9. Pole Placement and Model Matching

- ✓ Output feedback Control Configurations
- ✓ Unity feedback configuration –Pole Placement
- ✓ Regulation and Tracking
- ✓ Implementable Transfer Functions
- ✓ Model Matching-Two-Parameter Configuration

Output feedback Control Configurations

Output feedback Control Configurations

$$u(s) = C(s)\left[pr(s) - y(s) \right]$$

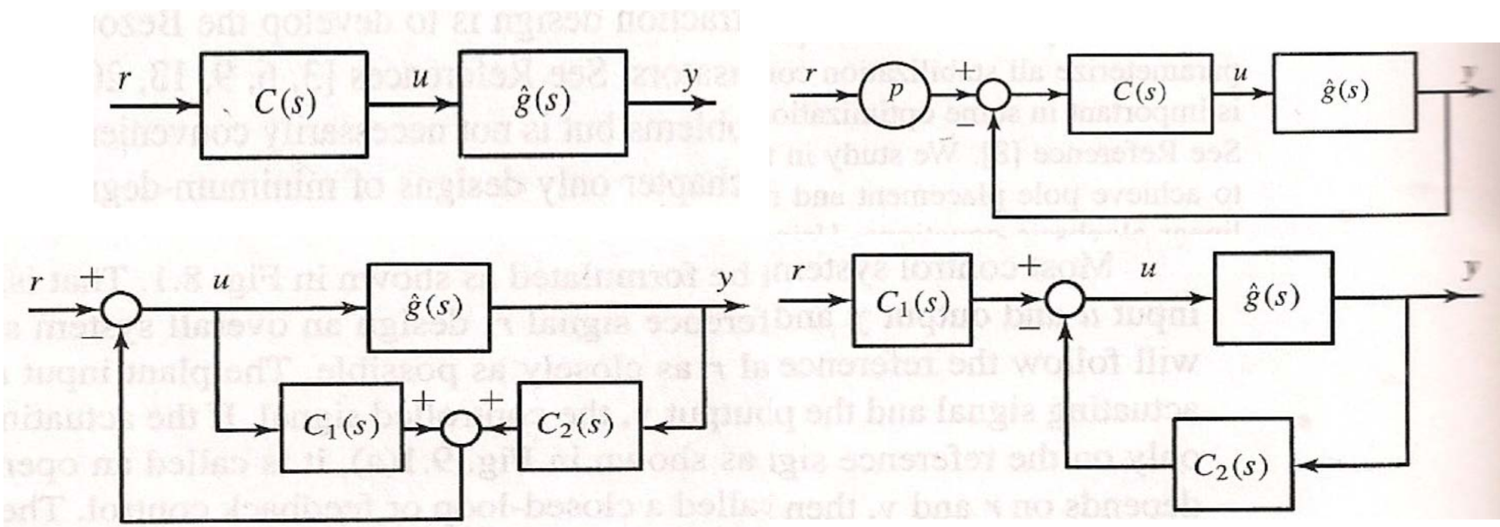
$$u(s) = \frac{1}{1+C_1(s)} r(s) - \frac{C_2(s)}{1+C_1(s)} y(s)$$

$$u(s) = C_1(s) r(s) - C_2(s) y(s)$$

Minimum phase

Nonminimum phase

Hurwitz polynomial



Unity feedback configuration

Unity feedback configuration –Pole Placement

Unity feedback:

$$u(s) = C(s)[pr(s) - y(s)]$$

$$y(s) = g(s)u(s)$$

Let

$$C(s) = B(s)/A(s), \quad g(s) = N(s)/D(s)$$

Then

$$g_o(s) = \frac{y(s)}{r(s)} = \frac{pC(s)g(s)}{1 - C(s)g(s)}$$

$$= \frac{pB(s)N(s)}{A(s)D(s) + B(s)N(s)}$$

Let $F(s)$ be desired characteristic polynomial, then

$$A(s)D(s) + B(s)N(s) = F(s)$$

which is called *compensator equation.*

Unity feedback configuration

Theorem 9.1

Given polynomials $D(s)$ and $N(s)$, polynomial solutions $A(s)$ and $B(s)$ exist in comoensator equation for any polynomial $F(s)$ if and only if $D(s)$ and $N(s)$ are coprime.

Proof :

Suppose $D(s)$ and $N(s)$ are not coprime and contain the same factor $(s+a)$, then $F(s)$ should contain $(s+a)$. This is contracts to any polynomial $F(s)$. This is proof of the necessity.

Unity feedback configuration

The proof of the sufficiency:

If $D(s)$ and $N(s)$ are coprime, there exists $\bar{A}(s)$ and $\bar{B}(s)$ such that

$$\bar{A}(s)D(s)+\bar{B}(s)N(s)=1$$

Its matrix version is called *Bezout Identity*. This equation can be expressed by Sylvester resultant form as

$$S\theta=n$$

where S Sylvester resultant, θ is vector composed of coefficients of $\bar{A}(s)$ and $\bar{B}(s)$, and $n=[1\ 0\ 0\ ....]'$.

S is nonsingular if $D(s)$ and $N(s)$ are coprime.

Then for any $F(s)$,

$$F(s)\bar{A}(s)D(s)+F(s)\bar{B}(s)N(s)=F(s)$$

Thus $A(s)=F(s)\bar{A}(s),\ B(s)=\bar{B}(s)N(s)$ is the solution.

Unity feedback configuration

If $\hat{A}(s)$ and $\hat{B}(s)$ are solution of

$$\hat{A}(s)D(s)+\hat{B}(s)N(s)=0$$

(for example $\hat{A}(s)=-N(s)$, $\hat{B}(s)=D(s)$ are solutions. Then

$$A(s)=\bar{A}(s)F(s)+Q(s)\hat{A}(s) \qquad B(s)=\bar{B}(s)F(s)+Q(s)\hat{B}(s)$$

are solutions of the compensator equations.

Example 9.1

Given $D(s)=s^2-1, N(s)=s-2,$ and

$F(s)=s^3+4s^2+6s+4,$ then

$$A(s)=\frac{1}{3}\left(s^3+4s^2+6s+4\right)+Q(s)(-s+2)$$

$$B(s)=-\frac{1}{3}(s+2)\left(s^3+4s^2+6s+4\right)+Q(s)\left(s^2-1\right)$$

$$A(s)=s+34/3 \qquad B(s)=(-22s-23)/3 \text{ for } Q(s)=(s^2+6s+15)/3$$

Unity feedback configuration

$$A(s)D(s)+B(s)N(s)=F(s)$$

$$D(s)=D_0+D_1s+D_2s^2+\cdots+D_ns^n \qquad D_n\neq 0$$

$$N(s)=N_0+N_1s+N_2s^2+\cdots+N_ns^n$$

$$A(s)=A_0+A_1s+A_2s^2+\cdots+A_ms^m$$

$$B(s)=B_0+B_1s+B_2s^2+\cdots+B_ms^m$$

$$F(s)=F_0+F_1s+F_2s^2+\cdots+F_{n+m}s^{n+m}$$

$$A_0D_0+B_0N_0=F_0$$

$$A_0D_1+B_0N_1+A_1D_0+B_1N_0=F_1$$

$$\vdots$$

$$A_mD_n+B_mN_n=F_{n+m}$$

$$\begin{bmatrix}A_0 & B_0 & A_1 & B_1 & \cdots & A_m & B_m\end{bmatrix}\mathbf{S}_m=\begin{bmatrix}F_0 & F_1 & F_2 & \cdots & F_{n+m}\end{bmatrix}$$

Unity feedback configuration

$$
\mathbf{S}_m : \begin{bmatrix}
D_0 & D_1 & \cdots & D_n & 0 & \cdots & 0 \\
N_0 & N_1 & \cdots & N_n & 0 & \cdots & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & D_0 & \cdots & D_{n-1} & D_n & \cdots & 0 \\
0 & N_0 & \cdots & N_{n-1} & N_n & \cdots & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\vdots & \vdots & & \vdots & \vdots & & \vdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & \cdots & 0 & D_0 & \cdots & D_n \\
0 & 0 & \cdots & 0 & N_0 & \cdots & N_n
\end{bmatrix}
$$

$$
2(m+1) \geq n+m+1 \quad \text{or} \quad m \geq n-1
$$

Unity feedback configuration

Theorem 9.2

Consider the unity-feedback system shown in Fig. 9.1(b). The plant is described by a strictly proper transfer function $g(s) = N(s)/D(s)$ with $N(s)$ and $D(s)$ coprime and $N(s) < \deg D(s) = n$. Let $m \geq n-1$. Then for any polynomial $F(s)$ of degree $(n+m)$, there exists a proper compensator $C(s) = B(s)/A(s)$ of degree m such that the overall transfer function equals

$$g_o(s) = \frac{pN(s)B(s)}{A(s)D(s) + B(s)N(s)} = \frac{pN(s)B(s)}{F(s)}$$

Furthermore, the compensator can be obtained by solving the linear algebraic equation in (9.13).

Regulation and Tracking

Regulation and Tracking

$$y(s) = g_o(s)r(s) = g_o(s)\frac{a}{s}$$

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sy(s) = g_o(0)a$$

$$g_o(0) = p\frac{N(0)B(0)}{F(0)} = p\frac{B_0N_0}{F_0}$$

$$p = \frac{F_0}{B_0N_0} \to \text{ Achieve the tracking.}$$

Regulation $\Leftrightarrow \mathrm{g}_0(s)$ *BIBO* stable

Tracking step reference $\Leftrightarrow \mathrm{g}_0(s)$ *BIBO* stable and $g_o(0) = 1$

Tracking ramp reference $\Leftrightarrow \mathrm{g}_0(s)$ *BIBO* stable, $g_o(0) = 1,\ g_o'(0) = 0$

Regulation and Tracking

Example 9.2

$$g(s) = (s-2)/(s^2-1)$$

Choose $m = n-1 = 1,\ \deg F = m+n = 2$

$$F(s) = (s+2)(s+1+j1)(s+1-j1) = (s+2)(s^2+2s+2)$$

$$= s^3 + 4s^2 + 6s + 4$$

$$\begin{bmatrix} A_0 & B_0 & A_1 & B_1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 1 & 0 \\ -2 & 1 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & -1 & 0 & 1 \\ 0 & -2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 6 & 4 & 1 \end{bmatrix}$$

$$A_0 = 1 \quad B_0 = 34/3 \quad A_1 = -22/3 \quad B_1 = -23/3$$

Regulation and Tracking

$$A(s) = s + 34/3 \qquad B(s) = (-22/3)s - 23/3 = (-22s - 23)/3$$

$$C(s) = \frac{B(s)}{A(s)} = \frac{-(23+22s)/3}{34/3+s} = \frac{-22s-23}{3s+34}$$

$$p = \frac{F_0}{B_0 N_0} = \frac{4}{(-23/3)(-2)} = \frac{6}{23}$$

$$g_o(s) = \frac{6}{23}\frac{\left[-(22s+23)/3\right](s-2)}{(s^3+4s^2+6s+4)} = \frac{-2(22s+23)(s-2)}{23(s^3+4s^2+6s+4)}$$

Regulation and Tracking

Robust Tracking and Disturbance Rejection

In example 9.2, if the system is perturbed into

$\bar{g}(s) = (s-2.1)/(s^2-0.95)$

Then the overall feedback system becomes

$$\bar{g}_o(s) = \frac{pC(s)\bar{g}(s)}{1+C(s)\bar{g}(s)} = \frac{6}{23}\frac{\frac{-22s-23}{3s+34}\frac{s-2.1}{s-0.95}}{1+\frac{-22s-23}{3s+34}\frac{s-2.1}{s-0.95}}$$

$$= \frac{-6(22s+23)(s-2.1)}{23(3s^3+12s^2+20.35s+16}$$

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sy(s) = \bar{g}_o(0)a = 0.7875a \to \textit{not} \text{ robust}$$

Regulation and Tracking

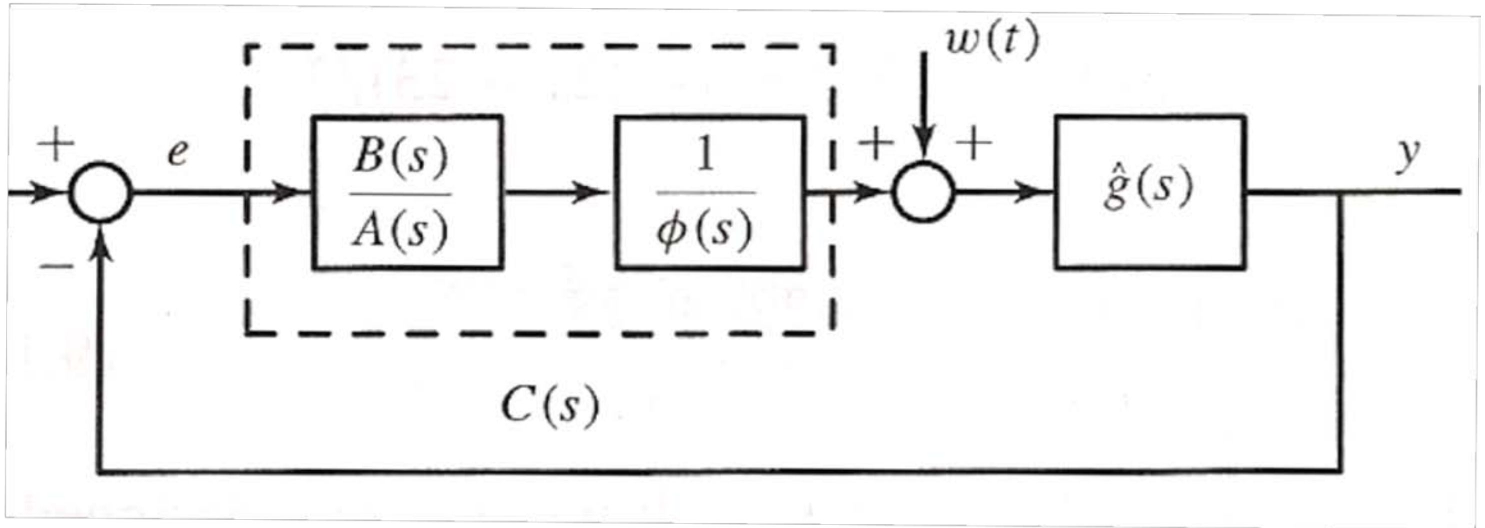
Robust Tracking and Disturbance Rejection

$$r(s) = [r(t)] = \frac{N_r(s)}{D_r(s)} \qquad w(s) = [w(t)] = \frac{N_w(s)}{D_w(s)}$$

$\phi(s)$: Least common denominator of the unstable poles of $r(s)$ and $w(s)$

$r(s) = a/s,\ \ w(s) = N_w(s)/s(s^2+\omega^2)$ for $w(t) = b + c\sin(\omega t + d)$

$\rightarrow \phi(s) = s(s^2+\omega^2)$



Regulation and Tracking

Theorem 9.3

Consider the unity-feedback system shown in Fig. 9.2(a) with a strictly proper plant transfer function $g(s) = N(s)/D(s)$. It is assumed that $D(s)$ and $N(s)$ are coprime. The reference signal $r(t)$ and disturbance $w(t)$ are modeled as $r(s) = N_r(s)/D_r(s)$ and $w(s) = N_w(s)/D_w(s)$.

Let $\phi(s)$ be the least common denominator of the unstable poles of $r(s)$ and $w(s)$. If no roots of $\phi(s)$ is a zero of $g(s)$, then there exists a proper compensator such that the overall system will track $r(t)$ and reject $w(t)$, both asymptotically and robustly.

Regulation and Tracking

Proof :

$$A(s)D(s)\phi(s)+B(s)N(s)=F(s)$$

$$C(s)=\frac{B(s)}{A(s)\phi(s)}$$

$$g_{yw}(s)=\frac{N(s)/D(s)}{1+(B(s)/A(s)\phi(s))(N(s)/D(s))}$$

$$=\frac{N(s)A(s)\phi(s)}{A(s)D(s)\phi(s)+B(s)N(s)}=\frac{N(s)A(s)\phi(s)}{F(s)}$$

The output excited by $w(t)$ equals

$$y_w(s)=g_{yw}(s)w(s)=\frac{N(s)A(s)\phi(s)}{F(s)}\frac{N_w(s)}{D_w(s)}$$

The unstable poles of $D_w(s)$ are cancelled by $\phi(s)$,

thus we have $y_w(t)\to 0$ as $t\to\infty$.

Regulation and Tracking

The output excited by $r(t)$:

$$y_r(s) = g_{yr}(s)r(s) = \frac{B(s)N(s)}{A(s)D(s)\phi(s) + B(s)N(s)} r(s)$$

$$e(s) := r(s) - y_r(s) = (1 - g_{yr}(s))r(s)$$

$$= \frac{A(s)D(s)\phi(s)}{F(s)} \frac{N_r(s)}{D_r(s)}$$

The unstable roots of $D_r(s)$ are cancelled by $\phi(s)$, then

$r(t) - y_r(t) \to 0$ as $t \to \infty$.

This shows asymptotic tracking and disturbance rejection.

This is refered to as *internal model principle*.

Regulation and Tracking

Example 9.3

$$g(s) = (s-2)/(s^2-1)$$

$$A(s)D(s)\phi(s) + B(s)N(s) = F(s)$$

For tracking to a step reference,

we intruduce the internal model $\phi(s) = 1/s.$

$\deg D(s)\phi(s) = 3 = n,$ we select $\deg A(s) = m = n-1 = 2$

Then $\deg F(s) = 5,$ If we select closed loop poles as

$-2, -2 \pm j1, -1 \pm j2.$

$$F(s) = (s+2)(s^2+4s+5)(s^2+2s+5)$$

$$= s^5 + 8s^4 + 30s^3 + 66s^2 + 85s + 50$$

$$D(s)\phi(s) = (s^2-1)s = 0 - s + 0s^2 + s^3$$

$$N(s) = -2 + s + 0s^2 + 0s^3$$

Regulation and Tracking

$$[A_0\ B_0\ A_1\ B_1\ A_2\ B_2]\begin{bmatrix} 0 & -1 & 0 & 1 & 0 & 0 \\ -2 & 1 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & -2 & 1 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & -2 & 1 & 0 & 0 \end{bmatrix}$$

$$= [50 \quad 85 \quad 66 \quad 30 \quad 8 \quad 1]$$

$$\frac{B(s)}{A(s)} = \frac{-96.3s^2 - 118.7s - 25}{s^2 + 127.3}$$

$$C(s) = \frac{B(s)}{A(s)\phi(s)} = \frac{-96.3s^2 - 118.7s - 25}{(s^2 + 127.3)s}$$

Regulation and Tracking

Embedding Internal Models

$$A(s)D(s)+B(s)N(s)=F(s)$$

If deg $D(s)=n$ and deg $A(s)=n-1$,
the solution is unique.
If we increase the deg $A(s)$ by one,
the solution is not unique. There is one free parameter.
We can choose one parameter in $A(s)=A_0+A_1 s+\ldots$
Here if we choose $A_0=0$, $A(s)$ inlcudes the root at $s=0$.

Regulation and Tracking

Example 9.4

$$A(s)D(s)+B(s)N(s)=F(s)$$

Deg $D(s)=2$, we choose deg $A(s)=2$ instead of $n-1=1$

If we choose $F(s)=(s^2+4s+5)(s^2+2s+5)$

$$=s^4+6s^3+18s^2+30s+25$$

$$\begin{bmatrix} A_0 & B_0 & A_1 & B_1 & A_2 & B_2 \end{bmatrix}\begin{bmatrix} -1 & 0 & 1 & 0 & 0 \\ -2 & 1 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & -1 & 0 & 1 & 0 \\ 0 & -2 & 1 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & -2 & 1 & 0 \end{bmatrix}=\begin{bmatrix} 25 & 30 & 18 & 6 & 1 \end{bmatrix}$$

Regulation and Tracking

By letting $A_0 = 0$,

$$C(s) = \frac{B_0 + B_1 s + B_2 s^2}{A_0 + A_1 s + A_2 s^2}$$

has $1/s$ as a factor, the remaining solution is unique.

$$\begin{bmatrix} A_0 & B_0 & A_1 & B_1 & A_2 & B_2 \end{bmatrix} = \begin{bmatrix} 0 & -12.5 & 34.8 & -38.7 & 1 & -28.8 \end{bmatrix}$$

$$C(s) = \frac{B(s)}{A(s)} = \frac{-28.8s^2 - 38.7s - 12.5}{s^2 + 34.8s}$$

This compensator can achieve robust tracking.

This is a better design of Example 9.3.

Regulation and Tracking

Example 9.4

$$g(s) = 1/s$$

For step tracking and rejection of disturbance $w(t) = a\sin(2t+\theta)$.

Then should include s^2+4 and does not need include s.

$$A(s)D(s) + B(s)N(s) = F(s)$$

$$\deg D(s) = n = 1 \rightarrow \deg A(s) \geq n-1 = 0$$

$$A(s) = \tilde{A}_0(s^2+4) \qquad B(s) = B_0 + B_1 s + B_2 s^2$$

$$\tilde{D}(s) = D(s)(s^2+4) = \tilde{D}_0 + \tilde{D}_1 s + \tilde{D}_2 s^2 + \tilde{D}_3 s^3 = 0 + 4s + 0\cdot s^2 + s^3$$

$$\tilde{A}_0\tilde{D}(s) + B(s)N(s) = F(s)$$

$$\begin{bmatrix} \tilde{A}_0 & B_0 & B_1 & B_2 \end{bmatrix} \begin{bmatrix} \tilde{D}_0 & \tilde{D}_1 & \tilde{D}_2 & \tilde{D}_3 \\ N_0 & N_1 & 0 & 0 \\ 0 & N_0 & N_1 & 0 \\ 0 & 0 & N_0 & N_1 \end{bmatrix} = \begin{bmatrix} F_0 & F_1 & F_2 & F_3 \end{bmatrix}$$

Regulation and Tracking

If we select

$$F(s) = (s+2)(s^2+2s+2) = s^3+4s^2+6s+4$$

Then

$$\begin{bmatrix} \tilde{A}_0 & B_0 & B_1 & B_2 \end{bmatrix} \begin{bmatrix} 0 & 4 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 6 & 4 & 1 \end{bmatrix}$$

$$C(s) = \frac{B(s)}{A(s)} = \frac{4s^2+2s+4}{1\times(s^2+4)} = \frac{4s^2+2s+4}{s^2+4}$$

This achieves the tracking to step reference and rejection of disturbance $a\sin(2t+\theta)$ asymptotically and robustly.

Implementable Transfer Functions

Implementable Transfer Functions

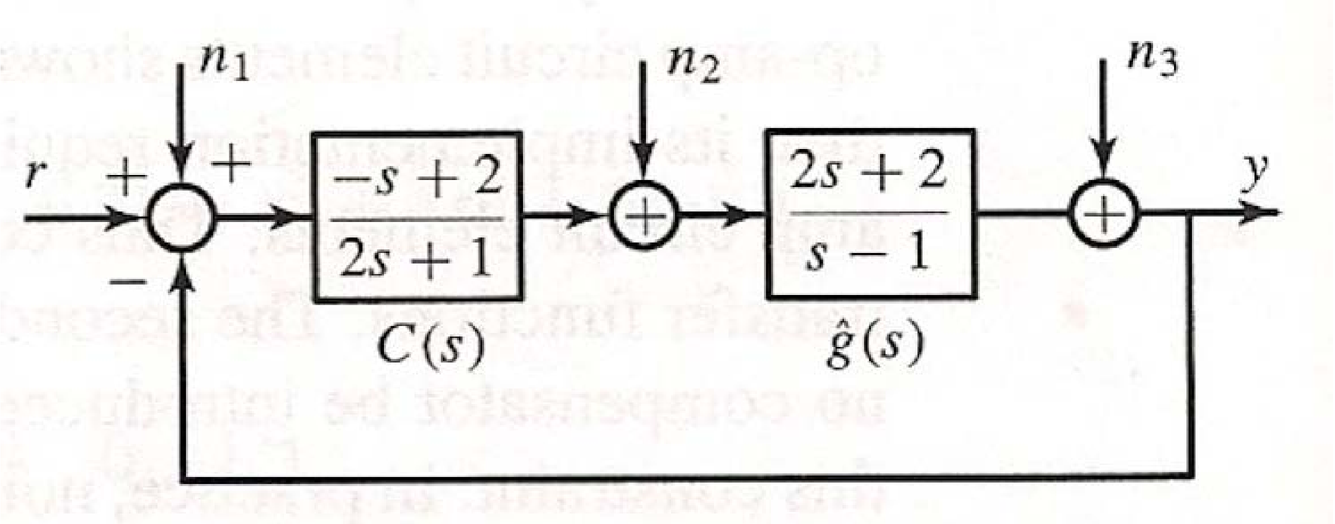
Design constraints:

1. All compensators used have proper rational transfer function.
2. The configuration selected has *no plant leakage* in the sense that all forward paths from r to y pass through the plant.
3. The closed loop transfer function of every possible input-output pair is proper (well posed) and BIBO stable(totally stable).

Implementable Transfer Functions

Implementable Transfer Functions

$$g_{yr}(s) = \frac{(-s+2)(2s+2)}{s+3} \rightarrow not \ \ proper(not \ \ well \ \ posed)$$



Implementable Transfer Functions

Definition 9.1

Given a plant with proper transfer function $g(s)$,
an overall transfer function $g_o(s)$ is said to be implementable
if there exists a no-plant-leakage configuration and
proper compensators so that the $g_o(s)$ is well posed and totally stable.

Theorem 9.4

Consider a plant with proper transfer function $g(s)$.
Then $g_o(s)$ is implementable if and only if $g_o(s)$ and

$$t(s) := \frac{g_o(s)}{g(s)}$$

are proper and BIBO stable.

Implementable Transfer Functions

Corollary 9.4

Consider a plant with proper transfer function $g(s) = N(s)/D(s)$.
Then $g_o(s) = E(s)/F(s)$ is implementable if and only if

1. All roots of $F(s)$ have negative real parts ($F(s)$ is Hurwitz).
2. Deg $F(s) - \deg E(s) \geq \deg D(s) - \deg N(s)$
   (pole-zero excess inequality).
3. All zeros of $N(s)$ with zero or positive real parts are retained in $E(s)$ (retainment of nonminimum-phase zeros).

Implementable Transfer Functions

Verification of Corollary 9.4

We design so that $g_o(s) = E(s)/F(s)$ is BIBO stable,

then all roots of $F(s)$ have negative real parts $\rightarrow$ *condition (1).*

In Theorem 9.4,

$$t(s) = \frac{g_o(s)}{g(s)} = \frac{E(s)D(s)}{F(s)N(s)}$$

should be proper in order for $g_o(s)$ to be implementable.

The condition for $t(s)$ to be proper is

$$\deg F(s) + \deg N(s) \geq \deg E(s) + \deg D(s) \rightarrow \textit{condition (2).}$$

In order for $t(s)$ to be BIBO stable, all roots of $N(s)$ with zero or positive real parts must be cancelled by the roots of $E(s)$. Thus $E(s)$ must contain the minimum phase zero of $N(s) \rightarrow$ *condition (3).*

Implementable Transfer Functions

Proof of Theorem 9.4

The necessity of Theorem 9.4

If the configuration with no plant leakage, then we have

$$y(s) = g_o(s)r(s) = g(s)u(s)$$

$$u(s) = \frac{g_o(s)}{g(s)} r(s) = t(s)r(s)$$

$t(s)$ is the closed loop transfer function from r to u. Then $t(s)$ should be proper and BIBO stable for the implementable condition (3).

The sufficiency of Theorem 9.4

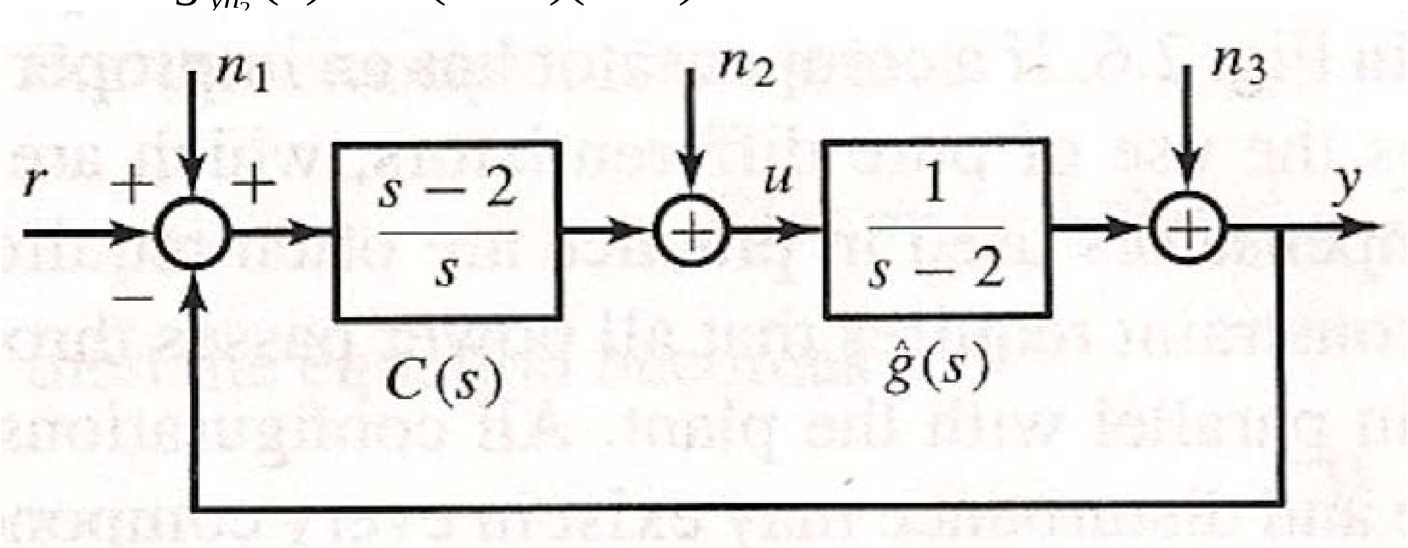
will be established constructively in the following lecture.

Implementable Transfer Functions

Problem of Unity Feedback Configuration

$$g_{yr}(s)=\frac{\frac{\cancel{s-2}}{s}\frac{1}{\cancel{s-2}}}{1+\frac{\cancel{s-2}}{s}\frac{1}{\cancel{s-2}}}=\frac{1}{s+1}\rightarrow not\ totally\ stable$$

$$g_{yn_2}(s)=s/(s-2)(s+1): not\ BIBO\ stable$$



Implementable Transfer Functions

Example 9.6

We want to design by unity feedback

$$g(s) = \frac{(s-2)}{s^2-1} \to g_o(s) = \frac{-(s-2)}{s^2+2s+2}$$

$$g_o(s) = \frac{C(s)g(s)}{1+C(s)g(s)} \to C(s) = \frac{g_o(s)}{g(s)\left[1-g_o(s)\right]} = \frac{-(s^2-1)}{s(s+3)}$$

$$g_o(s) = \frac{\dfrac{-(s^2-1)}{s(s+3)}\dfrac{(s-2)}{s^2-1}}{1+\dfrac{-(s^2-1)}{s(s+3)}\dfrac{(s-2)}{s^2-1}} = \frac{-(s^2-1)(s-2)}{(s^2-1)(s^2+2s+2)}$$

This implementation involves the pole-zero cancellation of unstable pole $s-1$ and so is not totally stable and not acceptable.

Model Matching-Two-Parameter Configuration

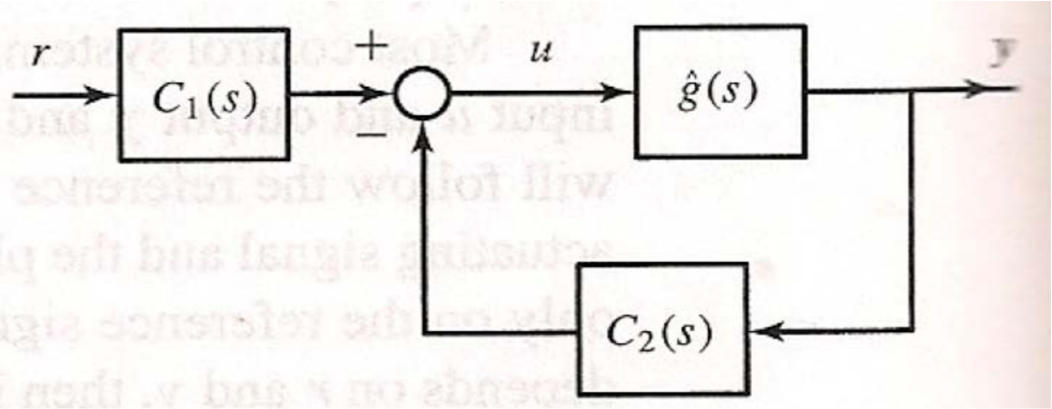
Model Matching-Two-Parameter Configuration

$$C_1(s) = \frac{L(s)}{A_1(s)} \qquad C_2(s) = \frac{M(s)}{A_2(s)}$$

Even if $A_1(s)$ & $A_2(s)$ are the same,

it can achieve any model matching. Then

$$C_1(s) = \frac{L(s)}{A(s)} \qquad C_2(s) = \frac{M(s)}{A(s)}$$



Model Matching-Two-Parameter Configuration

Model Matching-Two-Parameter Configuration

$$g_o(s) = C_1(s)\frac{g(s)}{1+g(s)C_2(s)} = \frac{L(s)}{A(s)}\frac{\frac{N(s)}{D(s)}}{1+\frac{N(s)}{D(s)}\frac{M(s)}{A(s)}}$$

$$= \frac{L(s)N(s)}{A(s)D(s)+M(s)N(s)}$$

$$g_o(s) = \frac{E(s)}{F(s)} = \frac{L(s)N(s)}{A(s)D(s)+M(s)N(s)} \left(cf.\frac{B(s)N(s)}{A(s)D(s)+B(s)N(s)}\right)$$

Problem

Given $g(s) = N(s)/D(s)$, where $N(s)$ & $D(s)$ are coprime and $\deg N(s) < \deg D(s) = n$, and given an implementable $g_o(s) = E(s)/F(s)$, find proper $L(s)/A(s)$ & $M(s)/A(s)$.

Model Matching-Two-Parameter Configuration

Procedure 9.1

1. Compute

$$\frac{g_o(s)}{N(s)} = \frac{E(s)}{F(s)N(s)} = \frac{\bar{E}(s)N^c(s)}{F(s)N^r(s)N^c(s)} =: \frac{\bar{E}(s)}{\bar{F}(s)}$$

where $\bar{E}(s)$and $\bar{F}(s)$ are coprime. Cancel all common factors between $E(s)$ and $N(s)$. Then

$$g_o(s) = \frac{\bar{E}(s)N(s)}{\bar{F}(s)} = \frac{L(s)N(s)}{A(s)D(s)+M(s)N(s)}$$

From this equation, we may be tempted to set $L(s)=\bar{E}(s)$ and solve for $A(s)$ and $M(s)$ from $\bar{F}(s)=A(s)D(s)+M(s)N(s)$. However, no proper $C_2(s)=M(s)/A(s)$ may exist in the equation. Thus we need some additional manipulation.

Model Matching-Two-Parameter Configuration

2. Introduce an arbitrary Hurwitz polynomial $\hat{F}(s)$

   such that the degree of $\bar{F}(s)\hat{F}(s)$ is $2n-1$ or higher to match

   the degree of denominators in both side of

$$g_o(s) = \frac{\bar{E}(s)\hat{F}(s)N(s)}{\bar{F}(s)\hat{F}(s)} = \frac{L(s)N(s)}{A(s)D(s)+M(s)N(s)} \quad (9.31)$$

   where $\deg A(s) \geq \deg D(s) - 1,$ whereas, $\deg A(s)\ D(s) \geq 2n-1.$

   In other words, if $\deg \bar{F}(s) = p,$ then $\deg \hat{F}(s) \geq 2n-1-p.$

   Because the polynomial $\hat{F}(s)$ will be canceled in the design,

   its roots should be chosen to lie inside the sector in Fig.8.3(a).

Model Matching-Two-Parameter Configuration

3. From

$$g_o(s) = \frac{\bar{E}(s)\hat{F}(s)N(s)}{\bar{F}(s)\hat{F}(s)} = \frac{L(s)N(s)}{A(s)D(s)+M(s)N(s)}$$

we set

$$L(s) = \bar{E}(s)\hat{F}(s)$$

and solve $A(s)$ and $M(s)$ from

$$A(s)D(s)+M(s)N(s) = \bar{F}(s)\hat{F}(s)$$

If we write

$$A(s) = A_0 + A_1 s + A_2 s^2 + \cdots + A_m s^m$$

$$M(s) = M_0 + B_1 s + M_2 s^2 + \cdots + M_m s^m$$

$$\bar{F}(s)\hat{F}(s) = F_0 + F_1 s + F_2 s^2 + \cdots + F_{n+m} s^{n+m}$$

with $m \geq n-1$.

Model Matching-Two-Parameter Configuration

Then $A(s)$ and $M(s)$ can be obtained by solving

$$[A_0\ M_0\ A_1\ M_1\ \cdots\ A_m\ M_m]\mathbf{S}_m = [F_0\ F_1\ F_2\ \cdots\ F_{n+m}]$$

$$\mathbf{S}_m := \begin{bmatrix} D_0 & D_1 & \dots & D_n & 0 & \dots & 0 \\ N_0 & N_1 & \dots & N_n & 0 & \dots & 0 \\ 0 & D_0 & \dots & D_{n-1} & D_n & \dots & 0 \\ 0 & N_0 & \dots & N_{n-1} & N_n & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 & D_0 & \dots & D_n \\ 0 & 0 & \dots & 0 & D_0 & \dots & D_n \end{bmatrix}$$

$M(s)/A(s)$ must be proper. In order for $L(s)/A(s)$ to be proper,

$$\deg L(s) \le \deg A(s) = \deg\left(\bar{F}(s)\hat{F}(s)\right) - \deg D(s)$$

$$\deg\left(\bar{F}(s)\hat{F}(s)\right) - \deg N(s) - \deg L(s) \ge \deg D(s) - \deg N(s)$$

relative degree of $g_o(s) \ge$ relative degree of $g(s)$.

Model Matching-Two-Parameter Configuration

Example 9.7

$$g(s)=\frac{s-2}{s^2-1}\rightarrow g_o(s)=\frac{-(s-2)}{s^2+2s+2}$$

$$\frac{g_o(s)}{N(s)}=\frac{-(s-2)}{(s^2+2s+2)(s-2)}=\frac{-1}{s^2+2s+2}=:\frac{\bar{E}(s)}{\bar{F}(s)}$$

$$L(s)=\bar{E}(s)\hat{\hat{F}}(s)=-(s+4),\ \ F(s)=(s+4)$$

$$A(s)D(s)+M(s)N(s)=\bar{F}(s)\hat{F}(s)=(s^2+2s+2)(s+4)$$

$$=s^3+6s^2+10s+8$$

$$D(s)=s^2+0s-1,\ \ N(s)=s-2.$$

Model Matching-Two-Parameter Configuration

Example 9.7 (cont)

$$[A_0\ M_0\ A_1\ M_1]\begin{bmatrix} -1 & 0 & 1 & 0 \\ -2 & 1 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & -1 & 0 & 1 \\ 0 & -2 & 1 & 0 \end{bmatrix} = [8\ 10\ 6\ 1]$$

Solution is $[18 \quad -13 \quad 1 \quad -12]$.

$$C_1(s) = \frac{L(s)}{A(s)} = \frac{-(s+4)}{s+18} \qquad C_2(s) = \frac{M(s)}{A(s)} = \frac{-(12s+13)}{s+18}$$

Model Matching-Two-Parameter Configuration

Example 9.8

$$g(s)=\frac{s-2}{s^2-1}\rightarrow g_o(s)=\frac{-(s-2)(4s+2)}{(s^2+2s+2)(s+2)}=\frac{-4s^2+6s+4}{s^3+4s^2+6s+4}$$

$$g(0)=1, g'(0)=0 \rightarrow \textit{tracking to } \text{step and ramp reference.}$$

$$\frac{g_o(s)}{N(s)}=\frac{-(s-2)(4s+2)}{(s^2+2s+2)(s+2)(s-2)}=\frac{-(4s+2)}{s^3+4s^2+6s+4}=:\frac{\bar{E}(s)}{\bar{F}(s)}$$

$$L(s)=\hat{F}(s)\bar{E}(s)=-(4s+2),\ \ where\ \hat{F}(s)=1$$

$$[A_0\ M_0\ A_1\ M_1]\begin{bmatrix}-1 & 0 & 1 & 0\\ -2 & 1 & 0 & 0\\ \cdots & \cdots & \cdots & \cdots\\ 0 & -1 & 0 & 1\\ 0 & -2 & 1 & 0\end{bmatrix}=[4\ 6\ 4\ 1]$$

$$C_1(s)=\frac{-(4s+2)}{s+34/3} \qquad C_2(s)=\frac{-(22s+23)}{3s+34}$$

Model Matching-Two-Parameter Configuration

Implementation of Two-Parameter Compensators

$$u(s) = C_1(s)r(s) - C_2(s)y(s) = \frac{L(s)}{A(s)}r(s) - \frac{M(s)}{A(s)}y(s)$$

$$= A^{-1}(s)\begin{bmatrix} L(s) & -M(s) \end{bmatrix}\begin{bmatrix} r(s) \\ y(s) \end{bmatrix}$$

$$\mathbf{C}(s) = \begin{bmatrix} C_1(s) & -C_2(s) \end{bmatrix} = A^{-1}(s)\begin{bmatrix} L(s) & -M(s) \end{bmatrix}$$

This is a transfer function matrix with 2-inputs, 1-output which can be realized by a $m-dimensional$ state equation.

Model Matching-Two-Parameter Configuration

Example 9.9

The implemenation of compensator in *Example* 9.8:

$$u(s) = C_1(s)r(s) - C_2(s)y(s) = \begin{bmatrix} \frac{-(4s+2)}{s+11.33} & \frac{7.33s+7.67}{s+11.33} \end{bmatrix} \begin{bmatrix} r(s) \\ y(s) \end{bmatrix}$$

$$= \left(\begin{bmatrix} -4 & 7.33 \end{bmatrix} + \frac{1}{s+11.33} \begin{bmatrix} 43.33 & -75.38 \end{bmatrix} \right) \begin{bmatrix} r(s) \\ y(s) \end{bmatrix}$$

$$\dot{x} = -11.33x + \begin{bmatrix} 43.33 & -75.38 \end{bmatrix} \begin{bmatrix} r \\ y \end{bmatrix}$$

$$u = x + \begin{bmatrix} -4 & 7.33 \end{bmatrix} \begin{bmatrix} r \\ y \end{bmatrix}$$

Future Study

- Optimization Techniques
  - ✓ Convex optimization
  - ✓ Newton methods, ...
  - ✓ Formulation of optimization problem
- Optimal Control
  - ✓ Find optimal gain for state feedback
  - ✓ Recatti equation
  - ✓ LQG/LTR techniques for Multivariable system
- Estimation Theory
  - ✓ Find optimal gain for state estimator
  - ✓ Kalman Recatti equation
  - ✓ Kalman filter design
- Random process
- Linear Systems

Future Study

- Adaptive Control
  - ✓ Automatic on-line design of compensators
  - ✓ On-line parameter estimation of compensators
- Nonlinear Filtering
  - ✓ Particle filter
  - ✓ Extended Kalman filter
- Applications
  - ✓ Control applications
  - ✓ Communication/power/vision/robot systems
  - ✓ Intelligent/Learning systems

Closing Remarks

T Thanks for your sincere listening to my lecture

S Sorry for me not to give good presentations.

L Love you