Generealized Hamilton's Principle

$$\int_{t_1}^{t_2} [\delta T - \delta U_1^{\ M} + \delta U_1^{\ E} + \delta W_1^{\ M} - \delta W_1^{\ E}]\mathrm{dt} = 0$$

Generalized Hamilton's Principle for Elastoelectric Bodies

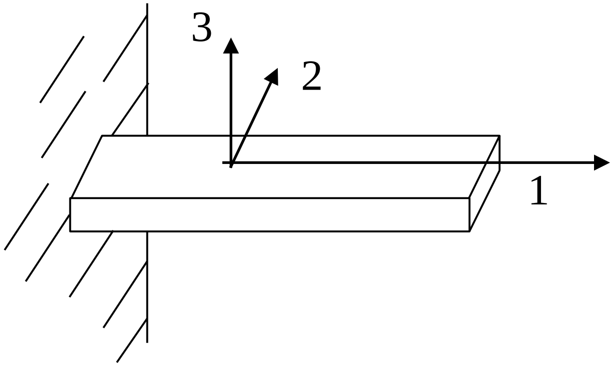
$$\int_V \delta E^T D dV$$

$$S = L_u \vec{U}(x,y,z) \Rightarrow \begin{Bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{Bmatrix} = \underbrace{\begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \\ 0 & \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial z} & 0 \end{bmatrix}}_{L_U} \begin{Bmatrix} U_1(x,y,z) \\ U_2(x,y,z) \\ U_3(x,y,z) \end{Bmatrix}$$

Generealized Hamilton's Principle

$$\begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = \underbrace{\begin{bmatrix} -\dfrac{\partial}{\partial x} \\ -\dfrac{\partial}{\partial y} \\ -\dfrac{\partial}{\partial z} \end{bmatrix}}_{L_P} \phi(x)$$

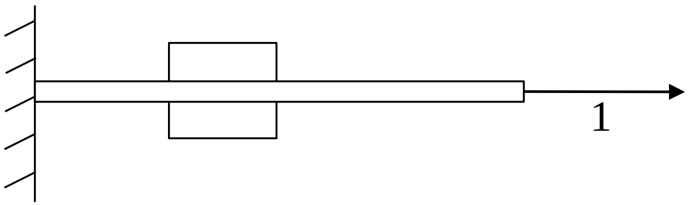
For Euler- Bernoulli beam,



$$\begin{Bmatrix} U_1(x,y,z) \\ U_2(x,y,z) \\ U_3(x,y,z) \end{Bmatrix} = \underbrace{\begin{bmatrix} 1 & -y\dfrac{\partial}{\partial x} & -z\dfrac{\partial}{\partial x} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{L_{u2}} \begin{bmatrix} \bar{U}_1(x) \\ \bar{U}_2(x) \\ \bar{U}_3(x) \end{bmatrix}$$

$$\Rightarrow L_u L_{u2}$$

Generealized Hamilton's Principle



$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{bmatrix} = \begin{bmatrix} \dfrac{\partial}{\partial x} & -y\dfrac{\partial^2}{\partial x^2} & -z\dfrac{\partial^2}{\partial x^2} \\ & & \\ & \underset{\sim}{0} & \\ & & \end{bmatrix} \begin{bmatrix} \bar{U}_1(x) \\ \bar{U}_2(x) \\ \bar{U}_3(x) \end{bmatrix}$$

Generealized Hamilton's Principle

- Classical plate

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{bmatrix} = \begin{bmatrix} \dfrac{\partial}{\partial x} & 0 & -z\dfrac{\partial^2}{\partial x^2} \\ 0 & \dfrac{\partial}{\partial y} & -z\dfrac{\partial^2}{\partial y^2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial x} & -2z\dfrac{\partial^2}{\partial x \partial y} \end{bmatrix} \begin{bmatrix} U_1(x,y) \\ U_2(x,y) \\ U_3(x,y) \end{bmatrix}$$

$$\vec{U}(\vec{x},t) = \psi_r(\vec{x})\,\vec{\mathrm{r}}(t)$$

Generalized Coordinate

$$\Uparrow$$

$$= \begin{bmatrix} \psi_{r1}(\vec{x}) & \cdots & \psi_{r_n}(\vec{x}) \end{bmatrix} \begin{bmatrix} r_1(t) \\ \vdots \\ r_n(t) \end{bmatrix}$$

Generealized Hamilton's Principle

- Classical Plate

$$\varphi(\vec{x},\mathrm{t}) = \psi_v(\vec{x})\vec{V}(t)$$

$$= \begin{bmatrix} \psi_{V_1}(\vec{x}) & \cdots \psi_{V_m}(\vec{x}) \end{bmatrix} \begin{bmatrix} V_1(t) \\ \vdots \\ V_m(t) \end{bmatrix}$$

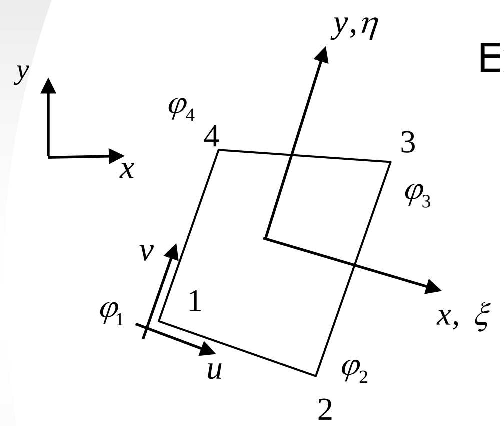
$$\vec{U}(\vec{x},t) = \psi_r(\vec{x})\,\vec{\mathrm{r}}(t)$$

$$\varphi(\vec{x},t) = \psi_V(\vec{x})\vec{V}(t)$$

$$\text{Rayleigh} - \text{Ritz} \Rightarrow \text{over the whole domain}$$

Generalized Hamilton's Principle

- Finite Elements
  - Displacement – voltage based finite elements are directly analogous to Rayleigh-Ritz procedure



Example - 2-D quadrilateral
- 3 DOF's per node 2 mechanical, 1 electrical

$$\left[u(x,y)\ \ v(x,y)\ \ \varphi(x,y)\right] = \frac{1}{4}\sum_{i=1}^{4}(1-\xi_i\xi)(1-\eta_i\eta)\left[u_i\ \ v_i\ \ \varphi_i\right]$$

ξ_i, η_i are ξ - η coordinate for the nodes

Combining shapes with differential operators

$$S(x,t) = \underbrace{L_u\psi_r(x)}_{N_r(x)}\vec{r}(t)$$

$$E(x,t) = \underbrace{L_\varphi\psi_V(x)}_{N_V(x)}V(\mathrm{t})$$