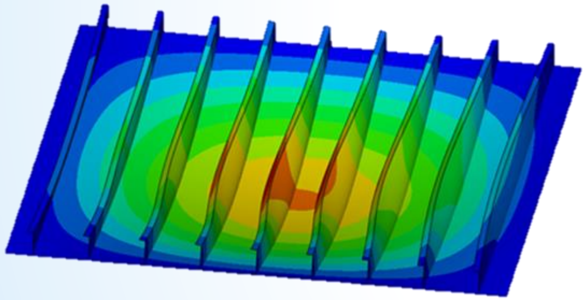
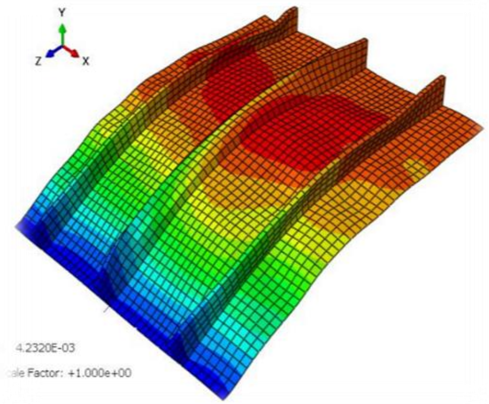
Topics in Ship Structures

(Advanced Local Structural Design & Analysis of Marine Structures)





* Buckling of Stiffened Panels (Topic 10)

(Post-buckling Behaviour)

Do Kyun Kim

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Adv. Local Structural Design & Analysis of Marine Structures (Overview)

[Theory of Plates and Grillages]

[Part I] Plastic Design of Structures

- Plastic theory of bending (Topic 1)
- Ultimate loads on beams (Topic 2)
- Collapse of frames and grillage structures (Topic 3)

[Part II] Elastic Plate Theory under Pressure

- Basic (Topic 4)
- Simply supported plates under Sinusoidal Loading (Topic 5)
- Long clamped plates (Topic 6)
- Short clamped plates (Topic 7)
- Low aspect ratio plates, strength & permanent set (Topic 7A)

[Part III] Buckling of Stiffened Panels

- Failure modes (Topic 8)
- Tripping (Topic 9) + Post-buckling strength of plate (Topic 9A)
- Post-buckling behaviour (Topic 10)

Objectives

The aim of this lecture is:

- To equip you with the knowledge and understanding of ultimate buckling strength of a flat plate.

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Ultimate strength of initially deflected plate under longitudinal compression: Part I = An advanced empirical formulation

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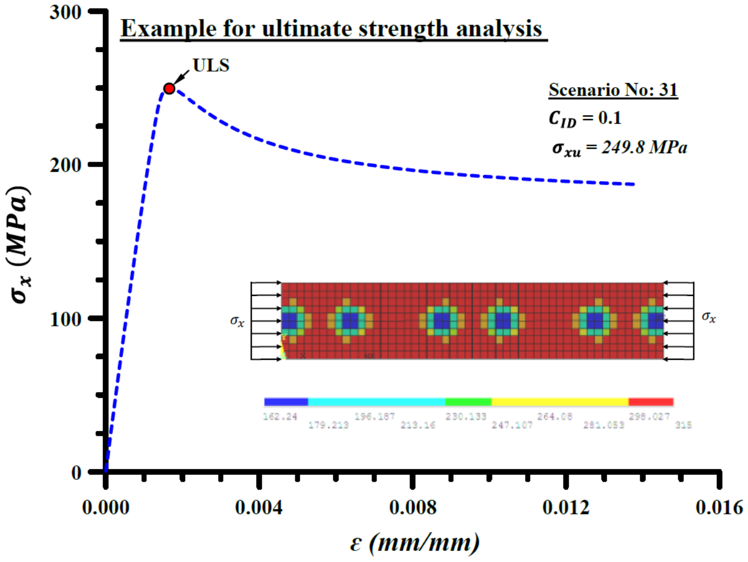
Abstract. In this study (Part I), an advanced empirical formulation was proposed to predict the ultimate strength of initially deflected steel plate subjected to longitudinal compression. An advanced empirical formulation was proposed by adopting Initial Deflection Index (IDI) concept for plate element which is a function of plate slenderness ratio (β) and coefficient of initial deflection. In case of initial deflection, buckling mode shape, which is mostly assumed type in the ships and offshore industry, was adopted. For the numerical simulation by ANSYS nonlinear finite element method (NLFEM), with a total of seven hundred 700 plate scenarios, including the combination of one hundred (100) cases of plate slenderness ratios with seven (7) representative initial deflection coefficients, were selected based on obtained probability density distributions of plate element from collected commercial ships. The obtained empirical formulation showed good agreement (R^2 = 0.99) with numerical simulation results. The obtained outcome with proposed procedure will be very useful in predicting the ultimate strength performance of plate element subjected to longitudinal compression.



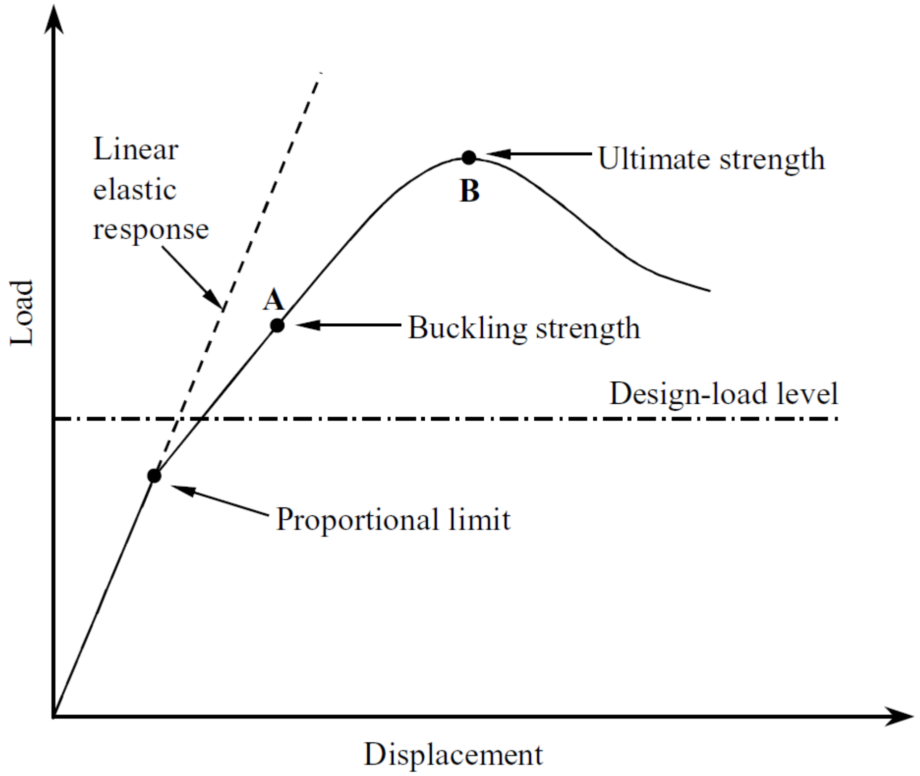
Learning Outcome

At the end of this lecture, you should be able to:

- Be aware of post buckling behaviour.
- Distinguish between initial buckling and collapse of plates under in-plane compression.
- Be familiar with the effective width of plate in relation to long plate strength and wide plate strength against buckling.
- Evaluate longitudinal and transverse compression strengths of plates.
- Discuss the factors affecting ultimate strength of plate.

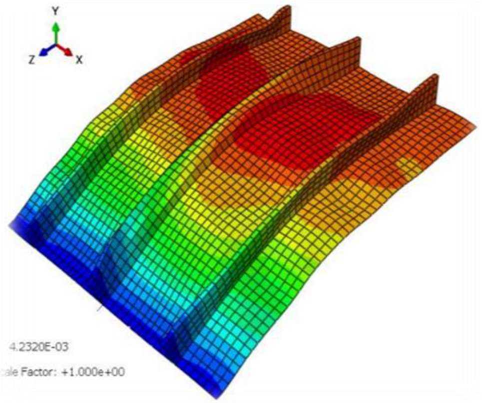
Introduction

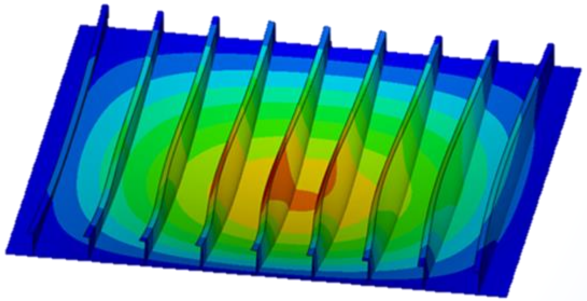
- A plate element with rigidly clamped edges is able to withstand lateral pressure much greater than that causes yielding, as membrane tension can develop in the plate under large deflection, adding appreciable stiffness to the plate.



[Ref.] Paik and Thayamballi (2007)

- Now, we should think what will happen when plates are subjected to in-plane compression beyond their initial critical buckling stresses.





*Lecture 10.1: Buckling of Stiffened Panels

Post-buckling Behaviour (Topic 10)

After buckling of the flat plate has occurred,

- The lateral deflections of grow rapidly under continued loading
- These deflections become the order of magnitude of thickness of the plate
- This indicates that deflections can no longer be considered small as used in linear theory.

As consequence of large deflections, the in-plane forces deviate significantly from their uniform values before buckling occurs.

In large deflection theory, the strain-displacement relations due to middle surface stretching becomes:

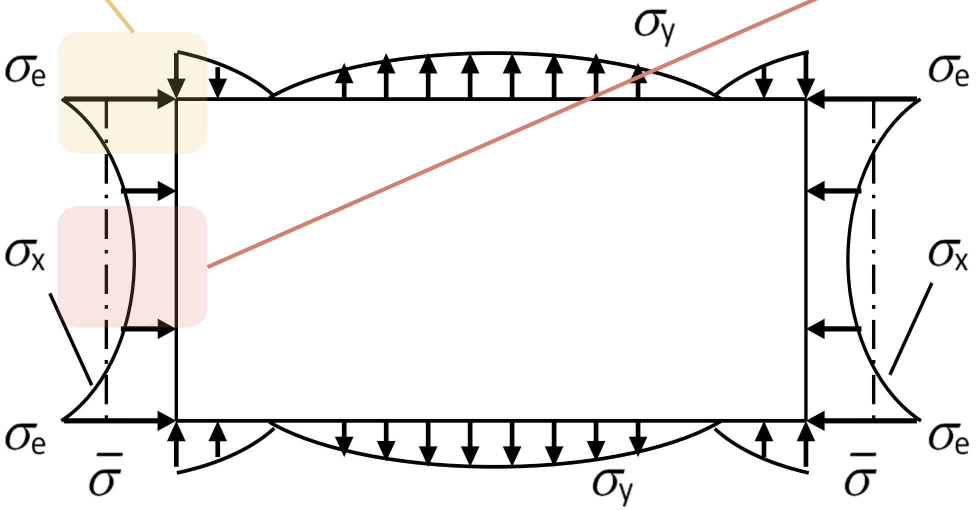
$$\varepsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \quad \varepsilon_y = \frac{\partial v}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 \quad \gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}$$

and hence M_x, M_y, M_{xy} etc., are different from that derived in small deflection theory.

Post-buckling Behaviour (1/3)

From below figure,

- We can observe that the axial compressive stress (σ_x) is no longer uniformly distributed over the loaded edge as it is before buckling occurs.
- It has maximum at simply supported edges (σ_e) and decreases to centre.



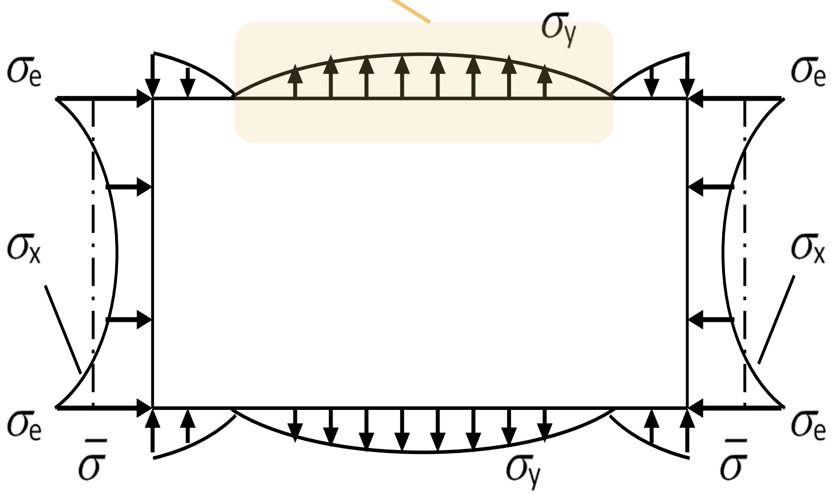
Post-buckling Behaviour (2/3)

- The middle-surface stresses (σ_y) that arise in the post buckling region for plates whose simply-supported unloaded edges are constrained to remain straight.

- In the central region of the plate, the σ_y stresses are tensile in character. These stresses tend to stiffen the plate against further lateral deflection and thus permit the plate to carry excess load beyond buckling load.

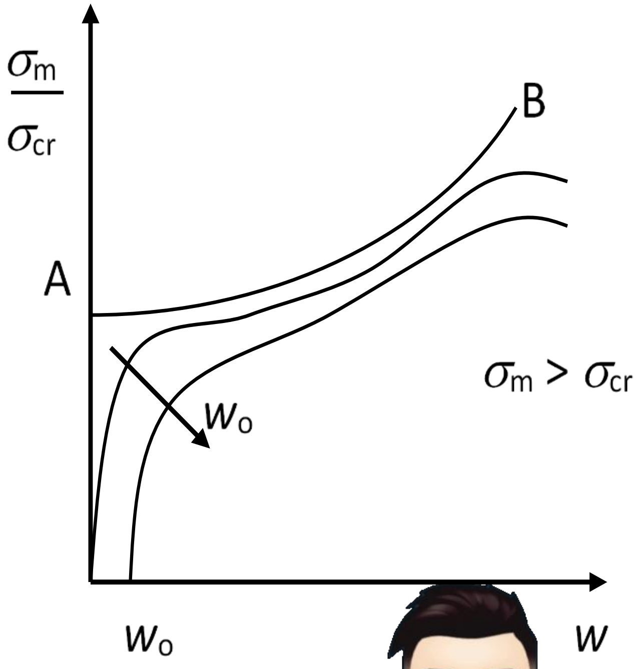
- By contrast, no such middle-surface forces arise in buckling of a column.
  (Main difference between 1D & 2D structure)

- Therefore, the load carrying ability of a column essentially terminates at buckling.



Post-buckling Behaviour (3/3)

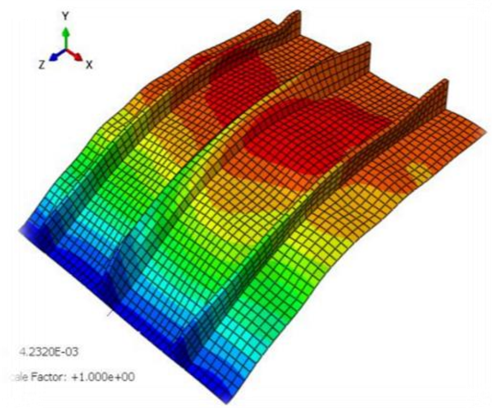
Most plates are able to carry load beyond the elastic buckling load, these being a considerable increase in buckled deformation w as shown.

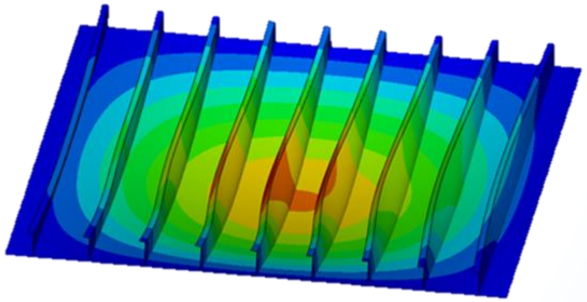


- Practical plates seldom experience the classical "bifurcation", where a rapid growth in buckled form appear at 'A' and increase indefinitely to 'B'.

- There is usually an increase in initial deformation 'δ' until 'A' and a more rapid increase afterwards. A maximum load is reached at stress σ_m, which is a function of plate slenderness β.

$$\beta = \frac{b}{t}\sqrt{\frac{\sigma_{yield}}{E}}$$



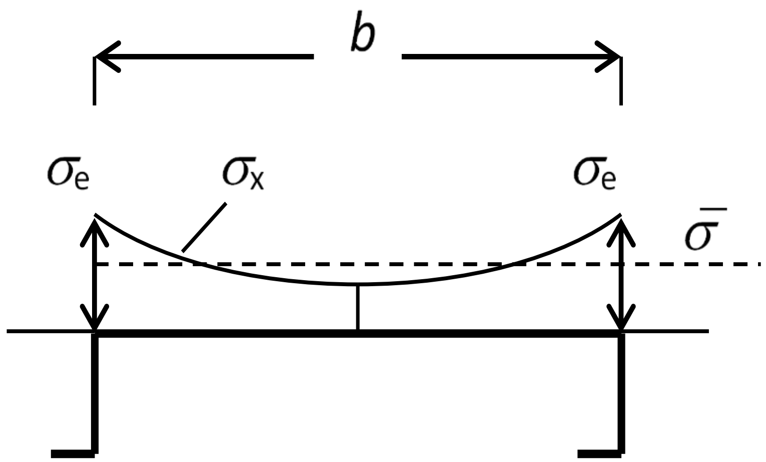


*Lecture 10.2: Buckling of Stiffened Panels

Effective width & Long plate strength

Effective Width & Long Plate Strength (1/6)

Effective width and long plate strength



When load increases beyond the initial elastic plate buckling stress

$$\sigma_E = k\frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2$$

For long plate under in-plane compression at short sides,

- The middle portion of the plate element does not take any further stress
- But the edges will carry the increased load

If these edges are supported sufficiently to sustain yield stress in compression, there will be a load shedding action away from the middle towards the edges as shown in the figure.

Effective Width & Long Plate Strength (2/6)

Such increase in load carrying capacity occurs from

- Initial deflections or
- Post-buckling actions.

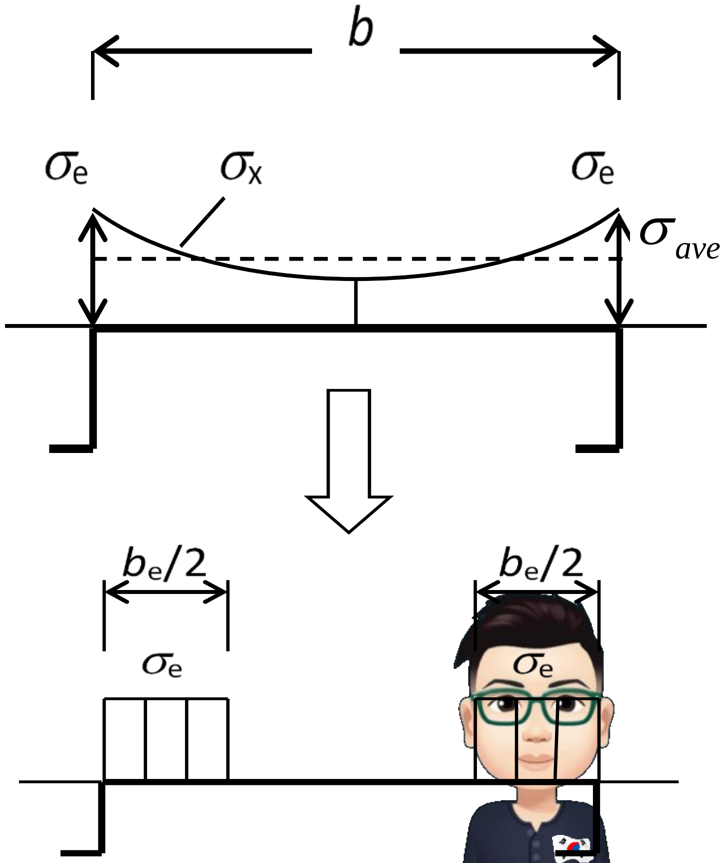
The stress distribution can be conveniently idealised into edge zones of uniform intensity σ_e (edge stress) and the effective width b_e.

$$b_e \sigma_e = \int \sigma_x dy = b \sigma_{ave}$$

where σ_{ave} is average stress.

The effective width be depends upon

- Level of compression load
- Initial distortion
- Welding residual stresses
- Plate slenderness β and in particular thickness t.



Effective Width & Long Plate Strength (3/6)

In predicting ultimate plate strength, it is the maximum average stress σ_m at which the plates will finally collapse when their edge stresses σ_e reach the yield stress σ_{yield} of material.

$$b_{em}\sigma_{yield} = b\sigma_m$$

where b_{em} is the minimum effective width for the predicted collapse load $b_{em}\, t\sigma_{yield}$. Then

$$\sigma_e = \sigma_{yield} = k\frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b_{em}}\right)^2$$

$$\therefore \quad b_{em} = \pi t\sqrt{\frac{E}{3(1-\nu^2)\sigma_{yield}}}$$

Effective Width & Long Plate Strength (4/6)

For simply supported plate, k = 4 and assuming υ = 0.3 in the above equation gives

$$\therefore \quad b_{em} = 1.9t\sqrt{\frac{E}{\sigma_Y}}$$

- b_{em} = 57t for mild steel E = 208000 N/mm^2 and σ_{yield} = 230 N/mm^2.
- Thickness is seen to be important.
- Values of b_{em} from 30t to 50t have been recommended.

Furthermore, we have

$$\frac{b_{em}}{b} = 1.9\frac{t}{b}\sqrt{\frac{E}{\sigma_{yield}}} = \frac{1.9}{\beta}$$

Hence,

$$\frac{\sigma_m}{\sigma_{yield}} = \frac{b_{em}}{b} = \frac{1.9}{\beta}$$

Effective Width & Long Plate Strength (5/6)

This plate strength equation given by von-Karman served the aircraft industry very well, where b/t ratios are generally large (in the range of 200 to 1000).

Moreover,

$$\sigma_E \text{ or } \sigma_{cr\_\min} = k\frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2 = 3.62E\left(\frac{t}{b}\right)^2$$ for k = 4 and v = 0.3

$$\therefore \quad \frac{\sigma_{cr}}{\sigma_{yield}} = 3.62\frac{E}{\sigma_{yield}}\left(\frac{t}{b}\right)^2$$

$$\therefore \quad \frac{\sigma_{cr}}{\sigma_{yield}} = \frac{3.62}{\beta^2}$$

Effective Width & Long Plate Strength (6/6)

For ship the equation, $\sigma_m / \sigma_{yield}$ = 1.9/beta was found to be too optimistic at low b/t ratios (30 to 120). After an exhaustive examination of many effective width equations and several hundred test data, Faulkner proposed the following plate strength equation

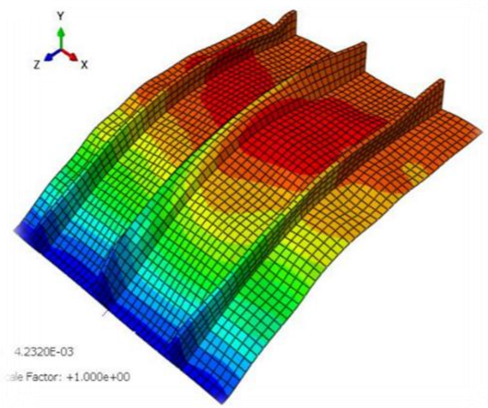
$$\frac{\sigma_u}{\sigma_Y} = \frac{\sigma_m}{\sigma_{yield}} = \frac{b_{em}}{b} = \begin{cases} \frac{2}{\beta} - \frac{1}{\beta^2} & \text{for } \beta \geq 1 \\ 1 & \beta < 1 \end{cases}$$

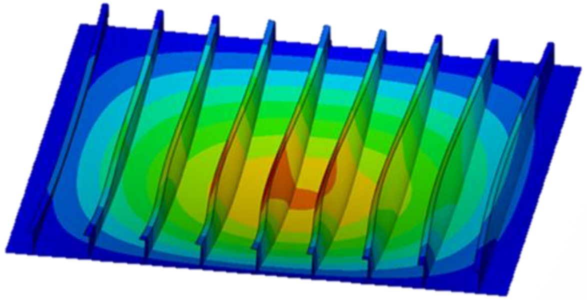
Important!

This plate strength equation for simply supported long plate under in-plane compression at short sides can be used:

- To determine maximum average stress σ_m for a given plate slenderness β.
- To determine stiffener spacing b or thickness t for a given factor of safety against maximum average stress σ_m.

Want to learn more? https://www.sciencedirect.com/science/article/pii/S0029801818310357
https://www.sciencedirect.com/science/article/pii/S0029801817302846





*Lecture 10.3: Buckling of Stiffened Panels

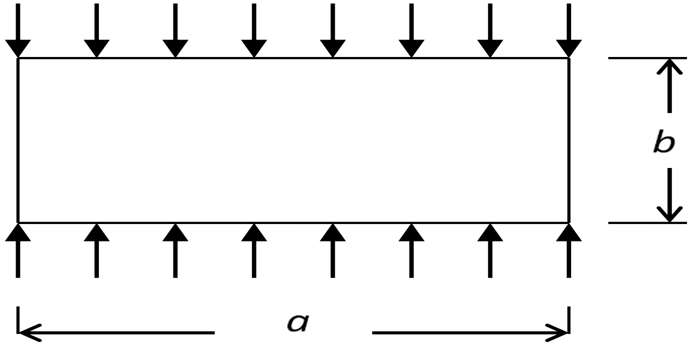
Wide plate strength

Wide Plate Strength (1/2)

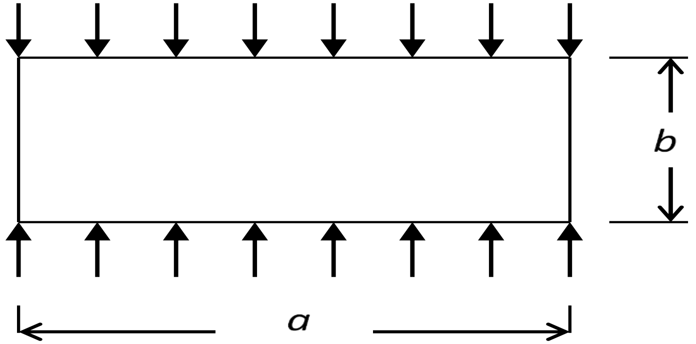
Wide plate strength

We can now apply the effective width concept to define the strength of wide plates on the following bases.

1. At failure the edge stress σ_e is limited by the yield stress.
2. There is a middle zone of the plate effectively carrying the buckling stress σ_{cr} for an infinitely wide plate.
3. The edge stress σ_e must be in equilibrium with the average failure load σ_m.



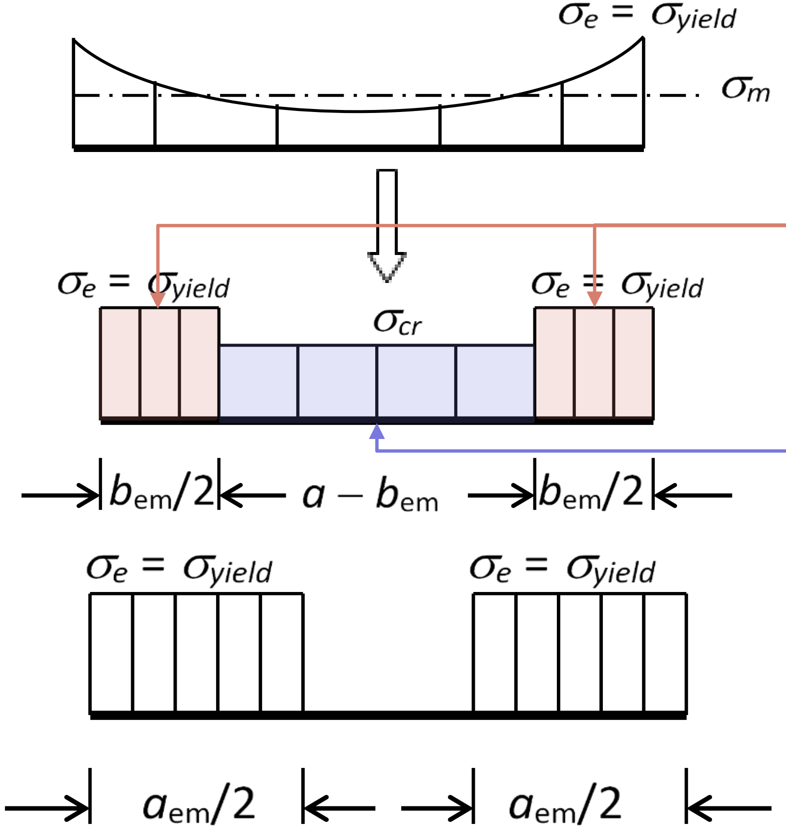
Wide Plate Strength (2/2)



For width plate the minimum critical buckling stress is

$$\min \sigma_{cr} = \frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2 \;\Rightarrow\; \therefore \min \sigma_{cr} = \frac{0.904}{\beta^2}\sigma_{yield}$$

for υ = 0.3



By Von Karman expression for plate strength

$$\frac{b_{em}}{b} = \frac{1.9}{\beta}$$

For equilibrium, we have,

$$\sigma_m a = b_{em}\sigma_{yield} + (a - b_{em})\sigma_{cr}$$

$$\therefore \frac{\sigma_m}{\sigma_{yield}} = \frac{b_{em}}{a} + \left(1 - \frac{b_{em}}{a}\right)\frac{\sigma_{cr}}{\sigma_{yield}} = \frac{1.9}{\beta}\frac{b}{a} + \left(1 - \frac{1.9}{\beta}\frac{b}{a}\right)\frac{0.904}{\beta^2}$$

Then,

$$\frac{\sigma_m}{\sigma_{yield}} = \frac{a_{em}}{a} = \frac{1.9}{\beta}\frac{b}{a}\left(1 - \frac{0.904}{\beta^2}\right) + \frac{0.904}{\beta^2}$$

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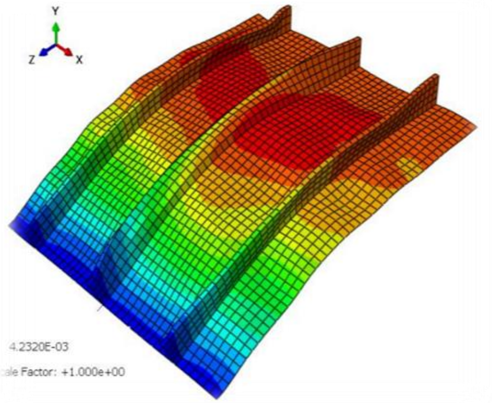
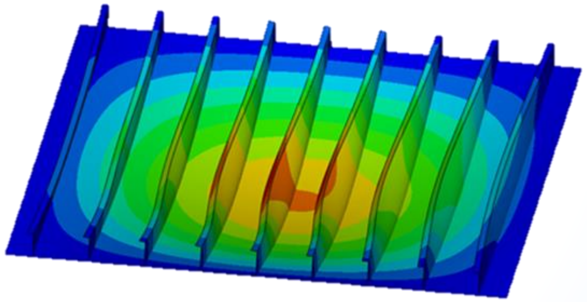




*Lecture 10.4: Buckling of Stiffened Panels

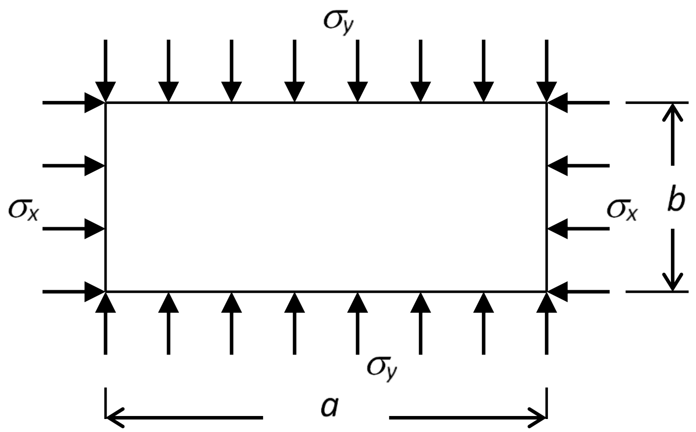
Bi-axially loaded plate

Bi-axially Loaded Plate

Bi-axially loaded plate

The failure conditions for bi-axially loaded plate would be best described by an interaction equation:

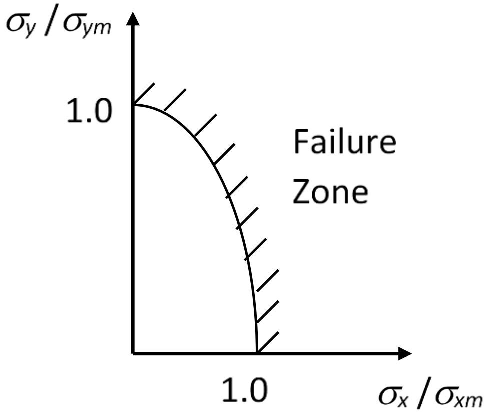
$$\frac{\sigma_x}{\sigma_{xm}} + \left(\frac{\sigma_y}{\sigma_{ym}}\right)^2 = 1$$

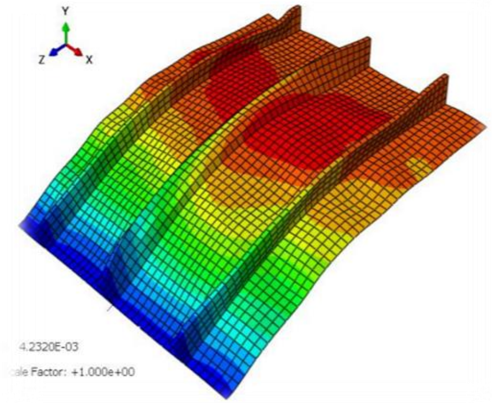


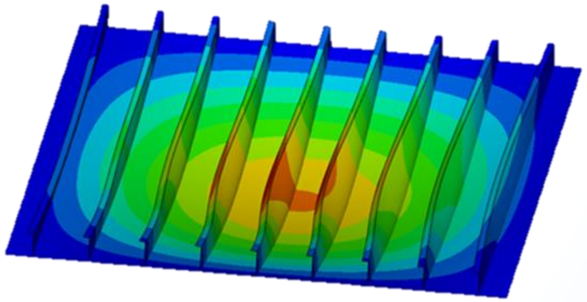
where

σ_{xm} is maximum average stress in x direction.

σ_{ym} is maximum average stress in y direction.







*Lecture 10.5: Buckling of Stiffened Panels

Factor affecting compression strength of long plate

→ BC & Initial imperfections

Factors affecting ULS Compression Strength of Long Plate (1/6)

a) Boundary conditions

- The buckling coefficient K for minimum critical elastic buckling stress in a clamped long plate is about 7, which is 75% greater than that for simply supported long plate. The wave lengths of the buckled pattern are also shorter.

- However, the reserve of strength in the post-buckling stage is reduced over most of the slender range and typically the strength of clamped plate is only 10 to 25% greater than for simply supported plate.

- A satisfactory expression for clamped plate strength is given by

$$\frac{\sigma_m}{\sigma_{yield}} = \frac{2.5}{\beta} - \frac{1.5625}{\beta^2} \quad \text{for } \beta > 1.25$$

$$\frac{\sigma_{xu}}{\sigma_Y} = \frac{b_e}{b} = \begin{cases} 1.0 & for \quad \beta < C_2 \\ C_1/\beta - C_2/\beta^2 & for \quad \beta \geq C_2 \end{cases}$$

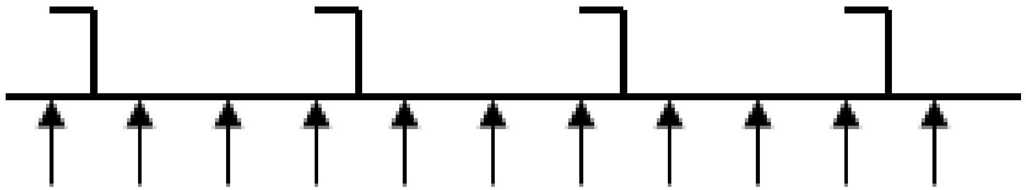
$$\begin{array}{lll} Simply\,supported & \rightarrow & C_1 = 2.0 \;\; \& \;\; C_2 = 1.0 \\ Clamped & \rightarrow & C_1 = 2.5 \;\; \& \;\; C_2 = 1.5625 \end{array}$$

Factors affecting ULS Compression Strength of Long Plate (2/6)

This may be used for compression strength estimation where

i) The rotation of the plate elements at their long edges is restricted by closed section, stiffeners of high torsional rigidity.

ii) The lateral pressure exists at high enough level to force a “clamped” mode of failure, i.e. to achieve zero slope over the stiffeners.



Factors affecting ULS Compression Strength of Long Plate (3/6)

b) Initial deformation

- Normal (lateral) deformation w_p which are same or close to the lower natural buckling forms, i.e. $m = a/b$, will lower the strength of flat plates because of magnification effects $w_p / (1 - \sigma / \sigma_{cr})$.

- However, for most plates where $a > b$, these natural modes do not occur through welding which induces a predominant $m = 1$ deformation.

- Hence, it behaves essentially as a flat plate because σ_{cr} is so high and magnification factor = 1.

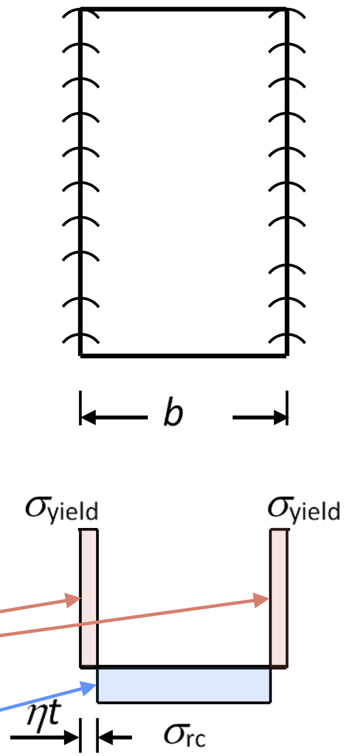
- In general, the effects of w_p are small and are incorporated in the empirically validated plate strength equations.

Factors affecting ULS Compression Strength of Long Plate (4/6)

c) Welding stresses

- When the long edges of a plate are welded (e.g. at the longitudinal stiffeners) a yield tension zone of width η_t arises as a residual longitudinal tensile force at each edge.

- These must be balanced by residual compression stresses σ_{rc} in the middle zone of the plate.



For equilibrium

$$2\eta t\sigma_{yield} = (b - 2\eta t)\sigma_{rc}$$

$$\therefore \frac{\sigma_{rc}}{\sigma_{yield}} = \frac{2\eta}{b/t - 2\eta}$$

Factors affecting ULS Compression Strength of Long Plate (5/6)

c) Welding stresses

The value of non-dimensional tension zero width parameter η depends upon

- The welding process MIG, TIG, etc.
- Whether welding is continuous, intermittent, staggered, etc.
- The rate of heat input to the weld, which in turn depends upon electric power input, weld speed, etc.
- The number of weld runs and in particular the cross-sectional area of the final weld deposit.

For ship with continuous welding η is about 3 to 6, but allowing for shake out of residual stress at sea due to alternate sag and hog bending, values of 1.5 to 4.5 are typically used.

Factors affecting ULS Compression Strength of Long Plate (6/6)

c) Welding stresses

For ship with continuous welding η is about 3 to 6, but allowing for shake out of residual stress at sea due to alternate sag and hog bending, values of 1.5 to 4.5 are typically used.

In any event the compression residual stresses lead to

- Premature plate element buckling at $\sigma = \sigma_{cr} - \sigma_{rc}$
- Loss in plate strength typically 10 to 15%

The two effects of welding stresses are

- To lower the maximum strength.
- To soften the suddenness of the unloading curve beyond σ_m.

Learning Outcomes (Review)

- We have investigated the Post-buckling behaviour.

- Now we are able to:
  - Be aware of post buckling behavior.
  - Distinguish between initial buckling and collapse of plates under in-plane compression.
  - Be familiar with the effective width of plate in relation to long plate strength and wide plate strength against buckling.
  - Evaluate longitudinal and transverse compression strengths of plates.
  - Discuss the factors affecting ultimate strength of plate.

- Details can be referred to topics 10 in the lecture notes.

[Theory of Plates and Grillages]

[Part I] Plastic Design of Structures

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- **Short clamped plates (Topic 7)**
- Additional (Low aspect ratio plates, strength & permanent set)

[Part III] Buckling of Stiffened Panels

- Failure modes (Topic 8)
- Tripping (Topic 9) + Post-buckling strength of plate (Topic 9A)
- Post-buckling behaviour (Topic 10)

Kam Sa Hab Ni Da

감사합니다

Thank you!

Questions?

Aerial View of Korean Presidential Archives in Sejong city
(Construction Completed in 2014)