

✓ Electrokinetic flows

- Mixing / Taylor dispersion
- Elastocapillarity
- Soft matter

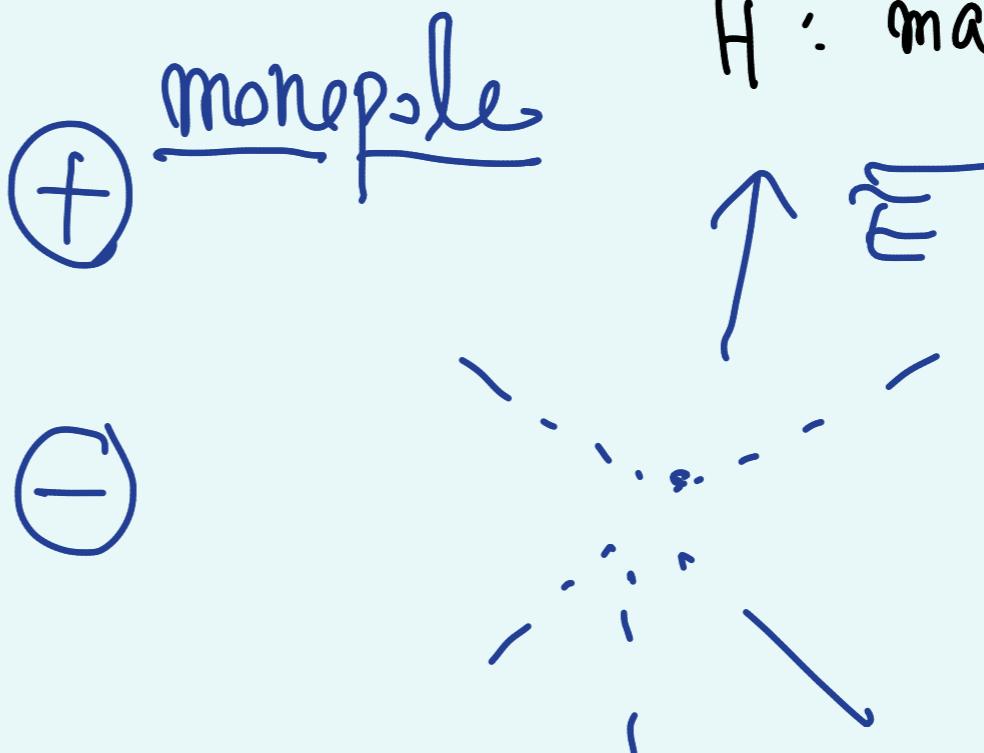
Electromagnetism
(freshman physics)

Maxwell's equations

in free space

$$\left\{ \begin{array}{l} \nabla \cdot \epsilon_0 \vec{E} = \rho_E \\ \frac{\partial \vec{E}}{\partial t} = \frac{1}{\epsilon_0} (\nabla \times \vec{H}) - \vec{J} \\ \nabla \cdot \mu_0 \vec{H} = 0 \\ \frac{\partial \vec{H}}{\partial t} = - \frac{1}{\mu_0} (\nabla \times \vec{E}) \end{array} \right.$$

ϵ_0 : permittivity
 \vec{E} : electric field
 ρ_E : charge density
 \vec{H} : magnetic field



- : Gauss' law
- : Ampere's law
- $\leftarrow \partial$
- : Faraday's law

Electroquasistatics

$$\nabla \times \vec{E} = - \frac{\partial \mu_0 \vec{H}}{\partial t} = 0$$

$$\nabla \cdot \epsilon_0 \vec{E} = \rho_E$$

$$\nabla^2 \Phi = - \frac{\rho_E}{\epsilon_0}$$

$$\vec{E} = - \nabla \Phi$$

Φ : electric potential [V]

Poisson's eq.

$$\vec{F} = \rho_E \vec{E} \quad [\text{N/m}^3]$$

$$= m \vec{a}$$

$$\frac{\partial}{\partial t} \sim f$$

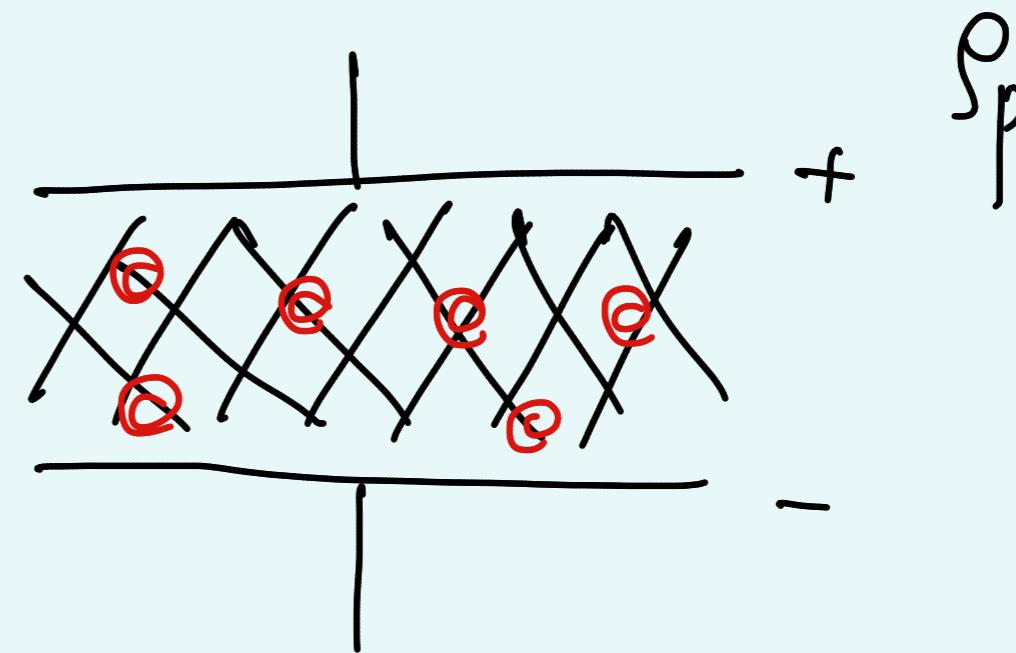
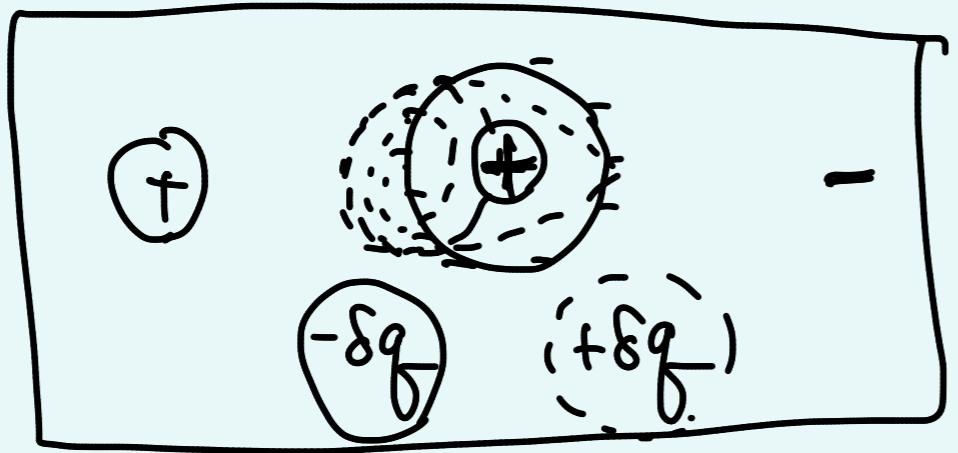
DC

AC.

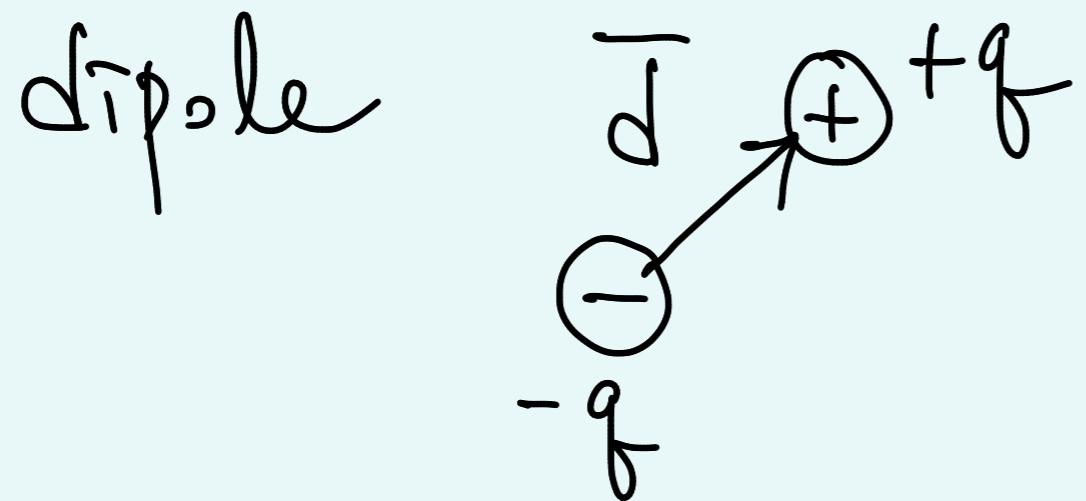
- Insulating (dielectric) materials

charge = unpaired charge + polarization charge

ρ_u



$$\nabla \cdot \overline{E_0 E} = \rho_u + \rho_p$$



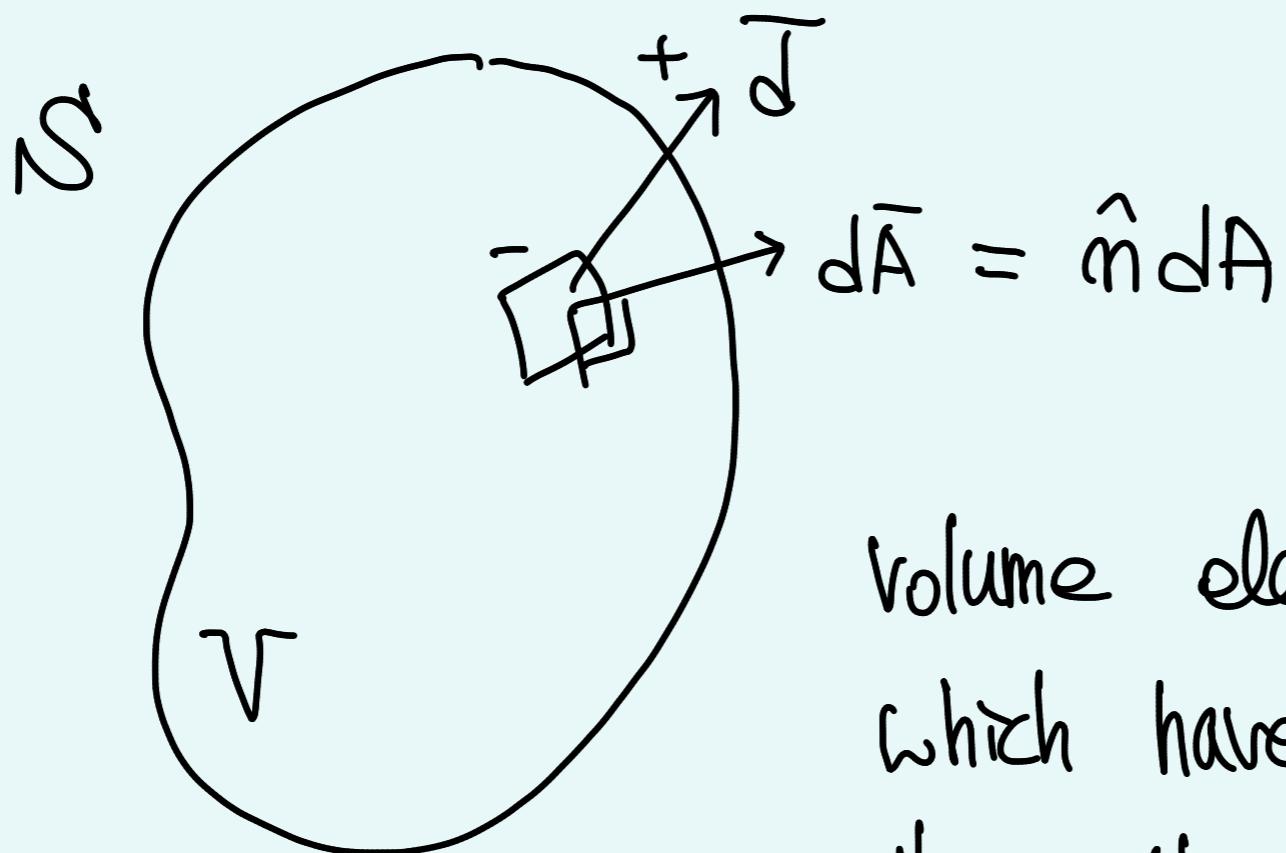
dipole moment

$$\overline{p} = q \overline{d}$$

polarization density

$$\overline{P} = N q \overline{d}$$

N : # of dipoles (polarized particles) per unit volume



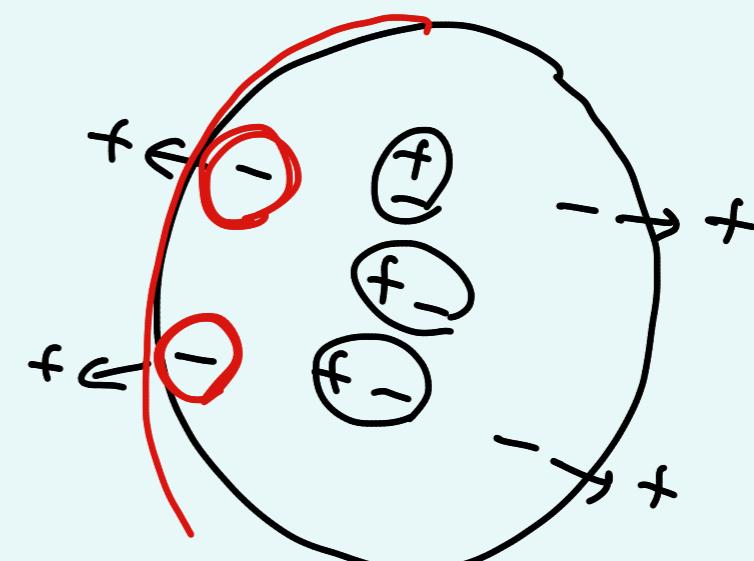
volume element containing positive charges
which have left negative charges on
the other side of surface S'

- net change left behind in V

$$Q = - \oint_{S'} (\underbrace{q N \bar{J}}_{= \bar{P}}) \cdot d\bar{A} = - \int_V \nabla \cdot \bar{P} dV$$

$$\textcircled{Q} = \int \rho_p dV$$

$$\therefore \rho_p = - \nabla \cdot \bar{P}$$



• Displacement flux density

$$\overline{D} = \epsilon_0 \overline{E} + \overline{P}$$

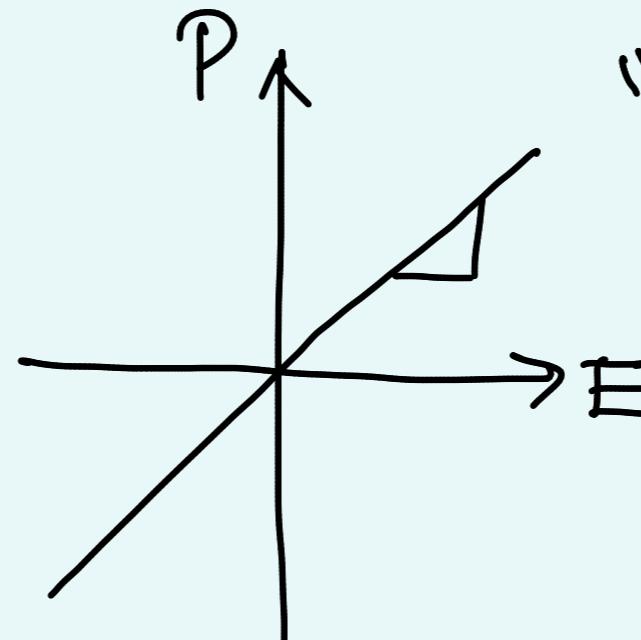
$$\nabla \cdot \epsilon \overline{E} = "f_E" = f_u + \overline{\rho_p} = f_u - \nabla \cdot \overline{P}$$

$$\nabla \cdot (\frac{\epsilon \overline{E} + \overline{P}}{\overline{D}}) = f_u$$

$$\nabla \cdot \overline{D} = f_u$$

• Constitutive law

$$\overline{P} = \overline{P}(\overline{E})$$



"electrically linear
medium"

$$\overline{P} = \epsilon_0 \chi_e \overline{E}$$

χ_e : dielectric susceptibility

$$\overline{D} = \epsilon \overline{E} + \epsilon \chi_e \overline{E} = \underbrace{\epsilon(1+\chi_e) \overline{E}}_{\epsilon'}$$

$$\overline{D} = \epsilon \overline{E}$$

$$\underline{\epsilon} \equiv \epsilon(1+\chi_e)$$

: permittivity or dielectric constant

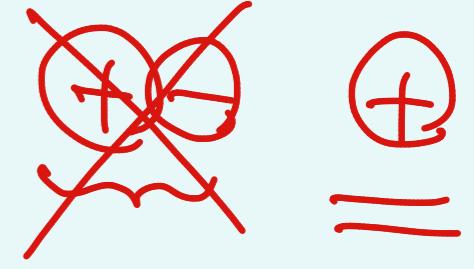
$$\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$$

$$\frac{\epsilon}{\epsilon_0} = 1 + \chi_e = \epsilon_r : \text{relative dielectric constant}$$

$(\epsilon = \epsilon_r \epsilon_0)$

Medium	χ_e
<u>air (0°C)</u>	0.00059
<u>water</u>	79.1
<u>diamond</u>	15.5
<u>paraffin</u>	1.1

EQS again



$$\nabla \cdot \bar{E} = \rho_u$$

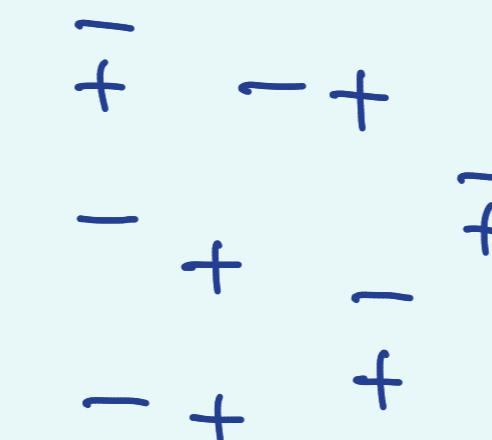
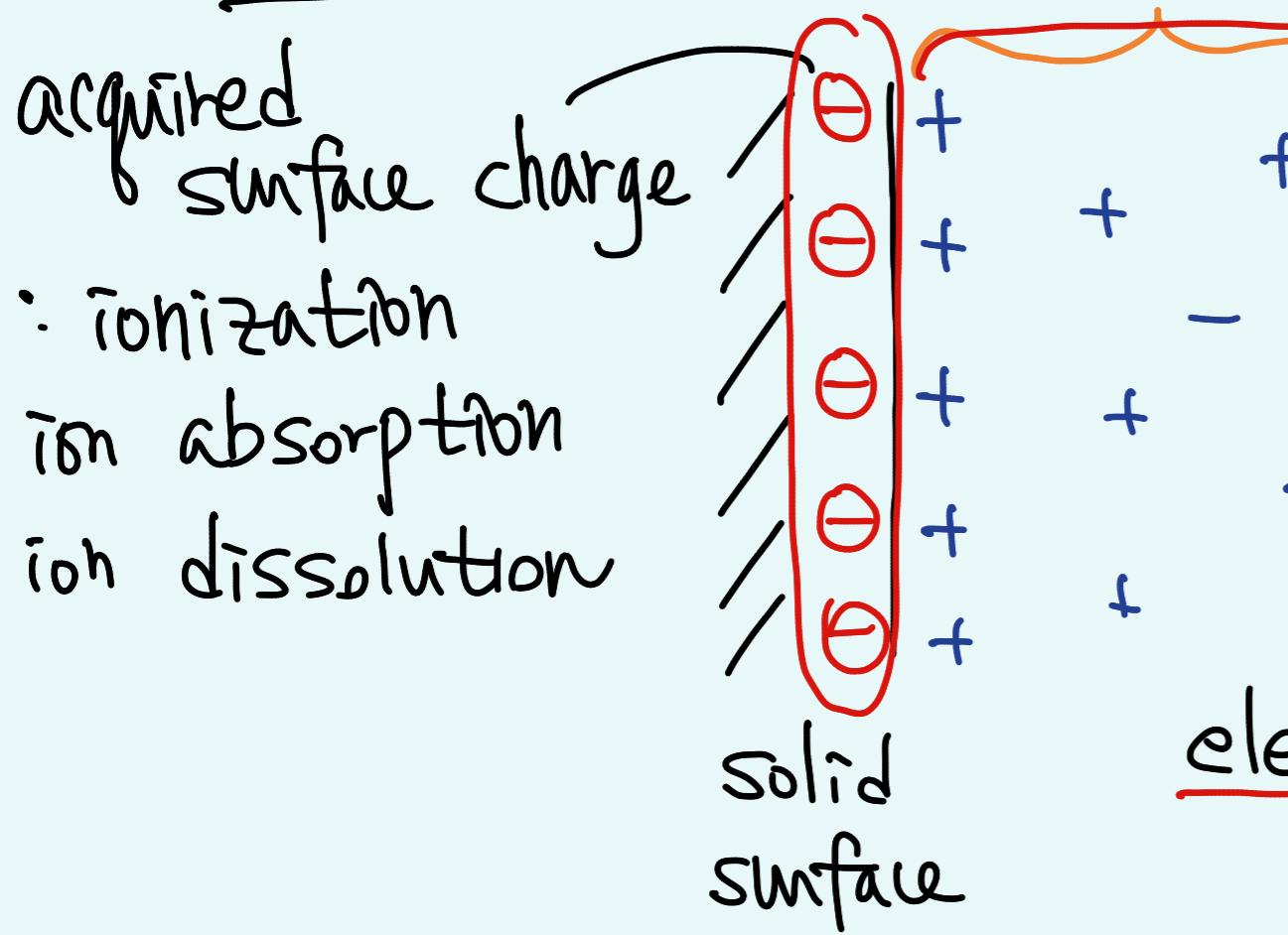
$$\nabla \times \bar{E} = 0 : \quad \bar{E} = -\nabla \Phi$$

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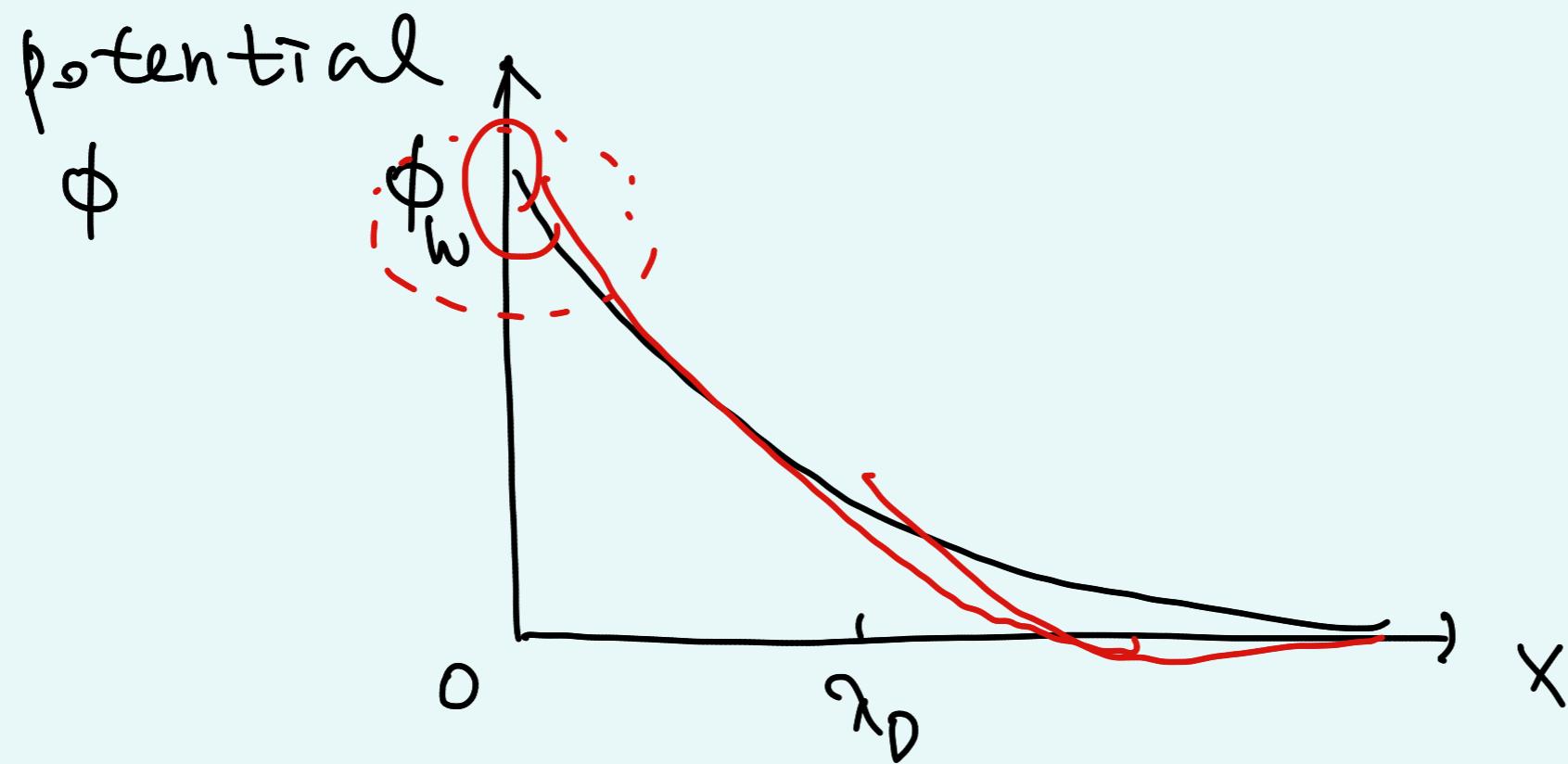
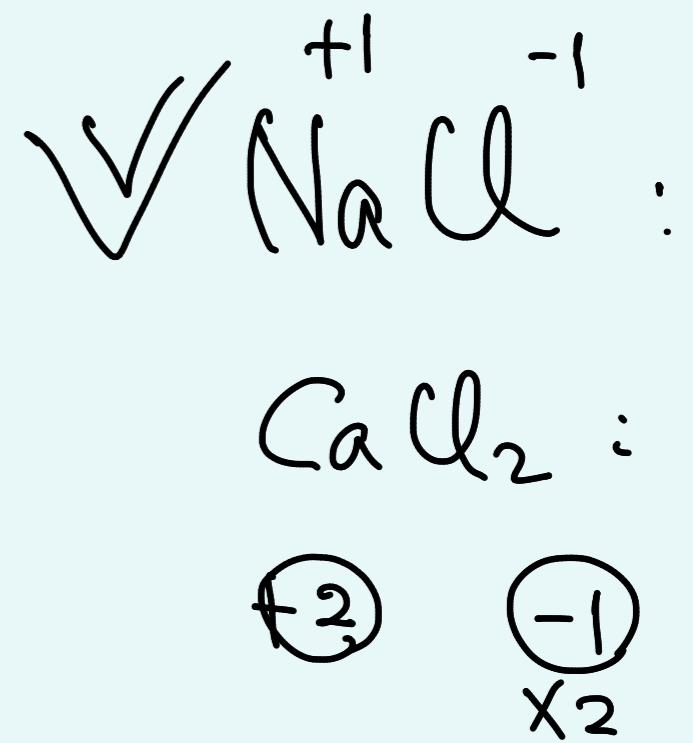
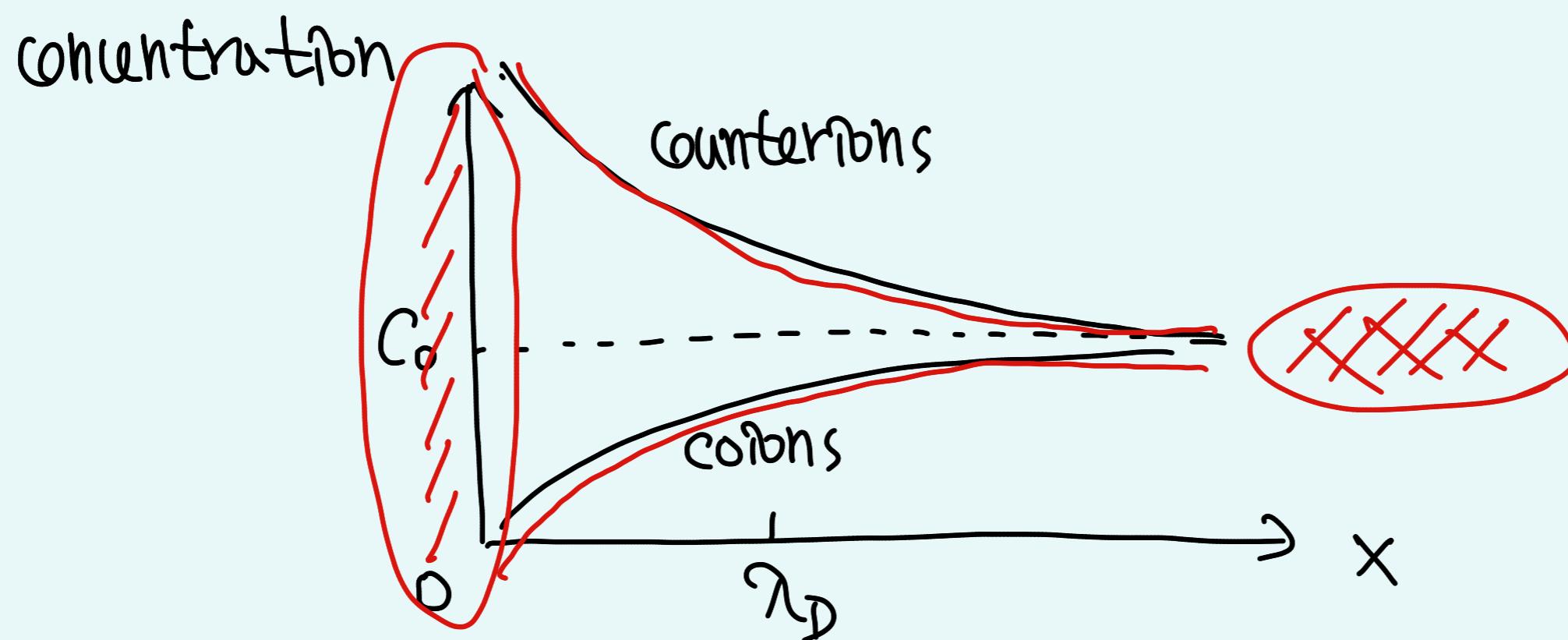
$$\nabla^2 \Phi = -\frac{\rho_u}{\epsilon}$$

Poisson's eq.

ELECTRICAL "DOUBLE" LAYER (EDL)



electrolyte



- Thickness of the double layer
(very rough estimate)
- Assume ① symmetric salt in solution

$$z_+ = -z_- = z$$

$$\textcircled{2} \quad \overline{F} = E \hat{i}_x$$

* $\textcircled{3}$ No charges in the layer

$$\left\{ \begin{array}{l} E_k = \frac{1}{2}mv^2 \\ E_g = mgh \\ E_e = QV \end{array} \right.$$

* Balancing : electric potential energy

~ thermal energy (kT)

Poisson's eq. (EDL)

$$\nabla^2 \phi = - \frac{\rho_E}{\epsilon}$$

$$\rho_E = F z c$$

↑ ↑

$$\left. \begin{array}{c} \nabla^2 \phi \\ \rho_E = F z c \end{array} \right\} \rightarrow \frac{\nabla^2 \phi}{dx^2} = \frac{F z c}{\epsilon}$$

concentration
of counterions

F : Faraday's constant (charge of 1 mole of singly ionized molecules)

$$= N_A e = 9.65 \times 10^4 \text{ C/mole}$$

$$[N_A = 6.022 \times 10^{23} / \text{mole}]$$

$$[e = 1.602 \times 10^{-19} \text{ C}]$$

$$\Delta\phi \sim \frac{FzC}{2\epsilon} x^2$$

electric potential energy $W = -Fz\phi$

$$\Delta W = -\frac{F^2 z^2 C x^2}{2\epsilon} \quad [J/mole]$$

thermal energy

$$E_k \approx RT \quad [J/mole]$$

$$\Delta W = E_k$$

$$x \equiv \lambda_D = \left(\frac{\epsilon R T}{2 F^2 z^2 C} \right)^{1/2} : \text{Debye length}$$

for an aqueous sol. of a sym. electrolyte at 25°C.

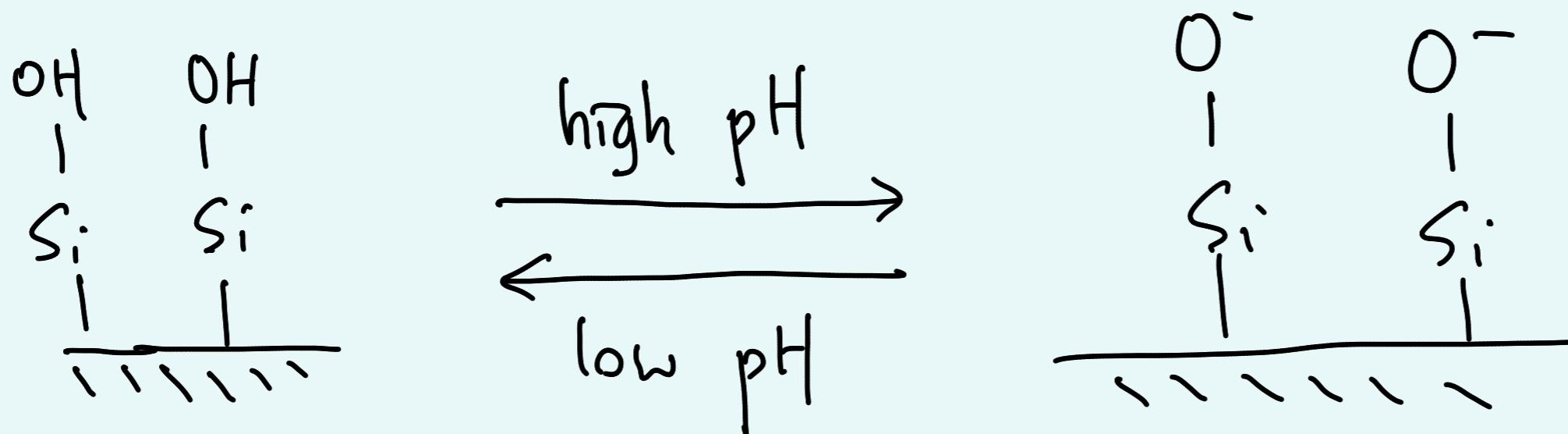
$$\lambda_D = \frac{9.61 \times 10^{-9}}{(z^2 C)^{1/2}} \quad [m]$$

$z=1$ (univalent)

$$\begin{cases} C = 10^2 \text{ mol/cm}^3 \\ C = 1 \text{ mol/cm}^3 \end{cases} \Rightarrow \begin{cases} \lambda_D = \frac{1 \text{ mm}}{10 \text{ mm}} \\ \lambda_D = \underline{\underline{10 \text{ mm}}} \end{cases}$$

- Charging of a solid surface in a liquid

e.g. silanol dissociation process



ionization of amino acid

