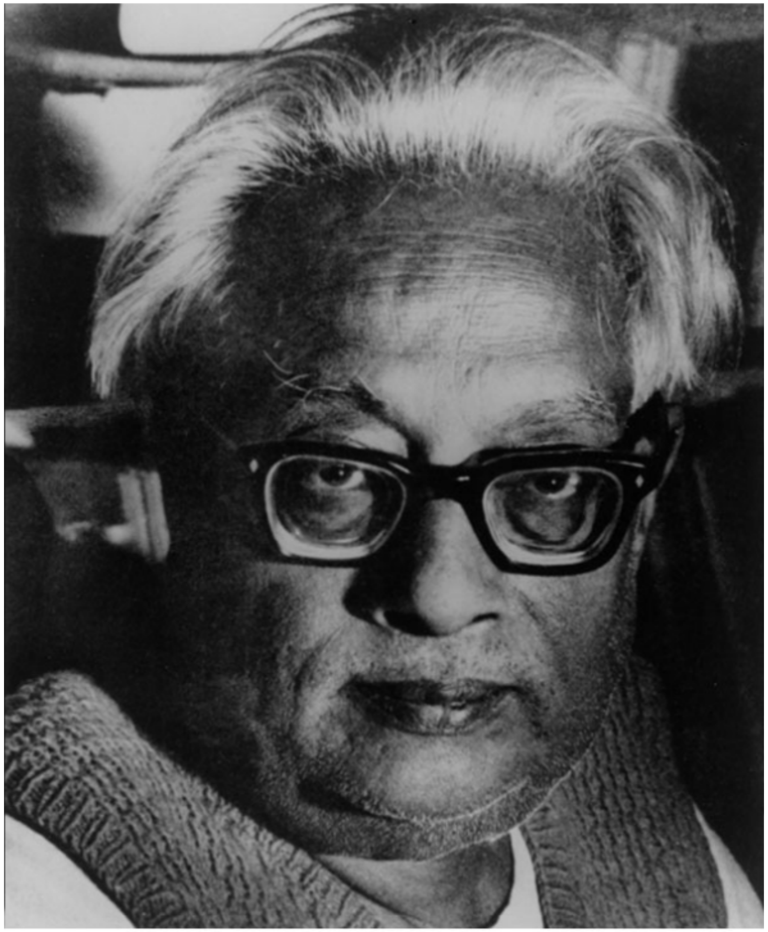
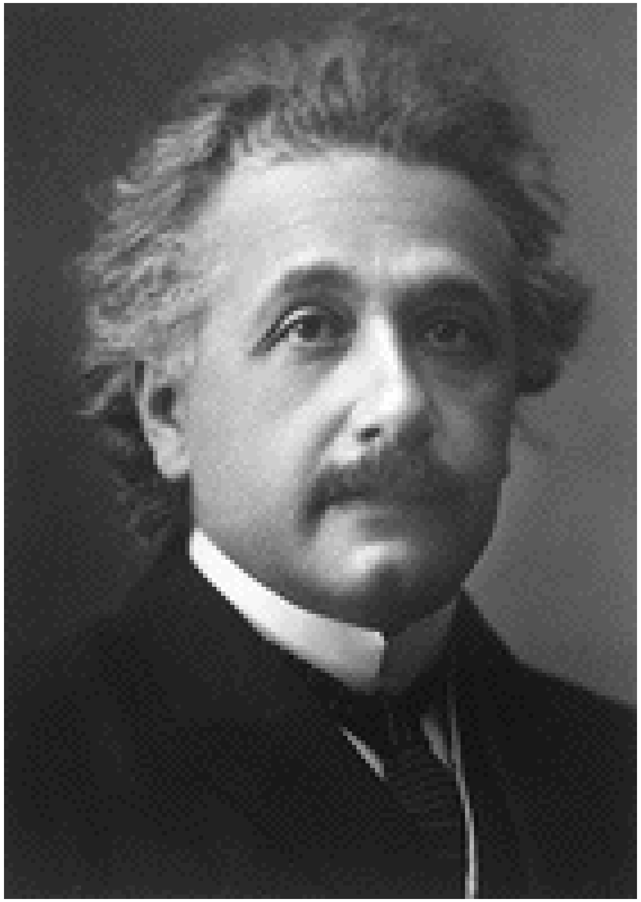
Bose



Satyendranath Bose
(1894-1974)

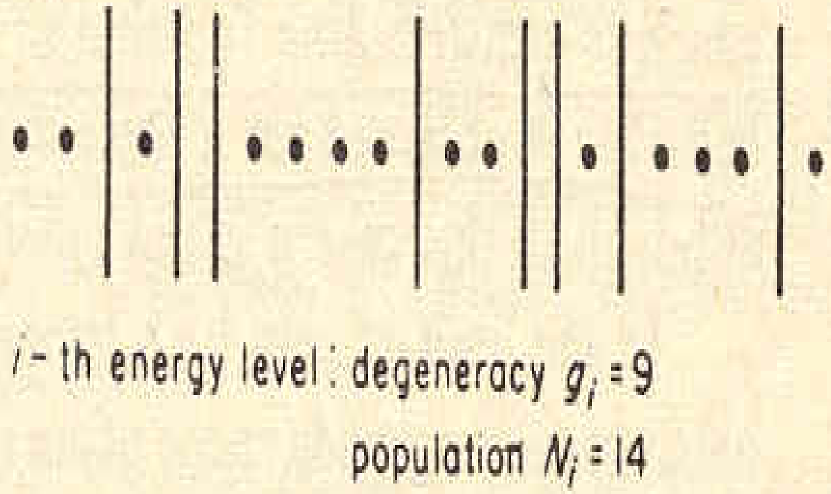
Einstein



Albert Einstein
(1879-1955)

Bose-Einstein Distribution

FIGURE 5.13. A possible distribution of particles among quantum states in the ith energy level of a system to which the Pauli exclusion principle does not apply.



Bose-Einstein Distribution

$$N = N_1 + N_2 + \cdots + N_n$$

$$U = E_1 N_1 + E_2 N_2 + \cdots + E_n N_n$$

$$Q(N_1, N_2, \cdots, N_n) = \prod_{i=1}^{n} \frac{\left(N_i + g_i - 1\right)!}{N_i!\left(g_i - 1\right)!}$$

Bose-Einstein Distribution

Lagrange's multiplier method

$$\frac{\partial \ln Q}{\partial N_i} + \alpha \frac{\partial f}{\partial N_i} + \beta \frac{\partial h}{\partial N_i} = 0$$

Stirling's approximation

$$\ln\left(n!\right) \approx n \ln n - n \quad \text{for } n >> 1$$

$$\ln\left(N_i + g_i - 1\right) - \ln N_i + \alpha + \beta E_i = 0$$

$$f_{BE}\left(E_i\right) = \frac{N_i}{g_i} = \frac{1}{e^{-\alpha} e^{-\beta E_i} - 1} \qquad \beta = -\frac{1}{k_B T}$$

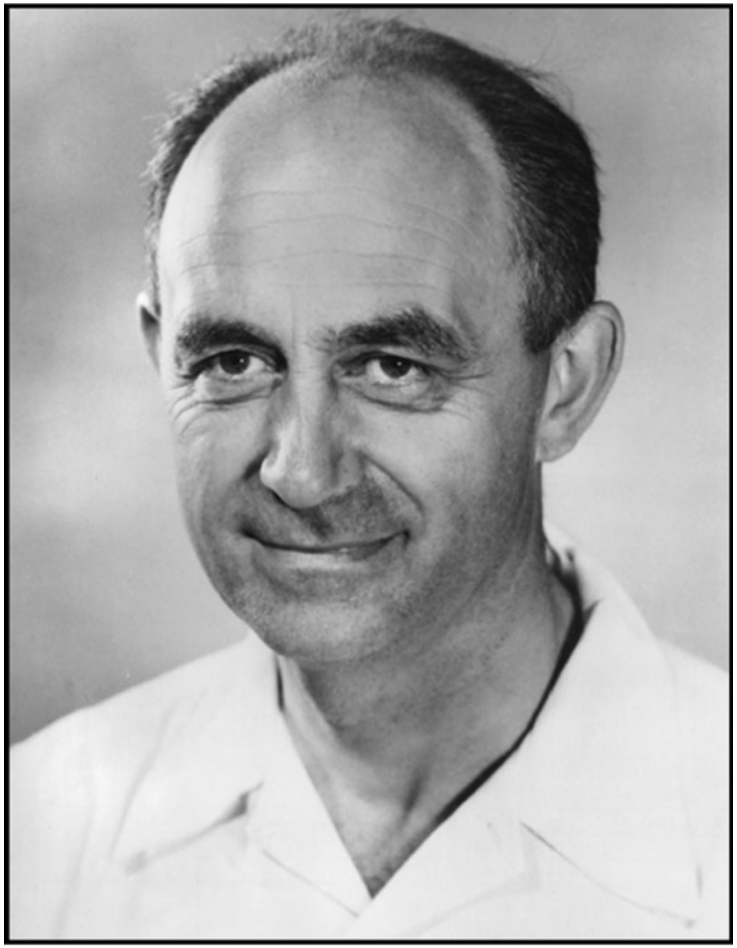
Bose-Einstein Distribution

$$f_{BE}\left(E\right)=\frac{1}{Be^{E_i/k_BT}-1} \qquad \left(\frac{N}{V}\right)_{bosons}=\int_0^{\infty} g(E) f_{BE}\left(E\right) dE$$

If there is no constraint of the conservation of partile number, then,

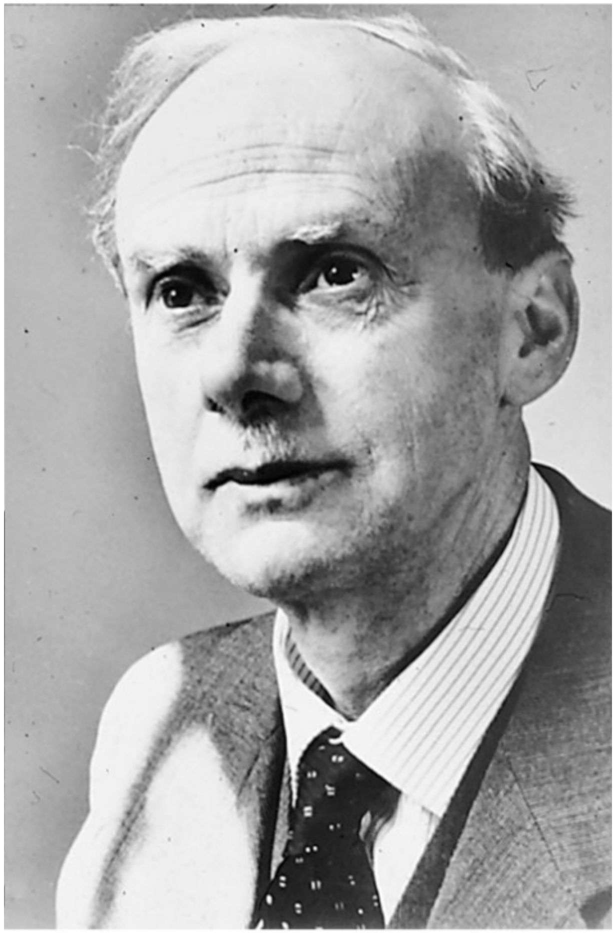
$$f_{BE}\left(E\right)=\frac{1}{e^{E_i/k_BT}-1}$$ (for photons or phonons)

Fermi



Enrico Fermi
(1901-1954)

Dirac



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Paul Adrien Maurice Dirac
(1902-1984)

Fermi-Dirac Distribution

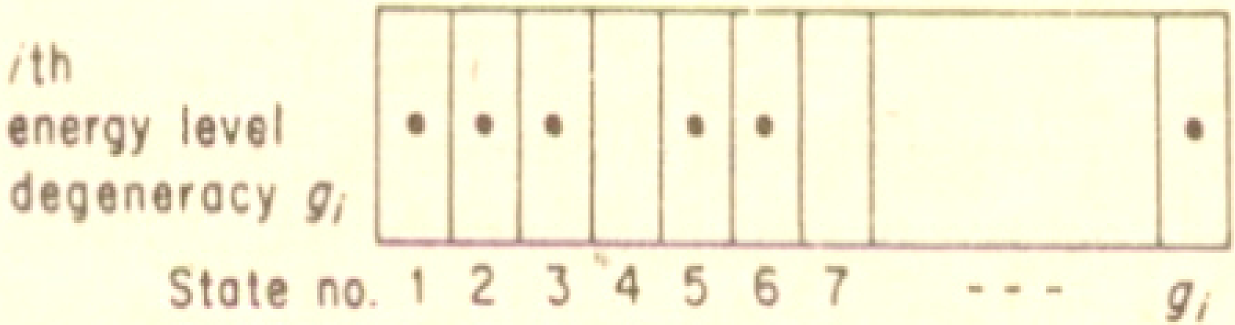


FIGURE 5.8. A possible distribution of particles among quantum states in the *i*th energy level of a system wherein the Pauli exclusion principle is applicable.

Fermi-Dirac Distribution

$$N = N_1 + N_2 + \cdots + N_n$$

$$U = E_1 N_1 + E_2 N_2 + \cdots + E_n N_n$$

$$Q(N_1, N_2, \cdots, N_n) = \prod_{i=1}^{n} \frac{g_i\,!}{N_i\,!\left(g_i - N_i\right)!}$$

Fermi-Dirac Distribution

$$-\ln N_i + \ln\left(g_i - N_i\right) + \alpha + \beta E_i = 0$$

$$f_{FD}\left(E_i\right) = \frac{N_i}{g_i} = \frac{1}{e^{-\alpha} e^{-\beta E_i} + 1} \qquad \beta = -\frac{1}{k_B T}$$

$$f_{FD}\left(E\right) = \frac{1}{e^{\left(E - E_F\right)/k_B T} + 1}$$

E_F : Fermi energy, Fermi level, chemical potential

$$\left(\frac{N}{V}\right)_{fermions} = \int_0^\infty g(E) f_{FD}(E) dE$$

Comparison

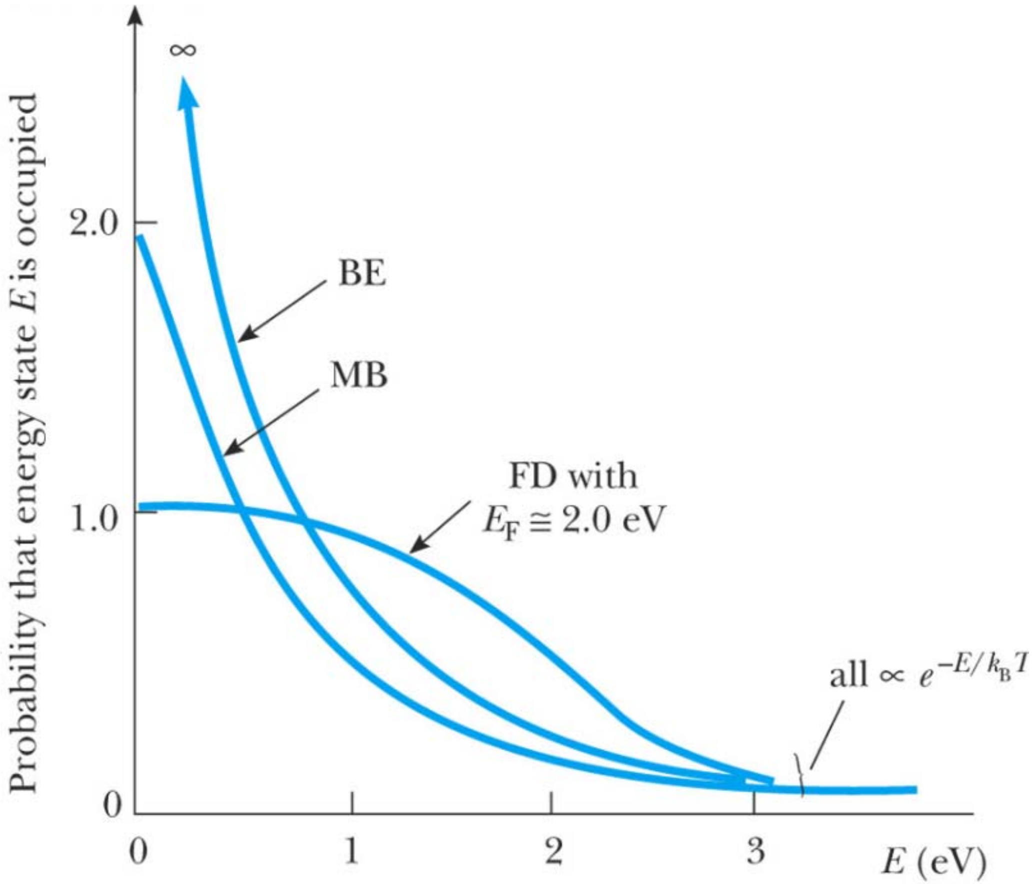


Fig. 10-8, p.350

Example of BE Distribution
Cavity Modes I

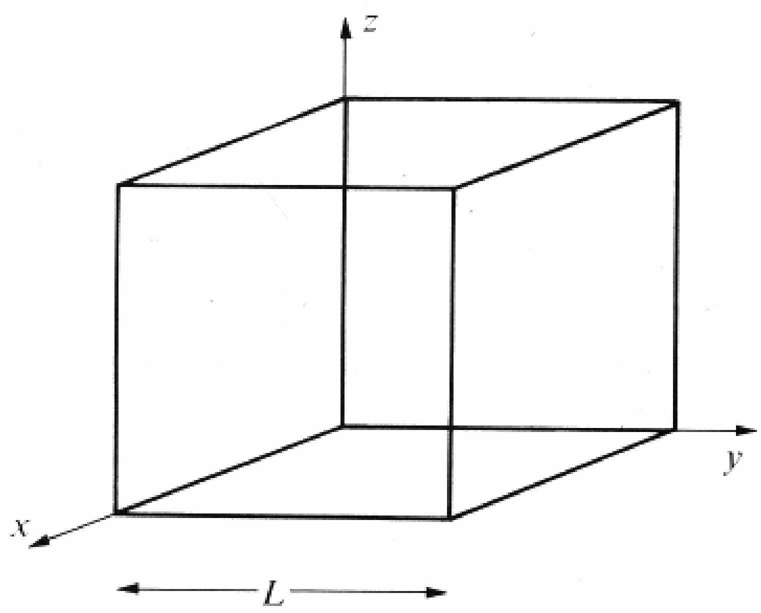
$$\nabla^2 \mathbf{E}(\mathbf{r},t) = \frac{1}{c^2}\frac{\partial^2 \mathbf{E}(\mathbf{r},t)}{\partial t^2}$$

$$\nabla \cdot \mathbf{E}(\mathbf{r},t) = 0 \qquad k = \frac{2\pi}{\lambda} = \frac{2\pi f}{c} = \frac{\omega}{c}$$

$$E_x(\mathbf{r},t) = E_x(t)\cos(k_x x)\sin(k_y y)\sin(k_z z)$$

$$E_y(\mathbf{r},t) = E_y(t)\sin(k_x x)\cos(k_y y)\sin(k_z z)$$

$$E_z(\mathbf{r},t) = E_z(t)\sin(k_x x)\sin(k_y y)\cos(k_z z)$$

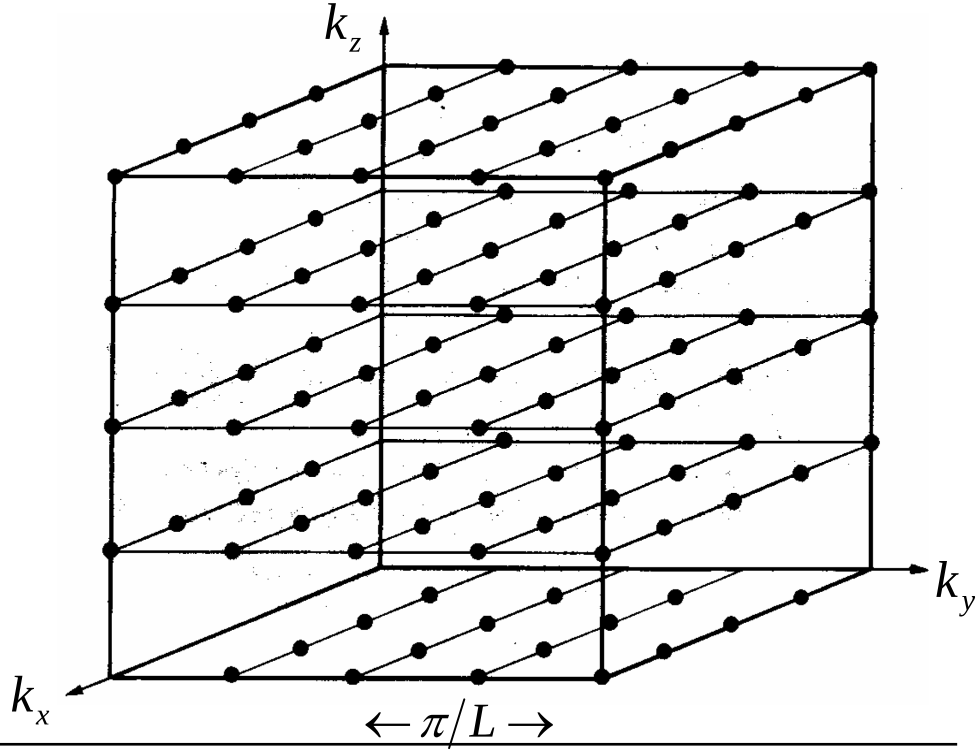


Cavity Modes II

$$k_x = \pi n_x / L, \quad k_y = \pi n_y / L, \quad k_z = \pi n_z / L$$

$$n_x, n_y, n_z = 0, 1, 2, 3, \ldots$$

$$\mathbf{k} \cdot \mathbf{E}(t) = 0$$



Mode Density

$$\frac{\frac{1}{8}\left(4\pi k^2 dk\right)}{\left(\pi/L\right)^3}\times 2$$

$$\rho(k)dk = k^2\, dk/\pi^2$$

$$f = \frac{ck}{2\pi}$$

$$\rho(f)df = \rho(k)dk$$

$$\rho(f) = \rho(k)\frac{dk}{df} = \frac{k^2}{\pi^2}\frac{2\pi}{c} = \frac{8\pi f^2}{c^3}$$

Planck's Blackbody Radiation Law

$$u(f,T) = \frac{8\pi h f^3}{c^3} \cdot \frac{1}{\exp\left(\dfrac{hf}{k_B T}\right) - 1}$$

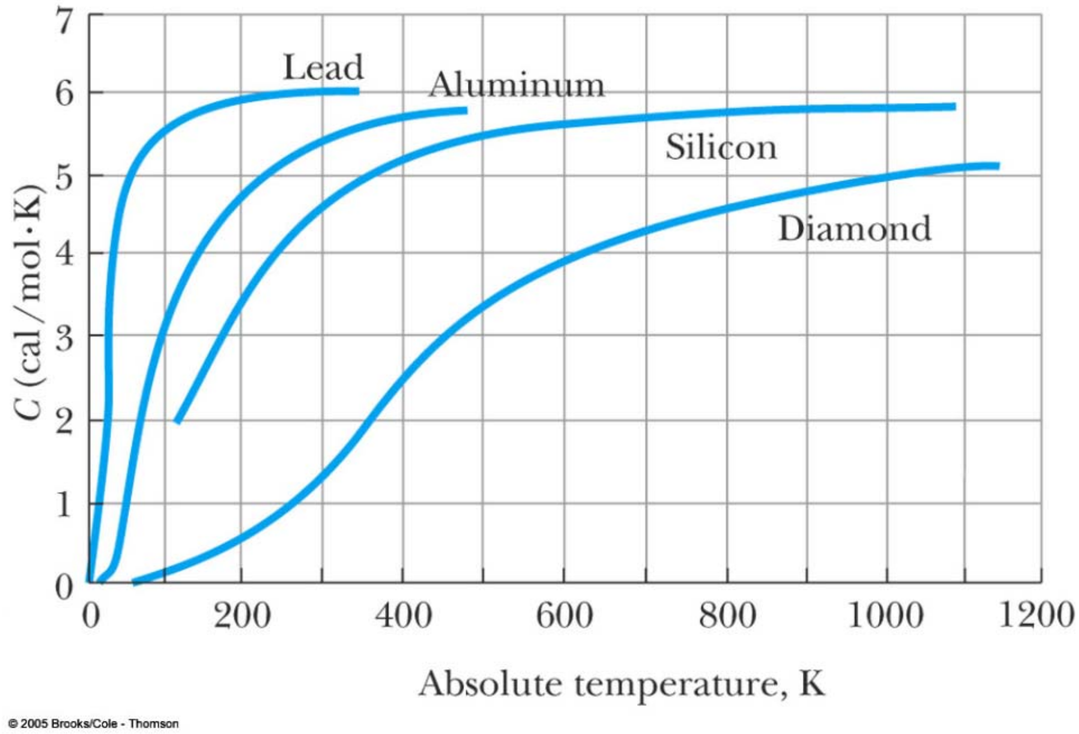
$$h = 6.626 \times 10^{-34}\ \mathrm{J \cdot s}$$ Planck's constant

$$\hbar = \frac{h}{2\pi}$$

After All... Due to Einstein

$$u(f,T) = \rho(f)\langle n\rangle hf = \frac{8\pi f^2}{c^3}\cdot\frac{1}{\exp\left(hf/k_B T\right)-1}\cdot hf$$

Specific Heat of Solids



$$C = \frac{dU}{dT}$$

$$U = 3N_A k_B T = 3RT$$

$$C = \frac{d(3RT)}{dT} = 3R$$

Fig. 10-9, p.353

Einstein's Theory of Specific Heat of Solids

$$\bar{E} = \frac{\hbar\omega}{e^{\hbar\omega/k_B T} - 1}$$

$$U = 3N_A\bar{E}$$

$$C = \frac{dU}{dT} = 3R\left(\frac{\hbar\omega}{k_B T}\right)^2 \frac{e^{\hbar\omega/k_B T}}{\left(e^{\hbar\omega/k_B T} - 1\right)^2}$$

Debye's Theory of Specific Heat of Solids

Coupled harmonic oscillators

Phonons

Fermi-Dirac Distribution

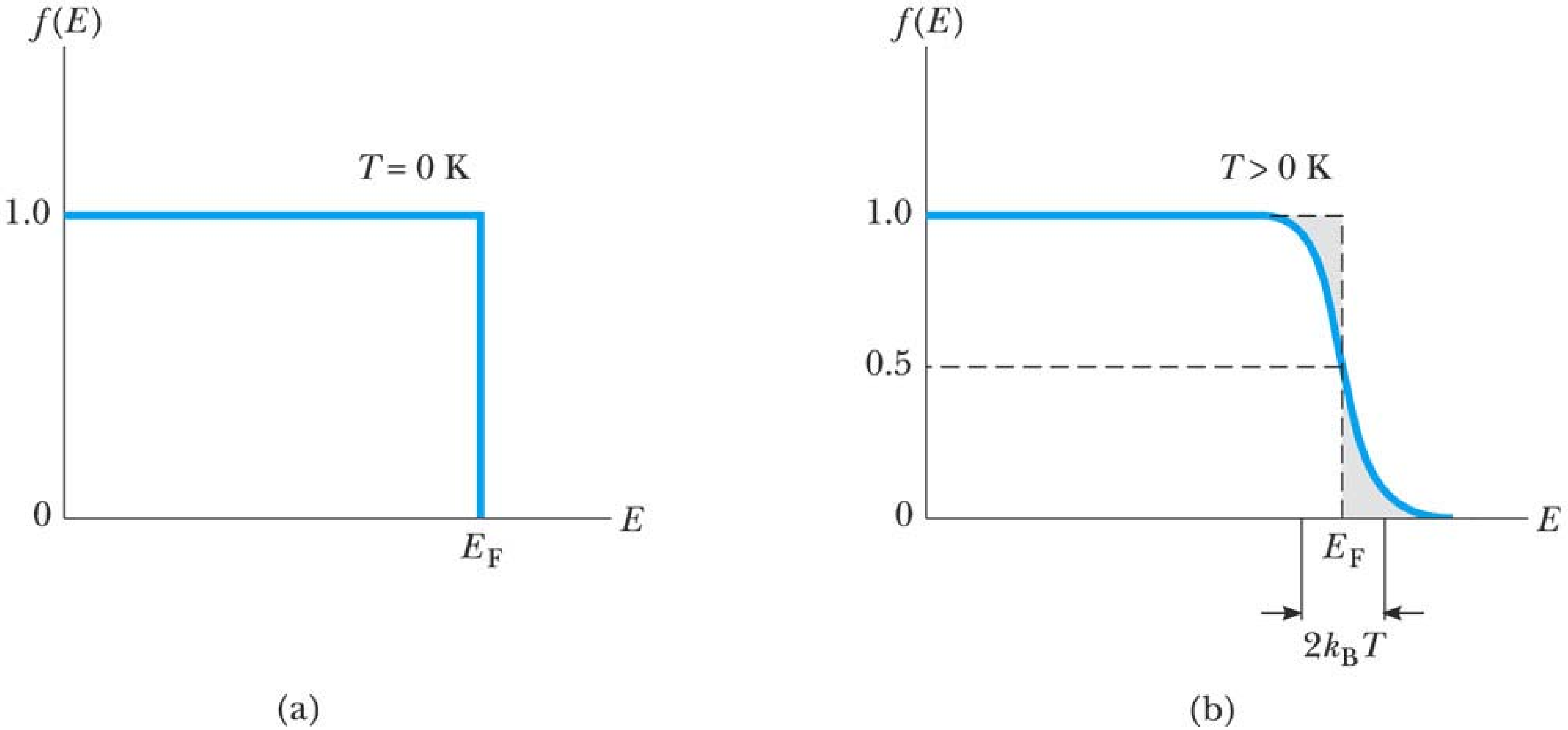


Fig. 10-11, p.357

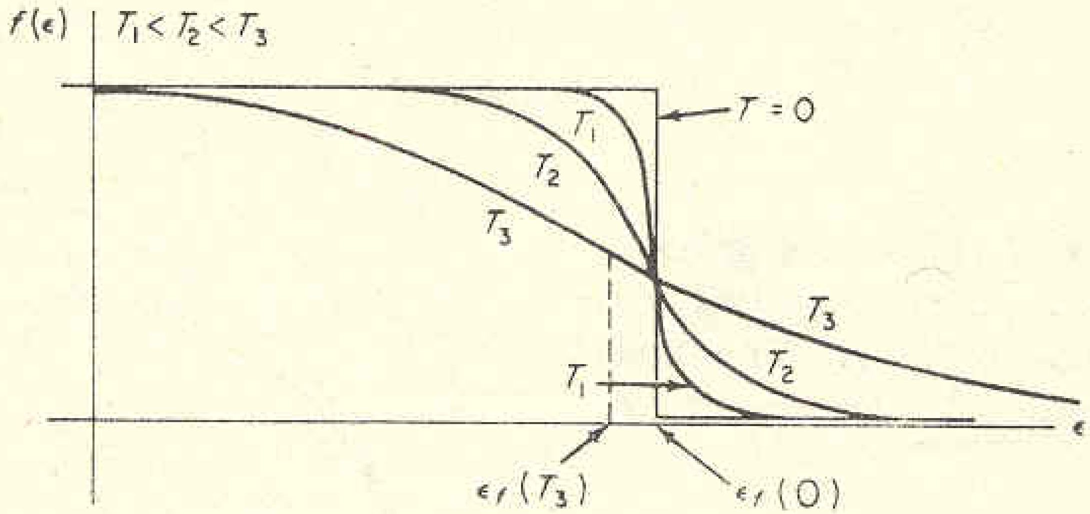


FIGURE 5.10. Schematic representation of the Fermi distribution function for four different temperatures. Note the variation of the Fermi energy with temperature. The temperature dependence of the Fermi energy depicted here is typical of a three-dimensional free-electron gas, but the actual variation in any particular system will depend critically upon the density of states function (or level degeneracies) for that system.

Fermi-Dirac Distribution in 3D Structures

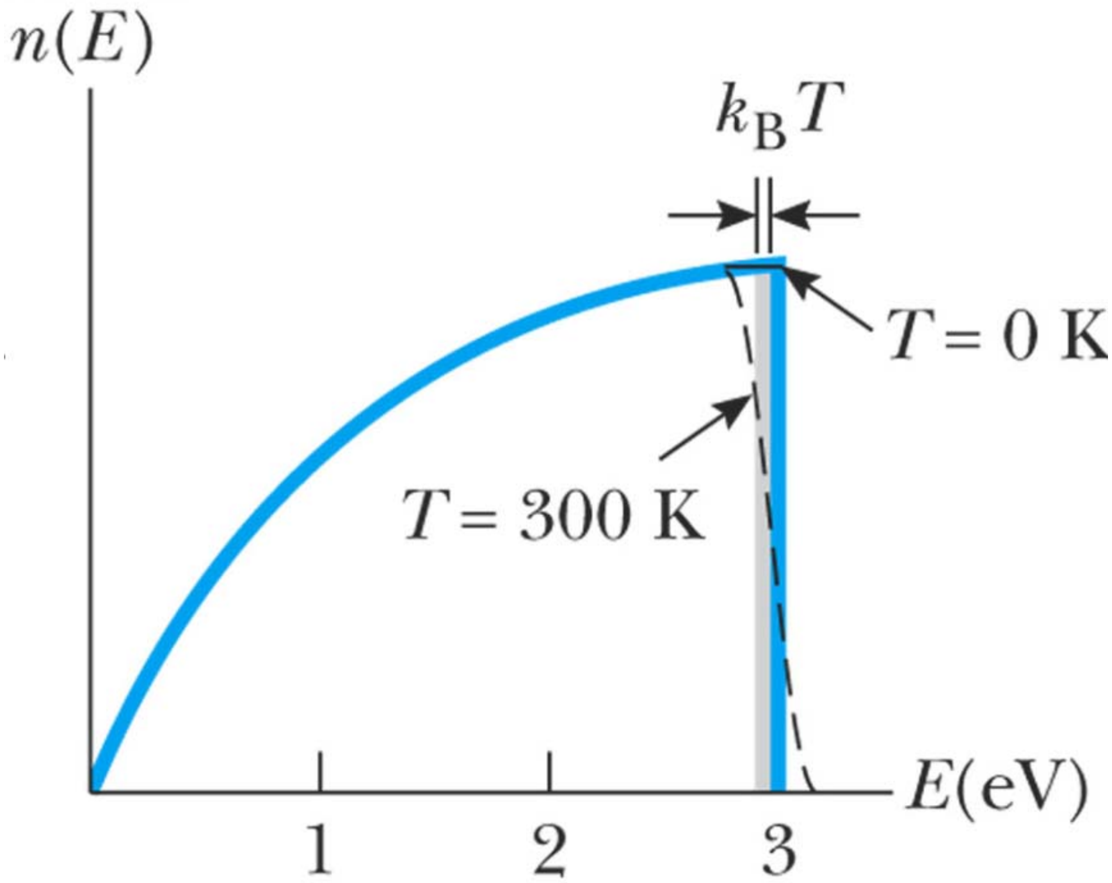
$$g(E)dE = D\sqrt{E}\,dE$$

$$D = \frac{8\sqrt{2}\pi m_e^{3/2}}{h^3}$$

$$n(E)dE = \frac{D\sqrt{E}\,dE}{e^{(E-E_F)/k_BT}+1}$$

$$\frac{N}{V} = \int_0^\infty n(E)dE = D\int_0^\infty \frac{\sqrt{E}\,dE}{e^{(E-E_F)/k_BT}+1}$$

Fermi Energy



At $T = 0K$

$$\frac{N}{V} = D\int_0^{E_F} \sqrt{E}\,dE = \frac{2}{3}DE_F^{3/2}$$

$$E_F(0) = \frac{h^2}{2m_e}\left(\frac{3N}{8\pi V}\right)^{2/3}$$

Fig. 10-12, p.358

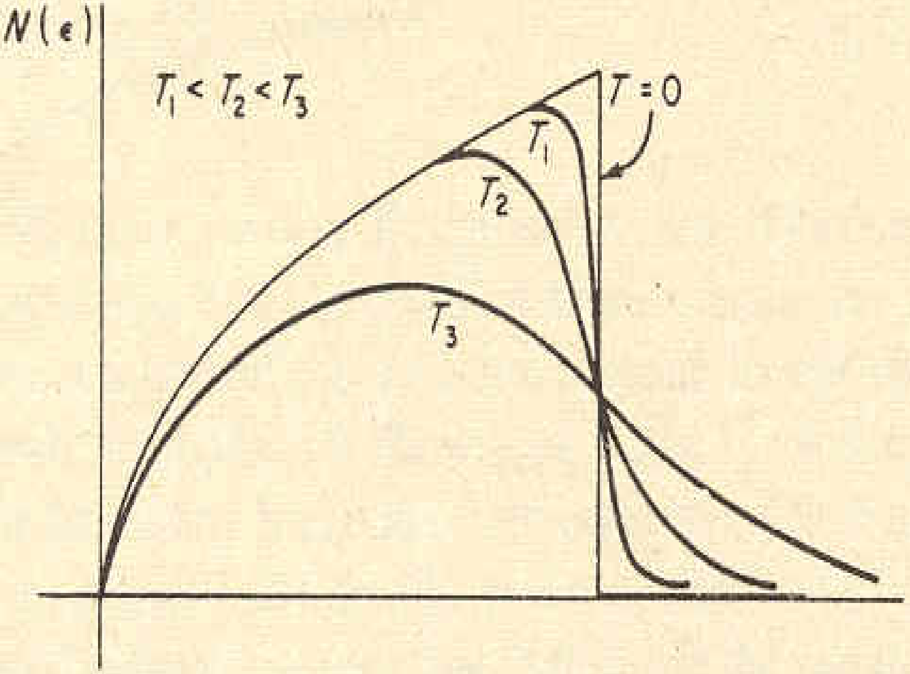


FIGURE 5.11. Schematic representation of electron density as a function of energy for a three-dimensional free-electron gas.