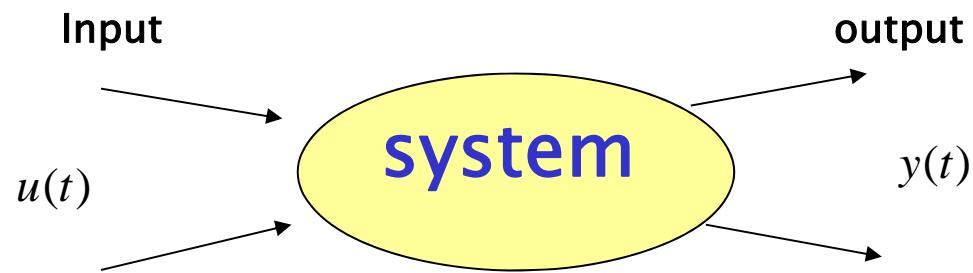


# Lecture 12

# Control Systems

# System, Input and Output



Output depends on inputs !

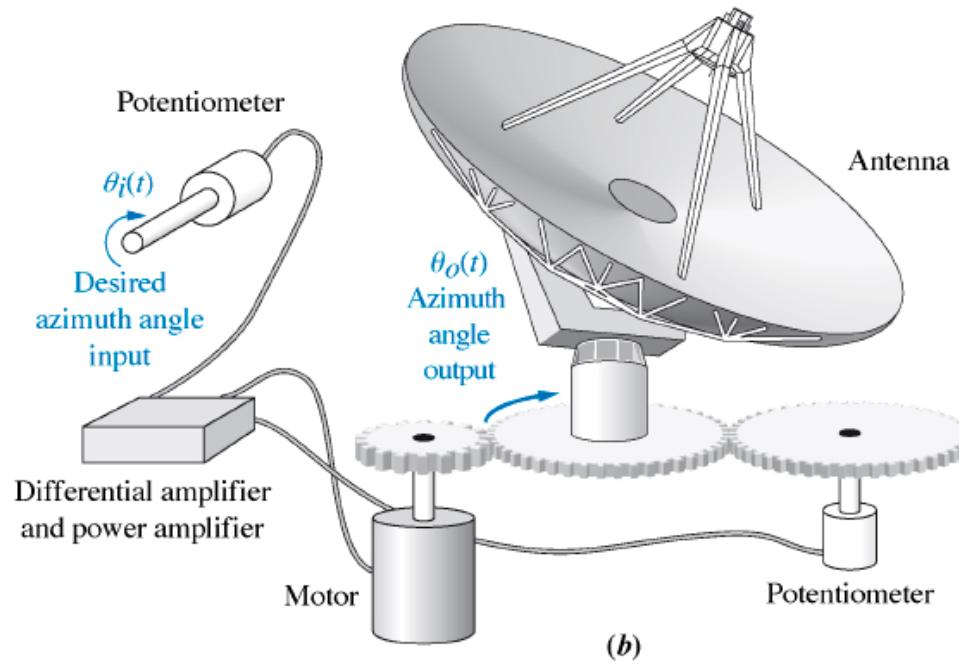
# Control systems

Open loop control

Closed loop control

Closed loop transfer function

# Antenna azimuth position control system



(b)

# a DC Servomotor System

## servo-motor system

$R_a$  : armature resistance,  $\Omega$

$i_a$  : armature current, A

$i_f$  : field current, A

$e_a$  : applied armature voltage, V

$e_b$  : back emf, V

$\theta_1$  : angular displacement of the motor shaft, rad

$\theta_2$  : angular displacement of the load shaft, rad

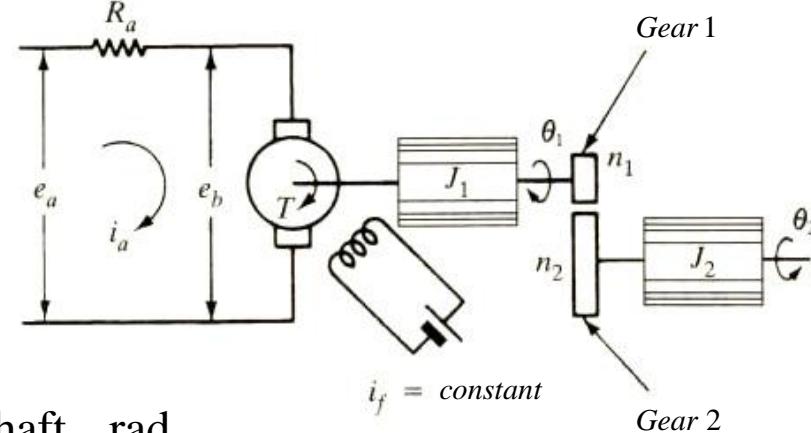
$T$  : torque developed by the motor, N- m

$J_1$  : equivalent moment of inertia of the motor,  $\text{kg}\cdot\text{m}^2$

$J_2$  : equivalent moment of inertia of the load,  $\text{kg}\cdot\text{m}^2$

The torque of motor :  $T = K i_a$

For a constant flux, the induced voltage :  $e_b = K_b \frac{d\theta}{dt}$        $K_b$  : back emf constant



# DC Servomotor System

Armature circuit D.E :  $R_a i_a + e_b = e_a$

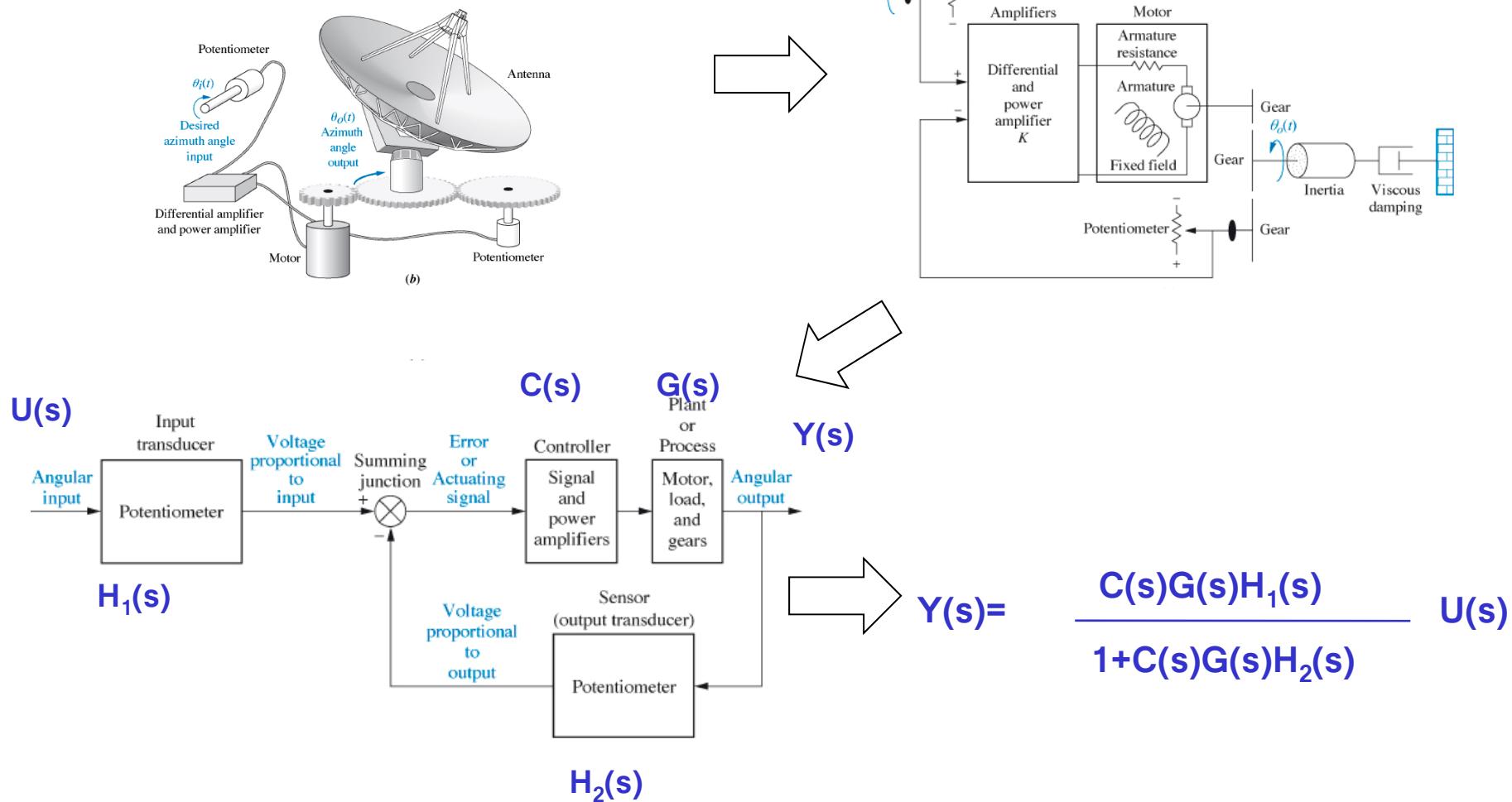
Inertia and friction :  $J_{1eq} = J_1 + \left( \frac{n_1}{n_2} \right)^2 J_2$

Laplace transforms of these equations :

$$K_b s \Theta(s) = E_b(s), \quad (L_a s + R_a) I_a(s) + E_b(s) = E_a(s), \quad (J s^2 + b s) \Theta(s) = T(s) = K I_a(s)$$

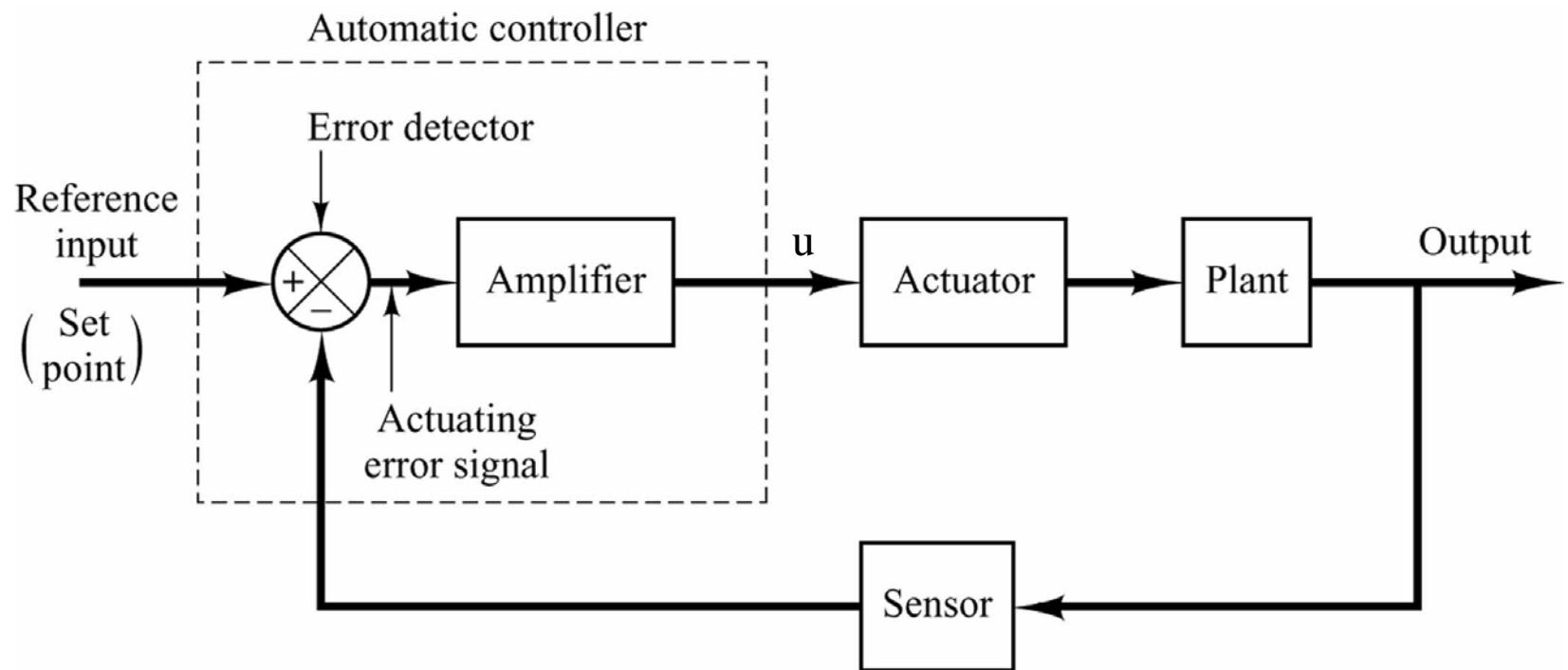
$$\begin{aligned} T.F &= \frac{\Theta(s)}{E_a(s)} = \frac{K}{s(R_a J s + R_a b + K K_b)} = \frac{\frac{K}{R_a J}}{s \left( s + \frac{R_a b + K K_b}{R_a J} \right)} \\ &= \frac{K_m}{s(T_m s + 1)} \end{aligned}$$

# System → Schematic → Block Diagram → Transfer Functions



# Antenna azimuth position control system

# Control Systems



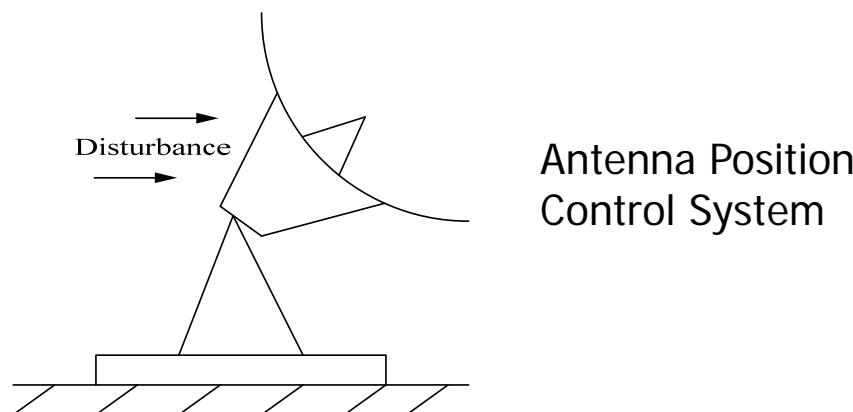
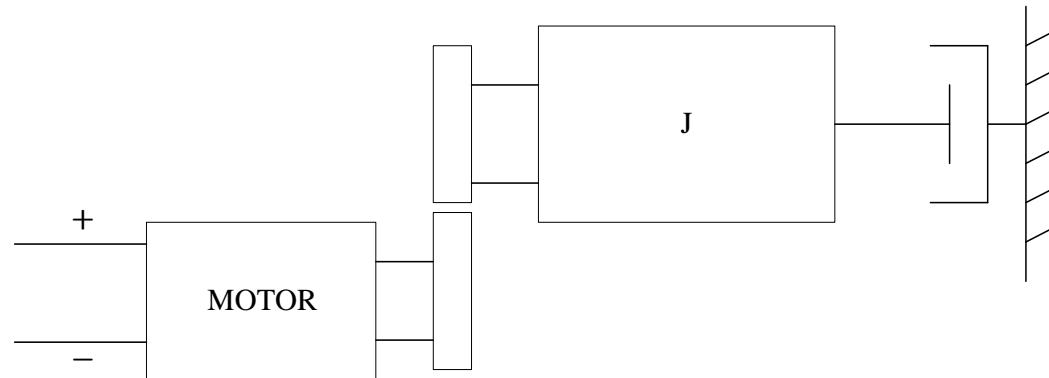
$$u=f(r,c)$$

# PID controllers

## Proportional Control

$$U = k_p e$$

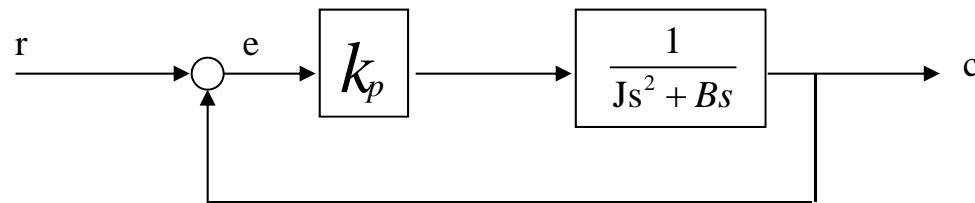
$k_p$  : The proportional Gain



# controllers

## Proportional Control

$$U = k_p e \quad k_p : \text{The proportional Gain}$$



$$\frac{c(s)}{r(s)} = \frac{\frac{K_p}{Js^2 + Bs}}{1 + \frac{K_p}{Js^2 + Bs}} = \frac{K_p}{Js^2 + Bs + K_p}$$

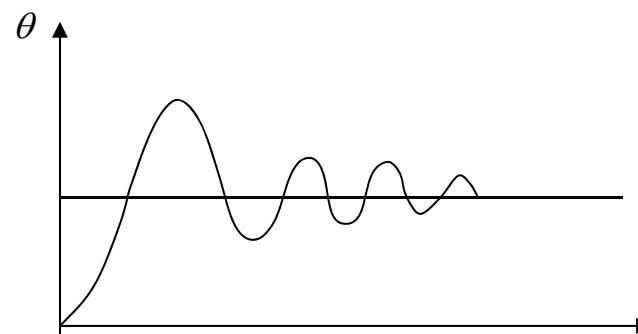
# controllers

## Proportional Control

=

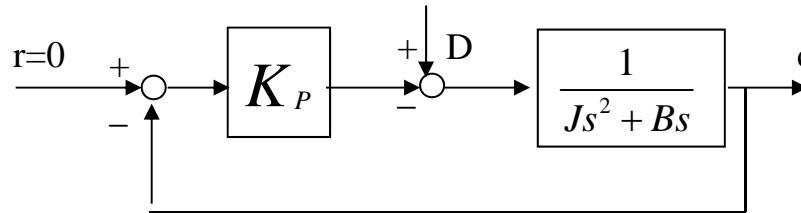
- Step input response  $r(s) = \frac{1}{s}$

- steady state response  $c(t) = \lim_{s \rightarrow 0} sC(s)$   
 $= \lim_{s \rightarrow 0} s \frac{K_p}{Js^2 + Bs + K_p} \frac{1}{s} = 1$



## Proportional Control

- Response to torque disturbance



$$C(s) = \frac{K_p}{J s^2 + B s + K_p} r(s) + \frac{1}{J s^2 + B s + K_p} D(s)$$

Assume that  $D(s) = \frac{1}{s}$  steady state response

$$c(t) = \lim_{s \rightarrow \infty} s \cdot \frac{1}{J s^2 + B s + K_p} \cdot \frac{1}{s} = \frac{1}{K_p} : \text{steady-state error}$$

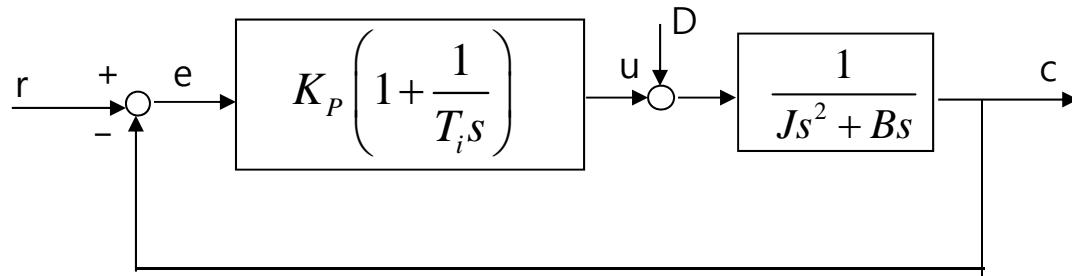
Large  $K_p$  → small steady-state error  
→ large motor power is needed  
→ oscillations  
→ large  $w_n$   $\left( w_n = \sqrt{\frac{K_p}{J}} \right)$

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$$\rightarrow \text{small damping ratio } \zeta = \frac{B}{2\sqrt{JK_p}}$$

## Proportional-Integral Control (PI control)

- Response to torque disturbance



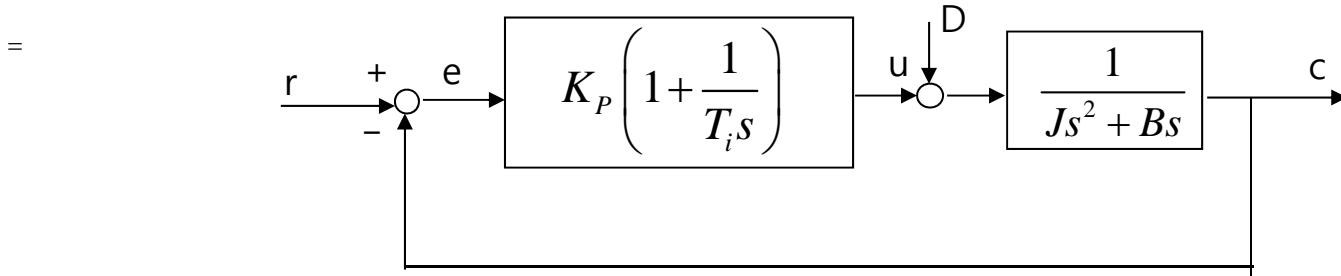
$$\frac{U(s)}{E(s)} = K_p \left( 1 + \frac{1}{T_i s} \right)$$

$$u(t) = K_p e(t) + \frac{K_p}{T_i} \int_0^t e(t) dt$$

$$\frac{C(s)}{R(s)} = \frac{K_p s + \frac{1}{T_i} K_p}{J s^3 + B s^2 + K_p s + \frac{K_p}{T_i}}$$

No steady-state error for reference input

# Proportional–Integral Control (PI control)



$$\frac{C(s)}{D(s)} = \frac{\frac{1}{Js^2 + Bs}}{1 + K_P \left(1 + \frac{1}{T_i s}\right) \frac{1}{Js^2 + Bs}}$$

$$= \frac{s}{Js^3 + Bs^2 + K_P s + \frac{K_P}{T_i}}$$

$$D(s) = \frac{1}{s}$$
$$C(t) = \lim_{s \rightarrow 0} s \frac{s}{Js^3 + Bs^2 + K_P s + \frac{K_P}{T_i}} \frac{1}{s} = 0$$

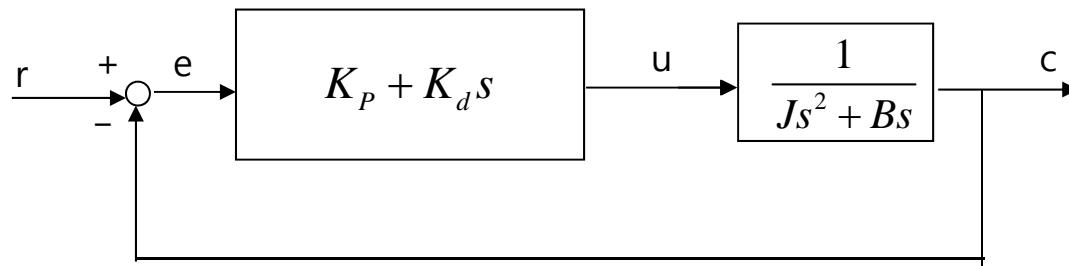
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No steady-state error for step disturbance

Spring 2014

## Proportional-Derivative Control (PD control)

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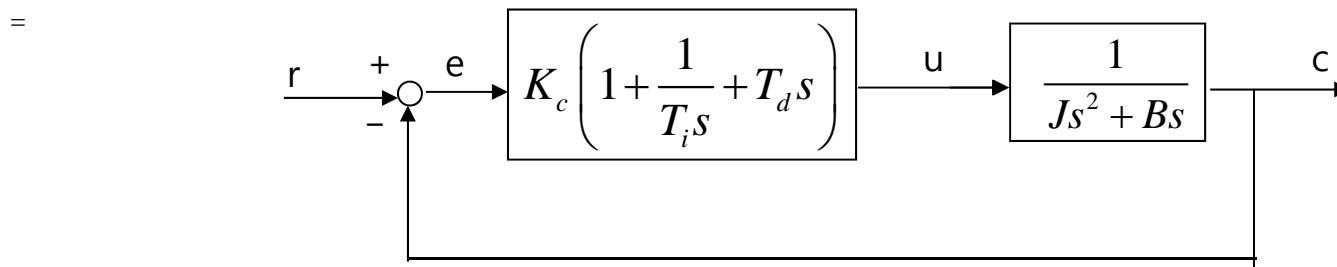
$$u(t) = K_p e(t) + K_d \frac{d}{dt} e(t)$$

$$\frac{C(s)}{R(s)} = \frac{K_p + K_d s}{J s^2 + (B + K_d) s + K_p}$$

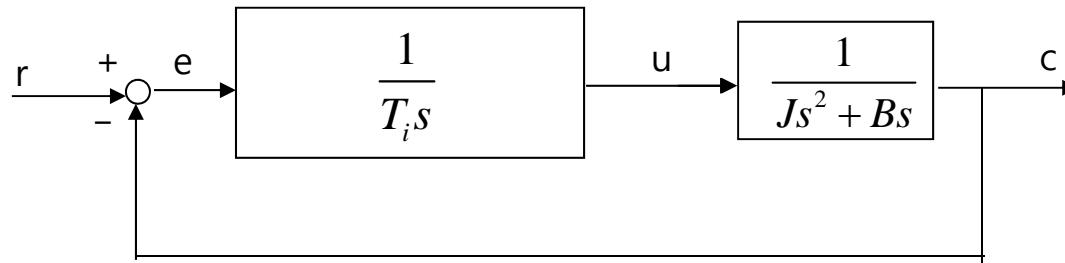
$$\zeta = \frac{B + K_d}{2\sqrt{J K_p}} : \text{increased effective damping}$$

# Classifications of controllers

## Proportional-Integral-Derivative Control (PID control)

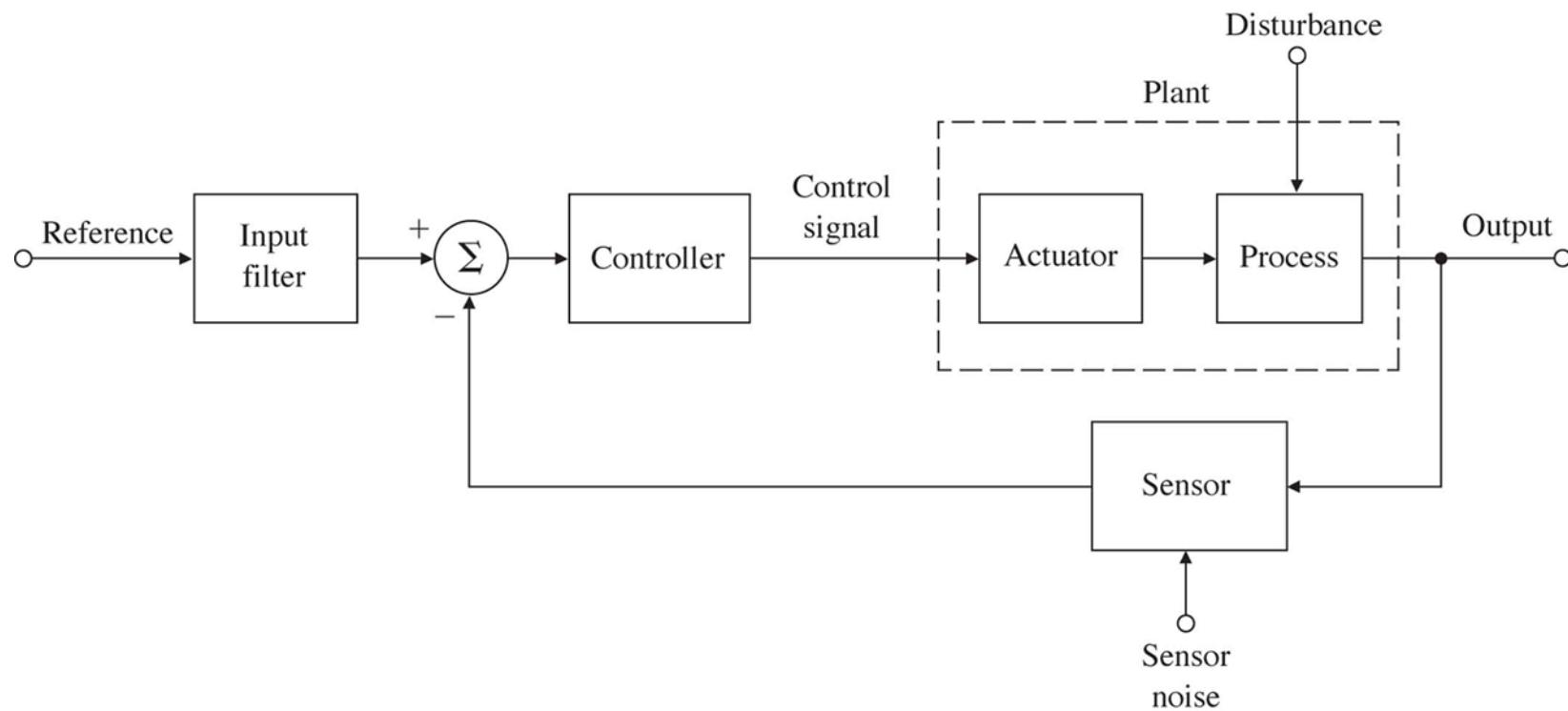


## I control



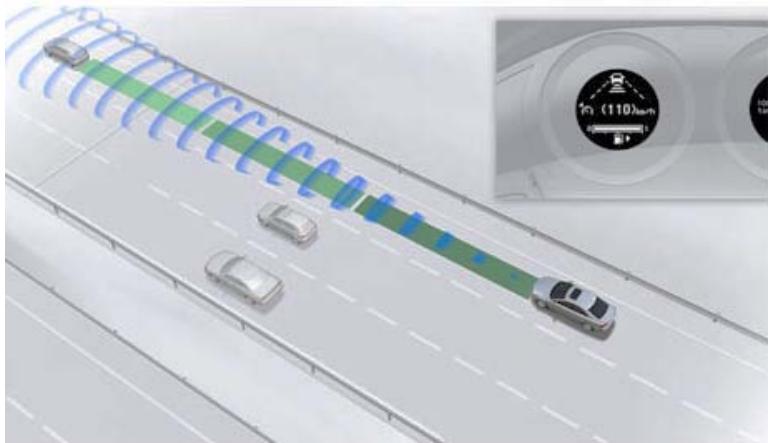
$$\frac{C(s)}{R(s)} = \frac{\frac{1}{T_i s} \frac{1}{J s^2 + B s}}{1 + \frac{1}{T_i s} \frac{1}{J s^2 + B s}} = \frac{\frac{1}{T_i}}{J s^3 + B s^2 + \frac{1}{T_i}}$$

# Component block diagram of an elementary feedback control



# Vehicle control systems

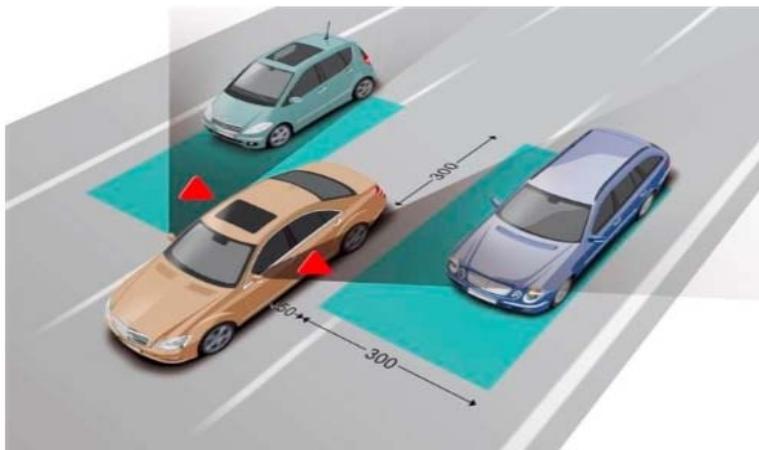
# Driver Assistance Systems



*Smart Cruise Control System*

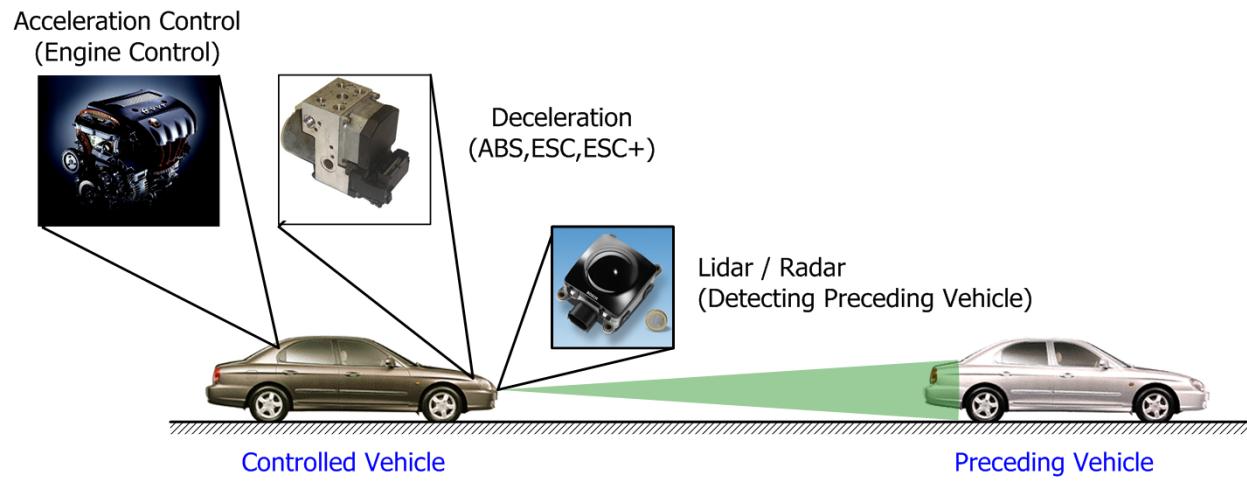


*Lane Keeping Assist System*



*Blind Spot Detection System*

# Hyundai Motor Company, GENESIS, in 2008



# Vehicle Control systems

Cruise Control

Automated Emergency Braking Systems

# **System Control (next semester)**

## *Controller design*

- feedback control systems (closed-loop control systems)
- Root Locus method: pole placement
- Frequency Response method : lead/lag compensators
- analysys and design Using MATLAB
- control system simulation using MATLAB/SIMULINK
  
- PID Control
- state space method

**End of Introduction to Control Systems**