Risk Management and Decision Analysis

Seokho Chi*

*Professor, Department of Civil and Environmental Engineering, Seoul National University

PART I

MODELING UNCERTAINTY

- Monte Carlo Simulation

Simulation

- Simulation may be defined as a technique that imitates the operation of a real-world system as it evolves over time.
  - This is normally done by developing a simulation model
- A simulation model takes the form of a set of assumptions about the operation of the system, expressed as mathematical or logical relations between the objects of interest in the system.
  - In contrast to the exact mathematical solutions available with most analytical models, the simulation process involves executing or running the model through time, usually on a computer, to generate representative samples may be seen as a sampling experiment on the real system, with the results being sample points.
  - To obtain the best estimate of the mean of the measure of performance, we average the sample results. Clearly, the sample points we generate, the better our estimate will be.
  - However, other factors, such as the starting conditions of the simulation, the length of the period being simulated, and the accuracy of the model itself, all have a bearing on how good our final estimate will be.

Advantages and Disadvantages

- The major advantage of simulation is that simulation theory is relatively straightforward.

- In general, simulation methods are easier to apply than analytical methods.
  - Whereas analytical models may require us to make many simplifying assumptions, simulation models have few such restrictions, thereby allowing much greater flexibility in representing the real system.

- Once a model is a built, it can be used repeatedly to analyze different policies, parameters, or designs.

Simulation Process and Steps

① Statement of Objectives

② Model Development

③ Data Collection and Preparation

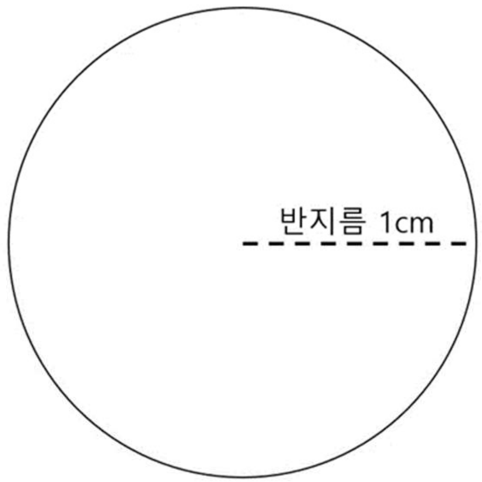
④ Solving the Model using Computer Program

⑤ Verification

⑥ Validation

⑦ Documentation

Monte Carlo Simulation (MCS)



반지름 1cm 인 원이 있다.

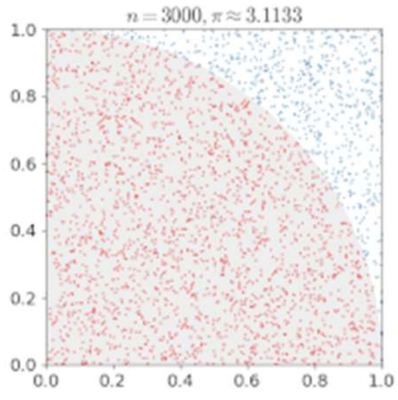
원자 폭탄 개발의 시뮬레이션 수행에는 확률이라는 수학적 지식이 반영되었다. 실제로 맨해튼 프로젝트에 몬테카를로 방법이 원자 폭탄의 확률을 계산하는 데 사용되었다. 임의의 수를 발생시켜 함수 값을 확률 계산하는 알고리즘을 의미하는 용어로, 중성자들이 무작위로 충돌하고 분열하는 현상이 도박과 유사했기 때문에 도박의 대명사인 모나코 북부 도시 몬테카를로라는 이름을 붙인 것으로 추정된다.

Monte Carlo Simulation (MCS)

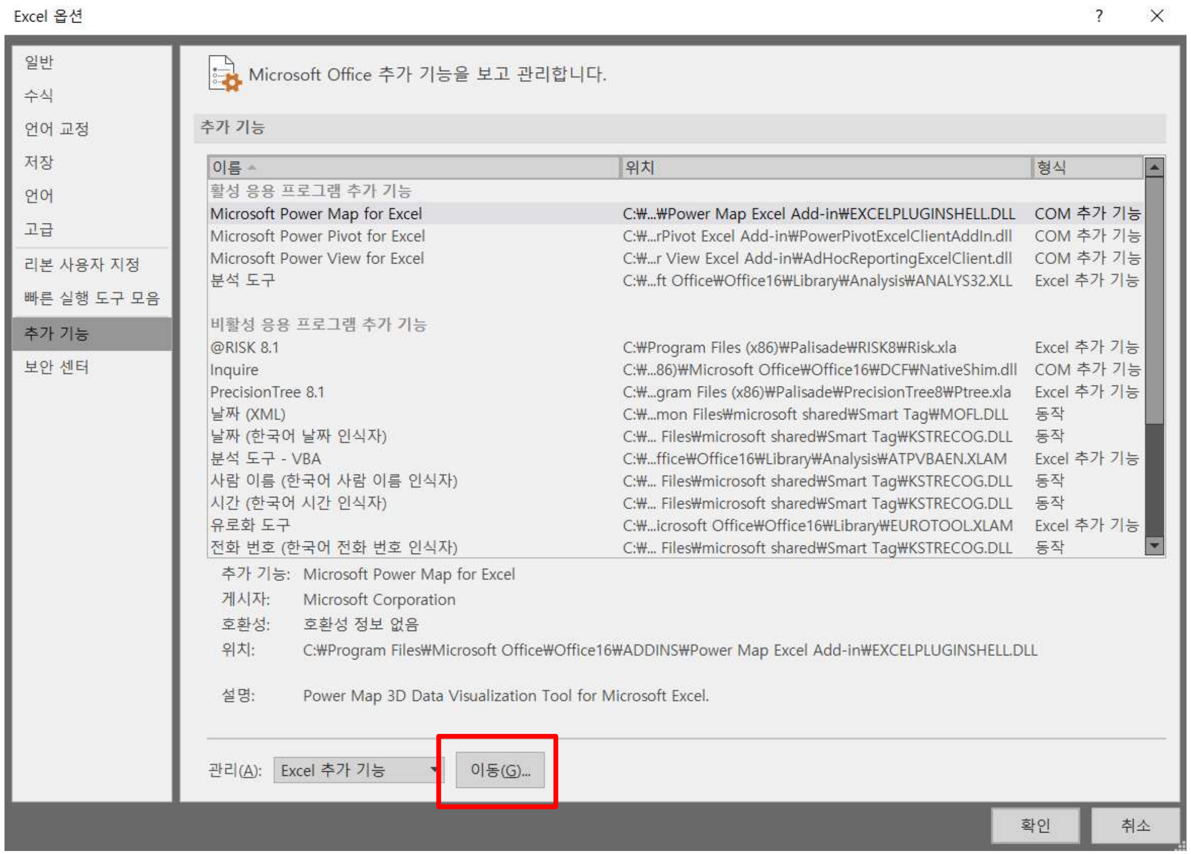
- Code name of technique used in WWII project(맨해튼) to invent atomic bomb
  - Technique originated by 20C U.S. mathematician J. von Neumann
- The principle behind the methods is to develop an analytical model, which is computer based, that predicts the behavior of a system.
- Then, the model is evaluated, and therefore the behavior is predicted, several times.
- Each evaluation (or called simulation cycle) is based on some randomly selected conditions for the input parameters of the system.

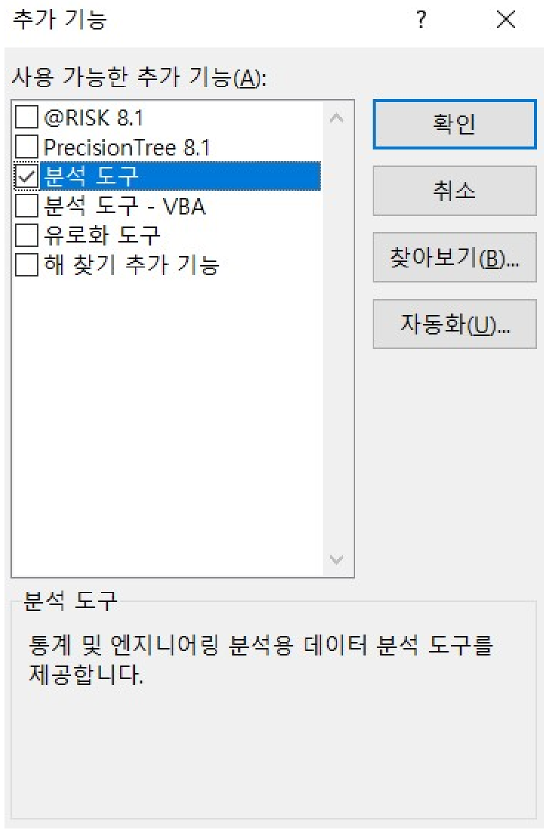
Monte Carlo Simulation을 활용하여 원주률(π) 구하기

- [1,0] x [0,1]에서 점 (x,y)를 표집
- 표집한 점이 중심이 (0,0)에 있고 반지름이 1인 원에 속하는지 계산
  (이는 원의 정의에 따라 $x^2 + y^2$과 1을 비교함으로써 계산)
- 이 과정을 반복하여 원에 속한 점들의 개수를 계산
- 원의 영역은 π/4의 넓이를 가지며 원에 속한 점 개수를 전체 점 개수로 나눈 비율이 이 값을 근사화



Monte Carlo Simulation (MCS)





Monte Carlo Simulation (MCS)

- **General Procedure**

① Definition of the system

② Generation of random numbers

③ Generation of random variables

④ Evaluation of the model N times

⑤ Statistical analysis of the resulting behavior

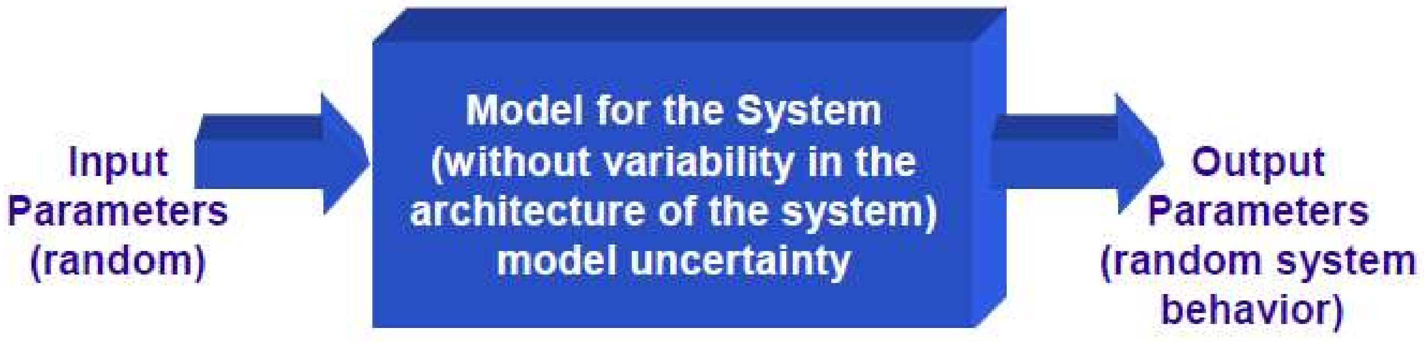
⑥ Study of efficiency and convergence

Monte Carlo Simulation (MCS)

- **General Procedure**

① Definition of the system

- The definition of the system should includes its boundaries, input parameters, output (or behavior) measures, architecture, and models that relate the input parameters and architecture to the output parameters.



Monte Carlo Simulation (MCS)

- **General Procedure**

  ② Generation of random numbers

  - Random numbers are real values, if normalized using the largest possible value, result in real values in the range [0 ,1].
  - Random numbers have a uniform distribution on the range [0 ,1].
  - A set of random numbers should also satisfy the condition of non-correlation for the purpose of simulation use.

Monte Carlo Simulation (MCS)

- **Table of Random Numbers in the Range [0,1]**

0.538246	0.181648	0.172614	0.450166	0.293027	0.030195	0.757836	0.915061
0.663357	0.368934	0.516388	0.656254	0.284258	0.906335	0.329788	0.054487
0.035771	0.053784	0.424573	0.942479	0.293872	0.326815	0.862351	0.358055
0.51356	0.165508	0.667312	0.878444	0.414203	0.100839	0.555287	0.685601
0.880006	0.069305	0.85441	0.371911	0.751341	0.128446	0.678679	0.514995
0.880006	0.069305	0.85441	0.371911	0.751341	0.128446	0.678679	0.514995
0.748794	0.902497	0.629615	0.662531	0.932879	0.018376	0.683876	0.55481
0.115441	0.207278	0.887853	0.812124	0.082143	0.939258	0.666874	0.582525
0.953369	0.543997	0.806486	0.707493	0.503949	0.489926	0.774467	0.248617
0.2436	0.537111	0.181388	0.619277	0.131852	0.131876	0.361814	0.582682
0.610186	0.41158	0.339972	0.080869	0.429448	0.82277	0.63269	0.863227
0.848375	0.043973	0.071429	0.713405	0.56201	0.71605	0.53662	0.357681
0.102922	0.201752	0.61727	0.416471	0.371492	0.633301	0.857578	0.483474
0.009326	0.912932	0.11385	0.3316	0.852807	0.626191	0.035676	0.581386
0.801494	0.365068	0.54875	0.480788	0.032959	0.906331	0.291263	0.706212
0.682049	0.946008	0.960047	0.830463	0.186225	0.123762	0.674147	0.012839

Monte Carlo Simulation (MCS)

- **General Procedure**

③ Generation of random variables

- A random number u is first generated in the range [0 ,1]

- Then the value (x) of a generated continuous random variable, X, is determined as follows:

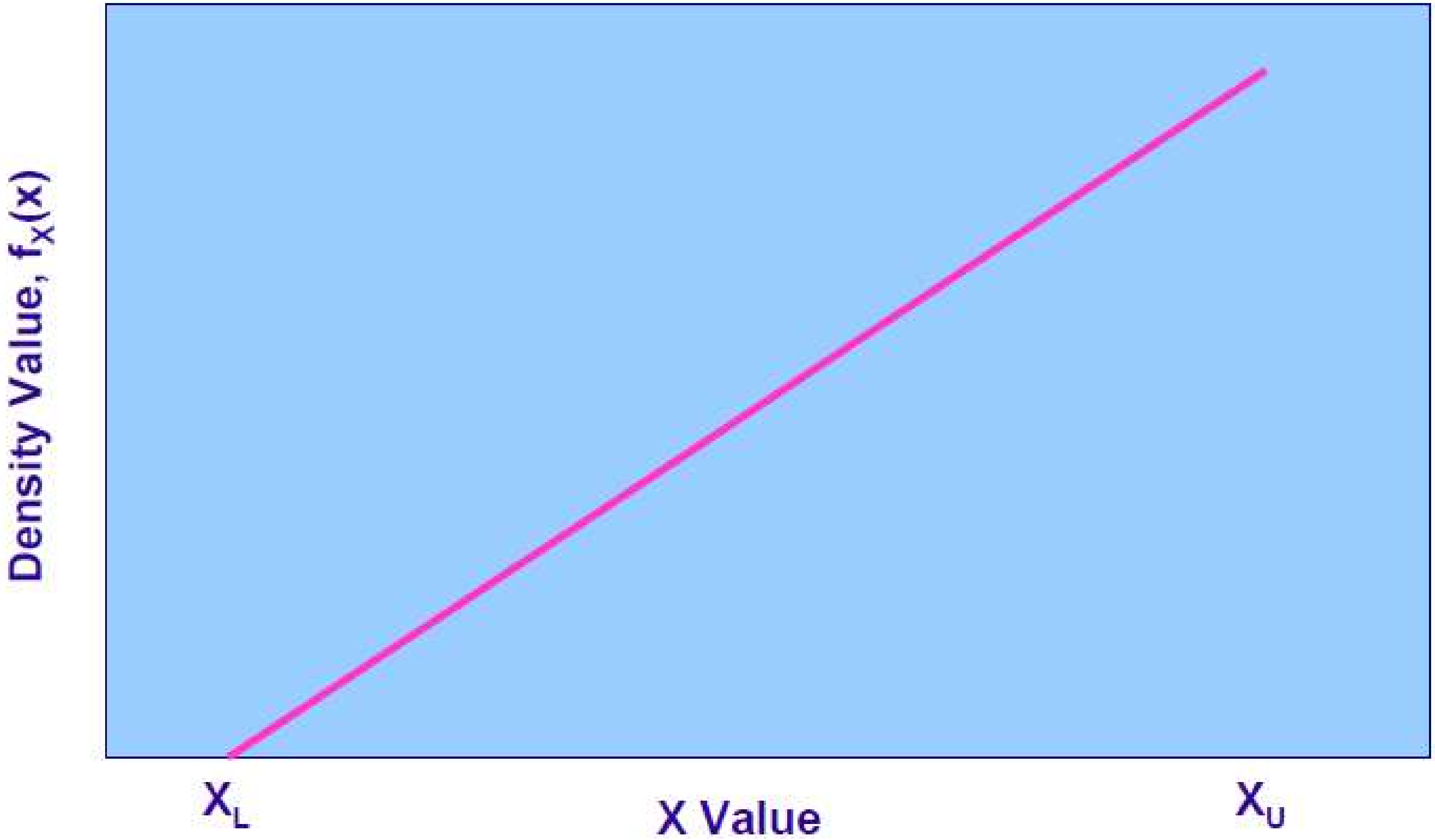
$$X = F_X^{-1}(u)$$

the inverse of the cumulative distribution function of the random variable X evaluated at u

- Since the range of $F_X(X)$ is in the range [0,1], a unique value for X is obtained all the time in each simulation cycle.

Monte Carlo Simulation (MCS)

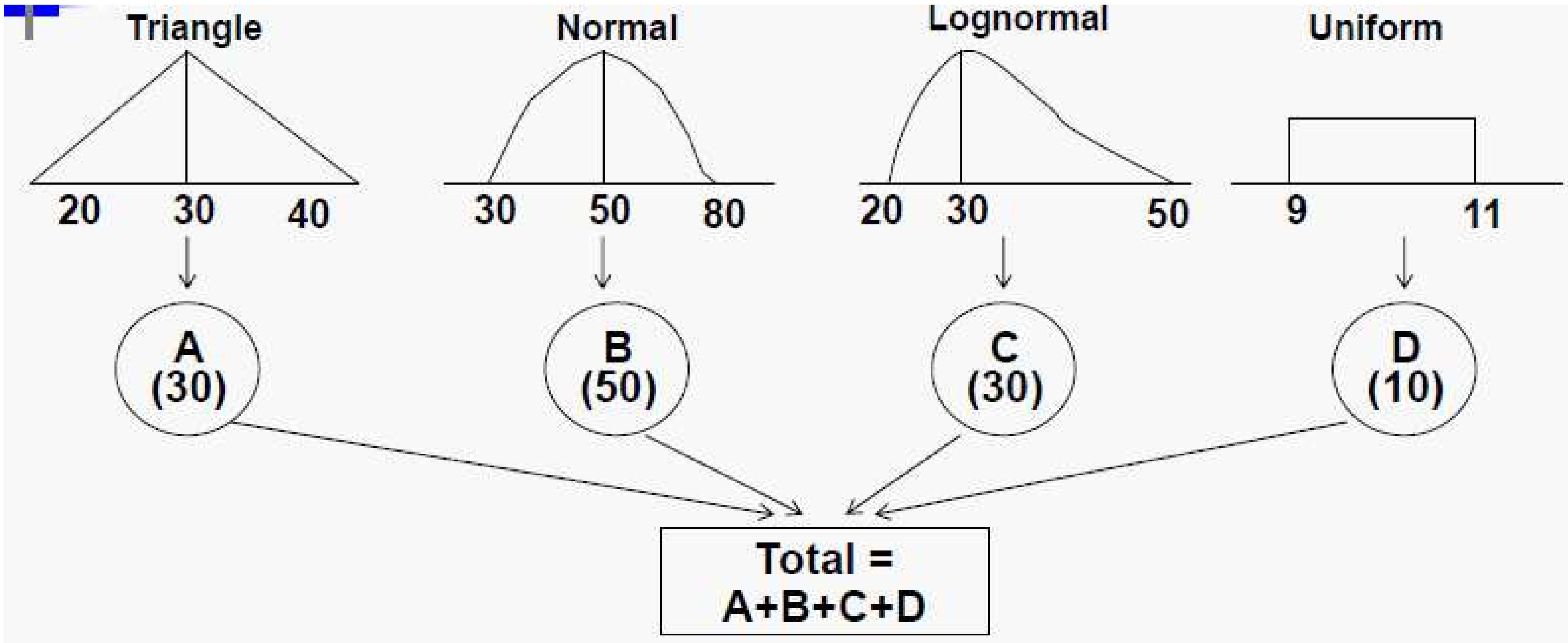
- **Triangular Density Function**



Monte Carlo Simulation (MCS)

- **Advantage**
  - When it is not obvious how to calculate an EMV, we can use this approach.
  - After running the simulation many times randomly, we have an approximation of the probability distribution for the outcomes from the different alternatives.
  - Finally, the results both cumulative curve of payoffs and average values can be used in decision analysis to choose an appropriate option.
  - Commercial software: Oracle Crystal ball, @Risk, etc.

Monte Carlo Simulation (MCS)



1. 1st Try: 30+50+30+10 =120
2. 2nd Try: 20+30+20+ 9 = 79
3. 3rd Try: 40+80+50+11 =189

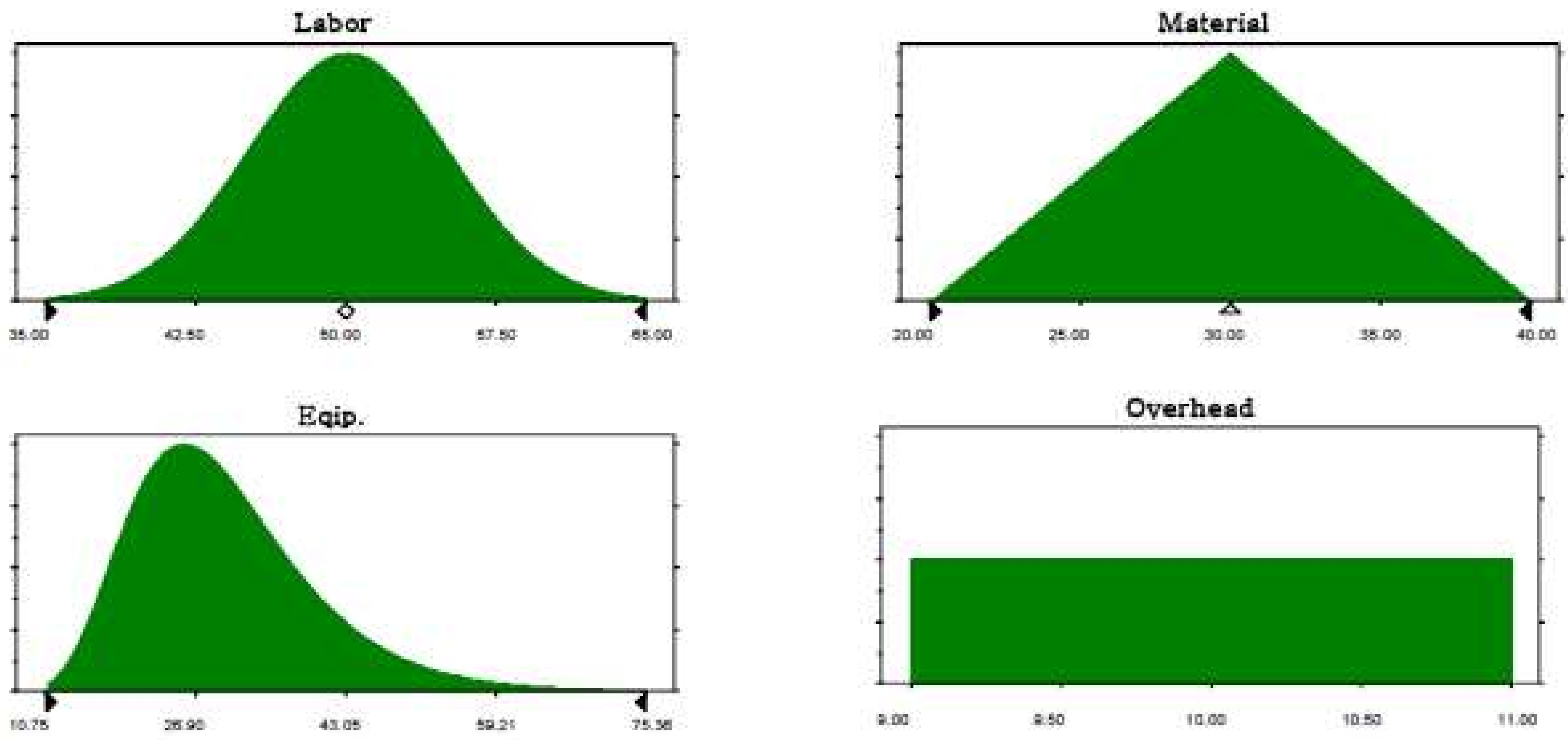
Random Selections →10,000 times → approach to the numerical analysis

Case 1: forecasting cost variance

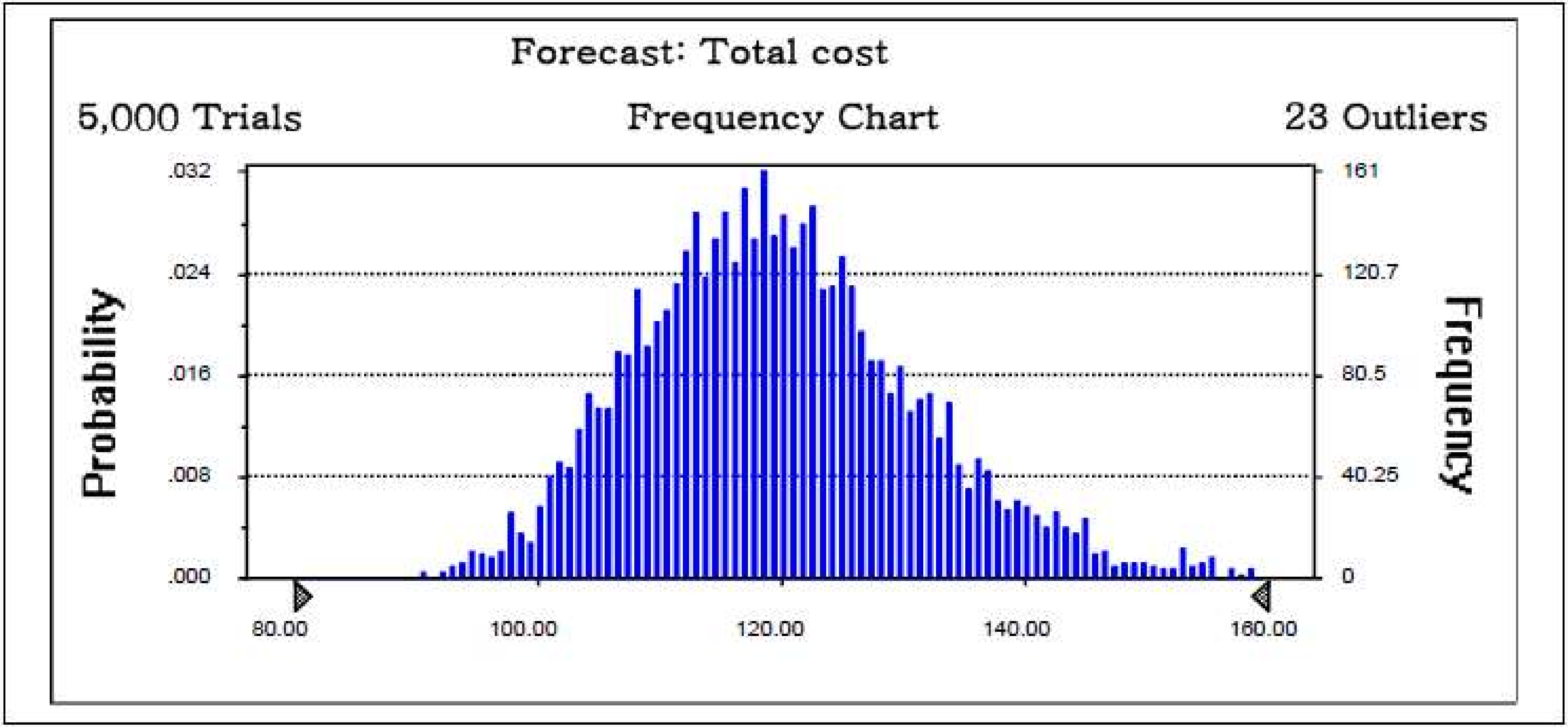
- What forecasting? → cost variance
- Identify the cost elements and probabilistic distribution of these elements
- Then, randomly estimate the range of total cost based on the distribution shapes of each cost element

	Material	Labor	Eqip.	Overhead	Total
	30	50	30	10	120
	Triangle	Norma	Lognormal	Uniform	
Mean	30	50	30	10	
Min	20	35	10	9	
Max	40	65	75	11	
SD	4.01	5	10	0.58	

1. Define the distribution shapes from data fittings



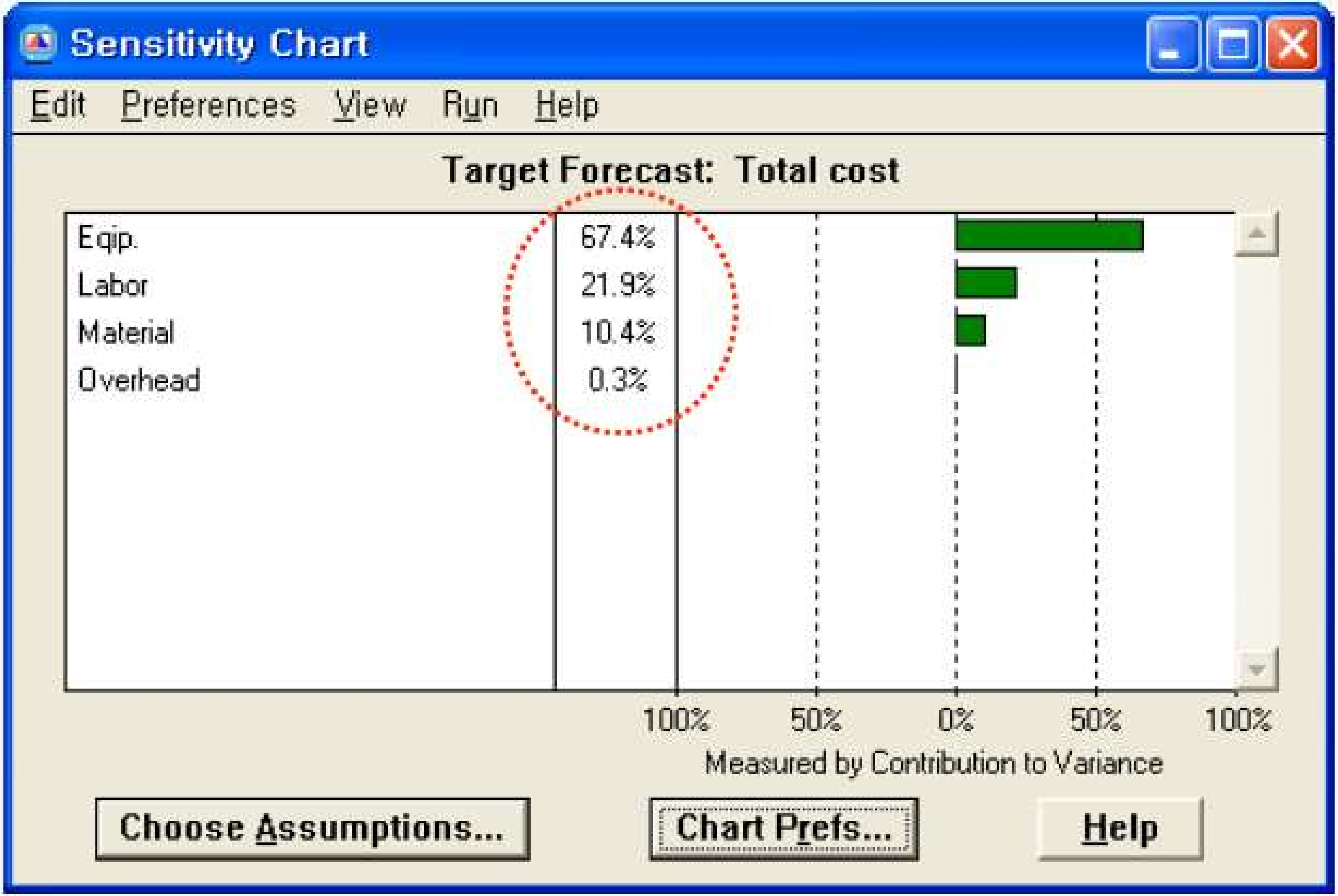
2. Perform the simulation (ex, 5000, 10000 times)



3. Forecasting cost based on the simulated results

Statistics:	Value
Trials	5000
Mean	120.08
Median	122.82
Mode	---
Standard Deviation	11.89
Variance	141.38
Skewness	0.61
Kurtosis	3.98
Coeff. of Variability	0.10
Range Minimum	84.17
Range Maximum	185.65
Range Width	101.48
Mean Std. Error	0.17

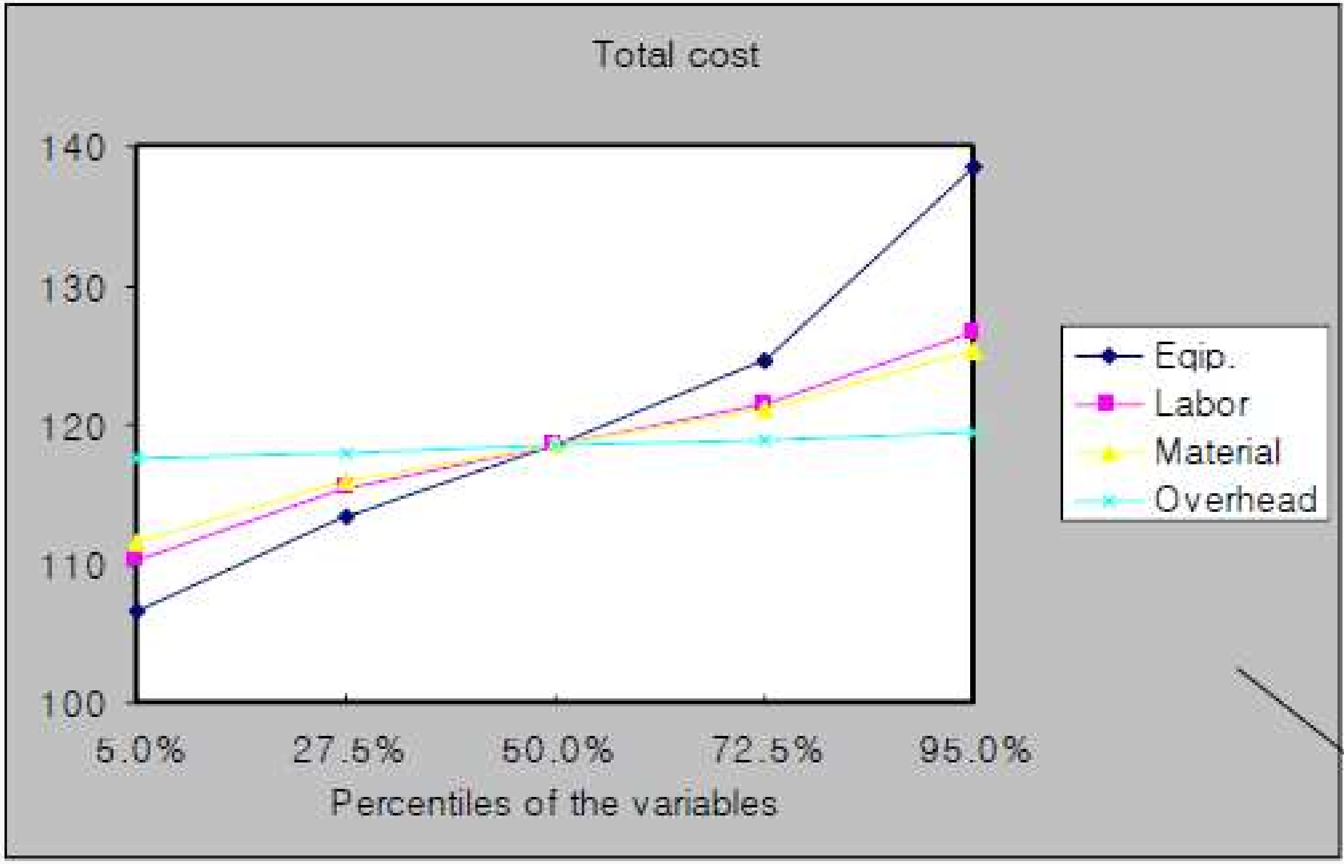
4. Sensitivity analysis: contribution to variance



- Sensitivity: Tornado diagram

Variable	Total cost Downside	Total cost Upside	Total cost Range	Input Downside	Input Upside	Input Base Case
Eqip.	106.69	138.54	31.86	16.69	48.54	28.46
Labor	110.24	126.68	16.45	41.78	58.22	50
Material	111.62	125.30	13.68	23.16	36.84	30
Overhead	117.56	119.36	1.8	9.1	10.9	10

Sensitivity: spider diagram

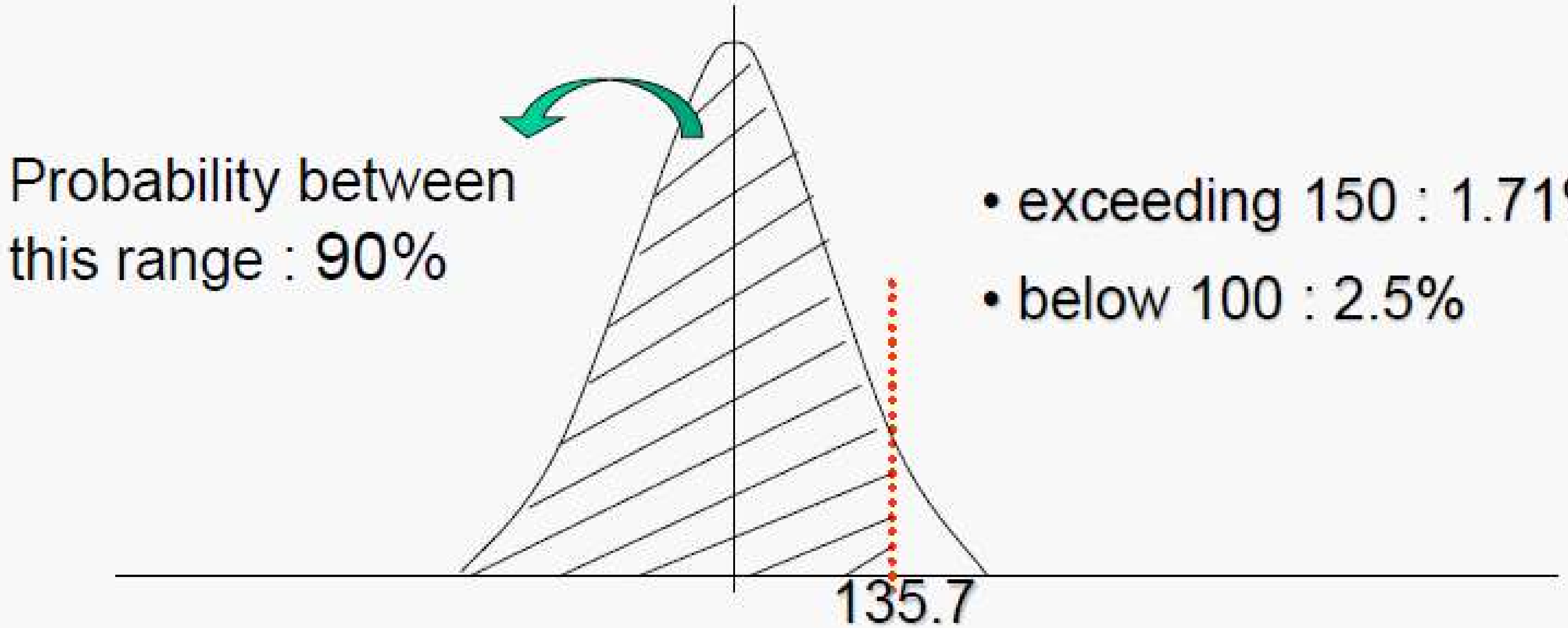


	Total cost				
Variable	5.0%	27.5%	50.0%	72.5%	95.0%
Eqip.	106.6866567	113.441043	118.4604989	124.5547764	138.5417789
Labor	110.2362307	115.4716983	118.4604989	121.4492996	126.6847671
Material	111.6227766	115.8766974	118.4604989	121.0443005	125.2982213
Overhead	117.5604989	118.0104989	118.4604989	118.9104989	119.3604989

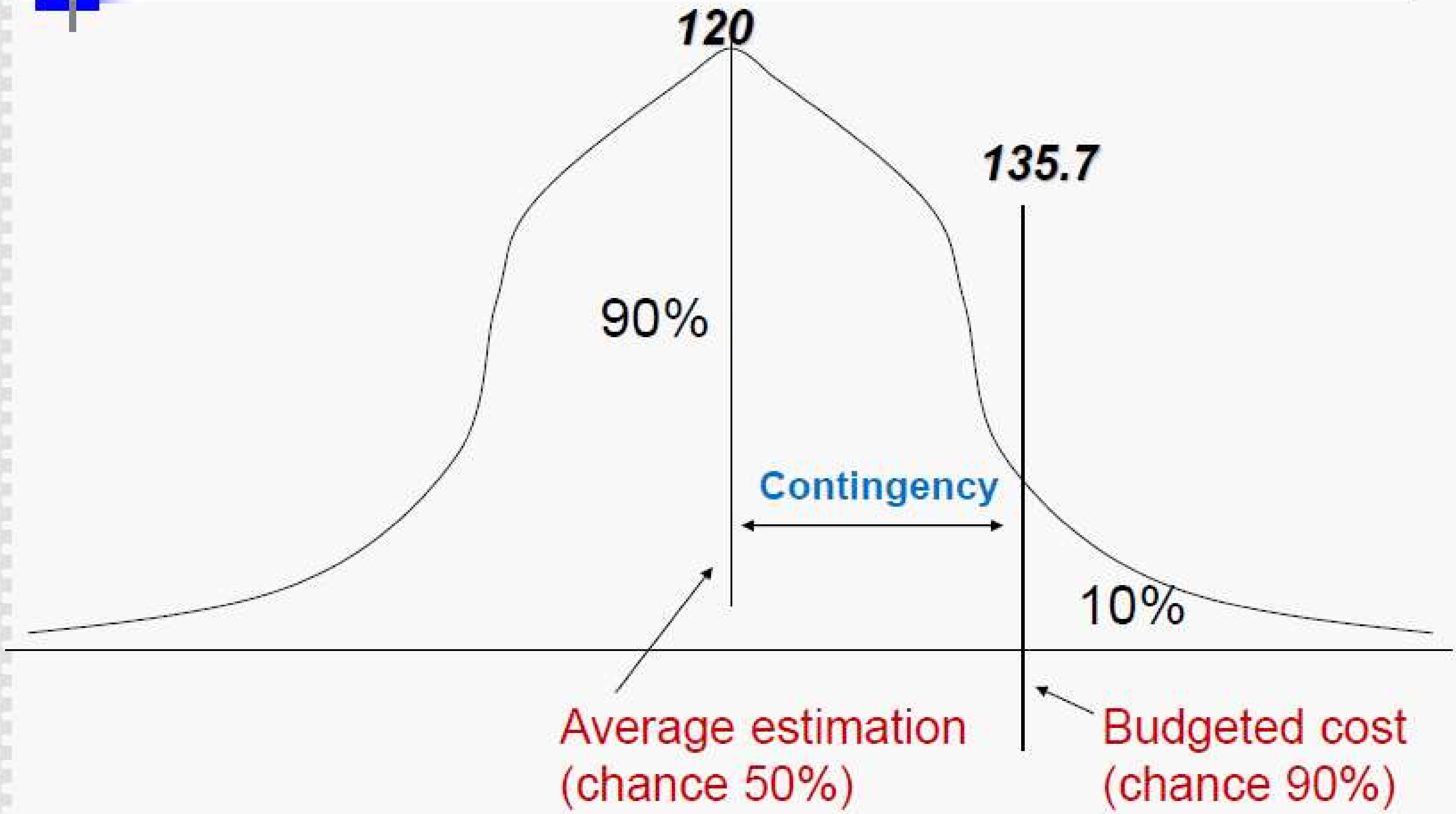
What's Difference?

- Traditional approach : single estimate → cost 12 billion
- Risk analysis approach : multiple estimates

→ average cost 120.08, SD 11.89, worst case 185.65, best case 84.17



If 90% Confidence Level



Case 3: Feasibility Study

- Evaluating BOT Highway project based on cost and benefit analysis
- Comparing traditional approach and risk analysis approach

Traditional : fixed discount rate, deterministic cost & benefits → needs sensitivity analysis

Risk analysis approach: probabilistic discount rate, variable cost and benefit to reflect the realistic decision situations

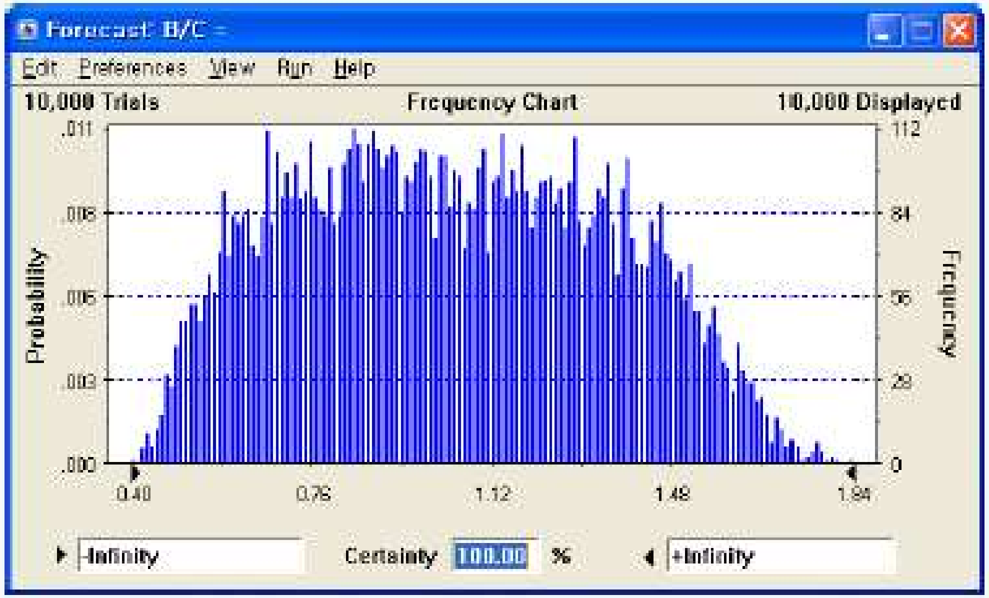
- <u>Traditional:</u> It is a good project to pursue

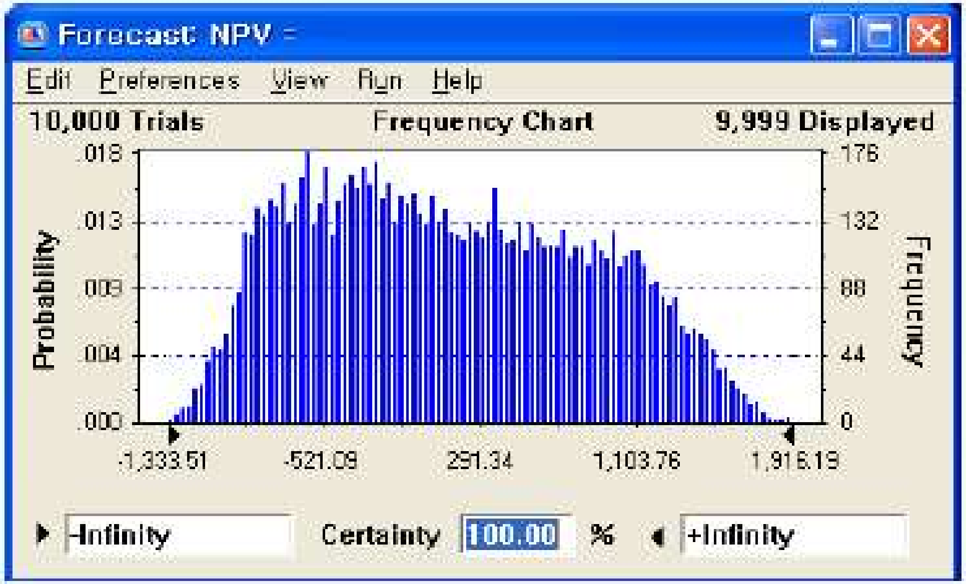
interest rate	10%
# of period	35 years
cost	3,000 (5 years)
O&M	1.9 billion /year from 6th year
benefit	15.1~701.8 billion /year from 6th year

전통적 방법 : 타당성 있음

NPV =	1,254.34
B/C =	1.54
IRR =	12.61%
Payback =	16

Risk analysis by Monte Carlo: uncertain project with high risks





B/C: average 1.04

B/C: SD 0.31

NPV: average 131

NPV: SD 724

Q & A


Street Calculus
Risk factors
black
male
aggressive body language
baseball cap on backwards
short hair
baggy jacket
wrong neighborhood
RF = 3
Mitigating Factors
female
over 40
loafers
tie and coat
whistling Sondheim
Fed Ex envelope
polo shirt
MF = 4
4>3
Risk: Acceptable.
Risk Factors
black
white
male
long hair
wrong neighborhood
police officer
baseball cap
Rf = 3
Mitigating factors
female
white
briefcase
groceries
humming Motown
tie + coat
over 40
MF = 4
4>3
Risk: Acceptable.
Good evening.
Good evening.