## Mechanics in Energy Resources Engineering - Chapter 4 Shear Forces and Bending Moments

Ki-Bok Min, PhD

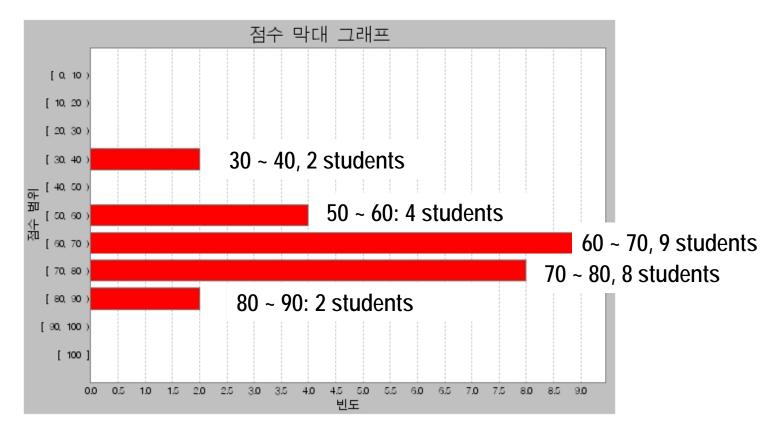
Assistant Professor Energy Resources Engineering Seoul National University







- Mean: 65.3, standard deviation: 12.9
- Max: 86.0, Min: 30.0







- In general, you demonstrated your understanding to a reasonable extent and you are in good positions to study further.
- Try to thoroughly understand the home assignments. I encourage discussion with your peers.
- This time only, partial point was around 10% 70%. However, it will be minimized next time. Max partial point will be 30%.
- Level of difficulty will be similar in the 2<sup>nd</sup> and 3<sup>rd</sup> exam.
- 2<sup>nd</sup> exam: Ch. 4, 5 & 12
- 3<sup>rd</sup> exam: entire chapters.





- Introduction
- Torsional Deformations of a circular bar (원형봉의 비틀림 변형)
- Circular bars of linearly elastic materials (선형탄성 원형봉)
- Nonuniform torsion (불균질 비틀림)
- Stresses and Strains in Pure Shear (순수전단에서의 응력과 변형율)
- Relationship Between Moduli of Elasticity E and G (탄성계수 E와 G의 관계)
- Statically Indeterminate Torsional Members (부정정 비틀림 부재)
- Strain Energy in Torsion and Pure Shear (비틀림과 순수전단에서의 변형에너지)

## Change of schedule



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- 5 April (Ch.4)
- 12 April (Ch.4)
- 19 April (Ch.5), hw#5 due
- 26 April (Review),

7 April (Ch.12) by Jae-Won Lee
14 April (Ch.5), hw#4 due
21 April (Ch.5)
28 April (2<sup>nd</sup> Exam), hw#6 due



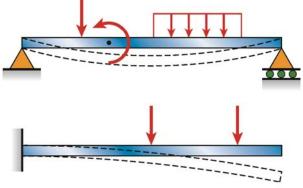


- Introduction
- Types of Beams, Loads, and Reactions
- Shear Forces and Bending Moments
- Relationships Between Loads, Shear Forces and Bending Moments
- Shear-Force and Bending-Moment Diagrams





- Structural members
  - Axially loaded bar (봉): forces along the axis
  - A bar in torsion: torques along the axis (moment vectors)
  - Beam (보): lateral loads
- Planar structure lie in a single plane
  - Loads and deflections occurs in the *plane of bending*



## Types of Beams, Loads, and Reactions beams



- Assumptions
  - Loads act in the plane of the figure: force vectors in the plane of figure & bending moments have their moments vectors perpendicular to the plane of the figure
  - Beam is symmetric about that plane → deflect only in the plane of bending

#### Types of Beams, Loads, and Reactions Types of Beams

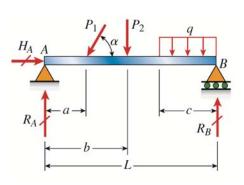


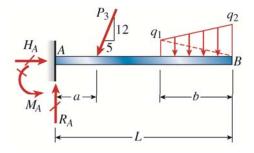
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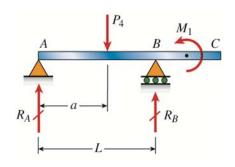
• Simple beam (단순보)

• Cantilever beam (캔틸레버보)

• Beam with an overhang (돌출보)

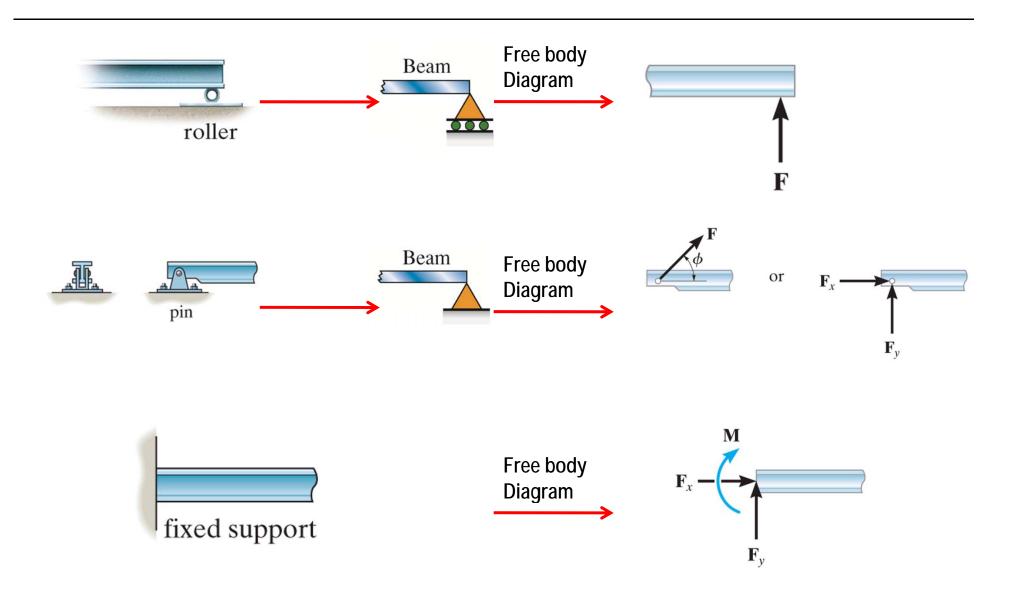






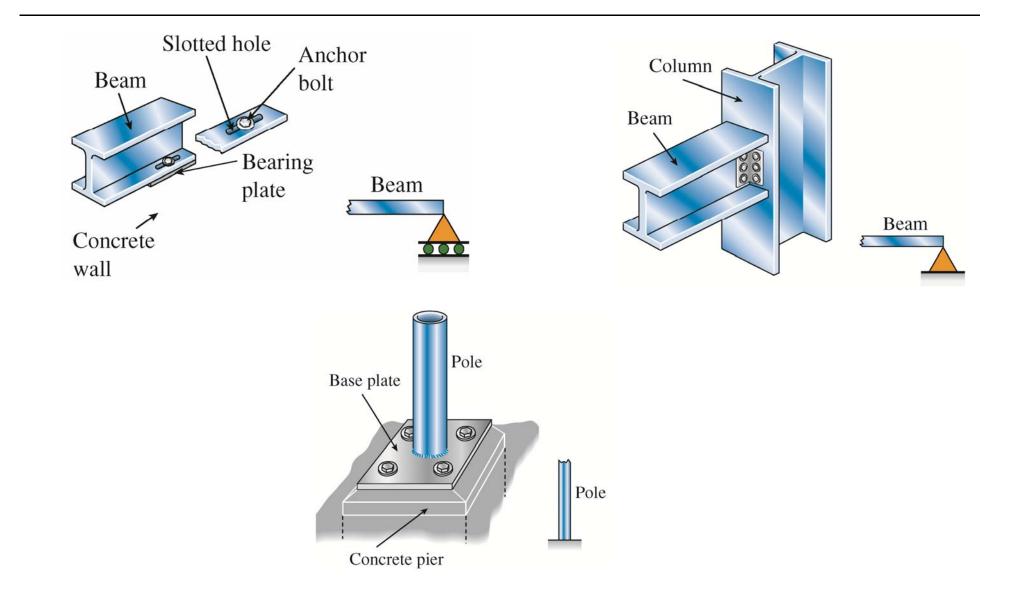
### Types of Beams, Loads, and Reactions Types of supports





### Types of Beams, Loads, and Reactions Actual Examples

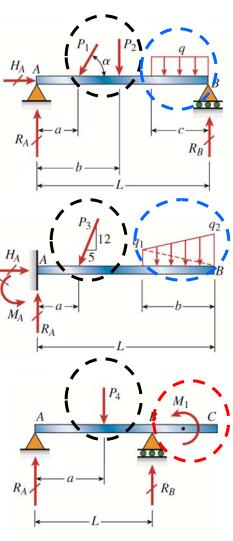




# Types of Beams, Loads, and Reactions Loads



- Concentrated load
  - applied over a very small area
- Distributed load
  - Spread along the axis of a beam
  - Measured by their intensity (Force/unit distance)
  - Uniformly distributed & linearly varying load
- Couple
  - The couple of moment  $M_1$  acting on the overhang

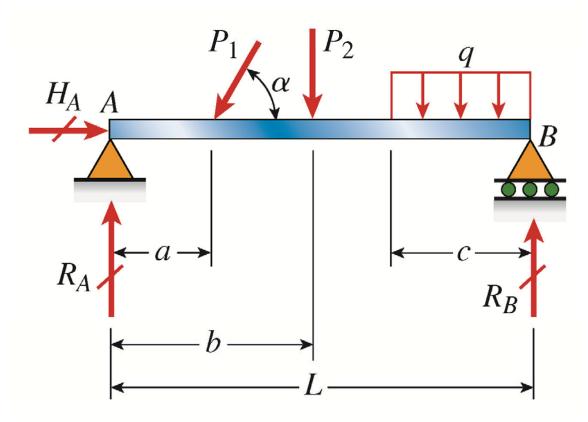


#### Types of Beams, Loads, and Reactions Reactions



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• Simple beam

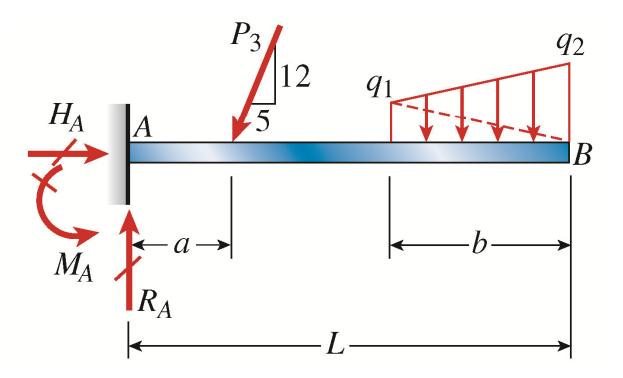


#### Types of Beams, Loads, and Reactions Reactions



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Cantilever beam

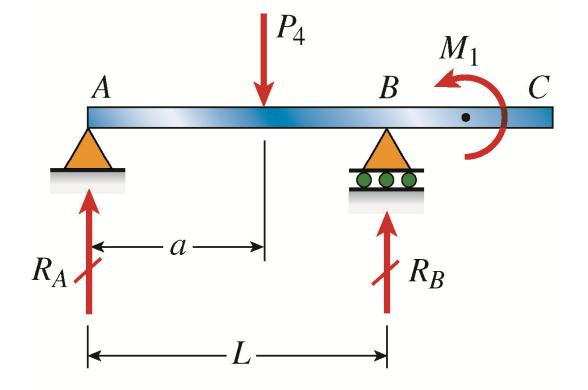


#### Types of Beams, Loads, and Reactions Reactions



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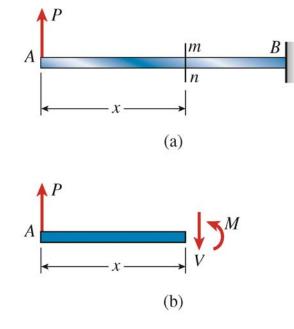
• Beam with an overhang

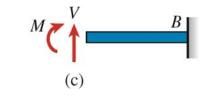


# Shear Forces and Bending Moments basic concepts

- Beams under forces or moment → stresses and strains are created throughout the interior of the beam.
- We first find the internal forces and couple (bending moment) on the cross section.
- Free Body Diagram isolate left or right hand part.
- Stress resultant (합응력): resultants of stresses distributed over the cross section.









# Shear Forces and Bending Moments methodology



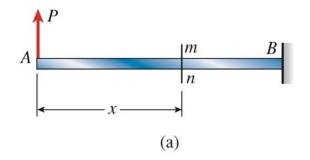
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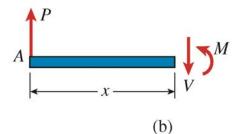
• Equilibrium Equation

$$\sum F_{ver} = 0 \qquad P - V = 0$$

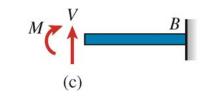
$$V = P$$

$$\sum M = 0 \qquad M - Px = 0$$





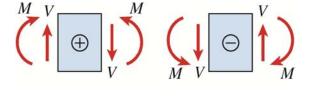
$$M = Px$$



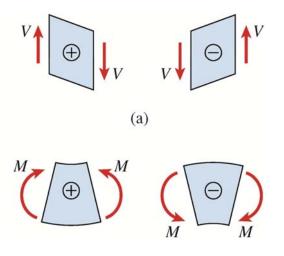
# Shear Forces and Bending Moments sign conventions for stress resultants



- (+) shear force: acts clockwise
- (-) shear force: .....counter-clockwise



- (+) bending moment: compress upper part
- (-) bending moment: ..... lower part
- 'deformation sign convention'
  - Based on how the material is deformed.
- 'static sign convention'
  - Forces/moments are (+) or (-) according to their directions



#### Shear Forces and Bending Moments Example 4-1



- Shear force V & bending moment M at the right and left of mid point?
  - $R_A \& R_B?$
  - Free Body Diagram.

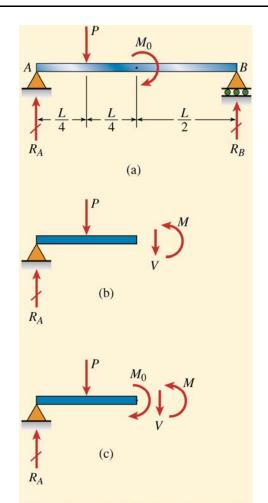


FIG. 4-11 Example 4-1. Shear forces and bending moment in a simple beam (parts (a) and (b) repeated)

#### Shear Forces and Bending Moments Example 4-2

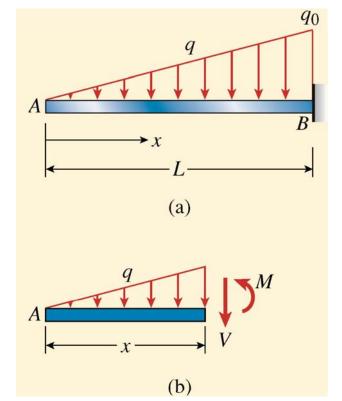


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• Shear force V & Bending moment M?

$$q = \frac{q_0}{L}x$$
$$V = -\frac{q_0 x^2}{2L}$$
$$M = -\frac{q_0 x^3}{6L}$$

$$-\frac{dV}{dx} = q \qquad \frac{dM}{dx} = V$$

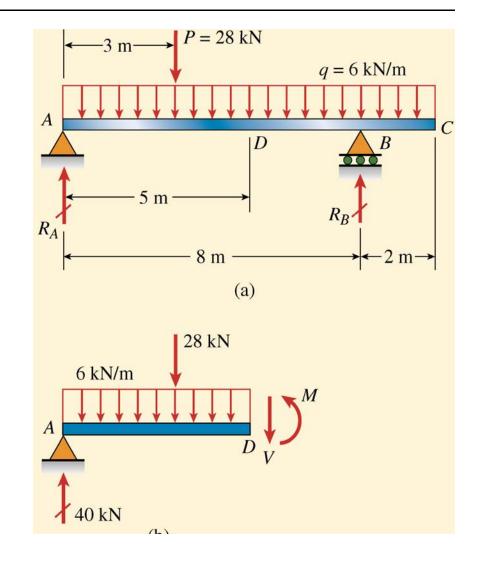


#### Shear Forces and Bending Moments Example 4-3



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• Shear force V & bending moment M at D?



# Relationships Between Loads, Shear Forces and Bending Moments



- Useful for investigating the shear forces and bending moments throughout the entire length of a beam
- Helpful when constructing shear-force and bending-moment diagrams





- Introduction
- Types of Beams, Loads, and Reactions
- Shear Forces and Bending Moments
- Relationships Between Loads, Shear Forces and Bending Moments
- Shear-Force and Bending-Moment Diagrams

Next Monday

## This Wednesday



- Introduction
- Centroids of Plane Areas
- Centroids of Composite Areas
- Moments of Inertia of Plane Areas
- Parallel-Axis Theorem for Moments of Inertia
- Polar Moments of Inertia

