Robust Design

4013.315 Architectural Engineering System Design

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Robust Design: Experiments for Better Products

Teaching materials to accompany:

Product Design and Development
Chapter 13

Karl T. Ulrich and Steven D. Eppinger
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1. Introduction
2. Development Processes and Organizations
3. Product Planning
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9. Product Architecture
10. Industrial Design
11. Design for Manufacturing
12. Prototyping
13. Robust Design
14. Patents and Intellectual Property
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Robust Design and Quality in the Product Development Process

- Planning
- Concept Development
- System-Level Design
- Detail Design
- Testing and Refinement
- Production Ramp-Up

- Robust Concept and System Design
- Robust Parameter Design
- Quality efforts are typically made here, when it is too late.
Goals for Designed Experiments

• Modeling
  – Understanding relationships between design parameters and product performance
  – Understanding effects of noise factors

• Optimizing
  – Reducing product or process variations
  – Optimizing nominal performance
Robust Designs

A robust product or process performs correctly, even in the presence of noise factors.

Noise factors may include:

– parameter variations
– environmental changes
– operating conditions
– manufacturing variations
Robust Design Example: Seat Belt Experiment
Who is the better target shooter?

Sam

John
Who is the better target shooter?

Sam can simply adjust his sights.

John requires lengthy training.
Exploiting Non-Linearity to Achieve Robust Performance

Response = \( f_A(A) + f_B(B) \)

What level of factor B gives the robust response?
How do we use factor A?
Robust Design Procedure
Step 1: Parameter Diagram

Step 1: Select appropriate controls, response, and noise factors to explore experimentally.

- Control factors (input parameters)
- Noise factors (uncontrollable)
- Performance metrics (response)
The “P” Diagram

Control Factors → Product or Process → Performance Metrics

Noise Factors
Parameter Diagram

**Control Factors**
- Belt webbing stiffness
- Belt webbing friction
- Lap belt force limiter
- Upper anchorage stiffness
- Buckle cable stiffness
- Front seatback bolster
- Tongue friction
- Attachment geometry

**Passenger Restraint Process**

**Noise Factors**
- Shape of rear seat
- Type of seat fabric
- Severity of collision
- Wear of components
- Positioning of passenger
- Positioning of belts on body
- Size of passenger
- Type of clothing fabric
- Web manufacturing variations
- Latch manufacturing variations

**Performance Metrics**
- Back angle
- Slip of buttocks
- Hip rotation
- Forward knee motion
Example: Brownie Mix

• Control Factors
  – Recipe Ingredients (quantity of eggs, flour, chocolate)
  – Recipe Directions (mixing, baking, cooling)
  – Equipment (bowls, pans, oven)

• Noise Factors
  – Quality of Ingredients (size of eggs, type of oil)
  – Following Directions (stirring time, measuring)
  – Equipment Variations (pan shape, oven temp)

• Performance Metrics
  – Taste Testing by Customers
  – Sweetness, Moisture, Density
Robust Design Procedure

Step 2: Objective Function

Step 2: Define an objective function (of the response) to optimize.

- maximize desired performance
- minimize variations
- target value
- signal-to-noise ratio
Types of Objective Functions

Larger-the-Better
- e.g. performance
  \[ \eta = \mu^2 \]

Nominal-the-Best
- e.g. target
  \[ \eta = 1/(\mu-t)^2 \]

Smaller-the-Better
- e.g. variance
  \[ \eta = 1/\sigma^2 \]

Signal-to-Noise
- e.g. trade-off
  \[ \eta = 10\log[\mu^2/\sigma^2] \]
Robust Design Procedure

Step 3: Plan the Experiment

Step 3: Plan experimental runs to elicit desired effects.

• Use full or fractional factorial designs to identify interactions.

• Use an orthogonal array to identify main effects with minimum of trials.

• Use inner and outer arrays to see the effects of noise factors.
Experiment Design: Full Factorial

• Consider $k$ factors, $n$ levels each.
• Test all combinations of the factors.
• The number of experiments is $n^k$.
• Generally this is too many experiments, but we are able to reveal all of the interactions.

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2 factors, 3 levels each:

$$n^k = 3^2 = 9 \text{ trials}$$

4 factors, 3 levels each:

$$n^k = 3^4 = 81 \text{ trials}$$
Experiment Design: One Factor at a Time

- Consider $k$ factors, $n$ levels each.
- Test all levels of each factor while freezing the others at nominal level.
- The number of experiments is $nk+1$.
- BUT this is an unbalanced experiment design.

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Experiment Design: Orthogonal Array

- Consider $k$ factors, $n$ levels each.
- Test all levels of each factor in a balanced way.
- The number of experiments is order of $1+k(n-1)$.
- This is the smallest balanced experiment design.
- BUT main effects and interactions are confounded.

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4 factors, 3 levels each:

$1+k(n-1) = 1+4(3-1) = 9$ trials
Using Inner and Outer Arrays

- Induce the same noise factor levels for each combination of controls in a balanced manner

4 factors, 3 levels each: L9 inner array for controls

3 factors, 2 levels each: L4 outer array for noise

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inner x outer = L9 x L4 = 36 trials
Robust Design Procedure

Step 4: Run the Experiment

Step 4: Conduct the experiment.
• Vary the control and noise factors
• Record the performance metrics
• Compute the objective function
## Paper Airplane Experiment

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Robust Design Procedure
Step 5: Conduct Analysis

Step 5: Perform analysis of means.

- Compute the mean value of the objective function for each factor setting.
- Identify which control factors reduce the effects of noise and which ones can be used to scale the response. (2-Step Optimization)
Analysis of Means (ANOM)

• Plot the average effect of each factor level.

Factor Effects on S/N Ratio

Choose the best levels of these factors

Scaling factor?

Prediction of response:

\[ E[\eta(A_i, B_j, C_k, D_l)] = \mu + a_i + b_j + c_k + d_l \]
Robust Design Procedure

Step 6: Select Setpoints

Step 6: Select control factor setpoints.

• Choose settings to maximize or minimize objective function.

• Consider variations carefully. (Use ANOM on variance to understand variation explicitly.)

Advanced use:

• Conduct confirming experiments.

• Set scaling factors to tune response.

• Iterate to find optimal point.

• Use higher fractions to find interaction effects.

• Test additional control and noise factors.
Confounding Interactions

- Generally the main effects dominate the response. BUT sometimes interactions are important. This is generally the case when the confirming trial fails.
- To explore interactions, use a fractional factorial experiment design.
Alternative Experiment Design Approach: Adaptive Factor One at a Time

- Consider $k$ factors, $n$ levels each.
- Start at nominal levels.
- Test each level of each factor one at a time, while freezing the previous ones at best level so far.
- The number of experiments is $nk+1$.
- Since this is an unbalanced experiment design, it is generally OK to stop early.
- Helpful to sequence factors for strongest effects first.
- Generally found to work well when interactions are present.

4 factors, 2 levels each:

\[ nk+1 = 2 \times 4 + 1 = 9 \text{ trials} \]

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Ref: Forthcoming paper by Dan Frey
Key Concepts of Robust Design

• Variation causes quality loss
• Two-step optimization
• Matrix experiments (orthogonal arrays)
• Inducing noise (outer array or repetition)
• Data analysis and prediction
• Interactions and confirmation
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DOE Plan and Data

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