Chap 6. Probabilistic Design of Breakwaters

6.1 Uncertainty of Design Values

6.1.1 Overview

• Deterministic design:
- One characteristic (e.g., mean) value for each design variable
- Most design variables contain uncertainty
- Deterministic design use a safety factor to cover up the problem of uncertainty

• Probabilistic design:
- Consider uncertainties (or stochastic properties) of design variables
- Reliability-based design ← design a structure such that reliability is above a certain level (or probability of failure is below a certain level)
- Performance-based design ← design a structure such that performance of a structure is above certain level (e.g. expected sliding distance of a caisson is less than a certain value)

• Sources of uncertainty
- Uncertainty related to natural processes (waves, tides,…)
- Errors associated with measurements
- Uncertainty in extreme wave analysis
- Accuracy of numerical models
- Uncertainty related to empirical formulas
- Uncertainty related to structural parameters

6.1.2 Examples of uncertainty of design parameters for breakwater design

Uncertainty is given by a probability distribution. True distribution is rarely known.
→ Assume a normal distribution and use the characteristic ratio ($=\mu / X$) and coefficient of variation ($=V$)

$$\frac{\mu}{X} = \frac{\text{mean value}}{\text{characteristic value}} \quad \rightarrow \text{ ratio of mean value to characteristic value}$$

(cf. bias = difference between mean value and characteristic value)

$$V = \frac{\text{standard deviation}}{\text{mean value}} \rightarrow \text{degree of scattering relative to the mean value}$$

See Table 6.1 for uncertainties of design variables for breakwater design. See also Table VI-6-1 of CEM

6.2 Reliability-Based Design of Breakwaters

6.2.1 Classification of reliability-based design method

• Different levels of RBD depending on the level of probabilistic concepts being employed

- Level I: overall safety factor method (single safety factor; conventional method)
  partial safety factor method (safety factors for each design variable)
- Level II: correlated and non-normal distribution of variables $\rightarrow$ uncorrelated and standard normal distribution; Calculate reliability index (failure probability)
- Level III: Actual distribution functions for the variables are used

6.2.2 Evaluation of external safety by Level II method

Failure: Damage that results in structure performance and functionality below the minimum anticipated by design, but not a total or partial collapse of a structure

There are many failure modes for a structure. Each failure mode must be described by a formula, and the interaction (correlation) between the failure modes must be known. However, little is known about the real correlation between the failure modes. Therefore, the failure modes are frequently assumed to be independent each other.

Consider a single failure mode, “hydraulic stability of armor layer” described by Hudson formula:

$$D_n^3 = \frac{H_s^3}{K_D \Delta^3 \cot\alpha}$$

where

D_n = nominal block size ($= V^{1/3}$)

$\Delta = \rho_s / \rho_w - 1$

ρ_s = block density
ρ_w = water density
α = armor slope angle
H_s = significant wave height
K_D = stability coefficient

Limit state function (failure function, performance function):

$$Z = g(X_1, X_2, \cdots, X_n) = R - S$$

where R = resistance and S = loading. Usually R and S are functions of many random variables, i.e.,

$$R = R(X_1^{res}, X_2^{res}, \cdots, X_m^{res}) \quad \text{and} \quad S = S(X_{m+1}^{load}, \cdots, X_n^{load}) \quad \text{or} \quad g = g(\overline{X})$$

Probability of failure:

$$P_f = \text{Probability}(g \le 0)$$

Reliability:

$$R_f = \text{Probability}(g > 0) = 1 - P_f$$

For Hudson formula,

$$R = R(A, \Delta, D_n, \cot\alpha), \quad S = S(H_s)$$
$$g = g(A, \Delta, D_n, \cot\alpha, H_s) = g(\overline{X})$$

$$g = A \cdot \Delta \cdot D_n (K_D \cot\alpha)^{1/3} - H_s = \begin{cases} < 0 & \text{failure (unsafe region)} \\ = 0 & \text{limit state} \\ > 0 & \text{no failure (safe region)} \end{cases}$$

where A = stochastic variable signifying the uncertainty of the formula.

For the case of sliding of a caisson,

$$g = f(W - U) - P$$

Level II method (FORM; First-Order Reliability Method)

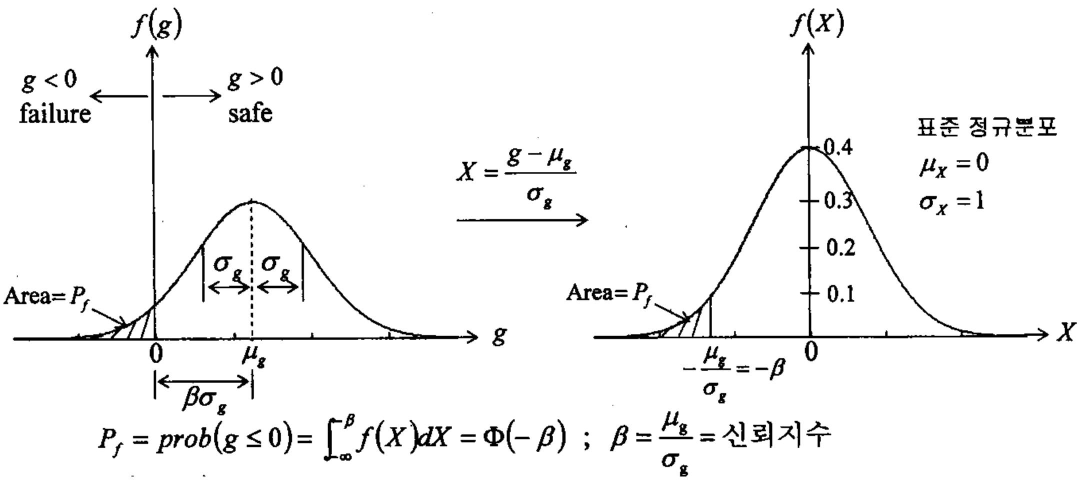
• Linear failure functions of normally-distributed random variables

Assume that S and R are independent normally-distributed variables with known means and standard deviations, i.e.,

R = normally distributed with mean μ_R and standard deviation σ_R

S = normally distributed with mean μ_S and standard deviation σ_S

Then, the linear failure function, $g = R - S$, is also normally distributed with mean $\mu_g = \mu_R - \mu_S$ and standard deviation $\sigma_g = \sqrt{\sigma_R^2 + \sigma_S^2}$. The probability density function, $f(g)$, can be transformed to a standard normal density function of $X = (g - \mu_g)/\sigma_g$:



The probability of failure is

$$P_f = prob(g \le 0) = \int_{-\infty}^{0} f(g)dg = \int_{-\infty}^{-\beta} f(X)dX = \Phi(-\beta)$$

where

$$\beta = \frac{\mu_g}{\sigma_g}$$

is the **reliability index** measuring the probability of failure. Note that $P_f \downarrow$ as $\beta \uparrow$. Also note that β is the inverse of the coefficient of variation, and it is the distance (in terms of number of σ_g) from the most probable value of g (in this case the mean) to the failure surface, $g = 0$.

If R and S are correlated, σ_g is given by

$$\sigma_g = \sqrt{\sigma_R^2 + \sigma_S^2 + 2\rho_{RS}\sigma_R\sigma_S}$$

where ρ_{RS} is the correlation coefficient:

$$\rho_{RS} = \frac{\mathrm{Cov}(R,S)}{\sigma_R\sigma_S} = \frac{E\left[(R-\mu_R)(S-\mu_S)\right]}{\sigma_R\sigma_S}$$

Additional geometrical interpretation of β:

Transform R and S into a normalized coordinate system of $R' = (R-\mu_R)/\sigma_R$ and $S' = (S-\mu_S)/\sigma_S$. Then, the failure surface $g = R - S = 0$ is transformed into $R'\sigma_R - S'\sigma_S + \mu_R - \mu_S = 0$. Then $\beta = \mu_g/\sigma_g = (\mu_R - \mu_S)/\sqrt{\sigma_R^2 + \sigma_S^2}$ is the shortest distance from the origin to the failure surface.

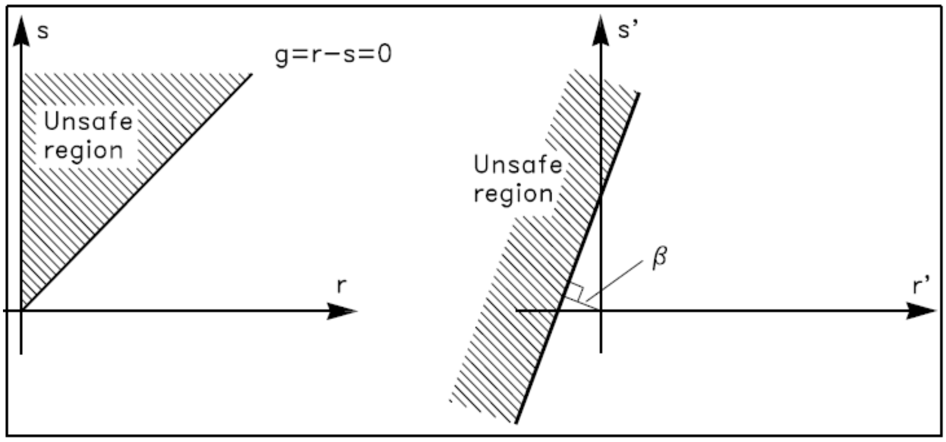


Figure VI-6-4. Illustration of β in normalized coordinate system

$$\beta = \frac{\mu_g}{\sigma_g} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} = \text{ shortest distance from origin to failure surface}$$

In Goda's book, μ_Z and σ_Z are used instead of μ_g and σ_g.

• Nonlinear failure functions of normally-distributed random variables

If the failure function is nonlinear, the approximate values for μ_Z and σ_Z can be obtained by using a linearized failure function (FORM, first-order reliability method). Linearization is generally performed by retaining only the linear terms of a Taylor series expansion about the design point, $x^* = (x_1^*, x_2^*, \cdots, x_n^*)$.

$$Z \simeq g(x_1^*, x_2^*, \cdots, x_n^*) + \sum_{i=1}^{n} (X_i - x_i^*) \left.\frac{\partial g}{\partial X_i}\right|_{x^*}$$

Iterative method to determine the design point (or the most probable failure point, MPFP):

0. Step 0. Assume the design point to be the mean point where each design value has its mean value.
1. Step 1. Calculate the mean and standard deviation of the limit state function by

   $$\mu_Z = g(\mu_{x_1}, \mu_{x_2}, \cdots, \mu_{x_n}); \qquad \sigma_Z = \sqrt{\sum_{i=1}^{n} \left(\sigma_{X_i} \left.\frac{\partial g}{\partial X_i}\right|_{\mu_{x_i}} \right)^2}$$

2. Step 2. Calculate sensitivity factor $\alpha_i (i = 1, 2, \cdots, n)$ by

   $$\alpha_i = \frac{\sigma_{X_i} \left.\frac{\partial g}{\partial X_i}\right|_{x^*}}{\sigma_Z}$$

3. Step 3. Determine a better estimate of x^* by

   $$x_i^* = \mu_{X_i} - \alpha_i \frac{\mu_Z}{\sigma_Z} \sigma_{X_i}$$

4. Step 4. Calculate the mean and standard deviation of the limit state function at x^* by

   $$\mu_Z = \sum_{i=1}^{n} (\mu_{X_i} - x_i^*) \left.\frac{\partial g}{\partial X_i}\right|_{x^*}; \qquad \sigma_Z = \sqrt{\sum_{i=1}^{n} \left(\sigma_{X_i} \left.\frac{\partial g}{\partial X_i}\right|_{x^*} \right)^2}$$

5. Step 5. Repeat Steps 2 to 4 to achieve convergence

• Nonlinear failure functions of non-normal random variables (e.g., extreme distribution of wave heights; Weibull or Gumbel distribution)

Substitute the non-normal distribution of the basic variable X_i by a normal distribution in such a way that the density and distribution functions f_{X_i} and F_{X_i} are unchanged at the design point (see CEM). Then use the above-mentioned iteration method to calculate μ_Z and σ_Z.

6.2.3 Design of breakwaters with partial factor system

Level II method calculates the probability of failure quantitatively. However, there remains some ambiguity in statistical characteristics of various design variables, and consensus is not established on the acceptable probability of failure. Moreover, the Level II method is quite different from the conventional safety factor method and is complicated to apply. The design engineers prefer the Level I (partial safety factor) method, which is similar to the conventional method.

Consider the limit state function formulated with resistance $R(\vec{X})$ and load $S(\vec{Y})$

$$Z = g(\vec{X},\vec{Y}) = R(\vec{X}) - S(\vec{Y}) \qquad (1)$$

The limit state function including the partial safety factors, γ, is given by

$$Z = \gamma_R R_c(\vec{X}_c) - \gamma_S S_c(\vec{Y}_c) \geq 0 \qquad (2)$$

where the subscript c stands for the characteristic value. For example, the limit state function for the sliding of a caisson is given by

$$Z = \gamma_f f(\gamma_W W - \gamma_U U) - \gamma_P P \geq 0 \qquad \text{(6.20) in Goda's book}$$

The reliability index, β, is defined as

$$\beta = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \qquad (3)$$

If the target reliability index is β_T, we need

$$\beta \geq \beta_T \qquad (4)$$

Substituting Eq. (3) into Eq. (4) and using the relationship $\sqrt{\sum_i \sigma_i^2} = \sum_i \alpha_i \sigma_i$ (definition of the sensitivity factor α), we have

$$\mu_R \geq \mu_S + \beta_T(\alpha_R \sigma_R + \alpha_S \sigma_S) \qquad (5)$$

Using $V_R = \sigma_R / \mu_R$ and $V_S = \sigma_S / \mu_S$,

$$(1 - \alpha_R \beta_T V_R)\mu_R \geq (1 - \alpha_S \beta_T V_S)\mu_S \qquad (6)$$

Comparing Eqs. (2) and (6), the partial safety factors for resistance and load are

$$\gamma_R = (1 - \alpha_R \beta_T V_R)\frac{\mu_R}{R_c(\vec{X}_c)}; \qquad \gamma_S = (1 - \alpha_S \beta_T V_S)\frac{\mu_S}{S_c(\vec{Y}_c)}$$

A general form for a random variable X :

$$\gamma_X = (1 - \alpha_X \beta_T V_X)\frac{\mu_X}{X_c}$$ ← Eq. (6.21) in Goda's book

6.3 Performance-Based Design of Breakwaters

6.3.1 Outline of performance-based design method

Reliability-based design method: $\beta \geq \beta_T$ or $P_f \leq (P_f)_T$

Performance-based design method: deformation of a structure (e.g. sliding distance of caisson) is less than allowable value.

Measure of deformation for different types of structures:

1) rubble mound breakwaters: damage level of armor units
2) vertical breakwaters: sliding distance of caisson

Performance requirement of a structure:

1) Serviceability = serviceability limit state in ISO 2394
2) Restorability
3) Safety = ultimate limit state in ISO 2394
4) Usability $\leftarrow$ at the stage of layout plan of port and harbor facilities

The allowable value of deformation is different for respective performance requirements: serviceability < restorability < safety

Performance-based design is to predict the magnitude of deformation and determine the structural dimensions so that the predicted deformation is less than the allowable value.

6.3.2 Performance-based design with expected sliding distance method

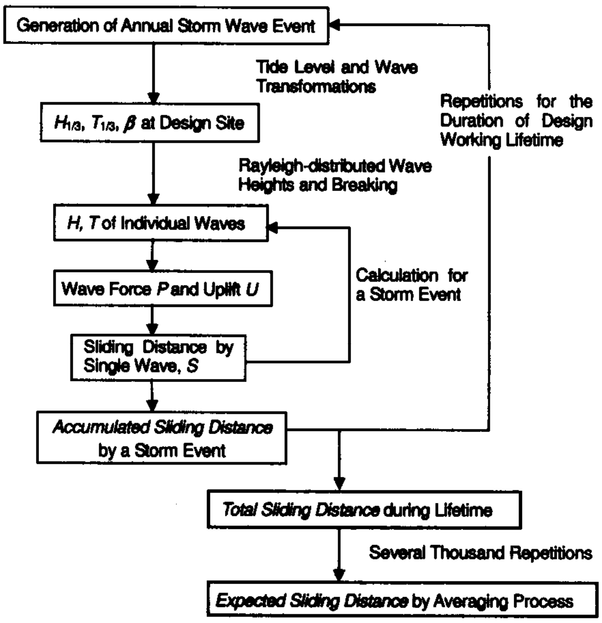


Fig. 6.1 Flow diagram for calculation of the expected sliding distance taken from Shimosako and Takahashi[18] with some modification.

The statistical characteristics (mean and coefficient of variation) and probability density function of each design variable are given as in Table 6.6.

→ The design variable is randomly sampled from the pdf in each simulation.

→ Different result in each simulation, S

→ Ensemble averaging of several thousand simulations

→ Expected sliding distance, S_E

• Two proposals for acceptable sliding distance:

Acceptable amount of accumulated sliding distance by one storm event: Table 6.7 gives a matrix for different return periods, different limit states, and different importance of the breakwater. For example, for a very important breakwater (A in Table 6.7) such as nuclear power plant breakwater, $S_E \le 0.03\,\text{m}$ for 500-year return period storm, and $S_E \le 0.1\,\text{m}$ for 5000-year return period storm.

Acceptable exceedance rate of total sliding distance during the lifetime of breakwater: Table 6.8 for different importance of breakwater and different total sliding distance. For example, $\Pr[S \ge 0.3\text{ m}] \le 10\%$ for the breakwater of medium importance.

6.3.3 Vertical breakwater design with modified Level I method

Partial safety factors in Table 6.5 were developed with constant $\beta_T = 2.4$. The partial safety factor method is easy to use, but it gives no idea on how much sliding would occur.

Eq. (6.22) calculates β_{SLT} considering sliding performance, which was developed by linear multiple regression analysis of existing breakwaters such that the expected total sliding distance is 0.3 m (Read text for detailed procedure). The partial safety factors are calculated by Eq. (6.21) with $\beta_T = \beta_{SLT}$. This method calculates different reliability index, but the expected sliding distance would be close to 0.3 m if it is calculated by using the performance-based design method.

Future design: preliminary design using modified Level I method → detailed design using expected sliding distance method