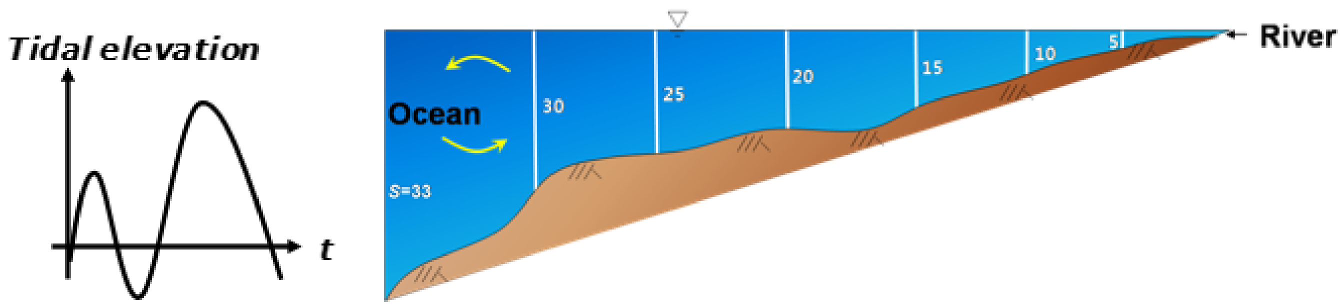
Chapter 8

Mixing in Estuaries



Chapter 8 Mixing in Estuaries

Contents

8.1 Introduction and Classification

○ Estuary: semienclosed coastal body of water which has a free connection with the open sea and within which sea water is measureably diluted with fresh water derived from land drainage (Pritchard, 1967)

- Mixing in estuaries: mixing in a flow driven by the slope of the tidal wave, by wind stresses and by internal density variations + flow oscillations

→ complex, unsteady, spatially varying flow

[Cf]

Mixing in rivers: mixing in a flow driven by the slope of the water surface

Mixing in reservoirs: mixing in a flow driven by wind stresses and by internal density variations

8.1 Introduction and Classification

○ Hydrodynamic categories (Bowden, 1967)

1) Sharply stratified estuary: salt-wedge estuary

2) Partially stratified estuary: significant vertical density gradient

3) Well mixed estuary ← mixed by strong tidal action

○ Geomorphological categories

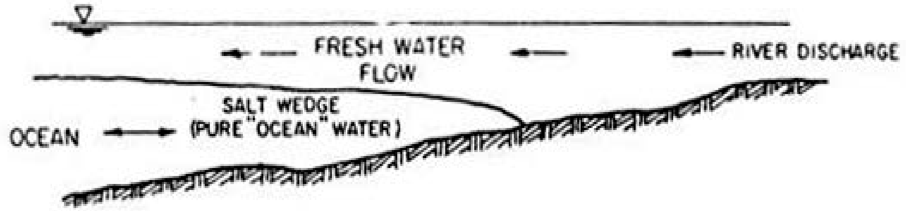
1) Coastal-plain estuary: formed by gradual drowning of a river system

2) Fjord type estuary: formed by glacial action

3) Bar-built estuary: formed by the closing off of an embayment by a sand bar

4) Rest

8.1 Introduction and Classification



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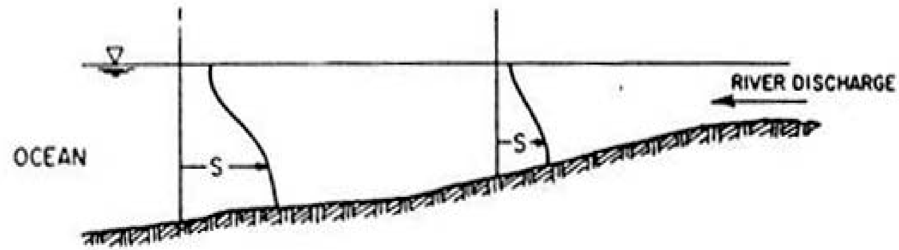

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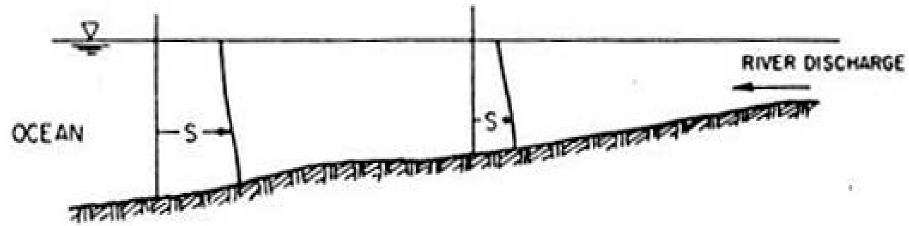


(c)

8.1 Introduction and Classification

○ Analytical classification (Hansen& Rattray, 1966)

- method based on the vertical variation of salinity and strength of the internal density-driven circulation

○ Geometrical classification

- method based on the ratios of length L, width W, and mean depth D

8.2 The Causes of Mixing in Estuaries

○ Mixing in estuary

- combination of small-scale turbulent diffusion and a larger scale variation of advective mean velocities

[Cf] Mixing in rivers

•advective mean velocities: a set of steady stream lines

•small-scale turbulent diffusion: mass transfer between stream lines

•dispersion: caused by flow along different stream lines with different speeds

8.2 The Causes of Mixing in Estuaries

1) Turbulent fluctuations: fluctuations with a period of less than a few minutes

2) Diffusive transport: transport whose scale is larger than a few minutes

3) Advective transport: varying in time, space, and direction

- semidiurnal and diurnal tidal variations

- wind-induced variations of almost any period

- inertial frequency caused by earth's rotation

- fluctuations of longer periods caused by monthly and longer term variation of tidal cycle and by seasonal variations of meteorological influences and tributary inflows

- Flow is going in different directions at different depths

8.2 The Causes of Mixing in Estuaries

○ Separate mechanisms

① mixing caused by wind

② mixing caused by tide

③ mixing caused by river

8.2.1 Mixing Caused by the Wind

- Wind: - dominant source of energy in large lakes, the open ocean and coastal areas

  - may or may not play a major role in estuary

8.2 The Causes of Mixing in Estuaries

- Breaking waves caused by wind have little to do with large scale dispersion

1) long, narrow estuary: - flow may be predominantly tidal

- wind has little chance to generate much current

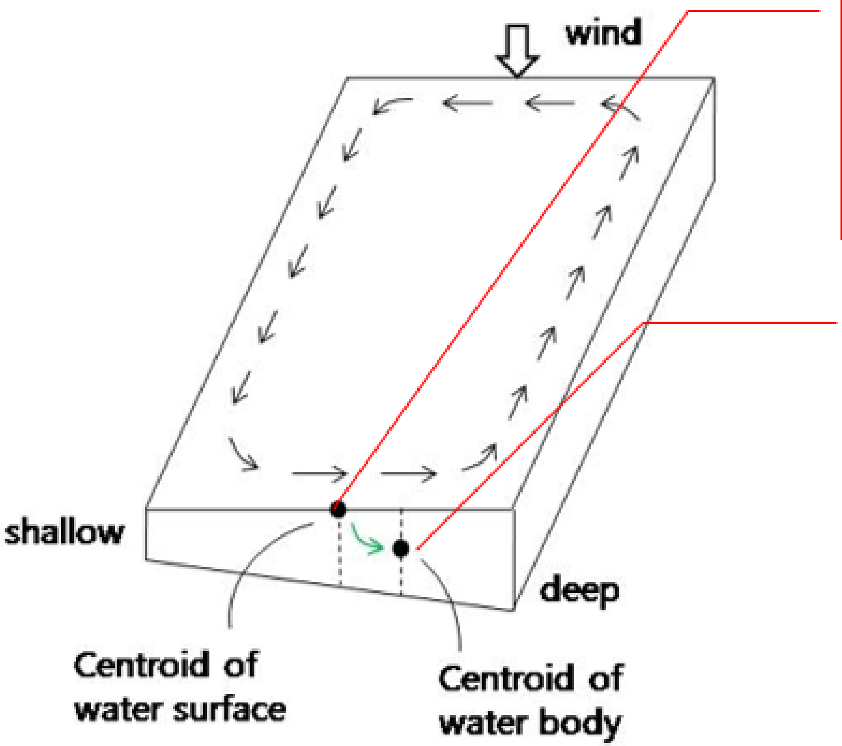
2) wide estuary: - wind stresses can generate currents of considerable importance

- Effect of wind

① drag on the water surface: dispersion of oil spill is directly affected by the local wind

② wind induced currents: dispersion of dissolved substances

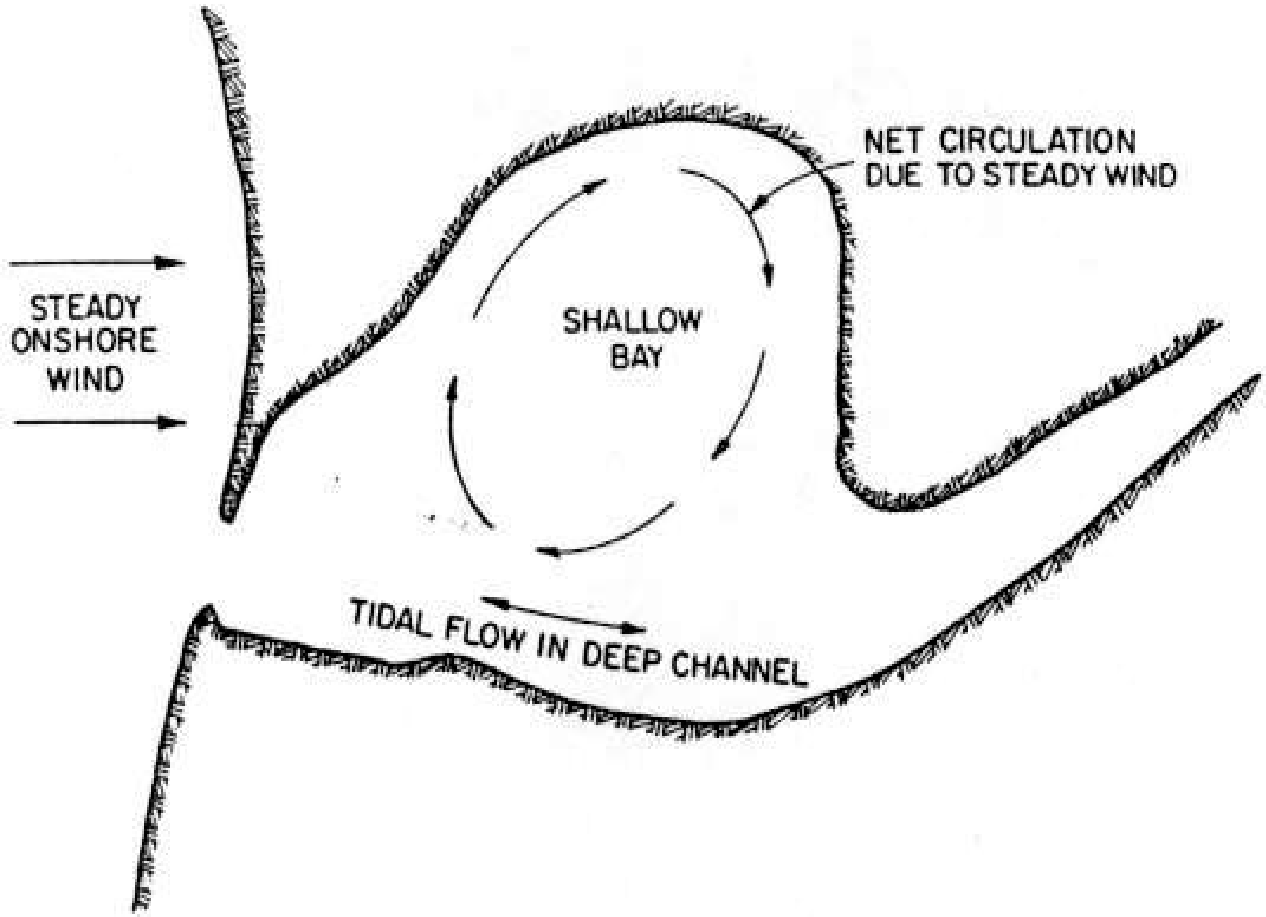
8.2 The Causes of Mixing in Estuaries



Line of action of the wind-induced force is through the centroid of the water surface.

Hence the line of action of the wind-induced force passes on the shallow side of the center of mass of the water, and a torque is induced causing the water mass to rotate.

8.2 The Causes of Mixing in Estuaries



8.2 The Causes of Mixing in Estuaries

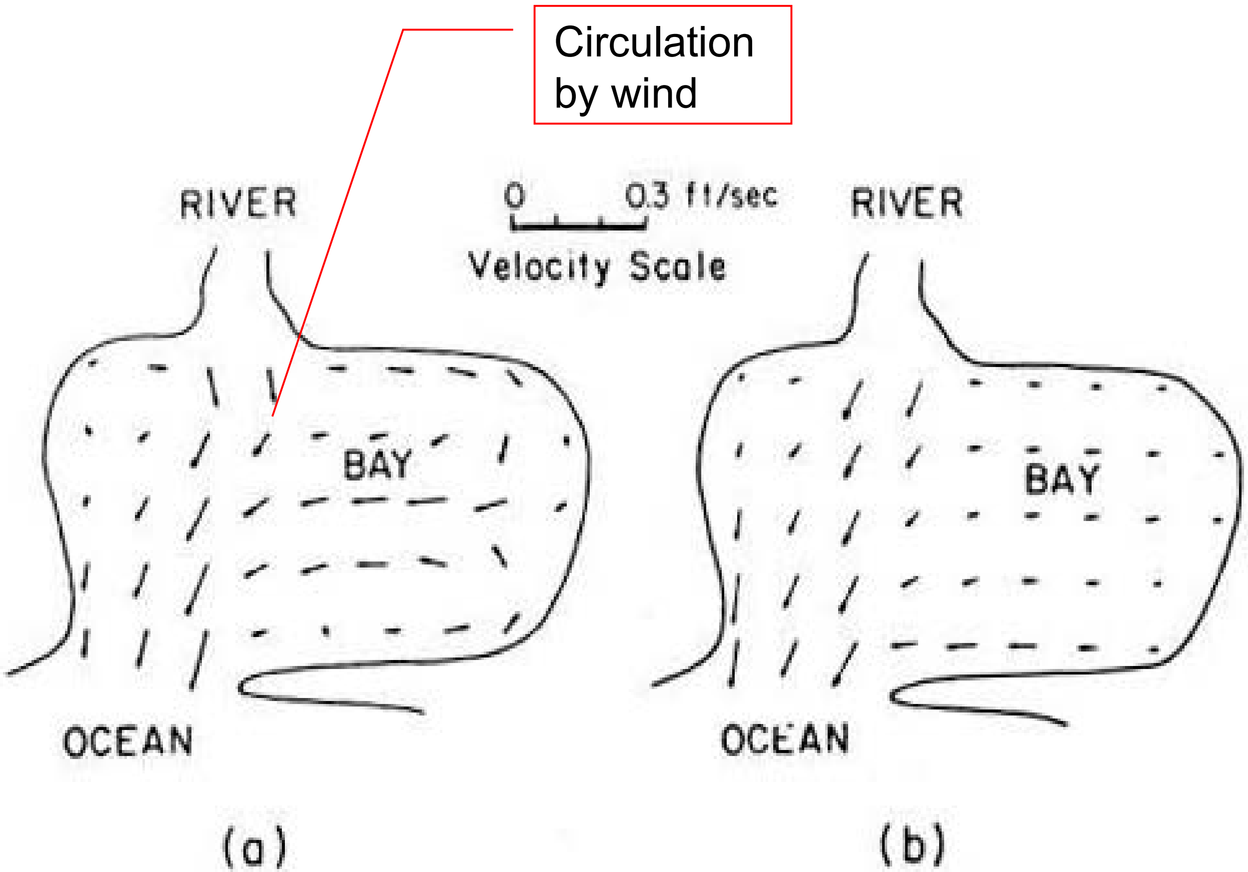
- Rotational current caused by a uniform wind blowing over a basin of variable depth

- Prediction of wind-induced current

- Depth-integrated 2D equations of motion

- Numerical solution (Leendertse, 1967): 2HT model (2D depth-averaged tidally varying model)

8.2 The Causes of Mixing in Estuaries



8.2 The Causes of Mixing in Estuaries

8.2.2 Mixing Caused by the Tide

• Tide generates mixing in two ways:

① turbulent mixing due to turbulence generated by friction of tidal flow running over channel bottom → Section 8.3

② large scale currents generated by the interaction of the tidal wave with the bathymetry

→ Shear flow dispersion → Ch. 4

Tidal pumping

Tidal trapping

8.2 The Causes of Mixing in Estuaries

8.2.2.1 Shear Effects in Estuaries and Tidal Rivers

Flow oscillation - Flow goes back and forth.

• Effect of oscillation on the longitudinal dispersion coeff. → Section 4.3

$$K = K_0 f(T') \qquad (8.1a)$$

where $f(T')$ is plotted in Fig. 8.6.

$T' = T / T_c$ = dimensionless time scale for cross-sectional mixing

$T =$ tidal period ~12 hrs

$T_C =$ cross-sectional mixing time = W^2 / ε_t

$K_0 =$ dispersion coefficient if $T \gg Tc$

8.2 The Causes of Mixing in Estuaries

- For wide and shallow cross section with no density effects

$$K_0 = I\overline{u'^2}T_C \qquad (8.1b)$$

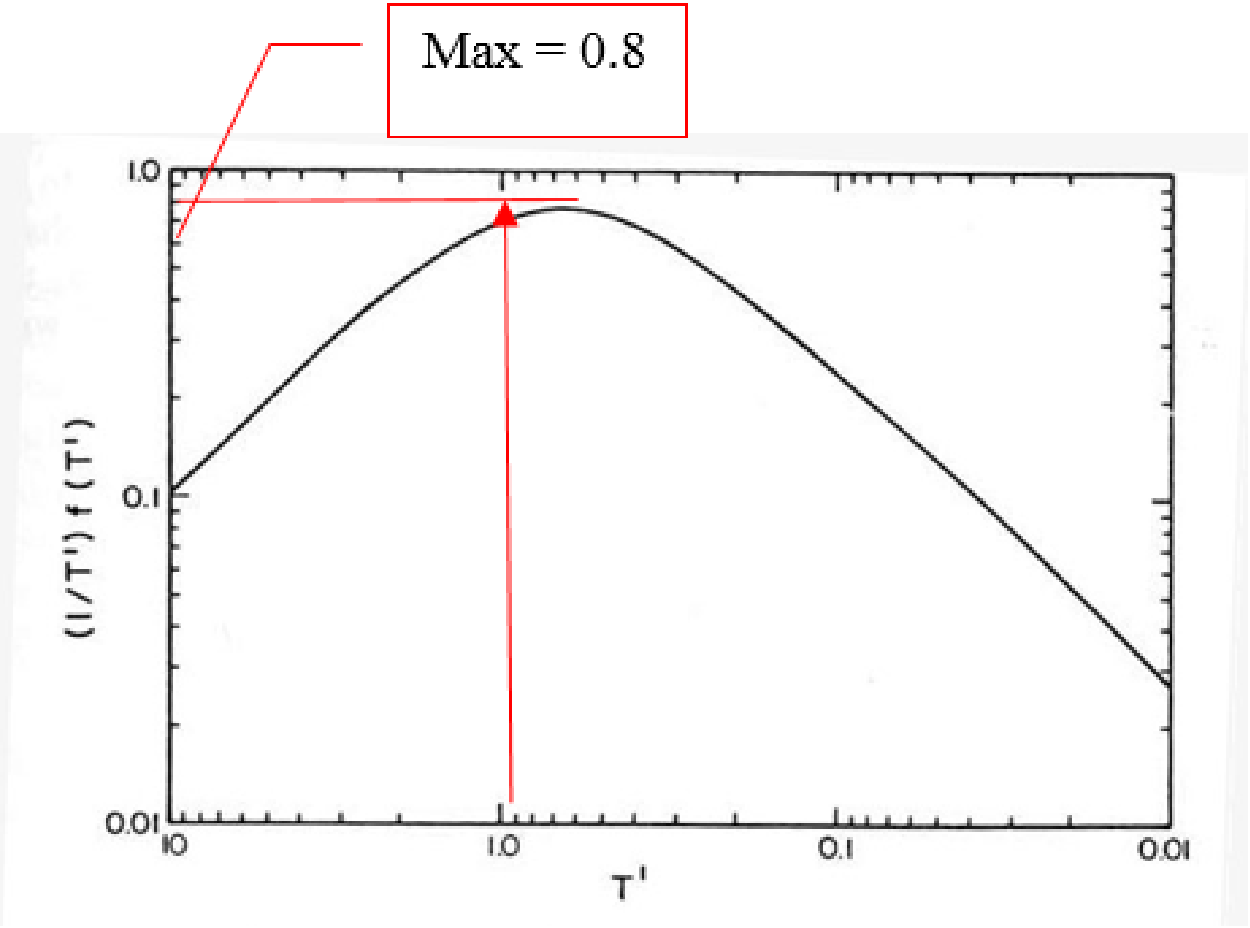
where I = dimensionless triple integral ≈ 0.1 (Table 4.1)

Combine Eq. (8.1a) and Eq. (8.1b)

$$\boxed{K = 0.1\overline{u'^2}T\left[\left(1/T'\right)f\left(T'\right)\right]} \qquad (8.2)$$

where the function $\left[\left(1/T'\right)f\left(T'\right)\right]$ is plotted in Fig. 8.5

8.2 The Causes of Mixing in Estuaries



8.2 The Causes of Mixing in Estuaries

In general T is fixed $\approx$ 12.5 hrs

i) T_C is small (narrow estuary) $T_C = \dfrac{W^2}{\varepsilon_t}$

$$T' = \frac{T}{T_C} >> 1$$

$\therefore K$ is small

ii) T_C is very large (very wide estuary)

$$T' = \frac{T}{T_C} << 1$$

$\therefore K$ is smallest

8.2 The Causes of Mixing in Estuaries

iii) $T' = \dfrac{T_C}{T} \approx 1 \quad : \left[\left(1/T' \right) f\left(T' \right) \right] \approx 0.8$

$$\therefore \; K = 0.08\overline{u'^2}T$$

Ch. 5

[Ex] $T = 12.5 \text{ hrs}, \quad \bar{u} = 0.3 \text{ m/s}, \quad \overline{u'^2} = 0.2\bar{u}^2$

Maximum $K = 0.08 \times 0.2(0.3)^2 \times (12.5 \times 3600)$

$\approx 60 \ \text{m}^2/\text{s}$

8.2 The Causes of Mixing in Estuaries

- Limitations of Eq. (8.2)

1) The channel must be relatively uniform over a long reach.

2) The channel must be much wider than it is deep.

3) The water must be of uniform density.

4) The shear flow appears to be the dominant mechanism for dispersion.

← neglect all other mechanism which can increase the dispersion coefficient

5) It gives a first estimate of the dispersion coefficient in constant-density portions of an estuary.

8.2 The Causes of Mixing in Estuaries

→ Eq. (8.2) predicts much smaller dispersion coefficients in tidal flows than in similar steady river flows.

- Potomac River (tidal reach): $K = 6 \sim 20\mathrm{m}^2/\mathrm{s}$
- Missouri River (steady flow reach): $K = 1{,}500\mathrm{m}^2/\mathrm{s}$

[Cf] Revisit of Eq. (4.55) - linear velocity profile in vertical direction

i) $T = 0.01T_C$ $\quad K = 0.0002(\frac{U^2h^2}{240D}) = 0.0002K_0$

ii) $T = 1.0T_C$ $\quad K = 0.8K_0$

iii) $T = 10T_C$ $\quad K = K_0$

8.2 The Causes of Mixing in Estuaries

[Example 8.1] Dispersion coefficient in a tidal slough

L= 20 km ; W=100 m ; h = 3 m; T = 12.5 hr

tidal range = 1 m; saline water all the way in the slough ← no fresh water inflow

(Sol) Since tidal range =1/3 of water depth,

tidal excursion length $\approx \frac{1}{3} \times 20\text{ km} = 7\text{ km}$

mean tidal velocity $\bar{u} = 7\text{ km} / 6.25\text{ hr} = 0.31\text{ m/s}$

Assume $u^* = 0.1\bar{u} = 0.031\text{ m/s};\ \varepsilon_t = 0.6hu^* = 0.6(3)(0.031) = 0.059\text{ m}^2/\text{s}$

8.2 The Causes of Mixing in Estuaries

$$T_C = W^2 / \varepsilon_t = (100)^2 / 0.059 = 170{,}000\,\mathrm{sec} = 47.22 hrs$$

$$T' = T / T_C = 0.27; \quad \text{From Fig.8.5,} \quad \left[(1/T')\, f\,(T')\right] \approx 0.5$$

$$K = 0.1\overline{u'^2}T(0.5) = 0.1(0.2\overline{u}^2)(12.5 \times 3{,}600)(0.5) = 44\ \mathrm{m}^2/\mathrm{s}$$

8.2.2.2 Tidal Pumping

• Residual circulation:

- net, steady circulation

- resulting velocity field obtained by averaging the velocity over the tidal cycle

- no tidal cycle is identical

- cause of the residual circulation:

8.2 The Causes of Mixing in Estuaries

the earth's rotation

interaction of the tidal flow with the irregular bathymetry

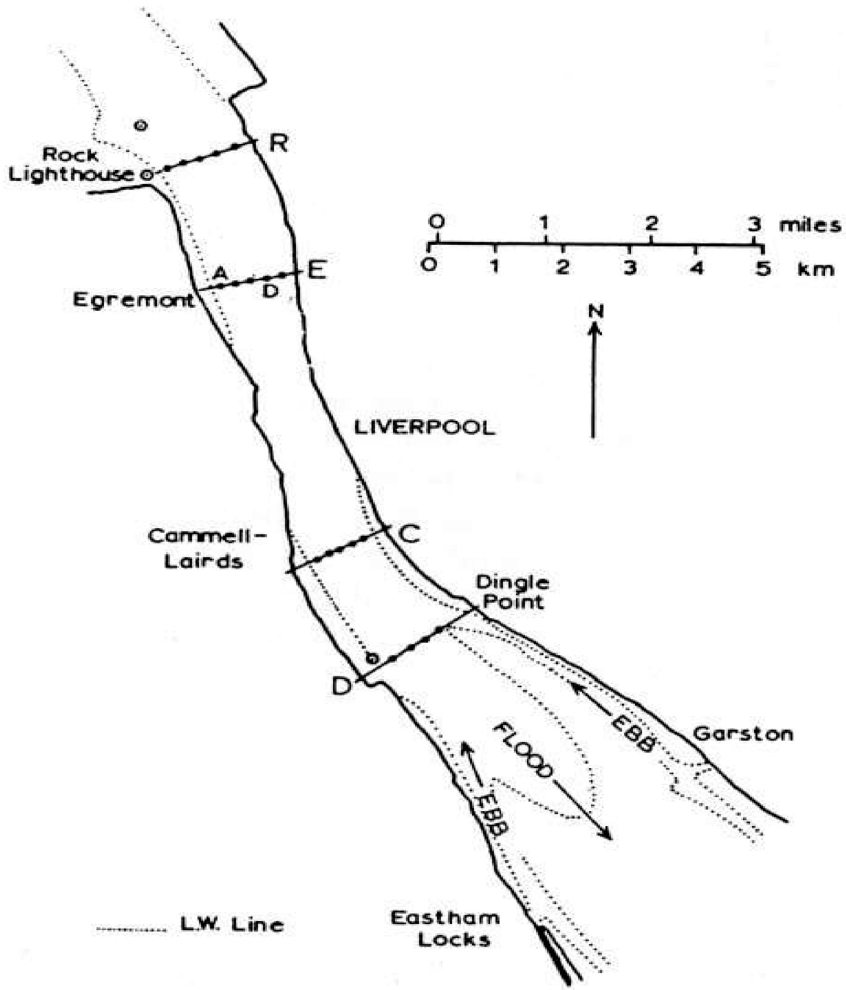
- circulation driven by the tide → additional to circulations driven by the wind and the river → tidal pumping
- important part of the flow distribution that increases longitudinal dispersion
- usually calculated using the 2D numerical model

[Ex 1] Mersey Estuary (Bowden and Gilligan, 1971)

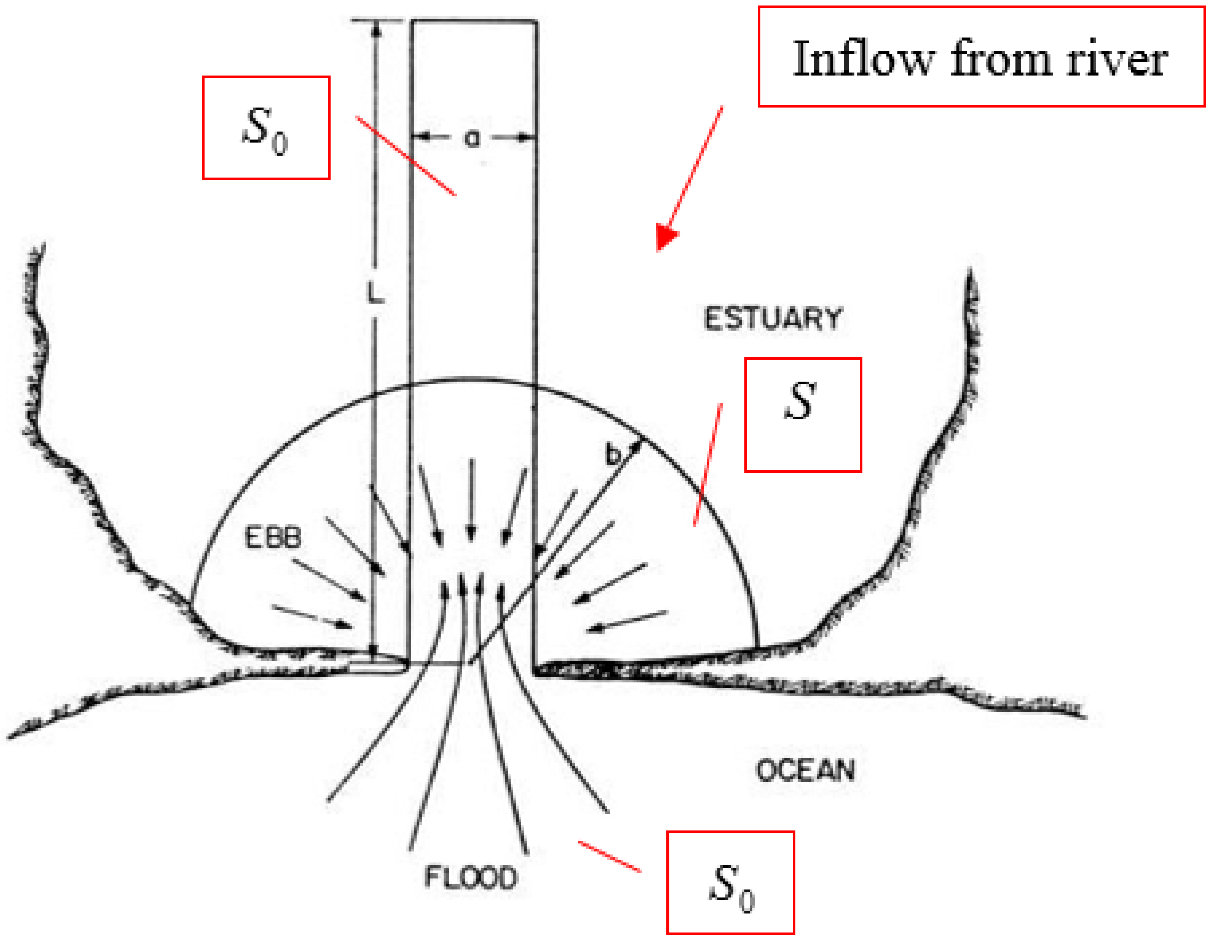
Flood channel: flood current is stronger than ebb current

Ebb channel: ebb current is stronger than flood current

8.2 The Causes of Mixing in Estuaries



8.2 The Causes of Mixing in Estuaries



8.2 The Causes of Mixing in Estuaries

i) flood (inward) flow: confined jet-like flow having width equal to width of the mouth, a

ii) ebb (outward) flow: sink flow coming uniformly from a semicircle of radius b

Assume volume of ebb flow equal to the volume of flood flow plus fresh water inflows from river

$$V = \frac{1}{2}\pi b^2 d = aLd + Q_f T \qquad (8.3)$$

where Q_f = river inflow ; d = water depth; T = duration of tidal cycle

8.2 The Causes of Mixing in Estuaries

Consider salinity transport for steady state

$$aLdS_0 = \frac{1}{2}\pi b^2 dS + (S_0 - S)abd \tag{8.4}$$

where S = salinity of ebb flow ; S_0 = ocean salinity

Combine Eq. (8.3) and Eq. (8.4)

$$S/S_0 = 1 - Q_f T \Big/ \left(V - a\sqrt{2Vd/\pi}\right) \tag{8.5}$$

8.2 The Causes of Mixing in Estuaries

[Ex 3] Pumped circulation at South San Francisco Bay

- net flow around islands or in braided channels

- oscillatory tidal current flowing over an irregular bottom topography, such as a series of shoals, induces residual vortices

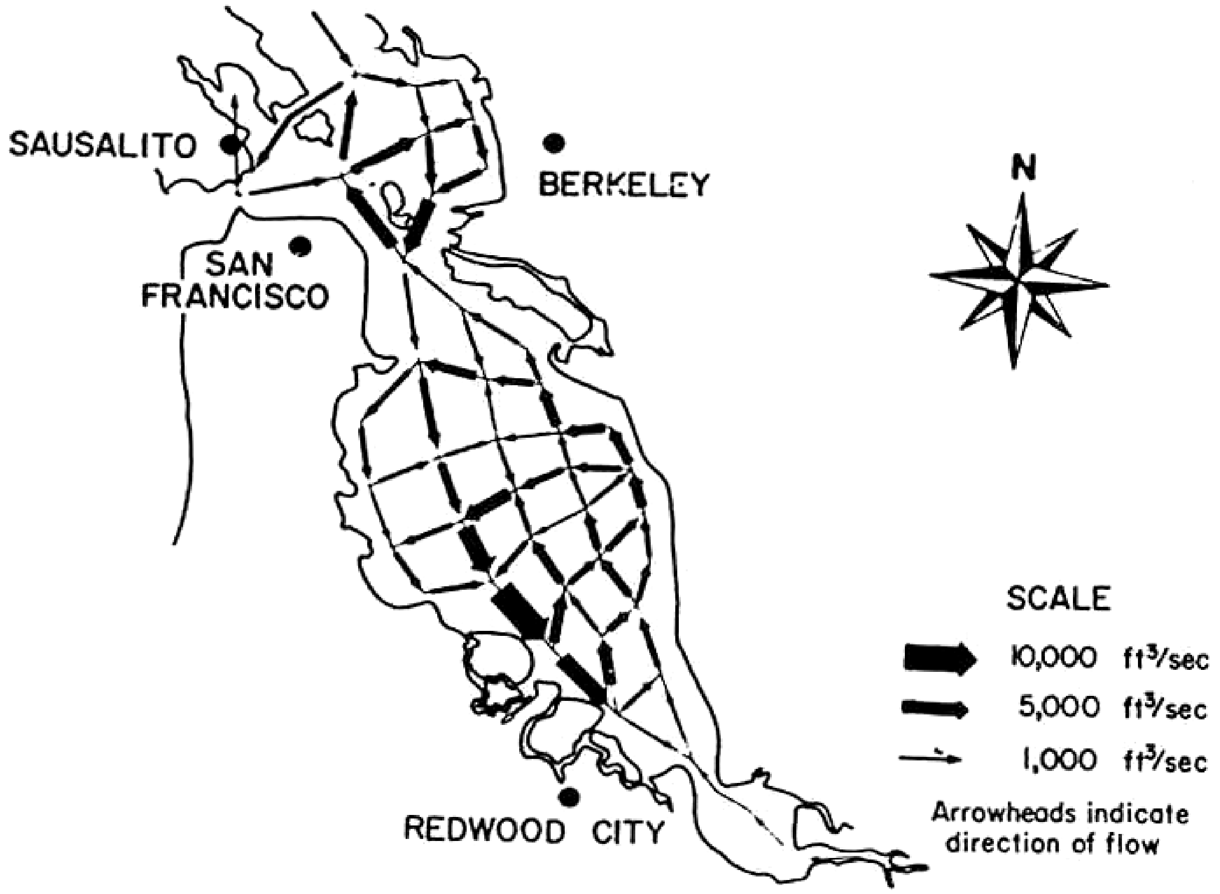
- counterclockwise gyre predicted by a 2D model

[Ex4] Dutch Wadden Sea (Zimmerman, 1978)

Longitudinal dispersion coefficient resulting from the residual vortices

~ 800 m^2/s

8.2 The Causes of Mixing in Estuaries



8.2 The Causes of Mixing in Estuaries

[Cf] effects of dead zones in rivers

8.2.2.3 Tidal Trapping

• Trapping of low velocity water along the sides of an estuary

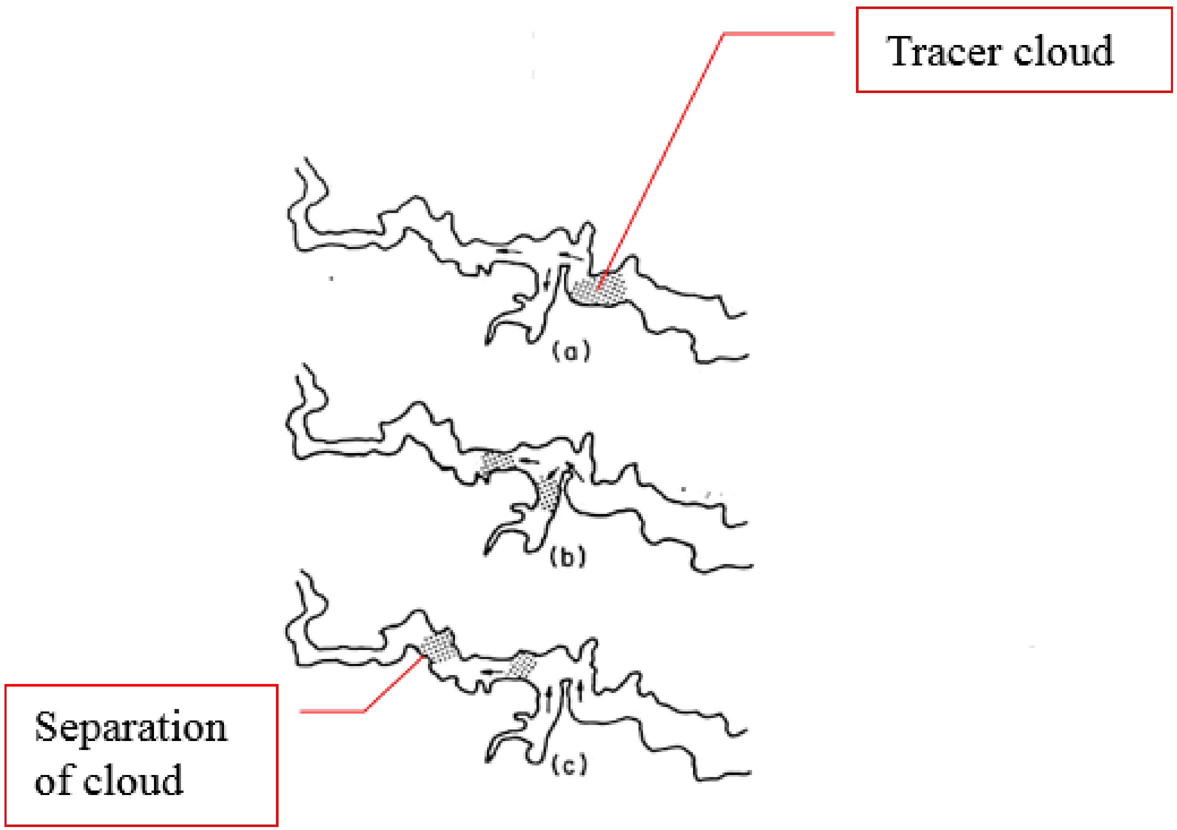
~ due to effects of side embayments and small branching channels

- The storage effects are enhanced by tidal action.

[Ex] Dutch estuaries analyzed by Schijf and Schonfeld (1953)

- They called “Storing basin” mechanism
- responsible for all diffusive salt flux

8.2 The Causes of Mixing in Estuaries



8.2 The Causes of Mixing in Estuaries

- Trapping mechanism

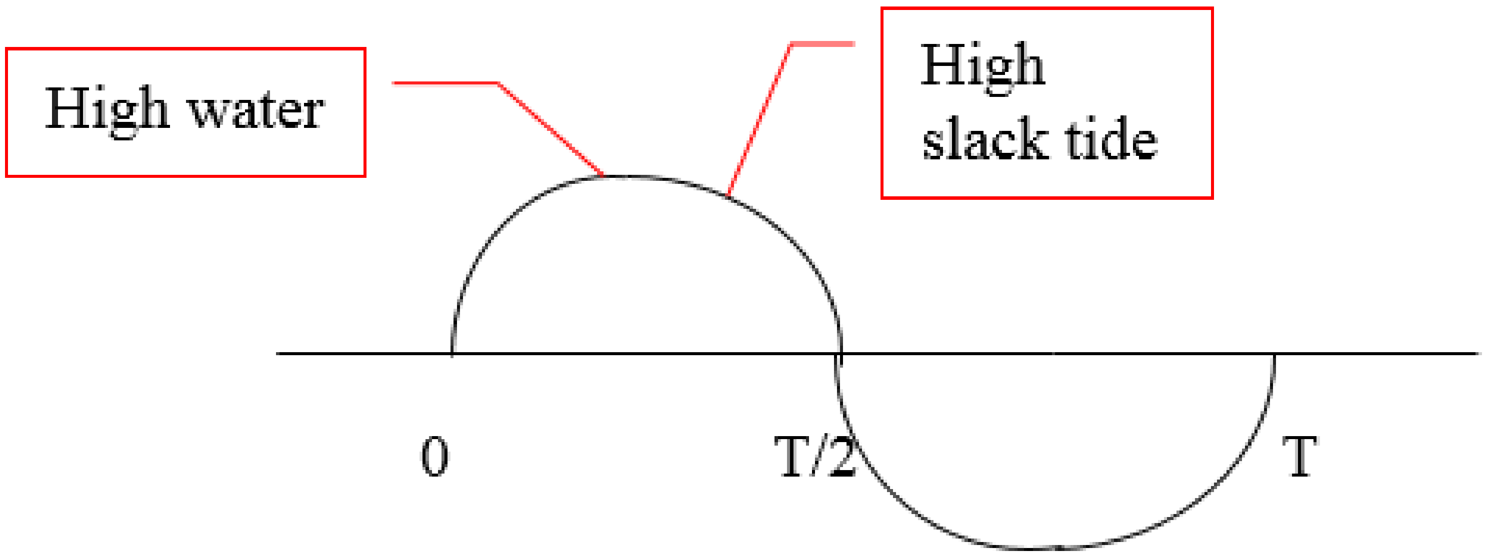
- The propagation of the tide in an estuary represents a balance between the inertia of the water mass, the pressure force due to the slope of the tidal waves, and the retarding force of bottom friction.

- In the main channel, tidal elevations and velocities are usually not in phase; high water occurs before high slack tide and low water before lower slack tide.

- This is because of the momentum of the flow in the main channel, which causes the current to continue to flow against an opposing pressure gradient.

- The side channel, in contrast, has less momentum and the current direction changes when the water level begins to drop.

8.2 The Causes of Mixing in Estuaries



8.2 The Causes of Mixing in Estuaries

- Longitudinal diffusivity by trapping mechanism (Okubo, 1973)

Assume uniform velocity in the main channel; $u = u_0 \cos \sigma t$

Assume uniform distribution of traps along the sides

r = ratio of trap volume to channel volume

$1/k$ = characteristic exchange time between trap and main flow

$$\boxed{K = \frac{K'}{1+r} + \frac{r u_0^{\,2}}{2k(1+r)^2(1+r+\frac{\sigma}{k})}} \qquad (8.6)$$

where K = effective longitudinal diffusivity; K' = longitudinal diffusivity in the main cahnnel

8.2 The Causes of Mixing in Estuaries

[Ex] Mersey estuary: $u_0 = 1.5\ \text{m/s}$; $\sigma = 1.4 \times 10^{-4}\ \text{sec}^{-1}$; $r = 0.1$; $k^{-1} = 10^4\ \text{sec}$

$$\boxed{\therefore\ K = 0.9K' + \underline{360\ \text{m}^2/\text{s}}}$$

[Cf] longitudinal dispersion coefficient

Shear flow dispersion: ~ 60 m^2/s

Tidal pumping: 800 m^2/s

Tidal trapping: > 360 m^2/s

8.2 The Causes of Mixing in Estuaries

8.2.3 Mixing Caused by the River

River delivers a discharge of fresh water Q_f.

Assume all the fresh water comes from a single upstream source.

i) Salt wedge estuary: Fig. 8.1 a)

If a river discharges into an estuary connected to a nearly <u>tideless sea</u>

→ the fresh water overrides the salt water

→ flows as a nearly undiluted layer into the sea

→ salt water intrudes underneath the fresh water layer in the form of a wedge

8.2 The Causes of Mixing in Estuaries

ii) Partially stratified estuary: Fig. 8.1 b)

If there is some tide

→ the wedge moves back and forth

→ the more the wedge motion the more kinetic energy is available to break down the interface and turbulently mix the fresh and saline layers

→ ① The river is a source of buoyancy,

$$\Delta\rho g Q_f$$

② The tide is a source of kinetic energy to overcome the buoyancy,

$$WU_t^3$$

where $\Delta\rho$ = the difference in density between the river and ocean water;

8.2 The Causes of Mixing in Estuaries

Q_f = discharge of fresh water; U_t = the rms tidal velocity;

W = the channel width

○ Estuarine Richardson Number, R

$$R = \frac{\left(\frac{\Delta\rho}{\rho}\right) g Q_f}{W U_t^3}$$

i) R large → the estuary is strongly stratified and flow is dominated by density currents

ii) R small → the estuary is well mixed, density effects are negligible.

iii) $0.08 < R < 0.8$ → transition from a well mixed to a strongly stratified estuary occurs

8.2 The Causes of Mixing in Estuaries

○ Isohalines for partially stratified estuary

- Isohaline = lines of constant salinity

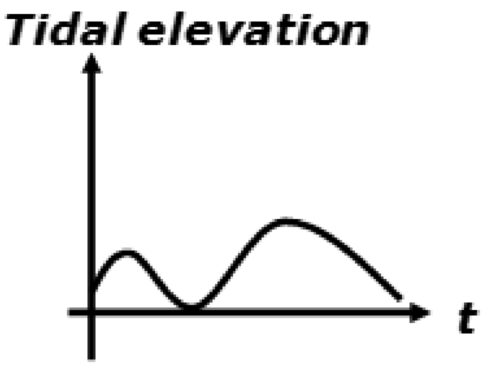
- tends to become horizontal because that is the condition of a stratified water body at rest

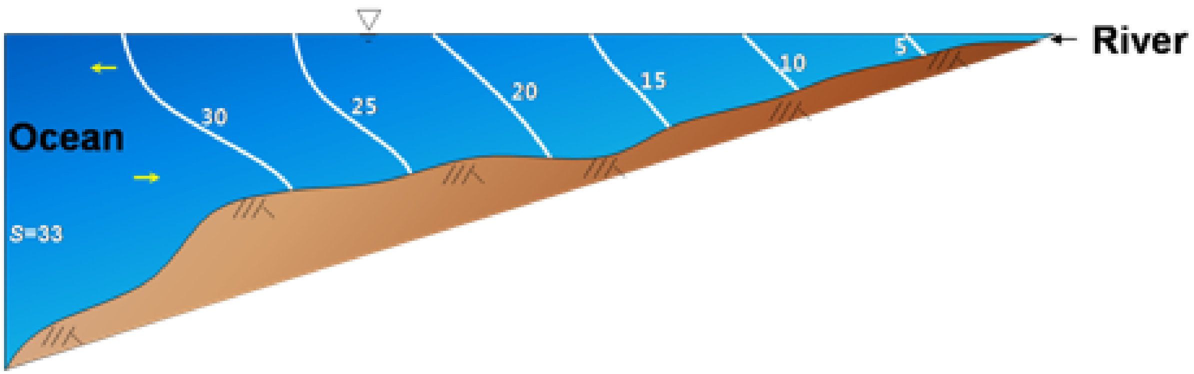
- the sloping isohalines imply pressure gradients which will drive a current tending to bring the isohalines to the horizontal.

→ The necessary currents are a flow landward along the bottom and seaward at the surface

→ Estuarine circulation (gravitational circulation)

8.2 The Causes of Mixing in Estuaries





8.2 The Causes of Mixing in Estuaries

baroclinic circulation: internal flows driven by density variations; river-driven flows

barotropic circulation: flows of constant density; tide-driven flows

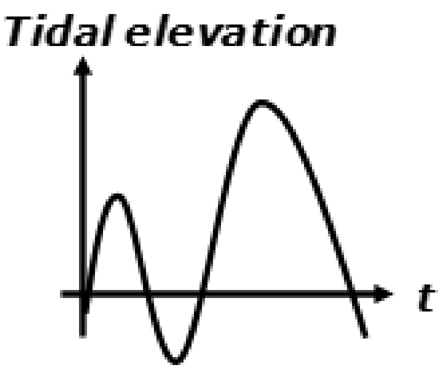
• Well mixed estuary: Fig. 8.1 c)

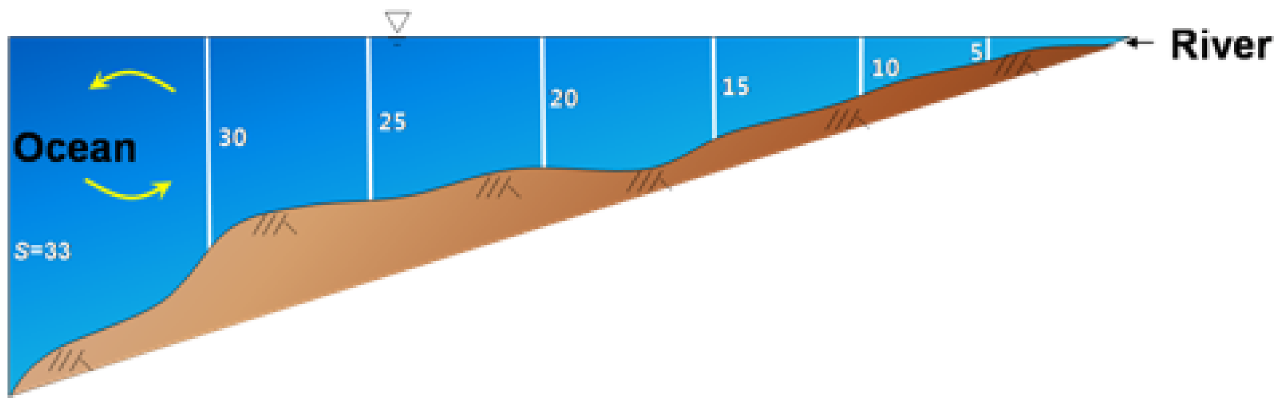
~ isohalines are vertical

→ still has a horizontal density gradient

→ internal baroclinic circulation is driven by a longitudinal density gradient

8.2 The Causes of Mixing in Estuaries





8.2 The Causes of Mixing in Estuaries

◎ Analysis of density-driven circulation

- Hansen and Rattray (1965, 1966)
- circulation in a vertical two-dimensional plane assuming no variation across the channel
- Assume steady flow
- tidal effect was to induce vertical and longitudinal turbulent mixing

→ Fig. 8.15:

vertical distribution of velocity and salinity ~ f (river flow, depth, width of channel)

8.2 The Causes of Mixing in Estuaries

- densimetric Froude number

$$F_m = \frac{Q_f}{A\left[\left(\Delta\rho/\rho\right)gd\right]^{\frac{1}{2}}} \tag{8.7}$$

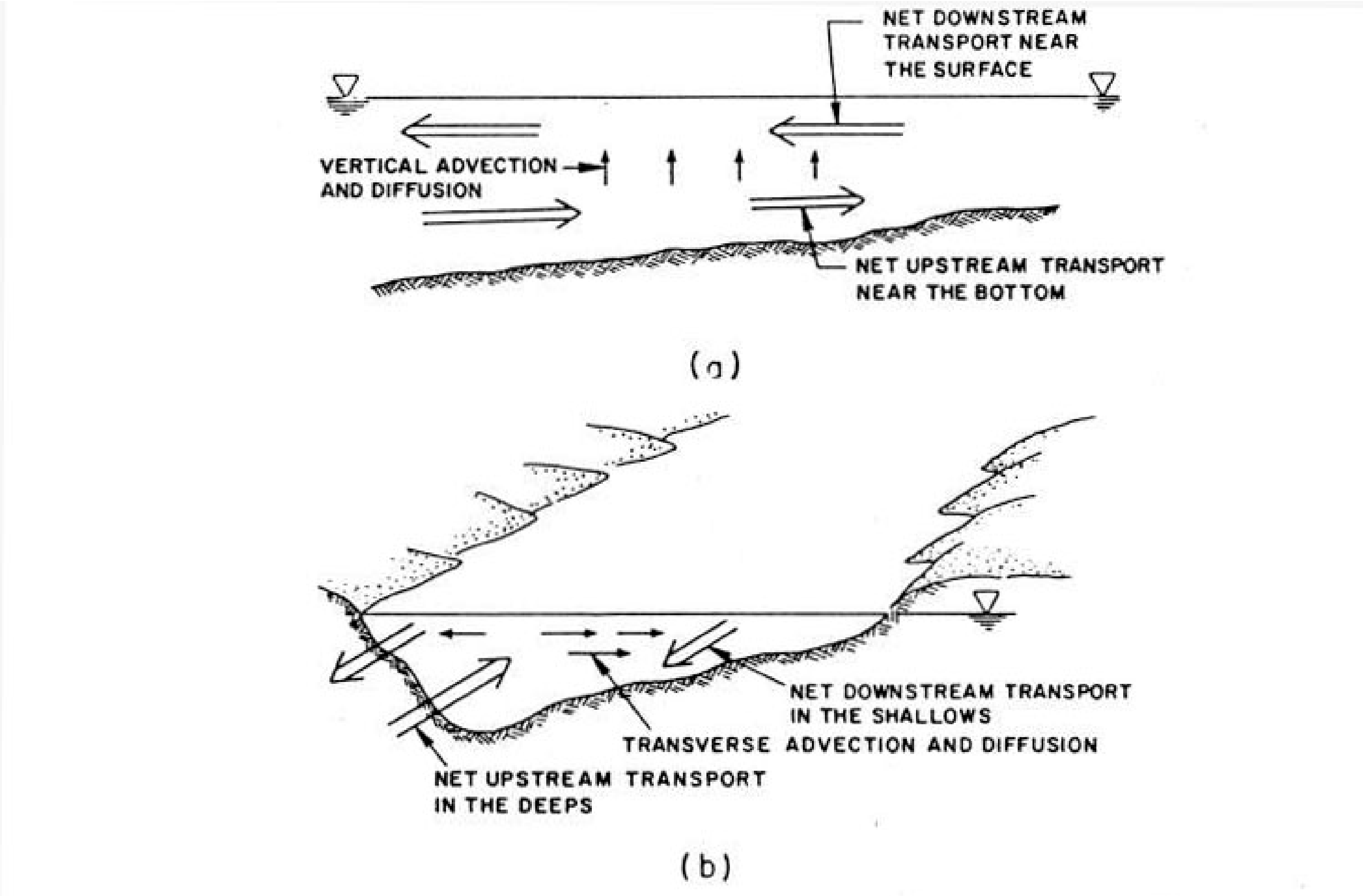
where A = cross-sectional area

$\frac{\delta S}{\bar{S}} =$ salinity difference between surface and bottom divided by the mean salinity

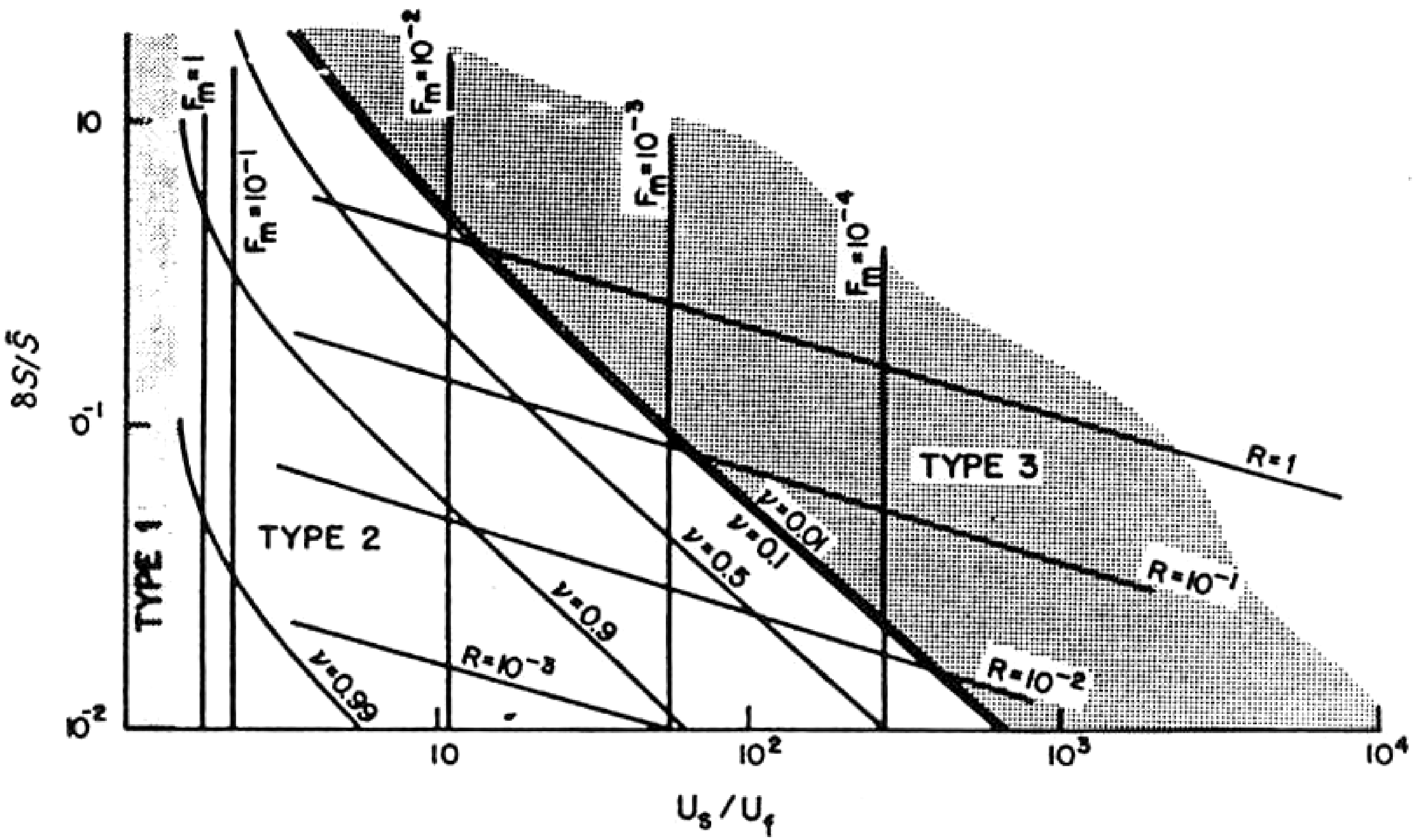
$U_s =$ residual velocity at the surface; $U_f = Q_f / A$

$\nu =$ fraction of landward transport of salinity caused by all dispersion mechanisms other than the density-driven circulation (Fischer, 1972)

8.2 The Causes of Mixing in Estuaries



8.2 The Causes of Mixing in Estuaries



8.2 The Causes of Mixing in Estuaries

[Ex] San Francisco Bay

$$Q_f = 100 \text{ m}^3/\text{s}, \quad d = 8 \text{ m}, \quad W = 3{,}125 \text{ m}, \quad \frac{\Delta\rho}{\rho} = 0.025,$$

$$U_f = 0.75 \text{ m/s}$$

$$\rightarrow R = 0.019, \;\; F_m = 0.0029$$

From Fig. 8.15;

$$\delta S / \overline{S} = 0.08$$

→ salinity difference = 2 ppt; mean salinity = 25 ppt

$$\nu = 0.7$$

8.2 The Causes of Mixing in Estuaries

→ 70% of the salt balance in this bay is maintained by mechanisms other than density driven circulations

→ Experimental results by Fischer & Dudley (1975), Fig. 8.21, give similar salinity intrusion

◎ Transverse gravitational circulation: Fischer (1972)

- Limitations of Hansen and Rattray's analysis because of assumption of two dimensionality

- The actual baroclinic circulation is complicated by the variation of depth across the channel.

8.2 The Causes of Mixing in Estuaries

- The channel geometry turns a vertically 2-D circulation cell into a horizontal circulation cell as shown in Fig. 8. 14b).
- The horizontal velocity gradients generally lead to much larger dispersion coefficients than do vertical ones, because in real channels the widths are so much greater than the depths.

○ Longitudinal dispersion coefficient for triangular channel of depth d and width W

$$\boxed{K = 1.9 \times 10^{-5} \left(\frac{g}{\rho} \frac{\partial \rho}{\partial x} \right)^2 \frac{d^6 W^2}{E_0^{\,2} \varepsilon_t}} \qquad (8.8)$$

where $E_0 =$ vertical mixing coefficient for momentum

8.2 The Causes of Mixing in Estuaries

[Ex] Mersey estuary:

$$K = 360\ m^2/s$$

◎ Laboratory studies of baroclinic circulation

① Waterways Experiment station in Vicksburg, MI

- conducted by Ippen and Harleman (1961)

② Delft Hydraulics Lab, The Netherlands

- Rigter (1973)

③ UC Berkeley

- By Abraham et al. (1975)

width of flume = 11 ft

length of flume = 600 ft

8.2 The Causes of Mixing in Estuaries

○ Relation of observed intrusion of salinity to estuarine Richardson number

• Modified estuarine Richardson number

$$R' = \frac{\Delta\rho}{\rho}\frac{gQ_f}{Wu^{*3}} \qquad (8.9)$$

where u^* = shear velocity

• Ordinate: K / du^*

→ shear velocity is used to include the effect of varying bottom friction

$$L_i U_f \sim K$$

8.2 The Causes of Mixing in Estuaries

where L_i = length of salinity intrusion

→ most results are from narrow rectangular flumes, where tide- and wind-driven mixing mechanisms are absent. → minimum length

8.2 The Causes of Mixing in Estuaries

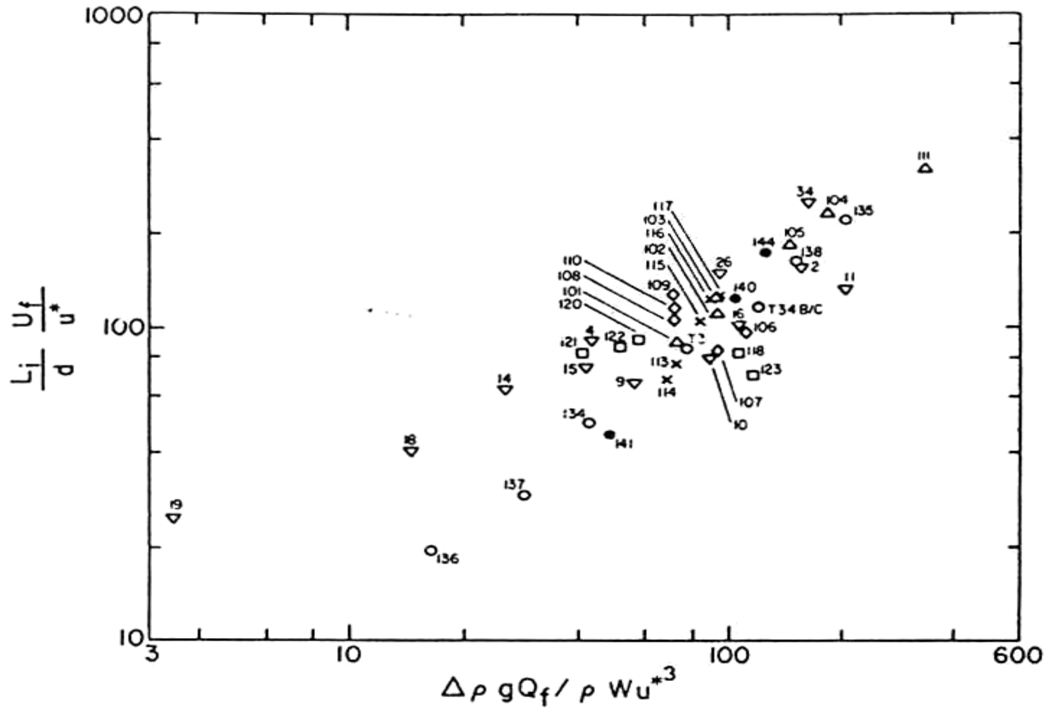


Figure 7.13 A correlation of observed salinity intrusions in laboratory flume experiments. Numbers beside points are run numbers given in the references. ▽ Ippen and Harleman (1961); other data from Rigter (1973) using one base condition and varying one parameter at a time identified as follows: ○ tidal amplitude, × channel roughness, △ channel length, □ fresh water discharge, ◇ mean depth, ● ocean salinity.

8.2 The Causes of Mixing in Estuaries

8.2.4 Synthesis and Summary

○ Three main causes of mixing

- wind
- tide
- river

- Motions resulting from all three causes are superposed.

- The main cause may change from season to season, or even from week to week.

- Many estuaries change from partially stratified or salt wedge in the wet season to well mixed in the dry season.

8.2 The Causes of Mixing in Estuaries

- A flood of a week or so may stratify a well-mixed estuary.

- A passing hurricane changes a stratified estuary into a well-mixed one.

→ The simple steady-state analysis is not adequate to explain these seasonal and catastrophic events.

○ Practical engineering studies of estuaries

① Physical modeling

② Numerical modeling

→ neglect certain mechanisms

③ Analytical model (1-D approach)

→ lump all of mixing mechanisms into a single longitudinal dispersion coefficient

▪ Longitudinal dispersion coefficient

- Based on an analysis of one mechanism at the neglect of others

8.2 The Causes of Mixing in Estuaries

Mixing mechanism		K	Range (m^2/s)
Mixing Caused by the Tide	Shear flow dispersion	$K = 0.1\overline{u'^2}T\left[(1/T')f(T')\right]$	< 60 (Maximum)
	Tidal pumping		800 (Dutch Wadden Sea)
	Tidal trapping	$K = \frac{K'}{1+r} + \frac{r u_0^2}{2k(1+r)^2(1+r+\frac{\sigma}{k})}$	>360 (Mersey estuary)
Mixing Caused by the River		$K = 1.9\times10^{-5}\left(\frac{g}{\rho}\frac{\partial\rho}{\partial x}\right)^2\frac{d^6W^2}{E_0^2\varepsilon_t}$	36 (Mersey estuary)

8.3 Cross-sectional Mixing in Estuaries

8.3.1 Vertical Mixing

i) Constant-density tidal flow:

- Vertical mixing is caused by turbulence generated by bottom shear stresses.

$$\varepsilon_v = 0.067 d u^*$$

- u^* varies from nearly zero at slack tide to a maximum at the time of highest velocity.

→ Use the average value of u^*

8.3 Cross-sectional Mixing in Estuaries

○ Bowden (1967)

$$\varepsilon_v = 0.0025 d U_a \tag{8.10}$$

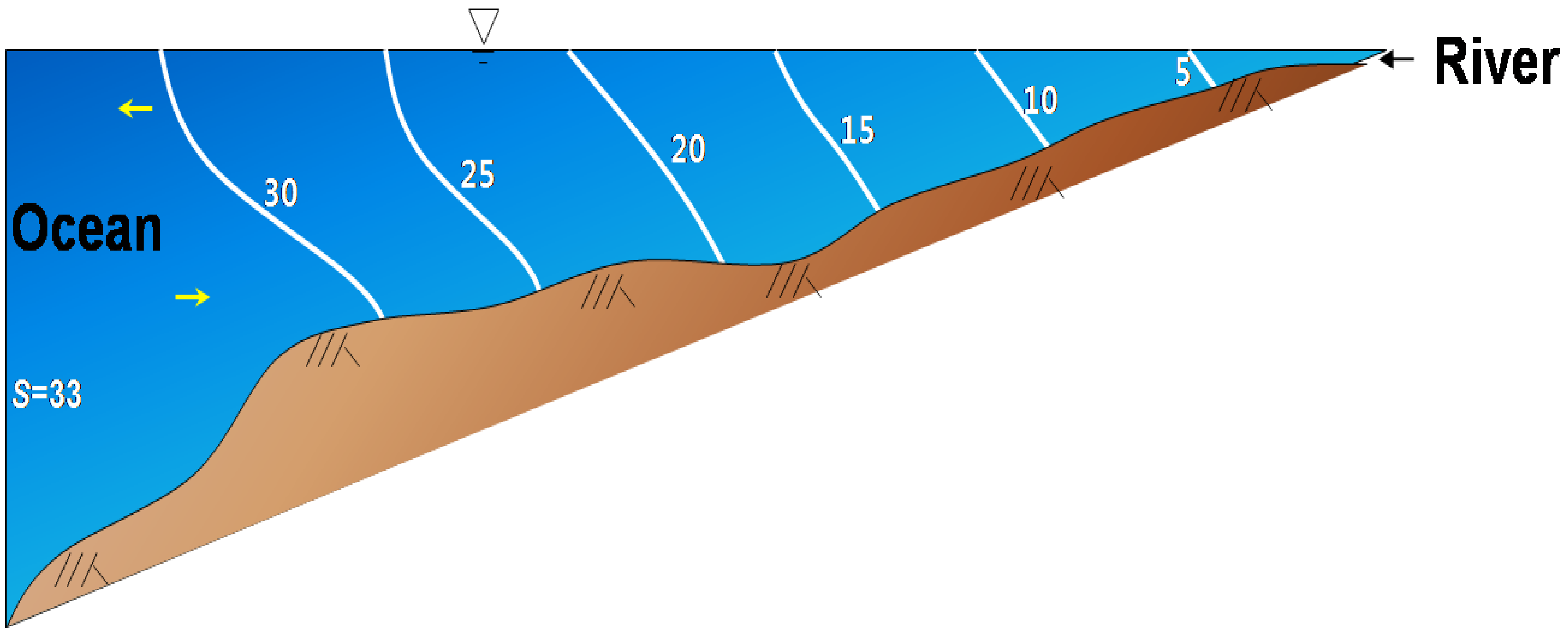
where $U_a =$ depth mean amplitude of the current

ii) Stratified flow

→ Turbulent mixing requires that some of the tidal energy be used to raise the potential energy of the water column.

→ The most of the energy for mixing is extracted from the bottom and internal shear.

8.3 Cross-sectional Mixing in Estuaries



8.3 Cross-sectional Mixing in Estuaries

○ Munk and Anderson (1948)

$$\varepsilon_v = \varepsilon_0 (1 + 3.33 R_i)^{\frac{-3}{2}} \tag{8.11}$$

where ε_0 = value of ε_v for neutral stability

R_i = gradient Richardson number $= g(\frac{\partial \rho}{\partial z}) / \rho (\frac{\partial u}{\partial z})^2$

[Cf] Richardson number = buoyancy/kinetic energy

○ Pritchard (1960)

$$\varepsilon_v = 8.59 \times 10^{-3}\ U_t \left[z^2 (d - z)^2 / d^3 \right] \left[1 + 0.276 R_i \right]^{-2} \tag{8.12}$$

where U_t = the rms tidal velocity

8.3 Cross-sectional Mixing in Estuaries

◎ Field data for ε_v

○ Mersey estuary (Bowden, 1963)

$$\varepsilon_v = \left[\begin{array}{ll} 5 \ \text{cm}^2/\text{s} & \text{at surface} \\ 71 \ \text{cm}^2/\text{s} & \text{at middepth} \end{array} \right.$$

→ These fit Munk & Anderson's formula, Eq. (8.11) reasonably well.

For Mersey estuary,

$$R_i = 0.1 \sim 1.0$$

$$\varepsilon_0 = 500 \ cm^2 / s$$ ← Eq. (8.10)

8.3 Cross-sectional Mixing in Estuaries

$$\frac{\varepsilon_v}{\varepsilon_0} = 0.064 \quad at \;\; R_i = 0.5$$

$$\rightarrow \frac{\varepsilon_v}{\varepsilon_0} = 1/100 \sim 1/10$$

○ Duwamish waterway near Seattle (Partch & Smith, 1978)

- saltwedge estuary

Higher rate of mixing during ebb was caused by an internal hydraulic jump.

$$\varepsilon_v = \left[\begin{array}{ll} 0.5 \text{ cm}^2/\text{s} & \text{during most of the cycle} \\ 5 \text{ cm}^2/\text{s} & \text{during the period of maximum ebb flow} \end{array} \right.$$

8.3 Cross-sectional Mixing in Estuaries

$\varepsilon_0 = 55\ cm^2/s$ ← Eq. (8.10)

$$\rightarrow \frac{\varepsilon_v}{\varepsilon_0} = 1/100 \sim 1/10$$

○ Blumberg (1975)

- numerical modeling
- suggest relationship between ε_v and R_i different in form from (8.11) and (8.12)

8.3 Cross-sectional Mixing in Estuaries

8.3.2 Transverse Mixing

i) For rivers, bottom-generated turbulence is a main cause of transverse mixing.

- For straight, rectangular section

$$\rightarrow \varepsilon_t = 0.15du^*$$

- For irregular section, much larger values are induced by sidwall irregularities and channel curvature.

$$\rightarrow \varepsilon_t = 0.6du^*$$

8.3 Cross-sectional Mixing in Estuaries

ii) For estuaries

- Mixing mechanisms are very complex.

- All mixing mechanisms are in part transverse mixing mechanisms:

→ Flow into and out of traps

Pumped circulation in bays

Wind-driven gyre (Fig. 8.3)

Transverse baroclinic circulation (Fig. 11b)

○ Transverse mixing coefficient

= small-scale turbulent fluctuations

+ transverse shear flow dispersion due to whatever transverse velocity profile that is caused by the superposition of all the mechanisms

8.3 Cross-sectional Mixing in Estuaries

◎ Measured values for ε_t

i) Well-mixed estuary (constant density reaches)

○ Ward (1974, 1976)

- San Francisco Bay: $\varepsilon_t = 1.0du^*$
- Cordova Bay, British Columbia: $\varepsilon_t = 0.42du^*$
- Gironde estuary, France: $\varepsilon_t = 1.03du^*$
- Fraser estuary, British Columbia: $\varepsilon_t = \begin{bmatrix} 0.44du^* - slack\,tide \\ 1.61du^* - ebb\,tide \end{bmatrix}$

○ Fischer (1974)

- Delaware estuary: $\varepsilon_t = 1.2du^*$

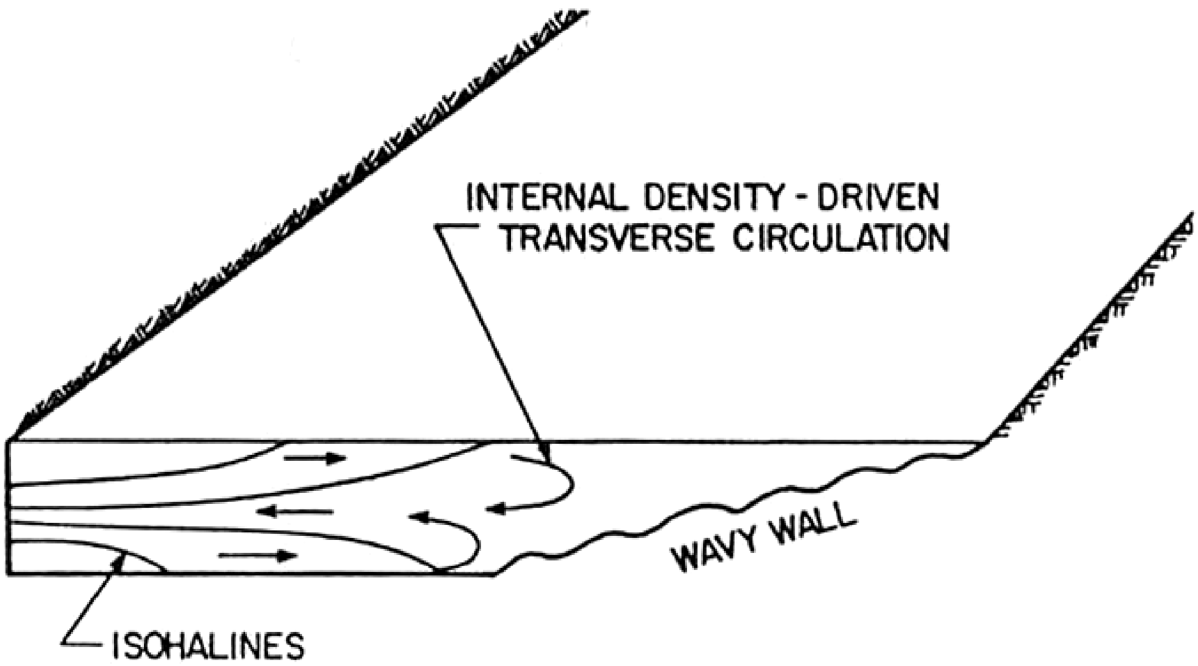
8.3 Cross-sectional Mixing in Estuaries

ii) Stratified-flow

○ Experiments by Sumer & Fischer (1977)

- Laboratory channel: $W = 3.5\ \mathrm{m}$, trapezoidal section
- 2 sets of experiments
  - uniform side
  - wavy side

8.3 Cross-sectional Mixing in Estuaries



8.3 Cross-sectional Mixing in Estuaries

→ The sidewall waviness generated vertical mixing on the wavy side, which in turn established transverse density gradients which drove a transverse baroclinic circulation as shown in Fig. 8.18.

→ The rate of transverse mixing was greatly enhanced in the case of the wavy wall.

→ Stratification in estuaries may greatly enhance the rate of transverse mixing by driving transverse circulations.

8.4 Longitudinal Dispersion and Salinity Intrusion

○ 1D longitudinal dispersion:

- longitudinal dispersion of pollutants along the channel axis
- intrusion of ocean-derived salinity up the channel axis by dispersive mechanisms

→ combine the result of all mechanisms into a single dispersion coefficient K

→ applied to relatively long, narrow estuary: Delaware, US; Thames, UK

○ Salt balance in an estuary in steady state

$$U_f S = K \frac{\partial S}{\partial x} \tag{8.13}$$

8.4 Longitudinal Dispersion and Salinity Intrusion

where $U_f = Q_f / A =$ net downstream velocity caused by the freshwater discharge;

$K =$ dispersion coefficient in which result of all mechanisms is combined

→ Eq. (8.13) states that the downstream advection of salt by the mean flow is in balance with upstream transport by all other mechanisms.

[Re] Total rate of mass transport by advective flux and dispersive flux

$$q = U_f S + \left(-K \frac{\partial S}{\partial x} \right)$$

8.4 Longitudinal Dispersion and Salinity Intrusion

Mass conservation equation: $\frac{\partial S}{\partial t}+\frac{\partial q}{\partial x}=0$

Combine (A) and (B)

$$\frac{\partial S}{\partial t}+\frac{\partial}{\partial x}\left(U_f S-K\frac{\partial S}{\partial x}\right)=0 \rightarrow \frac{\partial S}{\partial t}+U_f\frac{\partial S}{\partial x}=\frac{\partial}{\partial x}\left(K\frac{\partial S}{\partial x}\right)$$

8.4 Longitudinal Dispersion and Salinity Intrusion

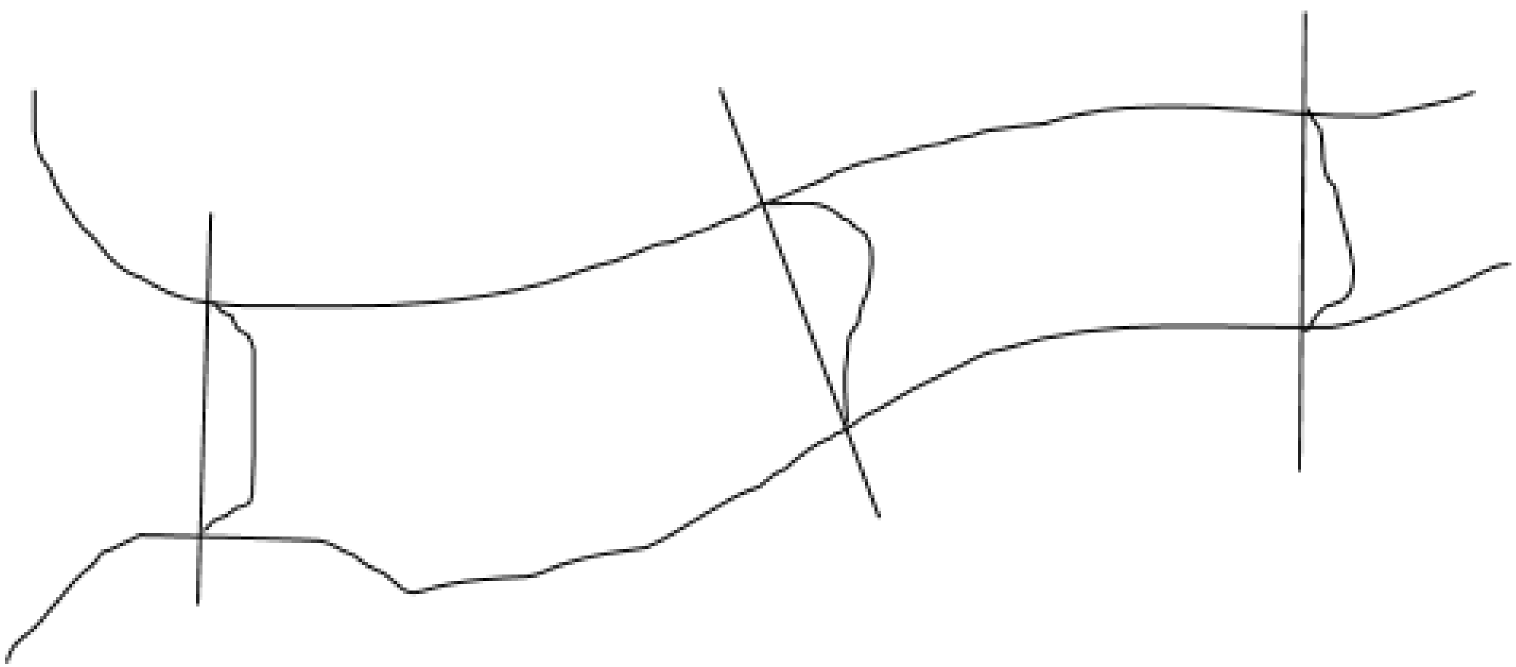
8.4.1 Decomposition of the Salinity and Velocity Profiles

○ Divide the observed fluctuations in velocity and salinity into components

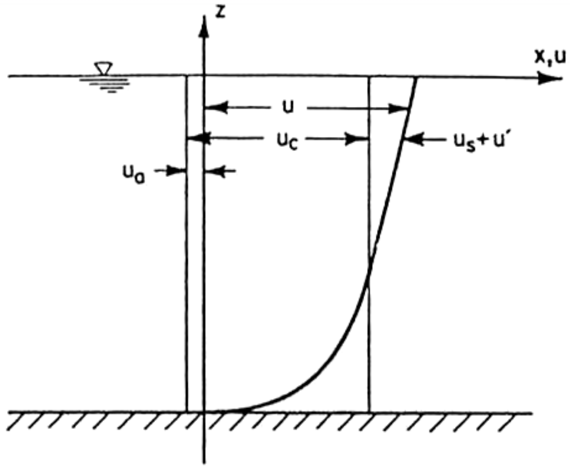
- timewise variations: periodic variations at the frequency of tidal cycle
- spacewise variations: caused by the variations of depth across a cross section, and by the variation of cross sectional shape along the axis of the estuary

Longer period storm and seasonal fluctuations are also important.

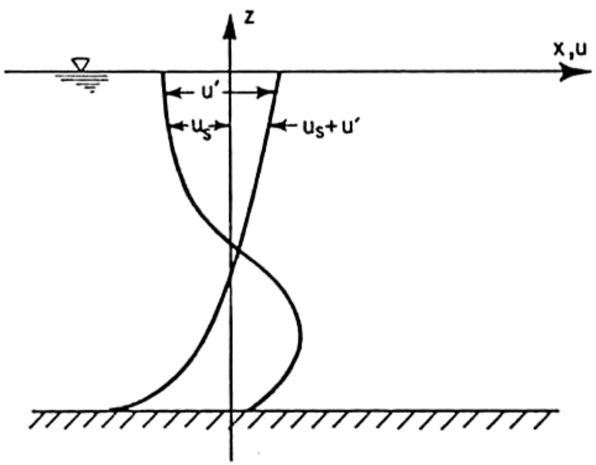
8.4 Longitudinal Dispersion and Salinity Intrusion



8.4 Longitudinal Dispersion and Salinity Intrusion

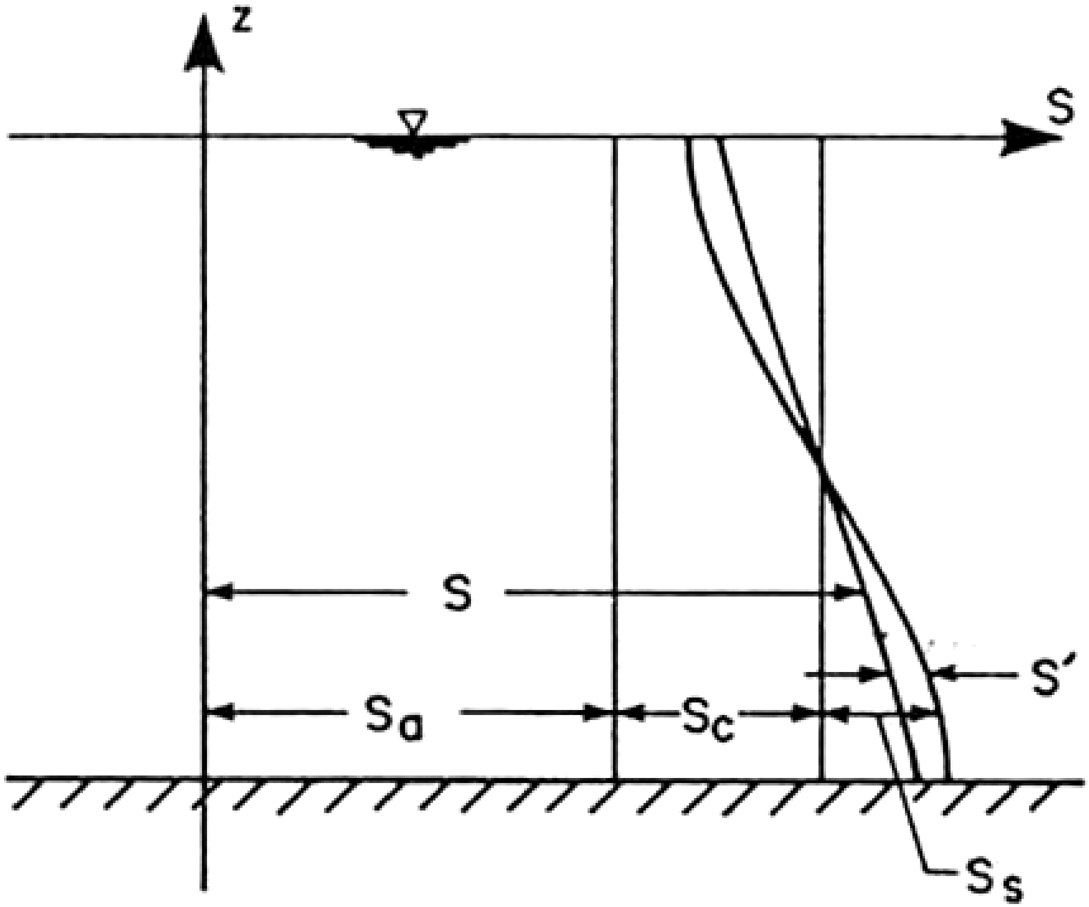


(a)



(b)

8.4 Longitudinal Dispersion and Salinity Intrusion



8.4 Longitudinal Dispersion and Salinity Intrusion

○ Decomposition by Fischer (1972)

The velocity and salinity at any point can be divided into four components

$$u(x,y,z,t) = u_a + u_c(x,t) + u_s(x,y,z) + u'(x,y,z,t) \qquad (8.14)$$

$$S(x,y,z,t) = S_a + S_c(x,t) + S_s(x,y,z) + S'(x,y,z,t) \qquad (8.15)$$

1) u_a, S_a = average over the cress section and tidal cycle

$$u_a = \langle \bar{u} \rangle = \frac{1}{T}\int_0^T \frac{1}{A}\int_0^W \int_0^d u \, dz dy dt$$

$$S_a = \langle \bar{S} \rangle$$

8.4 Longitudinal Dispersion and Salinity Intrusion

where < > = tidal cycle average; overbar = cross-sectional average

2) $u_c, S_c =$ cross-sectional averages at any point during the tidal cycle, minus the tidal cycle averages

$$u_c = \bar{u} - u_a$$

$$S_c = \bar{S} - S_a \qquad (8.16)$$

3) $u_s, S_s =$ the tidal cycle averages at any point, minus cross-sectionally averaged tidal cycle averages

$$u_s = < u > - u_a$$

$$S_s = < S > - S_a \qquad (8.17)$$

8.4 Longitudinal Dispersion and Salinity Intrusion

4) $u', S' =$ the remainder that are left over when the various averages are subtracted from the observed velocity

○ Total transport of salinity through a cross section during a tidal cycle

$$\dot{M} = A < \overline{uS} > = Q_f S_a + A\left\{ < u_c S_c > + \overline{u_s S_s} + < \overline{u' S'} > \right\} \qquad (8.18)$$

$$\boxed{\overline{u_c S_s} = 0}$$

where $Q_f =$ tributary discharge $= < \bar{u} A >$

$$\rightarrow \dot{M} = \underbrace{\left\langle \bar{u} A \right\rangle S_a}_{\text{mean advection}} + \underbrace{A\left\{ < u_c S_c > + \overline{u_s S_s} + \overline{< u' S' >} \right\}}_{\text{dispersion}}$$

8.4 Longitudinal Dispersion and Salinity Intrusion

○ If the estuary is in steady state

– the salinity distribution is the same at the beginning and end of the tidal cycle

Combine Eq. (8.13) and Eq. (8.18)

$$-K\frac{\partial S}{\partial x} = < u_c S_c > + \overline{u_s S_s} + \overline{\langle u'S' \rangle} \qquad (8.19)$$

i) $< u_c S_c > \rightarrow$ tidal cycle correlation of the cross-sectional averages

$$u_c = u_{cm} \sin \omega t$$

$$S_c = S_{cm} \cos \omega t$$

8.4 Longitudinal Dispersion and Salinity Intrusion

where $\omega = 2\pi / T$

$$< u_c S_c > = u_{cm} S_{cm} \int_0^T \sin\omega t \ \cos\omega t \, dt = 0$$

Thus, some researchers assumed it was zero.

However, in real estuaries, peak salinity occurs before high slack water and the minimum salinity before low slack water. → the correlation gives a net landward transport of salt → <u>trapping mechanism</u>

8.4 Longitudinal Dispersion and Salinity Intrusion

ii) $\overline{u_s S_s} \rightarrow$ residual circulation

- We still have no adequate way of predicting its effects in general

- The difficulty is that wind-driven gyres, bathymetric tidal pumping, and density-driven currents all contribute, and observations may result from any combination thereof.

[Ex] Fig. 11b

$\begin{cases} \text{residual baroclinic circulation – seaward in the shallow portions of a channel} \\ \text{tidal pumping/wind driven currents – opposite circulation} \end{cases}$

8.4 Longitudinal Dispersion and Salinity Intrusion

iii) $\overline{\langle u'S' \rangle}$ → the result of

- oscillatory shear flow
- random motions on time scales shorter than the tidal cycle
- short-term variations in the wind
- trapping mechanisms due to dead zones along the side of a channel

8.4.2 The Relative Magnitudes of the Terms

○ Some observations in real estuaries

→ The relative magnitude of the terms in Eq. (8.19) can be measured by establishing an observation transect and measuring the velocity at all points in the cross section throughout a typical tidal cycle.

8.4 Longitudinal Dispersion and Salinity Intrusion

① Dyer's data (1974)

Vellar estuary in India

Southampton Water and Mersey estuary in UK

② Murray et al. (1975)

Guayas estuary in Equador

○ u_s and u' should be separated into transverse and vertical variations

$$u_s = u_{st} + u_{sv}$$

$$u' = u_t' + u_v' \qquad (8.20)$$

8.4 Longitudinal Dispersion and Salinity Intrusion

where u_{st} = transverse variation of the vertical mean

u_{sv} = vertical deviation from the vertical mean

Then, Eq. (8.19) becomes

$$\overline{u_s S_s} + \overline{u' S'} = \overline{u_{st} S_{st}} + \overline{u_{sv} S_{sv}} + \overline{< u_t' S_t' >} + \overline{< u_v' S_v' >} \qquad (8.21)$$

○ Table 8.1

- Vellar estuary: a strongly stratified estuary

→ vertical residual circulation dominates transverse or shear effects

→ trapping term $< u_c S_c >$ is also important

- Southampton Water: Partially stratified estuary

→ transverse and vertical residual circulation dominates

8.4 Longitudinal Dispersion and Salinity Intrusion

SV

	$A_a\langle u_e S_e\rangle$	$A_a\overline{u_{st}S_{st}}$	$A_a\overline{u_{sv}S_{sv}}$	$A_a\langle\overline{u_t'S_t'}\rangle$	$A_a\langle\overline{u_v'S_v'}\rangle$
Vellar estuary, 9/2/77	−75	−14	−105	−4	−42
Southampton Water					
Transect A	−25	250	102	−20	−50
Transect B	0	−220	−200	−20	−12
Mersey estuary	2200	−300	−350	−220	−280

[a] Data from Dyer (1974); positive indicates downstream salt flux; values in kg/sec.

8.4 Longitudinal Dispersion and Salinity Intrusion

○ Guayas estuary: Fig. 8.17 ~ 8.19

- velocity and salinity distributions/residual velocity and salinity distributions

- relative contribution to upstream advective transport

  transverse variation: 53%

  vertical variation: 35%

  cross-product terms: 12%

◈ General conclusion

1)Strongly stratified estuary

Gravitational current

→ vertical residual circulation dominates the other cross-sectional variations

→ trapping mechanism is also important

8.4 Longitudinal Dispersion and Salinity Intrusion

2) Partially stratified estuary

Shape-induced current

→ transverse residual circulation becomes more important as stratification decreases

→ trapping mechanism is also important

8.4.3 Observed values of the Longitudinal Dispersion Coefficient

○ K are obtained by observing a longitudinal salinity gradient and fresh water outflow.

$$K = \frac{U_f S}{\partial S / \partial x}$$

8.4 Longitudinal Dispersion and Salinity Intrusion

- The result depends on whether S is observed at high slack water, low slack water, or is an average over the tidal cycle.

○ Table 8.2

- Many of the values are in the range of $\boxed{100 \sim 300 \text{ m}^2/\text{s}}$ which is notably smaller than the values observed in moderately sized rivers (Table 5.3).

→ The reason is that the shear flow mechanism in rivers is limited in estuaries as discussed in Sec. 8.2.2.1.

- In the constant density portions of estuaries, lower values of K are reported.

→ K are in the range of $\boxed{10 \sim 50 \text{ m}^2/\text{s}}$

→ resulting from shear flow dispersion only; maximum ~ $60 \text{ m}^2/\text{s}$

8.4 Longitudinal Dispersion and Salinity Intrusion

Estuary	Characteristic value or range of observed values of dispersion coeff. (m^2/sec)	Source	Comments
Hudson	160	Thatcher and Harleman (1972)	Values given are K in Thatcher and Harleman's model. Their K differs from the one defined by Eq. (7.13) by a factor usually not greater than three.
Rotterdam Waterway	280		
Potomac	55		
Delaware	500–1500		
San Francisco Bay	200	Glenne and Selleck (1969)	
San Francisco Bay	200	Cox and Macola (1967)	Approx. value used in numerical model
Severn	10–100	Stommel (1953)	
Potomac	20–100	Hetling and O'Connell (1966)	From dye experiment
Delaware	100	Paulson (1969)	
Mersey	160–360	Bowden (1963)	
Rio Quayas, Equador	760		Computed from data given by Murray *et al.* (1975)
Severn (summer)	54–122	Bowden (1963)	
Severn (winter)	124–535	Bowden (1963)	
Thames (low river flow)	53–84	Bowden (1963)	
Thames (high river flow)	338	Bowden (1963)	

8.5 One-Dimensional Analysis of Dispersion of Wastes

○ For long, narrow, unstratified estuaries, 1-D analysis is enough.

→ 1-D analysis is a firmly established engineering tool because it is convenient, relatively simple, and capable of giving practical answers.

→ In the 1-D model, the effect of cross-sectional variations and all the mixing mechanisms

are lumped into the longitudinal dispersion coefficient K.

8.5 One-Dimensional Analysis of Dispersion of Wastes

8.5.1 Tidal Exchange at the Mouth

○ Volume of water entering an estuary during flood tide

= water that left the estuary on the previous ebbs + new ocean water

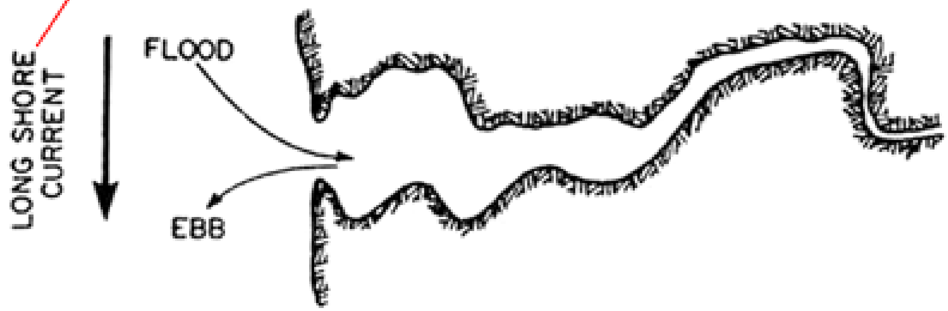
$$V_f = V_{fe} + V_0 \tag{8.25}$$

○ Tidal exchange ratio R

$$R = \frac{V_0}{V_f} \tag{8.26}$$

8.5 One-Dimensional Analysis of Dispersion of Wastes

The longshore current deflects the ebb flow downcoast and delivers the supply of new ocean water for the flood.



8.5 One-Dimensional Analysis of Dispersion of Wastes

where $V_0 =$ volume of new ocean water entering the estuary during the flood tide;

$V_f =$ total water volume entering the estuary on the flood tide;

$V_{fe} =$ part of V_f which flowed out of the estuary on the previous ebb

○ Total salt and water contents of the estuary are to remain constant

$$S_f V_f = S_e V_e \tag{8.22}$$

$$V_f + V_Q = V_e \tag{8.23}$$

$$S_f V_f = S_e V_{fe} + S_0 V_0 \tag{8.24}$$

8.5 One-Dimensional Analysis of Dispersion of Wastes

Combine the above equations

$$R = \frac{S_f - S_e}{S_0 - S_e} \qquad (8.27)$$

○ Accurate determination of average flood and ebb salinities requires complete cross-sectional measurements of both salinity and velocity throughout the tidal cycle.

$$S_e = \int_0^{Te} \int_A Su\, dAdt \Big/ \int_0^{Te} \int_A u dAdt \qquad (8.28)$$

8.5 One-Dimensional Analysis of Dispersion of Wastes

where S and u are point values of salinity and velocity;

$T_e =$ duration of the ebb flow; A = time-varying cross-sectional area

$$R = \frac{S_e}{S_0 - S_e} \frac{V_Q}{V_f} \tag{8.29}$$

8.5 One-Dimensional Analysis of Dispersion of Wastes

[Proof] Eq. (8.27) & (8.29)

1) Subtract (8.25)xSe from (8.24)

$$S_f V_f = S_e V_{fe} + S_0 V_0 \qquad (8.24)$$

$$-\quad S_e V_f = S_e V_{fe} + S_e V_0 \qquad (8.25)$$

$$\left(S_f - S_e\right) V_f = \left(S_0 - S_e\right) V_0$$

$$\therefore \frac{V_0}{V_f} = R = \frac{S_f - S_e}{S_0 - S_e}$$

8.5 One-Dimensional Analysis of Dispersion of Wastes

2) Substitute (8.22) into (8.27)

$$R = \frac{S_e \dfrac{V_e}{V_f} - S_e}{S_0 - S_e} = \frac{S_e}{S_0 - S_e}\left(\frac{V_e}{V_f} - 1\right)$$

Substitute (8.23)

$$R = \frac{S_e}{S_0 - S_e}\left(\frac{V_e - V_f}{V_f}\right) = \frac{S_e}{S_0 - S_e}\frac{V_Q}{V_f}$$

8.5 One-Dimensional Analysis of Dispersion of Wastes

○ Measurements of tidal exchange at San Francisco Bay

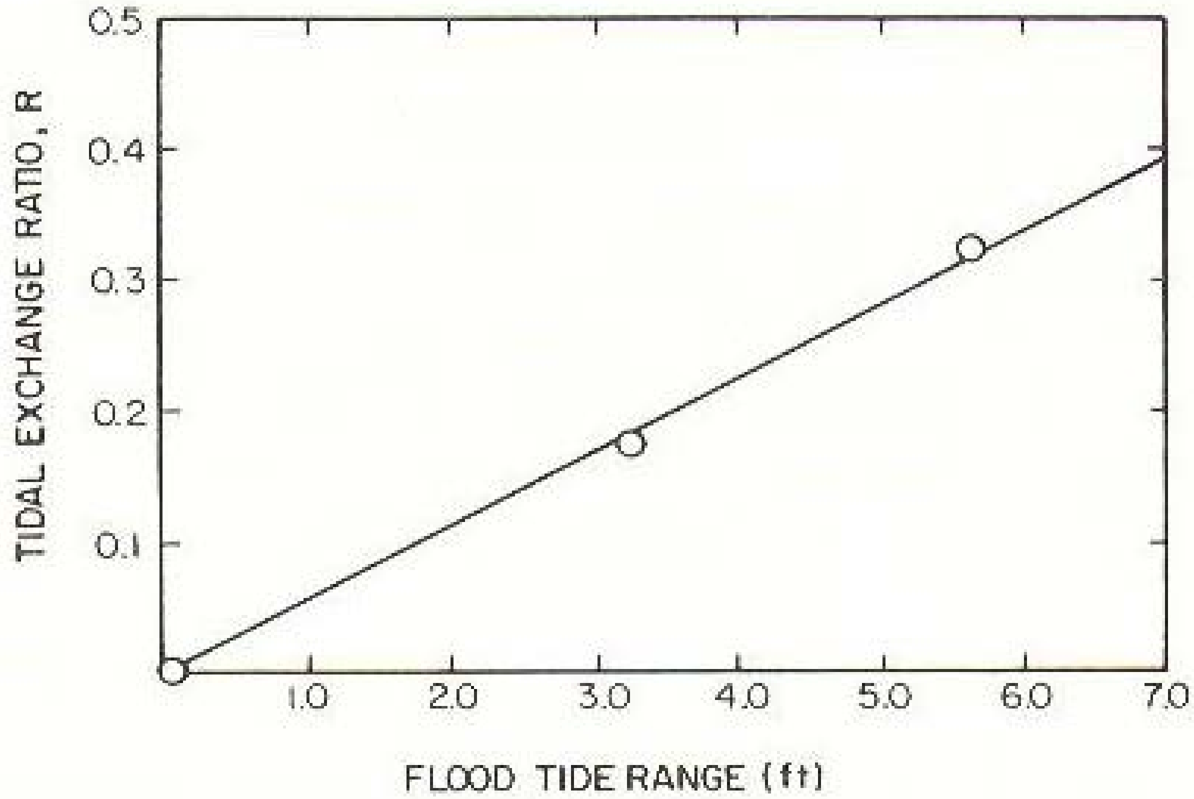
- Nelson and Lerseth (1972)

- Salinity and velocity were measured throughout the tidal cycle at a number of points on a transect at the Golden Gate.

- Fig. 8. 23: tidal exchange ratio $\propto$ flood tide range

- Numerical modeling showed that increasing the model values of R from 0.20 to 0.30 decreased the pollutant concentrations near the Golden Gate by 30%.

8.5 One-Dimensional Analysis of Dispersion of Wastes



8.5 One-Dimensional Analysis of Dispersion of Wastes

8.5.2 Tidal Exchange within the Estuary

○ A prediction of concentration at points up and downstream is needed when a given loading of effluent is discharged at a given point.

→ use the 1-D analysis using the distribution of ambient salinity as a guide.

○ Salt balance in the estuary

$$Q_0 S_0 = (Q_0 + Q_e + Q_f) S \tag{8.30}$$

8.5 One-Dimensional Analysis of Dispersion of Wastes

where Q_0 = circulating flow of ocean water;

Q_f = tributary discharge from all tributaries upstream of the effluent discharge point;

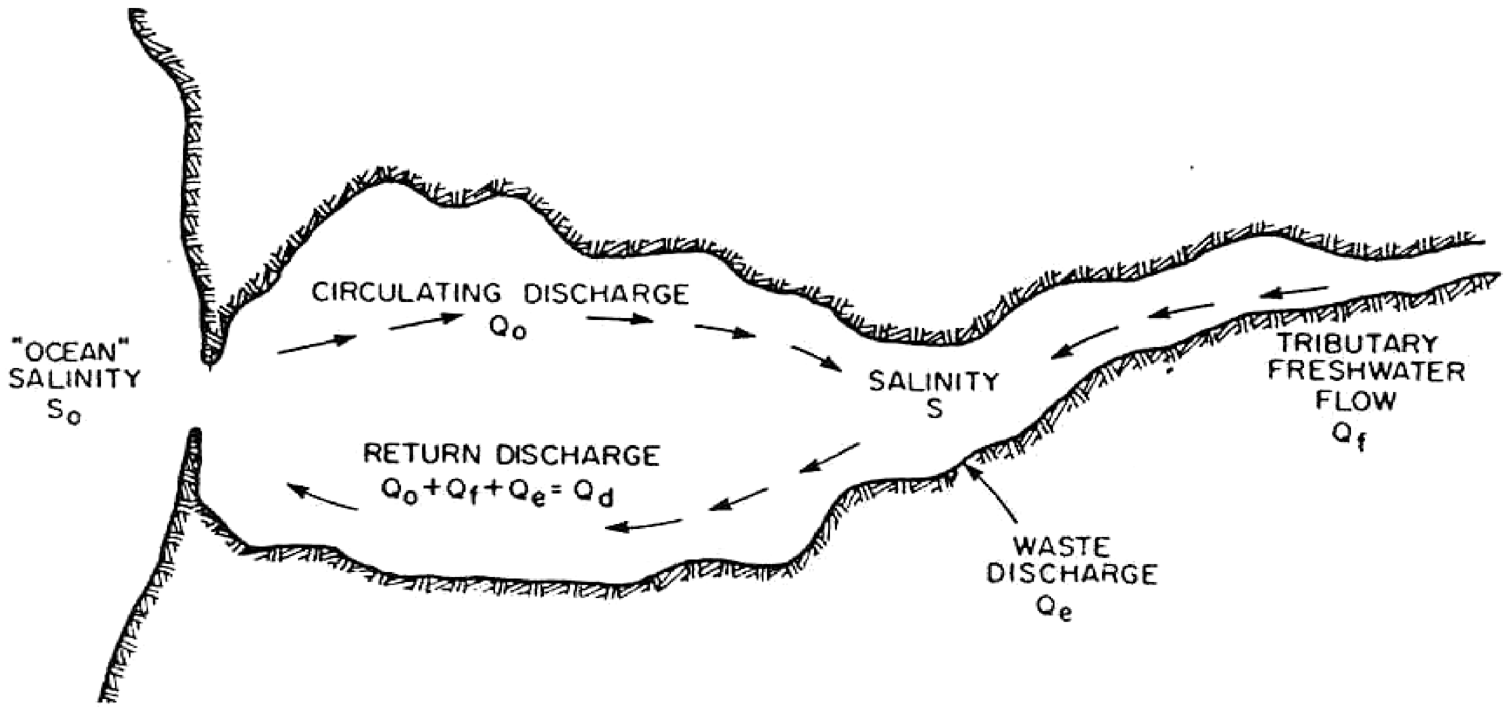
Q_e = effluent flow;

S_0 =ocean salinity; S = salinity in the estuary

Rearranging Eq. (8.30) yields

$$Q_0 = \frac{(Q_e + Q_f)S}{S_0 - S} \tag{8.31}$$

8.5 One-Dimensional Analysis of Dispersion of Wastes



8.5 One-Dimensional Analysis of Dispersion of Wastes

The total flow available for diluting the effluent is

$$Q_d = Q_0 + Q_e + Q_f = \frac{(Q_e + Q_f)S_o}{S_0 - S} \qquad (8.32)$$

where Q_d = return discharge

Assume complete mixing near the discharge point in short time

○ Mean concentration of effluent near the point of discharge

$$C_d = \frac{\dot{M}}{Q_d} \qquad (8.33)$$

where $\dot{M}$ = discharge rate of material (mass/time)

8.5 One-Dimensional Analysis of Dispersion of Wastes

○ Tidal exchange analysis of Sec. 8.5.1 gives

$$Q_0 T = RV_f$$

$$R = \frac{S_e}{S_0 - S_e}\frac{V_Q}{V_f}$$

$$\therefore Q_0 = \frac{RV_f}{T} \qquad \text{(A)}$$

Introducing Eq. (8.29) into (A) yields

$$Q_0 = \frac{S_e}{S_0 - S_e}\frac{V_Q}{T} = \frac{S_e}{S_0 - S_e}(Q_e + Q_f) \qquad (8.34)$$

$$\frac{V_Q}{T} = Q_Q = Q_e + Q_f$$

→ discharge of river water

8.5 One-Dimensional Analysis of Dispersion of Wastes

→ the same as Eq. (8.31) if S is taken to be the average ebb flow salinity, S_e.

○ In bays with low tributary inflow, the salinity may be near oceanic

$$Q_0 = \frac{RP}{T} \tag{8.35}$$

where P = tidal prism $\approx V_f$

○ R decreases rapidly as one moves upstream from the mouth of the estuary:

Ex: South San Francisco Bay

R = 0.076 → 0.039 → 0.031

8.5 One-Dimensional Analysis of Dispersion of Wastes

[Ex. 7.2] Discharge to an estuary with tributary inflow

$Q_e = 100$ cfs (industrial STP)

$C_e = 10$ ppt (toxic material)

Parts per thousand

$Q_f = 1000$ cfs (tributary inflow)

$S_e = 19$ ppt

$S_0 = 33$ ppt

Estimate the average concentration of the toxic material in the estuary in the vicinity of the discharge point.

8.5 One-Dimensional Analysis of Dispersion of Wastes

Solution: First, use salinity data to obtain Q_d

$$Q_d = Q_0 + Q_e + Q_f = (Q_e + Q_f) S_0 / \ (S_0 - S) \qquad (8.32)$$

$$= (100 + 1{,}000) \frac{33}{(33 - 19)} = 2596 \text{ cfs}$$

$$\therefore C_d = \frac{\dot{M}}{Q_d} \qquad (8.33)$$

where

$$\dot{M} = mass \,/\, time = Q_e C_e$$

$$\therefore \ C_d = \frac{100(10)}{2596} = 0.385 \text{ ppt}$$

8.5 One-Dimensional Analysis of Dispersion of Wastes

[Example 8.3] Discharge to an estuary with no inflow; use of a dye study

$Q_f = 0$ (no tributary discharge)

$Q_e = 100\ \text{cfs}$

$C_e = 10\ \text{ppt}$

$Q_{dye} = 33\ \text{cc / min}$ (continuous dye release for a period of 15 tidal cycle)

$C_{dye} = 200{,}000{,}000\ \text{ppb}$

Parts per billion

$C_d\big]_{dye} = 8.5\ \text{ppb}$

8.5 One-Dimensional Analysis of Dispersion of Wastes

Assuming no dye decay and that 15 tidal cycle is sufficient to reach equilibrium, estimate the concentration in the bay.

<u>Solution</u>:

First, for dilution of the dye, the dilution discharge is found by solving for Q_d in Eq. (8.33)

$$Q_d = \frac{\dot{M}}{C_d}$$

$$= \frac{33\ \text{cc/min}\ \times\ 2\times10^8\ \text{ppb}}{8.5\ \text{ppb}} = 7.76\times10^8\ \text{cc/min} = 459\ \text{cfs} \approx Q_0$$

→ This discharge can be used for effluent analysis.

8.5 One-Dimensional Analysis of Dispersion of Wastes

$$\therefore C_d\Big]_{effluent} = \frac{\dot{M}\Big]_{eff}}{Q_d} = \frac{\dot{M}\Big]_{eff}}{Q_0 + Q_e} = \frac{10\ \text{ppt} \times 100\ \text{cfs}}{459\ \text{cfs} + 100\ \text{cfs}} = 1.79\ \text{ppt}$$

○ For a conservative material,

i) Downstream of the outfall

→ the effluent continues to be diluted as it approaches the ocean, just as fresh water is diluted.

$$C_x = C_d \frac{S_0 - S_x}{S_0 - S_d}, \qquad \text{downstream} \qquad (8.36)$$

8.5 One-Dimensional Analysis of Dispersion of Wastes

ii) Upstream of the discharge point

→ concentration of the effluent is reduced in the same way that the salinity is reduced by nixing with the tributary fresh water.

$$C_x = C_d \frac{S_x}{S_d}, \qquad \text{upstream} \qquad (8.37)$$

◎ Assumptions

(i) The salinity observed at some instant is representative of steady-state conditions.

(ii) The results are valid only for conservative substances

(iii) The results give cross-sectional averages, not peak concentrations.

8.5 One-Dimensional Analysis of Dispersion of Wastes

[Example 8.4] Distribution along the estuary

The same conditions as for worked Example 8.2. Find the concentration downstream of the outfall at a point where the salinity is 24 ppt and upstream of the outfall at a point where the salinity is 5 ppt.

$$S_x\Big|_{up} = 5 \text{ ppt}; \quad S_x\Big|_{down} = 24 \text{ ppt}$$

$$C_x\Big|_{down} = 0.385\,\frac{33-24}{33-19} = 0.248 \text{ ppt}$$

$$C_x\Big|_{up} = 0.385\frac{5}{19} = 0.101 \text{ ppt}$$

8.5 One-Dimensional Analysis of Dispersion of Wastes

8.5.3 Dispersion of Decaying Substances

○ 1-D analysis for dispersion of pollutants based on the time and space averaged equation

$$A\frac{\partial C}{\partial t}+Q_f\frac{\partial C}{\partial x}=\frac{\partial}{\partial x}\left(KA\frac{\partial C}{\partial x}\right)+(\text{source}/\text{sink}) \qquad (8.38)$$

where the time derivative means the change per tidal cycle; K expresses the result of all the mixing processes that occur <u>within the tidal cycle</u>;

A = cross-sectional area at mean tide.

→ Eq. (8.38) is a postulated model (empirical model) subject to verification.

8.5 One-Dimensional Analysis of Dispersion of Wastes

○ Numerical modeling of Eq. (8.38)

- seven-segment model of San Francisco Bay by Cox and Macola (1967)

- Fig. 8.26

○ Analytical solution of Eq. (8.38)

- steady-state distribution

- a tracer undergoing first order decay

- a channel with constant cross section and dispersion coefficient

$$U_f \frac{\partial C}{\partial x} = K \frac{\partial^2 C}{\partial x^2} - kC \qquad (8.39)$$

8.5 One-Dimensional Analysis of Dispersion of Wastes

where k = rate coefficient

This equation has the two solutions: Sec. 5.5

$$\boxed{C = C_0 \exp\left[x'\left(1 \pm \sqrt{\alpha + 1}\right)\right]} \qquad (8.40)$$

where $x' = \dfrac{U_f x}{2K}$; $\alpha = \dfrac{4Kk}{U_f^{\,2}}$

○ Suppose that a pollutant is introduced into an estuary at a rate $\dot{M}$ (mass/time) at a point $x = L$, where $x = 0$ is the mouth of the estuary, and x positive landward.

→ Since U_f is a seaward velocity, x' is negative everywhere in the estuary.

8.5 One-Dimensional Analysis of Dispersion of Wastes

I. Case 1: estuary mouth is far from the discharge point

Boundary conditions:

$$C = 0 \qquad at \qquad x' = \pm\infty$$

(i) Solution for upstream of the pollutant source

$$C = C_0 \exp\left[\left(x' - L' \right) \left(1 + \sqrt{1 + \alpha} \right) \right] \qquad (8.41)$$

where $L' = U_f L / 2K$

8.5 One-Dimensional Analysis of Dispersion of Wastes

(ii) Solution for downstream reach

$$C = C_0 \exp\left[\left(x' - L'\right)\left(1 - \sqrt{1+\alpha}\right)\right] \tag{8.42}$$

Concentration at the injection point is given as

$$C_0 = \frac{\dot{M}}{Q_f} \frac{1}{\sqrt{1+\alpha}}$$

because $\dot{M} = \int_{-\infty}^{\infty} kCA\,dx$

8.5 One-Dimensional Analysis of Dispersion of Wastes

○ In estuary, U_f is often small and α often large. For case of tidal inlet with no fresh water inflow, $U_f \to 0; \ \alpha \to \infty$

$$C = \frac{\dot{M}}{A\sqrt{4Kk}} \exp\left[\pm (x - L)\sqrt{k / K} \right] \tag{8.43}$$

II. Case 2: complete removal of the pollutant at the estuary mouth

Boundary condition: $c = 0 \ \ at \ \ x = 0$

$$c = 0 \ \ at \ \ x = +\infty$$

(i) upstream solution is the same as Eq. (8.40)

(ii) downstream solution

8.5 One-Dimensional Analysis of Dispersion of Wastes

$$C = C_0 \frac{\exp\left[\left(1-\sqrt{1+\alpha}\right)x'\right]-\exp\left[\left(1+\sqrt{1+\alpha}\right)x'\right]}{\exp\left[\left(1-\sqrt{1+\alpha}\right)L'\right]-\exp\left[\left(1+\sqrt{1+\alpha}\right)L'\right]} \tag{8.44}$$

C_0 (concentration at the injection point) is also affected by the loss at the mouth, and is given by

$$C_0 = \dot{M}\ \frac{1-\exp\left(2L'\sqrt{1+\alpha}\right)}{Q_f\sqrt{1+\alpha}} \tag{8.45}$$

8.5 One-Dimensional Analysis of Dispersion of Wastes

Homework Assignment # 1

Due: 2 weeks from today

1. Derive Eq. (8.40).

2. Derive Eq. (8.44) & (8.45).

3. a) Solve Example 8.5.

   b) Plot C-x curve for both cases with various values of K and k.

   For example, K = 30, 60, 120, 240 m^2/s; k = 0.1, 0.2, 0.4, 0.8 1/day.

8.5 One-Dimensional Analysis of Dispersion of Wastes

[Example 8.5] Waste distribution in an estuary

A sewage treatment plant discharges BOD into an estuary.

$\dot{M} = 2\ \mathrm{kg/s}$ of BOD from STP, $k = 0.2\ \mathrm{/day}$

$A = 600\ \mathrm{m}^2$, $Q_f = 10\ \mathrm{m}^3\mathrm{/s}$ (fresh water discharge), $K = 60\ \mathrm{m}^2\mathrm{/s}$

Plot the longitudinal distribution of mean concentration for two cases:

(a) $L = 30\ \mathrm{km}$ from the mouth; and (b) $L = 5\ \mathrm{km}$ from the mouth

Solution:

$$U_f = Q_f / A = -10/600 = -1/60\ \mathrm{m/s}$$

$$x' = U_f x / 2K = -\frac{1}{60} \cdot x / 2(60) = -1.39 \times 10^{-4} x\ (\mathrm{m})$$

8.5 One-Dimensional Analysis of Dispersion of Wastes

$$L' = U_f L / 2K = \begin{cases} -0.69 & Case\, b \\ -4.14 & Case\, a \end{cases}$$

$$\alpha = \frac{4Kk}{{U_f}^2} = \frac{4\left(60m^2/s\right)\left(0.2/86{,}400s\right)}{\left(\frac{1}{60}m/s\right)^2} = 1.99$$

$$\frac{U_f}{2K} = -1.39\times10^{-4}$$

$$1+\sqrt{1+\alpha} = 1+\sqrt{1+1.99} = 2.729$$

$$1-\sqrt{1+\alpha} = 1-\sqrt{1+1.99} = -0.729$$

8.5 One-Dimensional Analysis of Dispersion of Wastes

i) Case (a) :

L' is large enough that the BOD will almost completely decay before reaching the mouth.

→ use Eqs. (8.40) - (8.42)

- Upstream

$$C = C_0 \exp\left[\left(x' - L' \right) \left(1 + \sqrt{1+\alpha} \right) \right] = C_0 \exp\left[\frac{U_f}{2K} \left(1 + \sqrt{1+\alpha} \right) \left(x - L \right) \right]$$

$$= C_0 \exp\left[-3.79 \times 10^{-4} \left(x - L \right) \right]$$

8.5 One-Dimensional Analysis of Dispersion of Wastes

- Downstream

$$C = C_0 \exp\left[(x' - L')\left(1 - \sqrt{1+\alpha}\right)\right] = C_0 \exp\left[\frac{U_f}{2K}\left(1 - \sqrt{1+\alpha}\right)(x - L)\right]$$

$$= C_0 \exp\left[1.01 \times 10^{-4}(x - L)\right]$$

$$C_0 = \frac{\dot{M}}{Q_f}\frac{1}{\sqrt{1+\alpha}} = \frac{2\ \mathrm{kg}/s}{10\ \mathrm{m}^3/s}\frac{1}{\sqrt{2.99}} = 0.116\ \mathrm{kg/m^3} = 116\ \mathrm{mg/l}\,(\mathrm{ppm})$$

ii) Case (b):

Solution is affected by the nearness of the mouth.

→ use Eqs. (8.44) - (8.45)

8.5 One-Dimensional Analysis of Dispersion of Wastes

- Upstream

$$C = C_0 \exp\left[(x' - L')\left(1 + \sqrt{1+\alpha}\right) \right] = C_0 \exp\left[-3.79 \times 10^{-4} (x - L) \right]$$

- Downstream

$$C = C_0 \frac{\exp\left[\left(1 - \sqrt{1+\alpha}\right) x' \right] - \exp\left[\left(1 + \sqrt{1+\alpha}\right) x' \right]}{\exp\left[\left(1 - \sqrt{1+\alpha}\right) L' \right] - \exp\left[\left(1 + \sqrt{1+\alpha}\right) L' \right]}$$

$$= C_0 \frac{\exp\left(1.01 \times 10^{-4} x\right) - \exp\left(-3.79 \times 10^{-4} x\right)}{(1.66 - 0.15)}$$

$$C_0 = 116 \text{ mg/l} \left[1 - \exp\left(2(-0.69)\sqrt{2.99} \right) \right] = 105 \text{ mg/l}$$

8.5 One-Dimensional Analysis of Dispersion of Wastes

[Re]

$$\left(1-\sqrt{1+\alpha}\right)L' = (-0.729)(-0.69) = 0.503$$

$$\left(1+\sqrt{1+\alpha}\right)L' = (2.729)(-0.69) = -1.883$$

$$\exp\left[\left(1-\sqrt{1+\alpha}\right)L' = \right] = \exp(0.503) = 1.654$$

$$\exp\left[\left(1+\sqrt{1+\alpha}\right)L' = \right] = \exp(-1.883) = 0.152$$

8.5 One-Dimensional Analysis of Dispersion of Wastes

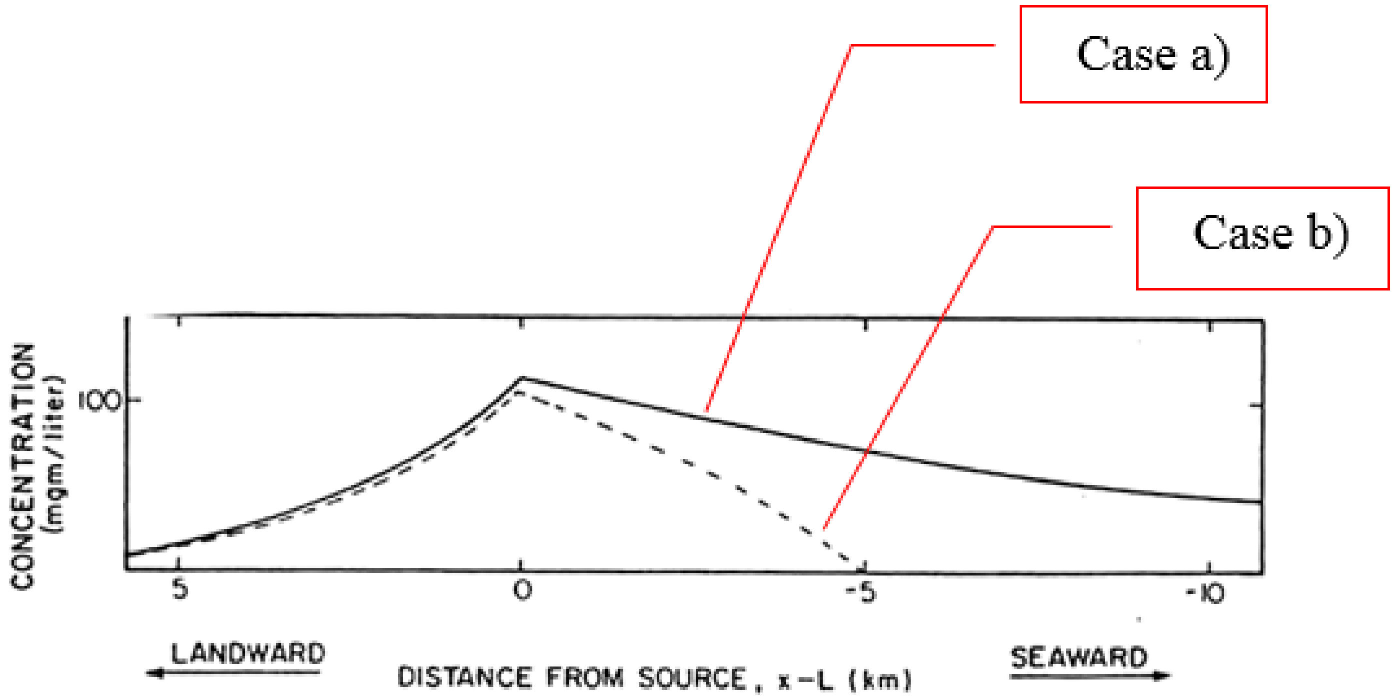


Figure 7.24 Longitudinal concentration distributions for Example 7.5: ——— discharge 30 km from the mouth; – – – discharge 5 km from the mouth.

8.5 One-Dimensional Analysis of Dispersion of Wastes

○ Concentration distributions obtained from Eqs. (8.40) - (8.45) are averaged concentrations over the tidal cycle.

○ Actually, high concentrations build up near the outfall at slack tide because there is little current to provide dilution.

→ Then, during the following flood or ebb, the peak formed at slack tide is advected and dispersed.

○ 1-D model using tidally varying quantities (Li, 1974)

$$\frac{\partial}{\partial t}(AC)+\frac{\partial}{\partial x}(\bar{u}AC)=\frac{\partial}{\partial x}\left(K_t A\frac{\partial c}{\partial x}\right)+\text{ source/sink terms} \qquad (8.46)$$

8.5 One-Dimensional Analysis of Dispersion of Wastes

where K_t = dispersion coefficient appropriate to a tidally varying analysis;

$\bar{u}$ = tidally varying cross-sectional mean velocity.

→ Li's analysis is limited to oscillating flow in a uniform channel sufficiently narrow that cross-sectional mixing is immediate.

○ In wide and irregular estuaries,

the cross-sectional mixing time is much longer than the tidal cycle.

→ use either tidally averaged model, Eq. (8.38) or 2-D numerical model

8.5 One-Dimensional Analysis of Dispersion of Wastes

[Ex] Delaware Estuary near Marcus Hook

W = 1,200 m

Time for transverse mixing $(T_c = W^2 / \varepsilon_t)$ ~ 10 days

8.5.4 Calculation of the Flushing Time

○ Flushing time = mean detention time

~ mean time that a particle of tracer remains inside an estuary

○ Freshness = fraction of any sample of water that is pure fresh water

$$f = \frac{S_0 - S}{S_0} \tag{8.47}$$

8.5 One-Dimensional Analysis of Dispersion of Wastes

where S_0 = salinity of pure ocean water

$f = \begin{cases} 1 \text{ for pure fresh water} \\ 0 \text{ for pure ocean water} \end{cases}$

○ Total amount of fresh water in an estuary between $x = L$ and $x = 0$ (mouth)

$$V = \int_0^L \int_A f\, dA dx \tag{8.48}$$

○ Flushing time between $x = L$ and $x = 0$

$$T_f = \frac{V}{Q_f} \tag{8.49}$$

8.5 One-Dimensional Analysis of Dispersion of Wastes

T_f = time required for the freshwater discharge (river), Q_f to completely replace the fresh water in the estuarine volume

○ Replacement time

~ time required for replacement of a tracer throughout a distance L landward from the mouth in an estuary with dispersion coefficient K

~ time required for complete mixing between $x = L$ and $x = 0$

→ similar to cross-sectional mixing time in Sec. 5.1.3

$$T = 0.4 \frac{L^2}{K} \qquad (8.50)$$

8.5 One-Dimensional Analysis of Dispersion of Wastes

where T = time required for a slug of material initially concentrated at one end of basin to reach an approximately uniform concentration throughout the basin;

L = length of the basin; K = dispersion coeff.

[Example 8.6] An estuary has a constant cross-sectional area.

$$A = 10{,}000 \text{ m}^2$$

$$K = 100 \text{ m}^2/\text{s}$$ (longitudinal dispersion coeff.)

$$Q_f = 30 \text{ m}^3/\text{s}$$ (fresh water inflow)

$$L = 30 \text{ km} = 30{,}000 \text{ m}$$

8.5 One-Dimensional Analysis of Dispersion of Wastes

Find the mean detention time by Eqs. (8.49) and (8.50)

Solution:

○ Analytical solution for 1D equation of conservative pollutant for steady state (Ch. 6)

$$0 = -U_f \frac{\partial C}{\partial x} + K \frac{\partial^2 C}{\partial x^2}$$

$$C = C_0 \exp(\frac{U_f}{K} x)$$

8.5 One-Dimensional Analysis of Dispersion of Wastes

○ Salinity distribution

$$S = S_0 \exp\left[-\left(\frac{Q_f}{A}\right)\left(\frac{x}{K}\right)\right]$$

$$= S_0 \exp\left[-\left(\frac{30}{10,000}\right)\left(\frac{x}{100}\right)\right] = S_0 \exp(-3\times10^{-5} x)$$

○ Freshness distribution

$$f = \frac{(S_0 - S)}{S_0} = 1 - \exp(-3\times10^{-5} x)$$

8.5 One-Dimensional Analysis of Dispersion of Wastes

○ Volume of fresh water in the bounded estuary volume

(between $x = 0$ and L)

$$V_f = \int_0^L fA dx$$

$$= A\int_0^{30,000} \left[1 - \exp\left(-3\times10^{-5} x\right)\right] dx$$

$$= 1.02\times10^8 \ m^3$$

○ Flushing time by Eq. (8.49)

$$T_f = \frac{V_f}{Q_f} = \frac{1.02\times10^8 \ \mathrm{m}^3}{30 \ \mathrm{m}^2/\mathrm{s}} = 3.41\times10^6 \ s = 39.4 \ \mathrm{day}$$

8.5 One-Dimensional Analysis of Dispersion of Wastes

○ Replacement time by Eq. (8.50)

$$T = 0.4\frac{L^2}{K} = 0.4\frac{(30{,}000\ \mathrm{m})^2}{100\ \mathrm{m}^2/\mathrm{s}} = 3.6\times10^6\ \mathrm{s} = 41.7\ \mathrm{day}$$

8.5.5 Uses and Limitations of One-Dimensional Analysis

○ Conditions required for 1-D model application

(1) The time scale for mixing across the estuary is significantly less than the time required for the effluent to pass out of the estuary, or for the substance to decay.

8.5 One-Dimensional Analysis of Dispersion of Wastes

$$T_c = 0.4\frac{W^2}{\varepsilon_t}$$

$$T_t = \frac{L}{U_f}$$

$$\frac{T_c}{T_t} << 1$$

(2) The estuary is not significantly stratified, so that the effluent can be expected to mix uniformly over the depth. If the estuary is strongly stratified, it is possible to use separate one-dimensional analysis in each layer.

8.5 One-Dimensional Analysis of Dispersion of Wastes

(3) Allowance is made in the analysis for the higher concentrations expected near the source, before cross-sectional mixing takes place, and for distributed sources and sinks in the case of a naturally occurring substances such as nutrients, dissolved oxygen, etc.

[Example 8.7] Use of 1-D analysis for effluent discharge

An industry plans to discharge conservative constituent into San Pablo Bay (Fig. 8.26).

$C_e = 1{,}000 \text{ ppm}$ (conservative pollutant from STP)

$Q_e = 0.5 \text{ m}^3/\text{s}$

8.5 One-Dimensional Analysis of Dispersion of Wastes

$A = 25,000 \text{ m}^2$ (cross-sectional area)

$d = 8 \text{ m}$ (mean depth)

$W = 3.125 \text{ m}$ (mean width)

$U_t = 0.75 \text{ m/s}$ (rms tidal velocity)

$u^* = 0.075 \text{ m/s}$ (rms shear velocity)

$Q_f = 100 \text{ m}^3\text{/s}$ (tributary net outflow)

$$S = \begin{cases} 24 \text{ ppt} & \text{at the surface} \\ 26 \text{ ppt} & \text{at the bottom} \end{cases}$$

$S_0 = 33 \text{ ppt}$

8.5 One-Dimensional Analysis of Dispersion of Wastes

Find the resulting concentration in the bay.

Solution:

○ Dilution discharge computed by Eq. (8.32)

$$Q_d = \frac{\left(Q_e + Q_f\right) S_0}{S_0 - S}$$

$$= (0.5 + 100\ \mathrm{m^3/s}) \frac{33}{33 - 25} = 415\ \mathrm{m^3/s}$$

8.5 One-Dimensional Analysis of Dispersion of Wastes

○ Tidal exchange ratio

tidal prism, $P \cong U_t A \dfrac{T}{2}$

$$R = \frac{Q_0 T}{P} \cong \frac{2(Q_d - Q_e - Q_f)}{U_t A} = \frac{2(415 - 0.5 - 100)}{0.75 \times 25{,}000} = 0.034$$

○ Mean concentration in San Pablo Bay is computed by Eq. (8.33)

$$C_d = \frac{\dot{M}}{Q_d}$$

$$= \frac{0.5\ \mathrm{m^3/s} \times 1000\ \mathrm{ppm}}{415\ \mathrm{m^3/s}} = 1.2\ \mathrm{ppm}$$

8.5 One-Dimensional Analysis of Dispersion of Wastes

○ Influence of stratification on vertical mixing

$$R_i = \frac{\Delta\rho}{\rho}\frac{gd}{U_t^2}$$

$$= 0.0014 \cdot \frac{9.8\ \text{m/s}^2 \times 8\ \text{m}}{(0.75\ \text{m/s})^2} = 0.2$$

→ R_i is large enough to suggest to reduce vertical turbulent mixing.

$$\varepsilon_v = 0.005 du^* \qquad < cf > \ \varepsilon_v = 0.067 du^*$$

$$= 0.005(8)(0.075)$$

$$= 0.003\ \text{m}^2/\text{s}$$

8.5 One-Dimensional Analysis of Dispersion of Wastes

- Time for vertical mixing

$$T_v = 0.4\frac{d^2}{\varepsilon_v}$$

$$= \ 0.4\frac{8^2}{0.0003} \ = \ 8{,}500 \text{ s} \ \approx \ 2.5 \text{ hr}$$

- Travel distance for effluent

$$L_t = U_t T_v$$

$$= \ 0.75(8{,}500) \ = \ 6{,}400 \text{ m} \ = \ 6.4 \text{ km}$$

8.5 One-Dimensional Analysis of Dispersion of Wastes

- Transverse mixing

$$\varepsilon_t = 0.6du^*$$

$$= 0.6(8)(0.075) = 0.36 \text{ m}^2/\text{s}$$

- Transverse extent of effluent after 2.5hr

$$4\sigma_t = 4\sqrt{2\varepsilon_t T_v}$$

$$= 4\sqrt{2 \times 0.36 \times 8500} = 313 \text{ m}$$

8.5 One-Dimensional Analysis of Dispersion of Wastes

- Peak concentration computed by Eq. (5.7)

$$C_p = \frac{\dot{M}}{dU_t\sqrt{4\pi\varepsilon_t\left(x/U_t\right)}}$$

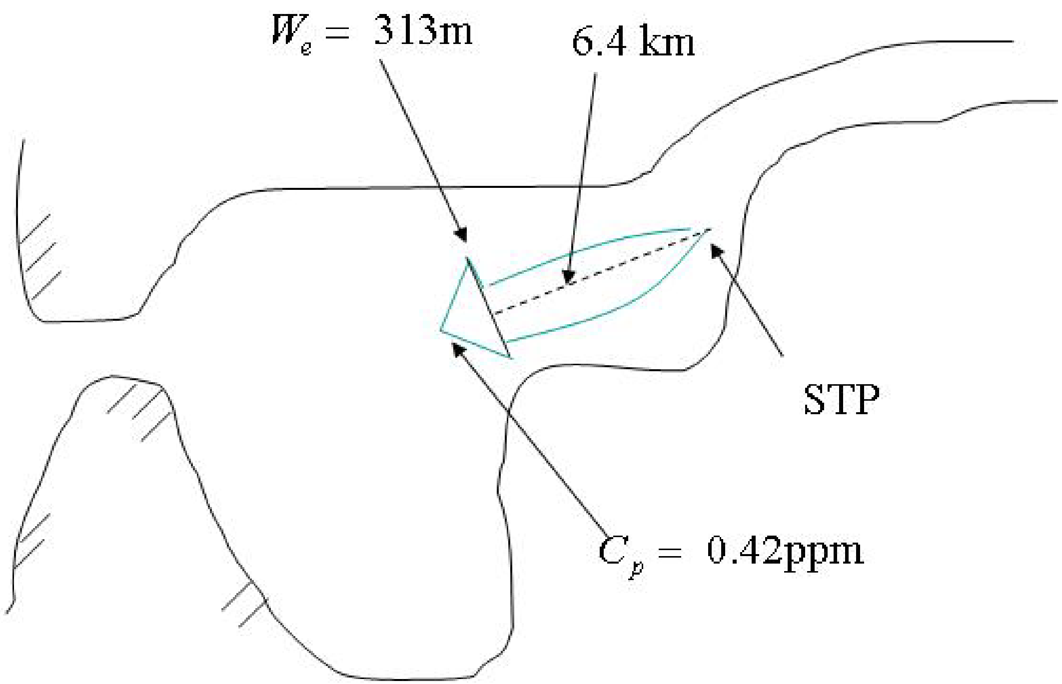
-line source of $\dot{M}$

-point source of $\dot{M}/d$ in 2-D flow

$$= \frac{0.5\ \mathrm{m^3/s}\ \times 1000\ \mathrm{ppm}}{8\ \mathrm{m} \times 0.75\ \mathrm{m/s}\ \sqrt{4\pi \times 0.36\ \mathrm{m^2/s}\ \times\ 8500\ \mathrm{s}}}$$

$$= 0.42\ \mathrm{ppm}$$

8.5 One-Dimensional Analysis of Dispersion of Wastes



8.5 One-Dimensional Analysis of Dispersion of Wastes

○ Comparison of C_d and C_p

$$C_d > C_p$$

where C_d = background buildup concentration of effluent over many tidal cycles

C_p = peak concentration by instant source

○ Time required for complete transverse mixing.

$$T_t = 0.4\frac{W^2}{\varepsilon_t}$$

$$= 0.4\frac{(3125)^2}{0.36} = 125 \text{ day}$$

→ Actual transverse mixing time is much less than 125 days because of larger scale circulations.