

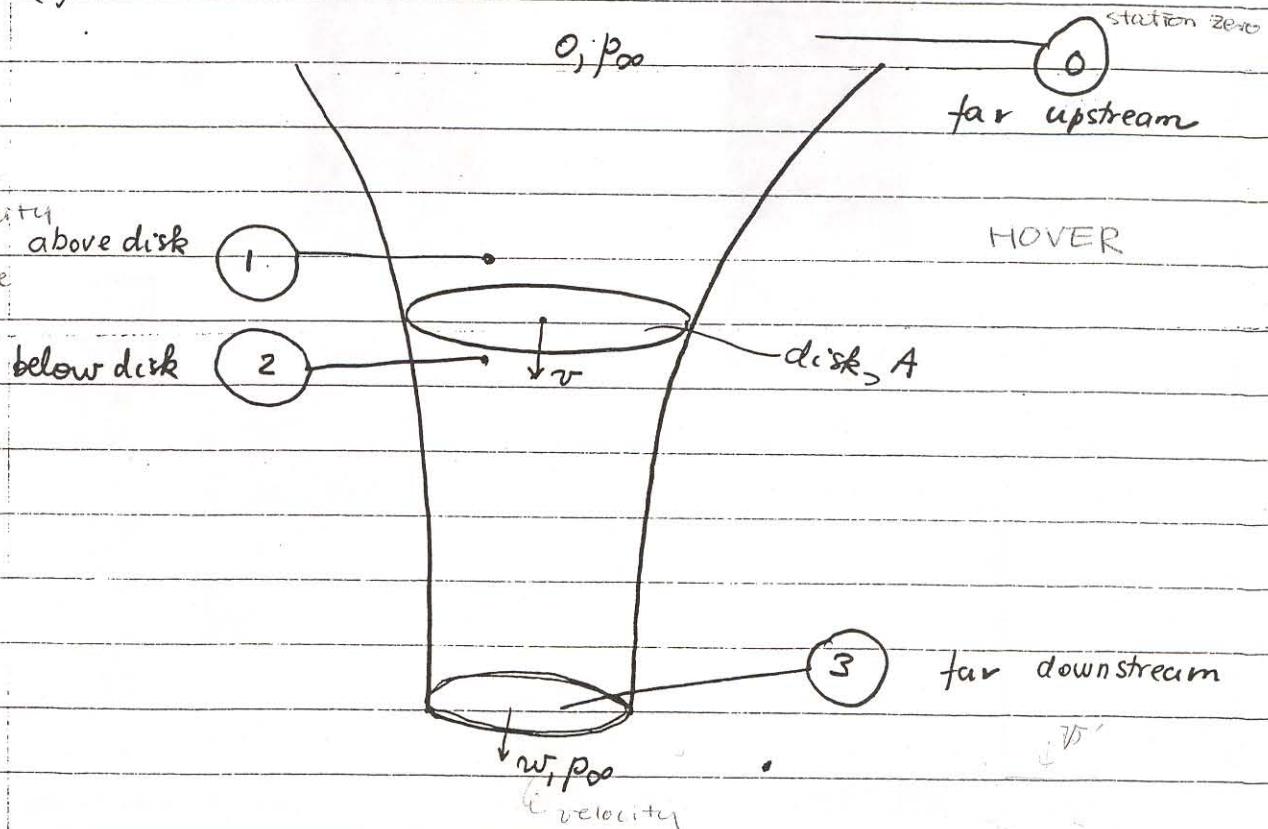
Introduction to Rotary Wing Aerodynamics

(1)

1. Momentum Theory of Rotors (Actuator Disk Theory)

Assumptions :

- (1) Low Disk loading
- (2) No rotational effects (No swirl effect)
- (3) Incompressible flow (Inflow velocity is uniform over the disk)
- (4) Assume v -inflow velocity, uniformly distributed over the disk (equivalent to assuming infinite number of blades)
- (5) Hover.



$$\text{Mass flow through rotor} = \rho A v = m \quad (1)$$

Thrust from momentum equation is given by, considering

15.

(2)

stations (0) — (3)

Momentum Theory

$$T = \rho A v (w - 0) = \rho A v w \quad (2)$$

Applying Bernoulli's eq. from station (0) — (1)

$$p_0 = p_\infty = p_1 + \frac{\rho v^2}{2} \quad (3)$$

between (2) — (3)

$$p_2 + \frac{\rho v^2}{2} = p_0 + \frac{\rho w^2}{2} \quad (4)$$

$$p_2 - p_0 = \frac{\rho}{2} (w^2 - v^2) \quad (4a)$$

From (3) & (4a)

$$p_2 - p_1 = \Delta p = p_0 + \frac{\rho}{2} (w^2 - v^2) - p_0 + \frac{\rho}{2} v^2 = \frac{\rho w^2}{2} \quad (5)$$

From (2) & (5)

$$T = \Delta p A = (p_2 - p_1) A = \frac{\rho}{2} w^2 A = \rho A v w$$

$$\boxed{v = \frac{w}{2}} \quad (6)$$

From (2) & (6)

$$T = m \omega = (\rho A v) \omega = \rho A v^2 \cdot \frac{w}{2}$$

$$T = \rho A v^2 \cdot \frac{w}{2} \quad \text{or}$$

(7)

16

(3)

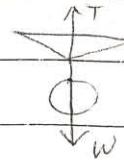
$$v = \sqrt{\frac{T}{2\rho A}}$$

$$\lambda = \text{inflow ratio} = \frac{v}{\Omega R} = \sqrt{\frac{T}{2\rho A \Omega^2 R^2}} = \sqrt{\frac{C_T}{2}}$$

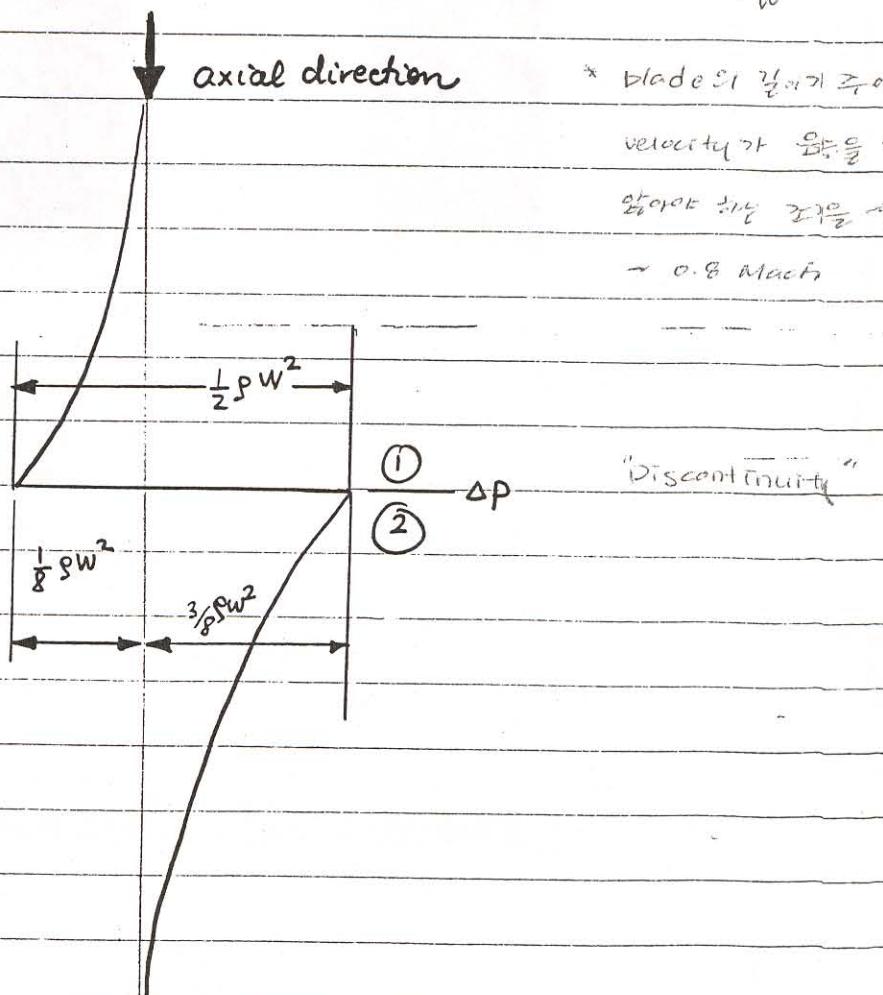
(7)

where $C_T = \frac{T}{\rho A \Omega^2 R^2}$ = Thrust coefficient of the rotor

* Effect of uniform inflow $\lambda = 1$ will be discussed later!
Weight!



Pressure variation plot in axial direction



- chord $c = \text{constant}$
- Hover or Axial flight
- Incompressible
- slightly loaded disks
- Rotational effects are neglected

2. Blade Element Theory

Consider the case of hovering flight or axial flight.

Assumptions

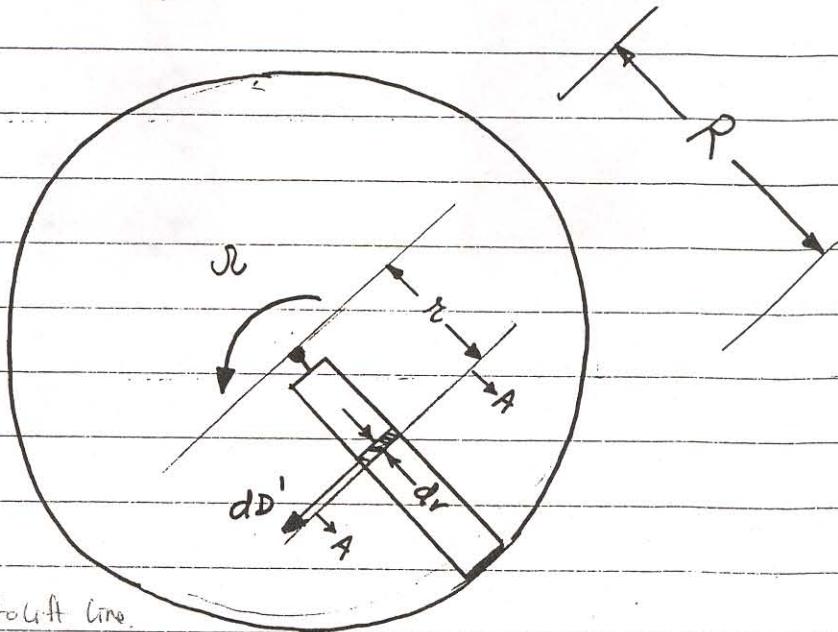
(1) Blade chord $c = \text{const.}$, pitch setting $\theta_0 = \text{const}$

$v = \text{inflow velocity}$ constant over the blade

(2) Axial flight (radial flow effects are negligible)

(3) Incompressible flow

(4) Low disk loading



zero lift line

Z.L.L. dT dl

$F_{2(r)}$

In rotor blade, sym / ex NACA 40012

is widely used.

Axial

effective

$\alpha(r)$

v

$\psi(r)$

θ_0

Ωr

S/r

inflow ratio

View A-A

Denote by V the velocity of vertical flight.

From the Figure blade element angle of attack

$$\alpha(r) = \theta_0 - \tan^{-1} \left(\frac{V+v}{\Omega r} \right) \quad (8)$$

For low axial velocities $V+v \ll \Omega r$, and a radial position sufficiently outboard from the axis of rotation.

$$\alpha(r) \approx \theta_0 - \left(\frac{V+v}{\Omega r} \right) \quad (8a)$$

The elemental lift, associated with a segment of width dr is

$$\text{Lift per unit span } dL = \frac{1}{2} \rho \left[(\Omega r)^2 + (V+v)^2 \right] c_e c dr \stackrel{\text{chord } C \text{-constant}}{\approx} \frac{1}{2} \rho (\Omega r)^2 c_e c dr \quad (9)$$

lift curve slope

because $1 \gg \left(\frac{V+v}{\Omega r} \right)^2$ Assumption: r is far from the root of the blade

Recall $c_e = a \alpha(r)$ ($a = \text{lift curve slope}$)

$$dL = \frac{1}{2} \rho \Omega^2 r^2 c \alpha(r) a dr = \frac{1}{2} \rho \Omega^2 r^2 a c \left[\theta_0 - \frac{V+v}{\Omega r} \right] dr \quad (10)$$

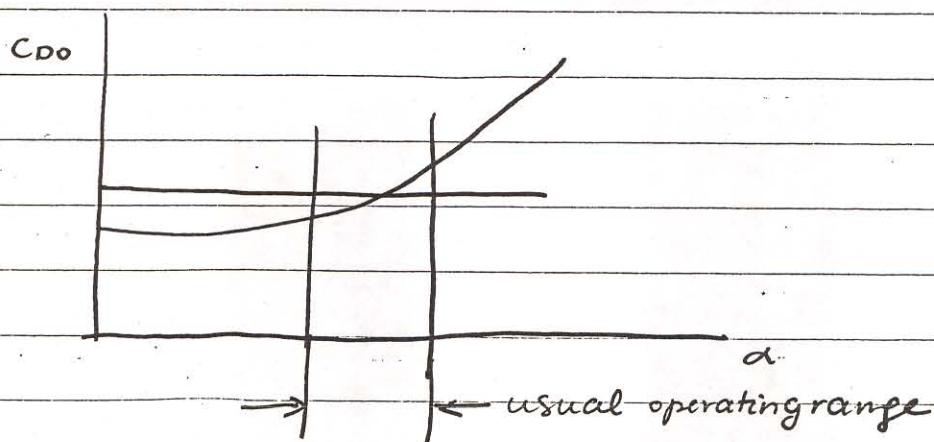
Similarly, one may write the following expression for drag

$$dD_o = \frac{1}{2} \rho \Omega^2 r^2 c_{D0} c dr \quad (11)$$

c_{D0} is assumed to be some representative constant profile drag coefficient, in reality

$$C_{D0} = \delta_0 + \delta_1 \alpha(r) + \delta_2 \alpha^2(r) \quad (12)$$

$\delta_0, \delta_1, \delta_2$ - are some constants which can be determined.



In rotor dynamics it is customary to resolve things into components perpendicular and parallel to the plane of rotation (hub plane)

$$\varphi(r) = \text{inflow angle} = \tan^{-1} \left(\frac{V+v}{\omega r} \right) \approx \frac{v+V}{\omega r} \Rightarrow \text{small}$$

Referring back to Fig. and taking components

$$dT = dL \cos \varphi(r) - dD_0 \sin \varphi(r) \quad (13)$$

$$dD' = dL \sin \varphi(r) + dD_0 \cos \varphi(r) \quad (14)$$

negligible

The lift L is approximately one order of magnitude larger than the drag D ($L/D \approx 6-9$)

thus last term in (13) is negligible

Using small angle approximation for $\varphi(r)$

C

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$$\cos \varphi(r) \approx 1.0 ; \sin \varphi(r) \approx \varphi(r)$$

Thus

$$dT \approx dL \quad (13a)$$

$$dD' = \varphi(r) dL + dD_o \quad (14a)$$

this term is called induced drag

For a rotor with b -blades, the total thrust is given by, using (13a) and Eq.(10), with $\bar{r} = r/R$ as integration variable

$$T = b \int_0^R \frac{dT}{dr} dr = \frac{1}{2} \rho v^2 R^3 acb \int_0^1 \bar{r}^2 \left[\theta_0 - \frac{v+V}{2\bar{r}R} \right] d\bar{r}$$

$$= \frac{1}{2} \rho acb \bar{v}^2 R^3 \left[\frac{\theta_0}{3} - \frac{v+V}{2\bar{r}R} \right] \quad (15)$$

$$\text{Recall } C_T = \frac{T}{\rho \pi R^2 (\bar{v} R)^2} = \frac{\frac{1}{2} \frac{\rho}{\pi R} bc}{\frac{\pi R^2}{2}} \left[\frac{\theta_0}{3} - \frac{\lambda}{2} \right]$$

$$\text{Define Solicity} = \frac{\text{Blade Area}}{\text{Disk Area}} = \frac{bcR}{\pi R^2} = \frac{bc}{\pi R} = C$$

$$\text{Inflow ratio} \quad \lambda = \frac{v+V}{\bar{v} R}$$

$$\lambda = \sqrt{\frac{C}{2}}$$

Thus Eq (15) can be rewritten as $C_T = \frac{ac}{2} \left[\frac{\theta_0}{3} - \frac{\lambda}{2} \right]$

$$C_T = \frac{ca}{2} \left[\frac{\theta_0}{3} - \frac{\lambda}{2} \right] \quad (16)$$

$$= \frac{ac}{2} \left[\frac{\theta_0}{3} - \frac{1}{2} \sqrt{\frac{C}{2}} \right]$$

Induced drag lift

$$dD' = dD_o + dL \varphi(r)$$

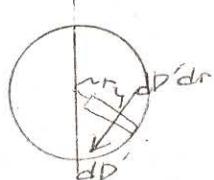
Rotor Torque

Another important quantity associated with rotor behavior is the torque, or the moment needed to overcome the drag, and keep the rotor turning at a certain RPM in steady state conditions. One can express the elemental torque as

$$dQ = r dD' \quad (16) \quad dQ = r dD' dr$$

and the total torque as

$$\begin{aligned} Q &= b \int_0^R r dD' = b \int_0^R (dL \varphi + dD_0) r dr = \\ &= b \int_0^R r \left[\frac{1}{2} \rho ac \Omega^2 r^2 \left(\theta_0 - \frac{V+v}{\Omega r} \right) \frac{V+v}{\Omega r} + \frac{1}{2} \rho c C_{D_0} \Omega^2 r^2 \right] dr \\ &= \int_0^R \frac{1}{2} \rho abc \Omega^2 c \left[\theta_0 \frac{V+v}{\Omega} r^2 - \frac{(V+v)^2}{\Omega^2} r \right] + \frac{1}{2} \rho c C_{D_0} \Omega^2 r^3 dr \\ &= \frac{1}{2} \rho abc \Omega^2 \left[\theta_0 \frac{V+v}{\Omega} \frac{R^3}{3} - \frac{(V+v)^2 R^2}{\Omega^2} \frac{R}{2} \right] + \frac{1}{2} \rho c C_{D_0} b \Omega^2 \frac{R^4}{4} \\ &= \frac{1}{2} \rho abc \Omega^2 \left[\left(\frac{\theta_0}{3} - \frac{\lambda}{2} \right) \lambda R^4 \right] + \frac{1}{2} \rho bc C_{D_0} \Omega^2 \frac{R^4}{4} \quad (17) \end{aligned}$$



Went through

Define the torque coefficient, similarly to the thrust coefficient thus

$$C_Q = \text{torque coefficient} = \frac{Q}{\rho \pi R^2 (\Omega R)^2 R} \quad (18)$$

Combining Eqs (17) and (18) one has

$$C_Q = \frac{1}{2} \frac{abc}{\pi R} \left[\left(\frac{\theta_0}{3} - \frac{\lambda}{2} \right) \lambda + \frac{bc}{\pi R} \frac{1}{2} C_{D_0} \frac{1}{4} \right]$$

$$C_Q = \frac{6a}{2} \left[\left(\frac{\theta_0}{3} - \frac{\lambda}{2} \right) \lambda \right] + \frac{C_{D_0}}{8} \quad (19)$$

Combining Eqs (16) and (19) one has

$$C_Q = C_T \lambda + \frac{C_{Q0}}{8} \quad (20)$$

The first term in Eq(20) is usually called the induced torque (because it is due to the induced drag) and the second term is called the profile torque.

Using the expression for the torque, Eq (17) it is also easy to define the power required

$$P = Q \nu r \quad (21)$$

The power coefficient is defined as

$$C_P = \frac{P}{\rho \pi R^2 (\nu R)^3} = \frac{Q \nu r}{\rho \pi R^2 \nu^2 R^3} = C_Q \quad (21)$$

$C_P = C_Q$ when hover or Axial flight

Next it is interesting to connect the expressions which have been derived with momentum theory, in hovering flight. Recall for this case power is given by

$$P = T \nu$$

$$C_{P_i} = \frac{T \nu}{\rho \pi R^2 (\nu R)^3} = \frac{T}{\rho \pi R^2 (\nu R)^2} \cdot \frac{\nu}{\nu R} = C_T \lambda$$

Since for hover $\lambda = \frac{\nu}{\nu R}$ and thus

$$C_{Q_i} = C_P = \lambda C_T \quad (22)$$

i.e. ideal torque coefficient, in absence of friction is identical from both, blade element theory and momentum theory.

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Hover or Axial Flight

$$C_T = \frac{G}{2} \left[\frac{\theta_0}{3} - \frac{\lambda}{2} \right] + C_{T\text{ hover}}$$

$$\lambda = \sqrt{\frac{C_T}{2}}$$

In hovering flight

$$\lambda = \frac{v}{\sigma R} = \sqrt{\frac{T}{2\sigma A \sigma^2 R^2}} = \sqrt{\frac{C_T}{2}} \quad (23)$$

Combining Eqs (16) and (23) one has

$$C_T = \frac{G}{2} \left[\frac{\theta_0}{3} - \frac{1}{2} \sqrt{\frac{C_T}{2}} \right] \quad (24)$$

@ Reasonable value

from which

$$\left(\frac{2 C_T}{G} \right) = \frac{\theta_0}{3} - \frac{1}{2} \sqrt{\frac{C_T}{2}}$$

$$\frac{1}{2} \sqrt{\frac{C_T}{2}} = \frac{\theta_0}{3} - \frac{2 C_T}{G}$$

$$\frac{C_T}{8} = \left(\frac{\theta_0}{3} - \frac{2 C_T}{G} \right)^2 = \frac{\theta_0^2}{9} - \frac{4 \theta_0 C_T}{3 G} + \frac{4 C_T^2}{(G)^2}$$

$$\frac{\theta_0^2}{9} - \left(\frac{\theta_0}{3} \frac{4}{G} + \frac{1}{8} \right) C_T + \frac{4 C_T^2}{G^2} = 0$$

$$C_T^2 - \frac{G^2}{4} \left(\frac{4 \theta_0}{3 G} + \frac{1}{8} \right) C_T + \frac{G^2}{36} \theta_0^2 = 0 \quad (25)$$

Equation (25) is sometimes a useful quadratic equation for C_T , when the value of collective pitch θ_0 is given.

Also recall Eq(20) and combine it with (23)

thus

$$C_Q = \frac{G C_{Q0}}{8} + \frac{C_T}{\sqrt{2}} = C_{Q0} + C_{Qi} \quad (26)$$

where C_{Q0} - is the profile torque coefficient from Eq (22)

$C_{Qi} = \frac{C_T^{3/2}}{\sqrt{2}}$ is the ideal torque coefficient.

USUAL RANGE $\sim (3^\circ < \theta_0 < 15^\circ)$

$$\alpha(r) = \theta_0 - \frac{v+r}{\sqrt{2} \cdot r} < 13, 14^\circ$$

- ideal torque

Another useful quantity often used in helicopter engineering is the Rotor Figure of Merit (F.M) which is defined as

$$F.M. = \frac{C_{Q\text{ideal}}}{C_Q} = \frac{1}{\sqrt{2}} \frac{C_T^{3/2}}{(C_{Q0} + C_{Qi})} \quad (27)$$

Measure of goodness of rotor

so that the ideal rotor Figure of Merit is

$$\frac{1}{\sqrt{2}} \frac{C_T^{3/2}}{C_{Qi}} = 1$$

For an actual rotor the Figure of Merit indicates the magnitude of the losses due to non-uniformity of flow, tip loss, and profile drag for a particular rotor. For a good rotor $F.M. \approx 0.75$, where "good" implies a well designed rotor.

In the equations used above C_{d0} has appeared a number of times, a good approximate relation for C_{d0} is given by

$$C_{d0} = 0.0081 - 0.0216\alpha + 0.4\alpha^2 \quad (28)$$

(α - in radians, angle of attack), for a reasonable angle of attack α , this normally yields a value of $C_{d0} \approx 0.012$. Assume of C_{d0}

Vertical Climb

R/C is slow — neglect fuselage effect

At high rates of vertical climb the drag of the fuselage has to be included. However for the relatively simple situation discussed in the class this effect will be neglected.

Recall that for a rotor in hover, the torque coefficient is

$$C_{QH} = C_{Q0} + \lambda_H C_T \quad (28) \quad \text{Hover}$$

↑ profile drag ↓ induced power

for climb or axial flight

$$C_Q = C_{Q0} + \lambda C_T \quad (29) \quad \text{climb}$$

roughly same

assuming that C_{Q0} does not change in an appreciable manner for vertical flight at constant axial velocity, also from equilibrium $W = T$ (still true)

Therefore one can combine Eqs (28) and (29)

$$\Delta C_Q = C_Q - C_{QH} = (\lambda - \lambda_H) C_T = \begin{matrix} \text{Change in Torque} \\ \text{to climb from Hover} \end{matrix}$$

$$\Delta \frac{C_Q}{C_T} = \lambda - \lambda_H = \lambda - \sqrt{\frac{C_T}{2}}$$

$$\text{Therefore } \lambda = \frac{\Delta C_Q}{C_T} + \sqrt{\frac{C_T}{2}} \quad (30) \quad \text{inflow ratio}$$

Also recall that by definition

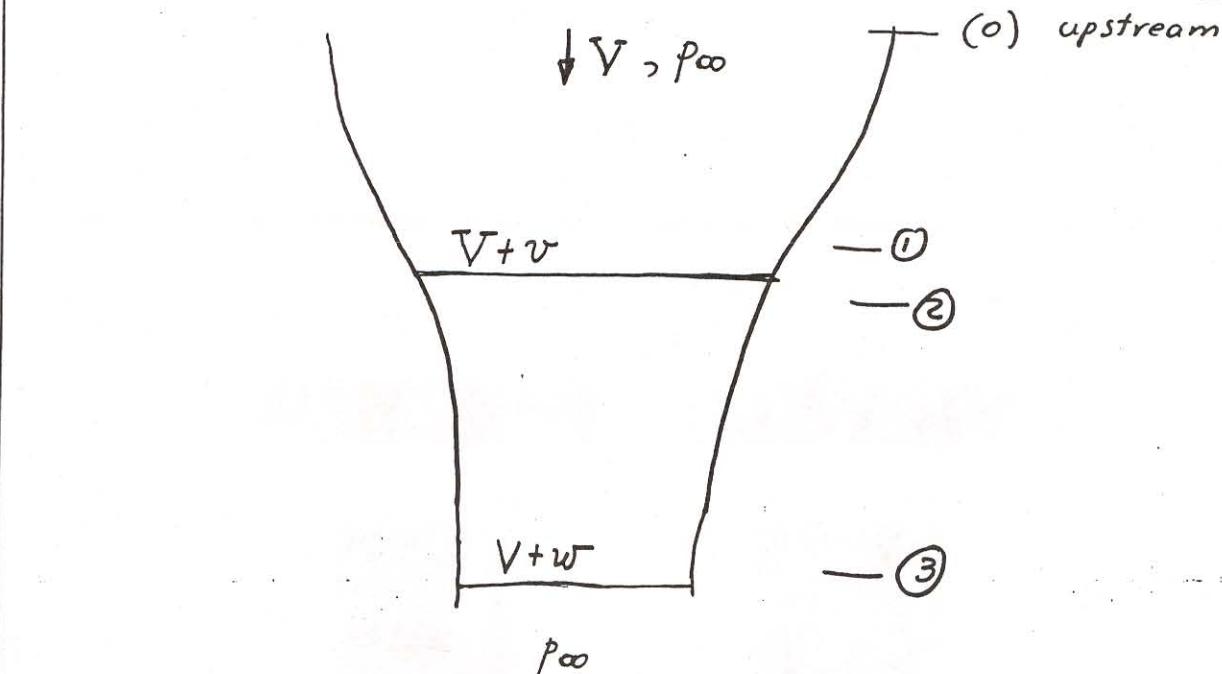
$$\lambda = \frac{V+v}{\sigma R} = \text{inflow ratio} = \frac{V}{\sigma R} + \frac{v}{\sigma R}$$

$$\text{thus } \frac{V}{\sigma R} = \lambda - \frac{v}{\sigma R} \quad (31)$$

For the case of axial flight one can go through momentum theory in a manner similar to our initial derivation for the case of hover.

(14)

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Same assumptions as before.

$$\text{Mass flow through the rotor} = \rho A(v+V) = \dot{m}$$

Thrust from momentum equation is given by considering

stations (0) - (3) $T = \dot{m} \Delta \vec{V} = (\rho A(v+V)) \cdot w = 2\rho A(v+V) \cdot w$

$$T = \rho A(v+V) w$$

Applying Bernoulli's equation between station (0) - (1) and (2) - (3) yields again

$$2v = w$$

for horizontal flight Axial Flight

$$\text{Thus } T = 2\rho A(v+V) v$$

$$\text{Therefore } C_T = \frac{T}{\rho A(\frac{v}{2R})^2} = 2 \left(\frac{v+V}{\frac{v}{2R}} \right) \frac{v}{\frac{v}{2R}}$$

$$\frac{C_T}{2\lambda} = \frac{v}{\frac{v}{2R}} \quad (32)$$

Combining Eqs (30), (31) and (32)

$$\frac{V}{2R} = \lambda - \frac{C_T}{2\lambda} = \left(\frac{\Delta C_Q}{C_T} + \sqrt{\frac{C_T}{2}} \right) - \frac{C_T/2}{\left(\frac{\Delta C_Q}{C_T} + \sqrt{\frac{C_T}{2}} \right)} =$$

$$\frac{V}{2R} = \lambda - \frac{v}{\frac{v}{2R}}$$

(15)

$$\frac{V}{\sqrt{R}} = \frac{\left(\frac{\Delta C_Q}{C_T}\right)^2 + \frac{2\Delta C_Q}{C_T} \sqrt{\frac{C_T}{2}} + \frac{C_T}{2} - \frac{C_T}{2}}{\left(\frac{\Delta C_Q}{C_T} + \sqrt{\frac{C_T}{2}}\right)} =$$

$$\frac{V}{\sqrt{R}} = \frac{\Delta C_Q \left(\frac{\Delta C_Q}{C_T} + 2\sqrt{\frac{C_T}{2}} \right)}{\left(\frac{\Delta C_Q}{C_T} + \sqrt{\frac{C_T}{2}} \right)} \approx 2 \frac{\Delta C_Q}{C_T} \quad (32)$$

Vel C_T의
한계는

$$\frac{V}{\sqrt{R}} \approx 2 \frac{\Delta C_Q}{C_T} \quad (32)$$

• 기울기면 풍속 Axial velocity를 고려해 드는 thrust가 헬리콥터에 걸친다.
the required differential torque coefficient is provided by Eq (32)

Equation (32) is based on the assumption that

$\frac{\Delta C_Q}{C_T}$ is small compared to $\sqrt{\frac{C_T}{2}}$, which implies a low rate of climb (R/c) of less than

$$V \leq 10 \text{ ft/sec} \quad \text{or} \quad V < 600 \text{ ft/min}$$

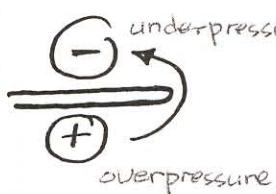
+ R/C slow
⇒ Assume
 $\frac{\Delta C_Q}{C_T} \ll \sqrt{\frac{C_T}{2}}$

Typical Power losses for hovering rotors (single rotor)

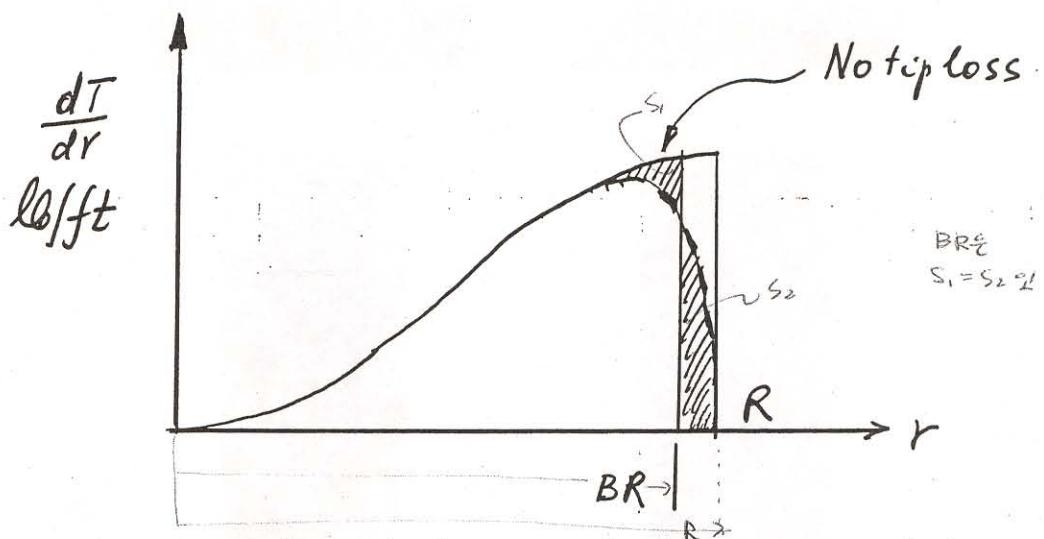
| | | |
|-------------------|---------------|----|
| <u>Mechanical</u> | Gear friction | 3% |
| | Tail Rotor | 7% |

| | | |
|--|--|------------------------------|
| <u>Aerodynamic</u> | Interference (download on fuselage) | 2% |
| | Tip loss | 3% |
| | Slip stream rotation | 0.2% |
| | non uniform inflow | 7% |
| $(\text{no twist } 4\%, \text{ or taper } 3\%)$ | | Free wake model $v(z, r)$ |
| \rightarrow explain why twist and taper are useful | | |

Rotor Tip Losses



pressure loss from high pressure to low pressure region



$$BR^2 \\ S_1 = S_2 \text{ if } \frac{S_1}{S_2} > BR$$

BR determined from balance of the two areas shown.

For lightly loaded rotors (Betz) came up with a simple relation

$$\text{tip loss factor: } B = 1 - \frac{\sqrt{2 C_T}}{b}$$

Theory for lightly loaded rotors (propellers,

$b = \# \text{ of blades}$

$B = \text{typical } 0.97$

$$T = b \int_{AR}^{BR} dT = \int_0^{BR}$$

Area of Root (AR) $\frac{dT}{dr}$ \rightarrow dT \rightarrow $\int dT$
" " area of tip $\frac{dT}{dr}$ \rightarrow dT \rightarrow $\int dT$

$$Q = b \int_0^R dQ_0 + b \int_{AR}^{BR} dQ_i$$

only induced torque needs to be corrected

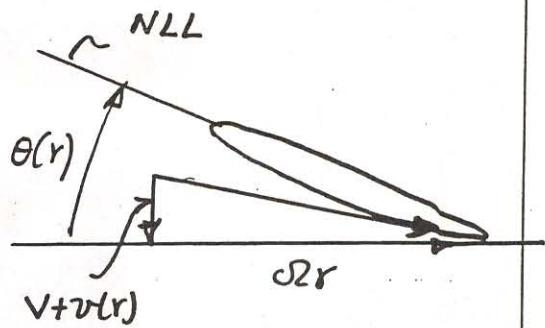
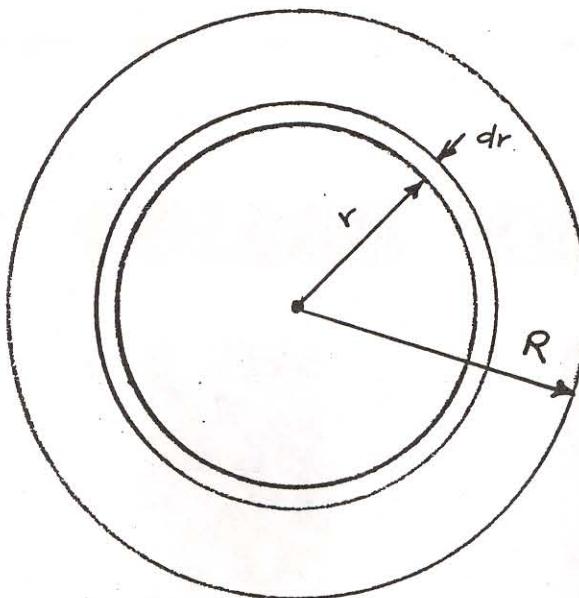
Note also that correction in the root region not very important because moment arm is small

Extended Blade Element Theory

Momentum Theory &

Blade Element Theory &

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Combine the features of blade e.t. & momentum theory so as to obtain a more accurate representation of the inflow distribution. Consider an actuator disc, in axial flight, and select an annular element of the disc as shown in the figure above. The elemental thrust of the b blade elements contained in the annular ring based on blade element theory is given by

$$\text{B.E.T.} \rightarrow dT = \frac{1}{2} \rho a b c(r) \Omega^2 r^2 \left[\theta(r) - \frac{V + v(r)}{\Omega r} \right] dr \quad (1)$$

For the same annular ring shown, the elemental thrust based on momentum theory is given by

$$\text{M.T.} \rightarrow dT = \rho 2\pi r dr [V + v(r)] 2 v(r) \quad (2)$$

Equating these two expressions yields

$$\begin{aligned} \frac{1}{2} \rho a b c(r) \Omega^2 r^2 \left[\theta(r) - \frac{V + v(r)}{\Omega r} \right] dr = \\ = \rho 2\pi r [V + v(r)] 2 v(r) dr \end{aligned} \quad (3)$$

which is a quadratic equation for $v(r)$

(18)

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Rewrite the equation in nondimensional form
by introducing

$$x = \frac{r}{R} \quad \text{and using a solidity}$$

$$\sigma(x) = \frac{bc(x)}{\pi R}$$

$$abc(x) \sqrt{2} R^2 x^2 \left[\theta(x) - \frac{V + v(x)}{\sqrt{2} x R} \right]$$

$$= 4 \sqrt{\pi} x R [V + v(x)] z v(x)$$

$$abc(x) \sqrt{2} R x \left[\theta(x) - \frac{V + v(x)}{\sqrt{2} x R} \right]$$

$$= 4 \pi [V + v(x)] z v(x)$$

$$a \frac{bc(x)}{\pi R} \sqrt{2} R^2 x \left[\theta(x) - \frac{V + v(x)}{\sqrt{2} x R} \right] = 4 [V + v(x)] z v(x)$$

$$a \sigma(x) \sqrt{2} R^2 x \left[\theta(x) - \frac{V}{\sqrt{2} x R} \right] - a \sigma(x) \sqrt{2} R^2 x \frac{v(x)}{\sqrt{2} x R}$$

$$= 8 V v(x) + 8 v^2(x)$$

$$8 v^2(x) + v(x) \left[8 V + a \sigma(x) \sqrt{2} R \right] - a \sigma(x) \sqrt{2} R^2 x \left[\theta(x) - \frac{V}{\sqrt{2} x R} \right] = 0$$

$$v^2(x) + v(x) \left[V + \frac{a \sigma(x) \sqrt{2} R}{8} \right] - \frac{a \sigma(x) \sqrt{2} R^2 x}{8} \left[\theta(x) - \frac{V}{\sqrt{2} x R} \right] = 0$$

solution of this quadratic equation yields

$$v(x) = \left(\frac{V}{2} + \frac{\sigma(x) a \sqrt{2} R}{16} \right) \left(-1 + \sqrt{1 + \frac{2 [\theta(x) x \sqrt{2} R - V]}{\frac{4 V^2}{\sigma(x) a \sqrt{2} R} + V + \frac{\sigma(x) a \sqrt{2} R}{16}}} \right) \quad (4)$$

$$x_{1,2} = -\frac{b}{2} \pm \sqrt{\left(\frac{b}{2}\right)^2 - ac} = \left(\frac{b}{2}\right) \left[-1 \pm \sqrt{1 - \frac{ac}{\left(\frac{b}{2}\right)^2}} \right] \Leftarrow$$

Equation (4) is an important and useful equation for determining the inflow in hover or axial flight. Once the induced velocity is known, the inflow angle at the blade element can be determined from

$$\varphi = \frac{v + V}{\sigma R x} \quad (5)$$

Expression (4) is a completely general expression which allows one to determine the inflow velocity for any blade planform and pitch distribution. A number of special forms of this equation are quite useful.

CASE (a) For example when $c = \text{constant}$ and the blade twist is inversely proportional to x , $\theta(x) = \frac{\theta_t}{x}$, where θ_t is blade twist at the tip, one obtains $v(x)$ constant over the disc.

$$v = v(x) = \left(\frac{V}{2} + \frac{\sigma a \sigma R}{16} \right) \left(-1 + \sqrt{1 + \frac{2 [\theta_t \sigma R - V]}{\frac{4V^2}{\sigma a \sigma R} + V + \frac{\sigma a \sigma R}{16}}} \right) \quad (5)$$

there is no "x" anymore
CONSTANT INFLOW

CASE (b) Another useful relation is obtained for the case of hover $V=0$ and constant chord $c(x)=c$. Assuming that the inflow velocity at $3/4 R$ is representative of a uniformly distributed inflow for $\theta=\text{const}$ one has

$$v(x) = \frac{\sigma a \sigma R}{16} \left(1 + \sqrt{1 + \frac{2 \theta \frac{3}{4} \sigma R}{\frac{\sigma a \sigma R}{16}}} \right)$$

$$v(x) = \frac{C_a \sqrt{R}}{16} \left(\sqrt{1 + \frac{24\theta}{C_a}} - 1 \right)$$

$$\lambda = \frac{v(x)}{\sqrt{R}} = \frac{C_a}{16} \left(\sqrt{1 + \frac{24\theta}{C_a}} - 1 \right) \quad (6)$$

which is an approximate relation for uniform inflow frequently used in aeroelastic calculations

Optimum Rotor for Hover

Here we are interested in the optimum rotor for hover including real fluid effects. We are seeking α_{opt} for

$$\max(L/D) \quad \left[\alpha \left(C_e/C_d \right)_{\text{max, with friction}} \right]$$

$$\alpha_{opt} = \frac{C_{e,0}}{a} \quad \text{for all } r.$$

Still want $v = \text{const}$ over the disk. Returning to Eq(3) for $V=0$ one has

$$\frac{1}{2} ab^2 C_D \sqrt{R^2 r^2} \left[\theta(r) - \frac{v(r)}{\sqrt{r}} \right] = 4\pi / v(r)^2 \quad (7)$$

$\underbrace{\alpha(r)}$

we want $\alpha(r) = \alpha_{opt}$, $A = \text{const}$

$$\text{Let } \theta(r) = \alpha_{opt} + \frac{v(r)}{\sqrt{r}} \quad (8)$$

Therefore $abc \sqrt{R^2 r} \alpha_{opt} = 8\pi v^2(r)$

$v(r) = \text{const}$ if $c(r) = c_0 \frac{R}{r}$, i.e. tapered blade

$$\frac{R}{R} ab^2 abc \frac{R}{r} \alpha_{opt} = 8\pi v^2$$

$$\left(\frac{v}{\sqrt{R}} \right)^2 = \frac{abc \alpha}{8\pi R};$$

$$\boxed{\frac{v}{\sqrt{R}} = \sqrt{\frac{abc \alpha}{8\pi R}}} \quad (9)$$