Optimal Design of Energy Systems

Chapter 10 Dynamic Programming

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Chapter 10. Dynamic Programming

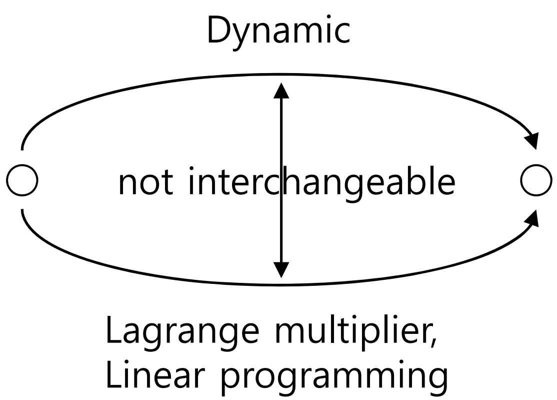
10.1 (Uniqueness) Introduction

- applicable to staged process

  to continuous functions which can be approximated by staged processes

- related to the calculus of variations

  → optimal function rather than an optimal state point



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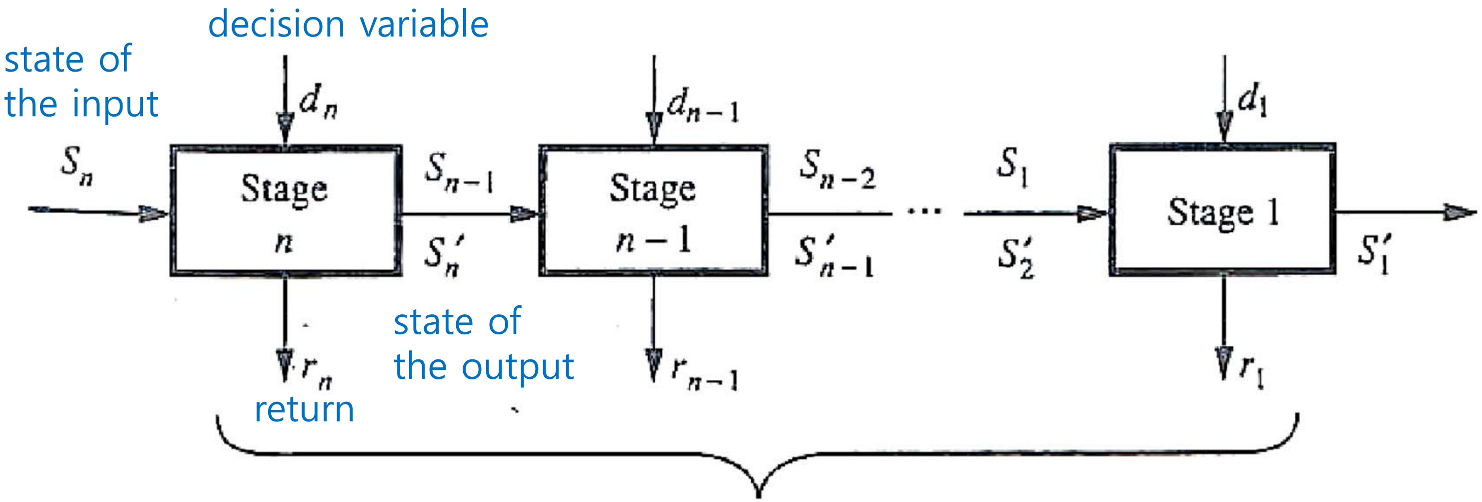
10.2 Symbolic Description



optimize $\sum_{i=1}^{n} r_i$

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10.3 Characteristics of dynamic programming solution

<Example 10.1> optimal route

Example 10.1. A minimum-cost pipeline is to be constructed between points A and E, passing successively through one node of each, B, C, and D, as shown in Fig. 10-2. The costs from A to B and from D to E are shown in Fig. 10-2, and the costs between B and C and between C and D are given in Table 10.1.

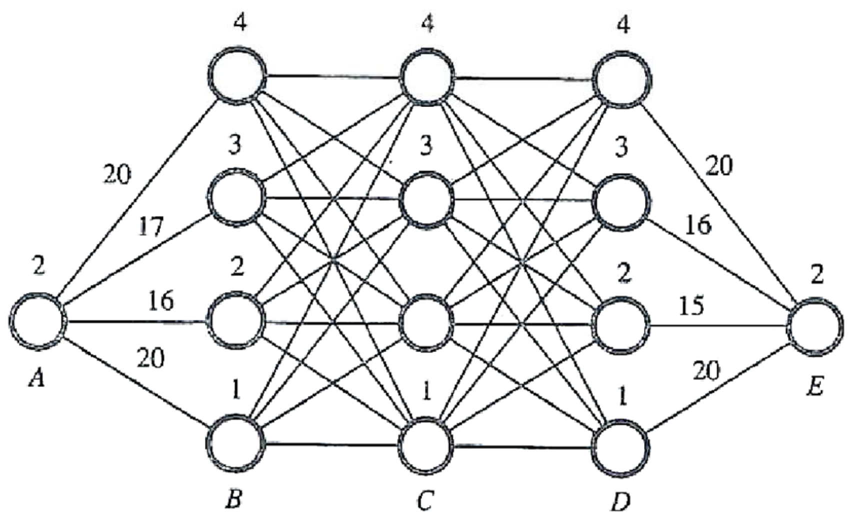

TABLE 10.1
Costs from B to C and C to D in Fig. 10-2

	To			
From	1	2	3	4
1	12	15	21	28
2	15	16	17	24
3	21	17	16	15
4	28	24	15	12

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10.3 Characteristics of dynamic programming solution

<solution>

TABLE 10.2
Example 10.1, D to E

From	Through	Cost	Optimum
$D1$	–	20	x
$D2$	–	15	x
$D3$	–	16	x
$D4$	–	20	x

TABLE 10.3
Example 10.1, C to E

		Cost			
From	Through	C to D	D to E	Total	Optimum
$C4$	$D4$	12	20	32	
	$D3$	15	16	31	x
	$D2$	24	15	39	
	$D1$	28	20	48	
$C3$	$D4$	15	20	35	
	$D3$	16	16	32	x
	$D2$	17	15	32	x
	$D1$	21	20	41	
$C2$	$D4$	24	20	44	
	$D3$	17	16	33	
	$D2$	16	15	31	x
	$D1$	15	20	35	
$C1$	$D4$	28	20	48	
	$D3$	21	16	37	
	$D2$	15	15	30	x
	$D1$	12	20	32	

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10.3 Characteristics of dynamic programming solution

<solution>

TABLE 10.4
Example 10.1, B to E

		Cost			
From	Through	B to C	C to E	Total	Optimum
$B4$	$C4$	12	31	43	x
	$C3$	15	32	47	
	$C2$	24	31	55	
	$C1$	28	30	58	
$B3$	$C4$	15	31	46	x
	$C3$	16	32	48	
	$C2$	17	31	48	
	$C1$	21	30	51	
$B2$	$C4$	24	31	55	
	$C3$	17	32	49	
	$C2$	16	31	47	
	$C1$	15	30	45	x
$B1$	$C4$	28	31	59	
	$C3$	21	32	53	
	$C2$	15	31	46	
	$C1$	12	30	42	x

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10.3 Characteristics of dynamic programming solution

<solution>

TABLE 10.5
Example 10.1, A to E

To E from	Through	Cost: A to B	Cost: B to E	Cost: Total	Optimum
$A2$	$B4$	20	43	63	
	$B3$	17	46	63	
	$B2$	16	45	61	x
	$B1$	20	42	62	

Optimum route : $A2 \rightarrow B2 \rightarrow C1 \rightarrow D2 \rightarrow E2$

Key feature

After an optimal policy has been determined from an intermediate state to the final state, future calculation passing through that state use only the optimal policy.

10.4 Efficiency of Dynamic Programming

(dynamic programming)

Total 40 calculations in example 10.1

Additional stage

(+16) = 56 calculations

(exhaustive examination)

# of total possible routes 64 ← 4 x 4 x 4 x 1

(x 4) = 256 calculations

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10.6 Apparently constrained problems

constrained, but converted into unconstrained form (example 10.1)

<Example 10.3> An evaporator which boils liquid inside tubes consists of four banks of tubes. Each bank consists of a number of tubes in parallel, and the banks are connected in series, as shown in Fig. 10-5. A mixture of liquid and vapor enters the first bank with a fraction of vapor $x = 0.2$, and the fluid leaves the evaporator as saturated vapor, $x = 1.0$. The flow rate is 0.5 kg/s, and each tube is capable of vaporizing 0.01 kg/s and thus of increasing x by 0.02.

Forty tubes are to be arranged in the banks so that the minimum total pressure drop prevails in the evaporator. The pressure drop in a bank is approximately proportional to the square of the velocity, and a satisfactory expression for the pressure drop Δp is

$$\Delta P = 720\left(\frac{x_i}{n}\right)^2$$

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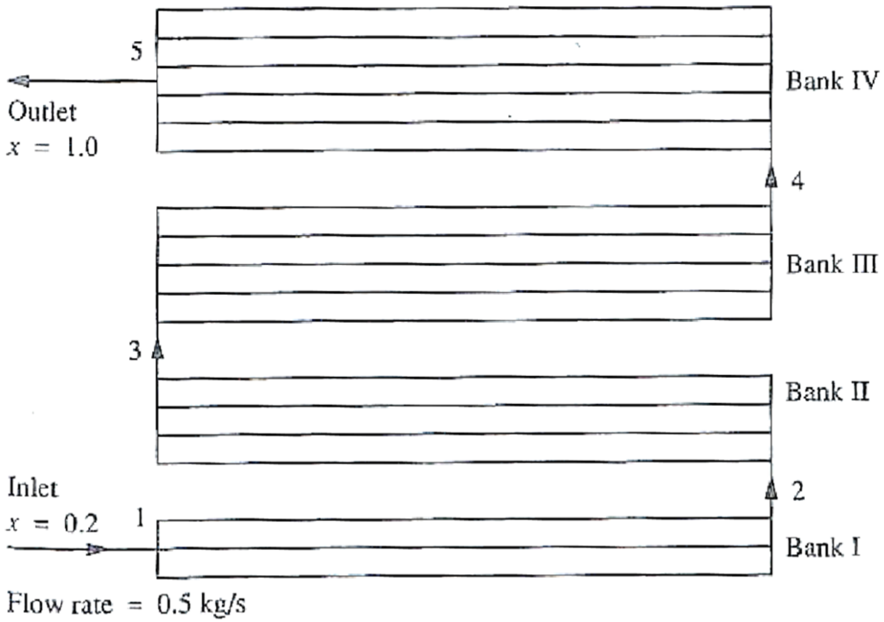
10.6 Apparently constrained problems

<Example 10.3>

where x_i = vapor fraction entering bank
n = number of tubes in bank

Use dynamic programming to determine the distribution of the 40 tubes so that the total pressure drop in the evaporator is minimum.



10.6 Apparently constrained problems

quality entering bank

$$\Delta P = 720\left(\frac{x_i}{n}\right)^2$$

40 tubes, (each tube Δx=0.02)

# of tubes in bank

Selection of # of tubes in a stage is not productive

→ state variable : cumulative tubes

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10.6 Apparently constrained problems

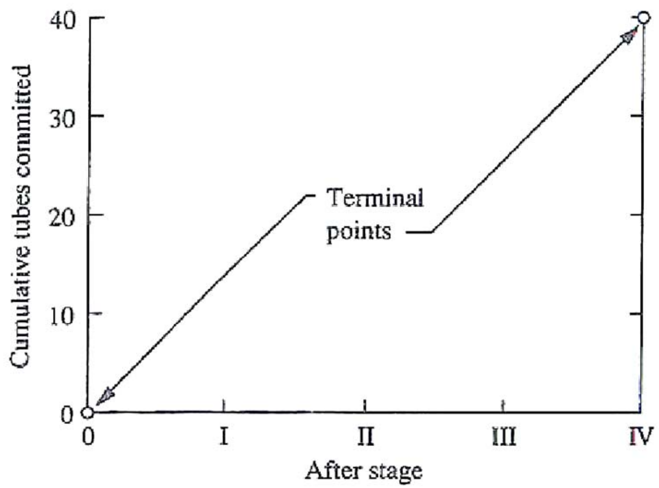

TABLE 10.13
Example 10.3, stage I

Total tubes committed	Tubes in stage I	Total Δp, kPa
2	2	7.20
3	3	3.20
4	4	1.80
5	5	1.15
6	6	0.80

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10.6 Apparently constrained problems

TABLE 10.14
Example 10.3, stages I and II

Total tubes	Tubes in II	Total Δp, kPa
11	5	0.80 + 2.95 = 3.75
	6	1.15 + 1.80 = 2.95*
	7	1.80 + 1.15 = 2.95*
	8	3.20 + 0.76 = 3.96
	9	7.20 + 0.51 = 7.70
12	6	0.80 + 2.05 = 2.85
	7	1.15 + 1.32 = 2.47*
	8	1.80 + 0.88 = 2.68
	9	3.20 + 0.60 = 3.68
13	7	0.80 + 1.50 = 2.30
	8	1.15 + 1.01 = 2.16*
	9	1.80 + 0.73 = 2.53
	10	3.20 + 0.49 = 3.69
14	7	0.59 + 1.70 = 2.29
	8	0.80 + 1.15 = 1.95*
	9	1.15 + 0.80 = 1.95*
	10	1.80 + 0.56 = 2.36
15	8	0.59 + 1.30 = 1.89
	9	1.15 + 0.80 = 1.71*
	10	1.15 + 0.65 = 1.80

TABLE 10.15
Example 10.3, stages I and III

Total tubes	Tubes in III	Total Δp, kPa
22	9	2.16 + 1.88 = 4.04
	10	2.47 + 1.39 = 3.86*
	11	2.95 + 1.05 = 4.00
23	9	1.95 + 2.05 = 4.00
	10	2.16 + 1.52 = 3.68
	11	2.47 + 1.15 = 3.62*
	12	2.95 + 0.88 = 3.83
24	10	1.95 + 1.66 = 3.61
	11	2.16 + 1.26 = 3.42*
	12	2.47 + 0.97 = 3.44
	13	2.95 + 0.75 = 3.70
25	10	1.71 + 1.80 = 3.51
	11	1.95 + 1.37 = 3.32
	12	2.16 + 1.06 = 3.22*
	13	2.47 + 0.82 = 3.29
26	11	1.71 + 1.49 = 3.20
	12	1.95 + 1.15 = 3.00*
	13	2.16 + 0.90 = 3.06

10.6 Apparently constrained problems

TABLE 10.16
Example 10.3, stages I to IV

Total tubes	Tubes in IV	Total Δp, kPa
40	13	2.93 + 2.33 = 5.26
	14	3.00 + 1.90 = 4.90
	15	3.22 + 1.57 = 4.79
	16	3.42 + 1.30 = 4.72
	17	3.62 + 1.09 = 4.71*
	18	3.86 + 0.91 = 4.77

The optimal distribution of tubes : 5, 7, 11, 17

Total pressure drop of 4.71 kPa