Ch3. Power-spectrum estimation for sensing the environment (1/2)

in Cognitive Dynamic Systems, S. Haykin

Course: Autonomous Machine Learning

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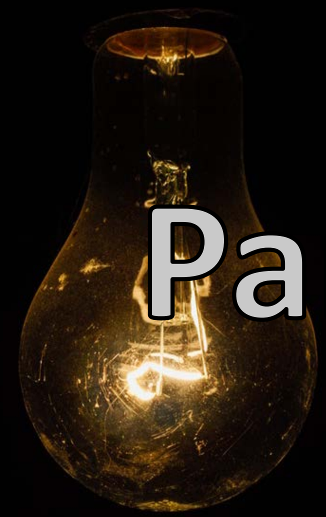


Part A

Review : Sensing the environment

Ill-posed inverse problem

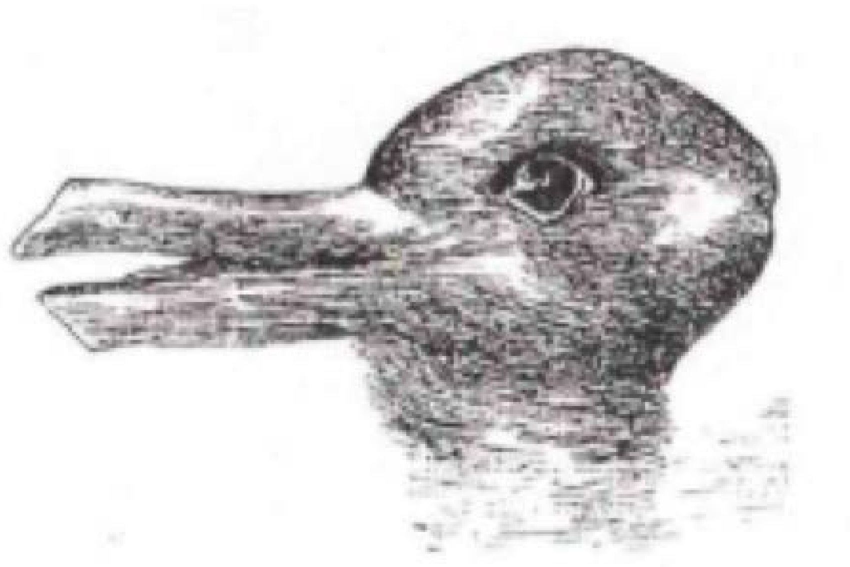
- Two views of perception : ill-posed inverse problem, Bayesian inference

- Ill-posed problem : have lack of well-posed conditions
  (existence, uniqueness, continuity)

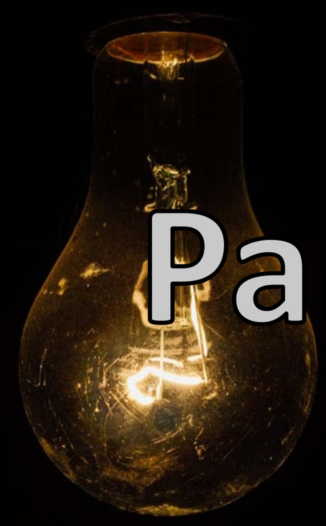
- Inverse problem : uncover underlying physical laws from the stimuli
  = find a mapping from stimuli to the state

Sensing the environment

- Solving an ill-posed inverse problem



- Power spectrum estimation is an example of sensing the environment



Part B

Power spectrum

Part B

What is power spectrum?

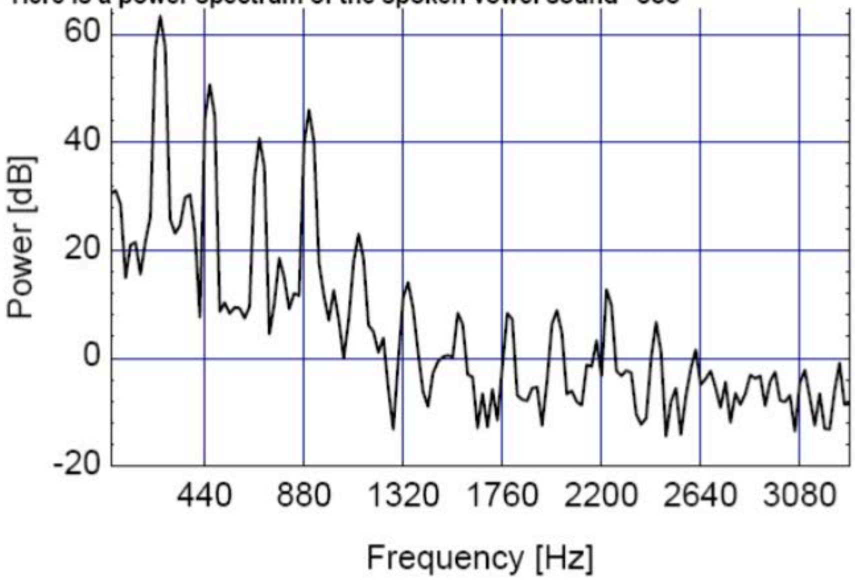
- The average power of the incoming signal expressed as a function of frequency
- Signal is measured by sensors as a time series(time domain)
- Power spectrum focus on signal power on frequency(frequency domain)

Why power spectrum estimation is useful?

Part B

- Impossible to predict future signal
- Estimate the shape of the signal by only finite set of data
- Ex : Radar, Radio, Speech Recognition

Figure 1:
Here is a power spectrum of the spoken vowel sound "ooo"



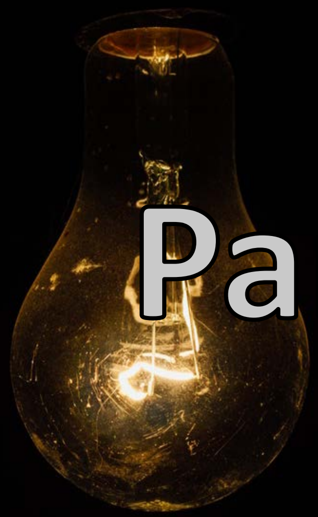
Part B

Power spectrum in a theory

$$S(f) = \int_{-\infty}^{\infty} s(t) \cdot e^{-i2\pi ft} dt.$$

Fourier transform

- Incoming signals are picked up as a stochastic(random) process
- Only finite set of data is used(applying theory is impossible)
- Power spectrum is an ill-posed inverse problem



Part C

Power spectrum estimation

Power spectrum Estimation

- Parametric methods(model-based)

- Non-parametric methods(model-free)

Parametric method example

Build a model $H(e^{j2\pi f})$

- H is a function to convert time domain signals to frequency domain signals

- Apply prior knowledge of Fourier transform(assume $e^{j2\pi f}$ relationship)

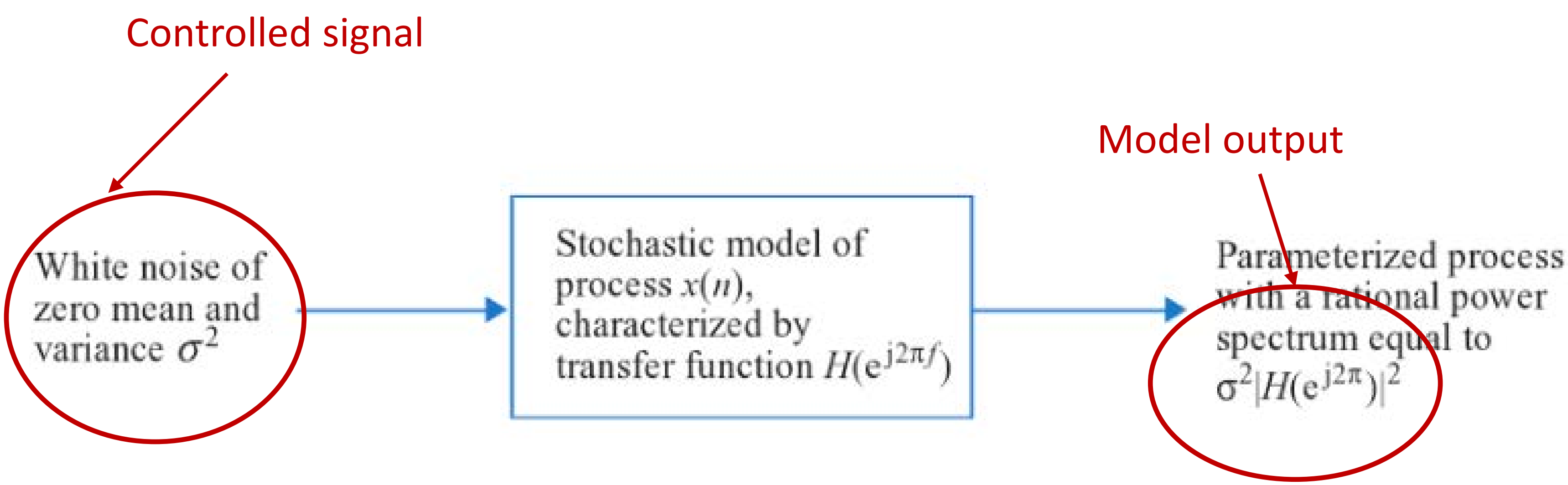
- H contains some parameters/coefficients

Parametric method example

Controlled input signal

- Controlled signal is given -> input signal power on frequency is known value

- Model output is only depends on the model, H



Parametric method example

Problem : Finding parameters/coefficient of H

- Finding parameters of H to produce acceptable output

- Finding parameters is a typical machine learning problem

Part C

Parametric method example

After finding good parameters of H

- Model is built! , problem is solved

- Use the model as an estimator

Power spectrum Estimation

- Parametric methods(model-based)

  If model is good : produce good quality estimates with less sample data

  If model is bad : mislead conclusion

- Non-parametric methods(model-free)

Nonparametric method(periodogram)

$x(n) \rightarrow X_N(f) \rightarrow S(f)$ (Fourier transform)

$$S(f) = \lim_{n\to\infty} \frac{1}{N} E[|XN(f)|]^{2}$$

Only finite set of signal data is used in practice

Use estimates of $S(f)$, $\hat{S}(f) = \frac{1}{N}|X_N(f)|2$

Part C

Nonparametric method(periodogram)

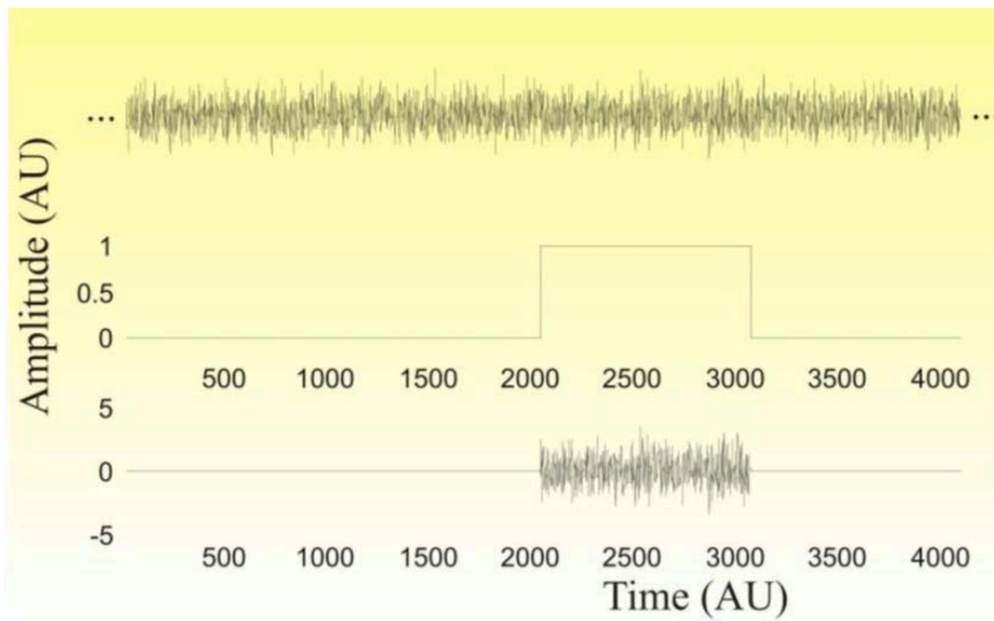
$$\hat{S}(f) = \frac{1}{N}|X_N(f)|2$$

Same effect as

$$Define. a(n) = 1 \; if \; n \; is \; in \; bound \; of \; N, 0 \; otherwise$$

$$x(n) \times a(n) \rightarrow x^*(n) \rightarrow X_N{}^*(f) \rightarrow \hat{S}(f)$$ (Fourier transform)

$a(n)$ is taper



Nonparametric method(periodogram)

Not require any model to develop

Directly calculate the value using Fourier transform with taper

Part C

Bias-variance dilemma with taper

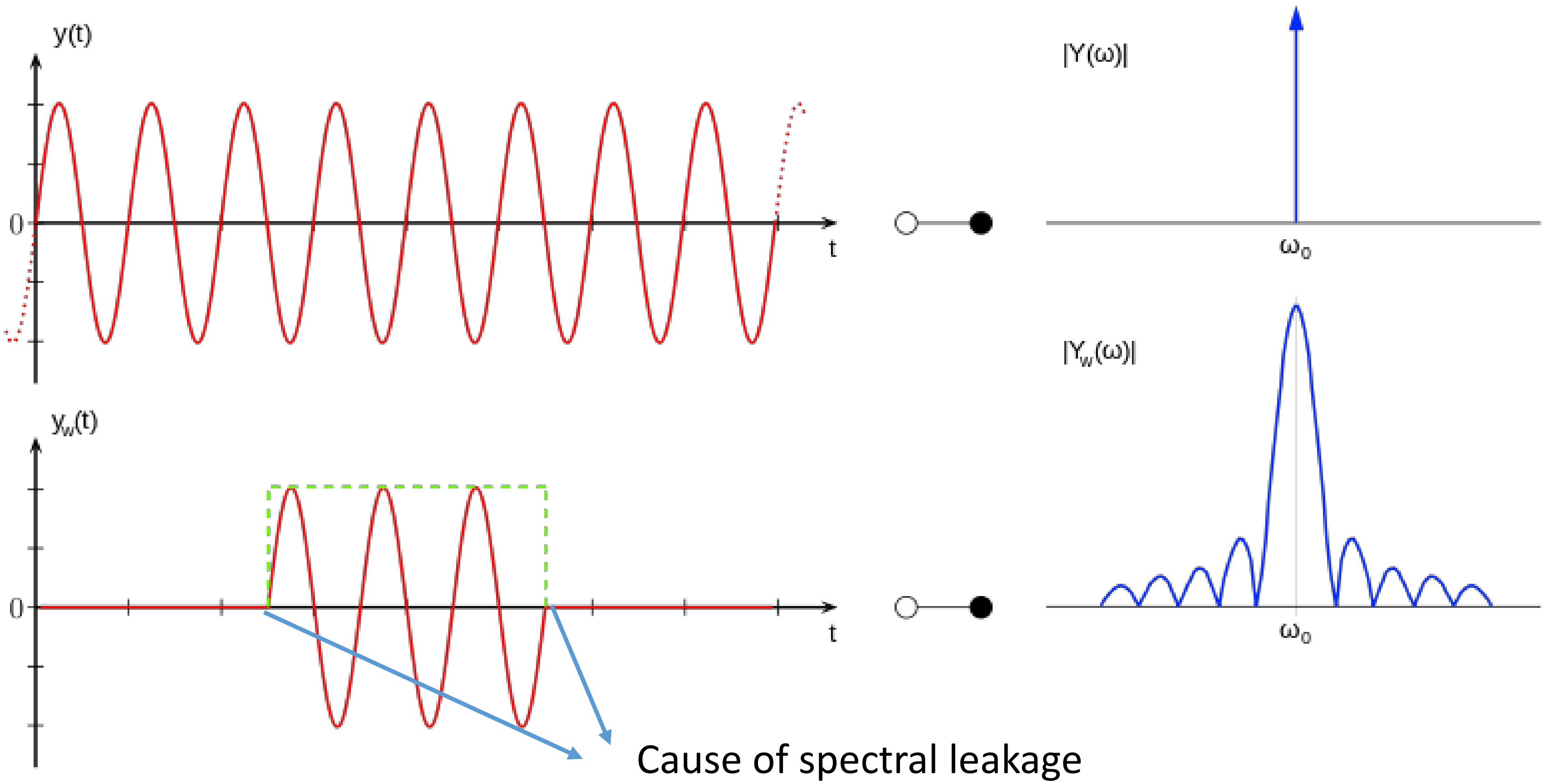
2 Conditions for good estimator

1. Low bias

2. Low variance(not noisy)

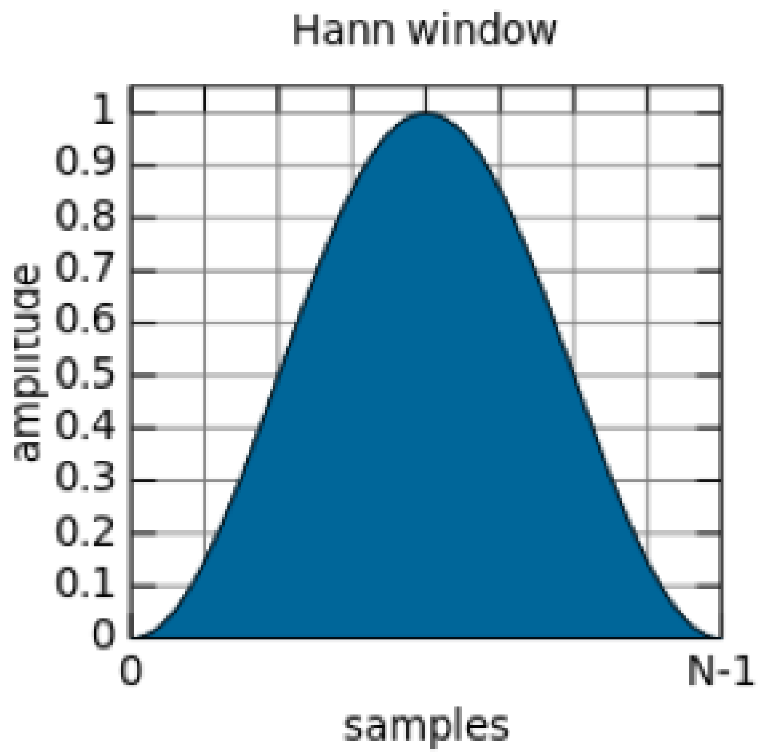
Bias-variance dilemma with taper

Bias from spectral leakage



Bias-variance dilemma with taper

Spectral leakage↓ by taper(not rectangle) -> bias↓

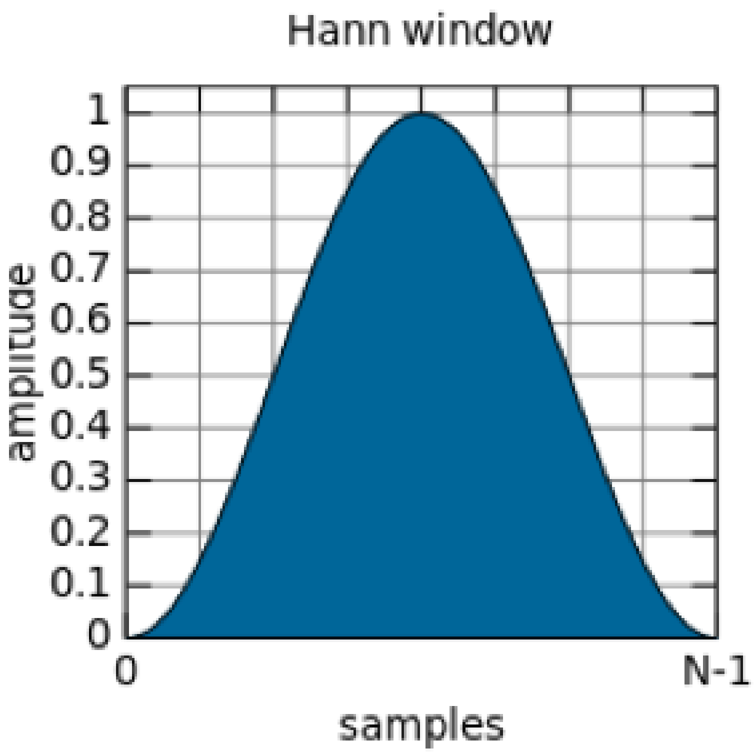


Bias-variance dilemma with taper

bias↓ -> variance↑

Taper-> reduce effective sample size -> variance ↑

Bias-variance trade-off!



Part D

Multitaper method

Multitaper method

Address the trade-off using multiple tapers instead of one

K tapers are used on the same sample data

K set of estimates are created

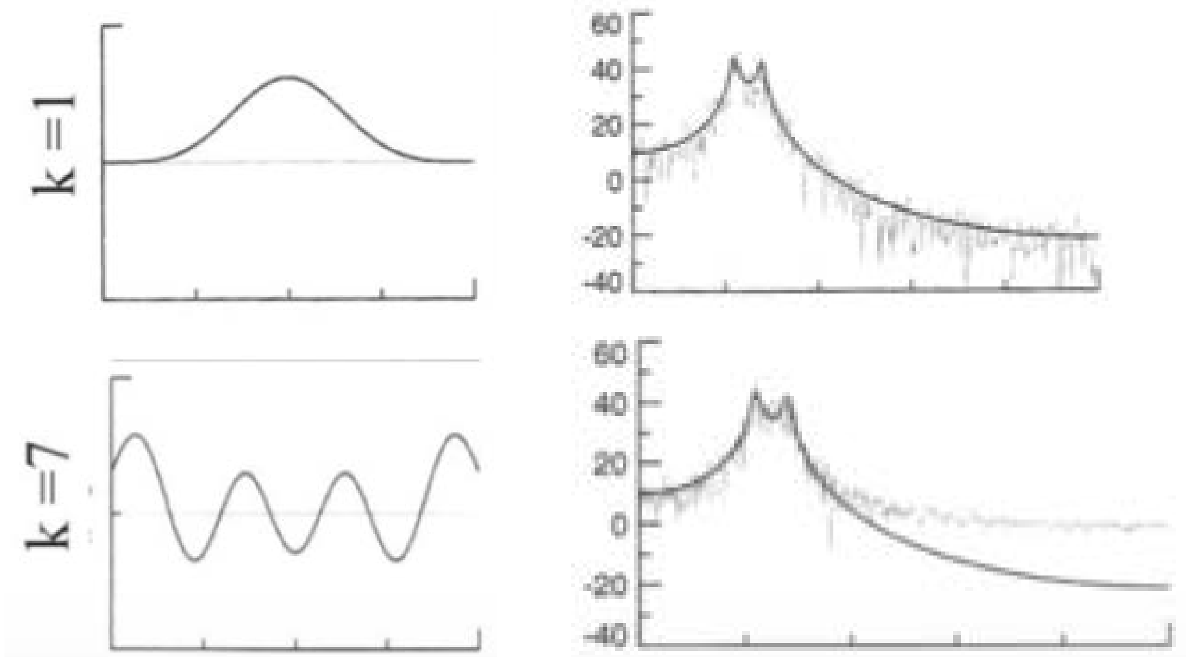
Average K set of estimates

$$\bar{S}(f) = \frac{1}{K} \sum_{k=1}^{K} \hat{S}_k(f).$$

Tapers are Slepian sequences

Tapers following Slepian sequences

kth taper : $k \uparrow \rightarrow bias \uparrow, variance \downarrow$



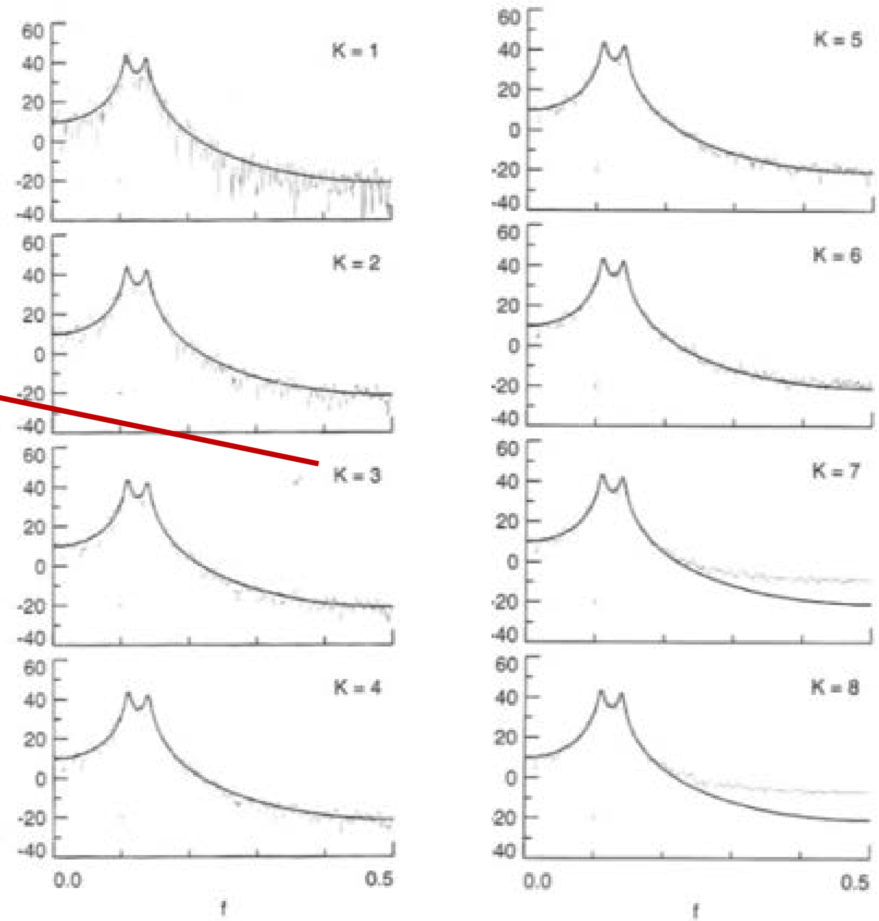
$\widehat{S}_1(f)$

$\widehat{S}_7(f)$

Multitaper method

All K estimates are averaged

Average of
$\widehat{S}_1(f), \widehat{S}_2(f), \widehat{S}_3(f),$



Part D

Multitaper method

All K estimates are averaged

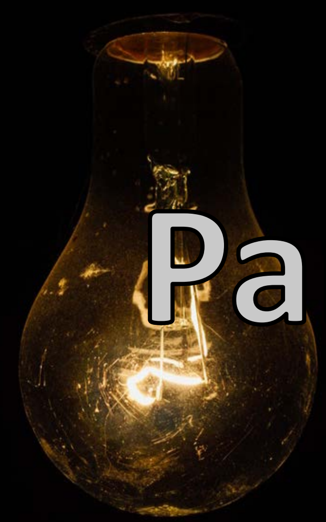
Find appropriate K to compromise bias and variance

Why multitaper method is good?

Reduce bias by tapers

Reduce variance by increasing effective sample size(N X K)

Easy to solve bias-variance trade-off by choosing K



Part E

Multitaper method in space

Multitaper method in space

Part E

Previous methods consider power of signal on frequency

Extended multitaper method to get power of signal on frequency and space

Multitaper method in space

$$\mathbf{A}(f)=\begin{bmatrix} a_0^{(0)}X_0^{(0)}(f) & a_1^{(0)}X_1^{(0)}(f) & \dots & a_{K-1}^{(0)}X_{K-1}^{(0)}(f) \\ a_0^{(1)}X_0^{(1)}(f) & a_1^{(1)}X_1^{(1)}(f) & \dots & a_{K-1}^{(1)}X_{K-1}^{(1)}(f) \\ \vdots & \vdots & & \vdots \\ a_0^{(M-1)}X_0^{(M-1)}(f) & a_1^{(M-1)}X_1^{(M-1)}(f) & \dots & a_{K-1}^{(M-1)}X_{K-1}^{(M-1)}(f) \end{bmatrix},$$

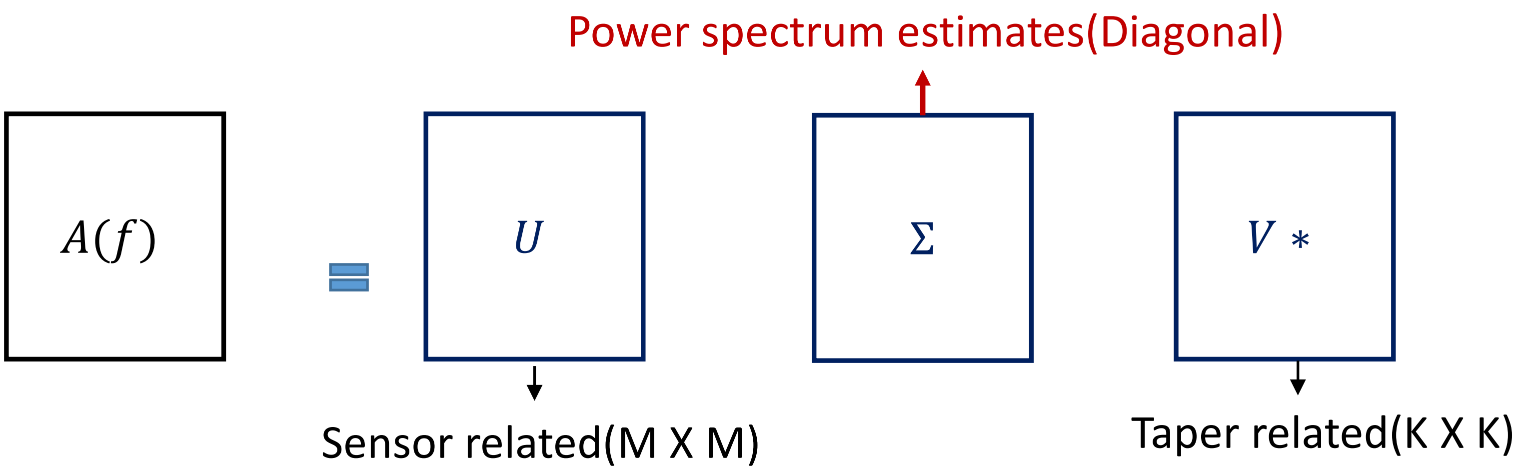
${X^m}_k(f)$: Fourier transform of input signal by m-th sensor

${a^m}_k$: coefficients accounting for different localized area around the gridpoints

Multitaper method in space

$$\mathbf{A}(f)=\begin{bmatrix} a_0^{(0)}X_0^{(0)}(f) & a_1^{(0)}X_1^{(0)}(f) & \dots & a_{K-1}^{(0)}X_{K-1}^{(0)}(f) \\ a_0^{(1)}X_0^{(1)}(f) & a_1^{(1)}X_1^{(1)}(f) & \dots & a_{K-1}^{(1)}X_{K-1}^{(1)}(f) \\ \vdots & \vdots & & \vdots \\ a_0^{(M-1)}X_0^{(M-1)}(f) & a_1^{(M-1)}X_1^{(M-1)}(f) & \dots & a_{K-1}^{(M-1)}X_{K-1}^{(M-1)}(f) \end{bmatrix},$$

Apply singular value decomposition to matrix A



Thank You!