Ch 5 Numerical Models for Buoyant jets

5.1 Analytical Solutions

5.2 Jet Integral Model

5.3 3D Hydrodynamic Model

5.4 Example Application of Numerical Model

Objectives:

- Review analysis methods for buoyant jets
- Introduce numerical models of

5.1 Analytical Solutions

5.1.1 Asymptotic solution

- unknowns:

  maximum values at the jet centerline (w_m , C_m),

  jet half width

- dimensional analysis

  Buckingham π theorem → dimensionless variables

- For Asymptotic cases consider only dominant process (mechanism) → include important parameters

$$w_m \frac{Q}{M} \rightarrow a_1 \left(\frac{l_Q}{z} \right)^1$$

$$\frac{w_m}{W} = 7.0 \frac{l_Q}{z} = 6.2 \frac{D}{z}$$

$$\frac{b}{l_Q} = f\left(\frac{z}{l_Q} \right)$$

$$\frac{b}{l_Q} = a_2 \left(\frac{z}{l_Q} \right)^1 \rightarrow b = a_2 z^1$$

$$\frac{b_w}{z} = 0.107 \quad (w = 0.37 w_m)$$

$$\frac{b_T}{z} = 0.127 \quad (c = 0.37c_m)$$

Velocity profile

- Similarity assumption

→ Gaussian profile, analytical solution, top-hat profile

$$w = w_m \exp\left[-k_w\left(\frac{x}{z}\right)^2\right] \tag{1}$$

5.1.2 Solution developed from equation of motion

1) continuity eq.

$$\frac{\partial \bar{u}}{\partial x}+\frac{\partial \bar{v}}{\partial y}+\frac{\partial \bar{w}}{\partial z}=0 \qquad (2)$$

2) time-averaged momentum eq. for steady flow (Reynolds eq.)

x -dir.:

$$\frac{\partial}{\partial x}\left(\bar{u}^2+\bar{u}'^2+\frac{\bar{p}}{\rho_0}\right)+\frac{\partial}{\partial y}(\overline{u'v'})+\frac{\partial}{\partial z}(\overline{uw}+\overline{u'v'})=0 \qquad (5.63)$$

3) z -dir.:

$$\frac{\partial}{\partial x}(\overline{uw}+\overline{u'v'})+\frac{\partial}{\partial y}(\overline{w'v'})+\frac{\partial}{\partial z}\left(\bar{w}^2+\bar{w}'^2+\frac{\bar{p}}{\rho_0}\right)=g_z'$$

$$=\frac{\Delta\rho}{\rho}g=\frac{\rho_a-\bar{\rho}}{\rho_0}g \qquad (5.64)$$

- Integrate Eq. (5.64) over jet cross section

(1) vertical region

$$\int_{A(z)}\frac{\partial}{\partial z}\bar{w}^2 dxdy=\int_{A(z)}\left(\frac{\rho_a-\bar{\rho}}{\rho_0}\right)dxdy \qquad (5.70)$$

→ Rate of change of vertical flow force in vertical direction is equal to buoyancy force.

$$\int_{A(z)} \frac{\partial}{\partial z}\left[\bar{w}(\rho_a - \bar{\rho})\right] dxdy = 0 \qquad (5.71)$$

→ Vertical flux of buoyancy is conserved.

(2) Bent over region

Integrate of Eq. (5.64) and Eq. (5.66) across a vertical plane, $A(z)$ with making same kind simplications.

Then we get

$$\int_{A(z)} \frac{\partial}{\partial x}(\overline{uw}) dydz = \int_{A(z)} \left(\frac{\rho_a - \bar{\rho}}{\rho_0}\right) g dydz \qquad (5.72)$$

$$\int_{A(z)} \frac{\partial}{\partial x}\left[\bar{u}(\rho_a - \bar{\rho})\right] dydz = 0 \qquad (5.73)$$

Eq.(5.72): horizontal flux of vertical momentum = buoyancy force acting in a vertical plane

Eq.(5.73): Horizontal flux of buoyancy is conserved.

1. Jet behavior in a crossflow

1-1. Jet vertical region (J.V.)

① Maximum(centerline) velocity, $w_m(\bar{z})$

For z □ l_Q, consider only Q, M ; neglect buoyancy B (or g')

Then, Eq. (5.70) becomes

$$\int_{A(z))} \frac{\partial \overline{w}^2}{\partial z} dxdy = 0 \quad \text{(A)}$$

Assume that velocity and tracer conc. profiles are similar in ZEF

→ use similarity solution

$$\frac{\overline{w}(x,y,z)}{w_m(\overline{z})} = \Phi\left(\frac{x}{\overline{z}}, \frac{y}{\overline{z}}\right) \tag{5.74}$$

$$\frac{(\rho_a - \overline{\rho})/\rho_0}{\theta(\overline{z})} = \psi\left(\frac{x}{\overline{z}}, \frac{y}{\overline{z}}\right) \tag{5.75}$$

substitute Eq. (5.74) into (A)

$$\int_{A(z)} \frac{\partial}{\partial z}\left[\overline{z}^2 \overline{w}_m^2(\overline{z})\phi^2\right] d\left(\frac{x}{\overline{z}}\right) d\left(\frac{y}{\overline{z}}\right) = 0$$

Leibnitz rule:

$$\therefore \frac{d}{dz}\int_{A(z)} \overline{z}^2 \overline{w}_m^2(\overline{z})\phi^2 d\left(\frac{x}{\overline{z}}\right) d\left(\frac{y}{\overline{z}}\right) = 0 \tag{5.76}$$

Since w_m and $\overline{z}$ don't vary over $A(z)$ at a particular $\overline{z}$ position,

$$\therefore \frac{d}{dz}\left\{\overline{z}^2 \overline{w}_m^2(\overline{z})\int_{A(z)} \phi^2 d\left(\frac{x}{\overline{z}}\right) d\left(\frac{y}{\overline{z}}\right)\right\} = 0$$

$$\therefore \frac{d}{dz}\left[\overline{z}^2 \overline{w}_m^2(\overline{z})\right] = 0 \quad \text{(B)}$$

$$\overline{z}^2 w_m^2(\overline{z}) = const$$

$$\overline{z}^2 w_m^2(\overline{z}) \sim M$$

Then $\dfrac{w_m(\bar{z})}{U} = c \cdot \dfrac{z_m}{\bar{z}}$ (5.82)

② Centerline concentration

Substitute Eq. (5.74) and Eq. (5.75) into Eq. (5.71)

$$\therefore \int_{A(z)} \frac{\partial}{\partial z}\left[\rho_0 \bar{z}^2 w_m(\bar{z})\theta(\bar{z})\Phi\psi\right] d\left(\frac{x}{\bar{z}}\right) d\left(\frac{y}{\bar{z}}\right) = 0 \qquad (5.77)$$

$$\frac{d}{dz}\left\{\bar{z}^2 w_m(\bar{z})\theta(\bar{z}) \int_{A(z)} \Phi\psi d\left(\frac{x}{\bar{z}}\right) d\left(\frac{y}{\bar{z}}\right)\right\} = 0$$

$$\therefore \frac{d}{dz}\left\{\bar{z}^2 w_m(\bar{z})\theta(\bar{z})\right\} = 0$$

$$\therefore \bar{z}^2 w_m(\bar{z})\theta(\bar{z}) = const. \quad \rightarrow \quad \left[L^3T^{-4}\right] = \left[Q\right](volume\ flux)$$ (volume flux)

$$\therefore \bar{z}^2 w_m(\bar{z})\theta(\bar{z}) = const.\ \frac{B}{g} \qquad (9.79)$$

$$\frac{Mg}{UB}\theta(z) = const. \frac{M^{1/2}}{U\bar{z}} = const. \frac{\bar{z}_m}{\bar{z}}$$

$$\therefore \ \frac{Mg}{UB}\theta(z) = D_1 \frac{\bar{z}_m}{\bar{z}} \qquad (5.83)$$

③ Jet trajectory

- A reasonable assumption is

$$\frac{d\bar{z}}{dx} = \frac{w_m(\bar{z})}{U} \qquad (5.84)$$

→ slope of jet trajectory

Substitute Eq. (5.84) into Eq. (5.82)

Eq.(5.82): $\dfrac{w_m(\bar{z})}{U} = const. \cdot \dfrac{z_m}{\bar{z}}$

$$\therefore \frac{d\bar{z}}{dx} = const. \cdot \frac{z_m}{\bar{z}}$$

$$\bar{z}dz = const.\, z_m dx$$

$$\frac{1}{2} d\left(\bar{z}^2\right) = const.\, z_m dx$$

Integrate once

$$\int \frac{1}{2} d\left(\bar{z}^2\right) = const.\, z_m \int dx$$

$$\frac{1}{2}\bar{z}^2 = const.\, z_m x + const.$$

$$\frac{\bar{z}^2}{z_m^2} = const.\ \frac{x}{z_m}$$

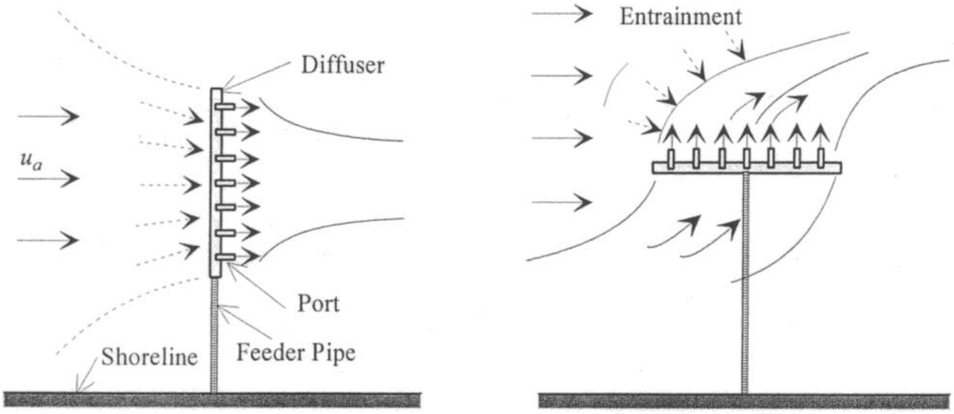
$$\therefore \frac{\bar{z}}{z_m} = C_1 \left(\frac{x}{z_m}\right)^{1/2} \qquad (5.85)$$

5.1.3 Empirical Models

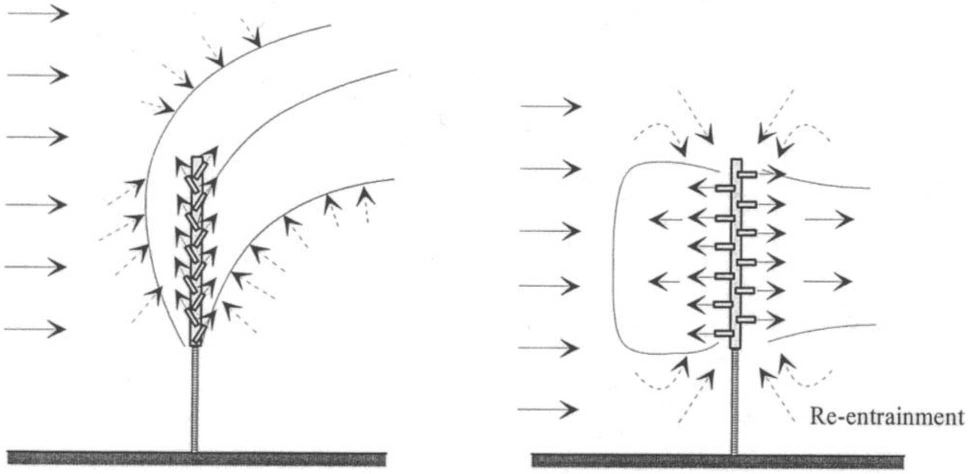
(1) Empirical Equations

- Description of multiport diffusers

Design goal of diffusers is to minimize detrimental effects of the discharge on the environment. Submerged multiport diffuser are shown in Figure 5.3.



(a) Co-flowing Diffuser (b) Tee Diffuser



(c) Staged Diffuser (d) Alternating Diffuser

Figure 5.3 Classification of thermal diffusers

- Dilution of multiport diffusers

i) T-Diffuser

Experimental and empirical equations for near field dilution are given below.

a. Dilution for stagnant ambient, S_0

$$S_0 = \sqrt{\frac{H}{2B}} = 0.71\sqrt{\frac{H}{B}}$$

where H = water depth, B = width of equivalent slot diffuser

Assuming merging of individual jets

$$BL_D = n(\pi D_0{}^2 / 4)$$

$$B = \frac{\pi D_0{}^2 / 4}{L_D / n} = \frac{\pi D_0{}^2}{4l}$$

where D_0 = port diameter, L_D = diffuser length, n = number of port, l = port spacing

b. Dilution for flowing ambient, S_t

$$\frac{S_0}{S_t} = \frac{1}{1-[60\exp(-5.0 m_r{}^{0.2})]m_r}$$

where m_r = momentum ratio between ambient flow and effluent discharge

$$m_r = \frac{m_a}{m_0} = \frac{U_a{}^2 H}{U_0{}^2 B}$$

ii) Staged Diffuser

a. Dilution for stagnant ambient, S_0

$$S_0 = 0.43\sqrt{\frac{H}{B}}$$

b. Dilution for flowing ambient, S_t

Weak cross flow: $\frac{S}{S_0} = 1.35$

Strong cross flow: $\frac{S}{S_0} = 1.35\sqrt{m_r}$

Table 5.3 Dilution equations of multiport diffusers

	Stagnant Ambient	Flowing Ambient	
	$m_r = 0$	$m_r < 1$	$m_r > 1$
Tee Diffuser	$S_0 = 0.71\sqrt{\frac{H}{B}}$	$\frac{S}{S_0} = 1 - [60\exp(-5.0 m_r^{0.2})] m_r$	
Staged Diffuser	$S_0 = 0.43\sqrt{\frac{H}{B}}$	$\frac{S}{S_0} = 1.35$	$\frac{S}{S_0} = 1.35\sqrt{m_r}$
Alternating Diffuser	$S_0 = 0.54\sqrt{\frac{H}{B}}$	$\frac{S}{S_0} = 0.82$	$\frac{S}{S_0} = 0.82\sqrt{m_r}$

iii) Dilution of single-port diffuser

(1) Stagnant water

For horizontal discharge into un-stratified ambient, minimum dilation at water surface is given below.

$$S_0 = 0.089 g'^{1/3} \frac{H^{5/3}}{Q_0^{2/3}}$$

$$Q_0 = \frac{\pi}{4} D_0^2 U_0$$

$$g' = g\frac{\Delta\rho}{\rho_0} = g\frac{\rho_a - \rho_0}{\rho_0}$$

(2) Ambient crossflow

For strong deflection,

$$S = 0.32\frac{U_a H^2}{Q_0}$$

(2) CORMIX

1) Introduction

The CORMIX modeling system is a comprehensive software system for the analysis, prediction, and design of outfall mixing zones resulting from discharge of aqueous pollutants into diverse water bodies. It contains mathematical models of point source discharge mixing within an intelligent computer-aided-design (CAD) interface. Its focus is environmental impact assessment and regulatory management. CORMIX has been developed under several cooperative funding agreements between U.S. EPA, U.S. Bureau of Reclamation, Cornell University, Oregon Graduate Institute (OGI), University of Karlsruhe, Portland State University, and MixZon Inc. during the period of 1985-2007.

CORMIX is a recommended analysis tool on the permitting of industrial, municipal, thermal, and other point source discharges to receiving waters. Although the system's major emphasis is predicting the geometry and dilution characteristics of the initial mixing zone so that compliance with water quality regulatory constraints may be judged, the system also predicts the behavior of the discharge plume at larger distances. CORMIX contains four core hydrodynamic simulation models and two post-processor simulation models. The simulation models are:

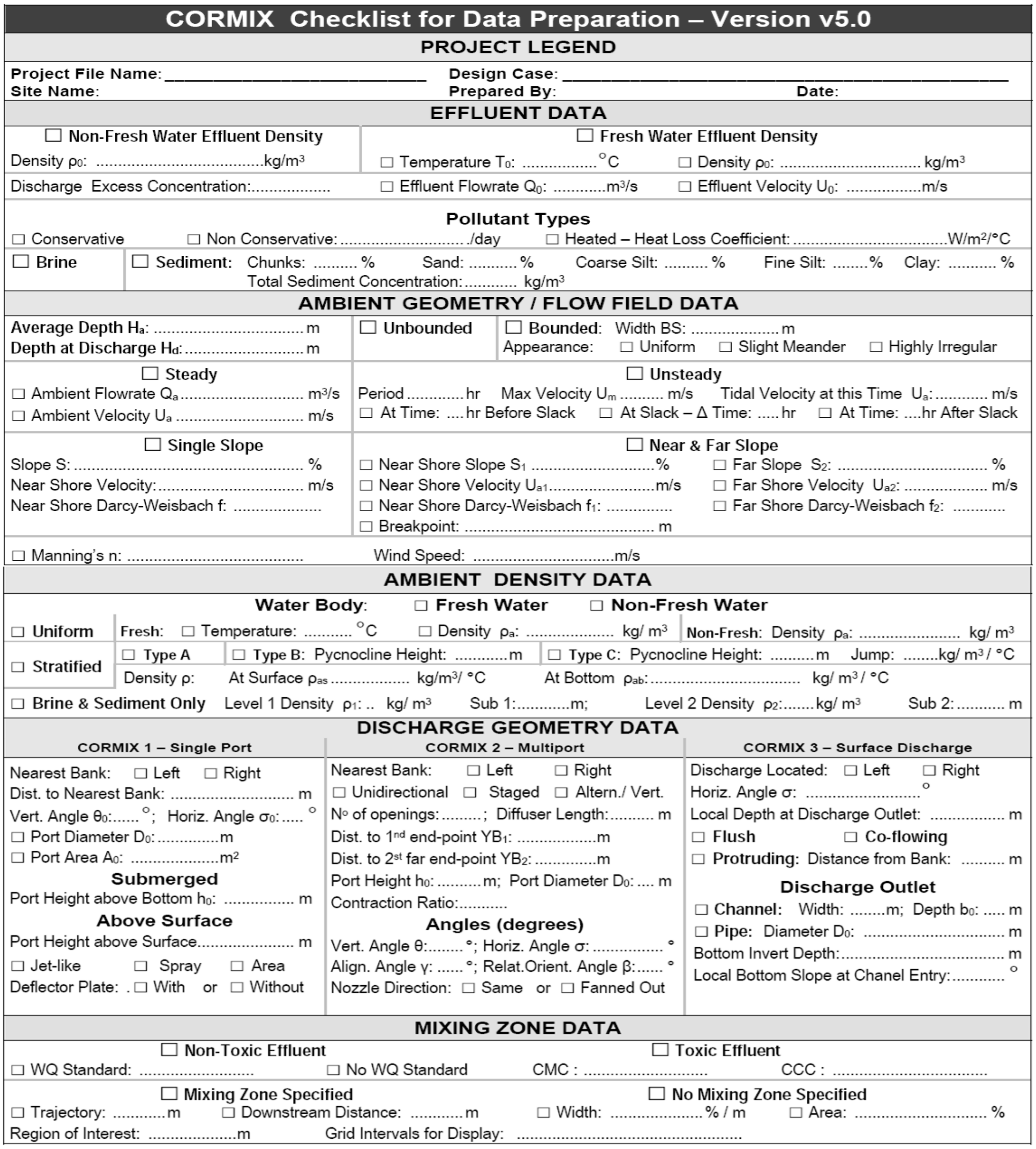
- Simulation models for single port discharges (CORMIX1).
- Simulation models for submerged multiport diffusers (CORMIX2).
- Simulation models for buoyant surface discharges (CORMIX3).
- Simulation models for dense brine and/or sediment discharges from single port, submerged multiport, or surface discharges in laterally unbounded coastal environments (DHYDRO).
- Post-processor simulation models for detailed near-field mixing of submerged single port and multiport diffusers in unbounded environments (CorJet).
- Post-processor simulation model for far-field plume analysis (FFL).

CORMIX1 predicts the geometry and dilution characteristics of the effluent flow resulting from a submerged single port diffuser discharge, of arbitrary density (positively, neutrally, or negatively buoyant) and arbitrary location and geometry, into an ambient receiving water body that may be stagnant or flowing and have ambient density stratification of different types.

CORMIX2 applies to three commonly used types of submerged multiport diffuser discharges under the same general effluent and ambient conditions as CORMIX1. It analyzes unidirectional, staged, and alternating designs of multiport diffusers and allows for arbitrary alignment of the diffuser structure within the ambient water body, and for arbitrary arrangement and orientation of the individual ports. For complex hydrodynamic cases, CORMIX2 uses the "equivalent slot diffuser" concept and thus neglects the details of the individual jets issuing from each diffuser port and their merging process, but rather assumes that the flow arises from a long slot discharge with equivalent dynamic characteristics. Hence, if details of the effluent flow behavior in the immediate diffuser vicinity are needed, an additional CORMIX1 simulation for an equivalent partial effluent flow may be recommended. CORMIX3 analyzes buoyant surface discharges that result when an effluent enters a larger water body laterally, through a canal, channel, or near-surface pipe. In contrast to CORMIX1 and 2, it is limited to positively or neutrally buoyant effluents. Different discharge geometries and orientations can be analyzed including flush or protruding channel mouths, and orientations normal, oblique, or parallel to the bank.

(1) Data Input

Input data groups are arranged in six topical tabs which are: Project descriptions, Effluent properties, Ambient conditions, Discharge conditions, Mixing Zone definitions, and Output control. All the data input requirements of CORMIX are included in the Checklist for Data Preparation.



CORMIX Checklist for Data Preparation – Version v5.0

PROJECT LEGEND

Project File Name: ____________________________ **Design Case:** ____
Site Name: **Prepared By:** **Date:**

EFFLUENT DATA

☐ Non-Fresh Water Effluent Density
Density ρ_0: ....kg/m³

☐ Fresh Water Effluent Density
☐ Temperature T_0: .... °C ☐ Density ρ_0: .... kg/m³

Discharge Excess Concentration:.................
☐ Effluent Flowrate Q_0: ....m³/s ☐ Effluent Velocity U_0: ....m/s

Pollutant Types
☐ Conservative ☐ Non Conservative: ..../day ☐ Heated – Heat Loss Coefficient: ....W/m²/°C

☐ **Brine**
☐ **Sediment:** Chunks: .... % Sand: .... % Coarse Silt: .... % Fine Silt: ....% Clay: .... %
Total Sediment Concentration:............ kg/m³

AMBIENT GEOMETRY / FLOW FIELD DATA

Average Depth H_a: .... m
Depth at Discharge H_d: .... m

☐ **Unbounded**

☐ **Bounded:** Width BS: .... m
Appearance: ☐ Uniform ☐ Slight Meander ☐ Highly Irregular

☐ Steady
☐ Ambient Flowrate Q_a .... m³/s
☐ Ambient Velocity U_a .... m/s

☐ Unsteady
Period ....hr Max Velocity U_m .... m/s Tidal Velocity at this Time U_a: .... m/s
☐ At Time: ....hr Before Slack ☐ At Slack – Δ Time: .....hr ☐ At Time: ....hr After Slack

☐ Single Slope
Slope S: .... %
Near Shore Velocity:................................ m/s
Near Shore Darcy-Weisbach f: ....

☐ Near & Far Slope
☐ Near Shore Slope S_1 ....% ☐ Far Slope S_2: .... %
☐ Near Shore Velocity U_{a1}.......................m/s ☐ Far Shore Velocity U_{a2}: .... m/s
☐ Near Shore Darcy-Weisbach f_1: .... ☐ Far Shore Darcy-Weisbach f_2: ....
☐ Breakpoint: .... m

☐ Manning's n: .... Wind Speed: ....m/s

AMBIENT DENSITY DATA

Water Body: ☐ **Fresh Water** ☐ **Non-Fresh Water**

☐ **Uniform** Fresh: ☐ Temperature: .... °C ☐ Density ρ_a: .... kg/ m³ Non-Fresh: Density ρ_a: .... kg/ m³

☐ **Stratified** ☐ **Type A** ☐ **Type B:** Pycnocline Height: ....m ☐ **Type C:** Pycnocline Height: ....m Jump: ....kg/ m³ / °C
Density ρ: At Surface ρ_{as} .... kg/m³/ °C At Bottom ρ_{ab}:................................ kg/ m³ / °C

☐ **Brine & Sediment Only** Level 1 Density ρ_1: .. kg/ m³ Sub 1:...........m; Level 2 Density ρ_2:.......kg/ m³ Sub 2:.......... m

DISCHARGE GEOMETRY DATA

CORMIX 1 – Single Port

Nearest Bank: ☐ Left ☐ Right
Dist. to Nearest Bank: .... m
Vert. Angle θ_0:...... °; Horiz. Angle σ_0: ..... °
☐ Port Diameter D_0: ....m
☐ Port Area A_0: ....m²

Submerged
Port Height above Bottom h_0: .... m

Above Surface
Port Height above Surface..................... m
☐ Jet-like ☐ Spray ☐ Area
Deflector Plate: . ☐ With or ☐ Without

CORMIX 2 – Multiport

Nearest Bank: ☐ Left ☐ Right
☐ Unidirectional ☐ Staged ☐ Altern./ Vert.
N° of openings:.........; Diffuser Length:.......... m
Dist. to 1nd end-point YB_1: ....m
Dist. to 2st far end-point YB_2: ....m
Port Height h_0: ....m; Port Diameter D_0: .... m
Contraction Ratio:..........

Angles (degrees)
Vert. Angle θ:........ °; Horiz. Angle σ: .... °
Align. Angle γ: ...... °; Relat.Orient. Angle β:...... °
Nozzle Direction: ☐ Same or ☐ Fanned Out

CORMIX 3 – Surface Discharge

Discharge Located: ☐ Left ☐ Right
Horiz. Angle σ: .... °
Local Depth at Discharge Outlet: .... m
☐ **Flush** ☐ **Co-flowing**
☐ **Protruding:** Distance from Bank: .... m

Discharge Outlet
☐ **Channel:** Width: ....m; Depth b_0: ..... m
☐ **Pipe:** Diameter D_0: .... m
Bottom Invert Depth:.................................... m
Local Bottom Slope at Chanel Entry:........... °

MIXING ZONE DATA

☐ Non-Toxic Effluent ☐ Toxic Effluent
☐ WQ Standard: .... ☐ No WQ Standard CMC : .... CCC : ....

☐ Mixing Zone Specified ☐ No Mixing Zone Specified
☐ Trajectory: ....m ☐ Downstream Distance: ....m ☐ Width: .... % / m ☐ Area: .... %
Region of Interest: ....m Grid Intervals for Display: ....

Figure A1. COMIX checklist sheet

(2) Data Output

The ‘Output’ tab form has radio control buttons to control CORMIX output in a simulation. The user can display, print, display and print, or have no output of the prediction file (fn.prd), session report (fn.ses), flow class description (fn.flw), design recommendations (fn.rec), and processing record (fn.jrn). In addition, the user can select radio buttons to show the rule-tree stem and leaf display of the rules used in data processing.

(3) Flow Classification

The table lists and describes categories of flow classes available in CORMIX1 and CORMIX2, consider 70 and 62 distinct flow classifications, respectively. Each flow class identifications consists of an alphanumeric label corresponding to the flow category and a number.

Model	Flow Class	Description
CORMIX1 (70 classes)	Classes S	Near bottom discharge flows trapped in a layer within linear stratification.
	Classes IS	Near surface discharge flow trapped in a layer within linear stratification.
	Classes V, H	Near bottom discharge positively buoyant flows in a uniform density layer.
	Classes IV, IH	Near surface discharge positively buoyant flows in a uniform density layer.

	Classes NV, NH	Near bottom discharge negatively buoyant flows in uniform density layer.
	Classes IPV, IPH	Near surface discharge positively buoyant flows in uniform density layer.
	Classes A	Near bottom discharge flows affected by dynamic bottom attachment
	Classes IA	Near surface discharge flows affected by dynamic surface attachment
CORMIX2 (62 class)	Classes MS	Near bottom discharge flows trapped in a layer within linear ambient stratification.
	Classes IMS	Near surface discharge flow trapped in a layer within linear ambient stratification.
	Classes MU	Near bottom discharge positively buoyant flows in a uniform density layer.
	Classes IMU	Near surface discharge positively buoyant flows in a uniform density layer.
	Classes MNU	Near bottom discharge Negatively buoyant flows in uniform density layer.
	Classes IMPU	Near surface discharge positively buoyant flows in uniform density layer.

Table A1. Flow specification in CORMIX

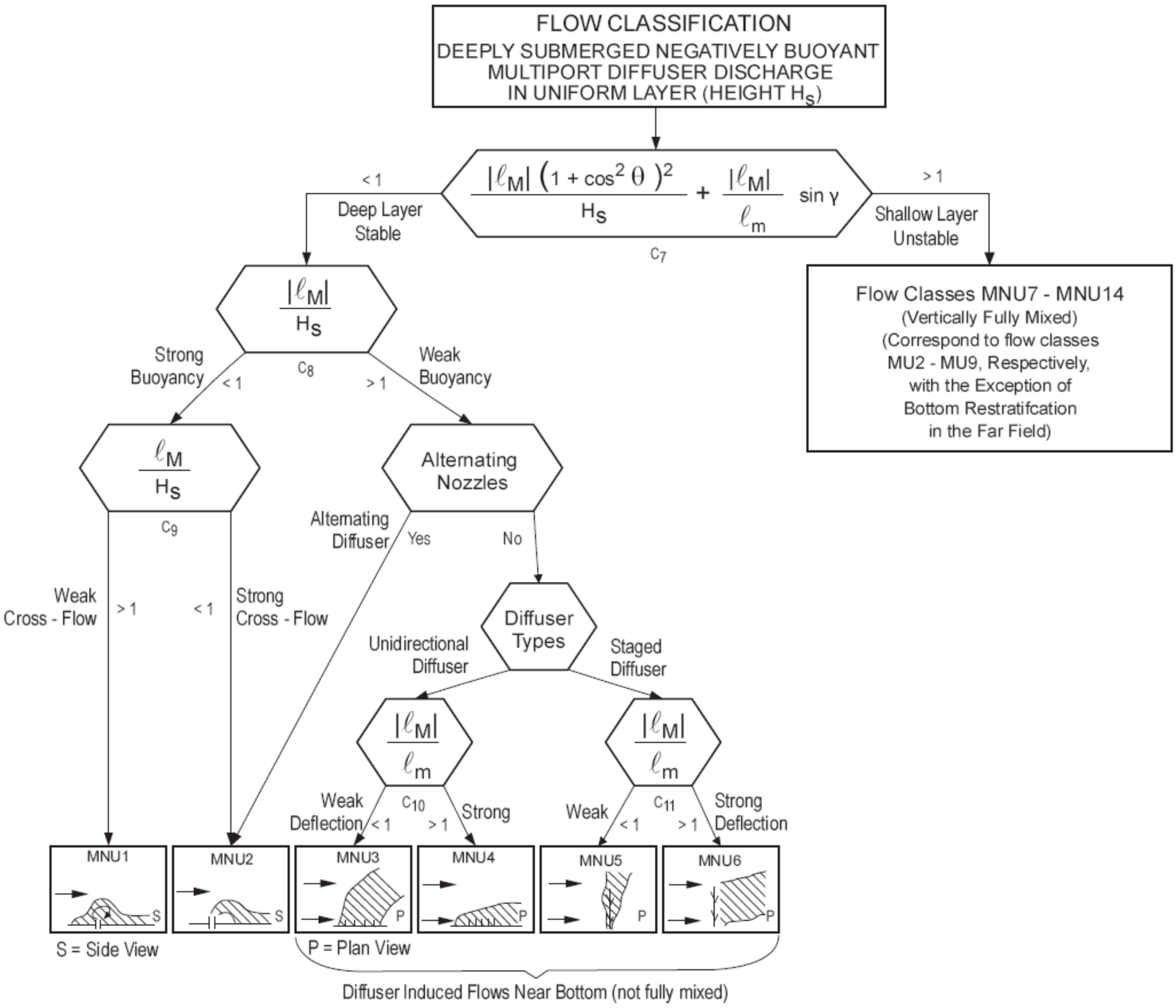


Figure A2. Flow specification of negative buoyant multiport diffuser (MNU1-6)

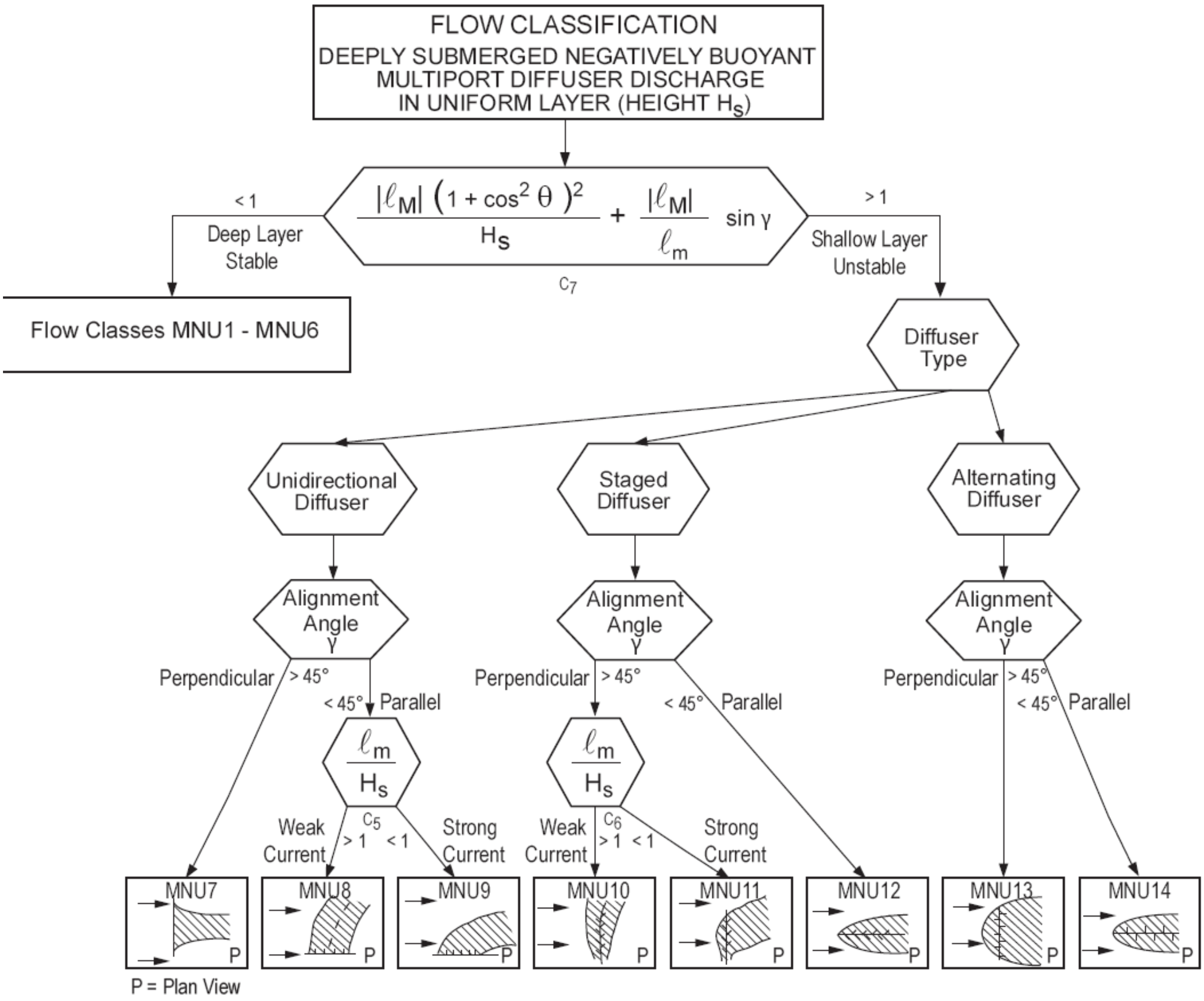
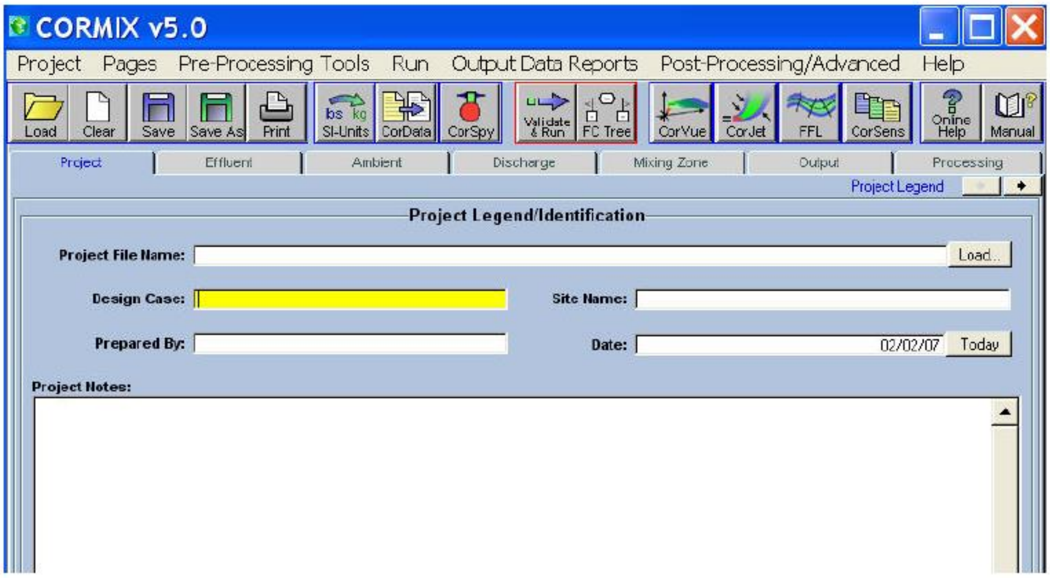


Figure A3. Flow specification of negative buoyant multiport diffuser (MNU7-14)



Project Legend/Identification

Project File Name:

Design Case:

Site Name:

Prepared By:

Date: 02/02/07

Project Notes:

Figure A4. CORMIX GUI window

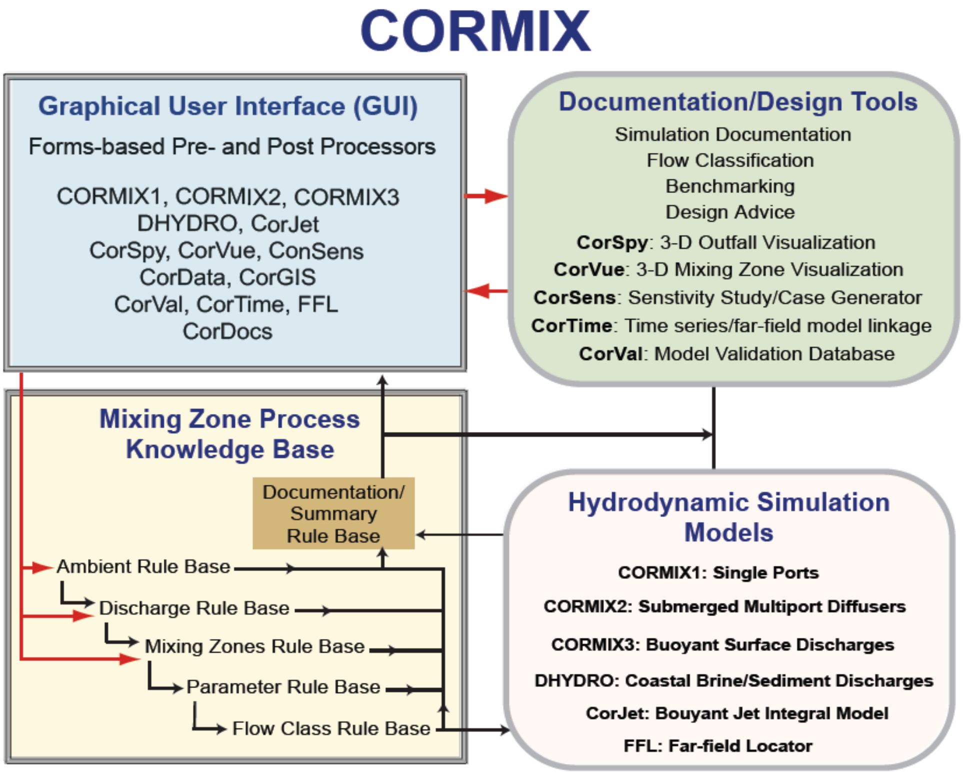


Figure A5. CORMIX system elements and conceptual linkages

5.2 Jet Integral Model

5.2.1 Governing Equation

Equation of motion for a vertical buoyant jet in a density-stratified ambient

1) volume flux: $\frac{d\mu}{dz} = -\lim_{r \to b(z)} (2\pi r \bar{u})$

(5.123)

2) momentum flux: $\frac{dm}{dz} = \int_0^{b(z)} 2\pi r g \theta dr$ (5.124)

3) buoyancy flux: $\frac{d\beta}{dz} = -g \frac{d\varepsilon}{dz} \mu$ (5.125)

where

$$\mu = \int_0^{b(z)} 2\pi r \bar{w} dr \qquad (5.126)$$

$$m = \int_0^{b(z)} 2\pi r \bar{w}^2 dr \qquad (5.127)$$

$$\beta = \int_0^{b(z)} 2\pi r g \bar{w} \theta dr \qquad (5.128)$$

Combine Eq. (5.107) and Eq. (5.123)

$$-\lim_{r \to b(z)} 2\pi r \bar{u} = 2\pi \alpha b_w w_m \qquad (5.129)$$

Adopt Gaussian distributions for $\bar{w}$ and θ

$$\bar{w} = w_m \exp\left[-\left(\frac{r}{b_w} \right)^2 \right] \qquad (5.130)$$

$$\theta = \theta_m \exp\left[-\left(\frac{r}{b_T} \right)^2 \right] \qquad (5.131)$$

Adopt constant value for ratio of half-width, b_T / b_w

$$b_T / b_w = \lambda = 1.2 \qquad (5.132)$$

Substituting Eqs. (5.129) ~ (5.132) into Eqs. (5.123) ~ (5.128), and assuming $b(z) \to \infty$ gives a set of 3 ordinary differential equations for w_m, θ_m, b_W

$$\frac{d}{dz}(\pi b_W^2 w_m) = 2\pi\alpha b_W w_m \qquad (5.133)$$

$$\frac{d}{dz}\left(\frac{\pi}{2} b_W^2 w_m^2 \right) = \pi g \lambda^2 b_W^2 \theta_m \qquad (5.134)$$

$$\frac{d}{dz}\left(\frac{\pi g \lambda^2 b_W^2 w_m \theta_m}{1+\lambda^2} \right) = -g \frac{d\varepsilon}{dz} \pi b_W^2 w_m \qquad (5.135)$$

Initial condition for w_m, θ_m, b_W are given as

$$\left[\pi b_W^2 w_m \right]_0 = Q \qquad (5.136)$$

$$\left[\frac{\pi}{2} b_W^2 w_m^2 \right]_0 = M \qquad (5.137)$$

$$\left[\pi g \frac{\lambda^2}{1+\lambda^2} w_m b_W^2 \theta_m \right]_0 = B \qquad (5.138)$$

General cases:

Governing Equations:

x-momentum eq.

z-momentum eq.

buoyancy flux eq.

geometric eqs - 3 eqs.

closure model: spreading eq. / entrainment eq.

Unknowns: $b_w, w_m, \theta_m, x, y, z, \gamma$

-> system of ODE

-> 4th order Runge Kutta method

5.2.2 Visjet

(1) Introduction

VISJET is a flow visualization tool to portray the evolution and interaction of multiple buoyant jets discharged at different angles to the ambient current. The modeling engine is a robust Lagrangian model, JETLAG, which has been tested extensively against theory, basic laboratory experimental data, field verification studies and applications. It is aimed to facilitate the environmental impact assessment and outfall design studies. It is able to predict the initial mixing of buoyant wastewater discharges in a current, and communicate the predicted impact effectively to the user . The model provides 3D flow visualization of the predicted path and mixing of an arbitrarily inclined buoyant plume in a moving receiving water which may be density-stratified. VISJET can be used to study the impact of either a single or a group of inclined buoyant jets in three-dimensional space. It can be used for outfall design, impact assessment and risk analysis of polluting or natural environmental discharges (e.g. deep sea hydrothermal vents). It can also be used as an educational tool to introduce concepts such as mixing and transport, and assimilative capacity of the receiving water.

(2) Governing equations

In solving arbitrarily-inclined buoyant jet in a crossflow with JETLAG model, there are 7 unknowns with 7 ordinary differential equations, which constitute a closed system. The unknowns are b (radius of the jet), w (velocity of the jet), θ (angle between discharge port and x-axis), x, y, z (center of the disk) and γ_u (angle between excess velocity vector and x-axis)

Table A2. Governing equations used in VISJET

x-momentum		$\frac{d}{ds}\left[\cos\gamma_u\left(I_m\overline{U_g^2}b^2+\overline{u_a}\cos\gamma_u I_q\overline{U_g}b^2\right)\right]=0$
z-momentum		$\frac{d}{ds}\left[\sin\gamma_u\left(I_m\overline{U_g^2}b^2+\overline{u_a}\cos\gamma_u I_q\overline{U_g}b^2\right)\right]=I_\Delta b^2\overline{\Delta_c}$
buoyancy flux		$\frac{d}{ds}\left[\overline{u_a}\cos\gamma_u I_\Delta\overline{\Delta_c}b^2+\overline{U_g}I_{q\Delta}\overline{\Delta_c}b^2\right]=0$
diffusion assumption		$db/ds=k_g\overline{U_g}\left(\overline{u_a}\cos\gamma_u+\overline{U_g}\right)$
geometric relationships	x	$dx/ds=\left(\overline{u_a}+\overline{U_g}\cos\gamma_u\right)/\left(\overline{u_a}\cos\gamma_u+\overline{U_g}\right)$
	y	$dy/ds=\left(\overline{U_g}\cos\gamma_u\sin\beta\right)/\left(\overline{u_a}\cos\gamma_u+\overline{U_g}\right)$
	z	$dz/ds=\overline{U_g}\sin\gamma_u/\left(\overline{u_a}\cos\gamma_u+\overline{U_g}\right)$

(3) Input parameters/output results

VISJET simulates the mixing of single or multiple buoyant jets discharged from one or more risers mounted on an ocean outfall. In a particular application, the input parameters for the ambient condition, the outfall, riser, and jet characteristics are needed. The following Table 2 is a input parameters required to run VISJET.

Table A3. Input parameters in VISJET

parameter	assignment	attribute	meaning
ambient parameter	specifying the vertical structure of the ambient water:	depth	depth below surface
		salinity/density	ambient salinity or density
		temperature	ambient temperature
		current	horizontal current speed
outfall parameter	specifying the properties of the outfall	depth	depth below surface
		salinity/density	effluent salinity or density
		temperature	effluent temperature
		length	length of outfall
		diameter	Diameter of the outfall
riser parameter	specifying the properties of the riser	flow	sum of the effluent flow of all the ports mounted on the riser
		distance	distance from the offshore end of the outfall
		bottom radius	radius at the bottom of the riser
		top radius	radius at the top of the riser
		height	the height of the riser
jet parameter	specifying the properties of the jet	flow	effluent flow from the port
		diameter	port diameter
		port height	port height

		vertical angle	vertical jet discharge angle relative to Horizontal plane
		horizontal angle	horizontal angle of current direction with respect to jet discharge
		salinity/density	effluent salinity or density
		temperature	effluent temperature

After the simulation, you can obtain disk/cross-section information. With the concentration results, dilution rate at a given trajectory can be acquired. The following Table 3 is about output results in VISJET.

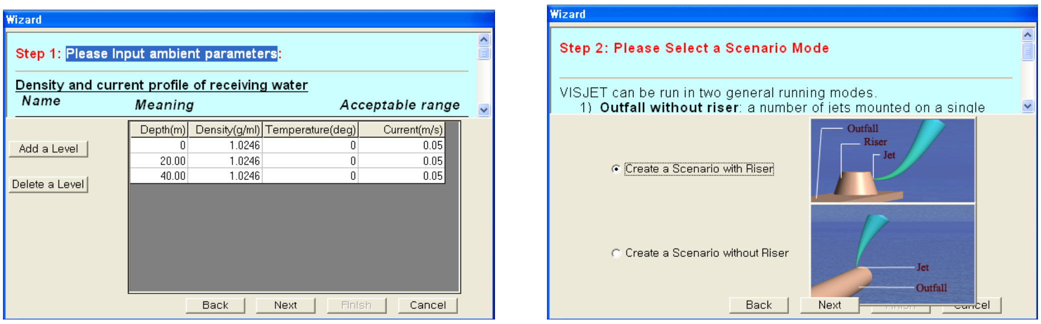
Table A4. Output results in VISJET

result	attribute	meaning
disk information	center position	the (x, y, z) co-ordinates of the center of the selected disk, which is the computed jet trajectory
	radius	jet half width of the selected disk
	thickness	thickness of the selected disk
	angle	vertical angle is the angle between the jet axis and the horizontal plane; horizontal angle is the angle between the x-axis and the projection of the jet axis on the horizontal plane
	velocity	jet velocity of the selected disk
	concentration	maximum/average concentration of the selected disk

cross-section information	position	the (x, y, z) co-ordinates at the position of the point selected by the mouse or pointing device
	total area	total projected area of the jets on the cutting plane
	sum of areas	sum of all the projected areas of the individual(selected) jets
	horizontal/vertical span	the horizontal/vertical span of the projected region for the selected jet
	concentration	average concentration at the above position

(4) GUI-window and result

VISJET is a Windows-based flow visualization tool to predict initial mixing of buoyant wastewater discharges in a current. The simulation wizard and visual toolbox allow the user to input the field condition easily and manipulate the view of the graphic outputs and the cutting plane. The following figures are captured frames of simulation procedures and 3-D result view in VISJET.



Step 1: Please Input ambient parameters:

Density and current profile of receiving water

Name	Meaning	Acceptable range

Depth(m)	Density(g/ml)	Temperature(deg)	Current(m/s)
0	1.0246	0	0.05
20.00	1.0246	0	0.05
40.00	1.0246	0	0.05

Step 2: Please Select a Scenario Mode

VISJET can be run in two general running modes.
1) **Outfall without riser**: a number of jets mounted on a single

● Create a Scenario with Riser

○ Create a Scenario without Riser

Figure A6. Simulation procedures in VISJET

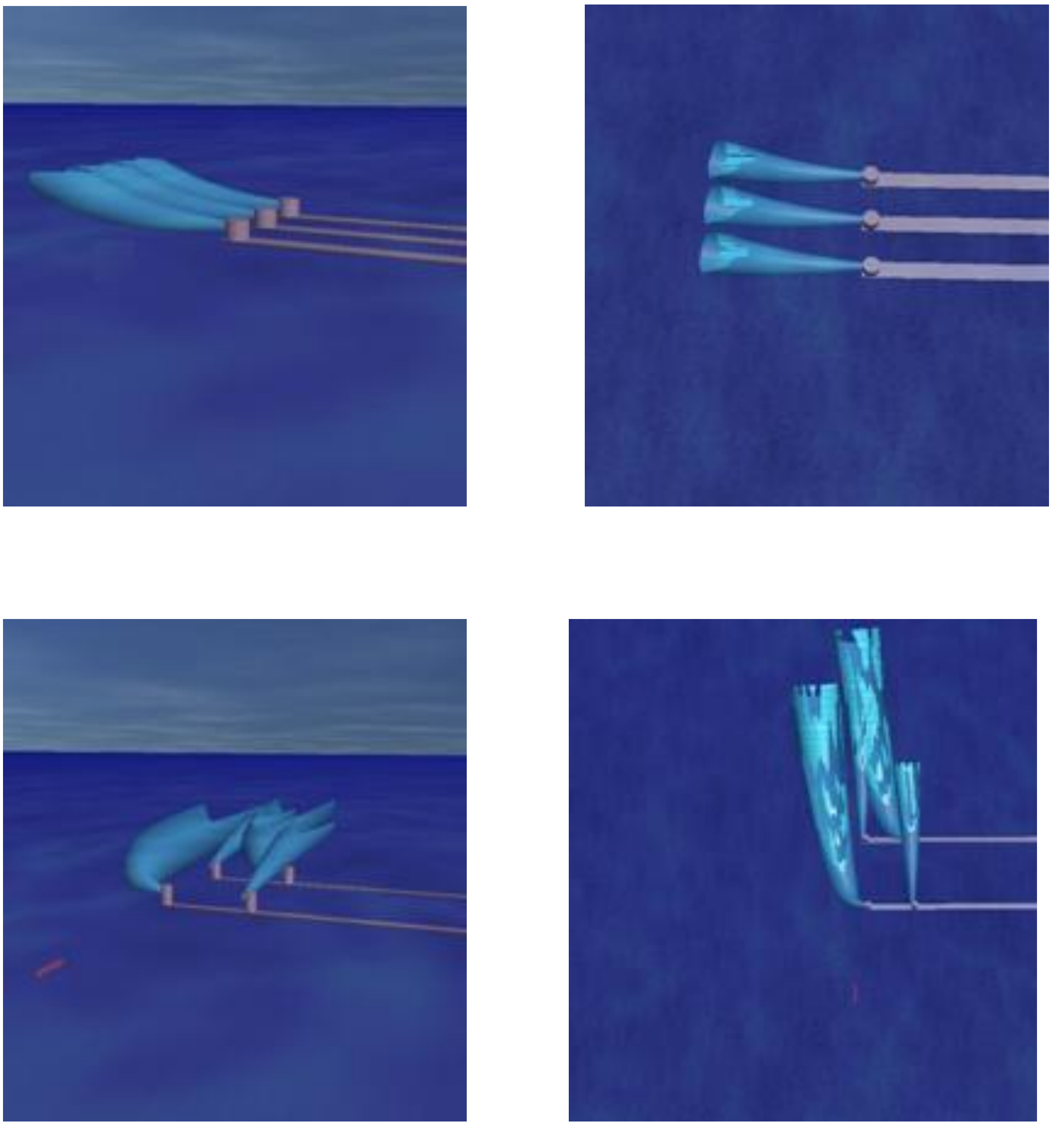


Figure A7. 3-D result view in VISJET

5.3 3D Hydrodynamic Model

5.3.1 Mathematical Model

5.3.2 FLUENT

5.3.3 FLOW-3D

5.4 Example Application of Numerical Model

Model Parameter Specifications

B1. CORMIX

(1) Diffuser type

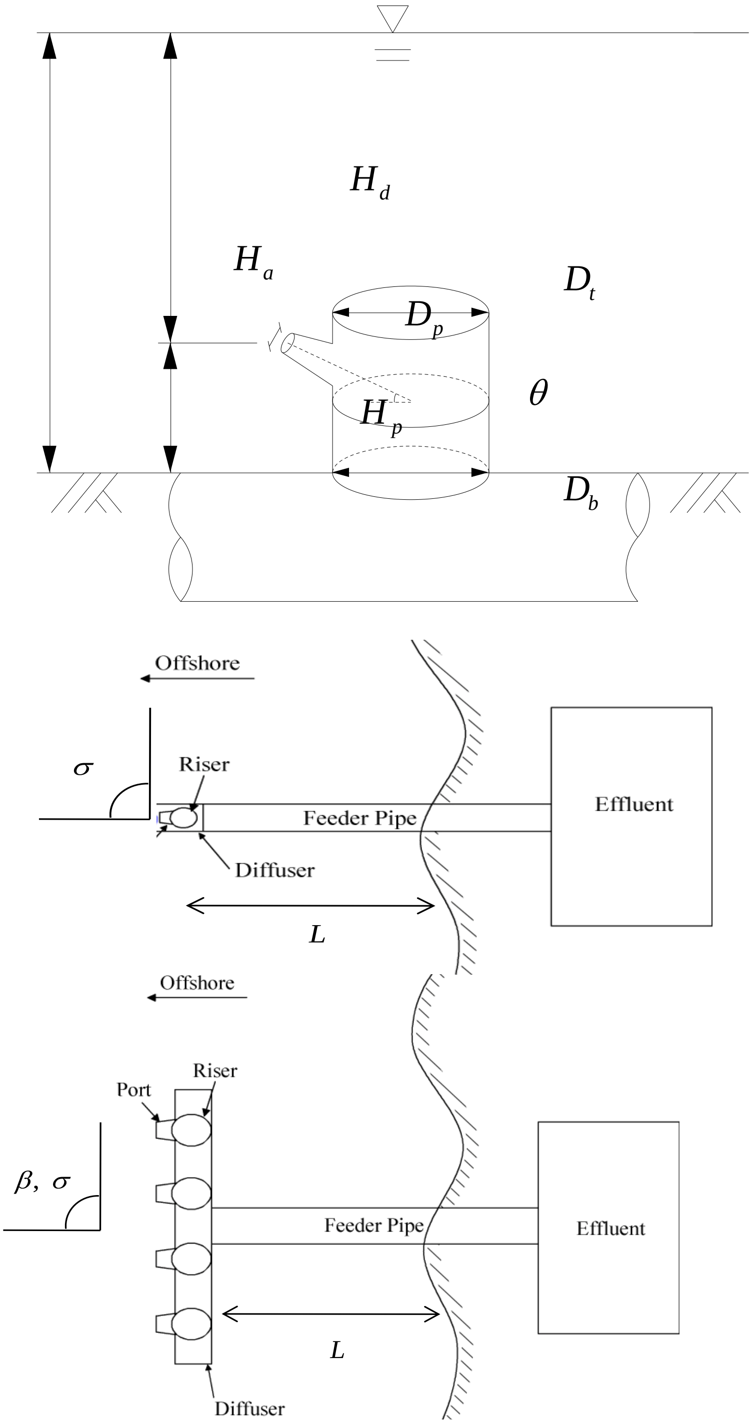


Figure B1. Definition of port geometry *a. Single port*

Table B1. Model Parameter Specifications (Single port)

Parameter	*Value*
Ambient Data	
Average Depth (Ha)	*7.5 m*
Discharge Depth (Hd)	*6.7 m (= Ha – 0.8 m)*
Manning coefficient (n)	*0.031*
Wind Speed (Uw)	*2.19 m/s*
Ambient density (ρa)	*Non-fresh water 1022.17 kg/m^3*
Effluent Data	
Discharge density (ρ0)	*Non-fresh water 1024.23 kg/m^3*
Temperature difference (ΔT)	*7 ℃*
Heat loss coefficient	*25 W/m^2/℃*
Discharge Geometry Data	
Nearest bank	*Left / Right*
Distance to nearest bank	*22 m*
Port height (h0)	*0.8 m*
Port diameter (D)	*1.0 m*
Vertical angle (θ)	*30°*
Horizontal angle (σ)	*90°*

b. Tee diffuser

Table B2. Model Parameter Specifications (Tee diffuser)

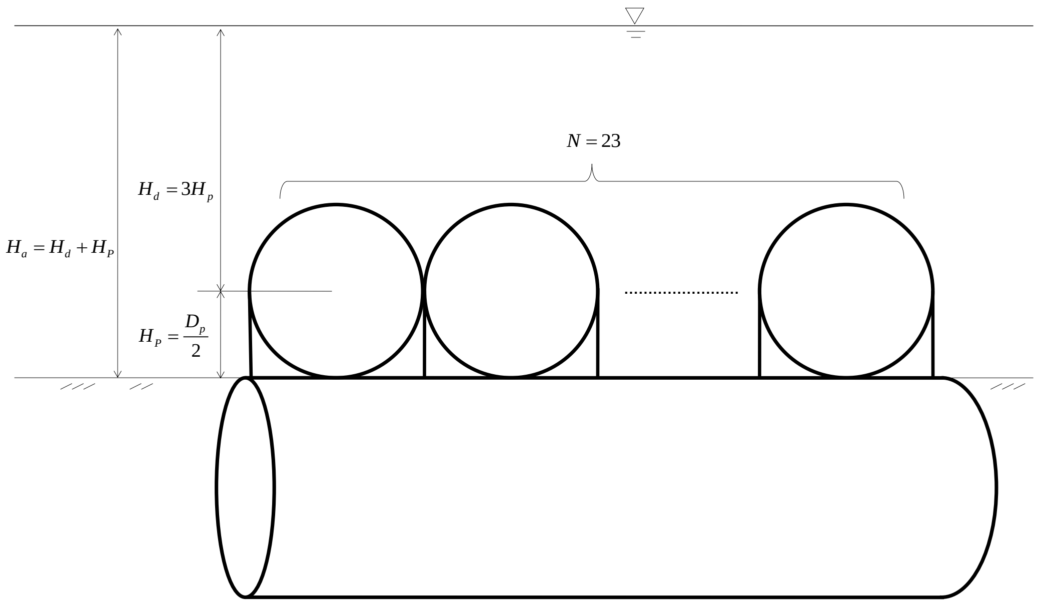
Parameter	*Value*
Ambient Data	
Average Depth (Ha)	*7.5 m*
Discharge Depth (Hd)	*6.7 m* *(= Ha – 0.8 m)*
Manning coefficient (n)	*0.031*
Wind Speed (Uw)	*2.19 m/s*
Ambient density (ρa)	*Non-fresh water* *1022.17* kg/m^3
Effluent Data	
Discharge density (ρ0)	*Non-fresh water* *1024.23* kg/m^3
Temperature difference (ΔT)	*7 ℃*
Heat loss coefficient	*25* W/m^2*/℃*
Discharge Geometry Data	
Nearest bank	*Left / Right*
Diffuser length (Ld)	*10 m*
Distance to on/ other end point	*22 / 22 m*
Port height (h0)	*0.8 m*
Port diameter (D)	*0.3 m*
Contraction ratio	*1 (rounded)*
Total number of opening (N)	*4*
Nozzles per riser	*Single*
Alignment angle (γ)	*0 °*
Diffuser arrangement	$\theta = 30° / \beta = 90° / \sigma = 90°$

c. Staged diffuser

Table B3. Model Parameter Specifications (Staged diffuser)

Parameter	*Value*
Ambient Data	
Average Depth (Ha)	*5.358 m* *7.358 m*
Discharge Depth (Hd)	*4.558 m* *6.558 m* *(= Ha – 0.8 m)*
Manning coefficient (n)	*0.031*
Wind Speed (Uw)	*2.19 m/s*
Ambient density (ρa)	*Non-fresh water* *1022.17 kg/m^3*
Effluent Data	
Discharge density ($\rho 0$)	*Non-fresh water* *1024.23 kg/m^3*
Temperature difference (ΔT)	*4.5 ℃ / 16 ℃*
Heat loss coefficient	*25 W/m^2/℃*
Discharge Geometry Data	
Nearest bank	*Left / Right*
Diffuser length (Ld)	*10 m*
Distance to on/ other end point	*160.48 / 170.48 m*
Port height (h0)	*0.8 m*
Port diameter (D)	*0.3 m*
Contraction ratio	*1 (rounded)*
Total number of opening (N)	*4*
Nozzles per riser	*Single*
Alignment angle (γ)	*0 °*
Diffuser arrangement	*$\theta = 30° / \beta = 0° / \sigma = 90°$*

(2) Outfall pit type



Assumption 1: Port Series (N = 23)

Table B4. Model Parameter Specifications (Outfall pit_ Assumption1)

Parameter	*Value*
Ambient Data	
Average Depth (Ha)	*0.585 m* *(= Hd + 30%)*
Discharge Depth (Hd)	*0.76 m (= 3 h0)*
Manning coefficient (n)	*0.031*
Wind Speed (Uw)	*2.19 m/s*
Ambient density (ρa)	*Non-fresh water 1022.17* kg/m^3
Effluent Data	
Discharge density (ρ0)	*Non-fresh water* *1024.23 kg/m3*
Temperature difference (ΔT)	*7 ℃*
Heat loss coefficient	*25* W/m^2*/℃*
Discharge Geometry Data	
Nearest bank	*Left / Right*
Diffuser length (Ld)	*11.5 m*
Distance to on/ other end point	*22 / 22 m*
Port height (h0)	*0.25 m (= D/2)*
Port diameter (D)	*0.497 m*
Contraction ratio	*1 (rounded)*
Total number of opening (N)	*23*
Nozzles per riser	*Single*
Alignment angle (γ)	*0 °*
Diffuser arrangement	*θ = 30° / β = 0° / σ = 90°*

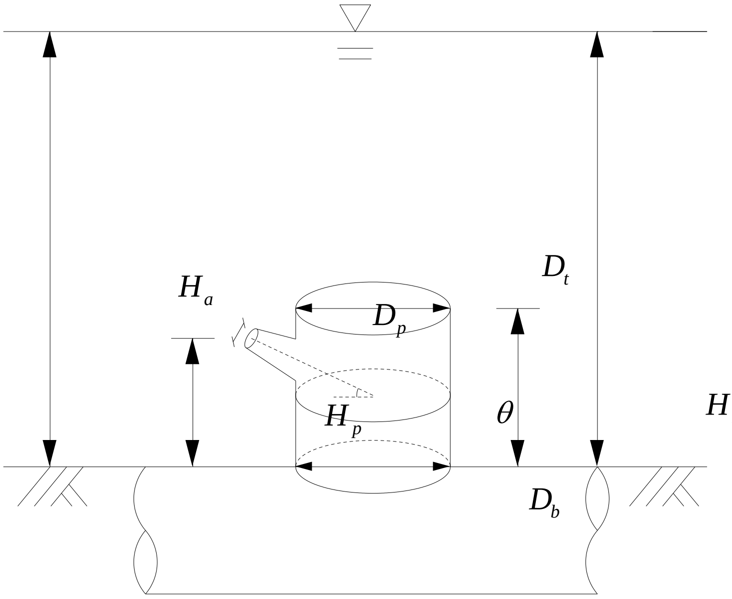
Assumption 2: Neutral buoyancy

Table B5. Model Parameter Specifications (Outfall pit_ Assumption2)

Parameter		Value
Ambient Data		
Average Depth (Ha)		0.646 m (= Hd + 30%)
Discharge Depth (Hd)		0.497 m
Manning coefficient (n)		0.031
Wind Speed (Uw)		2.19 m/s
Ambient density (ρa)		Non-fresh water 1024.23 kg/m^3
Effluent Data		
Discharge density (ρ0)		Non-fresh water 1024.23 kg/m^3
Temperature difference (ΔT)		7 ℃
Heat loss coefficient		25 W/m^2/℃
Discharge Geometry Data		
Discharge located on		Left / Right
Horizontal angle (σ)		90°
Bottom slope		2.58°
Depth at discharge (Hd0)		0.497 m
Channel	Width (B0)	9 m
	Depth (h0)	0.497 m

B2. VISJET

(1) Diffuser



parameter	assignment	attribute	meaning
effluent parameter	specifying the properties of the jet	flow rate (m^3/s)	effluent flow from the port
		depth (m)	depth below surface (He)
		salinity (psu) density (g/ml)	effluent salinity or density
		temperature (℃)	effluent temperature
ambient parameter	specifying the vertical structure of the ambient water	depth (m)	depth below surface (Ha)
		salinity (psu) density (g/ml)	ambient salinity or density
		temperature (℃)	ambient temperature
		current (m/s)	horizontal current speed
diffuser geometry parameter	specifying the properties of the outfall	length (m)	length of outfall
		diameter (m)	diameter of the outfall
	specifying the properties of the riser	distance (m)	distance from the offshore end of the outfall
		bottom diameter (m)	diameter at the bottom of the riser (Db)
		top diameter (m)	diameter at the top of the riser (Dt)
		height (m)	the height of the riser (Hr)
		diameter (m)	port diameter (Dp)
		port height (m)	port height (Hp)
		vertical angle (˚)	vertical jet discharge angle relative to Horizontal plane (θ)
		horizontal angle (˚)	horizontal angle of current direction with respect to jet discharge

a. Single port

Table B6. Model Parameter Specifications (Single port)

Parameter	*Value*
Ambient Data	
Ambient Depth	*7.5 m*
Current Velocity	*0 m/s, 1.0 m/s*
Current angle	*90°*
Ambient Salinity	*34.44 psu*
Ambient Temperature	*27.5 ℃*
Effluent Data	
Effluent Depth	*7.5 m*
Effluent Temperature	*20.5 ℃*
Effluent Discharge	*0.944 cms, 1.578 cms*
Effluent Salinity	*34.44 psu*
Discharge Geometry Data	
Riser height	*1 m*
Top radius of riser	*0.5 m*
Bottom radius of riser	*0.75 m*
Port height	*0.8 m*
Port diameter	*1.0 m*
Vertical angle	*30°*
Horizontal angle	*90°*

b. Tee diffuser

Table B7. Model Parameter Specifications (Tee diffuser)

Parameter	*Value*
Ambient Data	
Ambient Depth	*5.358 m, 7.358 m, 7.5 m*
Current Velocity	*0 m/s, 1.0 m/s*
Current Angle	*90°*
Ambient Salinity	*34.44 psu*
Ambient Temperature	*27.5 ℃*
Effluent Data	
Effluent Depth	*5.358 m, 7.358 m, 7.5 m*
Effluent Temperature	*20.5 ℃*
Effluent Discharge	*0.944 cms, 4.722 cms*
Distance between nearby risers	*2.5 m*
Effluent Salinity	*34.44 psu*
Discharge Geometry Data	
Riser height	*1 m*
Top radius of riser	*0.4 m*
Bottom radius of riser	*0.3 m*
Port height	*0.8 m*
Port diameter	*0.3 m*
Vertical angle	*30°*
Horizontal angle	*90°*

c. Staged diffuser

Table B8. Model Parameter Specifications (Staged diffuser)

Parameter	*Value*
Ambient Data	
Ambient Depth	*7.5 m*
Current Velocity	*0 m/s, 1.0 m/s*
Current Angle	*90°*
Ambient Salinity	*34.44 psu*
Ambient Temperature	*27.5 ℃*
Effluent Data	
Effluent Depth	*7.5 m*
Effluent Temperature	*20.5 ℃*
Effluent Discharge	*0.944 cms, 4.722 cms*
Distance between nearby outfall	*2.5 m*
Effluent Salinity	*34.44 psu*
Discharge Geometry Data	
Riser Height	*1 m*
Top Radius of Riser	*0.4 m*
Bottom Radius of Riser	*0.3 m*
Port Height	*0.8 m*
Port Diameter	*0.3 m*
Vertical Angle	*30°*
Horizontal Angle	*90°*

(2) Outfall pit type

Table B9. Model Parameter Specifications (Outfall pit type)

Parameter	*Value*
Ambient Data	
Ambient Depth (Ha)	*0.497 m*
Current Velocity	*0 m/s, 1.0 m/s*
Current Angle	*90°*
Ambient Salinity	*34.44 psu*
Ambient Temperature	*27.5 ℃*
Effluent Data	
Effluent Depth (He)	*0.697 m*
Effluent Temperature	*20.5 ℃*
Effluent Discharge	*0.944 cms, 1.578 cms, 4.722 cms*
Distance between nearby outfall (l_0)	*0.497 m*
Effluent Salinity	*34.44 psu*
Discharge Geometry Data	
Riser Height (Hr)	*0.697 m*
Top Diameter of Riser (Dt)	*0.497 m*
Bottom Diameter of Riser (Db)	*0.497 m*
Port Height (Hp)	*0.4485 m*
Port Diameter (Dp)	*0.497 m*
Vertical Angle	*30°*
Horizontal Angle	*90°*