457.644 Advanced Bridge Engineering

Aerodynamic Design of Bridges

Part III: Wind Loads

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Steady(Static) Wind Loads

- **The mean drag, lift, and moment per unit span**
  - $\bar{D} = \frac{1}{2}\rho U^2 C_D B$
  - $\bar{L} = \frac{1}{2}\rho U^2 C_L B$
  - $\bar{M} = \frac{1}{2}\rho U^2 C_M B^2$
- **Steady-state drag, lift, and moment coefficients as the function of incident angle(or attack angle) α**
  - $C_D(\alpha) = \frac{\bar{D}}{\frac{1}{2}\rho U^2 B}$
  - $C_L(\alpha) = \frac{\bar{L}}{\frac{1}{2}\rho U^2 B}$
  - $C_M(\alpha) = \frac{\bar{M}}{\frac{1}{2}\rho U^2 B^2}$

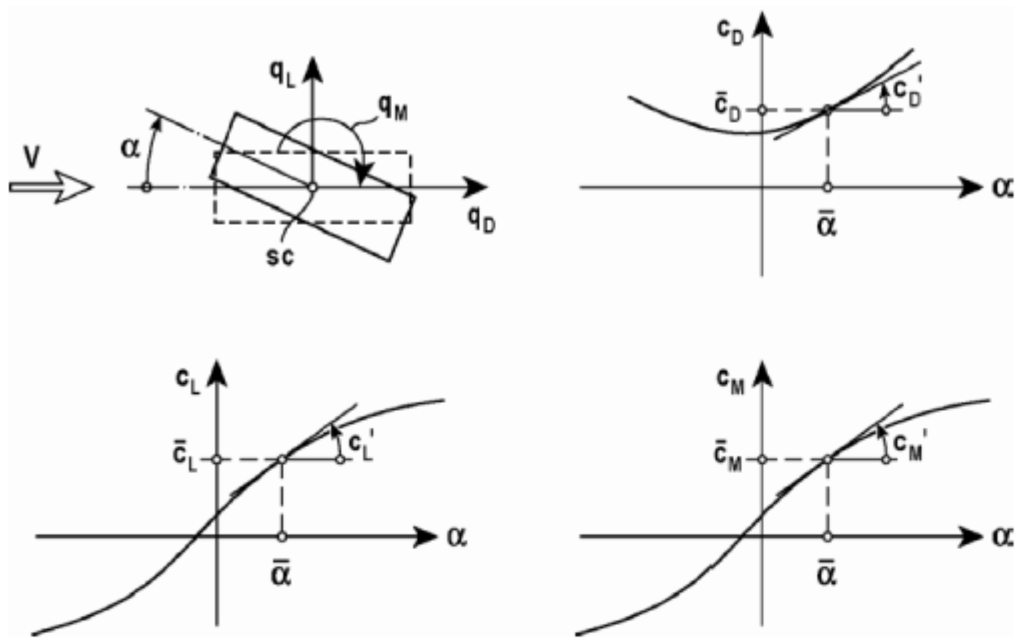
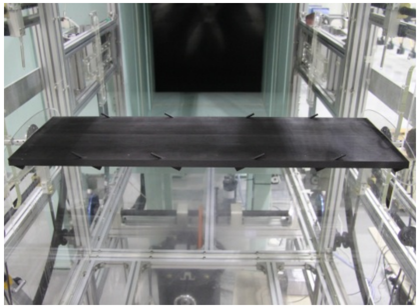
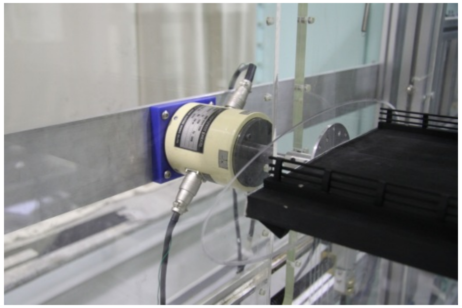


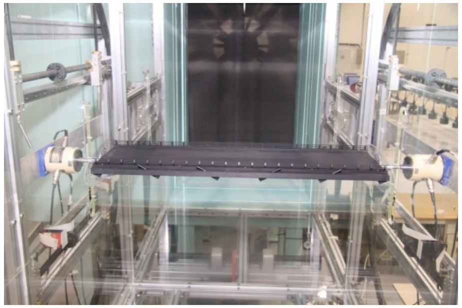
Fig. 5.2 *Load coefficients obtained from static tests*

Static Force Coefficients

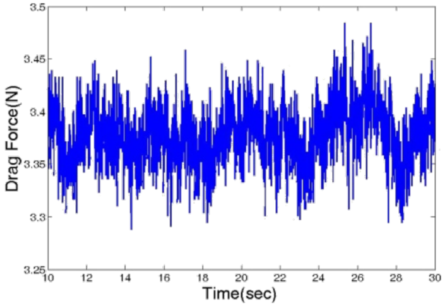
▶ Cross section, Setting in wind tunnel

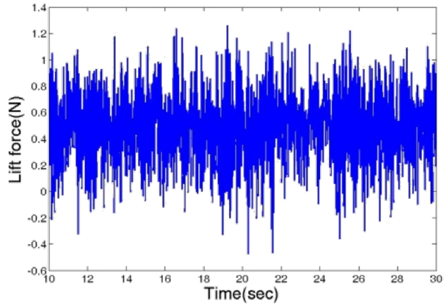


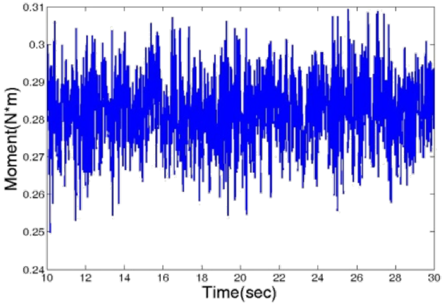




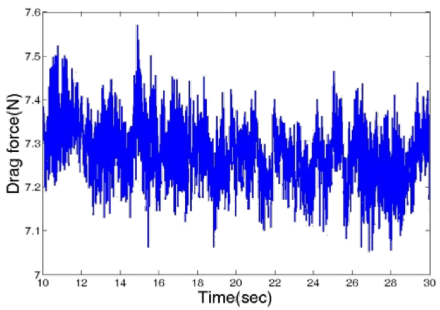
▶ Measure time histories (U=10m/s)

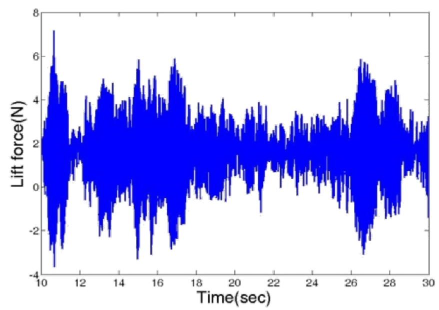


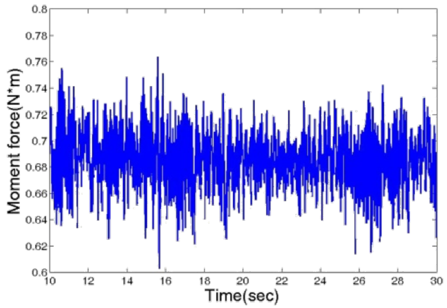




▶ Measure time histories (U=15m/s)

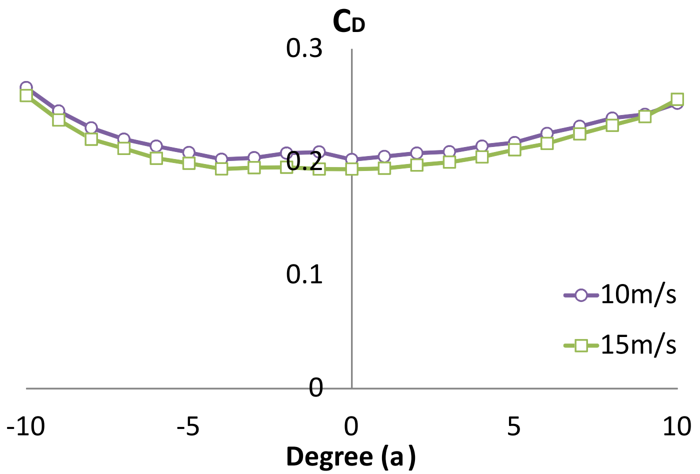




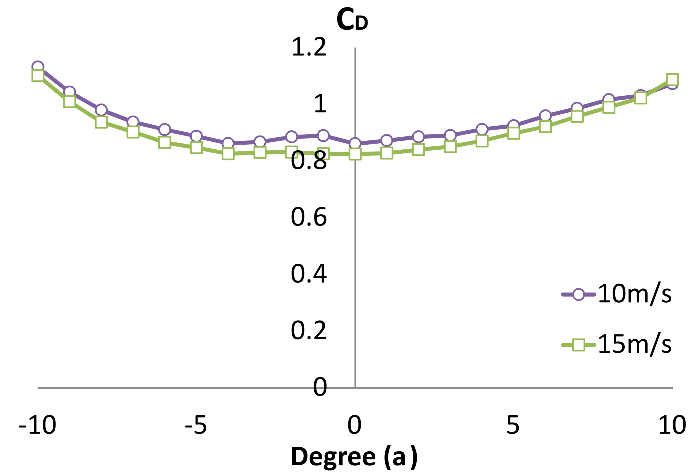


Examples: Static Force Coefficients

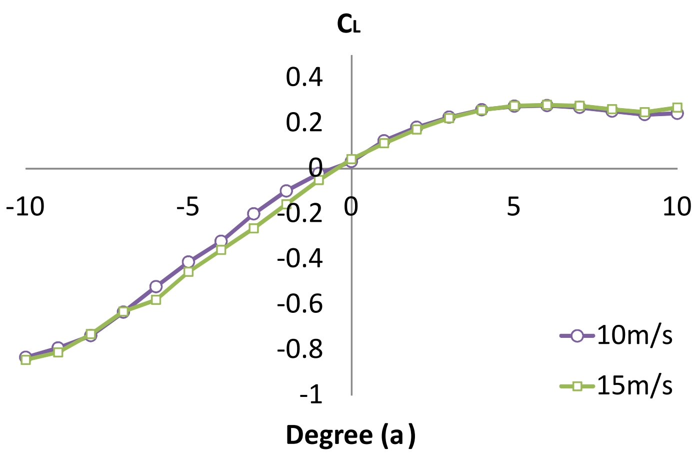
Drag coefficient(normalized by B)



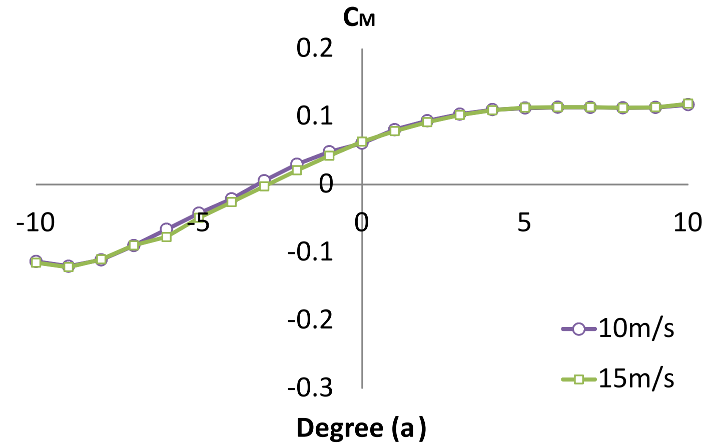
Drag coefficient(normalized by H)



Lift coefficient(normalized by B)



Moment coefficient(normalized by B)



Quasi-Steady Aerodynamic Forces

- **Fundamental assumptions**
  - Structural displacements and cross sectional rotations are small.
  - Fluctuation components of wind velocity are small as compared to U.
- **Relative velocity formulations**
  - Fluctuation velocity components inducing buffeting are included.
  - Motional velocity is considered in terms of relative velocity.

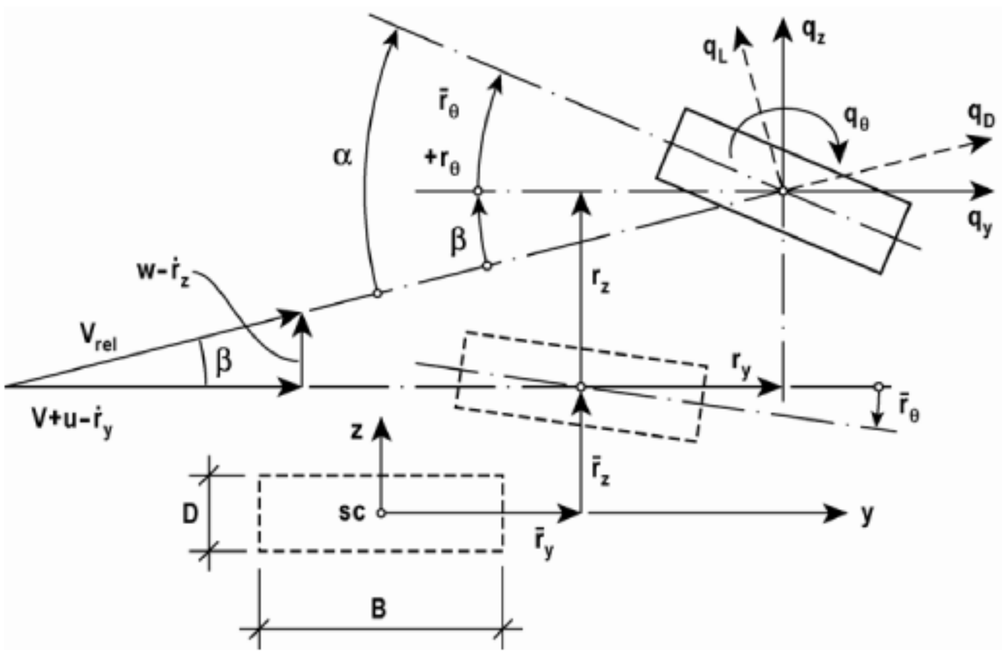


Fig. 5.1 *Instantaneous flow and displacement quantities*

$$\begin{cases} V_{rel}^2 = \left(V + u - \dot{r}_y\right)^2 + (w - \dot{r}_z)^2 \\ \alpha = \bar{r}_\theta + r_\theta + \beta \end{cases}$$

$$\begin{bmatrix} q_D(x,t) \\ q_L(x,t) \\ q_M(x,t) \end{bmatrix} = \frac{1}{2}\rho V_{rel}^2 \cdot \begin{bmatrix} B \cdot C_D(\alpha) \\ B \cdot C_L(\alpha) \\ B^2 \cdot C_M(\alpha) \end{bmatrix}$$

Linearization

Transformation into global coordinates

$$\begin{bmatrix} q_y \\ q_z \\ q_\theta \end{bmatrix} = \begin{bmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} q_D(x,t) \\ q_L(x,t) \\ q_M(x,t) \end{bmatrix} \quad \text{where, } \beta = \arctan\left(\frac{w - \dot{r}_z}{V + u - \dot{r}_y}\right)$$

First linearization

- Fluctuating flow components are small as compared to mean wind velocity, and that structural displacements (as well as cross sectional rotation) are also small.

$$\begin{cases} \cos\beta \approx 1 \\ \sin\beta \approx \tan\beta \approx \beta \approx (w - \dot{r}_z)/(V + u - \dot{r}_y) \approx (w - \dot{r}_z)/V \end{cases}$$

$$\rightarrow \begin{cases} V_{rel}^2 = \left(V + u - \dot{r}_y\right)^2 + (w - \dot{r}_z)^2 \approx V^2 + 2Vu - 2V\dot{r}_y \\ \alpha = \bar{r}_\theta + r_\theta + \beta \approx \bar{r}_\theta + r_\theta + \dfrac{w}{V} - \dfrac{\dot{r}_z}{V} \end{cases}$$

Linearization

- **Static load coefficients**

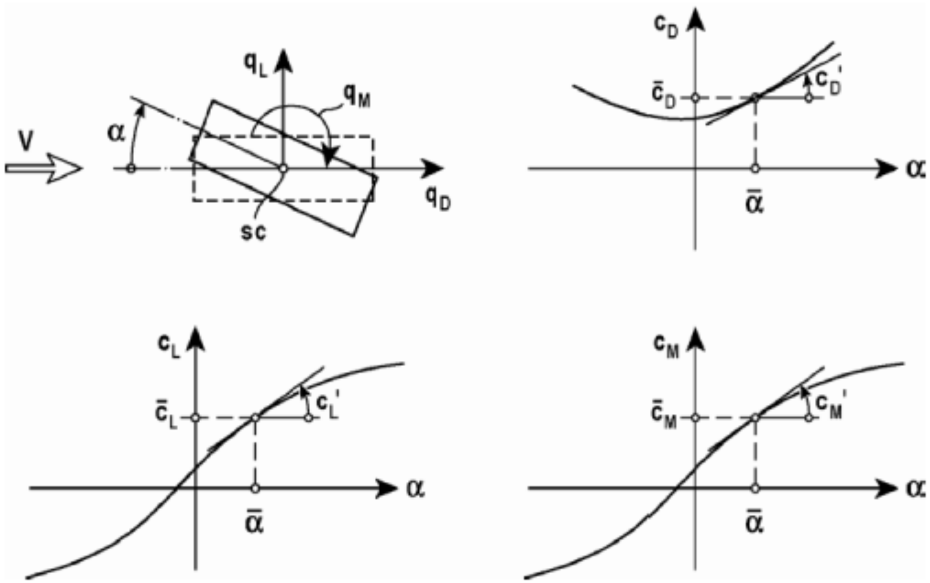


Fig. 5.2 *Load coefficients obtained from static tests*

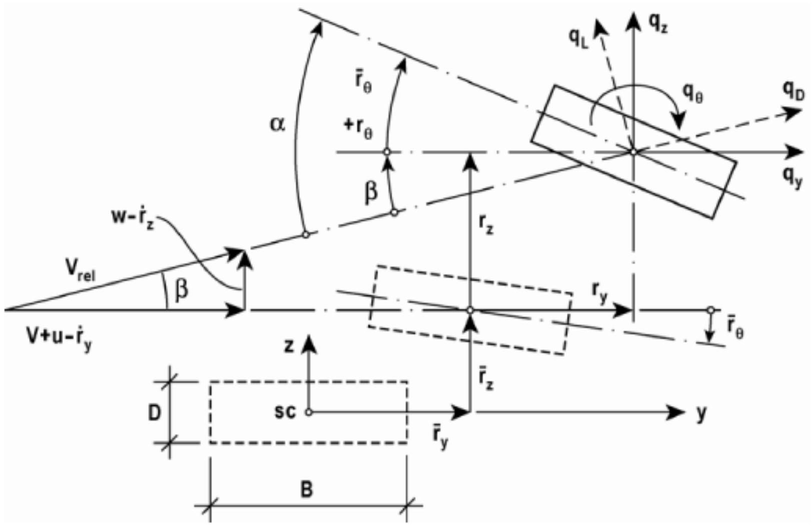
- **Second linearization in static coefficients**

$$\begin{bmatrix} C_D(\alpha) \\ C_L(\alpha) \\ C_M(\alpha) \end{bmatrix} = \begin{bmatrix} C_D(\bar{\alpha}) \\ C_L(\bar{\alpha}) \\ C_M(\bar{\alpha}) \end{bmatrix} + \alpha_f \cdot \begin{bmatrix} C'_D(\bar{\alpha}) \\ C'_L(\bar{\alpha}) \\ C'_M(\bar{\alpha}) \end{bmatrix}$$

$\bar{\alpha}$=mean angle of incidence

α_f=fluctuating angle of incidence

Combining all equations



$$\begin{bmatrix} q_y \\ q_z \\ q_\theta \end{bmatrix} = \frac{1}{2}\rho V_{rel}^2 \begin{bmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} B \cdot C_D(\alpha) \\ B \cdot C_L(\alpha) \\ B^2 \cdot C_M(\alpha) \end{bmatrix}$$

$$= \frac{1}{2}\rho\left(V^2 + 2Vu - 2V\dot{r}_y\right) \cdot \begin{bmatrix} 1 & -\left(\frac{w}{V} - \frac{\dot{r}_z}{V}\right) & 0 \\ \frac{w}{V} - \frac{\dot{r}_z}{V} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \left[\begin{bmatrix} BC_D \\ BC_L \\ B^2C_M \end{bmatrix} + \left(r_\theta + \frac{w}{V} - \frac{\dot{r}_z}{V}\right) \cdot \begin{bmatrix} BC_D' \\ BC_L' \\ B^2C_M' \end{bmatrix}\right]$$

$$= \frac{1}{2}\rho\left(V^2 + 2Vu - 2V\dot{r}_y\right) \cdot \left(\begin{bmatrix} BC_D \\ BC_L \\ B^2C_M \end{bmatrix} + \left(r_\theta + \frac{w}{V} - \frac{\dot{r}_z}{V}\right) \cdot \begin{bmatrix} BC_D' \\ BC_L' \\ B^2C_M' \end{bmatrix} + \left(\frac{w}{V} - \frac{\dot{r}_z}{V}\right) \cdot \begin{bmatrix} -BC_L \\ BC_D \\ 0 \end{bmatrix}\right)$$

Linearized Quasi-Steady Aerodynamic Forces

Discarding higher order terms

$$\begin{bmatrix} q_y \\ q_z \\ q_\theta \end{bmatrix} = \frac{1}{2}\rho\left(V^2 + 2Vu - 2V\dot{r}_y\right)\cdot\left(\begin{bmatrix} BC_D \\ BC_L \\ B^2C_M \end{bmatrix} + \left(r_\theta + \frac{w}{V} - \frac{\dot{r}_z}{V}\right)\cdot\begin{bmatrix} BC_D' \\ BC_L' \\ B^2C_M' \end{bmatrix} + \left(\frac{w}{V} - \frac{\dot{r}_z}{V}\right)\cdot\begin{bmatrix} -BC_L \\ BC_D \\ 0 \end{bmatrix}\right)$$

$$= \frac{\rho}{2}\left(V^2 + 2Vu - 2V\dot{r}_y\right)\begin{bmatrix} BC_D \\ BC_L \\ B^2C_M \end{bmatrix} + \frac{\rho}{2}\left(V^2 r_\theta + Vw - V\dot{r}_z\right)\cdot\begin{bmatrix} BC_D' \\ BC_L' \\ B^2C_M' \end{bmatrix} + \frac{\rho}{2}\left(Vw - V\dot{r}_z\right)\cdot\begin{bmatrix} -BC_L \\ BC_D \\ 0 \end{bmatrix}$$

$$= \frac{\rho V^2}{2}\begin{bmatrix} BC_D \\ BC_L \\ B^2C_M \end{bmatrix} + \frac{\rho V}{2}\begin{bmatrix} 2BC_D \\ 2BC_L \\ 2B^2C_M \end{bmatrix}\cdot u + \frac{\rho V}{2}\begin{bmatrix} BC_D' - BC_L \\ BC_L' + BC_D \\ B^2C_M' \end{bmatrix}\cdot w$$

$$- \frac{\rho V}{2}\begin{bmatrix} 2BC_D \\ 2BC_L \\ 2B^2C_M \end{bmatrix}\cdot\dot{r}_y - \frac{\rho V}{2}\begin{bmatrix} BC_D' - BC_L \\ BC_L' + BC_D \\ B^2C_M' \end{bmatrix}\cdot\dot{r}_z + \frac{\rho V^2}{2}\begin{bmatrix} BC_D' \\ BC_L' \\ B^2C_M' \end{bmatrix}\cdot r_\theta$$

- **Total load vector comprises a time invariant static part and a dynamic part**

$$\begin{bmatrix} q_y \\ q_z \\ q_\theta \end{bmatrix} = \frac{\rho V^2 B}{2}\begin{bmatrix} C_D \\ C_L \\ BC_M \end{bmatrix} + \frac{\rho VB}{2}\begin{bmatrix} 2C_D & C_D' - C_L \\ 2C_L & C_L' + C_D \\ 2BC_M & BC_M' \end{bmatrix}\cdot\begin{bmatrix} u \\ w \end{bmatrix} + \frac{\rho VB}{2}\begin{bmatrix} -2C_D & -C_D' + C_L & 0 \\ -2C_L & -C_L' - C_D & 0 \\ -2BC_M & -BC_M' & 0 \end{bmatrix}\cdot\begin{bmatrix} \dot{r}_y \\ \dot{r}_z \\ \dot{r}_\theta \end{bmatrix} + \frac{\rho V^2 B}{2}\begin{bmatrix} 0 & 0 & C_D' \\ 0 & 0 & C_L' \\ 0 & 0 & BC_M' \end{bmatrix}\cdot\begin{bmatrix} r_y \\ r_z \\ r_\theta \end{bmatrix}$$

Static part

Dynamic part

$$= \frac{\rho V^2 B}{2}\begin{bmatrix} C_D \\ C_L \\ BC_M \end{bmatrix} + \mathbf{B}_q \cdot \mathbf{u} + \mathbf{C}_{ae} \cdot \dot{\mathbf{r}} + \mathbf{K}_{ae} \cdot \mathbf{r}$$

Mean wind forces (Aerostatic)

Turbulence-induced buffeting forces (Aerodynamic)

Motion-induced self-excited forces (Aeroelastic)

Unsteady Self-Excited Forces

- **Self-excited forces with so-called *Flutter Derivatives***
  - Proposed by Scanlan and Tomko, “Airfoil and Bridge Deck Flutter Derivatives”, Journal of Engineering Mechanics Division, *J. Eng. Mech.*, 1971
  - For elastically supported section models for 2-dof (or 3-dof) for heaving and torsion under the steady-state harmonic motions with the exciting frequency of ω, self-excited forces can be expressed with experimentally-obtained flutter derivatives as

$$L_{se} = \frac{1}{2}\rho U^2 B\left[KH_1^*\frac{\dot{h}}{U} + KH_2^*\frac{B\dot{\alpha}}{U} + K^2H_3^*\alpha + K^2H_4^*\frac{h}{B} + KH_5^*\frac{\dot{p}}{U} + K^2H_6^*\frac{p}{B}\right]$$

$$M_{se} = \frac{1}{2}\rho U^2 B^2\left[KA_1^*\frac{\dot{h}}{U} + KA_2^*\frac{B\dot{\alpha}}{U} + K^2A_3^*\alpha + K^2A_4^*\frac{h}{B} + KA_5^*\frac{\dot{p}}{U} + K^2A_6^*\frac{p}{B}\right]$$

$$D_{se} = \frac{1}{2}\rho U^2 B\left[KP_1^*\frac{\dot{p}}{U} + KP_2^*\frac{B\dot{\alpha}}{U} + K^2P_3^*\alpha + K^2P_4^*\frac{p}{B} + KP_5^*\frac{\dot{h}}{U} + K^2P_6^*\frac{h}{B}\right]$$

  - in which, ρ = air density ; $K = \omega B/U$ = reduced frequency ; H^*, A^*, P^* = the flutter derivatives (the function of reduced frequency), ; B = the deck width ; U = the mean wind speed in the oncoming flow at the deck elevation

Unsteady aerodynamic forces

- The flutter derivatives L_{se}, M_{se}, D_{se} are described as unsteady lift force, unsteady moment and unsteady drag force, respectively.
- Since all the right sides of these formula consist of velocity and displacement terms of deck motion, unsteady aerodynamic force can be represented as:

$$F_{ae} = \begin{bmatrix} D_{se} \\ L_{se} \\ M_{se} \end{bmatrix}$$

$$= \frac{1}{2}\rho U B \begin{bmatrix} KP_1^* & KP_5^* & KP_2^* B \\ KH_5^* & KH_1^* & KH_2^* B \\ KA_5^* & KA_1^* B & K^2 A_2^* B^2 \end{bmatrix} \cdot \begin{bmatrix} \dot{p} \\ \dot{h} \\ \dot{\alpha} \end{bmatrix} + \frac{1}{2}\rho U^2 B \begin{bmatrix} K^2 P_4^* \frac{1}{B} & K^2 P_6^* \frac{1}{B} & K^2 P_3^* \\ K^2 H_6^* \frac{1}{B} & K^2 H_4^* \frac{1}{B} & K^2 H_3^* \\ K^2 A_6^* & K^2 A_4^* & K^2 A_3^* B \end{bmatrix} \cdot \begin{bmatrix} p \\ h \\ \alpha \end{bmatrix}$$

Velocity-dependent aerodynamic damping   Displacement-dependent aerodynamic stiffness

$$= \mathbf{C}_{ae} \cdot \dot{\mathbf{r}} + \mathbf{K}_{ae} \cdot \mathbf{r}$$

Flutter Derivatives in Quasi-steady Theory

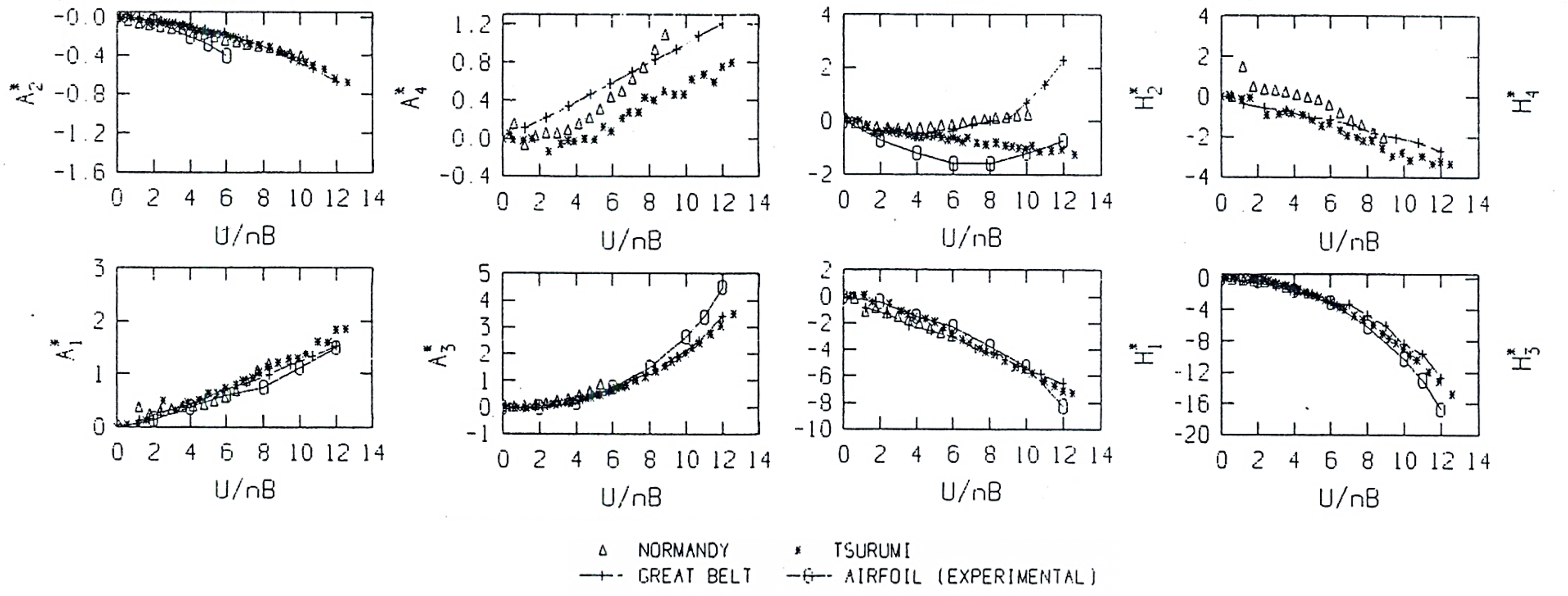
Unsteady aerodynamic forces $\mathbf{C}_{ae}, \mathbf{K}_{ae}$ *vs.* Quasi-steady $\mathbf{C}_{ae}, \mathbf{K}_{ae}$

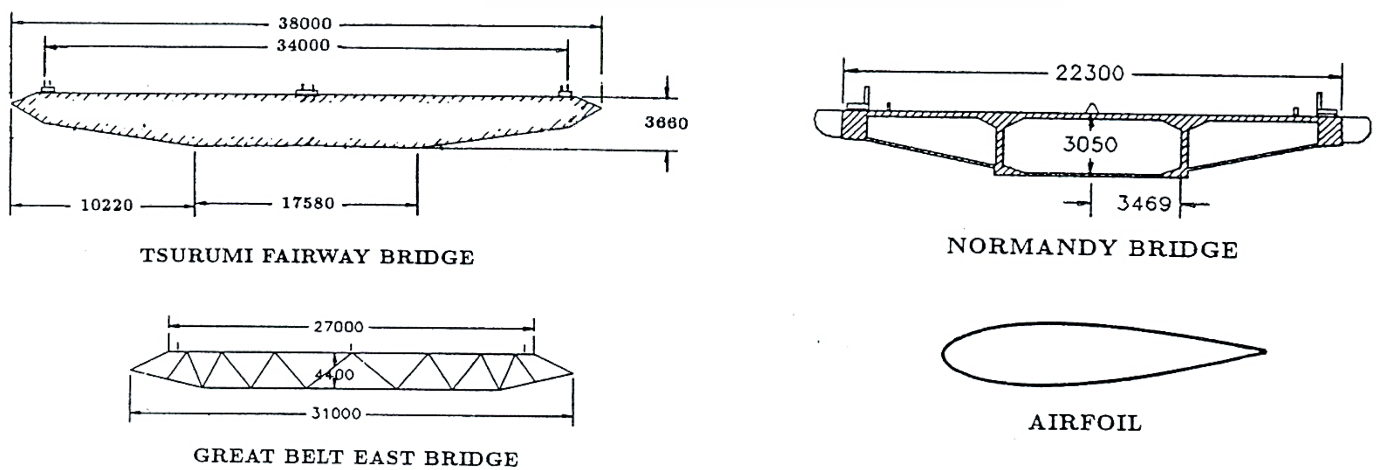
- The unsteady flutter derivatives can be represented in terms of static coefficients and its derivatives

$$\begin{bmatrix} KH_1^* = -C_L' - C_D \\ K^2H_3^* = C_L' \\ KH_5^* = -2C_L \end{bmatrix} \quad \begin{bmatrix} KA_1^* = -C_M' \\ K^2A_3^* = C_M' \\ KA_5^* = -2C_M \end{bmatrix} \quad \begin{bmatrix} KP_1^* = -2C_D \\ K^2P_3^* = C_D' \\ KP_5^* = -C_D' + C_L \end{bmatrix}$$

- All the others = 0

Box decks for three bridges and airfoils



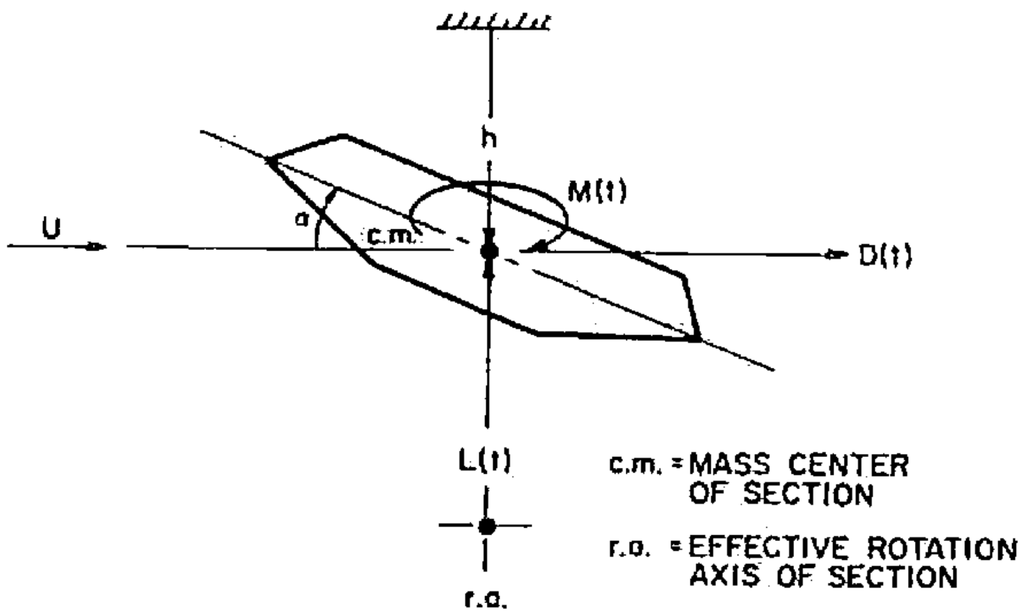


Linear aerodynamic forces

$$L_b = \frac{1}{2}\rho U^2 B\left[2C_L\frac{u(x,t)}{U} + \left(\frac{dC_L}{d\alpha} + C_D\right)\frac{w(x,t)}{U}\right]$$

$$M_b = \frac{1}{2}\rho U^2 B^2\left[2C_M\frac{u(x,t)}{U} + \left(\frac{dC_M}{d\alpha}\right)\frac{w(x,t)}{U}\right]$$

$$D_b = \frac{1}{2}\rho U^2 B\left[2C_D\frac{u(x,t)}{U}\right]$$



THANK YOU

for your attention!