cdot F_x)\ =0$$
$$+\mathbf{k}\cdot M_z\ +\mathbf{k}(x\cdot\cancel{F_y}-y\cdot F_x)$$

F′: reaction force

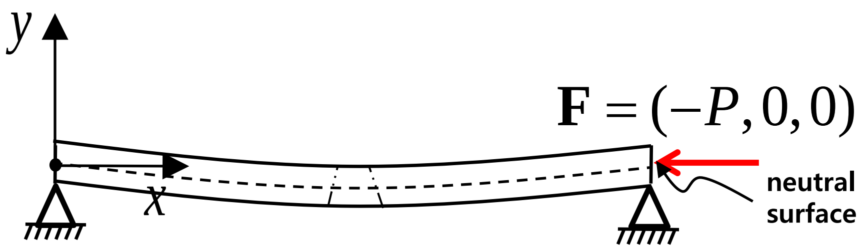


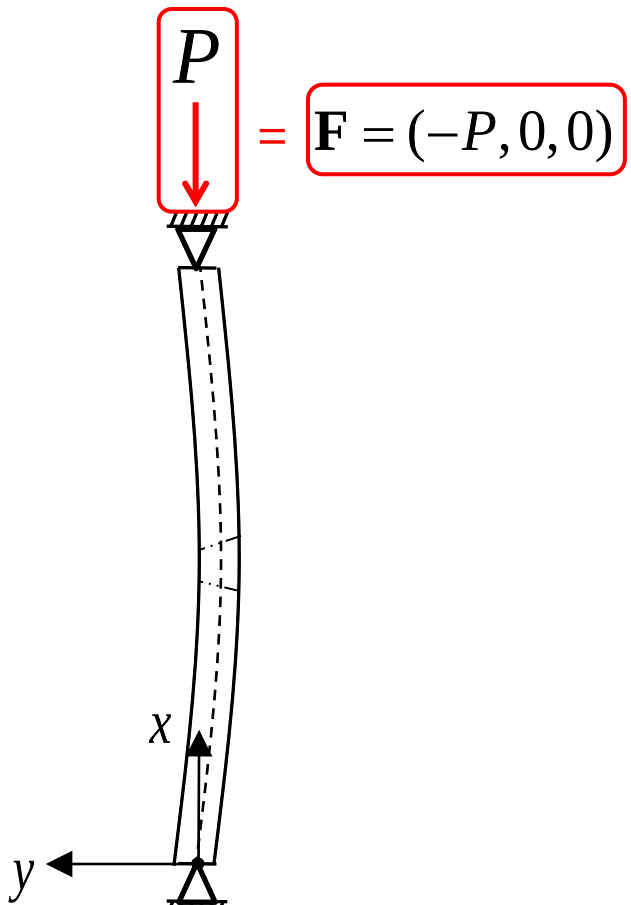
- Differential Equation of the deflection curve of a beam: $M=EI\dfrac{d^2y}{dx^2}$

$$\mathbf{k}\cdot M_z+\mathbf{k}\left(-y\cdot F_x\right)=0$$
$$\downarrow\ F_x=-P$$
$$\mathbf{k}\cdot M_z+\mathbf{k}\left(y\cdot P\right)=0$$
$$\downarrow\ \text{Drop the vector } \mathbf{k}$$
$$\boxed{M+y\cdot P=0}$$

$$\mathbf{r}\times\mathbf{F}=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\x&y&z\\F_x&F_y&F_z\end{vmatrix}\quad\begin{aligned}&\mathbf{i}(y\cdot F_z-z\cdot F_y)\\&=+\mathbf{j}(-x\cdot F_z+z\cdot F_x)\\&\quad+\mathbf{k}(x\cdot F_y-y\cdot F_x)\end{aligned}$$

Buckling of a Thin Vertical Column





- **Differential Equation of the deflection curve of a beam:**

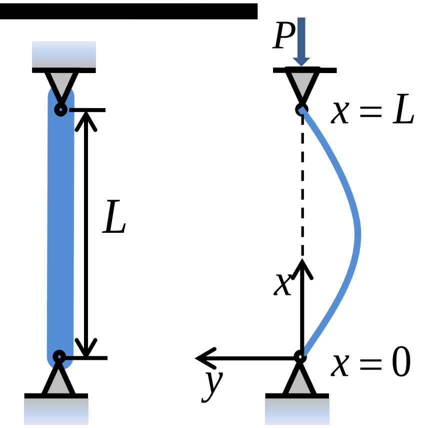
$$M = EI\frac{d^2y}{dx^2} \longrightarrow EI\frac{d^2y}{dx^2} - M = 0$$

- **Equilibrium of moments about z-axis:**

$$M + y \cdot P = 0 \longrightarrow -M = y \cdot P$$

$$\boxed{EI\frac{d^2y}{dx^2} + y \cdot P = 0}$$

Buckling of a Thin Vertical Column



Consider a long slender vertical column of uniform cross-section and length L.

$$EI\frac{d^2y}{dx^2}+Py=0$$

The boundary value problem to be solved is

$$EI\frac{d^2y}{dx^2}+Py=0 \qquad , y(0)=0 \quad , y(L)=0$$

For what values of P will the column bend?

→ In mathematical terms: For what values of P does the given boundary-value problem possess nontrivial solution?

$$y''+\lambda y=0 \quad , y(0)=0 \quad , y(L)=0 \qquad\qquad , where\ \lambda = P/EI$$

The deflection curves are $y_n(x)=c_2\sin(n\pi x/L)$ **, corresponding to the eigenvalues** $\lambda_n = P_n/EI = n^2\pi^2/L^2, n=1,2,3\ldots$

1) Column Buckling

James M. Gere, Mechanics of Materials 6th Edition, Thomson, pp. 748~762

- The equation of the deflection curve

$$\frac{P}{EI} = k^2, k = \frac{n\pi}{l}$$

- **Differential equation for column buckling:** $\boxed{EIy'' + Py = 0}$

Using the notation $k^2 = \dfrac{P}{EI}$, $\boxed{y'' + k^2 y = 0}$

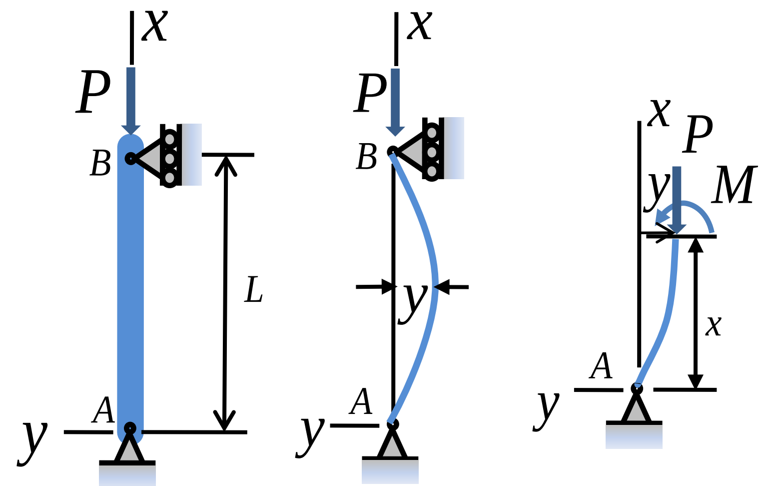
General solution of the equation: $\boxed{y = C_1 \sin kx + C_2 \cos kx}$

Boundary conditions:

$$y(0) = 0,\ y(l) = 0$$

$$y(0) = C_2 = 0$$

$$y(l) = \underline{C_1}\ \underline{\sin kL} = 0$$



1) If $C_1=0$, $y=0$ **(trivial solution).**

2) If $\sin kl=0$, **(**$\sin kl=0$**: buckling equation)**

① **If** $kl=0$, $y=0$ **(trivial solution).**

② **If** $kl=n\pi$ **(n=1,2,3) or** $P = \left(\dfrac{n\pi}{l}\right)^2 EI$, **it is nontrivial solution.**

$$\boxed{\therefore y = C_1 \sin kx = C_1 \sin \frac{n\pi x}{L}, n = 1, 2, 3\ldots}$$

E = modulus of elasticity
I = 2nd moment of the section area
EI = flexural rigidity
P = axial load
v = deflection of column
L = length of column

1) Column Buckling

- Critical stress

$$\frac{P}{EI} = k^2, k = \frac{n\pi}{l}$$

- **Differential equation for column buckling:** $EIy'' + Py = 0$

The equation of the deflection curve:

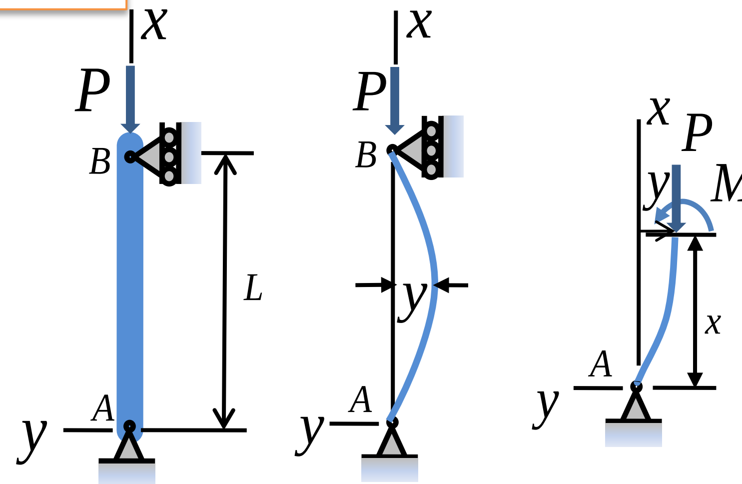
$$y = C_1 \sin\frac{n\pi x}{l}, n = 1, 2, 3\ldots$$

The critical loads :

$$P = k^2 EI = \left(\frac{n\pi}{l}\right)^2 EI$$

The lowest critical load(n=1) :

$$P_{cr} = \left(\frac{\pi}{l}\right)^2 EI = \frac{\pi^2 EI}{l^2}$$



The corresponding critical stress:

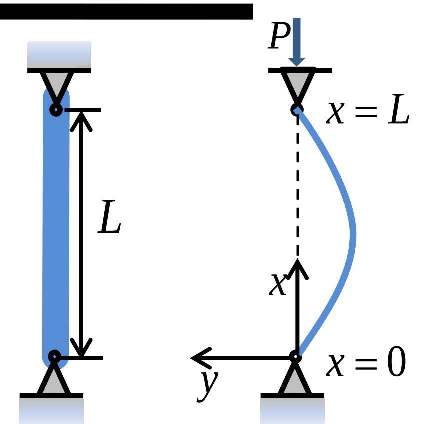
$$\boxed{\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 EI}{Al^2}}$$

Euler's formula

E = modulus of elasticity
I = 2nd moment of area
EI = flexural rigidity
P = axial load
v = deflection of column
A = area of column
L = length of column

Buckling of a Thin Vertical Column

$$y'' + \lambda y = 0 \quad , y(0) = 0 \quad , y(L) = 0 \quad , where\ \lambda = P / EI$$



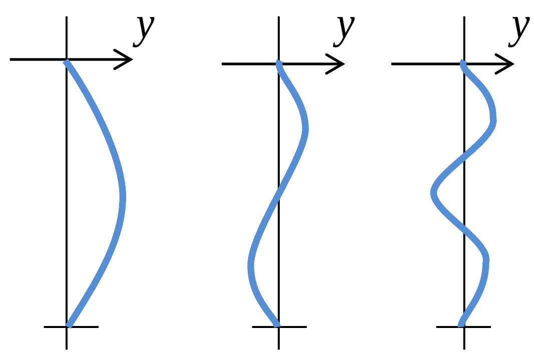
The deflection curves are $y_n(x) = c_2 \sin(n\pi x / L)$ **, corresponding to the eigenvalues** $\lambda_n = P_n / EI = n^2\pi^2 / L^2, n = 1, 2, 3...$

Physically this means that the column will buckle or deflect only when the compressive force is one of the values

$$P_n = n^2\pi^2 EI / L^2, n = 1, 2, 3... :$$ **Critical loads.**

The smallest critical load $P_1 = \pi^2 EI / L^2$ called **Euler load**

The deflection curves corresponding to n=1, n=2, and n=3 are shown in the right figures.



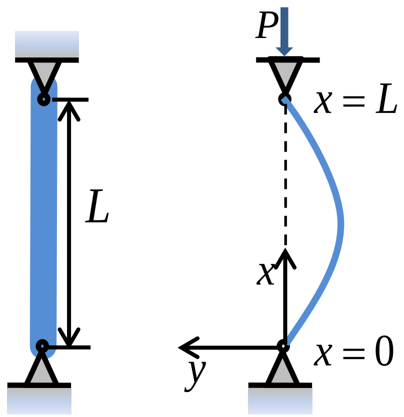
Note that if the original column has some sort of physical restraint put on it at x=L/2, then the smallest critical load will be $P_2 = 4\pi^2 EI / L^2$

1) Column Buckling
- Critical load

- **Differential equation for column buckling:** $y'' + \lambda y = 0 \ , y(0) = 0 \ , y(L) = 0$

$, where \ \lambda = P / EI$

The equation of the deflection curve: $y_n(x) = c_2 \sin(n\pi x / L)$



The critical loads : $P_n = n^2\pi^2 EI / L^2, n = 1, 2, 3...$

The lowest critical load(n=1) : $P_{cr} = P_1 = \pi^2 EI / L^2$

E = modulus of elasticity
I = 2nd moment of area
EI = flexural rigidity
P = axial load
y = deflection of column
A = area of column
L = length of column

1) Column Buckling
- Critical stress

A critical buckling stress is often used instead of a buckling load and it can be derived by dividing P_{cr} by A, the cross sectional area of the column.

Euler's formula

The corresponding critical stress:

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 EI}{Al^2} = \pi^2 E\left(\frac{k}{l}\right)^2$$

E = modulus of elasticity
I = 2nd moment of area
EI = flexural rigidity
P = axial load
y = deflection of column
A = area of column
l = length of column

, where $k\left(k^2 = I/A\right)$ is the radius of gyration[1)] of the section of the column.

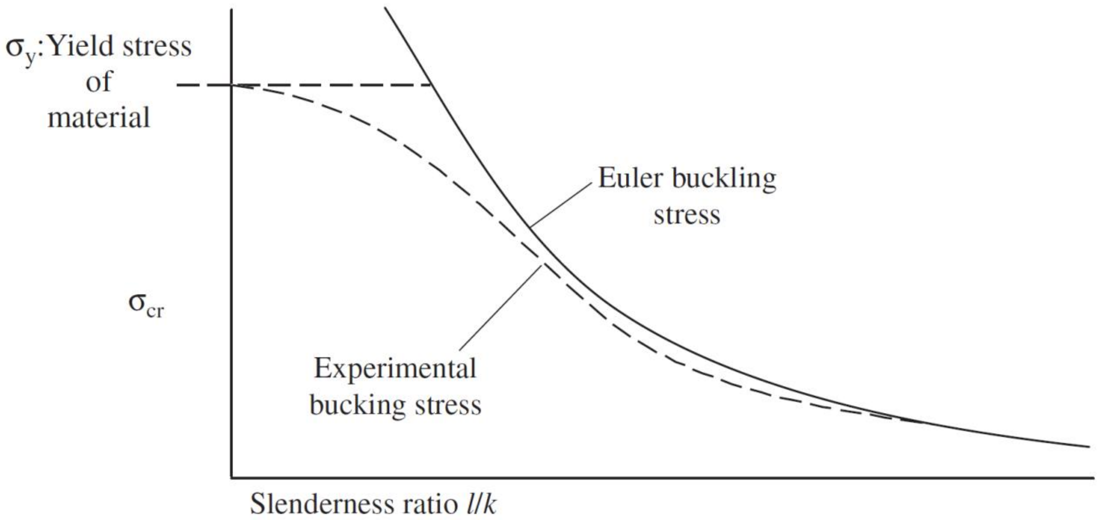
The ratio l/k , often called the slenderness ratio, is the main factor which governs the critical stress

For large value of l/k the critical stress tends toward zero, and at small values of l/k it tends to infinity. In Euler's formula, the buckling stress may become infinite for a small value of l/k, however, buckling stress never goes up above the yield stress of the material in actual conditions, because the material would fail if the stress exceeded the yield stress.

1) the radius of gyration: describes a circular ring whose area is the same as the area of interest.

1) Column Buckling

- Curve of buckiling stress



by theoretical consideration, a horizontal line of yield stress connected to Euler buckling stress is specified as an upper limit of Euler's buckling curve.

$$\sigma_{cr} = a - b\left(\frac{l}{k}\right) \quad \text{Tetmayer's formula}$$

$$\sigma_{cr} = a - b\left(\frac{l}{k}\right)^2 \quad \text{Johnson's formula}$$

$$\sigma_{cr} = \frac{a}{1 + b\left(l/k\right)^2} \quad \text{Rankine's formula}$$

For example, one of the Classification Societies, ABS (American Bureau of Shipping) specifies the permissible load of a pillar or strut of mild steel material in the following equation:

$$\sigma_{cr} = 1.232 - 0.00452\left(\frac{l}{k}\right) \; tonf/cm^2$$

From the above equation, we can see that the ABS formula is theoretically based on Tetmayer's experimental result.

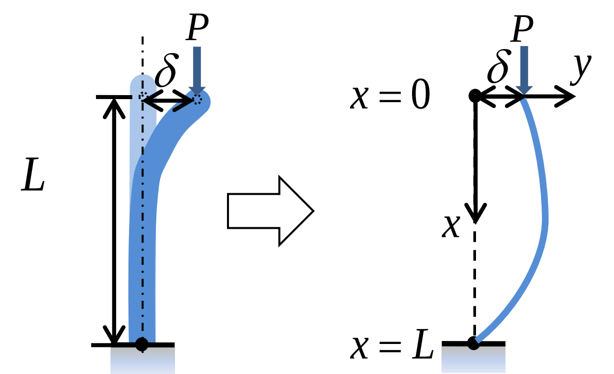
1) Column Buckling

– Buckling of thin vertical column embedded at its base and free at its top

Suppose that a tin vertical homogeneous column is embedded at its base $(x=0)$ **and free at its top** $(x=L)$ **and that a constant axial load** P **is applied to its free end.**

The load either causes a small deflection δ **, or does not cause such a deflection. In either case the differential equation for the deflection** $y(x)$ **is**

$$EI\frac{d^2y}{dx^2} = P(\delta - y) \quad \Rightarrow \quad EI\frac{d^2y}{dx^2} + Py = P\delta \quad \cdots(1)$$



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(1) What is the predicted deflection when $\delta = 0$ **?**

- **The general solution of the differential equation (1) is**

$$y = c_1\cos\sqrt{\frac{P}{EI}}x + c_2\sin\sqrt{\frac{P}{EI}}x + \delta$$

- **The boundary conditions of the differential equation (1) are**

$$y(0) = y'(0) = 0$$

- **If** $\delta = 0$**, this implies that** $c_1 = c_2 = 0$ **and** $y(x) = 0$**. That is, there is no deflection.**

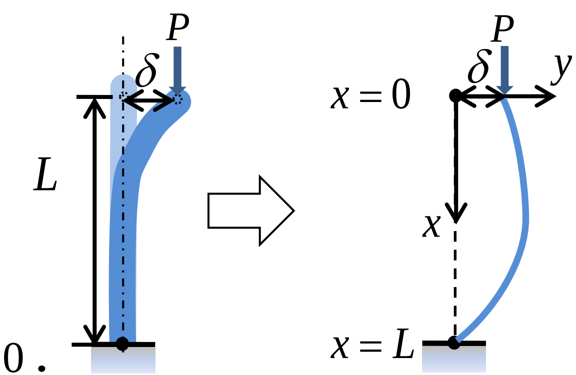
Zill, D.G., Advanced Engineering Mathematics, 3rd edition, 2006, p.166-174

1) Column Buckling

– Buckling of thin vertical column embedded at its base and free at its top

Suppose that a tin vertical homogeneous column is embedded at its base $(x=0)$ **and free at its top** $(x=L)$ **and that a constant axial load** P **is applied to its free end.**
The load either causes a small deflection δ **, or does not cause such a deflection. In either case the differential equation for the deflection** $y(x)$ **is**

$$EI\frac{d^2y}{dx^2} = P(\delta - y) \quad \Rightarrow \quad EI\frac{d^2y}{dx^2} + Py = P\delta \quad \cdots(1)$$



(2) When $\delta \neq 0$ **, show that the Euler load for this column is one-fourth of the Euler load for the hinged column?**

- If $\delta \neq 0$**, the boundary conditions give, in turn,** $c_1 = -\delta,\ c_2 = 0$ **.**
Then

$$y = \delta\left(1 - \cos\sqrt{\frac{P}{EI}}x\right)$$

- In order to satisfy the boundary condition $y(L) = \delta$**, we must have**

$$\delta = \delta\left(1 - \cos\sqrt{\frac{P}{EI}}L\right) \longrightarrow \cos\sqrt{\frac{P}{EI}}L = 0 \longrightarrow \sqrt{\frac{P}{EI}}L = n\pi/2$$

One-fourth of the Euler load

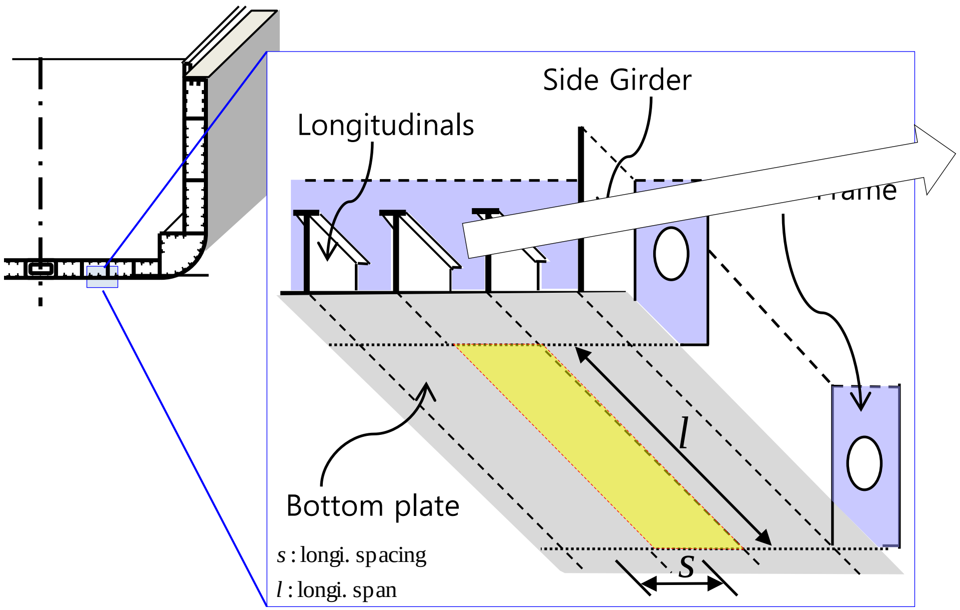
- The smallest value of P_n **, the Euler load, is then**

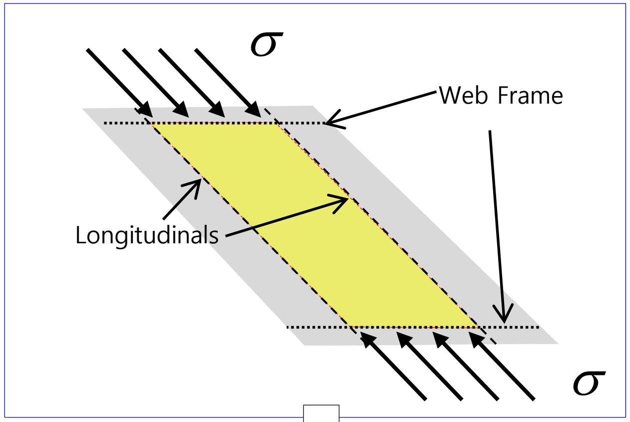
$$\sqrt{\frac{P_1}{EI}}L = \frac{\pi}{2} \quad \text{or} \quad P_1 = \boxed{\frac{1}{4}}\underbrace{\left(\frac{\pi^2 EI}{L^2}\right)}_{\text{Euler load}}$$

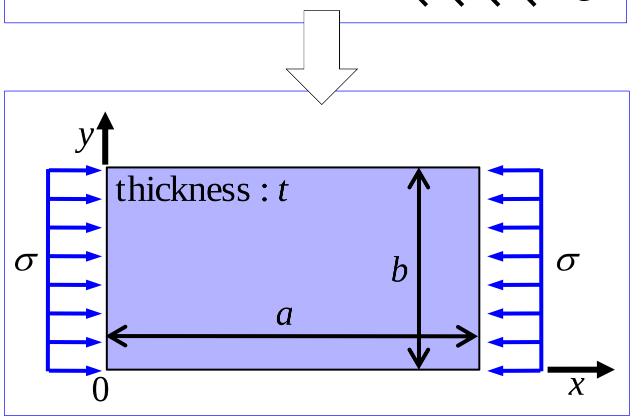
Zill, D.G., Advanced Engineering Mathematics, 3rd edition, 2006, p.166-174

BUCKLING STRENGTH OF PLATE

3) Buckling Strength of Plate





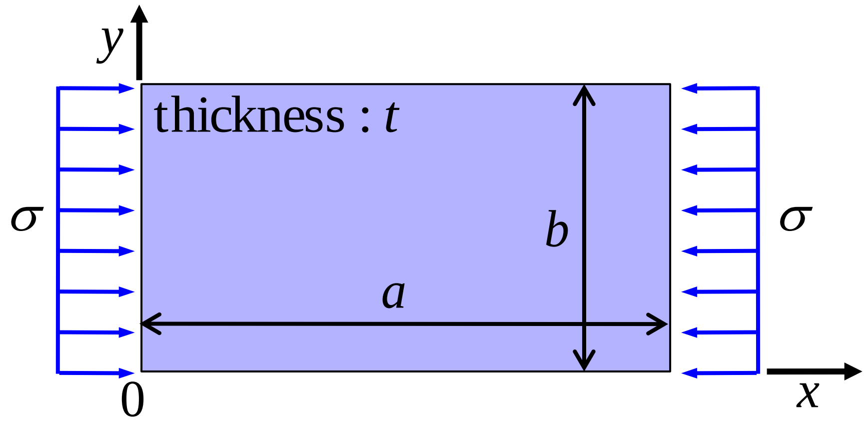


- A ship hull is a stiffened-plate structure, the plating supported by a system of transverse or longitudinal stiffeners.

- For practical design purpose, it is often assumed that **the plate is simply supported at the all edges**, since it gives the least critical stress and is on the safe side.

3) Buckling Strength of Plate

Let us consider the rectangular plate with only supported edges as shown in this figure.



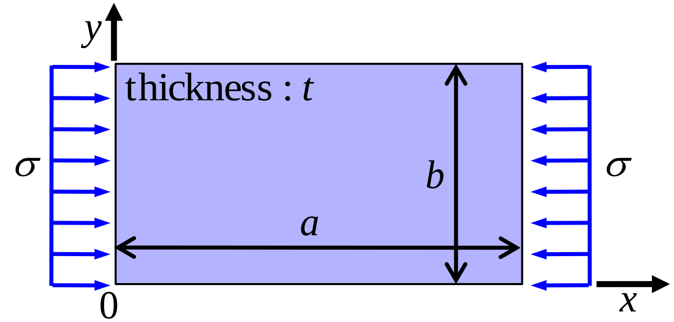
σ : the uni-axial compressive stress
ν : Poisson's ratio
E : Modulus of elasticity
a: plate length
b : plate width
t : thickness of the plate

- **The equation of elastic buckling stress of the plate under uni-axial compressive stress:**

$$\boxed{\frac{Et^3}{12(1-\nu^2)}\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)+\sigma t\frac{\partial^2 w}{\partial x^2}=0} \cdots (1)$$

where, $w = w(x, y)$: deflection of the plate

3) Buckling Strength of Plate



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where, $w = w(x, y)$ **: deflection of the plate**

- **Because all four edges are simply supported, the boundary condition can be expressed in the form:**

$$w(0, y) = w(a, y) = 0$$
$$w(x, 0) = w(x, b) = 0$$

deformation at the edges are zero

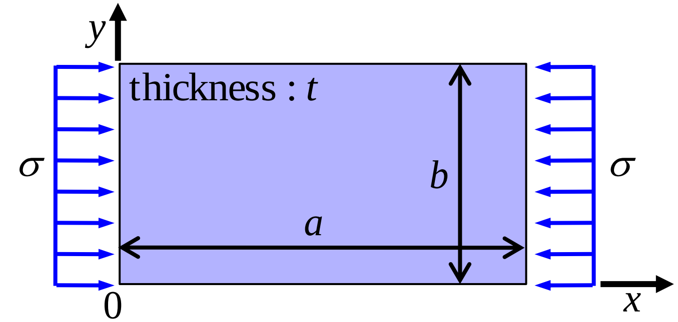
- **Let us assume the following formula for the solution of the equation (1), so that the solution <u>satisfies the boundary conditions</u>.**

$$w = f \sin\left(\frac{m\pi x}{a}\right)\cdot\sin\left(\frac{n\pi y}{b}\right) \cdots(2)$$

where, m, n : integers presenting the number of half-wave of buckles

Okumoto, Y., Design of Ship Hull Structures, 2009, pp.57-60

3) Buckling Strength of Plate



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$$\boxed{\frac{Et^3}{12(1-\nu^2)}\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)+\sigma t\frac{\partial^2 w}{\partial x^2}=0} \cdots(1)$$

where, $w = w(x, y)$ **: deflection of the plate**

- **Substituting the formula (2) into the equation (1),**

$$w = f\sin\left(\frac{m\pi x}{a}\right)\cdot\sin\left(\frac{n\pi y}{b}\right) \quad \cdots(2)$$

$$\boxed{\sigma = \frac{Et^3}{12(1-\nu^2)}\frac{\pi^2}{b^2 t}\left(\frac{m}{\alpha}+n^2\frac{\alpha}{m}\right)^2} \cdots(3)$$

where, $\alpha = \dfrac{a}{b}$

- **Elastic buckling stress is a minimum critical stress, therefore, we put n=1 in the equation (3),**

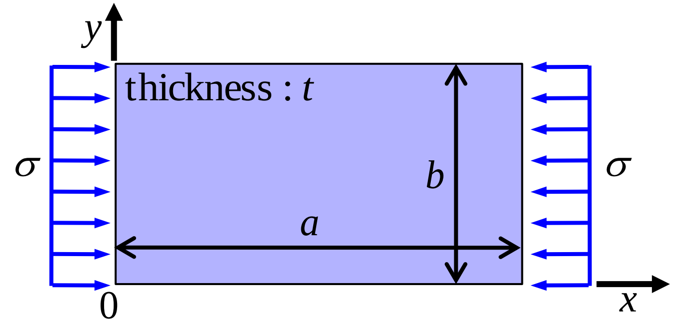
Ideal elastic(Euler) compressive buckling stress:

$$\boxed{\sigma_{el} = \frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2 K}$$

where, K =Minimum value of k, $k = \left(\dfrac{m}{\alpha}+\dfrac{\alpha}{m}\right)^2$

Okumoto, Y., Design of Ship Hull Structures, 2009, pp.57-60

3) Buckling Strength of Plate



σ : the uni-axial compressive stress
ν : Poisson's ratio
E : Modulus of elasticity

a: plate length
b : plate width
t : thickness of the plate

Ideal elastic(Euler) compressive buckling stress:

$$\boxed{\sigma_{el} = \frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2 K}$$

where, $K =$ Minimum value of k

$$k = \left(\frac{m}{\alpha} + \frac{\alpha}{m}\right)^2, \alpha = \frac{a}{b}$$

- For the small b in comparison with t, the elastic buckling stress becomes more than the yield stress of the plate material.
- Therefore, it is usual to use Johnson's modification factor η_p and the critical buckling stress σ_c for the full range of value of t/b as follows:

◆ Bryan's formula[1)]

$$\boxed{\frac{\sigma_c}{\eta_p} = \sigma_{el} = \frac{\pi^2 E}{12\left(1-\nu^2\right)} \cdot \left(\frac{t}{b}\right)^2 \cdot K}$$

σ_c : the critical compressive buckling stress
σ_{el} : the ideal elastic(Euler) compressive buckling stress
K : plate factor (corresponding to the boundary conditions and a/b)
η_p : plasticity reduction factor

$$\eta_p = 1 \qquad , \text{when } \sigma_{el} < \frac{\sigma_y}{2}$$

$$\eta_p = \frac{\sigma_y}{\sigma_{el}}\left(1 - \frac{\sigma_y}{4\sigma_{el}}\right), \text{when } \sigma_{el} \geq \frac{\sigma_y}{2}$$

σy = upper yield stress in N/mm²

ex) Coefficient K when all four edges are simply supported

$$K = 4.0 \quad a/b \geq 1.0$$

$$K = (a/b + b/a)^2, \; a/b < 1.0$$

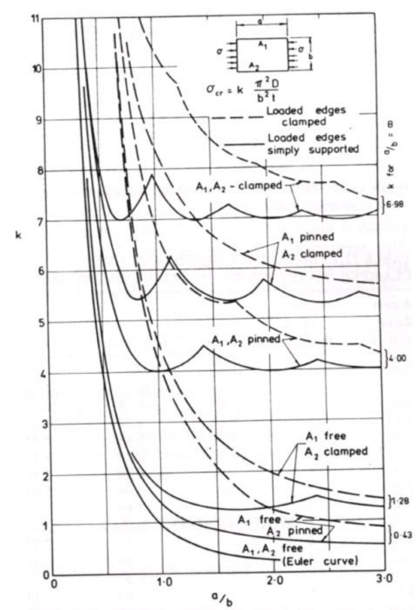


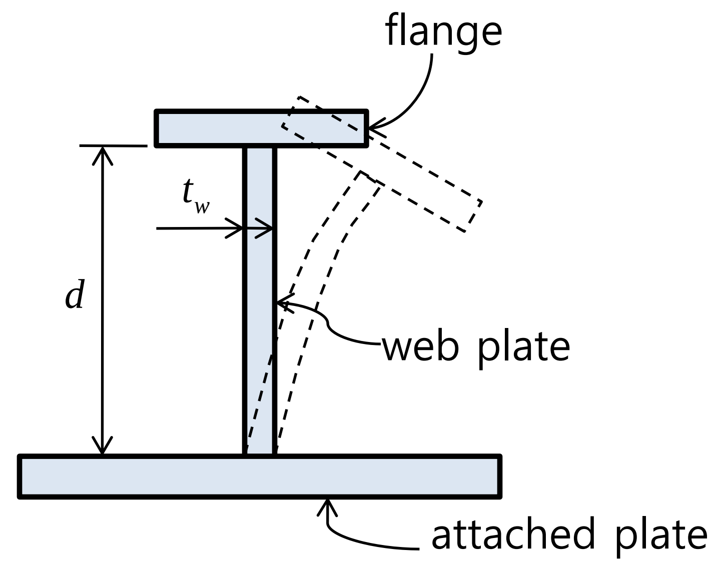
Figure 12.5a Buckling stress coefficient k for flat plates in uni-axial compression.

1) DSME, “선박구조설계” 13-18 Buckling,

3) Buckling Strength of Plate
- The buckling strength of web plate

Web plate of stiffener have to be checked about buckling.

In case of T-bar, it is assumed that the <u>web plate of stiffener is the plate simply supported by flange and attached plate</u>.



$$\frac{\sigma_c}{\eta_p} = \sigma_{el} = \frac{\pi^2 E}{12\left(1-\nu^2\right)} \cdot \left(\frac{t}{d}\right)^2 \cdot K \quad \text{,(Bryan's formula)}$$

, K=4.0

$$\rightarrow \frac{d}{t_w} \le \sqrt{\frac{\pi^2 EK}{12\left(1-\nu^2\right)} \frac{1}{\sigma_{el}}}$$

σ_{cr}: the critical compressive buckling stress

σ_{el} : the ideal elastic(Euler) compressive buckling stress

ν : Poisson’s ratio

K : Plate factor (corresponding to the boundary conditions and a/b)

d : depth of web plate

t : thickness of web plate

E : Modulus of elasticity

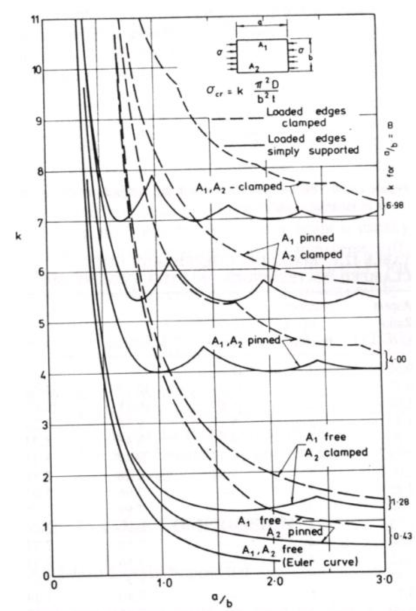


Figure 12.5a Buckling stress coefficient k for flat plates in uni-axial compression.

3) Buckling Strength of Plate
- The buckling strength of flange plate

Flange of stiffener have to be checked about buckling.

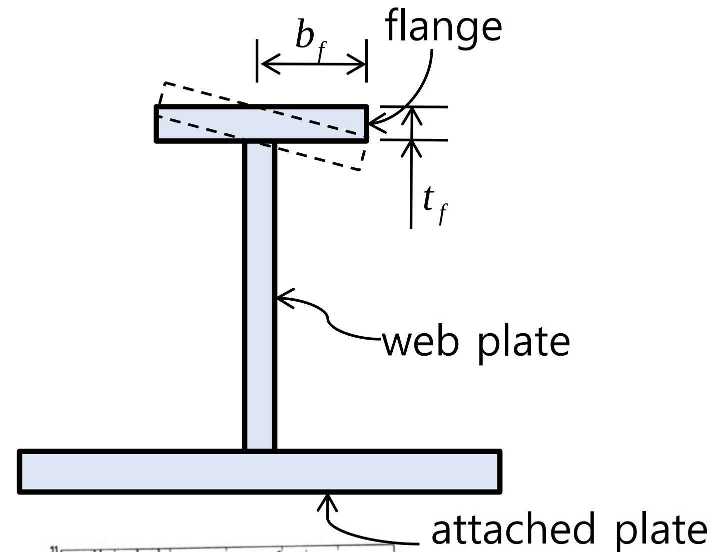
It is assumed that the <u>flange of stiffener is the rectangular plate simply supported on one end by web plate</u>.

$$\frac{\sigma_c}{\eta_p} = \sigma_{el} = \frac{\pi^2 E}{12\left(1-\nu^2\right)} \cdot \left(\frac{t_f}{b_f}\right)^2 \cdot K$$,(Bryan’s formula) , K=0.5

$$\rightarrow \frac{b}{t_f} \le \sqrt{\frac{K\pi^2 E}{12\left(1-\nu^2\right)} \frac{1}{\sigma_{el}}}$$

In general, b/t_f does not exceed 15.

- σ_c : the critical compressive buckling stress
- σ_{el} : the ideal elastic(Euler) compressive buckling stress
- ν : Poisson’s ratio
- K : Plate factor (corresponding to the boundary conditions and a/b)
- b_f : breadth of flange plate
- t_f : thickness of flange plate
- E : Modulus of elasticity



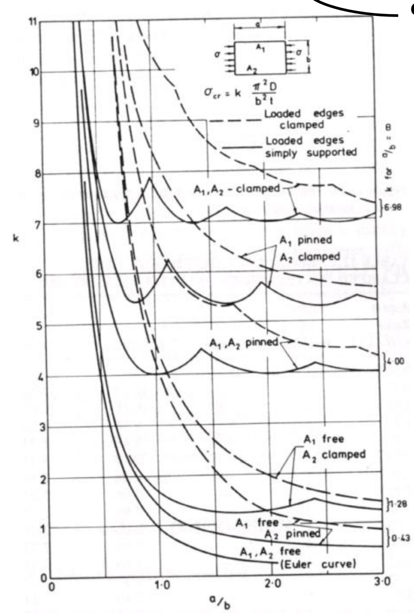


Figure 12.5a Buckling stress coefficient k for flat plates in uni-axial compression.

3) Buckling Strength of Plate by DNV Rule

- Plate panel in uni-axial compression

◆ Criteria for Buckling Strength

[1]Rules for classification of ships, Det Norske Veritas, January 2004, Pt.3 Ch.1 Sec.13, pp.92~93

$$\boxed{\sigma_c > \frac{\sigma_a}{\eta}}$$

σ_c = critical buckling stress in N/mm²
σ_a = calculated actual compressive stress in N/mm²
η = usage factor

Usage Factor (η)

η = 1.0	Deck, Single bottom & Side shell (long stiff)
η = 0.9	Bottom, Inner bottom & Side shell (trans stiff)
η = 1.0	Extreme loads (Q = 10^{-8})
η = 0.8	Normal loads (Q = 10^{-4})

◆ Critical buckling stress σ_c

$$\sigma_c = \sigma_{el} \quad \text{, when } \sigma_{el} < \frac{\sigma_f}{2}$$

$$= \sigma_f \left(1 - \frac{\sigma_f}{4\sigma_{el}}\right) \quad \text{, when } \sigma_{el} > \frac{\sigma_f}{2}$$

σ_{el} = ideal compressive buckling stress
σ_f = upper yield stress in N/mm²
'σ_{el}' is determined according to specific load.

From Bryan's formula,

$$\boxed{\frac{\sigma_c}{\eta_p} = \sigma_{el} = \frac{\pi^2 E}{12(1-\nu^2)} \cdot \left(\frac{t}{b}\right)^2 \cdot K}$$

when $\sigma_{el} < \frac{\sigma_f}{2}$, $\eta_p = 1$

$$\sigma_c = \eta_p \sigma_{el} \rightarrow \sigma_c = \sigma_{el}$$

when $\sigma_{el} \geq \frac{\sigma_f}{2}$, $\eta_p = \frac{\sigma_f}{\sigma_{el}}\left(1 - \frac{\sigma_f}{4\sigma_{el}}\right)$

$$\sigma_c = \eta_p \sigma_{el} \rightarrow \sigma_c = \sigma_f \left(1 - \frac{\sigma_f}{4\sigma_{el}}\right)$$

4) Buckling Strength by DNV Rule

[1] Rules for classification of ships, Det Norske Veritas, January 2004, Pt.3 Ch.1 Sec.13, pp.92~93

◆ Criteria for Buckling Strength

$$\sigma_c > \frac{\sigma_a}{\eta}$$

σ_c = critical buckling stress in N/mm²
σ_a = calculated actual stress in N/mm²
η = usage factor

◆ Critical buckling stress σ_c

- σ_f is yield stress of material in N/mm²

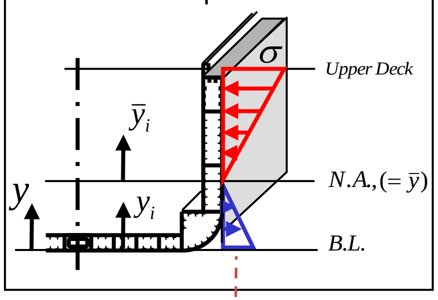
$$\sigma_c = \sigma_{el} \quad , \text{when} \quad \sigma_{el} < \frac{\sigma_f}{2}$$
$$= \sigma_f \left(1 - \frac{\sigma_f}{4\sigma_{el}}\right) \quad , \text{when} \quad \sigma_{el} > \frac{\sigma_f}{2}$$

◆ Calculated actual stress σ_a

- σ_a is calculated actual stress in general
- In plate panels subject to longitudinal stress, σ_a is given by

$$\sigma_a = \frac{Ms + Mw}{I_{N.A.}}(z_n - z_a)10^5 \quad , (N/mm^2)$$
$$= \text{minimum } 30\ f_1\ \text{N/mm}^2 \text{ at side}$$

<Midship section>



consider each different stress according to location

◆ σ_{el} for Plate in uni-axial compression[1]

Plate

$$\sigma_{el} = 0.9kE\left(\frac{t - t_k}{1000s}\right)^2$$

Stiffener

◆ σ_{el} for stiffener in uni-axial compression[1]

$$\sigma_{el} = 3.8E\left(\frac{t_w - t_k}{h_w}\right)^2$$

◆ σ_{el} for stiffener in lateral buckling mode

$$\sigma_{el} = 0.001 \cdot E \cdot \frac{I_A}{Al^2}$$

M_S = still water bending moment as given in Sec.5
M_W = wave bending moment as given in Sec.5
I_{NA} = moment of inertia in cm⁴ of the hull girder
σ_{el} = ideal compressive buckling stress
'σ_{el}' is determined according to specific load.
σ_c = critical buckling stress
σ_f = upper yield stress in N/mm²
t = thickness (mm)
t_k =corrosion addition
t_w = web thickness, h_w = web height
E = modulus of elasticity
s = stiffener spacing (m)
I_A = moment of inertia in cm⁴ about the axis perpendicular to the expected direction of buckling
A = cross-sectional area in cm²
l = length of member in m

B 100 General

101 Local plate panels between stiffeners may be subject to uni-axial or bi-axial compressive stresses, in some cases also combined with shear stresses. Methods for calculating the critical buckling stresses for the various load combinations are given below.

102 Formulae are given for calculating the ideal compressive buckling stress σ_{el}. From this stress the critical buckling stress σ_c may be determined as follows:

$$\sigma_c = \sigma_{el} \quad \text{when} \quad \sigma_{el} < \frac{\sigma_f}{2}$$

$$= \sigma_f \left(1 - \frac{\sigma_f}{4\sigma_{el}}\right) \quad \text{when} \quad \sigma_{el} > \frac{\sigma_f}{2}$$

103 Formulae are given for calculating the ideal shear buckling stress τ_{el}. From this stress the critical buckling stress τ_c may be determined as follows:

$$\tau_c = \tau_{el} \quad \text{when} \quad \tau_{el} < \frac{\tau_f}{2}$$

$$= \tau_f \left(1 - \frac{\tau_f}{4\tau_{el}}\right) \quad \text{when} \quad \tau_{el} > \frac{\tau_f}{2}$$

τ_f = yield stress in shear of material in N/mm^2

$$= \frac{\sigma_f}{\sqrt{3}}.$$

5) Buckling Strength of Stiffener by DNV Rule
: Stiffener in uni-axial compression

[1]Rules for classification of ships, Det Norske Veritas, January 2004, Pt.3 Ch.1 Sec.13, pp.92~93

◆ Criteria for Buckling Strength

(in the same way of plate)

$$\sigma_c > \frac{\sigma_a}{\eta}$$

σ_c = critical buckling stress in N/mm^2
σ_a = calculated actual compressive stress in N/mm^2
η = usage factor

Usage Factor (η)

η = 1.0	Deck, Single bottom & Side shell (long stiff)
η = 0.9	Bottom, Inner bottom & Side shell (trans stiff)
η = 1.0	Extreme loads ($Q = 10^{-8}$)
η = 0.8	Normal loads ($Q = 10^{-4}$)

◆ Critical buckling stress σ_c

$$\begin{aligned}\sigma_c &= \sigma_{el} &&, \text{when } \sigma_{el} < \frac{\sigma_f}{2} \\ &= \sigma_f\left(1 - \frac{\sigma_f}{4\sigma_{el}}\right) &&, \text{when } \sigma_{el} > \frac{\sigma_f}{2}\end{aligned}$$

σ_{el} = ideal compressive buckling stress
'σ_{el}' is determined according to specific load.

σ_f : yield stress of material in N/mm^2

◆ Calculated actual stress σ_a

(Uni-axial compression)

- σ_a is calculated actual compressive stress in general
- In plate panels subject to longitudinal stress, σ_a is given by

$$\begin{aligned}\sigma_a &= \frac{Ms + Mw}{I_{N.A.}}(z_n - z_a)10^5 \quad ,\left(N/mm^2\right) \\ &= \text{minimum } 30\ f_1\ \text{N/mm}^2 \text{ at side}\end{aligned}$$

(※ Hull girder bending moment is acting on the cross section of the ship as moment from the point view of global deformation.
And it is acting on the each stiffener as axial load from the point view of local deformation.)

M_S = still water bending moment as given in Sec.5
M_W = wave bending moment as given in Sec.5
I_N = moment of inertia in cm^4 of the hull girder

205 The critical buckling stress calculated in 201 shall be related to the actual compressive stresses as follows:

$$\sigma_c \geq \frac{\sigma_a}{\eta}$$

σ_a = σ_a calculated compressive stress in plate panels. With linearly varying stress across the plate panel, shall be taken as the largest stress.

In plate panels subject to longitudinal stresses, σ_a is given by:

$$\sigma_{al} = \frac{M_S + M_W}{I_N}(z_n - z_a)10^5 \quad (\mathrm{N/mm^2})$$

$$= \text{minimum } 30\ f_1\ \mathrm{N/mm^2} \text{ at side}$$

η = 1.0 for deck, single bottom and longitudinally stiffened side plating
= 0.9 for bottom, inner bottom and transversely stiffened side plating
= 1.0 for local plate panels where an extreme load level is applied (e.g. impact pressures)
= 0.8 for local plate panels where a normal load level is applied
M_S = stillwater bending moment as given in Sec.5
M_W = wave bending moment as given in Sec.5
I_N = moment of inertia in cm^4 of the hull girder.

For reduction of plate panels subject to elastic buckling, see 207.

M_S and M_W shall be taken as sagging or hogging values for members above or below the neutral axis respectively.

For local plate panels with cut-outs, subject to local compression loads only, σ_a shall be taken as the nominal stress in panel without cut-outs.

An increase of the critical buckling strength may be necessary in plate panels subject to combined in-plane stresses, see 400 and 500.

5) Buckling Strength of Stiffener by DNV Rule
: Stiffener in uni-axial compression

◆ Critical buckling stress σ_c

[1]Rules for classification of ships, Det Norske Veritas, January 2004, Pt.3 Ch.1 Sec.13, pp.92~93

$$\boxed{\begin{aligned} \sigma_c &= \sigma_{el} && ,\text{when } \sigma_{el} < \frac{\sigma_f}{2} \\ &= \sigma_f\left(1-\frac{\sigma_f}{4\sigma_{el}}\right) && ,\text{when } \sigma_{el} > \frac{\sigma_f}{2} \end{aligned}}$$

σ_f : yield stress of material in N/mm^2

'σ_{el}' is determined according to specific load.

◆ Ideal compressive buckling stress σ_{el} of stiffener in uni-axial compression[1)]

$$\boxed{\sigma_{el} = 3.8E\left(\frac{t_w - t_k}{h_w}\right)^2}$$

- **Derivation of the coefficient '3.8'**

From Bryan's formula $\dfrac{\sigma_{cr}}{\eta} = \sigma_e = \dfrac{\pi^2 E}{12(1-\nu^2)}\cdot\left(\dfrac{t}{b}\right)^2 \cdot K$,

$$\frac{\pi^2}{12(1-\nu^2)} = 0.9038(\fallingdotseq 0.9)$$

And substituting K=4(for simply supported plate), the coefficient is approximately equal to 3.8.

σ_{el} = ideal compressive buckling stress
σ_c = critical buckling stress
σ_s = minimum upper yield stress
t_w = web thickness, h_w = web height
E = modulus of elasticity
s = stiffener spacing (m)
ν = 0.3 (Poisson's ratio of steel)

◆ Ideal compressive buckling stress σ_{el} of stiffener in lateral buckling mode

$$\boxed{\sigma_{el} = 0.001 \cdot E \cdot \frac{I_A}{Al^2}}$$

- **Derivation of the coefficient '0.001'**

From Euler's formula $\sigma_{cr} = \dfrac{\pi^2 EI}{Al^2}\ \ \dfrac{\pi^2\ N/mm^2\ cm^4}{cm^2\ m^2}$,

$$\frac{\pi^2\ N/mm^2\ cm^4}{cm^2\ m^2} = \frac{\pi^2\ N/mm^2\,(10mm)^4}{(10mm)^2\,(1000\,mm)^2} \fallingdotseq 0.001\ N/mm^2$$

◆ Thickness of flange

For flanges on angles and T-sections of longitudinals and other highly compressed stiffeners, the thickness shall not be less than

$$t_f = 0.1b_f + t_k \quad (mm)$$

b_f = flange width in mm for angles, half the flange width for T-Section(m)
t_k = corrosion addition(DNV Rule : Pt.3 Ch.1 Sec.2 - Page15)

Pt.3 Ch.1 Sec.13, B102, B103 2011

102 Formulae are given for calculating the ideal compressive buckling stress σ_{el}. From this stress the critical buckling stress σ_c may be determined as follows:

$$\sigma_c = \sigma_{el} \quad \text{when} \quad \sigma_{el} < \frac{\sigma_f}{2}$$

$$= \sigma_f\left(1 - \frac{\sigma_f}{4\sigma_{el}}\right) \quad \text{when} \quad \sigma_{el} > \frac{\sigma_f}{2}$$

103 Formulae are given for calculating the ideal shear buckling stress τ_{el}. From this stress the critical buckling stress τ_c may be determined as follows:

$$\tau_c = \tau_{el} \quad \text{when} \quad \tau_{el} < \frac{\tau_f}{2}$$

$$= \tau_f\left(1 - \frac{\tau_f}{4\tau_{el}}\right) \quad \text{when} \quad \tau_{el} > \frac{\tau_f}{2}$$

τ_f = yield stress in shear of material in N/mm^2

$= \frac{\sigma_f}{\sqrt{3}}$.

6) Buckling Strength of Plate by DNV Rule

- Plate panel in uni-axial compression

◆ Criteria for Buckling Strength

[1]Rules for classification of ships, Det Norske Veritas, January 2004, Pt.3 Ch.1 Sec.13, pp.92~93

$$\sigma_c > \frac{\sigma_a}{\eta}$$

σ_c = critical buckling stress in N/mm^2
σ_a = calculated actual compressive stress in N/mm^2
η = usage factor

Usage Factor (η)

η = 1.0	Deck, Single bottom & Side shell (long stiff)
η = 0.9	Bottom, Inner bottom & Side shell (trans stiff)
η = 1.0	Extreme loads (Q = 10^{-8})
η = 0.8	Normal loads (Q = 10^{-4})

◆ Critical buckling stress σ_c

$$\sigma_c = \sigma_{el} \quad \text{, when } \sigma_{el} < \frac{\sigma_f}{2}$$

$$= \sigma_f \left(1 - \frac{\sigma_f}{4\sigma_{el}}\right) \quad \text{,when } \sigma_{el} > \frac{\sigma_f}{2}$$

σ_{el} = ideal compressive buckling stress
σ_f = upper yield stress in N/mm^2
'σ_{el}' is determined according to specific load.

◆ Calculated actual stress σ_a

(Uni-axial compression)

- σ_a is calculated actual compressive stress in general
- In plate panels subject to longitudinal stress, σ_a is given by

$$\sigma_a = \frac{Ms + Mw}{I_{N.A.}}(z_n - z_a)10^5 \quad ,(N/mm^2)$$

$$= \text{minimum } 30 \text{ f}_1 \text{ N/mm}^2 \text{ at side}$$

(※ Hull girder bending moment is acting on the cross section of the ship as moment from the point view of global deformation.
And it is acting on the each plate as axial load from the point view of local deformation.)

M_S = still water bending moment as given in Sec.5
M_W = wave bending moment as given in Sec.5
I_N = moment of inertia in cm^4 of the hull girder

6) Buckling Strength of Plate by DNV Rule

- Plate panel in uni-axial compression

◆ Critical buckling stress σ_c

[1]Rules for classification of ships, Det Norske Veritas, January 2004, Pt.3 Ch.1 Sec.13, pp.92~93

$$\begin{aligned} \sigma_c &= \sigma_{el} && ,\text{when } \sigma_{el} < \frac{\sigma_f}{2} \\ &= \sigma_f\left(1-\frac{\sigma_f}{4\sigma_{el}}\right) && ,\text{when } \sigma_{el} > \frac{\sigma_f}{2} \end{aligned}$$

σ_f : minimum upper yield stress of material in N/mm²

'σ_{el}' is determined according to specific load.

◆ Ideal compressive buckling stress σ_{el} in uni-axial compression[1)]

$$\sigma_{el} = 0.9kE\left(\frac{t-t_k}{1000s}\right)^2$$

- **Derivation of the coefficient '0.9'**

From Bryan's formula $\frac{\sigma_{cr}}{\eta} = \sigma_e = \frac{\pi^2 E}{12(1-\nu^2)}\cdot\left(\frac{t}{b}\right)^2\cdot K$,

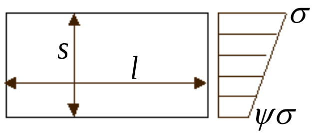
$$\frac{\pi^2}{12(1-\nu^2)} = \frac{3.141593^2}{12(1-0.3^2)} = 0.9038\ \ (\fallingdotseq 0.9)$$

σ_{el} = ideal compressive buckling stress
σ_c = critical buckling stress
σ_f = upper yield stress in N/mm²
t = thickness (mm)
t_k =corrosion addition
E = modulus of elasticity
s = stiffener spacing (m)
ν = 0.3 (Poisson's ratio of steel)

◆ factor k

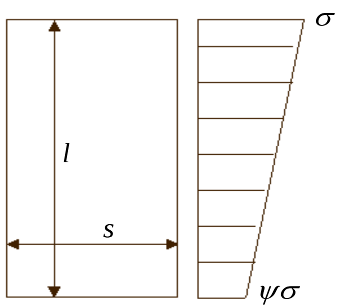
- For plating with longitudinal stiffeners (in direction of compression stress) :

$$k = k_l = \frac{8.4}{\psi + 1.1}$$



- For plating with transverse stiffeners (perpendicular to compression stress) :

$$k = k_s = c\left[1+\left(\frac{s}{l}\right)^2\right]^2 \frac{2.1}{\psi+1.1}$$



ψ = ratio between the smaller and the larger compressive stress (positive value) $(0 \le \psi \le 1)$

c=1.21 when stiffeners are angles or T sections
=1.10 when stiffeners are bulb flats
=1.05 when stiffeners are flat bars
=1.3 when plating is supported by deep girders

6) Buckling Strength of Plate by DNV Rule
- Plate panel in uni-axial compression

◆ Critical buckling stress σ_c

[1] Rules for classification of ships, Det Norske Veritas, January 2004, Pt.3 Ch.1 Sec.13, pp.92~93

$$\sigma_c = \sigma_{el} \qquad \text{, when } \sigma_{el} < \frac{\sigma_f}{2}$$
$$= \sigma_f \left(1 - \frac{\sigma_f}{4\sigma_{el}}\right) \qquad \text{, when } \sigma_{el} > \frac{\sigma_f}{2}$$

σ_f : minimum upper yield stress of material in N/mm²

'σ_{el}' is determined according to specific load.

◆ Ideal compressive buckling stress σ_{el} in uni-axial compression[1]

$$\sigma_{el} = 0.9kE\left(\frac{t - t_k}{1000s}\right)^2$$

- **Derivation of the coefficient '0.9'**

From Bryan's formula $\frac{\sigma_{cr}}{\eta} = \sigma_e = \frac{\pi^2 E}{12(1-\nu^2)} \cdot \left(\frac{t}{b}\right)^2 \cdot K$,

$$\frac{\pi^2}{12(1-\nu^2)} = \frac{3.141593^2}{12(1-0.3^2)} = 0.9038 \ (≒ 0.9)$$

σ_{el} = ideal compressive buckling stress
σ_c = critical buckling stress
σ_f = upper yield stress in N/mm²
t = thickness (mm)
t_k = corrosion addition
E = modulus of elasticity
s = stiffener spacing (m)
ν = 0.3 (Poisson's ratio of steel)

◆ factor k

- For plating with longitudinal stiffeners (in direction of compression stress) : $k = k_l = \frac{8.4}{\psi + 1.1}$
- For plating with transverse stiffeners (perpendicular to compression stress) : $k = k_s = c\left[1 + \left(\frac{s}{l}\right)^2\right]^2 \frac{2.1}{\psi + 1.1}$

Example) If $\psi = 1.0, c = 1.05, s/l = 1/10$

$$k = k_l = \frac{8.4}{1.0 + 1.1} = \underline{4}$$

$$k = k_s = c\left[1 + \left(\frac{s}{l}\right)^2\right]^2 \frac{2.1}{\psi + 1.1} = 1.05\left[1 + \left(\frac{1}{10}\right)^2\right]^2 \frac{2.1}{1.0 + 1.1} = \underline{1.071}$$

Thus, the plate with longitudinal stiffeners can endure much stress than the plate with transverse stiffeners

201 The ideal elastic buckling stress may be taken as:

$$\sigma_{el} = 0.9\,k\,E\left(\frac{t - t_k}{1000\,s}\right)^2 \qquad (\text{N/mm}^2)$$

For plating with longitudinal stiffeners (in direction of compression stress):

$$k = k_l = \frac{8.4}{\psi + 1.1} \qquad \text{for } (0 \le \psi \le 1)$$

For plating with transverse stiffeners (perpendicular to compression stress):

$$k = k_s = c\left[1 + \left(\frac{s}{l}\right)^2\right]^2 \frac{2.1}{\psi + 1.1} \qquad \text{for } (0 \le \psi \le 1)$$

c = 1.21 when stiffeners are angles or T-sections
= 1.10 when stiffeners are bulb flats
= 1.05 when stiffeners are flat bars
c = 1.3 when the plating is supported by floors or deep girders.

For longitudinal stiffened double bottom panels and longitudinal stiffened double side panels the c-values may be multiplied by 1.1.

ψ is the ratio between the smaller and the larger compressive stress assuming linear variation, see Fig.1.

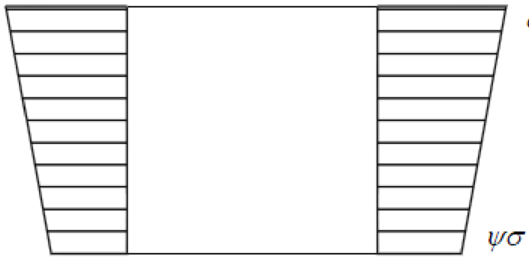


Fig. 1
Buckling stress correction factor

The above correction factors are not valid for negative ψ-values.

The critical buckling stress is found from 102.

Example of Buckling Check

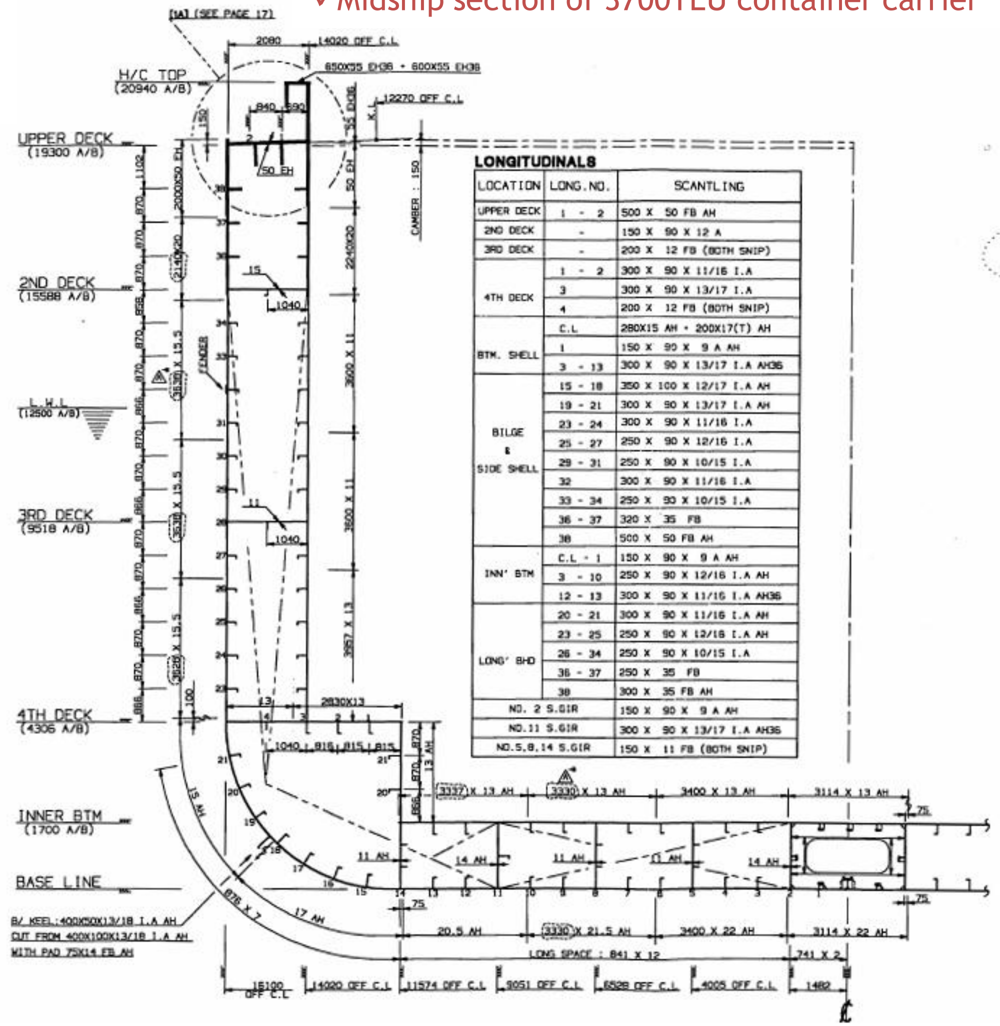
✓ Basis ship: 3700TEU Container Carrier

✓ Arrangement of structure member, longi. spacing, seam line of design ship are same with those of basis ship.

✓ Design ship in this example is the same with the ship considered in the example of local scantling.

Main particulars of design ship	
LOA(m)	259.64
LBP(m)	247.64
L_scant(m)	245.11318
B(m)	32.2
D(m)	19.3
Td(m)	11
Ts(m)	12.6
Vs(knt)	24.5
C_b	0.6563

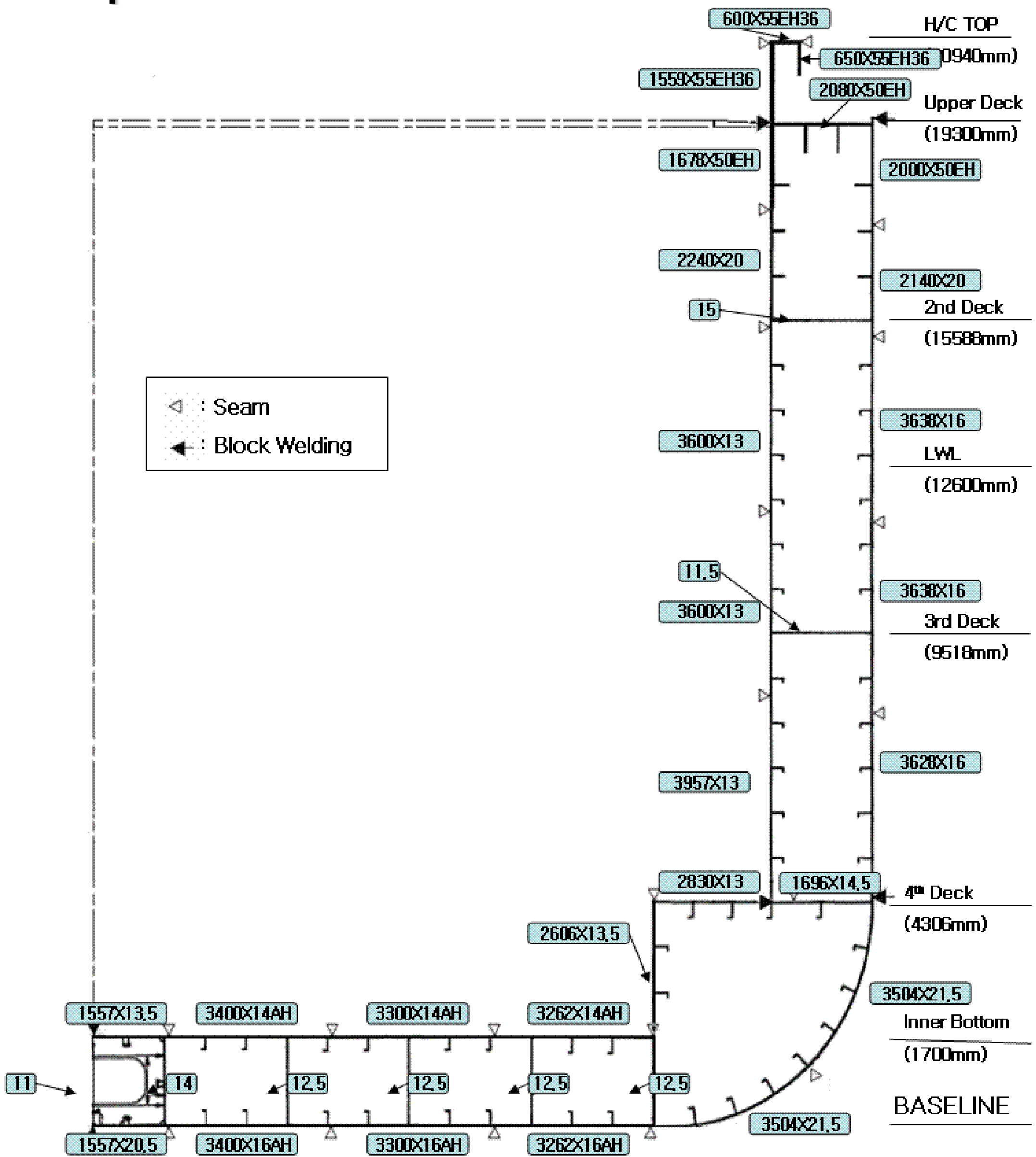
✓Midship section of 3700TEU container carrier



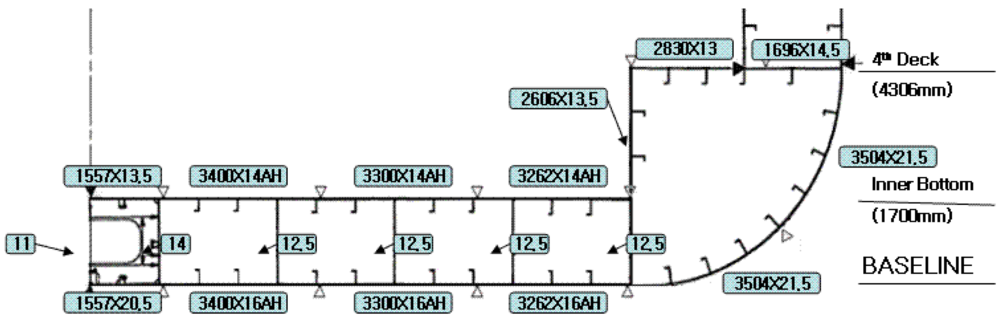
5) Example of Buckling Check by DNV Rule[1)]
: Midship Section

◆ Midship Section

[1)]Rules for classification of ships, Det Norske Veritas, January 2004, Pt.3 Ch.1 Sec.13, pp.92~98



5) Example of Buckling Check by DNV Rule[1)]
: Bottom plate



◆ Criteria for Buckling Strength

$$\sigma_c > \frac{\sigma_a}{\eta}$$

◆ Critical buckling stress σ_c

$$\sigma_c = \sigma_{el} \qquad ,\text{when} \quad \sigma_{el} < \frac{\sigma_f}{2}$$

$$= \sigma_f\left(1-\frac{\sigma_f}{4\sigma_{el}}\right) \quad ,\text{when} \quad \sigma_{el} > \frac{\sigma_f}{2}$$

◆ Calculated actual stress σ_a

$$\sigma_a = \frac{Ms+Mw}{I_N}(z_n - z_a)10^5 \quad ,(N/mm^2)$$

$= \text{minimum } 30\ f_1\ N/mm^2$ at side

◆ σ_{el} for PLATE in uni-axial compression[1)]

Plate $\sigma_{el} = 0.9kE\left(\frac{t-t_k}{1000s}\right)^2$

◆ σ_{el} for stiffener in uni-axial compression[1)]

$$\sigma_{el} = 3.8E\left(\frac{t_w - t_k}{h_w}\right)^2$$

Stiffener

◆ σ_{el} for stiffener in Lateral buckling mode

$$\sigma_{el} = 0.001 \cdot E \cdot \frac{I_A}{Al^2} \cdot \left(\frac{1}{n^2}\right)$$

◆ Calculation of elastic buckling stress

Name		$t(mm)$	$s(m)$	ψ	k	σ_{el} (N/mm²)	$\sigma_f/2$ (N/mm²)
Bottom Plate	KP	20.5	0.741	1	4	513.573	117.5
	BP1	16	0.841	1	4	235.918	117.5
	BP2	16	0.841	1	4	235.918	117.5
	BP3	16	0.841	1	4	235.918	117.5
	BP4	21.5	0.876	0.97	4.058	412.020	117.5
	BP5	21.5	0.876	0.97	4.058	412.020	117.5

Because $\sigma_{el} > \frac{\sigma_f}{2}$ for all bottom plates, critical buckling stress is calculated as follows:

$$\sigma_c = \sigma_f\left(1-\frac{\sigma_f}{4\sigma_{el}}\right)$$

$$\sigma_c = \sigma_{el} \qquad when \quad \sigma_{el} < \frac{\sigma_f}{2}$$

$$\sigma_c = \sigma_f\left(1-\frac{\sigma_f}{4\sigma_{el}}\right) \qquad when \quad \sigma_{el} > \frac{\sigma_f}{2}$$

σ_c = critical buckling stress

σ_f = minimum yield stress

$= 235\ N/mm^2$ for mild steel

$= 315\ N/mm^2$ for AH, DH, EH steel

$= 355\ N/mm^2$ for $AH36, DH36, EH36$ steel

σ_{el} = elastic buckling stress

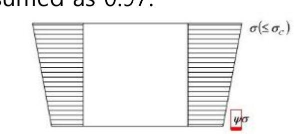
$$= 0.9kE\left(\frac{t-t_k}{1000s}\right)^2 \qquad (N/mm^2)$$

$E = 2.06\times10^5 \quad N/mm^2$

$$k = k_l = \frac{8.4}{\psi+1.1} \qquad for\ (0 \le \psi \le 1)$$

s :shortest side of plate panel in m

Ψ: the ratio between the smaller and the larger compressive stress assuming linear variation
In this example, Ψ at bottom and deck are assumed as 1, Ψ at side plate is assumed as 0.97.



[1)]Rules for classification of ships, Det Norske Veritas, January 2004, Pt.3 Ch.1 Sec.13, pp.92~98

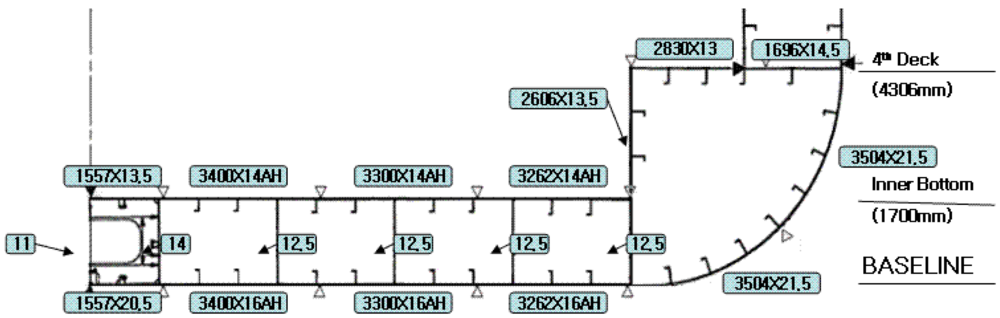
301 The ideal elastic buckling stress may be taken as:

$$\tau_{el} = 0.9 k_t E \left(\frac{t - t_k}{1000 s}\right)^2 \quad (\mathrm{N/mm^2})$$

$$k_t = 5.34 + 4\left(\frac{s}{l}\right)^2$$

The critical shear buckling stress is found from 103.

5) Example of Buckling Check by DNV Rule[1]
: Bottom plate



◆ Calculation of elastic buckling stress

Because $\sigma_{el} > \dfrac{\sigma_f}{2}$ for all bottom plates, critical buckling stress is calculated as follows:

$$\sigma_c = \sigma_f \left(1 - \frac{\sigma_f}{4\sigma_{el}}\right)$$

Name		σ_{el} (N/mm²)	σ_f (N/mm²)	σ_c (N/mm²)
Bottom Plate	KP	513.573	235	208.117
	BP1	235.918	235	176.478
	BP2	235.918	235	176.478
	BP3	235.918	235	176.478
	BP4	412.020	235	201.491
	BP5	412.020	235	201.491

[1]Rules for classification of ships, Det Norske Veritas, January 2004, Pt.3 Ch.1 Sec.13, pp.92~98

◆ Criteria for Buckling Strength

$$\sigma_c > \frac{\sigma_a}{\eta}$$

◆ Critical buckling stress σ_c

$$\sigma_c = \sigma_{el} \quad ,\text{when } \sigma_{el} < \frac{\sigma_f}{2}$$
$$= \sigma_f\left(1 - \frac{\sigma_f}{4\sigma_{el}}\right) \quad ,\text{when } \sigma_{el} > \frac{\sigma_f}{2}$$

◆ Calculated actual stress σ_a

$$\sigma_a = \frac{Ms + Mw}{I_N}(z_n - z_a)10^5 \;, (N/mm^2)$$
$$= \text{minimum } 30\, f_1 \ \text{N/mm}^2 \text{ at side}$$

◆ σ_{el} for PLATE in uni-axial compression[1)]

Plate

$$\sigma_{el} = 0.9kE\left(\frac{t - t_k}{1000s}\right)^2$$

◆ σ_{el} for stiffener in uni-axial compression[1)]

$$\sigma_{el} = 3.8E\left(\frac{t_w - t_k}{h_w}\right)^2$$

Stiffener

◆ σ_{el} for stiffener in Lateral buckling mode

$$\sigma_{el} = 0.001 \cdot E \cdot \frac{I_A}{Al^2} \cdot \left(\frac{1}{n^2}\right)$$

$$\sigma_c = \sigma_{el} \qquad when \quad \sigma_{el} < \frac{\sigma_f}{2}$$

$$\sigma_c = \sigma_f\left(1 - \frac{\sigma_f}{4\sigma_{el}}\right) \qquad when \quad \sigma_{el} > \frac{\sigma_f}{2}$$

σ_c = critical buckling stress

σ_f = minimum yield stress

$= 235\ N/mm^2$ for mild steel

$= 315\ N/mm^2$ for AH, DH, EH steel

$= 355\ N/mm^2$ for AH36, DH36, EH36 steel

σ_{el} = elastic buckling stress

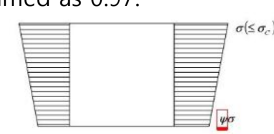
$$= 0.9kE\left(\frac{t - t_k}{1000s}\right)^2 \qquad (N/mm^2)$$

$E = 2.06 \times 10^5 \quad N/mm^2$

$$k = k_l = \frac{8.4}{\psi + 1.1} \qquad for\ (0 \le \psi \le 1)$$

s :shortest side of plate panel in m

Ψ: the ratio between the smaller and the larger compressive stress assuming linear variation
In this example, Ψ at bottom and deck are assumed as 1, Ψ at side plate is assumed as 0.97.



201 For longitudinals subject to longitudinal hull girder compressive stresses, supporting bulkhead stiffeners, pillars, cross ties, panting beams etc., the ideal elastic lateral buckling stress may be taken as:

$$\sigma_{el} = 0.001\ E\ \frac{I_A}{A l^2} \quad (N/mm^2)$$

I_A = moment of inertia in cm^4 about the axis perpendicular to the expected direction of buckling
A = cross-sectional area in cm^2.

When calculating I_A and A, a plate flange equal to 0.8 times the spacing is included for stiffeners. For longitudinals supporting plate panels where elastic buckling is allowed, the plate flange shall not be taken greater than the effective width, see B207 and Appendix A.

Where relevant t_k shall be subtracted from flanges and web plates when calculating I_A and A.

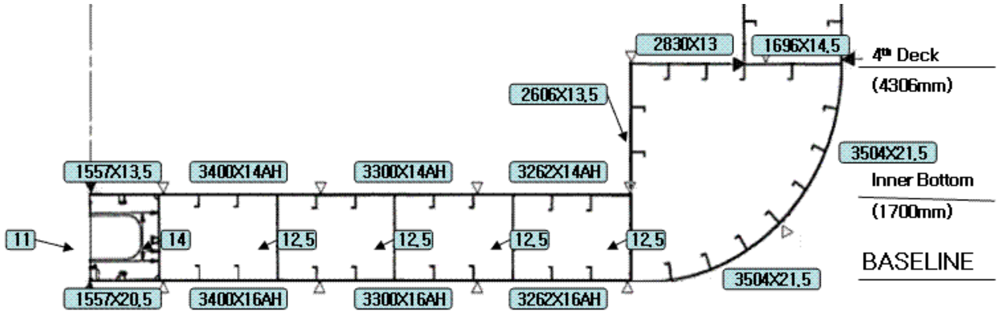
The critical buckling stress is found from 101.

The formula given for σ_{el} is based on hinged ends and axial force only.

If, in special cases, it is verified that one end can be regarded as fixed, the value of σ_{el} may be multiplied by 2. If it is verified that both ends can be regarded as fixed, the value of σ_{el} may be multiplied by 4.

In case of eccentric force, additional end moments or additional lateral pressure, the strength member shall be reinforced to withstand bending stresses.

5) Example of Buckling Check by DNV Rule[1] : Bottom plate



◆ **Criteria for Buckling Strength**

$$\sigma_c > \frac{\sigma_a}{\eta}$$

◆ **Critical buckling stress** σ_c

$$\sigma_c = \sigma_{el} \quad ,\text{when } \sigma_{el} < \frac{\sigma_f}{2}$$

$$= \sigma_f\left(1 - \frac{\sigma_f}{4\sigma_{el}}\right) \quad ,\text{when } \sigma_{el} > \frac{\sigma_f}{2}$$

◆ **Calculated actual stress** σ_a

$$\sigma_a = \frac{Ms + Mw}{I_N}(z_n - z_a)10^5 \quad ,(N/mm^2)$$

$$= \text{minimum } 30\ f_1\ \ N/mm^2 \text{ at side}$$

◆ σ_{el} **for PLATE in uni-axial compression**[1)]

Plate

$$\sigma_{el} = 0.9kE\left(\frac{t - t_k}{1000s}\right)^2$$

◆ σ_{el} **for stiffener in uni-axial compression**[1)]

$$\sigma_{el} = 3.8E\left(\frac{t_w - t_k}{h_w}\right)^2$$

Stiffener

◆ σ_{el} **for stiffener in Lateral buckling mode**

$$\sigma_{el} = 0.001 \cdot E \cdot \frac{I_A}{Al^2} \cdot \left(\frac{1}{n^2}\right)$$

◆ Comparison between critical buckling stress and actual stress

$$\sigma_c \geq \frac{\sigma_a}{\eta}$$

Name		z_a (m)	η	σ_a	σ_a min.	σ_a	σ_a/η (N/mm²)	σ_c (N/mm²)
Bottom Plate	KP	0.000	0.9	173.608	30	173.608	192.898	208.117
	BP1	0.000	0.9	173.608	30	173.608	192.898	176.478
	BP2	0.000	0.9	173.608	30	173.608	192.898	176.478
	BP3	0.000	0.9	173.608	30	173.608	192.898	176.478
	BP4	0.660	0.9	163.902	30	163.902	182.114	201.491
	BP5	2.810	0.9	132.284	30	132.284	146.982	201.491

In this example, buckling check for BP1~BP3 are not satisfied. To satisfy that, the change such as increase of plate thickness or change of material from mild to high tensile steel is needed.

$$\sigma_c \geq \frac{\sigma_a}{\eta}$$

$$\sigma_a = \sigma_{al} = \frac{M_S + M_W}{I_N}(z_n - z_a)10^5 \qquad (N/mm^2)$$

$$= minimum \quad 30 f1 \;\; N/mm^2 \qquad at \;\; side$$

$$for \;\; mild \;\; steel \;\; 30 \; N/mm^2$$

$$for \;\; AH, DH, EH \;\; 38.4 \; N/mm^2$$

$$for \;\; AH36, DH36, EH36 \;\; 41.7 \; N/mm^2$$

η = 1.0 for deck, single bottom, longitudinally stiffened side plating
= 0.9 for bottom, inner bottom, transversely stiffened side plating
= 1.0 for local plate panels where an extreme load level is applied. (e.g impact pressures)
= 0.8 for local plate panels where an normal load level is applied.

z_n : vertical distance in m from the baseline or deckline to the neutral axis of the hull girder, whichever is relevant
z_a : vertical distance in m from the baseline or deckline to the point in question below or above the neutral axis, Respectively
I : Moment of Inertia (cm4)
Ms,Mw : Still water bending moment, vertical wave bending moment (kNm)

[1)]Rules for classification of ships, Det Norske Veritas, January 2004, Pt.3 Ch.1 Sec.13, pp.92~98

5) Example of Buckling Check by DNV Rule[1)] : Bottom plate

◆ Criteria for Buckling Strength

$$\sigma_c > \frac{\sigma_a}{\eta}$$

◆ Critical buckling stress σ_c

$$\sigma_c = \sigma_{el} \quad ,\text{when} \quad \sigma_{el} < \frac{\sigma_f}{2}$$

$$= \sigma_f\left(1 - \frac{\sigma_f}{4\sigma_{el}}\right) \quad ,\text{when} \quad \sigma_{el} > \frac{\sigma_f}{2}$$

◆ Calculated actual stress σ_a

$$\sigma_a = \frac{Ms + Mw}{I_N}(z_n - z_a)10^5 \quad ,(N/mm^2)$$

$$= \text{minimum } 30\ f_1\ N/mm^2 \text{ at side}$$

◆ σ_{el} for PLATE in uni-axial compression[1)]

Plate $\sigma_{el} = 0.9kE\left(\frac{t - t_k}{1000s}\right)^2$

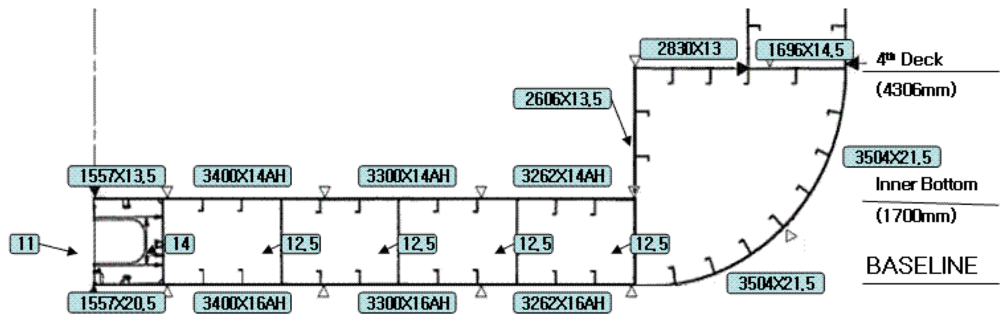
Stiffener

◆ σ_{el} for stiffener in uni-axial compression[1)]

$$\sigma_{el} = 3.8E\left(\frac{t_w - t_k}{h_w}\right)^2$$

◆ σ_{el} for stiffener in Lateral buckling mode

$$\sigma_{el} = 0.001 \cdot E \cdot \frac{I_A}{Al^2} \cdot \left(\frac{1}{n^2}\right)$$



◆ Determination of the way of buckling reinforcement

$$\sigma_c \geq \frac{\sigma_a}{\eta}$$

Name		$t(mm)$	Steel grade	σ_{el} (N/mm²)	$\sigma_f/2$ (N/mm²)	σ_c (N/mm²)
Bottom Plate	KP	20.5	mild	513.57341	117.5	208.117
	BP1	21.5	AH	235.91755	157.5	209.852
	BP2	21.5	AH	235.91755	157.5	209.852
	BP3	21.5	AH	235.91755	157.5	209.852
	BP4	21.5	mild	412.01989	117.5	201.491
	BP5	21.5	mild	412.01989	117.5	201.491

In this example, we increase the thickness of plate.

$$\sigma_c \geq \frac{\sigma_a}{\eta}$$

$$\sigma_a = \sigma_{al} = \frac{M_S + M_W}{I_N}(z_n - z_a)10^5 \qquad (N/mm^2)$$

$$= minimum \quad 30 f1 \ N/mm^2 \quad at \ side$$

$$for \ mild \ steel \ \ 30 \ N/mm^2$$

$$for \ AH, DH, EH \ \ 38.4 \ N/mm^2$$

$$for \ AH36, DH36, EH36 \ \ 41.7 \ N/mm^2$$

η = 1.0 for deck, single bottom, longitudinally stiffened side plating
= 0.9 for bottom, inner bottom, transversely stiffened side plating
= 1.0 for local plate panels where an extreme load level is applied. (e.g impact pressures)
= 0.8 for local plate panels where an normal load level is applied.

z_n : vertical distance in m from the baseline or deckline to the neutral axis of the hull girder, whichever is relevant
z_a : vertical distance in m from the baseline or deckline to the point in question below or above the neutral axis, Respectively
I : Moment of Inertia (cm4)
Ms,Mw : Still water bending moment, vertical wave bending moment (kNm)

[1)]Rules for classification of ships, Det Norske Veritas, January 2004, Pt.3 Ch.1 Sec.13, pp.92~98

Reference:
Buckling of a Thin Vertical Column

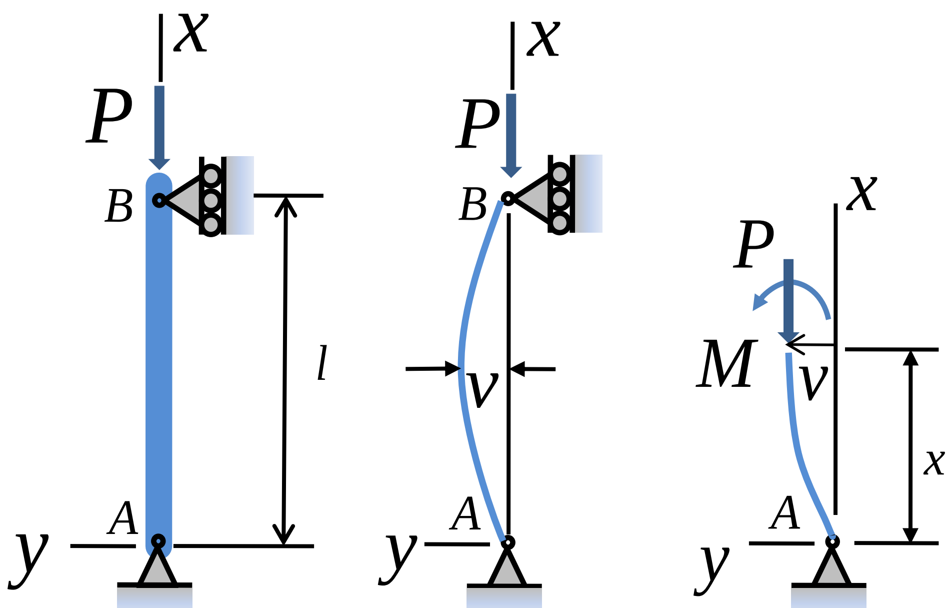
Buckling of a Thin Vertical Column

The differential equations of the deflection curve of a beam are applicable to a buckled column because the column bends as though it were a beam.

- **Differential Equation of the deflection curve of a beam:** $EIv'' = M$
- **Equilibrium of moments about point A:** $M + P \cdot v = 0$

$$\boxed{EIv'' + Pv = 0}$$

: Differential equation for column buckling

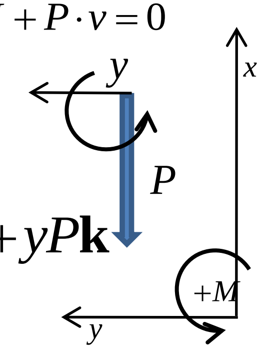


•Sign convention of moments

$$\sum M_z = M + P \cdot v = 0$$

$$+M\mathbf{k}$$

$$\mathbf{y} \times \mathbf{P} = +yP\mathbf{k}$$



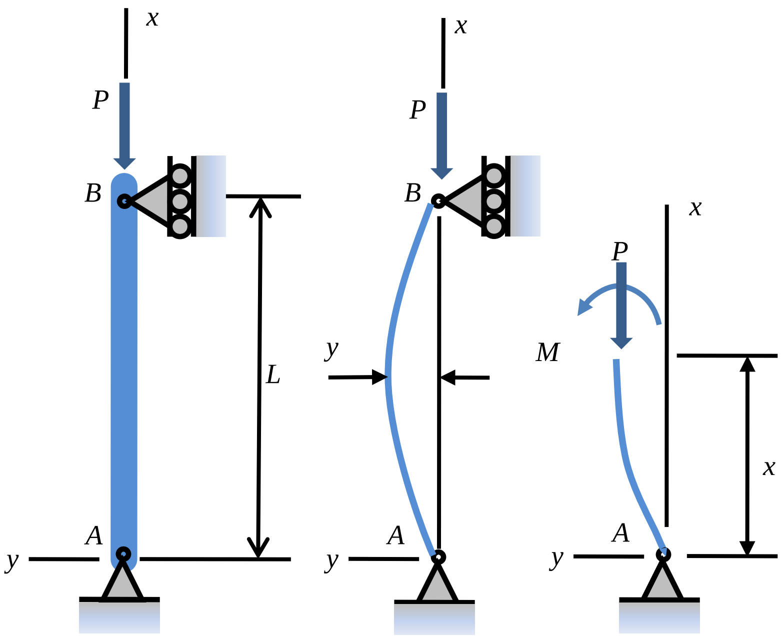
E = modulus of elasticity
I = 2^{nd} Moment of Area
EI = flexural rigidity
P = axial load
v = deflection of column

James M. Gere, Mechanics of Materials 6^{th} Edition, Thomson, pp. 748-762
Zill D.G., Cullen M.R., Advanced Engineering Mathematics, 9^{th} edition, Jones and Bartlett, 2006, p169

Buckling of a Thin Vertical Column

Buckling of a Thin Vertical Column

$$EI\frac{d^2y}{dx^2}+Py=0$$



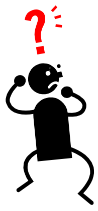
When both end side of Vertical Column, Boundary condition is

$$y(0)=0,\ \ y(L)=0$$

- By writing $\lambda = k^2 = \dfrac{P}{EI}$

$$y''+\lambda y=0,\ y(0)=0,\ \ y(L)=0$$

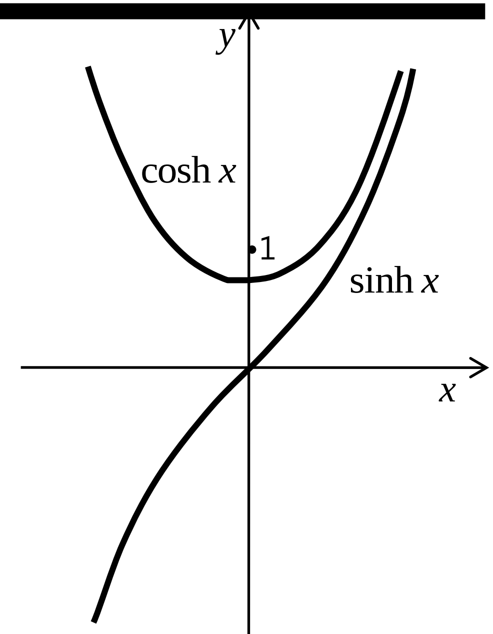
$$y''+k^2y=0$$



$$\lambda = k^2 = \frac{P}{EI} > 0$$

Buckling of a Thin Vertical Column

$$\lambda = k^2 = \frac{P}{EI} > 0 \qquad y(0)=0,\ \ y(L)=0$$



Case I : $\lambda = 0$

$$y'' = 0$$

$$y = c_1 x + c_2$$

integral

From boundary condition:

$$c_1 = 0,\ c_2 = 0$$

$\therefore y = 0$ (trivial solution)

Case II : $\lambda < 0$ Write $\lambda = -k^2,\ k > 0$

$$y'' - k^2 y = 0$$

Let: $y = e^{mx}$

$$\rightarrow (m^2 - k^2)e^{mx} = 0$$

$$\rightarrow m_1 = k,\ m_2 = -k$$

$$y = c_1 e^{kx} + c_2 e^{-kx}$$

From boundary condition:

$$y(0) = c_1 + c_2 = 0, c_2 = -c_1$$

$$y(L) = c_1 e^{kL} + c_2 e^{-kL}$$

$$= c_1(e^{kL} - e^{-kL}) = 0$$

If $(e^{kL} - e^{-kL}) = 0$

$$\rightarrow e^{2kL} = 1,$$

$$\rightarrow k = 0\ (\because L \neq 0)$$ ⇨ Case I : trivial sol.

If $c_1 = 0 \rightarrow c_2 = 0$ ⇨ trivial sol.

Linear combination of e^{kx} and e^{-kx}

(basis transformation)

$$\frac{e^{kx} + e^{-kx}}{2} = \cosh kx, \quad \frac{e^{kx} - e^{-kx}}{2} = \sinh kx$$

$$y = c_1 \cosh kx + c_2 \sinh kx$$

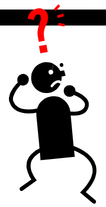
From boundary condition:

$$y(0) = c_1 = 0$$

$$y(L) = c_2 \sinh kL = 0$$

- If $k = 0\ (\because L \neq 0)$ ⇨ Case I : trivial sol.
- If $c_2 = 0$ ⇨ trivial sol.

Buckling of a Thin Vertical Column



$$\lambda = k^2 = \frac{P}{EI} > 0 \qquad y(0) = 0,\ \ y(L) = 0$$

Case III : $\lambda > 0$ Write $\lambda = k^2,\ k > 0$

$$y'' + k^2 y = 0$$

Let: $y = e^{mx}$

Then roots of auxiliary equation is

$$\longrightarrow \quad (m^2 + k^2)e^{mx} = 0$$

$$\longrightarrow \quad m_1 = ik,\ m_2 = -ik$$

$$\therefore y = c_1 \cos kx + c_2 \sin kx$$

$$y(0) = C_1 = 0$$

$$y(L) = \underline{C_2}\, \underline{\sin kL} = 0$$

If $c_2 = 0 : y = 0$ 이므로 trivial solution.

$$\therefore c_2 \neq 0\ , \sin kL = 0$$

$$\therefore kL = 0,\ \pi,\ 2\pi, \ldots$$

If $kL = 0 : k = 0\ (\because L \neq 0)$ ⇨ Case I : trivial sol.

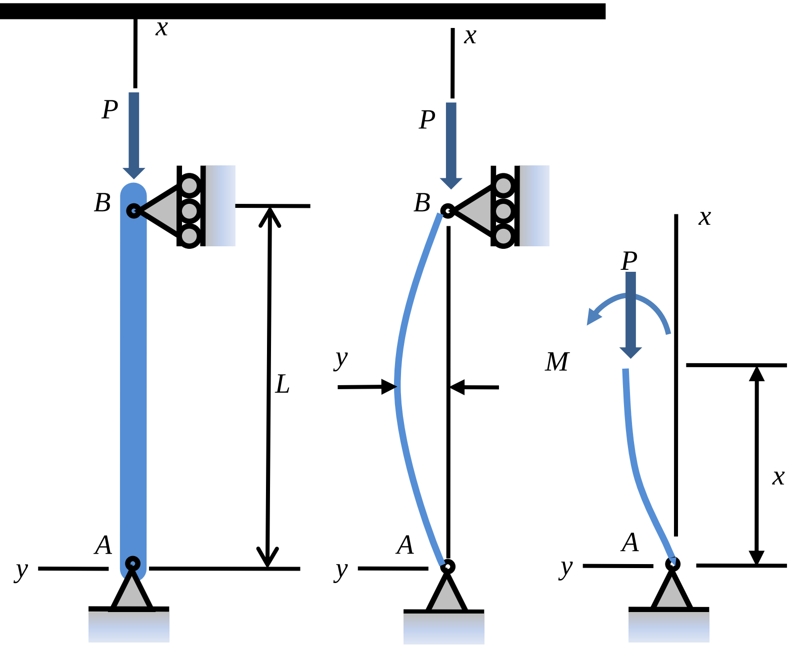
$$\therefore kL = n\pi,\ n = 1, 2, 3\ldots$$

$$\therefore y_n = C_2 \sin kx = C_2 \sin \frac{n\pi x}{L},\ n = 1,\ 2,\ 3\ldots$$

Buckling of a Thin Vertical Column

$$y'' + k^2 y = 0,\ y(0) = 0,\ \ y(L) = 0$$

$$\lambda = k^2 = \frac{P}{EI}$$



Homogeneous solution: $y = C_1 \sin kx + C_2 \cos kx$

Boundary conditions: $y(0) = 0,\ y(L) = 0$

$$y(0) = C_2 = 0$$

$$y(L) = \underline{C_1}\ \underline{\sin kL} = 0$$

If $c_1 = 0 : y = 0$ 이므로 trivial solution.

$$\therefore c_1 \neq 0\ , \sin kL = 0$$

$$\therefore kL = 0,\ \pi,\ 2\pi, \ldots$$

If $kL = 0 : k = 0\ (\because L \neq 0)$ ⇨ trivial sol.

$$\therefore kL = n\pi,\ n = 1, 2, 3\ldots$$

$$\therefore y_n = C_1 \sin kx = C_1 \sin \frac{n\pi x}{L},\ n = 1,\ 2,\ 3\ldots$$

:Deflection curve

Eigenvalues:

$$\lambda_n = k_n^{\ 2} = \left(\frac{n\pi}{L}\right)^2$$

Buckling of a Thin Vertical Column

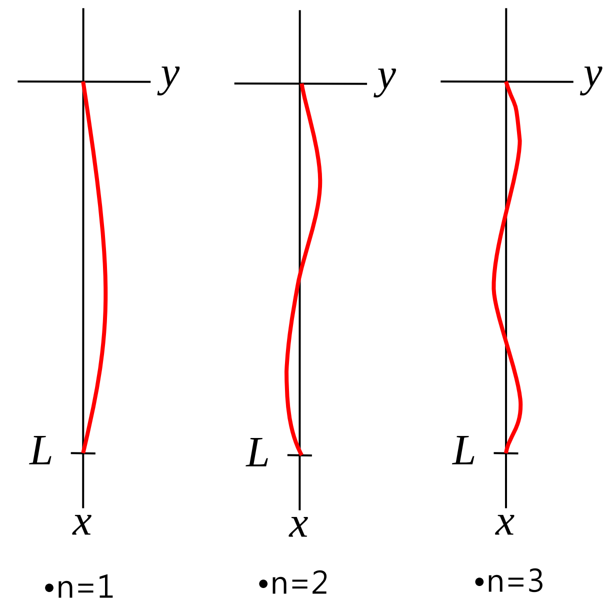
$$y_n(x) = c_1 \sin(\frac{n\pi x}{L})$$

$$\frac{P_n}{EI} = \frac{n^2\pi^2}{L^2}$$

$$P_n = \frac{n^2\pi^2 EI}{L^2},\ n = 1, 2, 3, ..$$ (Critical loads)

When n=1 $P_1 = \dfrac{\pi^2 EI}{L^2}$ (Euler load)

$$y_1(x) = c_1 \sin(\frac{\pi x}{L})$$ (first buckling mode)



Chapter 17. Grillage Analysis for Midship Cargo Hold

u : Axial Displacement G : Shear Modulus A : Sectional Area G : Shear Modulus E : Young's Modulus

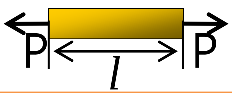
w : Vertical Displacement θ : Angle of Twist l : Length J : Polar Moment of Inertia I : Moment of Inertia

Summary of Approximation Methods

Element | Differential Equation → Weighted Residual → (•Discretization •Approximation) → Finite Element Method | •Superposition of Stiffness Matrix •Coordinates Transformation **Behavior → Structure**

$$\mathbf{M}\ddot{\mathbf{x}} = \sum \mathbf{F} \text{ , where } \ddot{\mathbf{x}} = 0 \qquad \mathbf{Kd} = \mathbf{F}$$

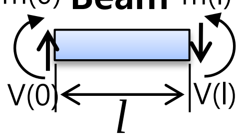
Bar



$$EA\frac{d^2u(x)}{dx^2} = 0 \;\Rightarrow\; \int_0^L W_l\left(EA\frac{d^2\hat{u}(x)}{dx^2}\right)dx = 0 \;\Rightarrow\; \hat{u}(x) = a_0 + a_1x, \quad \frac{EA}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} -P(0) \\ P(l) \end{bmatrix}$$

Tension → **Truss**, **Frame**

Beam

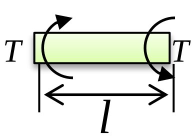


$$EI\frac{d^4y(x)}{dx^4} = 0 \;\Rightarrow\; \int_0^L W_l\left(EI\frac{d^4\hat{y}(x)}{dx^4}\right)dx = 0 \;\Rightarrow\; \hat{y}(x) = b_0 + b_1x + b_2x^2 + b_3x^3,$$

$$\frac{2EI}{l^3}\begin{bmatrix} 6 & 3l & -6 & 3l \\ 3l & 2l^2 & -3l & l^2 \\ -6 & -3l & 6 & -3l \\ 3l & l^2 & -3l & 2l^2 \end{bmatrix}\begin{bmatrix} d_{y1} \\ \phi_1 \\ d_{y2} \\ \phi_2 \end{bmatrix} = \begin{bmatrix} V(0) \\ -m(0) \\ -V(l) \\ m(l) \end{bmatrix}$$

Bending → **Frame**, **Grillage**

Shaft



$$GJ\frac{d^2\theta(x)}{dx^2} = 0 \;\Rightarrow\; \int_0^L W_l\left(GJ\frac{d^2\hat{\theta}(x)}{dx^2}\right)dx = 0 \;\Rightarrow\; \hat{\theta}(x) = c_0 + c_1x, \quad \frac{GJ}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} -T(0) \\ T(l) \end{bmatrix}$$

Torsion → **Grillage**

Application

Midship Cargo Hold → Engineering Concept ! → **Grillage Modeling** → **Solution**

Grillage Modeling
- **3D→ 2D**
- **Equivalent Force & Moment**
- **Boundary condition**

Solution
- **programming**
- **visulaization**

Beam Theory : Sign Convention, Deflection of Beam

Elasticity : Displacement, Strain, Stress, Force Equilibrium, Compatibility, Constitutive Equation

Grillage Analysis for Midship Cargo Hold

- Background

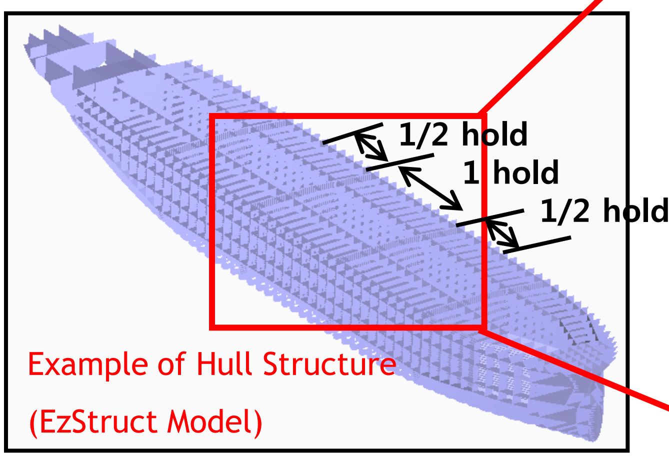
  to determine the distribution of deflection and stress

FEM Approach	Grillage Analysis Approach
- Could calculate the accurate deflection and stress distribution but - Time Consuming for Model Preparation - Analysis Model may not be available before the design completed	- Could estimate the overall deflection and stress distribution comparatively in a short time and even the design is not over - A simplified and practical approach

Midship Cargo Hold Analysis

- Analysis Region : 2 Holds (½ Hold + 1 Hold + ½ Hold)
  - ① **1 Hold : Analysis is not correct because of the boundary condition**
  - ② **All Holds : It takes much time to preparing the analysis model**
  - ③ **2 Holds : Comparatively correct considering the time for model preparation**

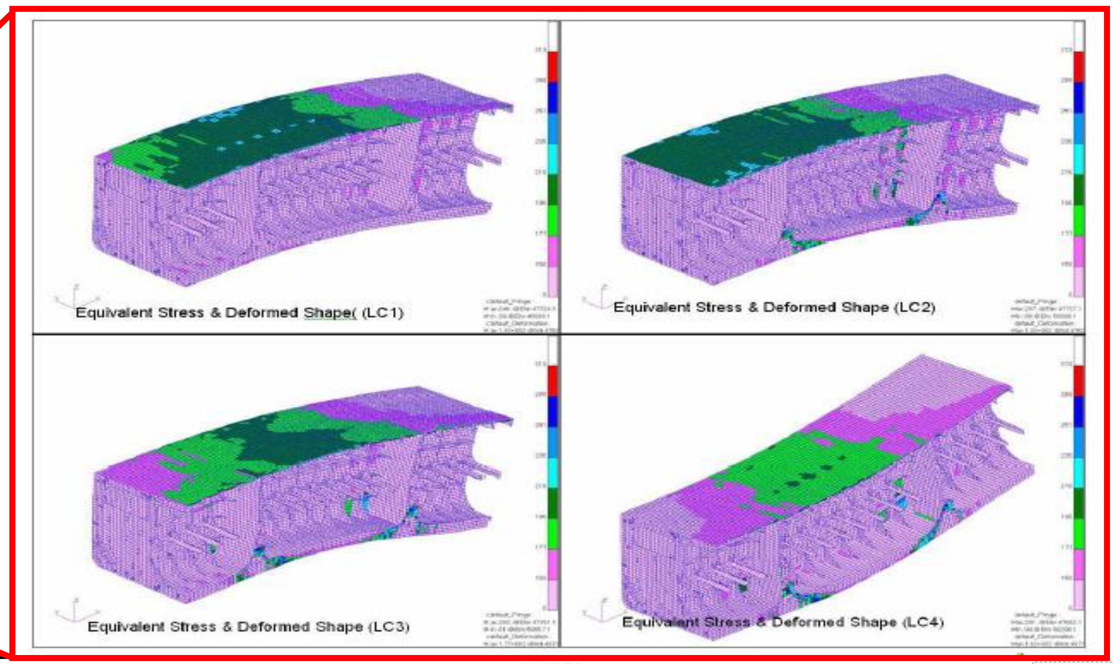
VLCC (Very Large Crude oil Carrier)



Example of Hull Structure

(EzStruct Model)

Structural Analysis Result (MSC.Patran)



Stress acting on a Ship

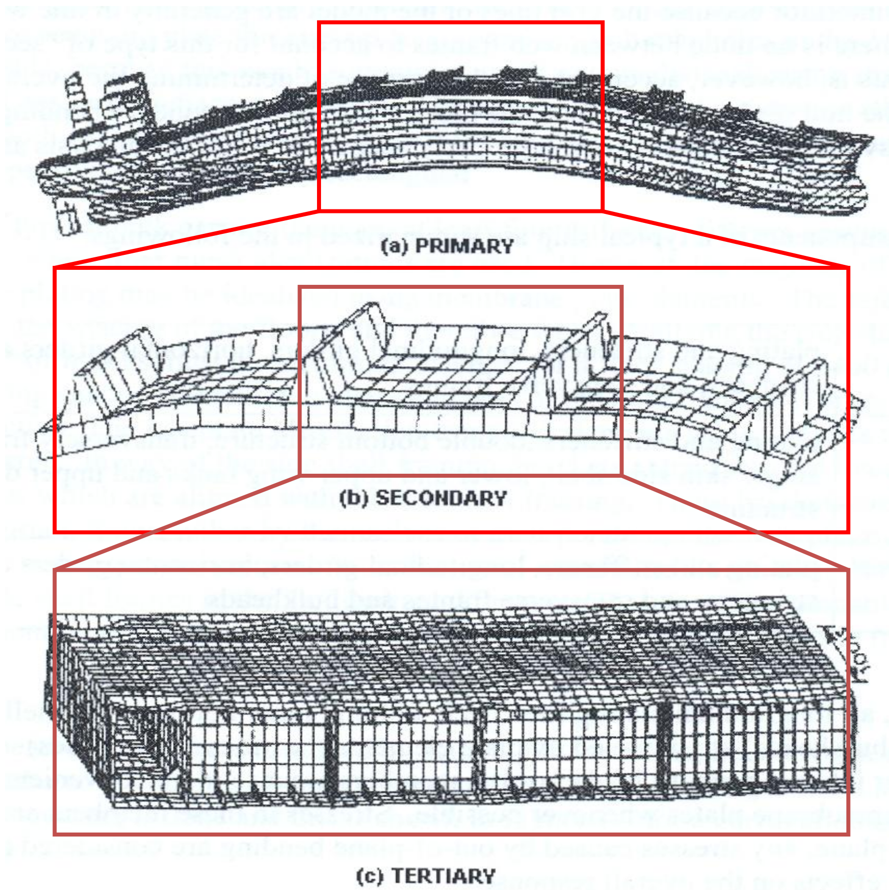
▪ Stress and Deflection Components

The structural response of the hull girder and the associated members can be subdivided into three components

• <u>Primary response</u> is the response of the entire hull, when the ship bends as a beam under the longitudinal distribution of load.

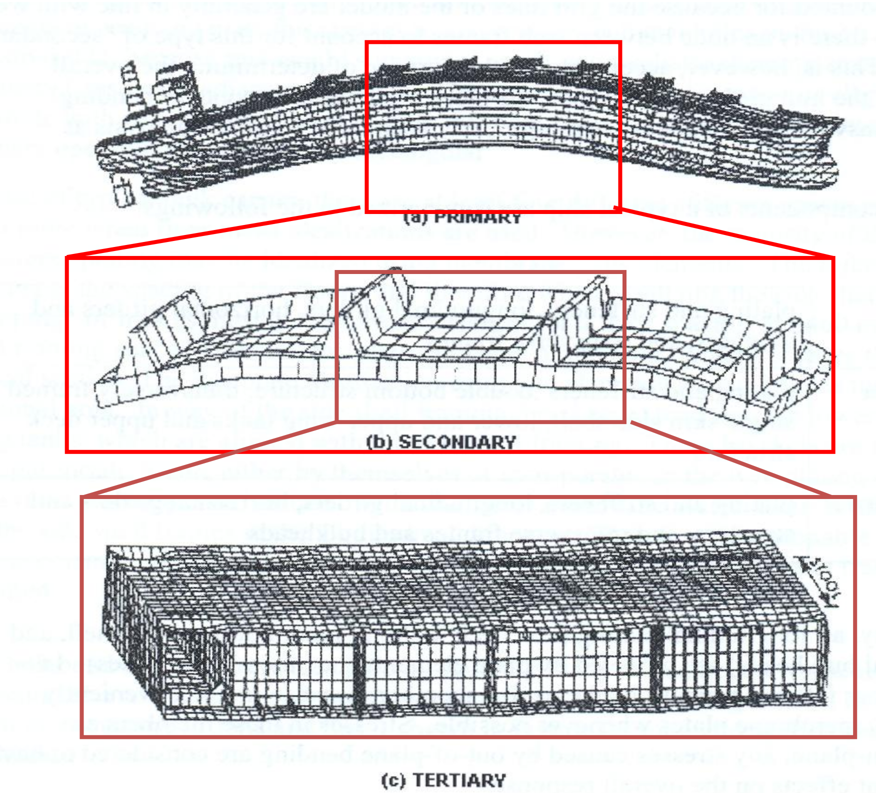
• <u>Secondary response</u> relates to the global bending of stiffened panels (for single hull ship) or to the behavior of double bottom, double sides, etc., for double hull ships

• <u>Tertiary response</u> describes the out-of-plane deflection and associated stress of an individual unstiffened plate panel included between 2 longitudinals and 2 transverse web frames.

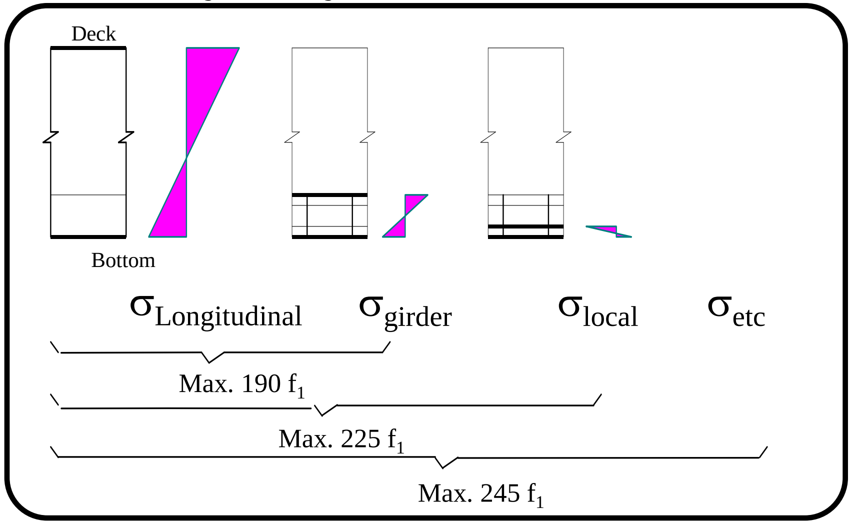


*Philippe RIGO and Enrico Rizzuto,, Ship Design and Construction,, 2002, chapter 18. Analysis and design of ship structure, p.18-21

Local Strength & Allowable Stresses



The sum of $\sigma_{Longitudinal}, \sigma_{girder}, \sigma_{local}$ is not to exceed 245 f1 N/mm2



Loads and Stresses

1) Hogging or Sagging

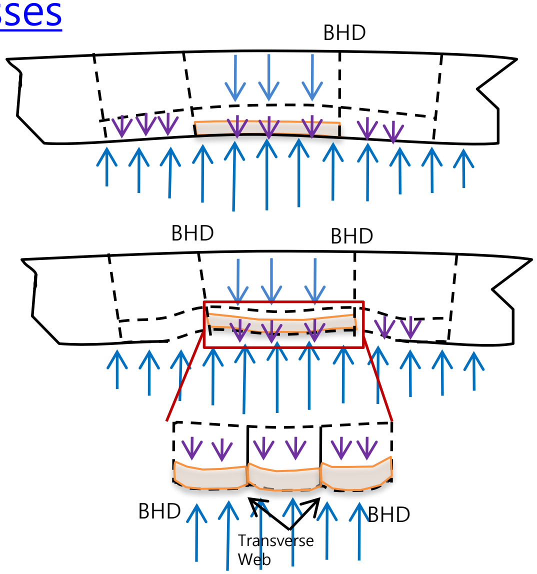
$$\sigma_1 = \sigma_{Longitudinal}$$

2) Cargo Load

$$\sigma_2 = \sigma_{girder}$$

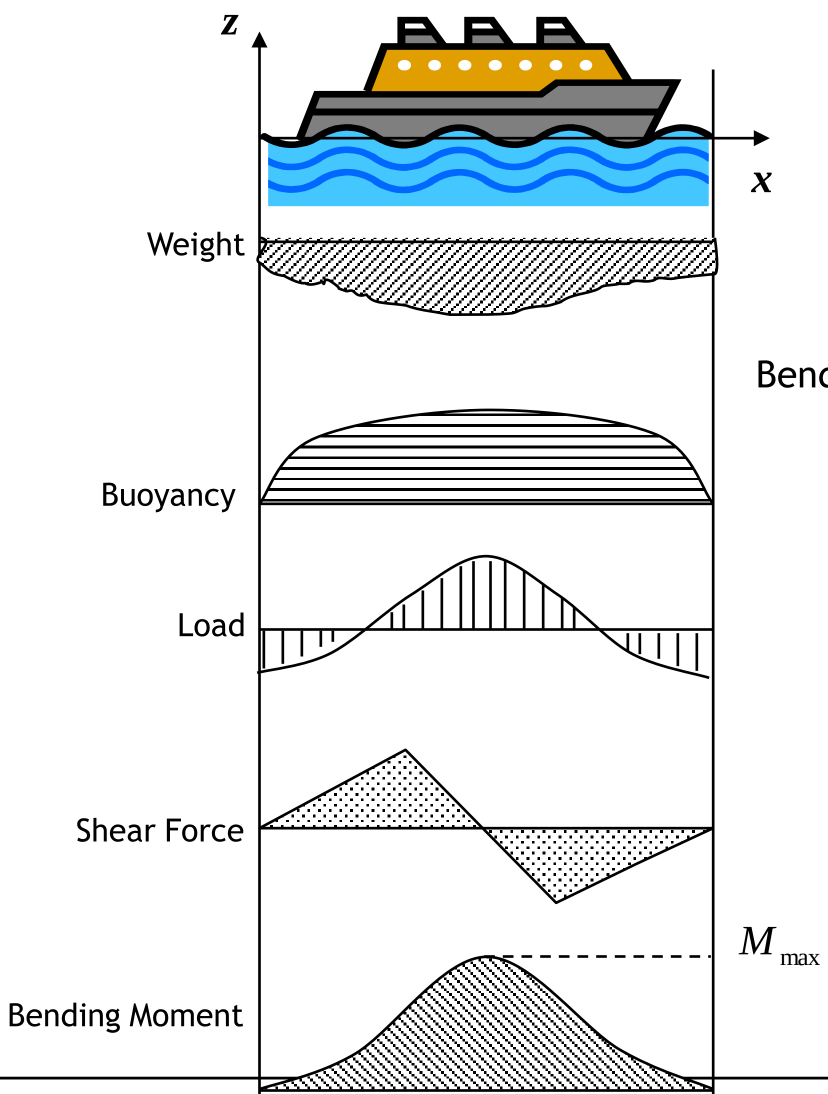
3) Ballasting Load

$$\sigma_3 = \sigma_{local}$$



Primary Stress (σ_1) and Longitudinal Strength of a Ship

- Longitudinal Stress Analysis



Load= Weight + Buoyancy

⇩

Shear Force = ∫(Load) dx

⇩

Bending Moment = ∫(Shear Force) dx

⇩

$$\sigma_L = \frac{M_{max}}{Z}$$ (Section Modulus $Z = \frac{I}{y}$)

⇩

$$\sigma_L \le 175 f_1 \; N/mm^2$$

(f_1 : Material constant
ex) mild Steel : f_1=1.0)

Beam Theory

w: Load

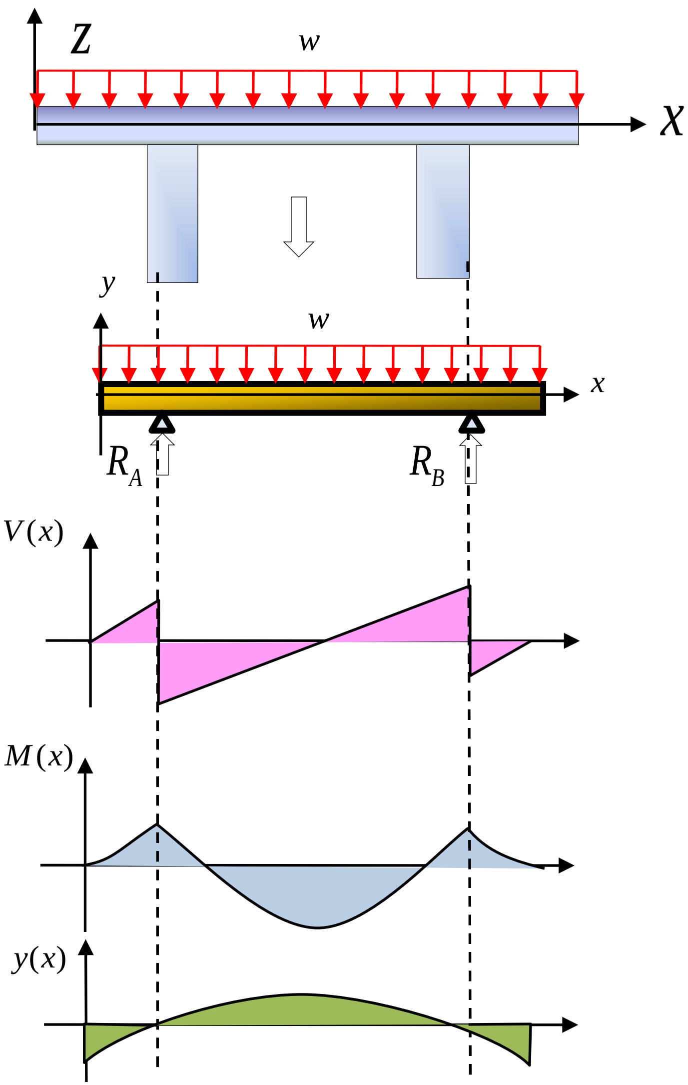
Shear Force:
$$V = -\int wdx$$

Bending Moment:
$$M = \int Vdx$$

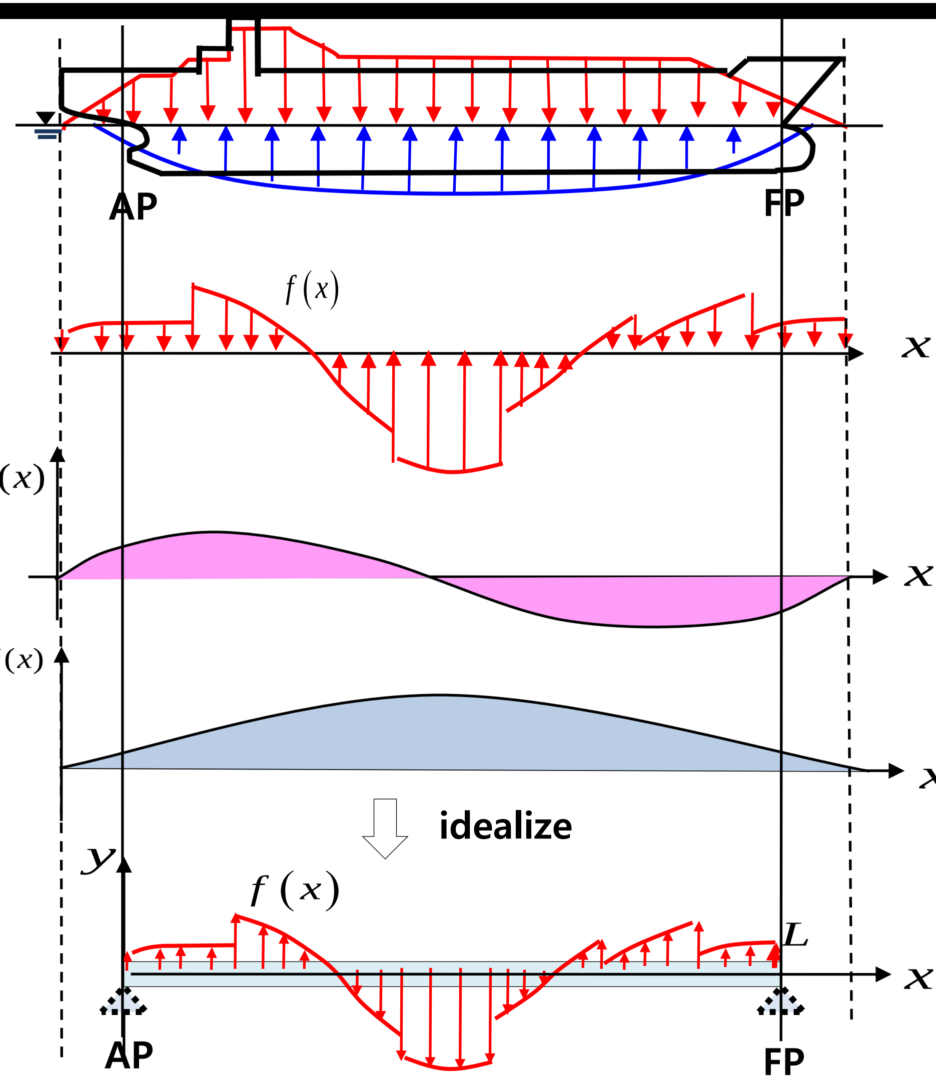
Primary Stress

☞ $\sigma_L = \sigma_1$

Applying Beam Theory on a Ship



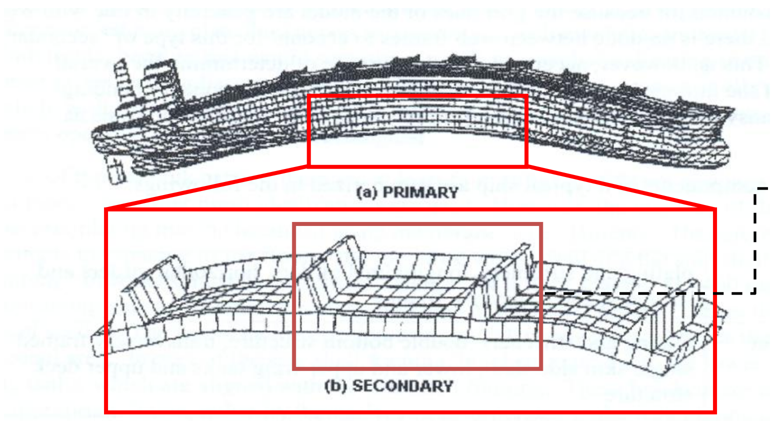
Deflection exist at the end point*

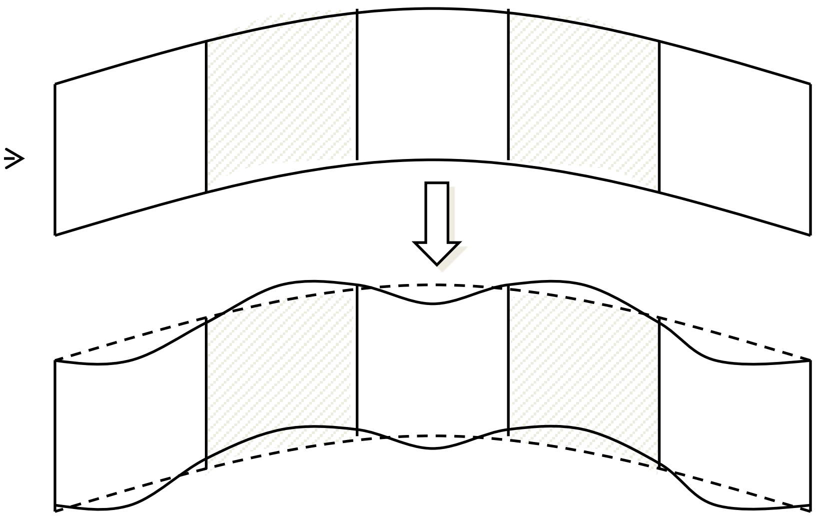


Assume : small deflection at the ends of the ship and imaginary supports at the AP and FP

*James M. Gere, Mechanics of Materials 6th Edition, Thomson, Chap.4, p.292

Grillage Analysis and Secondary Stress (σ_2)



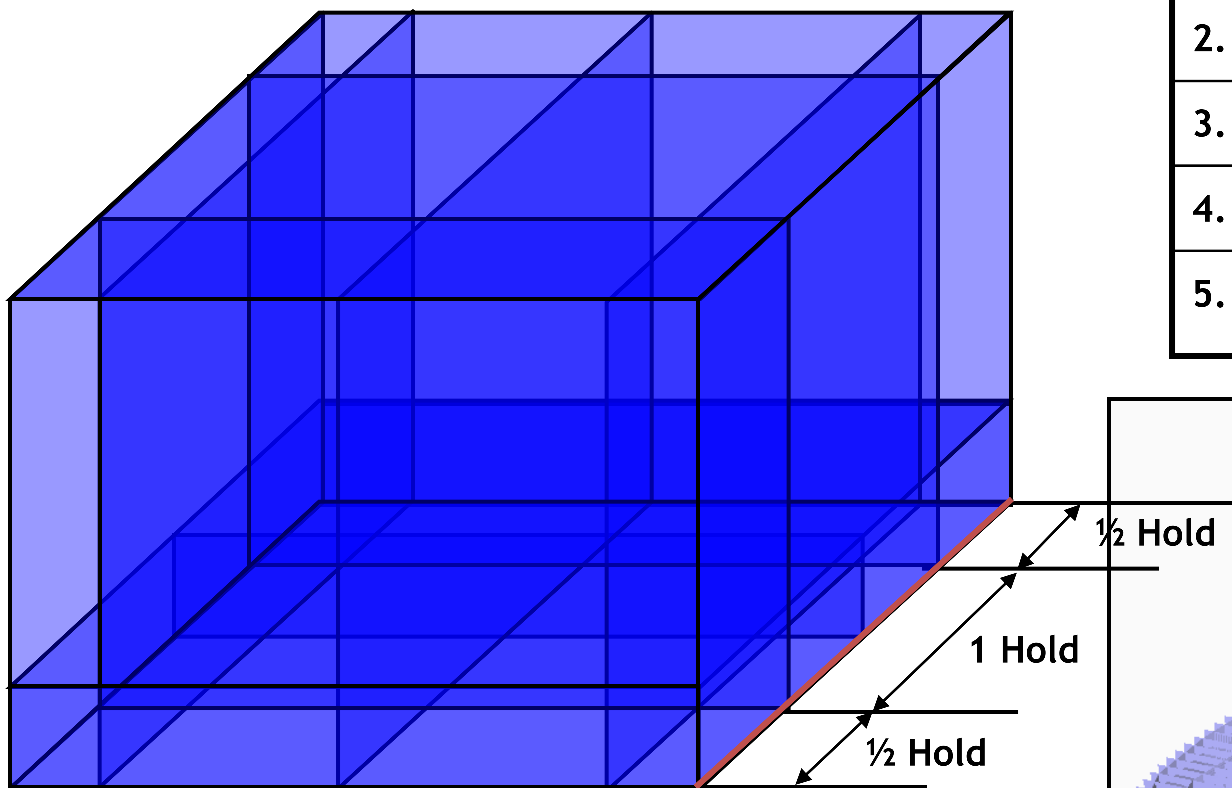


For a stiffened panel, there is the stress (σ2) and deflection of the global bending of the orthotropic stiffened panels, for example, the panel of bottom structure contained between two adjacent transverse bulkheads. The stiffener and the attached plating bend under the lateral load and the plate develops additional plane stresses since the plate acts as a flange with the stiffeners. In longitudinally framed ships there is also a second type of secondary stresses which corresponds to the bending under the hydrostatic pressure of the longitudinals between transverse frames (web frames). For transversally framed panels, this stress may also exist and would correspond to the bending of the equally spaced frames between two stiff longitudinal girder*

- Grillage Analysis : an analysis approach which models the cross-stiffened panel as a system of discrete intersecting beams, each beam being composed of stiffener and associated effective plating
- Object : to determine the distribution of deflection and stress over the length and width dimensions of the stiffened panel

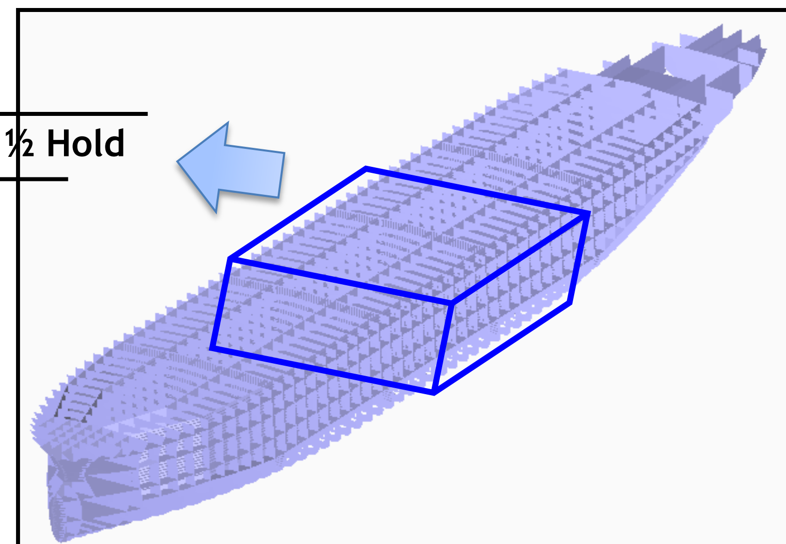
*Philippe RIGO and Enrico Rizzuto,, Ship Design and Construction,, 2002, chapter 18. Analysis and design of ship structure, p.18-21

Grillage Analysis : Midship Cargo Hold



Grillage Analysis

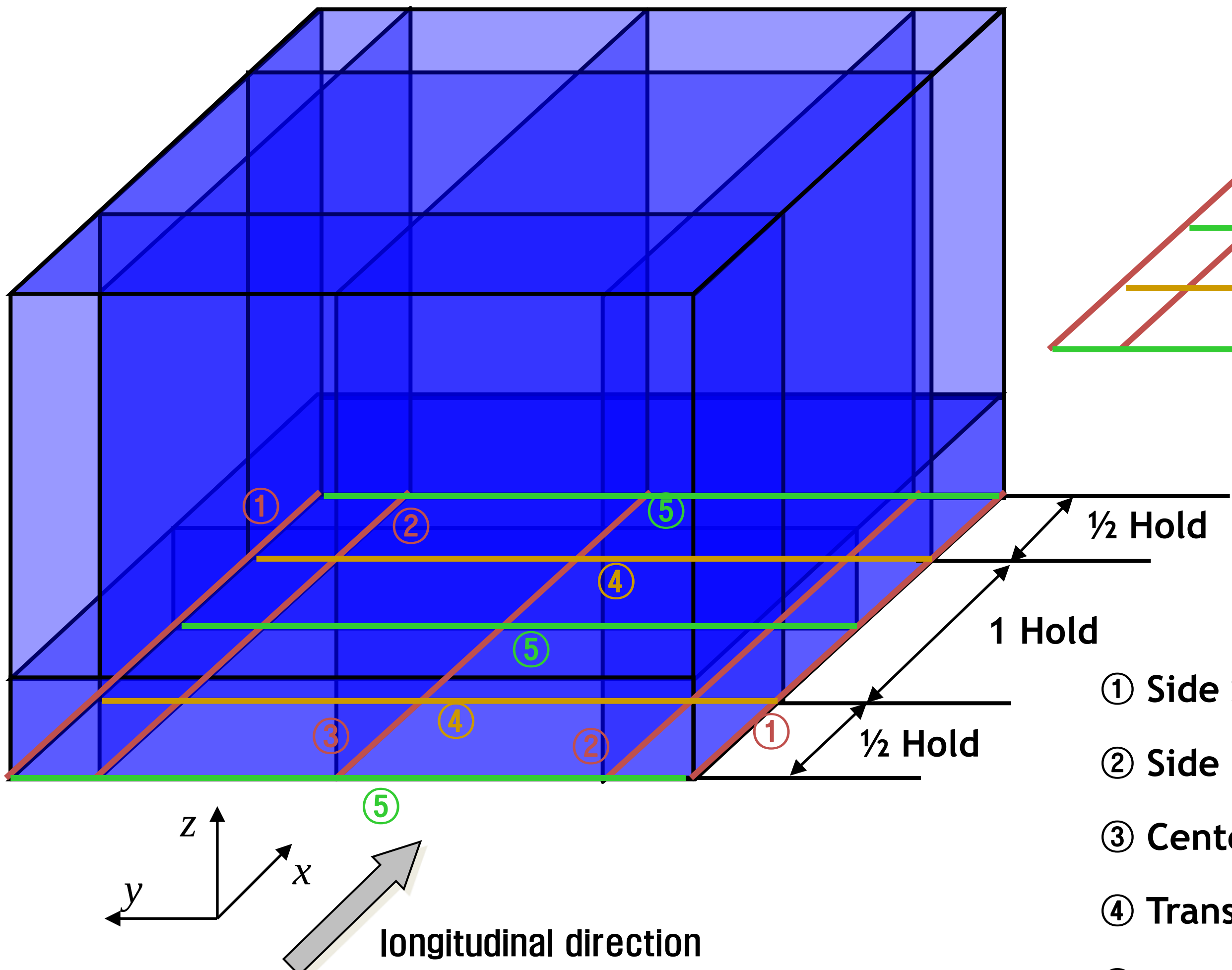
1. Grillage Model
2. Element Properties
3. Loading
4. Boundary Conditions
5. Solution

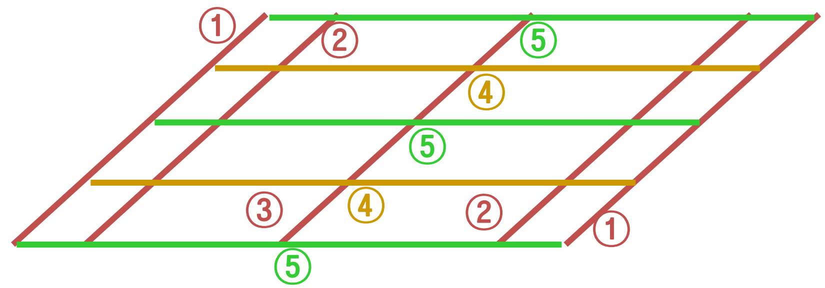


- Analysis Region : ½ Hold + 1 Hold + ½ Hold

Grillage Analysis : Midship Cargo Hold

Step1. **Grillage Model**





① **Side Shell**

② **Side Longitudinal Bulkhead**

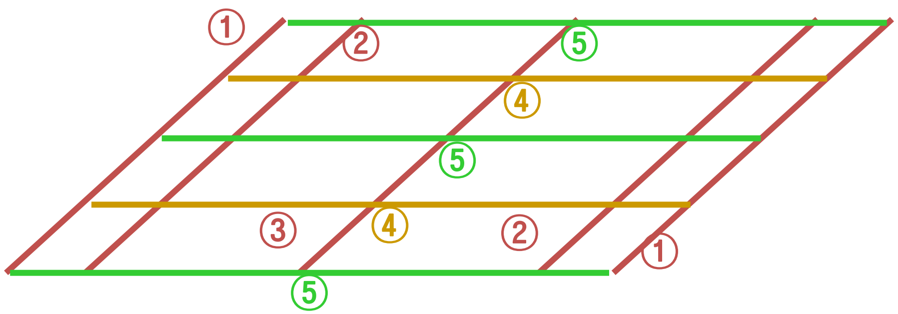
③ **Center Girder + Longitudinal Bulkhead**

④ **Transverse Bulkhead**

⑤ **Transverse Floor**

Grillage Analysis : Midship Cargo Hold

Step2. Properties for the Elements



NOMENCLATURE

D_T – Depth of Tank

$I_{\otimes}$ – Vertical Moment of Inertia of Full Midship Section

l – Spacing of Transverse Bulkheads

t_B – Thickness of Bottom Shell

t_D – Thickness of Deck Plating

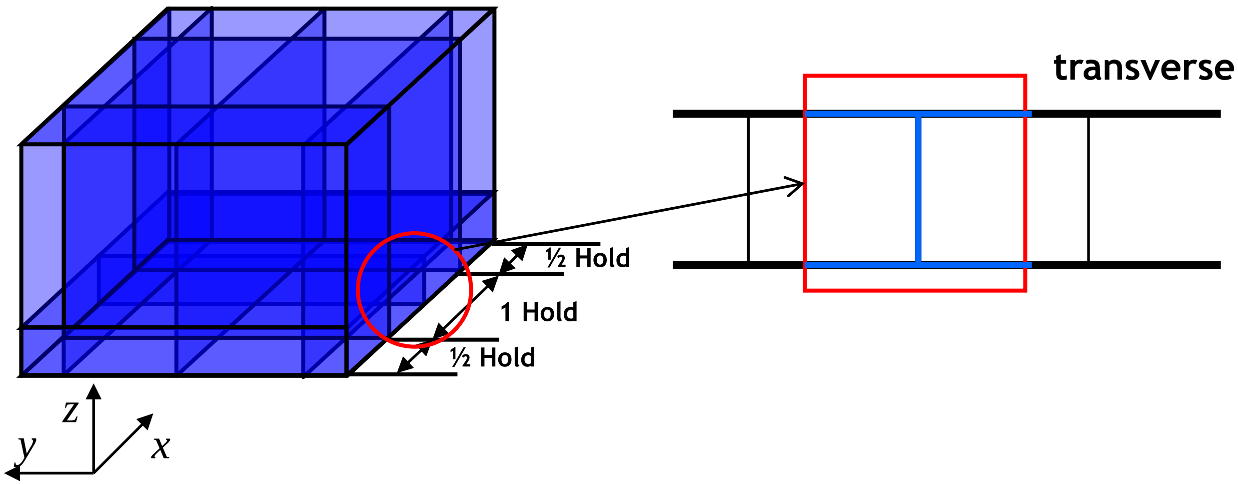
BAR TYPE (See Idealization)	TORSION CONSTANT (J)	INERTIA (I)
1.Center Longi. Bulkhead	$5 \times I_{\otimes}$	$0.11 \times I_{\otimes}$
2. Longitudinal Bulkhead	$5 \times I_{\otimes}$	$0.22 \times I_{\otimes}$
3. Side Shell	$5 \times I_{\otimes}$	$0.17 \times I_{\otimes}$
4. Bottom Transv. floor	10^{-5}	Moment of Inertia of I- type Beam element
5.Oil-tight Bulkhead	$l \cdot D_T^{\ 2} \cdot (t_B + t_D)/4$	Not less than $0.3 \times I_{\otimes}$

Grillage Analysis : Midship Cargo Hold

Step2. Element Properties

I : Moment of Inertia

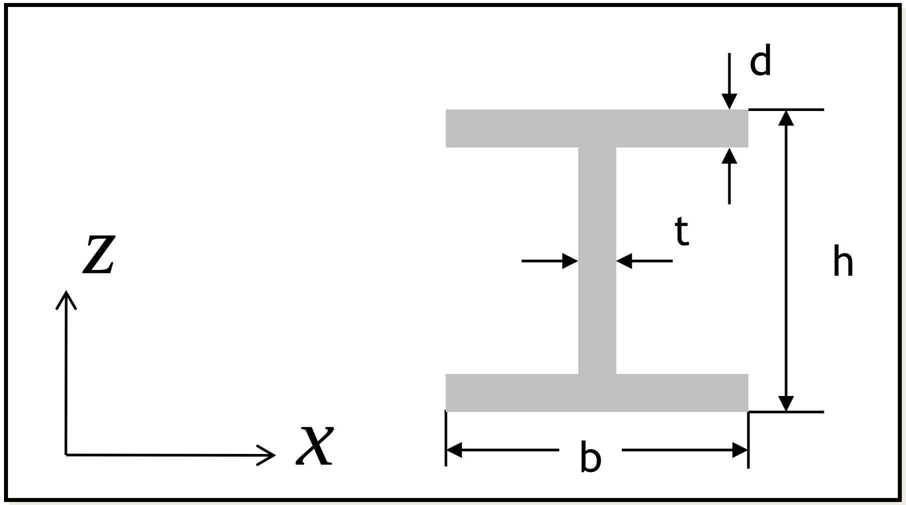
J : Polar Moment of Inertia



$$I_x = \frac{bh^3}{12} - \frac{(b-t)(h-2d)^3}{12}$$

$$I_z = \frac{2db^3}{12} + \frac{(h-2d)t^3}{12}$$

$$J = I_x + I_z$$



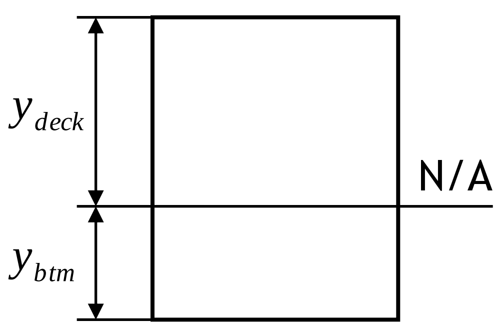
Grillage Analysis : Midship Cargo Hold

Vertical moment of inertia of the midship Section $I_{\otimes}$ is calculated by using the midship section modulus

<ex. given section modulus (cm³)>

	Rule Requirement	Design
Deck	18,274,500	22,036,400
Bottom	18,274,500	26,933,300

sol.)



① $y_{deck} + y_{btm} = Depth$

② $Z_{deck} = \dfrac{I_{\otimes}}{y_{deck}} \Rightarrow y_{deck} = \dfrac{I_{\otimes}}{Z_{deck}}$

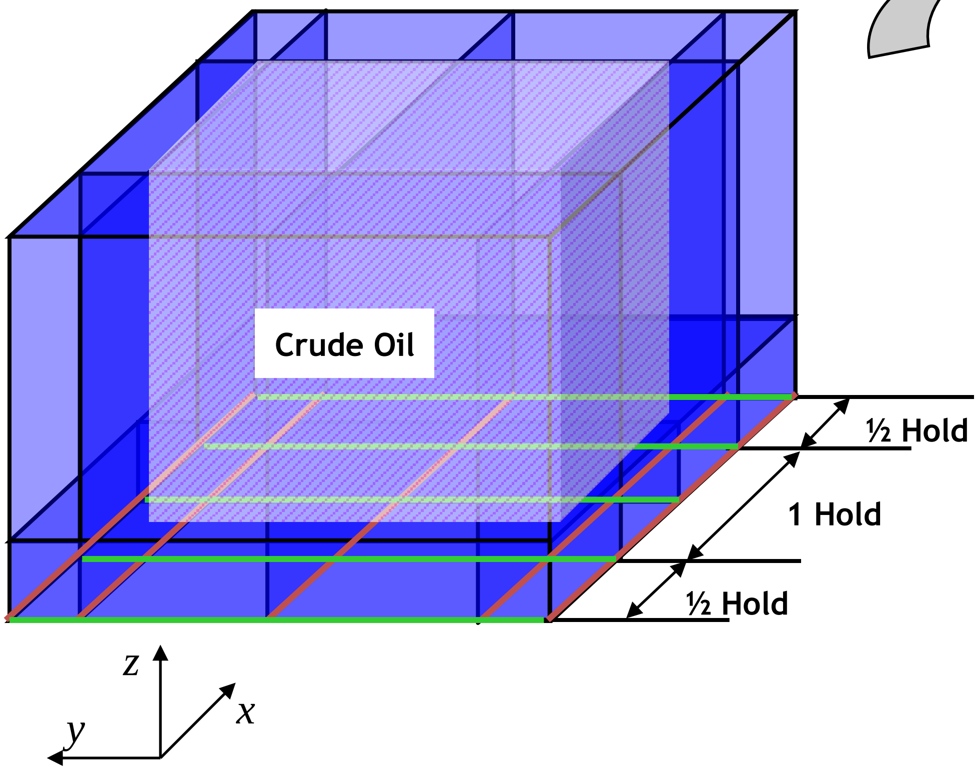
③ $Z_{btm} = \dfrac{I_{\otimes}}{y_{btm}} \Rightarrow y_{btm} = \dfrac{I_{\otimes}}{Z_{btm}}$

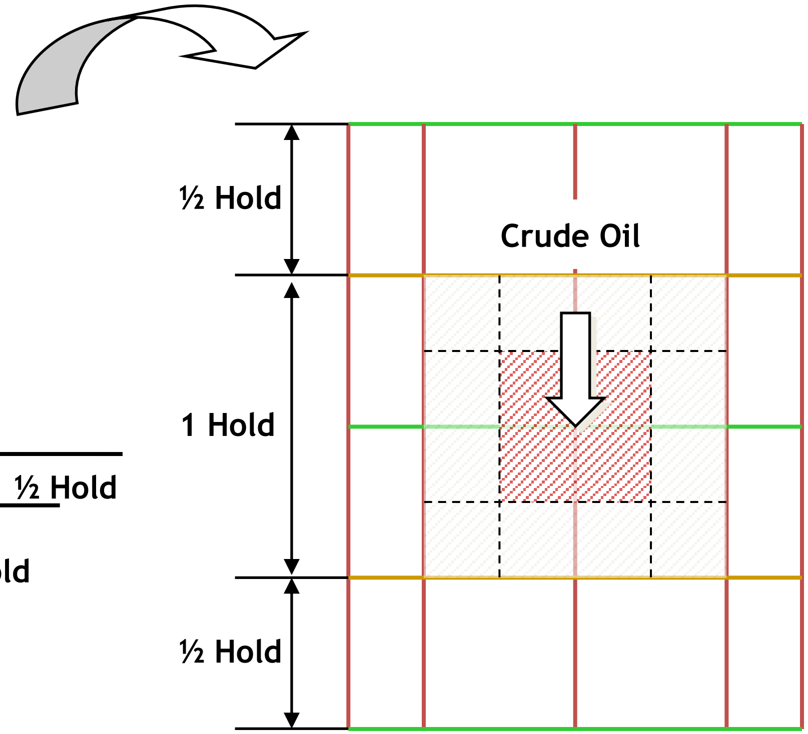
④ $\dfrac{I_{\otimes}}{Z_{deck}} + \dfrac{I_{\otimes}}{Z_{btm}} = Depth \Rightarrow I_{\otimes} = \dfrac{Depth \times (Z_{deck} Z_{btm})}{Z_{deck} + Z_{btm}}$

Grillage Analysis : Midship Cargo Hold

Step3. Loading

1. Cargo Load

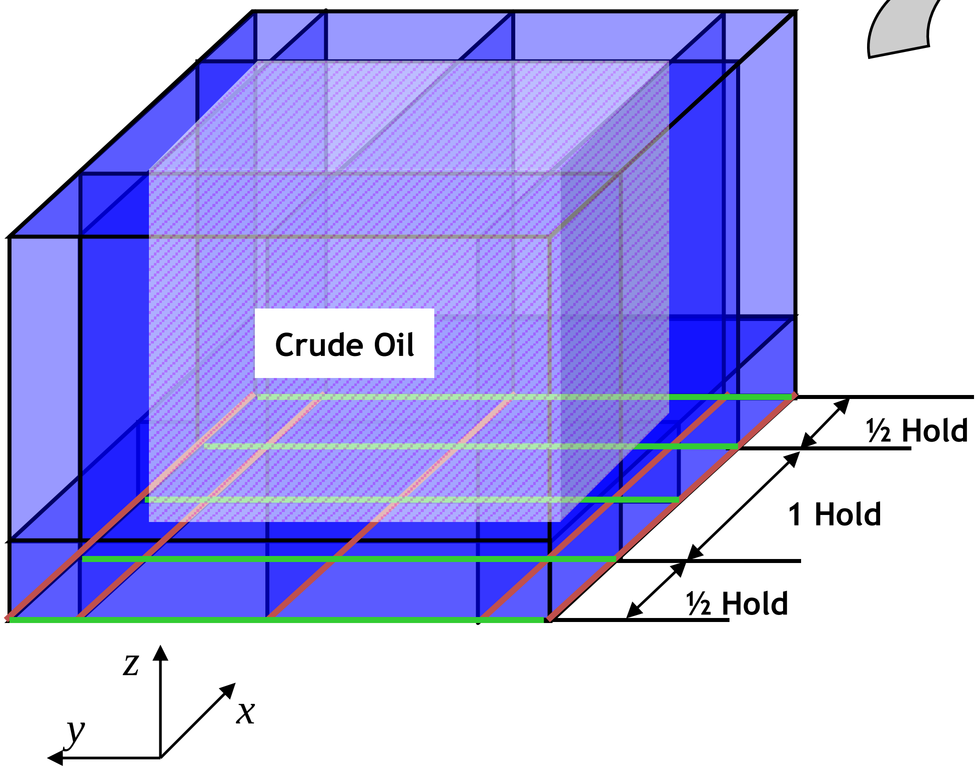


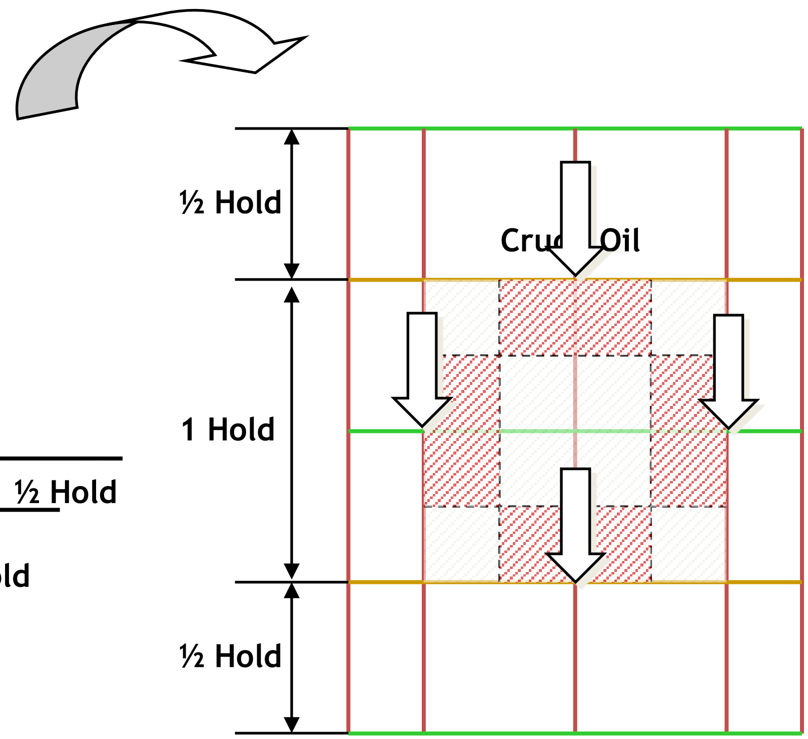


Grillage Analysis : Midship Cargo Hold

Step3. Loading

1. Cargo Load

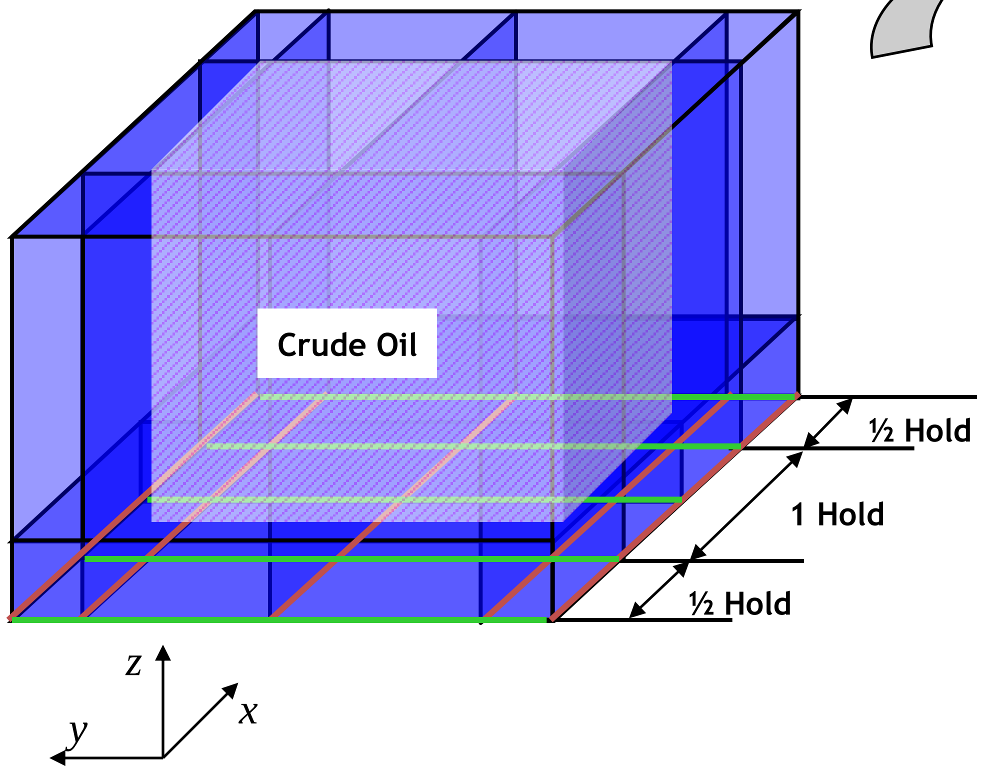


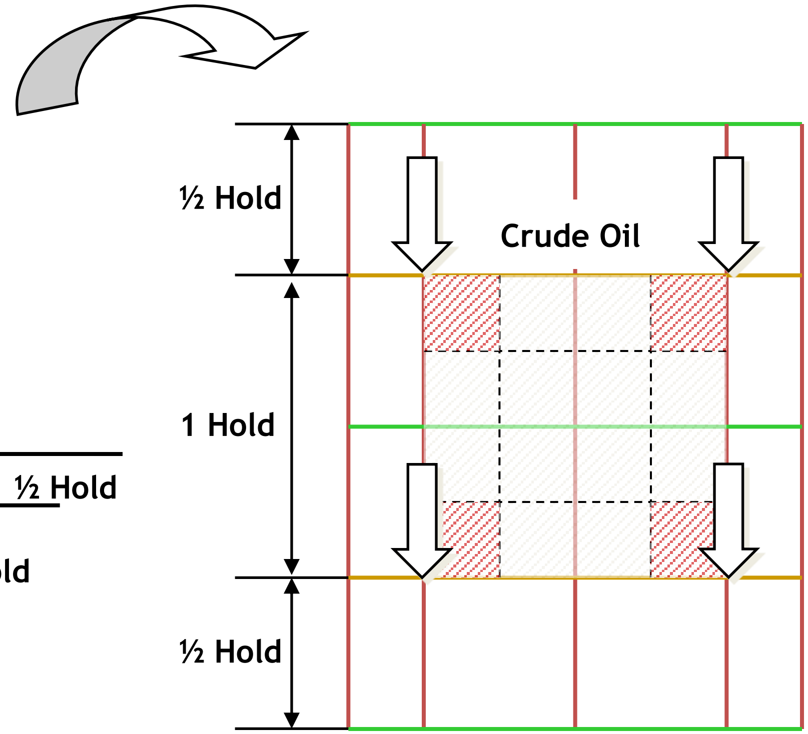


Grillage Analysis : Midship Cargo Hold

Step3. Loading

1. Cargo Load

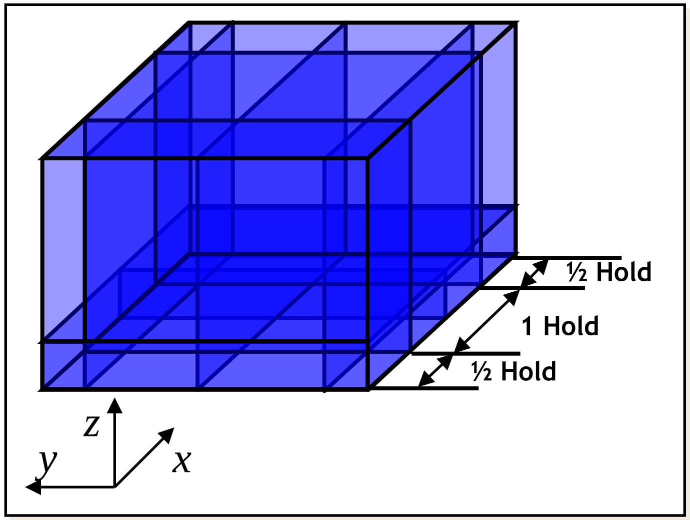


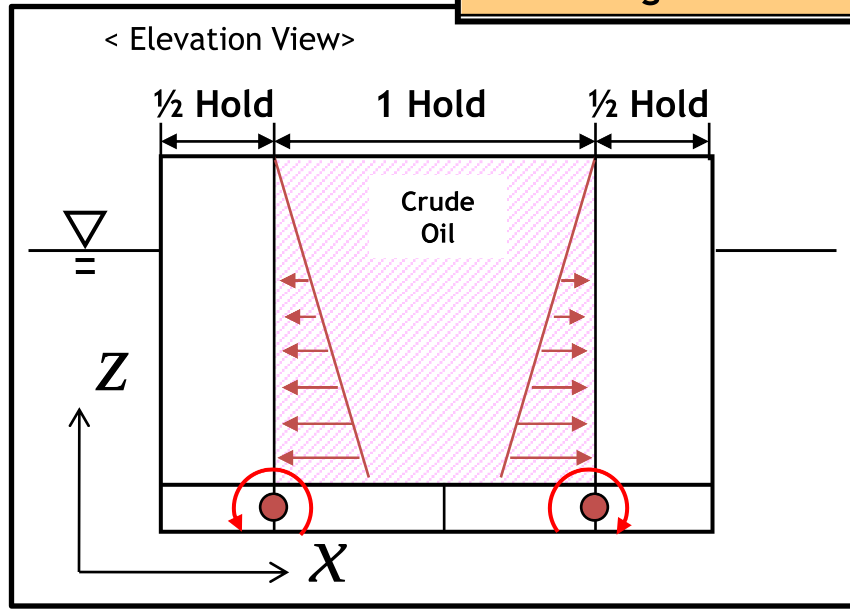


Grillage Analysis : Midship Cargo Hold

Step3. Loading

2. Moment : caused by the water pressure and cargo Load



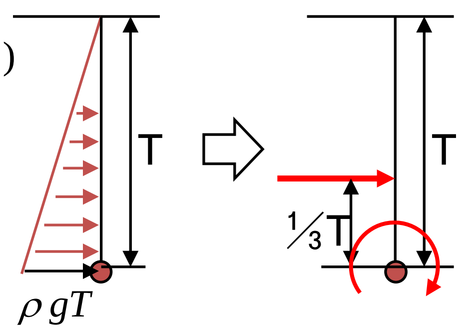


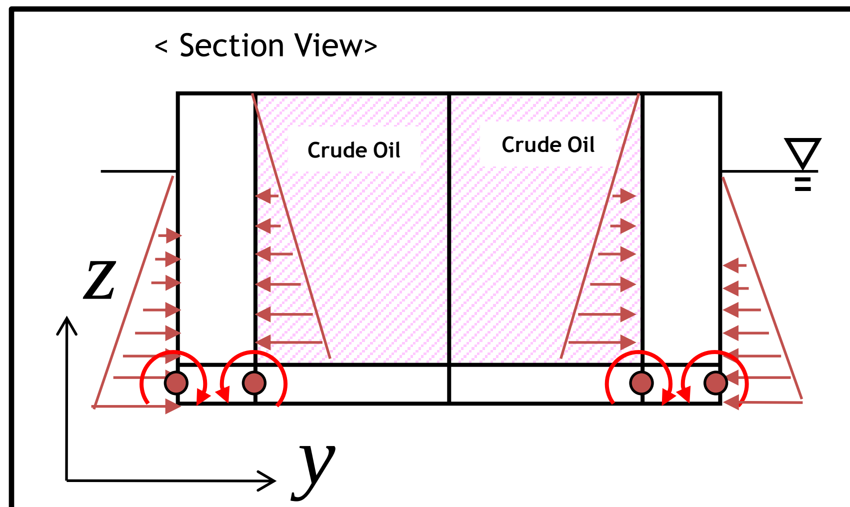
※ **Moment**

$$M = (Total\ \ Force) \times (Distance\ to\ the\ center)$$

$$= (\frac{1}{2} \times \rho g T \times T) \times (\frac{1}{3} T)$$

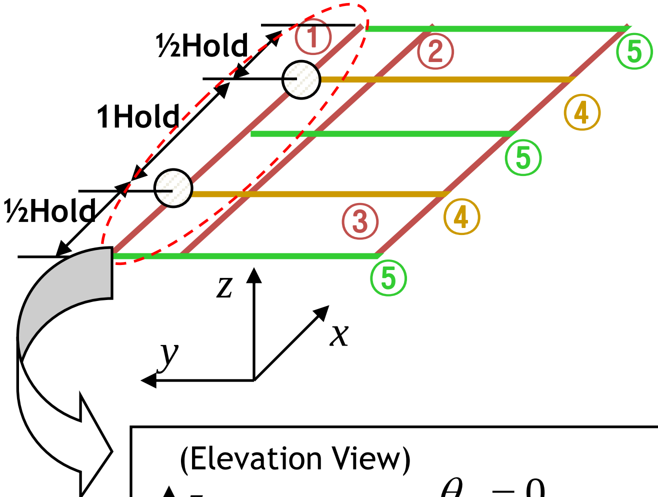
$$= \frac{1}{6} \rho g T^3$$





Grillage Analysis : Midship Cargo Hold

Step4. Boundary Conditions



① Side Shell

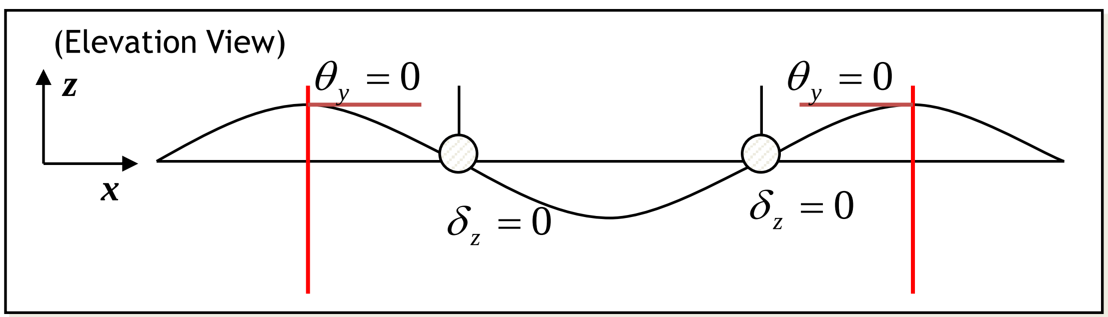
② Side Longitudinal Bulkhead

③ Center Girder + Longitudinal Bulkhead

④ Transverse Bulkhead

⑤ Transverse Floor

θ_x, θ_y : deformation angle about x-axis and y-axis



(1) Transversal Symmetry : $\theta_x = 0$ at Center Girder, because only half-width is considered

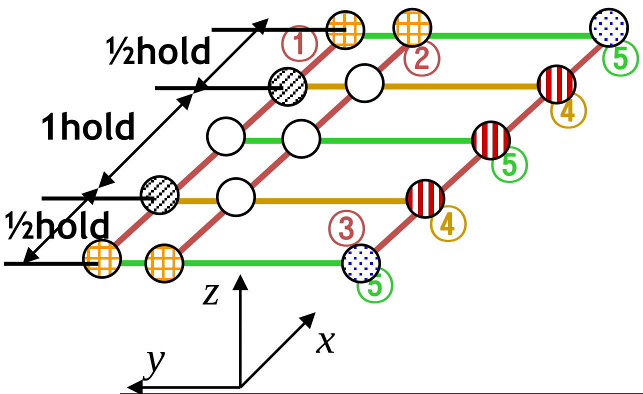
(2) Constraint : $\delta_z = 0$ at the intersection of Side Shell and T.BHD

(3) Longitudinal Symmetry : $\theta_y = 0$ at the end point of ½ Hold

※ $\theta_y = \frac{dz}{dx} = 0$

Grillage Analysis : Midship Cargo Hold

Step4. Boundary Conditions



① Side Shell

② Side Longitudinal Bulkhead

③ Center Girder + Longitudinal Bulkhead

④ Transverse Bulkhead

⑤ Transverse Floor

Grillage Constraints

$\delta_x = 0$

$\delta_y = 0$

$\theta_z = 0$

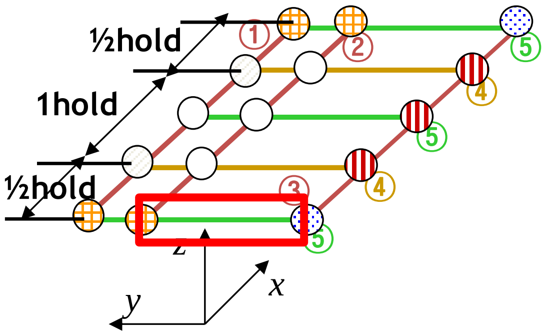
	Remark	θ_x	θ_y	δ_z	known (0 or Given)	unknown
	Constraints	−	−	0	M_x, M_y, δ_z	θ_x, θ_y, F_z
	Longitudinal Symmetry	−	0	−	M_x, θ_y, F_z	θ_x, M_y, δ_z
	Longitudinal and Transversal Symmetry	0	0	−	θ_x, θ_y, F_z	M_x, M_y, δ_z
	Transversal Symmetry	0	−	−	θ_x, M_y, F_z	M_x, θ_y, δ_z
	No Conditions	−	−	−	M_x, M_y, F_z	$\theta_x, \theta_y, \delta_z$

구속되지 않은 부분에 가해지는 힘과 구속된 부분의 변위가 주어짐. 예를 들어 첫 줄을 살펴보면 δ_z가 구속되어 있으며, θ_x θ_y 는 구속되어 있지 않으므로 변위 δ_z 및 모멘트 M_x, M_y 주어져야 하는 것이고, 이를 통해 θ_x, θ_y, F_z 를 계산하는 것임

Grillage Analysis : Midship Cargo Hold

1. Grillage Model
2. Element Properties
3. Loading
4. Boundary Conditions
5. Solution

step5. Displacement



① Side Shell
② Side Longitudinal Bulkhead
③ Center Girder + Longitudinal Bulkhead
④ Transverse Bulkhead
⑤ Transverse Floor

G : Shearing Modulus
E : Modulus of elasticity
I : Moment of Inertia
J : Polar Moment of Inertia

<Stiffness Matrix of Grillage*>

$$\begin{bmatrix} M_{x1} \\ M_{y1} \\ f_{z1} \\ M_{x2} \\ M_{y2} \\ f_{z2} \end{bmatrix} = \boxed{\begin{bmatrix} \frac{GJ}{L} & 0 & 0 & -\frac{GJ}{L} & 0 & 0 \\ 0 & \frac{4EI}{L} & -\frac{6EI}{L^2} & 0 & \frac{2EI}{L} & \frac{6EI}{L^2} \\ 0 & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & 0 & -\frac{6EI}{L^2} & -\frac{12EI}{L^3} \\ -\frac{GJ}{L} & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\ 0 & \frac{2EI}{L} & -\frac{6EI}{L^2} & 0 & \frac{4EI}{L} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & 0 & \frac{6EI}{L^2} & \frac{12EI}{L^3} \end{bmatrix}} \begin{bmatrix} \theta_{x1} \\ \theta_{y1} \\ \delta_{z1} \\ \theta_{x2} \\ \theta_{y2} \\ \delta_{z2} \end{bmatrix}$$

$$\Rightarrow [\mathbf{K}_{pq}]$$

<Coordinates Transformation>

$$[\mathbf{T}] = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\theta & \sin\theta & 0 \\ 0 & 0 & 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

<Stiffness Equation>

$$[\mathbf{F}_{xy}] = [\mathbf{T}]^T [\mathbf{K}_{pq}][\mathbf{T}][\delta_{xy}]$$

$$\Downarrow$$

$$[\mathbf{F}_{xy}] = [\mathbf{K}_{xy}][\delta_{xy}]$$

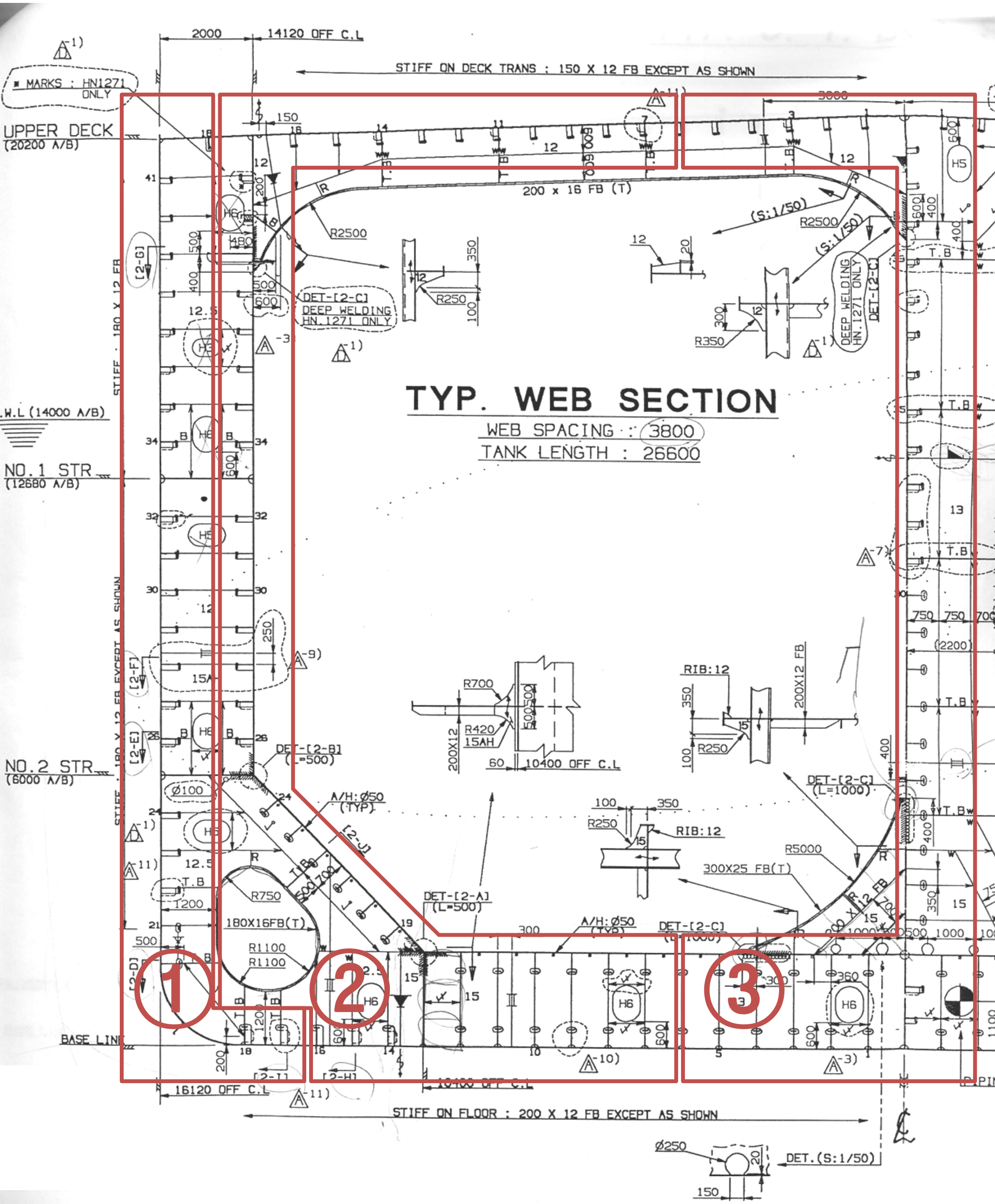
22 equations

⇩

superposition

***Refer to the Lecture Note on "Computer Aided Ship Design", Fall 2011, Kyu Yeul Lee**

Grillage Analysis : Midship Cargo Hold



72.5K Oil Tanker Principal Dimensions
LOA : 228.50m
LBP : 219.00m
Breadth : 32.24m
Depth : 20.20m
Draft Scantling : 14.00m
Draft Design :12.20m

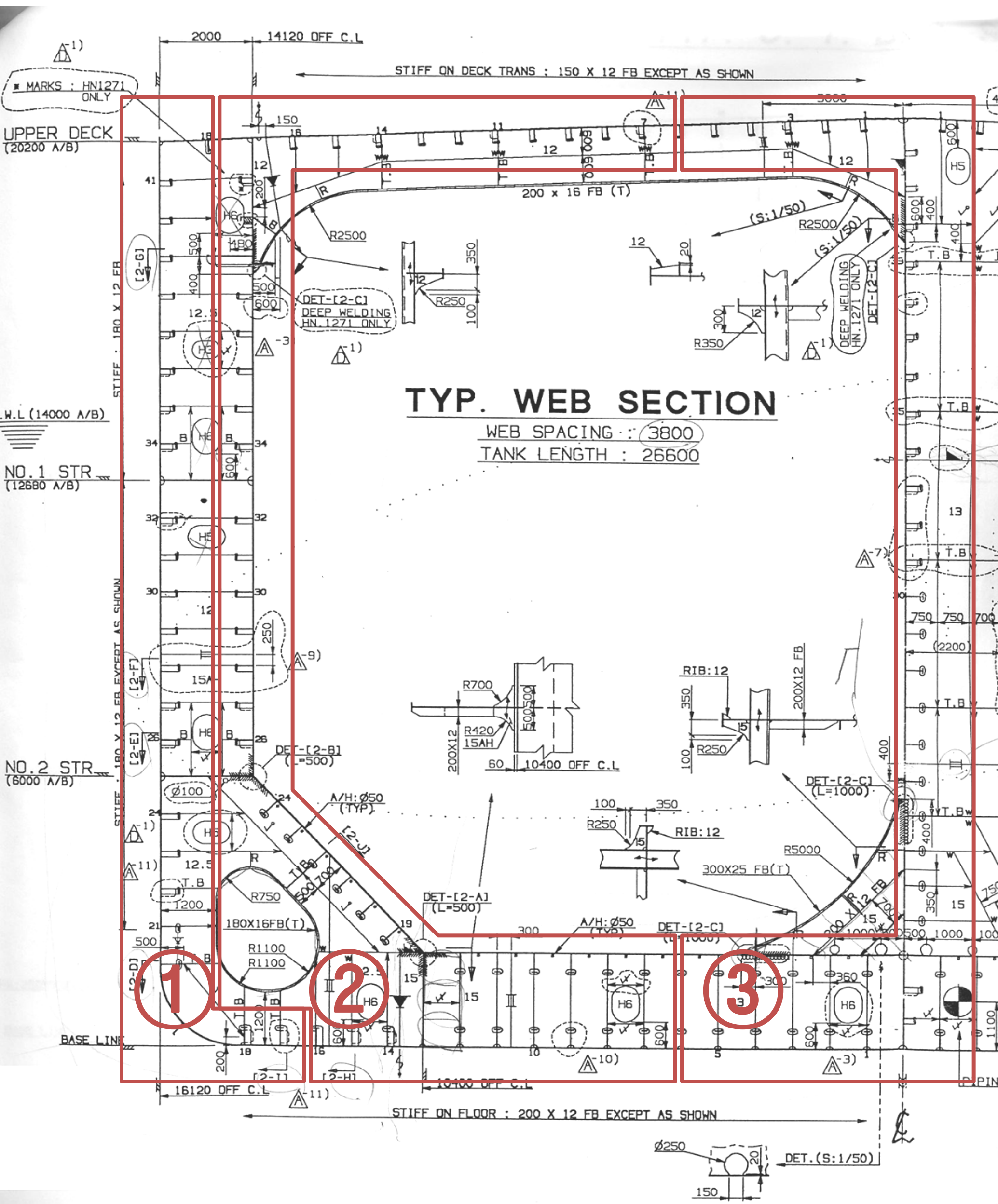
Web Frame Space : 3,800mm
Cargo Tank length : 26,600mm
Number of Web between Transverse Bulkhead : 6

② Side Longitudinal Bulkhead

① Side Shell

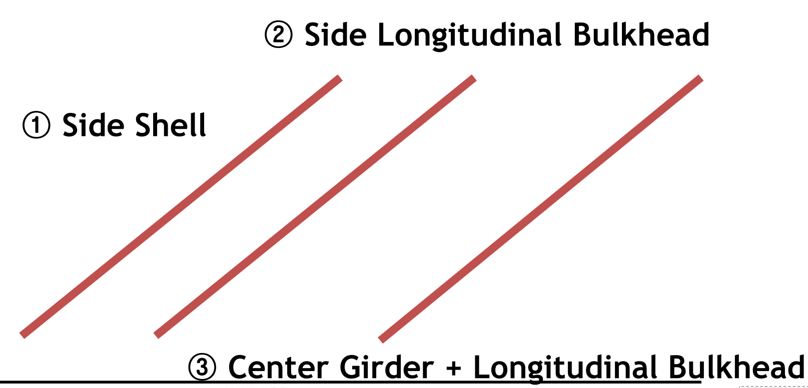
③ Center Girder + Longitudinal Bulkhead

Ex.) Grillage Analysis



72.5K Oil Tanker Principal Dimensions
LOA : 228.50m
LBP : 219.00m
Breadth : 32.24m
Depth : 20.20m
Draft Scantling : 14.00m
Draft Design :12.20m

Web Frame Space : 3,800mm
Cargo Tank length : 13,300mm
Number of Web between Transverse Bulkhead : 3

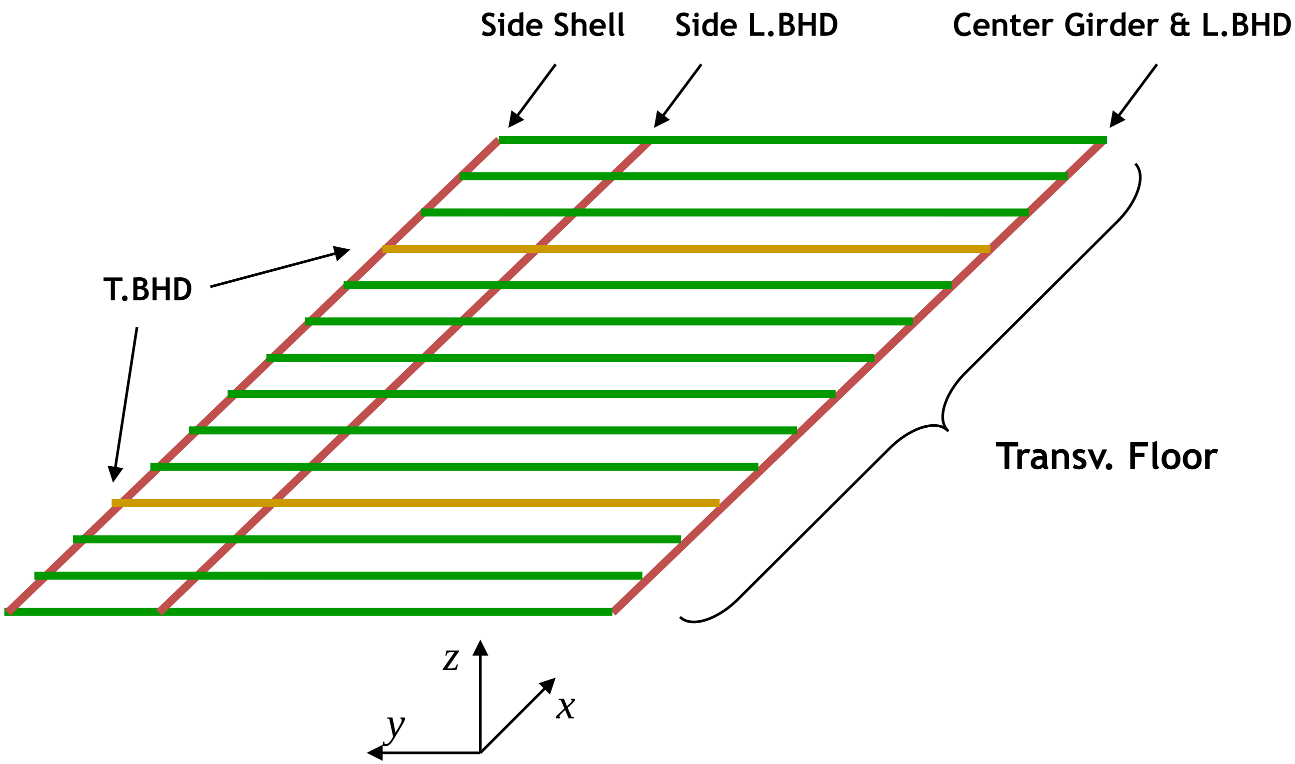


Ex.) Grillage Analysis

1. Grillage Model
2. Element Properties
3. Loading
4. Boundary Conditions
5. Solution

Step1. Grillage Model

- Analysis Region : ½ Hold + 1 Hold + ½ Hold



Ex.) Grillage Analysis

Step2. Element Properties

< Section Modulus[cm²-m] >

	Rule REQ	Design
Deck	18,274,500	22,036,400 [cm^3]= 22.0364 [m^3]
Bottom	18,274,500	26,933,300 [cm^3]= 26.9333 [m^3]

$$\therefore I_{\otimes} = \frac{Depth \times (Z_{deck} Z_{btm})}{Z_{deck} + Z_{btm}} = \frac{20.20 \times (22.3464 \times 26.9333)}{22.3464 + 26.9333} = 244.824[m^4]$$

BAR TYPE	TORSION CONSTANT (J)	INERTIA (I)
1.Center Longi. Bulkhead	$5 \times I_{\otimes}$ =1224.12 [m^4]	$0.11 \times I_{\otimes}$ = 26.93 [m^4]
2. Longitudinal Bulkhead	$5 \times I_{\otimes}$ =1224.12 [m^4]	$0.22 \times I_{\otimes}$ = 53.86 [m^4]
3. Side Shell	$5 \times I_{\otimes}$ =1224.12 [m^4]	$0.17 \times I_{\otimes}$ = 41.62 [m^4]
4. Bottom Transv. floor	10^{-5} [m^4]	0.1335 [m^4]
5.Oil-tight Bulkhead	$l \cdot D_T^{\ 2} \cdot (t_B + t_D)/4$ = 65.36 [m^4]	Not less than $0.3 \times I_{\otimes}$ = 73.45 [m^4]

NOMENCLATURE

D_T – Depth of Tank

$I_{\otimes}$ – Vertical Moment of Inertia of Full Midship Section

l – Spacing of Transverse Bulkheads

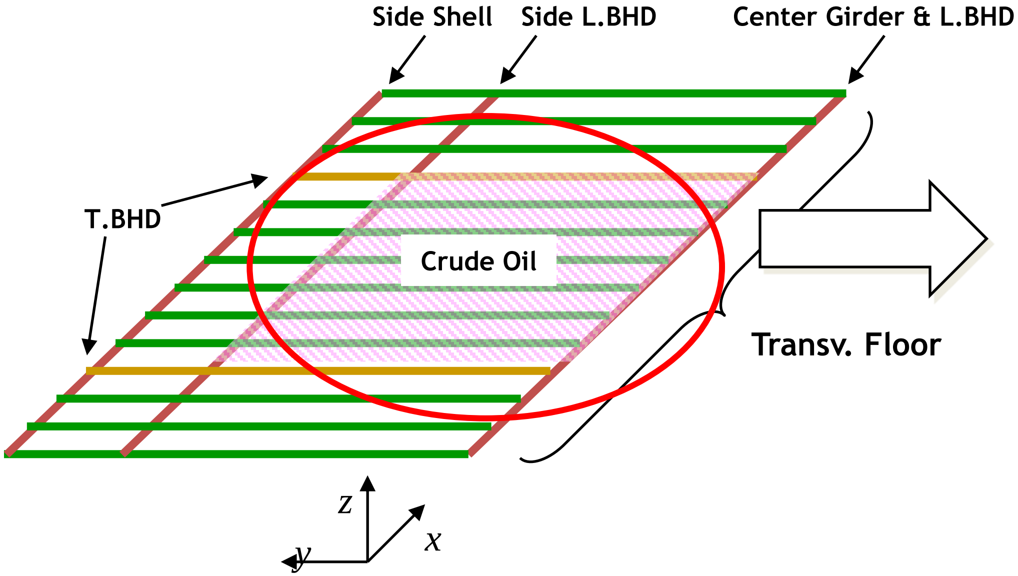
t_B – Thickness of Bottom Shell

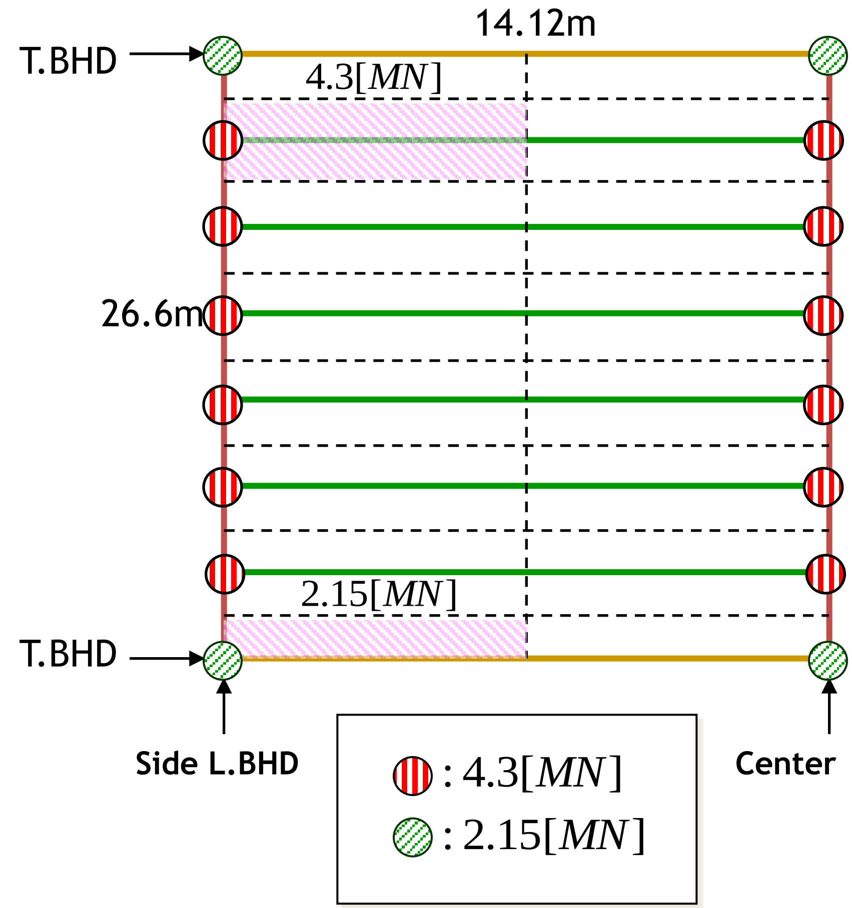
t_D – Thickness of Deck Plating

$$l \cdot D_T^{\ 2} \cdot (t_B + t_D)/4$$
$$= 26.6 \cdot 18.1 \cdot (0.015 + 0.015)/4$$
$$= 65.36$$

Ex.) Grillage Analysis

Step3. Loading





- Cargo Load

$$W_{oil} = \rho_{oil} \times (\text{Cargo Volume}) = 0.90 \times 9.81 \times 26.6 \times 14.12 \times 18.1$$
$$= 60{,}021\ [KN] = 60\ [MN]$$

- Nodal Load

① at the intersection of L.BHD and Transverse Floor

$$W_{oil} / 14 \approx 4.3[MN]$$

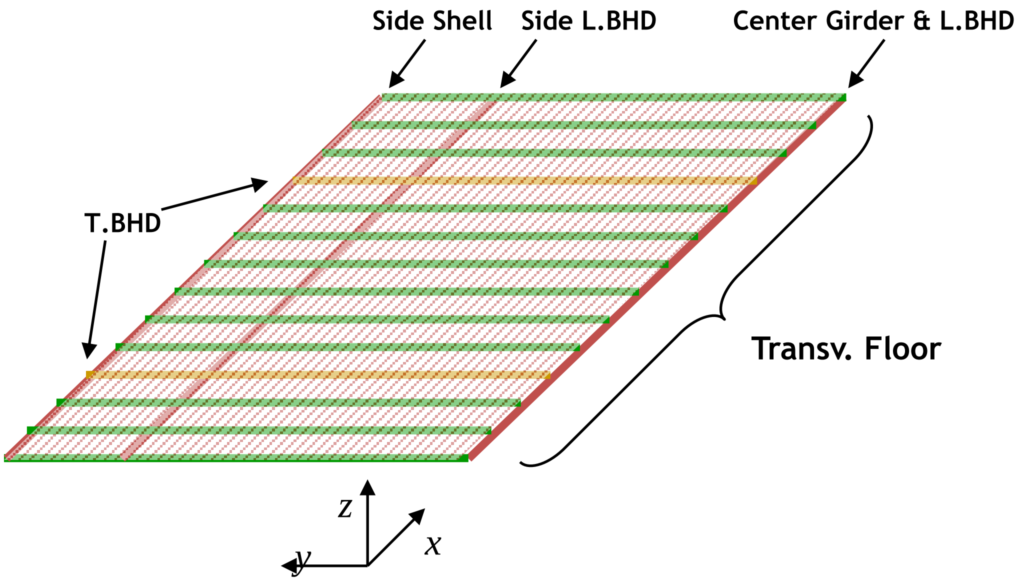
② at the intersection of L.BHD and T.BHD Floor

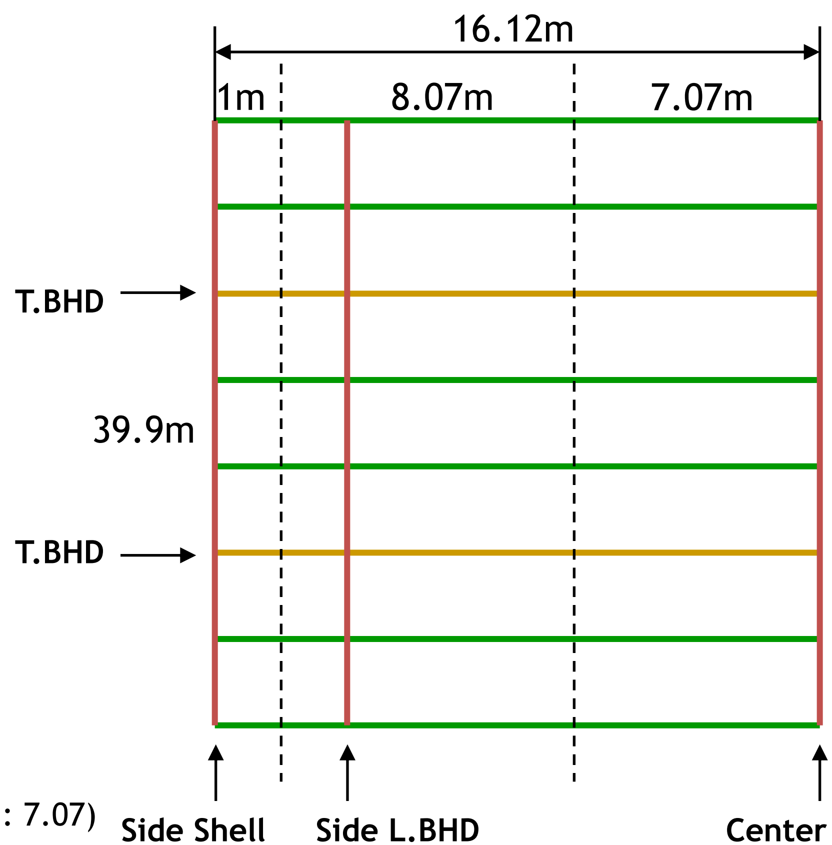
$$4.3 / 2 = 2.15[MN]$$

Ex.) Grillage Analysis

1. Grillage Model
2. Element Properties
3. Loading
4. Boundary Conditions
5. Solution

Step3. Loading





- Load by Water Pressure

$$B = \rho_{sea} \times T \times (area\ of\ \ cargo\ hold\ \ bottom)$$
$$= 1.024 \times 9.81 \times 14 \times (16.12 \times 39.9)$$
$$= 90,456[KN] = 90.46[MN]$$

- Nodal Load

① distribute the load depending on the ratio of node width (1 : 8.07 : 7.07)

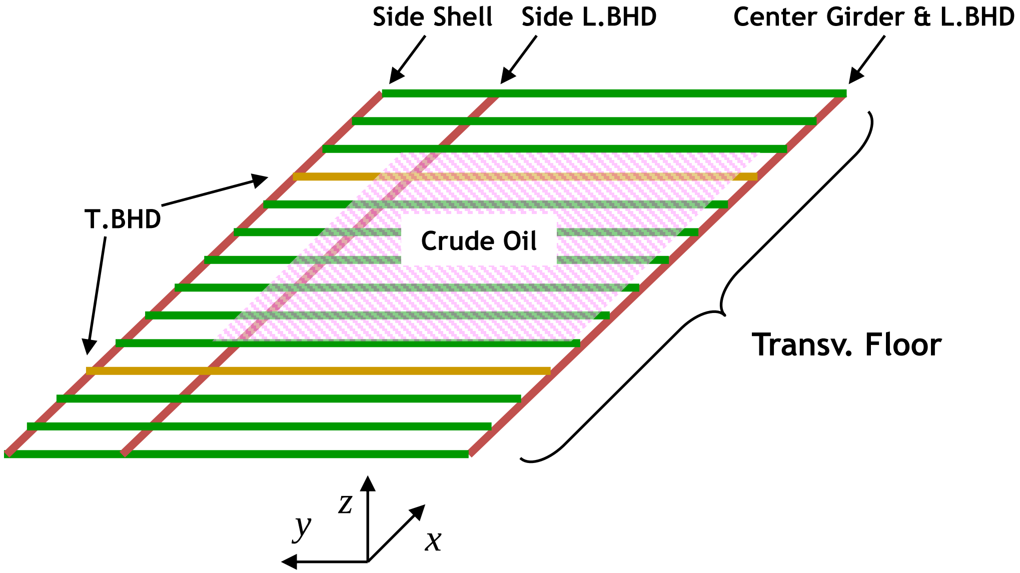
② divide the distributed load by the number of nodes(14).

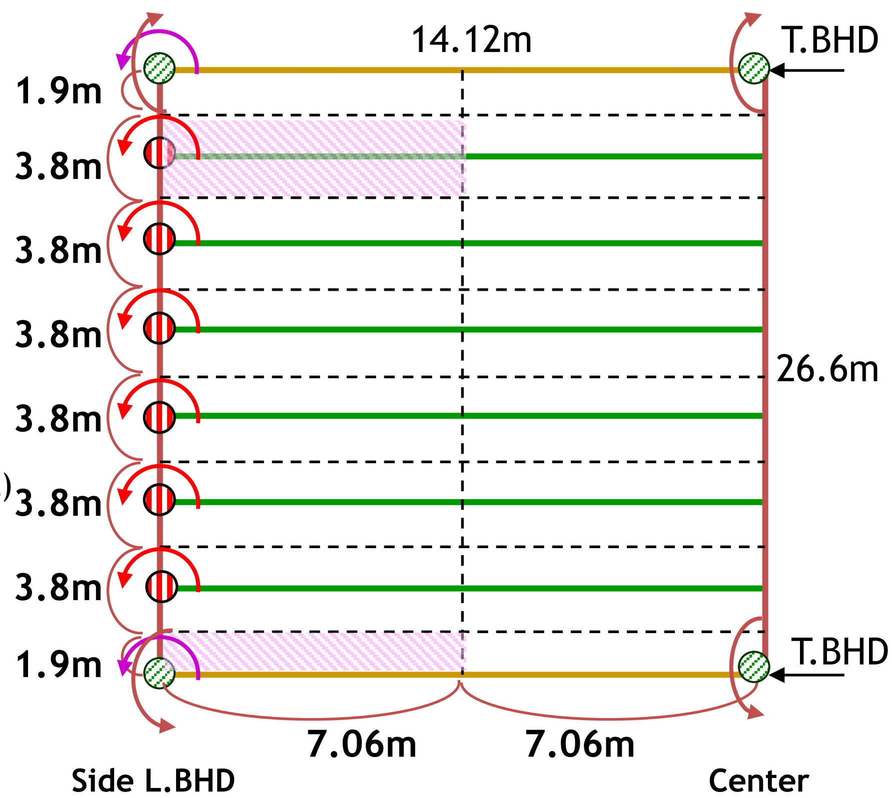
Ex.) Grillage Analysis

1. Grillage Model
2. Element Properties
3. Loading
4. Boundary Conditions
5. Solution

Step3. Load

- moment by cargo load





- Moment per Length (D: cargo hold height , d: double bottom height)

$$= \rho_{oil} \cdot \frac{D^2}{2} \cdot \left(\frac{D}{3} + \frac{d}{2} \right)$$

$$= 0.9 \cdot 9.81 \cdot \frac{18.1^2}{2} \left(\frac{18.1}{3} + \frac{2.1}{4} \right)$$

$$= 9,484 [KN \cdot m / m] = 9.48 [MN \cdot m / m]$$

- Nodal Moment

$$M_{x1} = (moment\ \ per\ \ length) \times 3.8 = 35.08 [MN \cdot m]$$

$$M_{x2} = (moment\ \ per\ \ length) \times 1.9 = 17.54 [MN \cdot m]$$

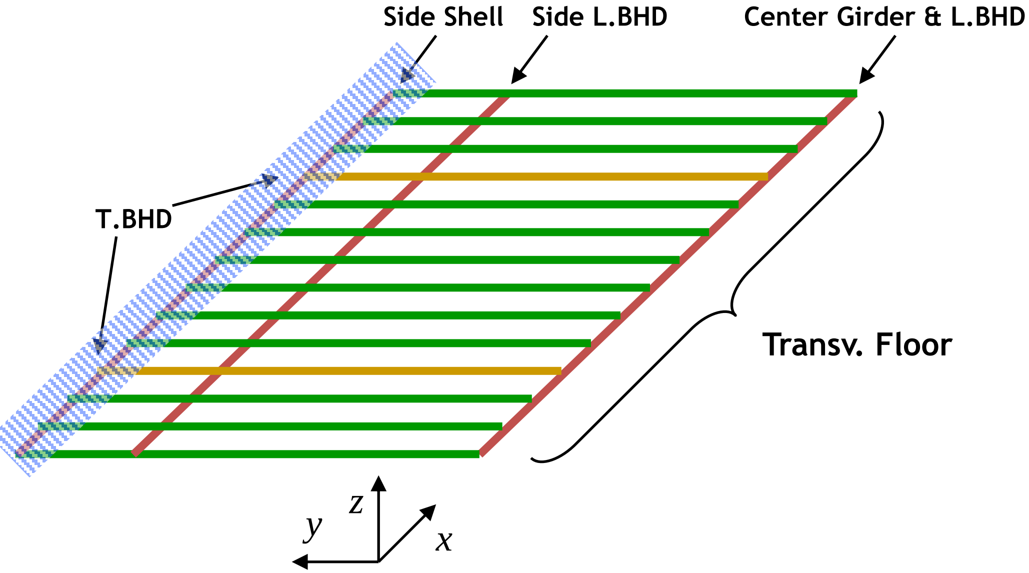
$$M_{y} = (moment\ \ per\ \ length) \times 7.06 = 65.18 [MN \cdot m]$$

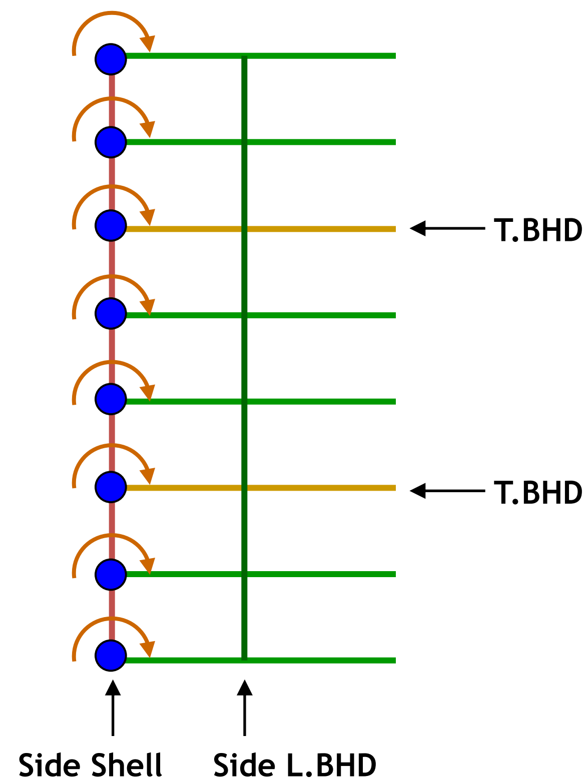
Ex.) Grillage Analysis

1. Grillage Model
2. Element Properties
3. Loading
4. Boundary Conditions
5. Solution

Step3. Load

- moment by water pressure





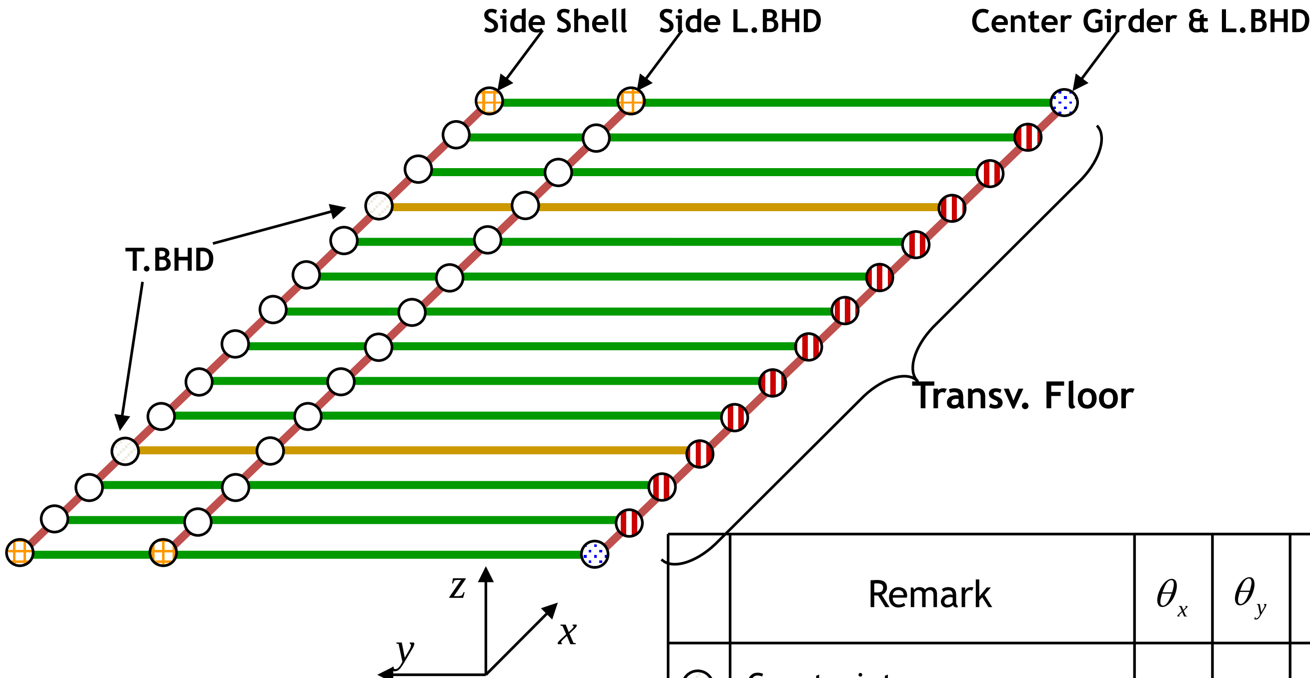
- Moment per Length(T: draft , d: double bottom height)

$$= \rho_{sea} \cdot \frac{(T-d/2)^2}{2} \cdot \frac{(T-d/2)}{3} = \rho_{sea} \cdot \frac{(T-d/2)^3}{6}$$

- Nodal Moment

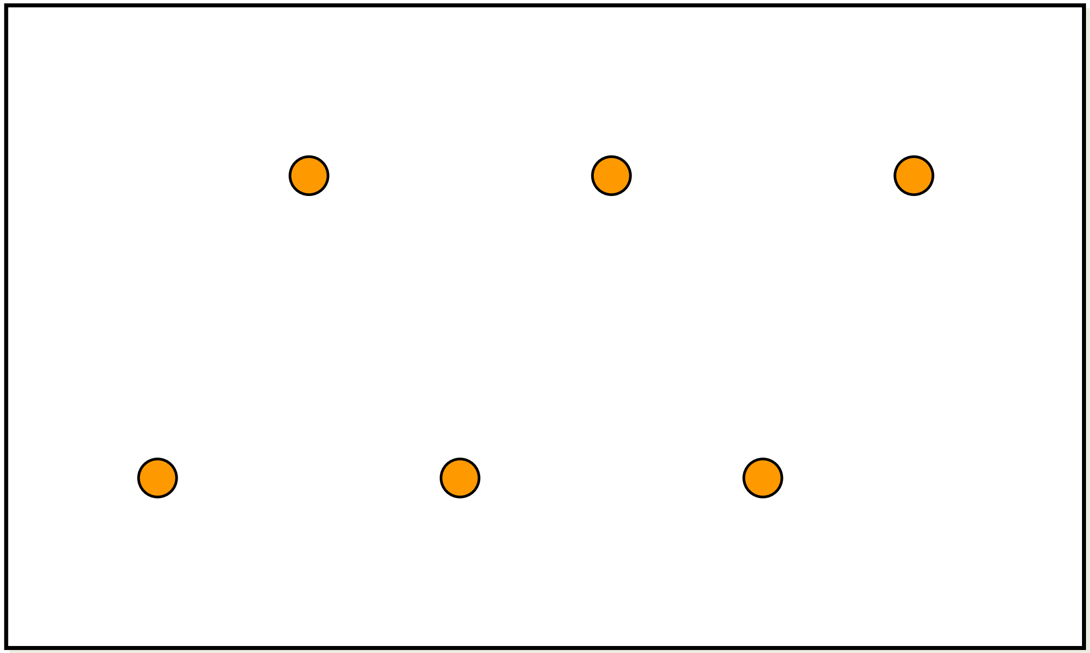
$$M_x = \rho_{sea} \cdot \frac{(T-d/2)^3}{6} \times 3.8 = 82.90[MN \cdot m]$$

Ex.) Grillage Analysis



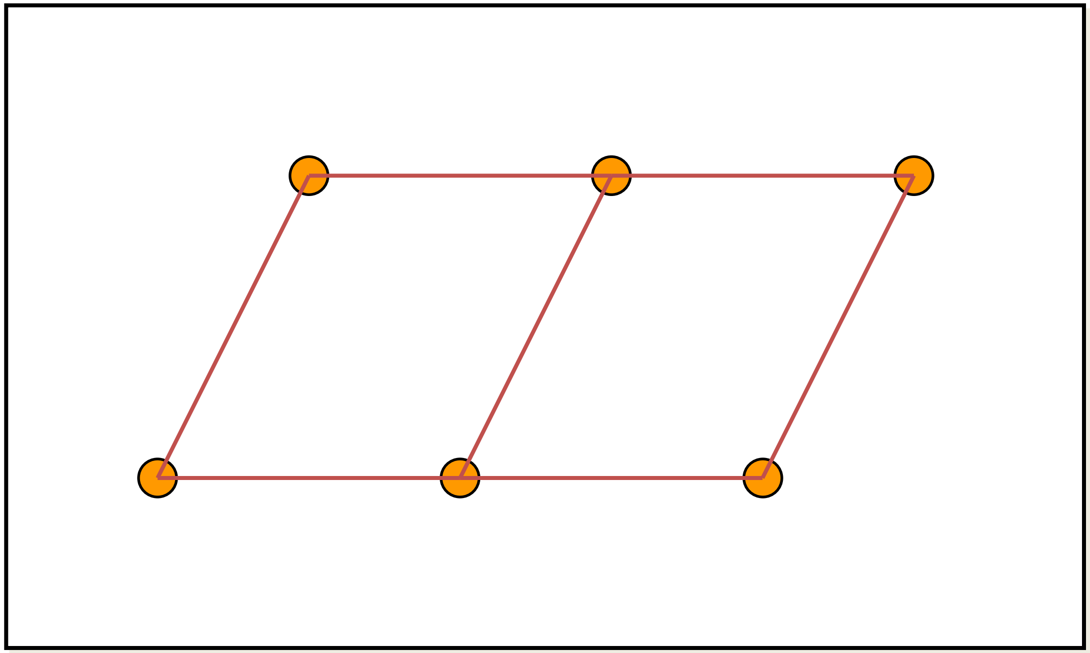
	Remark	θ_x	θ_y	δ_z	known (0 or Given)	unknown
	Constraints	–	–	0	M_x, M_y, δ_z	$\theta_x,\ \theta_y,\ F_z$
	Longitudinal Symmetry	–	0	–	$M_x,\ \theta_y,\ F_z$	$\theta_x,\ M_y, \delta_z$
	Longitudinal and Transversal Symmetry	0	0	–	$\theta_x,\ \theta_y,\ F_z$	M_x, M_y, δ_z
	Transversal Symmetry	0	–	–	$\theta_x,\ M_y, F_z$	$M_x,\ \theta_y,\ \delta_z$
	No Conditions	–	–	–	M_x, M_y, F_z	$\theta_x,\ \theta_y,\ \delta_z$

Grillage Analysis Program



Step1. Input : Nodes

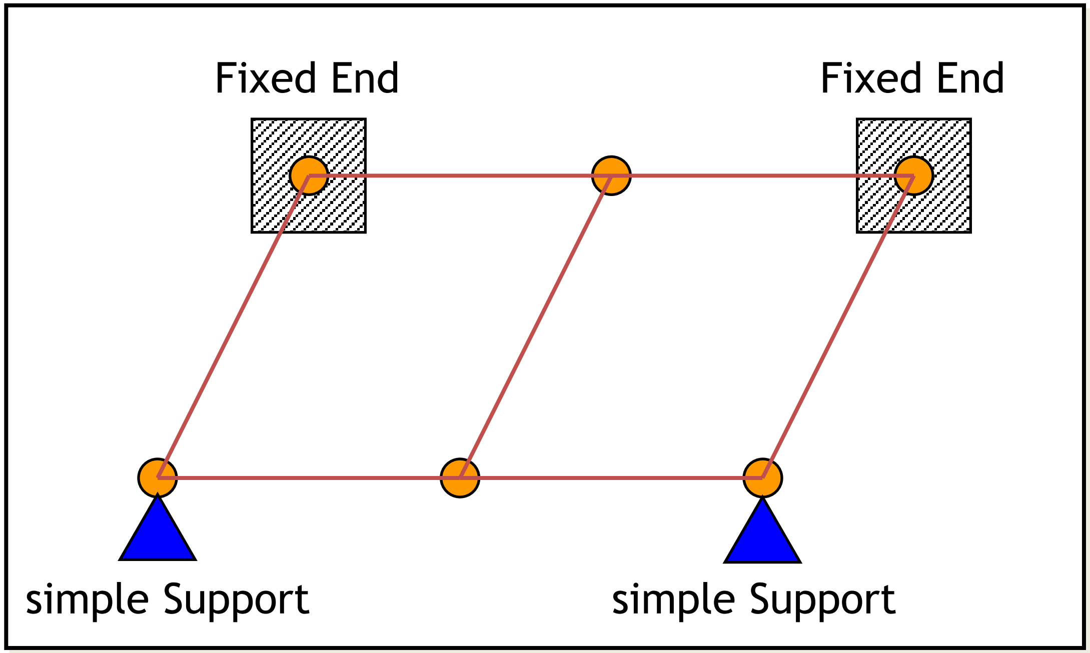
Grillage Analysis Program



Step1. Input : Nodes

Step2. Link : Between Nodes

Grillage Analysis Program

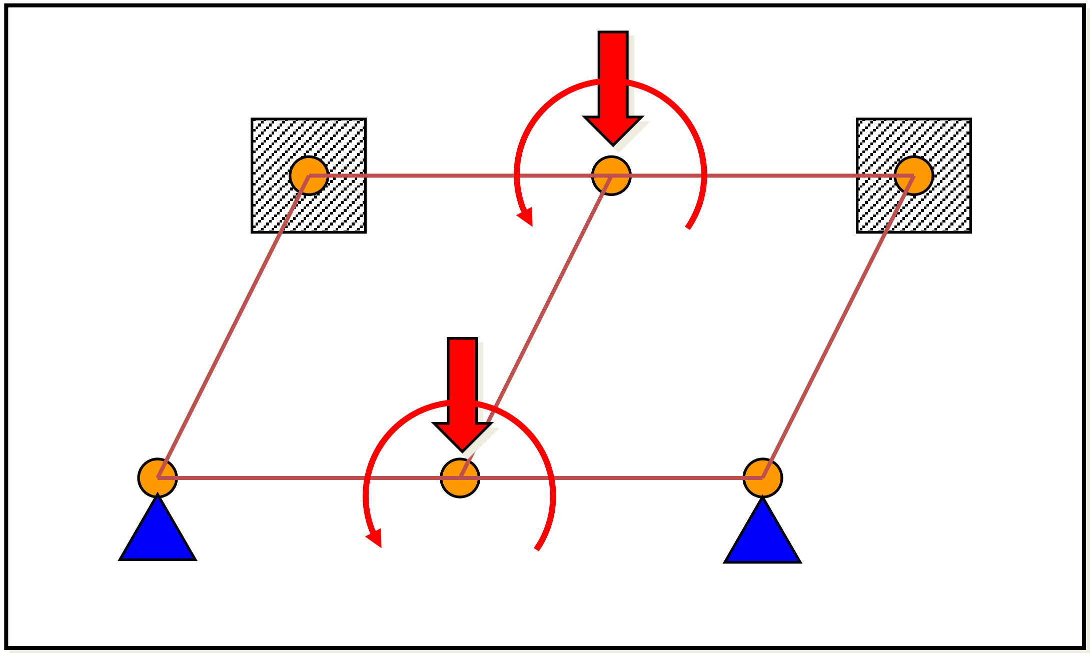


Step1. Input : Nodes

Step2. Link : Between Nodes

Step3. Input : Boundary Conditions

Grillage Analysis Program



Step1. Input : Nodes

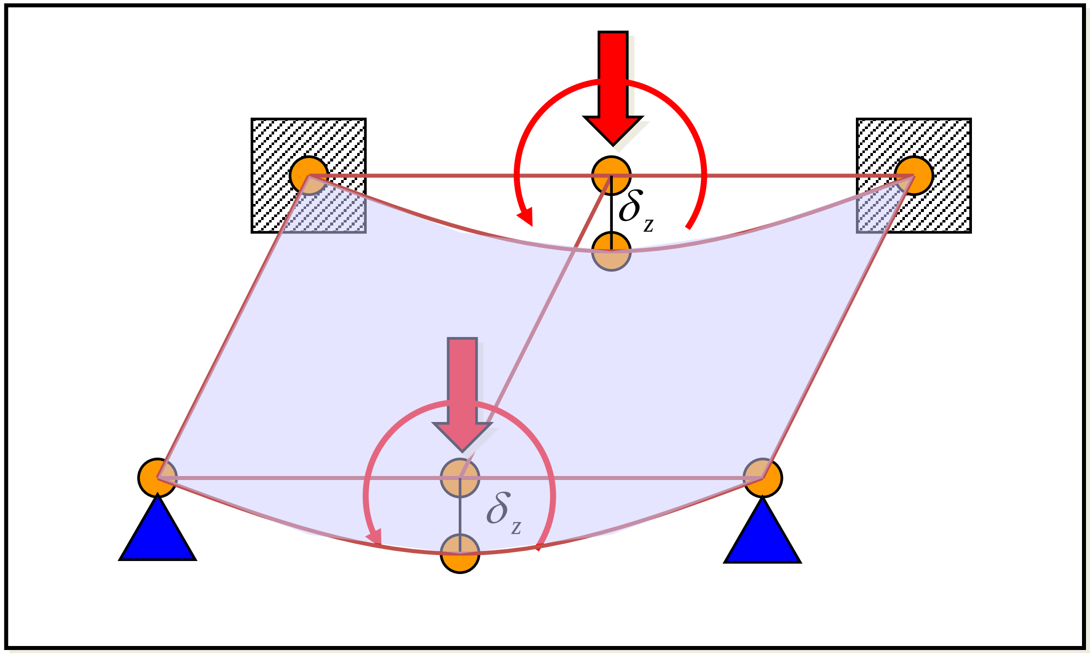
Step2. Link : Between Nodes

Step3. Input : Boundary Conditions

Step4. Input : Force and Moment

Step5. Grillage Analysis

Grillage Analysis Program



Step1. Input : Nodes

Step2. Link : Between Nodes

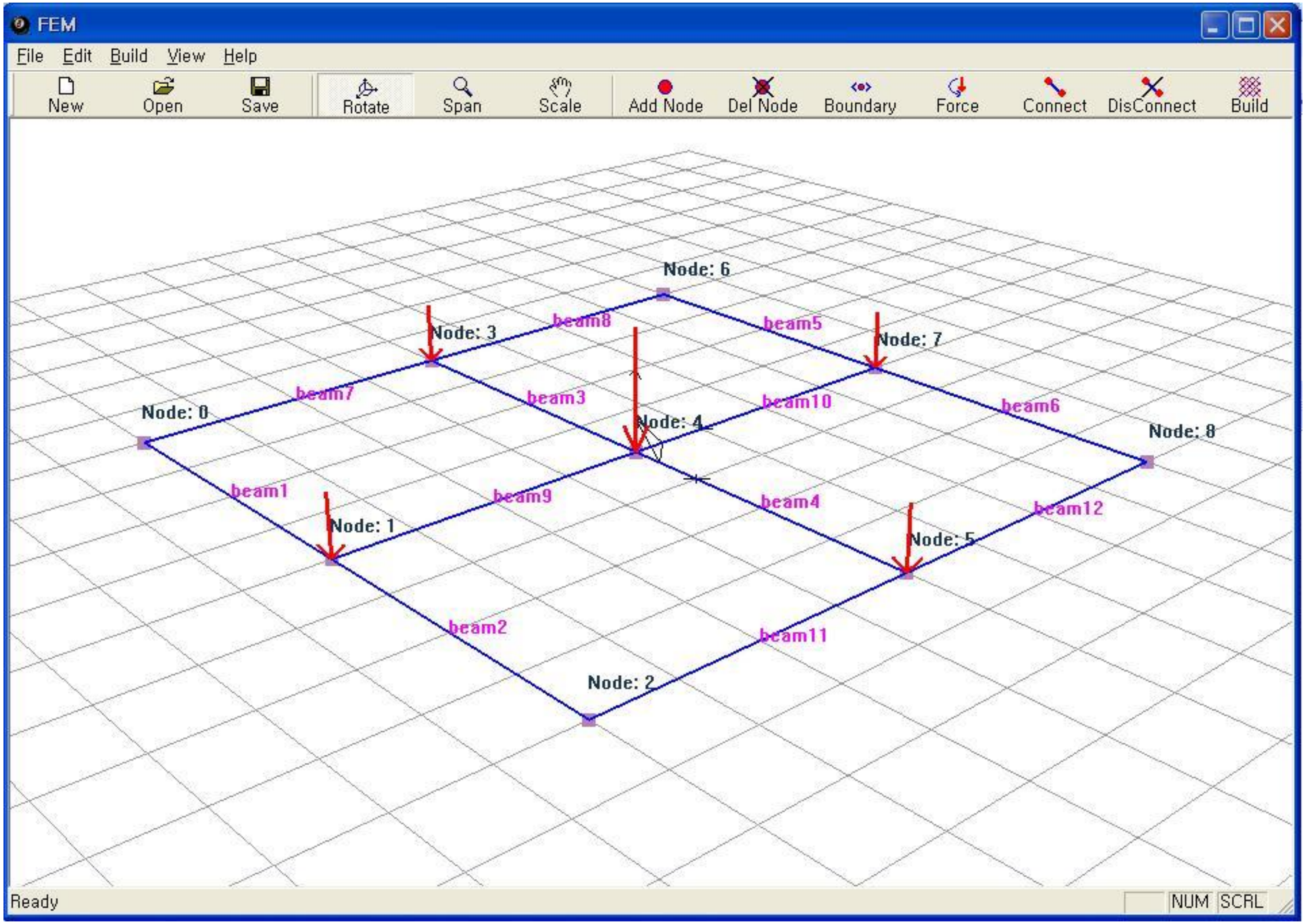
Step3. Input : Boundary Conditions

Step4. Input : Force and Moment

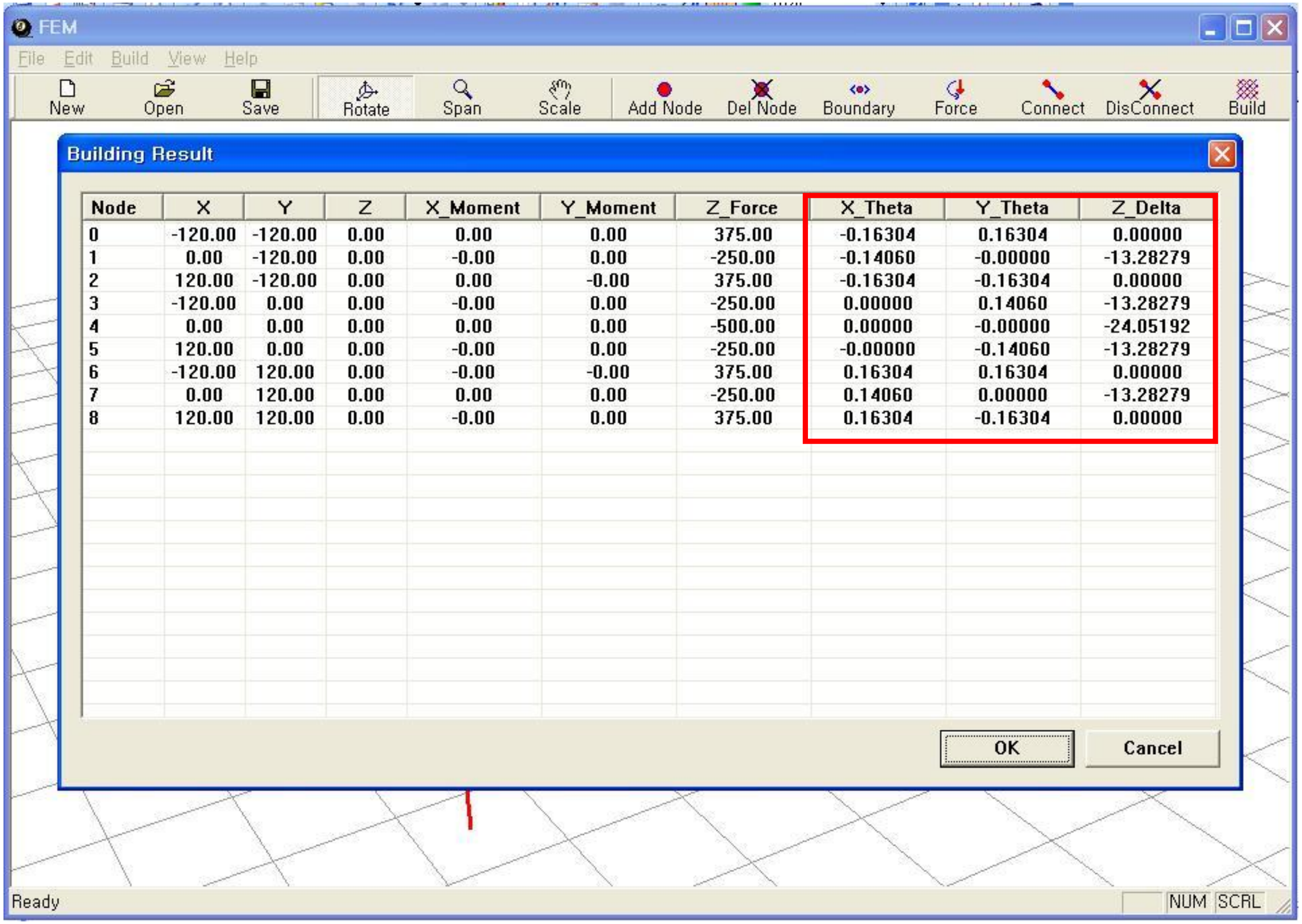
Step5. Grillage Analysis

Step6. Nodal Deflection
→Visualization by B-spline Surface

Example 1 : Grillage Analysis

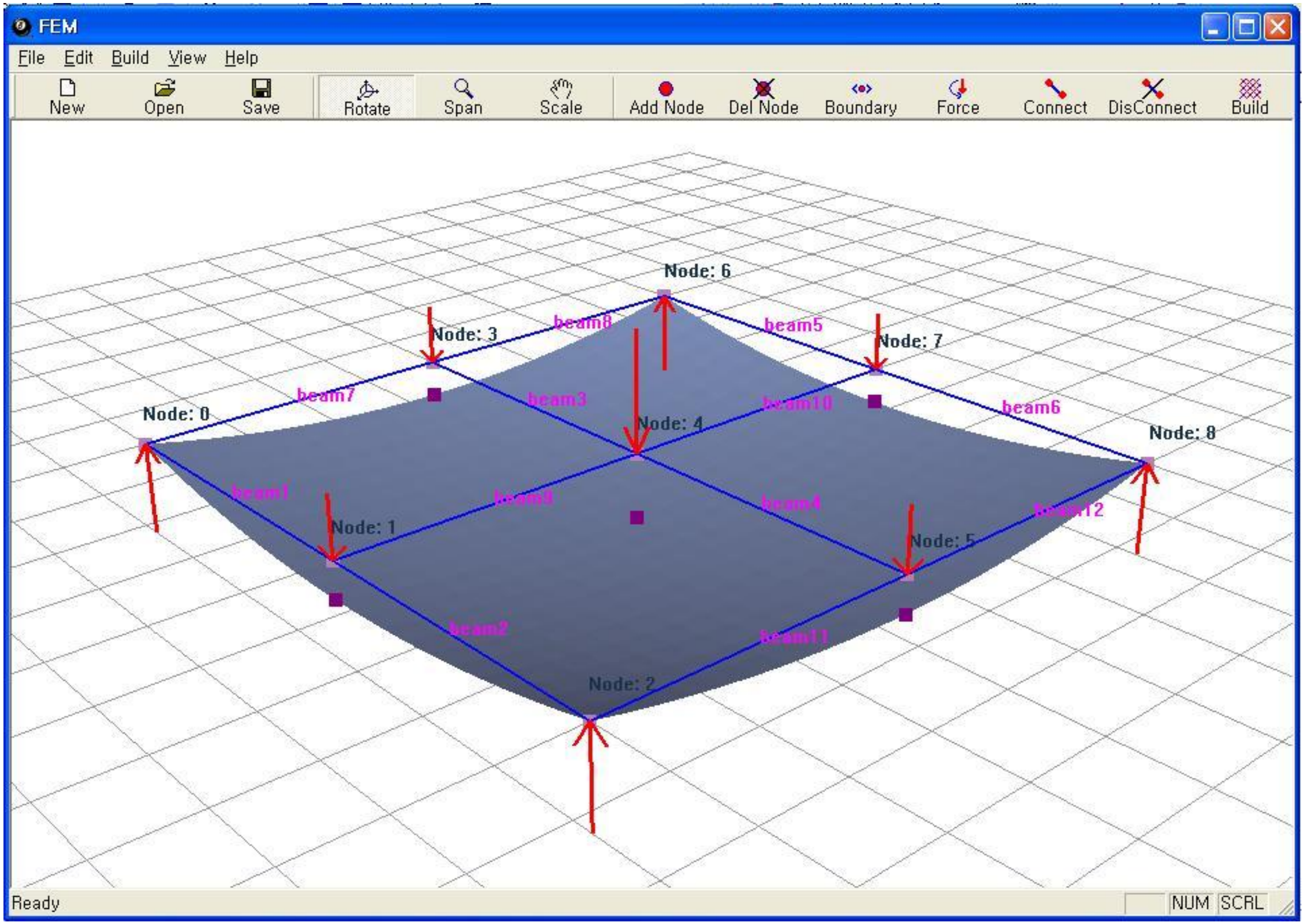


Example 1 : Grillage Analysis

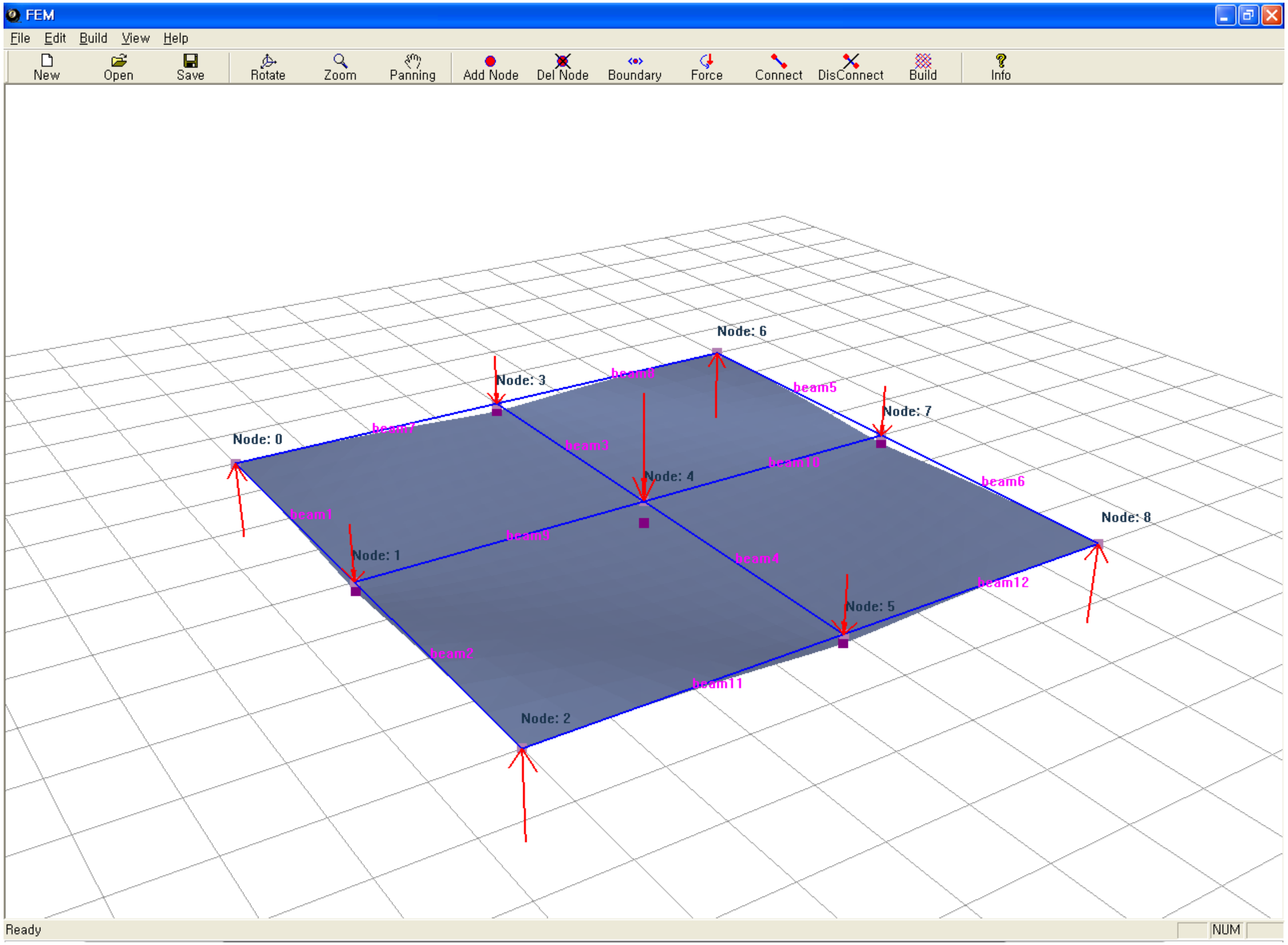


Node	X	Y	Z	X_Moment	Y_Moment	Z_Force	X_Theta	Y_Theta	Z_Delta
0	-120.00	-120.00	0.00	0.00	0.00	375.00	-0.16304	0.16304	0.00000
1	0.00	-120.00	0.00	-0.00	0.00	-250.00	-0.14060	-0.00000	-13.28279
2	120.00	-120.00	0.00	0.00	-0.00	375.00	-0.16304	-0.16304	0.00000
3	-120.00	0.00	0.00	-0.00	0.00	-250.00	0.00000	0.14060	-13.28279
4	0.00	0.00	0.00	0.00	0.00	-500.00	0.00000	-0.00000	-24.05192
5	120.00	0.00	0.00	-0.00	0.00	-250.00	-0.00000	-0.14060	-13.28279
6	-120.00	120.00	0.00	-0.00	-0.00	375.00	0.16304	0.16304	0.00000
7	0.00	120.00	0.00	0.00	0.00	-250.00	0.14060	0.00000	-13.28279
8	120.00	120.00	0.00	-0.00	0.00	375.00	0.16304	-0.16304	0.00000

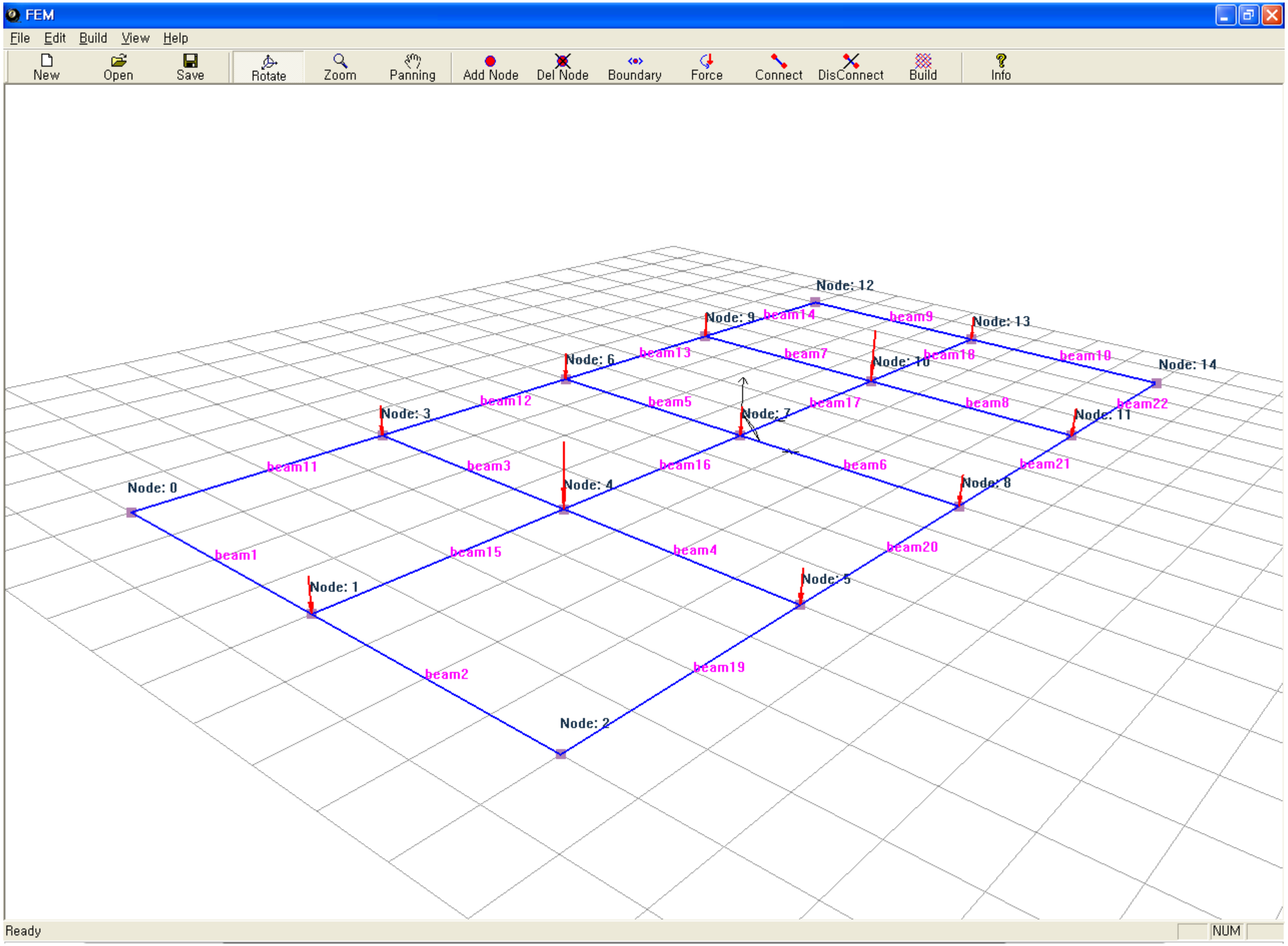
Example 1 : Grillage Analysis



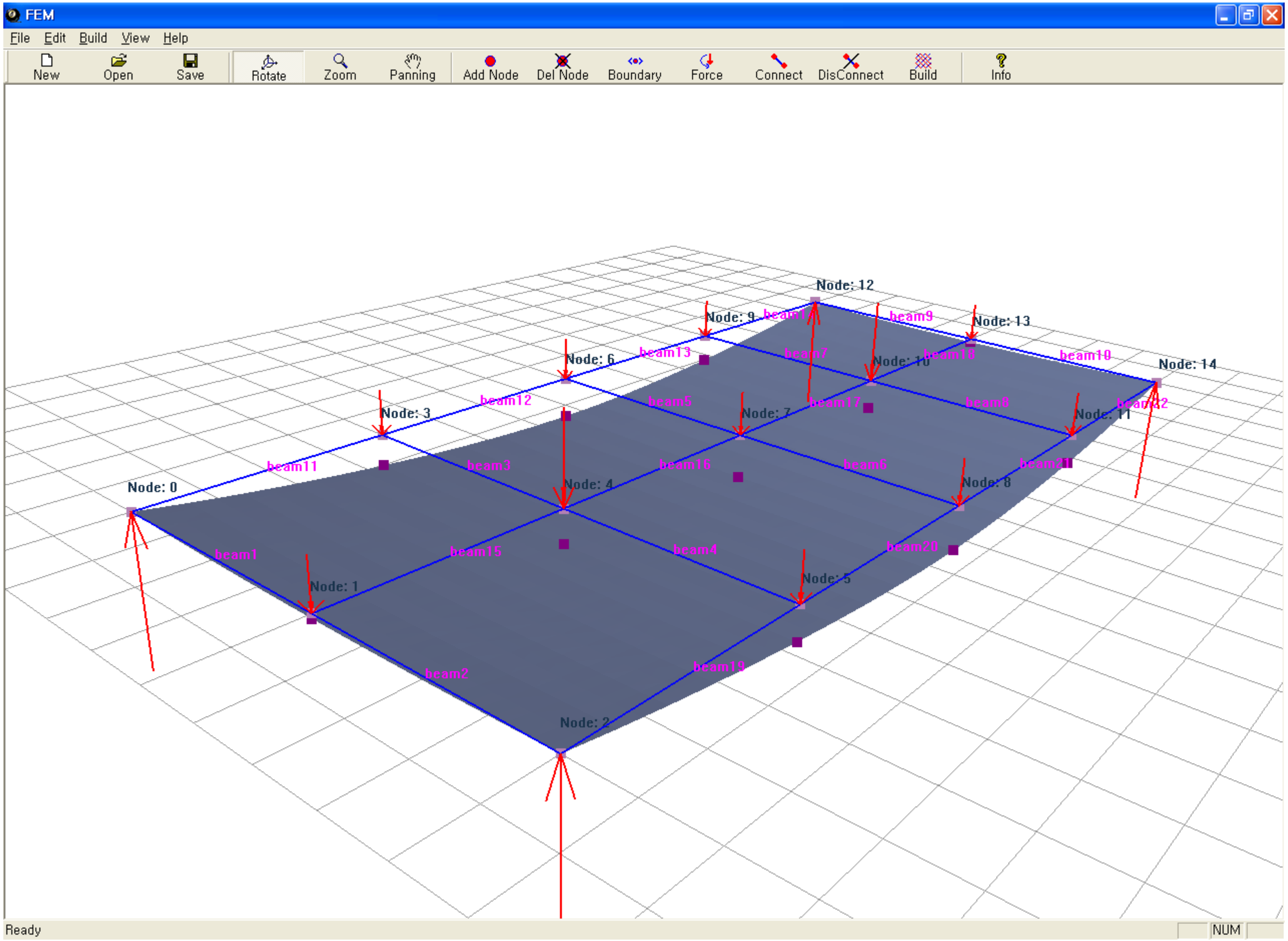
Example 1 : Grillage Analysis



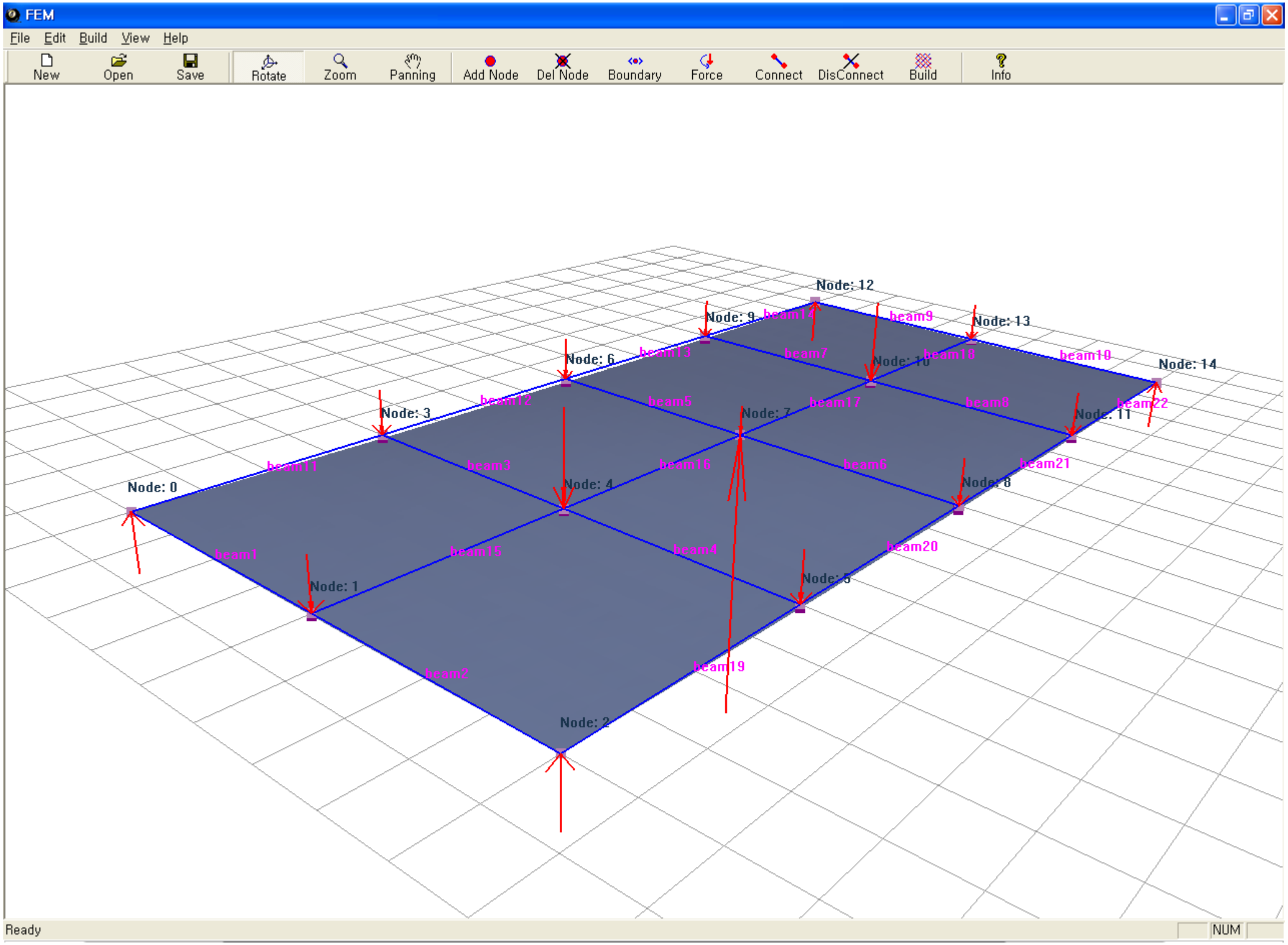
Example 1 : Grillage Analysis



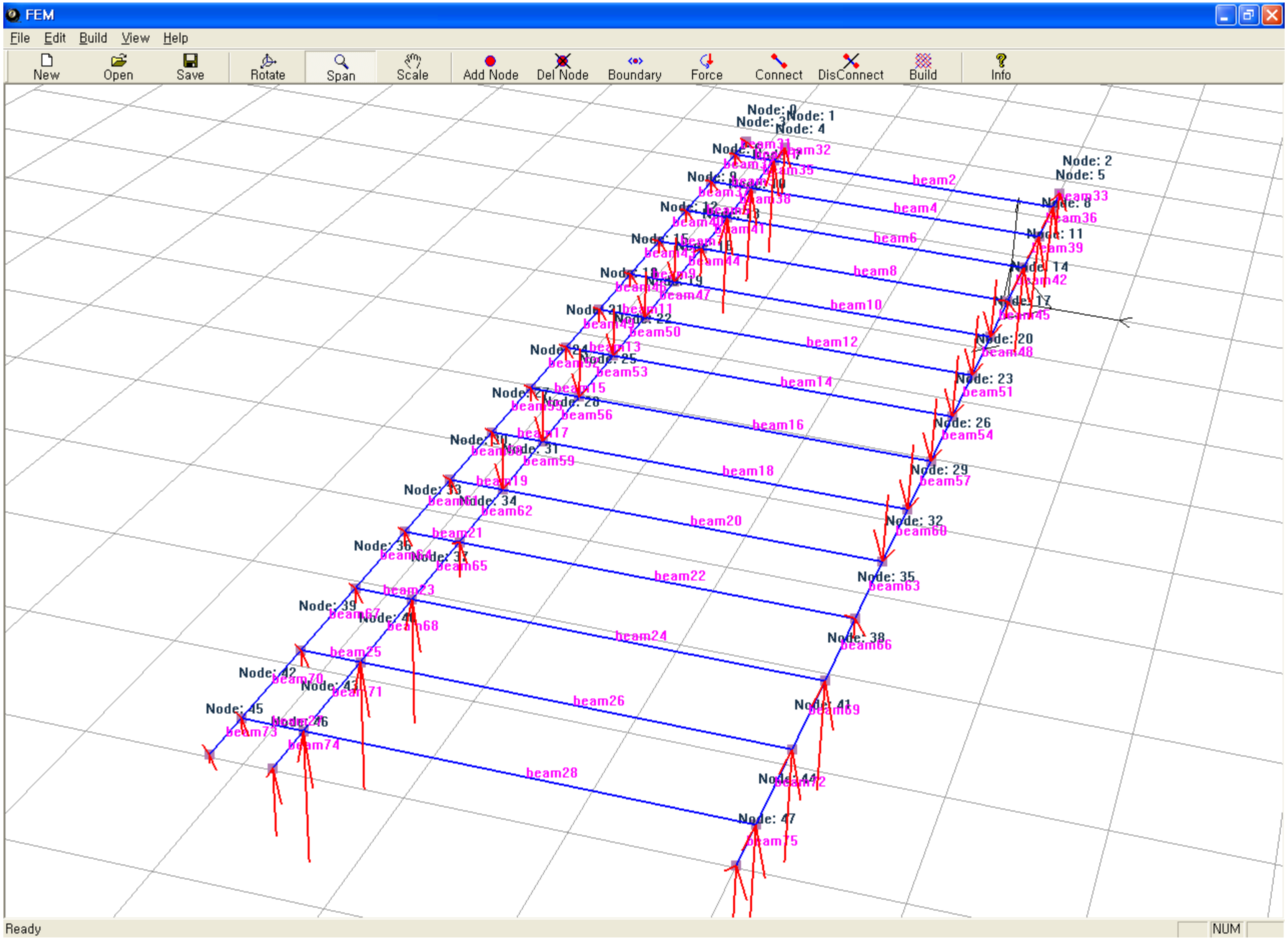
Example 1 : Grillage Analysis



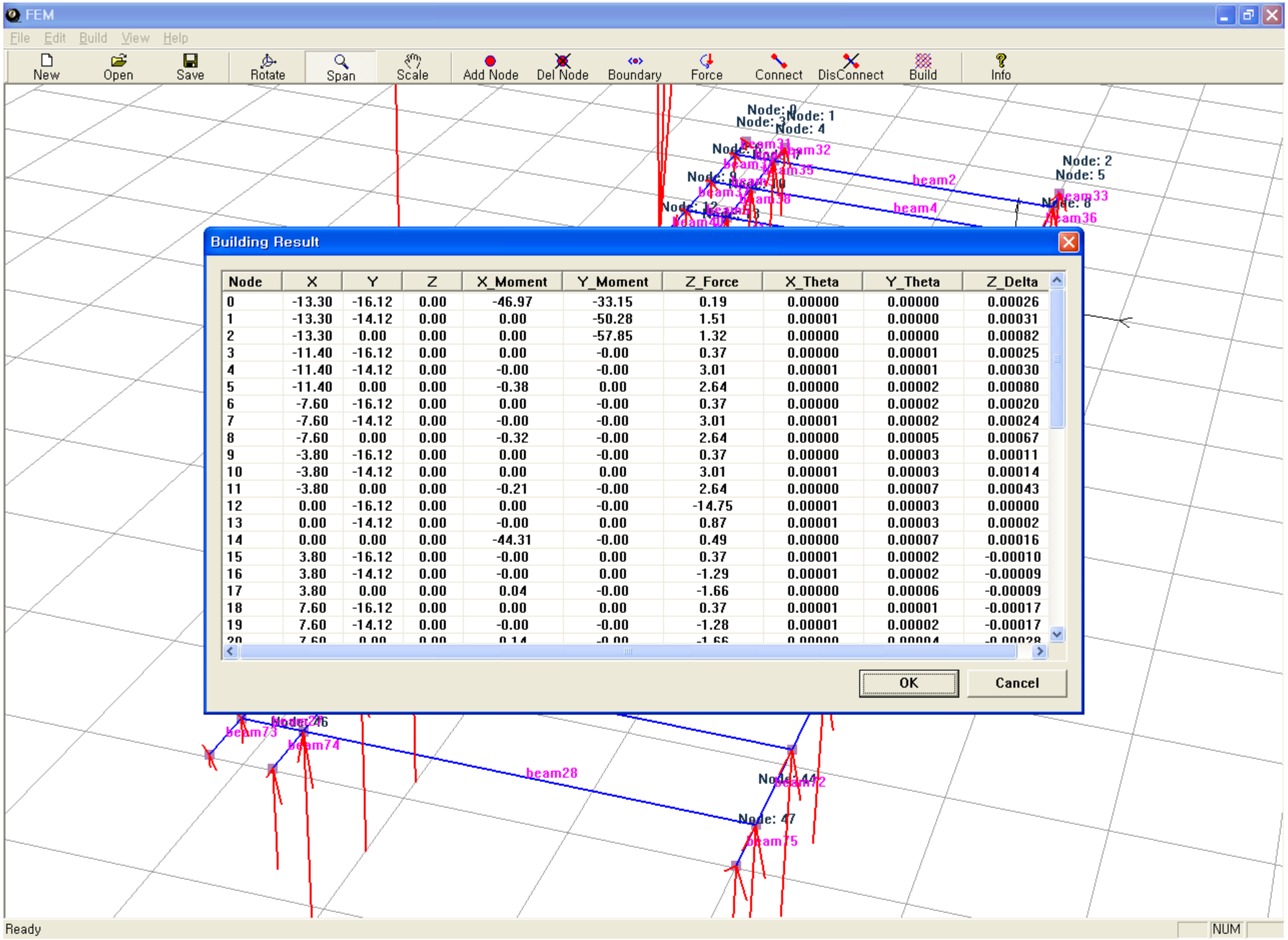
Example 2 : Grillage Analysis



Example 2 : Midship Cargo Hold Grillage Analysis

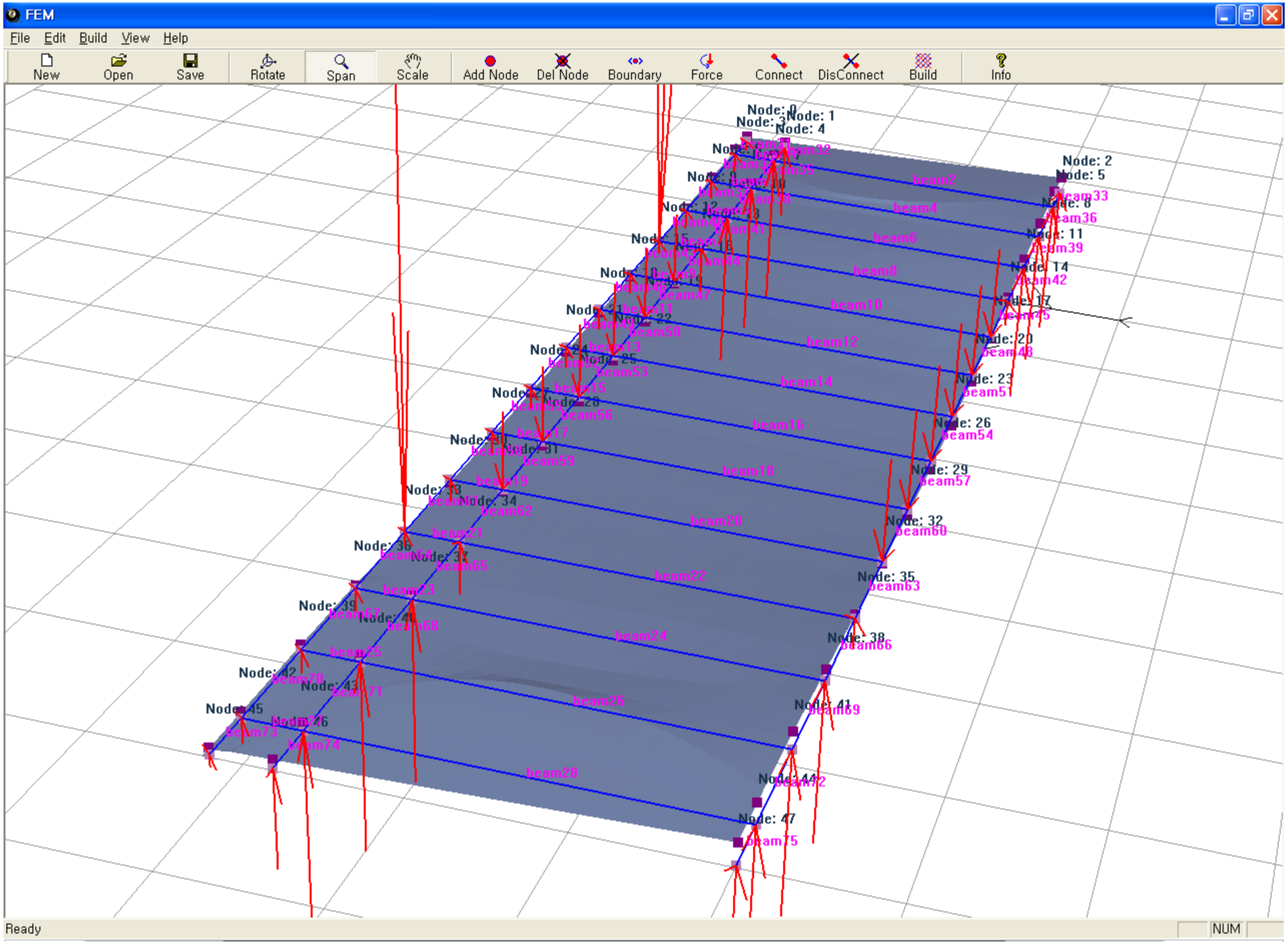


Example 2 : Midship Cargo Hold Grillage Analysis



Node	X	Y	Z	X_Moment	Y_Moment	Z_Force	X_Theta	Y_Theta	Z_Delta
0	-13.30	-16.12	0.00	-46.97	-33.15	0.19	0.00000	0.00000	0.00026
1	-13.30	-14.12	0.00	0.00	-50.28	1.51	0.00001	0.00000	0.00031
2	-13.30	0.00	0.00	0.00	-57.85	1.32	0.00000	0.00000	0.00082
3	-11.40	-16.12	0.00	0.00	-0.00	0.37	0.00000	0.00001	0.00025
4	-11.40	-14.12	0.00	-0.00	-0.00	3.01	0.00001	0.00001	0.00030
5	-11.40	0.00	0.00	-0.38	0.00	2.64	0.00000	0.00002	0.00080
6	-7.60	-16.12	0.00	0.00	-0.00	0.37	0.00000	0.00002	0.00020
7	-7.60	-14.12	0.00	-0.00	-0.00	3.01	0.00001	0.00002	0.00024
8	-7.60	0.00	0.00	-0.32	-0.00	2.64	0.00000	0.00005	0.00067
9	-3.80	-16.12	0.00	0.00	-0.00	0.37	0.00000	0.00003	0.00011
10	-3.80	-14.12	0.00	0.00	0.00	3.01	0.00001	0.00003	0.00014
11	-3.80	0.00	0.00	-0.21	-0.00	2.64	0.00000	0.00007	0.00043
12	0.00	-16.12	0.00	0.00	-0.00	-14.75	0.00001	0.00003	0.00000
13	0.00	-14.12	0.00	-0.00	0.00	0.87	0.00001	0.00003	0.00002
14	0.00	0.00	0.00	-44.31	-0.00	0.49	0.00000	0.00007	0.00016
15	3.80	-16.12	0.00	-0.00	0.00	0.37	0.00001	0.00002	-0.00010
16	3.80	-14.12	0.00	-0.00	0.00	-1.29	0.00001	0.00002	-0.00009
17	3.80	0.00	0.00	0.04	-0.00	-1.66	0.00000	0.00006	-0.00009
18	7.60	-16.12	0.00	0.00	0.00	0.37	0.00001	0.00001	-0.00017
19	7.60	-14.12	0.00	-0.00	-0.00	-1.28	0.00001	0.00002	-0.00017
20	7.60	0.00	0.00	0.14	-0.00	-1.66	0.00000	0.00004	-0.00028

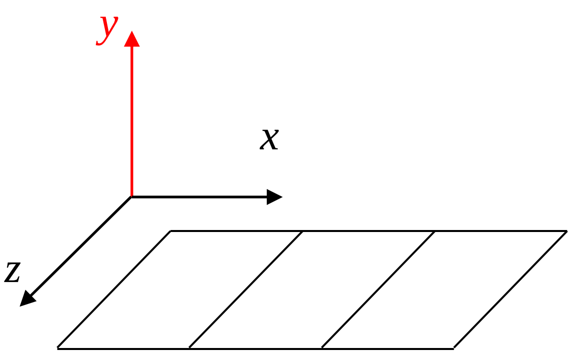
Example 2 : Midship Cargo Hold Grillage Analysis

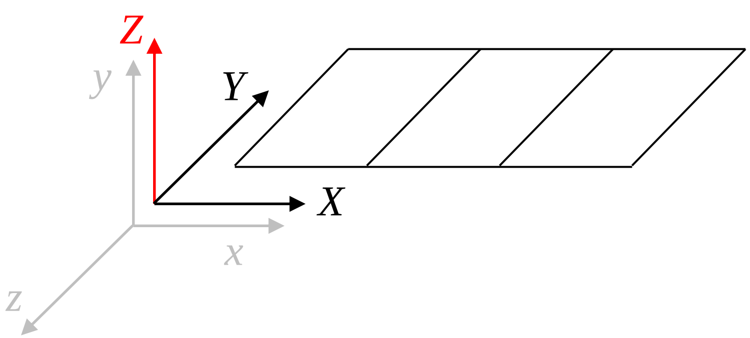


GRILLAGE STIFFNESS MATRIX IN THE LEFT-HAND ORIENTED SPACE-FIXED COORDINATE SYSTEM

Grillage Stiffness Matrix in the Left-Hand Oriented Space-fixed Coordinate System

- Formulation of Grillage Matrix in the left-hand orientated space-fixed Coordinate System





$$
\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(-90) & -\sin(-90) \\ 0 & \sin(-90) & \cos(-90) \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}
$$

- **Grillage Stiffness Matrix in xyz coordinate system*:**

$$
\begin{bmatrix} M_{x1} \\ f_{y1} \\ M_{z1} \\ M_{x2} \\ f_{y2} \\ M_{z2} \end{bmatrix} =
\begin{bmatrix}
\frac{GJ}{L} & 0 & 0 & -\frac{GJ}{L} & 0 & 0 \\
0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\
0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\
-\frac{GJ}{L} & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\
0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\
0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L}
\end{bmatrix}
\begin{bmatrix} \theta_{x1} \\ \delta_{y1} \\ \theta_{z1} \\ \theta_{x2} \\ \delta_{y2} \\ \theta_{z2} \end{bmatrix}
$$

***Refer to the Lecture Note on "Computer Aided Ship Design", Fall 2011, Kyu Yeul Lee**

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(-90) & -\sin(-90) \\ 0 & \sin(-90) & \cos(-90) \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

- **Grillage Stiffness Matrix in xyz coordinate system:**

$$\begin{bmatrix} M_{x1} \\ f_{y1} \\ M_{z1} \\ M_{x2} \\ f_{y2} \\ M_{z2} \end{bmatrix} = \begin{bmatrix} \frac{GJ}{L} & 0 & 0 & -\frac{GJ}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{GJ}{L} & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{bmatrix} \theta_{x1} \\ \delta_{y1} \\ \theta_{z1} \\ \theta_{x2} \\ \delta_{y2} \\ \theta_{z2} \end{bmatrix}$$

Coordinate transformation of force and moments

$$\begin{bmatrix} M_{x1} \\ f_{y1} \\ M_{z1} \\ M_{x2} \\ f_{y2} \\ M_{z2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \cos(-90) & -\sin(-90) & 0 & 0 & 0 \\ 0 & \sin(-90) & \cos(-90) & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cos(-90) & -\sin(-90) \\ 0 & 0 & 0 & 0 & \sin(-90) & \cos(-90) \end{bmatrix} \begin{bmatrix} M_{X1} \\ M_{Y1} \\ f_{Z1} \\ M_{X2} \\ M_{Y2} \\ f_{Z2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} M_{X1} \\ M_{Y1} \\ f_{Z1} \\ M_{X2} \\ M_{Y2} \\ f_{Z2} \end{bmatrix}$$

In the same manner

Coordinate transformation of displacements

$$\begin{bmatrix} \theta_{x1} \\ \delta_{y1} \\ \theta_{z1} \\ \theta_{x2} \\ \delta_{y2} \\ \theta_{z2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} \theta_{X1} \\ \theta_{Y1} \\ \delta_{Z1} \\ \theta_{X2} \\ \theta_{Y2} \\ \delta_{Z2} \end{bmatrix}$$

$$\begin{bmatrix} M_{x1} \\ f_{y1} \\ M_{z1} \\ M_{x2} \\ f_{y2} \\ M_{z2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} M_{X1} \\ M_{Y1} \\ f_{Z1} \\ M_{X2} \\ M_{Y2} \\ f_{Z2} \end{bmatrix}, \quad \begin{bmatrix} \theta_{x1} \\ \delta_{y1} \\ \theta_{z1} \\ \theta_{x2} \\ \delta_{y2} \\ \theta_{z2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} \theta_{X1} \\ \theta_{Y1} \\ \delta_{Z1} \\ \theta_{X2} \\ \theta_{Y2} \\ \delta_{Z2} \end{bmatrix}$$

substituting

$$\begin{bmatrix} M_{x1} \\ f_{y1} \\ M_{z1} \\ M_{x2} \\ f_{y2} \\ M_{z2} \end{bmatrix} = \begin{bmatrix} \frac{GJ}{L} & 0 & 0 & -\frac{GJ}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{GJ}{L} & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{bmatrix} \theta_{x1} \\ \delta_{y1} \\ \theta_{z1} \\ \theta_{x2} \\ \delta_{y2} \\ \theta_{z2} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} M_{X1} \\ M_{Y1} \\ f_{Z1} \\ M_{X2} \\ M_{Y2} \\ f_{Z2} \end{bmatrix}, = \begin{bmatrix} \frac{GJ}{L} & 0 & 0 & -\frac{GJ}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{GJ}{L} & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} \theta_{X1} \\ \theta_{Y1} \\ \delta_{Z1} \\ \theta_{X2} \\ \theta_{Y2} \\ \delta_{Z2} \end{bmatrix}$$

$$
\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}
\begin{bmatrix} M_{X1} \\ M_{Y1} \\ f_{Z1} \\ M_{X2} \\ M_{Y2} \\ f_{Z2} \end{bmatrix}
=
\begin{bmatrix}
\frac{GJ}{L} & 0 & 0 & -\frac{GJ}{L} & 0 & 0 \\
0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\
0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\
-\frac{GJ}{L} & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\
0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\
0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L}
\end{bmatrix}
\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}
\begin{bmatrix} \theta_{X1} \\ \theta_{Y1} \\ \delta_{Z1} \\ \theta_{X2} \\ \theta_{Y2} \\ \delta_{Z2} \end{bmatrix}
$$

$$
\begin{bmatrix} M_{X1} \\ M_{Y1} \\ f_{Z1} \\ M_{X2} \\ M_{Y2} \\ f_{Z2} \end{bmatrix}
=
\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}
\begin{bmatrix}
\frac{GJ}{L} & 0 & 0 & -\frac{GJ}{L} & 0 & 0 \\
0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\
0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\
-\frac{GJ}{L} & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\
0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\
0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L}
\end{bmatrix}
\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}
\begin{bmatrix} \theta_{X1} \\ \theta_{Y1} \\ \delta_{Z1} \\ \theta_{X2} \\ \theta_{Y2} \\ \delta_{Z2} \end{bmatrix}
$$

$$
\begin{bmatrix} M_{X1} \\ M_{Y1} \\ f_{Z1} \\ M_{X2} \\ M_{Y2} \\ f_{Z2} \end{bmatrix}
=
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
\frac{GJ}{L} & 0 & 0 & -\frac{GJ}{L} & 0 & 0 \\
0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\
0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\
-\frac{GJ}{L} & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\
0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\
0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L}
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & -1 & 0
\end{bmatrix}
\begin{bmatrix} \theta_{X1} \\ \theta_{Y1} \\ \delta_{Z1} \\ \theta_{X2} \\ \theta_{Y2} \\ \delta_{Z2} \end{bmatrix}
$$

$$
\begin{bmatrix} M_{X1} \\ M_{Y1} \\ f_{Z1} \\ M_{X2} \\ M_{Y2} \\ f_{Z2} \end{bmatrix}
=
\begin{bmatrix}
\frac{GJ}{L} & 0 & 0 & -\frac{GJ}{L} & 0 & 0 \\
0 & -\frac{6EI}{L^2} & -\frac{4EI}{L} & 0 & \frac{6EI}{L^2} & -\frac{2EI}{L} \\
0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\
-\frac{GJ}{L} & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\
0 & -\frac{6EI}{L^2} & -\frac{2EI}{L} & 0 & \frac{6EI}{L^2} & -\frac{4EI}{L} \\
0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2}
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & -1 & 0
\end{bmatrix}
\begin{bmatrix} \theta_{X1} \\ \theta_{Y1} \\ \delta_{Z1} \\ \theta_{X2} \\ \theta_{Y2} \\ \delta_{Z2} \end{bmatrix}
$$

$$
\begin{bmatrix} M_{X1} \\ M_{Y1} \\ f_{Z1} \\ M_{X2} \\ M_{Y2} \\ f_{Z2} \end{bmatrix}
=
\begin{bmatrix}
\frac{GJ}{L} & 0 & 0 & -\frac{GJ}{L} & 0 & 0 \\
0 & \frac{4EI}{L} & -\frac{6EI}{L^2} & 0 & \frac{2EI}{L} & \frac{6EI}{L^2} \\
0 & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & 0 & -\frac{6EI}{L^2} & -\frac{12EI}{L^3} \\
-\frac{GJ}{L} & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\
0 & \frac{2EI}{L} & -\frac{6EI}{L^2} & 0 & \frac{4EI}{L} & \frac{6EI}{L^2} \\
0 & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & 0 & \frac{6EI}{L^2} & \frac{12EI}{L^3}
\end{bmatrix}
\begin{bmatrix} \theta_{X1} \\ \theta_{Y1} \\ \delta_{Z1} \\ \theta_{X2} \\ \theta_{Y2} \\ \delta_{Z2} \end{bmatrix}
$$

- **Stiffness Matrix in the left-hand coordinate system**

$$
\begin{bmatrix} M_{X1} \\ M_{Y1} \\ f_{Z1} \\ M_{X2} \\ M_{Y2} \\ f_{Z2} \end{bmatrix}
=
\begin{bmatrix}
\frac{GJ}{L} & 0 & 0 & -\frac{GJ}{L} & 0 & 0 \\
0 & \frac{4EI}{L} & -\frac{6EI}{L^2} & 0 & \frac{2EI}{L} & \frac{6EI}{L^2} \\
0 & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & 0 & -\frac{6EI}{L^2} & -\frac{12EI}{L^3} \\
-\frac{GJ}{L} & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\
0 & \frac{2EI}{L} & -\frac{6EI}{L^2} & 0 & \frac{4EI}{L} & \frac{6EI}{L^2} \\
0 & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & 0 & \frac{6EI}{L^2} & \frac{12EI}{L^3}
\end{bmatrix}
\begin{bmatrix} \theta_{X1} \\ \theta_{Y1} \\ \delta_{Z1} \\ \theta_{X2} \\ \theta_{Y2} \\ \delta_{Z2} \end{bmatrix}
$$

Naval Architecture & Ocean Engineering

Chapter 18. Hoisting System of Offshore Drilling Rig*

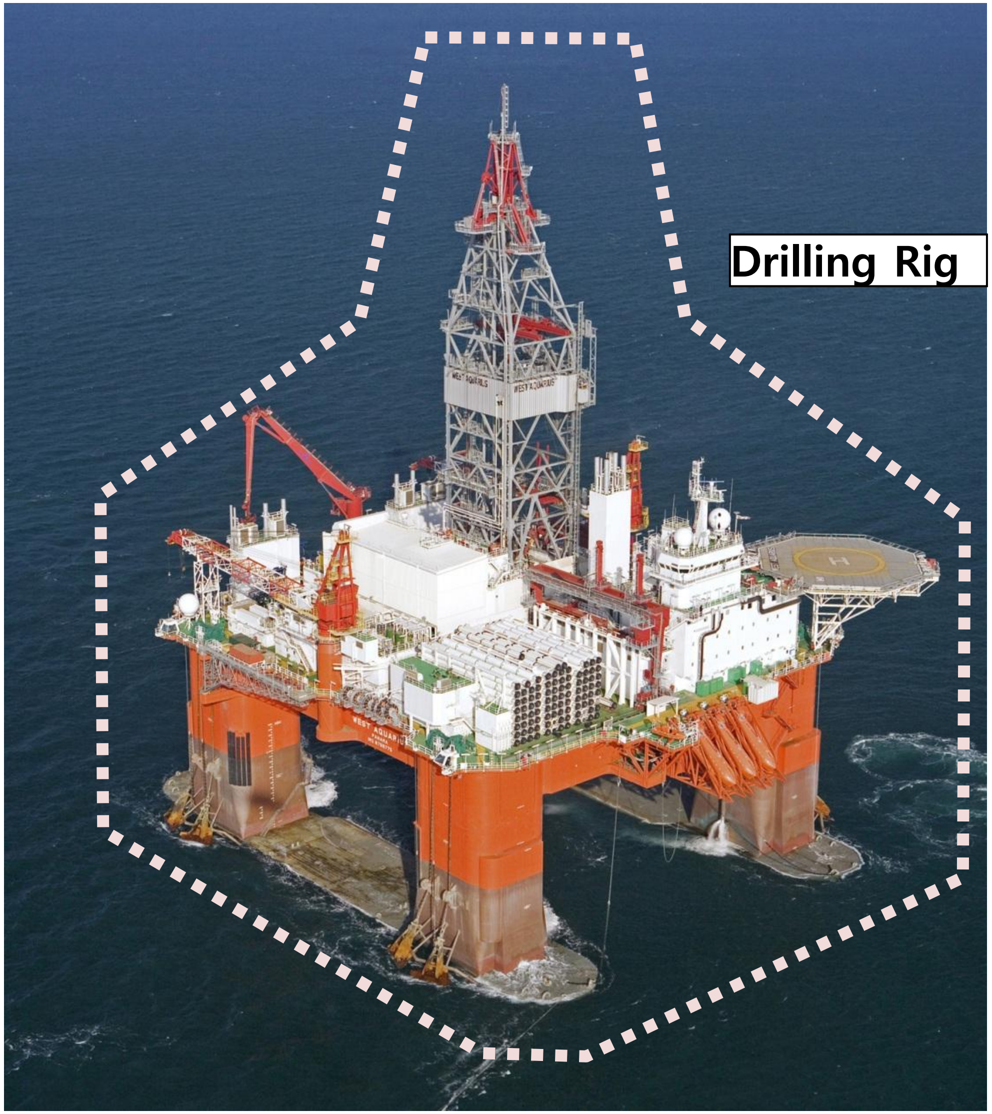
***Ref : "Dynamic Analysis and Control of Heave Compensation System for Offshore Drilling Operation based on Multibody Dynamics", Ku Nam Kuk, PhD Thesis, Feb., 2012, Dept. Naval Architecture & Ocean Enginnering, Seoul National University**

Seoul National Univ.

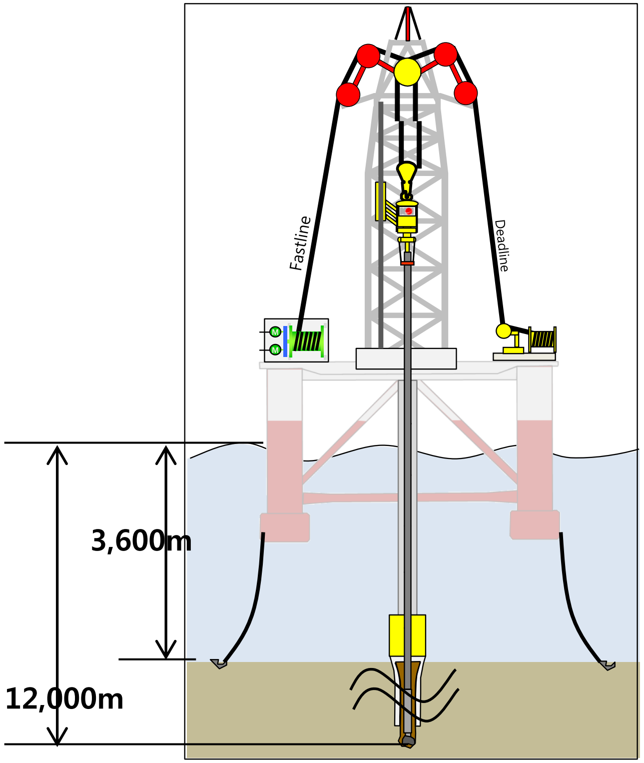
@SDAL
Advanced Ship Design Automation Lab.
http://asdal.snu.ac.kr

18-1. Introduction

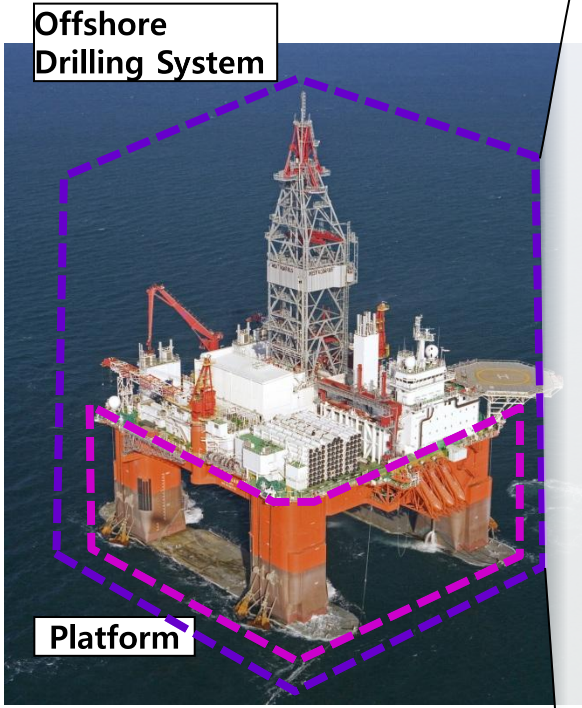
Offshore Drilling Rig

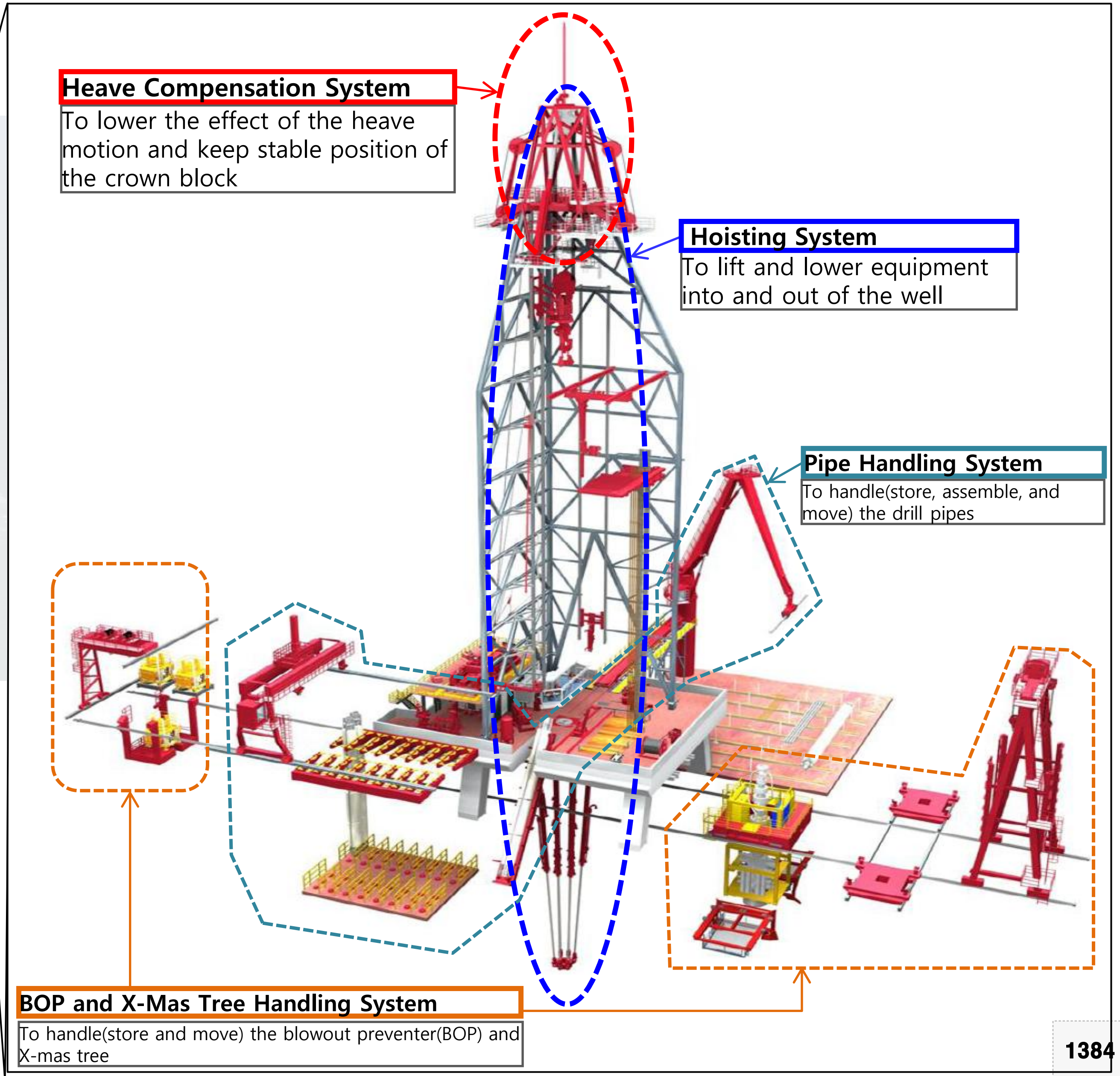


West Aquarius(2008)
Manufacturer : Seadrill (DSME in Korea)
Maximum Operating Depth : 3,600m
Maximum Drilling Depth : 12,000m

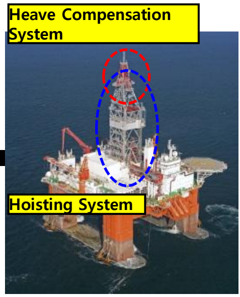


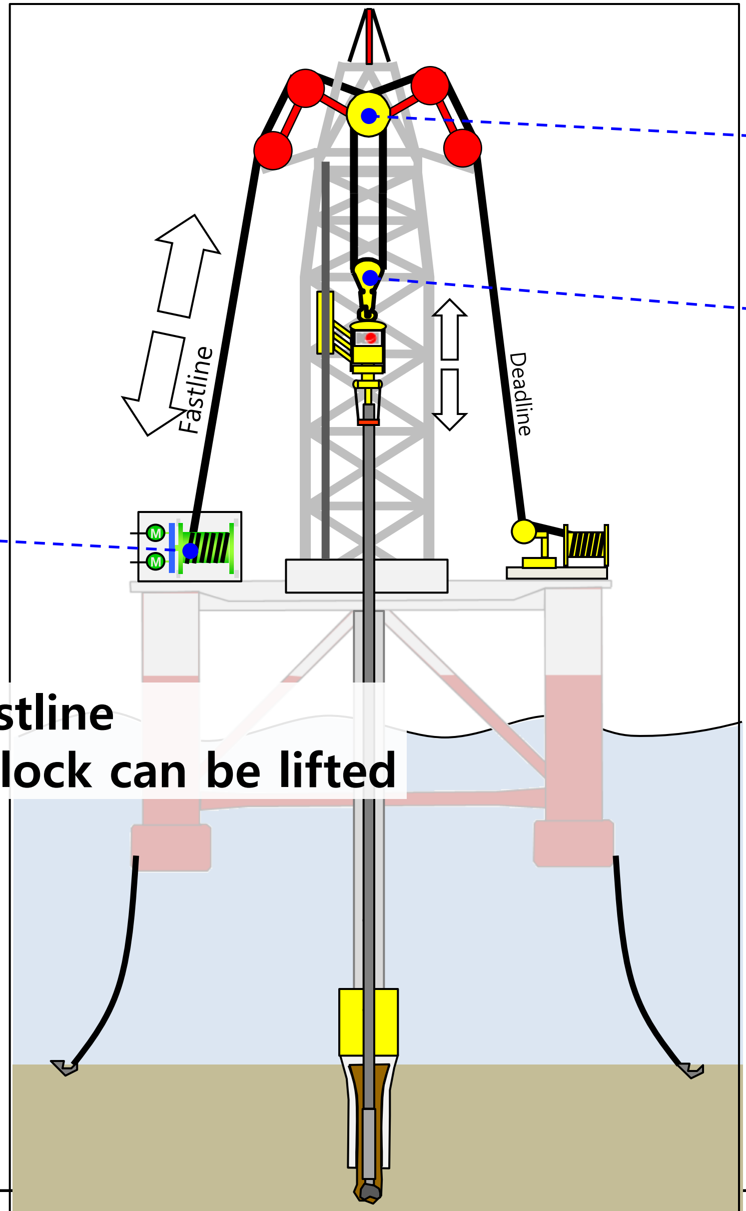
Offshore Drilling System





- How to hoist up the traveling block and drill string





Crown Block

Traveling Block

Drawwork

Drawwork pulls the fastline so that the traveling block can be lifted

Classification of Offshore Drilling Rig

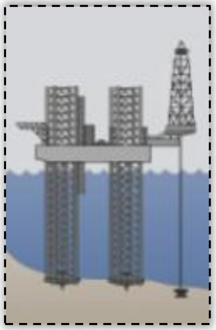
- Drilling Rig
  - Land rigs
  - Offshore rigs
    - Floating rigs
      - Semi-submersible

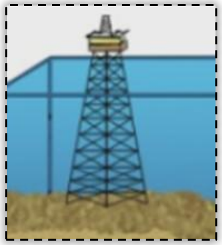
      - Drillship

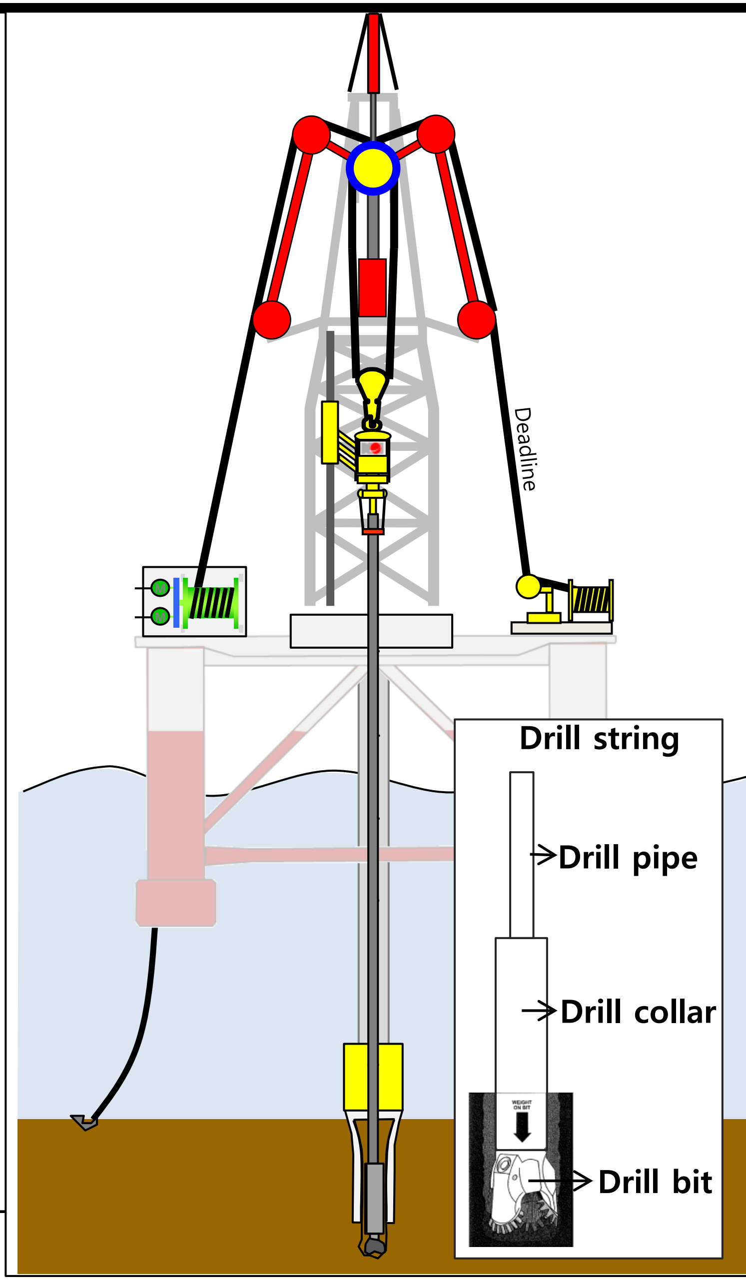
    - Bottom Supported Rigs
      - Jackup

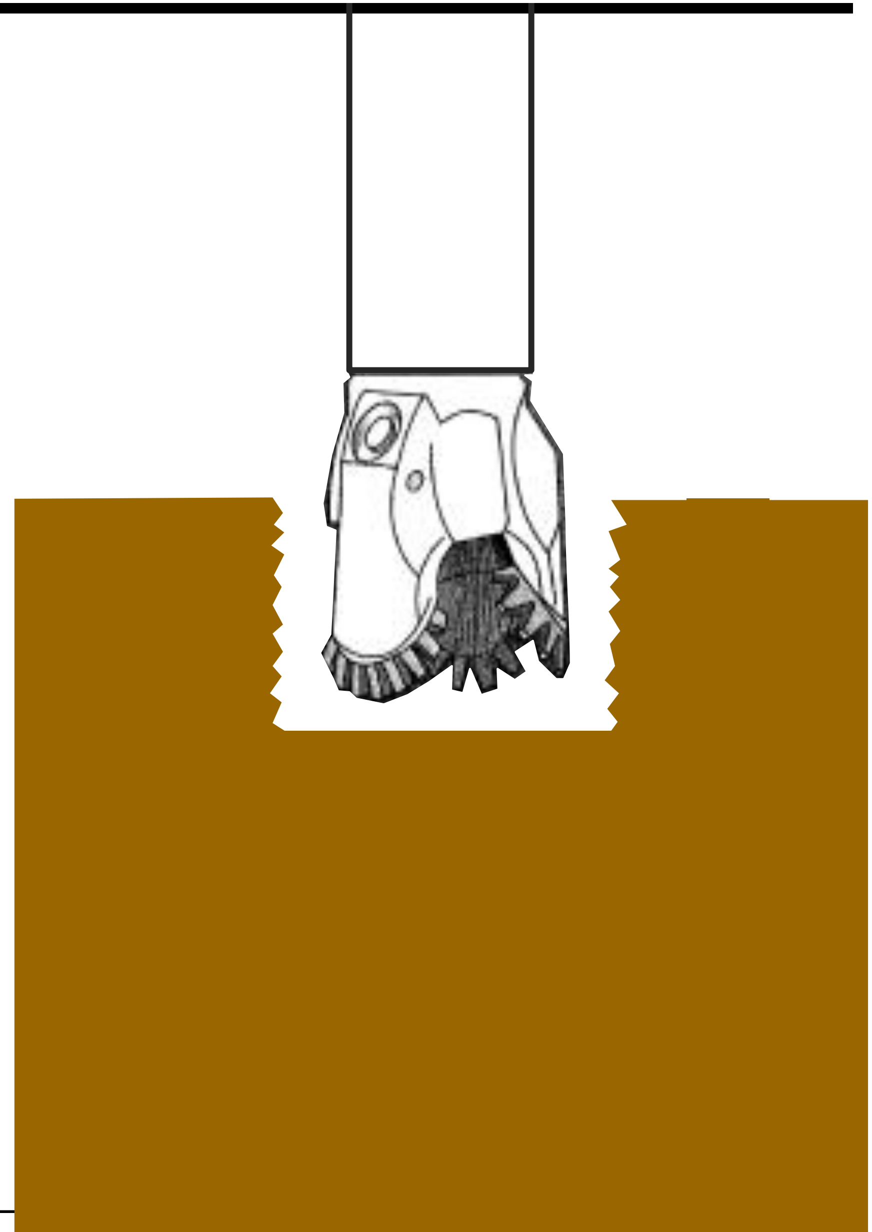
        

      - Fixed platform

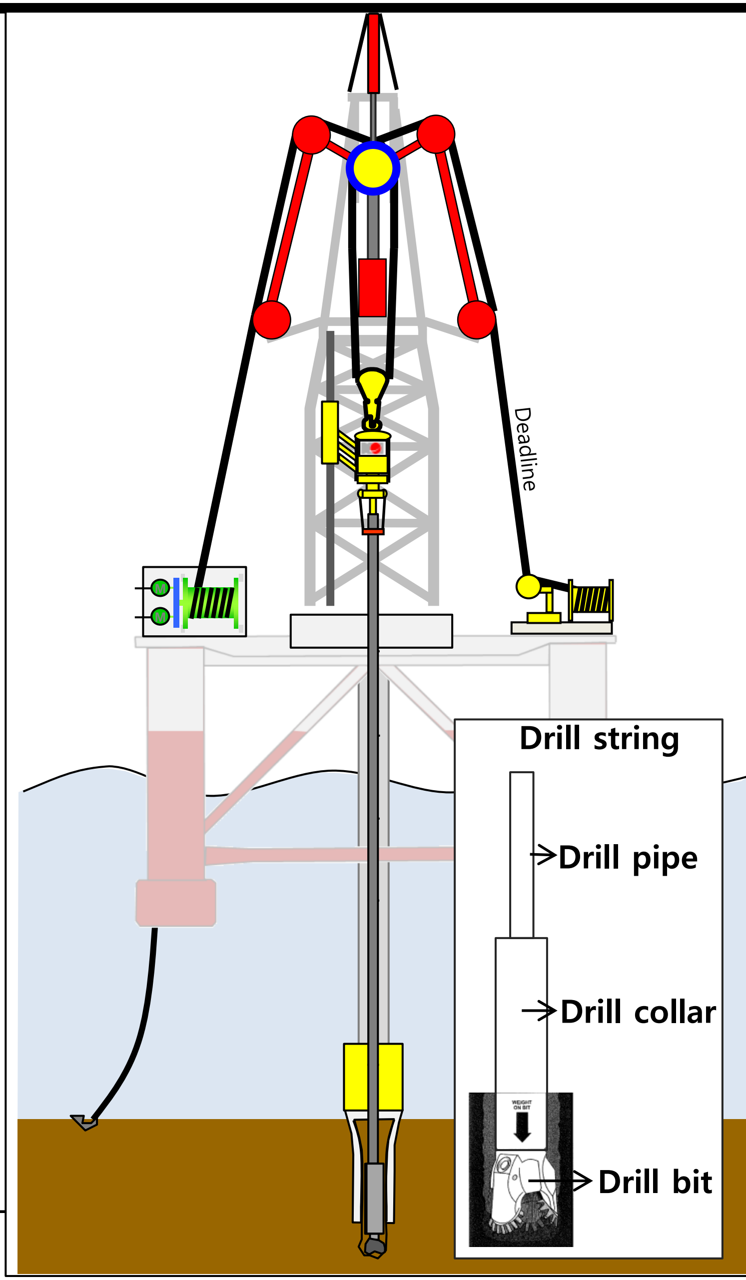
Drilling Procedure

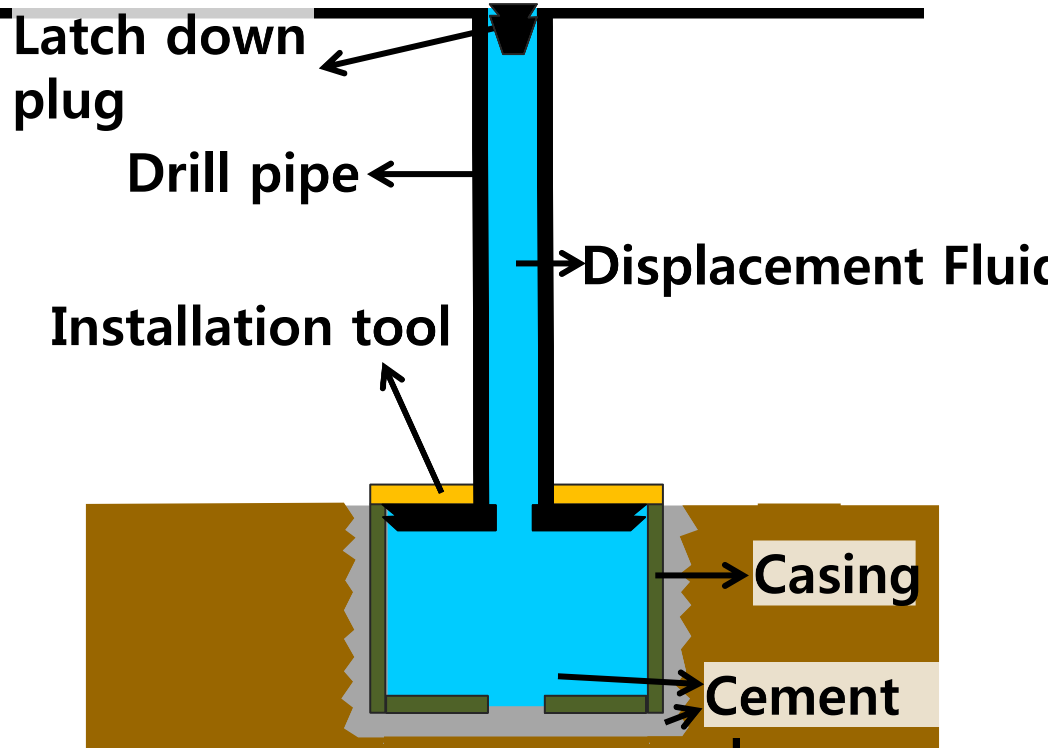




Drilling Procedure

	Diameter(cm)	Weight per unit length (kg/m)	Material	Thickness(mm)
Casing	12.7~76.2	15~197	Steel	5~16





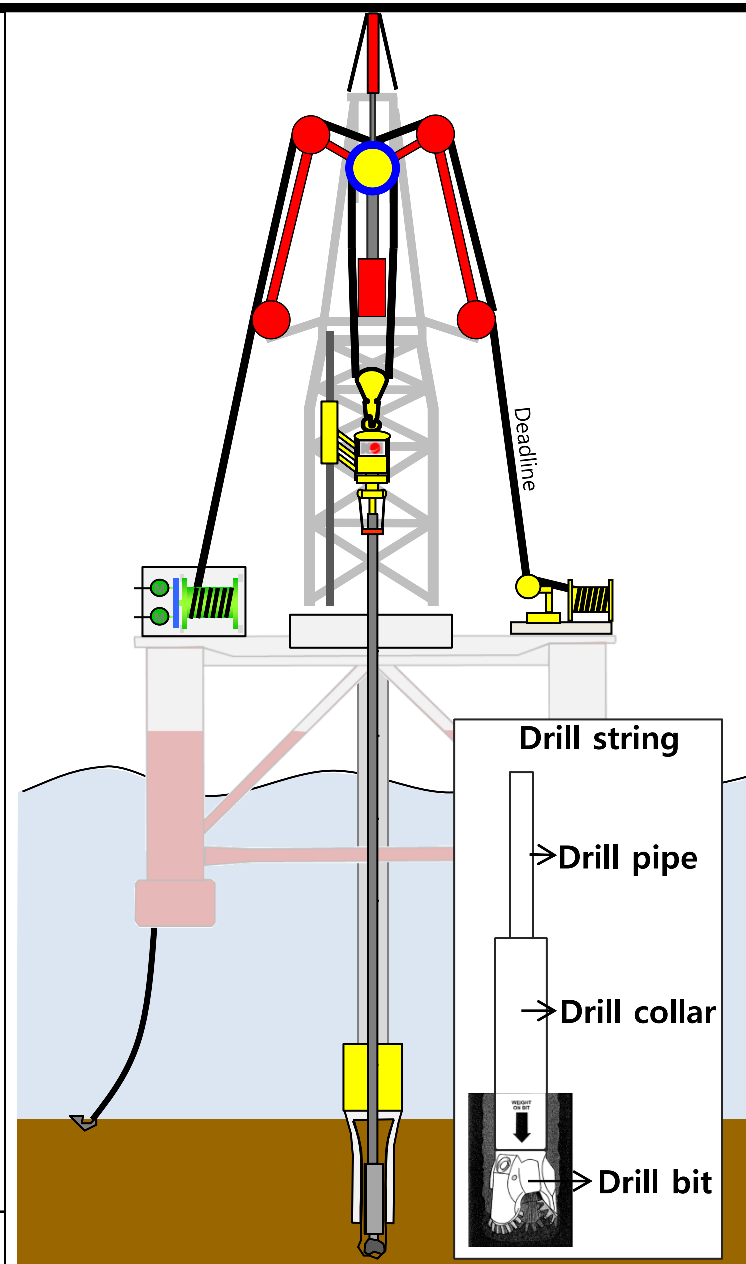
1. Running the casing into the hole.
   (The casing is connected with drill pipe using the installation tool)

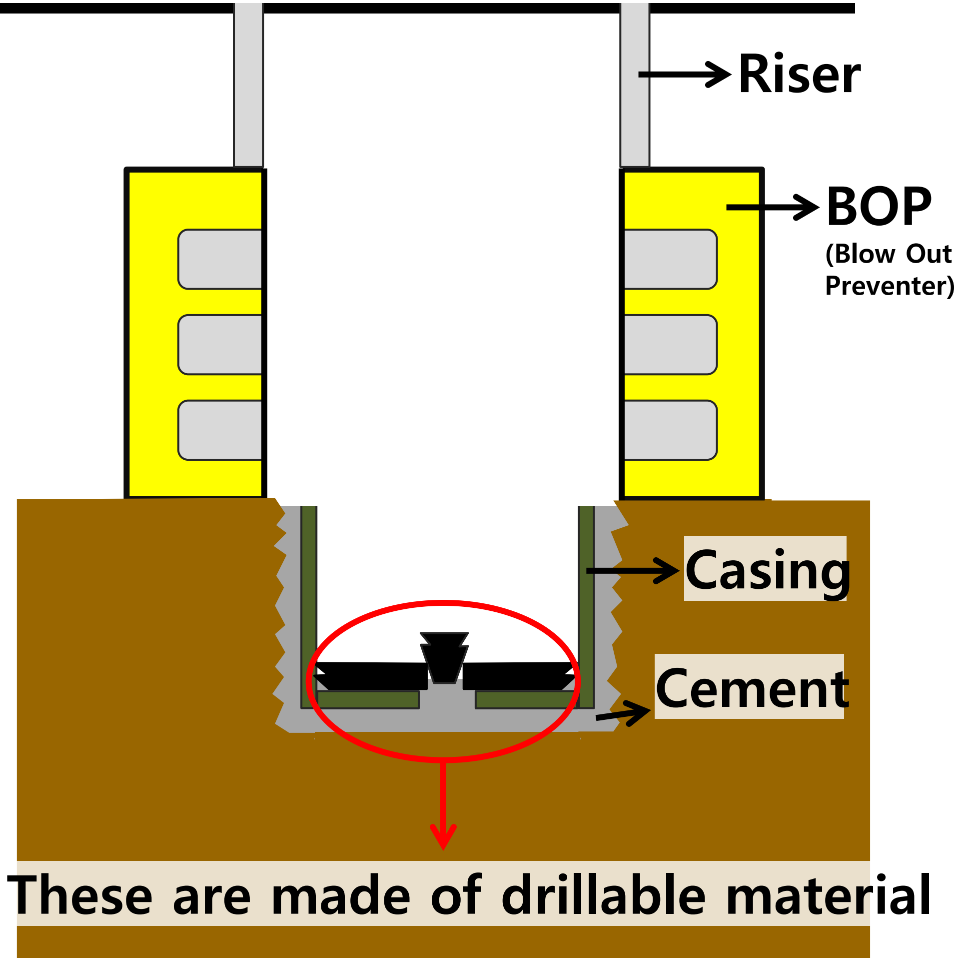
2. Pumping the cement slurry through the casing.

3. As the last of the cement slurry enters the casing,the latch down plug(wiper plug) is releases.
   (the latch down plug separates the cement slurry from the displacement fluid)

4. Displacement fluid moves the cement slurry down the casing.

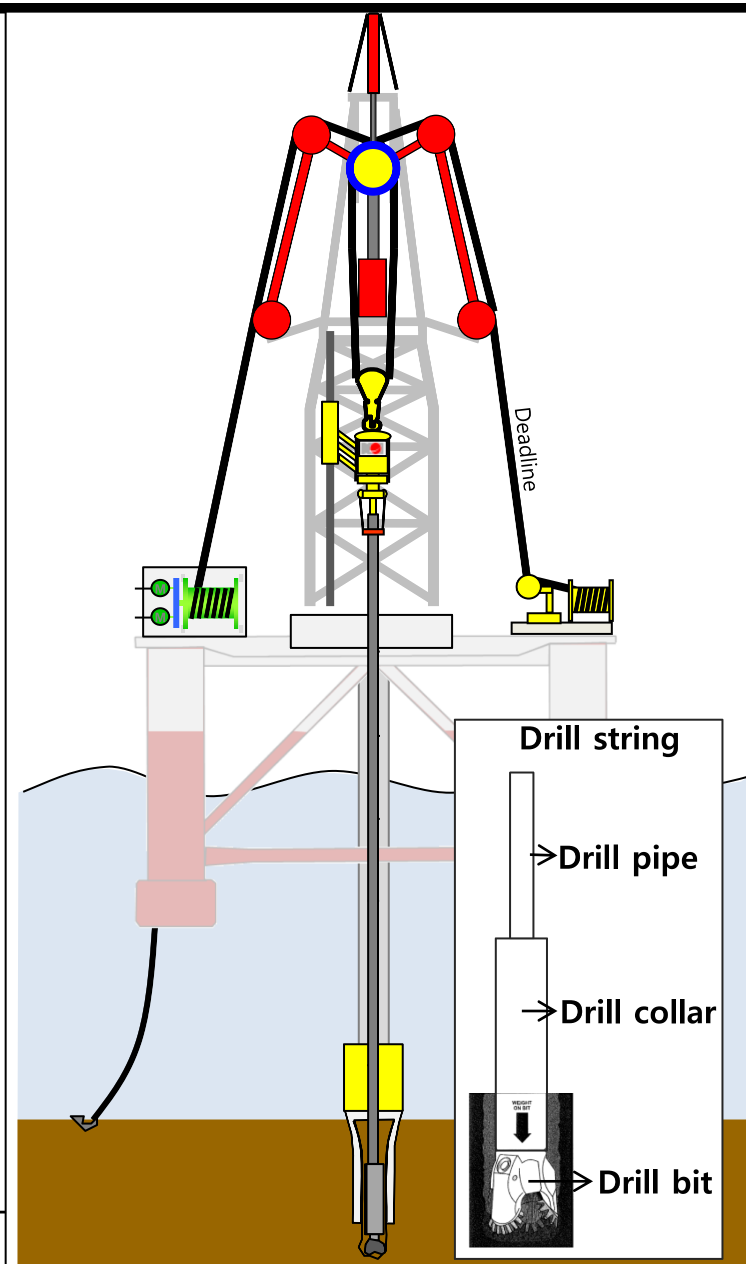
Drilling Procedure

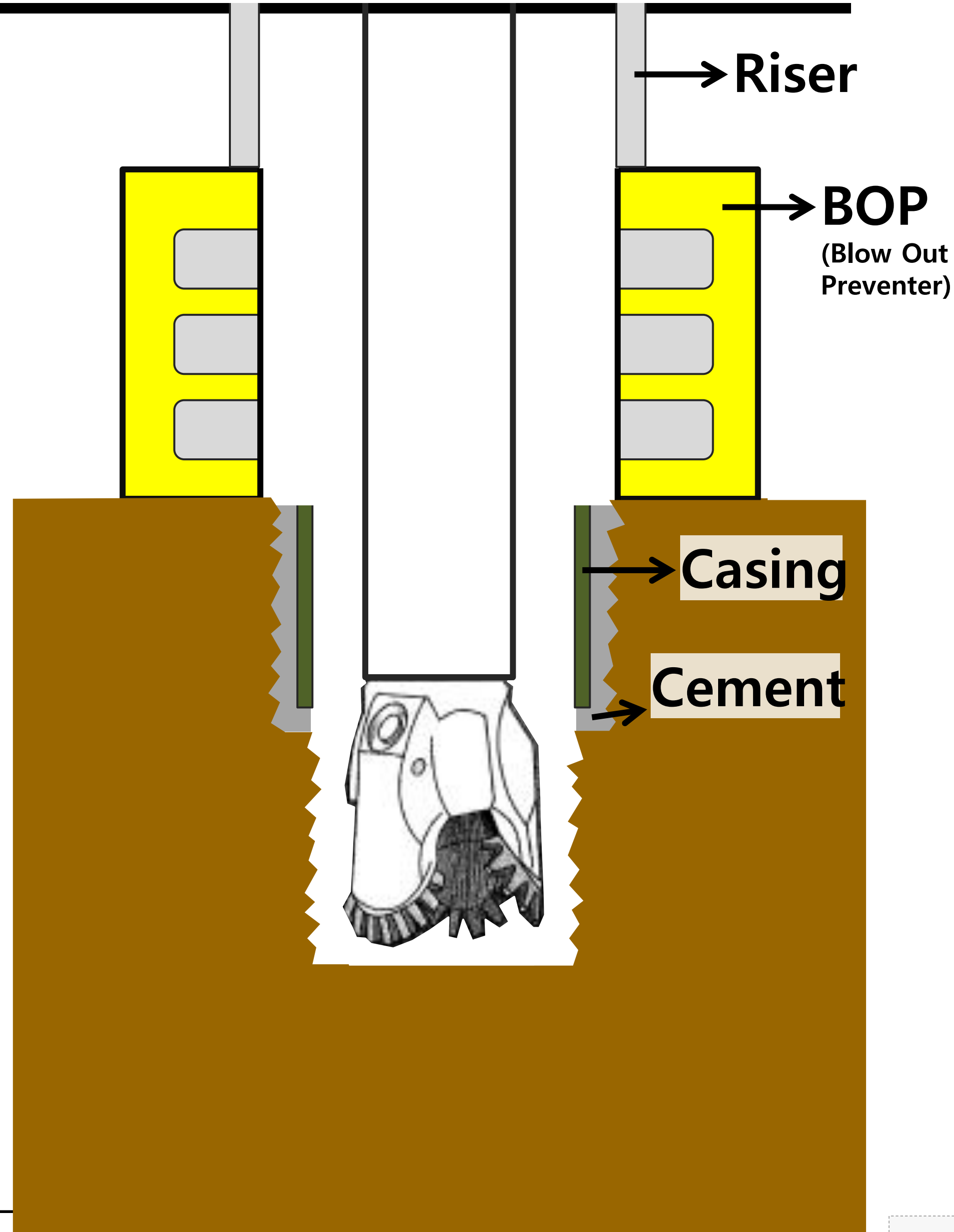




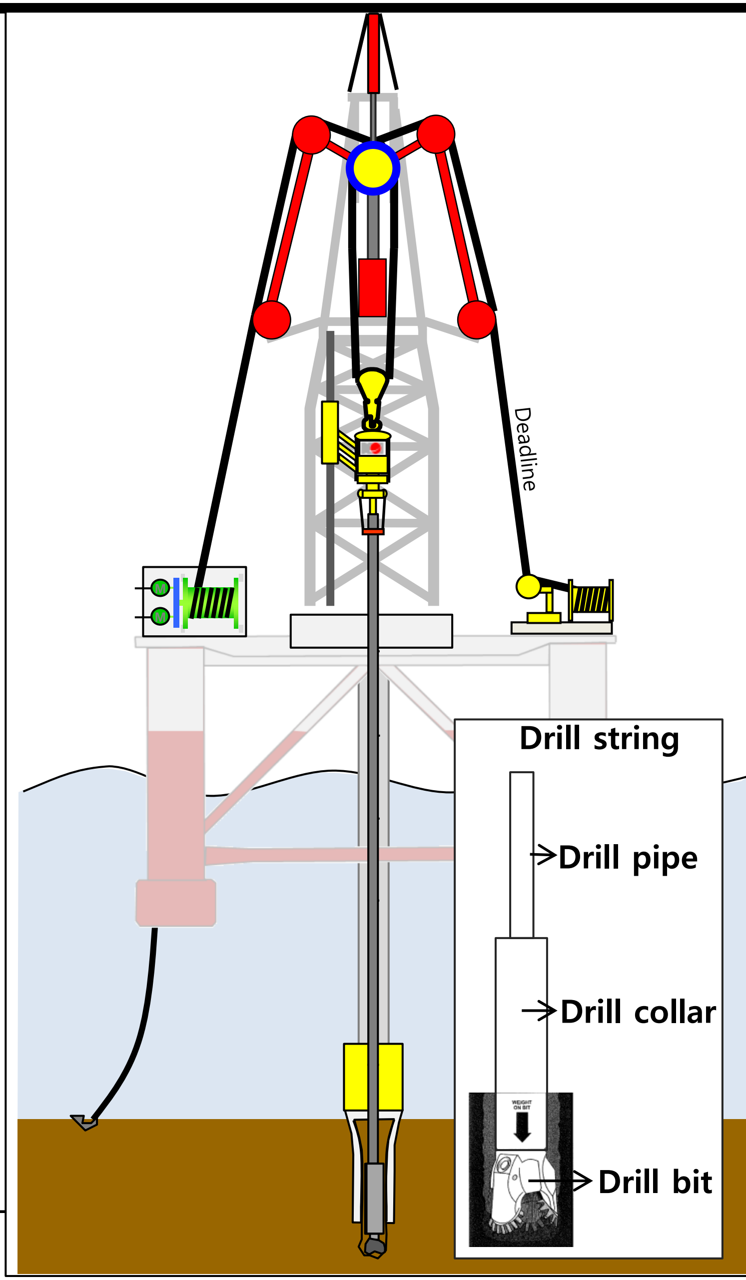
The operator wait until the cement to harden. This period of time is referred to as *waiting on cement*, or *WOC*. Usual WOC time is a few hours, depending on the cement formulation and the well temperature.

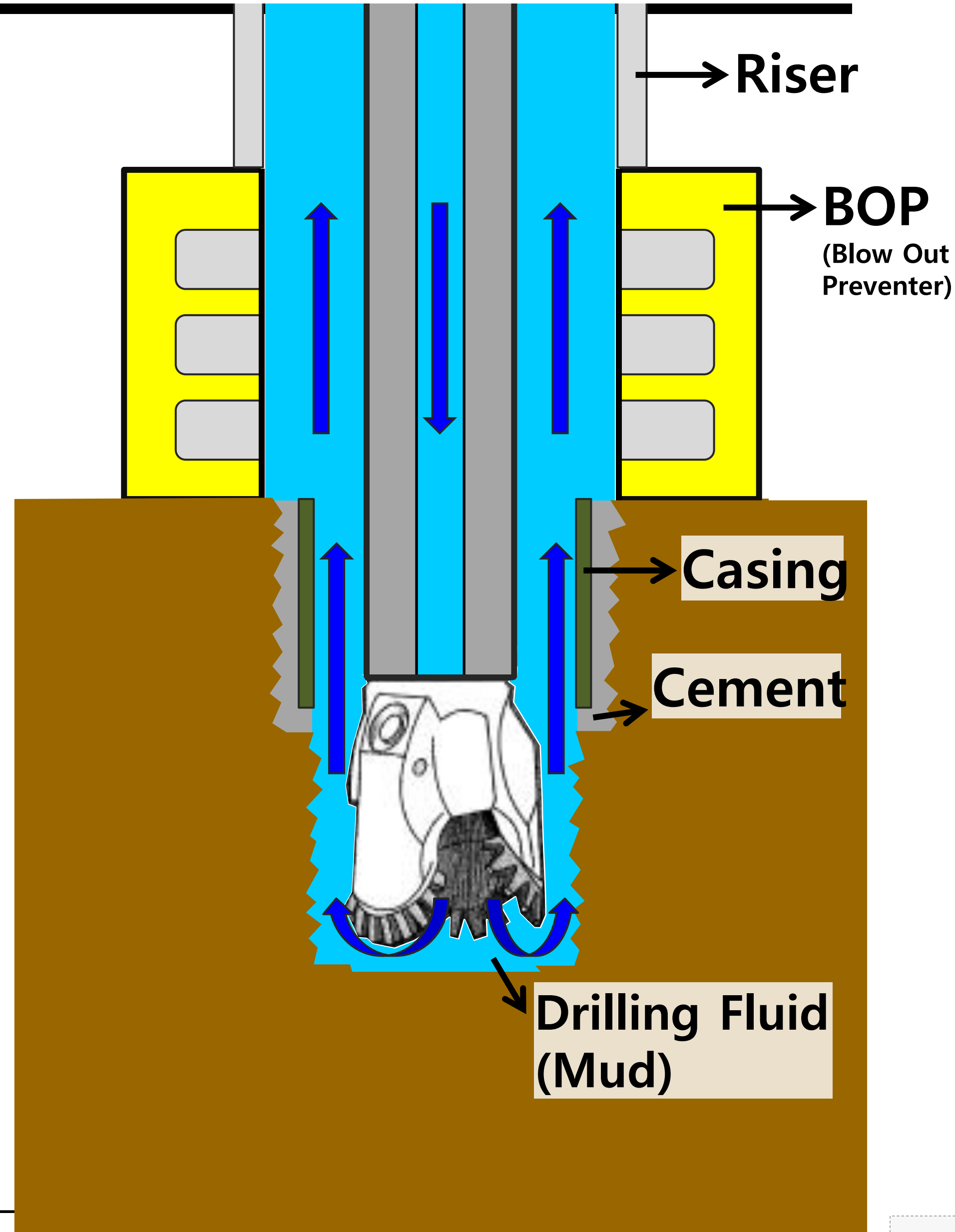
Drilling Procedure



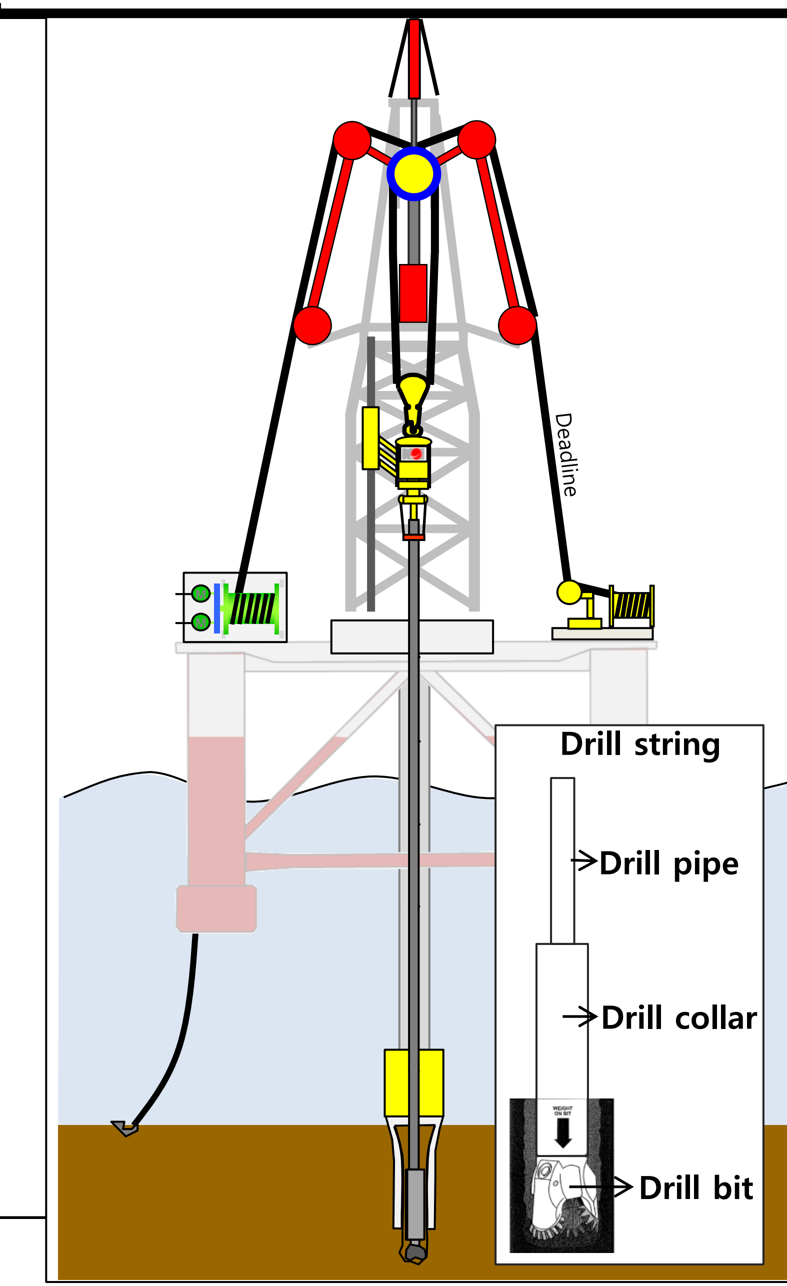


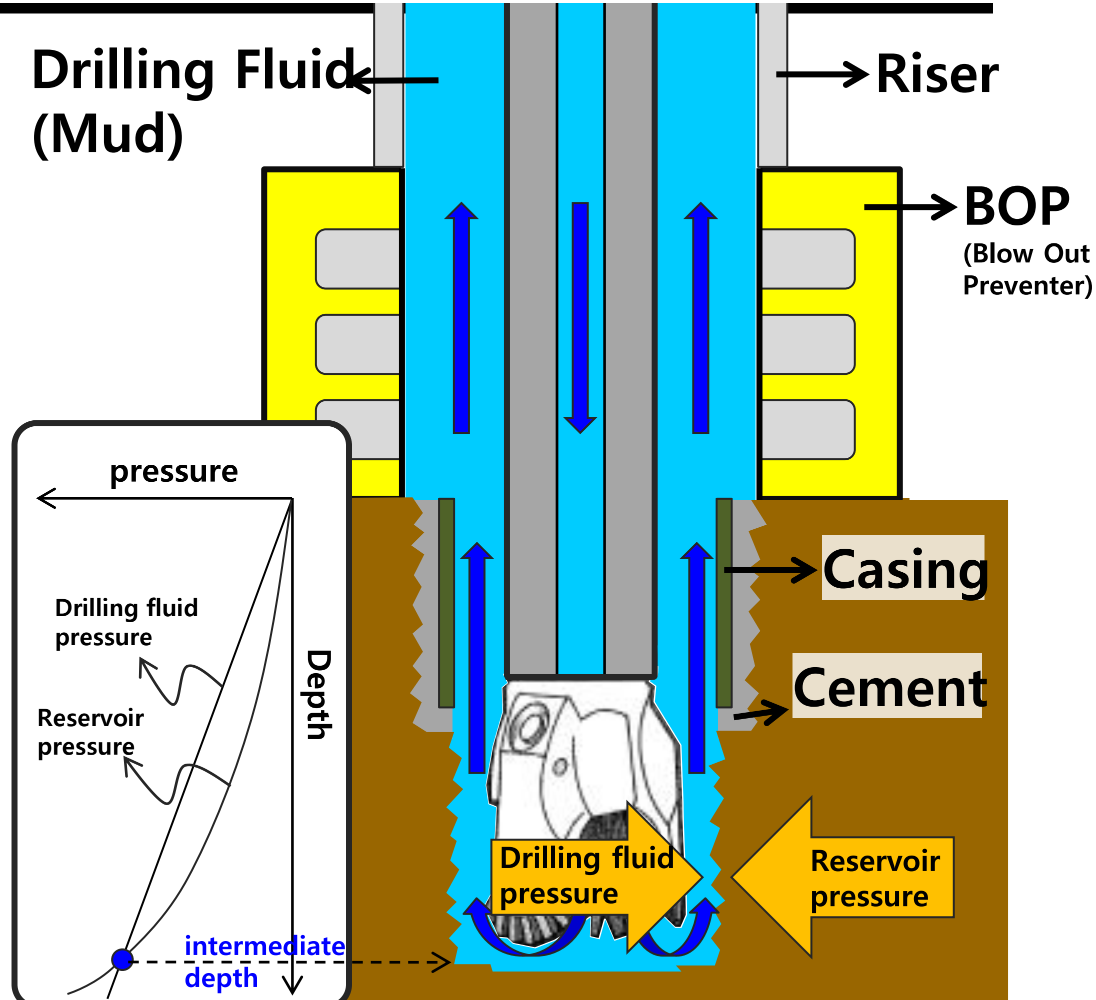
Drilling Procedure





Drilling Procedure

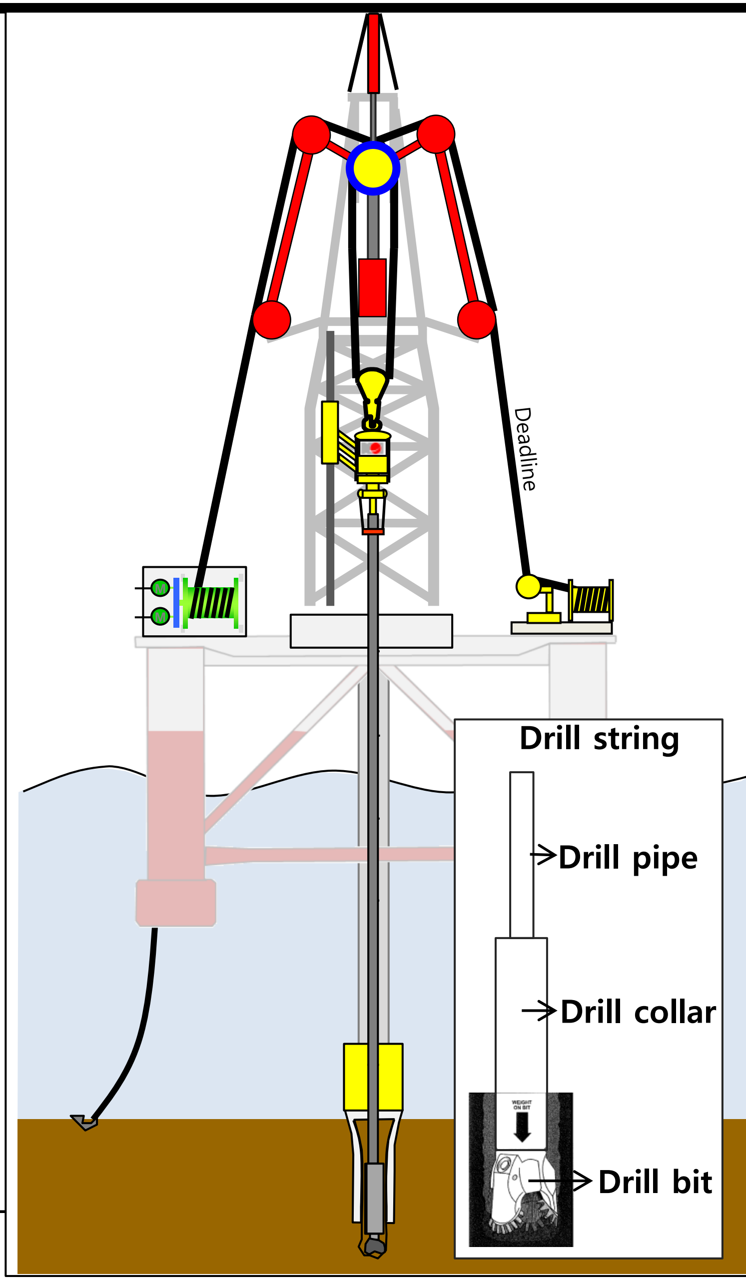


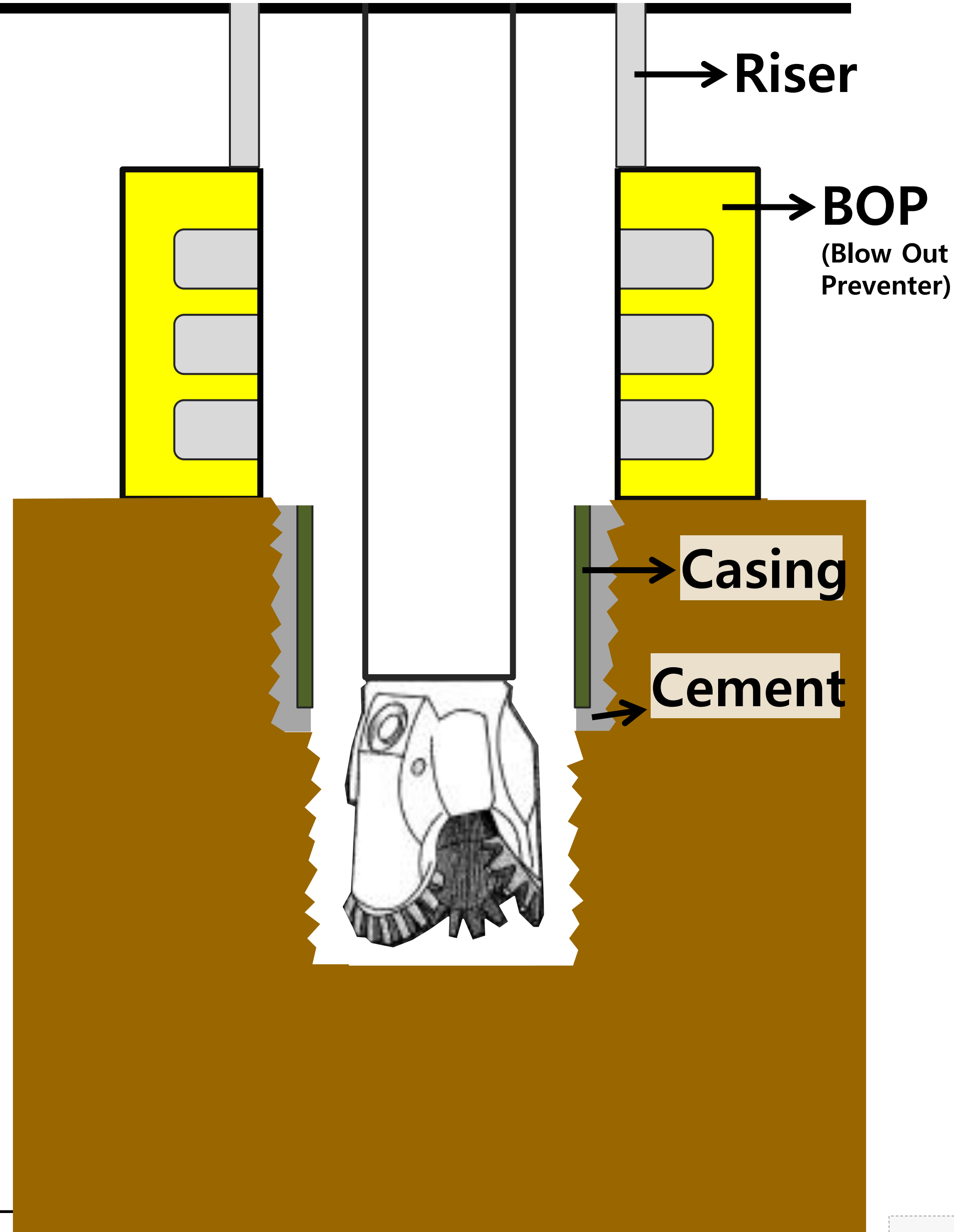


Determination of an intermediate depth of drilling

The drilling fluid prevents the flow of reservoir fluid by creating pressure inside the hole that is greater than the reservoir pressure. Therefore, the drilling proceeds to the intermediate depth, at which the reservoir pressure becomes greater than the pressure of the drilling fluid

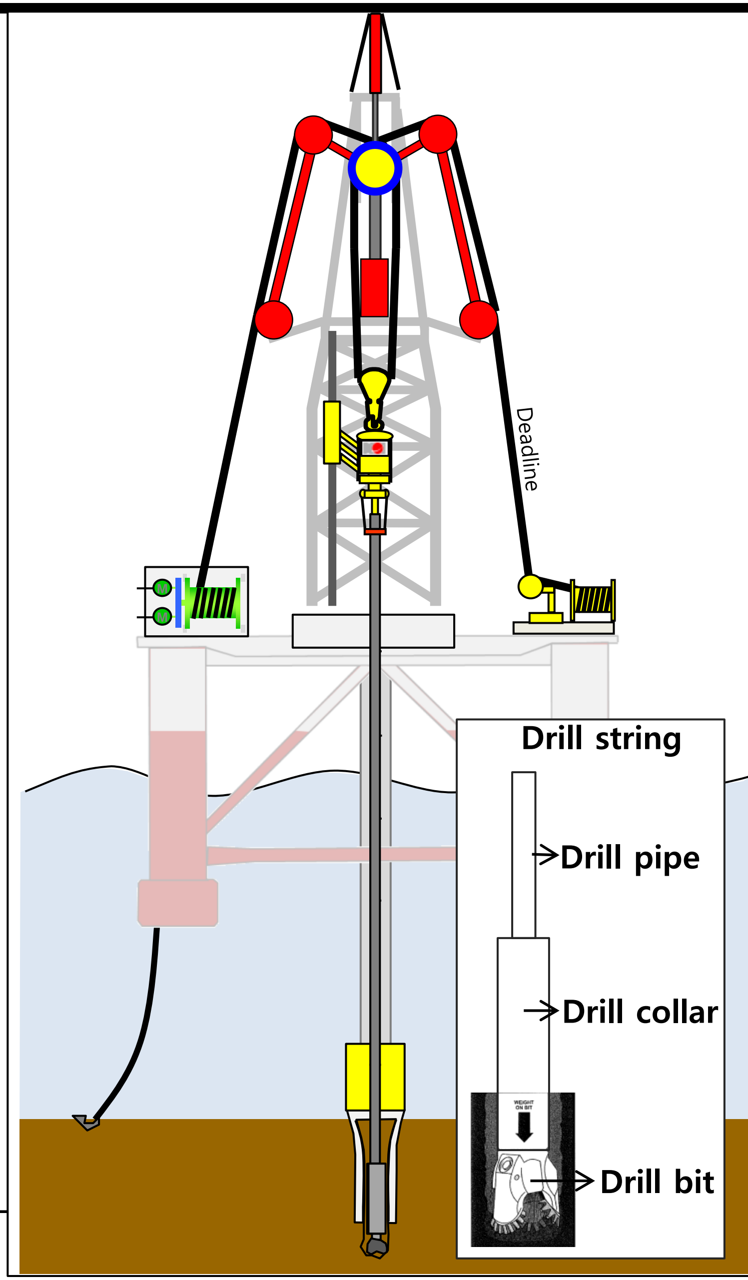
Drilling Procedure

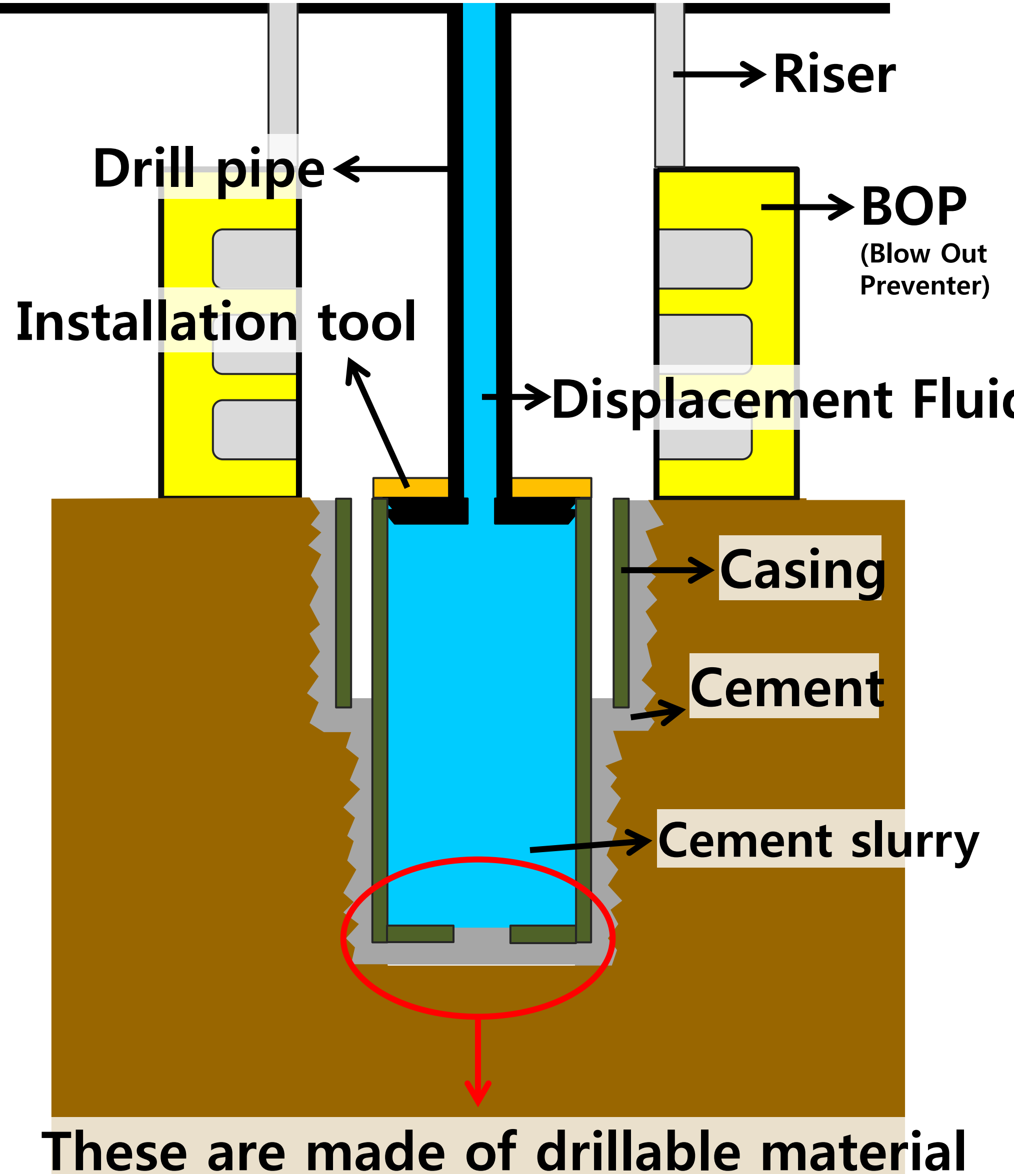




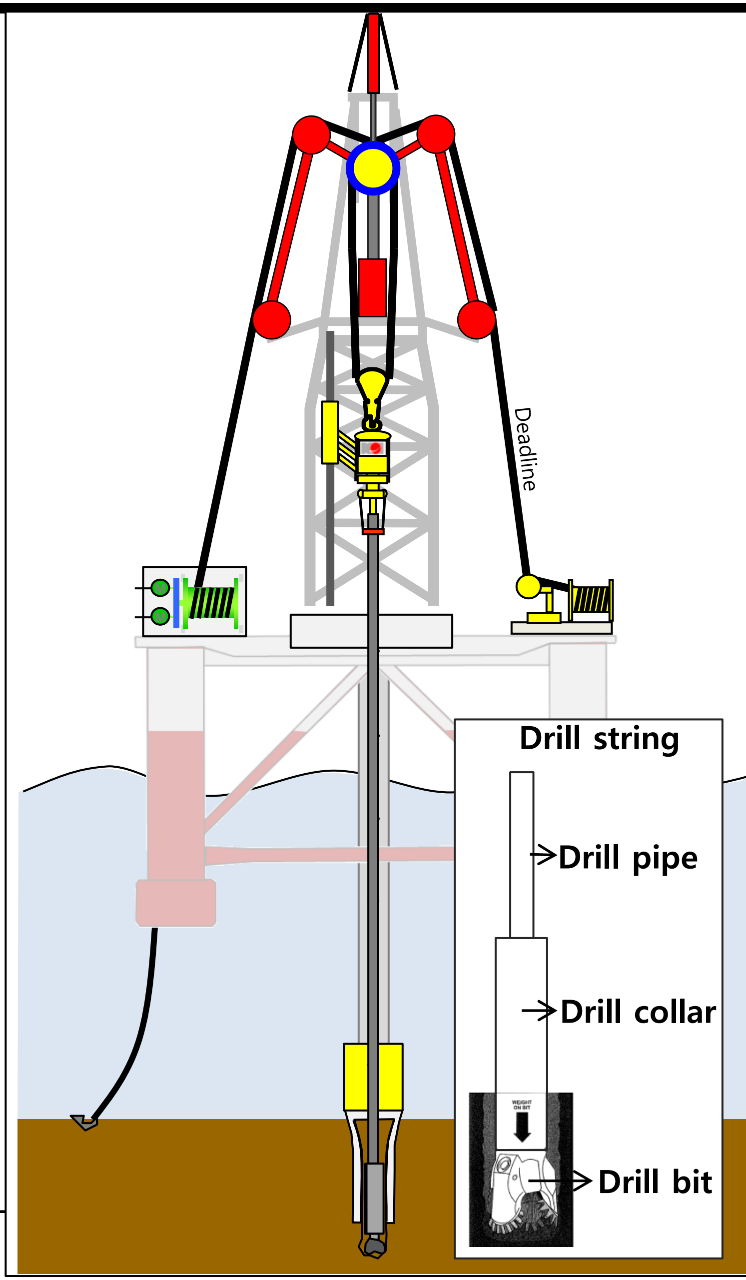
Drilling Procedure

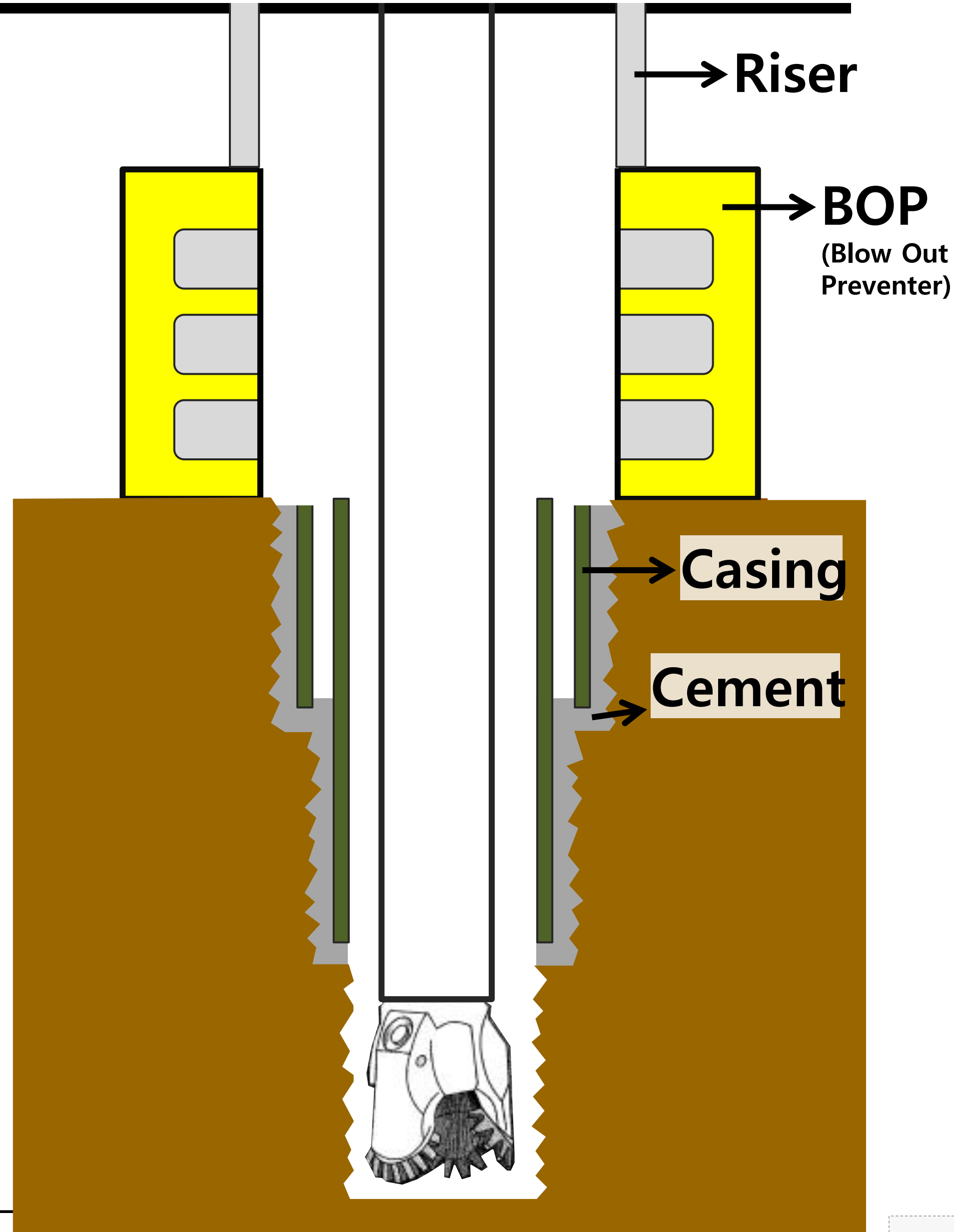
	Diameter(cm)	Weight per unit length (kg/m)	Material	Thickness(mm)
Casing	12.7~76.2	15~197	Steel	5~16





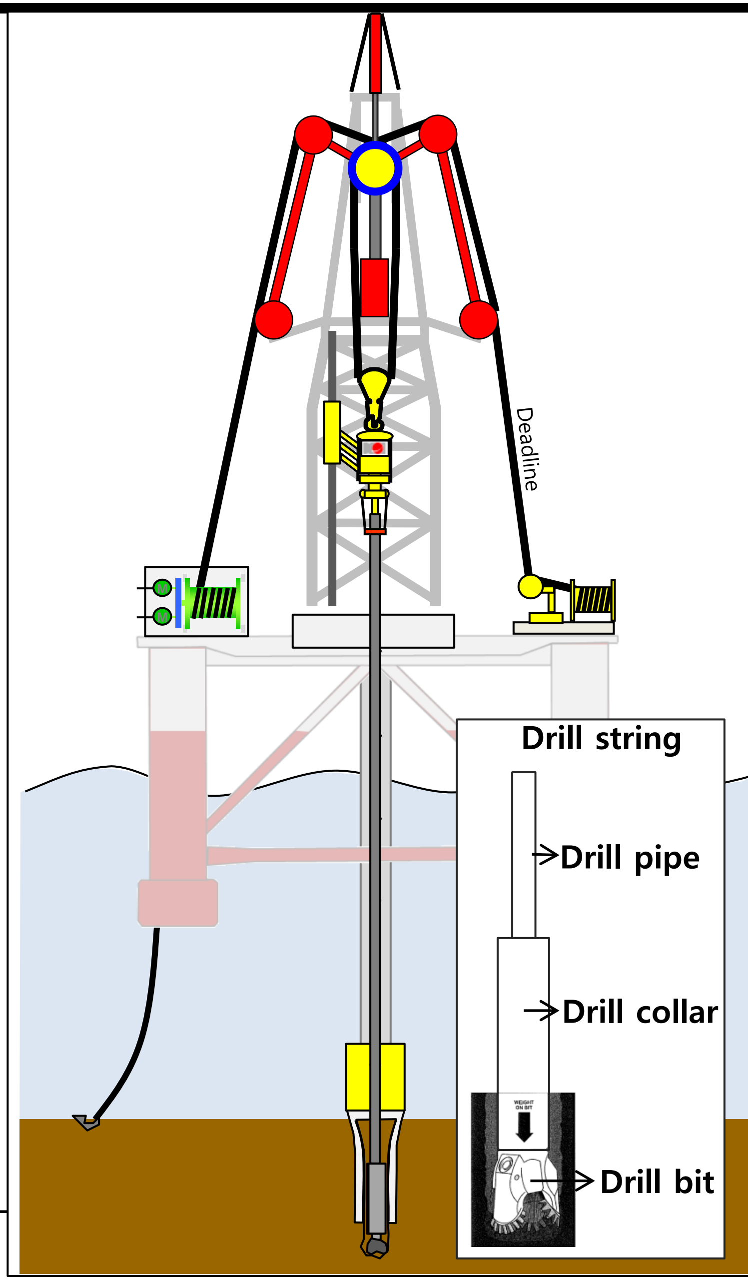
Drilling Procedure

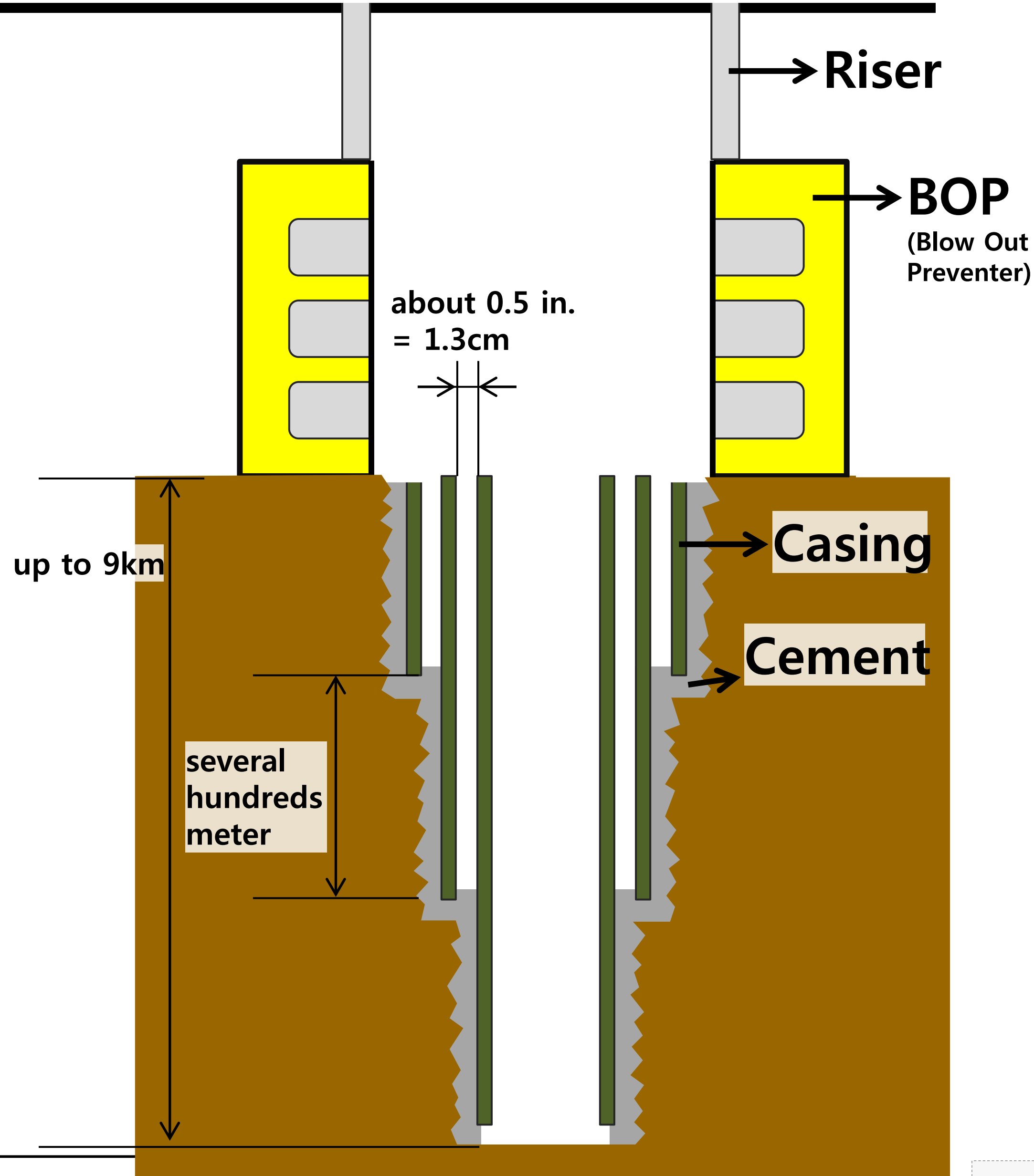




Drilling Procedure

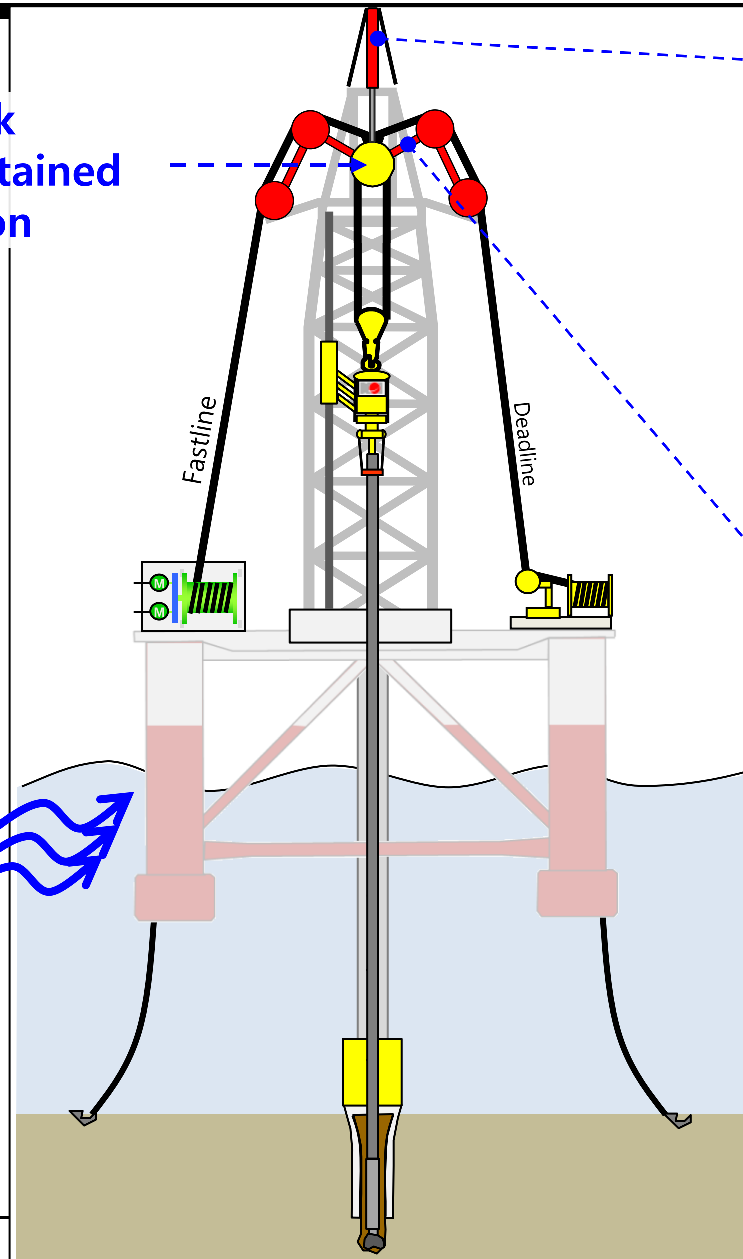
	Diameter(cm)	Weight per unit length (kg/m)	Material	Thickness(mm)
Casing	12.7~76.2	15~197	Steel	5~16





Function of the Drill String Compensator and Active Heave Compensator of Heave Compensation System

The crown block should be maintained constant position



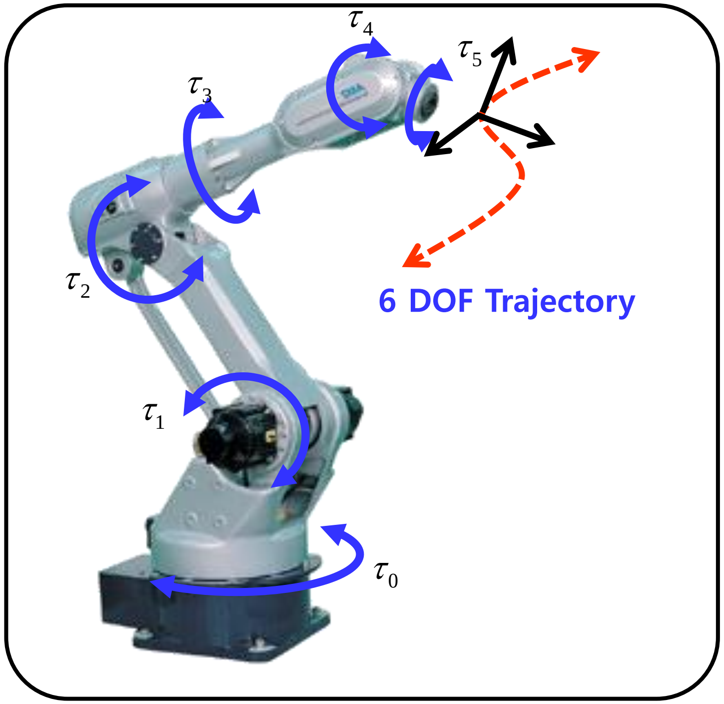
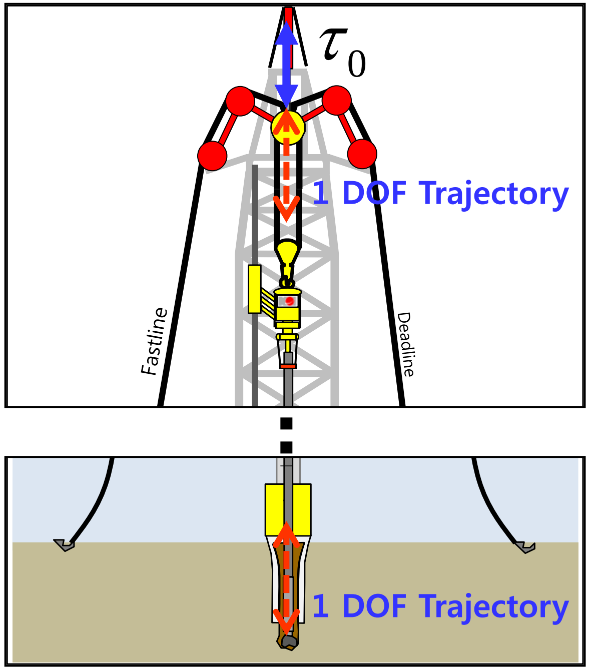
Hydrostatic force,
Hydrodynamic force

Active Heave Compensator(AHC)

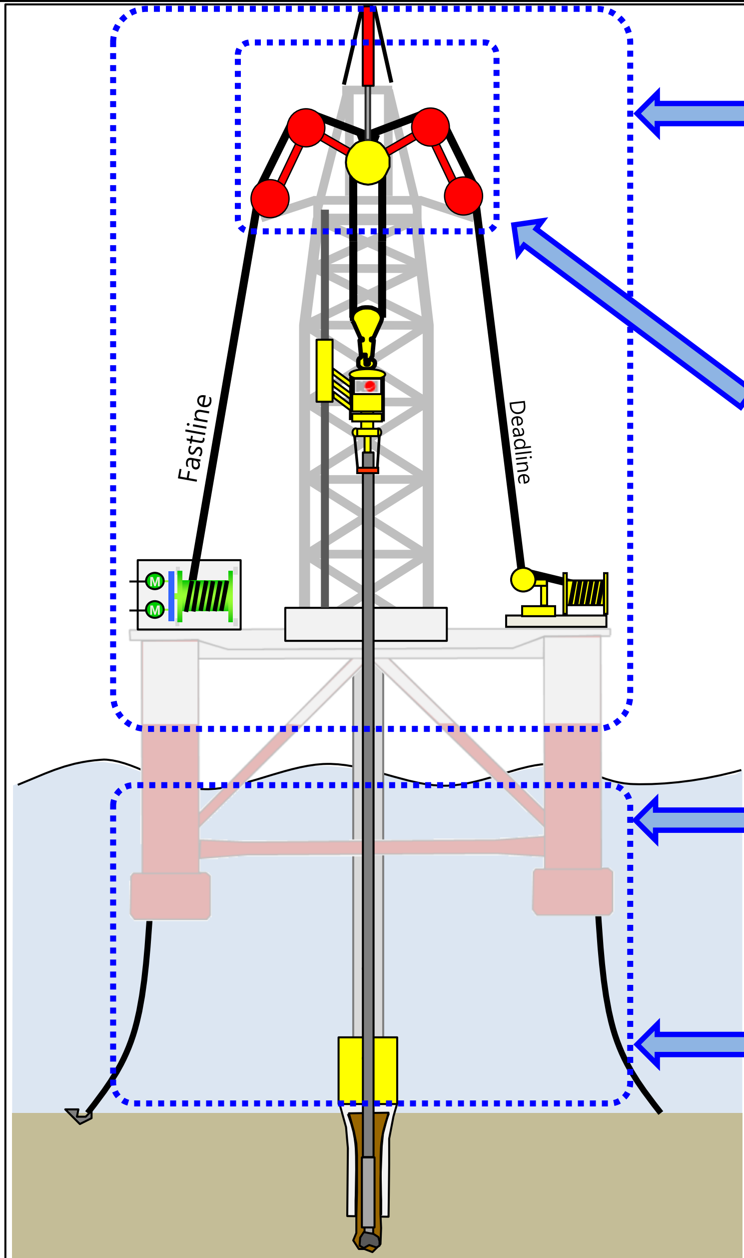
Drill String Compensator(DSC)

Heave compensation system

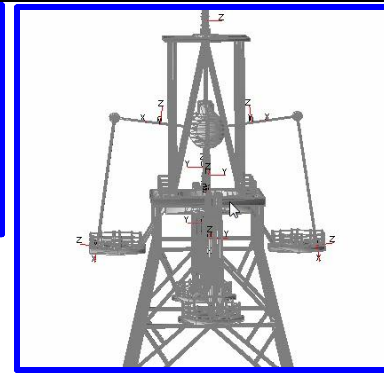
Comparison with Robotic System

	6 DOF Robot	Heave Compensation System (a kind of 1 DOF Robot)
		
Control Forces	Torque of the 6 joints must be determined	Force of the 1 joint(AHC) must be determined (Passive control is used for DSC)
Actuator	Electronic Power	Hydraulic and Pneumatic Power

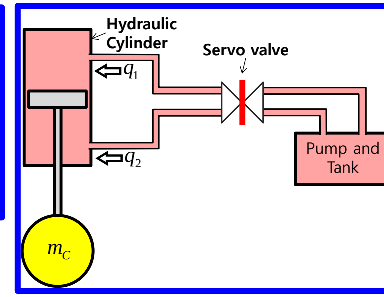
System Configuration



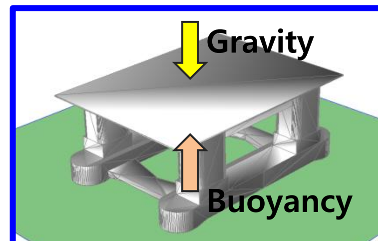
Multibody dynamics kernel is developed based on recursive formulation, and verified.



Hydraulic and Pneumatic control module is developed and verified.



Hydrostatic force and static equilibrium position can be calculated.

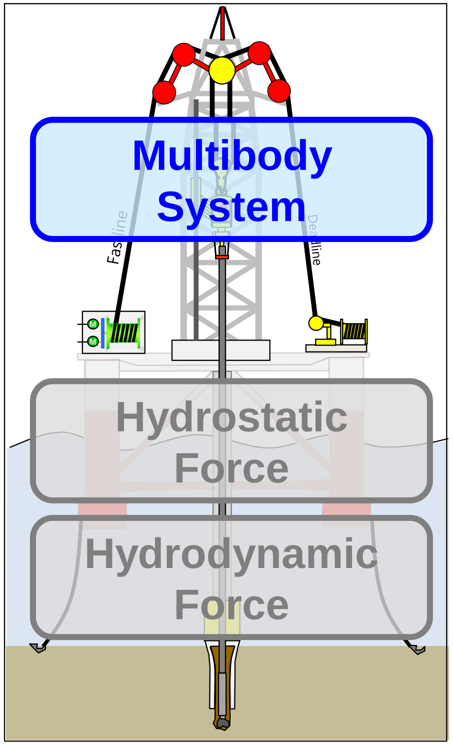


Hydrodynamic force in time domain

Hydrodynamic force in frequency domain

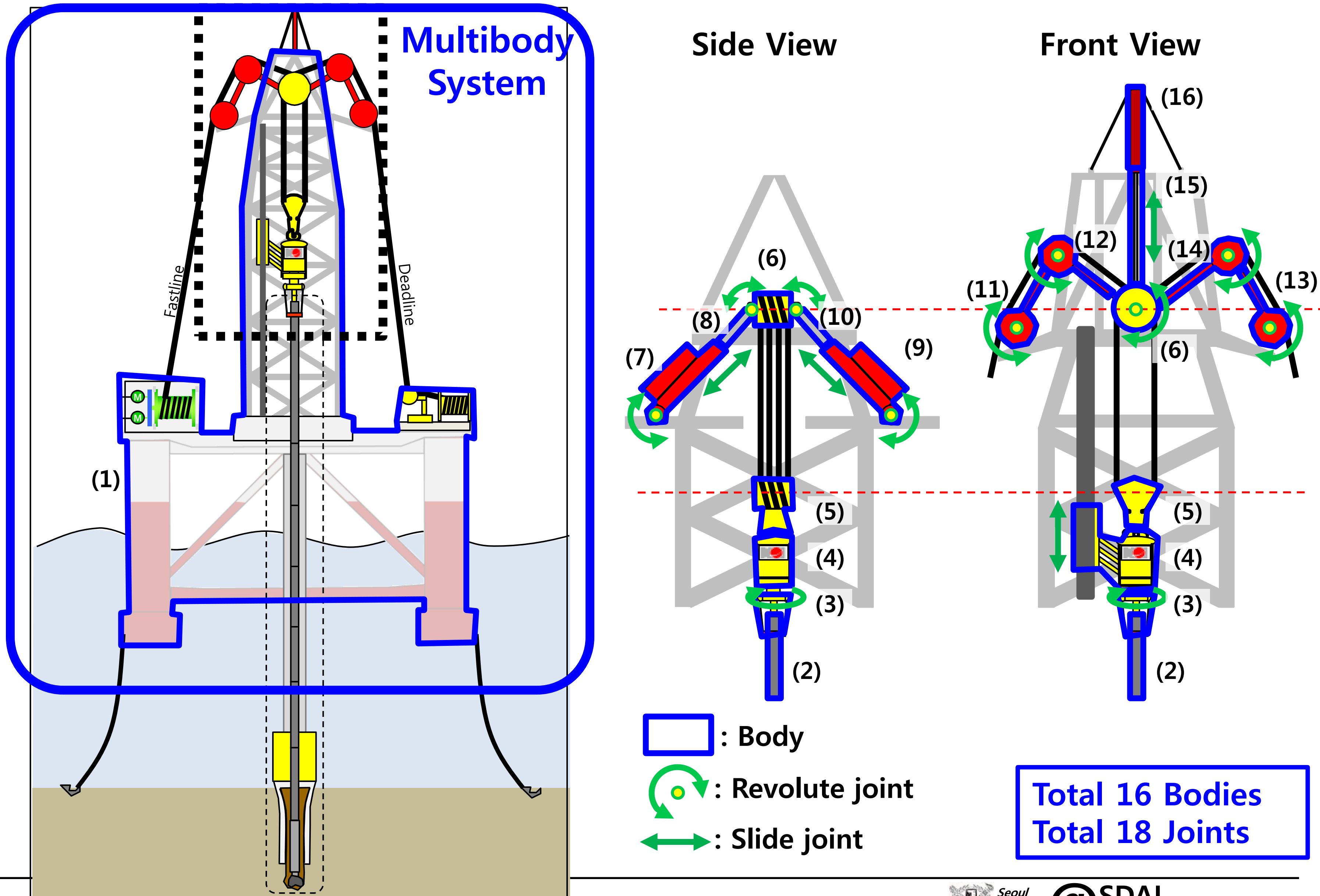
WADAM : frequency domain hydrodynamic analysis S/W developed by DNV

18-2. Mathematical Model of Hoisting and Heave Compensation System Based on Multibody Dynamics



MULTIBDOY SYSTEM

- Kinematic Relations between the Components of Hoisting System and Heave Compensation System



Total 16 Bodies
Total 18 Joints

Equations of motion of the Hoisting System

$$\mathbf{M}_1\ddot{\mathbf{s}}_1 + \mathbf{Q}_1\dot{\mathbf{s}}_1 = \mathbf{F}^e_{hydrostatc} + \mathbf{F}^e_{hydrodynamic} + \mathbf{F}^e_{gravity} + \mathbf{F}^c_{1+4} + \mathbf{F}^c_{1+7} + \mathbf{F}^c_{1+9} + \mathbf{F}^c_{1+11} + \mathbf{F}^c_{1+13} + \mathbf{F}^c_{1+16}$$

$$\mathbf{M}_2\ddot{\mathbf{s}}_2 + \mathbf{Q}_2\dot{\mathbf{s}}_2 = \mathbf{F}^e_{gravity} + \mathbf{F}^c_{2+3}$$

$$\mathbf{M}_3\ddot{\mathbf{s}}_3 + \mathbf{Q}_3\dot{\mathbf{s}}_3 = \mathbf{F}^e_{gravity} + \mathbf{F}^c_{3+2} + \mathbf{F}^c_{3+4}$$

$$\mathbf{M}_4\ddot{\mathbf{s}}_4 + \mathbf{Q}_4\dot{\mathbf{s}}_4 = \mathbf{F}^e_{gravity} + \mathbf{F}^c_{4+1} + \mathbf{F}^c_{4+3} + \mathbf{F}^c_{4+5}$$

$$\mathbf{M}_5\ddot{\mathbf{s}}_5 + \mathbf{Q}_5\dot{\mathbf{s}}_5 = \mathbf{F}^e_{wire} + \mathbf{F}^e_{gravity} + \mathbf{F}^c_{5+4}$$

$$\mathbf{M}_6\ddot{\mathbf{s}}_6 + \mathbf{Q}_6\dot{\mathbf{s}}_6 = \mathbf{F}^e_{wire} + \mathbf{F}^e_{gravity} + \mathbf{F}^c_{6+8} + \mathbf{F}^c_{6+10} + \mathbf{F}^c_{6+12} + \mathbf{F}^c_{6+14} + \mathbf{F}^c_{6+15}$$

$$\mathbf{M}_7\ddot{\mathbf{s}}_7 + \mathbf{Q}_7\dot{\mathbf{s}}_7 = \mathbf{F}^e_{wire} + \mathbf{F}^e_{gravity} + \mathbf{F}^c_{7+1} + \mathbf{F}^c_{7+8} + \mathbf{F}^u_{7+8}$$

$$\mathbf{M}_8\ddot{\mathbf{s}}_8 + \mathbf{Q}_8\dot{\mathbf{s}}_8 = \mathbf{F}^e_{gravity} + \mathbf{F}^c_{8+6} + \mathbf{F}^c_{8+7} + \mathbf{F}^u_{8+7}$$

$$\mathbf{M}_9\ddot{\mathbf{s}}_9 + \mathbf{Q}_9\dot{\mathbf{s}}_9 = \mathbf{F}^e_{gravity} + \mathbf{F}^c_{9+1} + \mathbf{F}^c_{9+10} + \mathbf{F}^u_{9+10}$$

$$\mathbf{M}_{10}\ddot{\mathbf{s}}_{10} + \mathbf{Q}_{10}\dot{\mathbf{s}}_{10} = \mathbf{F}^e_{gravity} + \mathbf{F}^c_{10+6} + \mathbf{F}^c_{10+9} + \mathbf{F}^u_{10+9}$$

$$\mathbf{M}_{11}\ddot{\mathbf{s}}_{11} + \mathbf{Q}_{11}\dot{\mathbf{s}}_{11} = \mathbf{F}^e_{gravity} + \mathbf{F}^c_{11+1} + \mathbf{F}^c_{11+12}$$

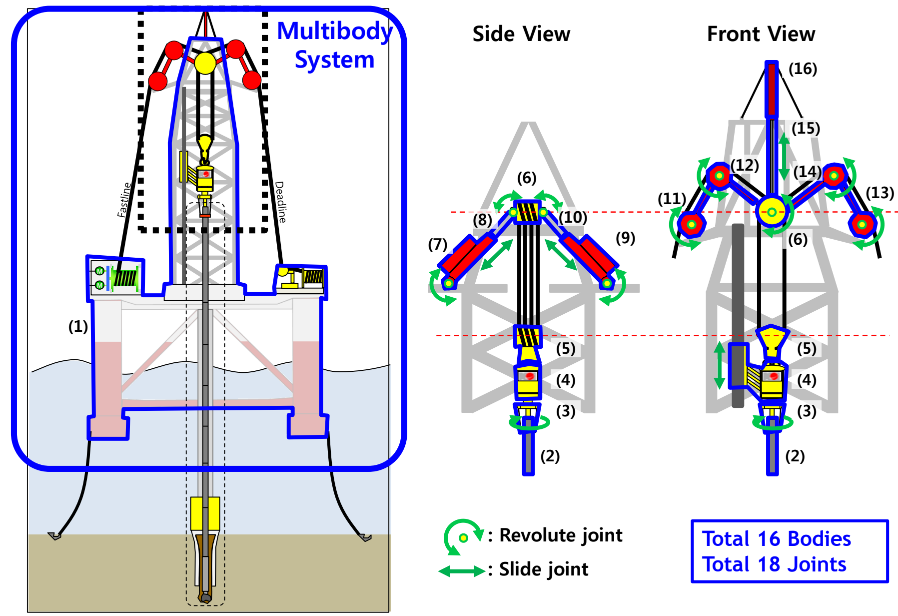
$$\mathbf{M}_{12}\ddot{\mathbf{s}}_{12} + \mathbf{Q}_{12}\dot{\mathbf{s}}_{12} = \mathbf{F}^e_{gravity} + \mathbf{F}^c_{12+11} + \mathbf{F}^c_{12+6}$$

$$\mathbf{M}_{13}\ddot{\mathbf{s}}_{13} + \mathbf{Q}_{13}\dot{\mathbf{s}}_{13} = \mathbf{F}^e_{gravity} + \mathbf{F}^c_{13+1} + \mathbf{F}^c_{13+14}$$

$$\mathbf{M}_{14}\ddot{\mathbf{s}}_{14} + \mathbf{Q}_{14}\dot{\mathbf{s}}_{14} = \mathbf{F}^e_{gravity} + \mathbf{F}^c_{14+13} + \mathbf{F}^c_{14+6}$$

$$\mathbf{M}_{15}\ddot{\mathbf{s}}_{15} + \mathbf{Q}_{15}\dot{\mathbf{s}}_{15} = \mathbf{F}^e_{gravity} + \mathbf{F}^c_{15+6} + \mathbf{F}^c_{15+16} + \mathbf{F}^u_{15+16}$$

$$\mathbf{M}_{16}\ddot{\mathbf{s}}_{16} + \mathbf{Q}_{16}\dot{\mathbf{s}}_{16} = \mathbf{F}^e_{gravity} + \mathbf{F}^c_{16+1} + \mathbf{F}^c_{16+15} + \mathbf{F}^u_{16+15}$$



$\mathbf{M}$: Mass and mass moment of inertia

$\mathbf{Q}\dot{\mathbf{s}}$: Coriolis and centrifugal forces

$\mathbf{s}$: Coordinates of each body

$\mathbf{F}^e$: External Forces (gravitational, hydrostatic, hydrodynamic, current, mooring forces

$\mathbf{F}^c$: Constraint Force

Equations of motion of the Hoisting System

$$\mathbf{M}_1\ddot{\mathbf{s}}_1 + \mathbf{Q}_1\dot{\mathbf{s}}_1 = \mathbf{F}^e_{hydrostatc} + \mathbf{F}^e_{hydrodynamic} + \mathbf{F}^e_{gravity} + \boxed{\mathbf{F}^c_{1+4} + \mathbf{F}^c_{1+7} + \mathbf{F}^c_{1+9} + \mathbf{F}^c_{1+11} + \mathbf{F}^c_{1+13} + \mathbf{F}^c_{1+16}}$$

$$\mathbf{M}_2\ddot{\mathbf{s}}_2 + \mathbf{Q}_2\dot{\mathbf{s}}_2 = \mathbf{F}^e_{gravity} + \boxed{\mathbf{F}^c_{2+3}}$$

$$\mathbf{M}_3\ddot{\mathbf{s}}_3 + \mathbf{Q}_3\dot{\mathbf{s}}_3 = \mathbf{F}^e_{gravity} + \boxed{\mathbf{F}^c_{3+2} + \mathbf{F}^c_{3+4}}$$

$$\mathbf{M}_4\ddot{\mathbf{s}}_4 + \mathbf{Q}_4\dot{\mathbf{s}}_4 = \mathbf{F}^e_{gravity} + \boxed{\mathbf{F}^c_{4+1} + \mathbf{F}^c_{4+3} + \mathbf{F}^c_{4+5}}$$

$$\mathbf{M}_5\ddot{\mathbf{s}}_5 + \mathbf{Q}_5\dot{\mathbf{s}}_5 = \mathbf{F}^e_{wire} + \mathbf{F}^e_{gravity} + \boxed{\mathbf{F}^c_{5+4}}$$

$$\mathbf{M}_6\ddot{\mathbf{s}}_6 + \mathbf{Q}_6\dot{\mathbf{s}}_6 = \mathbf{F}^e_{wire} + \mathbf{F}^e_{gravity} + \boxed{\mathbf{F}^c_{6+8} + \mathbf{F}^c_{6+10} + \mathbf{F}^c_{6+12} + \mathbf{F}^c_{6+14} + \mathbf{F}^c_{6+15}}$$

$$\mathbf{M}_7\ddot{\mathbf{s}}_7 + \mathbf{Q}_7\dot{\mathbf{s}}_7 = \mathbf{F}^e_{wire} + \mathbf{F}^e_{gravity} + \boxed{\mathbf{F}^c_{7+1} + \mathbf{F}^c_{7+8}} + \mathbf{F}^u_{7+8}$$

$$\mathbf{M}_8\ddot{\mathbf{s}}_8 + \mathbf{Q}_8\dot{\mathbf{s}}_8 = \mathbf{F}^e_{gravity} + \boxed{\mathbf{F}^c_{8+6} + \mathbf{F}^c_{8+7}} + \mathbf{F}^u_{8+7}$$

$$\mathbf{M}_9\ddot{\mathbf{s}}_9 + \mathbf{Q}_9\dot{\mathbf{s}}_9 = \mathbf{F}^e_{gravity} + \boxed{\mathbf{F}^c_{9+1} + \mathbf{F}^c_{9+10}} + \mathbf{F}^u_{9+10}$$

$$\mathbf{M}_{10}\ddot{\mathbf{s}}_{10} + \mathbf{Q}_{10}\dot{\mathbf{s}}_{10} = \mathbf{F}^e_{gravity} + \boxed{\mathbf{F}^c_{10+6} + \mathbf{F}^c_{10+9}} + \mathbf{F}^u_{10+9}$$

$$\mathbf{M}_{11}\ddot{\mathbf{s}}_{11} + \mathbf{Q}_{11}\dot{\mathbf{s}}_{11} = \mathbf{F}^e_{gravity} + \boxed{\mathbf{F}^c_{11+1} + \mathbf{F}^c_{11+12}}$$

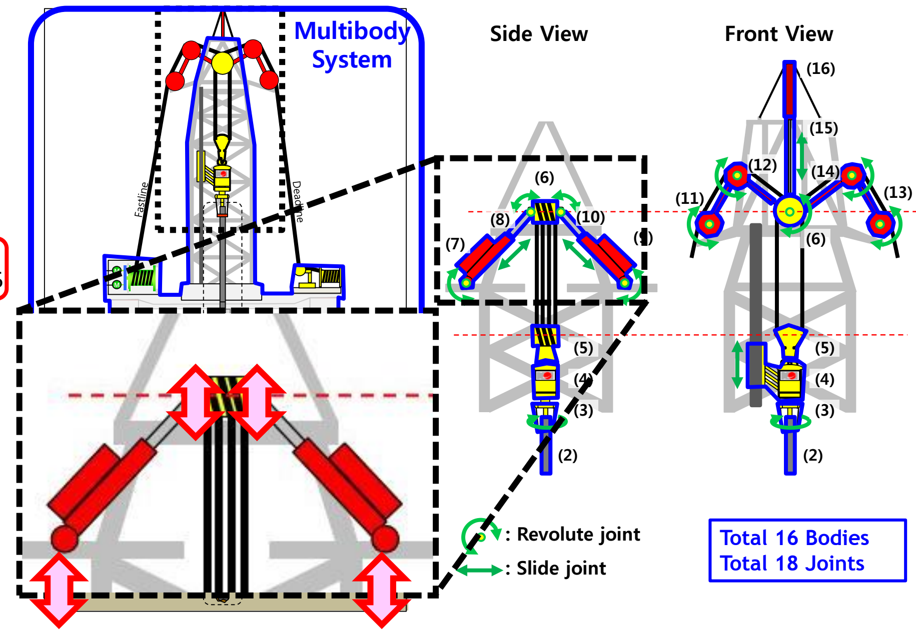
$$\mathbf{M}_{12}\ddot{\mathbf{s}}_{12} + \mathbf{Q}_{12}\dot{\mathbf{s}}_{12} = \mathbf{F}^e_{gravity} + \boxed{\mathbf{F}^c_{12+11} + \mathbf{F}^c_{12+6}}$$

$$\mathbf{M}_{13}\ddot{\mathbf{s}}_{13} + \mathbf{Q}_{13}\dot{\mathbf{s}}_{13} = \mathbf{F}^e_{gravity} + \boxed{\mathbf{F}^c_{13+1} + \mathbf{F}^c_{13+14}}$$

$$\mathbf{M}_{14}\ddot{\mathbf{s}}_{14} + \mathbf{Q}_{14}\dot{\mathbf{s}}_{14} = \mathbf{F}^e_{gravity} + \boxed{\mathbf{F}^c_{14+13} + \mathbf{F}^c_{14+6}}$$

$$\mathbf{M}_{15}\ddot{\mathbf{s}}_{15} + \mathbf{Q}_{15}\dot{\mathbf{s}}_{15} = \mathbf{F}^e_{gravity} + \boxed{\mathbf{F}^c_{15+6} + \mathbf{F}^c_{15+16}} + \mathbf{F}^u_{15+16}$$

$$\mathbf{M}_{16}\ddot{\mathbf{s}}_{16} + \mathbf{Q}_{16}\dot{\mathbf{s}}_{16} = \mathbf{F}^e_{gravity} + \boxed{\mathbf{F}^c_{16+1} + \mathbf{F}^c_{16+15}} + \mathbf{F}^u_{16+15}$$



$\mathbf{M}$: Mass and mass moment of inertia

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$\mathbf{s}$: Coordinates of each body

$\mathbf{F}^e$: External Forces (gravitational, hydrostatic, hydrodynamic, current, mooring forces

$\boxed{\mathbf{F}^c\text{: Constraint Force}}$

Action and Reaction force between bodies

Equations of motion of the Hoisting System

$$\mathbf{M}_1\ddot{\mathbf{s}}_1+\mathbf{Q}_1\dot{\mathbf{s}}_1=\mathbf{F}^e_{hydrostatc}+\mathbf{F}^e_{hydrodynamic}+\mathbf{F}^e_{gravity}+\mathbf{F}^c_{1+4}+\mathbf{F}^c_{1+7}+\mathbf{F}^c_{1+9}+\mathbf{F}^c_{1+11}+\mathbf{F}^c_{1+13}+\mathbf{F}^c_{1+16}$$

$$\mathbf{M}_2\ddot{\mathbf{s}}_2+\mathbf{Q}_2\dot{\mathbf{s}}_2=\mathbf{F}^e_{gravity}+\mathbf{F}^c_{2+3}$$

$$\mathbf{M}_3\ddot{\mathbf{s}}_3+\mathbf{Q}_3\dot{\mathbf{s}}_3=\mathbf{F}^e_{gravity}+\mathbf{F}^c_{3+2}+\mathbf{F}^c_{3+4}$$

$$\mathbf{M}_4\ddot{\mathbf{s}}_4+\mathbf{Q}_4\dot{\mathbf{s}}_4=\mathbf{F}^e_{gravity}+\mathbf{F}^c_{4+1}+\mathbf{F}^c_{4+3}+\mathbf{F}^c_{4+5}$$

$$\mathbf{M}_5\ddot{\mathbf{s}}_5+\mathbf{Q}_5\dot{\mathbf{s}}_5=\mathbf{F}^e_{wire}+\mathbf{F}^e_{gravity}+\mathbf{F}^c_{5+4}$$

$$\mathbf{M}_6\ddot{\mathbf{s}}_6+\mathbf{Q}_6\dot{\mathbf{s}}_6=\mathbf{F}^e_{wire}+\mathbf{F}^e_{gravity}+\mathbf{F}^c_{6+8}+\mathbf{F}^c_{6+10}+\mathbf{F}^c_{6+12}+\mathbf{F}^c_{6+14}+\mathbf{F}^c_{6+15}$$

$$\mathbf{M}_7\ddot{\mathbf{s}}_7+\mathbf{Q}_7\dot{\mathbf{s}}_7=\mathbf{F}^e_{wire}+\mathbf{F}^e_{gravity}+\mathbf{F}^c_{7+1}+\mathbf{F}^c_{7+8}+\mathbf{F}^u_{7+8}$$

$$\mathbf{M}_8\ddot{\mathbf{s}}_8+\mathbf{Q}_8\dot{\mathbf{s}}_8=\mathbf{F}^e_{gravity}+\mathbf{F}^c_{8+6}+\mathbf{F}^c_{8+7}+\mathbf{F}^u_{8+7}$$

$$\mathbf{M}_9\ddot{\mathbf{s}}_9+\mathbf{Q}_9\dot{\mathbf{s}}_9=\mathbf{F}^e_{gravity}+\mathbf{F}^c_{9+1}+\mathbf{F}^c_{9+10}+\mathbf{F}^u_{9+10}$$

$$\mathbf{M}_{10}\ddot{\mathbf{s}}_{10}+\mathbf{Q}_{10}\dot{\mathbf{s}}_{10}=\mathbf{F}^e_{gravity}+\mathbf{F}^c_{10+6}+\mathbf{F}^c_{10+9}+\mathbf{F}^u_{10+9}$$

$$\mathbf{M}_{11}\ddot{\mathbf{s}}_{11}+\mathbf{Q}_{11}\dot{\mathbf{s}}_{11}=\mathbf{F}^e_{gravity}+\mathbf{F}^c_{11+1}+\mathbf{F}^c_{11+12}$$

$$\mathbf{M}_{12}\ddot{\mathbf{s}}_{12}+\mathbf{Q}_{12}\dot{\mathbf{s}}_{12}=\mathbf{F}^e_{gravity}+\mathbf{F}^c_{12+11}+\mathbf{F}^c_{12+6}$$

$$\mathbf{M}_{13}\ddot{\mathbf{s}}_{13}+\mathbf{Q}_{13}\dot{\mathbf{s}}_{13}=\mathbf{F}^e_{gravity}+\mathbf{F}^c_{13+1}+\mathbf{F}^c_{13+14}$$

$$\mathbf{M}_{14}\ddot{\mathbf{s}}_{14}+\mathbf{Q}_{14}\dot{\mathbf{s}}_{14}=\mathbf{F}^e_{gravity}+\mathbf{F}^c_{14+13}+\mathbf{F}^c_{14+6}$$

$$\mathbf{M}_{15}\ddot{\mathbf{s}}_{15}+\mathbf{Q}_{15}\dot{\mathbf{s}}_{15}=\mathbf{F}^e_{gravity}+\mathbf{F}^c_{15+6}+\mathbf{F}^c_{15+16}+\mathbf{F}^u_{15+16}$$

$$\mathbf{M}_{16}\ddot{\mathbf{s}}_{16}+\mathbf{Q}_{16}\dot{\mathbf{s}}_{16}=\mathbf{F}^e_{gravity}+\mathbf{F}^c_{16+1}+\mathbf{F}^c_{16+15}+\mathbf{F}^u_{16+15}$$

In matrix form

$$\begin{bmatrix}
\mathbf{M}_1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&\mathbf{M}_2&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&\mathbf{M}_3&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&\mathbf{M}_4&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&\mathbf{M}_5&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&\mathbf{M}_6&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&\mathbf{M}_7&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&\mathbf{M}_8&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&\mathbf{M}_9&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&\mathbf{M}_{10}&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&\mathbf{M}_{11}&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&\mathbf{M}_{12}&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&\mathbf{M}_{13}&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&\mathbf{M}_{14}&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&\mathbf{M}_{15}&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&\mathbf{M}_{16}
\end{bmatrix}
\begin{bmatrix}\ddot{\mathbf{s}}_1\\\ddot{\mathbf{s}}_2\\\ddot{\mathbf{s}}_3\\\ddot{\mathbf{s}}_4\\\ddot{\mathbf{s}}_5\\\ddot{\mathbf{s}}_6\\\ddot{\mathbf{s}}_7\\\ddot{\mathbf{s}}_8\\\ddot{\mathbf{s}}_9\\\ddot{\mathbf{s}}_{10}\\\ddot{\mathbf{s}}_{11}\\\ddot{\mathbf{s}}_{12}\\\ddot{\mathbf{s}}_{13}\\\ddot{\mathbf{s}}_{14}\\\ddot{\mathbf{s}}_{15}\\\ddot{\mathbf{s}}_{16}\end{bmatrix}
+
\begin{bmatrix}
\mathbf{Q}_1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&\mathbf{Q}_2&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&\mathbf{Q}_3&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&\mathbf{Q}_4&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&\mathbf{Q}_5&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&\mathbf{Q}_6&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&\mathbf{Q}_7&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&\mathbf{Q}_8&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&\mathbf{Q}_9&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&\mathbf{Q}_{10}&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&\mathbf{Q}_{11}&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&\mathbf{Q}_{12}&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&\mathbf{Q}_{13}&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&\mathbf{Q}_{14}&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&\mathbf{Q}_{15}&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&\mathbf{Q}_{16}
\end{bmatrix}
\begin{bmatrix}\dot{\mathbf{s}}_1\\\dot{\mathbf{s}}_2\\\dot{\mathbf{s}}_3\\\dot{\mathbf{s}}_4\\\dot{\mathbf{s}}_5\\\dot{\mathbf{s}}_6\\\dot{\mathbf{s}}_7\\\dot{\mathbf{s}}_8\\\dot{\mathbf{s}}_9\\\dot{\mathbf{s}}_{10}\\\dot{\mathbf{s}}_{11}\\\dot{\mathbf{s}}_{12}\\\dot{\mathbf{s}}_{13}\\\dot{\mathbf{s}}_{14}\\\dot{\mathbf{s}}_{15}\\\dot{\mathbf{s}}_{16}\end{bmatrix}
=
\begin{bmatrix}
\mathbf{F}^e_{hydrostatc}+\mathbf{F}^e_{hydrodynamic}+\mathbf{F}^e_{gravity}\\
\mathbf{F}^e_{gravity}\\
\mathbf{F}^e_{gravity}\\
\mathbf{F}^e_{gravity}\\
\mathbf{F}^e_{wire}+\mathbf{F}^e_{gravity}\\
\mathbf{F}^e_{wire}+\mathbf{F}^e_{gravity}\\
\mathbf{F}^e_{wire}+\mathbf{F}^e_{gravity}+\mathbf{F}^u_{7+8}\\
\mathbf{F}^e_{gravity}+\mathbf{F}^u_{8+7}\\
\mathbf{F}^e_{gravity}+\mathbf{F}^u_{9+10}\\
\mathbf{F}^e_{gravity}+\mathbf{F}^u_{10+9}\\
\mathbf{F}^e_{gravity}\\
\mathbf{F}^e_{gravity}\\
\mathbf{F}^e_{gravity}\\
\mathbf{F}^e_{gravity}\\
\mathbf{F}^e_{gravity}+\mathbf{F}^u_{15+16}\\
\mathbf{F}^e_{gravity}+\mathbf{F}^u_{16+15}
\end{bmatrix}
+
\begin{bmatrix}
\mathbf{F}^c_{1+4}+\mathbf{F}^c_{1+7}+\mathbf{F}^c_{1+9}+\mathbf{F}^c_{1+11}+\mathbf{F}^c_{1+13}+\mathbf{F}^c_{1+16}\\
\mathbf{F}^c_{2+3}\\
\mathbf{F}^c_{3+2}+\mathbf{F}^c_{3+4}\\
\mathbf{F}^c_{4+1}+\mathbf{F}^c_{4+3}+\mathbf{F}^c_{4+5}\\
\mathbf{F}^c_{5+4}\\
\mathbf{F}^c_{6+8}+\mathbf{F}^c_{6+10}+\mathbf{F}^c_{6+12}+\mathbf{F}^c_{6+14}+\mathbf{F}^c_{6+15}\\
\mathbf{F}^c_{7+1}+\mathbf{F}^c_{7+8}\\
\mathbf{F}^c_{8+6}+\mathbf{F}^c_{8+7}\\
\mathbf{F}^c_{9+1}+\mathbf{F}^c_{9+10}\\
\mathbf{F}^c_{10+6}+\mathbf{F}^c_{10+9}\\
\mathbf{F}^c_{11+1}+\mathbf{F}^c_{11+12}\\
\mathbf{F}^c_{12+11}+\mathbf{F}^c_{12+6}\\
\mathbf{F}^c_{13+1}+\mathbf{F}^c_{13+14}\\
\mathbf{F}^c_{14+13}+\mathbf{F}^c_{14+6}\\
\mathbf{F}^c_{15+6}+\mathbf{F}^c_{15+16}\\
\mathbf{F}^c_{16+1}+\mathbf{F}^c_{16+15}
\end{bmatrix}$$

$$\mathbf{M}\ddot{\mathbf{s}} \quad + \quad \mathbf{Q}\dot{\mathbf{s}} \quad = \mathbf{F} + \mathbf{F}^c$$

Forward and inverse dynamics

Inverse dynamics

✓ The calculation of the force that must be applied to a given rigid-body system in order to produce a given acceleration response

✓ Inverse dynamics has a variety of uses:
motion control system, trajectory planning etc…

$$\boldsymbol{\tau} = ID(model, \mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})$$

ex) $\mathbf{M}\ddot{\mathbf{r}} = \mathbf{F}$

- Given: $\ddot{\mathbf{r}}$
- Find : $\mathbf{F}$

Forward dynamics

✓ The calculation of the acceleration response of a given rigid-body system to a given applied force

✓ Forward dynamics is used mainly in simulation

$$\ddot{\mathbf{q}} = FD(model, \mathbf{q}, \dot{\mathbf{q}}, \boldsymbol{\tau})$$

ex) $\ddot{\mathbf{r}} = \mathbf{M}^{-1}\mathbf{F}$

- Given: $\mathbf{F}$
- Find : $\ddot{\mathbf{r}}$

Derivation of Equations of Motion using Recursive Newton-Euler Formulation

$$\mathbf{M}_i\ddot{\mathbf{s}}_i + \mathbf{Q}_i\dot{\mathbf{s}}_i = \mathbf{F}_i^e + \mathbf{F}_i^c$$

Recursive Newton-Euler Formulation

$$\mathbf{v}_i = \mathbf{v}_{i-1} + \mathbf{S}_i\dot{q}_i$$

$$\mathbf{a}_i = \mathbf{a}_{i-1} + \mathbf{S}_i\ddot{q}_i + \dot{\mathbf{S}}_i\dot{q}_i$$

$$\mathbf{f}_i^B = \mathbf{I}_i\mathbf{a}_i + \mathbf{v}_i \times^* \mathbf{I}_i\mathbf{v}_i$$

$$\mathbf{f}_i = \mathbf{f}_i^B - \mathbf{f}_i^e + \mathbf{f}_{i-1}$$

$$\boldsymbol{\tau}_i = \mathbf{S}_i^T\mathbf{f}_i$$

$\mathbf{v}_i$: Velocity vector of body i (6 components)

$\mathbf{a}_i$: Acceleration vector of body i (6 components)

q_i : Generalized coordinate (joint values)

$\mathbf{S}_i$: Velocity transformation matrix

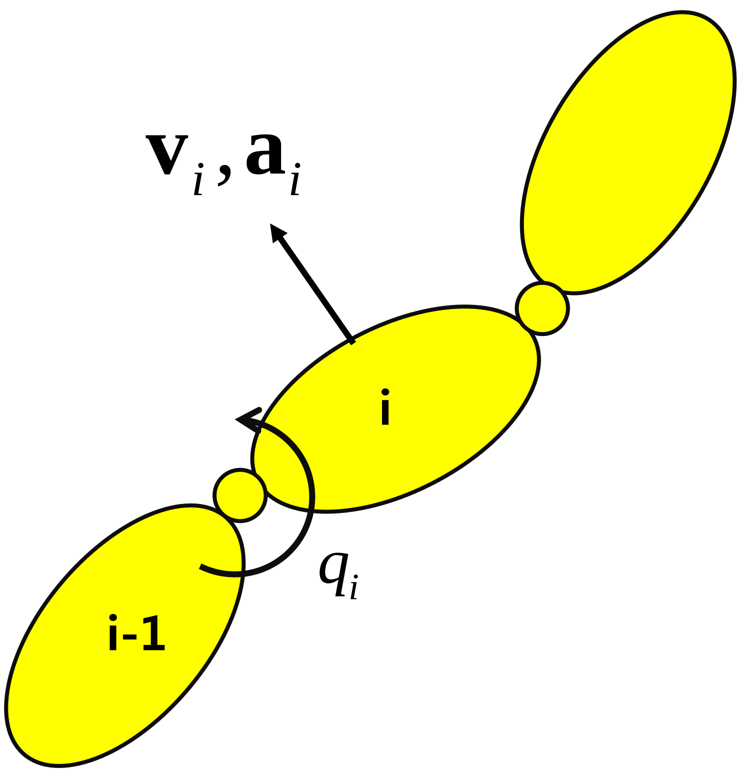
$\mathbf{I}_i$: Mass and mass moment of inertia of body i

$\mathbf{f}_i^B$: Resultant force exerted on body i

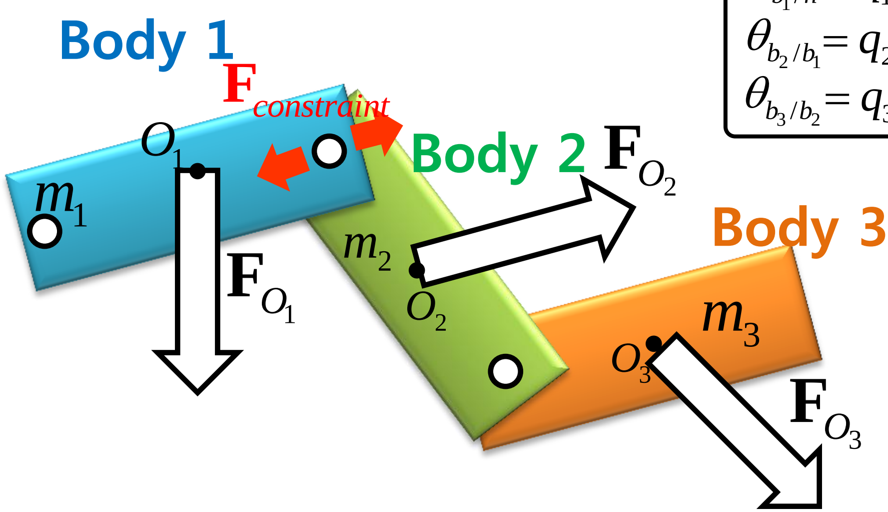
$\mathbf{f}_i^e$: External force exerted on body i

$\mathbf{f}_i$: Force exerted on the joint i which is on body i

$\boldsymbol{\tau}_i$: Force generated by joint i



Inverse Dynamics of 3-Link Arm



$\theta_{b_1/n} = q_1$	**Given:**	$\theta_{b_1/n}, \dot{\theta}_{b_1/n}, \ddot{\theta}_{b_1/n}$	**Find:**	τ_1
$\theta_{b_2/b_1} = q_2$		$\theta_{b_2/b_1}, \dot{\theta}_{b_2/b_1}, \ddot{\theta}_{b_2/b_1}$		τ_2
$\theta_{b_3/b_2} = q_3$		$\theta_{b_3/b_2}, \dot{\theta}_{b_3/b_2}, \ddot{\theta}_{b_3/b_2}$		τ_3

$$\mathbf{p} = \mathbf{p}\left(\theta, \dot{\theta}, \mathbf{F}\right)$$

$$\mathbf{c} = \mathbf{c}\left(\theta, \dot{\theta}\right)$$

$$\mathbf{S} : known$$

$$\mathbf{X} = \left(\theta\right)$$

Equations for link 1	**Equations for link 2**	**Equations for link 3**	
$\boxed{\mathbf{a}_1} = {}^1\mathbf{X}_0 \cdot \mathbf{a}_0 + \mathbf{S}_1 \cdot \ddot{q}_1 + \mathbf{c}_1$ (1)	$\boxed{\mathbf{a}_2} = {}^2\mathbf{X}_1 \cdot \mathbf{a}_1 + \mathbf{S}_2 \cdot \ddot{q}_2 + \mathbf{c}_2$ (2)	$\boxed{\mathbf{a}_3} = {}^3\mathbf{X}_2 \cdot \mathbf{a}_2 + \mathbf{S}_3 \cdot \ddot{q}_3 + \mathbf{c}_3$ (3)	→ **Forward Recursive**
$\boxed{\mathbf{f}_1^B} = \mathbf{I}_1 \cdot \mathbf{a}_1 + \mathbf{p}_1$ (4)	$\boxed{\mathbf{f}_2^B} = \mathbf{I}_2 \cdot \mathbf{a}_2 + \mathbf{p}_2$ (4)	$\boxed{\mathbf{f}_3^B} = \mathbf{I}_3 \cdot \mathbf{a}_3 + \mathbf{p}_3$ (4)	→ **Non Recursive**
$\boxed{\mathbf{f}_1} = \mathbf{f}_1^B + {}^1\mathbf{X}_2^* \cdot \mathbf{f}_2$ (7) $\boxed{\tau_1} = \mathbf{S}_1^T \cdot \mathbf{f}_1$	$\boxed{\mathbf{f}_2} = \mathbf{f}_2^B + {}^2\mathbf{X}_3^* \cdot \mathbf{f}_3$ (6) $\boxed{\tau_2} = \mathbf{S}_2^T \cdot \mathbf{f}_2$	$\boxed{\mathbf{f}_3} = \mathbf{f}_3^B + {}^3\mathbf{X}_4^* \cdot \mathbf{f}_4$ (5) $\boxed{\tau_3} = \mathbf{S}_3^T \cdot \mathbf{f}_3$	→ **Backward Recursive**

Derivation of Equations of Motion using Recursive Newton-Euler Formulation for Forward Dynamics

$$\mathbf{v}_i = \mathbf{v}_{i-1} + \mathbf{S}_i \dot{q}_i$$

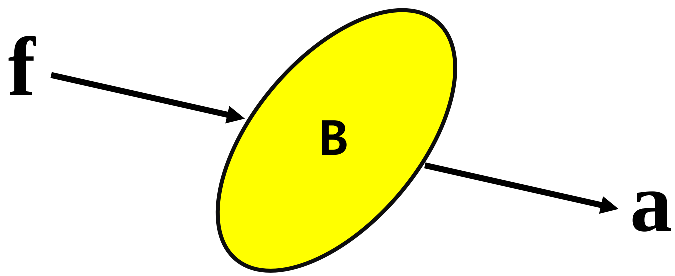
$$\mathbf{a}_i = \mathbf{a}_{i-1} + \mathbf{S}_i \ddot{q}_i + \dot{\mathbf{S}}_i \dot{q}_i$$
$$= \mathbf{a}_{i-1} + \mathbf{S}_i \cdot \ddot{q}_i + \mathbf{c}_i$$

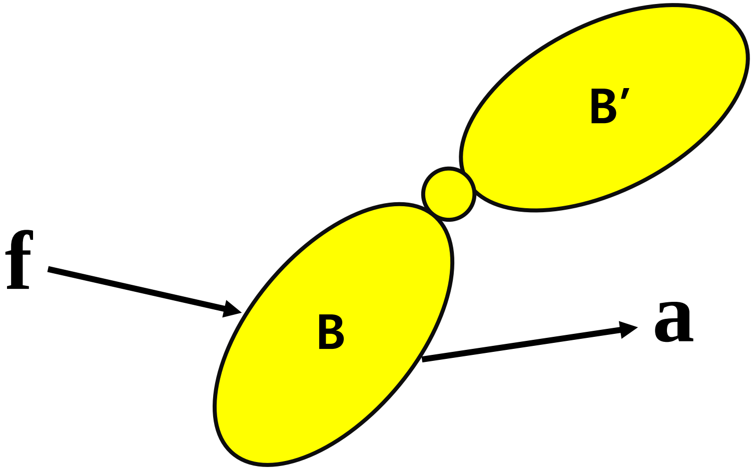
$$\mathbf{f}_i^B = \mathbf{I}_i \mathbf{a}_i + \mathbf{v}_i \times^* \mathbf{I}_i \mathbf{v}_i$$
$$= \mathbf{I}_i \cdot \mathbf{a}_i + \mathbf{p}_i$$

$$\mathbf{f}_i = \mathbf{f}_i^B - \mathbf{f}_i^e + \mathbf{f}_{i-1}$$

$$\boldsymbol{\tau}_i = \mathbf{S}_i^T \mathbf{f}_i$$

Comparing a rigid body (a) with an articulated body (b)





(a) $\mathbf{f} = \mathbf{I} \cdot \mathbf{a} + \mathbf{p}$

(b) $\mathbf{f} = \mathbf{I}^A \cdot \mathbf{a} + \mathbf{p}^A$

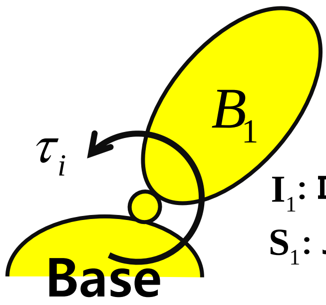
The relationship between the applied force, f, and the resulting acceleration, a, is given by the body' equation of motion of body B.

$$\mathbf{f} = \mathbf{I} \cdot \mathbf{a} + \mathbf{p}$$

Now consider the two-body system shown in diagram (b), in which a second body, B', has been connected to B via a joint. The effect of this second body is to alter the acceleration response of B, so that the relationship between f and a is now given by

$$\mathbf{f} = \mathbf{I}^A \cdot \mathbf{a} + \mathbf{p}^A$$

Derivation of Equations of Motion using Recursive Newton-Euler Formulation for Forward Dynamics



$\mathbf{I}_1$: Inertia Matrix

$\mathbf{S}_1$: Joint's Motion Subspace Matrix

Given : $q_1, \dot{q}_1, \tau_1$

$$\mathbf{a}_1 = \mathbf{a}_0 + \mathbf{S}_1 \ddot{q}_1 + \mathbf{c}_1$$

$$\mathbf{f}_1 = \mathbf{I}_1 \mathbf{a}_1 + \mathbf{p}_1$$

$$\mathbf{S}_1^T \mathbf{f}_1 = \tau_1$$

$$\mathbf{S}_1^T \left(\mathbf{I}_1 \left(\mathbf{a}_0 + \mathbf{S}_1 \ddot{q}_1 + \mathbf{c}_1 \right) + \mathbf{p}_1 \right) = \tau_1$$

$$\mathbf{S}_1^T \mathbf{I}_1 \left(\mathbf{a}_0 + \mathbf{S}_1 \ddot{q}_1 + \mathbf{c}_1 \right) + \mathbf{S}_1^T \mathbf{p}_1 = \tau_1$$

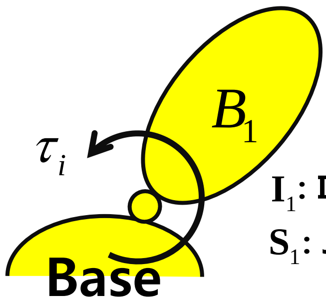
$$\mathbf{S}_1^T \mathbf{I}_1 \left(\mathbf{a}_0 + \mathbf{c}_1 \right) + \mathbf{S}_1^T \mathbf{I}_1 \mathbf{S}_1 \ddot{q}_1 + \mathbf{S}_1^T \mathbf{p}_1 = \tau_1$$

$$\mathbf{S}_1^T \mathbf{I}_1 \mathbf{S}_1 \ddot{q}_1 = \tau_1 - \mathbf{S}_1^T \mathbf{I}_1 \left(\mathbf{a}_0 + \mathbf{c}_1 \right) - \mathbf{S}_1^T \mathbf{p}_1$$

$$\ddot{q}_1 = \left(\boxed{\mathbf{S}_1^T \mathbf{I}_1 \mathbf{S}_1} \right)^{-1} \left(\boxed{\tau_1} - \boxed{\mathbf{S}_1^T \mathbf{I}_1} \left(\boxed{\mathbf{a}_0} + \boxed{\mathbf{c}_1} \right) - \boxed{\mathbf{S}_1^T \mathbf{p}_1} \right)$$

known

Derivation of Equations of Motion using Recursive Newton-Euler Formulation for Forward Dynamics



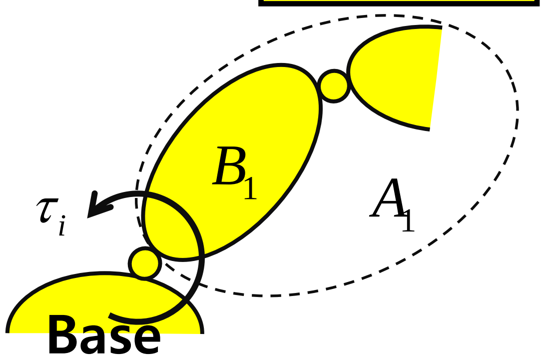
$\mathbf{I}_1$: Inertia Matrix

$\mathbf{S}_1$: Joint's Motion Subspace Matrix

Given : $q_1, \dot{q}_1, \tau_1$

Find : $\ddot{q}_1$

$\mathbf{I}_1, \mathbf{p}_1 \rightarrow \mathbf{I}_1^A, \mathbf{p}_1^A$



Given : $q_1, \dot{q}_1, \tau_1$

Find : $\ddot{q}_1$

$$\mathbf{a}_1 = \mathbf{a}_0 + \mathbf{S}_1 \ddot{q}_1 + \mathbf{c}_1$$

$$\mathbf{f}_1 = \boxed{\mathbf{I}_1^A} \mathbf{a}_1 + \boxed{\mathbf{p}_1^A}$$

$$\mathbf{S}_1^T \mathbf{f}_1 = \tau_1$$

$$\mathbf{S}_1^T \left(\mathbf{I}_1^A \left(\mathbf{a}_0 + \mathbf{S}_1 \ddot{q}_1 + \mathbf{c}_1 \right) + \mathbf{p}_1^A \right) = \tau_1$$

$$\mathbf{S}_1^T \mathbf{I}_1^A \left(\mathbf{a}_0 + \mathbf{S}_1 \ddot{q}_1 + \mathbf{c}_1 \right) + \mathbf{S}_1^T \mathbf{p}_1^A = \tau_1$$

$$\mathbf{S}_1^T \mathbf{I}_1^A \left(\mathbf{a}_0 + \mathbf{c}_1 \right) + \mathbf{S}_1^T \mathbf{I}_1^A \mathbf{S}_1 \ddot{q}_1 + \mathbf{S}_1^T \mathbf{p}_1^A = \tau_1$$

$$\mathbf{S}_1^T \mathbf{I}_1^A \mathbf{S}_1 \ddot{q}_1 = \tau_1 - \mathbf{S}_1^T \mathbf{I}_1^A \left(\mathbf{a}_0 + \mathbf{c}_1 \right) - \mathbf{S}_1^T \mathbf{p}_1^A$$

$$\ddot{q}_1 = \left(\mathbf{S}_1^T \mathbf{I}_1^A \mathbf{S}_1 \right)^{-1} \left(\tau_1 - \mathbf{S}_1^T \mathbf{I}_1^A \left(\mathbf{a}_0 + \mathbf{c}_1 \right) - \mathbf{S}_1^T \mathbf{p}_1^A \right)$$

known

unknown

$$\ddot{q}_i = \left(\mathbf{S}_i^T \mathbf{I}_i^A \mathbf{S}_i \right)^{-1} \left(\tau_i - \mathbf{S}_i^T \mathbf{I}_i^A \left(\mathbf{a}_{i-1} + \mathbf{c}_i \right) - \mathbf{S}_i^T \mathbf{p}_i^A \right)$$

Inverse Dynamics using Recursive Newton-Euler Formulation - Articulated Body Method

Given: $q_1, \dot{q}_1, \tau_1$

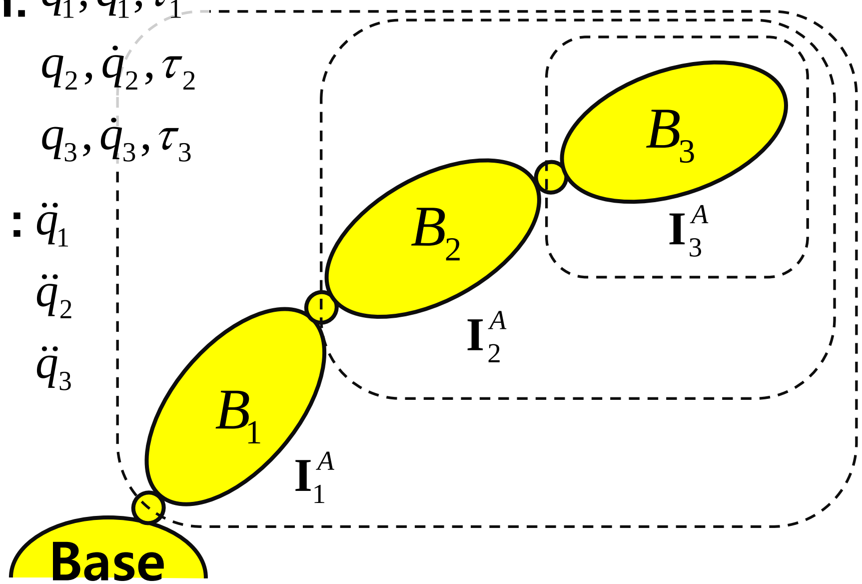
$q_2, \dot{q}_2, \tau_2$

$q_3, \dot{q}_3, \tau_3$

Find : $\ddot{q}_1$

$\ddot{q}_2$

$\ddot{q}_3$



$$\ddot{q}_i = \left(\mathbf{S}_i^T \mathbf{I}_i^A \mathbf{S}_i\right)^{-1} \left(\tau_i - \mathbf{S}_i^T \mathbf{I}_i^A \left(\mathbf{a}_{i-1} + \mathbf{c}_i\right) - \mathbf{S}_i^T \mathbf{p}_i^A\right)$$

$$\mathbf{I}_i^A = \mathbf{I}_i + \mathbf{I}_{i+1}^a$$

$$\mathbf{I}_{i+1}^a = \mathbf{I}_{i+1}^A - \mathbf{I}_{i+1}^A \cdot \mathbf{S}_{i+1} \left(\mathbf{S}_{i+1}{}^T \mathbf{I}_{i+1}^A \mathbf{S}_{i+1}\right)^{-1} \mathbf{S}_{i+1}{}^T \mathbf{I}_{i+1}^A$$

$$\mathbf{p}_i^A = \mathbf{p}_i + \mathbf{p}_{i+1}^a$$

$$\mathbf{p}_{i+1}^a = \mathbf{p}_{i+1}^A + \mathbf{I}_{i+1}^A \cdot \mathbf{c}_{i+1} + \mathbf{I}_{i+1}^A \cdot \mathbf{S}_{i+1} \left(\mathbf{S}_{i+1}{}^T \mathbf{I}_{i+1}^A \mathbf{S}_{i+1}\right)^{-1} \left(\boldsymbol{\tau}_{i+1} - \mathbf{S}_{i+1}{}^T \left(\mathbf{I}_{i+1}^A \mathbf{c}_{i+1} + \mathbf{p}_{i+1}^A\right)\right)$$

Forward Recursive

$$\mathbf{v}_1 = \mathbf{v}_0 + \mathbf{S}_1 \cdot \dot{q}_1$$

$$\mathbf{v}_2 = \mathbf{v}_1 + \mathbf{S}_2 \cdot \dot{q}_2$$

$$\mathbf{v}_3 = \mathbf{v}_2 + \mathbf{S}_3 \cdot \dot{q}_3$$

$$\mathbf{c}_1 = \dot{\mathbf{S}}_1 \cdot \dot{q}_1$$

$$\mathbf{c}_2 = \dot{\mathbf{S}}_2 \cdot \dot{q}_2$$

$$\mathbf{c}_3 = \dot{\mathbf{S}}_3 \cdot \dot{q}_3$$

Backward Recursive

$$\mathbf{I}_3^A = \mathbf{I}_3 + \mathbf{I}_4^a$$

$$\mathbf{I}_2^A = \mathbf{I}_2 + \mathbf{I}_3^a = \mathbf{I}_2 + \mathbf{I}_3^A - \mathbf{I}_3^A \cdot \mathbf{S}_3 \left(\mathbf{S}_3{}^T \mathbf{I}_3^A \mathbf{S}_3\right)^{-1} \mathbf{S}_3{}^T \mathbf{I}_3^A$$

$$\mathbf{I}_1^A = \mathbf{I}_1 + \mathbf{I}_2^a = \mathbf{I}_1 + \mathbf{I}_2^A - \mathbf{I}_2^A \cdot \mathbf{S}_2 \left(\mathbf{S}_2{}^T \mathbf{I}_2^A \mathbf{S}_2\right)^{-1} \mathbf{S}_2{}^T \mathbf{I}_2^A$$

$$\mathbf{p}_3^A = \mathbf{p}_3 + \mathbf{p}_4^a$$

$$\mathbf{p}_2^A = \mathbf{p}_2 + \mathbf{p}_3^a = \mathbf{p}_2 + \mathbf{p}_3^A + \mathbf{I}_3^A \cdot \mathbf{c}_3 + \mathbf{I}_3^A \cdot \mathbf{S}_3 \left(\mathbf{S}_3{}^T \mathbf{I}_3^A \mathbf{S}_3\right)^{-1} \left(\boldsymbol{\tau}_3 - \mathbf{S}_3{}^T \left(\mathbf{I}_3^A \mathbf{c}_3 + \mathbf{p}_3^A\right)\right)$$

$$\mathbf{p}_1^A = \mathbf{p}_1 + \mathbf{p}_2^a = \mathbf{p}_1 + \mathbf{p}_2^A + \mathbf{I}_2^A \cdot \mathbf{c}_2 + \mathbf{I}_2^A \cdot \mathbf{S}_2 \left(\mathbf{S}_2{}^T \mathbf{I}_2^A \mathbf{S}_2\right)^{-1} \left(\boldsymbol{\tau}_2 - \mathbf{S}_2{}^T \left(\mathbf{I}_2^A \mathbf{c}_2 + \mathbf{p}_2^A\right)\right)$$

Forward Recursive

$$\ddot{q}_1 = \left(\mathbf{S}_1^T \mathbf{I}_1^A \mathbf{S}_1\right)^{-1} \left(\tau_1 - \mathbf{S}_1^T \mathbf{I}_1^A \left(\mathbf{a}_0 + \mathbf{c}_1\right) - \mathbf{S}_1^T \mathbf{p}_1^A\right)$$

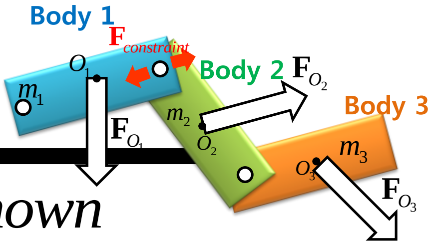
$$\mathbf{a}_1 = \mathbf{a}_0 + \mathbf{S}_1 \cdot \ddot{q}_1 + \dot{\mathbf{S}}_1 \cdot \dot{q}_1$$

$$\ddot{q}_2 = \left(\mathbf{S}_2^T \mathbf{I}_2^A \mathbf{S}_2\right)^{-1} \left(\tau_2 - \mathbf{S}_2^T \mathbf{I}_2^A \left(\mathbf{a}_1 + \mathbf{c}_2\right) - \mathbf{S}_2^T \mathbf{p}_2^A\right)$$

$$\mathbf{a}_2 = \mathbf{a}_1 + \mathbf{S}_2 \cdot \ddot{q}_2 + \dot{\mathbf{S}}_2 \cdot \dot{q}_2$$

$$\ddot{q}_3 = \left(\mathbf{S}_3^T \mathbf{I}_3^A \mathbf{S}_3\right)^{-1} \left(\tau_3 - \mathbf{S}_3^T \mathbf{I}_3^A \left(\mathbf{a}_2 + \mathbf{c}_3\right) - \mathbf{S}_3^T \mathbf{p}_3^A\right)$$

Forward Dynamics of 3-Link Arm



$\theta_{b_1/n} = q_1$ **Given:** $\theta_{b_1/n}, \dot{\theta}_{b_1/n}, \tau_1$ **Find:** $\ddot{\theta}_{b_1/n}$

$\theta_{b_2/b_1} = q_2$ $\theta_{b_2/b_1}, \dot{\theta}_{b_2/b_1}, \tau_2$ $\ddot{\theta}_{b_2/b_1}$

$\theta_{b_3/b_2} = q_3$ $\theta_{b_3/b_2}, \dot{\theta}_{b_3/b_2}, \tau_3$ $\ddot{\theta}_{b_3/b_2}$

$$\mathbf{p} = \mathbf{p}\left(\theta, \dot{\theta}, \mathbf{F}\right) \quad \mathbf{S} : known$$

$$\mathbf{c} = \mathbf{c}\left(\theta, \dot{\theta}\right) \quad \mathbf{X} = \left(\theta\right)$$

Equations for link 1

$$\boxed{\mathbf{a}_1} = {}^1\mathbf{X}_0 \cdot \mathbf{a}_0 + \mathbf{S}_1 \cdot \boxed{\ddot{q}_1} + \mathbf{c}_1$$

(3)

$$\mathbf{f}_1 = \mathbf{I}_1^A \cdot \mathbf{a}_1 + \mathbf{p}_1^A$$

$$\tau_1 = \mathbf{S}_1{}^T \cdot \mathbf{f}_1$$

$$\boxed{\ddot{q}_1} = \left(\mathbf{S}_1^T \mathbf{I}_1^A \mathbf{S}_1\right)^{-1} \left(\tau_1 - \mathbf{S}_1^T \left(\mathbf{I}_1^A \left({}^1\mathbf{X}_0 \mathbf{a}_0 + \mathbf{c}_1\right) + \mathbf{p}_1^A\right)\right)$$

(2)

Equations for link 2

$$\boxed{\mathbf{a}_2} = {}^2\mathbf{X}_1 \cdot \mathbf{a}_1 + \mathbf{S}_2 \cdot \boxed{\ddot{q}_2} + \mathbf{c}_2$$

(5)

$$\mathbf{f}_2 = \mathbf{I}_2^A \cdot \mathbf{a}_2 + \mathbf{p}_2^A$$

$$\tau_2 = \mathbf{S}_2{}^T \cdot \mathbf{f}_2$$

$$\boxed{\ddot{q}_2} = \left(\mathbf{S}_2^T \mathbf{I}_2^A \mathbf{S}_2\right)^{-1} \left(\tau_2 - \mathbf{S}_2^T \left(\mathbf{I}_2^A \left({}^2\mathbf{X}_1 \boxed{\mathbf{a}_1} + \mathbf{c}_2\right) + \mathbf{p}_2^A\right)\right)$$

(4)

Equations for link 3

$$\mathbf{a}_3 = {}^3\mathbf{X}_2 \cdot \mathbf{a}_2 + \mathbf{S}_3 \cdot \ddot{q}_3 + \mathbf{c}_3$$

$$\mathbf{f}_3 = \mathbf{I}_3 \cdot \mathbf{a}_3 + \mathbf{p}_3$$

$$\tau_3 = \mathbf{S}_3{}^T \cdot \mathbf{f}_3$$

$$\boxed{\ddot{q}_3} = \left(\mathbf{S}_3^T \mathbf{I}_3 \mathbf{S}_3\right)^{-1} \left(\tau_3 - \mathbf{S}_3^T \left(\mathbf{I}_3 \left({}^2\mathbf{X}_2 \boxed{\mathbf{a}_2} + \mathbf{c}_3\right) + \mathbf{p}_3\right)\right)$$

(6)

(1)

$$\mathbf{I}_1^A = \mathbf{I}_1 + {}^1\mathbf{X}_2^* \cdot \mathbf{I}_2^A \cdot {}^2\mathbf{X}_1 - {}^1\mathbf{X}_2^* \cdot \mathbf{I}_2^A \cdot \mathbf{S}_2 \cdot \left(\mathbf{S}_2^T \mathbf{I}_2^A \mathbf{S}_2\right)^{-1} \mathbf{S}_2^T \mathbf{I}_2^A \, {}^2\mathbf{X}_1$$

$$\mathbf{I}_2^A = \mathbf{I}_2 + {}^2\mathbf{X}_3^* \cdot \mathbf{I}_3 \cdot {}^3\mathbf{X}_2 - {}^2\mathbf{X}_3^* \cdot \mathbf{I}_3 \cdot \mathbf{S}_3 \cdot \left(\mathbf{S}_3^T \mathbf{I}_3 \mathbf{S}_3\right)^{-1} \mathbf{S}_3^T \mathbf{I}_3 \, {}^3\mathbf{X}_2$$

$$\mathbf{p}_1^A = \mathbf{p}_1 + {}^1\mathbf{X}_2^* \cdot \mathbf{p}_2^A + {}^1\mathbf{X}_2^* \cdot \mathbf{I}_2^A \cdot \mathbf{c}_2 + {}^1\mathbf{X}_2^* \cdot \mathbf{I}_2^A \cdot \mathbf{S}_2 \cdot \left(\mathbf{S}_2^T \mathbf{I}_2^A \mathbf{S}_2\right)^{-1} \left(\tau_2 - \mathbf{S}_2^T \left(\mathbf{I}_2^A \mathbf{c}_2 + \mathbf{p}_2^A\right)\right)$$

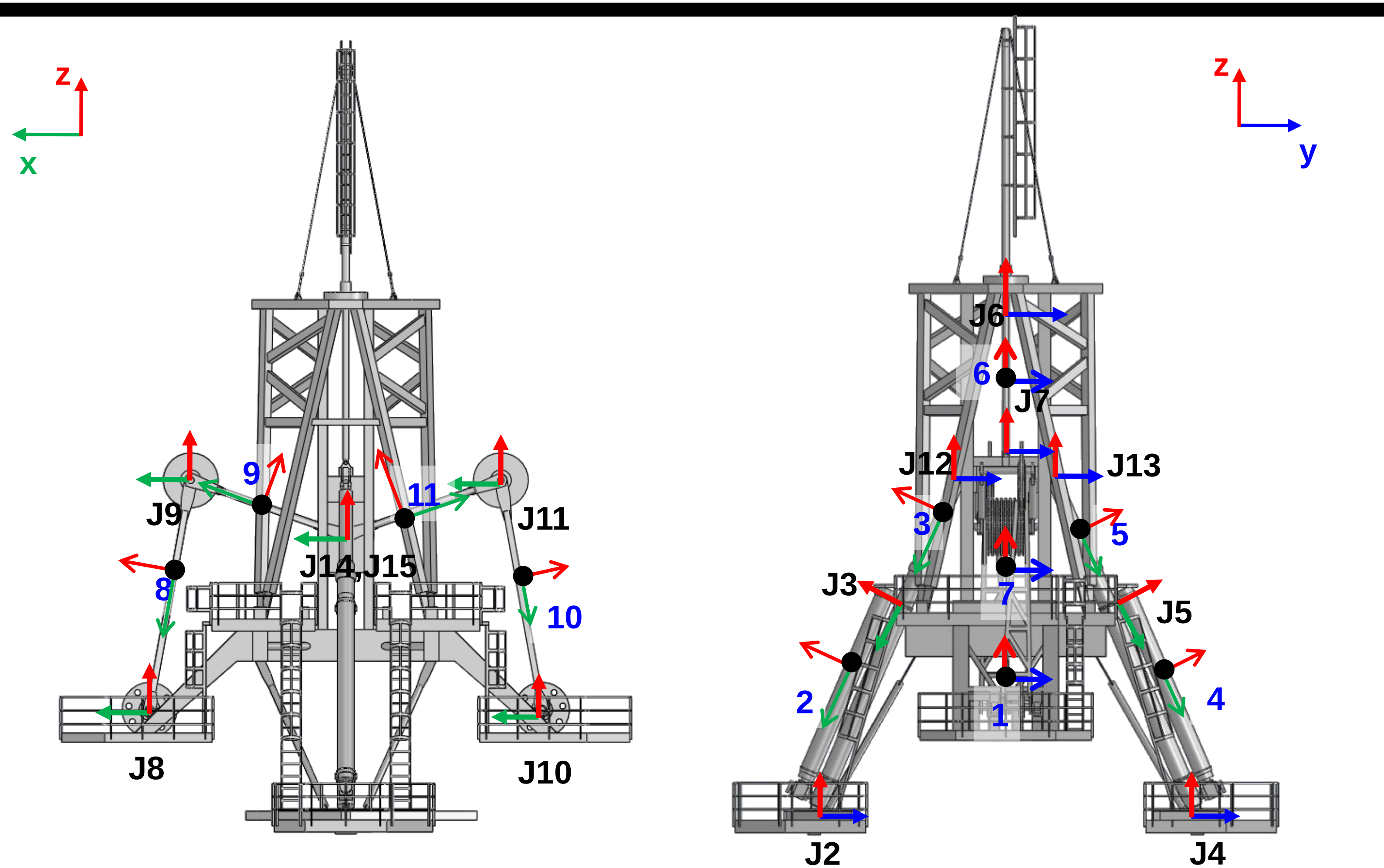
$$\mathbf{p}_2^A = \mathbf{p}_2 + {}^2\mathbf{X}_3^* \cdot \mathbf{p}_3 + {}^2\mathbf{X}_3^* \cdot \mathbf{I}_3 \cdot \mathbf{c}_3 + {}^2\mathbf{X}_3^* \cdot \mathbf{I}_3 \cdot \mathbf{S}_3 \cdot \left(\mathbf{S}_3^T \mathbf{I}_3 \mathbf{S}_3\right)^{-1} \left(\tau_3 - \mathbf{S}_3^T \left(\mathbf{I}_3 \mathbf{c}_3 + \mathbf{p}_3\right)\right)$$

Comparison of Augmented, Embedding, and Recursive Formulation

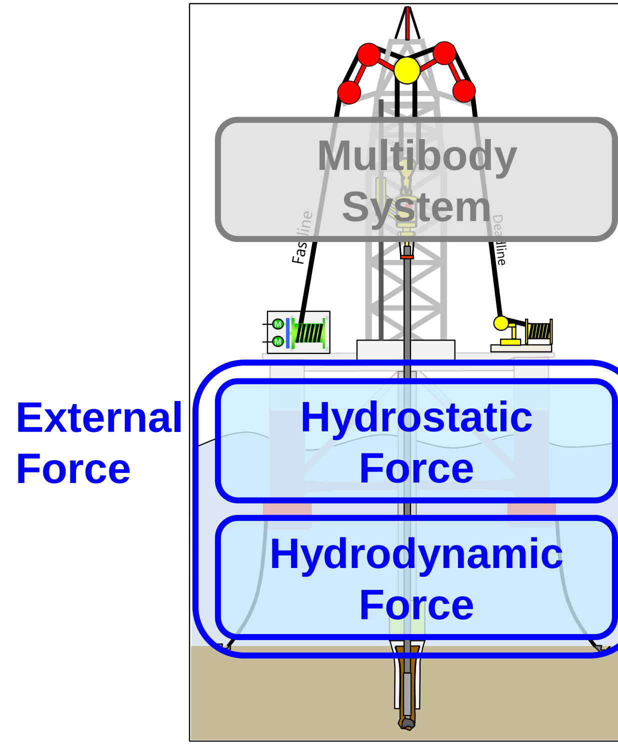
	Augmented formulation	Embedding formulation	Recursive formulation
	$\begin{bmatrix} \bar{\mathbf{M}}(\mathbf{q}) & \mathbf{C}_{\mathbf{q}}^T(\mathbf{q}) \\ \mathbf{C}_{\mathbf{q}}(\mathbf{q}) & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \bar{\mathbf{F}}(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}},t) - \bar{\mathbf{k}}(\mathbf{q},\dot{\mathbf{q}}) \\ -(\mathbf{C}_{\mathbf{q}}\dot{\mathbf{q}})_{\mathbf{q}}\dot{\mathbf{q}} \end{bmatrix}$	$\bar{\mathbf{M}}\ddot{\mathbf{q}} + \bar{\mathbf{k}} - \bar{\mathbf{F}} = 0$	$\mathbf{v}_i = \mathbf{v}_{i-1} + \mathbf{S}_i\dot{q}_i$ $\mathbf{a}_i = \mathbf{a}_{i-1} + \mathbf{S}_i\ddot{q}_i + \dot{\mathbf{S}}_i\dot{q}_i$ $\mathbf{f}_i^B = \mathbf{I}_i\mathbf{a}_i + \mathbf{v}_i \times^* \mathbf{I}_i\mathbf{v}_i$ $\mathbf{f}_i = \mathbf{f}_i^B - \mathbf{f}_i^e + \mathbf{f}_{i-1}$ $\boldsymbol{\tau}_i = \mathbf{S}_i^T\mathbf{f}_i$
Reference frame of Position and Orientation	Inertial frame	Body fixed frame	Body fixed frame
Constraint force	Calculated without Additional calculation	Additional calculation is needed	Additional calculation is needed
the complexity of computation	$O(n^3)$	$O(n^3)$	$O(n)$

In this study, the recursive formulation is used to develop the dynamics kernel.

Modeling of heave compensation system



Model of heave compensation system using the developed dynamics kernel



18-3.
External Forces Acting on the Platform

- HYDROSTATIC FORCE
- HYDRODYNAMIC FORCE

$$\mathbf{M}_1\ddot{\mathbf{s}}_1 + \mathbf{Q}_1\dot{\mathbf{s}}_1 = \boxed{\mathbf{F}^e_{hydrostatc} + \mathbf{F}^e_{hydrodynamic}}$$
$$+\mathbf{F}^e_{gravity} + \mathbf{F}^c_{1+4} + \mathbf{F}^c_{1+7} + \mathbf{F}^c_{1+9} + \mathbf{F}^c_{1+11} + \mathbf{F}^c_{1+13} + \mathbf{F}^c_{1+16}$$

Determination of initial position and attitude

Sequence of determining initial condition

(1) $\mathbf{q}_{HS}^{(i)} = \left[z^{(i)} \quad \phi^{(i)} \quad \theta^{(i)} \right]^T$

(2) Calculating the force and moment using volume integration

$$F\left(\mathbf{q}_{HS}^{(i)}\right), M_T\left(\mathbf{q}_{HS}^{(i)}\right), M_L\left(\mathbf{q}_{HS}^{(i)}\right)$$

(3)

$$\underbrace{\begin{bmatrix} -F\left(\mathbf{q}_{HS}^{(i)}\right) \\ -M_T\left(\mathbf{q}_{HS}^{(i)}\right) \\ -M_L\left(\mathbf{q}_{HS}^{(i)}\right) \end{bmatrix}}_{\text{Known}} = \underbrace{\begin{bmatrix} \frac{\partial F}{\partial z} & \frac{\partial F}{\partial \phi} & \frac{\partial F}{\partial \theta} \\ \frac{\partial M_T}{\partial z} & \frac{\partial M_T}{\partial \phi} & \frac{\partial M_T}{\partial \theta} \\ \frac{\partial M_L}{\partial z} & \frac{\partial M_L}{\partial \phi} & \frac{\partial M_L}{\partial \theta} \end{bmatrix}_{z^{(i)},\phi^{(i)},\theta^{(i)}}}_{\text{To be known}} \underbrace{\begin{bmatrix} \delta z^{(i)} \\ \delta \phi^{(i)} \\ \delta \theta^{(i)} \end{bmatrix}}_{\text{Find}}$$

Known **To be known** **Find**

(4)(8) Determining Derivatives using Volume integration

At this step, volume integration doesn't need to be carried out again, since the displacement is calculated at step (2) or (7).

$$\begin{bmatrix} -\rho g A_{WP} & -\rho g T_{WP} & \rho g L_{WP} \\ -\rho g T_{WP} & -\rho g \nabla^n z_{B/O} - \rho g I_T & \rho g I_P \\ \rho g L_{WP} & \rho g I_P & -\rho g \nabla^n z_{B/O} - \rho g I_L \end{bmatrix}$$

(5)

$$z^{(i+1)} = z^{(i)} + \delta z^{(i)}$$
$$\phi^{(i+1)} = \phi^{(i)} + \delta \phi^{(i)}$$
$$\theta^{(i+1)} = \theta^{(i)} + \delta \theta^{(i)}$$

Calculating the force and moment using volume integration

$$F\left(\mathbf{q}_{HS}^{(i+1)}\right), M_T\left(\mathbf{q}_{HS}^{(i+1)}\right), M_L\left(\mathbf{q}_{HS}^{(i+1)}\right)$$

(6)

$$F\left(\mathbf{q}_{HS}^{(i+1)}\right) \overset{!}{=} 0, M_T\left(\mathbf{q}_{HS}^{(i+1)}\right) \overset{!}{=} 0, M_L\left(\mathbf{q}_{HS}^{(i+1)}\right) \overset{!}{=} 0$$

NO

(7)

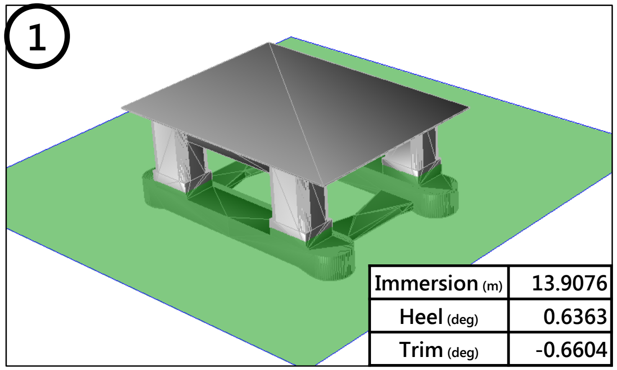
$$F\left(\mathbf{q}_{HS}^{(i)}\right) = F\left(\mathbf{q}_{HS}^{(i+1)}\right), M_T\left(\mathbf{q}_{HS}^{(i)}\right) = M_T\left(\mathbf{q}_{HS}^{(i+1)}\right),$$
$$M_L\left(\mathbf{q}_{HS}^{(i)}\right) = M_L\left(\mathbf{q}_{HS}^{(i+1)}\right)$$

$$i = i + 1$$

(9) YES $z^* = z^{(i+1)}, \phi^* = \phi^{(i+1)}, \theta^* = \theta^{(i+1)}$

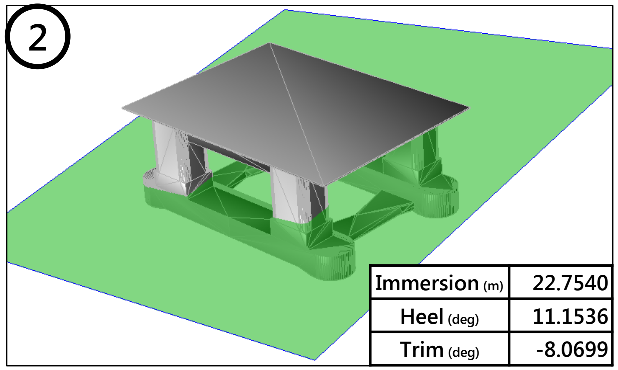
Determination of initial position and attitude
Examples: Center of mass: (-0.5, -0.5, 25.22)

1



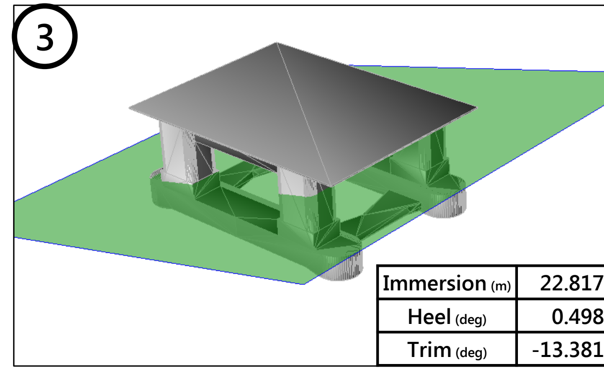
Immersion (m)	13.9076
Heel (deg)	0.6363
Trim (deg)	-0.6604

2



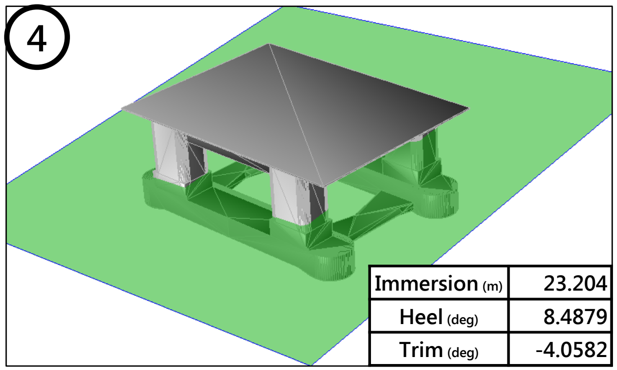
Immersion (m)	22.7540
Heel (deg)	11.1536
Trim (deg)	-8.0699

3



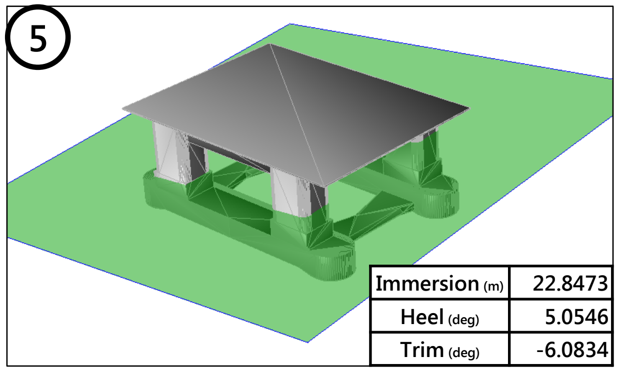
Immersion (m)	22.8179
Heel (deg)	0.4981
Trim (deg)	-13.3810

4



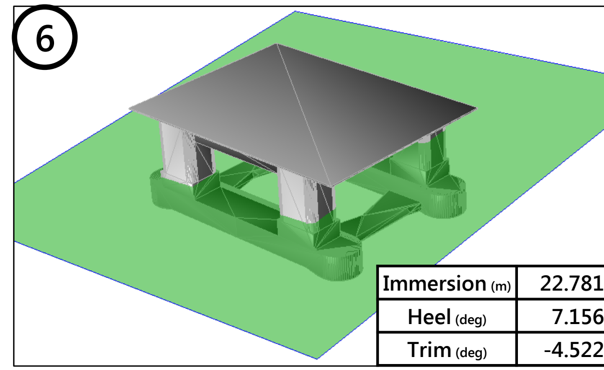
Immersion (m)	23.204
Heel (deg)	8.4879
Trim (deg)	-4.0582

5



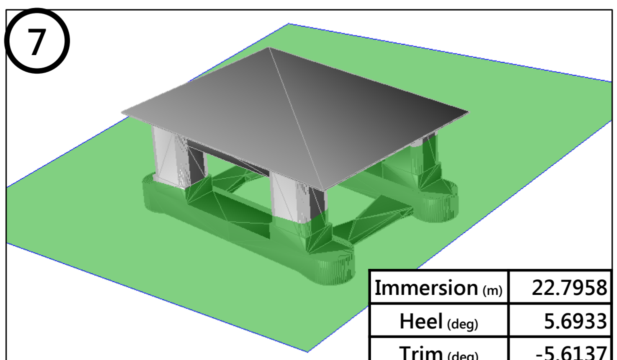
Immersion (m)	22.8473
Heel (deg)	5.0546
Trim (deg)	-6.0834

6



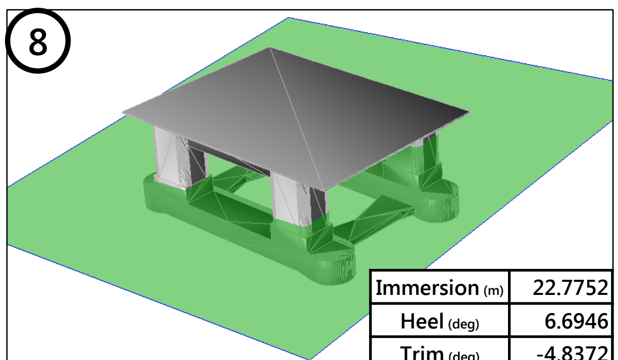
Immersion (m)	22.7816
Heel (deg)	7.1566
Trim (deg)	-4.5227

7



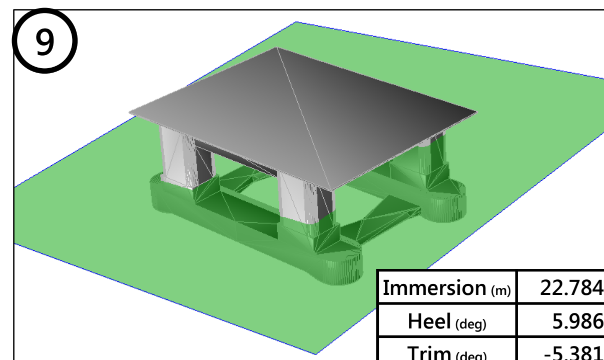
Immersion (m)	22.7958
Heel (deg)	5.6933
Trim (deg)	-5.6137

8



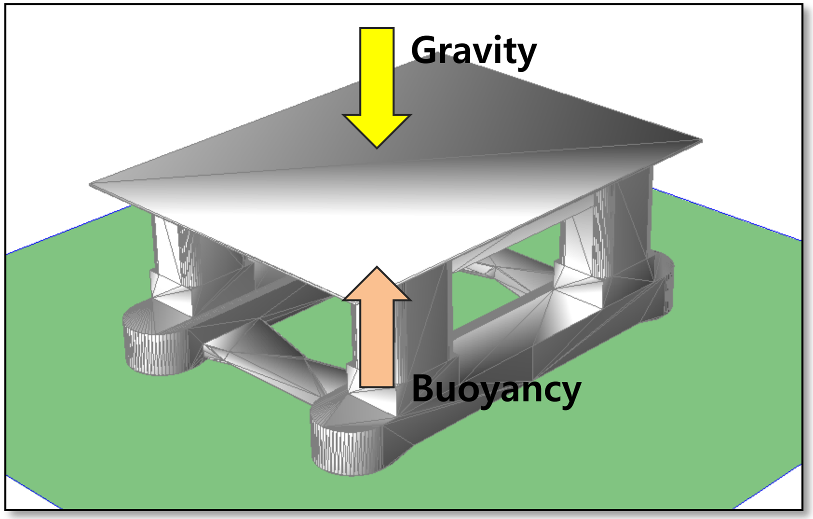
Immersion (m)	22.7752
Heel (deg)	6.6946
Trim (deg)	-4.8372

9



Immersion (m)	22.7840
Heel (deg)	5.9867
Trim (deg)	-5.3817

Static Analysis: Determination of initial position and attitude

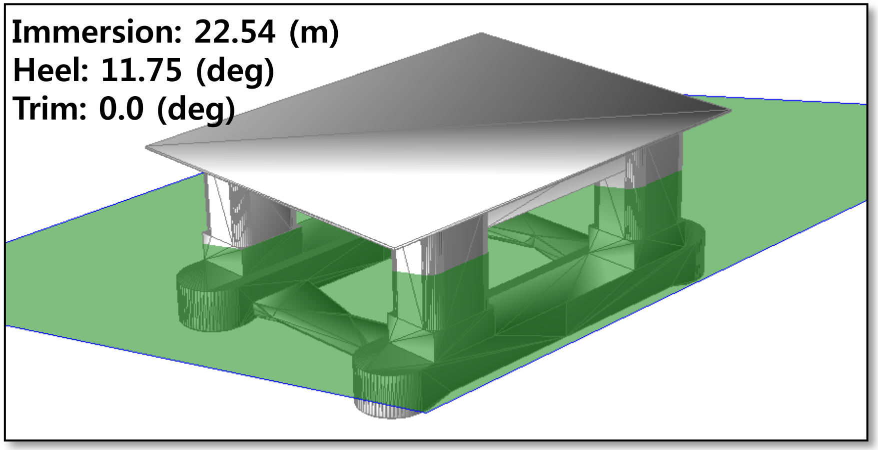


Weight: 55,411 tons

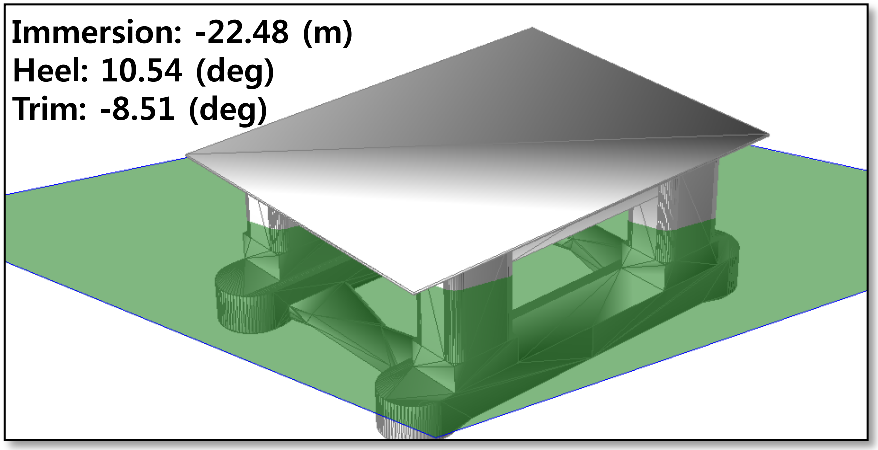
Center of mass can be changed by
1) adding an additional cargo and
2) flooding some compartments

Need to determine the initial position and attitude of floating structure for dynamic analysis.

Case #1: Center of mass: (0, -1.0, 25.22)



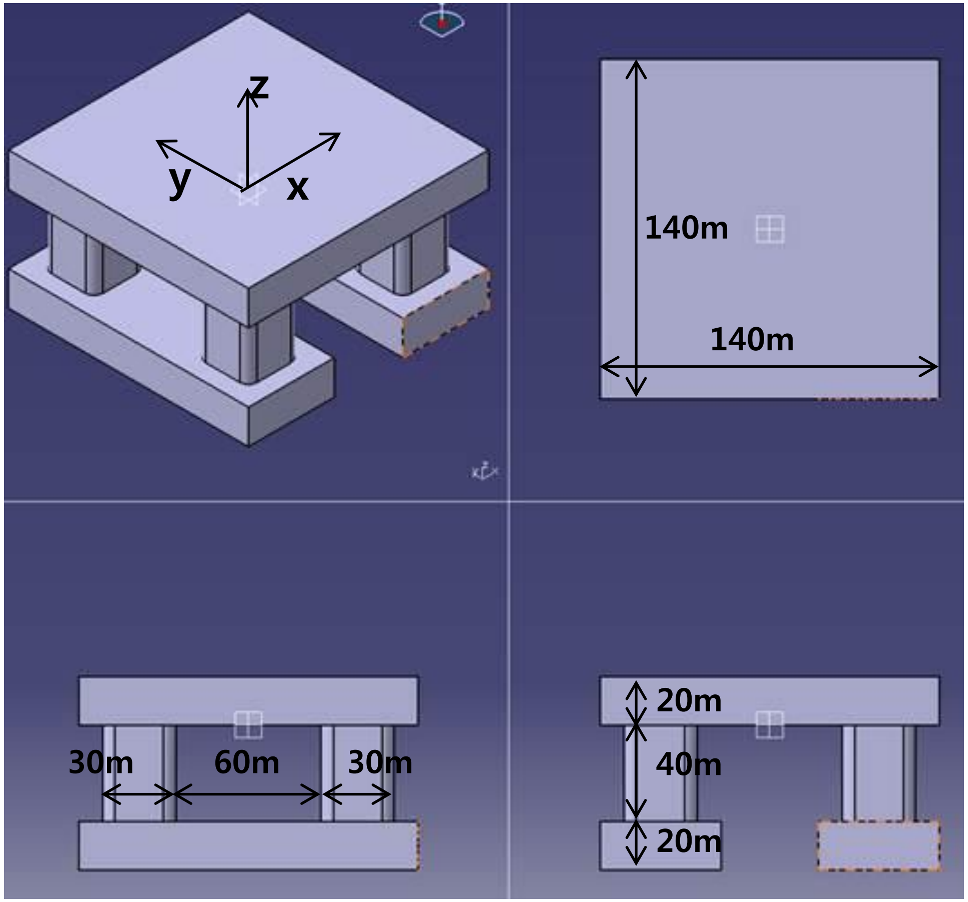
Case #2: Center of mass: (-1.0, -1.0, 25.22)



Calculation of hydrostatic force

- **Because the semi-submersible can have various types of complex bodies, the polyhedron model is used for the calculation of the volume.**

  To verify the calculation The volume and center of volume of this model is calculated by the developed module and a commercial CAD system[1)]



Volume calculation results of the Semi-submergible

The developed module

- Volume: 816,000m^3
- Center of Volume: (0,0,-15.882)

The commercial CAD system

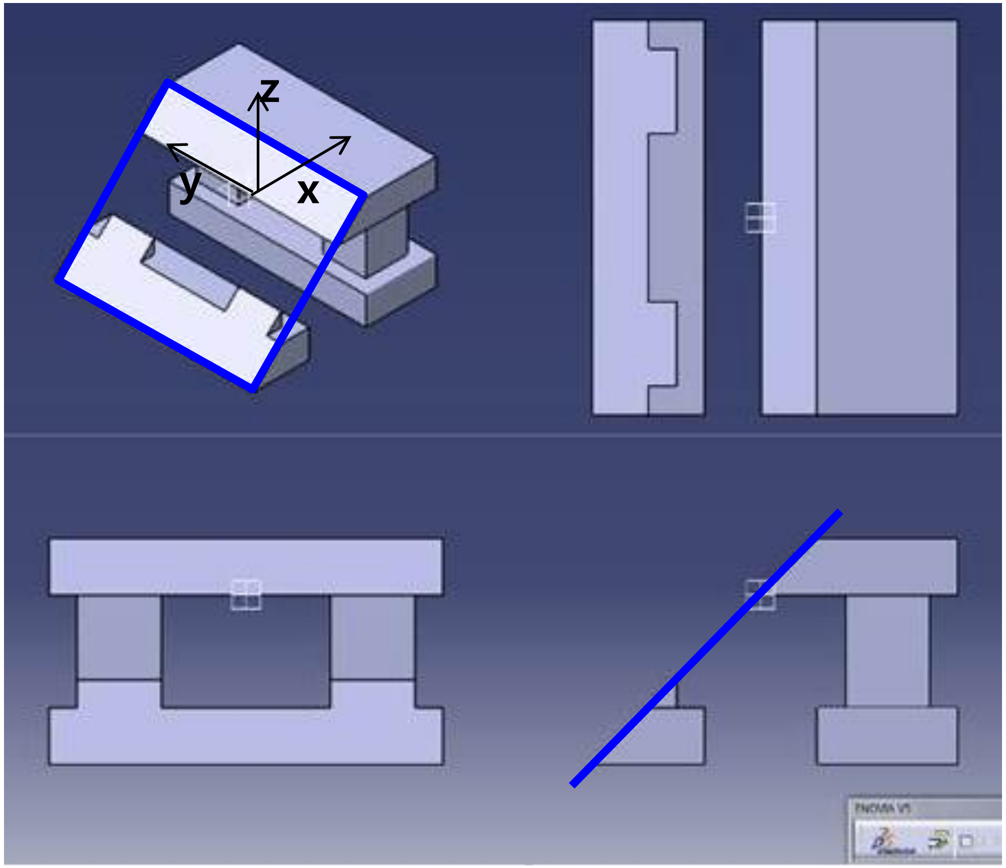
- Volume: 816,000m^3
- Center of Volume: (0,0,-15.882)

1) CATIA

Calculation of hydrostatic force

- **Because the semi-submersible can have various types of complex bodies, the polyhedron model is used for the calculation of the volume.**

To verify the calculation The volume and center of volume of this model is calculated by the developed module and a commercial CAD system[1)]



Volume calculation results of the Semi-submergible under the waterplane

The developed module

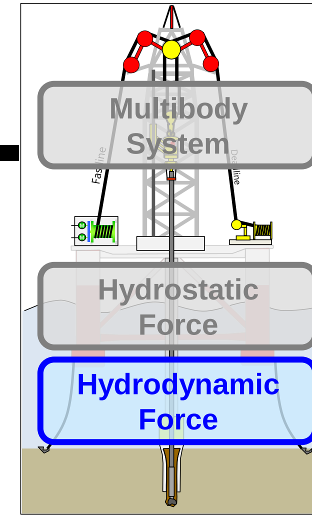
- Volume: 467,000m^3
- Center of Volume: (28.108, 0, -24.104)

The commercial CAD system

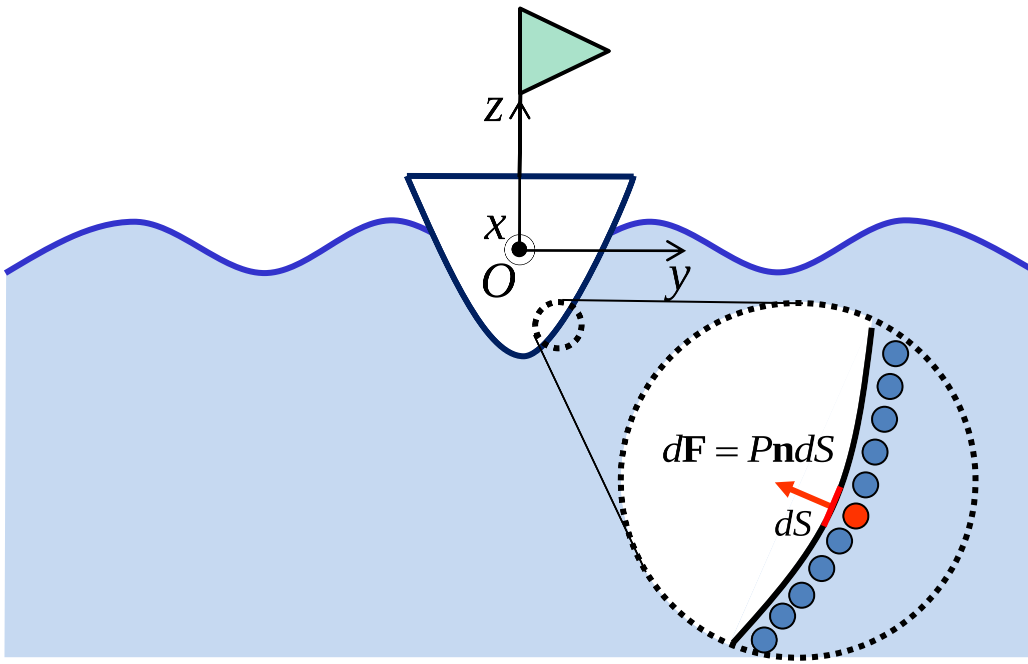
- Volume: 467,000m^3
- Center of Volume: (28.108, 0, -24.104)

1) CATIA

Calculation of hydrodynamic force



Hydrodynamic Force by Potential Theory in Frequency Domain



$$\mathbf{F}_{hydrodynamic} = \iint_{S_B} P\mathbf{n}dS$$

$$P = -\rho \frac{\partial \Phi}{\partial t}$$ **(From Bernoulli Eq.)**

Velocity Potential

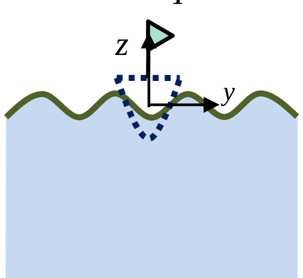
$$\Phi = \Phi_I + \Phi_D + \Phi_R$$

Governing Eq. (Laplace Eq.)

$$\nabla^2 \Phi = 0$$

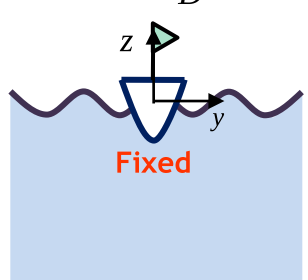
Incident wave velocity potential + **Diffraction velocity potential** + **Radiation velocity potential**

Φ_I



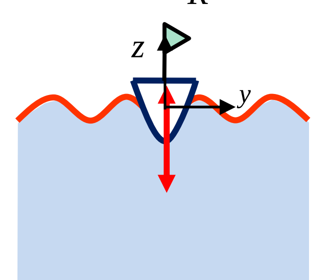
+

Φ_D

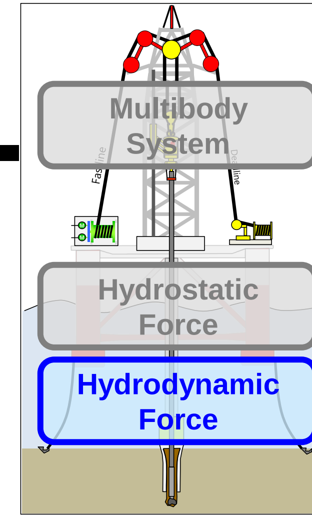


+

Φ_R



Calculation of hydrodynamic force



Hydrodynamic Force by Potential Theory in Frequency Domain

$$\boxed{\mathbf{F}_{hydrodynamic} = \iint_{S_B} P\mathbf{n}dS} \Leftarrow P = -\rho\frac{\partial \Phi}{\partial t}$$

Velocity Potential

$$\Phi = \left\{\phi_I(x,y,z) + \phi_D(x,y,z) + \phi_R(x,y,z)\right\} e^{i\omega t}$$

Governing Eq. (Laplace Eq.)

$$\boxed{\nabla^2 \phi = 0}$$

Frequency domain

$$\phi_I = \frac{igA}{\omega}\frac{\cosh(kz+h)}{\cosh kh}e^{-k(x\cos\theta + y\sin\theta)}$$

$$\phi_R = \iint_{S_B}\left(\phi_R\frac{\partial G}{\partial n} - G\frac{\partial \phi_R}{\partial n}\right)dS$$

$$F_{R,k} = \sum_{j=1}^{6} -\ddot{\xi}a_{jk} - \dot{\xi}b_{jk}$$

Hydrodynamic force in frequency domain

WADAM : frequency domain hydrodynamic analysis S/W developed by DNV

Convolution integral

Time domain

$$\mathbf{F}_I + \mathbf{F}_D = -\rho e^{i\omega t}\iint_{S_B}\left(\phi_I\frac{\partial \phi_{R,k}}{\partial n_k} - \phi_{R,k}\frac{\partial \phi_I}{\partial n_k}\right)dS,$$

$$\mathbf{F}_R = -\mathbf{A}\cdot\ddot{\mathbf{x}}(t) - \int_0^\infty \mathbf{B}(\tau)\cdot\dot{\mathbf{x}}(t-\tau)d\tau$$

$$\left(\begin{array}{l} B_{ij}(\tau) = \dfrac{2}{\pi}\displaystyle\int_0^\infty b_{ij}(\omega)\cos(\omega\tau)d\omega \\ A_{ij} = a_{ij}(\omega) + \dfrac{1}{\omega}\displaystyle\int_0^\infty B_{ij}(\tau)\sin(\omega\tau)d\tau \end{array}\right)$$

$$\boxed{\mathbf{M}\ddot{\mathbf{x}} = \sum \mathbf{F} = \mathbf{F}_{Gravity} + \mathbf{F}_{Hydrostatic} + \mathbf{F}_R + \mathbf{F}_I + \mathbf{F}_D}$$

* Cummins, WE (1962). "The Impulse Response Function and Ship Motions," *Schiffstechnik*, Vol 9, pp 101-109.

Ship motion analysis in time domain

$$\mathbf{M}\ddot{\mathbf{x}} = \sum \mathbf{F} = \underbrace{\mathbf{F}_{Gravity} + \mathbf{F}_{Hydrostatic}}_{\mathbf{F}_{\text{Restoring}}(=-\mathbf{C}\cdot\mathbf{x}(t))} + \mathbf{F}_R + \underbrace{\mathbf{F}_I + \mathbf{F}_D}_{\mathbf{F}_{\text{exciting}}}$$

$$\mathbf{F}_I + \mathbf{F}_D = -\rho e^{i\omega t} \iint_{S_B} \left(\phi_I \frac{\partial \phi_{R,k}}{\partial n_k} - \phi_{R,k} \frac{\partial \phi_I}{\partial n_k} \right) dS,$$

$$\mathbf{F}_R = -\mathbf{A}\cdot\ddot{\mathbf{x}}(t) - \int_0^\infty \mathbf{B}(\tau)\cdot\dot{\mathbf{x}}(t-\tau)\,d\tau$$

$$\left(\begin{aligned} B_{ij}(\tau) &= \frac{2}{\pi}\int_0^\infty \underline{b_{ij}(\omega)} \cos(\omega\tau)\,d\omega \\ A_{ij} &= \underline{a_{ij}(\omega)} + \frac{1}{\omega}\int_0^\infty B_{ij}(\tau)\sin(\omega\tau)\,d\tau \end{aligned} \right)$$

$a_{ij}(\omega), b_{ij}(\omega)$: added mass and damping data in the frequency domain

Substitute $\mathrm{F_{Restoring}}$ and $\mathrm{F_R}$ to the equation

$$\mathbf{M}\ddot{\mathbf{x}} = -\mathbf{C}\cdot\mathbf{x}(t) - \mathbf{A}\cdot\ddot{\mathbf{x}}(t) - \int_0^\infty \mathbf{B}(\tau)\cdot\dot{\mathbf{x}}(t-\tau)\,d\tau + \mathbf{F}_{exciting}$$

Newton's 2nd law yields the linear equation of motion in the time domain:

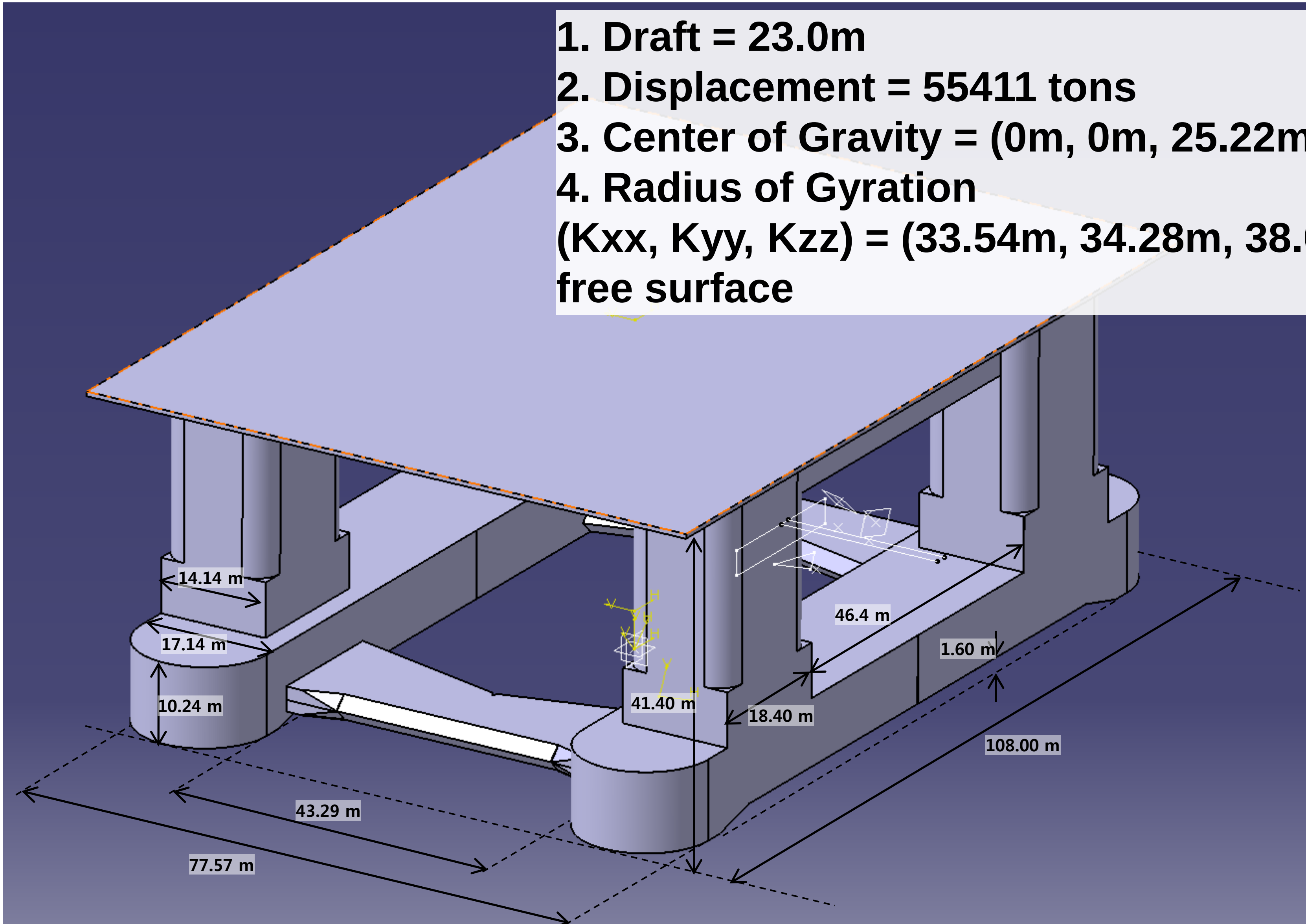
$$\boxed{(\mathbf{M}+\mathbf{A})\cdot\ddot{\mathbf{x}}(t) + \int_0^\infty \mathbf{B}(\tau)\cdot\dot{\mathbf{x}}(t-\tau)\,d\tau + \mathbf{C}\cdot\mathbf{x}(t) = \mathbf{F}_{exciting}}$$

: Cummins Equation

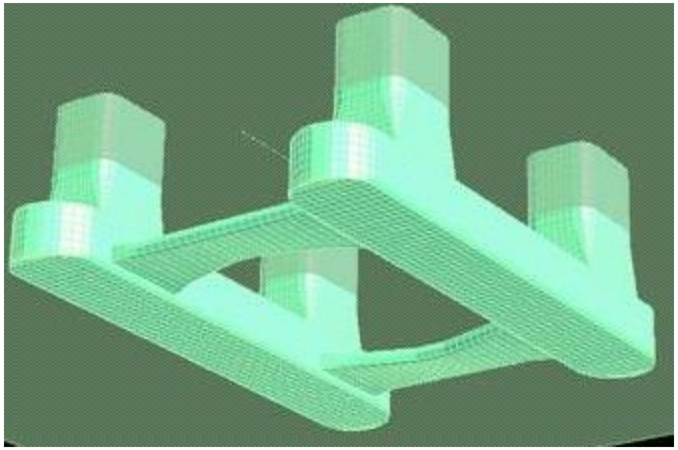
, where the components of the matrices A and B can be obtained by the added mass and damping data <u>in the frequency domain</u>

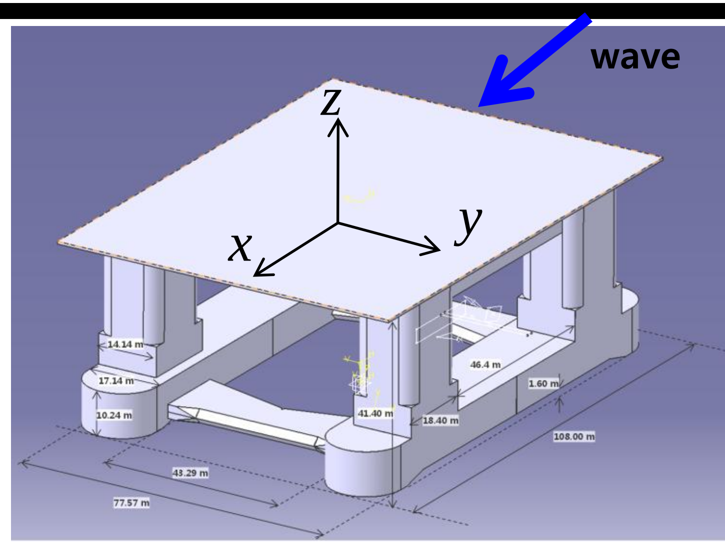
Dimension of the Semi-submersible rig

1. Draft = 23.0m
2. Displacement = 55411 tons
3. Center of Gravity = (0m, 0m, 25.22m)
4. Radius of Gyration
(Kxx, Kyy, Kzz) = (33.54m, 34.28m, 38.01m) at free surface



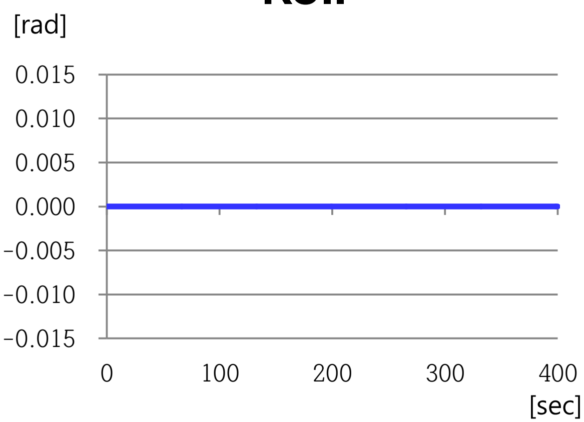
Calculating Hydrodynamic Force by using WADAM*: Following Sea





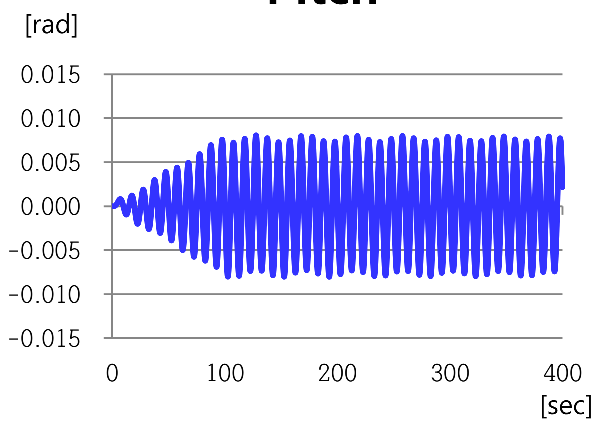
Wave
Amp. = 1.0 m
Period = 10 s
Direction = 0 deg

Roll



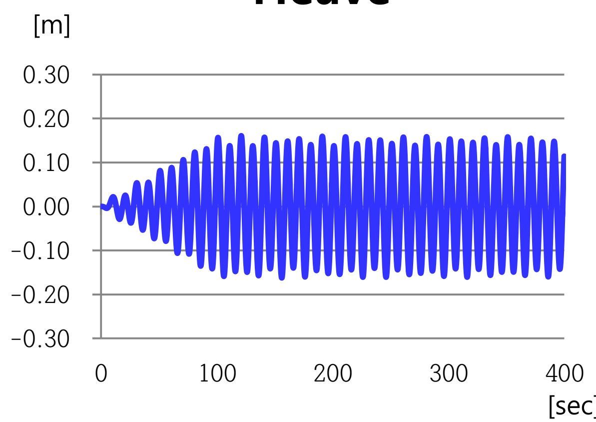
WADAM: 0.0000 [rad]

Pitch



WADAM: 0.007603 [rad]

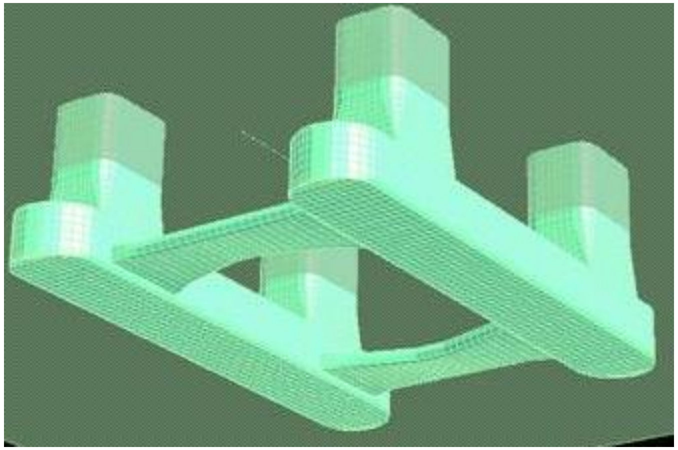
Heave

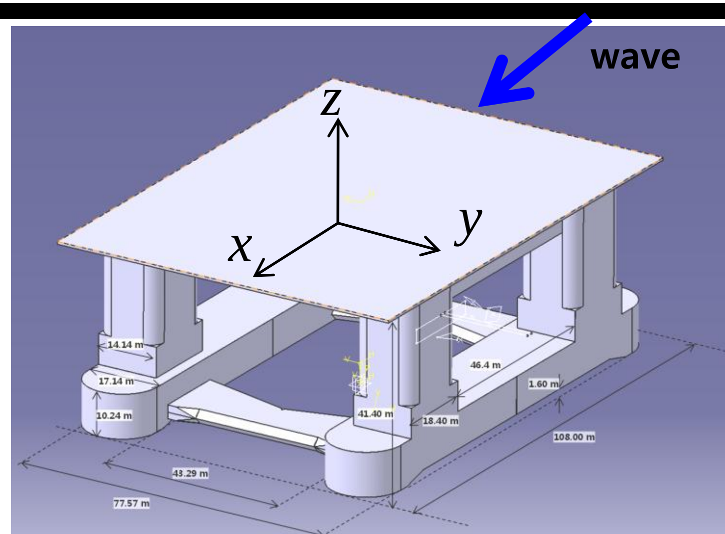


WADAM: 0.1713 [m]

*WADAM : frequency domain hydrodynamic analysis S/W developed by DNV (www.dnv.com)

Calculating of Hydrodynamic Force by using WADAM*: Following Sea - RAO



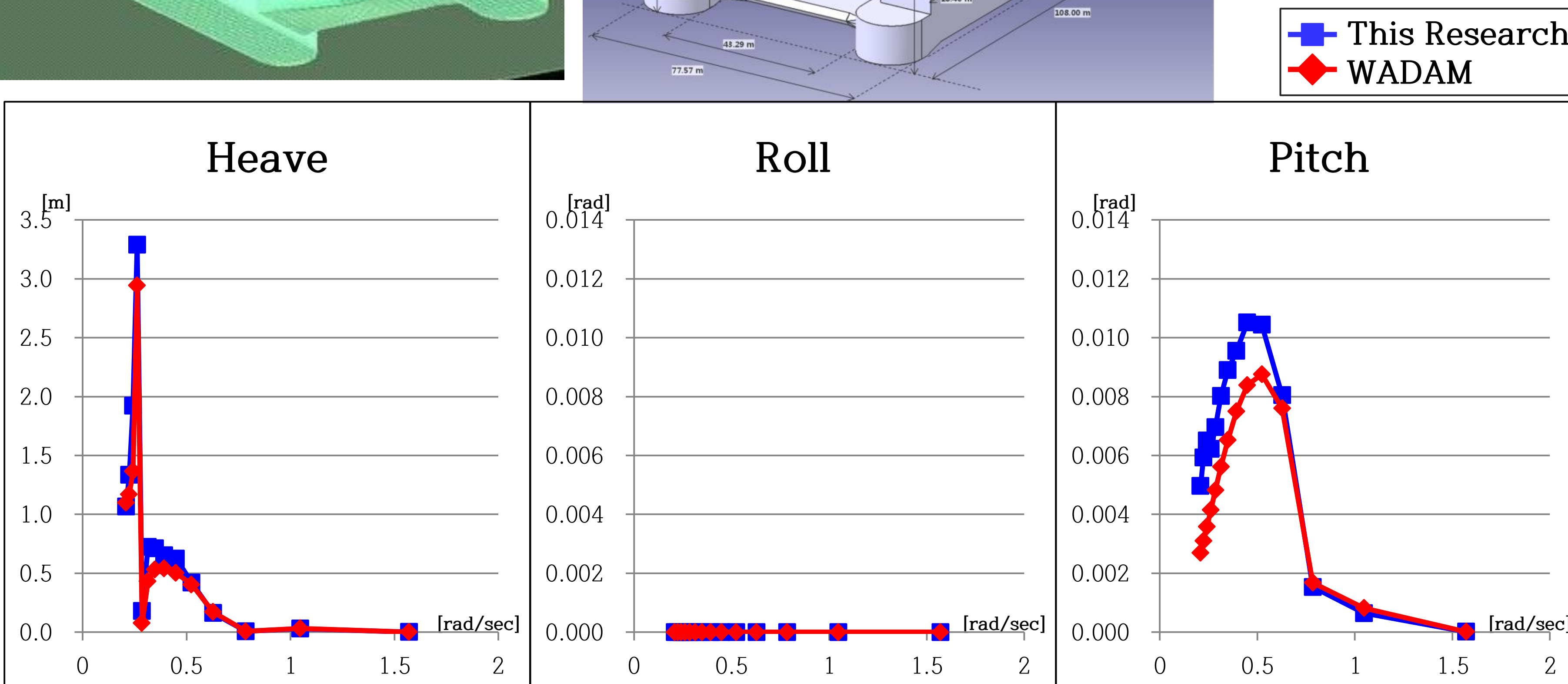


Wave

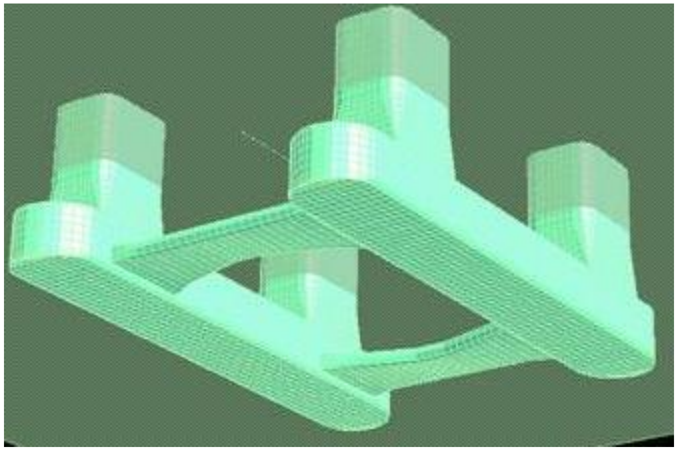
Amp. = 1.0 m

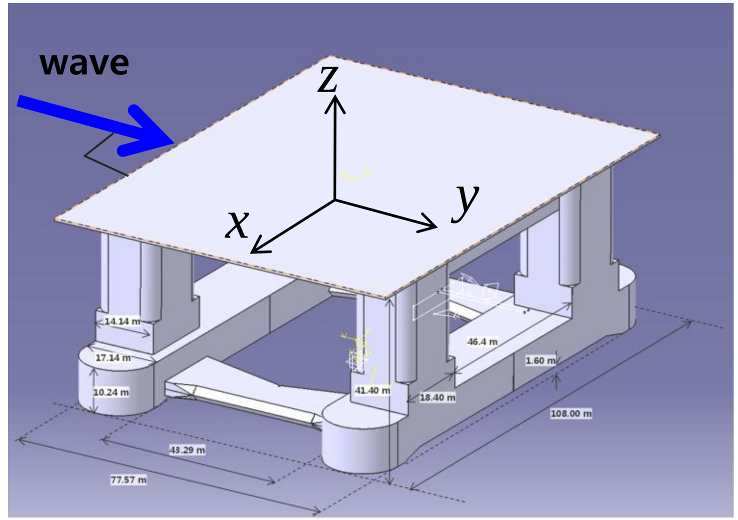
Period = 10 s

Direction = 0 deg



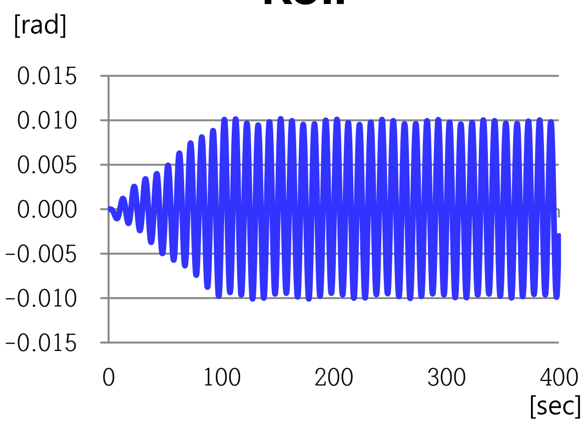
Calculating of Hydrodynamic Force by using WADAM*: Beam Sea





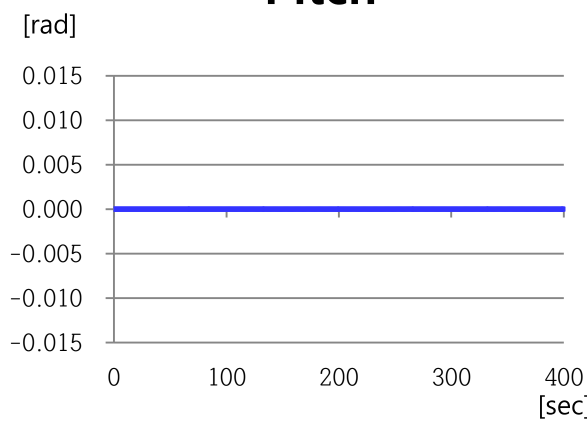
Wave
Amp. = 1.0 m
Period = 10 s
Direction = 90 deg

Roll



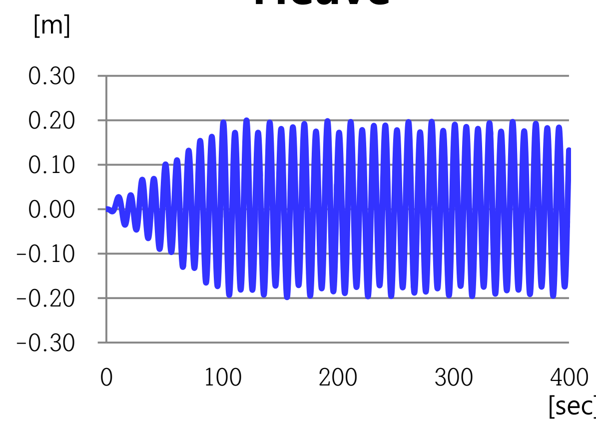
WADAM: 0.009072 [rad]

Pitch



WADAM: 0.0000 [rad]

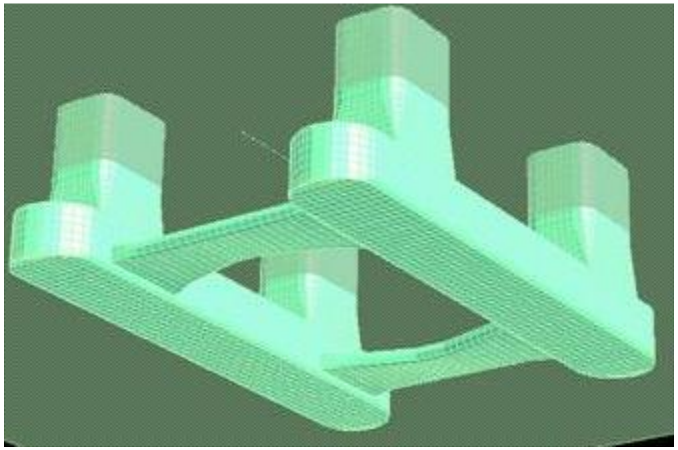
Heave

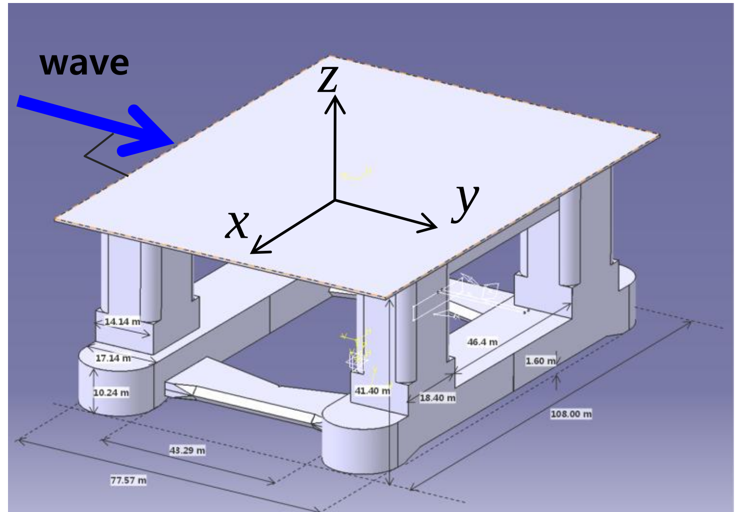


WADAM: 0.2116 [m]

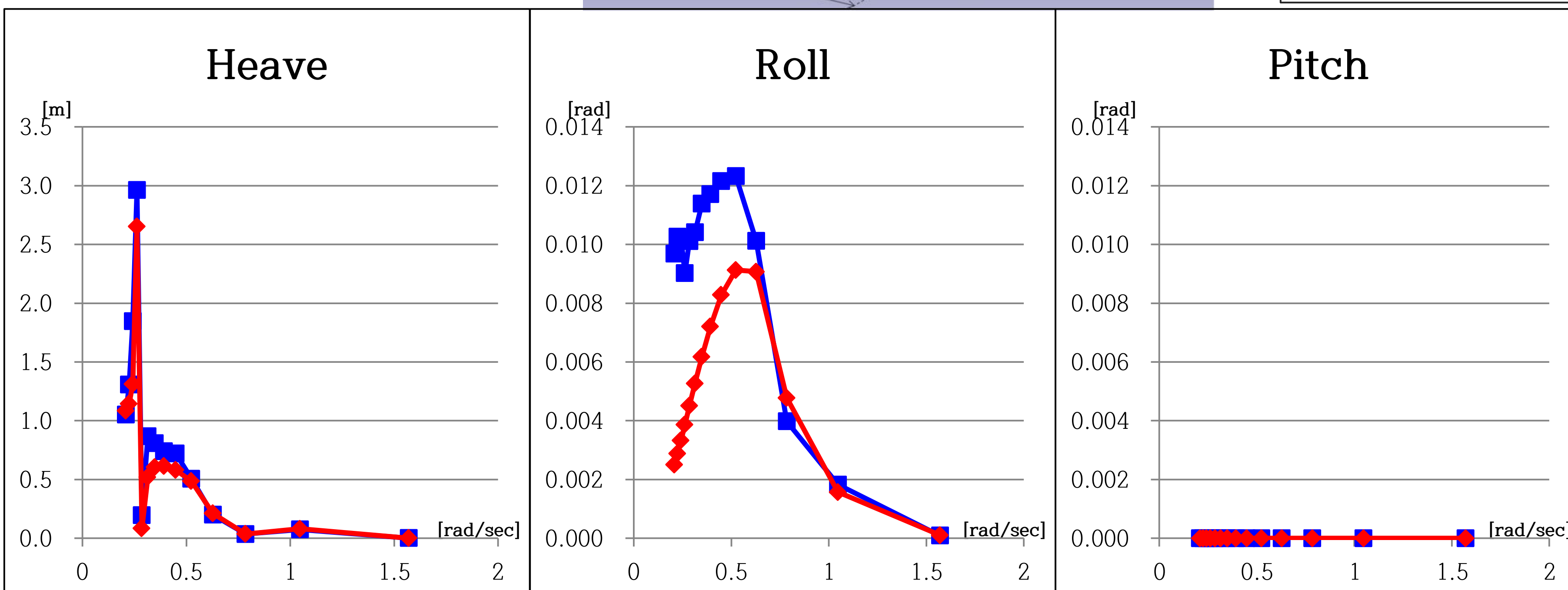
*WADAM : frequency domain hydrodynamic analysis S/W developed by DNV (www.dnv.com)

Calculating of Hydrodynamic Force by using WADAM*: Beam Sea

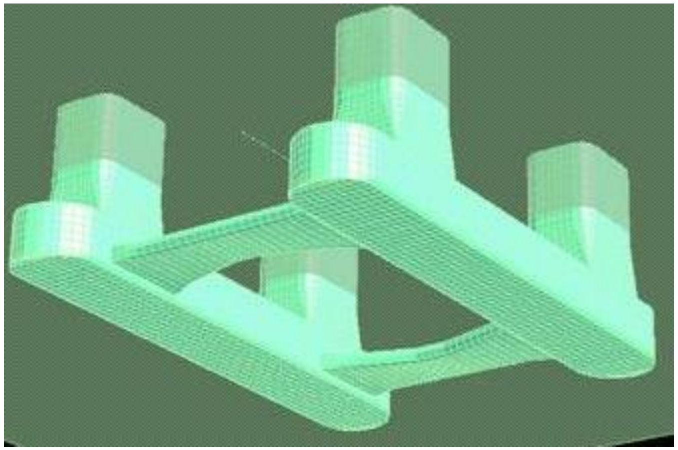


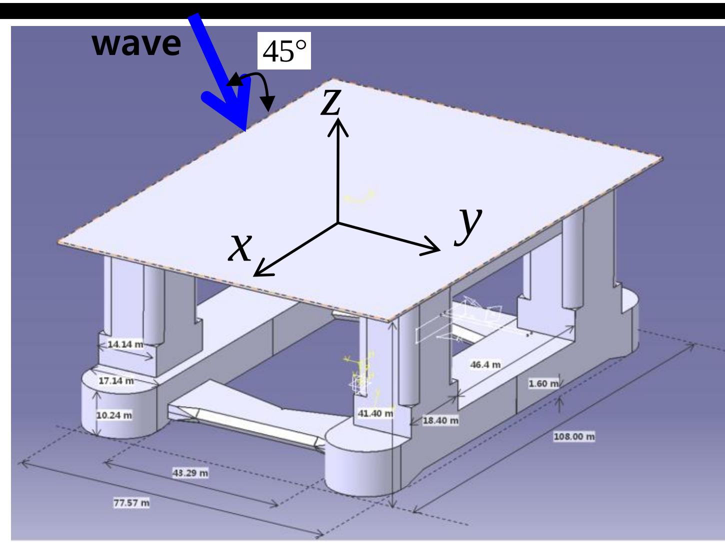


Wave
Amp. = 1.0 m
Period = 10 s
Direction = 90 deg



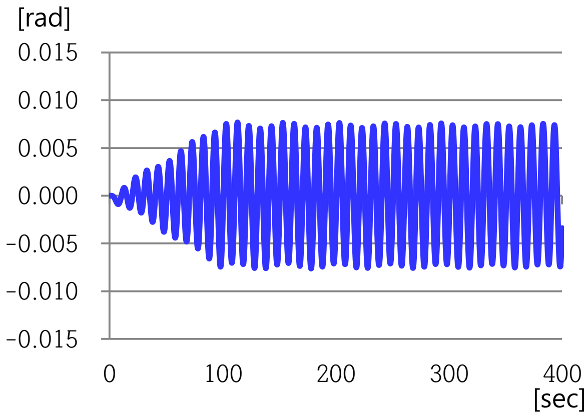
Calculating of Hydrodynamic Force by using WADAM*: Quatering Sea





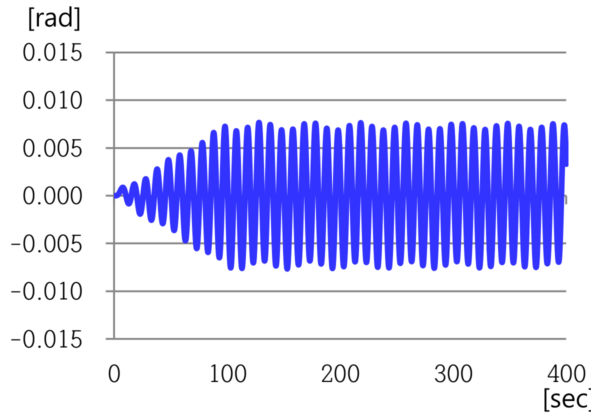
Wave
Amp. = 1.0 m
Period = 10 s
Direction = 45 deg

Roll



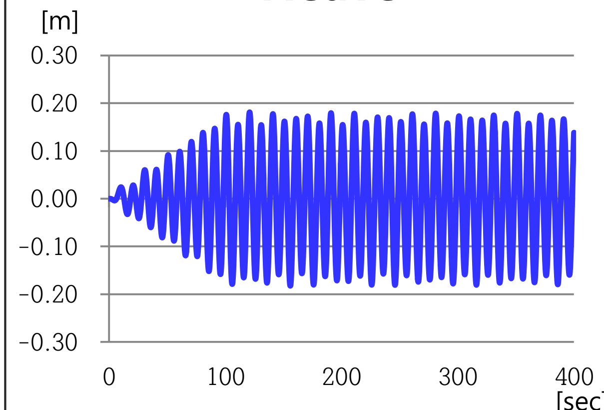
WADAM: 0.006802 [rad]

Pitch



WADAM: 0.007180 [rad]

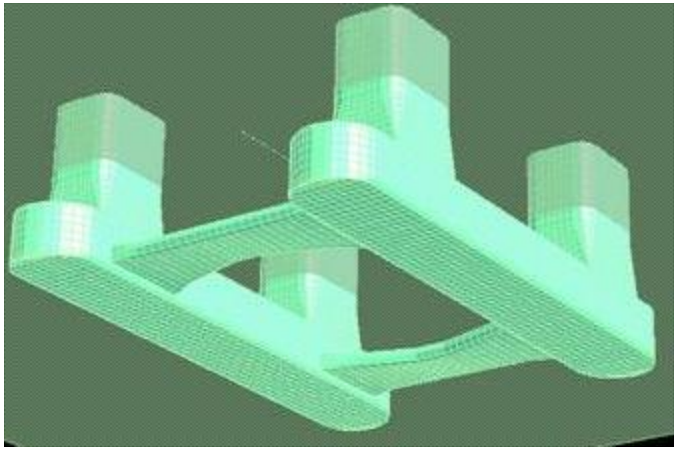
Heave

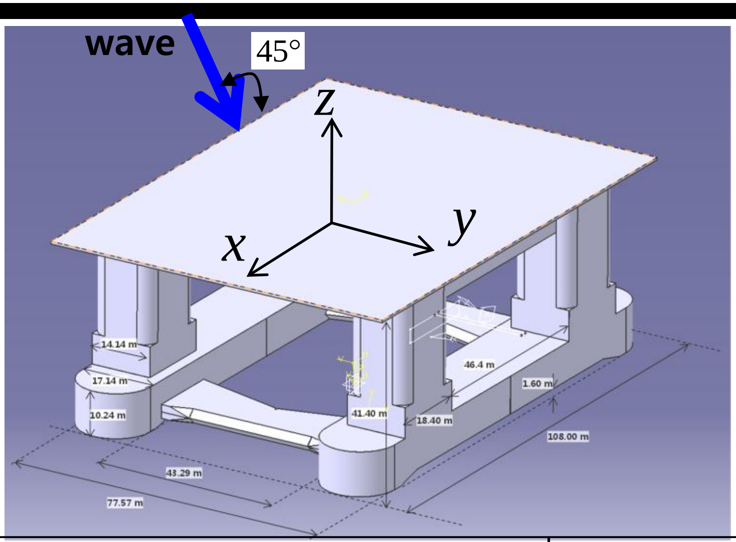


WADAM: 0.1925 [m]

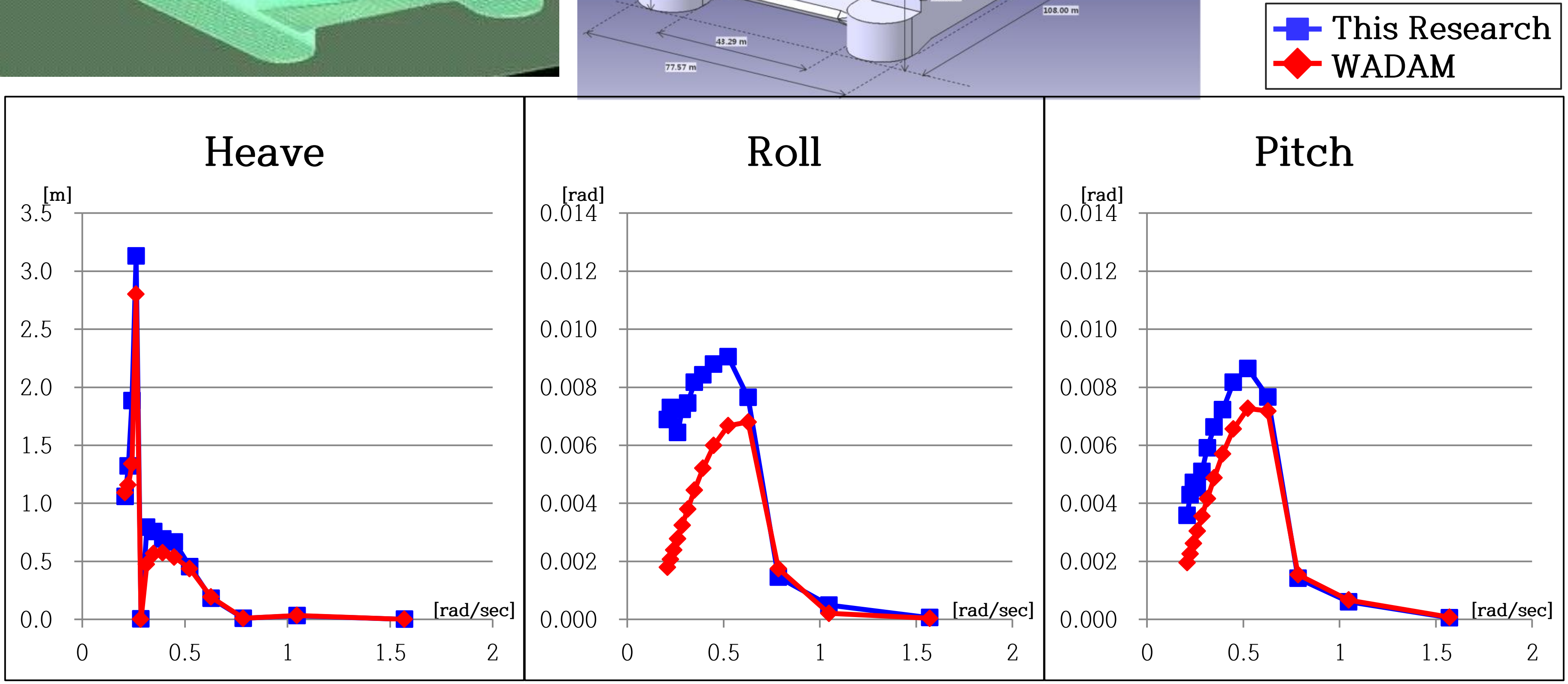
*WADAM : frequency domain hydrodynamic analysis S/W developed by DNV (www.dnv.com)

Calculating of Hydrodynamic Force by using WADAM*: Quatering Sea





Wave
Amp. = 1.0 m
Period = 10 s
Direction = 45 deg

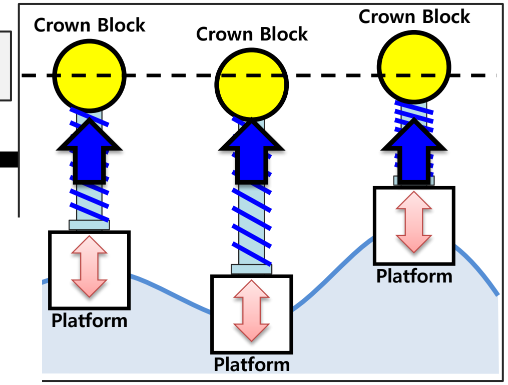


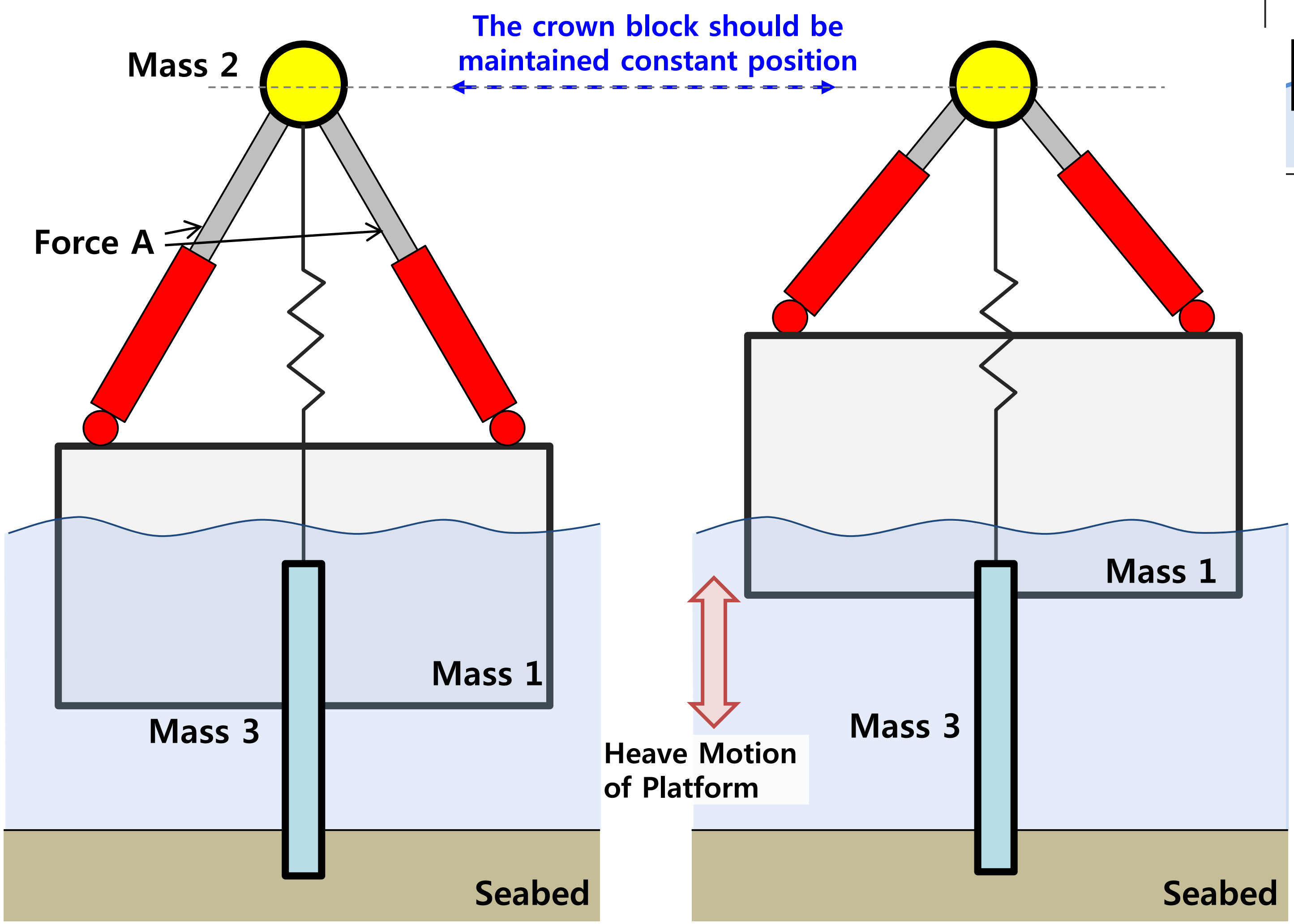
18-4.
Hydraulic and Pneumatic Control Force

Mathematical Model for Hoisting and Heave Compensation System

Considering Only the Drill String Compensator

Concept of Drill String Compensator



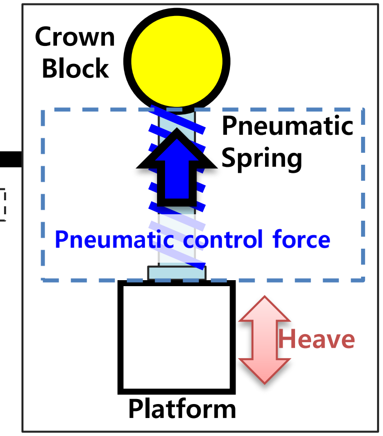


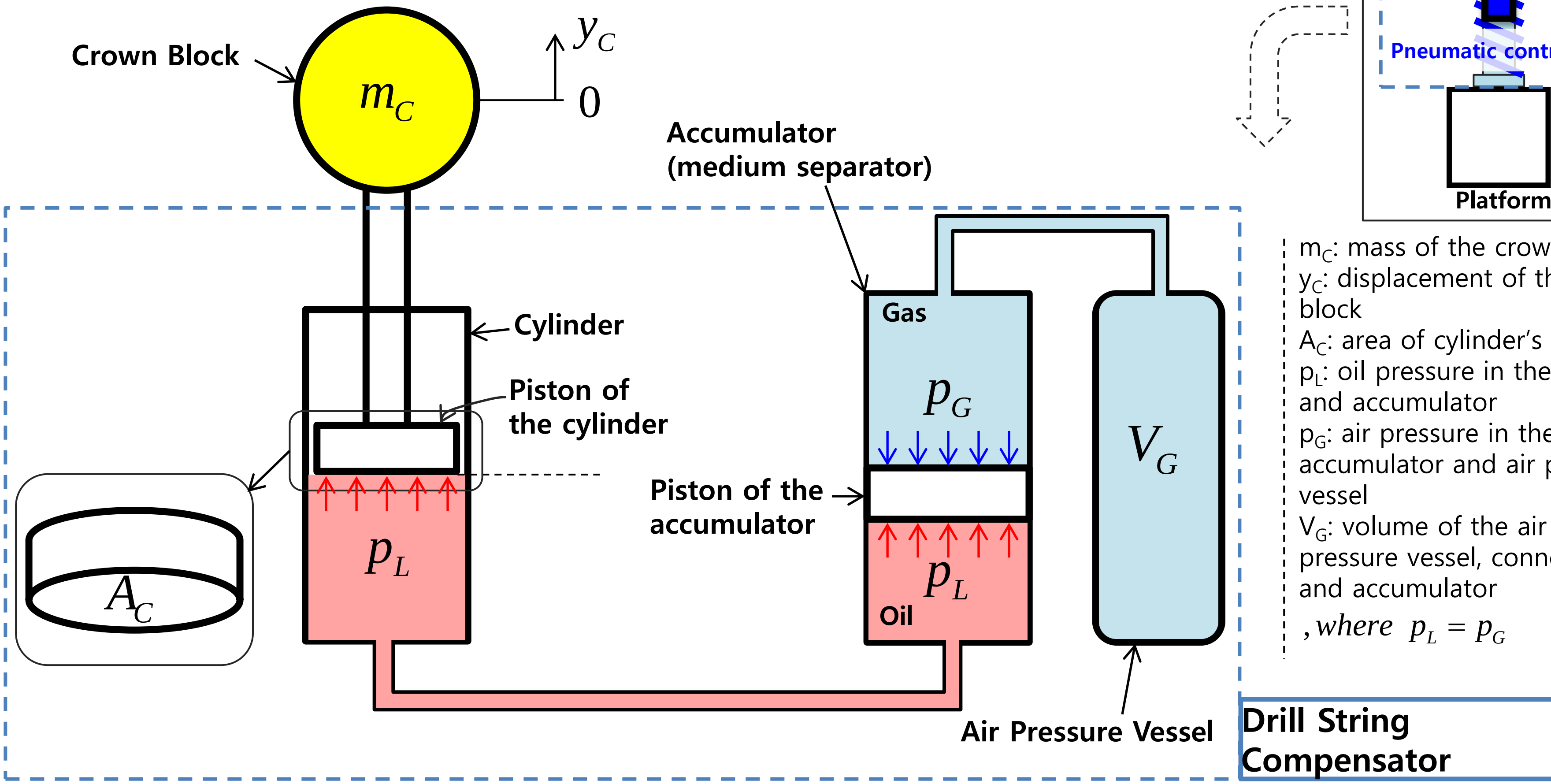
Mass 1: Platform
(including derrick, drawworks, deadline anchor, supply reel, drill string compensator and active heave compensator)
Mass 2: Crown block
(including wire rope drilling line, traveling block, and top drive)
Mass 3: Drill string

Force A: Force exerted by the drill string compensator
Force B: Force exerted by the active heave compensator
Force C: Spring force indicating the stiffness of the drill string
Force D: Damping force describing the contacts between the drill bit and seabed:

To examine the effect of the drill string compensator on the motion of the crown block, the forces exerted by the drill string compensator are divided into two kind of forces:
Force A-1: Spring force describing the low-rate pneumatic spring
Force A-2: Damping force describing the seal friction between the cylinder and piston

Realization of the Spring Mechanism of the Drill String Compensator By Introducing the Combined Hydraulic and Pneumatic System



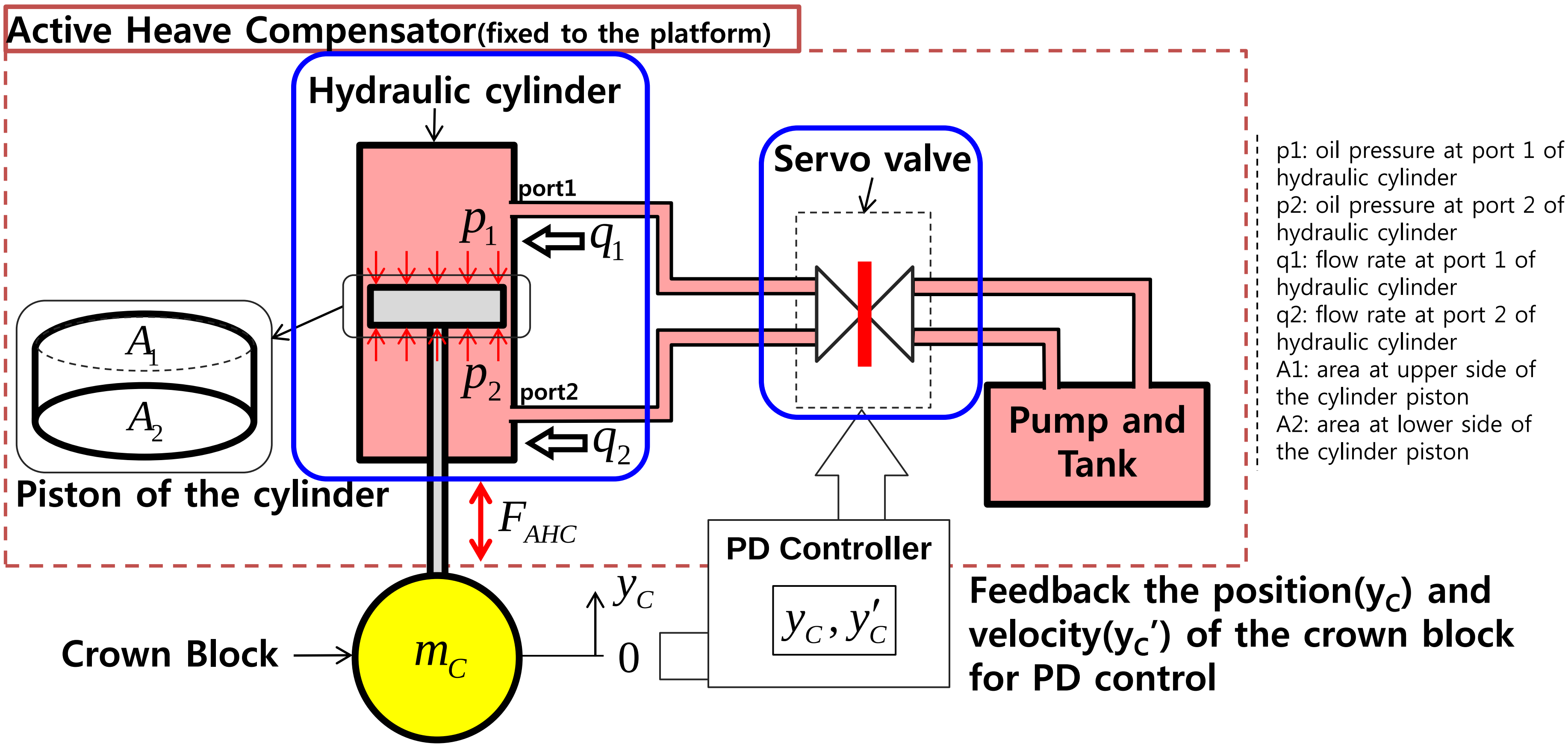


m_C: mass of the crown block
y_C: displacement of the crown block
A_C: area of cylinder's piston
p_L: oil pressure in the cylinder and accumulator
p_G: air pressure in the accumulator and air pressure vessel
V_G: volume of the air in the air pressure vessel, connecting pipe, and accumulator
$, where \; p_L = p_G$

- The combined hydraulic and pneumatic system, composed of cylinder, accumulator, and air pressure vessel, forms a spring mechanism of which both the pre-tension (the air pressure in the system) and the stiffness (the volume of the linked gas container) can be set
- The pressure of the air is controlled in order to vary the load that can be supported

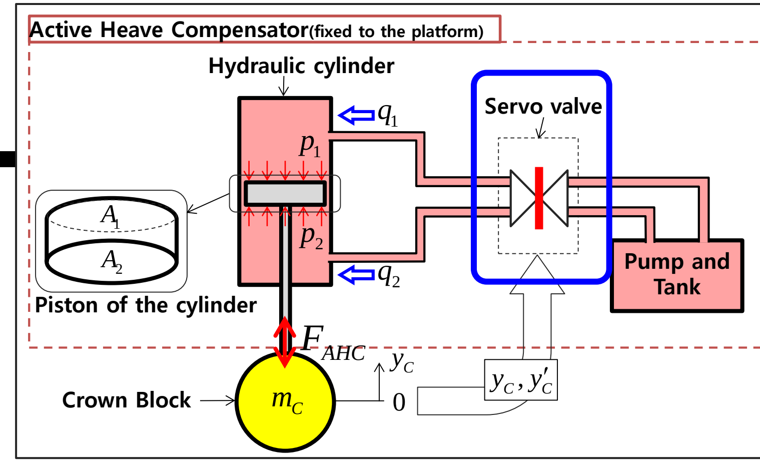
Reference) Peter Albers, Motion Control in Offshore and Dredging, 2010, pp.208-215

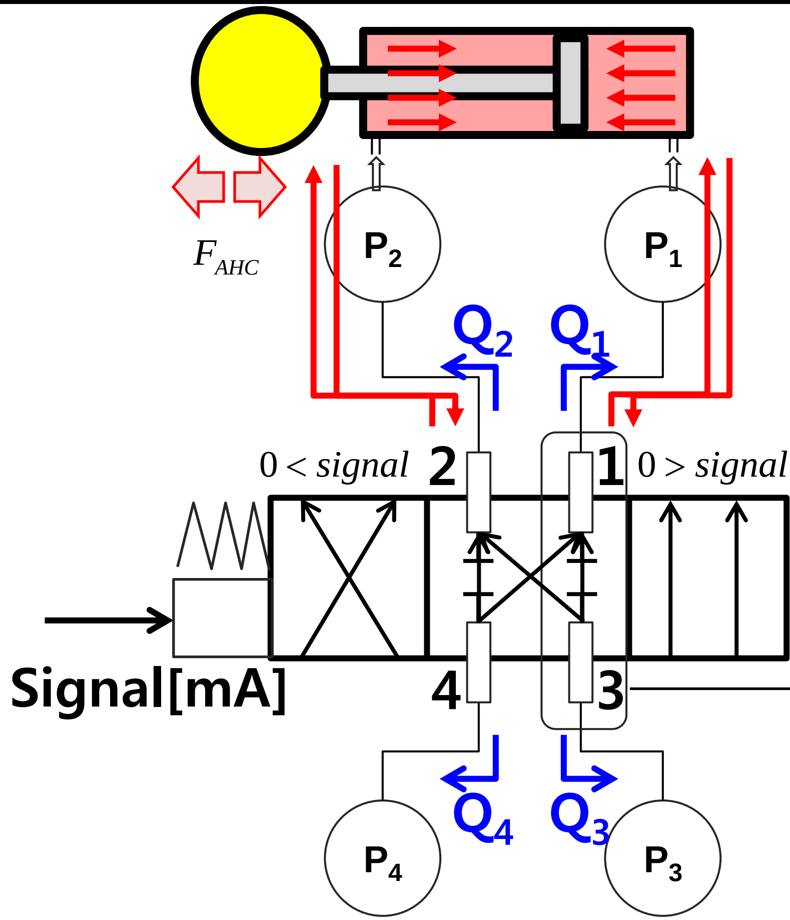
Realization of the Hydraulic Control Force of the Active Heave Compensator by Introducing Hydraulic Cylinder and Servo Valve

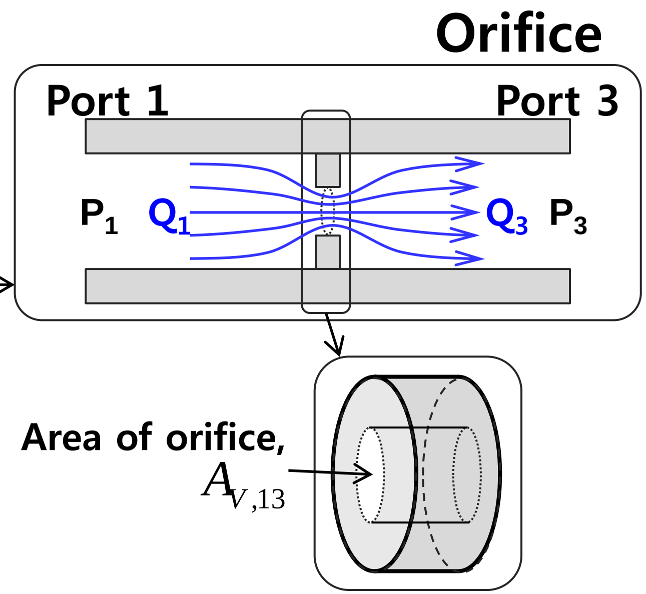


- In order to keep the position of the crown block constant, the PD control algorithm is applied
- Active heave compensator is mainly composed of hydraulic cylinder and servo valve Therefore, mathematical models for two modules are formulated:
  1) mathematical model for hydraulic cylinder module
  2) mathematical model for servo valve module

2) Mathematical Model for Servo Valve Module
- Parameters of the Servo Valve Module







P_1, P_2, P_3, P_4: oil pressures at port 1, 2, 3, and 4 of servo valve

Q_1, Q_2, Q_3, Q_4: flow rate at port 1, 2, 3, and 4 of servo valve

$A_{V,14}$, $A_{V,23}$, $A_{V,13}$, and $A_{V,24}$: areas of orifice connecting the ports 1 to

- **The flow part is determined according to the input signal**
- **There are 4 possible flow paths: 1 to 4, 2 to 3, 1 to 3, 2 to 4**

- **If the port 1 is connected to the port 3, the flow path is described as an orifice**
  Then the flow rates, Q_1 and Q_3 are represented by the area of orifice, $A_{V,13}$, and the pressures P_1 and P_3.

Reference 1) Merrit, H.E., Hydraulic Control Systems, John Wiley & Sons, Inc., 1967, pp.39-45, pp.79-88

18-5.
Mathematical model of Drill String

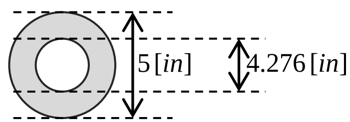
Properties of Drill String

The properties of drill pipe, HWDP, and BHA are illustrated in the figure.

Drill pipe

Length: $10,000 + 30,000 - 600 - 300 = 39,100[\,ft]$

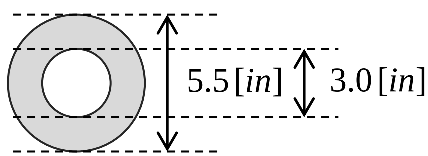
Outer/Inner Diameter: 5.0 in / 4.276 in



HWDP

Length: $600[\,ft]$

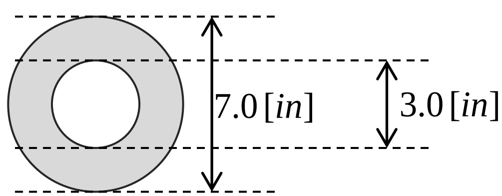
Outer/Inner Diameter: 5.5 in / 3.0 in

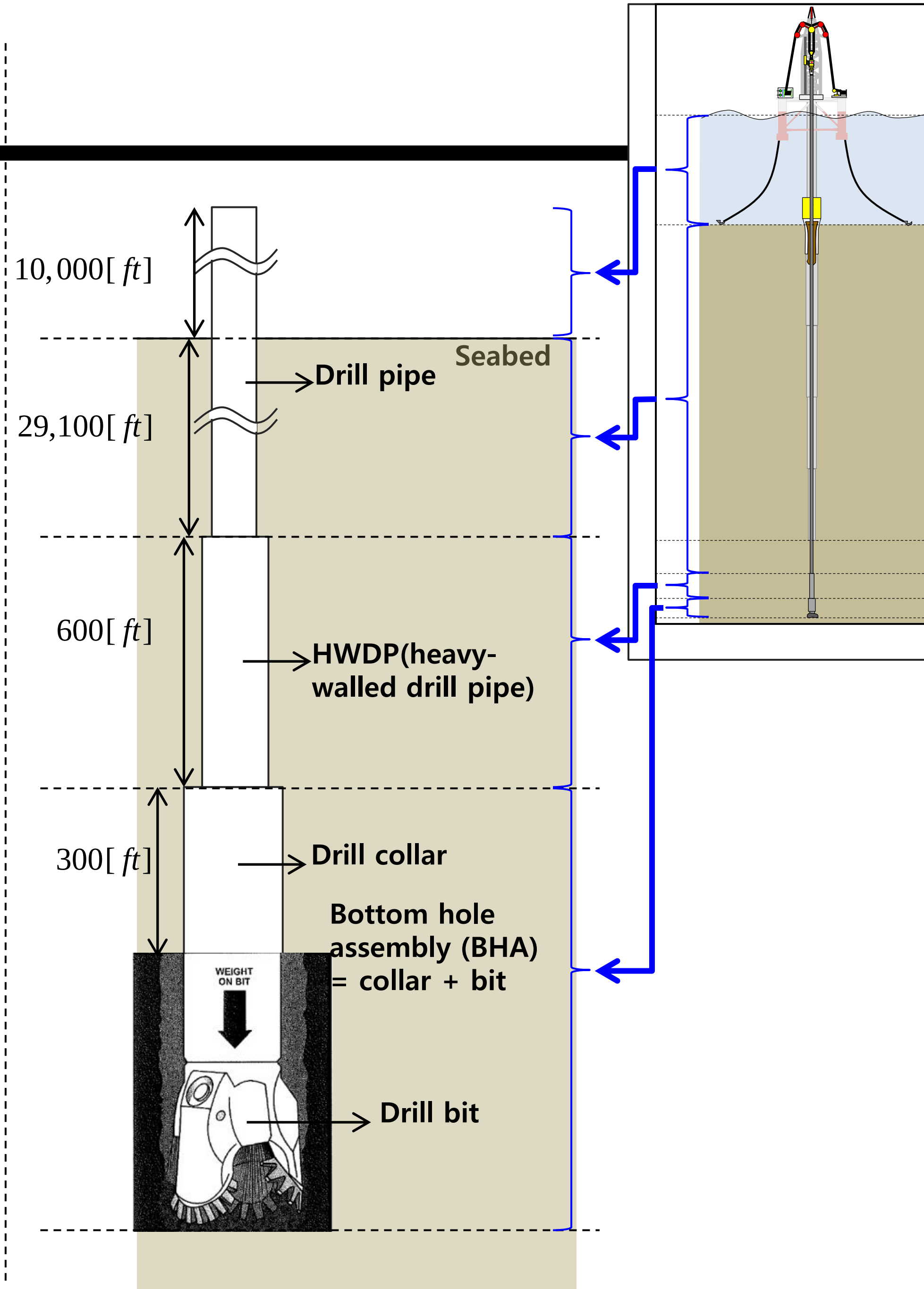


Drill collar

Length: $300[\,ft]$

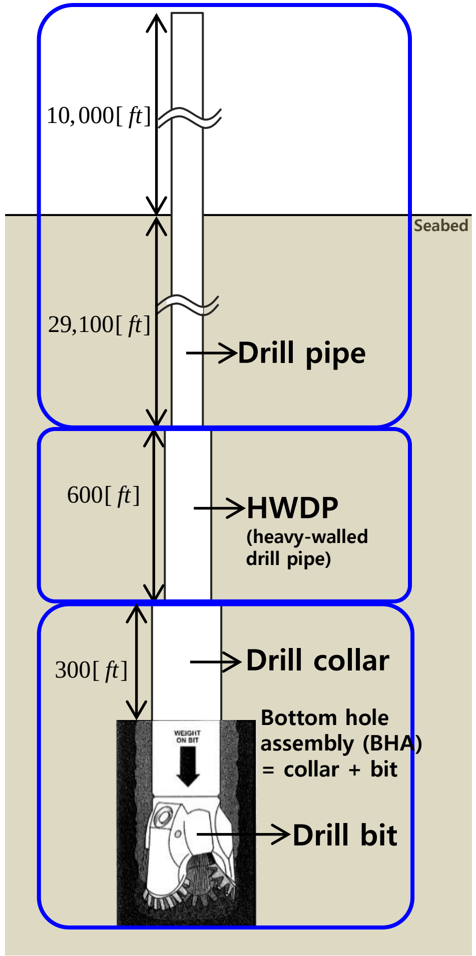
Outer/Inner Diameter: 7.0 in / 3.0 in

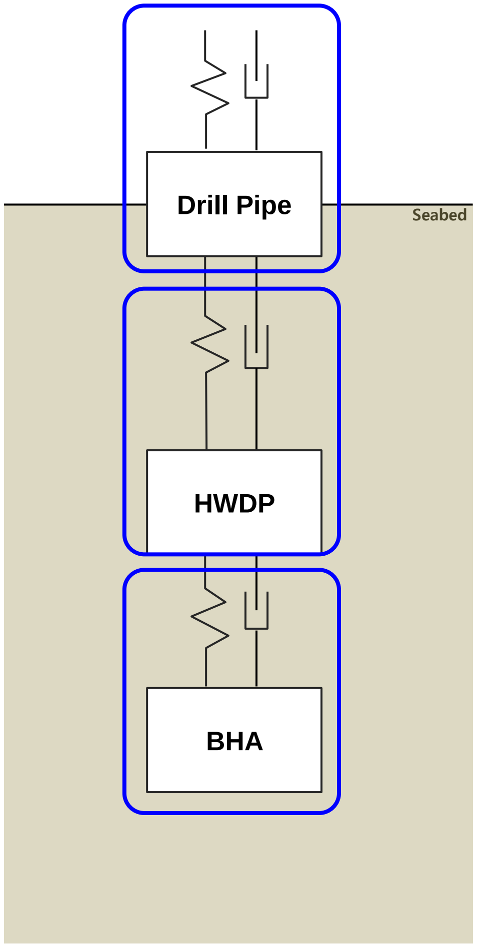




Reference) Jonggeun Choe, et al, Analyses and Procedures for Kick Detection in Subsea Mudlift Drilling, SPE Drilling & Completion, December 2007

Mathematical Modeling of Drill String
- Replace the Drill String with Equivalent Mass, Spring, and Damper

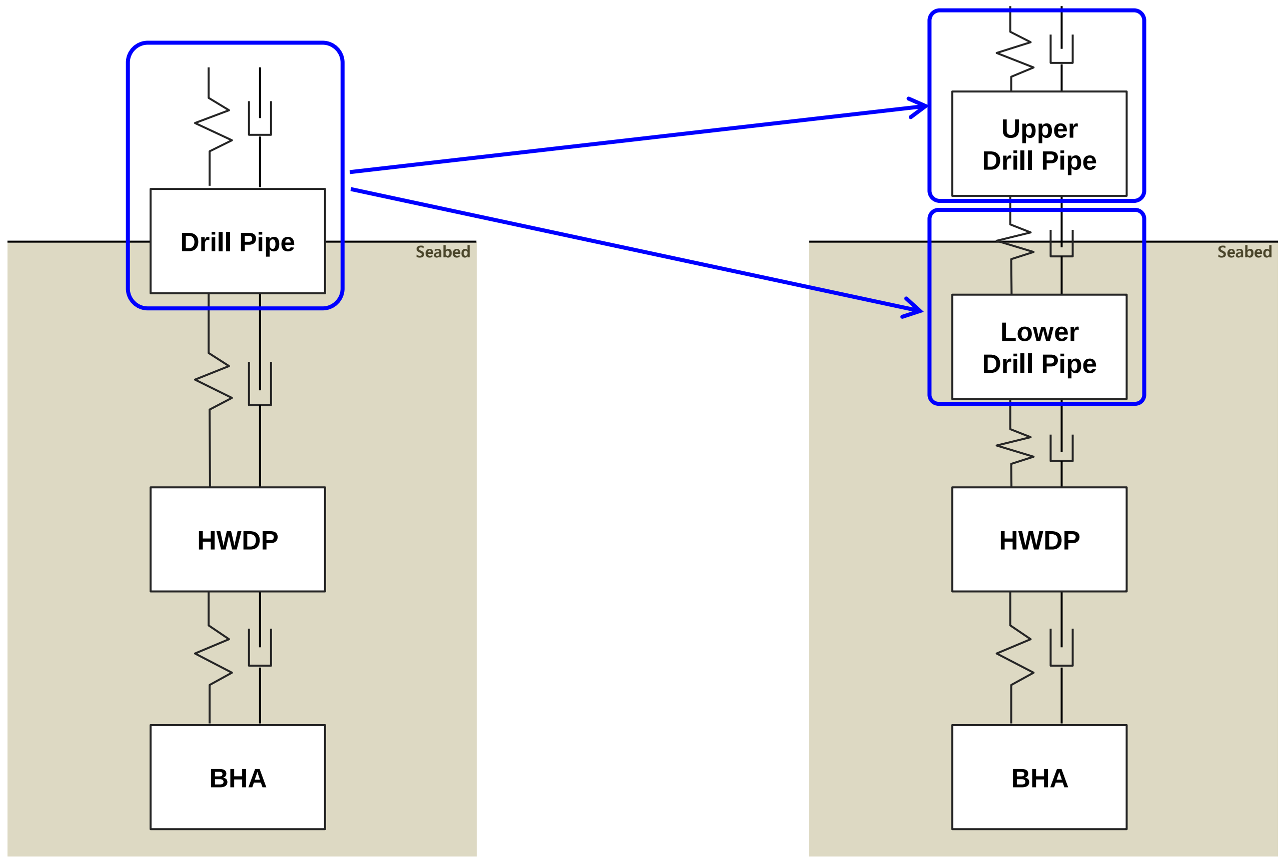




Since the drill pipe, HWDP, and BTA will stretch under its own weight modified by the buoyancy factor, and be subject to drag friction as they are pulled through the drilling fluid, they can be replaced by equivalent mass, spring, and damper.

Reference) Hatleskog, J.T., Dunnigan, M.W., Passive Compensator Load Variation for Deep-Water Drilling, Journal of Oceanic Engineering, Vol.32, No.3, July 2007

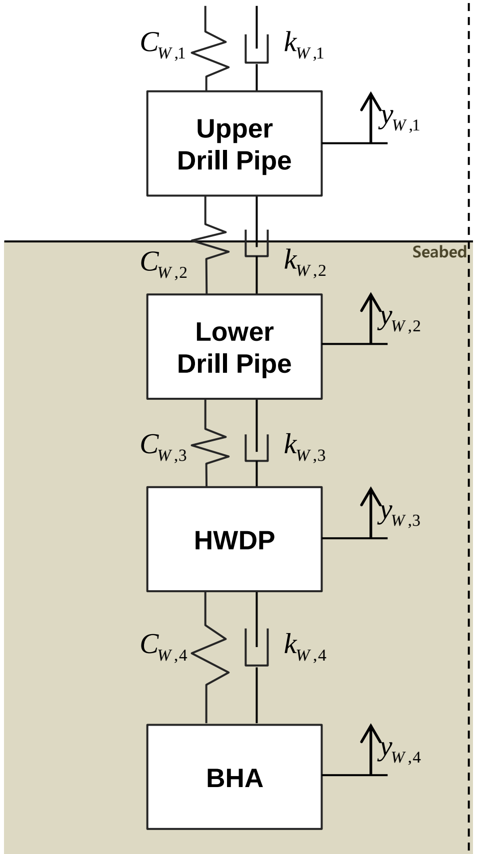
Mathematical Modeling of Drill String
- Replace the Drill String with Equivalent Mass, Spring, and Damper



The drill pipe is considered in two parts to accommodate the different friction regime; the friction experienced by the drill string in the upper section is predominantly viscous drag whereas the lower section will typically experience a fair degree of coulomb friction, which will change significantly if the drill string is turning.[Ref.1)] Also, in practice, it suffices to use a model where the drill pipe is split into two parts[Ref.2)].

Reference 1) Hatleskog, J.T., Dunnigan, M.W., Passive Compensator Load Variation for Deep-Water Drilling, Journal of Oceanic Engineering, Vol.32, No.3, July 2007

Mathematical Modeling of Drill String
- Replace the Drill String with Equivalent Mass, Spring, and Damper



The weights of the upper drill pipe, lower drill pipe, HWDP, and BHA are modified by the buoyancy factor.

$$Weight,_{\text{modified}} = B \cdot m \cdot g$$

, where

B: Buoyancy factor

$$= \frac{\rho_S - \rho_M}{\rho_S} = \frac{65.5 - 15.5}{65.5} = 0.763$$

ρ_S: density of pipe, 65.50[ppg]

ρ_M: density of mud, 15.50[ppg]

m: mass of the drill pipe

g: gravitational acceleration

	Length [m]	O.Dia. [m]	I.Dia. [in]	Volume [m3]	Weight [kg]	Weight*BF [kg]
Upper Drill Pipe	3,048.00	0.127	0.109	10.372	81,422	62,155
Lower Drill Pipe	8,869.68	0.127	0.109	30.183	236,938	180,872
HWDP	182.88	0.140	0.076	1.969	15,458	11,800
Drill collar	91.44	0.178	0.076	1.853	14,549	11,106

The spring coefficient is indicated by

$$k = \frac{E \cdot A}{L}$$

, where

E: modulus of elasticity=140GPa for the drill pipe

A: cross-sectional area of the drill pipe

L: length of the drill pipe

	Area [m^2]	Length [m]	Spring Coefficient [N/m]
Upper Drill Pipe	0.0034029582	3,048.00	156,303.857
Lower Drill Pipe	0.0034029582	8,869.68	53,712.666
HWDP	0.0107675339	182.88	8,242,862.808
Drill collar	0.0202682992	91.44	31,031,954.100

The damping coefficient is indicated by

$$d = C_1 \cdot S \cdot \rho$$

, where

C_1: drag coefficient, 0.95

S: surface area of the drill pipe

ρ: density of drilling fluid, i.e. mud, 15.50[ppg]

	Surface Area [m^2]	Length [m]	Damping Coefficient [N/m]
Upper Drill Pipe	1,216.098	3,048.00	21,030,492.241
Lower Drill Pipe	3,538.845	8,869.68	61,198,732.422
HWDP	80.262	182.88	1,388,012.488
Drill collar	51.076	91.44	883,280.674

ppg: pound per gallon

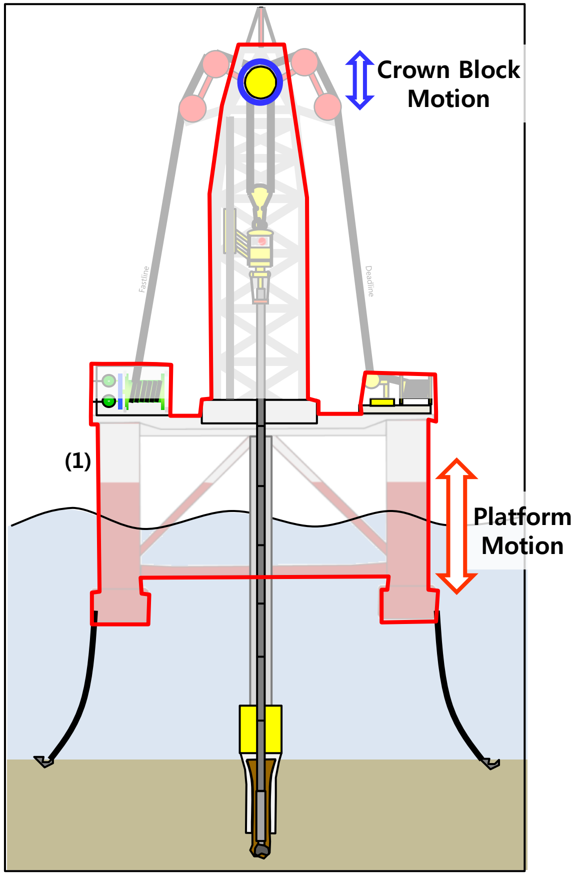
Reference 1) Jonggeun Choe, et al, Analyses and Procedures for Kick Detection in Subsea Mudlift Drilling, SPE Drilling & Completion, December 2007

Reference 1) Hatleskog, J.T., Dunnigan, M.W., Passive Compensator Load Variation for Deep-Water Drilling, Journal of Oceanic Engineering, Vol.32, No.3,

18-6.
Dynamic Response Analysis and Control of the Hoisting and Heave Compensation System

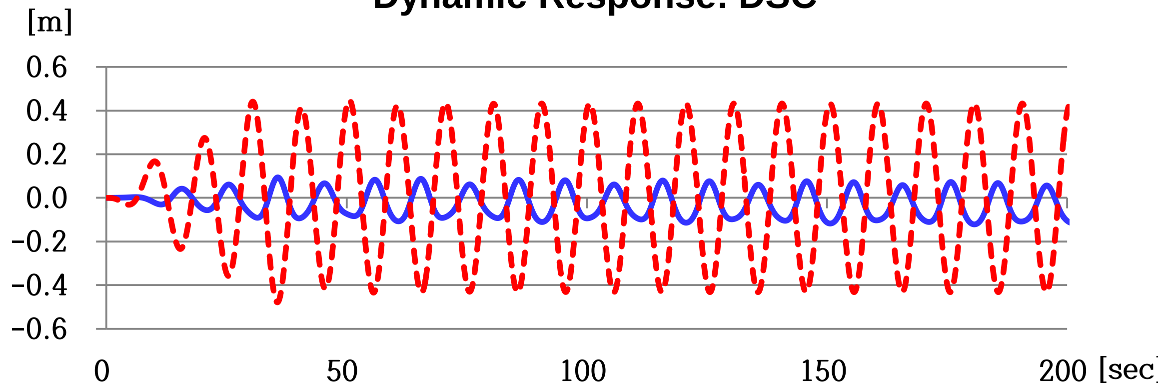
Dynamic Response Analysis
- Wave Amplitude: 3m

Crown Block
Platform



Wave	Amplitude	3 m
	Period	10 sec
	Angle	0 degree

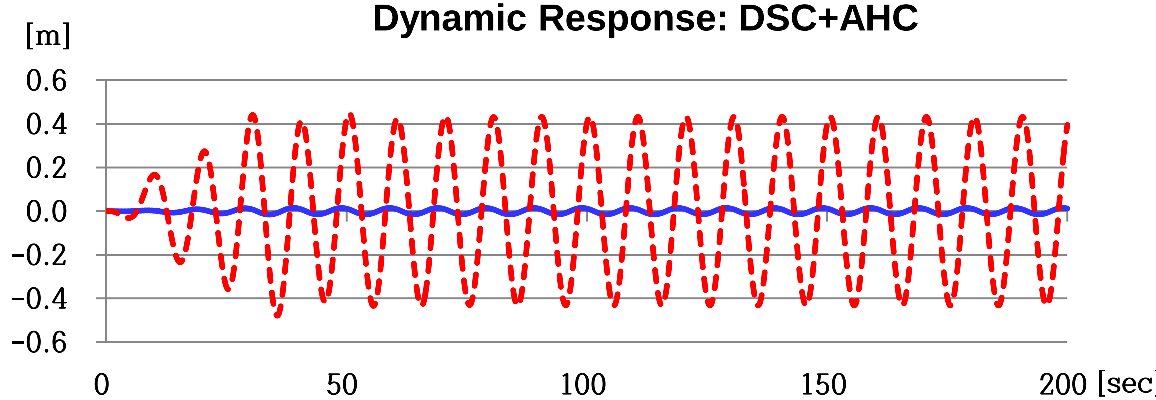
Dynamic Response: DSC



Platform Motion Amplitude: 0.483 (Max: 0.477, Min: -0.489)
Crown Block Motion Amplitude: 0.088 (Max: 0.085, Min: -0.091)

→ 82% of amplitude is decreased.

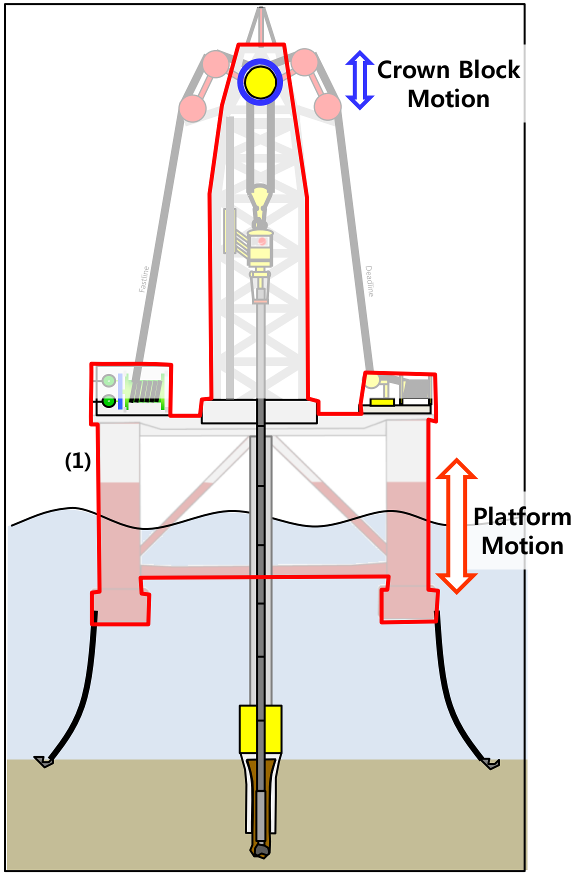
Dynamic Response: DSC+AHC



Platform Motion Amplitude: 0.463 (Max: 0.447, Min: -0.479)
Crown Block Motion Amplitude: 0.0153 (Max: 0.0151, Min: -0.0155)

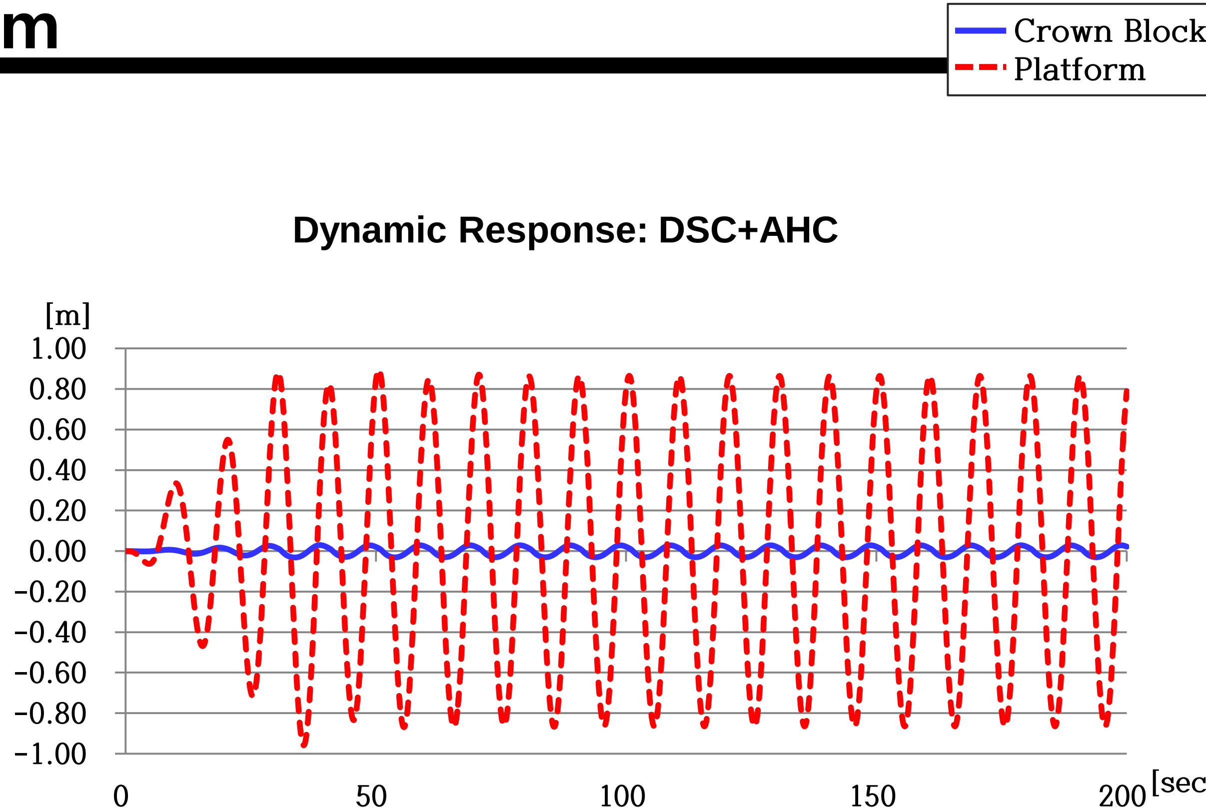
→ 97% of amplitude is decreased.

Dynamic Response Analysis
- Wave Amplitude: 6m



Wave	Amplitude	6 m
	Period	10 sec
	Angle	0 degree

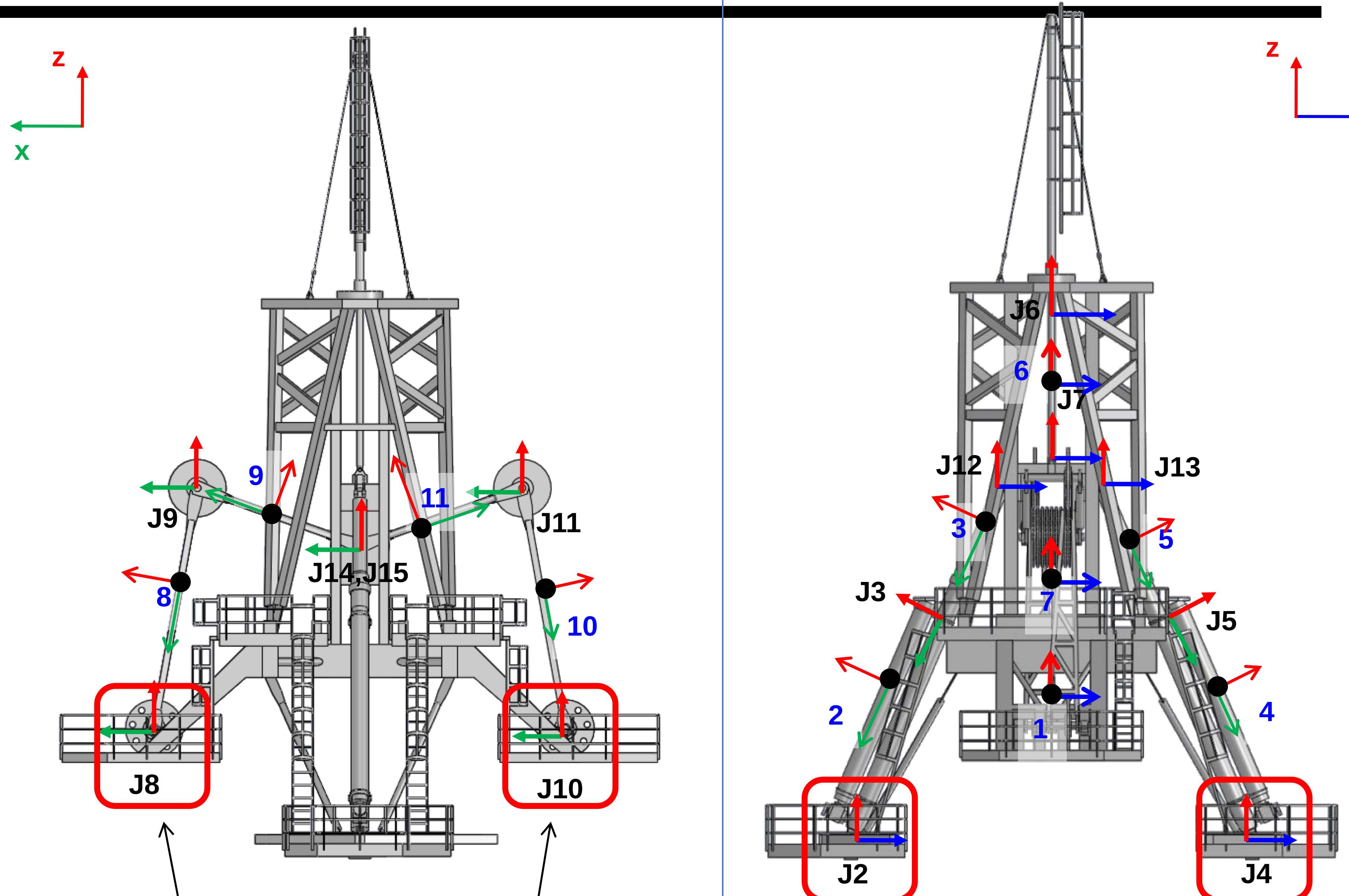
Dynamic Response: DSC+AHC



Platform Motion Amplitude: 0.926 (Max: 0.895, Min: -0.958)
Crown Block Motion Amplitude: 0.0306 (Max: 0.0297, Min: -0.0315)

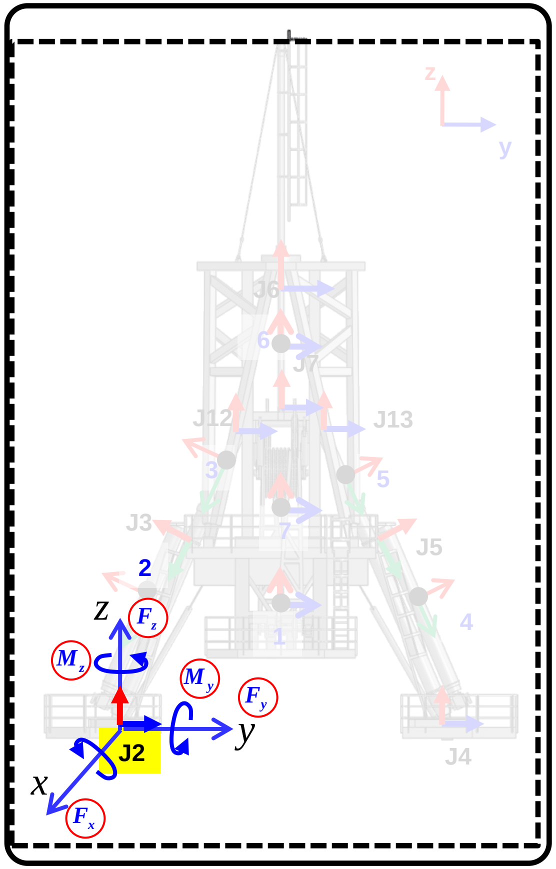
→ 97% of amplitude is decreased.

Coordinate systems of heave compensation system

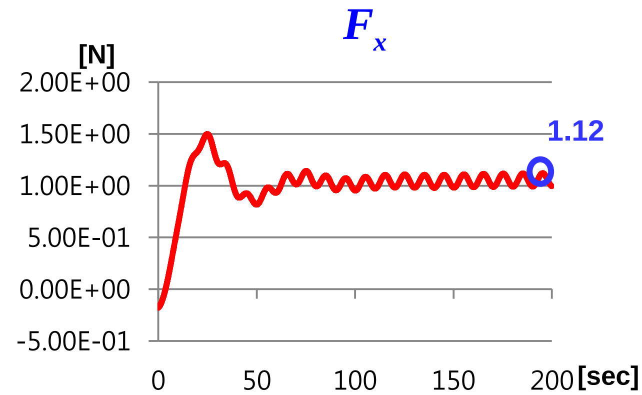


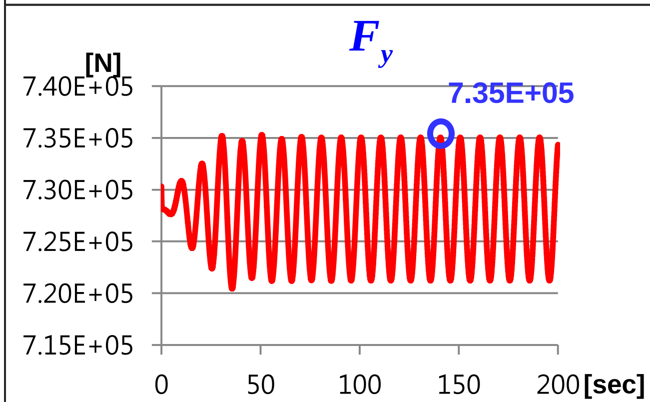
Calculation of constraint force

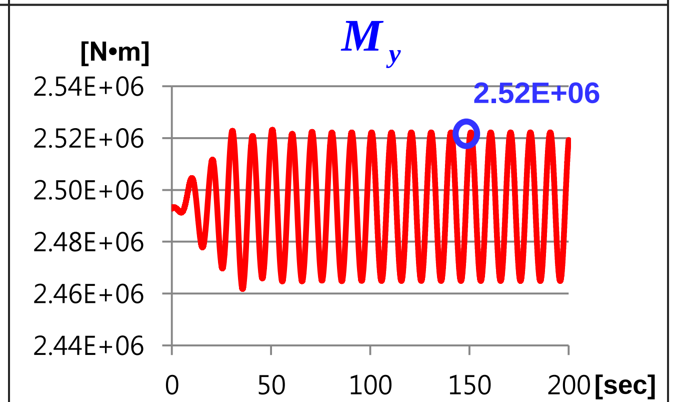
Constraint Forces and Moments: Joint 2

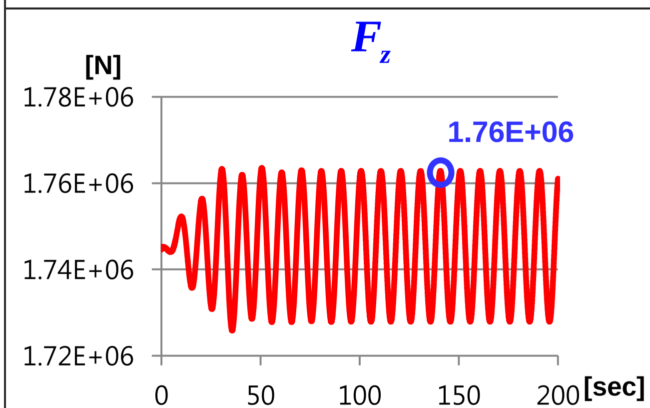


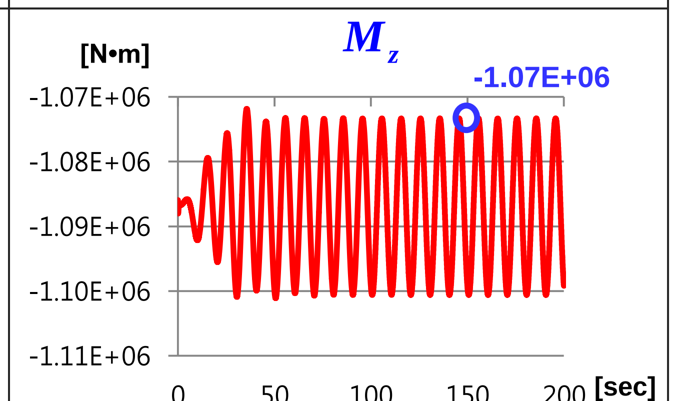
Wave	Amplitude	3 m
	Period	10 sec
	Angle	0 degree



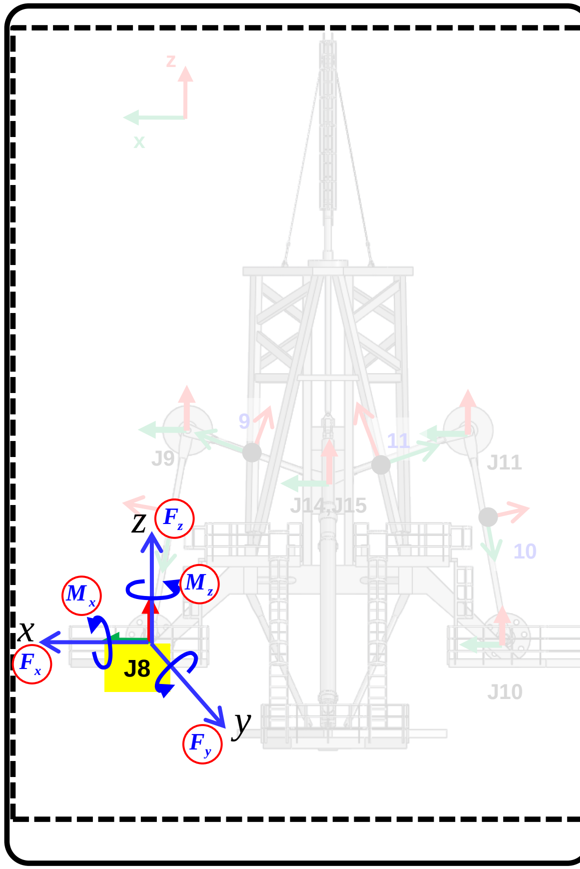




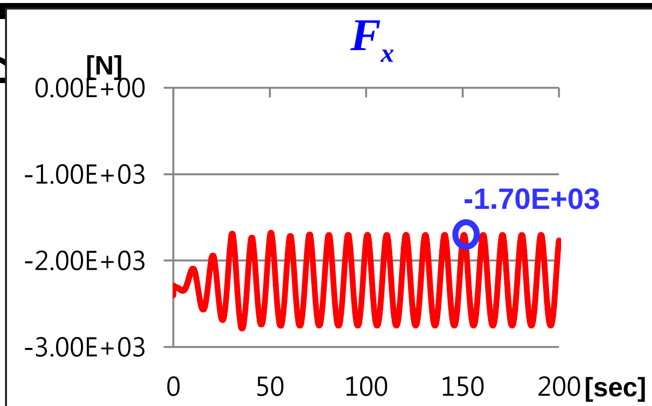


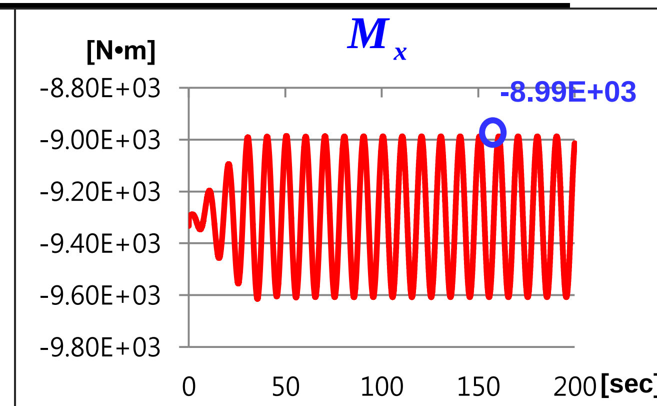


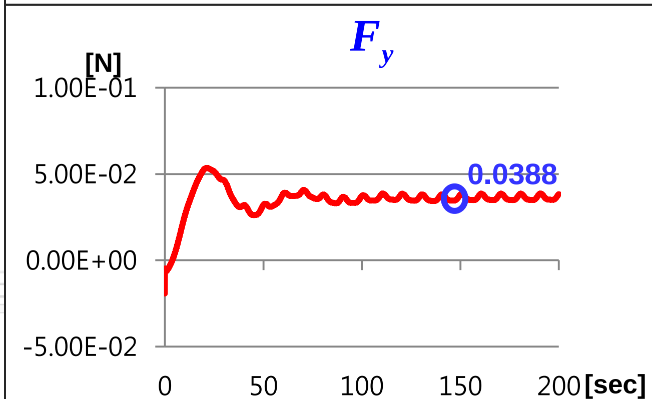
Constraint Forces and Moments: Joint 8

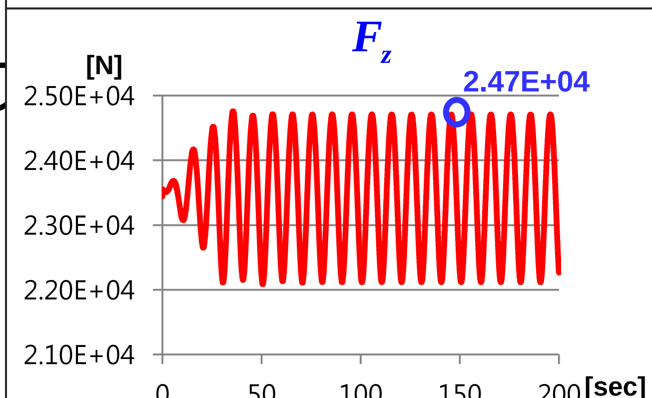


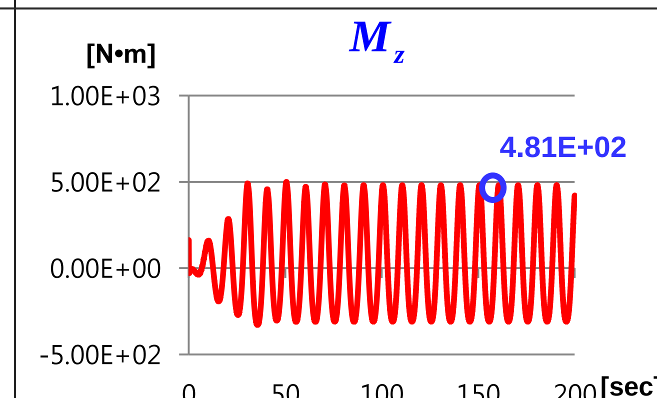
Wave	Amplitude	3 m
	Period	10 sec
	Angle	0 degree





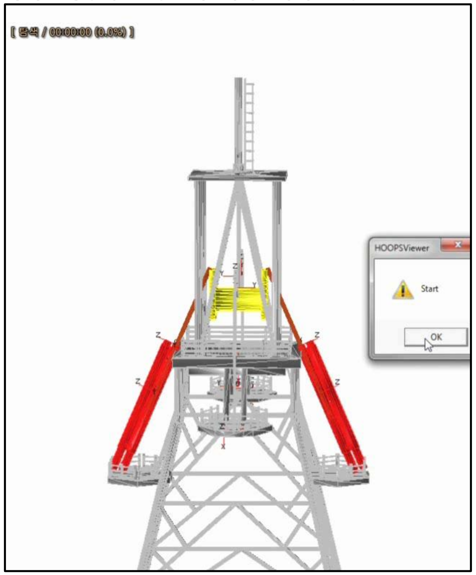




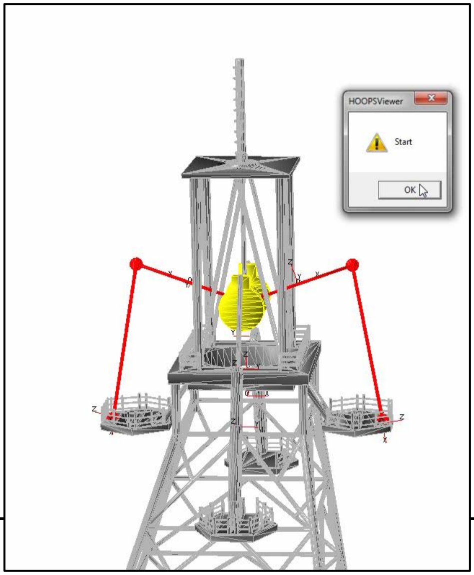


Comparison of Constraint Forces: F_Z of Joint 2 and Joint 8

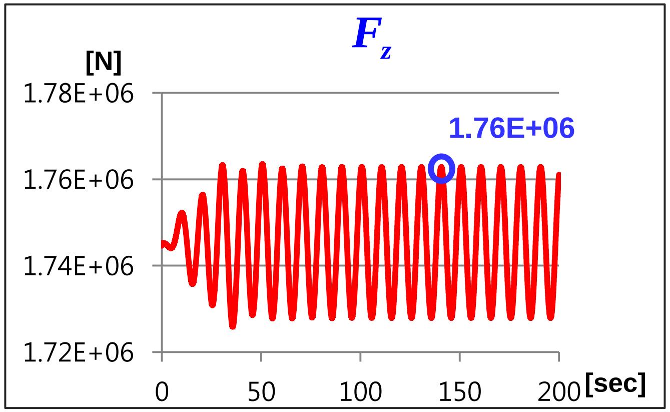
Constraint Force: Joint 2



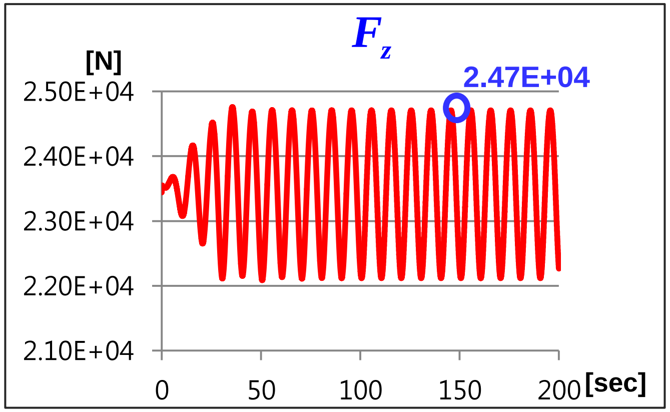
Constraint Force: Joint 8



Constraint Force: Joint 2



Constraint Force: Joint 8



Maximum dynamic constraint force of exerted on the joint 2: 1,760 kN
Maximum dynamic constraint force of exerted on the joint 8: 24.7 kN

→Constraint force of joint 2 is 70 times greater than that of the joint 8

18-7.
Risk Assessment of Heave Compensation System

Failure Mode, Effects, and Criticality Analysis(FMECA)

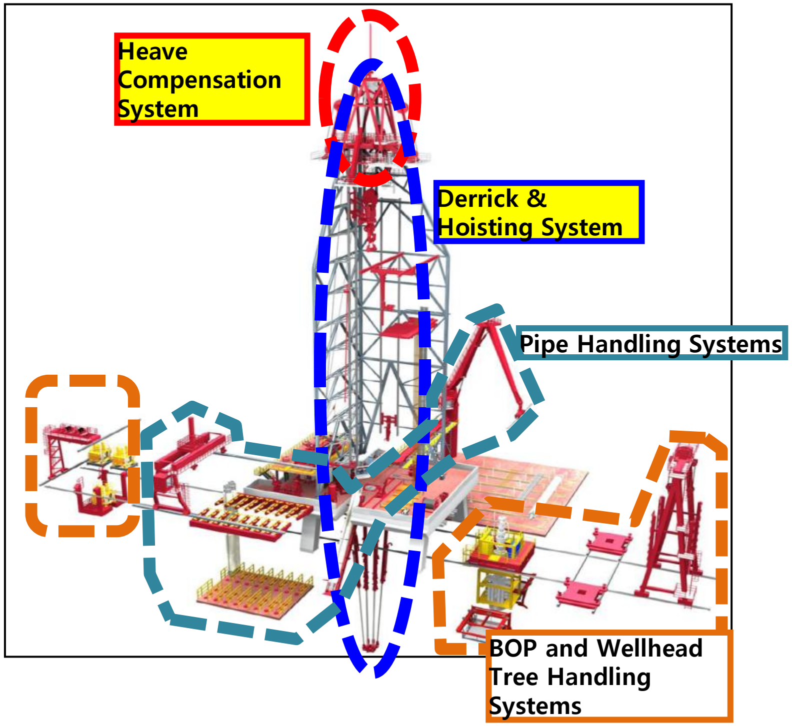
- **Definition:**
  FMECA is a is a technique used to identify, prioritize, and eliminate potential failures from the system, design or process before they reach the customer[1).

- **Scheme of the FMECA[2)]:**
  1. **Definition and delimitation of the system which components are within the boundaries of the system and which are outside.**
  2. **Definition of the main functions of the system.**
  3. **Description of the operational modes of the system.**
  4. **System breakdown into subsystems that can be handled effectively.**
  5. **Preparation of a complete component list for each subsystem.**
  6. **Description of the operational and environmental stresses that may affect the system and its operation. These are reviewed to determine the adverse effects that they could generate on the system and its components.**

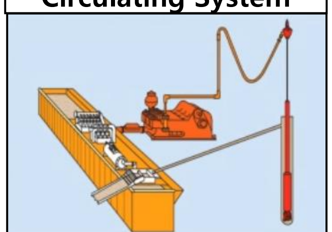
1) Omdahl, T.P., et al., Reliability, Availability and Maintainability (RAM) Dictionary, American Society for Quality Control (ASQC) Press, Milwaukee, WI, 1988
2) Hoyland, A., Rausand, M., System reliability theory : models and statistical methods, 2nd edition, John Wiley & Sons, 2004, pp.88-96

Failure Mode, Effects, and Criticality Analysis(FMECA)

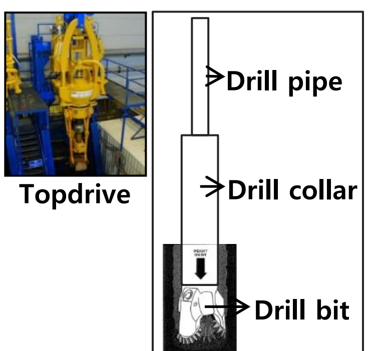
- Category of Components of Drilling System



Circulating System



Rotary & drilling system



Heave compensation systems	
Drill string compensator	HC100
Active heave compensator	HC200

Derrick & hosting system	
Derrick	DH100
Crown block	DH200
Traveling block	DH300
Deadline anchor	DH500
Drawwork	DH600
Drilling line	DH700

Pipe handling system	
Finger board	PH100
Elevator	PH200
Pipe racking arm	PH300
Stabbing board	PH400
Iron Roughneck	PH500
Carne (handler)	PH600
Power slip	PH700
Tong	PH800

Rotary & drilling system	
Drill pipe	RD100
Drill bit	RD200
Drill collar	RD300
Top drive	RD400

Mud system	
Mud pit	MD100
Agitator	MD200
Flow divider	MD300
Shale shaker	MD400
Degasser	MD500
Desander	MD600
Desilter	MD700
Centrifuge	MD800
Mud tank	MD900
Mud pump	MD1000
Stand pipe	MD1100
Rotary hose	MD1200
Trip tank	MD1300
Trip tank pump	MD1400
Diverter	MD1500

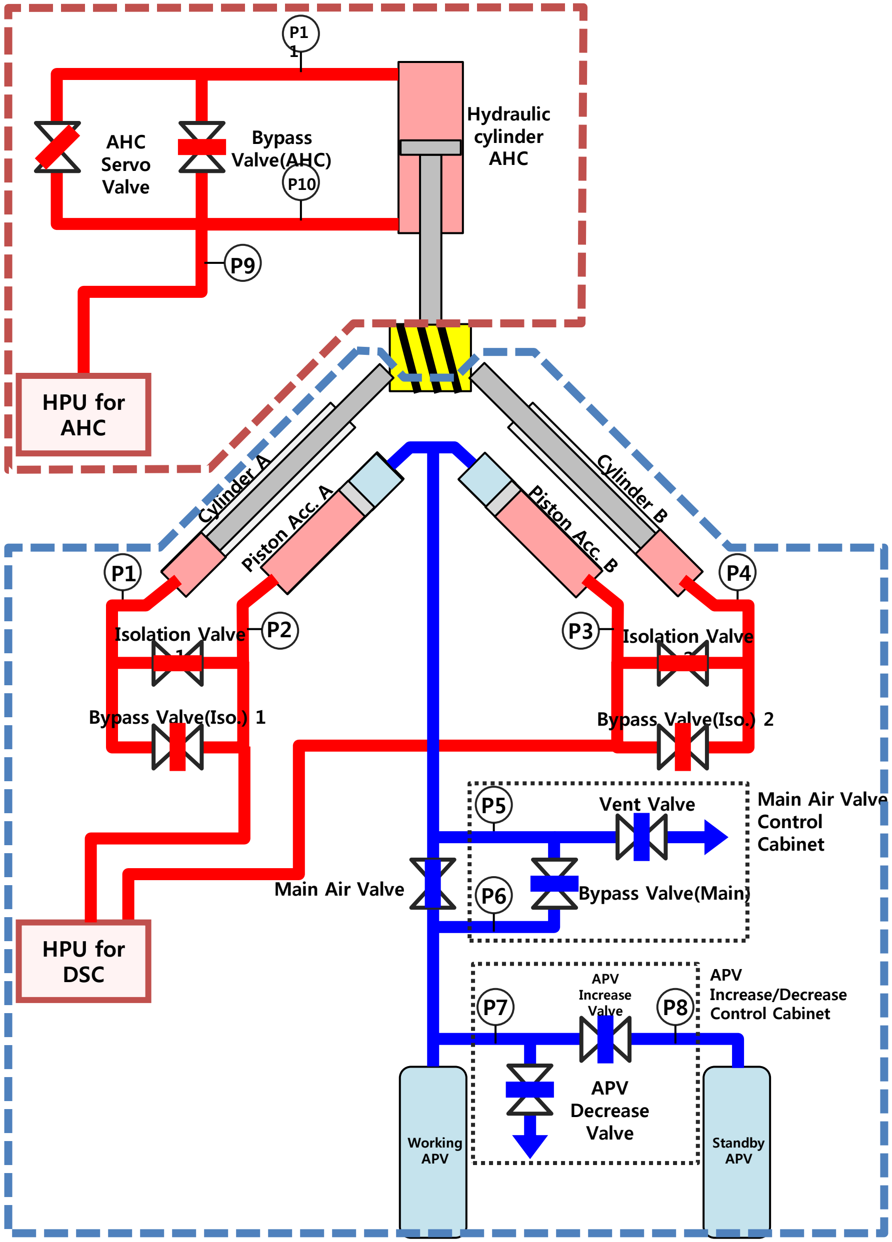
BOP and Wellhead tree Handling systems	
BOP	BW100
Wellhead tree	BW200
Crane	BW300

→ Level 1: 6 systems
→ Level 2: 38 components

Reference 1) 이예슬, FMECA of Offshore Drilling System, 해양플랜트 설계연구회 추계 워크샵, 2011

Failure Mode, Effects, and Criticality Analysis(FMECA)
- Category of Components of Heave Compensation Systems

Drill String Compensator
Active Heave Compensator
: Pneumatic
: Hydraulic
: Electric



Level 2:

Heave compensation systems	
Drill string compensator	HC100
Active heave compensator	HC200

Level 3:

Drill string compensator(DSC)	
Cylinder A	DSC100
Cylinder B	DSC200
Piston accumulator A	DSC300
Piston accumulator B	DSC400
Isolation valve 1	DSC500
Isolation valve 2	DSC600
Main air valve	DSC700
Vent valve	DSC800
Working APV	DSC900
APV increase valve	DSC1000
APV decrease valve	DSC1100
Standby APV(Air pressure Vessels)	DSC1200
HPU(Hydraulic Power Unit) for DSC	DSC1300

Active heave compensator(AHC)	
Hydraulic cylinder AHC	AHC100
AHC servo valve	AHC200
HPU for AHC	AHC300

Failure Mode, Effects, and Criticality Analysis(FMECA)

- FMECA Sheet (4)

Reference: 1) Hoyland, A., Rausand, M., System reliability theory : models and statistical methods, 2nd edition, John Wiley & Sons, 2004, pp.88-96

Description of item				Description of failure			Effect of failure		Failure rate ranking	Severity ranking	Detectability ranking	RPN
Ref. No	Item description	Function	Operational mode	Failure mode	Failure cause of mechanism	Detection of failure	On the subsystem	On the system				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
DSC900	Working APV	To provide the air spring volume to obtain a low "bit weight" variation	Downhole operation	The air spring volume to be provided is decreased and the pressure in the APV is also decreased.	Leak of the air in the APV due to damage	Pressure Measure ment	Fail to control the pressure in the piston accumulator.	Fail to obtain a low "bit weight" variation.				
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮				

(10)Failure rate ranking: The rank of the occurrence of the failure mode

(11) Severity ranking: The rank of the severity of the failure mode. The severity means the worst potential consequence of the failure, determined by the degree of injury, property damage, or system damage that could ultimately occur[1)].

(12) Detectability ranking: The rank of the likelihood the failure will be detected.

(13) Risk priority number(RPN) = Failure rate ranking(10) x Severity ranking(11) x Detectability ranking(12)

Using the dynamic analysis of the heave compensation system in the absence of the selected item, it can be measured how dose the selected item effects on the function of the system and sub system.

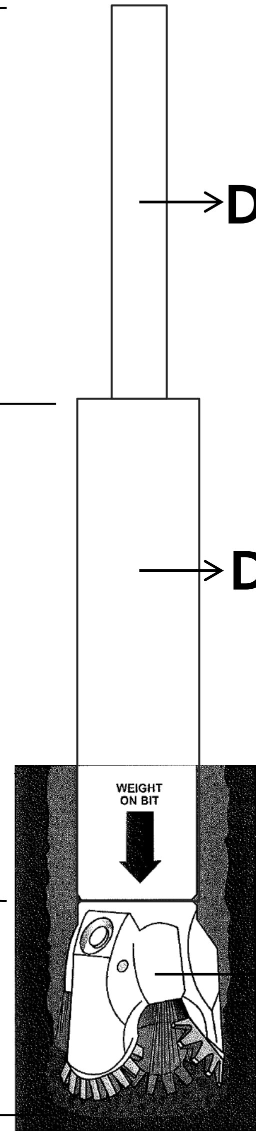
Supplementary Slide

Drill String

Drill String and Drill Bit

Drill string

The drill string can consist of many different tools. The simplest drill string consists of drill pipe and heavy-walled pipe called drill collars. Drill collars, like drill pipe, are steel tubes
which the drilling mud is pumped.



Drill pipe

Drill collar

Drill bit

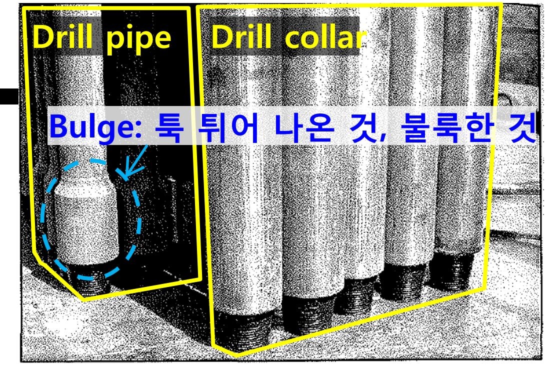


Figure 139. Drill collars racked in front of drill pipe on the rig floor

Drill pipe can be distinguished from drill collars because collars do not have the bulge at either end that characterizes the joints of drill pipe

Bottom hole assembly (BHA).

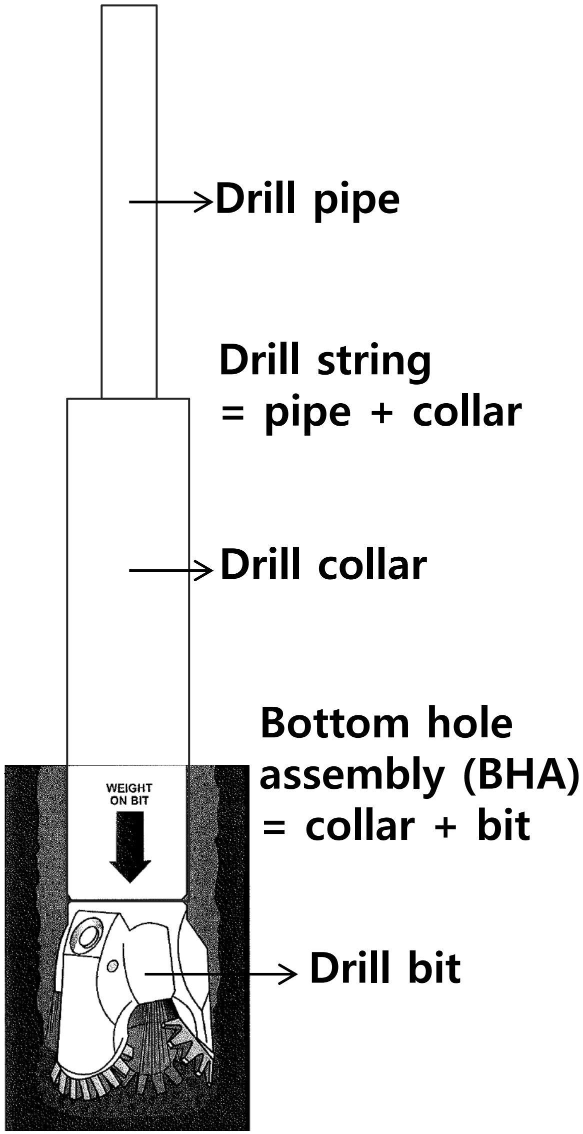
The drill collars, any other tools placed below drill pipe, and the bit are called the bottomhole assembly (BHA).

Drill collars are thicker than drill pipe and weigh more on pound-per-foot(kg/meter) basis than drill pipe. Drill collars are in the bottom part of the string and must be heavy

The drill bit is the cutting structure that mechanically breaks the rock at the bottom of the hole

Drill collars put weight
on the bit, which forces the bit mtters
into the fomzation to drill bit.

Drill String Design



Determination of length of bottom hole assembly (BHA) necessary for a desired weight on bit (WOB).

$$Length,\ ft = \frac{WOB \times f}{Wdc \times BF}$$

where
WOB = desired weight to be used while drilling
f = safety factor to place neutral point in drill collars
Wdc = drill collar weight, lb/ft
BF = buoyancy factor

Interpretation

$$\boxed{\boxed{Length,\ ft \times Wdc} \times BF} = WOB \times f$$

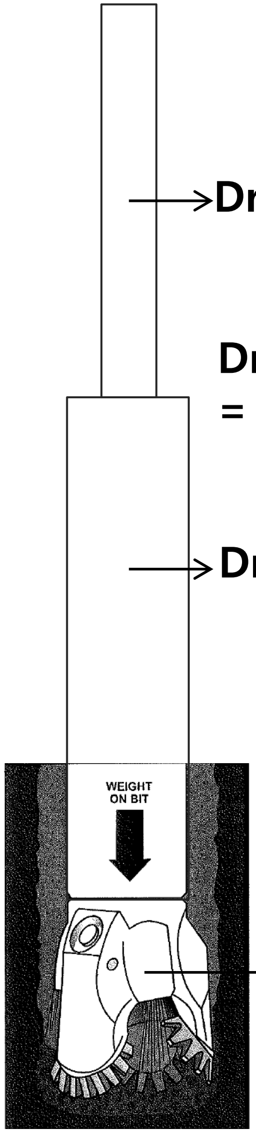
Drill collar weight in the air

Drill collar weight in the fluid(mud)

What is safety factor to place neutral point in drill collar
Don't we have to consider the weight of the drill pipe?

Reference) Lapeyrouse, N.J., Formulas and calculations for drilling, production, and workover, 2002

Drill String Design



Drill pipe

Drill string
= pipe + collar

Drill collar

Bottom hole
assembly (BHA)
= collar + bit

Drill bit

Determination of length of bottom hole assembly (BHA) necessary for a desired weight on bit (WOB).

$$Length,\ ft = \frac{WOB \times f}{Wdc \times BF}$$

where
WOB = desired weight to be used while drilling
f = safety factor to place neutral point in drill collars
Wdc = drill collar weight, lb/ft
BF = buoyancy factor

Example:
Desired WOB while drilling= 50,000lb
Safety factor = 15%
Mud weight = 12.0 ppg[1)]
Drill collar weight = $\boxed{\text{147 lb/ft}}$ (8 in. OD[2)] - 3 in. ID[3)])

This data can be confirm from "Drilling Data Handbook"

1) ppg: pound per gallon
2) OD: outer diameter
3) ID: inner diameter

Drill String Design

Drill collar weight = 147lb/ft (8 in. OD[2] - 3 in. ID[3])

1 lb/ft = 0.4536kg/0.3048m
=1.488kg/m

WEIGHT OF DRILL COLLARS (kg/m)

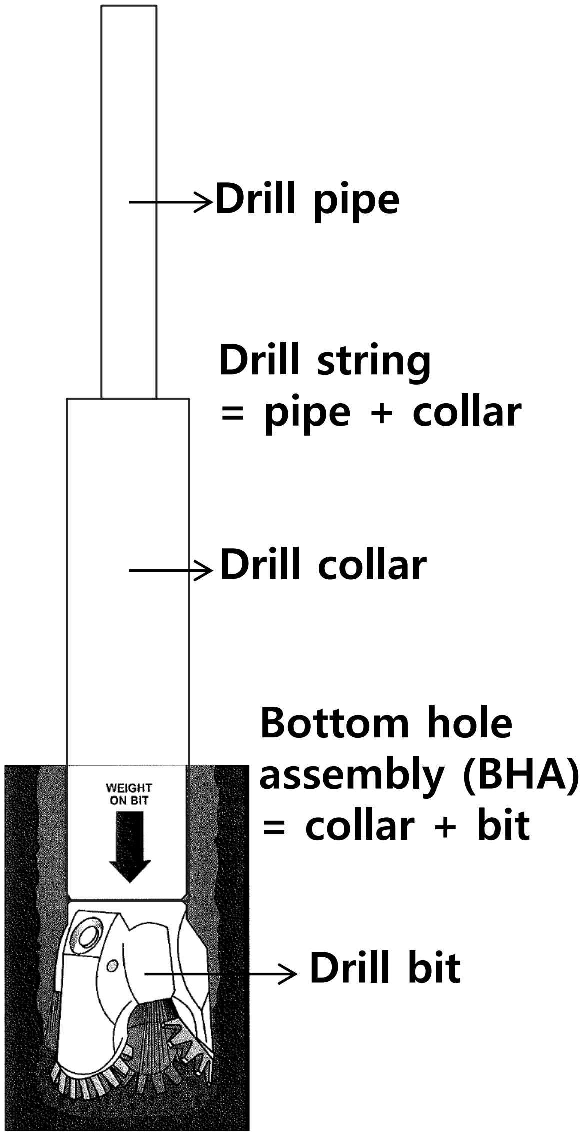
OD		Inside diameter (in and mm)														
		1	1 1/4	1 1/2	1 3/4	2	2 1/4	2 1/2	2 3/4	2 4/5	2 7/8	3	3 1/4	3 1/2	3 3/4	4
(in)	(mm)	25.40	31.75	38.10	44.45	50.80	57.15	63.50	69.86	71.44	73.03	76.20	82.55	88.90	95.25	101.60
2 7/8	73.03	28.90	26.66	23.93												
3	76.20	31.82	29.58	26.85												
3 1/8	79.38	34.87	32.63	29.89												
3 1/4	82.55	38.04	35.80	33.06												
3 1/2	88.90	44.75	42.51	39.78												
3 3/4	95.25	51.96	49.72	46.99	43.75											
4	101.60	59.66	57.43	54.69	51.46	47.73	43.51									
4 1/8	104.78	63.70	61.47	58.73	55.50	51.77	47.55									
4 1/4	107.95	67.87	65.63	62.90	59.66	55.94	51.71									
4 1/2	114.30	76.57	74.33	71.60	68.37	64.64	60.41									
4 3/4	120.65	85.77	83.53	80.80	77.56	73.84	69.61	64.89								
5	127.00	95.46		90.49	87.26	83.53	79.30	74.58	69.36							
5 1/4	133.35	105.66		100.68	97.45	93.72	89.50	84.77	79.55	78.17	76.76	73.84				
5 1/2	139.70	116.35		111.37	108.14	104.41	100.19	95.46	90.24	88.86	87.45	84.53				
5 3/4	146.05	127.53		122.56	119.33	115.60	111.37	106.65	101.43	100.05	98.63	95.71	89.50			
6	152.40	139.22		134.25	131.01	127.28	123.06	118.34	113.11	111.73	110.32	107.40	101.18	94.47	87.26	
6 1/4	158.75	151.40		146.43	143.20	139.47	135.24	130.52	125.30	123.91	122.50	119.58	113.36	106.65	99.44	
6 3/8	161.93	157.68		152.70	149.47	145.74	141.52	136.79	131.57	130.19	128.78	125.86	119.64	112.93	105.72	
6 1/2	165.10	164.08		159.11	155.87	152.15	147.92	143.20	137.97	136.59	135.18	132.26	126.04	119.33	112.12	
6 5/8	168.28	170.60		165.63	162.40	158.67	154.44	149.72	144.50	143.12	141.70	138.78	132.57	125.86	118.65	
6 3/4	171.45	177.25		172.28	169.05	165.32	161.09	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	132.51	125.30	117.59
7	177.80	190.93		185.96	182.72	178.99	174.77	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	146.18	138.97	131.26
7 1/4	184.15	205.10		200.13	196.89	193.16	188.94	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	160.35	153.14	145.43
7 1/2	190.50	219.77		214.79	211.56	207.83	203.61	198.88	193.66	192.28	190.87	[illegible]	181.73	175.02	167.81	160.10
7 3/4	196.85	234.93		229.96	226.73	223.00	218.77	214.05	208.83	207.44	206.03	[illegible]	196.89	190.18	182.97	175.27
8	203.20	250.59		245.62	242.39	238.66	234.43	229.71	224.49	223.11	221.69	218.77	212.56	205.84	198.63	190.93
8 1/4	209.55	266.75		261.78	258.55	254.82	250.59	245.87	240.65	239.27	237.85	234.93	228.72	222.00	214.79	207.09
8 1/2	215.90	283.41		278.44	275.20	271.47	267.25	262.53	257.30	255.92	254.51	251.59	245.37	238.66	231.45	223.74
8 3/4	222.25	300.56		295.59	292.36	288.63	284.40	279.68	274.46	273.08	271.66	268.74	262.53	255.81	248.60	240.90
9 1/4	234.95	336.36			328.16	324.43	320.20	315.48	310.26	308.87	307.46	304.54	298.32	291.61	284.40	276.70
9 1/2	241.30	355.01			346.80	343.07	338.85	334.12	328.90	327.52	326.11	323.18	316.97	310.26	303.05	295.34
9 3/4	247.65	374.15			365.94	362.22	357.99	353.27	348.04	346.66	345.25	342.33	336.11	329.40	322.19	314.48
10	254.00	393.79				381.85	377.63	372.91	367.68	366.30	364.89	361.97	355.75	349.04	341.83	334.12
10 1/2	266.70	434.56				422.63	418.40	413.68	408.46	407.07	405.66	402.74	396.52	389.81	382.60	374.89
10 3/4	273.05	455.69				443.76	439.53	434.81	429.59	428.20	426.79	423.87	417.65	410.94	403.73	396.03
11	279.40	477.32						456.44	451.22	449.83	448.42	445.50	439.28	432.57	425.36	417.65
11 1/4	285.75	499.44						478.56	473.34	471.96	470.54	467.62	461.41	454.70	447.49	439.78
12	304.80	568.80						547.92	542.70	541.32	539.90	536.98	530.77	524.06	516.85	509.14
14	355.60	775.64							749.54	748.16	746.74	743.82	737.61	730.89	723.68	715.98

218 kg/m = 147 lb/ft

Reference) Gabolde, G., Drilling Data Handbook, Editions Technip, 2006

Drill String Design

1 lb/ft = 0.4536kg/0.3048m
=1.488kg/m



Determination of length of bottom hole assembly (BHA) necessary for a desired weight on bit (WOB).

$$Length,\ ft = \frac{WOB \times f}{Wdc \times BF}$$

where
WOB = desired weight to be used while drilling
f = safety factor to place neutral point in drill collars
Wdc = drill collar weight, lb/ft
BF = buoyancy factor

Example:
Desired WOB while drilling= 50,000lb = 22,680kg
Safety factor = 15%
Mud weight = 12.0 ppg[1)]
Drill collar weight = 147 lb/ft (8 in. OD[2)] - 3 in. ID[3)])

1) ppg: pound per gallon
2) OD: outer diameter
3) ID: inner diameter

Solution:

a) Buoyancy factor (BF):

$$BF = \frac{65.5 - 12.0}{65.5} = 0.8168$$

This formula can be confirm from “Drilling Data Handbook”

Reference) Lapeyrouse, N.J., Formulas and calculations for drilling, production, and workover, 2002

Drill String Design

Buoyancy factor (BF):

Mud weight = 12.0 ppg[1)]

Steel density

$$BF = \frac{\boxed{65.5} - 12.0}{\boxed{65.5}} = 0.8168$$

Calculation of buoyed weight in one fluid

Buoyed weight = Weight in air - Buoyancy

$$\text{Buoyancy} = \text{Weight of displaced fluid}$$
$$= (\text{Volume of steel}) \cdot (\text{Fluid density})$$
$$= \frac{\text{Weight in air}}{\text{Steel density}} \cdot \text{Fluid density}$$

$$\text{Buoyed weight} = \text{Weight in air} - \frac{\text{Weight in air}}{\text{Steel density}} \cdot \text{Fluid density}$$
$$= \left(1 - \frac{\text{Fluid density}}{\text{Steel density}}\right) \cdot \text{Weight in air}$$

Let
Wa = Weight in air ; Wb = Buoyed Weight ;
ρ_S = Steel density, ρ_m = Fluid density.

$$\text{Wb} = \left(1 - \frac{\rho_m}{\rho_S}\right) \cdot \text{Wa} = \left(\frac{\rho_S - \rho_m}{\rho_S}\right) \cdot \text{Wa} = \text{BF} \cdot \text{Wa}$$

$$\text{Buoyancy Factor} = \text{BF} = \left(\frac{\rho_S - \rho_m}{\rho_S}\right)$$

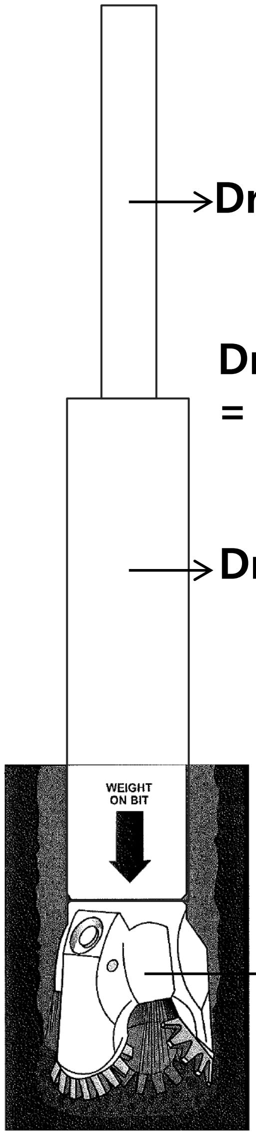
ppg

BUOYANCY FACTOR
(Steel density ρ_s = 7.85 kg/l) = 65.5 ppg

Mud density			BF	Mud density			BF
(kg/l)	(lb/gal)	(lb/ft³)	ρ_s = 7.85 kg/l	(kg/l)	(lb/gal)	(lb/ft³)	ρ_s = 7.85 kg/l
1.00	8.35	62.4	0.873	1.62	13.52	101.1	0.794
1.02	8.51	63.7	0.870	1.64	13.69	102.4	0.791
1.04	8.68	64.9	0.868	1.66	13.85	103.6	0.789
1.06	8.85	66.2	0.865	1.68	14.02	104.9	0.786
1.08	9.01	67.4	0.862	1.70	14.19	106.1	0.783
1.10	9.18	68.7	0.860	1.72	14.35	107.4	0.781
1.12	9.35	69.9	0.857	1.74	14.52	108.6	0.778
1.14	9.51	71.2	0.855	1.76	14.69	109.9	0.776
1.16	9.68	72.4	0.852	1.78	14.85	111.1	0.773
1.18	9.85	73.7	0.850	1.80	15.02	112.4	0.771
1.20	10.01	74.9	0.847	1.82	15.19	113.6	0.768
1.22	10.18	76.2	0.845	1.84	15.36	114.9	0.766
1.24	10.35	77.4	0.842	1.86	15.52	116.1	0.763
1.26	10.51	78.7	0.839	1.88	15.69	117.4	0.761
1.28	10.68	79.9	0.837	1.90	15.86	118.6	0.758
1.30	10.85	81.2	0.834	1.92	16.02	119.9	0.755
1.32	11.02	82.4	0.832	1.94	16.19	121.1	0.753
1.34	11.18	83.7	0.829	1.96	16.36	122.4	0.750
1.36	11.35	84.9	0.827	1.98	16.52	123.6	0.748
1.38	11.52	86.2	0.824	2.00	16.69	124.9	0.745
1.40	11.68	87.4	0.822	2.02	16.86	126.1	0.743
1.42	11.85	88.6	0.819	2.04	17.02	127.4	0.740
1.44	12.02	89.9	0.817	2.06	17.19	128.6	0.738
1.46	12.18	91.1	0.814	2.08	17.36	129.8	0.735
1.48	12.35	92.4	0.811	2.10	17.52	131.1	0.732
1.50	12.52	93.6	0.809	2.12	17.69	132.3	0.730
1.52	12.68	94.9	0.806	2.14	17.86	133.6	0.727
1.54	12.85	96.1	0.804	2.16	18.03	134.8	0.725
1.56	13.02	97.4	0.801	2.18	18.19	136.1	0.722
1.58	13.19	98.6	0.799	2.20	18.36	137.3	0.720
1.60	13.35	99.9	0.796	2.22	18.53	138.6	0.717

Reference) Gabolde, G., Drilling Data Handbook, Editions Technip, 2006

Drill String Design



→Drill pipe

Drill string
= pipe + collar

→Drill collar

Bottom hole
assembly (BHA)
= collar + bit

→Drill bit

Determination of length of bottom hole assembly (BHA) necessary for a desired weight on bit (WOB).

$$Length,\ ft = \frac{WOB \times f}{Wdc \times BF}$$

where
WOB = desired weight to be used while drilling
f = safety factor to place neutral point in drill collars
Wdc = drill collar weight, lb/ft
BF = buoyancy factor

Example:
Desired WOB while drilling= 50,000lb = 22,680kg
Safety factor = 15%
Mud weight = 12.0 ppg[1)]
Drill collar weight = 147 lb/ft (8 in. OD[2)] - 3 in. ID[3)])

1) ppg: pound per gallo
2) OD: outer diameter
3) ID: inner diameter

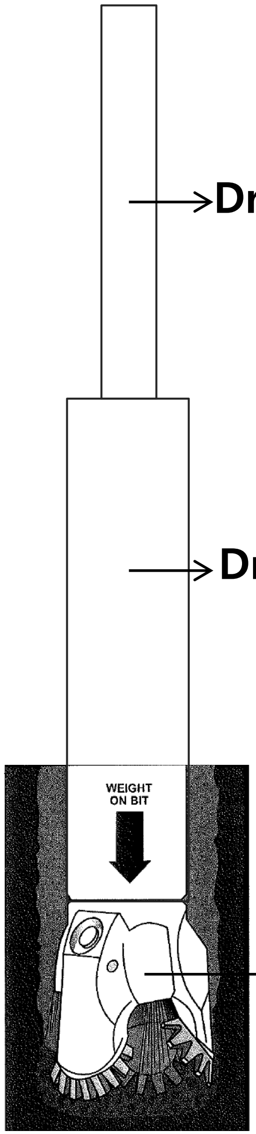
Solution:
a) Buoyancy factor (BF):

$$BF = \frac{65.5 - 12.0}{65.5} = 0.8168$$

b) Length of bottom hole assembly necessary:

$$Length,\ ft = \frac{WOB \times f}{Wdc \times BF}$$
$$= \frac{50,000 \times 1.15}{147 \times 0.8168} = 479\ ft \approx 146m$$

Drill String Design



Feet of drill pipe that can be used with a specific bottom hole assembly (BHA)

a) Determine buoyancy factor (BF): $BF = \frac{65.5 - mud\ weight,\ ppg}{65.5}$

b) Determine maximum length of drill pipe that can be run into the hole with a specific bottom hole assembly:

$$\text{Length}_{MAX} = \frac{[(\text{T} \times \text{f}) - \text{MOP} - \text{Wbha}] \times \text{BF}}{\text{Wdp}}$$

, where
T = tensile strength, lb for new pipe
f = safety factor to correct new pipe to no. 2 pipe
MOP = margin of overpull
Wbha = BHA weight in air, lb/ft
Wdp = drill pipe weight in air, lb/ft, including tool joint
BF = buoyancy factor

Interpretation

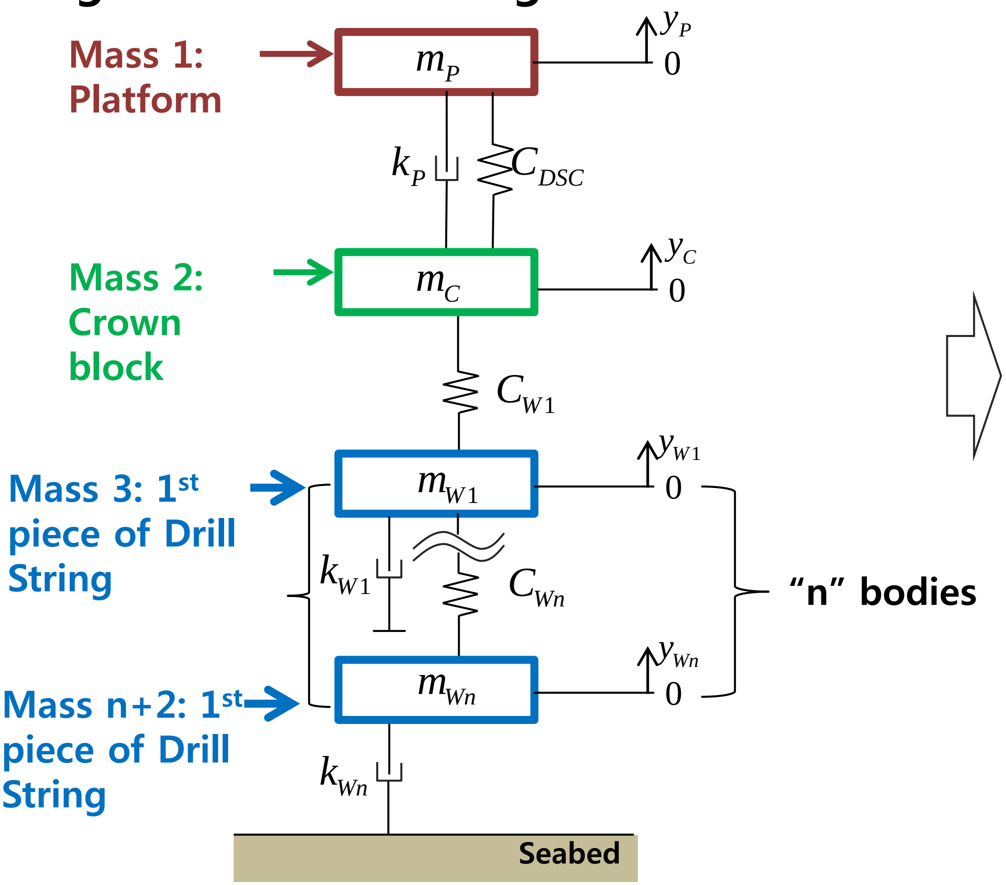
$$\text{Length}_{MAX} \times \text{Wdp} = [(\text{T} \times \text{f}) - \text{MOP} - \text{Wbha}] \times \text{BF}$$

$$\text{Length}_{MAX} \times \text{Wdp} + \text{MOP} \times \text{BF} + \text{Wbha} \times \text{BF} = \text{T} \times \text{f} \times \text{BF}$$

Reference) Lapeyrouse, N.J., Formulas and calculations for drilling, production, and workover, 2002

Effects of the Number of the Drill String
- Modeling of the Weight on Bit

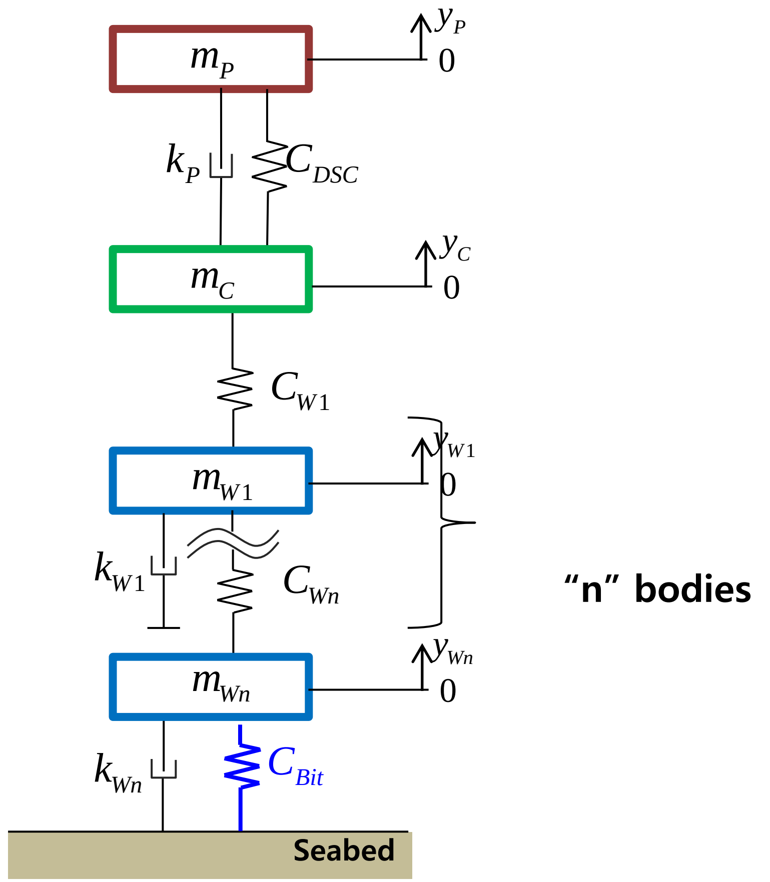
For the modeling of the weight on bit, a spring is added between the n^th segment of drill string and seabed.



$$C_{W1}=C_{W2}=\cdots=C_{Wn}=n\cdot C_W$$

$$k_{W1}=k_{W2}=\cdots=k_{Wn}=\frac{1}{n}\cdot k_W$$

$$m_{W1}=m_{W2}=\cdots=m_{Wn}=\frac{1}{n}\cdot m_W$$



$$C_{W1}=C_{W2}=\cdots=C_{Wn}=n\cdot C_W$$

$$k_{W1}=k_{W2}=\cdots=k_{Wn}=\frac{1}{n}\cdot k_W$$

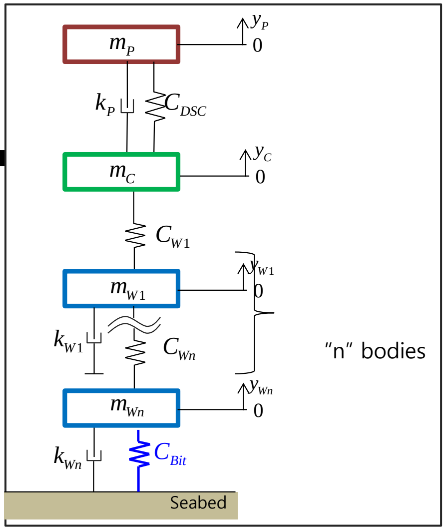
$$m_{W1}=m_{W2}=\cdots=m_{Wn}=\frac{1}{n}\cdot m_W$$

Effects of the Number of the Drill String
- Modeling of the Weight on Bit

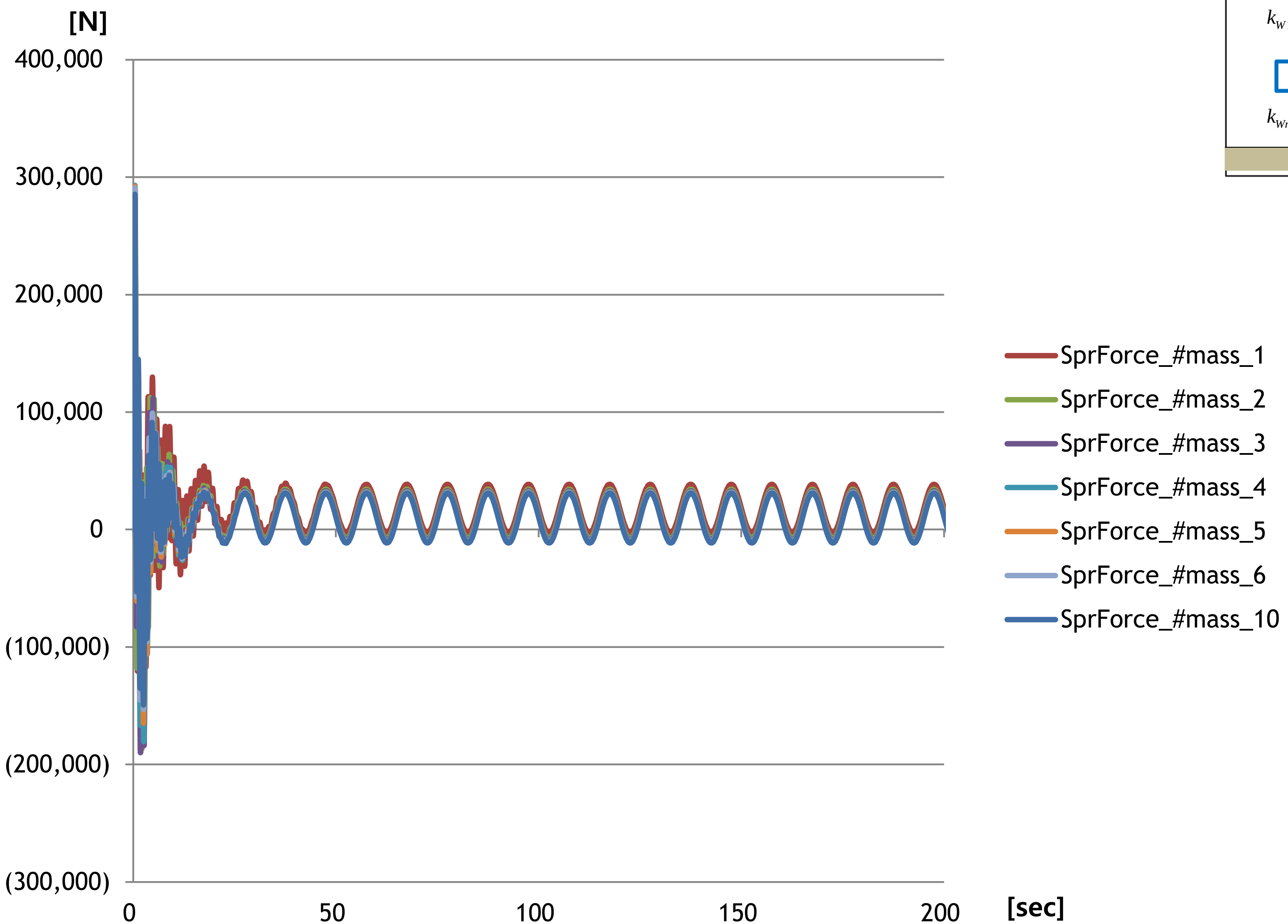
$$C_{W1} = C_{W2} = \cdots = C_{Wn} = n \cdot C_W$$

$$k_{W1} = k_{W2} = \cdots = k_{Wn} = \frac{1}{n} \cdot k_W$$

$$m_{W1} = m_{W2} = \cdots = m_{Wn} = \frac{1}{n} \cdot m_W$$

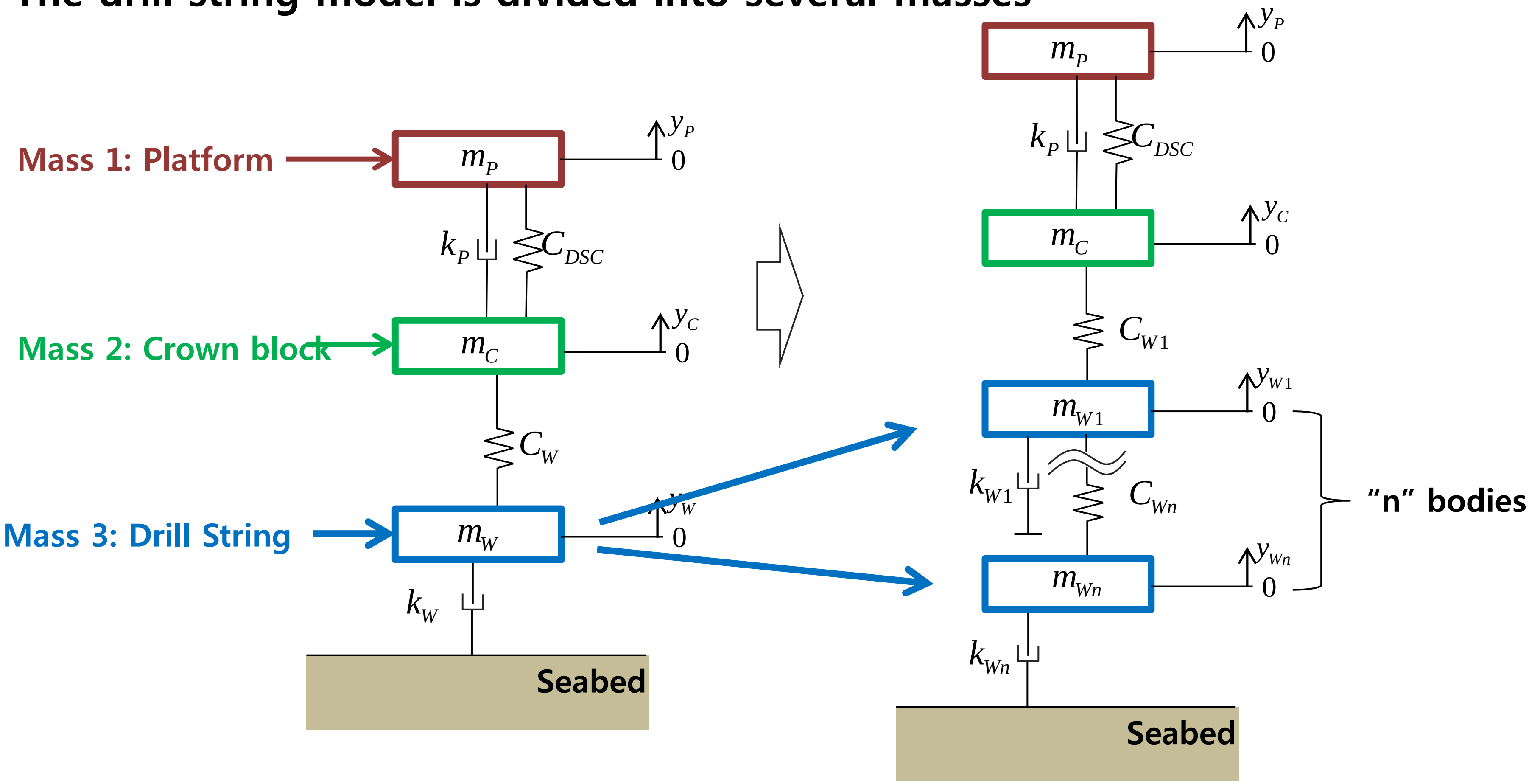


Dynamic response of the crown block
- Drill string compensator is applied to the system
- The amplitude of the platform's heave motion is 1m



Effects of the Number of the Drill String

The drill string model is divided into several masses



$$C_{W1} = C_{W2} = \cdots = C_{Wn} = n \cdot C_W$$

$$k_{W1} = k_{W2} = \cdots = k_{Wn} = \frac{1}{n} \cdot k_W$$

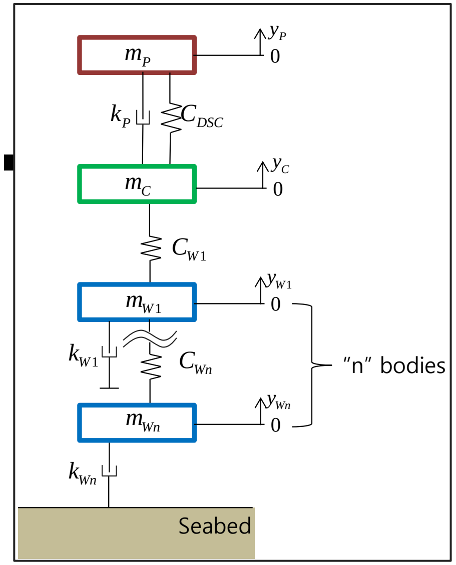
$$m_{W1} = m_{W2} = \cdots = m_{Wn} = \frac{1}{n} \cdot m_W$$

Effects of the Number of the Drill String

$$C_{W1} = C_{W2} = \cdots = C_{Wn} = n \cdot C_W$$

$$k_{W1} = k_{W2} = \cdots = k_{Wn} = \frac{1}{n} \cdot k_W$$

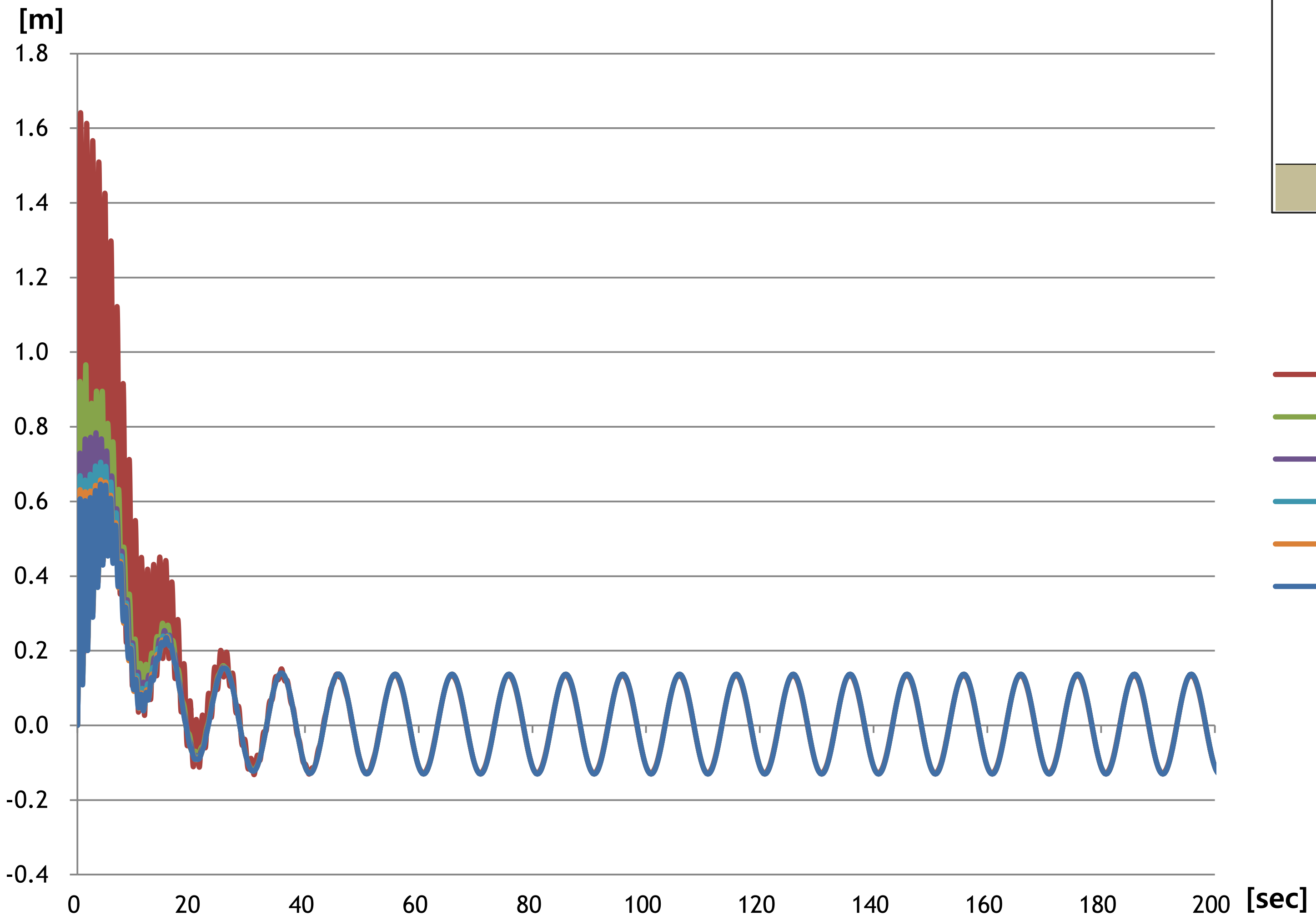
$$m_{W1} = m_{W2} = \cdots = m_{Wn} = \frac{1}{n} \cdot m_W$$



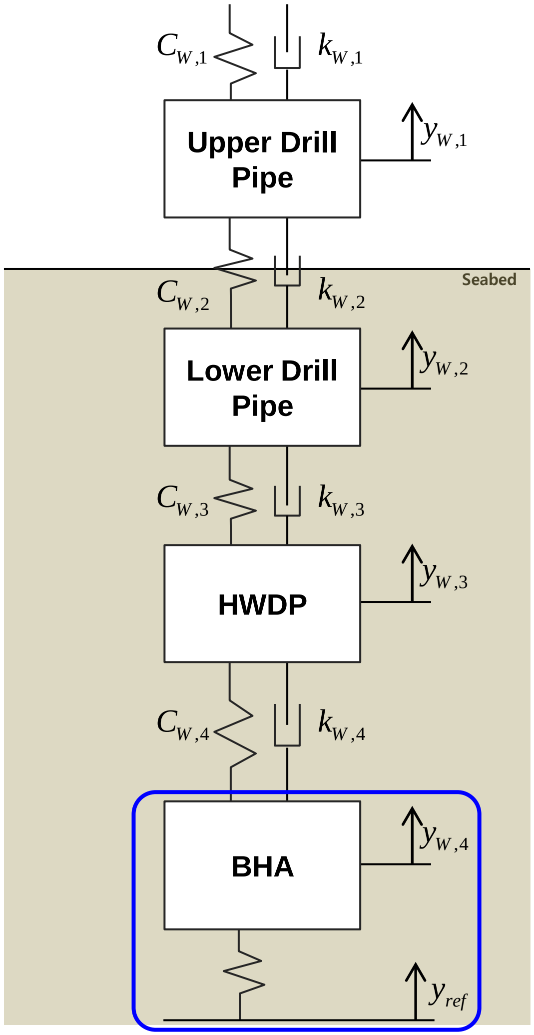
Dynamic response of the crown block

- Drill string compensator is applied to the system

- The amplitude of the platform's heave motion is 1m



Mathematical Modeling of Drill String
- Replace the Drill String with Equivalent Mass, Spring, and Damper



When the drill bit is in contact with the formation, there is an upward force which relates to the compliance of the bottom formation.
Since there is not any corresponding downwards force when the drill bit is lifted clear of the bottom, the equation of the load variation at the drill bit on the bottom formation is written by

$$-\frac{k_5}{2}\left[\left(y_5 - y_{ref}\right) - \left|\left(y_5 - y_{ref}\right)\right|\right]$$

,where

$k_5 = 500\,[\mathrm{kN/m}]$ **on soft formation**

$k_5 = 1,600\,[\mathrm{kN/m}]$ **on hard formation**

$k_5 = 5,000\,[\mathrm{kN/m}]$ **on very hard formation**

Reference 1) Hatleskog, J.T., Dunnigan, M.W., Passive Compensator Load Variation for Deep-Water Drilling, Journal of Oceanic Engineering, Vol.32, No.3,

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