Optimal Design of Energy Systems

Chapter 9 Search Method

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Chapter 9. Search Method

9.1 Overview

elimination

hill-climbing

- ultimate approach if other optimization methods fail
- no one systematic procedure

Chapter 9. Search Method

9.1 Overview

1. Single variable
   a. Exhaustive
   b. Efficient
      - Dichotomous
      - Finonacci
2. Multivariable, unconstrained
   a. Lattice
   b. Univariate
   c. Steepest ascent
3. Multivariable, constrained
   a. Penalty functions
   b. Search along a constraint

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9.2 Interval of Uncertainty

In search methods, precise point will never be known

The best that can be achieved → interval of uncertainty

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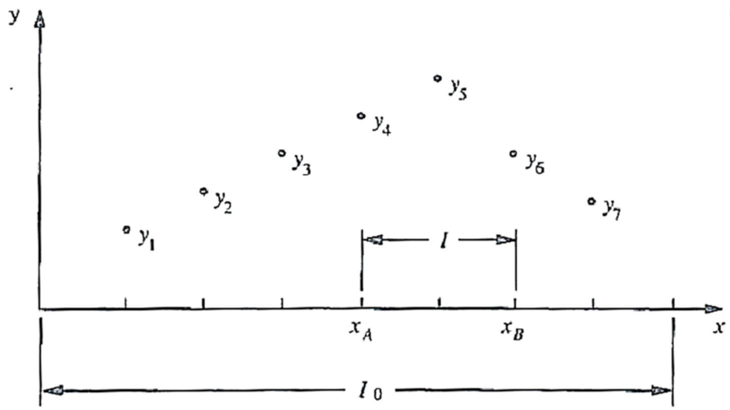
9.3 Exhaustive search (linear search)

$$y = y(x)$$

$\uparrow$

Uniformly spaced throughout the $\underbrace{\text{interval of interest}}_{I_o}$



Maximum lies $x_A < \quad < x_B$

Interval of uncertainty $I = \dfrac{2}{8} I_o = \dfrac{1}{4} I_o$ $\qquad I = \dfrac{2 I_o}{n+1}$

7 observations $\qquad$ n observations

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9.4 Unimodal functions

- Only one peak (or valley) in the interval of interest

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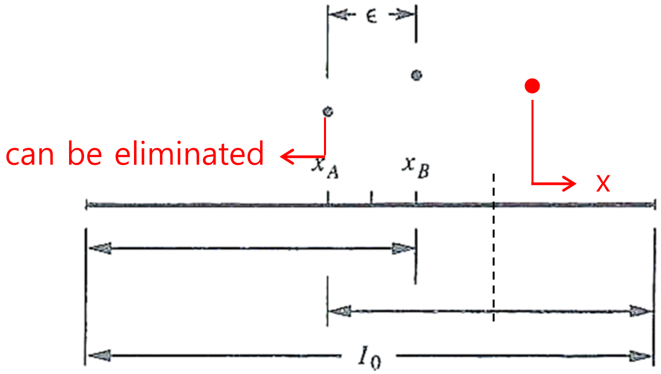
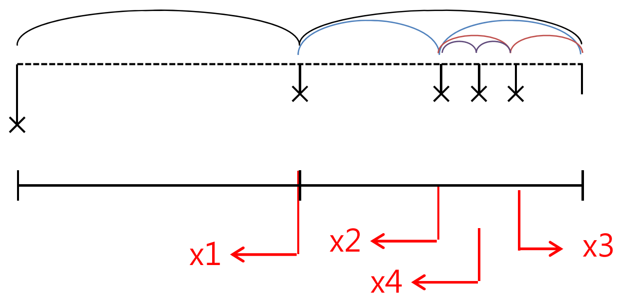
9.6 Dichotomous search





1st trial remaining interval $I = \dfrac{I_o + \varepsilon}{2}$

2nd trial $$I = \frac{\dfrac{I_o + \varepsilon}{2} + \varepsilon}{2} = \frac{I_o}{4} + \left(\varepsilon - \frac{1}{4}\varepsilon\right)$$

3rd trial $$I = \frac{\dfrac{\frac{I_o + \varepsilon}{2} + \varepsilon}{2} + \varepsilon}{2} = \frac{I_o}{8} + \left(\varepsilon - \frac{1}{8}\varepsilon\right)$$

n trial points (n=2,4,6,…) $$I = \frac{I_o}{2^{n/2}} + \varepsilon\left(1 - \frac{1}{2^{n/2}}\right)$$

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9.7 Fibonacci search (13th century mathematician)

$$F_0 = 1$$

$$F_1 = 1$$

$$F_i = F_{i-2} + F_{i-1} \quad i \geq 2$$

$$\underset{=F_2}{2},\ 3,\ 5,\ 8,\ 13,\ 21,\ 34,\ 55,\ \underset{=F_{10}}{89}$$

① Place the first observation in I_o at $I_o \dfrac{F_{n-1}}{F_n}$

② 2nd observation

- Symmetric in the interval of uncertainty

③ last observation – as close as possible to center point

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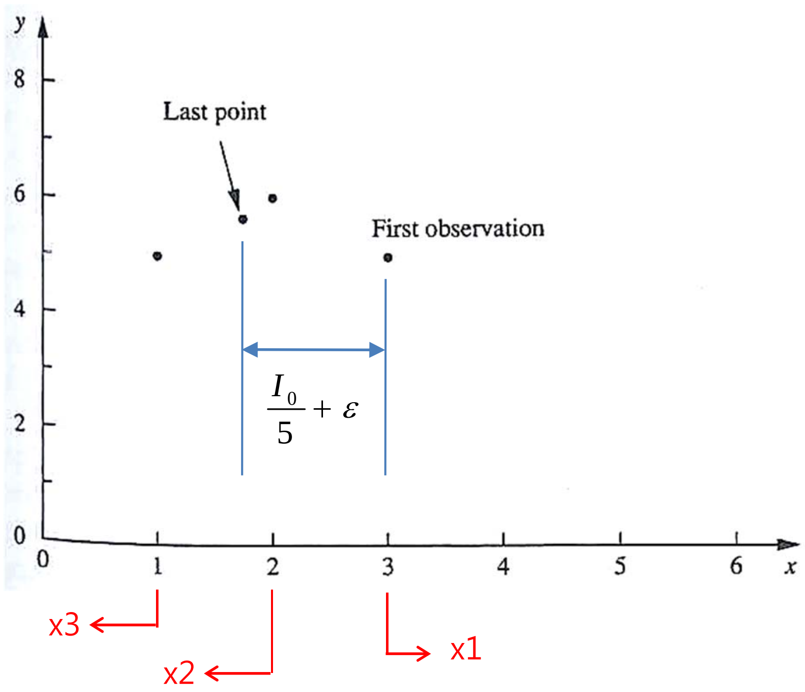
9.7 Fibonacci search

<Example 9.1> Find the maximum of the function $y = -x^2 + 4x + 2$

In the interval $0 \le x \le 5$.

<solution>



$n = 4, \quad F_4 = 5$

1st $\quad x = I_0 \dfrac{F_3}{F_4} = \dfrac{3}{5} I_0 = 3$

2nd symmetric $0 \sim 5$

$x = 2$ eliminate 3~5

3rd symmetric $0 \sim 3$

$x = 1$ eliminate 0~1

Final $x = 2 - \varepsilon$

Interval of uncertainty

$$2 - \varepsilon \le x \le 3 \qquad \frac{I_0}{5} + \varepsilon = \frac{I_0}{F_n} + \varepsilon$$

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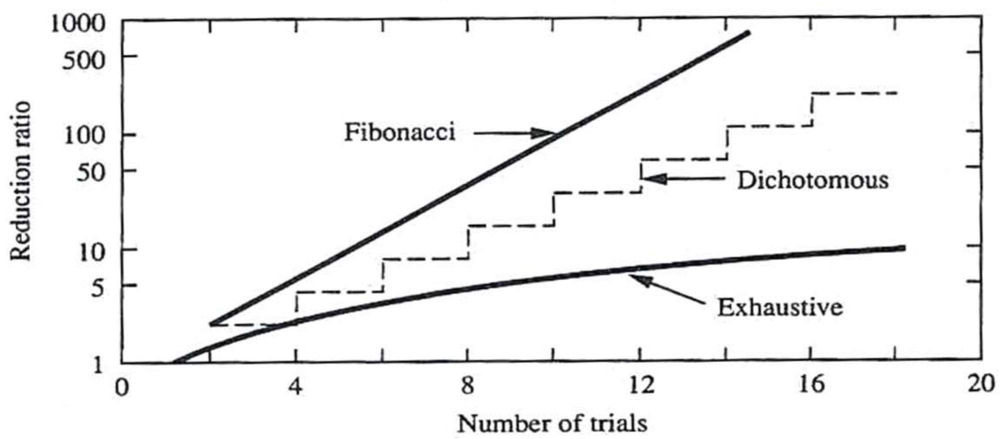
9.8 Comparative effectiveness

Reduction Ratio (RR) = $\dfrac{I_0}{I_n}$

			single variable search
$=$	$\dfrac{n+1}{2}$	exhaustive	O.K.
	$2^{\frac{n}{2}}$	dichotomous	good
	F_n	Fibonacci	good



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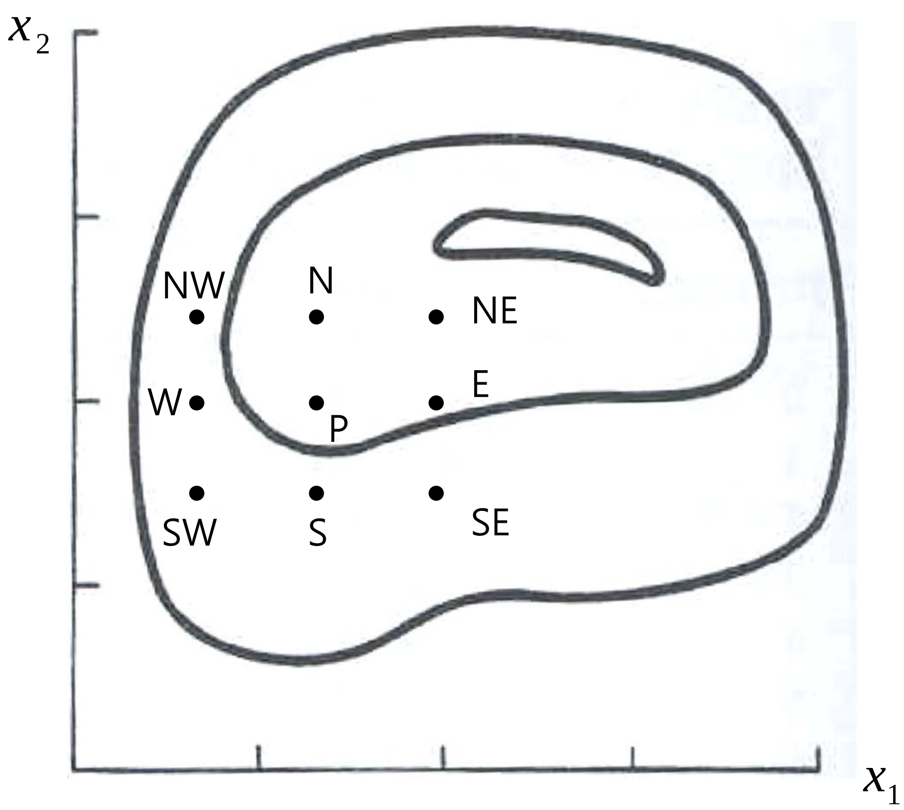
9.11 Lattice search

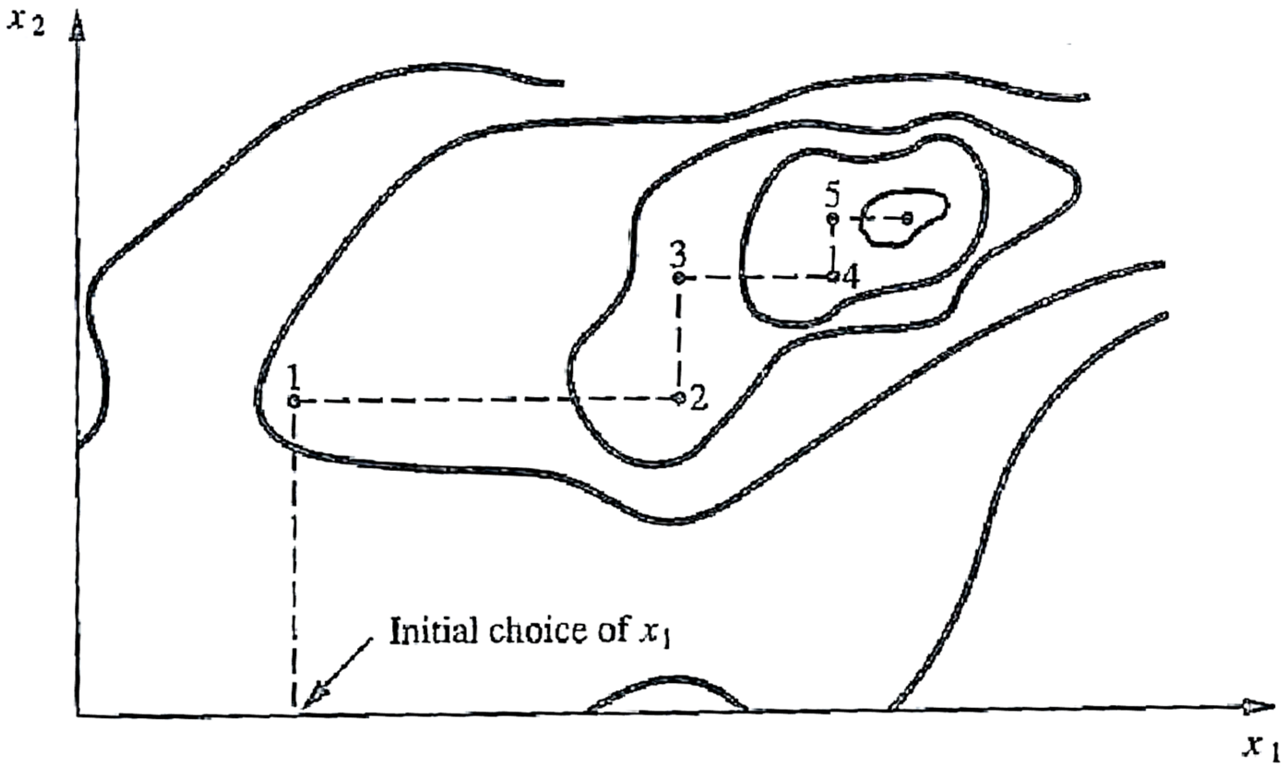


coarse grid

→ fine grid

9.12 Univariate search

- Optimization with respect to one variable at a time



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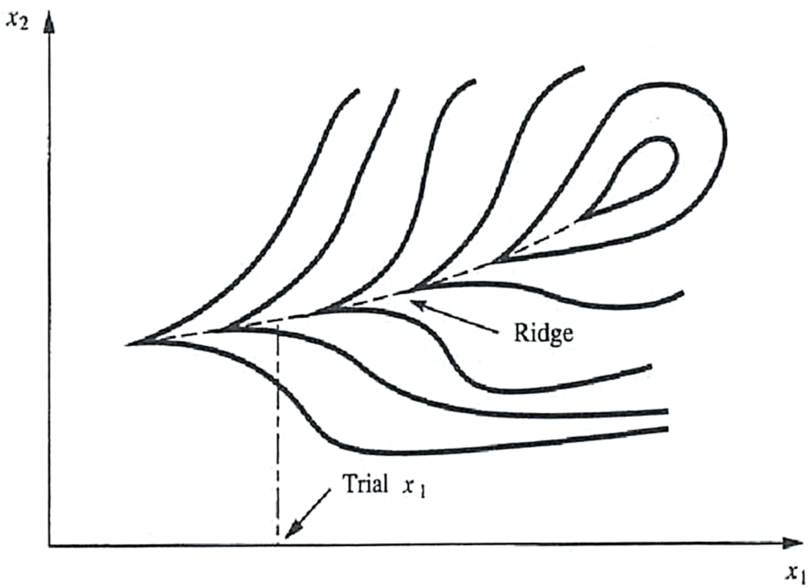
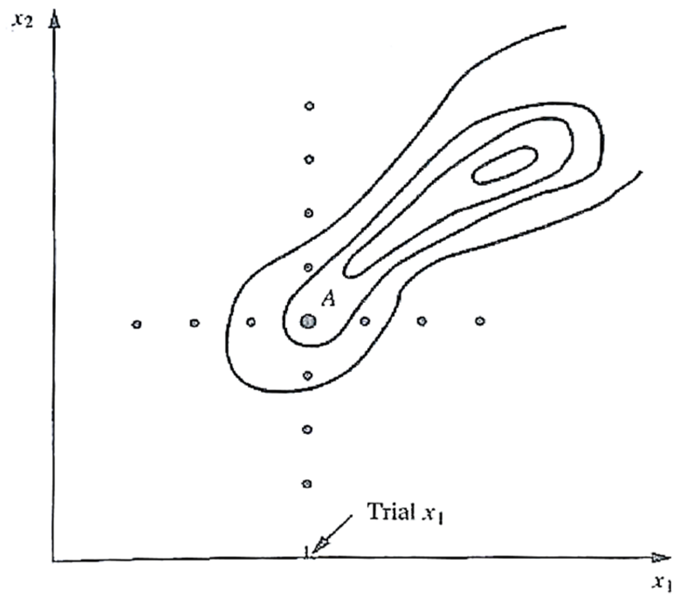
9.12 Univariate search

- Failure occurs

Ridge



Large interval

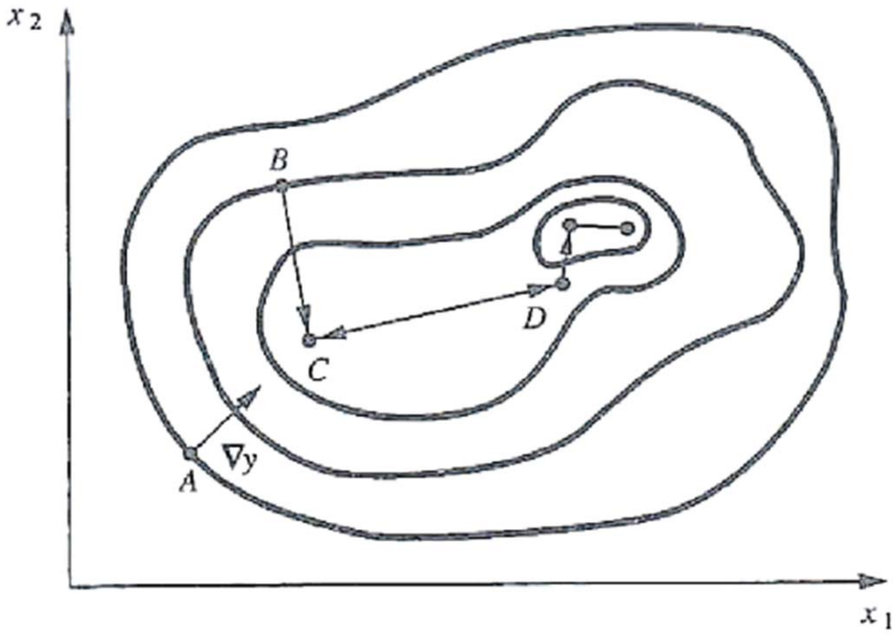


9.13 Steepest ascent method

$$\nabla y = \frac{\partial y}{\partial x_1}\hat{i}_1 + \frac{\partial y}{\partial x_2}\hat{i}_2$$

Direction
Distance

gradient vector (at A) is normal to the contour line (at A)



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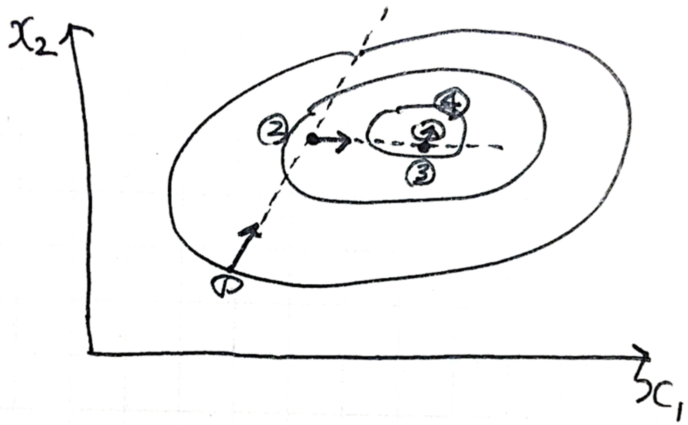
9.13 Steepest ascent method

① trial point as near to the optimum as possible (otherwise, arbitrarily chosen)

②
$$\frac{\partial y}{\partial x_i} = \frac{y(x_1,\cdots,x_i+\Delta x_i,\cdots,x_n) - y(x_1,\cdots,x_i,\cdots,x_n)}{\Delta x_i}$$

$$\frac{\Delta x_1}{\partial y / \partial x_1} = \cdots = \frac{\Delta x_i}{\partial y / \partial x_i}$$ distance (change in x_i) in the direction of the gradient

③ in the direction of gradient, move until optimum is reached



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9.13 Steepest ascent method

Example 9.2. An insulated steel tank storing ammonia, as shown in Fig. 9-12, is equipped with a recondensation system which can control the pressure and thus the temperature of the ammonia.[3] Two basic decisions to make in the design of the tank are the shell thickness and insulation thickness.

If the tank operates with a temperature near ambient, the pressure in the tank will be high and a heavy expensive vessel will be required. On the other hand, to maintain a low pressure in the tank requires more operation of the recondensation system because there will be more heat transferred from the environment unless the insulation is increased, which also adds cost.

Determine the optimum operating temperature and insulation thickness if the following costs and other data apply:

Vessel cost, $1000 + 2.2(p - 100)^{1.2}$ dollars for $p \geq 200$ kPa
Insulation cost for the 60 m^2 of heat-transfer area, $21x^{0.9}$ dollars

9.13 Steepest ascent method

<Example 9.2>

Recondensation cost, 2.5 cents per kilogram of ammonia
Lifetime hours of operation, 50,000 h
Ambient temperature, 25°C
Average latent heat of vaporization of ammonia, 1200 kJ/kg
Conductivity of insulation k, 0.04 W/(m · K)
Pressure-temperature relation for ammonia

$$\ln p = -\frac{2800}{t + 273} + 16.33$$

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9.13 Steepest ascent method

<Example 9.2>

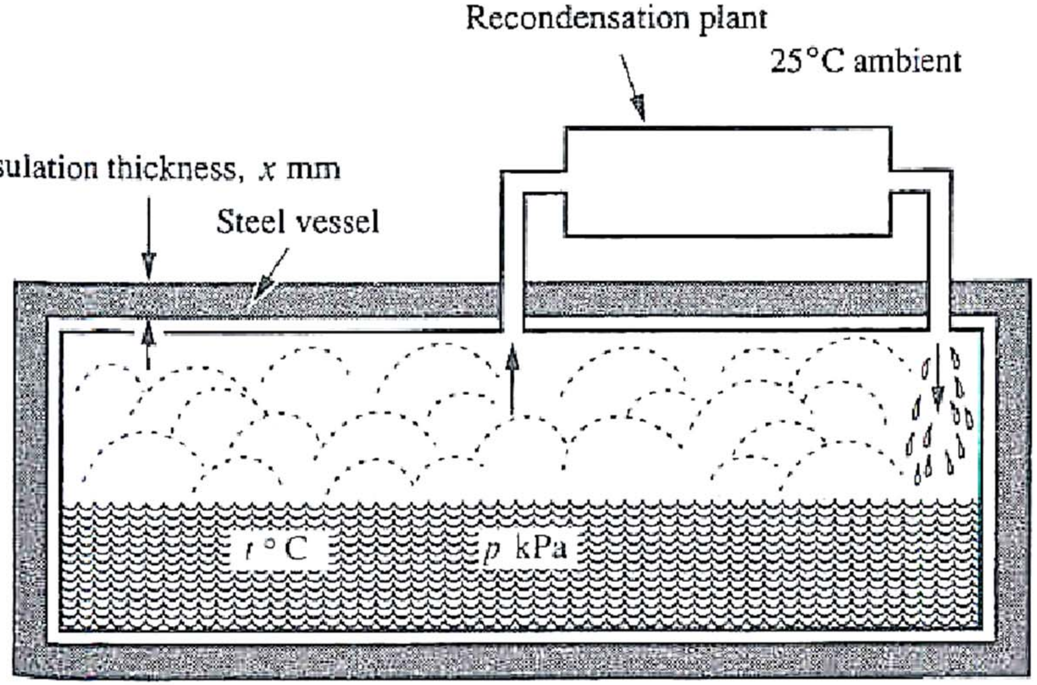

FIGURE 9-12
Ammonia storage tank in Example 9.2.

9.13 Steepest ascent method

<Solution>

Total life time cost = vessel +insulation + lifetime cost of recondensation

- ✓ insulation : $IC = 21x^{0.9}$
- ✓ vessel : $p = e^{-2800/(t+273)+16.33}$

$$VC = 1000 + 2.2(e^{-2800/(t+273)+16.33} - 100)^{1.2}$$

- ✓ recondensation : $RC = (w\ kg/s)(0.25\ \$/kg)(3600\ s/h)(50000\ h)$

$$w = \frac{q\ kW}{1200\ kJ/kg}$$

$$q\ kW = \frac{25-t}{(x\ mm)/1000}[0.00004\ kW/(m \cdot K)](60\ m^2)$$

$$RC = \frac{9000(25-t)}{x}$$

$$\rightarrow Total\ Cost = IC + VC + RC$$

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9.13 Steepest ascent method

<Solution>

$$\frac{\partial C}{\partial x}(0.9)(21)x^{-0.1} - \frac{9000(25-t)}{x^2}$$

$$\frac{\partial C}{\partial t} = (2.2)(1.2)(e^A - 100)^{0.2} e^A \frac{2800}{(t+273)^2} - \frac{9000}{x} \quad \text{(where} \quad A = -\frac{2800}{t+273} + 16.33\text{)}$$

- Arbitrary value : $x = 100\ \ mm,\ \ t = 5^o C$

$$C = \$7237.08,\ \ \partial C / \partial x = -6.075,\ \ \partial C / \partial t = 77.347$$

$$\frac{\Delta x}{-6.075} = \frac{\Delta t}{77.347} \quad \text{(direction)}$$

- Find optimum value in this direction

9.13 Steepest ascent method

<Solution>

TABLE 9.1
Steepest-descent search in Example 9.2

Iteration	x, mm	t, °C	C	$\partial C/\partial x$	$\partial C/\partial t$
0	100.00	5.00	$7237.08	−6.075	77.347
1	101.23	−10.66	6675.58	−19.409	−1.455
2	142.83	−7.54	6334.98	−2.845	37.505
3	143.62	−17.78	6152.85	−7.165	−0.634
4	168.92	−15.54	6067.68	−1.471	16.145
5	169.40	−20.80	6026.09	−3.051	−0.294
6	182.87	−19.50	6005.85	−0.749	7.477
...	...	...	...	...	...
32	196.56	−23.24	5986.07	−0.090	0.189
33	196.62	−23.36	5986.05	−0.112	−0.123
34	196.69	−23.28	5986.05	−0.086	0.111
35	196.77	−23.38	5986.04	−0.100	−0.143
36	196.83	−23.30	5986.04	−0.076	0.088

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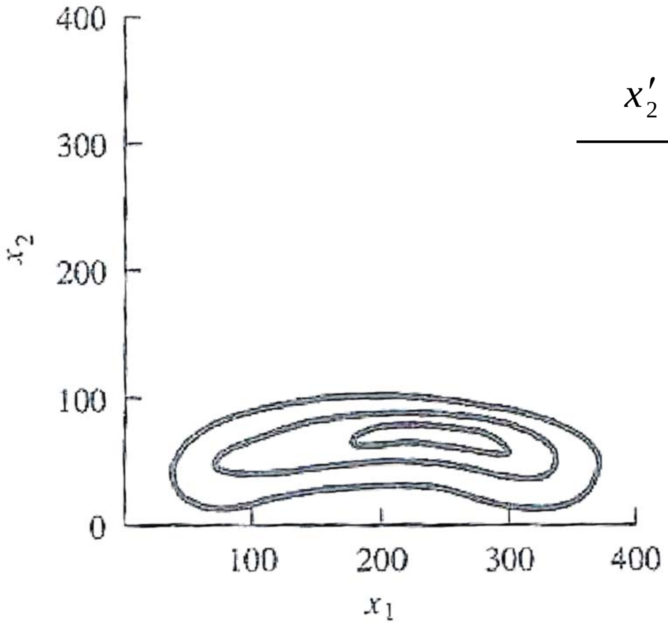
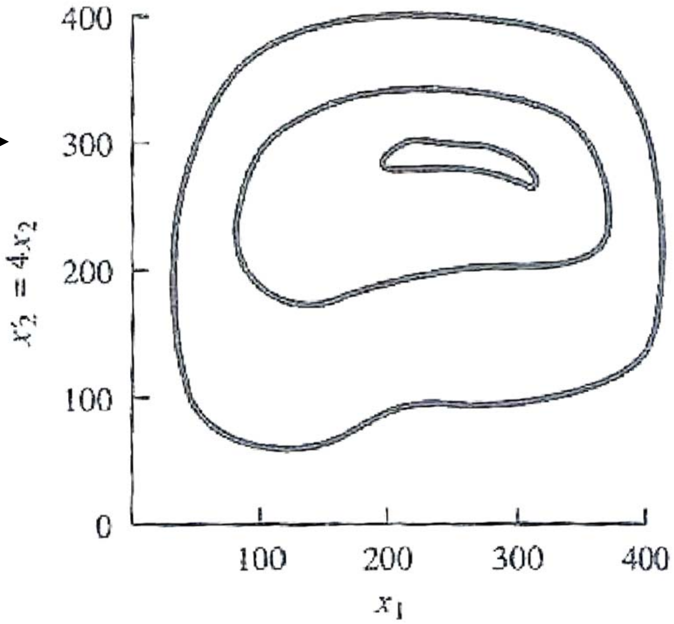
9.14 Scales of the independent variables

- Contours should be as spherical as possible to accelerate the convergence



$$x_2' = c x_2$$



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9.15 Constrained optimization

most frequent

most important

1) Conversion to unconstrained by use of penalty functions

2) Searching along the constraint

→ equality constraints only

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9.16 Penalty function

$$y = y(x_1, x_2, \cdots x_n) \quad \rightarrow \text{maximum}$$

Subject to

$$\phi_1 = y(x_1, x_2, \cdots x_n) = 0$$

$$\vdots$$

$$\phi_m = y(x_1, x_2, \cdots x_n) = 0$$

New unconstrained function

$$Y = y - P_1\phi_1^2 - \cdots - P_m\phi_m^2$$

if minimum

$$Y = y + P_1\phi_1^2 + \cdots + P_m\phi_m^2$$

P_i Relative weighting

too high – move very slowly

too small – terminate without satisfying the constraints

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9.17 Optimization by searching along a constraint - Hemstitching

① Choose a trial point

② Driving toward the constraint(s) (fixed x_1 or x_2)

③ On constraint(s), optimize along the constraint(s) (tangential move)

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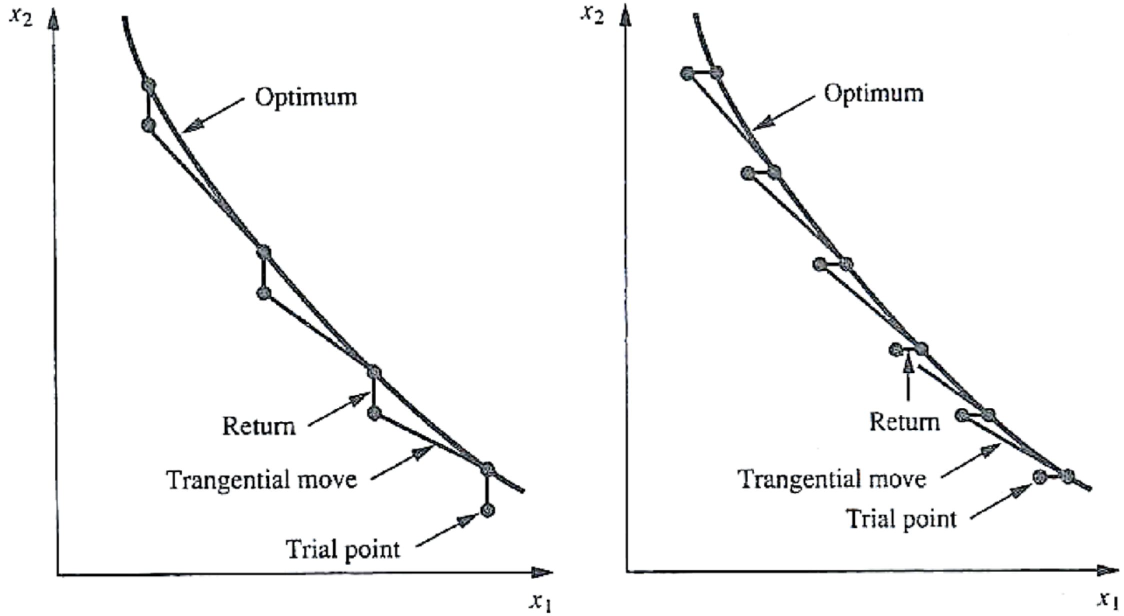
9.19 Hemstitching search

$$\left.\begin{array}{l}\#\text{ of constraints} = m \\ \#\text{ of variables} = n\end{array}\right\} \quad n - m = 1$$

$x_1^2 x_2 = 8$ $\qquad\qquad$ $\phi(x_1, x_2) = 0$



(a) $\qquad\qquad$ (b)

9.19 Hemstitching search

- constraint

$$\phi(x_1, x_2) = 0$$

$$\Delta\phi = \frac{\partial\phi}{\partial x_1}\Delta x_1 + \frac{\partial\phi}{\partial x_2}\Delta x_2 = 0$$

$$\frac{\Delta x_1}{\Delta x_2} = -\frac{\partial\phi / \partial x_1}{\partial\phi / \partial x_2}$$

- objective function

$$\Delta y \approx \frac{\partial y}{\partial x_1}\Delta x_1 + \frac{\partial y}{\partial x_2}\Delta x_2$$

$$= \left(-\frac{\partial y}{\partial x_1}\frac{\partial\phi / \partial x_2}{\partial\phi / \partial x_1} + \frac{\partial y}{\partial x_2} \right)\Delta x_2 = G\,\Delta x_2$$

In minimization, if $G>0$, $\Delta x_2<0$
if $G<0$, $\Delta x_2>0$

In maximization, if $G>0$, $\Delta x_2>0$
if $G<0$, $\Delta x_2<0$

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9.19 Hemstitching search

Example 9.4. The objective function associated with the constraint of Example 9.3, $x_1^2 x_2 - 8 = 0$, is

$$y = 3x_1^2 + x_2^2$$

Minimize this function starting with a trial value of $x_2 = 1.6$ choosing a step size $|\Delta x_2| = 0.05$, and returning to the constraint by holding x_2 constant.

9.19 Hemstitching search

<Solution>

trial value $x_2 = 1.6 \rightarrow x_1 = 2.236,\ y = 17.56,\ \phi = 0,\ G = -6.175$

TABLE 9.2
Hemstitching search in Example 9.4

Cycle	Before move	x_1	x_2	y	ϕ	G
1	tangent	2.236	1.60	17.560	0	−6.175
	return	2.271	1.65	18.194	0.509	
2	tangent	2.202	1.65	17.268	0	−5.515
	return	2.235	1.70	17.880	0.494	
3	tangent	2.169	1.70	17.008	0	−4.904
	return	2.201	1.75	17.598	0.479	
...						
16	tangent	1.886	2.25	15.729	0	−0.241
	return	1.907	2.30	16.195	0.361	
17	tangent	1.865	2.30	15.725	0	0.0631
	return	1.845	2.25	15.272	−0.343	
18	tangent	1.886	2.25	15.729	0	−0.241
	return	1.907	2.30	16.195	0.361	

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9.19 Hemstitching search

Three-variable problem where n=3, m=2

optimize $y = y(x_1, x_2, x_3)$

subject to $\phi_1(x_1, x_2, x_3) = 0$

$\phi_2(x_1, x_2, x_3) = 0$

On the constraints, (tangential move)

$$\left.\begin{aligned}\Delta\phi_1 &= \frac{\partial\phi_1}{\partial x_1}\Delta x_1 + \frac{\partial\phi_1}{\partial x_2}\Delta x_2 + \frac{\partial\phi_1}{\partial x_3}\Delta x_3 = 0\\ \Delta\phi_2 &= \frac{\partial\phi_2}{\partial x_1}\Delta x_1 + \frac{\partial\phi_2}{\partial x_2}\Delta x_2 + \frac{\partial\phi_2}{\partial x_3}\Delta x_3 = 0\end{aligned}\right\}$$

Eliminate Δx_1, Δx_2

$$\Delta y = \frac{\partial y}{\partial x_1}\Delta x_1 + \frac{\partial y}{\partial x_2}\Delta x_2 + \frac{\partial y}{\partial x_3}\Delta x_3$$

$$= G\Delta x_3$$

In minimization, if $G>0$, $\Delta x_3<0$
if $G<0$, $\Delta x_3>0$

In maximization, if $G>0$, $\Delta x_3>0$
if $G<0$, $\Delta x_3<0$

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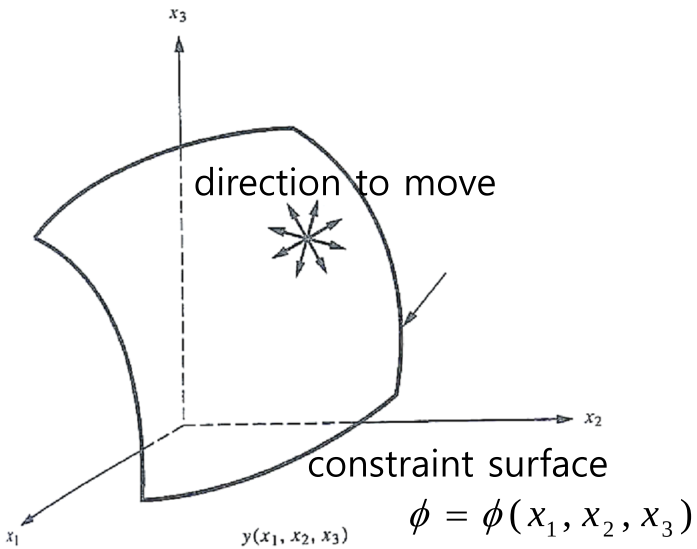
9.20 Moving tangent to a constraint

n=3, m=1



- maximum change of y

$$\Delta y = \frac{\partial y}{\partial x_1}\Delta x_1 + \frac{\partial y}{\partial x_2}\Delta x_2 + \frac{\partial y}{\partial x_3}\Delta x_3$$

- direction (tangent to a constraint)

$$\Delta \phi = \frac{\partial \phi}{\partial x_1}\Delta x_1 + \frac{\partial \phi}{\partial x_2}\Delta x_2 + \frac{\partial \phi}{\partial x_3}\Delta x_3 = 0$$

- distance

$$\Delta x_1^2 + \Delta x_2^2 + \Delta x_3^2 = r^2 = const.$$

- maximum

$$\Delta y = ?$$

9.20 Moving tangent to a constraint

Lagrange Multiplier Method

$$\frac{\partial y}{\partial x_1} - \lambda_1(2\Delta x_1) - \lambda_2 \frac{\partial \phi}{\partial x_1} = 0 \qquad ①$$

$$\frac{\partial y}{\partial x_2} - \lambda_1(2\Delta x_2) - \lambda_2 \frac{\partial \phi}{\partial x_2} = 0 \qquad ②$$

$$\frac{\partial y}{\partial x_3} - \lambda_1(2\Delta x_3) - \lambda_2 \frac{\partial \phi}{\partial x_3} = 0 \qquad ③$$

$$① \times \frac{\partial \phi}{\partial x_1} + ② \times \frac{\partial \phi}{\partial x_2} + ③ \times \frac{\partial \phi}{\partial x_3}$$

$$\frac{\partial y}{\partial x_1}\frac{\partial \phi}{\partial x_1} + \frac{\partial y}{\partial x_2}\frac{\partial \phi}{\partial x_2} + \frac{\partial y}{\partial x_3}\frac{\partial \phi}{\partial x_3} - \lambda_2\left[\left(\frac{\partial \phi}{\partial x_1}\right)^2 + \left(\frac{\partial \phi}{\partial x_2}\right)^2 + \left(\frac{\partial \phi}{\partial x_3}\right)^2\right] = 0$$

$$\rightarrow \lambda_2$$

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9.20 Moving tangent to a constraint

$$\frac{1}{2\lambda_1} = \frac{\Delta x_1}{\frac{\partial y}{\partial x_1} - \lambda_2 \frac{\partial \phi}{\partial x_1}} = \frac{\Delta x_2}{\frac{\partial y}{\partial x_2} - \lambda_2 \frac{\partial \phi}{\partial x_2}} = \frac{\Delta x_3}{\frac{\partial y}{\partial x_3} - \lambda_2 \frac{\partial \phi}{\partial x_3}}$$

$\Delta x_i =$ step size of on variable in the move