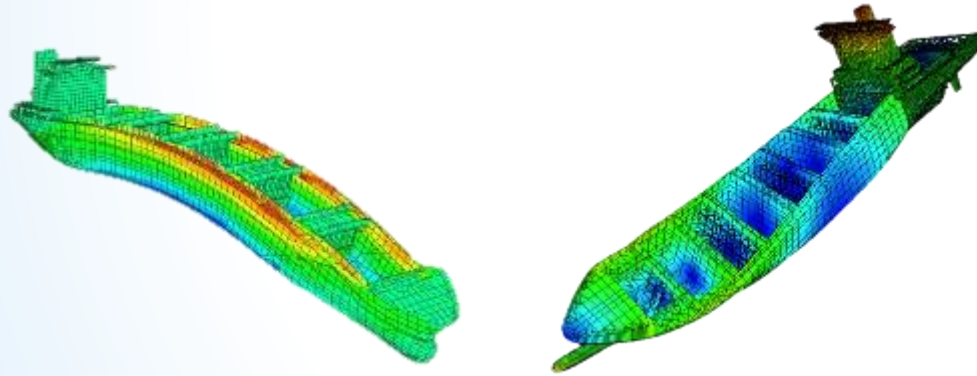


Topics in Ship Structures

(Advanced Local Structural Design & Analysis of Marine Structures)



* Long Clamped Plates (Topic 6)

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<https://sites.google.com/snu.ac.kr/ost>

[Theory of Plates and Grillages]

[Part I] Plastic Design of Structures

- Plastic theory of bending (Topic 1)
- Ultimate loads on beams (Topic 2)
- Collapse of frames and grillage structures (Topic 3)

[Part II] Elastic Plate Theory under Pressure

- Basic (Topic 4)
- Simply supported plates under Sinusoidal Loading (Topic 5)
- Long clamped plates (Topic 6)
- Short clamped plates (Topic 7)
- Low aspect ratio plates, strength & permanent set (Topic 7A)

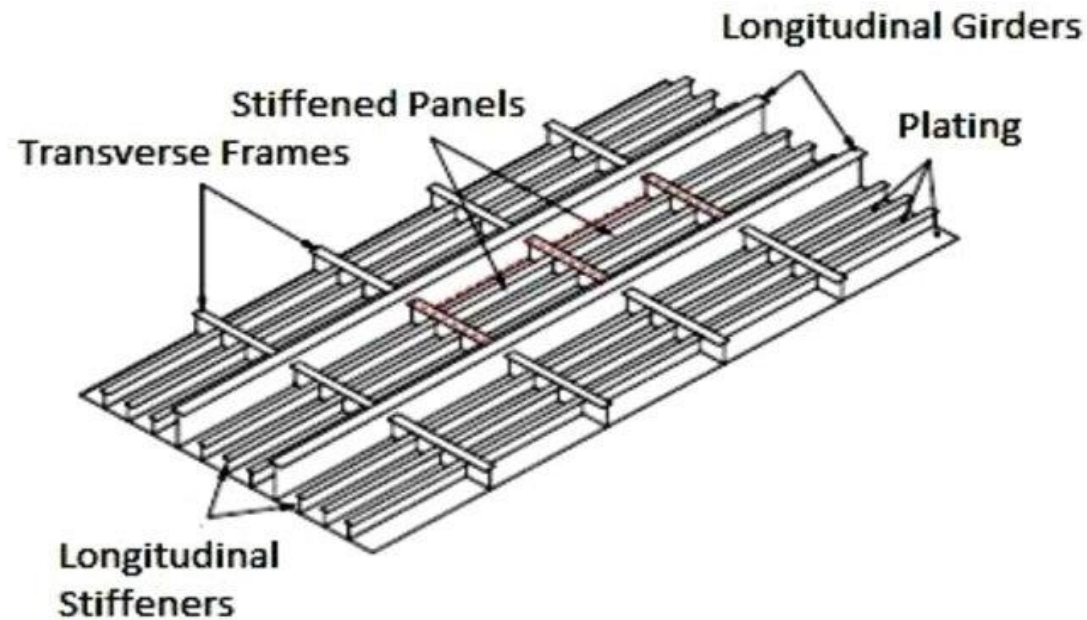
[Part III] Buckling of Plate & Stiffened Panels

- Failure modes (Topic 8)
- Tripping (Topic 9) + Post-buckling strength of plate (Topic 9A)
- Post-buckling behaviour (Topic 10)



The aim of this lecture is:

- To equip you with the knowledge and understanding of long plate theory



Picture from:

Raviprakash A. V., Bala P., and Natarajan A. (2012). "Effect of location and orientation of two short dents on ultimate compressive strength of a thin square steel plate", *International Journal of Computer Applications in Technology* 45(1) DOI: 10.1504/IJCAT.2012.050134

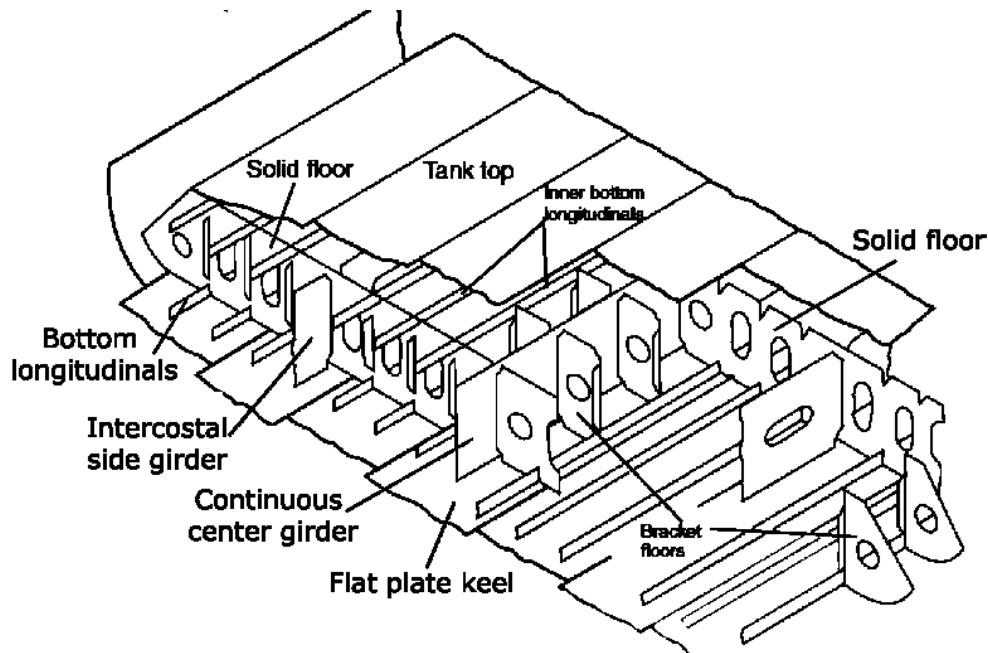


At the end of this lecture, you should be able to:

- Describe plate behaviour under lateral pressure and explain the effects of edge conditions on plate design.
- Perform elasto-plastic and plastic analysis of plates undergoing large deflections.
- Design under uniform pressure to meet various design criteria.
- Predict the performance of plate under uniform pressure.



- Ship plates are either **longitudinally framed** or **transversely framed**.
- The **spacing of secondary members** such as longitudinals, beams and ordinary frames are **smaller** than those of **primary members** such as deep transverses and girders.
- This results in the **aspect ratios** of the **plates** being **Greater than 1.0**.



Picture from:

<https://www.marineinsight.com/naval-architecture/design-of-ships-bottom-structure/>

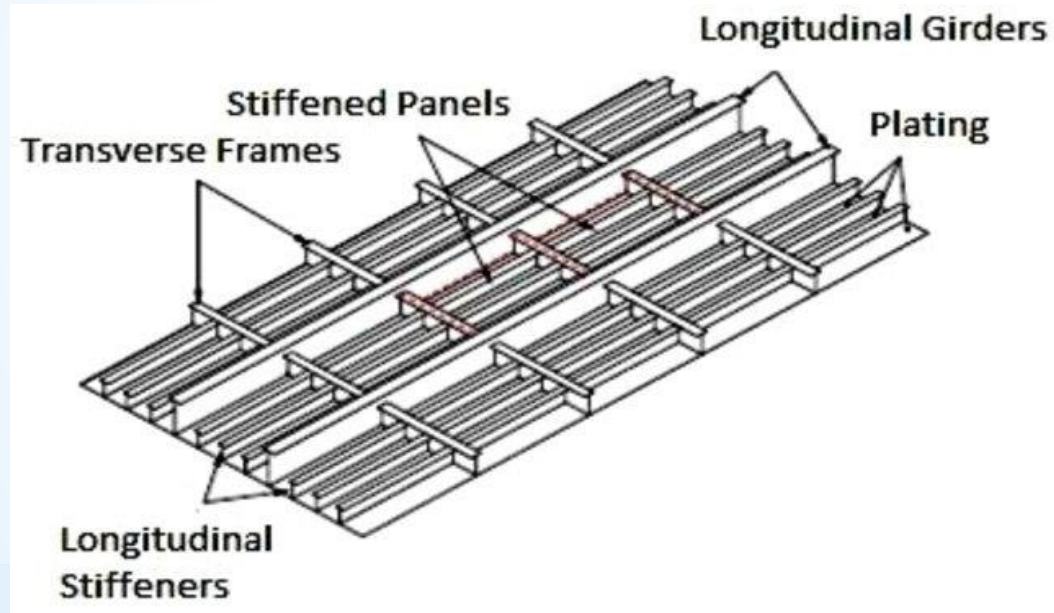


- In the case of **large aspect ratio**, i.e. “long” plate, the perturbation at the **short edges** is **ignored** and the **plate** is assumed to **bend in one plane** only as a beam.
- It follows therefore that **linear theory** will apply for rather **greater deflections** which need only to be considered **small** compared with the **width** of the **plate** so that the **simple linear expression** for **curvature** can be used with sufficient **accuracy**.
- The **first three assumptions** for **linear plate theory** are still made for **long plate theory**.



- The **advantages** of studying **long plate theory**, quite apart from its instructional value and simplicity, are:
 - **Non-linear effects** can be more readily **examined**, as can a **variety of boundary conditions**.
 - It is relatively easily **extended** to cover **elasto-plastic behaviour**.
 - It is **useful** in design in so far as **long plate data** is **easily presented**
 - The **reduction in thickness** required for short plates is available for **any aspect ratio** and a **variety of theories** and **boundary assumptions**.





*Topic 6.1:
Long Clamped Plates under Uniform Pressure



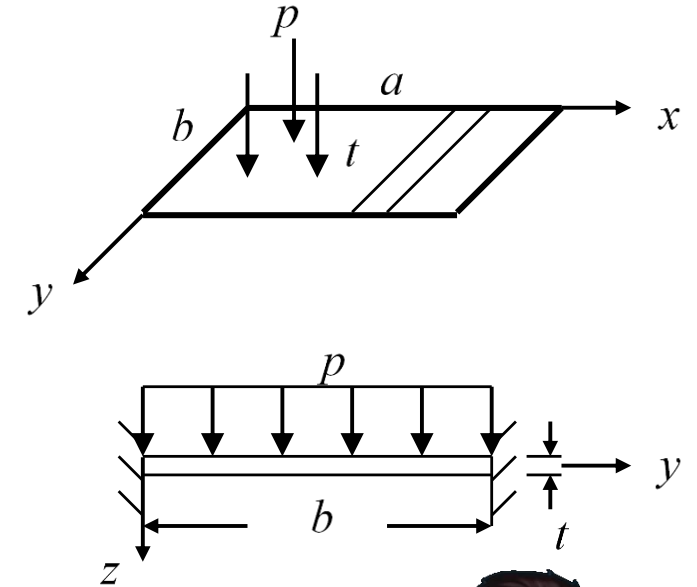
Long Clamped Plates: Uniform Pressure (1/5)

- A plate may be considered “long” if its **primary direction of bending** (i.e. that leading to the maximum stresses) leads to **moments** and **stresses** virtually **identical** to that of an **infinitely long plate** between parallel supports of distance b apart.
- The analysis is referred to as “**cylindrical bending**” and is identical (apart from **Poisson’s effects**) to **beam bending analysis**.
- If a is the length of the plate, the **maximum central deformation** is $w_{\max} = k(w_{\max})_{a=\infty}$
- When the **aspect ratio a/b** exceeds **2**, $k \rightarrow 1$, a **plate** may generally be **considered long** if its **aspect ratio** is **greater than** about
 2.5 to 3 for **elastic theories**
 2 to 2.5 for **elasto-plastic** or **plastic theories**
- For **long clamped plate** we can isolate a typical ‘**unit strip**’ and consider its behaviour under **uniform pressure p** .



Long Clamped Plates: Uniform Pressure (2/5)

- The edges are assumed to be **clamped** and **held rigidly**.
- The **rigid assumption** means that the plate strip **CANNOT** move in any direction and in particular **resists in-plane sliding** that may occur as **large deflection** built-up.
- In this case we will need to allow for “**membrane**” **tension** which arise because of the **rigid edge**.
- For any given moment M_y **per unit length**, the **bending stress** in y -direction is



- This is **maximum** at the edge where $M_y = pb^2/12$ (i.e. fixed end moment for UDL)

Moment-Curvature Relationships

$$\sigma_x = \frac{M_x z}{I^3/12} \quad \sigma_y = \frac{M_y z}{I^3/12}$$

• Maximum stresses occur on plate surface:

$$(\sigma_x)_{\max} = \pm \frac{6M_x}{t^2}$$

$$(\sigma_y)_{\max} = \pm \frac{6M_y}{t^2}$$

$$\sigma_y = \frac{6M_y}{t^2}$$

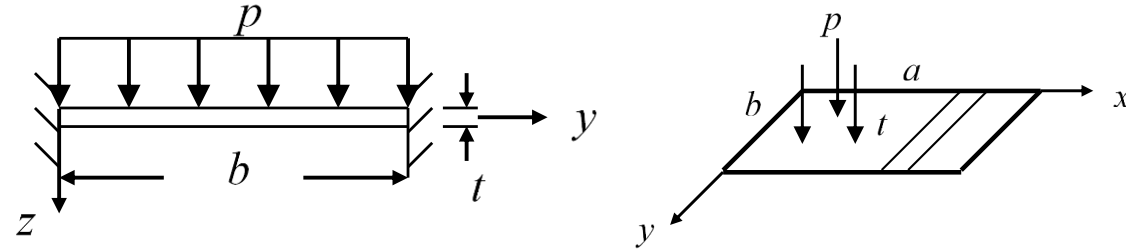
$$(\sigma_y)_{\max} = \frac{p}{2} \left(\frac{b}{t} \right)^2$$

Note: UDL = uniformly distributed load



Long Clamped Plates: Uniform Pressure (3/5)

$$(\sigma_y)_{\max} = \frac{p}{2} \left(\frac{b}{t} \right)^2$$



- This result can also be obtained from the expression for σ_m in **clamped plate** under **uniform pressure** when k_2 **equals** to 1 as a/b **exceeds** 2.
- The **yield pressure** at which the edge stress first equals to the yield stress is

$$P_{\text{yield}} = 2\sigma_{\text{yield}} \left(\frac{t}{b} \right)^2 \quad \text{without Poisson's effect.}$$

- Because of **Poisson's effect** there is also a stress in the **long x-direction** which is approximated by assuming the strain in x -direction is **zero**.

$$\varepsilon_x = \frac{1}{E} (\sigma_x - \nu\sigma_y) = 0$$

Then $\sigma_x = \nu\sigma_y$



Long Clamped Plates: Uniform Pressure (4/5)

- To find the **pressure** at which **yield first occurs**, using **von-Mises criteria** of yielding, requires

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \sigma_{yield}^2 \quad \text{where } \sigma_1 \text{ and } \sigma_2 \text{ are principal stresses.}$$

- Let $\sigma_1 = (\sigma_x)_{\max} = \nu(\sigma_y)_{\max}$ and $\sigma_2 = (\sigma_y)_{\max}$

- For **yield failure** $(\sigma_y)_{\max} (1 - \nu + \nu^2) = \sigma_{yield}^2$ \Rightarrow $\sigma_{y \max} = \frac{\sigma_{yield}}{\sqrt{1 - \nu + \nu^2}}$

- In case of **steel** $\nu = 0.3$, we have $(\sigma_y)_{\max} = 1.125\sigma_{yield}$

- Hence, the yield pressure should be **modified** to account for **12.5% increase** in the stress value at which **yielding really occurs**.

$$p_{yield} = 2 \times 1.125\sigma_{yield} \left(\frac{t}{b}\right)^2 = 2.25\sigma_{yield} \left(\frac{t}{b}\right)^2 \quad \text{with Poisson's effect}$$



- If the **central deflection** is required, this is given with **analogue to beam results**.

- Maximum deflection** $w_m = \frac{pb^4}{384 D}$ for **Plate**

where, $D = \frac{Et^3}{12(1-\nu^2)}$

$$w_m = \frac{qL^4}{384 EI} \text{ for beam}$$

$$\therefore w_m = \frac{(1-\nu^2)pb^4}{32Et^3} = 0.0284 \frac{pb^4}{Et^3} \text{ for steel}$$

- Comparing this result with the expression for the **maximum central deflection in clamped plate under uniform pressure**, the value of k_1 approaches to **0.0284** as a/b exceeds **2**.

Clamped Plates: Uniform Pressure

- Maximum deflection

$$w_m = k_1 \frac{p_o b^4}{Et^3}$$

- Maximum bending stress in y-direction

$$\sigma_m = k_2 \frac{p_o}{2} \left(\frac{b}{t}\right)^2$$

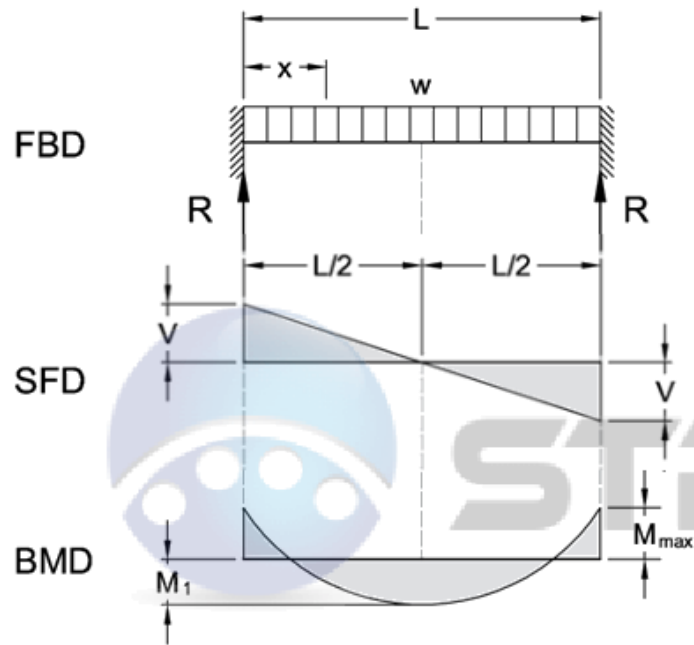
- Values of k_1 and k_2 according to **two reliable theories** are plotted on the following page for $\nu = 0.3$.

a/b	1.0	1.2	1.4	1.6	1.8	2.0	∞
k_2	0.3078	0.3834	0.4356	0.4680	0.4872	0.4974	0.5000
k_1	0.0138	0.0188	0.0226	0.0251	0.0267	0.0277	0.0284



Long Clamped Plates: For your study (1/2)

- (a) Sketch shear force diagram (SFD) and bending moment diagram (BMD).
- (b) Calculate maximum deflection of the beam.



$$R = V \dots \dots \dots = \frac{wL}{2}$$

$$V_x \dots \dots \dots = w \left(\frac{L}{2} - x \right)$$

$$M_{\max} \text{ (at ends)} \dots \dots \dots = \frac{wL^2}{12}$$

$$M_1 \text{ (at centre)} \dots \dots \dots = \frac{wL^2}{24}$$

$$M_x \dots \dots \dots = \frac{w}{12} (6Lx - L^2 - 6x^2)$$

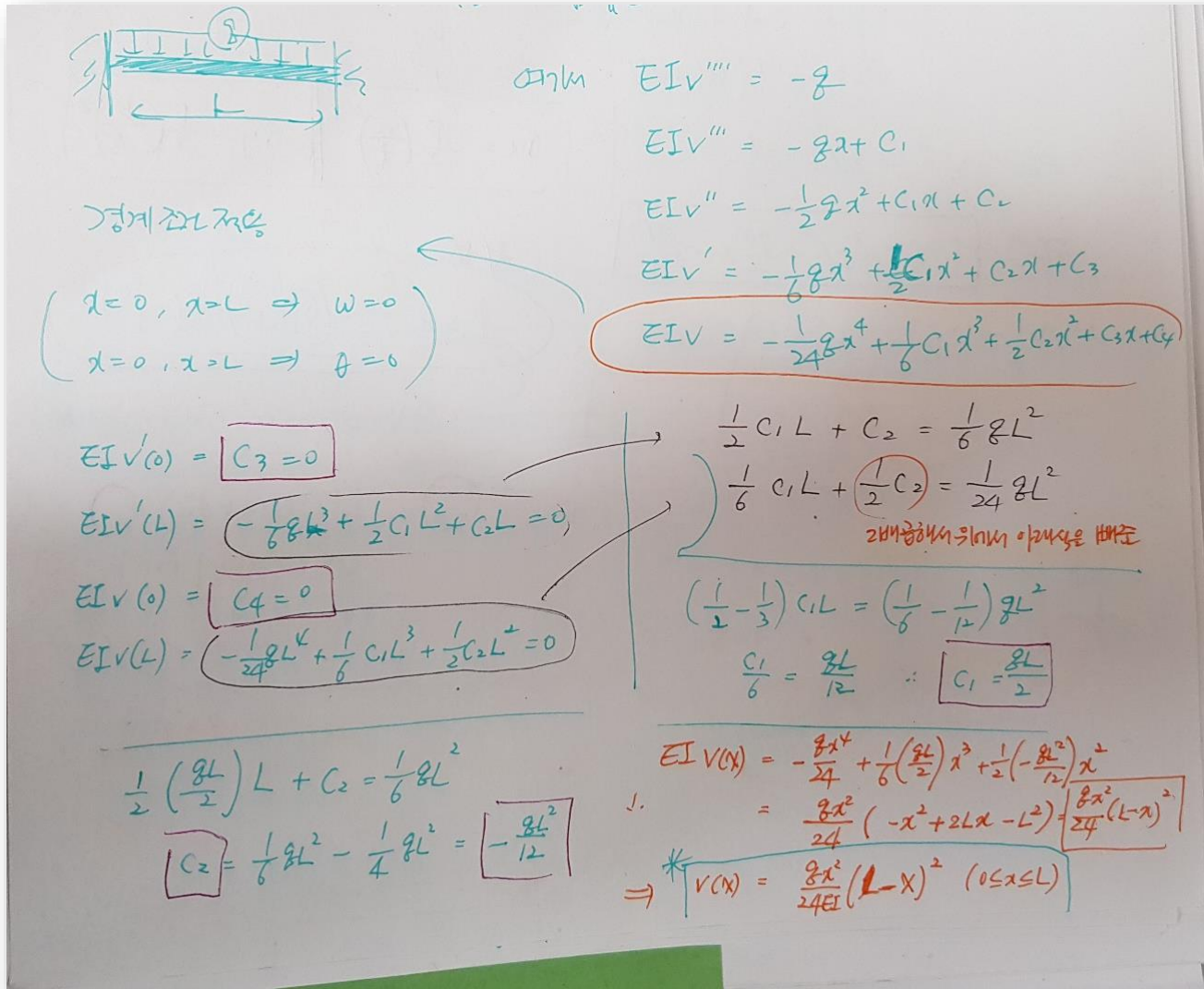
$$\Delta_{\max} \text{ (at centre)} \dots \dots \dots = \frac{wL^4}{384EI}$$

$$\Delta_x \dots \dots \dots = \frac{wx^2}{24EI} (L - x)^2$$

$$x \text{ (points of contraflexure)} \dots \dots \dots = (\sqrt{3} - 3)L$$



Long Clamped Plates: For your study (2/2)



$EIV'''' = -q$
 $EIV''' = -qx + C_1$
 $EIV'' = -\frac{1}{2}qx^2 + C_1x + C_2$
 $EIV' = -\frac{1}{6}qx^3 + \frac{1}{2}C_1x^2 + C_2x + C_3$
 $EIV = -\frac{1}{24}qx^4 + \frac{1}{6}C_1x^3 + \frac{1}{2}C_2x^2 + C_3x + C_4$

경계 조건 적용
 $x=0, x=L \Rightarrow w=0$
 $x=0, x=L \Rightarrow \theta=0$

$EIV(0) = C_4 = 0$
 $EIV(L) = -\frac{1}{24}qL^4 + \frac{1}{6}C_1L^3 + \frac{1}{2}C_2L^2 = 0$
 $EIV'(0) = C_3 = 0$
 $EIV'(L) = -\frac{1}{6}qL^3 + \frac{1}{2}C_1L^2 + C_2L = 0$

$\frac{1}{2}C_1L + C_2 = \frac{1}{6}qL^2$
 $\frac{1}{6}C_1L + \frac{1}{2}C_2 = \frac{1}{24}qL^2$
 $(\frac{1}{2} - \frac{1}{3})C_1L = (\frac{1}{6} - \frac{1}{12})qL^2$
 $\frac{C_1}{6} = \frac{qL}{12} \therefore C_1 = \frac{qL}{2}$

$\frac{1}{2}(\frac{qL}{2})L + C_2 = \frac{1}{6}qL^2$
 $C_2 = \frac{1}{6}qL^2 - \frac{1}{4}qL^2 = -\frac{qL^2}{12}$

$EIV(x) = -\frac{qx^4}{24} + \frac{1}{6}(\frac{qL}{2})x^3 + \frac{1}{2}(-\frac{qL^2}{12})x^2$
 $= \frac{qx^2}{24}(-x^2 + 2Lx - L^2) - \frac{qx^2(L-x)^2}{24}$
 $\Rightarrow V(x) = \frac{qx^2}{24EI}(L-x)^2 \quad (0 \leq x \leq L)$

$v(x)$ or $w(x)$ = deflection(m)

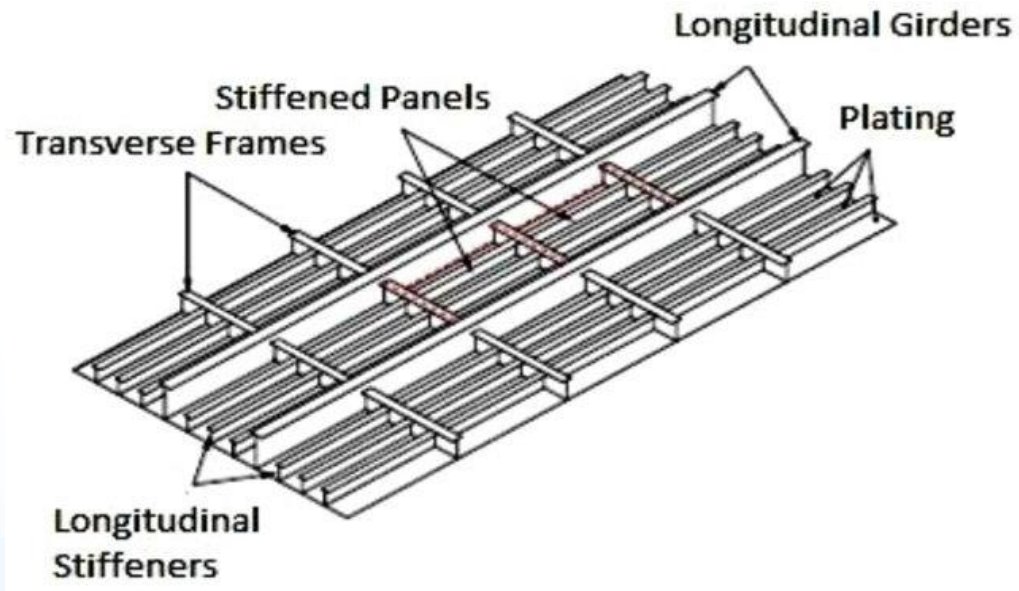
$\theta(x) = \frac{dv(x)}{dx} = v'(x)$ = slope(rad)

$M(x) = EI \frac{d^2v(x)}{dx^2} = EIv''(x)$ = moment(Nm)

$V(x) = EI \frac{d^3v(x)}{dx^3} = EIv'''(x)$ = shear(N)

$q(x) = EI \frac{d^4v(x)}{dx^4} = EIv''''(x)$ = Load(N/m)



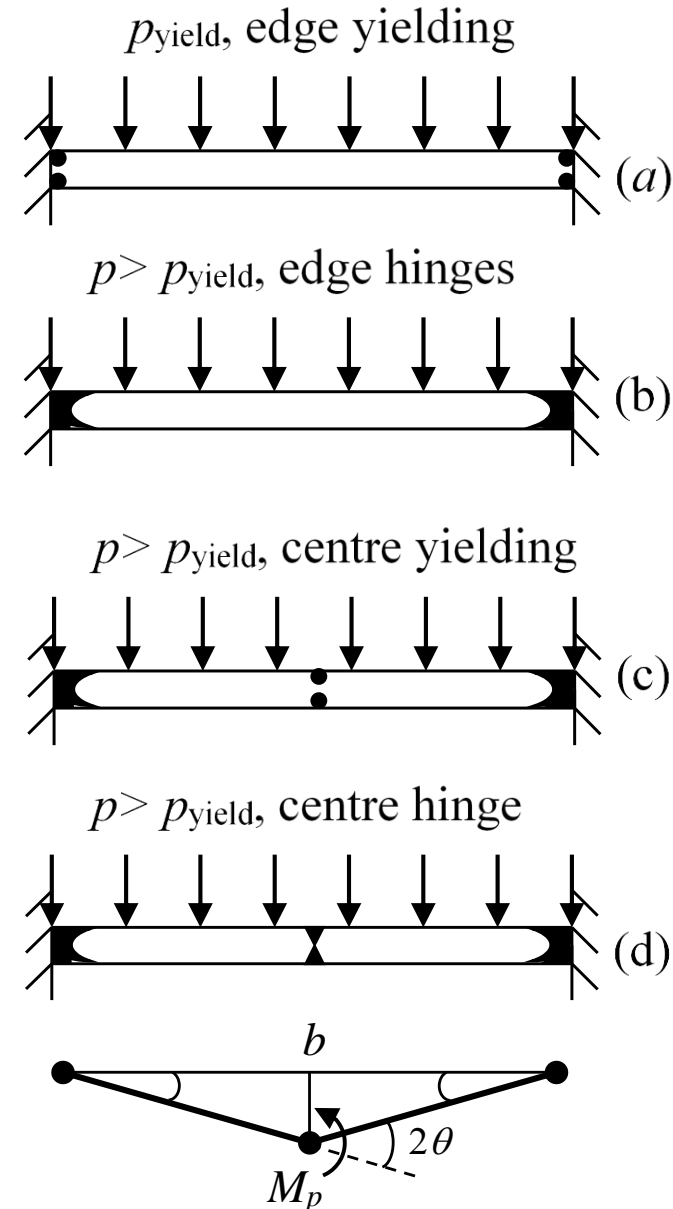


*Topic 6.2:
Elasto-plastic Bending of Long Clamped Plates

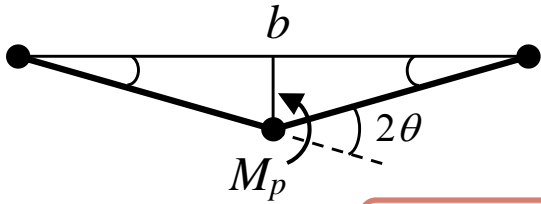


Long Clamped Plates: Elasto-plastic Bending (1/2)

- As **pressure increases**, the elastic bending moment diagram indicates that the **yield first occurs at the edges (STAGE a)** and **bending moment increase at the mid-span**.
- Then it **progresses until fully developed hinges occur at the edges (STAGE b)**.
- Pressure continues to **increase in the bending moment at mid-span until it yields (STAGE c)** and **finally forms a hinge (STAGE d)**.
- The above progress is just the same as a **plastic beam**. In the final **STAGE (d)**, **three hinges** have now formed a **collapse mechanism**.



Long Clamped Plates: Elasto-plastic Bending (2/2)



- Let p_u = collapse pressure
- M_p = plastic moment per unit length

- Internal work = $4 M_p \theta$

- External work = $2 \left(\frac{p_u b}{2} w_p \right) = 2 \left(\frac{p_u b}{2} \frac{b\theta}{4} \right) = \frac{p_u b^2 \theta}{4}$



- Equating them gives

$$p_u = \frac{16 M_p}{b^2}$$

- Recalling $M_p = \sigma_{yield} Z_p$

where the plastic section modulus

$$Z_p = \frac{t^2}{4} \text{ per unit length.}$$

- Hence,

$$p_u = 4 \sigma_{yield} \left(\frac{t}{b} \right)^2$$

$$p_u = 4.5 \sigma_{yield} \left(\frac{t}{b} \right)^2$$

W/O Poisson's effect.

WITH Poisson's effect.

- When Poisson's effect is present, the maximum stress will increase by 12.5%.
- This collapse pressure is derived on the basis that the clamped edges are free to slide.

Plastic Theory of Bending (Topic 1)

$$M_p = \sigma_y \frac{A}{2} (\bar{y}_t + \bar{y}_c)$$

- In fact, final bending collapse will occur at plastic state. Thus, the fully plastic moment M_p is the limiting bending strength of the cross-section.
- Write:

$$M_p = \sigma_y Z_p \tag{3}$$
 where

$$Z_p = \frac{A}{2} (\bar{y}_t + \bar{y}_c) \tag{4}$$
- Z_p is the plastic section modulus.
- The state of M_p , when rotation occurs without a change in the moment, is known as plastic hinge and fully yielded section.

Ultimate Loads on Beams

(c) Fixed end beam with uniform distributed load

- Internal work = $4 M_p \theta$

- External work =

$$2 \left(\frac{w_p L}{2} \delta \right) = 2 \left(\frac{w_p L}{2} \frac{L\theta}{4} \right) = \frac{w_p L^2}{4} \theta$$

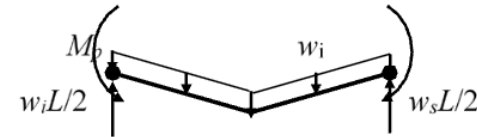
- By the principle of virtual work,

$$\frac{w_p L^2}{4} \theta = 4 M_p \theta$$

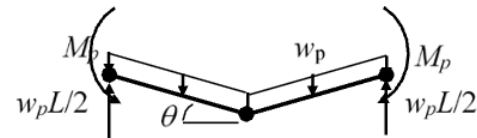
- Thus, the ultimate load

$$w_Y = \frac{12M_Y}{L^2}$$

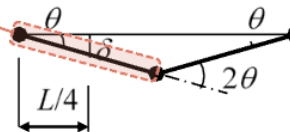
$$w_p = \frac{16M_p}{L^2}$$



First hinges at supports

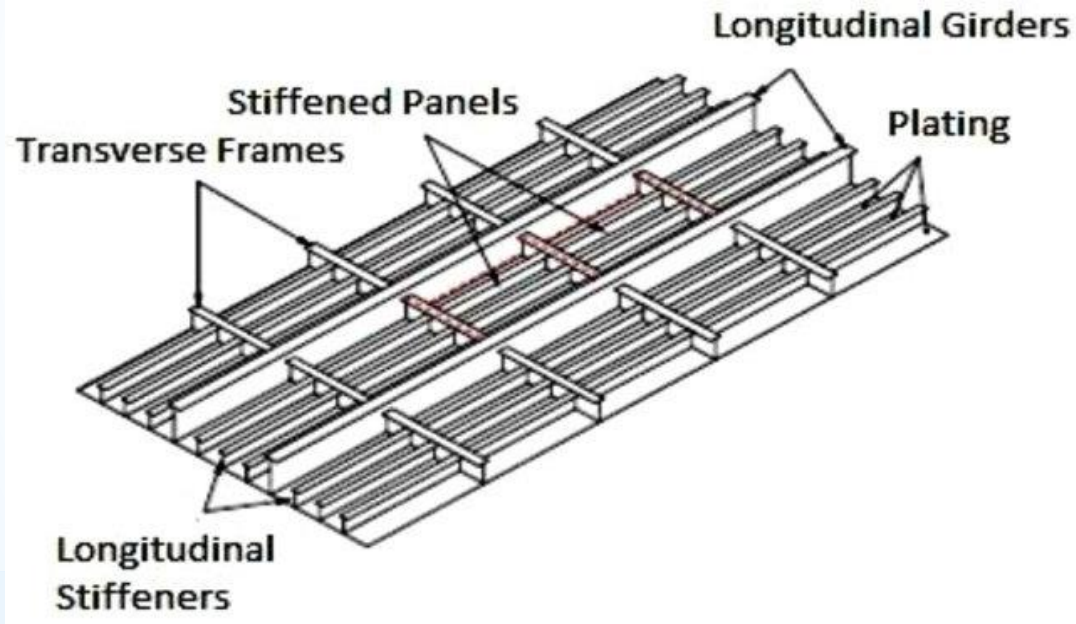


Plastic mechanism



Displacements





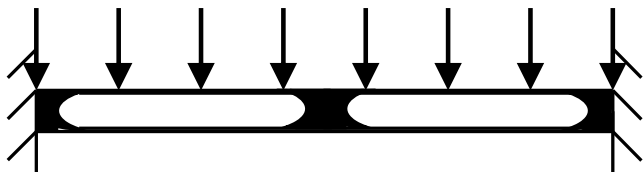
*Topic 6.3:

Long Plates at Rupture

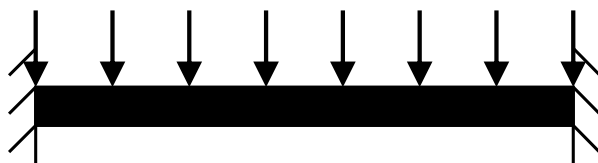


- If the edges are held rigidly, p_u derived above WILL NOT represent collapse.
- This is because membrane tension σ_T can develop in the plate, adding appreciable stiffness to the plate.
- In this case, plasticity will finally spread everywhere until $\sigma_T = \sigma_{yield}$.
- The plate will have become a fully plastic membrane in which it has NO REMAINING ability to carry a bending moment by sustaining the load by membrane alone.

$p > p_{yield}$, plasticity spreading



$p \gg p_{yield}$, plastic membrane



Elastic Membranes

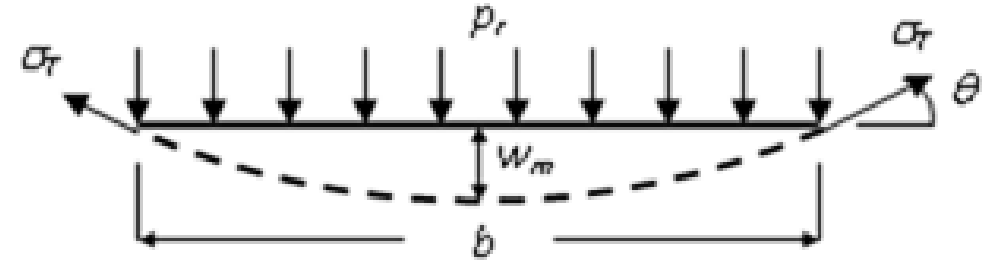
- Consider a **long thin plate** in which the **bending resistance** is **low**.
- We can gain some insight into behaviour by assuming **zero bending rigidity**
- Thus, the only **resistance to pressure** is through **membrane tension forces**.
- For **equilibrium**,
- For **small θ** ,
- Therefore,

$$p_r b = 2\sigma_T t \sin \theta$$

$$\sin \theta \approx \theta \approx \frac{b/2}{R}$$

$$p_r = \frac{\sigma_T t}{R}$$

where R is the **unknown radius of membrane**, which can be expressed in terms of the **central deflection w_m** from **geometry**.



Long Plates at Rupture (3/5): Elastic Membranes

- From the geometry of a circle as shown,

$$\frac{w_m}{b/2} = \frac{b/2}{2R - w_m} \approx \frac{b}{4R} \quad \text{for } R \gg w_m \quad \therefore \frac{1}{R} = \frac{8w_m}{b^2}$$

- Hence,

$$p_r = \frac{\sigma_T t}{R} = \frac{8tw_m}{b^2} \sigma_T$$

R = unknown radius of membrane which can be expressed by central deflection w_m from geometry

- From stress-strain relation, we have

$$\epsilon_T = \frac{\sigma_T}{E} (1 - \nu^2)$$

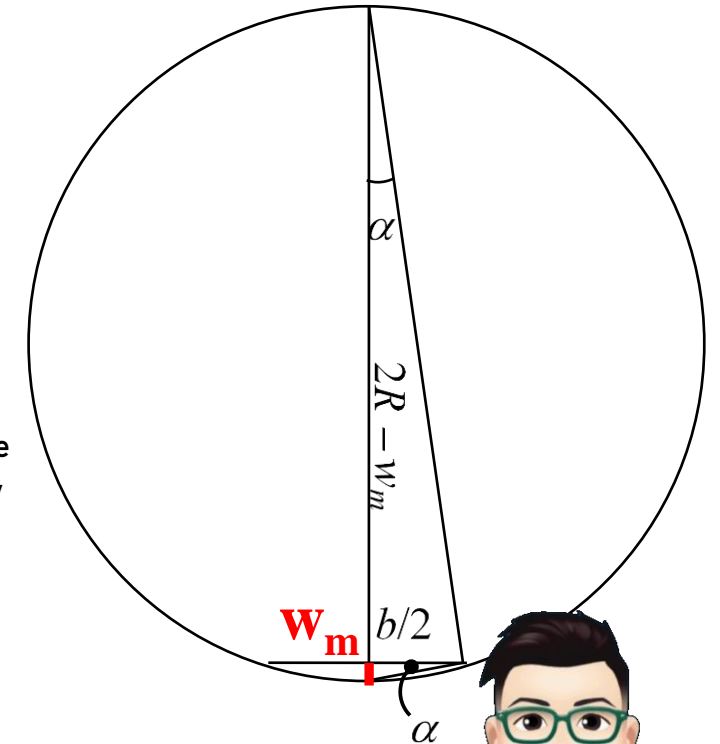
$$\sin \theta \approx \theta \approx \frac{b/2}{R}$$

$$b/2 = R \sin \theta$$

- From the geometry of a circle as shown,

$$\epsilon_T = \frac{R\theta - b/2}{b/2} = \frac{\theta - \sin \theta}{\sin \theta} = \frac{\theta - \theta + \theta^3/6 - \dots}{\theta - \theta^3/6 + \dots} \approx \frac{\theta^3/6}{\theta} \approx \frac{\theta^2}{6}$$

$$\therefore \epsilon_T = \frac{\theta^2}{6} = \frac{b^2}{24R^2} = \frac{8}{3} \left(\frac{w_m}{b} \right)^2$$



Long Plates at Rupture (4/5): Elastic Membranes

$$p_r = \frac{\sigma_T t}{R} = \frac{8t w_m}{b^2} \sigma_T \quad \text{②}$$

$$\varepsilon_T = \frac{\theta^2}{6} = \frac{b^2}{24R^2} = \frac{8}{3} \left(\frac{w_m}{b} \right)^2 = \frac{\sigma_T}{E} (1 - \nu^2)$$



$$p_r = \frac{8t w_m}{b^2} \sigma_T = \frac{8t w_m}{b^2} \frac{E \cdot \varepsilon_T}{(1 - \nu^2)} = \frac{8t w_m}{b^2} \frac{E}{(1 - \nu^2)} \frac{8}{3} \left(\frac{w_m}{b} \right)^2$$

- Hence, we have **rupture pressure** p_r as a function of **central deflection** w_m

$$p_r = \frac{8t w_m}{b^2} \frac{E}{(1 - \nu^2)} \frac{8}{3} \left(\frac{w_m}{b} \right)^2 \quad \therefore \quad p_r = \frac{64E}{3(1 - \nu^2)} \frac{t w_m^3}{b^4} \quad \text{①}$$

or as a function of **membrane stress** by **eliminating** w_m  $\frac{\text{②}^3}{\text{①}}$

$$p_r = \frac{8t \sigma_T}{b^2} b \sqrt{\frac{3\sigma_T (1 - \nu^2)}{8E}} \quad \therefore \quad p_r = \frac{t}{b} \sqrt{\frac{24(1 - \nu^2) \sigma_T^3}{E}}$$

- Taking $\sigma_T = \frac{2}{3} \sigma_{yield}$ as a **limiting stress** gives

$$p_r = \frac{8}{3} \sqrt{1 - \nu^2} \frac{t}{b} \sqrt{\frac{\sigma_{yield}^3}{E}}$$



Long Plates at Rupture (5/5): Plastic Membranes

$$p_r = \frac{\sigma_T t}{R} = \frac{8t w_m}{b^2} \sigma_T$$

Plastic Membranes

- If we ignore the elastic behaviour and assume the material is rigid plastic, then the pressure is only resisted by membrane action at constant yield stress

$$\sigma_T = \sigma_{yield}$$

- Hence, from the elastic membrane analysis, we have

$$p_r = \frac{8t w_m}{b^2} \sigma_{yield}$$

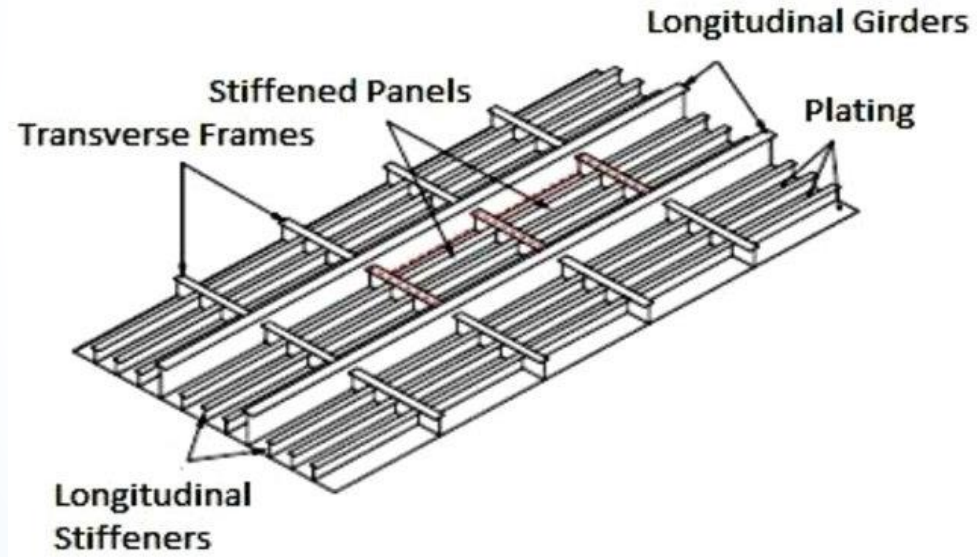
without Passion's effect

or

$$p_r = \frac{9t w_m}{b^2} \sigma_{yield}$$

with Passion's effect
(12.5% increased)





*Topic 6.4:

Clarkson's Criteria for Design Limits



Clarkson's Criteria for Design Limits (1/2)

- Working for the Royal Navy in the late 50's and early 60's **Clarkson** (Trans RINA 1956) developed an **Elasto-plastic analysis** for **long and square plates** having **clamped rigid boundaries**
- It allowed for the **growth of membrane stresses** and **plasticity**.
- The **effect of rotational constraints** was shown to have **little effect** once **permanent sets** developed.
- He suggested **three criteria** for design limits.



- i. For stocky plates $\beta \leq 2.5$ the formation of the **centre plastic hinge** with a **load factor of 1.0** (i.e. design load = failure load) **should be considered**, which clearly approaches to:

$$p_u = 4.5\sigma_{yield} \left(\frac{t}{b} \right)^2$$

- ii. For slender plates $\beta > 2.5$ the **membrane stress Does Not Exceed** $2/3 \sigma_{yield}$. This approaches to

$$p_r = \frac{8}{3} \sqrt{1-\nu^2} \frac{t}{b} \sqrt{\frac{\sigma_{yield}^3}{E}}$$

Since permanent sets arise from **welding**, and also from **concentrated loads** such as wheel loads (from aircraft, helicopters, fork-lift trucks, cars, lorries, etc), Clarkson also proposed an **alternative design criterion** based on an **allowable permanent set** δ (i.e. allowable initial deformation).

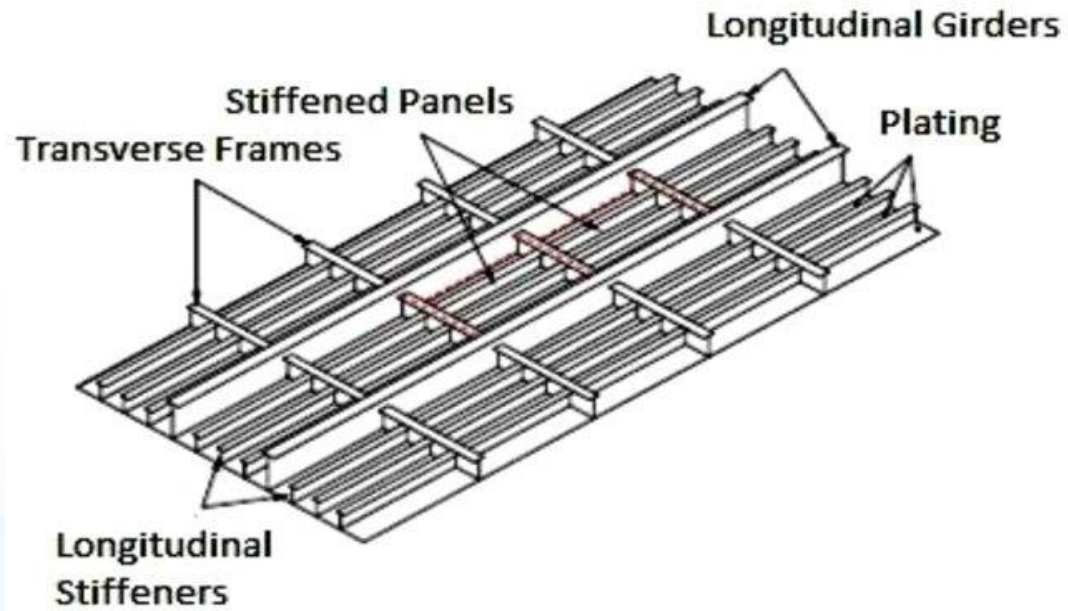
in which **plate slenderness**

$$\beta = \frac{b}{t} \sqrt{\frac{\sigma_{yield}}{E}}$$



- In most **stiffened plate** structures, the nature of the **in-plane conditions varies** from one plate element to another **throughout the grillage**.
- **Near deck edges** there is usually **little resistance to in-plane movement**.
- At the **centre of deck**, the **continuity of plating** suggests **edges-held-rigidly** apart might apply.
- However, **in-plane compressive forces**, which exist from **grillage bending** and/or **hull bending actions**, cause **shear slide** in the **edge** and **increase lateral deformation**.





*Topic 6.5:
Design Methods for Long Clamped Plates



Design Methods for Long Clamped Plates (1/6)

- We now consider **design methods** for **long clamped plates** under **uniform lateral pressure**. The problem is given the **uniform pressure** p , the **breadth** b and **material properties** (yield stress σ_{yield} , Young's modulus of elasticity E and Poisson's ratio ν) to determine the **plate thickness** t .
- For any **given** t , we can calculate the following quantities:

1

Calculate Elastic stress and deflection p , when the edges of the plate are free to slide

2

Calculate a value of p to cause initial yield

3

Calculate value of p to plastic collapse, if the edges are free to slide

4

Calculate the maximum permissible set δ

5

Calculate the value of p to cause rupture

$$\sigma_m = \frac{p}{2} \left(\frac{b}{t} \right)^2 \quad w_m = 0.0284 \frac{pb^4}{Et^3}$$

$$p_{\text{yield}} = 2\sigma_{\text{yield}} \left(\frac{t}{b} \right)^2$$

$$p_u = 4\sigma_{\text{yield}} \left(\frac{t}{b} \right)^2$$



Design Methods for Long Clamped Plates (2/6)

1

Calculate Elastic stress and deflection p , when the edges of the plate are free to slide

- [From 1] Determine a suitable thickness t , given a working stress σ_m .

$$t = b \sqrt{\frac{p}{2\sigma_m}}$$

2

Calculate a value of p to cause initial yield

- [From 2] Determine a suitable thickness t , given the yield stress σ_{yield} with a first yield condition

$$t = b \sqrt{\frac{p}{2\sigma_{yield}}}$$

3

Calculate value of p to plastic collapse, if the edges are free to slide

- [From 3] Determine a suitable thickness t given the yield stress s_{yield} with an ultimate strength condition

$$t = b \sqrt{\frac{p}{4\sigma_{yield}}}$$

4

Calculate the maximum permissible set δ

- [From 4] there are design charts.

5

Calculate the value of p to cause rupture

- The rupture value, in 5, is not often used because of uncertainties concerning the actual ductility.



- The thickness selected depends upon:
 1. **Boundary conditions** and especially the response of the supports to membrane forces.
 2. **The criterion of failure**. This depends upon the use to which the structure is put.
 - **Maximum stress**
 - **Yield stress**
 - **Plastic collapse**



Design Methods for Long Clamped Plates (4/6)

- A **blast wall** in an offshore structure for example would only have to **withstand** the **loading once** and for all
- If **rupture** were **avoided**, **large permanent deformations** would be **quite acceptable**.
- The **deck of a Ro-Ro ship** would have to **withstand** the same **loading many times**, **WITHOUT** any permanent deformation, large deflections, or overstressing.
- **Fatigue** would also have to be **considered**.
- **Different cases need different criteria**. These different cases **need different design curves**.
- It is useful and more general to **develop** such **curves** on a **non-dimensional basis**. Consider case 2 above.

• Then
$$\frac{b}{t} = \sqrt{\frac{2\sigma_{yield}}{p}}$$

• Hence,

$$\frac{b}{t} \sqrt{\frac{\sigma_{yield}}{E}} = \sqrt{\frac{2\sigma_{yield}^2}{pE}}$$

β = plate
slenderness ratio

• It is thus **convenient** to plot
$$\frac{pE}{\sigma_{yield}^2}$$
 against
$$\frac{b}{t} \sqrt{\frac{\sigma_{yield}}{E}}$$

- Such a plot can be applied to a **range of materials**.



[Summary] Bending of Long PlatesLoad

$$P_y = 2.25\sigma_y \left(\frac{t}{b}\right)^2 = 2\sigma_{yp} \left(\frac{t}{b}\right)^2$$

$$P_{EH} = 3.375\sigma_y \left(\frac{t}{b}\right)^2 = 3\sigma_{yp} \left(\frac{t}{b}\right)^2$$

$$P_C = 4.5\sigma_y \left(\frac{t}{b}\right)^2 = 4\sigma_{yp} \left(\frac{t}{b}\right)^2$$

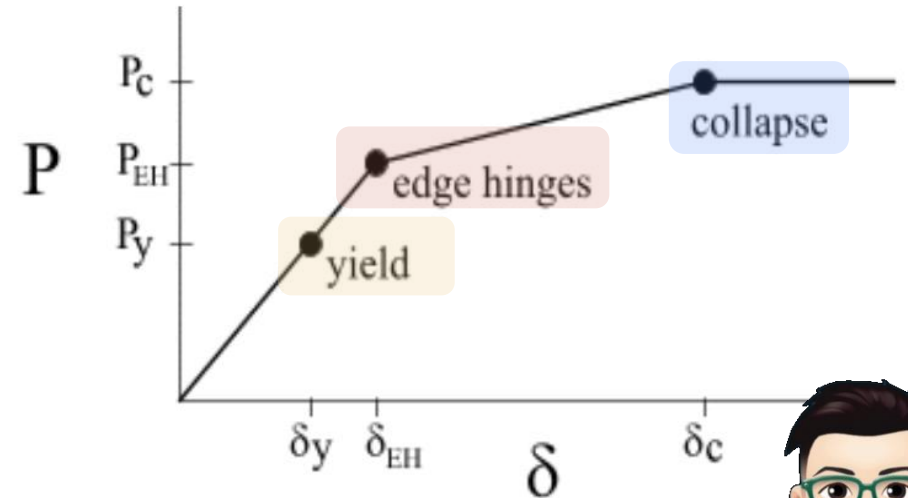
Deflection

$$\delta_y = \frac{1}{384} \frac{P_y b^4}{D}$$

$$\delta_{EH} = \frac{1}{384} \frac{P_{EH} b^4}{D}$$

$$\delta_c = \frac{2}{384} \frac{P_C b^4}{D}$$

- Plotting the load vs deflection, we get:



- Nomenclature**

P_y = First yield pressure

P_{EH} = Pressure to form first plastic hinges

P_C = Collapse pressure

σ_y = Normal material yield stress

$\sigma_{yp} = 1.125\sigma_y$ (enhanced material yield stress)

δ_y = plate central deflection = w_m



- If we **normalize** the **loads** P_c and the **deflections** by δ_c we get,

Load

$$\tilde{P}_y = \frac{P_y}{P_c} = 0.5$$

$$\tilde{P}_{EH} = \frac{P_{EH}}{P_c} = 0.75$$

$$\tilde{P}_c = \frac{P_c}{P_c} = 1$$

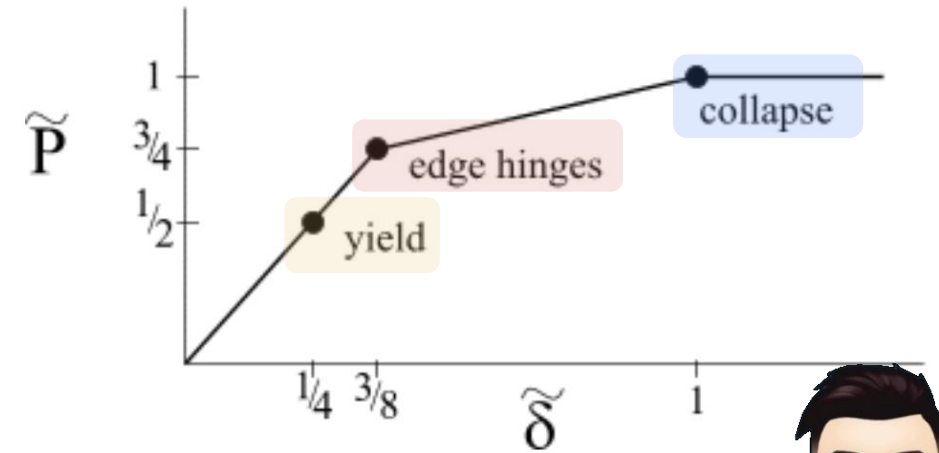
Deflection

$$\tilde{\delta}_y = \frac{\delta_y}{\delta_c} = \frac{1}{4}$$

$$\tilde{\delta}_{EH} = \frac{\delta_{EH}}{\delta_c} = \frac{3}{8}$$

$$\tilde{\delta}_c = \frac{\delta_c}{\delta_c} = 1$$

- Plotting the load vs deflection, we get:



- Nomenclature**

P_y = First yield pressure

P_{EH} = Pressure to form first plastic hinges

P_c = Collapse pressure

σ_y = Normal material yield stress

$\sigma_{yp} = 1.125\sigma_y$ (enhanced material yield stress)

δ_y = plate central deflection = w_m

- Thus, we see that:

P_y is 50% P_c & P_{EH} is 75% of P_c

δ_y is 25% δ_c & δ_{EH} is 37.5% of δ_c



- We have **investigated** the **Elastic Plate Theory**.
- Now we are able to:
 - **Describe plate behaviour** under **lateral pressure** and explain the effects of **edge conditions** on plate design.
 - **Perform elasto-plastic** and **plastic analysis** of plates undergoing **large deflections**.
 - **Design** under **uniform pressure** to meet various **design criteria**.
 - **Predict** the **performance of plate** under **uniform pressure**.
- Details can be referred to **topics 6** in the lecture notes.



[Theory of Plates and Grillages]

[Part I] Plastic Design of Structures

- Plastic theory of bending (Topic 1)
- Ultimate loads on beams (Topic 2)
- Collapse of frames and grillage structures (Topic 3)

[Part II] Elastic Plate Theory

- Basic (Topic 4)
- Simply supported plates under Sinusoidal Loading (Topic 5)
- Long clamped plates (Topic 6)
- Short Clamped plates (Topic 7)
- Additional (Low aspect ratio plates, strength & permanent set)

[Part III] Buckling of Stiffened Panels

- Failure modes (Topic 8)
- Tripping (Topic 9)
- Post-buckling behaviour (Topic 10)





Kam Sa Hab Ni Da

감사합니다

Thank you!

Questions?

Aerial View of Korean Presidential Archives in Sejong city
(Construction Completed in 2014)

