Chapter 3. Power-spectrum estimation for sensing the environment

in Cognitive Dynamic Systems, S. Haykin

Course: Autonomous Machine Learning

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Outline

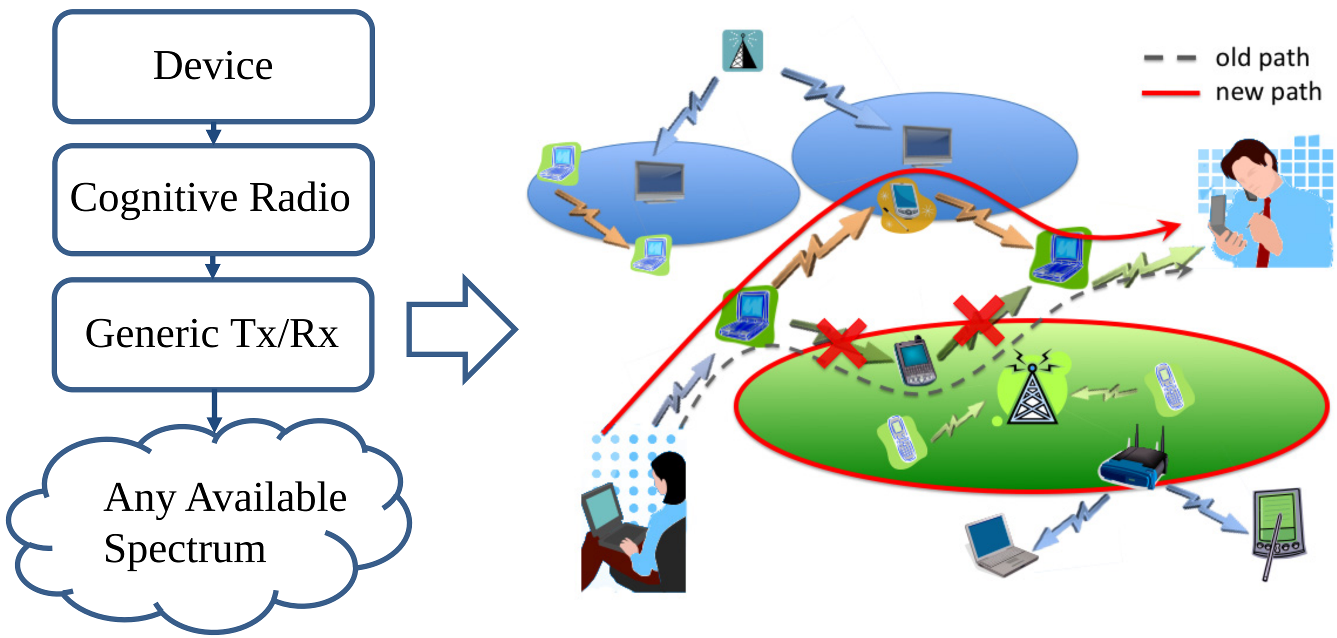
- Overview
  - Power Spectrum Sensing
- Time-Frequency Analysis
  - Loève spectral correlation framework
  - Fourier spectral correlation framework
- Loève Vs Fourier framework
- Summary

Background

Scarce Spectrum Problem

- Why do we need **power spectrum sensing**?
- Goal:

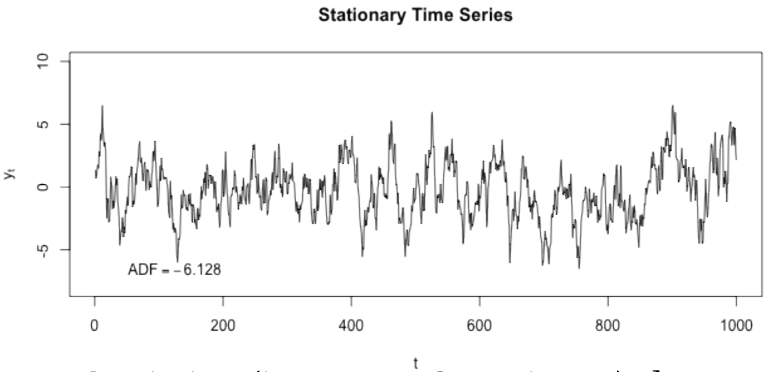
1. Use of vacant bandwidth
2. Sharing of bandwidth with adaption



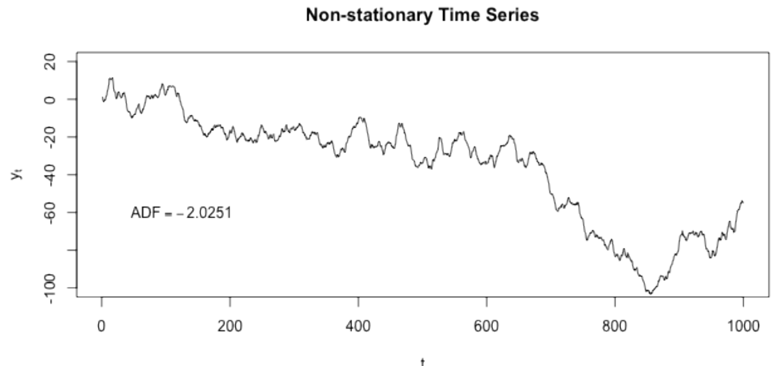
Power Spectrum Sensing

- Three essential dimensions of sensing:

Frequency	• To identify location of spectrum holes along the frequency axis
Space	• To “sniff” the RF environment along different directions
Time	• To describe the process whether stationary or non-stationary



Statistics (i.e. mean & variance) does not change over time! ☺



Statistics changes over time! ☹

Recap (Perception-Action cycle)

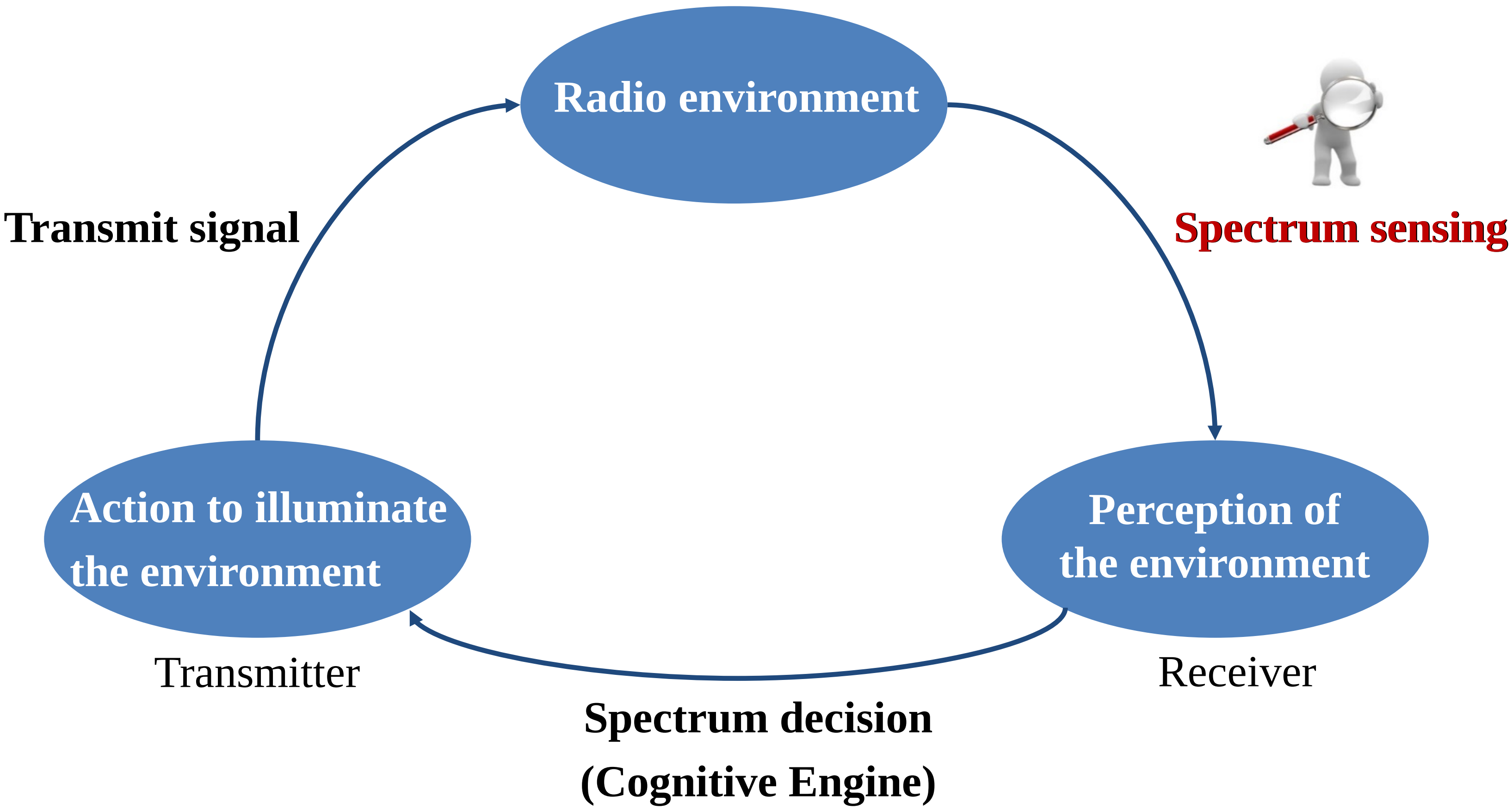
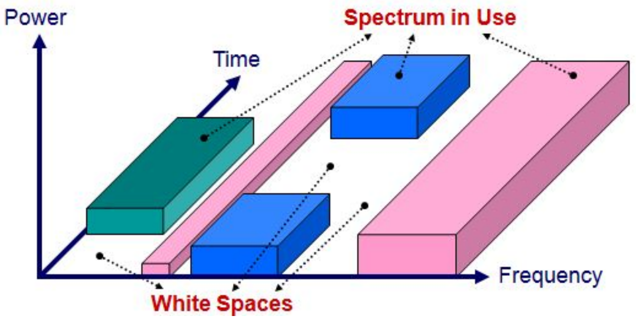


Figure. Role of power spectrum sensing in cognitive radio networks

Time-Frequency Analysis

- Three typical theories of sensing:
  - Stationary → Wiener-Kolmogorov theory
  - Non-stationary → Loève theory
  - Cyclostationary → Fourier theory



- Overview of the theories:

Wiener-Kolmogorov	• Compute a statistical estimate of underlying process using a related signal as an input
Loève	• Model the underlying process in probabilistic approach
Fourier	• Model the underlying process in either probabilistic or deterministic approach

Traditional Loève Theorm

- For a *complex continuous stochastic* process which is *harmonizable*, we can write the **sample function** as:

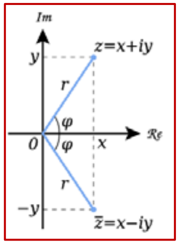
$$x(t) = \int_{-1/2}^{1/2} e^{j2\pi v t} dX(v)$$

frequency variable

polynomial for transfer function

correlation between two samples: $x(t_1)$, $x(t_2)$

complex conjugation



- **Covariance function** in the time domain is defined as:

$$\Gamma_L(t_1, t_2) = E[x(t_1)x^*(t_2)]$$

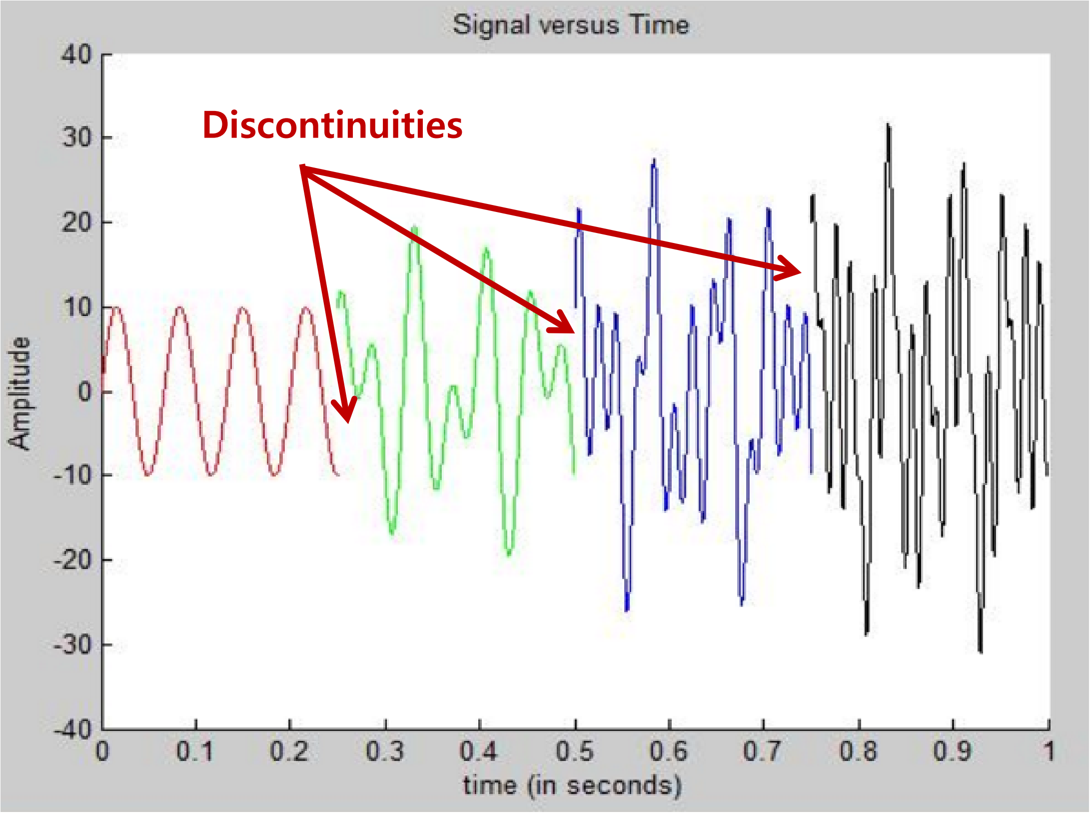
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{j2\pi(t_1 f_1 - t_2 f_2)}\, \gamma_L(f_1, f_2)\, df_1 df_2$$

- **Loève spectrum** for two frequencies in a non-stationary process is defined as:

$$\gamma_L(f_1, f_2)\, df_1 df_2 = E[dX(f_1)dX^*(f_2)]$$

Continuity Problem

- Both the spectrum $\gamma_L(f_1, f_2)$ and covariance function $\Gamma_L\ (t_1, t_2)$ may include discontinuities:



Dealing with the Continuity Problem (1/2)

- Solution – Rotate both the time and frequency coordinates by 45°.

① Define new time coordinates as center $\mathbf{t_0}$ and delay $\boldsymbol{\tau}$

$$t_1 + t_2 = 2t_0 \rightarrow t_1 = t_0 + \tau/2$$
$$t_1 - t_2 = \tau \rightarrow t_2 = t_0 - \tau/2$$

② Define new frequency coordinates as center **f** and delay **g**

$$f_1 + f_2 = 2f \rightarrow f_1 = f_0 + g/2$$
$$f_1 - f_2 = g \rightarrow f_2 = f_0 - g/2$$

- Now, the **covariance function** of the time domain can be redefined.

$$\Gamma_L(t_1, t_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{j2\pi(t_1 f_1 - t_2 f_2)}\,\gamma_L(f_1, f_2)\,df_1 df_2$$

$$\Gamma_L(t_0, \tau) = \int_{-\infty}^{\infty}\left\{\int_{-\infty}^{\infty} e^{j2\pi(t_0 g - \tau f)}\right\}\gamma_L(f, g)\,df\,dg$$

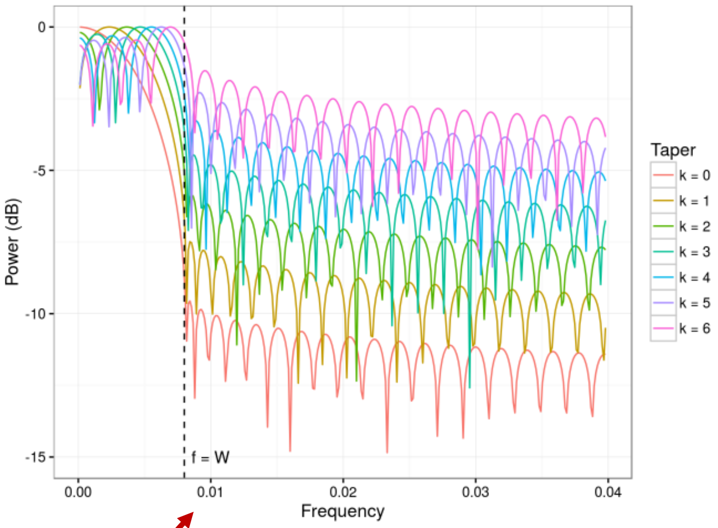
Dealing with the Continuity Problem (2/2)

- So far so good?
  - Covariance function $\Gamma_L\ (t_0, \tau)$: **Solved** with the shiftings! ☺
  - Spectrum $\gamma_L(f, g)$: Discontinuity **problem at g = 0** still remains! ☹

- Solution:
  - Transform rapid variations expected around g = 0 into a slowly varying function t_0.
  - Instead of generalized spectral density 'γ', use **dynamic spectrum** by applying the inverse Fourier transform to γ(f,g).

$$D\ (t_0, \mathrm{f}) = \int_{-\infty}^{\infty} e^{j2\pi t_0 g}\ \gamma_L(f, g)\ dg$$

Spectral coherences of non-stationary processes



- <u>Details:</u>
  - Let $X_k(f_1)$ and $X_k(f_2)$ denotes the multitaper Fourier transforms of the sample function x(t), where k = k^{th} Slepian taper.
  - Apply on traditional Loève theorem:

complex conjugation

$$\gamma_L(f_1, f_2) = \frac{1}{K} \sum_{k=0}^{K-1} X_k(f_1)\, X_k^*(f_2)$$

  - **Problem – Insufficient for a complete second-order description!**

Second-order statistics of Loève spectrum

- Key Idea:
  - Use “inner” and “outer” subscripts to distinguish between spectral correlations

I. $\gamma_{L,inner}$ = Loeve spectrum of the 1st kind

II. $\gamma_{L,outer}$ = Loeve spectrum of the 2nd kind (get rid of complex conjugation)

- Redefine the estimate of Loève spectral correlation:

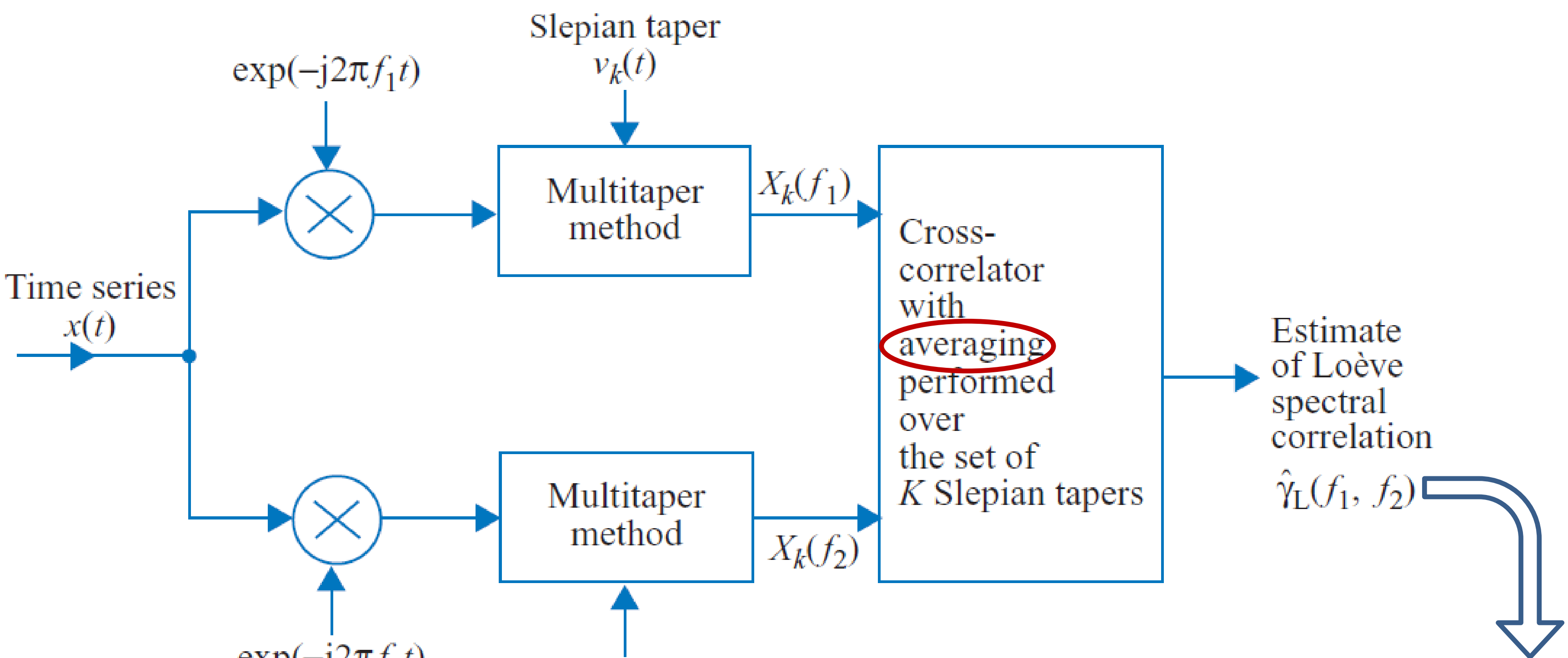
complex conjugation

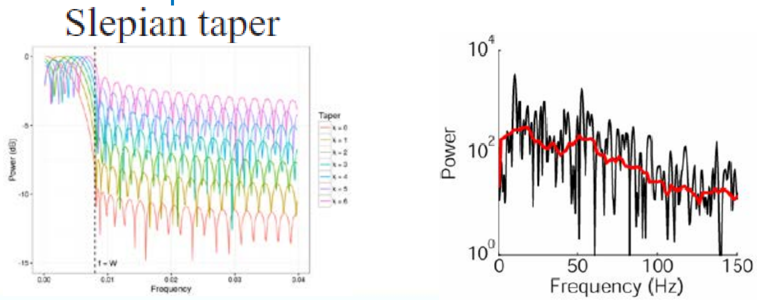
$$\gamma_{L,inner}(f_1, f_2) = \frac{1}{K}\sum_{k=0}^{K-1} X_k(f_1)\, X_k^*(f_2)$$

no complex conjugation

$$\gamma_{L,outer}(f_1, f_2) = \frac{1}{K}\sum_{k=0}^{K-1} X_k(f_1)\, X_k(f_2)$$

Framework for Loève spectral correlation





Cross-correlation	Function used
1st kind	$\gamma_{L,inner}(f_1, f_2)$
2nd kind	$\gamma_{L,outer}(f_1, f_2)$

Cyclostationarity

- A **non-stationary** process that does ***not*** include a **trend-like** behavior.
- The **statistics** of a stochastic process **vary *cyclically* with time**.

- <u>Details</u>:

$\Gamma_L\ (t_0, \tau)$

  - Let T_0 = the period of x(t).
  - Autocorrelation = *correlation* of a signal with itself at different points of time
  - Cyclostationary Process

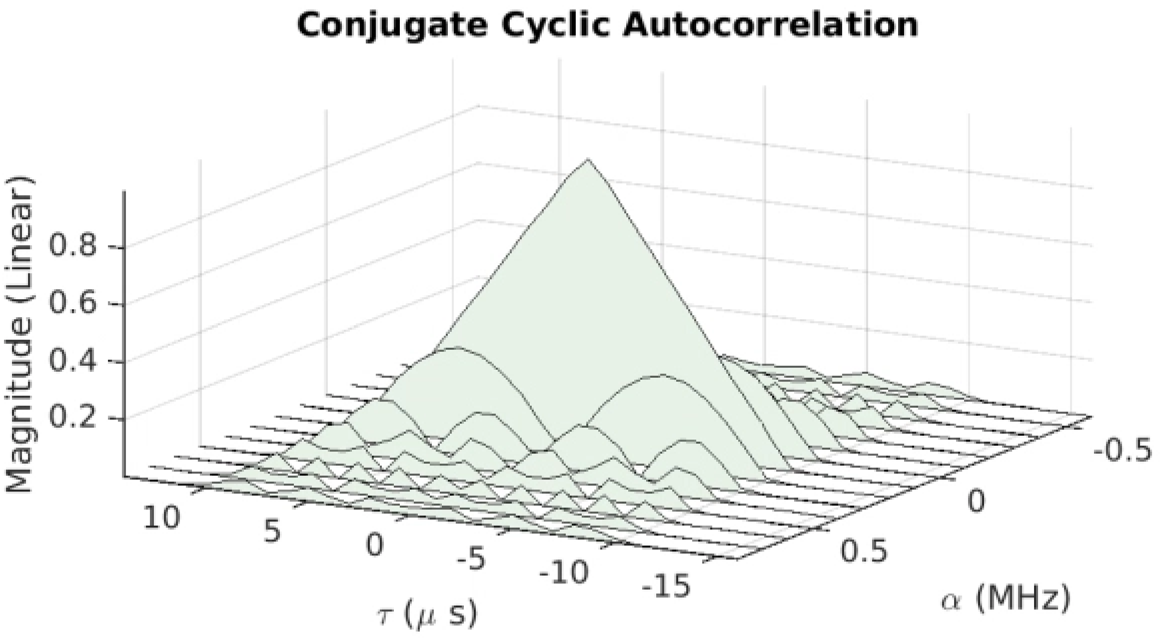
⇔The **autocorrelation** sequence $\mathbf{R_x(.)}$ is periodic with the same period T_0:

$$R_x\left(t + T_0 + {}^{\tau}/_{2}, t + T_0 - {}^{\tau}/_{2}\right) = R_x\left(t + {}^{\tau}/_{2}, t - {}^{\tau}/_{2}\right)$$

⇔ **Mean** is also periodic with the same period T_0.

An example of a Cyclostationary Process

- A cyclostationary process can be viewed as multiple interleaved stationary processes.



Fourier framework of cyclic statistics

- Define the *inner* and *outer* cyclic power spectrums using Fourier theory.

- Details:
  - $\alpha = n/T_0$ is infinite set of frequencies where T_0 = period, n = 0, 1, 2,….
  - Cyclic power spectrums:

polynomial for transfer function

$$S_{inner}(t,f) = \sum_{\propto} s_{inner}^{\propto}(f)\, e^{j2\pi\propto t}\,, \quad S_{outer}(t,f) = \sum_{\propto} s_{outer}^{\propto}(f)\, e^{j2\pi\propto t}$$

  - Fourier coefficients for varying α:

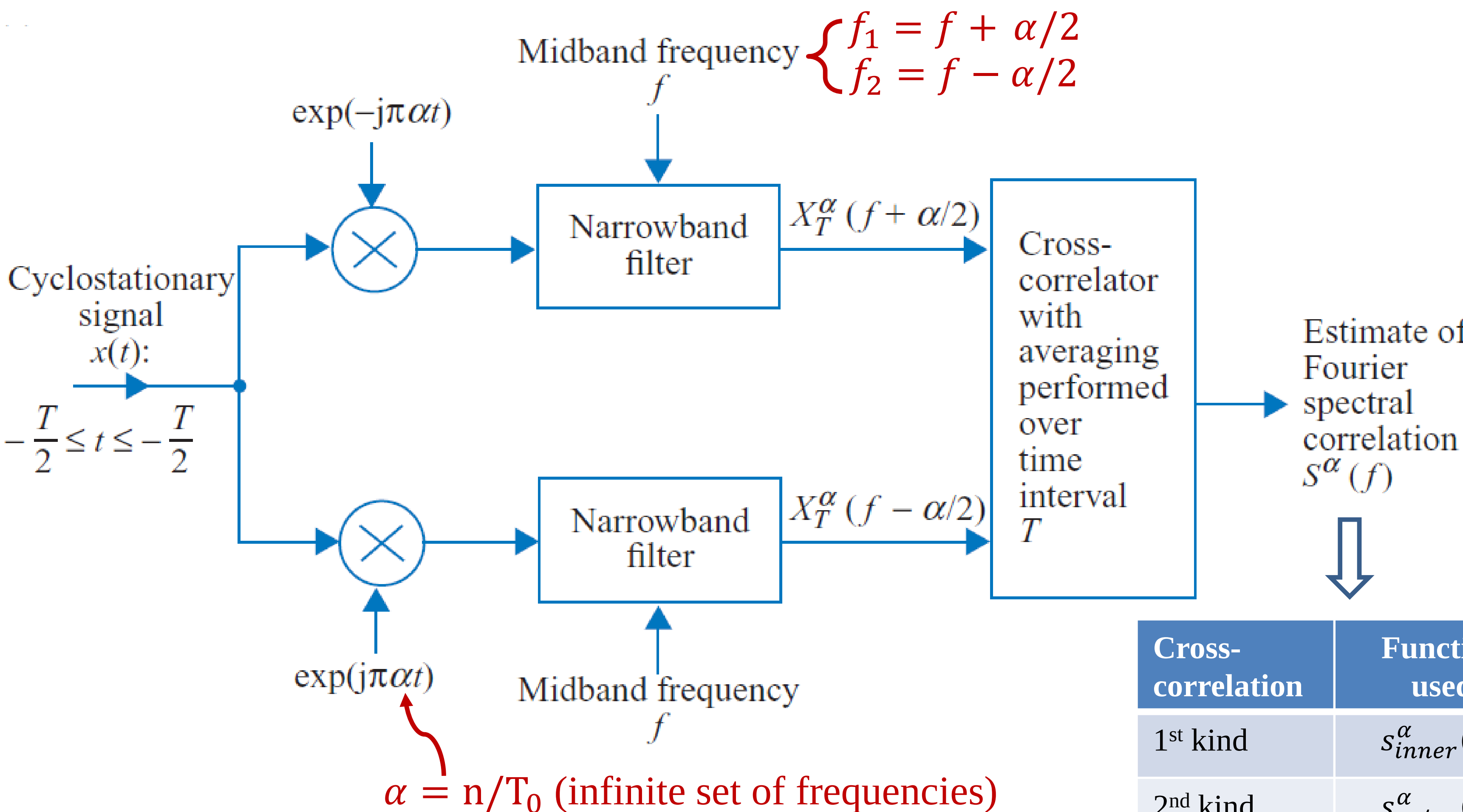
complex conjugation

$$s_{inner}^{\alpha}(f) = \lim_{T\to\infty}\lim_{\Delta t\to 0}\frac{1}{\Delta t}\int_{-\Delta t/2}^{\Delta t/2}\frac{1}{T}\{X_T(t,f+\alpha)\, X_T^{*}(t,f-\alpha)\}\quad dt$$

no complex conjugation

$$s_{outer}^{\alpha}(f) = \lim_{T\to\infty}\lim_{\Delta t\to 0}\frac{1}{\Delta t}\int_{-\Delta t/2}^{\Delta t/2}\frac{1}{T}\{X_T(t,f+\alpha)\, X_T(t,f-\alpha)\}\quad dt$$

Framework for Fourier spectral correlation



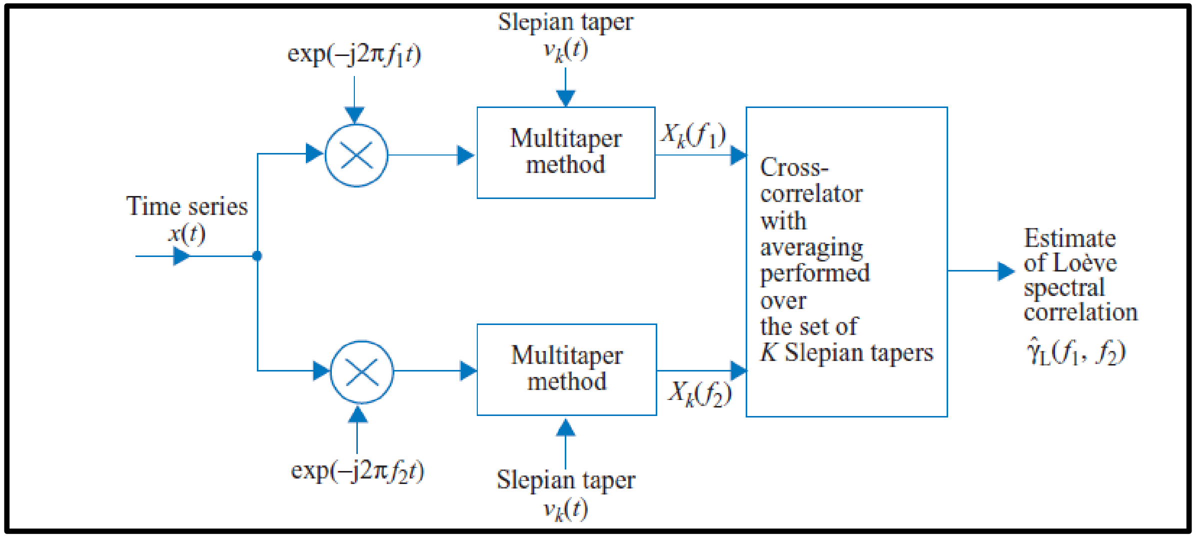
Cross-correlation	Function used
1st kind	$s_{inner}^{\alpha}(f)$
2nd kind	$s_{outer}^{\alpha}(f)$

Fourier Vs Loève Framework

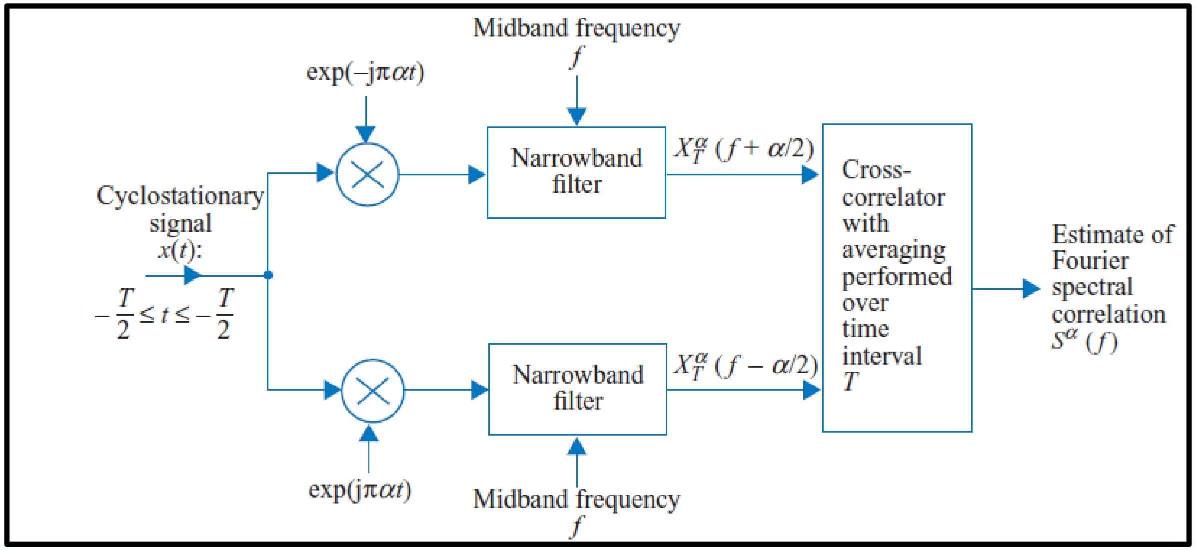
	Loève	Fourier
Type of process	Non-stationary	Cyclostationary
Multiplying factor	$e^{-j2\pi f_1 t}$ $e^{-j2\pi f_2 t}$	$e^{j2\pi\alpha t}$ $e^{-j2\pi\alpha t}$
Power Spectrum Estimation Method	Multi Taper Method (MTM)	Narrowband filter
Cross-correlator input	$X_k(f_1)$ $X_k(f_2)$	$X_T(f+\alpha/2)$ $X_T(f-\alpha/2)$
Cross-correlator output	$\gamma_L(f_1, f_2)$	$s^{\alpha}(f)\begin{cases} f_1 = f+\alpha/2 \\ f_2 = f-\alpha/2 \end{cases}$

Fourier Vs Loève Framework (2/2)

- **Loève** ⇨



- **Fourier** ⇨



Summary

- Spectrum becomes a **scarce resource** due to the rapid growth of demand in wireless communication.
- **Cognitive radio** introduces an efficient use of the unlicensed bands by **sensing the spectrum holes**.
- This chapter introduced **two theories** for sensing the environment:
  - Loève – Any non-stationary process
  - Fourier – Cyclostationary process

Thanks for your attention ☺