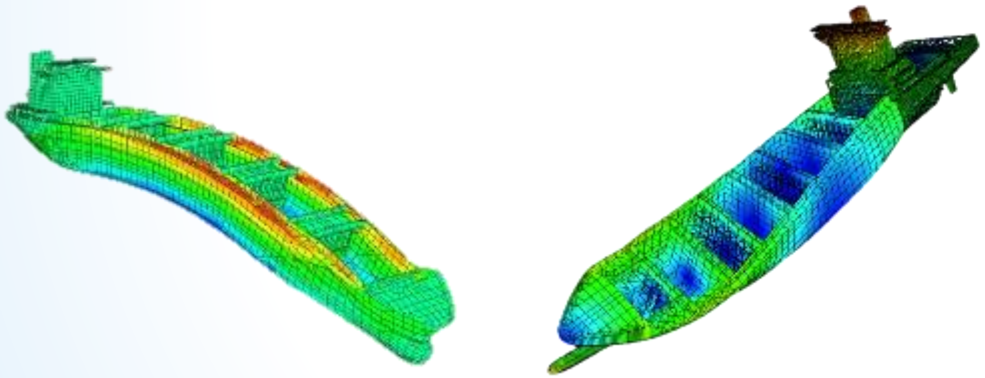
Topics in Ship Structures

(Advanced Local Structural Design & Analysis of Marine Structures)



* Short Clamped Plates (Topic 7)

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Adv. Local Structural Design & Analysis of Marine Structures (Overview)

[Theory of Plates and Grillages]

[Part I] Plastic Design of Structures

- Plastic theory of bending (Topic 1)
- Ultimate loads on beams (Topic 2)
- Collapse of frames and grillage structures (Topic 3)

[Part II] Elastic Plate Theory under Pressure

- Basic (Topic 4)
- Simply supported plates under Sinusoidal Loading (Topic 5)
- Long clamped plates (Topic 6)
- Short clamped plates (Topic 7)
- Additional (Low aspect ratio plates, strength & permanent set)

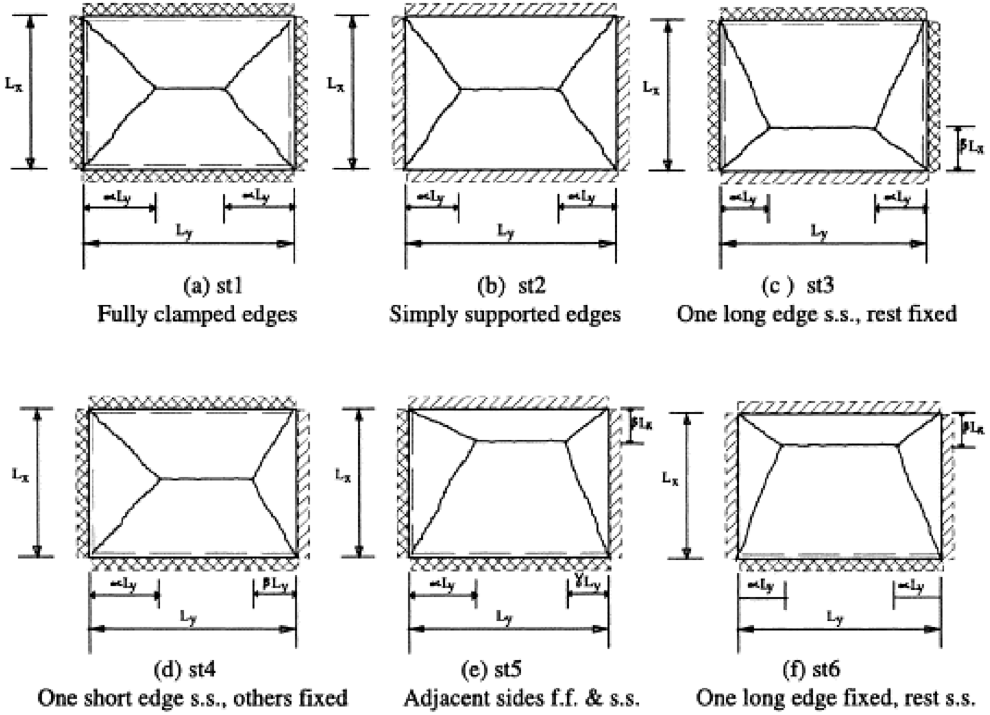
[Part III] Buckling of Stiffened Panels

- Failure modes (Topic 8)
- Tripping (Topic 9) + Post-buckling strength of plate (Topic 9A)
- Post-buckling behaviour (Topic 10)

Objectives

The aim of this lecture is:

- To equip you with the knowledge and understanding of yield-line theory, and with the necessary skills to design plates of finite aspect ratio.



Picture from:
Raviprakash A. V., Bala P., and Natarajan A. (2012). "Effect of location and orientation of two short dents on ultimate compressive strength of a thin square steel plate", *International Journal of Computer Applications in Technology* 45(1), DOI: 10.1504/IJCAT.2012.050134

Learning Outcome

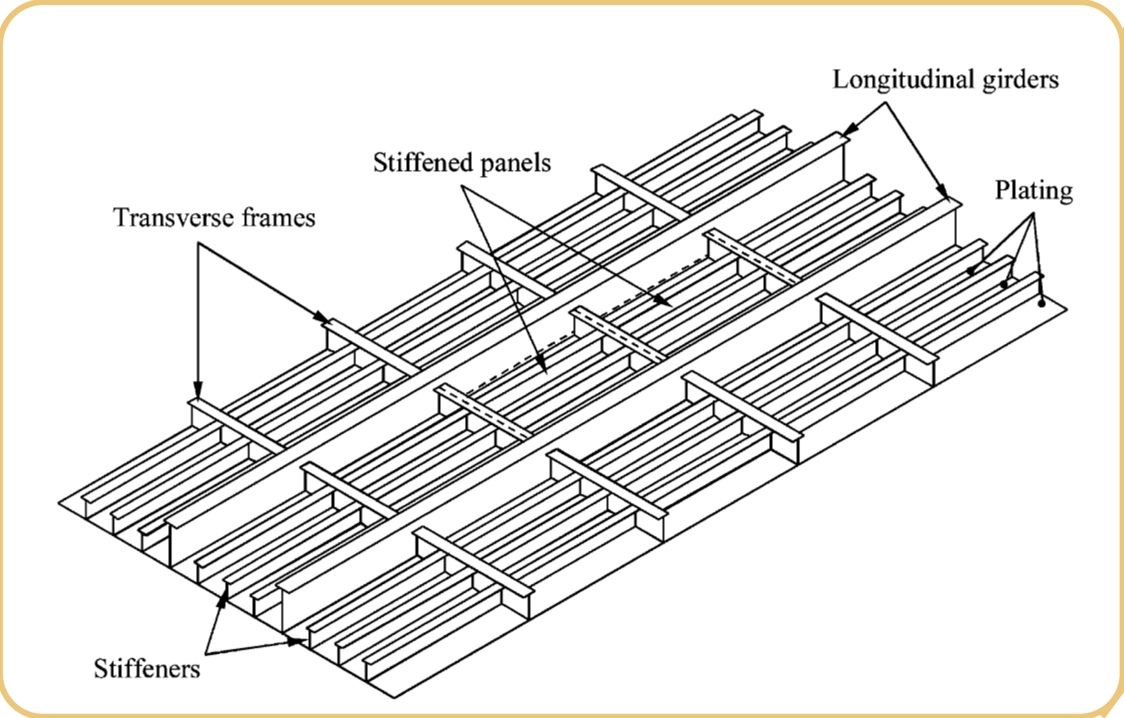
At the end of this lecture, you should be able to:

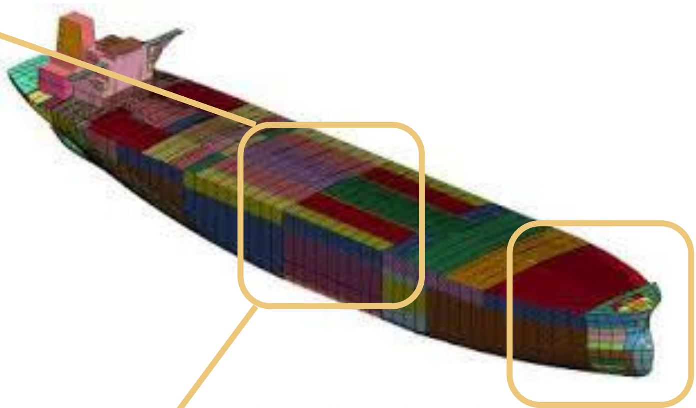
- Describe short clamped plates subjected to uniform pressure.
- Predict the performance of short clamped plates.
- Perform plastic analysis of short plate collapse.

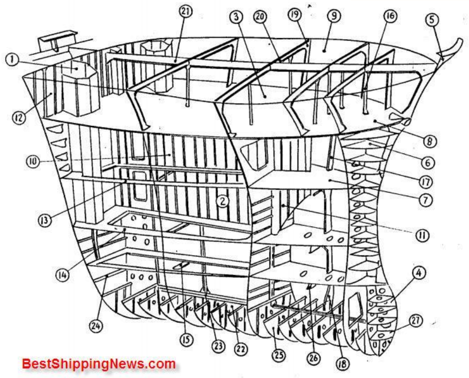
Introduction (1/2)

- When plates are supported by stiffeners of nearly equal spacing in both longitudinal and transverse directions, their aspect ratios become small.

- The span length of these “short plates” are SMALLER than those of their counter parts – “long” plates of the same width.



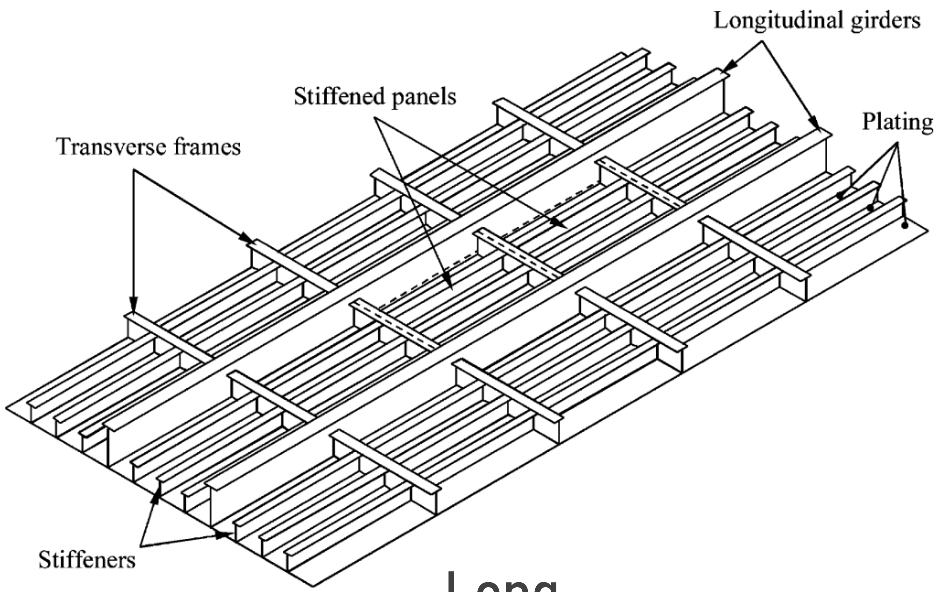




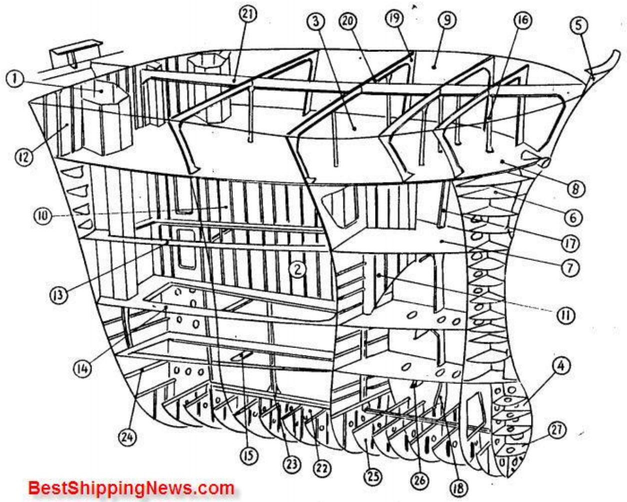
Picture from: Professor Jeom Kee Paik

Introduction (2/2)

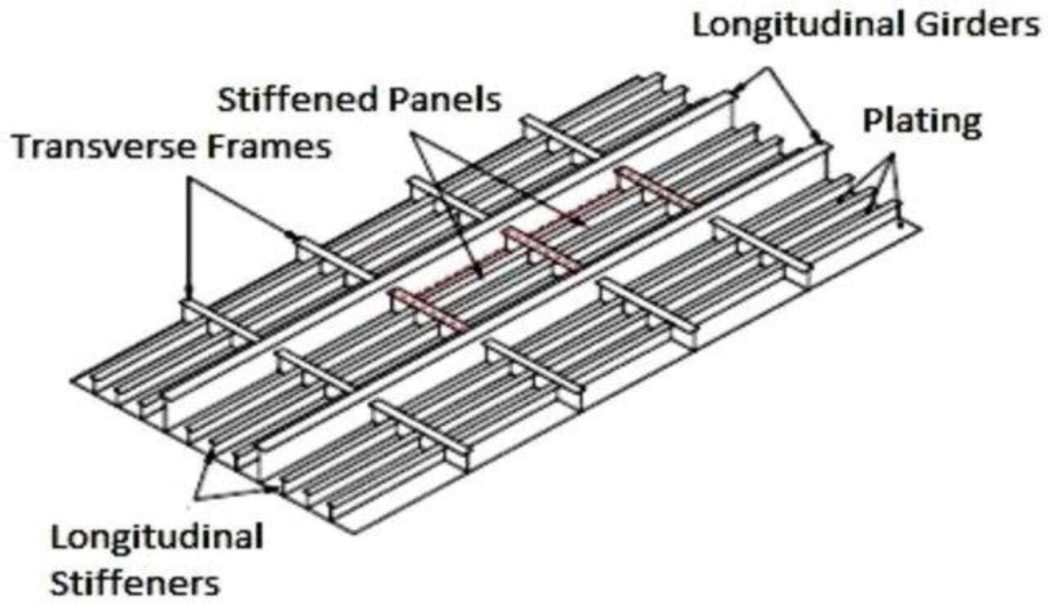
- Therefore, the short plates are much stronger than the long plates.
- Thus, the required thickness of short plate can be reduced when the aspect ratio decreases.
- Now, we should think how to design and analyse short plates under uniform pressure.



Long



Short



*Topic 7.1:

Design of Short Clamped Plates

Picture from:
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Design of Short Clamped Plates (1/4)

- For plates with an aspect ratio $\alpha = a/b$ less than about 2.5, the long plate theory is no longer valid.
- However, a straightforward approach is to design the plate (i.e. find t_L) as if it was a "long" plate and then modify the result to obtain t_s for a "short" plate.
- This depends upon the aspect ratio and whether the approach is elastic (e.g. first yielding) or elasto-plastic and plastic.
- The modification of the result is facilitated by

$$\frac{t_s}{t_L} = 0.11 + 0.9238\alpha - 0.2408\alpha^2$$

for elastic design of short plates of $\alpha \le 2$

$$\frac{t_s}{t_L} = 1 - \frac{32}{\pi^3}\operatorname{sech}\left(\frac{\pi}{2}\alpha\right)$$

for plastic design of short plates

Design of Short Clamped Plates (2/4)

Elastic Behaviour

- Based on the elastic plate theory, various approximate solutions for maximum deflection or stress on a clamped plate under uniform pressure p_o have been developed and given in the form:

$$\boxed{w_m = k_1 \frac{p_o b^4}{E t^3}} \quad \text{and} \quad \boxed{\sigma_m = k_2 \frac{p_o}{2}\left(\frac{b}{t}\right)^2}$$

- Values of k_1 and k_2 are plotted in a data sheet against the aspect ratio a/b.

- For a given pressure and aspect ratio, the thickness t from the above equations can be determined to ensure that the maximum working stress and/or deflection is smaller than an allowable value.

- For design purpose, if we first determine the thickness which would be required for a "long" plate as t_L, then the smaller thickness which would be required for a "short" plate t_s is given by the ratio t_s / t_L.

Design of Short Clamped Plates (3/4)

Elasto-plastic Behaviour

- The theory analytically gets very complicated, but fortunately for design we can use the same approach as for elastic plate with $t_s \,/\, t_L$ to find the required thickness t_s from the long plate analysis.

- Clarkson (1962) conducted a systematic series of pressure tests on rectangular plates to determine permanent sets δ with edges-free-to-slide conditions.

- A curve fitting study in which plate slenderness β and aspect ratio α were considered to be the independent variables revealed that the pressure p versus the acceptable permanent set δ data could be represented by the following approximations:

$$\left.\begin{aligned} p_u &= 6\frac{\sigma_{yield}}{\sqrt{\alpha}}\left(\frac{t}{b}\right)^2\left(1+\frac{2\delta}{\alpha t}\right) && \text{for } \beta < 2.5 \\ p_u &= 6\frac{\sigma_{yield}}{\sqrt{\alpha}}\left(\frac{t}{b}\right)^2\left(\frac{4}{3}+\frac{\delta}{\alpha t}\right) && \text{for } \beta \geq 2.5 \end{aligned}\right\} \quad 1 \leq \alpha \leq 5$$

- J. Clarkson (1962), Uniform pressure tests on plates with edges free to slide inwards, Trans. RINA, 104(1), 67-80.

Design of Short Clamped Plates (4/4)

Plastic Behaviour

- For sturdy plates the formation of plastic "hinges" will occur at relatively small deflections before membrane actions become significant, whilst for slender plates membrane actions will be significant before the hinges are fully developed.

(i) Sturdy Plates ($\beta < 2$)

- The failure mechanism analogous to three-hinge collapse for long plate is the "roof-top" mechanism for short plate, where yield-line hinges form with a central hinge linked to others running to the four corners.

- A yield-line theory for collapse of rigid clamped plates shows:

$$p_u = 12\sigma_{yield}\left(\frac{t}{b}\right)^2\left(\sqrt{3+1/\alpha^2}-1/\alpha\right)^{-2}$$

(ii) Slender Plates ($\beta \geq 2.5$)

- If we postulate the material to be rigid-plastic (i.e. ideally plastic but ignoring elastic portion) then it must follow that the plate will behave as a pure membrane with simply supported edges.

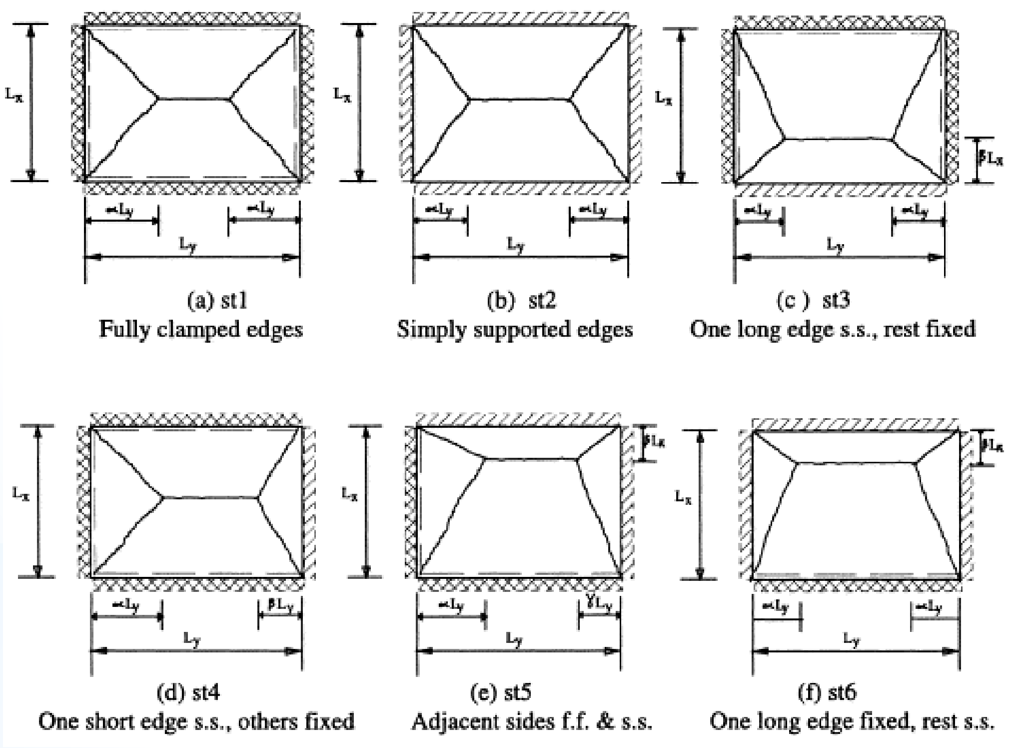
- The membrane stresses can only exist as "circulating" stresses tangential to the boundary.

- The centre is subject to tension actions balanced by a compressive surround which can cause buckling in slender plates.

- The permanent stretching of the plate into a non-developable surface is known as "shape hardening".

- A plastic membrane approach yields

$$p_r = 8\sigma_{yield}\left(\frac{t}{b}\right)^2\left(\frac{w_m}{t}\right)\left(1-\frac{32}{\pi^3}\operatorname{sech}\frac{\pi\alpha}{2}\right)^{-2}$$



*Topic 7.2:

Yield-line Theory (Roof-top Collapse)

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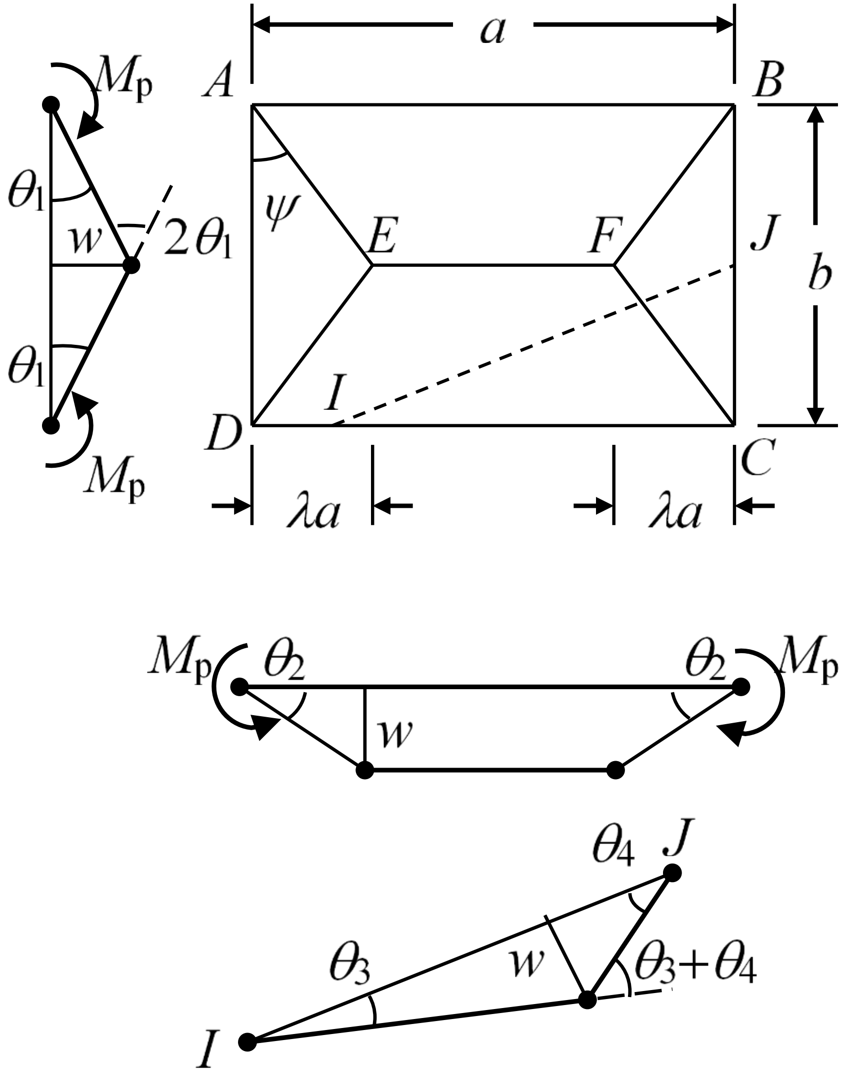
Yield-line Theory (Roof-top Collapse) (1/8)

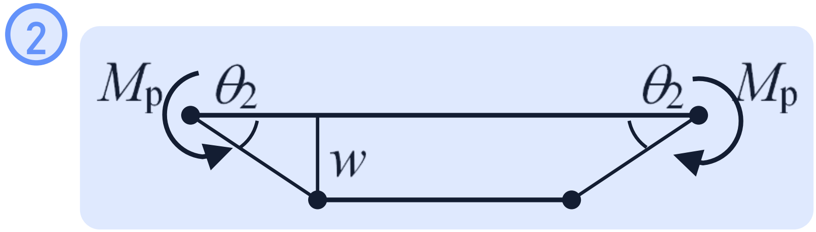
Assumption:

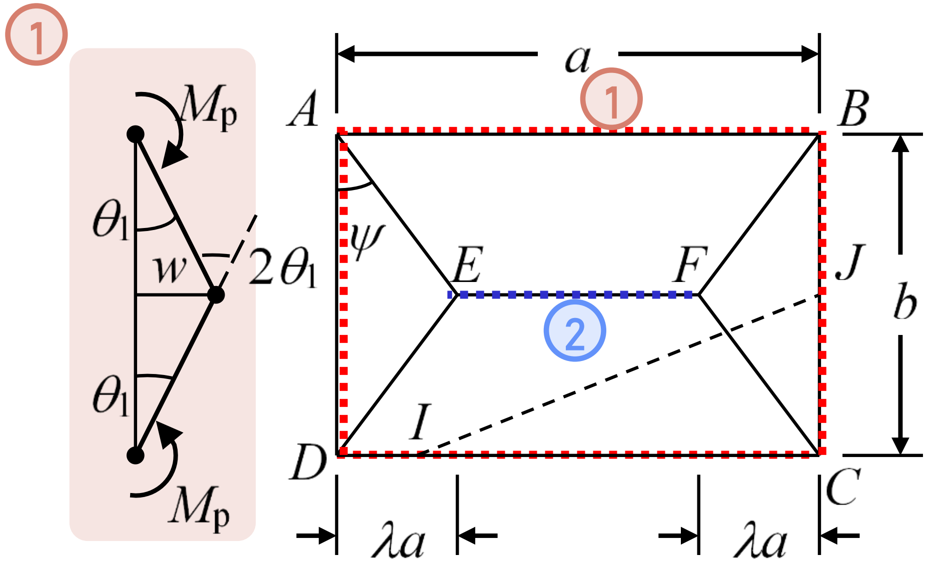
1. The material is ideally plastic.
2. Membrane actions are ignored.
3. Yield line hinges form as shown with a central hinge linked to others running to the four corners.



Yield-line Theory (Roof-top Collapse) (2/8)

- Let M_p is the plastic moment per unit width
- Consider the root top undergoes a small virtual displacement w
- Then internal work done by yield line hinges is:





1. Side hinges AB, BC, CD, DA

$$U_1 = 2M_p\theta_1 a + 2M_p\theta_2 b = 2M_p w\left(2\alpha + \frac{1}{\lambda\alpha}\right)$$

2. Central hinge EF

$$U_2 = 2M_p\theta_1(a - 2\lambda a) = 4M_p w\alpha(1 - 2\lambda)$$

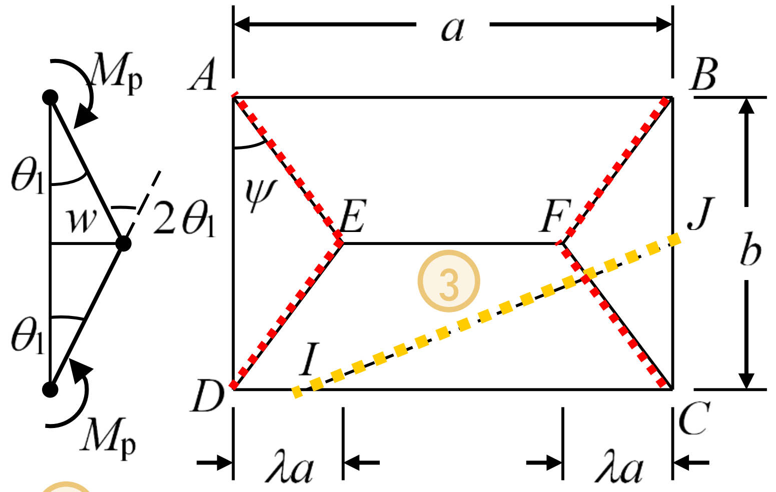
Yield-line Theory (Roof-top Collapse) (3/8)

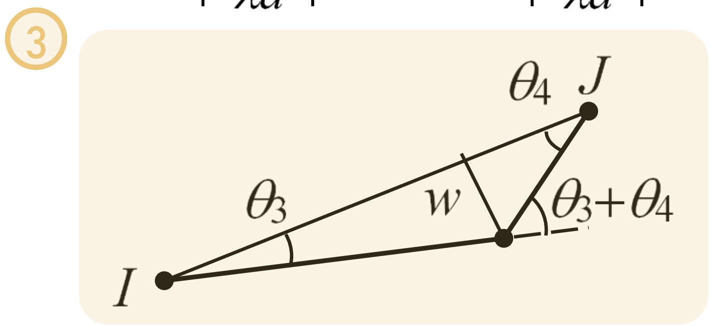
3. Diagonal hinges, AE, DE, BF, CF.

$$U_3 = 4M_p\left(\theta_3+\theta_4\right)AE$$

$$= 4M_p\left(\frac{2w}{b}\sin\psi + \frac{w}{\lambda a}\cos\psi\right)\frac{b}{2}\sec\psi$$

$$=2M_p w\left(2\tan\psi + \frac{b}{\lambda a}\right) = 2M_p w\left(4\lambda\alpha + \frac{1}{\lambda\alpha}\right)$$





- Hence, the total internal work done is

$$\boxed{U} = U_1 + U_2 + U_3 = \boxed{4M_p w\left(2\alpha + \frac{1}{\lambda\alpha}\right)}$$

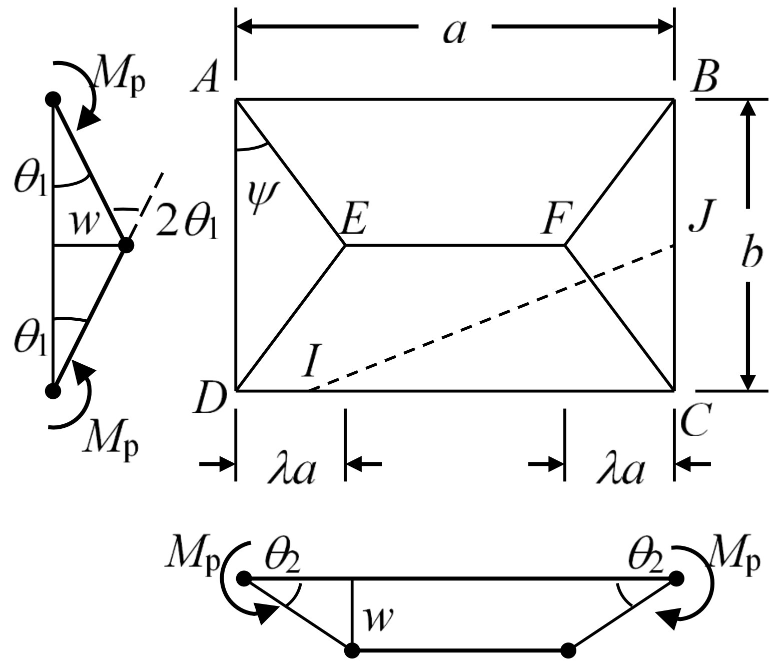
Yield-line Theory (Roof-top Collapse) (4/8)

- Virtual external work done by the pressure p_u is

$$\boxed{W} = p_u \times \text{Area} \times \text{depression}$$

$$= p_u \times \text{Volume of depression}$$

$$= \boxed{p_u \frac{abw}{6}(3-2\lambda)}$$



where Volume = 2 Pyramid AED + Ridge EF

$$= 2 \times \frac{1}{3} \times w \times \lambda a \times b + \frac{bw}{2}(a - 2\lambda a)$$

$$= \frac{abw}{6}(3-2\lambda)$$

Yield-line Theory (Roof-top Collapse) (5/8)

$$U = U_1 + U_2 + U_3 = 4M_p w\left(2\alpha + \frac{1}{\lambda\alpha}\right) \qquad W = p_u \frac{abw}{6}(3-2\lambda)$$

- By principle of virtual work, $\boldsymbol{U = W}$

$$\therefore 4M_p w\left(2\alpha + \frac{1}{\lambda\alpha}\right) = p_u \frac{abw}{6}(3-2\lambda) \Rightarrow \therefore \boxed{M_p} = p_u \frac{ab}{24}\frac{(3-2\lambda)}{\left(2\alpha + \frac{1}{\lambda\alpha}\right)} = \boxed{p_u \frac{ab}{24}\frac{(3-2\lambda)\lambda\alpha}{1+2\lambda\alpha^2}}$$

- This is the plastic moment required to resist hinge collapse under the pressure p_u.

Yield-line Theory (Roof-top Collapse) (6/8)

$$\therefore M_p = p_u \frac{ab}{24} \frac{(3-2\lambda)\lambda\alpha}{1+2\lambda\alpha^2}$$

- For maximum plastic moment M_p, $dM_p/d\lambda = 0$:

$$\therefore \frac{d}{d\lambda}\left\{(3-2\lambda)\Big/\left(2\alpha+\frac{1}{\lambda\alpha}\right)\right\}=0 \qquad \therefore \frac{-2}{2\alpha+\dfrac{1}{\lambda\alpha}}-\frac{3-2\lambda}{\left(2\alpha+\dfrac{1}{\lambda\alpha}\right)^2}\left(\frac{-1}{\lambda^2\alpha}\right)=0$$

$$\therefore 4\alpha^2\lambda^2+4\lambda-3=0$$

$$\boxed{\therefore \lambda=\frac{-4+\sqrt{16+48\alpha^2}}{8\alpha^2}=\frac{1}{2\alpha^2}\left(\sqrt{1+3\alpha^2}-1\right)}$$

Yield-line Theory (Roof-top Collapse) (7/8)

$$\therefore M_p = p_u \frac{ab}{24} \frac{(3-2\lambda)\lambda\alpha}{1+2\lambda\alpha^2}$$

$$\therefore \lambda = \frac{1}{2\alpha^2}\left(\sqrt{1+3\alpha^2}-1\right)$$

- Substituting this value into M_p expression, we have

$$\therefore M_p = p_u \frac{ab}{24} \frac{\left[3-\left(\sqrt{1+3\alpha^2}-1\right)/\alpha^2\right]\frac{1}{2\alpha}\left(\sqrt{1+3\alpha^2}-1\right)}{\sqrt{1+3\alpha^2}}$$

$$= p_u \frac{ab}{48} \frac{\left(3+\frac{1}{\alpha^2}-\frac{\sqrt{3+1/\alpha^2}}{\alpha}\right)\left(\sqrt{3+1/\alpha^2}-\frac{1}{\alpha}\right)}{\alpha\sqrt{3+1/\alpha^2}}$$

$$= p_u \frac{ab}{48\alpha}\left(\sqrt{3+1/\alpha^2}-1/\alpha\right)^2$$

Yield-line Theory (Roof-top Collapse) (8/8)

$$\therefore M_p = p_u \frac{ab}{48\alpha}\left(\sqrt{3+1/\alpha^2}-1/\alpha\right)^2$$

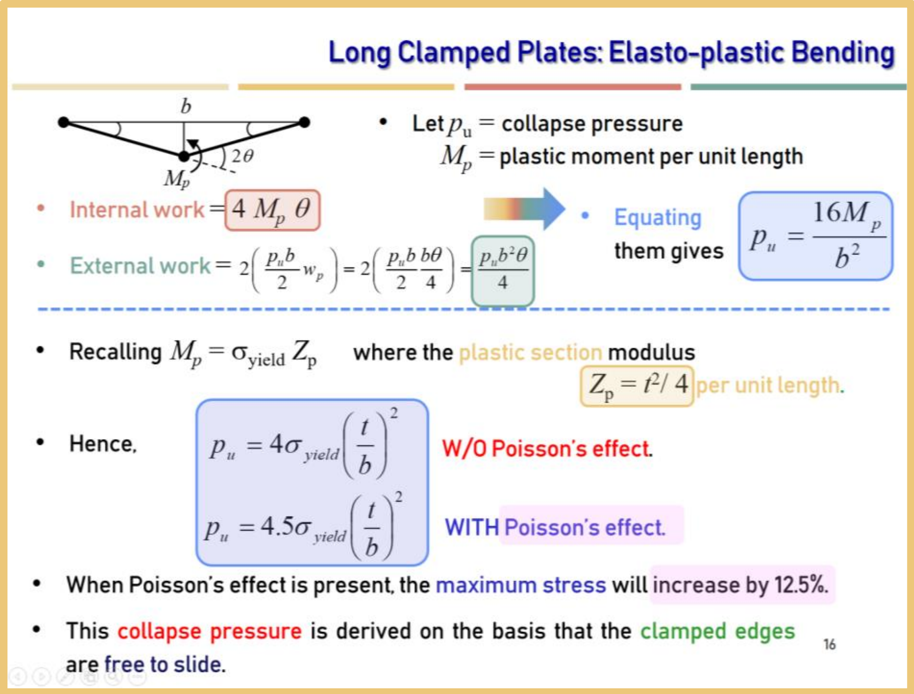
- For plate section, we have

$$M_p = \sigma_{yield}\, Z_p = \sigma_{yield}\, t^2/\, 4$$

- Hence,

$$p_u = 12\sigma_{yield}\left(\frac{t}{b}\right)^2\left(\sqrt{3+1/\alpha^2}-1/\alpha\right)^{-2}$$

- The first term is 3 times the “long” plate 3-hinge collapse and the last bracket term represents the effect of aspect ratio α.



Long Clamped Plates: Elasto-plastic Bending

- Let p_u = collapse pressure
  M_p = plastic moment per unit length
- Internal work = $\boxed{4\,M_p\,\theta}$
- External work = $2\left(\frac{p_u b}{2} w_p\right) = 2\left(\frac{p_u b}{2}\frac{b\theta}{4}\right) = \boxed{\frac{p_u b^2\theta}{4}}$
- Equating them gives $\boxed{p_u = \frac{16M_p}{b^2}}$
- Recalling $M_p = \sigma_{yield} Z_p$ where the plastic section modulus $\boxed{Z_p = t^2/\,4}$ per unit length.
- Hence, $p_u = 4\sigma_{yield}\left(\frac{t}{b}\right)^2$ W/O Poisson's effect.
  $p_u = 4.5\sigma_{yield}\left(\frac{t}{b}\right)^2$ WITH Poisson's effect.
- When Poisson's effect is present, the maximum stress will increase by 12.5%.
- This collapse pressure is derived on the basis that the clamped edges are free to slide.

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Learning Outcomes (Review)

- We have investigated the Elastic Plate Theory.

- Now we are able to:
  - Describe short clamped plates subjected to uniform pressure.
  - Predict the performance of short clamped plates.
  - Perform plastic analysis of short plate collapse.

- Details can be referred to topics 7 in the lecture notes.

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Kam Sa Hab Ni Da

감사합니다

Thank you!

Questions?

Aerial View of Korean Presidential Archives in Sejong city
(Construction Completed in 2014)