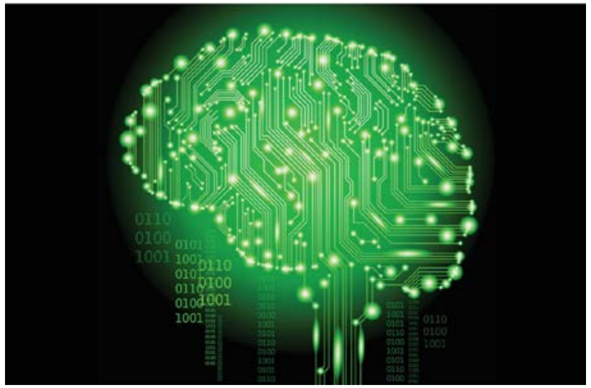
Chapter 6.

Cognitive Radar

From CDS written by S. Haykin



Jibeom Choi

Laboratory Behavioral Ecology & Evolution

School of Biological Science

Notation Notification

X_n = $\mathbf{x}_n$

S~_(n-1) = $\tilde{s}_{n-1}$

x^_n = $\hat{\mathbf{x}}_n$

What is Radar?

Function of radar : remote-sensing system

(1) Traditional active radar : it transmits and receives the signal sent to the environment. (*feedforward*)

(2) Fore-active radar : it distributes capacity(scantime, library of transmit waveform).

(3) Cognitive radar

What is Radar?

For a radar to be cognitive, it requires

(1) Perception action cycle

(2) Memory for predicting the consequences of actions

(3) Attention for prioritizing the allocation of resources

(4) Intelligence for decision-making

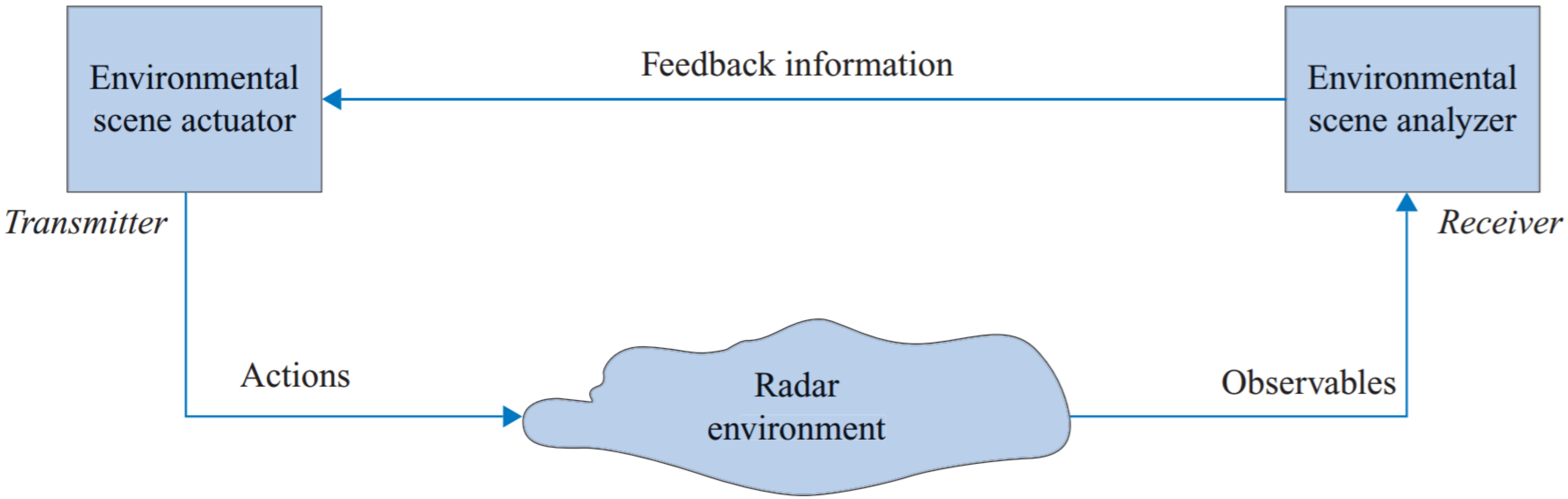


Figure 6.1. The perception–action cycle of radar with global feedback as the first step towards cognition.

Radar environment : electromagnetic medium where target of interest is embedded

Observable : “radar returns” produced by reflections from target

* State estimation serves as “perception” of the environment

Baseline Model for Signal Transmission

Payoff for CF & LFM

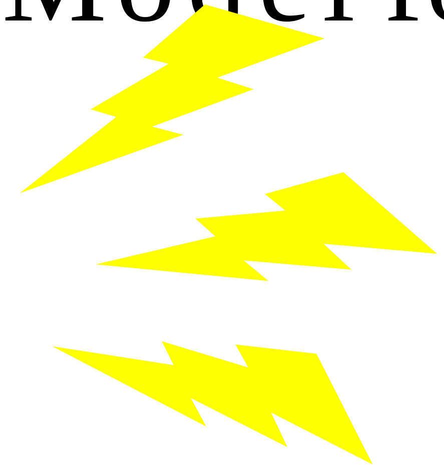
CF : *Constant-frequency* pulse, good for range-rate(velocity) resolution, poor for range(position) resolution

LFM : *Linear frequency-modulated* pulse, poor for range-rate resolution, good for range resolution

Uncertainty principle for radar

Solution : Combination two forms of modulations

Baseline Model for Signal Transmission



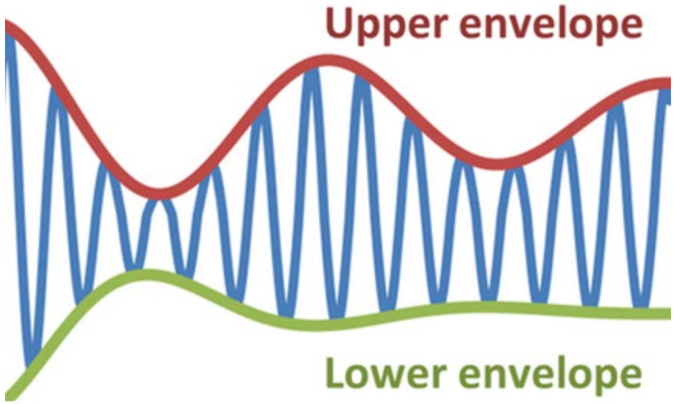
Transmitted signal toward target

$$s_T(t) = \sqrt{2E_T}\,\mathrm{Re}[\tilde{s}(t)\exp(j2\pi f_c t)]$$

Re : Real part

s~(t) : Envelop of s_T(t)

E_T : Energy of transmitted signal



Baseline Model for Signal Transmission

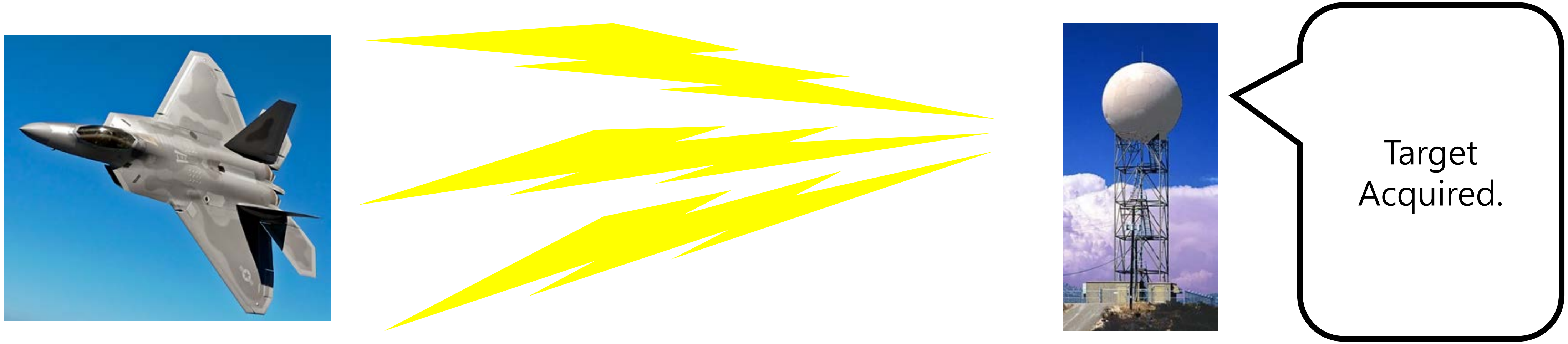
$$\tau = \frac{2\rho}{c}$$

Go-and-comeback time

$$f_{\mathrm{D}} = \frac{2\dot{\rho}}{c}$$

Shift by Doppler effect

$$r(t) = \sqrt{2E_{\mathrm{R}}}\,\mathrm{Re}[\tilde{s}(t-\tau)\exp[\,j2\pi(f_{\mathrm{c}}+f_{\mathrm{D}})t+\phi]+\tilde{n}(t)]$$



State-space model of the target

Systems equation

$$\overrightarrow{x_n} = \overrightarrow{a(x_{n-1})} + \overrightarrow{\omega_n}$$

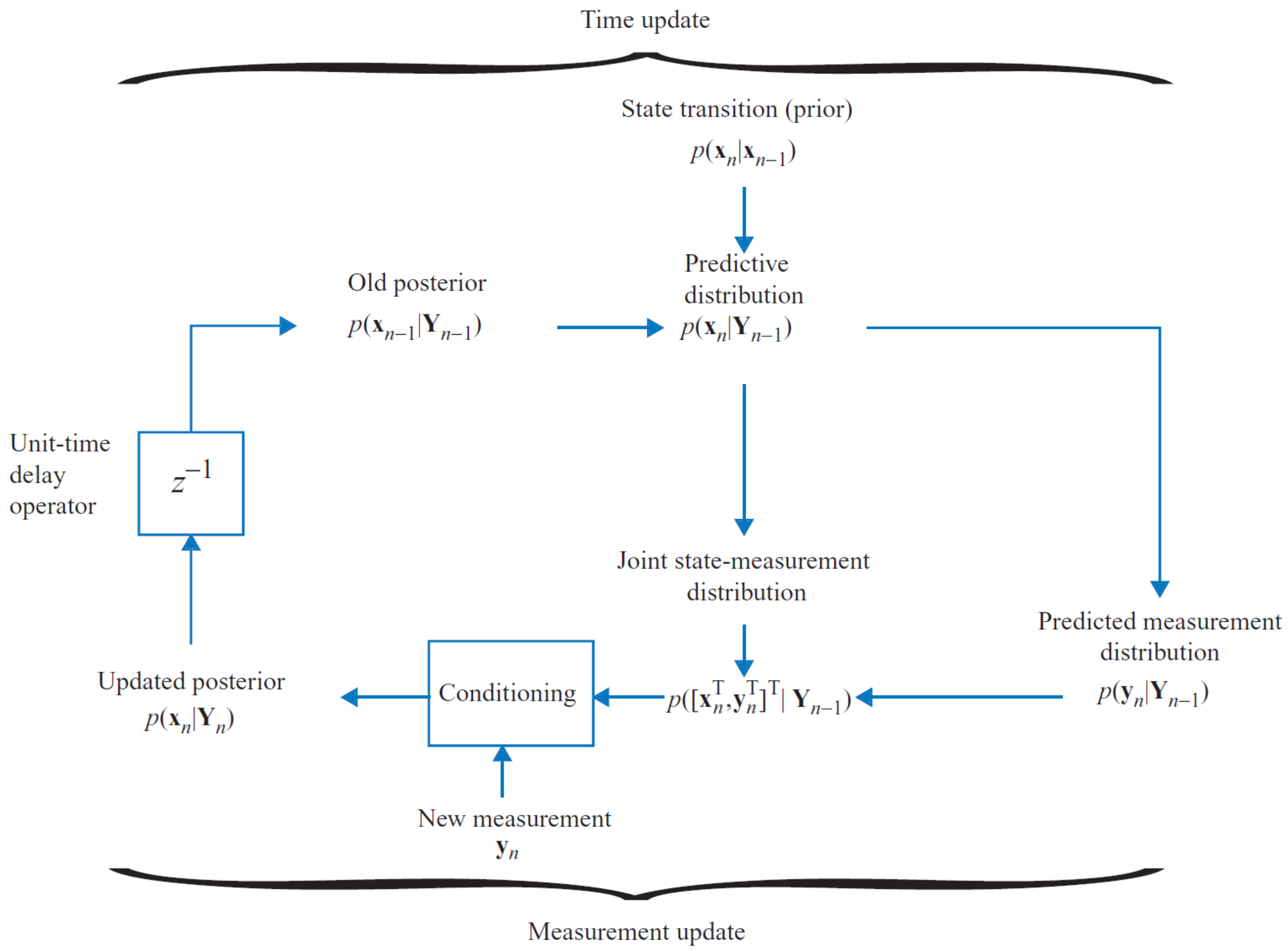
(true value)

Measurement equation

$$\overrightarrow{y_n} = \overrightarrow{b_n(x_n)} + \overrightarrow{v(\theta_{n-1})}$$

(observation value)

Probability-distribution Flow-graph



Probability-distribution Flow-graph

$p(\mathbf{x}_{n-1} | \mathbf{Y}_{n-1})$ Old posterior(old information)

$Y_k = [y_1, y_2, \ldots, y_k]$

$p(\mathbf{x}_n | \mathbf{x}_{n-1})$ State transition of the object

$p(\mathbf{x}_n | \mathbf{Y}_{n-1})$ Predicative distribution

$p(\mathbf{y}_n | \mathbf{Y}_{n-1})$ Predicted measurement distribution

$p(\mathbf{x}_n | \mathbf{Y}_n)$

Updated posterior(new information)

$p([\mathbf{x}_n^{\mathrm{T}}, \mathbf{y}_n^{\mathrm{T}}]^{\mathrm{T}} | \mathbf{Y}_{n-1})$ Joint state-measurement distribution

Conditional Probability for Multiple Conditions

Probability of A to happen on assumption that B already happened:

$$P(A|B) = \frac{P(A\cap B)}{P(B)}$$

Probability of A to happen on assumption that B,C already happened:

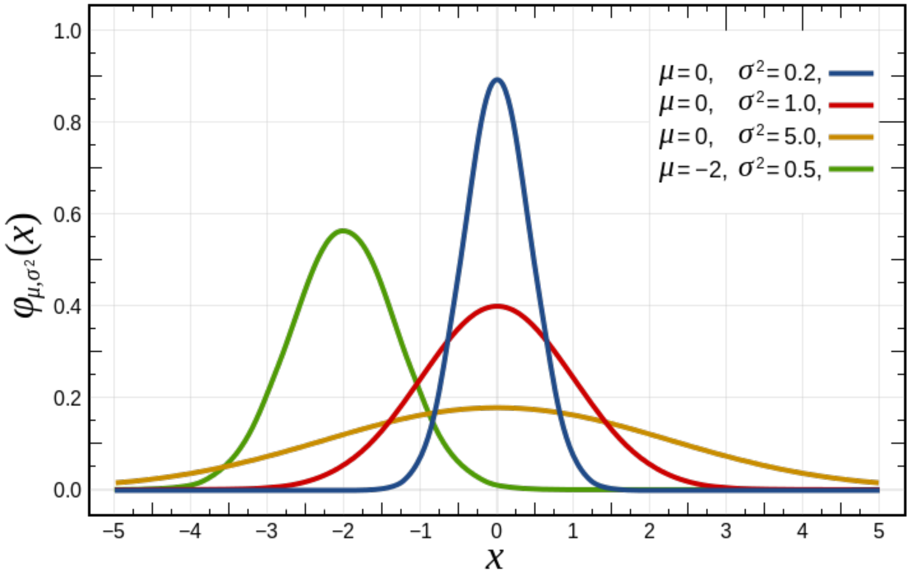
$$P(A|B,C) = \frac{P(A\cap B\cap C)}{P(B\cap C)}$$

Gaussian Distribution

Gaussian(Normal) distribution whose variable is x, average is u(mu) and sigma is the covariance.

$$N(x; \mu, \Sigma)$$

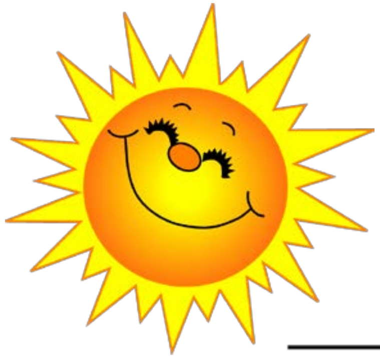
$$f(x \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\sigma^2\pi}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



Gaussian Distribution

Systems equation

$$\overrightarrow{x_n} = \overrightarrow{a(x_{n-1})} + \overrightarrow{\omega_n}$$



$$\overrightarrow{x_n} = \overrightarrow{a(x_{n-1})} + \overrightarrow{\omega_n}$$

$$\overrightarrow{x_{n-1}}$$

$$\overrightarrow{a(x_{n-1})}$$

$$\overrightarrow{\omega_n} \sim N(0,\sigma^2)$$

Linear vs. Non-Linear

For some operator L,

For arbitrary vectors a and b, and scalar k, L is said to be linear map if

L(ka) = kL(a), L(a+b) = L(a) + L(b)

cf. If L(a) is M*a where M is a matrix, then L(a) is linear

$$\overrightarrow{x_n} = \overrightarrow{a(x_{n-1})} + \overrightarrow{\omega_n} \qquad \overrightarrow{y_n} = \overrightarrow{b_n(x_n)} + \overrightarrow{v(\theta_{n-1})}$$

Time Update

$$\hat{\mathbf{x}}_{n|n-1} = \mathbb{E}[\mathbf{x}_n \mid \mathbf{Y}_{n-1}]$$

Estimation(expectation) of x_n given all the history

$$\hat{\mathbf{x}}_{n|n-1} = \mathbb{E}[(\mathbf{a}(\mathbf{x}_{n-1}) + \boldsymbol{\omega}_n) \mid \mathbf{Y}_{n-1}]$$

$$\hat{\mathbf{x}}_{n|n-1} = \mathbb{E}[\mathbf{a}(\mathbf{x}_{n-1}) \mid \mathbf{Y}_{n-1}]$$
$$= \int_{\mathrm{R}_{N_{\mathbf{x}}}} \mathbf{a}(\mathbf{x}_{n-1}) p(\mathbf{x}_{n-1} \mid \mathbf{Y}_{n-1})\, \mathrm{d}\mathbf{x}_{n-1}$$
$$= \int_{\mathrm{R}_{N_{\mathbf{x}}}} \mathbf{a}(\mathbf{x}_{n-1}) \mathcal{N}(\mathbf{x}_{n-1}; \hat{\mathbf{x}}_{n-1|n-1}, \mathbf{P}_{n-1|n-1})\, \mathrm{d}\mathbf{x}_{n-1}$$

For every possible x_(n-1), we multiply the probability to get the expected value

Probability for the certain x_(n-1) follows normal distribution whose average is x^_(n-1)

Additional Explanation

Suppose we play a dice game :

Rule : 1, 2 → 5 $ reward

3, 4 → no reward

5, 6 → 2 $ penalty

Expected value

= sum(result * p of that result to happen)

= 1/3 * 5 + 1/3 * 0 + 1/3 * (-2)

= 1 ($)

Additional Explanation

In generalized form, $E = \sum_{i=1}^{N} p_i R_i = p_1 R_1 + p_2 R_2 + \cdots + p_n R_n$

$$\hat{\mathbf{x}}_{n|n-1} = \mathbb{E}[\mathbf{a}(\mathbf{x}_{n-1}) \mid \mathbf{Y}_{n-1}]$$
$$= \int_{\mathbb{R}^{N_\mathbf{x}}} \mathbf{a}(\mathbf{x}_{n-1}) p(\mathbf{x}_{n-1} \mid \mathbf{Y}_{n-1}) \, d\mathbf{x}_{n-1}$$
$$= \int_{\mathbb{R}^{N_\mathbf{x}}} \mathbf{a}(\mathbf{x}_{n-1}) \mathcal{N}(\mathbf{x}_{n-1}; \hat{\mathbf{x}}_{n-1|n-1}, \mathbf{P}_{n-1|n-1}) \, d\mathbf{x}_{n-1}$$

Probability of x_(n-1) to happen

Result when x_(n-1) happened

Measurement Update

$$\hat{\mathbf{y}}_{n|n-1} = \int_{\mathrm{R}_{N_{\mathbf{x}}}} \mathbf{b}(\mathbf{x}_n)\mathcal{N}(\mathbf{x}_n; \hat{\mathbf{x}}_{n|n-1}, \mathbf{P}_{n|n-1})\, \mathrm{d}\mathbf{x}_n$$

$$p(\mathbf{y}_n \mid \mathbf{Y}_{n-1}) = \mathcal{N}(\mathbf{y}_n; \hat{\mathbf{y}}_{n|n-1}, \mathbf{P}_{\mathbf{yy},n|n-1})$$

$$\mathbf{P}_{\mathbf{yy},n|n-1} = \int_{\mathrm{R}_{N_{\mathbf{x}}}} \mathbf{b}(\mathbf{x}_n)\mathbf{b}^{\mathrm{T}}(\mathbf{x}_n)\mathcal{N}(\mathbf{x}_n; \hat{\mathbf{x}}_{n|n-1}, \mathbf{P}_{n|n-1})\, \mathrm{d}\mathbf{x}_n - \hat{\mathbf{y}}_{n|n-1}\hat{\mathbf{y}}^{\mathrm{T}}_{n|n-1} + \mathbf{R}(\boldsymbol{\theta}_{n-1})$$

Cost-to-go Function

Radar thought the object was placed at x^_n

The actual object however is located at x_n

$$\mathbf{x}_{n+1} - \hat{\mathbf{x}}_{n+1|n+1}(\boldsymbol{\theta}_n)$$

Cost-to-go Function

Using mean-square error

$$\boldsymbol{\varepsilon}_{n+1|n+1}(\boldsymbol{\theta}_n) = \mathbf{x}_{n+1} - \hat{\mathbf{x}}_{n+1|n+1}(\boldsymbol{\theta}_n)$$

$$g(\boldsymbol{\varepsilon}_{n+1|n+1}(\boldsymbol{\theta}_n)) = \mathbb{E}[\| \boldsymbol{\varepsilon}_{n+1|n+1}(\boldsymbol{\theta}_n) \|^2]$$

Using Shannon's entropy

$$g(\boldsymbol{\varepsilon}_{n+1|n+1}(\boldsymbol{\theta}_n)) = \underbrace{H(\boldsymbol{\varepsilon}_{n+1|n+1}(\boldsymbol{\theta}_n))}_{\text{Entropy}}$$

$$H = -\sum_i p_i \log_b p_i$$

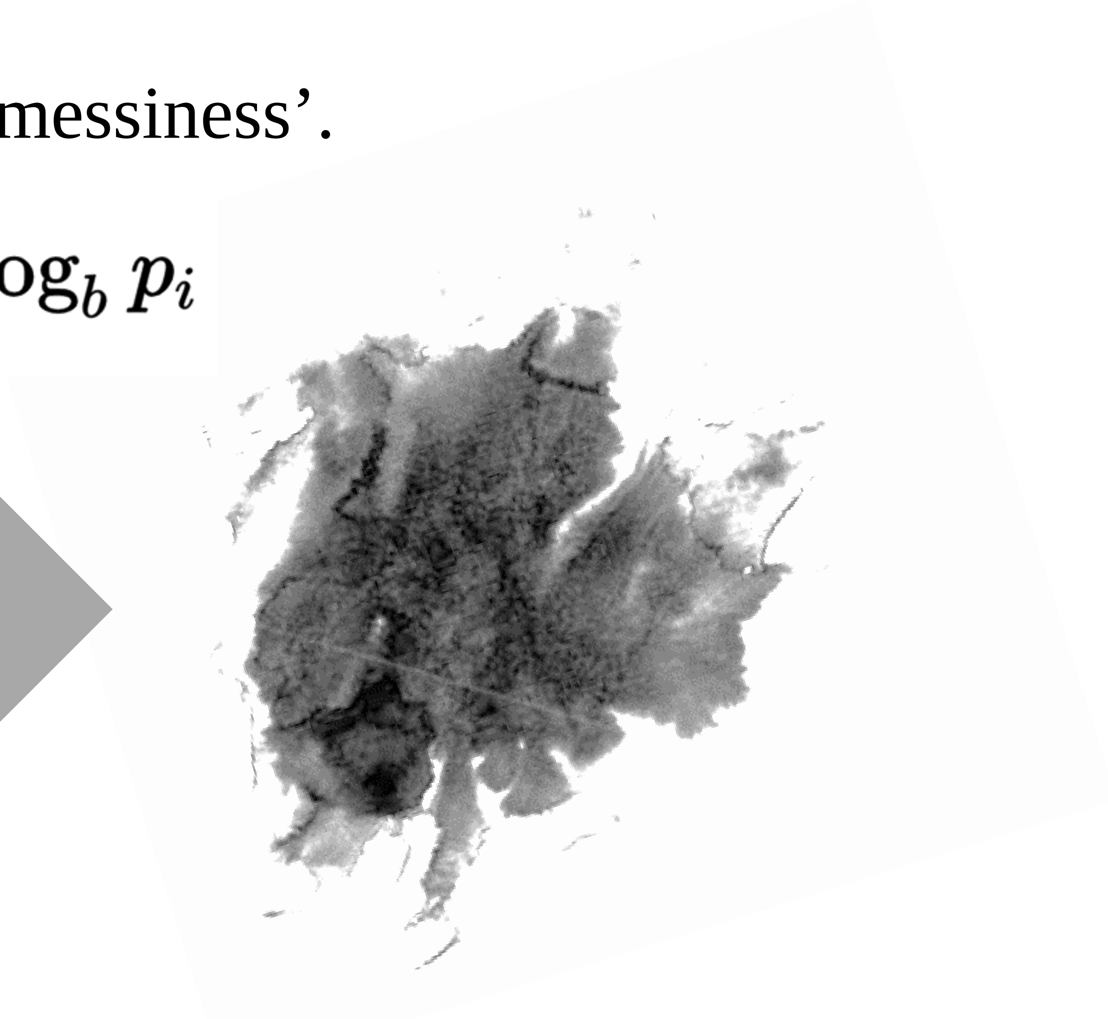
$$= \int_{-\infty}^{\infty} \underbrace{\boldsymbol{\varepsilon}_{n+1|n+1}(\boldsymbol{\theta}_n)}_{\substack{\text{Random error}\\ \text{vector}}} \underbrace{p(\boldsymbol{\varepsilon}_{n+1|n+1}(\boldsymbol{\theta}_n))}_{\substack{\text{Conditional probability}\\ \text{density function}}} \, d\boldsymbol{\varepsilon}_{n+1|n+1}(\boldsymbol{\theta}_n)$$

Additional Explanation

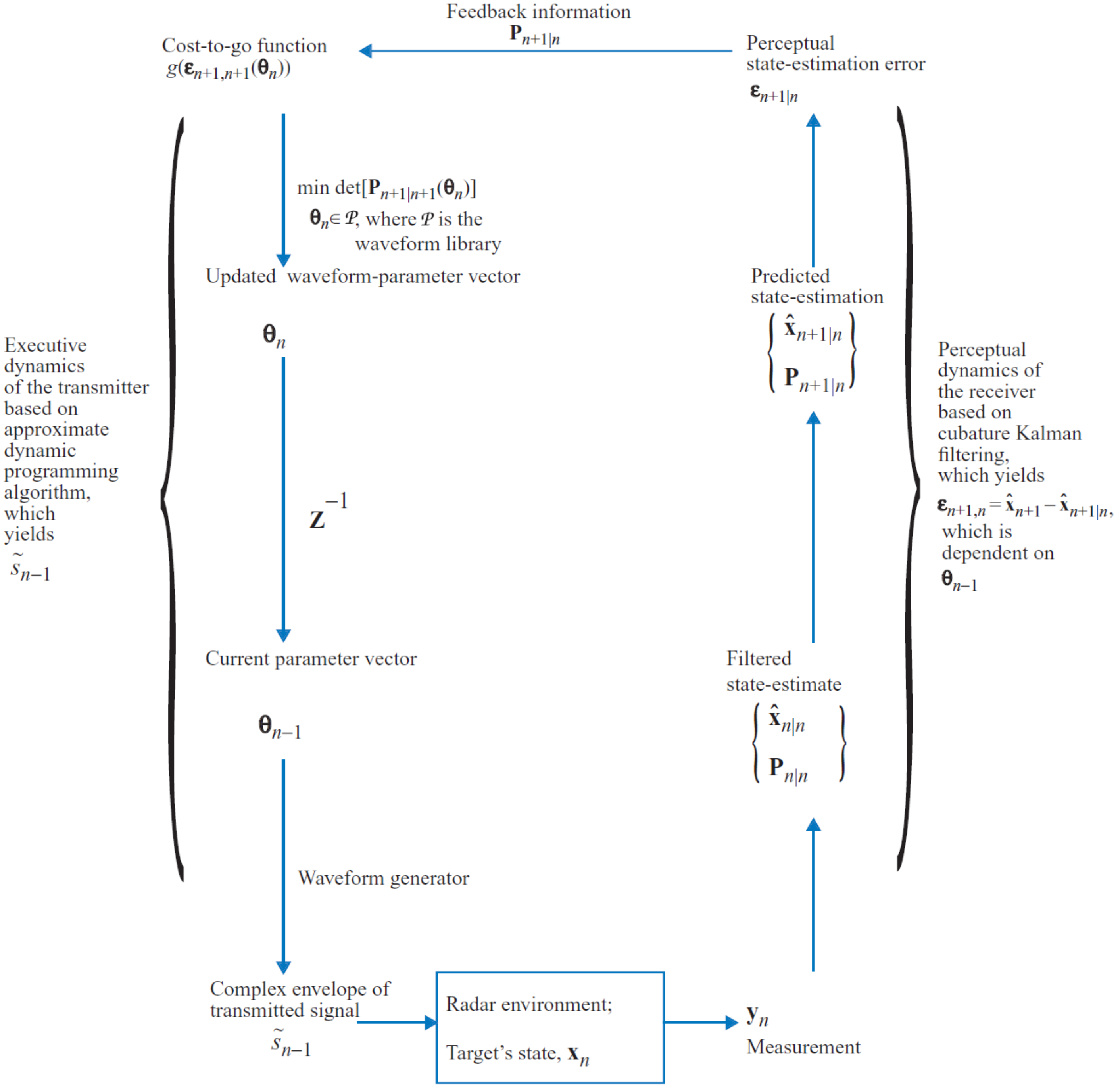
Q : What is entropy?

A : Roughly speaking, the amount of 'messiness'.

$$H = -\sum_i p_i \log_b p_i$$

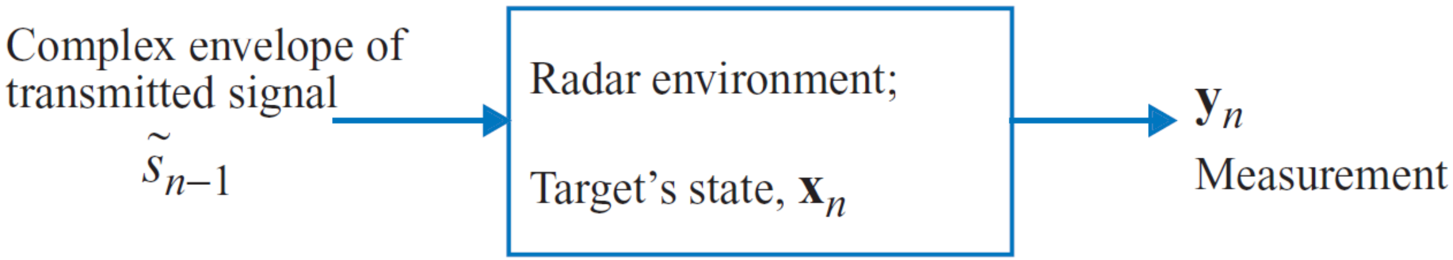


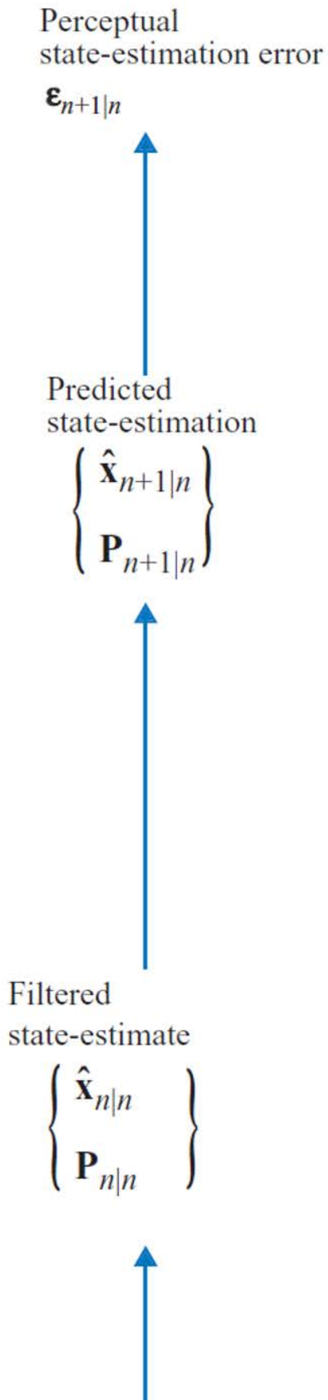
Directed Information Flow-graph



Directed Information Flow-graph

1. The radar receives the signal(electromagnetic wave, s~_(n-1)) and measure the target's property. (Perception part)

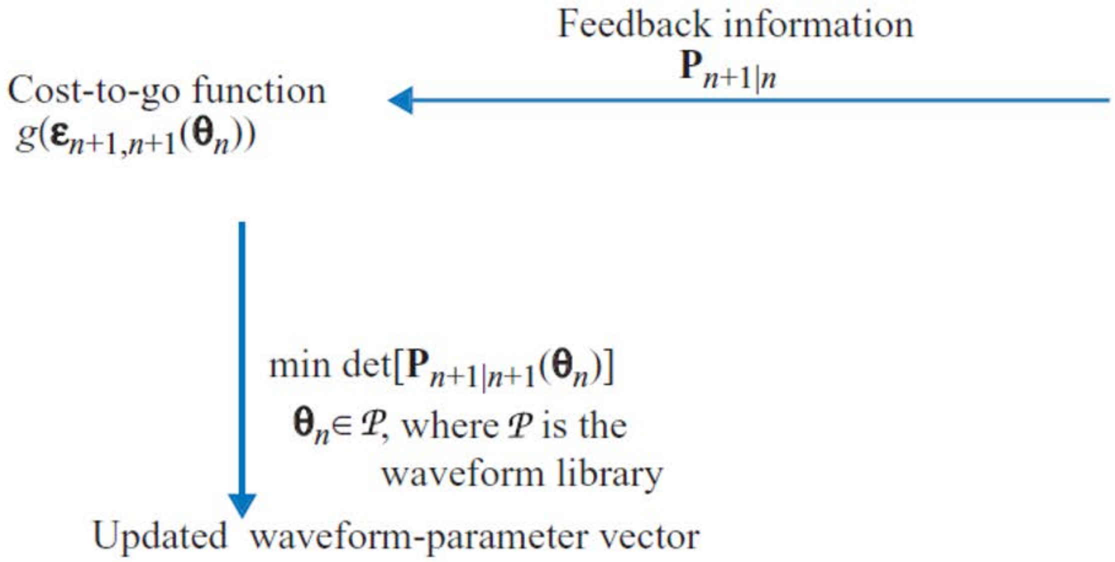




Directed Information Flow-graph

2. State of the object and corresponding errors are predicted .

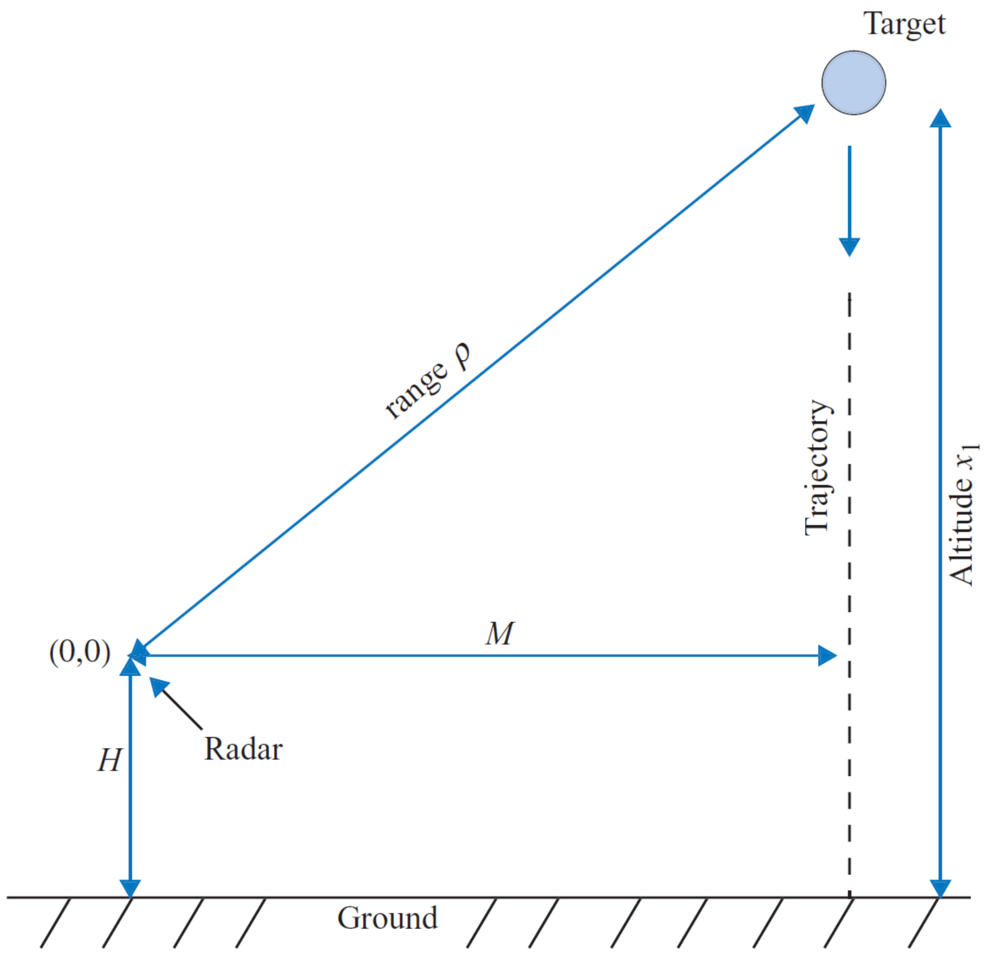
Directed Information Flow-graph



3. Feedback is performed to obtain the cost-to-go function. The cost-to-go function and control policy are updated to minimize the error.

(Action part)

Case Study: Tracking a Falling Object



$$\dot{x}_1 = -x_2$$

$$\dot{x}_2 = \underbrace{-\frac{\xi(x_1) g x_2^2}{2x_3}}_{\text{drag}} + g$$

$$\dot{x}_3 = 0$$

x_1 : altitude

x_2 : velocity

x_3 : ballistic coefficient

Case Study: Tracking a Falling Object

State part

$$\mathbf{x} = [x_1, x_2, x_3]^{\mathrm{T}}$$

Initial state

$$\dot{\mathbf{x}}_t = \mathbf{f}(\mathbf{x}_t)$$

$$\begin{aligned}\mathbf{x}_n &= \mathbf{x}_{n-1} + \delta \mathbf{f}(\mathbf{x}_{n-1}) \\ &= \mathbf{a}(\mathbf{x}_{n-1}),\end{aligned}$$

How states vary

$$\mathbf{x}_n = \mathbf{a}(\mathbf{x}_{n-1}) + \boldsymbol{\omega}_n$$

Generalized expression of the state

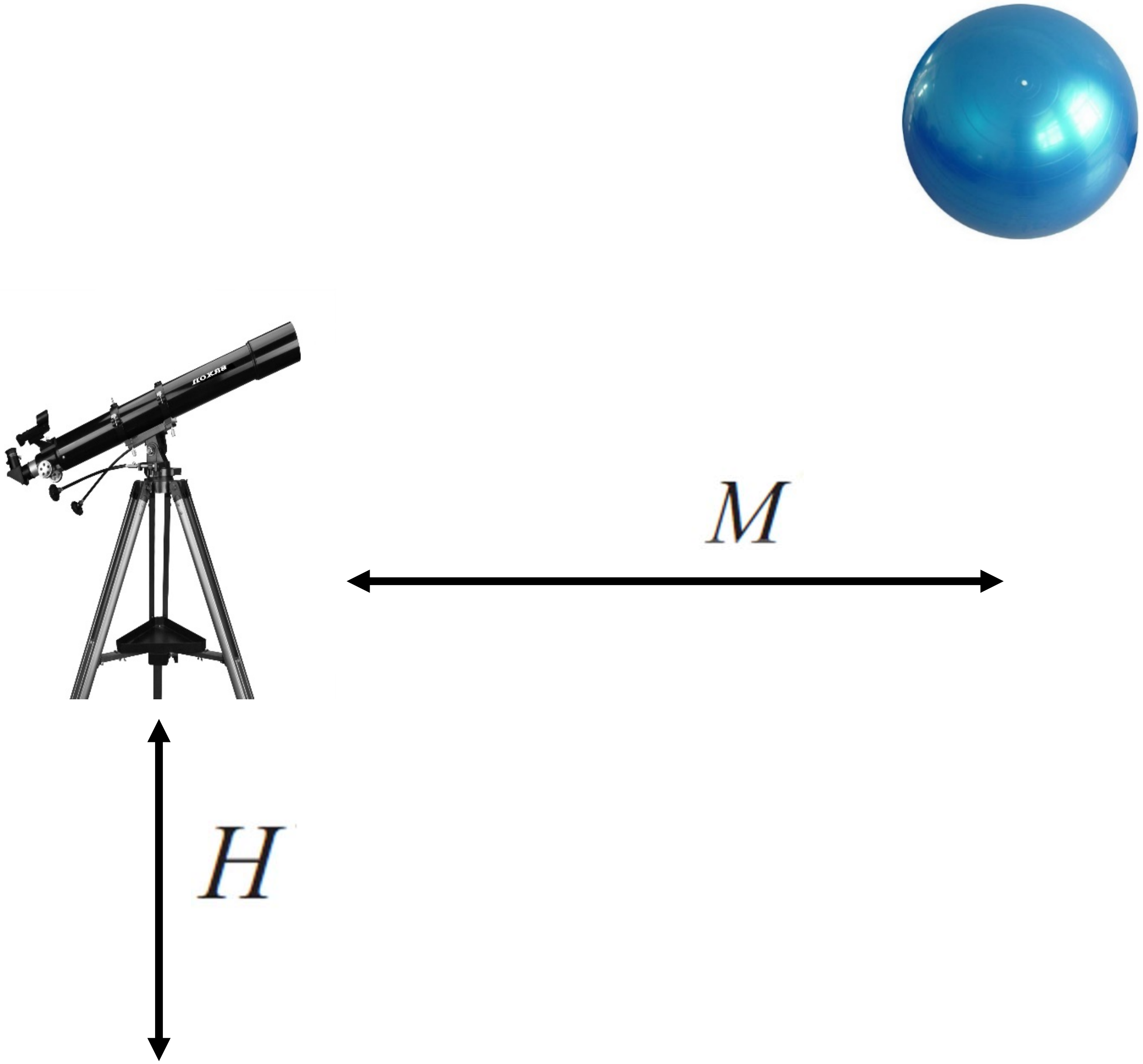
Case Study: Tracking a Falling Object

Measurement part

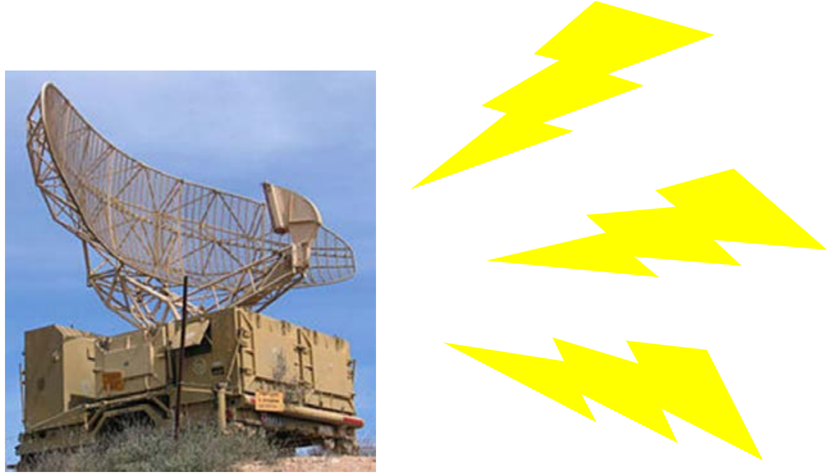
$$y_{1,n} = \sqrt{M^2 + (x_{1,n} - H)^2} + \nu_{1,n}$$

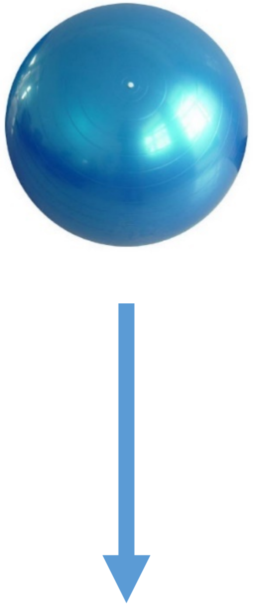
$$y_{2,n} = \frac{x_{2,n}(x_{1,n} - H)}{\sqrt{M^2 + (x_{1,n} - H)^2}} + \nu_{2,n}$$

$$\boldsymbol{\nu}_n \sim \mathcal{N}(\mathbf{0}, \mathbf{R}(\boldsymbol{\theta}_{n-1}))$$



Case Study: Tracking a Falling Object





$$\hat{\mathbf{x}}_{0|0} = [61.5\text{ km}, 3400\text{ m/s}, 19\,100]^T$$

$$\mathbf{x}_0 = [61\text{ km}, 3048\text{ m/s}, 19\,161]^T$$

Case Study: Tracking a Falling Object

Ensemble-Averaged Root Mean Square Error

$$\text{EA-RMSE}(n) = \sqrt{\frac{1}{N}\sum_{k=1}^{N}\left(\left(x_{1,n}^{(k)} - \hat{x}_{1,n}^{(k)}\right)^2 + \left(x_{2,n}^{(k)} - \hat{x}_{2,n}^{(k)}\right)\right)^2}$$

Case Study: Tracking a Falling Object

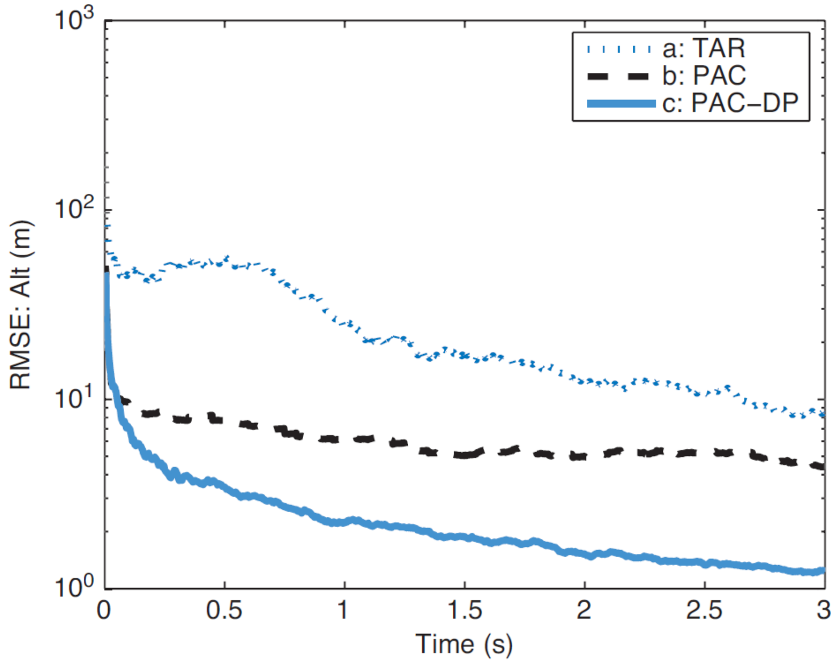


Figure 6.6. RMSE of altitude for: (a) traditional active radar with fixed waveform (TAR); (b) perception–action cycle with dynamic optimization (PAC); (c) perception–action cycle with dynamic programming (PAC- DP).

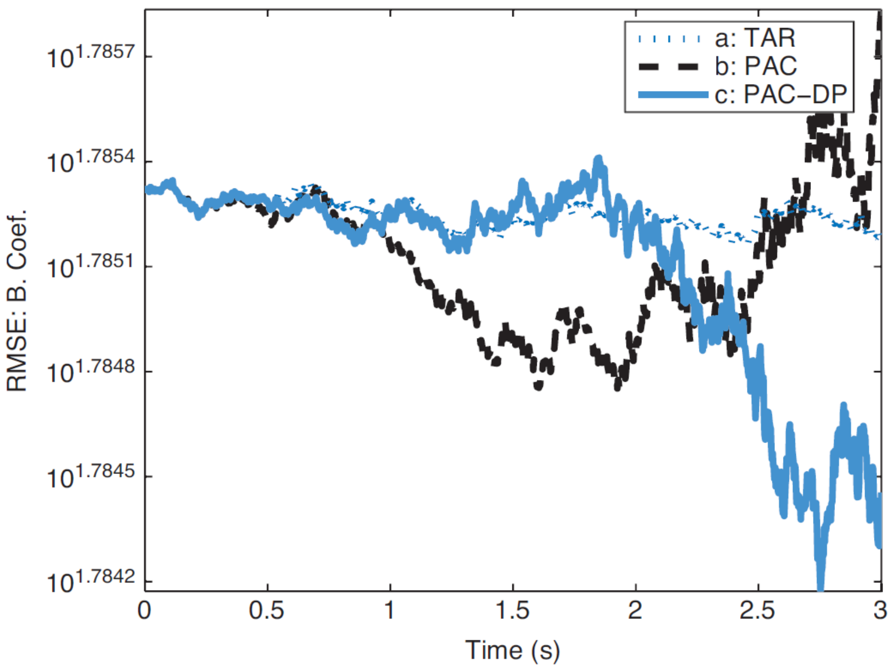


Figure 6.8. RMSE of the target ballistic coefficient for: (a) traditional active radar with fixed waveform (TAR); (b) perception–action cycle with dynamic optimization (PAC); (c) perception–action cycle with dynamic programming (PAC-DP).

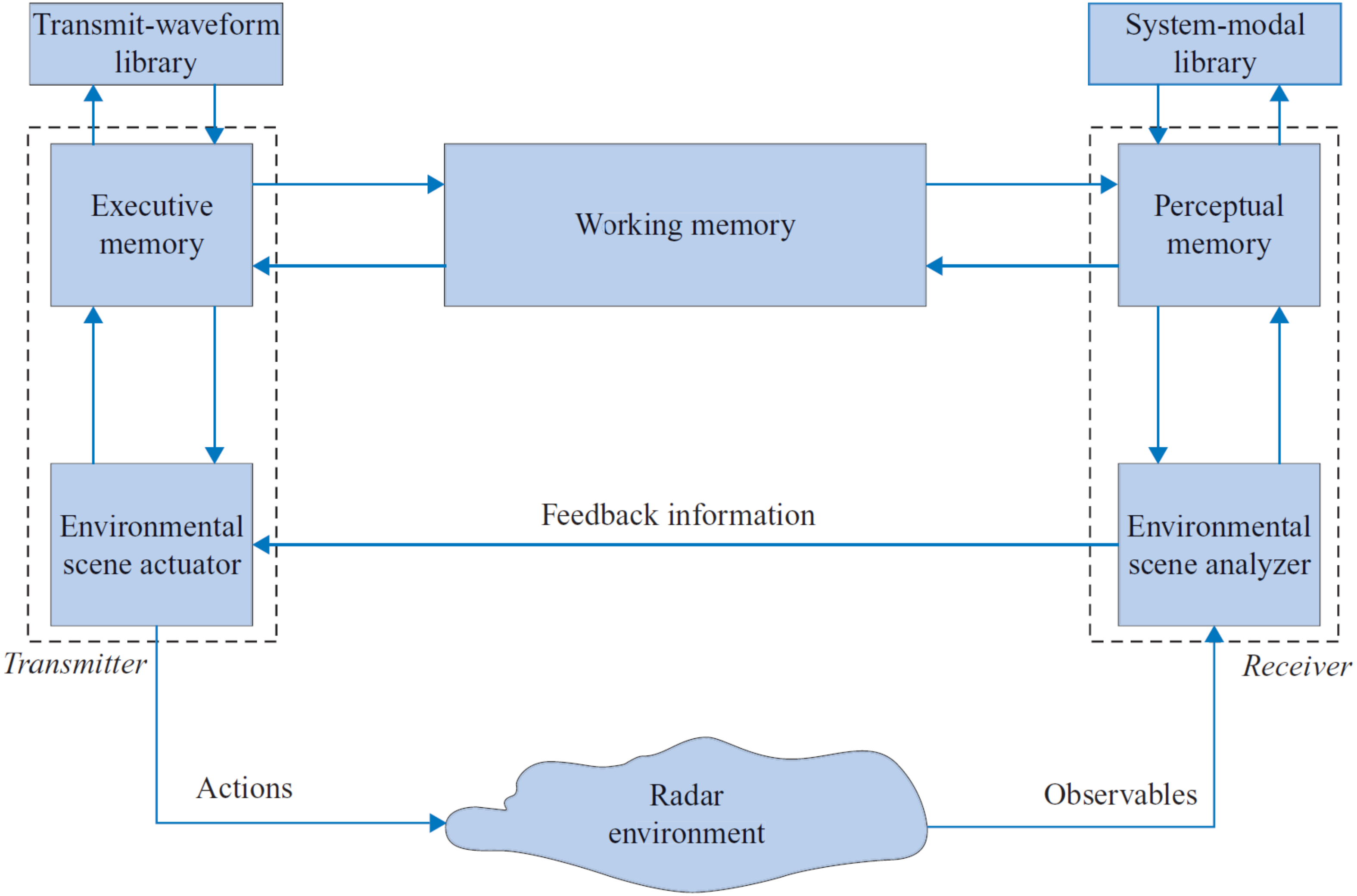
Cognitive Radar with Single Layer of Memory

Perceptual memory : general understanding toward the environment, coupled with environmental scene analyzer

Executive memory : memory related with the actuator’s actions

Working memory : short-span memory for prediction and attention

Cognitive Radar with Single Layer of Memory



Communication in Cognitive Radar

Step 1. Receiver sends measurement to the perceptual memory; the goal is to retrieve particular function a() that best matches.

Step 2. System equations are updated and new features of environment is sent to the perceptual memory as information A_in.

The information is temporarily stored as an working memory.

Communication in Cognitive Radar

Step 3. Feedback information is computed based on error vector and sent to the actuator. Kalman filter is used.

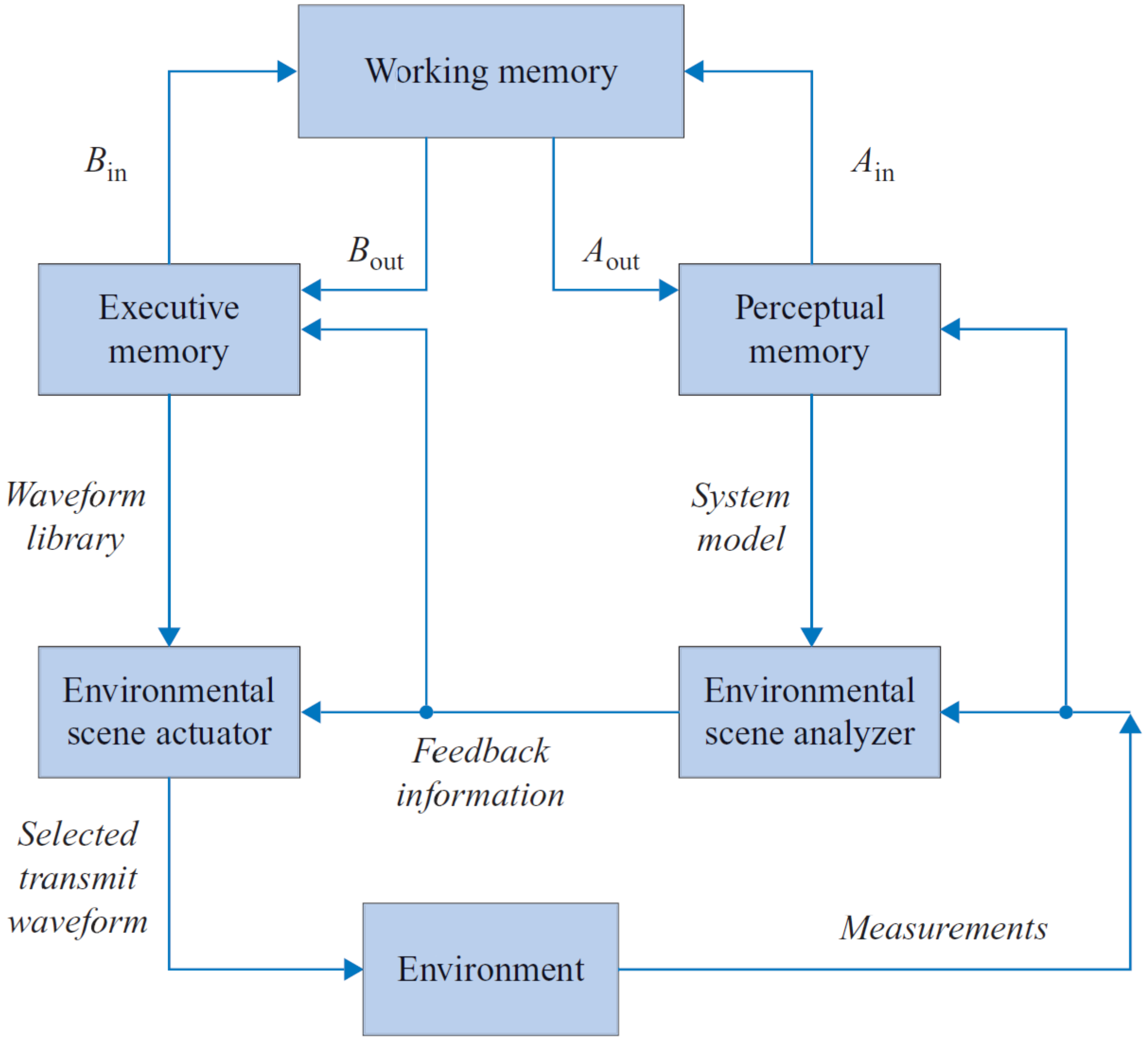
Step 4. The feedback information is sent to the executive memory. The goal is to retrieve particular subset of transmit-waveform library that best fit to the radar environment as information B_in.

Communication in Cognitive Radar

Step 5. The new subset of waveform library is fed back to the environmental scene actuator. Cost-to-go function is formulated to select optimum transmit waveform.

Step 6. Output A_out corresponding to A_in, and B_out corresponding to B_in is sent to the executive memory.

Communication in Cognitive Radar



Interpretation of Triple Memory System

What is that thing on the plant?
Aha, it is a moth!

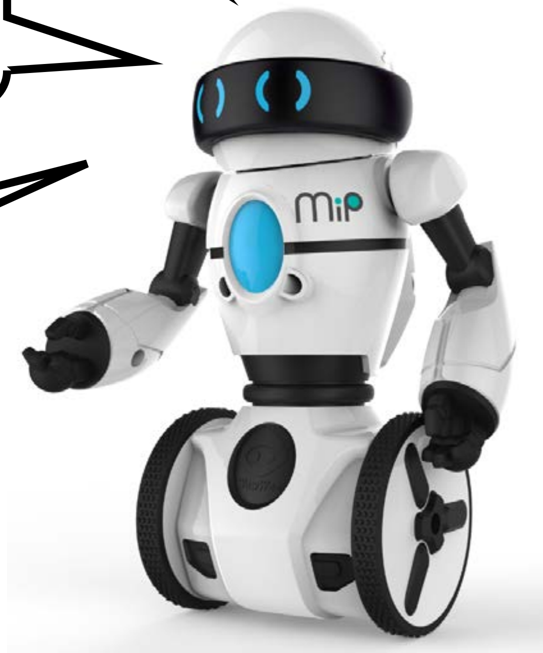
Perceptual memory

Moth, moth, moth.... What should I do?

Working memory

It would better if I pay more
attention to it!

Executive memory



The Composition of Percept

Percept : Snapshot of the perception process.

Component 1 : *Recognition* and, therefore, retrieval of a set of nonlinear functions and associated system noise variances, which are stored in the perceptual memory.

Component 2 : *Categorization*(classification) of features in the new measurements that are matched with the memory

The Composition of Execution

Execution : Snapshot of the decision-making process

Component 1 : *Recognition* and retrieval of a particular set in the waveform library, which is stored in the executive memory.

Component 2 : *Categorization* of the current feedback information that is matched with the memory.

Communications Within the Memory System

Perceptual memory - Receiver

Executive memory - Transmitter

Working memory acts as on the “mediator” between the two memories through a *matching process*

Cognitive Radar with Multiscale Memory

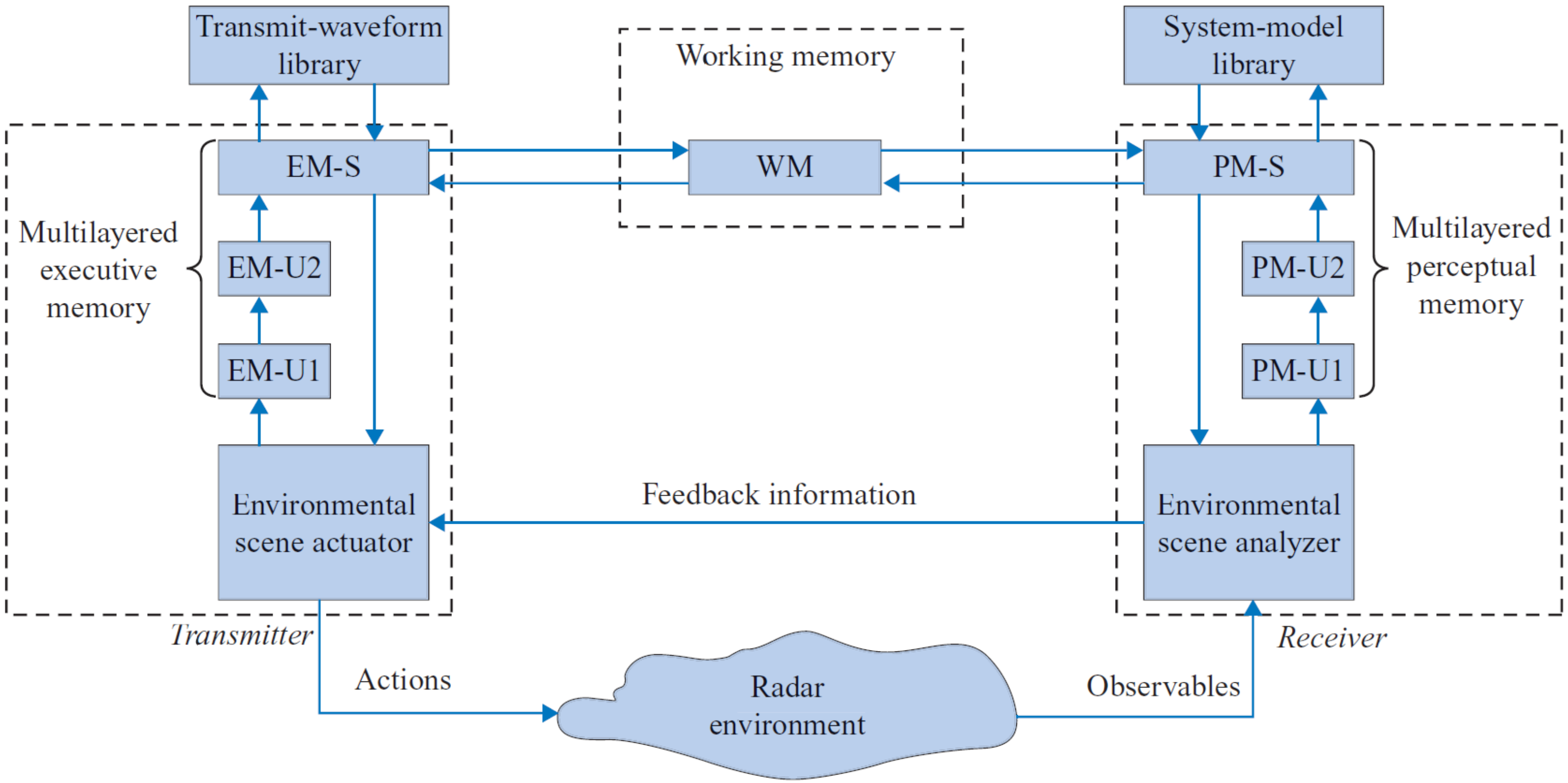


Figure 6.15. Cognitive radar with multiscale memory. Acronyms: (1) Perceptual memory: PM; unsupervised learning: PM-U1, and PM-U2; supervised learning: PM-S. (2) Executive memory: EM; unsupervised learning: EM-U1, and EM-U2; supervised learning: EM-S.

Cognitive Radar with Multiscale Memory

Benefits :

Radar's capabilities of perception, memory, attention, and intelligence are strengthened;

The Environmental disturbances become less effective.

Temporal discontinuity can be modified.

"The estimation accuracy of range and range-rate will progressively improve with increasing depth of the multiscale memory."

Sparse Coding

Goal : *Redundancy Reduction*

Analogy with a brain :

"The principle of sparse coding refers to a neural code, in which each sensory input of a nervous system is represented by the strong activation of a relatively small number of neurons out of a large population of neurons in the system."

- Olshausen and Field, 1996

Sparse Coding

Advantages of sparse coding :

Increased storage capacity in memories

Representation of complex signals in a manner easier to recognize at higher levels of memory

More robust to noise

Saving of energy

- Olshausen and Field, 1996