DYNAMIC PROGRAMMING FOR ACTION IN THE ENVIRONMENT

Cognitive Dynamic Systems, *Simon Haykin*

Presented by

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THE BIG PICTURE

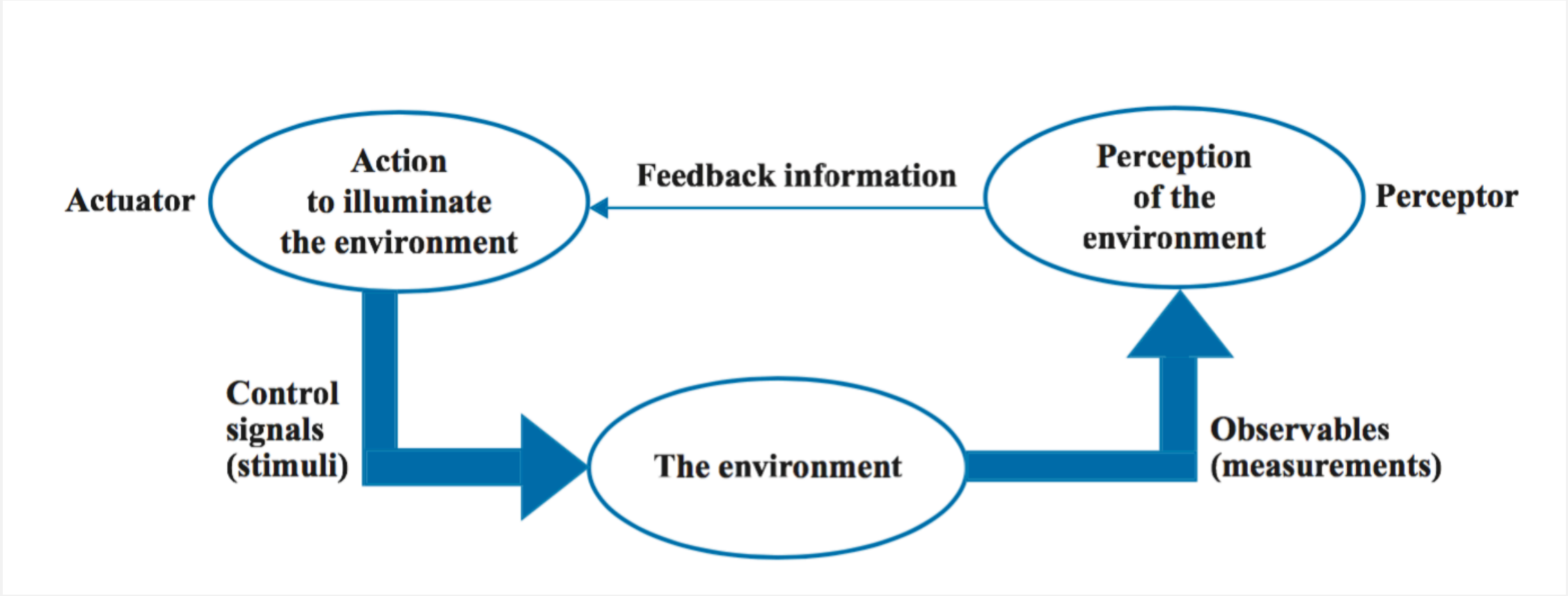
COGNITIVE DYNAMIC SYSTEMS

Build up rules of behavior over time through

learning from continuous experiential interactions with the environment,

and thereby deal with environmental uncertainties.

THE PERCEPTION-ACTION CYCLE



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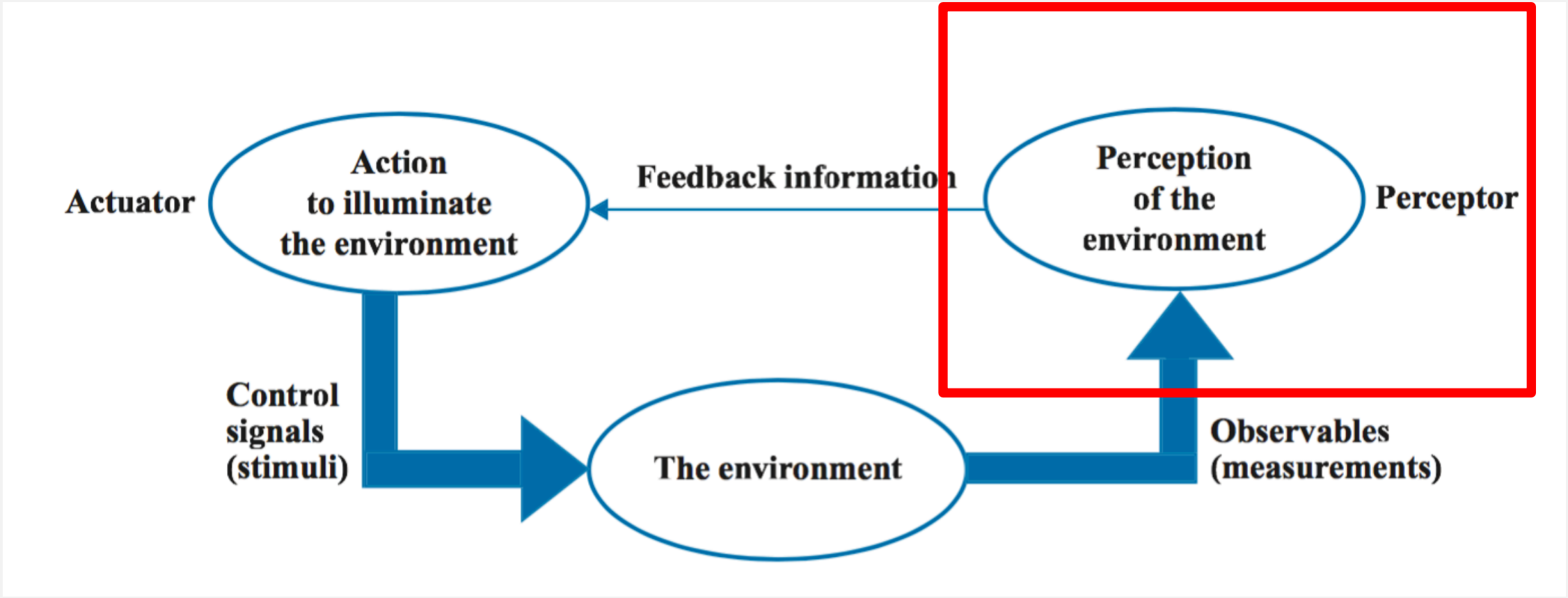

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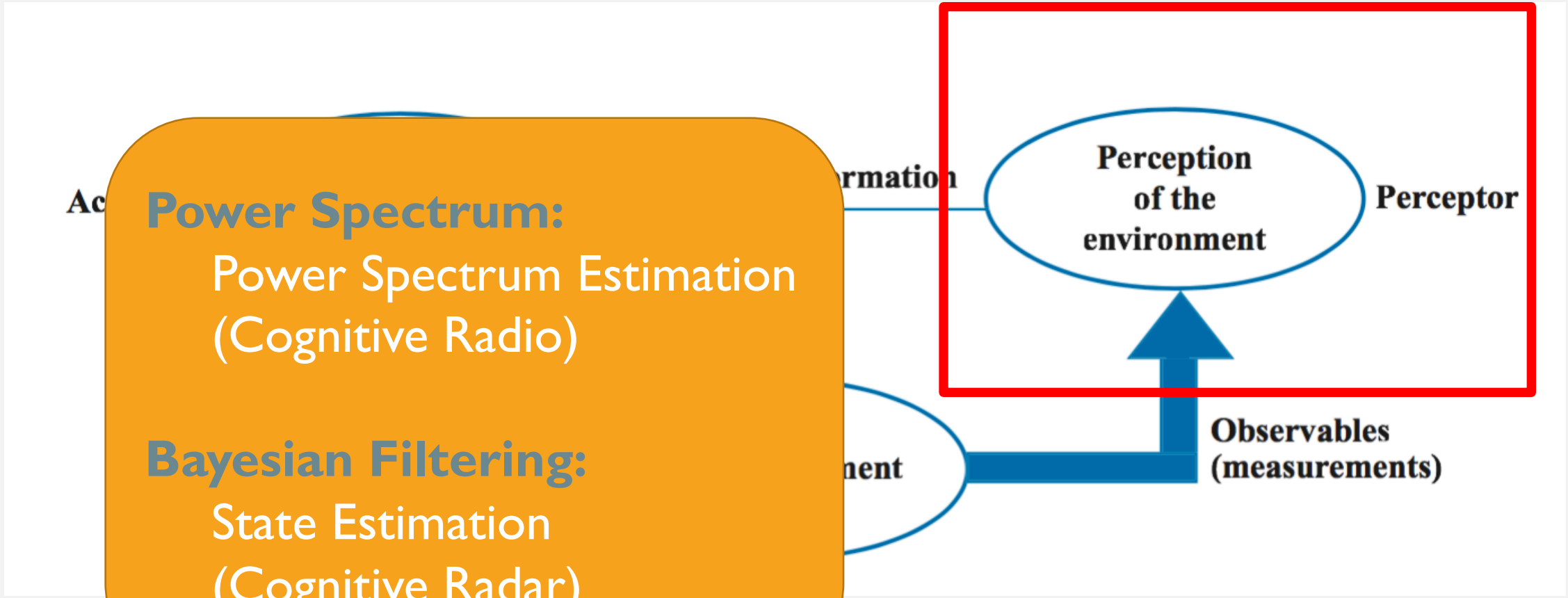


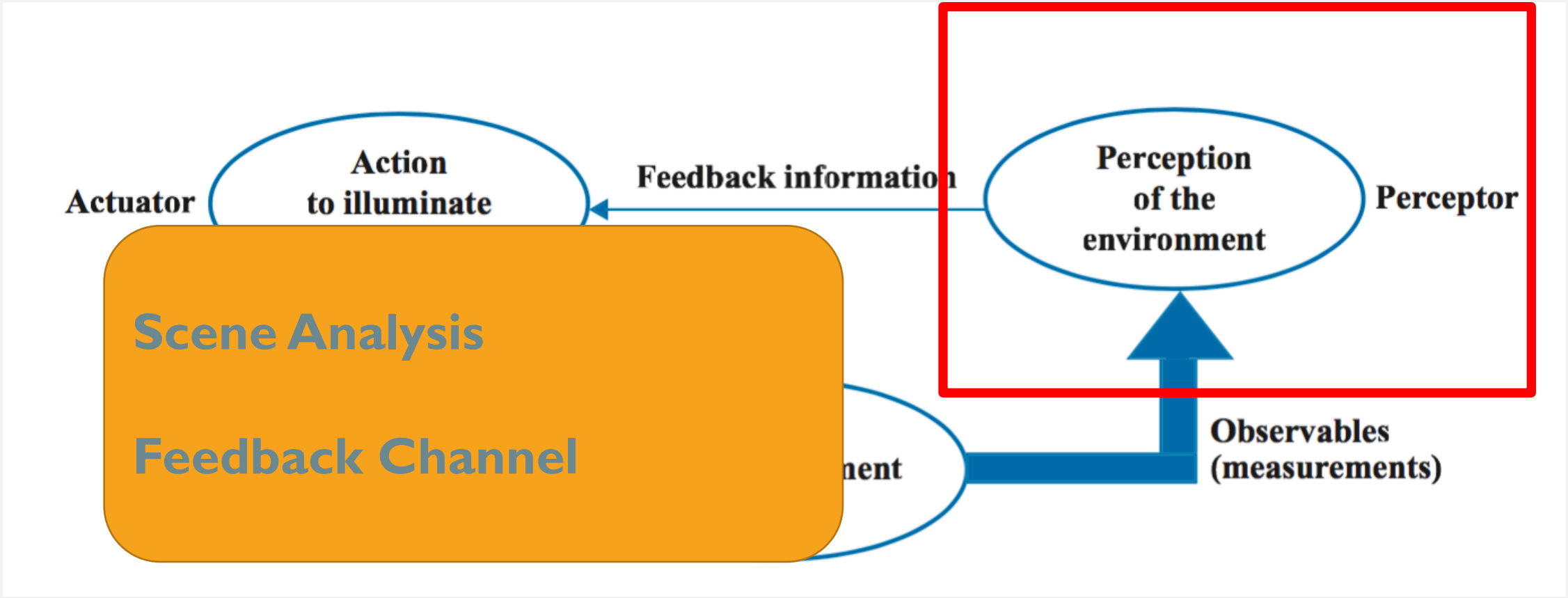
Power Spectrum:

Power Spectrum Estimation
(Cognitive Radio)

Bayesian Filtering:

State Estimation
(Cognitive Radar)

THE PERCEPTION-ACTION CYCLE



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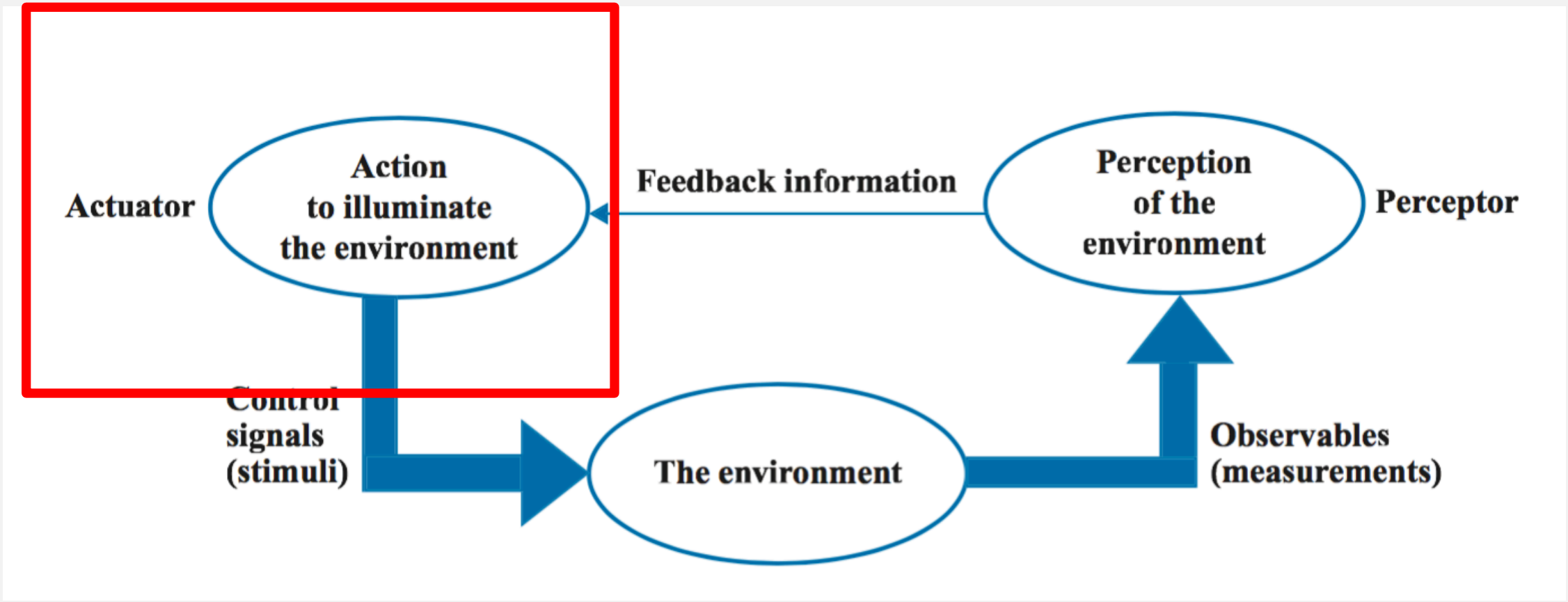


DYNAMIC PROGRAMMING

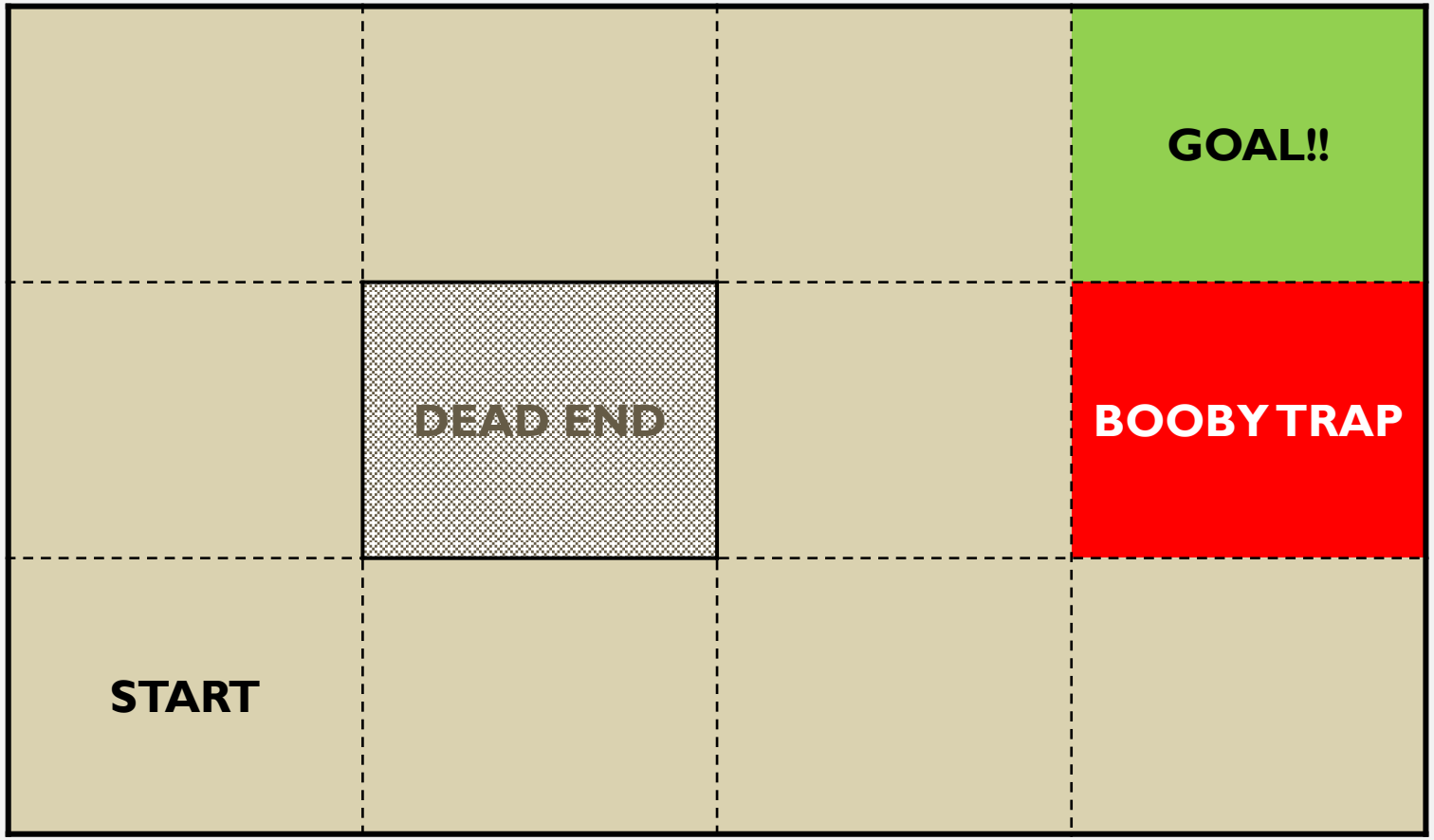
WHAT IS DYNAMIC PROGRAMMING?

- *Dynamic programming* is a technique that deals with situations where decisions are made in stages (i.e. different time steps), with the outcome of each decision being *predictable* to some extent before the next decision is made.
- A key aspect of such situations is that decisions cannot be made in isolation. Rather, the desire for a low cost at the present is balanced against the undesirability of a high cost in the future.

DYNAMIC PROGRAMMING

- Markov Decision Processes
- Bellman’s Optimality Criterion
- Policy Iteration
- Value Iteration

MARKOV'S DECISION PROCESS



You can move UP, DOWN, LEFT, RIGHT

DETERMINISTIC VS. STOCHASTIC

Deterministic

- Every state can be uniquely determined with model parameters and previous states.
- Always perform identically for a given set of initial conditions.

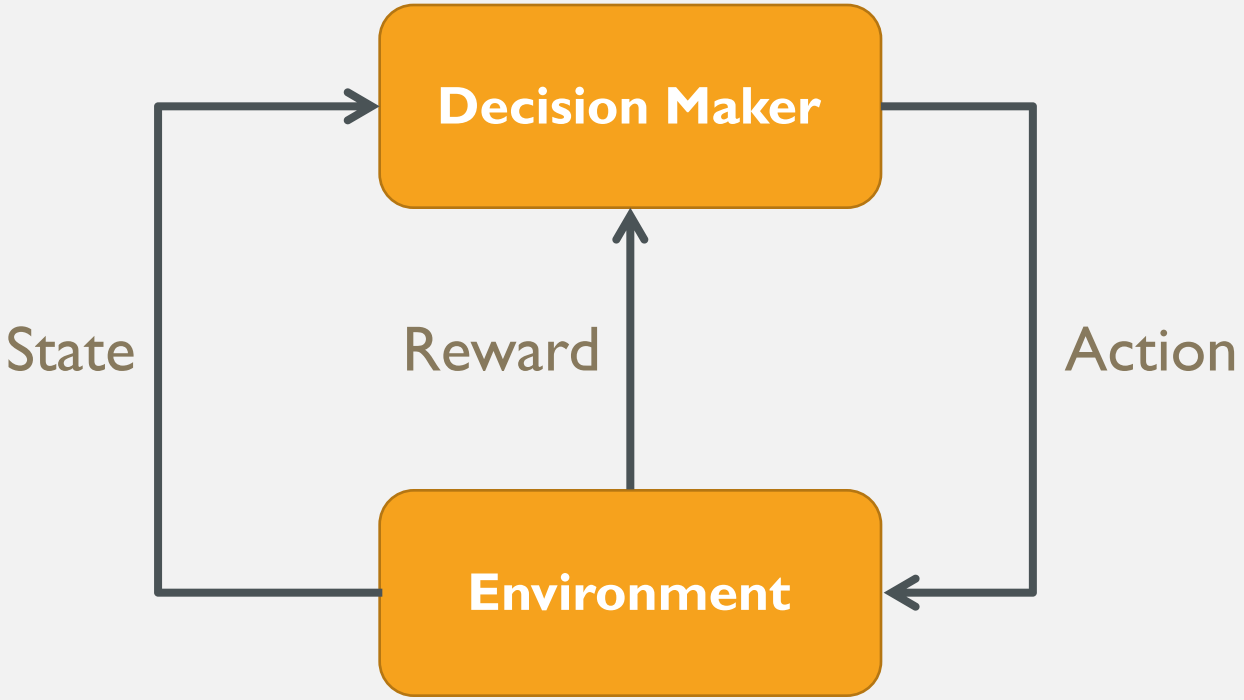
Stochastic

- States determined by probability distributions.
- Such probability distributions include a notion of randomness.

MARKOV'S DECISION PROCESSES

The Actuator's Decision Making Process

Discrete-time & Stochastic



- Set of States: $\boldsymbol{S} = \{\boldsymbol{s}\}$
- Set of Actions: $\boldsymbol{A} = \{\boldsymbol{a}\}, \boldsymbol{a}: \boldsymbol{S} \rightarrow \boldsymbol{S}$
- Reward Function: $\boldsymbol{R}(\boldsymbol{s})$
- Transition Model:

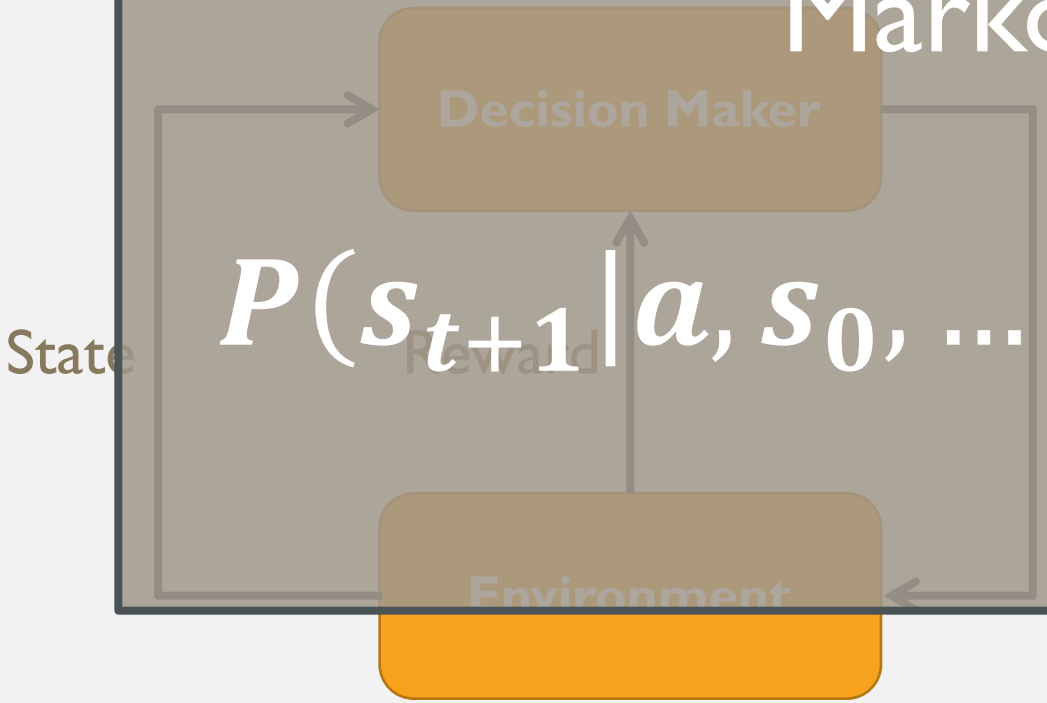
  $\boldsymbol{T}(\boldsymbol{s}, \boldsymbol{a}, \boldsymbol{s}')$ or $\boldsymbol{P}(\boldsymbol{s}'|\boldsymbol{s}, \boldsymbol{a})$
- Policy: $\boldsymbol{\pi}(\boldsymbol{s}) \rightarrow \boldsymbol{a}$

MARKOV'S DECISION PROCESSES

The Actuator's Decision Making Process

Discrete-time & Stochastic

Markov Property



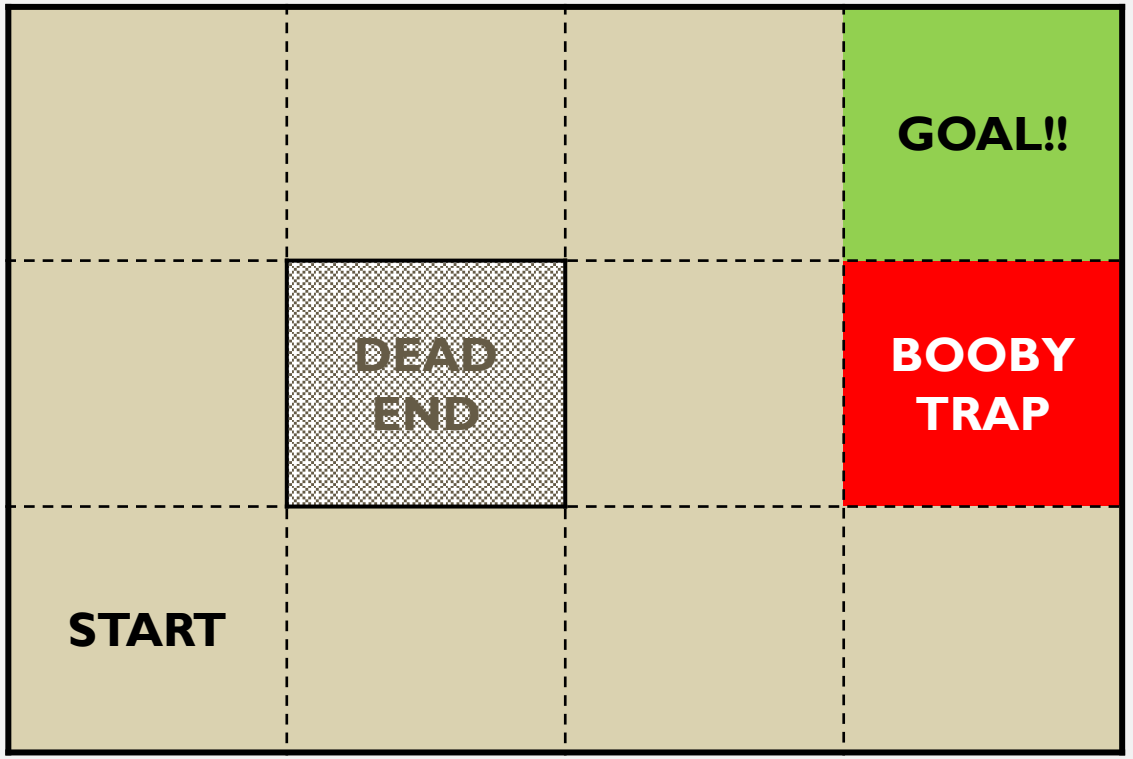
$$P(s_{t+1}|a, s_0, \dots, s_t) = P(s_{t+1}|a, s_t)$$

- Set of States: $S = \{s\}$
- Set of Actions: $A = \{a\}, a: S \to S$
- Reward Function: $R(\cdot)$
- Discount Factor: γ
- Initial State: s_1

MARKOV'S DECISION PROCESS

- Set of States: $\boldsymbol{S = \{s\}}$
- Set of Actions: $\boldsymbol{A = \{a\}, a: S \rightarrow S}$
- Reward Function: $\boldsymbol{R(s)}$
- Transition Model:

  $\boldsymbol{T(s, a, s')}$ or $\boldsymbol{P(s'|s, a)}$
- Policy: $\boldsymbol{\pi(s) \rightarrow a}$



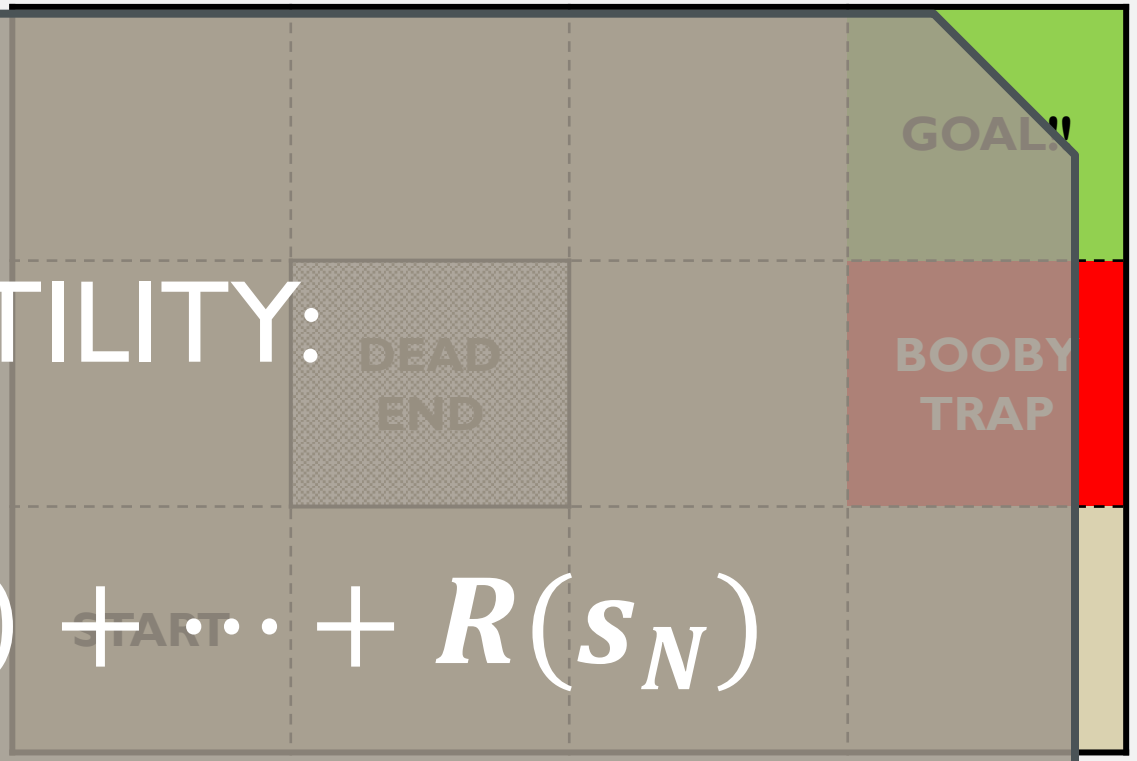
You can move UP, DOWN, LEFT, RIGHT

MARKOV'S DECISION PROCESS

- Set of States: $S = \{s\}$
- Set of Actions: $A = \{a\}, a: S \rightarrow S$
- Reward Function: $R(s)$
- Transition Model:
  $T(s, a, s')$ or $P(s' | s, a)$
- Policy: $\pi(s) \rightarrow a$

Define UTILITY:

$$U(s_0, \dots, s_N) = R(s_0) + R(s_1) + \cdots + R(s_N)$$



You can move UP, DOWN, LEFT, RIGHT

MARKOV'S DECISION PROCESS

$$U(s_0, \dots, s_N) = R(s_0) + R(s_1) + \cdots + R(s_N)$$

But this **N** can increase infinitely!

$$U(s_0, \dots) = R(s_0) + \gamma^1 R(s_1) + \cdots + \gamma^N R(s_N) + \cdots$$

Discount Factor: $0 < \gamma < 1$

MARKOV'S DECISION PROCESS

$$\boldsymbol{U(s_0, \dots) = R(s_0) + \gamma^1 R(s_1) + \cdots + \gamma^N R(s_N) + \cdots}$$

We move from s_t to s_{t+1} by **action** $\boldsymbol{\pi(s_t)}$

GOAL

Find a **sequence of actions,** $\boldsymbol{\pi(s_t)}$**,** such that the resulting sequence of states *maximizes* the total discounted reward, ***U***

MARKOV'S DECISION PROCESS

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GOAL

Find a **sequence of actions,** $\pi(s_t)$, such that the resulting sequence of states maximizes the total discounted reward, ***U***

Optimal Policy:

The policy π^* that *maximizes* the expected utility U of the sequence of states generated by π^* for all initial states s

MARKOV'S DECISION PROCESS

Question:

How do we compute the optimal policy π^*?

BELLMAN'S EQUATION

- If action $\boldsymbol{a}$ is taken in state $\boldsymbol{s}$,

$$\boldsymbol{U(s) = R(s,a) + \gamma \sum_{S'} T(s,a,s')\, U(s')}$$

$\boldsymbol{U(s)}$**:** Utility in state $\boldsymbol{s}$

$\boldsymbol{R(s,a)}$**:** Reward when taking action $\boldsymbol{a}$ from state $\boldsymbol{s}$

$\boldsymbol{\gamma}$**:** Discount factor

$\boldsymbol{T(s,a,s')}$: Transition probability from $\boldsymbol{s}$ to $\boldsymbol{s'}$ by taking $\boldsymbol{a}$

BELLMAN'S EQUATION

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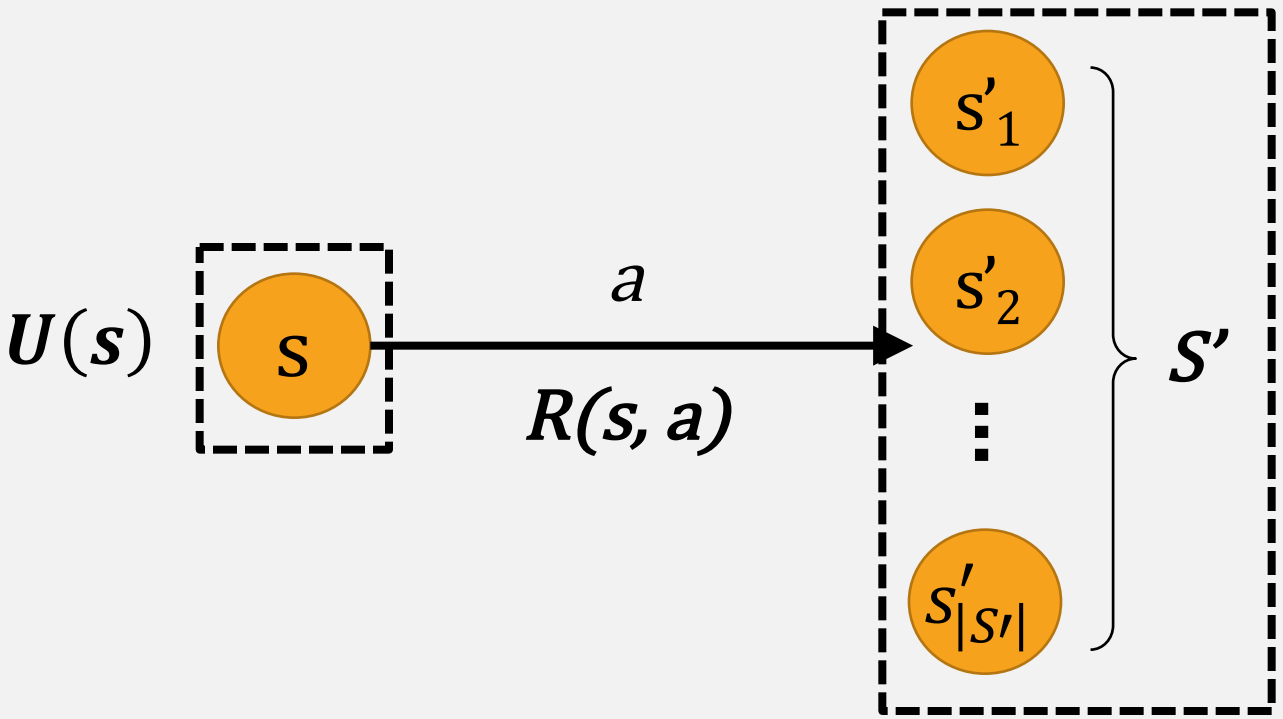
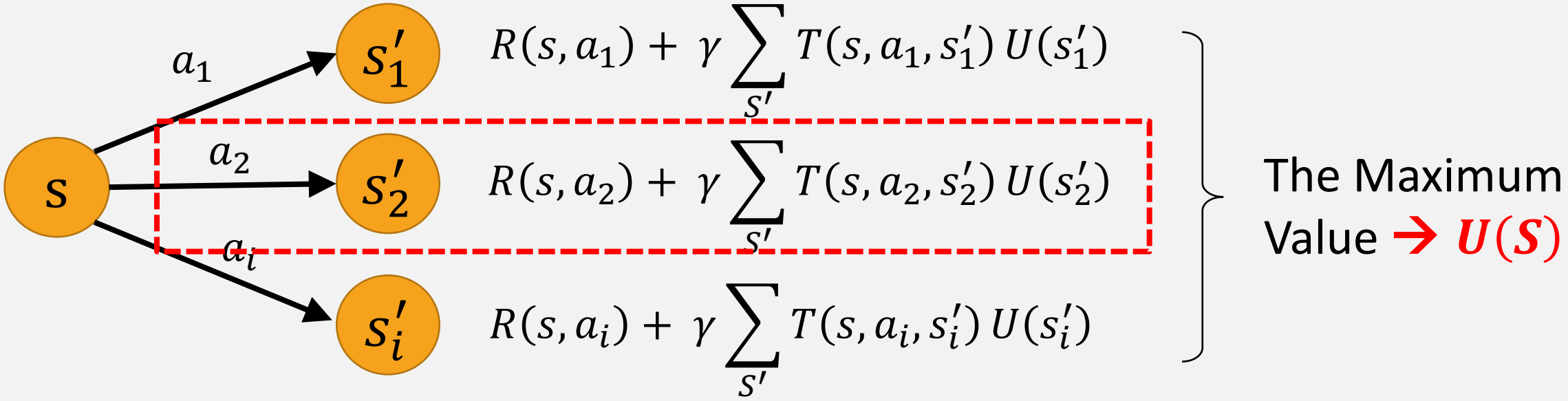


$$\boldsymbol{\gamma \sum_{S'} T(s,a,s')\, U(s')}$$

Expected value of future rewards from state s'

BELLMAN'S OPTIMALITY CRITERION

$$\boldsymbol{U(s) = \max_{a} \left(R(s,a) + \gamma \sum_{s'} T(s,a,s')\, U(s')\right)}$$



BELLMAN'S OPTIMALITY CRITERION

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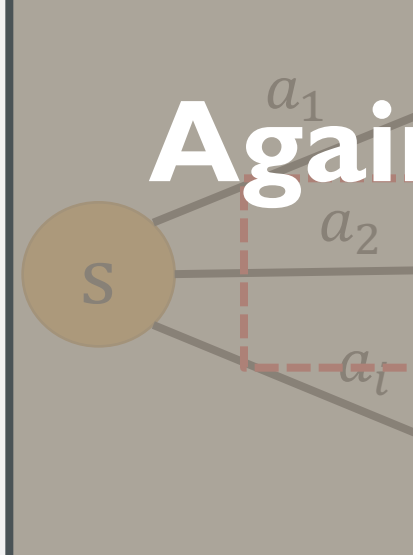


$$R(s,a_1) + \gamma \sum_{s'} T(s,a_1,s'_1)\, U(s'_1)$$

$$R(s,a_2) + \gamma \sum_{s'} T(s,a_2,s'_2)\, U(s'_2)$$

$$R(s,a_i) + \gamma \sum_{s'} T(s,a_i,s'_i)\, U(s'_i)$$

The Maximum Value → $U(S)$

Again, how do we find the optimal policy?

POLICY ITERATION

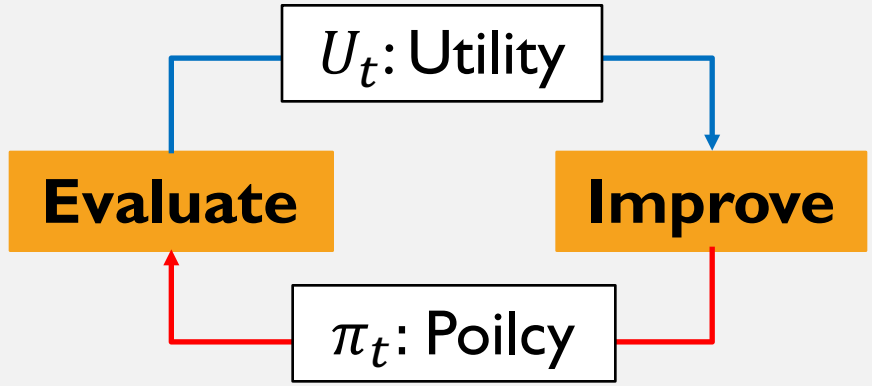
Initialize: $\boldsymbol{\pi_0} \leftarrow$ guess

Evaluate: given $\boldsymbol{\pi_t}$,

calculate $\boldsymbol{U_t(s)} = R(s) + \gamma \sum_{s'} T(s, \boldsymbol{\pi_t}(s), s')\, \boldsymbol{U_t(s')})$ (Bellman's eq)

Improve: $\boldsymbol{\pi_{t+1}} = \underset{a}{\operatorname{argmax}} \sum_{s'} T(s, \boldsymbol{\pi_t}(s), s')\, \boldsymbol{U_t(s')}$)

POLICY ITERATION



Initialize: $\boldsymbol{\pi_0}$ ← guess

Evaluate: given $\boldsymbol{\pi_t}$,

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Improve: $\boldsymbol{\pi_{t+1}} = \underset{a}{\operatorname{argmax}} \sum_{s'} T(s, \boldsymbol{\pi_t}(s), s')\, \boldsymbol{U_t(s')}$)

Repeat until the Utility converges → $\boldsymbol{U_t(s)} \approx \boldsymbol{U_{t+1}(s)}$

VALUE ITERATION

Initialize: Start with arbitrary utilities

Update utilities based on neighbors

$$\hat{\mathrm{U}}_{\boldsymbol{t+1}}(\boldsymbol{s}) = R(s) + \gamma \max_{a} \sum_{s'} T(s, \pi_{\boldsymbol{t}}(s), s')\, \hat{\mathrm{U}}_{\boldsymbol{t}}(\boldsymbol{s'})$$ (Bellman's eq)

VALUE ITERATION

Initialize: Start with arbitrary utilities

Update utilities based on neighbors

$$\hat{U}_{t+1}(s) = R(s) + \gamma \max_a \sum_{s'} T(s, \pi_t(s), s') \hat{U}_t(s'))$$ (Bellman's eq)

→ Repeat until converges.

The quality of Utility gets better as more *truth* gets involved

CHECKPOINT

- **Markov Decision Process**

  Decision making process in a cognitive dynamic system

- **Bellman's Optimality Criterion**

  Defines the optimization problem the decision-maker must solve

- **Policy Iteration**

  Finds the optimal policy by iteratively estimating the policy

- **Value Iteration**

  Finds the optimal policy by iteratively estimating the utility

INTERMISSION

REINFORCEMENT LEARNING

as Dynamic Programming

REINFORCEMENT LEARNING RECAP

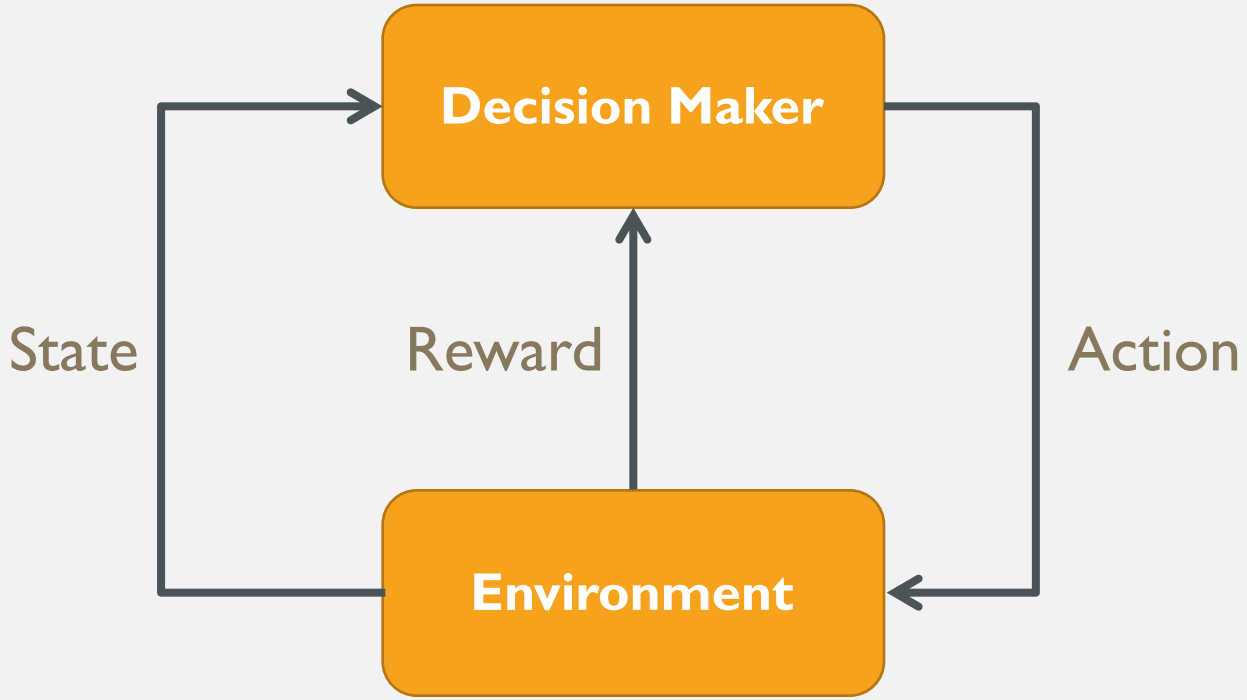
REINFORCEMENT LEARNING RECAP

Model-Based

Model-Free

REINFORCEMENT LEARNING RECAP

Model-Based (MDP)



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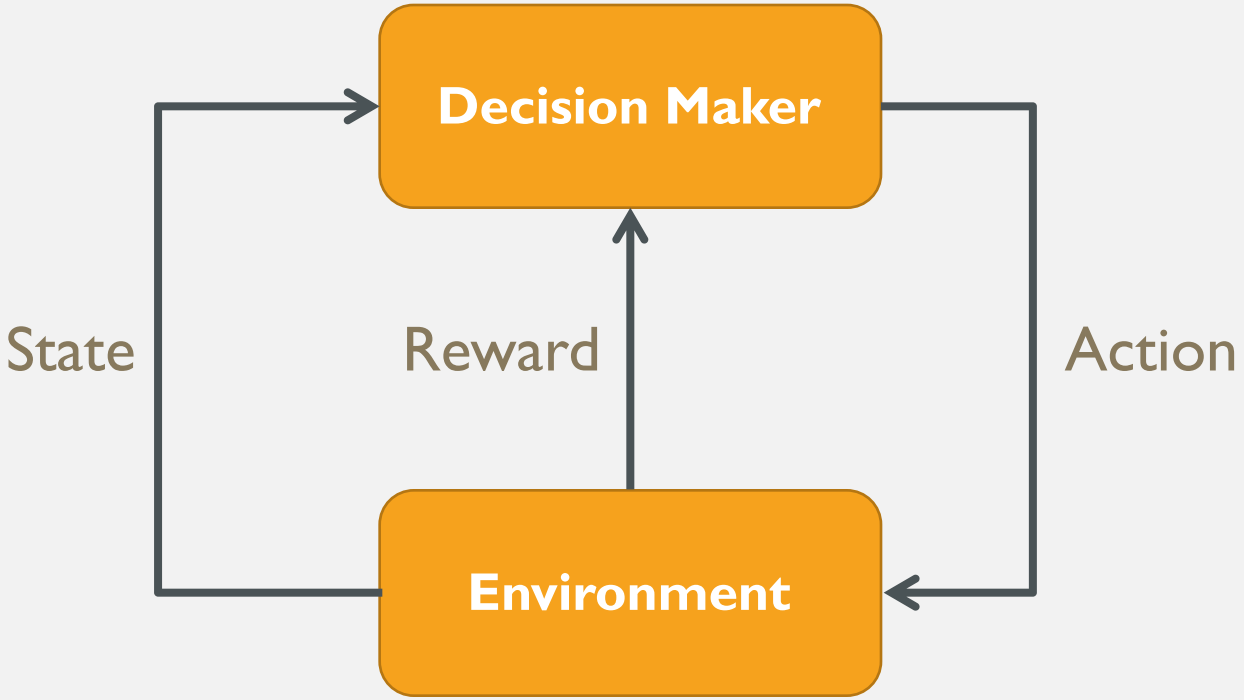
REINFORCEMENT LEARNING RECAP

Model-Free



- Set of States: $\boldsymbol{S} = \{\boldsymbol{s}\}$
- Set of Actions: $\boldsymbol{A} = \{\boldsymbol{a}\}, \boldsymbol{a}: \boldsymbol{S} \rightarrow \boldsymbol{S}$

There is no explicit model for ***R*** and ***T***
We instead have explicit transition data:
$< \boldsymbol{s}, \boldsymbol{a}, \boldsymbol{s}', \boldsymbol{r} >$

- Policy: $\boldsymbol{\pi}(\boldsymbol{s}) \rightarrow \boldsymbol{a}$

MODEL-FREE REINFORCEMENT LEARNING

- Temporal Difference Learning
- Q-Learning

TEMPORAL DIFFERENCE (TD) LEARNING

$$\boldsymbol{U(s) = R(s,a) + \gamma \sum_{s'} T(s,a,s')\, U(s')}$$

Upon an action $a = \pi(s)$

For all s', successor of s, $U(s)$ must be "in between"

a) the new value considering only s': $R(s) + \gamma\, U(s')$

b) the old value $U(s)$

TEMPORAL DIFFERENCE (TD) LEARNING

$$\boldsymbol{U(s) = R(s,a) + \gamma \sum_{s'} T(s,a,s')\, U(s')}$$

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b) the old value $U(s)$

The Notion of Temporal Difference

TEMPORAL DIFFERENCE (TD) LEARNING

$$\boldsymbol{U(s) \coloneqq (1 - \alpha)\, U(s) \; + \; \alpha\, (R(s) \; + \; \gamma\, U(s'))}$$

The new approximation of $\boldsymbol{U(s)}$ using $\boldsymbol{\alpha}$
when moving from a state $\boldsymbol{s}$ to another state $\boldsymbol{s'}$

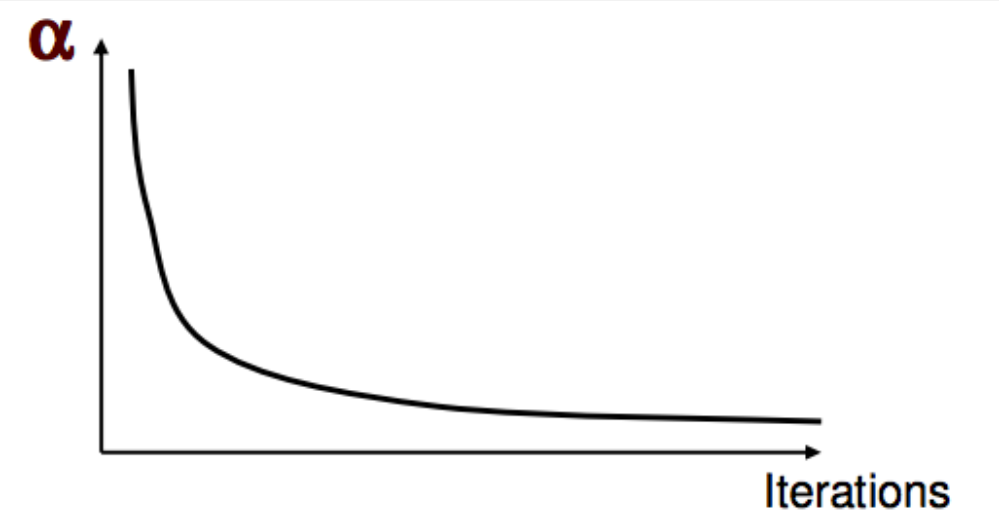
Rearrange the above equation to update $\boldsymbol{U(s)}$**:**

$$\boldsymbol{U(s) \coloneqq U(s) \; + \; \alpha\, (R(s) \; + \; \gamma\, U(s') - \; U(s))}$$

TEMPORAL DIFFERENCE (TD) LEARNING

$$U(s) := U(s) + \alpha(R(s) + \gamma U(s') - U(s))$$

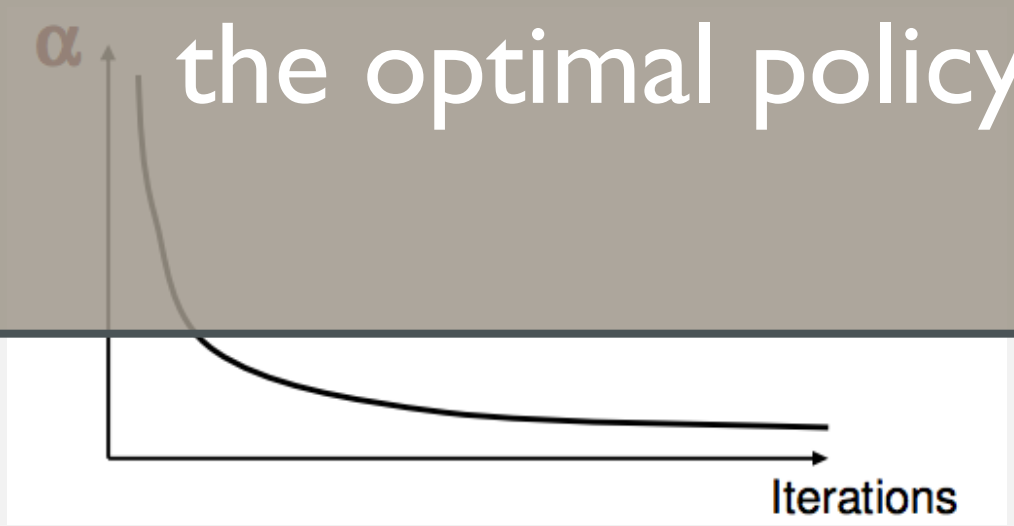
Learning Rate: $0 \leq \alpha < 1$



TEMPORAL DIFFERENCE (TD) LEARNING

$$U(s) := U(s) + \alpha(R(s) + \gamma U(s') - U(s))$$

But this says nothing about the optimal policy, π^*



Q-LEARNING

$$\boldsymbol{U(s) \coloneqq U(s) \; + \; \alpha \, (R(s) \; + \; \gamma \, U(s') - \; U(s))}$$

Policy is about taking an action $\boldsymbol{a}$ from a state $\boldsymbol{s}$
--> Let's introduce action to $\boldsymbol{U(s)}$!

$\boldsymbol{Q(s, a)}$**:** Expected sum of future discounted rewards
after taking action $\boldsymbol{a}$ at state **s**

$$\boldsymbol{Q(s, a) \coloneqq Q(s, a) + \alpha(R(s) + \gamma \; \max_{a'} Q(s', a') - Q(s, a))}$$

Q-LEARNING

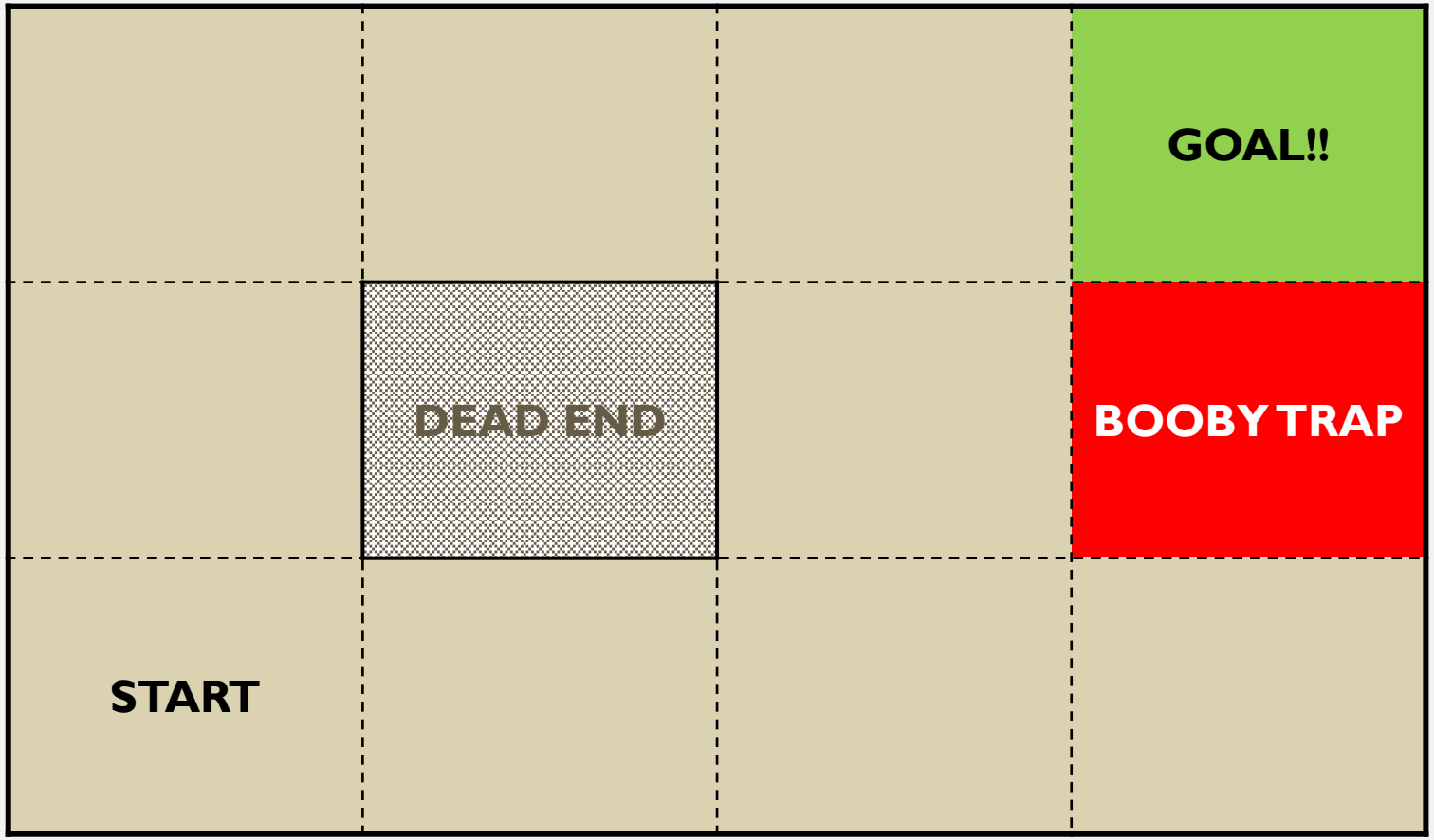
$$Q(s,a) \coloneqq Q(s,a) + \alpha(R(s) + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

Finding the optimal policy is now a matter of finding the actions for the states in maximizing the Q-value:

$$\pi(s) = \operatorname*{argmax}_{a} Q(s,a)$$

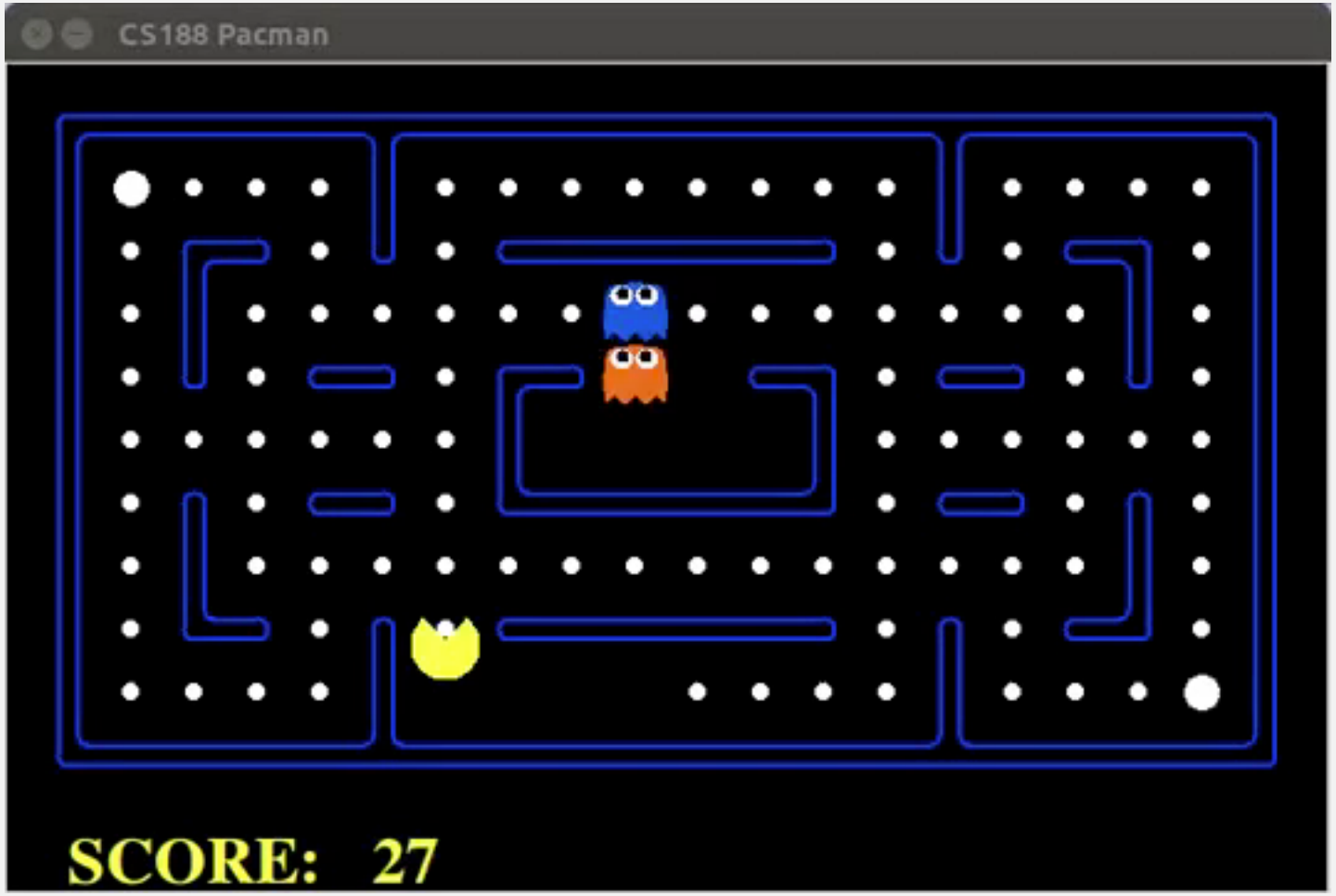
EXAMPLE

SCENARIO



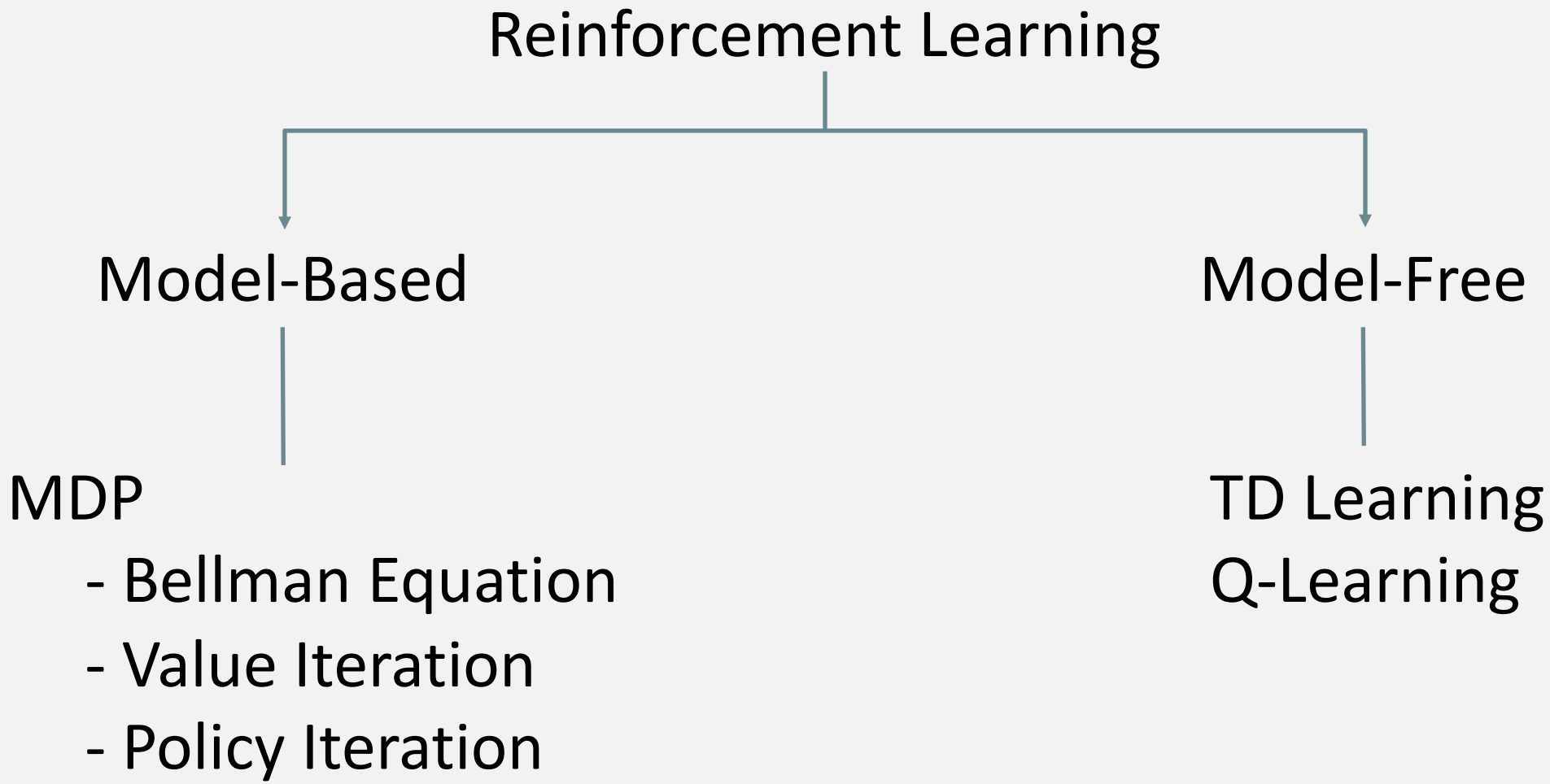
You can move UP, DOWN, LEFT, RIGHT

PACMAN



SUMMARY

SUMMARY


Reinforcement Learning
Model-Based
Model-Free
MDP
- Bellman Equation
- Value Iteration
- Policy Iteration
TD Learning
Q-Learning

DISCUSSION

QUESTIONS FOR THOUGHT

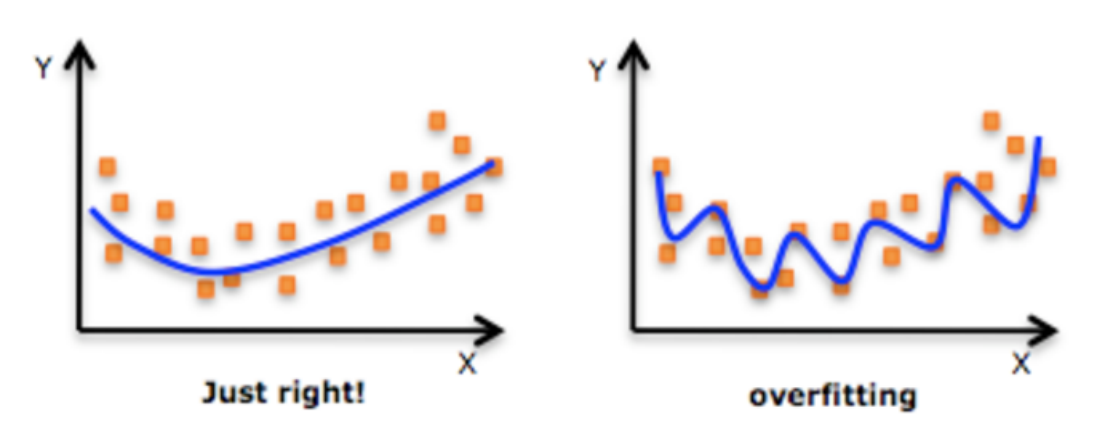
Q1. What if the problem we are trying to solve is large scale?

$\Rightarrow$ State-space is large!

$\Rightarrow$ ex. value iteration may take forever!

$\Rightarrow$ examining the entire training dataset is endless!

Q2. What if our solution "overfits" data given by the environment?



APPROXIMATE DYNAMIC PROGRAMMING

LINEAR APPROXIMATION APPROACH

The *curse-of-dimensionality problem*: the exponential growth of computational complexity with increasing dimensionality of the state space.

a) Abandon the idealized notion of optimality
and be content with a suboptimal solution ☺

b) Approximate functions for data: $< s, a, s', r >$
- can prevent overfitting

Q & A

APPENDIX

NOTATION & TERMINOLOGY

- We found that the notations used in the book are tricky to understand, and concluded to use the equivalent alternatives that were easier.

<TEXTBOOK>

- $i \sim X_n$: State
- $\mu(i)$: Policy
- $J^{\pi}\ (i)$: Cost-to-go function
- $g\left(X_n, \mu_{n,}\, X_{n+1}\right)$: Transition Cost
- $p_{ij}(\mu(i))$: Transition Probability

<THE SLIDES>

- $s \in S$: State
- $\pi(s)$: Policy
- $U(s)$: Utility
- $R(s, a)$: Reward (or R(s))
- $T(s, a, s')$: Transition Probability