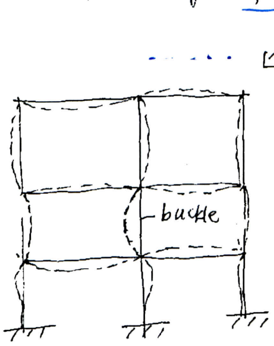
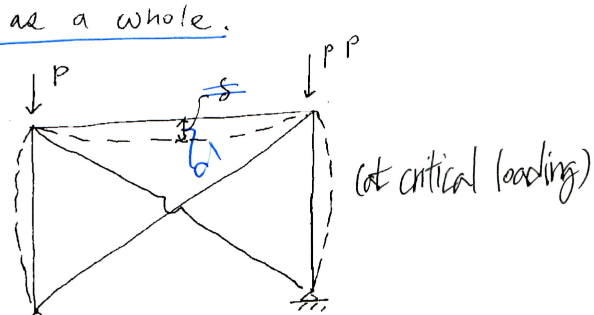
4.1 Introduction

(1) 3장까지의 주 내용 : "이상적 지점조건"을 갖는 "나홀로 기둥" (Isolated columns)을 중심으로 고찰 (대부분) → 일부 simple frames 취급

(hinged, fixed, or guided)

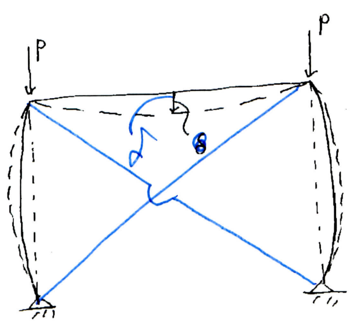
(2) In reality, most structural members are connected to other members to form a framework. To determine the P_{cr} of the column in a frame, it is necessary to investigate the stability of the frame as a whole.





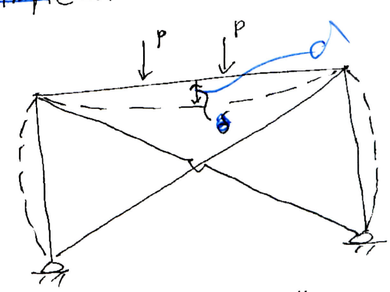
(a) perfect system

: ~~simple braced~~ portal frames

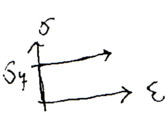


(b) Imperfect system (initially crooked columns) – load deflection problem

: simple braced portal frames



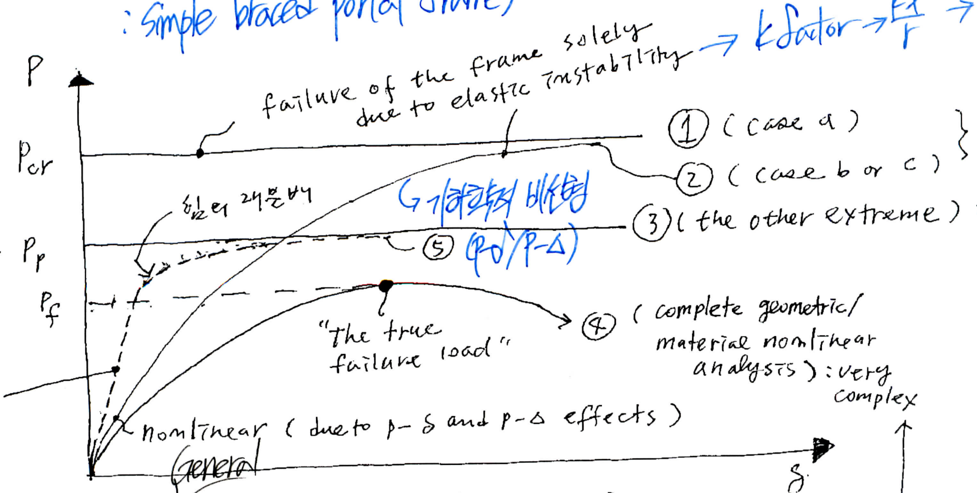
(c) "Imperfect" system (presence of primary moments)

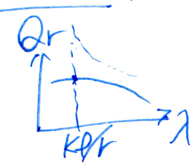


Simple plastic theory

↓

"Rigid" plastic collapse load level





"Elasto"-plastic analysis (통상의 재료적 비선형해석)

교재 Fig. 4.2 보완

Fig. 1 : Load-deflection behavior of frames (Macro 해석)

우리가 수행하는 해석의 위계를 개념적으로 보여주는 중요한 그림.

↑

「Instability에 따른 failure mode의 가능성은 배제하고 재료적 비선형만 고려하여 해석 시의 limit load level」

가령, ABAQUS 같은 computer code의 도움 필요

C.H. Lee 최근 연구 ← (만일 dynamic effect까지 추가되면 더욱 복잡)

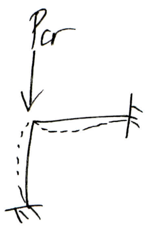
- "p. 238의 내용 정일독"

- 골조의 탄성 좌굴 하중 P_{cr}을 산정하는 방법 3가지 ← 매우 간단한 경우 취급
  - 미분방정식의 해에 의함 (4.2절)
  - 처짐각법에 의함 (4.3절) ★ → 가장 중요 (design implication 多) → "해석적 tractability"
  - matrix 변위법 (4.4절)

4.2 Elastic Critical loads by Differential Eq. Method

(1) Easy to follow ← 4.2.1 braced frame의 예 / 4.2.2 unbraced frame의 예

유사해석
여러 입장에서 경험



(2) 적어도 이론상으로 동일한 절차를 적용하여 (즉 각 부재의 경계조건과 절점에서의 변형의 적합조건을 이용) 좌굴하중을 산정할 수 있으나, 다층/다스팬을 넘어서는 경우 고유치의 산정이 너무 복잡해져서 실용성이 없음.

↳ slope-Deflection equation을 사용하는 것이 훨씬 간단.

향후
*직접해석법 (Direct analysis method) 보충예정

4.3 Slope-Deflection Equation Method에 의한 탄성 좌굴하중 산정법

↑ 미분방정식에 의한 풀이보다 simpler (근사적)

General procedure

(1) Numbering or naming of member ends, joint rotations or translations

(2) Write slope-deflection equations (적합조건 "자동" 반영)

axial force 의 영향이 반영된 Eqs. 3.7.11 – 3.7.16

(3) Apply joint moment equilibrium or story shear equilibrium equations

(4) obtain the characteristic equation "by setting the determinant of the coefficient matrix of the resulting set of equilibrium equations equal to zero."

↳ Cramer's rule 에 의해 nontrivial solution 을 얻기 위한 조건

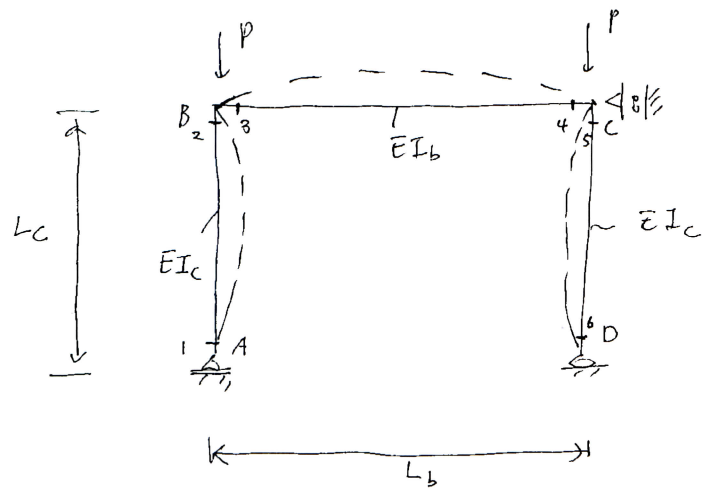
(5) Obtain the critical load as the eigenvalue of the characteristic equation

↑ 일반적으로 trial error 또는 graphical method 필요

Table 3.7 Stability functions 가 도움이 될 수 있음

P 값에 따른 s_{ii}, s_{ij} 값을 정리한 표

4.3.1 Sway-prevented Case ← 소위 Braced Frame



기둥부재 → beam-column member

보부재 → beam member / flexural member

∴ Nonsway (symmetric) buckling → $|\theta_B| = |\theta_C|$ or $\theta_B = -\theta_C$

(Solution)

(1) Numbering / Naming : done

(2) Beam-column

$$M_1 = \left(\frac{EI}{L}\right)_c (S_{iic}\theta_A + S_{ijc}\theta_B) = 0 \; ; \quad \theta_A = -\frac{S_{ijc}}{S_{iic}} \times \theta_B$$

대입

$$M_2 = \left(\frac{EI}{L}\right)_c (S_{iic}\theta_B + S_{ijc}\theta_A)$$

$$= \left(\frac{EI}{L}\right)_c \left(S_{iic} - \frac{S_{ijc}^2}{S_{iic}}\right) \times \theta_B \; ; \quad \text{타단 hinge 경우의 식 (Eq. 3.8.5)}$$

일부러 '$\theta_B = -\theta_C$' 라는 대칭성을 이용치 않고 풀어 봄

Beam

$$M_3 = \left(\frac{2EI}{L}\right)_b (2\theta_B + \theta_C)$$

$$M_4 = \left(\frac{2EI}{L}\right)_b (\theta_B + 2\theta_C)$$

Beam-column

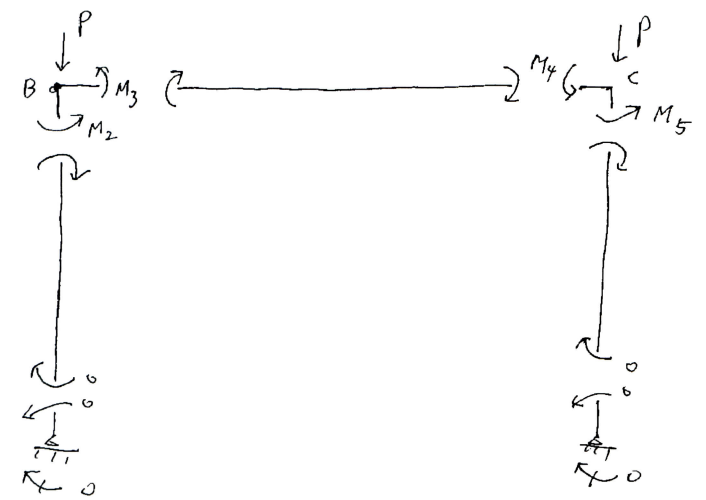
$$M_5 = \left(\frac{EI}{L}\right)_c (S_{iic}\theta_C + S_{ijc}\theta_D) = \left(\frac{EI}{L}\right)_c \left(S_{iic} - \frac{S_{ijc}^2}{S_{iic}}\right) \times \theta_C$$

$$M_6 = \left(\frac{EI}{L}\right)_c (S_{iic}\theta_D + S_{ijc}\theta_C) = 0 \; ; \quad \theta_D = -\frac{S_{ijc}}{S_{iic}}\theta_C$$

→ M_5에 대입

↑ rigid joint에 deformation compatibility는 어떻게 반영되고 있는가?

(3)



$$\overset{\curvearrowleft}{+}\ \Sigma M_B = M_2 + M_3 = \left(\frac{EI}{L}\right)_c \left(S_{iic} - \frac{S_{ijc}^2}{S_{iic}}\right)\theta_B + \left(\frac{2EI}{L}\right)_b (2\theta_B + \theta_C) = 0$$

$$\overset{\curvearrowleft}{+}\ \Sigma M_C = M_4 + M_5 = \left(\frac{EI}{L}\right)_c \left(S_{iic} - \frac{S_{ijc}^2}{S_{iic}}\right)\theta_C + \left(\frac{2EI}{L}\right)_b (\theta_B + 2\theta_C) = 0$$

or,

$$\underbrace{\begin{bmatrix} \left\{\left(\frac{EI}{L}\right)_c \left(S_{iic} - \frac{S_{ijc}^2}{S_{iic}}\right) + \left(\frac{4EI}{L}\right)_b\right\} & \left(\frac{2EI}{L}\right)_b \\ \left(\frac{2EI}{L}\right)_b & \left\{\left(\frac{EI}{L}\right)_c \left(S_{iic} - \frac{S_{ijc}^2}{S_{iic}}\right) + \left(\frac{4EI}{L}\right)_b\right\} \end{bmatrix}}_{2\times 2} \underbrace{\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix}}_{2\times 1} = \underbrace{0}_{2\times 1}$$

2×2 (the coefficient matrix)

(4) For a nontrivial solution,

$$\left\{\left(\frac{EI}{L}\right)_c \left(S_{iic} - \frac{S_{ijc}^2}{S_{iic}}\right) + \left(\frac{4EI}{L}\right)_b\right\}^2 - \left(\frac{2EI}{L}\right)_b^2 = 0$$

「앞에서 "눈감은" 대칭성이 여기서 나타나고 있음.」

$$\text{or,}\quad \left(\frac{EI}{L}\right)_c \left(S_{iic} - \frac{S_{ijc}^2}{S_{iic}}\right) + \left(\frac{4EI}{L}\right)_b = \left(\frac{2EI}{L}\right)_b$$

$$\left(\frac{EI}{L}\right)_c \left(S_{iic} - \frac{S_{ijc}^2}{S_{iic}}\right) + \left(\frac{2EI}{L}\right)_b = 0$$ --- (교재 4.3.7 식과 동일한 결과!)

Specialization : $\left(\frac{EI}{L}\right)_c = \left(\frac{EI}{L}\right)_b = \left(\frac{EI}{L}\right)$

$$\frac{EI}{L}\left[\, S_{iic} - \frac{S_{ijc}^2}{S_{iic}} + 2 \,\right] = 0 \quad \text{---- } (4.3.8)$$

See Table 3.7 ; Euler 기둥보다 큰 좌굴하중이 예상되므로

"활용법" $\quad kL = \pi \times \sqrt{\frac{P}{P_e}} \geq \pi = 3.14$ 이후 영역에서 해가 존재할 것임.

$$f(kL) = S_{iic} - \frac{S_{ijc}^2}{S_{iic}} + 2$$

Table 3.7 이용, 개략 $\quad S_{iic}(\pi) = 2.46$ etc

시행착오 작업.

$$f(kL = \pi) = 2.46 - \frac{2.47^2}{2.46} + 2 = +1.98$$

$$f(kL = 1.5\pi = 4.71) = -0.66 - \frac{3.98^2}{-0.66} + 2 = 25.34$$

$$f(kL = 1.2\pi = 3.77) = 1.62 - \frac{2.80^2}{1.62} + 2 = -1.22$$

$$f(kL = 1.1\pi = 3.45) = 2.08 - \frac{2.61^2}{2.08} + 2 = 0.80$$

$$f(kL = 1.15\pi \doteq 3.60) = 1.86 - \frac{2.70^2}{1.86} + 2 = -0.06$$

Stop

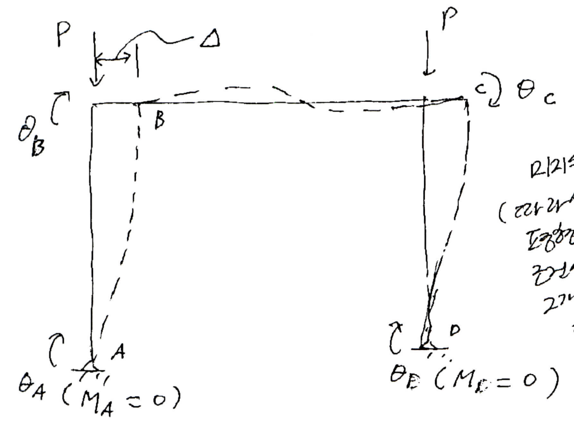
$$\therefore \quad \pi\sqrt{\frac{P_{cr}}{P_e}} = 3.61$$

$$P_{cr} = \frac{3.61^2}{\pi^2} \times P_e = 1.32 \times P_e = \frac{\pi^2 EI}{\left(\frac{1}{\sqrt{1.32}} L\right)^2} = \frac{\pi^2 EI}{(0.87\, L)^2}$$

K factor < 1.0

(effective length factor)

4.3.2 Sway-permitted case



미지수 2개 (따라서 평형 조건식은 2개만 필요)

☆ 원래의 "primary unknowns = 5개"

$M_A = 0$, $M_B = 0$ 에서 θ_A, θ_D 소거 가능

복곡률의 역대칭성 $\theta_B = \theta_C$

Notes:

보-기둥 부재 : member axis rotation ($\frac{\Delta}{L}$)이 있음에 유의 (시계방향이므로 "+")

$$\hookrightarrow M_n = \frac{EI}{L}\left[s_{ii}\theta_n + s_{ij}\theta_f - (s_{ii} + s_{ij})\frac{\Delta}{L} \right]$$

where n = near, f = far

보 부재 : $M_n = \frac{2EI}{L}(2\theta_n + \theta_f)$ ← 「부재축 회전 없음, 축력 효과 없음」

(Solution)

보-기둥부재

'+' 부호

$$M_{AB} = \left(\frac{EI}{L}\right)_c \left[s_{iic}\theta_A + s_{ijc}\theta_B - (s_{iic} + s_{ijc})\left(\frac{\Delta}{L_c}\right) \right] = 0$$

$$M_{BA} = \left(\frac{EI}{L}\right)_c \left[s_{iic}\theta_B + s_{ijc}\theta_A \cdots \right]$$

θ_A 소거

$$M_{BA} = \left(\frac{EI}{L}\right)_c \left[\left(s_{iic} - \frac{s_{ijc}^2}{s_{iic}} \right)\theta_B - \left(s_{iic} - \frac{s_{ijc}^2}{s_{iic}} \right)\left(\frac{\Delta}{L_c}\right) \right] \quad \text{--- (4.3.14)}$$

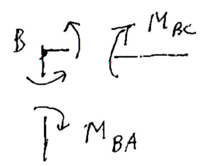
보부재

$$M_{BC} = \left(\frac{2EI}{L}\right)_b (2\theta_B + \theta_C)$$

역대칭성에 의해 $\theta_B = \theta_C$

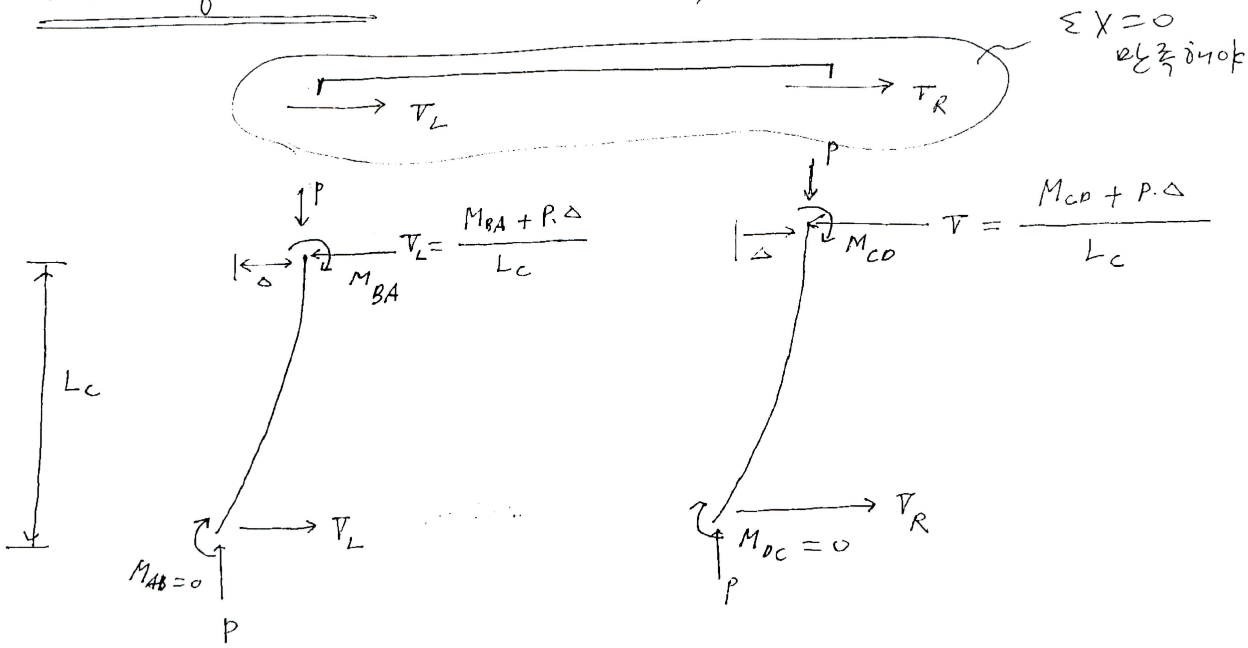
$$= \left(\frac{6EI}{L}\right)_b \times \theta_B \quad \text{------ (4.3.15)}$$

Moment Equilibrium at Joint B (첫번째 조건식)



$M_{BA} + M_{BC} = 0$ 에서 (4.3.19) 식이 얻어짐

Shear Equation (두번째 조건식)



∴ Free-body (정확히 그려야!)

$M_{CD} = M_{BA}$ (역대칭시)

$$\overset{+}{\rightarrow} \Sigma X_i = V_L + V_R = \frac{M_{BA} + P\cdot\Delta}{L_c} + \frac{M_{CD} + P\cdot\Delta}{L_c} = 0$$

or

$$\frac{M_{BA} + P\cdot\Delta}{L_c} = 0 \quad \text{-----} \quad (4.3.24)^{\star}$$

M_{BA} 대신 (4.3.14)식 대입하여 정리하면 (4.3.26)식이 얻어짐.

(4.3.19)식 및 (4.3.26)식을 matrix form으로 표시하면,

$\Sigma M = 0$ ---(1) $\quad \Sigma X_i = 0$ --(2)

$$\begin{bmatrix} S + 6\frac{I_b L_c}{I_c L_b} & -S \\ S & -S + (K_c L_c)^2 \end{bmatrix} \begin{bmatrix} \theta_B \\ \frac{\Delta}{L_c} \end{bmatrix} = \underline{0} \quad \cdots (4.3.27)$$

where $S = S_{iic} - \frac{S_{ijc}^2}{S_{iic}}$

Specilization : $I_b = I_c = I$, $L_b = L_c = L$,

$$\det \left| \begin{bmatrix} s+6 & -s \\ s & s+(KL)^2 \end{bmatrix} \right| = 0$$ 에서 특성방정식의 근을 구하면

(시행착오 plus Table 3.7 이용)

$KL = 1.35$

$$P_{cr} = 1.82 \frac{EI}{L^2}$$

$$= \frac{1.82}{\pi^2} \times \frac{\pi^2 EI}{L^2} = \frac{\pi^2 EI}{\left(\frac{\pi}{\sqrt{1.82}} L\right)^2}$$

$$= \frac{\pi^2 EI}{(2.33L)^2}$$

물론, 미분방정식의 풀이에 의한 결과와 동일함.

HW # 확인토

$K = 2.33 \gg 1.0$ ← unbraced frame

Note : 적어도 이론상으로는, 미분방정식의 풀이에 의한 방법과 마찬가지로, 모든 종류의 골조에 대해 P_{cr}을 계산할 수 있지만 독립된 자유도의 수효가 증가하면 특성방정식의 근을 계산하는데 소요되는 노력이 막대해짐.

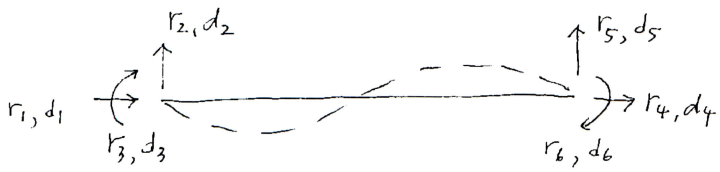
↓ ← 처짐각법의 확장

matrix stiffness method
(With the use of computer)

4.4 Elastic Critical Loads by Matrix Stiffness Method.

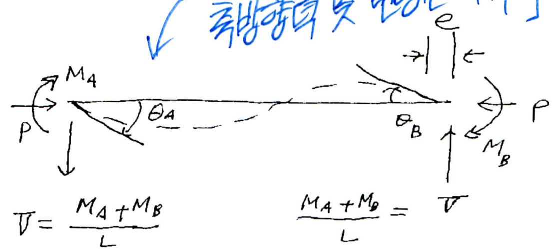
4.4.1 Element Stiffness Formulation

↑ from 「the slope-deflection equation」



(a) element end forces and end displacements of a frame member

「처짐각 방정식에 축방향력 및 변형만 추가」



(b) End forces and end displacements in the slope deflection equation. (non-sway mode or simple case)

Equilibrium Relationships ← '힘 변환 관계식'

$$r_1 = P$$
$$r_2 = -V = -\frac{M_A + M_B}{L}$$
$$r_3 = M_A$$
$$r_4 = -P$$
$$r_5 = V = \frac{M_A + M_B}{L}$$
$$r_6 = M_B$$

$$\longrightarrow \underset{6\times1}{\begin{Bmatrix} r_1 \\ \vdots \\ r_6 \end{Bmatrix}} = \underset{6\times3}{\begin{bmatrix} \underline{A} \end{bmatrix}} \underset{3\times1}{\begin{Bmatrix} P \\ M_A \\ M_B \end{Bmatrix}} \quad \cdots (4.4.10)$$

Kinematic Relationships ← '변위 변환 관계식'

$$e = -(d_4 - d_1)\ ; \text{ compression positive}$$
$$\theta_A = d_3 + \left(\frac{d_5 - d_2}{L}\right)$$
$$\theta_B = d_6 + \left(\frac{d_5 - d_2}{L}\right)$$

$$\longrightarrow \underset{3\times1}{\begin{Bmatrix} e \\ \theta_A \\ \theta_B \end{Bmatrix}} = \underset{3\times6}{\begin{bmatrix} \underline{B} \end{bmatrix}} \underset{6\times1}{\begin{Bmatrix} d_1 \\ \vdots \\ d_6 \end{Bmatrix}} \quad \cdots (4.4.11)$$

3) End forces vs. displ. relationship
(P, M_A, M_B) versus (e, θ_A, θ_B)

$$\begin{bmatrix} P \\ M_A \\ M_B \end{bmatrix} = \frac{EI}{L} \underbrace{\left[\begin{array}{c|c|c} \frac{A}{I} & 0 & 0 \\ \hline 0 & S_{ii} & S_{ij} \\ \hline 0 & S_{ji} & S_{jj} \end{array}\right]}_{\underline{C}\ \ 3\times3} \begin{bmatrix} e \\ \theta_A \\ \theta_B \end{bmatrix} \quad \cdots (4.4.15)$$

(4.4.11) → (4.4.15) 대입
→ (4.4.15) → (4.4.10) 대입

$$\underset{6\times1}{\underline{r}_{ns}} = \underset{6\times3}{\underline{A}}\ \underset{3\times3}{\underline{C}}\ \underset{3\times6}{\underline{B}}\ \underset{6\times1}{\underline{d}} \quad \cdots (4.16)$$

$$\therefore\ \underline{r}_{ns} = \underline{k}_{ns} \times \underline{d} \quad \cdots (4.4.17)$$

↑ non-sway mode

If the member is permitted to sway as,

$\frac{P\Delta}{L} = \frac{P}{L}(d_2 - d_5) = -r_2$

$\frac{P\Delta}{L} = \frac{P}{L}(d_2 - d_5) = r_5$

[additional shear due to swaying of the member]

∴ Equilibrium and kinematic relationships due to swaying of the member

$$\underset{6\times1}{\underset{\sim}{r}_s} = \underset{6\times6}{\underset{\sim}{k}_s} \cdot \underset{6\times1}{\underset{\sim}{d}} \quad \text{---- (4.4.19)} \leftarrow \text{for sway only}$$

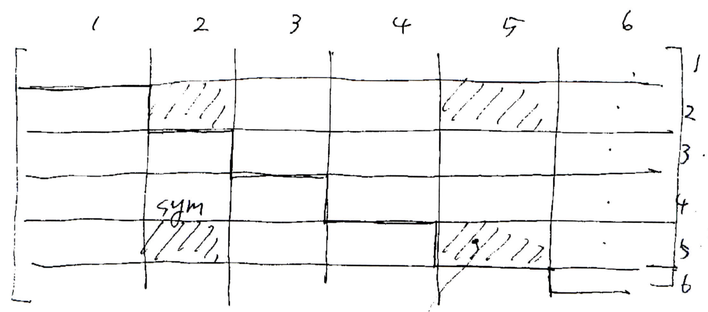
(4.4.20)

By combining Eqs. (4.4.17) and (4.4.20),

$$\underset{\sim}{r} = \underset{\sim}{k} \cdot \underset{\sim}{d} \quad \text{---- (4.4.21)}$$

where $\underset{\sim}{r} = \underset{\sim}{r}_{ns} + \underset{\sim}{r}_s$; $\underset{\sim}{k} = \underset{\sim}{k}_{ns} + \underset{\sim}{k}_s$ --- (4.4.22)

$\underset{\sim}{k} = \frac{EI}{L}$



---- (4.4.23)

$\frac{EI}{L} \times \frac{-(kL)^2}{L^2}$

$= -\frac{EI}{L}k^2$

$= -\frac{EI}{L}\cdot\frac{P}{EI}$

$= -\frac{P}{L}$ //

Sway mode 의 영향이 더해진 항들

(4.4.23)에 stability function (S_{ii}, S_{ij})을 대입하여 정리하면
(3.7.11) ~ (3.7.12) (p.184 참고)

$$\underset{\sim}{k} = \frac{EI}{L}\begin{bmatrix} \phi_1, \phi_2, & \\ \phi_3, \phi_4 & \end{bmatrix} \quad \text{---- (4.4.24)}$$

↳ The expressions for $\phi_1 \sim \phi_4$ are given in Table 4.1 (p. 259)

↳ function of P

Note : ① $P \to 0$ 이면 $\phi_i \ (i = 1 \sim 4) \to 1.0$ 이 되어 일반 frame element 의 stiffness matrix 로 귀착 됨 (로피탈의 정리에 의해 입증 가능)

② Table 4.1 의 급수전개식 (Goto - Chen 1987)의 장점 : 교과서 참조

↳ ϕ_i 의 값을 Taylor 급수 전개의 제 2항까지만 취하여 근사하면 (4.4.27) 및 (4.4.28) 식이 얻어 짐. linear term

↳ HW # 확인요

$f\left(\frac{P}{L}\right)^2 + \cdots$ 고차항 nonlinear terms

$$\underline{k} = \frac{EI}{L}\left[\quad\right] \mp \frac{P}{L}\left[\quad\right] \quad \cdots (4.4.27)$$

"−" ve : compression

"+" ve : tension

Note the stiffness reduction due to the presence of P

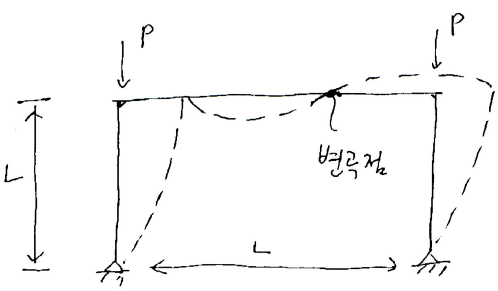
「Linearized buckling analysis」

$$= \underline{k}_0 + \underline{k}_g \quad \cdots (4.4.28)$$

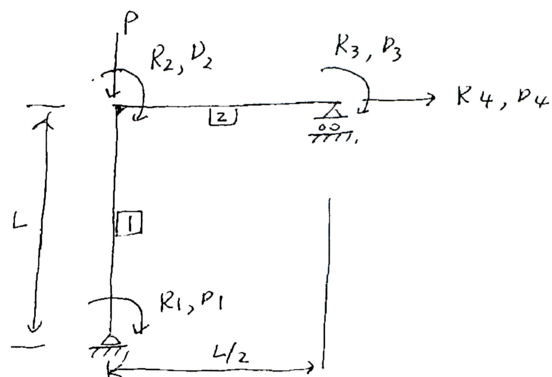
$\underline{k}_0$: the first order elastic stiffness matrix

$\underline{k}_g$: geometric (or initial stress) stiffness matrix accounting for the effect of the axial force P on the bending stiffness of the member

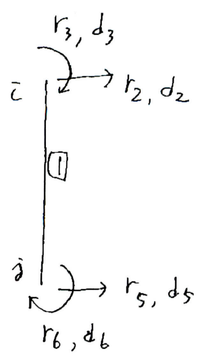
4.4.2 Sway Buckling of a Pinned-Base Portal Frame



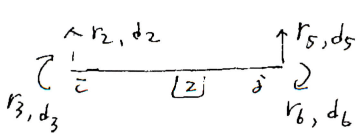
: original structure



: one half of the structure ($\frac{1}{2}$ part analysis) and global dof numbering



: element [1] (beam-column element)



: element [2] (beam element)

"axial deformation이 없는 것으로 단순화" → (flexural deformation만 고려)

6x6 행렬을 4x4 행렬로 축소가능 (제 1행 및 4행, 그리고 제 1열 및 4열 모두 delete)

① The element stiffness matrix for the column → Eq. (4.4.30)
(beam-column element)

② The element stiffness matrix for the beam → Eq. (4.4.31)
(beam element)
$p = 0$

以下는 assemble 과정 ← 통상 Direct Stiffness Method 사용

For element 1 ; $\underset{4\times1}{\underline{d}_1} = \underset{4\times4}{\underline{T}_1} \underset{4\times1}{\underline{D}}$ --- (4.4.32)
(local) ↳ 변위변환행렬   global

For element 2 ; $\underset{4\times1}{\underline{d}_2} = \underset{4\times4}{\underline{T}_2} \underset{4\times1}{\underline{D}}$ --- (4.4.33)

} 변위변환

For element 1 : $\underline{R}_1 = \underline{T}_1^T \underline{r}_1$ --- (4.4.36)

For element 2 : $\underline{R}_2 = \underline{T}_2^T \underline{r}_2$ --- (4.4.37)

} 힘변환

global   local   힘변환행렬 ← law of contragradience

From consideration of joint equilibrium,

$$\underline{R} = \underline{R}_1 + \underline{R}_2 \quad \text{---- (4.4.40)}$$

외력 vector   member end force vector

$$= \underline{T}_1^T \underline{r}_1 + \underline{T}_2^T \underline{r}_2$$

$$= \underline{T}_1^T (\underline{k}_1 \cdot \underline{d}_1) + \underline{T}_2^T (\underline{k}_2 \cdot \underline{d}_2)$$

$$= \underline{T}_1^T \underline{k}_1 (\underline{T}_1 \underline{D}) + \underline{T}_2^T \underline{k}_2 (\underline{T}_2 \underline{D})$$

$$\underline{R} = (\underline{T}_1^T \underline{k}_1 \underline{T}_1 + \underline{T}_2^T \underline{k}_2 \underline{T}_2) \times \underline{D} \quad \text{--- (4.4.45)}$$

$$= \underline{K} \cdot \underline{D} \quad \text{------- (4.4.46)}$$

In general, $\underline{K}$ = structure stiffness matrix

$$= \sum_i \underline{T}_i^T \underline{k}_i \underline{T}_i \quad \text{---- (4.4.48)}$$

$$\underset{4\times4}{\underline{K}} \cdot \underset{4\times1}{\underline{p}} = \underset{4\times1}{\underline{R}} = \underset{4\times1}{\underline{0}}$$

↑ (4.4.51)式

처짐각법 또는 미방해

$1.82 \dfrac{EI}{L^2}$ -- (4. .31)

미묘한 차이

why?

At bifurcation,

$$\det |\underline{K}| = 0 \longrightarrow P_{cr} = 1.83 \dfrac{EI}{L^2} \quad \text{--- (4.4.53)}$$

「(4.4.24) 식을 (4.4.27) 식으로 truncation 했기 때문.」

★ 미방의 풀이나 처짐각법 보다 훨씬 계산 노력이 많이 소요됨 → "계산기"(computer)의 도움 필요.

4.5 Second-order elastic analysis ← "직접해석법"의 주요 수단

(1) In an eigenvalue analysis (or bifurcation of equilibrium approach), the system is assumed to be perfect. ← 4.4절까지의 접근법

↳ No lateral deflections in the members until the load reaches the critical load P_{cr}. See Figure 1 in page ①. (curve 1)

(2) If the system is not perfect, lateral deflections will occur as soon as the load is applied. See Figure 1 in page ① (curve 2)

↳ A second-order elastic analysis is necessary to trace the load-deflection response of the frame.

(3) In a second order analysis, such secondary effects as the $\underbrace{P-\delta}_{\text{member effect } (B_1 \text{ factor})}$ and $\underbrace{P-\Delta}_{\text{system effect } (B_2 \text{ factor})}$ effects can be incorporated directly into the analysis procedure.

「통상의 실무용 프로그램에서도 이것이 포함된 경우는 많지 않아 보임.」

↓

(MIDAS GT-STRUDL SAP2000) O.K

↳ the use of B_1 and B_2 factors are not necessary.

↳ The equilibrium equations are formulated with respect to the deformed geometry of the structure which is constantly changing with the applied loads

AISC2016

"upload"

(6)

(4) Second-order analysis 의 일반적 절차 (the incremental load approach)

↳ p. 266 천일독

「deformed geometry 를 기준하여 평형 방정식을 구성하여 해석」

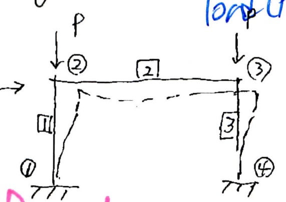
일반적으로 iteration 필요.

"out of plumbness"

initial imperfection simulation "by force" = Notional load (H)

"유인물 ①쪽의 case②에 상당

$\frac{\Sigma P}{500} = H$



* keywords : load increment & iteration (다음쪽 해설)

Notional load or initial imperfection

for sidesway → 「AISC 2010에 부록에서 본문으로 등장」

Step 1 : Discretize the frame into a number of beam-column elements

Step 2 : Formulate the element stiffness matrix $\underline{k}$ for each and every element.

effect of P on flexural stiffness 반영

(stability function)

Eq. (4.4.24) ← exact form, p. 257

or

Eq. (4.4.27) ← approx. form, p. 260 (Taylor series approx.)

Step 3 : Assemble the element stiffness matrices to form the structure stiffness matrix $\underline{K}$.

Step 4 : Solve for the incremental displacement vector using

$$\Delta \underline{R}_i = \underline{K}_i^1 \cdot \Delta \underline{D}_i^1 \quad \cdots (4.5.1)$$

① iteration

i load step

$$\Delta \underline{D}_i^1 = (\underline{K}_i^1)^{-1} \cdot \Delta \underline{R}_i \quad \cdots (4.5.2)$$

where $\Delta \underline{R}_i$ = prescribed incremental load vector of the ith load step.

$\underline{K}_i^1$ = structure secant stiffness matrix at the beginning of ith load step

$\Delta \underline{D}_i^1$ = incremental structure nodal displacement vector at ith load step.

Step 5 : Update the structure nodal displacement vector from

$$\underline{D}_i^1 = \underline{D}_i + \Delta \underline{D}_i^1 \quad \cdots (4.5.3)$$

where $\underline{D}_i^1$ = structure nodal displacement vector at the end of the first cycle of calculation at the ith load step

$\Delta \underline{D}_i^1 = \boxed{} \quad \cdots (4.5.2)$

Step 6. Extract the element end displacement vector $\underline{d}_i$ from $\underline{D}_i^1$ for each and every element in the structure.

Step 7. For each element, evaluate element axial displacement e and element end rotations θ_A, θ_B from Eqs. (4.4.7) to (4.4.9)

p. 254 하단

Step 8. For each element, evaluate element axial force P and element end moments M_A, M_B from Eqs. (4.4.12) to (4.4.14).

p. 255

Step 9. For each element, evaluate element end forces from Eq. (4.4.10).

p. 255

Step 10. Form the structure "internal" force vector $\underline{R}_i^1$ at the end of the first cycle of calculation at the ith load step by assembling the element end forces evaluated in step 9.

Step 11. Calculate the "external" force vector from

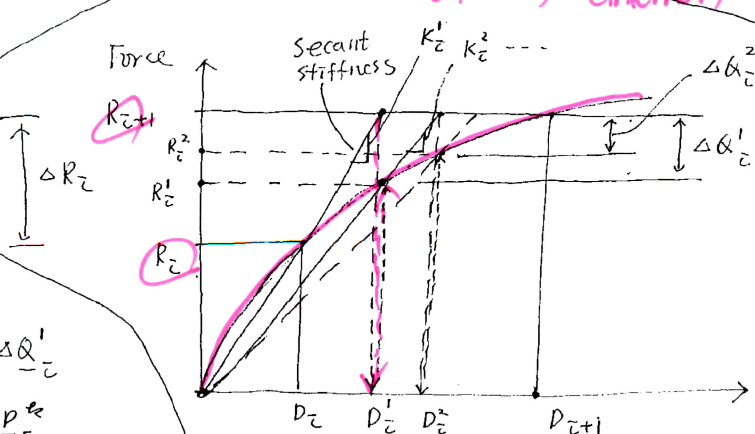
$$\underline{R}_{i+1} = \underline{R}_i + \Delta\underline{R}_i \quad \text{---} (4.5.4)$$

residual force (잔차응력)

Step 12. Evaluate the unbalanced force vector $\Delta\underline{Q}_i^1$ at the end of cycle,

$$\Delta\underline{Q}_i^1 = \underline{R}_{i+1} - \underline{R}_i^1 \quad \text{---} (4.5.5)$$

(external) (internal)



: First cycle of calculation

(for a one degree of freedom structure)

piece-wise linear analysis

「수렴조건을 만족시킬 때까지의 iteration 과정」

Steps 13 ~ 19 :

$$\Delta\underline{D}_i^2 = (\underline{K}_i^2)^{-1}\Delta\underline{Q}_i^1$$

$$\underline{D}_i^2 = \underline{D}_i + \sum_{k=1}^{2}\Delta\underline{D}_i^k$$

$$\Delta\underline{Q}_i^2 = \underline{R}_{i+1} - \underline{R}_i^2 \quad \text{----}$$

step 20.

Another load increment and repeat steps 2 ~ 19.

* 수렴조건: ex) Euclidian vector norm

unbalanced force vector $\|\Delta\underline{Q}_i^j\| \le \varepsilon$ (say, $\varepsilon = 10^{-8}$)

4.6 plastic collapse loads (생략)

↑ stability limit state 이 거동을 지배하지 않을 때

(Equilibrium and
kinematics "
moment ")

상계정리
하계정리
유일성정리 …

4.6.1 plastic hinge (소성힌지)

4.6.2 Hinge-by-hinge method

4.6.3 Mechanism method

권일두 ("철골구조소성학" 참조)

Sequential yield (or event-to-event type) analysis (항복이 발생할 때마다 가상의 힌지를 삽입 후 일련의 탄성해석 수행)

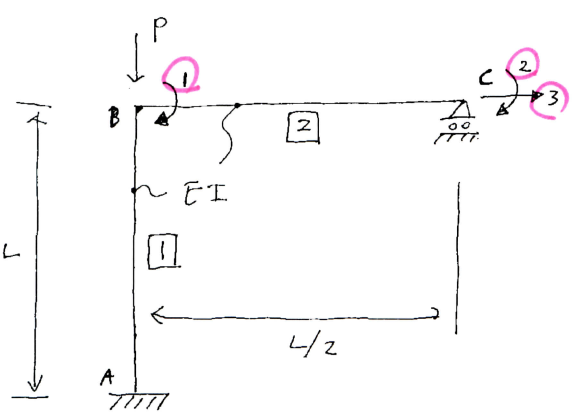
↓

linear analysis + book-keeping

↑ Energy balance at mechanism

$(W_{E,p} = W_{I,p})$; upper bound solution (최소 P선택)

HW #



↑「in stead of hinge support」

matrix stiffness method에 의해 P_{cr}을 산정할 것 (재축방향의 변형은 없는 것으로 가정, 총자유도수는 3개)

Note: $P_{cr} > 1.83 \cdot \frac{EI}{L^2}$

교과서 4.4.2절의 예제. (p.261)

4.7 Merchant – Rankine Interaction Equation

↳ P_f 산정의 어려움은 이미 언급한 바가 있음

(Introduction section의 P-δ 거동 논의 상기)

↳ In reality, the effects of instability and plasticity interact with each other.

geometric nonlinear

material nonlinear

- One simple/approximate interaction equation (linear)
  (by Horne-Merchant 1965) ← 지금은 강력한 numerical tool이 존재.

"Postulation"

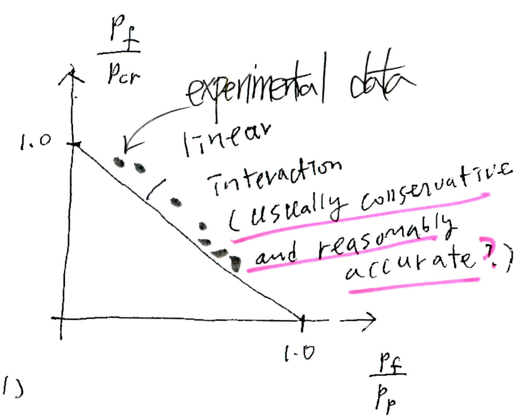
$$\frac{P_f}{P_{cr}} + \frac{P_f}{P_p} = 1$$

elastic critical load

plastic collapse load

or

$$P_f = \frac{1}{\left(\frac{1}{P_{cr}} + \frac{1}{P_p}\right)} = \frac{P_{cr} \times P_p}{P_{cr} + P_p} \quad \text{--- (4.7.1)}$$



「 It appears that the authors introduce the Merchant – Rankine equation (4.7.1) without fully clarifying the conditions under which the failure load P_f obtained from Eq. (4.7.1) is usually conservative and reasonably accurate. 」