

# **Optimal Design of Energy Systems**

## **Chapter 15 Dynamic Behavior of Thermal Systems**

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# Chapter 15. Dynamic Behavior of Thermal Systems

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- with respect to time
  - on (start up), off (shutdown), under control, disturbance

{ off-design }  
design

- focus on thermal systems
- automatic control
- block diagram



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# Chapter 15. Dynamic Behavior of Thermal Systems

## 15.3 Dynamic Simulation

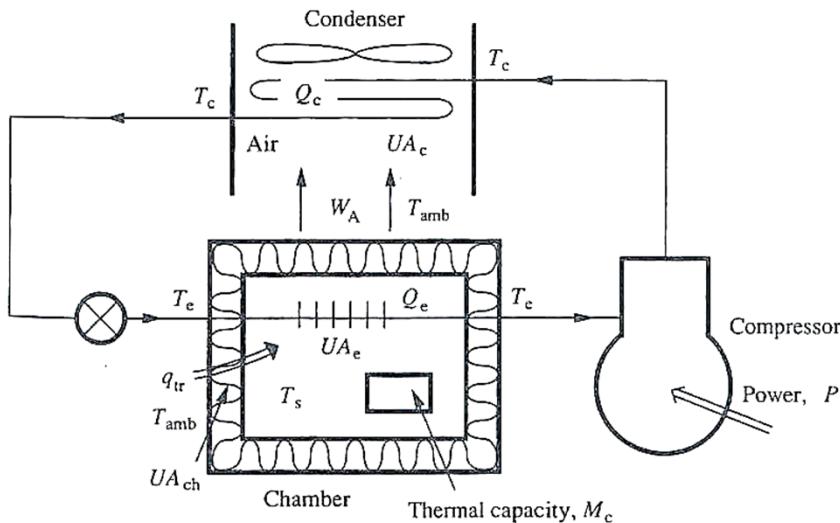


FIGURE 15-1  
System with one dynamic element—refrigeration plant serving a cold room.

compressor ref. capacity       $q_e = f_1(T_e, T_c)$

compressor power       $P = f_2(T_e, T_c)$

condenser       $q_c = \dot{m}c_{p,a}(T_c - T_{amb})(1 - e^{UA/\dot{m}c_{p,a}})$

evaporator       $q_e = (T_s - T_e)(UA_e)$

energy balance       $q_c = P + q_e$

heat transfer to chamber       $q_e = q_{tr} = UA_{ch}(T_{amb} - T_s)$

steady-state



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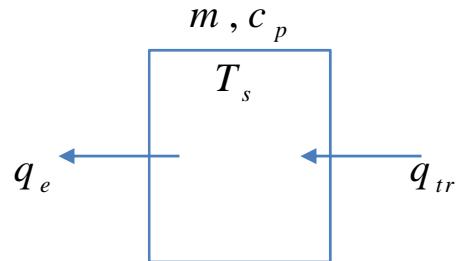
## 15.3 Dynamic Simulation

pull-down

$$q_{tr} = UA_{ch}(T_{amb} - T_s)$$

$$q_{tr} = q_e + mc_p \frac{dT_s}{dt}$$

dynamic : during pull-down  $q_{tr} \neq q_e$



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**15.4 Laplace transform**

- Powerful tool in predicting dynamic behavior
- One way to solve ODE

$$L\{F(t)\} = \int_0^{\infty} F(t)e^{-st} dt = f(s)$$

$$\begin{aligned} L\{F'(t)\} &= \int_0^{\infty} F'(t)e^{-st} dt \\ &= e^{-st} F(t) \Big|_0^{\infty} - \int_0^{\infty} F(t)(-s)e^{-st} dt \\ &= -F(0) + sf(s) \end{aligned}$$

$$\begin{aligned} L\{F''(t)\} &= \int_0^{\infty} F''(t)e^{-st} dt \\ &= e^{-st} F'(t) \Big|_0^{\infty} - \int_0^{\infty} F'(t)(-s)e^{-st} dt \\ &= -F'(0) + s[-F(0) + sf(s)] \\ &= -F'(0) - sF(0) + s^2 f(s) \end{aligned}$$



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**15.5 Inversion**  $L^{-1}\{f(s)\} = F(t)$

<example 15.4> Invert

$$\frac{s+10}{(s-2)^2(s+1)}$$

<solution>  $\frac{s+10}{(s-2)^2(s+1)} = \frac{A}{(s+1)} + \frac{B}{(s-2)^2} + \frac{B'}{s-2}$

$$constants: 10 = 4A + B - 2B'$$

$$s: 1 = -4A + B - B'$$

$$s^2: 0 = A + B'$$

$$A = 1, B = 4, B' = -1$$

$$\therefore L^{-1} \left\{ \frac{s+10}{(s-2)^2(s+1)} \right\} = e^{-t} + 4te^{2t} - e^{2t}$$



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## 15.5 Inversion $L^{-1}\{f(s)\} = F(t)$

<another solution for example 15.4>

- ✓ For non-repeated roots

$$\frac{N(s)}{D(s)} = \frac{A}{s-a} + \frac{B}{s-b} + \dots \quad A = \left. \frac{N(s)(s-a)}{D(s)} \right|_{s \rightarrow a} \quad B = \left. \frac{N(s)(s-b)}{D(s)} \right|_{s \rightarrow b}$$

- ✓ For repeated roots

$$\frac{N(s)}{D(s)} = \frac{A}{s-a} + \frac{B}{(s-b)^2} + \frac{B'}{s-b} \quad B = \left. \frac{N(s)(s-b)^2}{D(s)} \right|_{s \rightarrow b} \quad B' = \left. \frac{d}{ds} \left[ \frac{N(s)(s-b)^2}{D(s)} \right] \right|_{s \rightarrow b}$$

$$\rightarrow A = \left. \frac{s+10}{(s-2)^2} \right|_{s \rightarrow -1} = 1 \quad B = \left. \frac{s+10}{s+1} \right|_{s \rightarrow 2} = 4 \quad B' = \left. \frac{d}{ds} \left( \frac{s+10}{s+1} \right) \right|_{s \rightarrow 2} = -1$$



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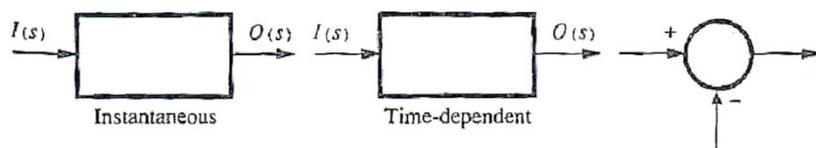
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## 15.7 Block diagram and transfer functions

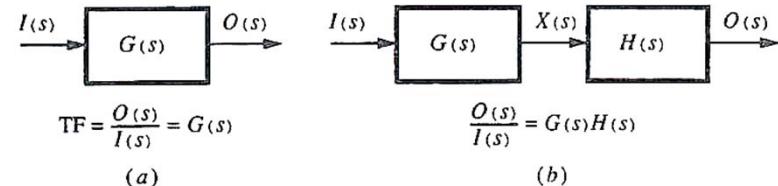
- Variable in S domain  
(not in time domain)

$$\text{Transfer function : } TF = \frac{L\{O(t)\}}{L\{I(t)\}} = \frac{O(s)}{I(s)}$$

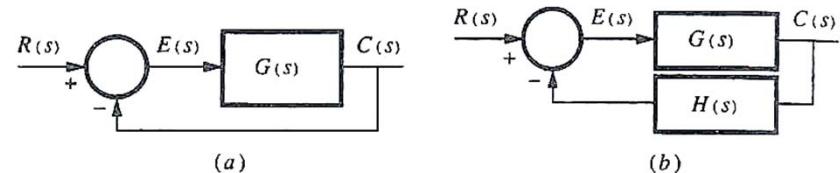
ratio of the output to the input



**FIGURE 15-2**  
Symbols used in block diagrams.



**FIGURE 15-3**  
Transfer function and cascading of blocks.



**FIGURE 15-4**  
(a) Unity feedback loop (b) nonunity feedback loop.



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## 15.8 Feedback control loop

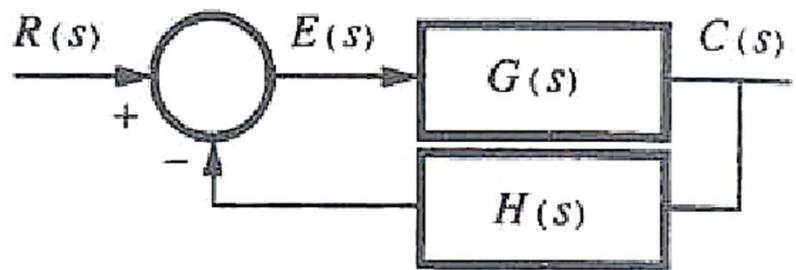


FIGURE 15-4 nonunity feedback loop.

$$C(s) = G(s)E(s)$$

$$E(s) = R(s) - H(s)C(s)$$

$$TF = \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$



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## 15.9 Time constant blocks

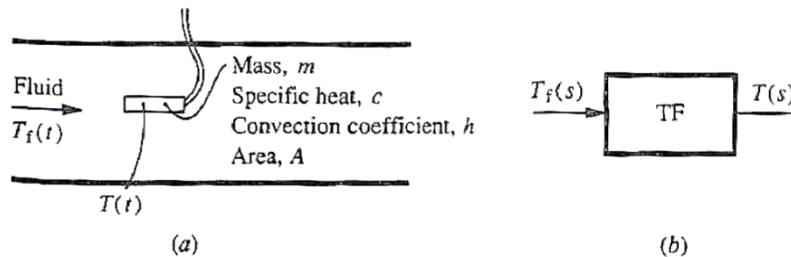


FIGURE 15-5

(a) Response of a temperature-sensing bulb to a change in fluid temperature (b) transfer function of this time-constant block.

Standard technique for developing transfer function

1. Write differential equation  $mc \frac{dT}{dt} = (T_f - T)hA$
2. Transform equation  $\frac{mc}{hA} [sL(T) - T(0)] = L(T_f) - L(T)$
3. Solve for transfer function ( $L(O) / L(I)$ )  
$$TF = \frac{T(s)}{T_f(s)} = \frac{1 + T(0)}{1 + Bs} \frac{\frac{B}{hA}}{\frac{1 + T(0)}{hA} \frac{T_f(s)}{hA}} \quad \left( B = \frac{mc}{hA} \right)$$

For special case  $T(0) = 0$ :  $TF = \frac{1}{1 + Bs}$



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## 15.9 Time constant blocks

$$mc \frac{d(T - T_0)}{dt} = [(T_f - T_0) - (T - T_0)]hA$$

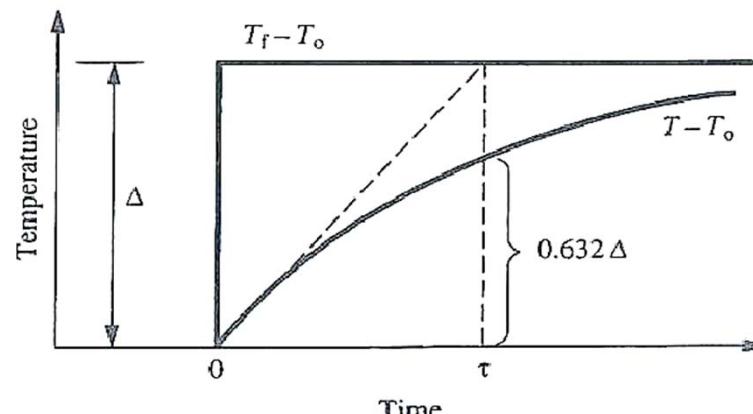
$$TF = \frac{L\{T - T_0\}}{L\{T_f - T_0\}} = \frac{1}{Bs + 1}$$

$$T_f : \text{unit step increase} \quad T_f(s) = \frac{\Delta}{s}$$

$$L\{T - T_0\} = L\{T_f - T_0\} \frac{1}{(Bs + 1)} = \frac{\Delta}{s(Bs + 1)} = \Delta \left( \frac{\alpha}{s} - \frac{\beta}{Bs + 1} \right) \stackrel{\alpha B - \beta = 0, \alpha = 1, \beta = B}{=} \Delta \left( \frac{1}{s} - \frac{B}{Bs + 1} \right)$$

$$T - T_0 = \Delta(1 - e^{-t/B})$$

$$\left( B = \frac{mc}{hA} \right) : \text{time constant}$$



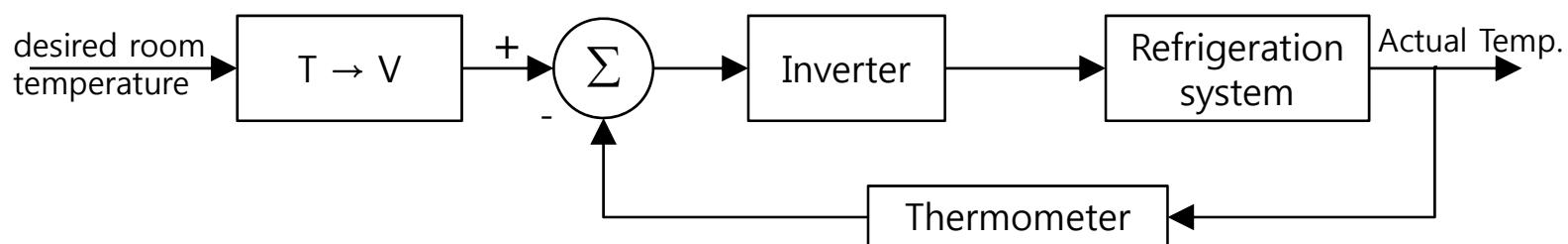
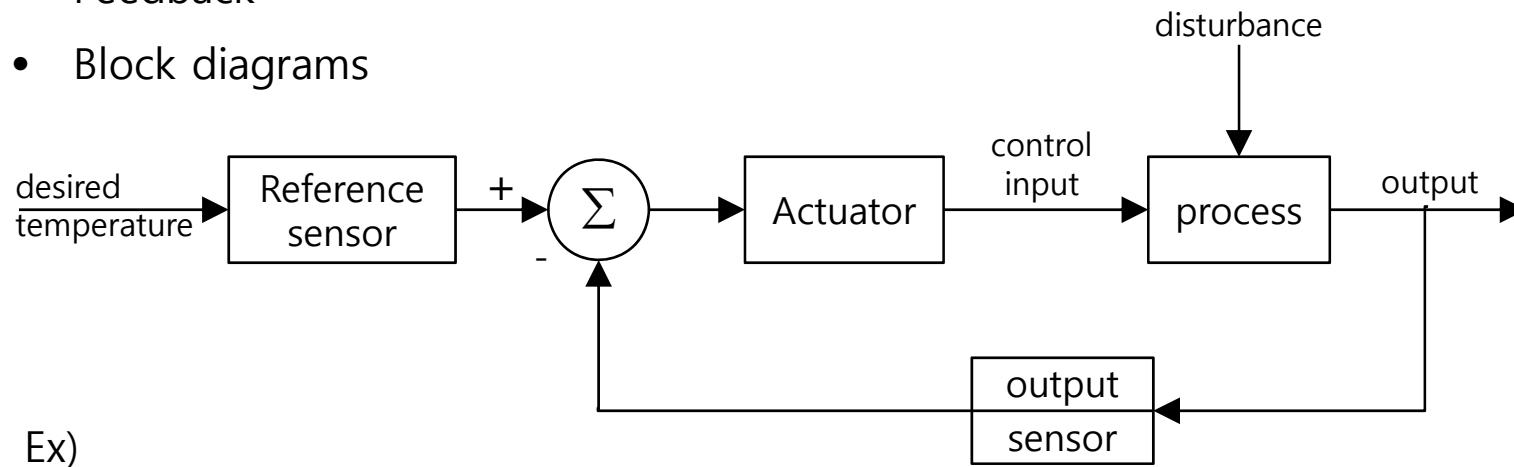
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## 15.9 Time constant blocks - additional

- Control
- Feedback
- Block diagrams



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## 15.9 Time constant blocks - additional

- Dynamic model – a set of differential equations to describe the dynamic behavior of the system

$$\dot{q} \leftarrow \boxed{m c_p^T} \quad \dot{q} = m c_p \frac{dT}{dt}$$

- Differential equation in state variable form (modern control)

$X$  : State of the system

$\dot{X} = f(X, u)$   $u$  : input

$y = h(X, u)$   $y$  : output

Linear case

$$\dot{X} = F X + G u \quad F : [n \times n], \quad G : [n \times 1]$$

$$H : [1 \times n], \quad J : scalar$$

$$y = H X + J u \quad X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$



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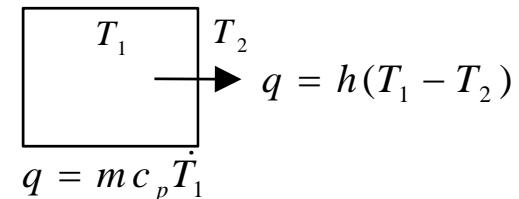
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## 15.9 Time constant blocks - additional

$$\text{Ex)} \quad q = h(T_1 - T_2)$$

$$q = m c_p \frac{dT_1}{dt}$$

$$\rightarrow \dot{T}_1 = \frac{h}{m c_p} (T_1 - T_2)$$



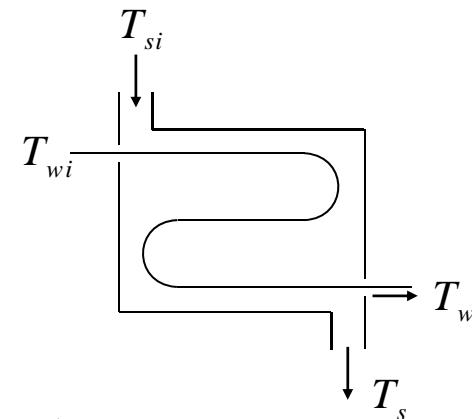
Ex) Heat exchanger

$$q_{in} = \dot{m}_s c_p (T_{si} - T_s)$$

$$q_{out} = \dot{m}_w c_p (T_w - T_{wi})$$

$$\text{Steam} \quad m_s c_{p,s} \frac{dT_s}{dt} = q_{in} - q_{steam \rightarrow water}$$

$$= \dot{m}_s c_{p,s} (T_{si} - T_s) - UA(T_s - T_w)$$



$$\text{Water} \quad m_w c_{p,w} \frac{dT_w}{dt} = \dot{m}_w c_{p,w} (T_{wi} - T_w) + UA(T_s - T_w)$$

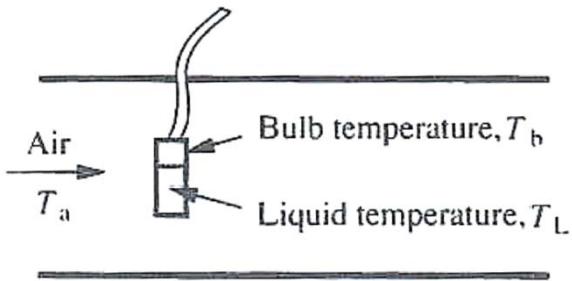


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# Chapter 15. Dynamic Behavior of Thermal Systems

## 15.10 Cascade time-constant blocks



Heat balance equation :

$$(T_a - T_b)h_1 A_1 = m c \frac{dT_b}{dt} + (T_b - T_L)h_2 A_2$$

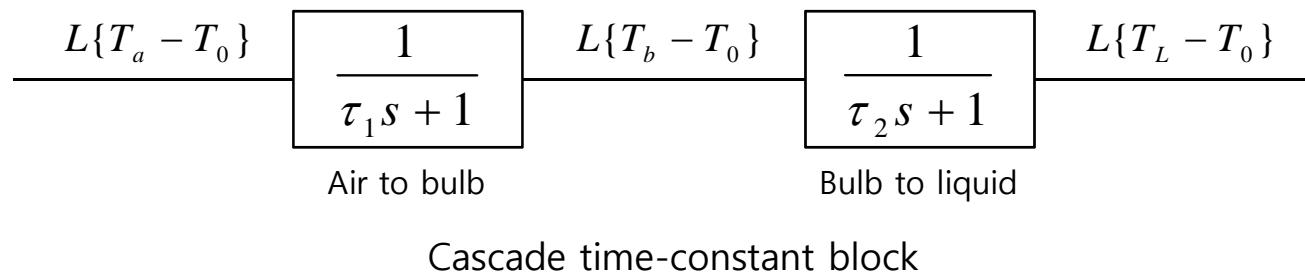
neglect

$$(T_b - T_L)h_2 A_2 = m c \frac{dT_L}{dt}$$

$$\text{Let } \tau_1 = \frac{m c}{h_1 A_1}, \quad \tau_2 = \frac{m c}{h_2 A_2}$$

subscript 1 : air to bulb

subscript 2 : bulb to liquid



Cascade time-constant block



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# Chapter 15. Dynamic Behavior of Thermal Systems

## 15.10 Cascade time-constant blocks

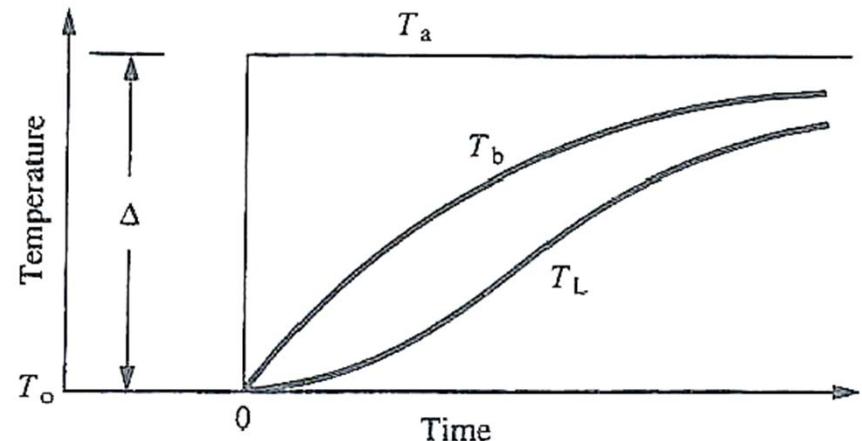
For unit step input

$$L\{T_L - T_0\} = \frac{\Delta}{s} \left( \frac{1}{\tau_1 s + 1} \right) \left( \frac{1}{\tau_2 s + 1} \right)$$

Inversion

$$\frac{T_L - T_0}{\Delta} = 1 - \frac{\tau_1}{\tau_1 - \tau_2} e^{-t/\tau_1} - \frac{\tau_2}{\tau_2 - \tau_1} e^{-t/\tau_2}$$

- ①  $t = 0, T_L - T_0 = 0$
- ②  $\frac{d(T_L - T_0)}{dt} = 0 \text{ at } t = 0$
- ③ if  $\tau_2 \ll \tau_1$   $T_L - T_0 = \Delta(1 - e^{-t/\tau_1})$
- ④ if  $\tau_2 = \tau_1$   $\frac{T_L - T_0}{\Delta} = 1 - e^{-t/\tau} - \frac{te^{-t/\tau}}{\tau}$



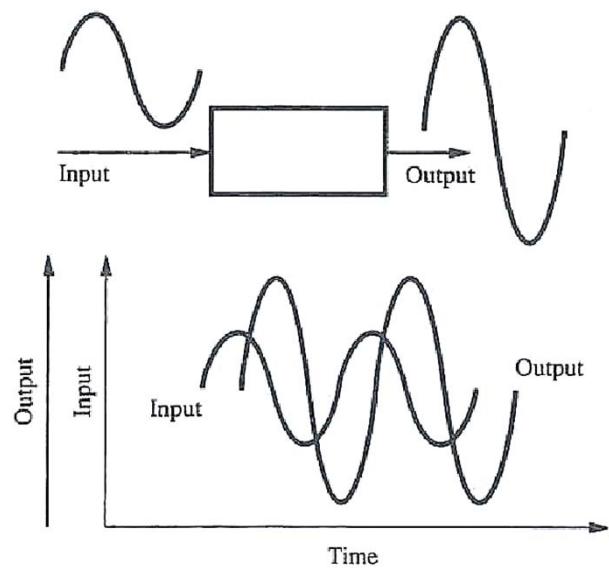
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# Chapter 15. Dynamic Behavior of Thermal Systems

## 15.11 Stability analysis

- Frequency response / Bode plot (diagram)
  - Response to sinusoidal input



Sinusoidal input :  $T_f(t) - T_0 = \Delta \sin(2\pi ft)$

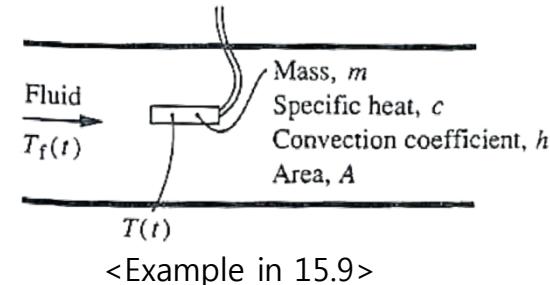
$$L\{\Delta \sin(2\pi ft)\} = \Delta \frac{a}{s^2 + a^2}, \quad a = 2\pi f$$

<From example in 15.9>

$$L(T - T_0) = \frac{1}{\tau s + 1} \cdot \frac{\Delta a}{s^2 + a^2} = \frac{A}{\tau s + 1} + \frac{Bs + C}{s^2 + a^2}$$

$$A = \frac{\Delta a}{1/\tau^2 + a^2}, \quad B = \frac{-\Delta a}{\tau a^2 + 1/\tau}, \quad C = \frac{\Delta a}{\tau^2 a^2 + 1}$$

$$L(T - T_0) = \frac{\Delta a \tau^2}{\tau s + 1} + \frac{-\Delta a}{\tau a^2 + 1/\tau} s + \frac{\Delta a}{s^2 + a^2}$$



<Example in 15.9>



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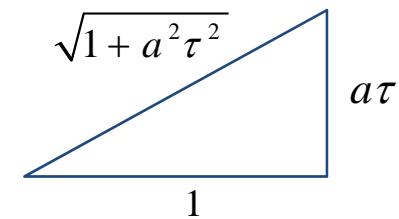
## 15.11 Stability analysis

$$\begin{aligned}T - T_0 &= \frac{\Delta a}{a^2 \tau^2 + 1} L^{-1} \left\{ \frac{\tau^2}{\tau s + 1} + \frac{-\tau s + 1}{s^2 + a^2} \right\} \\&= \frac{\Delta a}{a^2 \tau^2 + 1} \left\{ \tau e^{-t/\tau} - \tau \cos(at) + \frac{1}{a} \sin(at) \right\}\end{aligned}$$

as  $t \rightarrow \infty$

$$\begin{aligned}T - T_0 &= \frac{\Delta}{a^2 \tau^2 + 1} \{ \sin(at) - a\tau \cos(at) \} \\&= \frac{\Delta}{a^2 \tau^2 + 1} \left\{ \sqrt{a^2 \tau^2 + 1} \cdot \sin(at + \phi) \right\} = \frac{\Delta}{\sqrt{a^2 \tau^2 + 1}} \sin(at - \phi)\end{aligned}$$

$$\phi = \tan^{-1}(a\tau)$$



$$\text{Amplification ratio : } \frac{1}{\sqrt{1 + (2\pi f \tau)^2}}$$

$$\text{Phase lag : } \phi = \tan^{-1}(2\pi f \tau)$$

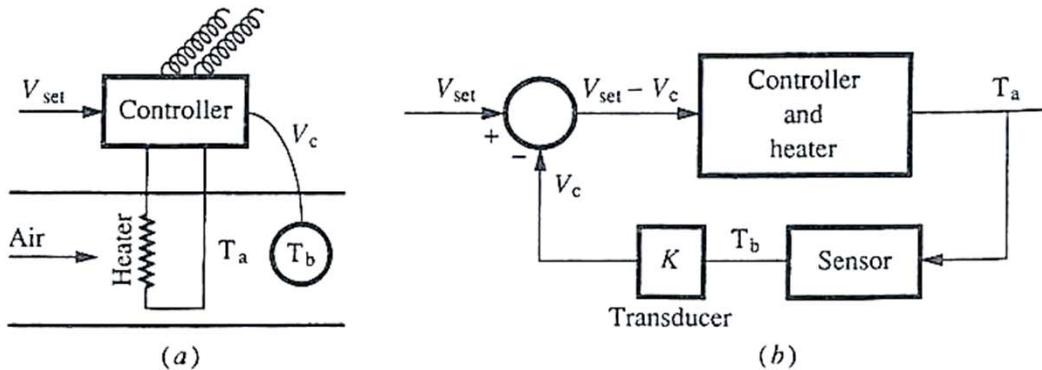


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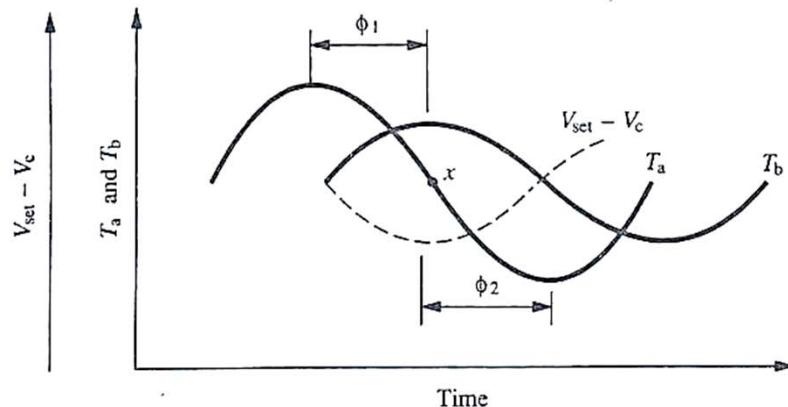
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# Chapter 15. Dynamic Behavior of Thermal Systems

## 15.11 Stability analysis



**FIGURE 15-11**  
(a) Air heater (b) control block diagram.



**FIGURE 15-12**  
Perpetuation of sinusoidal disturbances throughout the control loop of the air heater in Fig. 15-11.



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## 15.11 Stability analysis

- Stability criterion

At the frequency  $f$  where sum of phase lags =  $180^\circ$  Fig.(a)

product of amplitude ratio > 1 Fig.(b)

→ the loop is unstable

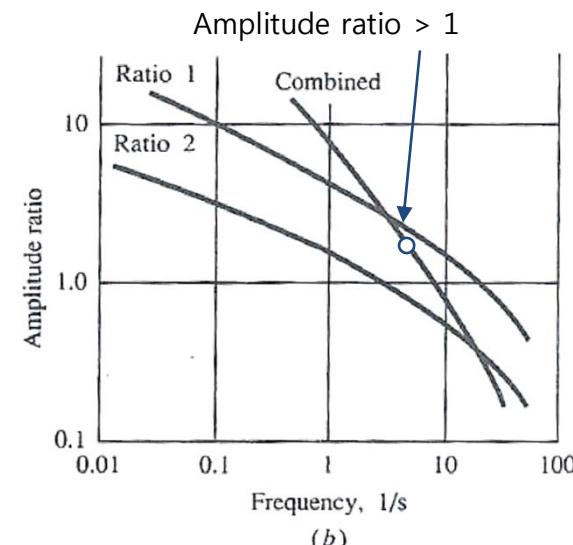
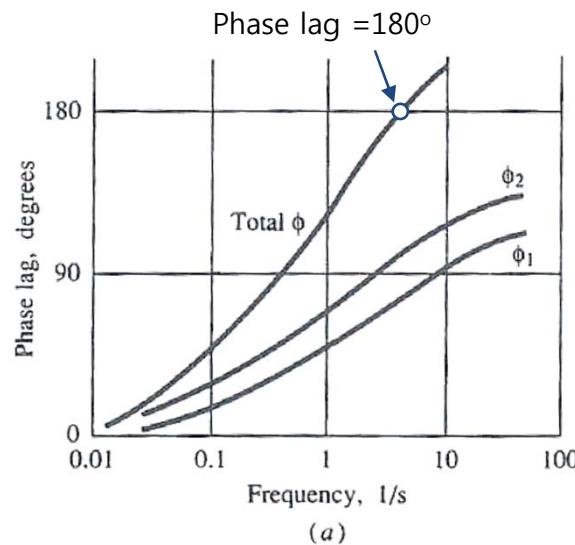


Fig. Bode diagram



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## 15.11 Stability analysis - additional

$$* \quad \frac{Y(s)}{U(s)} = G(s)$$

$u(t) = U_0 \sin \omega t$  : input

$$\begin{aligned} Y(s) &= G(s) \frac{U_0 \omega}{s^2 + \omega^2} \\ &= \frac{\alpha_1}{s - a_1} + \cdots + \frac{\alpha_n}{s - a_n} + \frac{\alpha_0}{s + j\omega} + \frac{\alpha_0^*}{s - j\omega} \end{aligned}$$

$$y(t) = \alpha_1 e^{a_1 t} + \cdots + \alpha_n e^{a_n t} + 2 |\alpha_0| \sin(\omega t + \phi), \quad \phi = \tan^{-1} \frac{\text{Im}(\alpha_0)}{\text{Re}(\alpha_0)}$$

$$\text{as } t \rightarrow \infty, \quad y(t) = U_0 A \sin(\omega t + \phi)$$

$$\text{magnitude } A = |G(j\omega)| = |G(s)| = \sqrt{\{\text{Re}[G(j\omega)]\}^2 + \{\text{Im}[G(j\omega)]\}^2}$$

$$\text{phase } \phi = \angle G(j\omega) = \tan^{-1} \frac{\text{Im}[G(j\omega)]}{\text{Re}[G(j\omega)]}$$

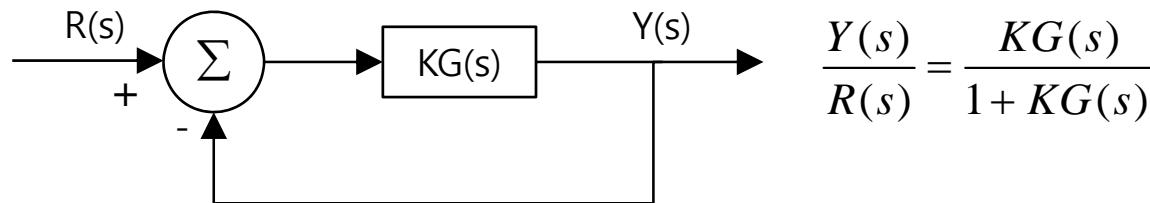


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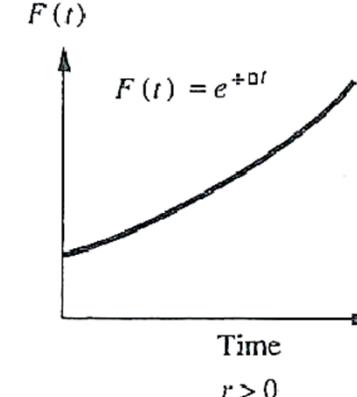
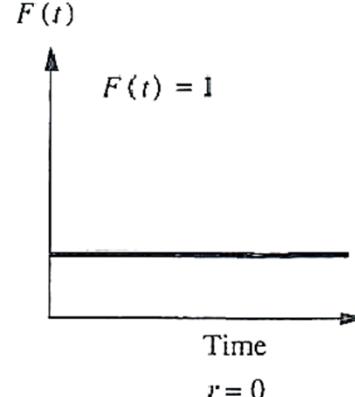
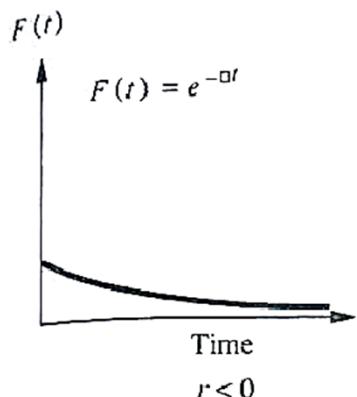
## 15.12 Stability analysis using loop transfer function



Roots are the poles of the transfer function  $1+KG(s)$

→ root locus : graph of all possible roots

root < 0	$e^{-at}$
" = 0	1
" > 0	$e^{at}$
" = $a+ib$	$e^{(a+ib)t}$



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## 15.12 Stability analysis using loop transfer function

$$KG(s) = \frac{Kb(s)}{a(s)} = \frac{K(s^m + b_1 s^{m-1} + \dots + b_m)}{s^n + a_1 s^{n-1} + \dots + a_n} \quad n > m$$

$$b(s) = (s - z_1)(s - z_2) \cdots (s - z_m)$$

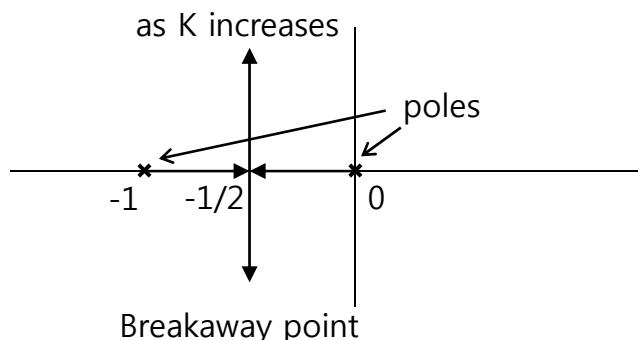
$$a(s) = (s - p_1)(s - p_2) \cdots (s - p_n)$$

Root locus form :  $1 + KG(s) = 1 + K \frac{b(s)}{a(s)} = 0 \rightarrow a(s) + Kb(s) = 0$

Ex)  $KG(s) = \frac{K}{s(s+1)}$    Denominator  $= 1 + \frac{K}{s(s+1)} \rightarrow s^2 + s + K = 0$

Root locus is a graph  
of the roots of the  
quadratic equation

$$r_1, r_2 = -\frac{1}{2} \pm \frac{\sqrt{1-4K}}{2}$$



$$0 \leq K \leq 1/4 \quad -1 < \quad < 0$$

$$K > 1/4 \quad complex$$



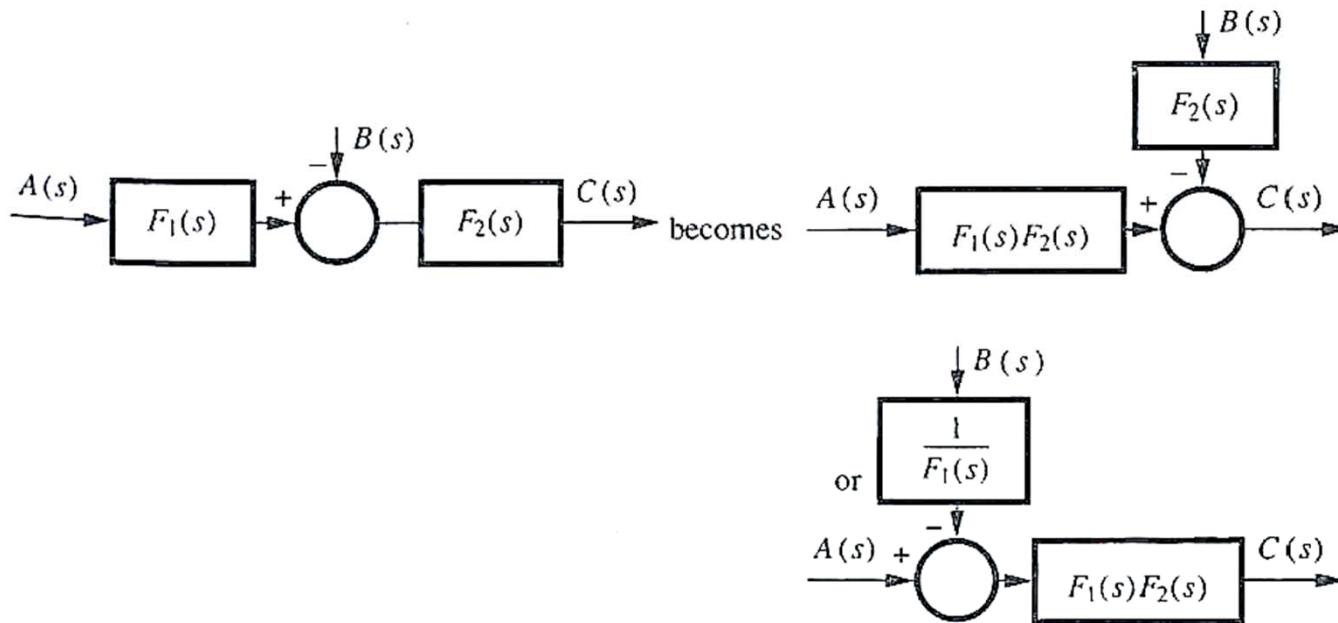
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## 15.14 Restructuring the block diagram

- └ converting complex block diagram → feedback loop
- └ simplifying



**FIGURE 15-22**  
Moving a summing point around a block.

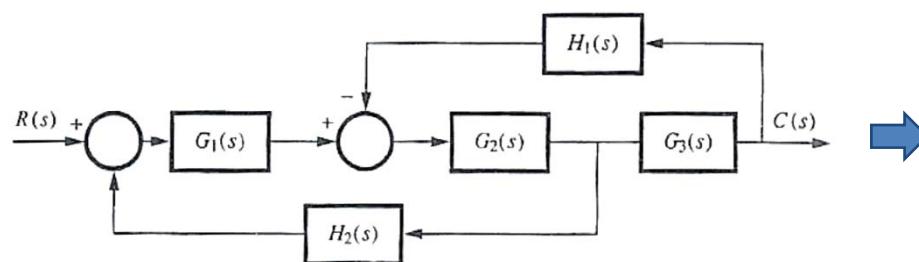


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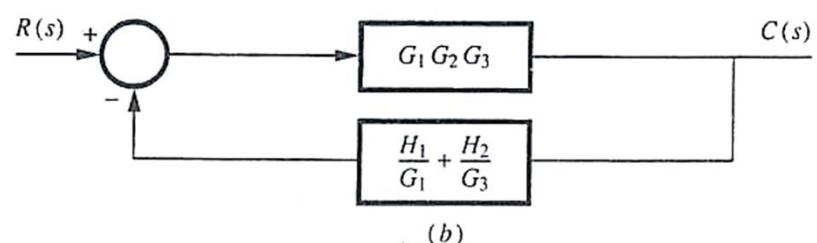
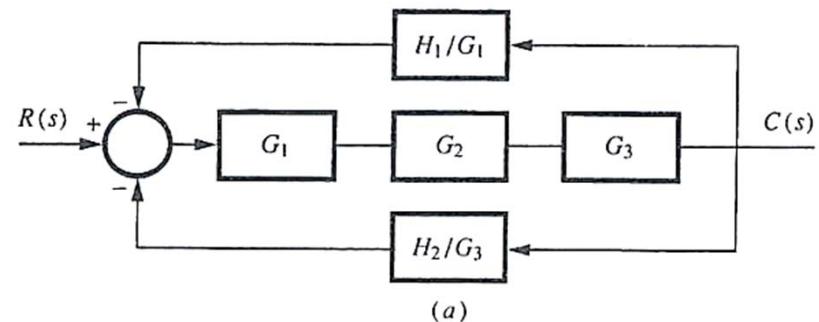
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# Chapter 15. Dynamic Behavior of Thermal Systems

## 15.14 Restructuring the block diagram



**FIGURE 15-24**  
Control block diagram in Example 15.10.



**FIGURE 15-25**  
Modifications of loop in Example 15.10.

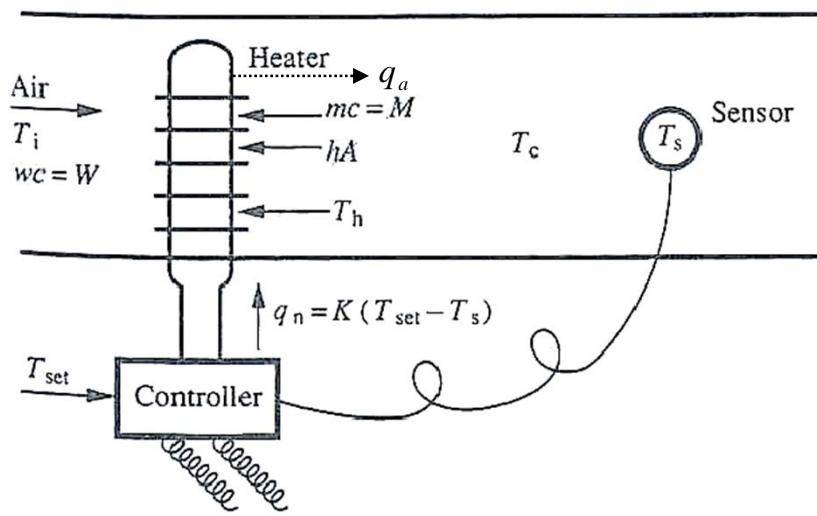


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# Chapter 15. Dynamic Behavior of Thermal Systems

## 15.15 Physical situation → block diagram



$$q_h = K(T_{set} - T_s)$$

$$q_a = w_c(T_h - T_i)(1 - e^{-\frac{hA}{w_c}}) = W(T_h - T_i)\varepsilon$$

heater → air

$w_c = W$

$$q_h - q_a = mc \frac{dT_h}{dt}$$

$$q_h(s) - q_a(s) = M [sT_h(s) - T_h(0)]$$

**FIGURE 15-26**  
Air heating system and its control.

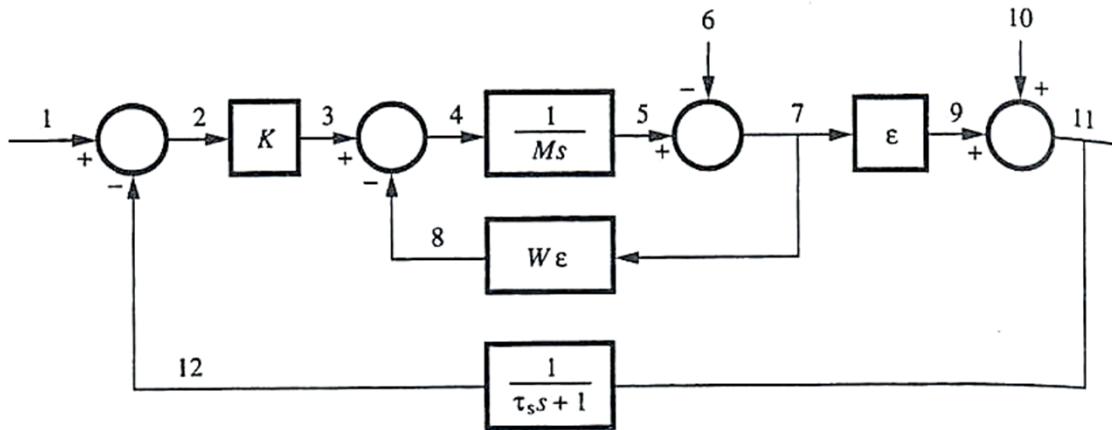


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# Chapter 15. Dynamic Behavior of Thermal Systems

## 15.15 Physical situation → block diagram



**FIGURE 15-27**  
Air heating system and its control.

**TABLE 15.2**  
Designations of variables in block diagram of Fig. 15-27.

Position	Nonnormalized	Normalized
1	$T_{\text{set}}$	$T_{\text{set}} - T_{\text{set}, 0}$
2	$T_{\text{set}} - T_s$	$(T_{\text{set}} - T_s) - (T_{\text{set}, 0} - T_{s, 0})$
3	$q_h$	$q_h - q_{h, 0}$
4	$q_h - q_a$	$(q_h - q_a) - (q_{h, 0} - q_{a, 0})$
5	$T_h$	$T_h - T_{h, 0}$
6	$T_i$	$T_i - T_{i, 0}$
7	$T_h - T_i$	$(T_h - T_i) - (T_{h, 0} - T_{i, 0})$
8	$q_a$	$q_a - q_{a, 0}$
9	$T_c - T_i$	$(T_c - T_i) - (T_{c, 0} - T_{i, 0})$
10	$T_i$	$T_i - T_{i, 0}$
11	$T_c$	$T_c - T_{c, 0}$
12	$T_s$	$T_s - T_{s, 0}$



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## 15.15 Physical situation → block diagram

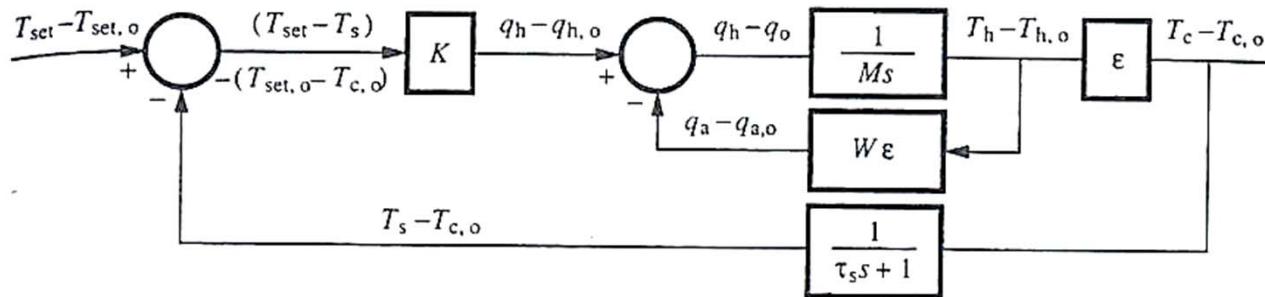


FIGURE 15-29

Diagram after elimination of two summing points.

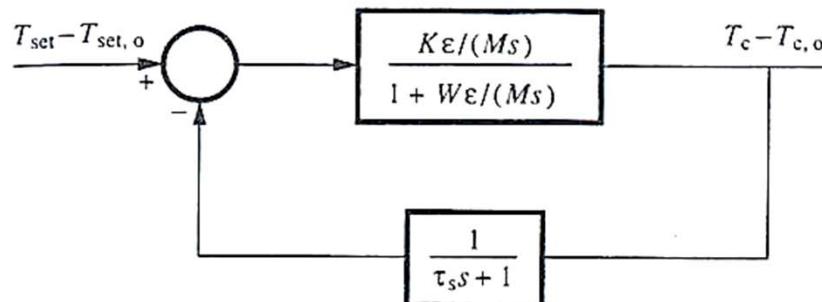


FIGURE 15-30

Simplified nonunity feedback loop for air heater controller.



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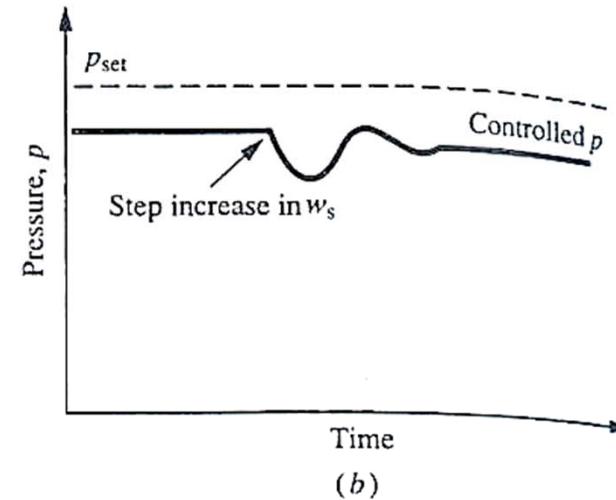
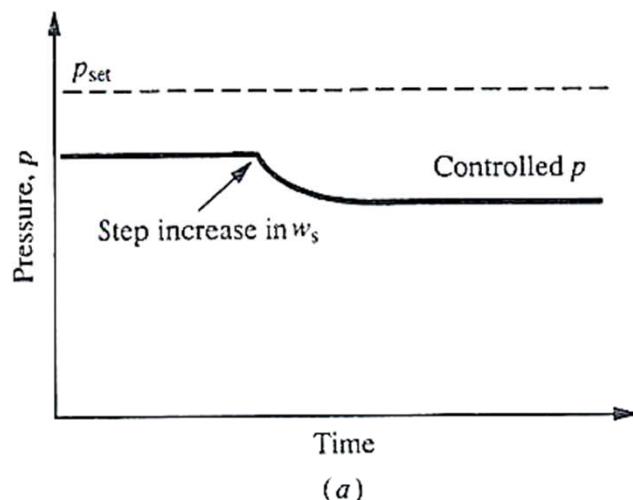
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# Chapter 15. Dynamic Behavior of Thermal Systems

## 15.16 Proportional control

$$q_h = K_p \underbrace{(T_{set} - T_s)}_{\text{error}}$$

$K \uparrow$  unstable  
 $K \downarrow$  offset



**FIGURE 15-31**  
Pressure controller (a) with low gain (b) with high gain.



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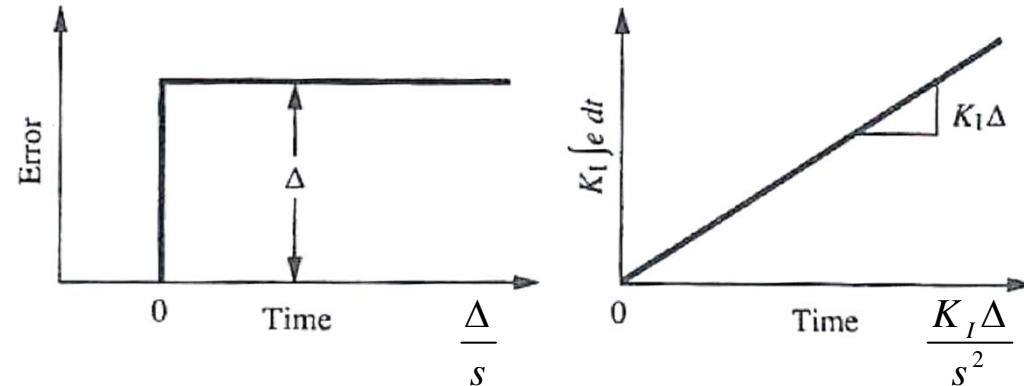
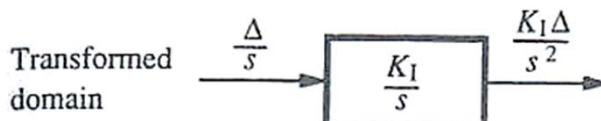
# Chapter 15. Dynamic Behavior of Thermal Systems

## 15.17 Proportional – Integral (PI) control

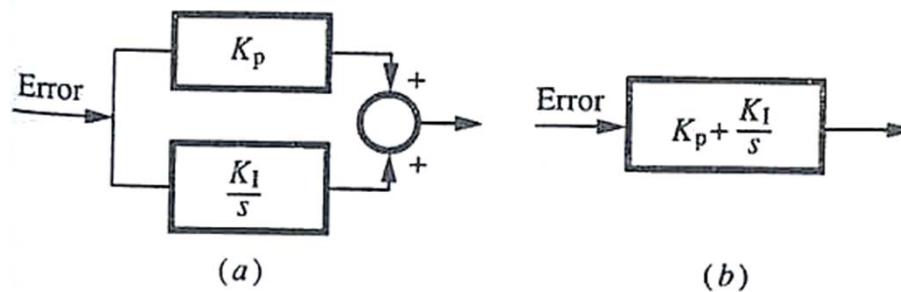
- to eliminate the offset

$$K_I \int (error) dt$$

$$TF = \frac{K_I}{s} \leftarrow \frac{K_I \Delta / s^2}{\Delta / s}$$



- PI control



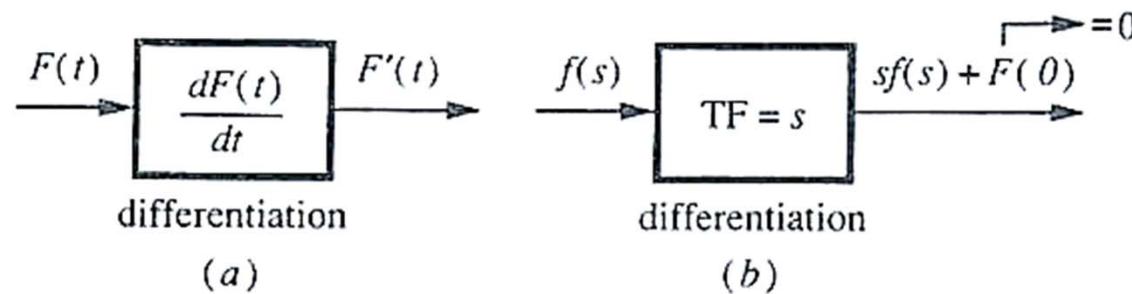
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# Chapter 15. Dynamic Behavior of Thermal Systems

## 15.18 PID control

$$K_P + \frac{K_I}{s} + K_D s$$



**FIGURE 15-36**

The differentiation process in (a) the time domain, (b) the transformed domain.



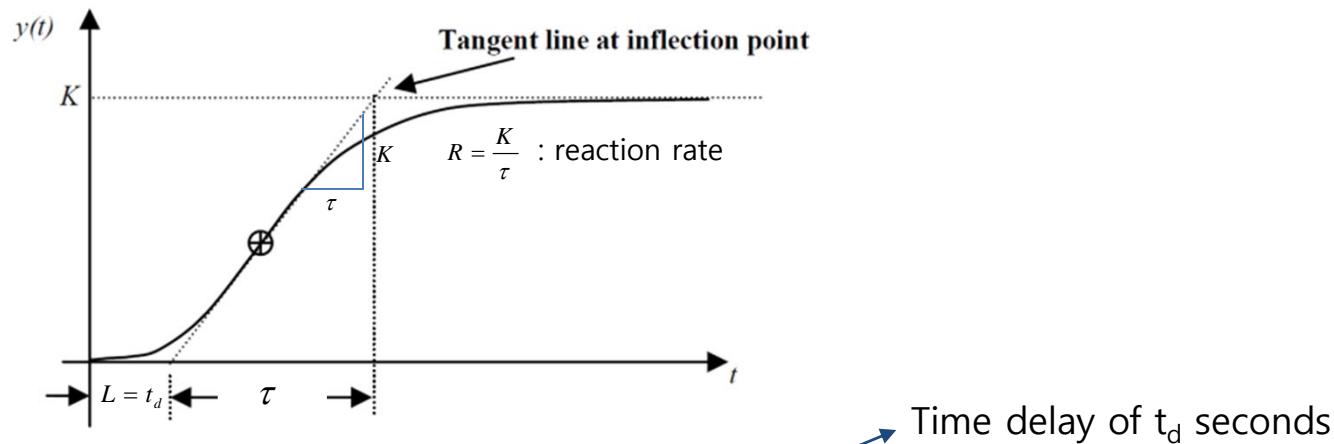
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# Chapter 15. Dynamic Behavior of Thermal Systems

## 15.18 PID control

- ❖ Ziegler-Nichols Tuning of PID controller (1942)



TF may be approximated by  $TF = \frac{Ke^{-t_d s}}{\tau s + 1}$

$$\begin{aligned} D(s) &= K \left( 1 + \frac{1}{T_I s} + T_D s \right) \\ &= K + \frac{K}{T_I s} + K T_D s = K_P + \frac{K_I}{s} + K_D s \end{aligned}$$



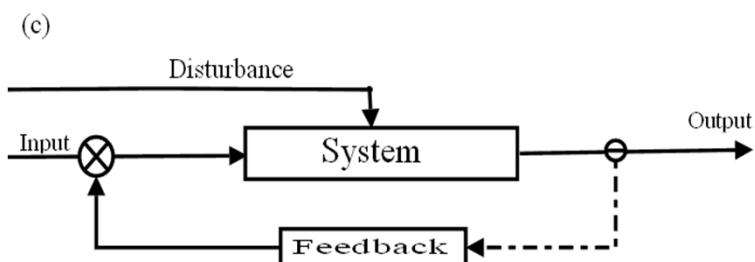
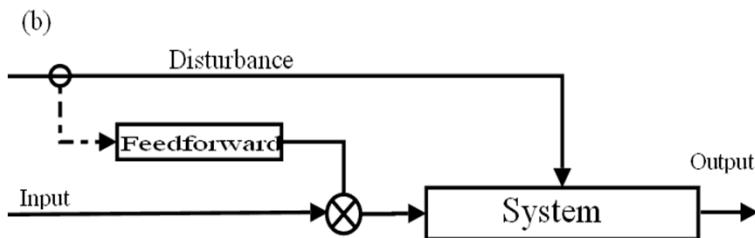
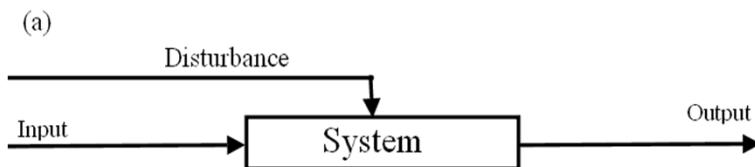
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# Chapter 15. Dynamic Behavior of Thermal Systems

## 15.19 Feed forward control

└ open loop control



(a) No control

(b) Feed forward control

(c) Feedback control



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