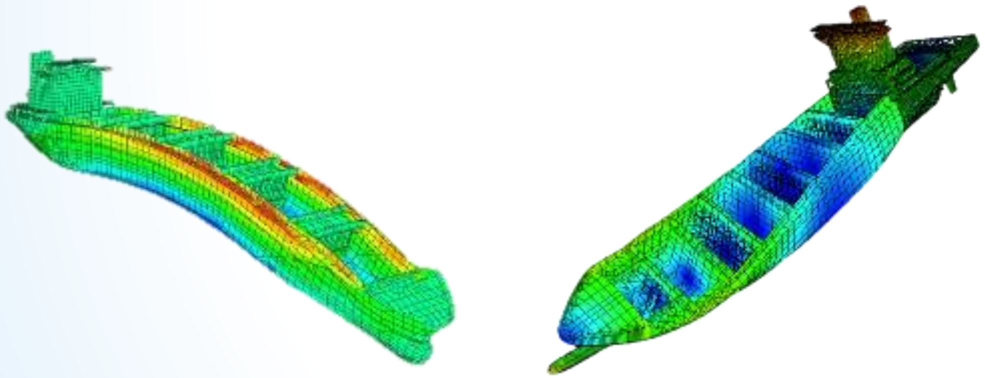
(Advanced Local Structural Design & Analysis of Marine Structures)



* Ultimate loads on beams (Topic 2)

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https://sites.google.com/snu.ac.kr/ost

Adv. Local Structural Design & Analysis of Marine Structures (Overview)

[Theory of Plates and Grillages]

[Part I] Plastic Design of Structures

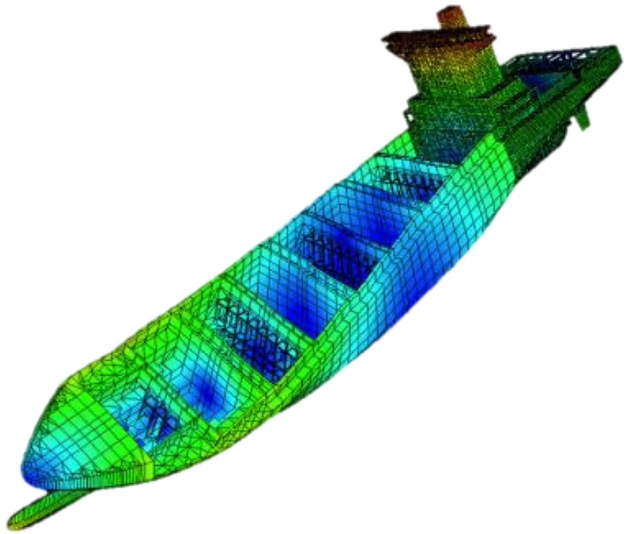
- Plastic theory of bending (Topic 1)
- Ultimate loads on beams (Topic 2)
- Collapse of frames and grillage structures (Topic 3)

[Part II] Elastic Plate Theory under Pressure

- Basic (Topic 4)
- Simply supported plates under Sinusoidal Loading (Topic 5)
- Long clamped plates (Topic 6)
- Short clamped plates (Topic 7)
- Low aspect ratio plates, strength & permanent set (Topic 7A)

[Part III] Buckling of Stiffened Panels

- Failure modes (Topic 8)
- Tripping (Topic 9) + Post-buckling strength of plate (Topic 9A)
- Post-buckling behaviour (Topic 10)



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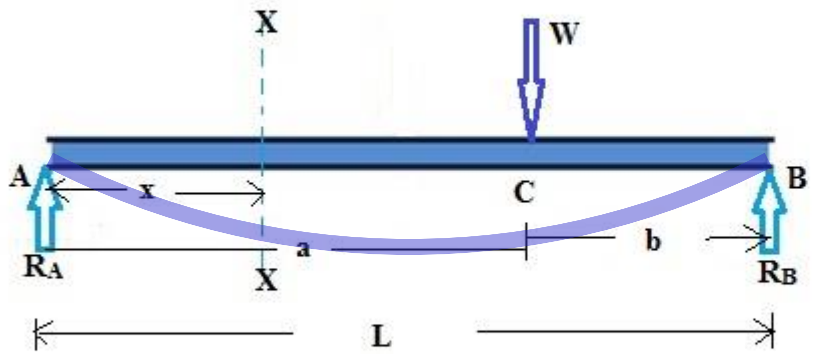
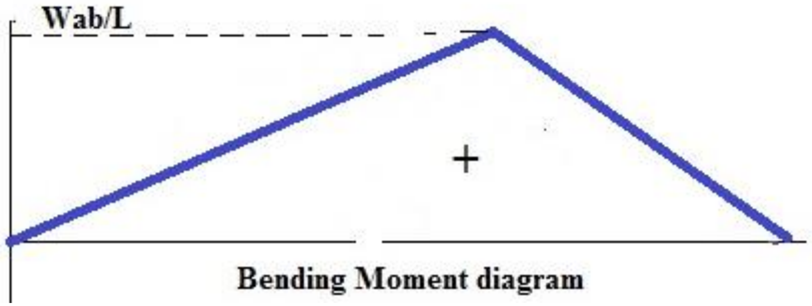
Objectives

The aim of this lecture is:

- To equip you with the necessary skills to carry out plastic analysis of beams.





Learning Outcome

At the end of this lecture, you should be able to:

- Discuss three conditions that must be satisfied in plastic analysis.
- Apply the mechanism method or the principle of virtual work to calculate ultimate load on a beam.
- Be aware of upper bound theorem and lower bound theorem.

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Introduction

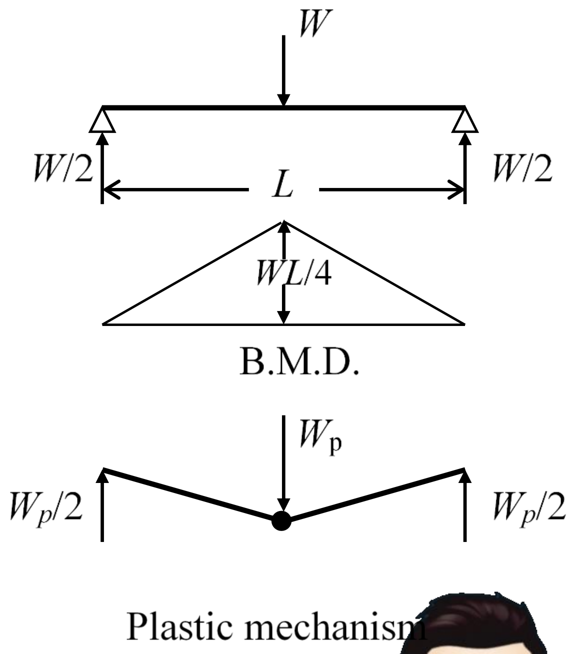
- In plastic design of structures, the criterion of failure is the collapse of structures.
- Therefore, it is necessary to find out the ultimate strength or load.
- These can be estimated by means of mechanism method or the principle of virtual work.

Ultimate Loads on Beams

(a) Simply supported beam with central point load

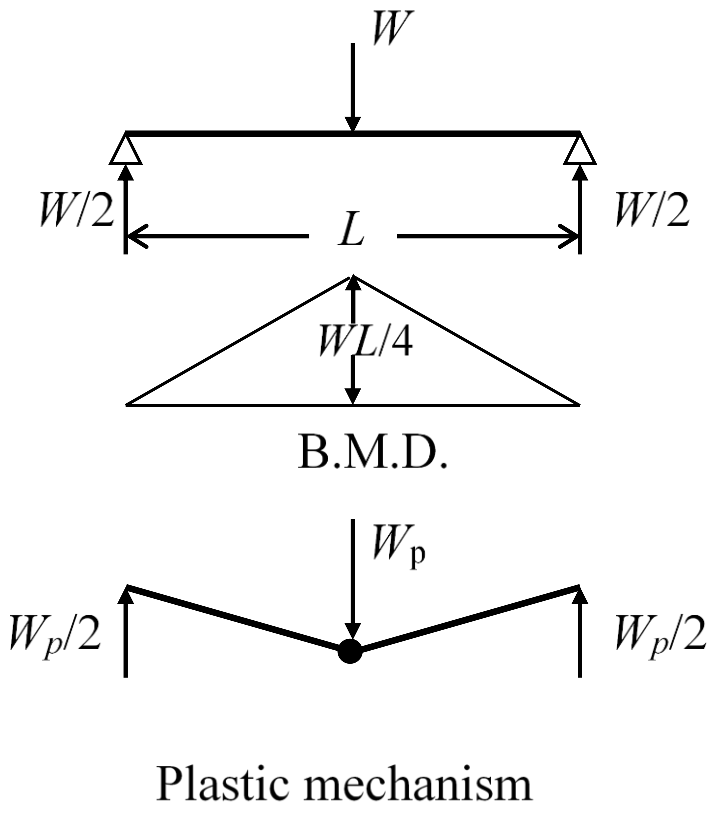
- The max bending moment occurs at mid-span and is $WL/4$.
- As the load increases, the yield first occurs at the mid-span
- The yield moment and yield load are, respectively,

$$M_Y = \frac{W_Y L}{4} \quad \& \quad W_Y = \frac{4M_Y}{L}$$



- Further increasing the load to Wp, a plastic hinge will form at the mid-span and the plastic moment is $M_p = \frac{W_p L}{4}$
- As there are three hinges: two at simple supports and one at the mid-span, no more load can be taken and so a collapse mechanism occurs.

(a) Simply supported beam with central point load



- Thus, the ultimate load is

$$W_p = \frac{4M_p}{L}$$

- Hence,

$$\frac{W_p}{W_Y} = \frac{M_p}{M_Y} = \varphi$$

$$W_Y = \frac{4M_Y}{L}$$

Ultimate Loads on Beams

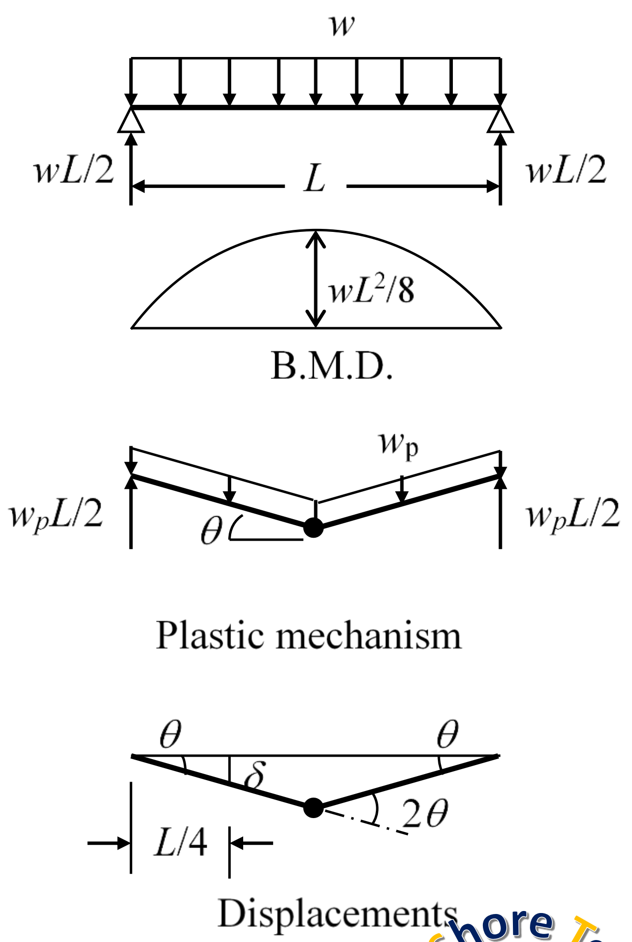
(b) Simply supported beam with uniform distributed load

- The B.M.D. indicates that the max bending moment occurs at mid-span and will reach the yield moment first

$$M_Y = \frac{w_Y L^2}{8}$$

- Thus, the yield load is

$$w_Y = \frac{8M_Y}{L^2}$$



Ultimate Loads on Beams

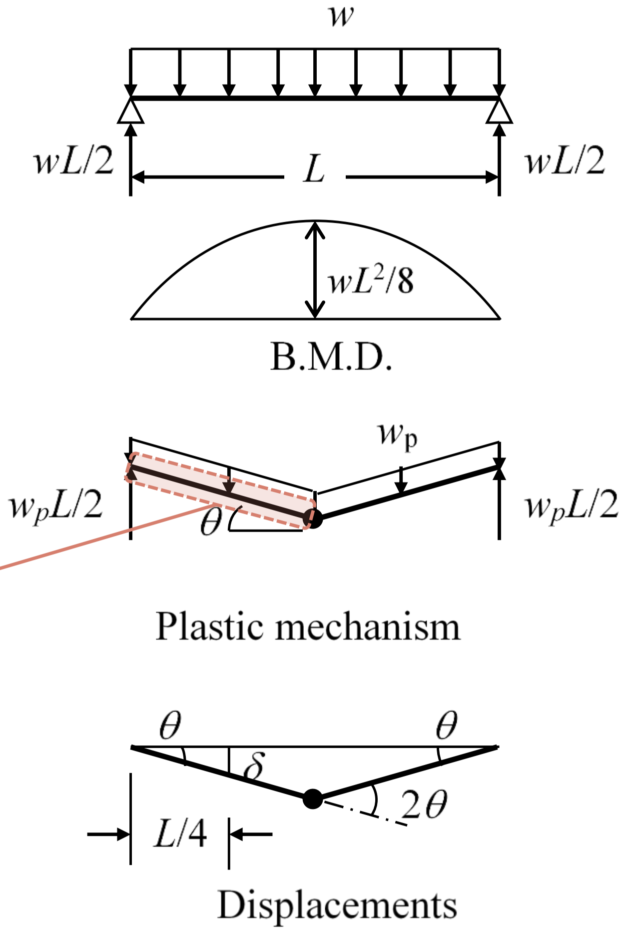
(b) Simply supported beam with uniform distributed load

- Further increase in load to w_p will made a plastic hinge at the mid-span and no more load can be taken. The beam will collapse.

- Internal work = $2\,M_p\,\theta$

- External work =

$$2\left(\frac{w_p L}{2}\delta\right) = 2\left(\frac{w_p L}{2}\frac{L\theta}{4}\right) = \frac{w_p L^2}{4}\theta$$



Ultimate Loads on Beams

(b) Simply supported beam with uniform distributed load

- By the principle of virtual work,

$$\frac{w_p L^2}{4}\theta = 2M_p\theta$$

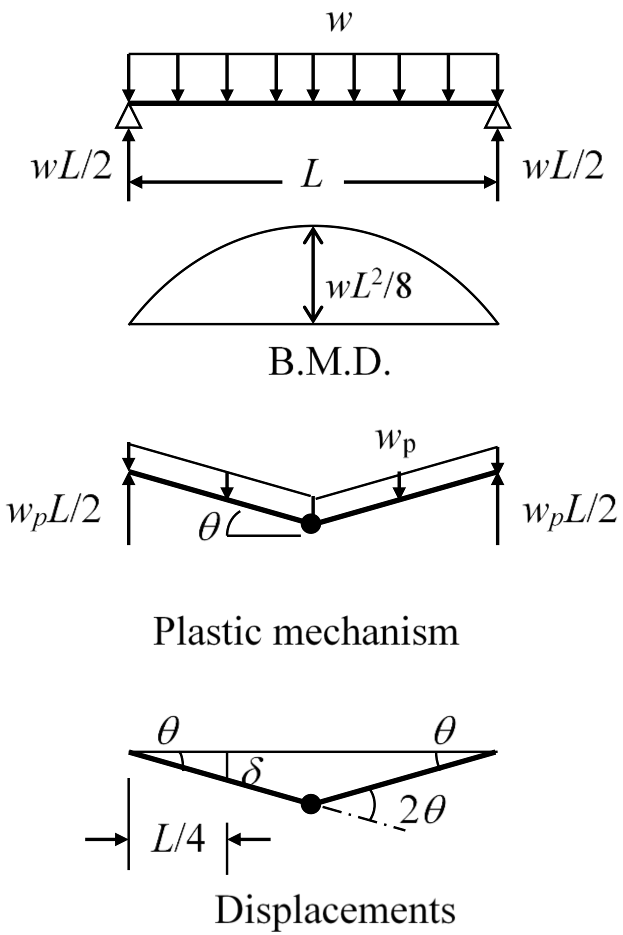
  - Thus, the ultimate load is

$$w_p = \frac{8M_p}{L^2}$$

- Hence,

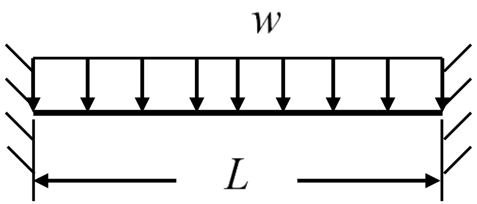
$$\frac{w_p}{w_Y} = \frac{M_p}{M_Y} = \varphi$$

$$w_Y = \frac{8M_Y}{L^2}$$



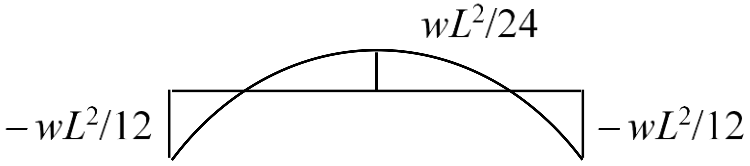
(c) Fixed end beam with uniform distributed load

- As w increases, first yield will occur at the extreme fibre at the supports (max. B.M.)



- Then, we have Yield moment

$$M_Y = \frac{w_Y L^2}{12}$$



B.M.D.

- And yield load

$$w_Y = \frac{12 M_Y}{L^2}$$

Refer to Chapter 6

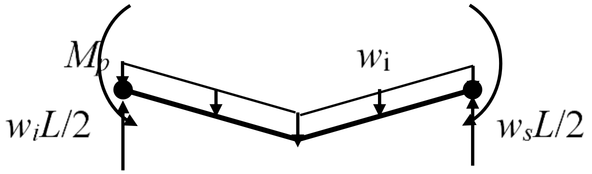
(c) Fixed end beam with uniform distributed load

- Further increase in w will increase the end moments up to M_p, at which loading hinges will form at the supports.

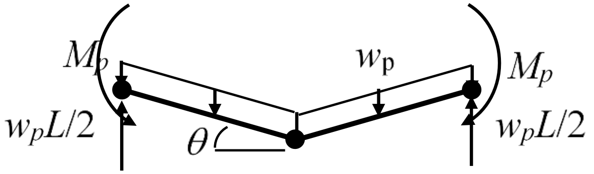
- With further loading, moment at the supports remains constant as the hinge rotates; it acts like a simply supported beam.

- Next cross-section with critical moment is the mid-span. First it suffers extreme fibre yield. Then as yield progresses, a hinge forms.

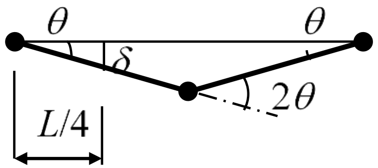
- Once 3 hinges have formed, collapse will occur.



First hinges at supports



Plastic mechanism



Displacements

(c) Fixed end beam with uniform distributed load

- Internal work = $4\ M_{\mathrm{p}}\ \theta$
- External work =

$$2\left(\frac{w_p L}{2}\delta\right) = 2\left(\frac{w_p L}{2}\frac{L\theta}{4}\right) = \frac{w_p L^2}{4}\theta$$

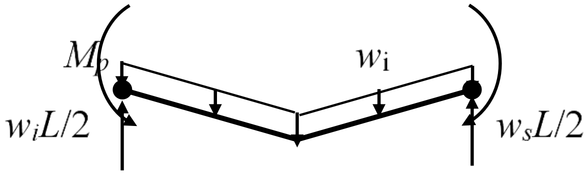
- By the principle of virtual work,

$$\frac{w_p L^2}{4}\theta = 4M_p\theta$$

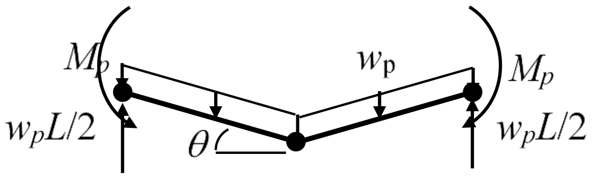
- Thus, the ultimate load

$$w_p = \frac{16M_p}{L^2}$$

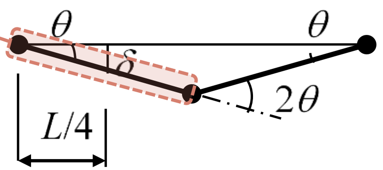
$$w_Y = \frac{12M_Y}{L^2}$$



First hinges at supports



Plastic mechanism



Displacements

Ultimate Loads on Beams

(c) Fixed end beam with uniform distributed load

- Hence,

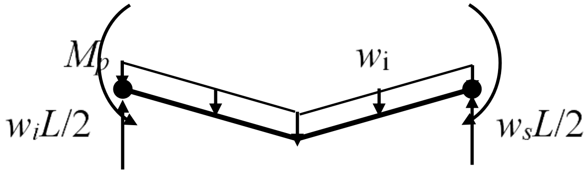
$$\frac{w_p}{w_Y} = \frac{16M_p}{12M_Y} = \frac{4}{3}\varphi$$

- Typical value of shape factor for stiffened plate is about 1.3, which gives

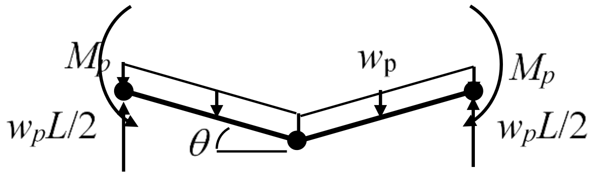
$$\frac{w_p}{w_y} = \frac{4}{3} \times 1.3 \approx 1.73$$

- The above example shows that plastic hinges form at the supports of the beam while the moment increases at the mid-span to maintain equilibrium with the load.

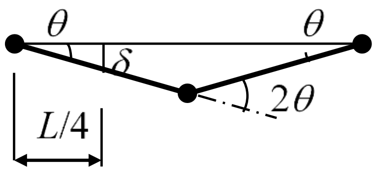
- This redistribution of moment continues through the attainment of yielding at the mid-span until the third plastic hinge is formed. Then a mechanism is formed.



First hinges at supports



Plastic mechanism



Displacements

Requirements of plastic analysis

- It has been pointed out that full yielding of a cross-section of a member results in a plastic hinge if no instability occurs.
- For statically indeterminate structures, the formation of a plastic hinge reduces the indeterminacy as shown in example (Page 11-14).
- After first plastic hinges have formed, further increases in load are possible until the formation of a sufficient number of plastic hinges
- This results in unrestricted plastic flow, that is, until a plastic mechanism of part, or of the whole structure has been reached.
- When the mechanism is formed, the structure CANNOT take any additional load.
- It is necessary to find the correct mechanism or its corresponding equilibrium moment diagram for strength evaluation.
- This is because the plastic strength of a beam depends on the formation of plastic hinges and on the redistribution of moments to form a mechanism,
- There are three conditions which must be satisfied in plastic analysis.

Requirements of plastic analysis

1 - Equilibrium: $\sum F = 0$ and $\sum M = 0$

2 - Mechanism: Sufficient plastic hinges form a plastic mechanism

3 - Plastic moment: $M = M_p$ is reached at each plastic hinge & $M < M_p$ at the remaining part of the beam.

Methods of plastic analysis

1. Mechanism Method

2. Principle of Virtual Work

1. Mechanism Method

- To fulfill all the conditions required for plastic analysis and to find the plastic strength, mechanism method may be used.

- The procedures of the mechanism method are:

  1. Assume a plastic mechanism.
  2. Find unknown reactions in terms of M_{p} by means of equilibrium condition.
  3. Finally find the ultimate load in terms of M_{p}.

2. Principle of virtual work

- Instead of equilibrium condition, the principle of virtual work can be used for alternative mechanism method

1. Assume a plastic mechanism.
2. Find displacement relationships from displacement diagram.
3. By the principle of virtual work to find ultimate load in terms of M_{p}.

- If the plastic moment condition $(M \leq M_{\mathrm{p}})$ everywhere is satisfied, the assumed mechanism is CORRECT and the load is the true ultimate load.

2. Principle of virtual work (Continued)

- However, if $M > M_p$ occurs at locations other than the assumed plastic hinges, the load is higher than the true load. This is the Upper Bound Theorem

- Upper bound theorem
  A load, computed on the basis of an assumed plastic mechanism, is at best equal to or greater than the true plastic limit load.

- Lower bound theorem
  A load, computed on the basis of a bending moment distribution in which no moment exceeds M_p, is at best equal to or less than the true plastic limit load.

Learning Outcomes (Review)

- We have investigated the Ultimate Loads on Beams.

- Now we are able to:

  - Discuss three condition which must be satisfied in plastic analysis.
  - Apply the mechanism method or the principle of virtual work to calculate ultimate load on a beam.
  - Be aware of upper bound theorem and lower bound theorem.

- Details can be referred to topics 2 in the lecture notes.

Adv. Marine Structures / Adv. Structural Design & Analysis (Next class)

[Theory of Plates and Grillages]

[Part I] Plastic Design of Structures

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[Part II] Elastic Plate Theory

- Basic (Topic 4)
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- Additional (Low aspect ratio plates, strength & permanent set)

[Part III] Buckling of Stiffened Panels

- Failure modes (Topic 8)
- Tripping (Topic 9)
- Post-buckling behaviour (Topic 10)

Kam Sa Hab Ni Da

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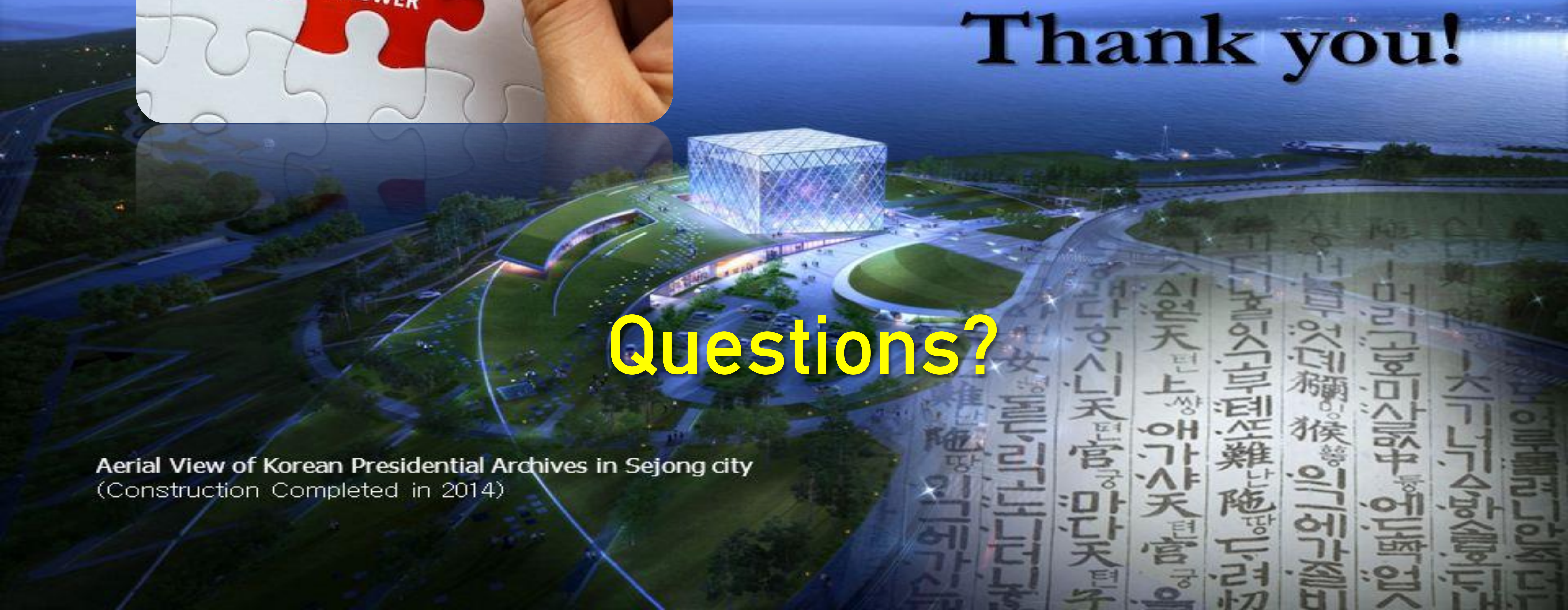
Thank you!

Questions?



Aerial View of Korean Presidential Archives in Sejong city
(Construction Completed in 2014)