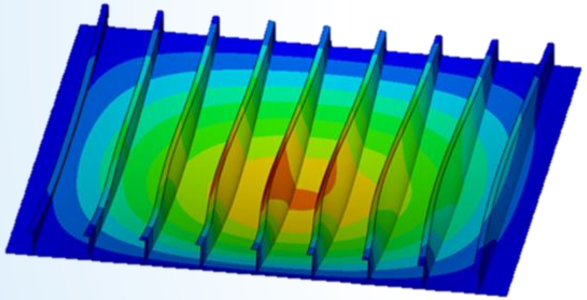
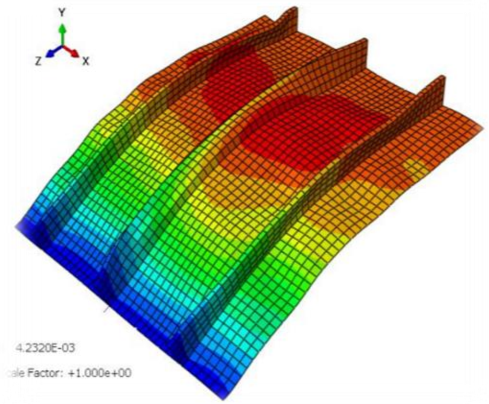
Topics in Ship Structures

(Advanced Local Structural Design & Analysis of Marine Structures)





* Buckling of Plates (Topic 8)

(Failure modes)

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NAOE
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Adv. Local Structural Design & Analysis of Marine Structures (Overview)

[Theory of Plates and Grillages]

[Part I] Plastic Design of Structures

- Plastic theory of bending (Topic 1)
- Ultimate loads on beams (Topic 2)
- Collapse of frames and grillage structures (Topic 3)

[Part II] Elastic Plate Theory under Pressure

- Basic (Topic 4)
- Simply supported plates under Sinusoidal Loading (Topic 5)
- Long clamped plates (Topic 6)
- Short clamped plates (Topic 7)
- Low aspect ratio plates, strength & permanent set (Topic 7A)

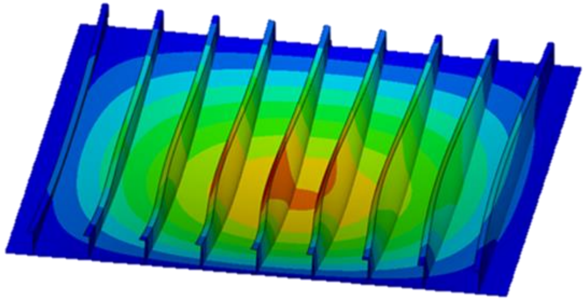
[Part III] Buckling of Plate & Stiffened Panels

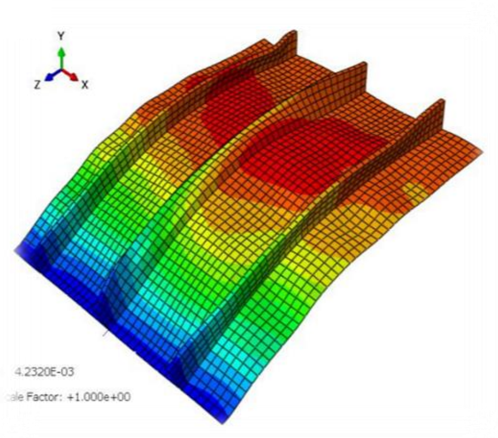
- Failure modes (Topic 8)
- Tripping (Topic 9) + Post-buckling strength of plate (Topic 9A)
- Post-buckling behaviour (Topic 10)

Objectives

The aim of this lecture is:

- To reinforce your understanding of instability of main structures.
- To equip you with the knowledge and understanding of the buckling of flat plates.





Learning Outcome

At the end of this lecture, you should be able to:

- Be aware of various modes of failure caused by in-plane compression of grillages.
- Be familiar with energy method for derivation of critical buckling stress of a flat plate.
- Evaluate the critical buckling stress of a flat plate.
- Design an effective stiffening system against in-plane compression.

How to Utilize the Outcome??

Structural Engineering and Mechanics, *Vol. 68, No. 2 (2018) 247-259*
DOI: https://doi.org/10.12989/sem.2018.68.2.247

Ultimate strength of initially deflected plate under longitudinal compression: Part I = An advanced empirical formulation

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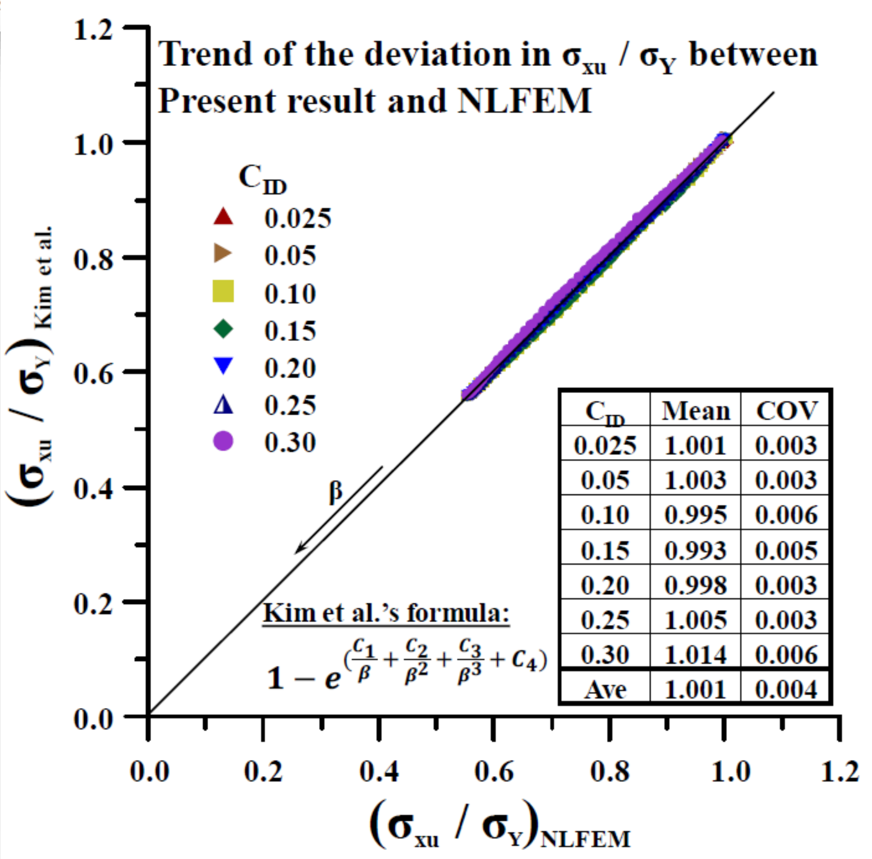
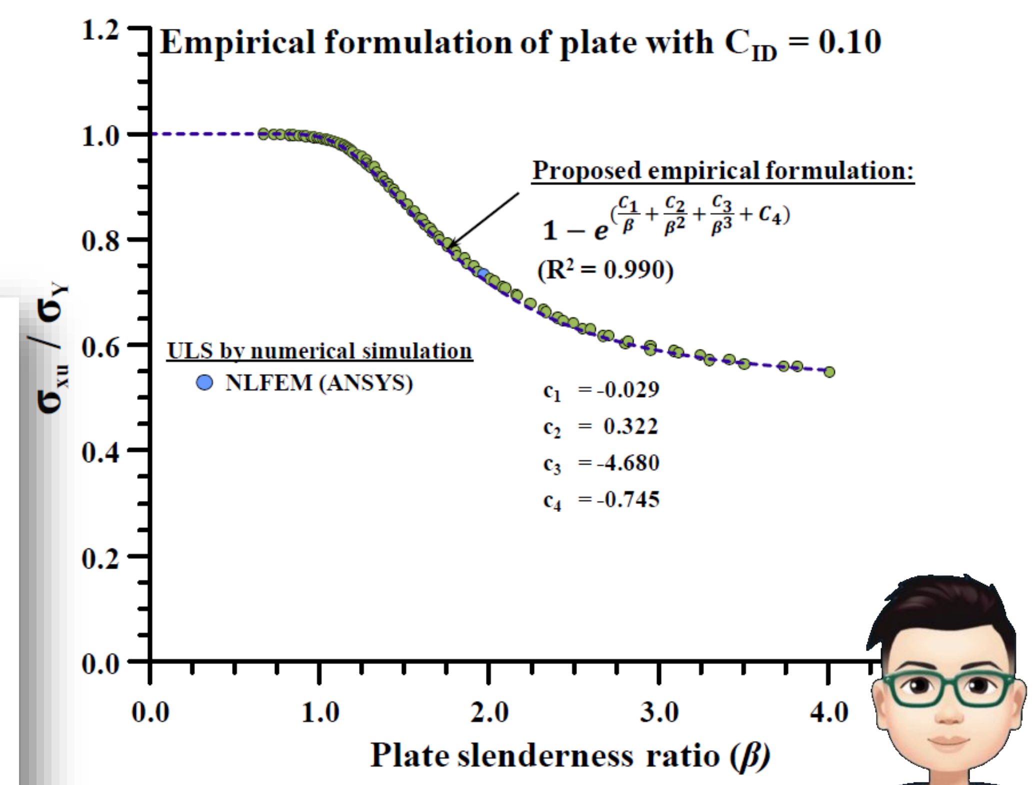


C_{ID}	Mean	COV
0.025	1.001	0.003
0.05	1.003	0.003
0.10	0.995	0.006
0.15	0.993	0.005
0.20	0.998	0.003
0.25	1.005	0.003
0.30	1.014	0.006
Ave	1.001	0.004



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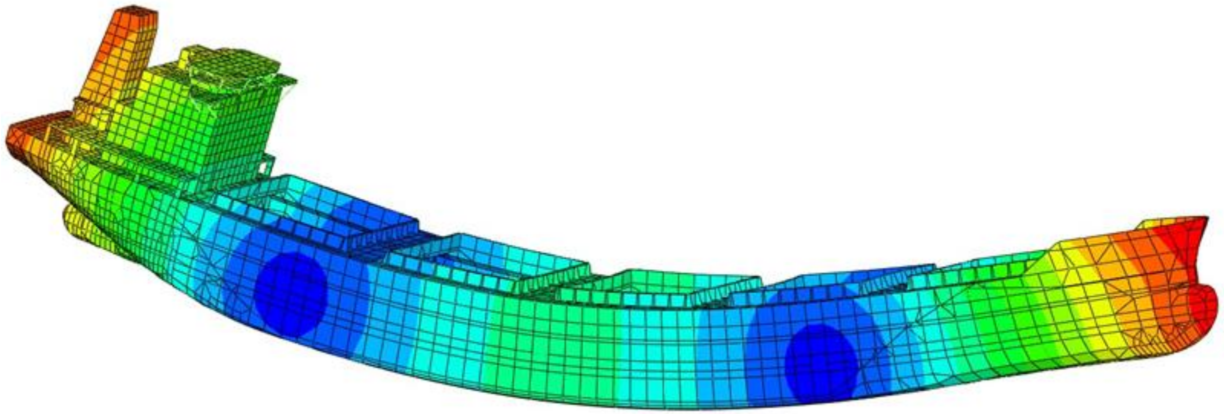
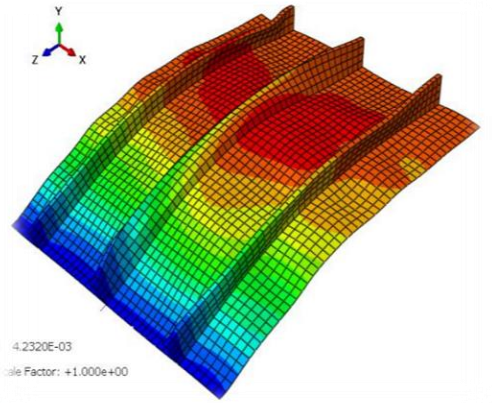
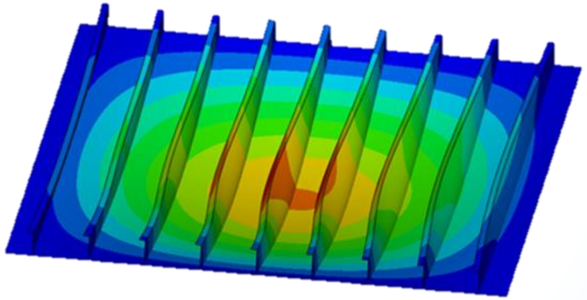
Introduction

- Deck and bottom structures suffers in-plane compression due to
  - Global longitudinal bending of the hull girder

- The in-plane compression may cause local and/or overall structural instability
  - Lead to collapse of the structure if the buckling strength is not sufficient.







*Lecture 8.1:

Failure Modes due to In-plane Compression of Grillage

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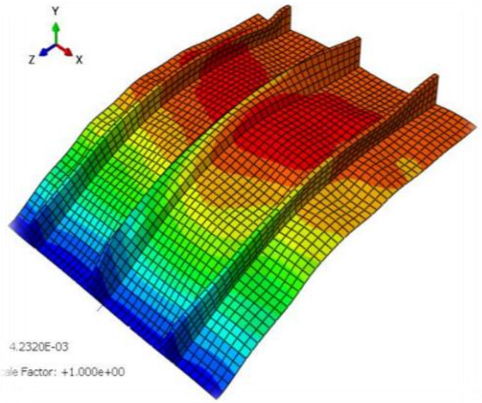
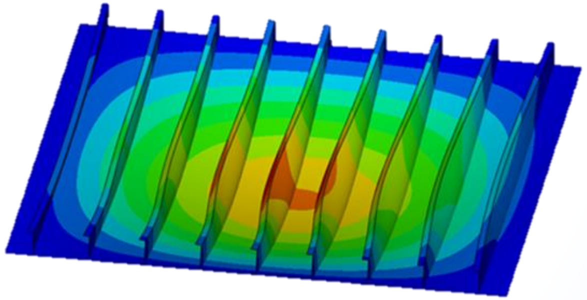
Failure Modes due to In-plane Compression of Grillage

- Failure modes of the grillage due to in-plane compression
  - Buckling of plate element
  - Column buckling of stiffened panel
  - Tripping of stiffeners
  - Overall collapse of the grillage

- Following are important to check
  - Plate buckling strength.
  - Strength of stiffened panel against column buckling.
  - Strength of stiffener against tripping.
  - Overall buckling strength of grillage.





Lecture 8.2:

Buckling of Flat Plates

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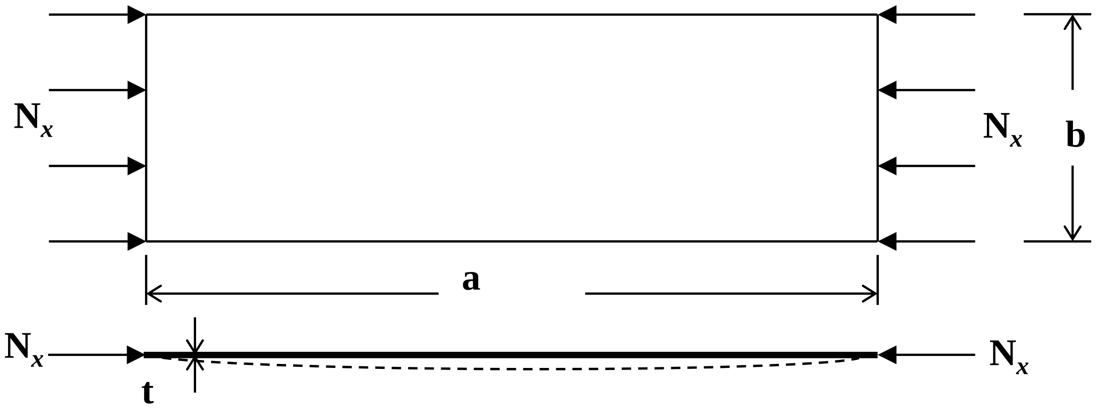
Buckling of Flat Plates

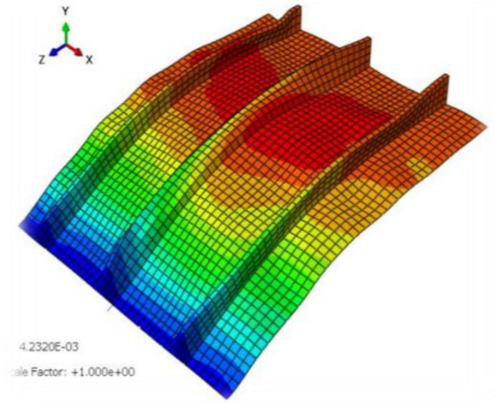
- Consider a flat plat $a \times b \times t$ subjected to in-plane compressive force N_x as shown below

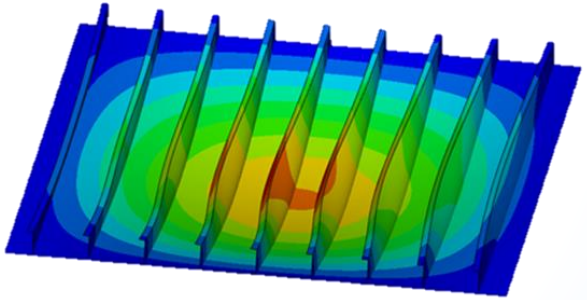


– If work done *(W)* by the applied stress is small than the strain energy *(U)*, the plate is stable.

– If work done *(W)* by the applied stress is equal to the strain energy *(U)*, the plate is in state of equilibrium and buckling is about to start.

– If work done *(W)* by the applied stress is larger than the strain energy *(U)*, the plate is unstable and buckling occurs.





*Lecture 8.3:

Strain energy of bending and twisting

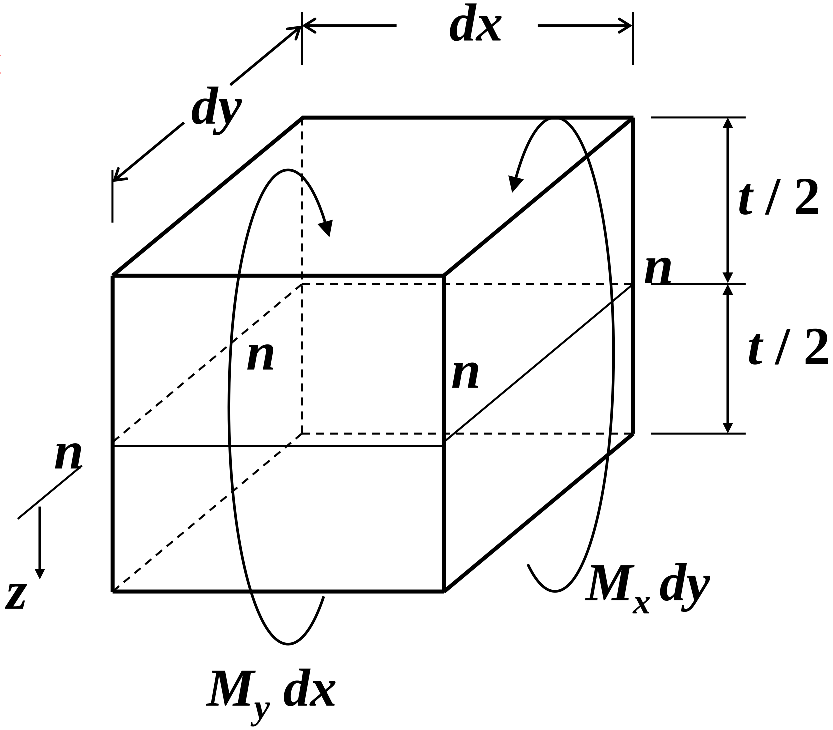
Strain Energy of Bending and Twisting (1/3)

- Strain energy due to bending moment M_x in small element $dx \times dy \times t$:

$$dU_x = \tfrac{1}{2} M_x dy \theta_x = \tfrac{1}{2} M_x dy \frac{dx}{R_x} = -\tfrac{1}{2} M_x \frac{\partial^2 w}{\partial x^2} dydx$$

$$\boxed{\frac{1}{R_x} = -\frac{\partial^2 w}{\partial x^2}}$$

where Θ_x is rotation due to M_x



- Similarly, strain energy due to M_y is

$$dU_y = -\tfrac{1}{2} M_y \frac{\partial^2 w}{\partial y^2} dydx$$

Strain Energy of Bending and Twisting (2/3)

$$dU_x = -\frac{1}{2} M_x \frac{\partial^2 w}{\partial x^2} dydx \qquad dU_y = -\frac{1}{2} M_y \frac{\partial^2 w}{\partial y^2} dydx$$

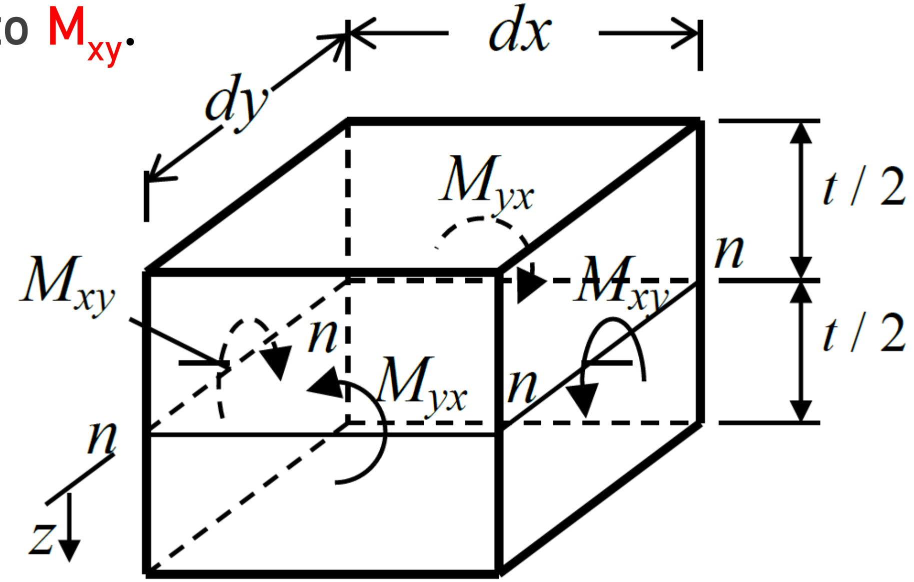
- Strain energy due to M_{xy} is

$$dU_{xy} = \tfrac{1}{2} M_{xy} dy \frac{\partial}{\partial x}\left(\frac{\partial w}{\partial y}\right) dx = \tfrac{1}{2} M_{xy} dy \frac{\partial^2 w}{\partial x \partial y} dx$$

where $\frac{\partial}{\partial x}\left(\frac{\partial w}{\partial y}\right) dx$ is rotation due to M_{xy}.

- Strain energy due to M_{yx} is

$$dU_{yx} = -\tfrac{1}{2} M_{yx} dx \frac{\partial^2 w}{\partial x \partial y} dy$$



Strain Energy of Bending and Twisting (3/3)

$$dU_x = -\tfrac{1}{2} M_x \frac{\partial^2 w}{\partial x^2} dydx$$

$$dU_y = -\tfrac{1}{2} M_y \frac{\partial^2 w}{\partial y^2} dydx$$

$$dU_{xy} = \tfrac{1}{2} M_{xy} dy \frac{\partial^2 w}{\partial x \partial y} dx$$

$$dU_{yx} = -\tfrac{1}{2} M_{yx} dx \frac{\partial^2 w}{\partial x \partial y} dy$$

- Total strain energy in the element due to bending and twisting is

$$dU = -\tfrac{1}{2}\left(M_x \frac{\partial^2 w}{\partial x^2} + M_y \frac{\partial^2 w}{\partial y^2} + \left(M_{yx} - M_{xy}\right)\frac{\partial^2 w}{\partial x \partial y}\right) dydx$$

- Substituting the moment-curvature relationships into the forging equation gives

$$dU = \tfrac{1}{2} D\left\{\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)^2 - 2(1-\nu)\left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right]\right\} dydx$$

$$M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2}\right)$$

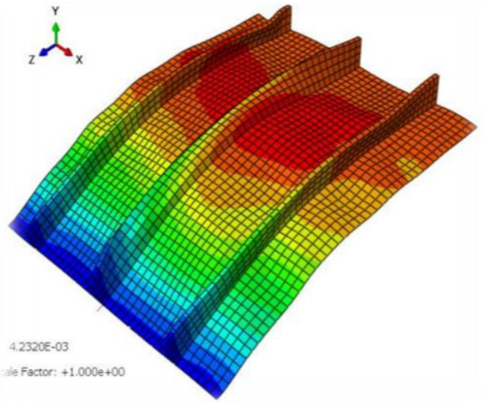
$$M_y = -D\left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2}\right)$$

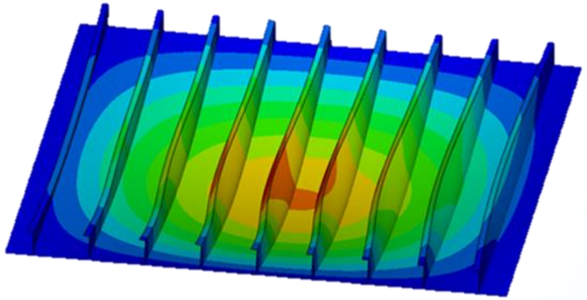
in which $D = \dfrac{Et^3}{12(1-\nu^2)}$ = plate flexural rigidity

$$M_{xy} = D(1-\nu)\frac{\partial^2 w}{\partial x \partial y}$$

- The total strain energy in the plate is

$$U = \tfrac{1}{2} D \int_0^a \int_0^b \left\{\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)^2 - 2(1-\nu)\left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right]\right\} dydx$$



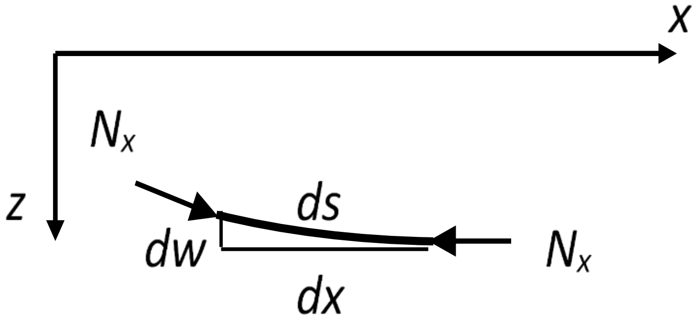


*Lecture 8.4:

Work done by in-plane direct & shear forces

Work Done by In-plane Direct & Shear Forces

- Work done by in-plane force N_x per unit width dy in x-direction is



$$dW_x = N_x dy du \quad = \tfrac{1}{2} N_x \left(\frac{\partial w}{\partial x}\right)^2 dydx$$

where $du = ds - dx = \left[(dx)^2 + (\partial w)^2\right]^{1/2} - dx$

$$= dx\left[1 + \left(\frac{\partial w}{\partial x}\right)^2\right]^{1/2} - dx = dx\left[1 + \tfrac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 + \cdots\cdots\right] - dx \quad = \tfrac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \mathrm{dx}$$

- Similarly, the work done by in-plane force N_y per unit width dx in y-direction is

$$dW_y = \tfrac{1}{2} N_y \left(\frac{\partial w}{\partial y}\right)^2 dydx$$

[Typo] See "Lecture Note (Topic 8)"

Buckling of Flat Plates

- The work done by shear force N_{xy} per unit width dy is $$dW_{xy} = \tfrac{1}{2} N_{xy}\gamma_{xy} dydx = \tfrac{1}{2} N_{xy}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) dydx$$

Since $$\frac{\partial u}{\partial y} = \tfrac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \frac{\partial x}{\partial y} = \tfrac{1}{2}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y} \quad \& \quad \frac{\partial v}{\partial x} = \tfrac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 \frac{\partial y}{\partial x} = \tfrac{1}{2}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}$$

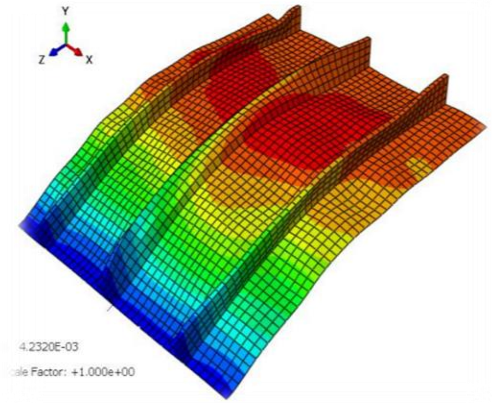
Then $$dW_{xy} = \tfrac{1}{2} N_{xy}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y} dydx$$

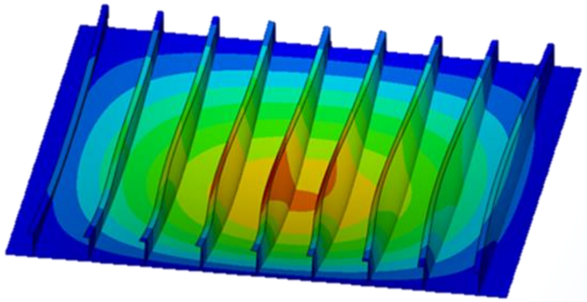
- Similarly, the work done by in-plane force N_{yx} per unit width dx is

$$dW_{yx} = \tfrac{1}{2} N_{yx}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y} dydx$$

- Hence, the total work done by in-plane direct forces and shear forces in the plate is

$$W = \frac{1}{2}\int_0^a\int_0^b \left[N_x\left(\frac{\partial w}{\partial x}\right)^2 + N_y\left(\frac{\partial w}{\partial y}\right)^2 + 2N_{xy}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right] dydx$$





*Lecture 8.5:

Simply supported rectangular plate compressed in short edges

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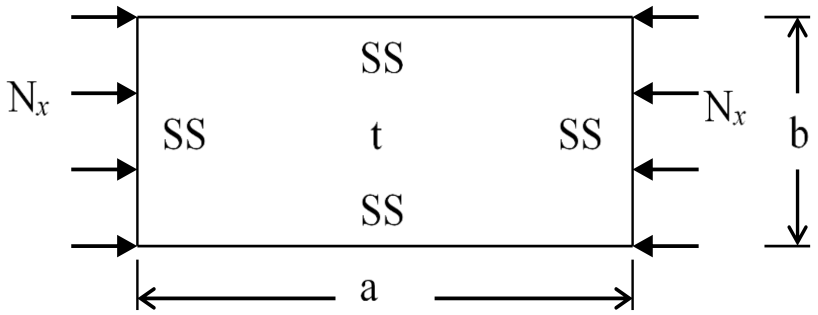
Simply supported plate under longitudinal compression (1/5)

Simply-supported rectangular plate compressed in short edges

- Consider a simply-supported rectangular plate of $a \times b \times t$ compressed in short edges of width b by N_x in the x-direction only, thus, $N_y = N_{xy} = 0$.



- The deflected shape of the plate can be represented by

Topic 4 and 5 [Reminder]

$$\boxed{w = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}}$$

which satisfies the boundary conditions of $w=0$, $M_x=M_y=0$ at the edges.

Simply supported plate under longitudinal compression (2/5)

$$U = \tfrac{1}{2} D \int_0^a \int_0^b \left\{ \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)^2 - 2(1-\nu) \left[\frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 \right] \right\} dydx$$

$$W = \tfrac{1}{2} \int_0^a \int_0^b \left[N_x \left(\frac{\partial w}{\partial x} \right)^2 + N_y \left(\frac{\partial w}{\partial y} \right)^2 + 2N_{xy} \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} \right] dydx$$

$$w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}$$

- Substitute this series into the strain energy equation gives

Lecture Note (Topic 14) or (Topic 8)

$$U = \frac{\pi^4 abD}{8} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn}^2 \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right)^2$$

- Similarly, the work done by N_x can be written as

$$W = \frac{\pi^2 b}{8a} N_x \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} m^2 a_{mn}^2$$

- Equating work done with the energy due to bending, we obtain an expression for N_x to cause buckling

$$N_x = \frac{\pi^2 a^2 D \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn}^2 \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right)^2}{\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} m^2 a_{mn}^2}$$

Simply supported plate under longitudinal compression (3/5)

- The above expression will become a minimum when all the coefficients a_{mn} except one, are zero. This condition leads to

$$N_x = \frac{\pi^2 a^2 D}{m^2}\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^2$$

- Irrespective of the value of m, N_x will be a minimum when n = 1. Hence

$$\left(N_x\right)_{\min} = \frac{\pi^2 D}{b^2}\left(\frac{mb}{a} + \frac{a}{mb}\right)^2$$

$$w = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}$$

$$N_x = \frac{\pi^2 a^2 D \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} a_{mn}^2 \left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^2}{\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} m^2 a_{mn}^2}$$

- Dividing the above equation by thickness (t) and taking the plate flexural rigidity

$$D = \frac{Et^3}{12\left(1-\nu^2\right)}$$

gives the elastic plate buckling stress

$$\boxed{\sigma_{E(plate)} = k\,\frac{\pi^2 E}{12\left(1-\nu^2\right)}\left(\frac{t}{b}\right)^2}$$

where the buckling coefficient

$$\boxed{k = \left(\frac{mb}{a} + \frac{a}{mb}\right)^2}$$

Simply supported plate under longitudinal compression (4/5)

buckling coefficient $k=\left(\frac{mb}{a}+\frac{a}{mb}\right)^2$ $\sigma_{E(plate)}=k\frac{\pi^2 E}{12\left(1-\nu^2\right)}\left(\frac{t}{b}\right)^2$

- The physical interpretation of n=1 is that all buckling modes will involve only a single half wave in the direction perpendicular to the direction of compression.

- Let's investigate the behaviour of the buckling coefficient k to see where the minimum value of N_x occurs, since this is when the plate will buckle. Let $\phi=\frac{mb}{a}$ then $k=\left(\varphi+\frac{1}{\varphi}\right)^2=\varphi^2+2+\frac{1}{\varphi^2}$

- On differentiating k with respect to ϕ and setting it to zero for a minimum, we obtain

$$\frac{dk}{d\varphi}=2\left(\varphi-\frac{1}{\varphi^3}\right)=0 \qquad \text{Thus,} \quad \phi=\frac{mb}{a}=1$$

When, $m=\frac{a}{b}$, k = 4 and the minimum critical (= Elastic) plate buckling stress is

$$\boxed{\min \quad \sigma_{E(plate)}=\frac{4\pi^2 E}{12\left(1-\nu^2\right)}\left(\frac{t}{b}\right)^2}$$

for a / b being integer

Simply supported plate under longitudinal compression (5/5)

The buckling pattern of a long plate is such that the length a will subdivide into 'm' half waves across the long edge and one half wave across the short edge.

- For aspect ratios other than integers, the buckling coefficient K is somewhat greater 4.
- The elastic buckling stress for plating depends on the plate width 'b' and is independent of the plate length 'a'.
- The critical buckling stress for plating can be obtained by plasticity correction.

$$\sigma_{E(plate)} = \frac{4\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2$$

Plasticity Correction

In Fig. 4.39, the dashed line is elastic buckling strength, σ_E, which is equal to σ_{cr} expressed by Eq. (4.1). On the other hand, the solid line represents the buckling strength with Johnson's plasticity correction, which is expressed as:

$$\boxed{\sigma_{cr} = \left(1 - \frac{\sigma_Y}{4\sigma_E}\right)\sigma_Y} \qquad (4.41)$$

Eq. (4.41) is applicable when σ_E is greater than $0.5 \times \sigma_Y$.

Johnson – Ostenfeld formulation

$$\sigma_{cr(plate)} = \begin{cases} \sigma_E & \text{for } \sigma_E \le 0.5\sigma_Y \\ \sigma_Y\left(1 - \dfrac{\sigma_Y}{4\sigma_E}\right) & \text{for } \sigma_E > 0.5\sigma_Y \end{cases}$$

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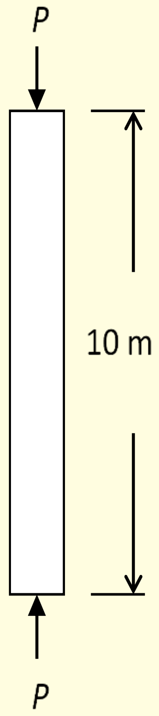
A column fabricated from steel plates of 10 mm thick has span of 10 m and square cross-section of 1m × 1m. Find the collapse mechanism and the collapse load if the column is assumed to be pinned ends. Yield stress = 230 N/mm^2, Young modulus E = 207000 N/mm^2 and Poisson's ration v = 0.3

Solution



There are two possible modes of failure.

- Local buckling of plate panel
- Column buckling of the column in its entirety.

Local plate buckling mode:

$$\sigma_{E(plate)} = \frac{4\pi^2 E}{12\left(1-\nu^2\right)}\left(\frac{t}{b}\right)^2$$

$$= \frac{4\pi^2 \times 207000}{12\left(1-0.3^2\right)}\left(\frac{10}{1000}\right)^2 = 74.8\ \mathbf{MPa}$$

Column buckling mode:

The second moment of area and area of the cross section are respectively

$$I = I_{out} + I_{in} = 1000(1000)^3/12 - 980(980)^3/12 = 6469320000 \text{ mm}^4$$

$$A = 1000^2 - 980^2 = 39600 \text{ mm}^2$$

Euler buckling stress for pinned ends column is:

$$\sigma_E = \frac{\pi^2 EI}{AL_e^2} = \frac{\pi^2 \times 207000 \times 6469320000}{39600 \times 10000^2} = \mathbf{3337.6 \text{ MPa}}$$

Based on linear interaction, the critical column buckling stress (Rankine-Gordon) is

$$\frac{1}{\sigma_{cr}} = \frac{1}{\sigma_E} + \frac{1}{\sigma_{yield}} = \frac{1}{\mathbf{3337.6}} + \frac{1}{230}$$

$$\therefore \sigma_{cr} = \mathbf{215.2 \text{ MPa or MN/ m}^2}$$

Therefore the column should collapse by local plate buckling rather than overall buckling.

Adv. Marine Structures / Adv. Structural Design & Analysis (Overview)

[Theory of Plates and Grillages]

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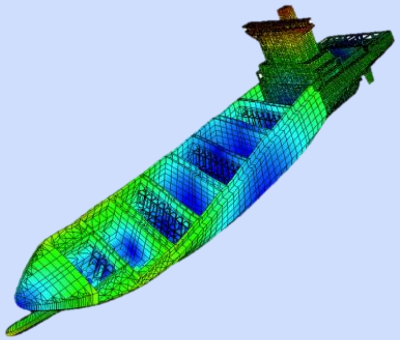
[Part III] Buckling of Plate & Stiffened Panels

- Failure modes (Topic 8) + Application
- Tripping (Topic 9) + Post-buckling strength of plate (Topic 9A)
- Post-buckling behaviour (Topic 10)

[Vibration]

[Part IV] Flexural Vibration

- Fundamentals (Topic 11)
- Vibration of Beam (Topic 12)
- Various Support Conditions (Topic 13)
- Vibration of Plate (Topic 14)



Buckling of Flat Plates (1/21)

Rectangular plates compressed in short edges with different edge conditions

- Based on energy method, the critical plate buckling stress in rectangular plate compressed in the short edges for different edge conditions can also be derived and presented in the form

$$\sigma_{E(plate)} = k\frac{\pi^2 E}{12\left(1-\nu^2\right)}\left(\frac{t}{b}\right)^2$$

where the buckling coefficient k varies with edge conditions and is shown in the following page.

$$k = \left(\frac{mb}{a} + \frac{a}{mb}\right)^2$$

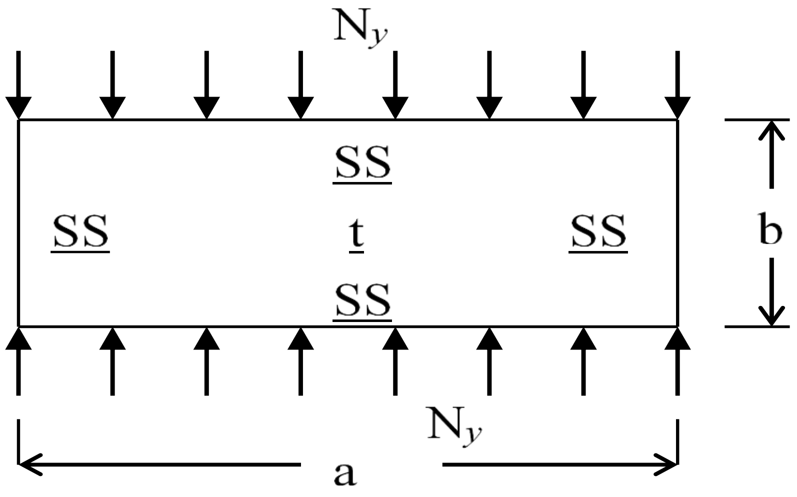
$$\frac{a}{b} \le \sqrt{m(m+1)}$$

m = buckling half-wave number

Buckling of Flat Plates (2/21)

Simply-supported rectangular plates compressed in long edges

- When a > b, then it is defined as long plate
- When the long edge is under load, it is defined as wide plate
- By using, plate buckling expression derived earlier, now for a wide plate, we interchange 'a' and 'b' in the equation



$$\sigma_{E(plate)} = \frac{\pi^2 D}{ta^2}\left(\frac{ma}{b}+\frac{b}{ma}\right)^2$$

- Rearrange to achieve $(t/b)^2$ outside the bracket gives

$$\sigma_{yE(plate)} = \frac{\pi^2 E}{12\left(1-\nu^2\right)}\left(\frac{t}{b}\right)^2\left(m+\frac{b^2}{ma^2}\right)^2$$

Buckling of Flat Plates (3/21)

- Since 'm' increases from 1 to ∞ and a > b, the minimum value of σ_E occurs when m = 1.

$$\min\ \sigma_{yE} = \frac{\pi^2 E}{12\left(1-\nu^2\right)}\left(\frac{t}{b}\right)^2\left(1+\frac{b^2}{a^2}\right)^2$$

- When a >> b, b/a can be neglected.

$$\min\ \sigma_{yE} = \frac{\pi^2 E}{12\left(1-\nu^2\right)}\left(\frac{t}{b}\right)^2$$

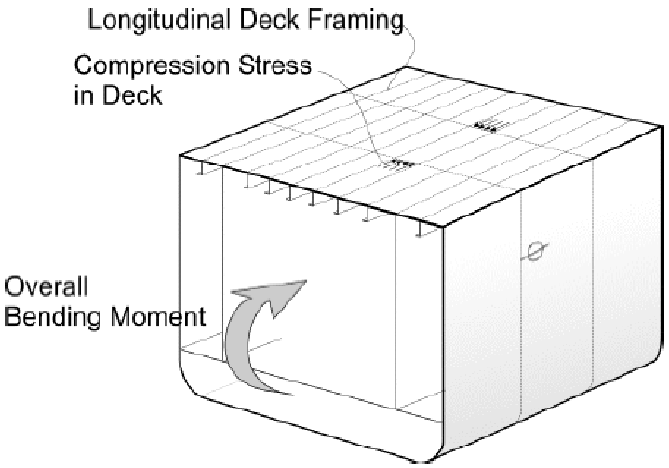
- For width plate loading, unloaded edge 'b' in direction of load is important.
- Minimum elastic buckling stress of rectangular plate compressed in long edges is 4 times less than that compressed in short edges.
- This indicates that longitudinal stiffening system gives higher buckling strength than transverse stiffening system.

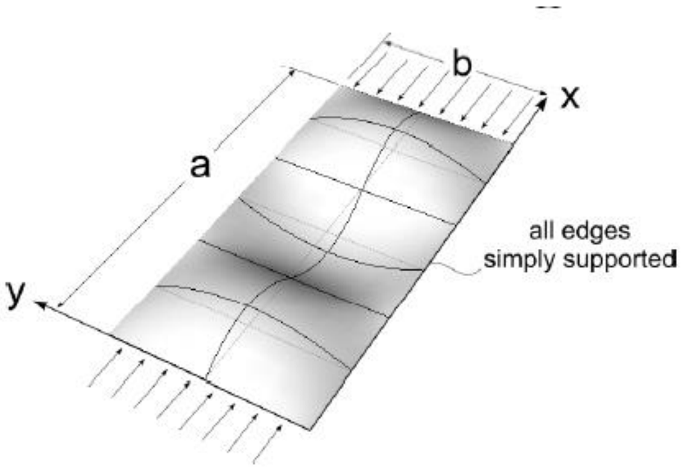
Buckling of Flat Plates (4/21)

Plates Buckling

Long Plates/Longitudinal Buckling (edges simply supported)

- When we consider a long plate loaded in the in-plane longitudinal direction.
- This will typically occur in the deck of a ship with longitudinal framing.
- In this configurations, the plate can buckle into a rippled pattern
- We can't use the simplifying assumption that we used with transverse plating. We have to consider the whole plate.

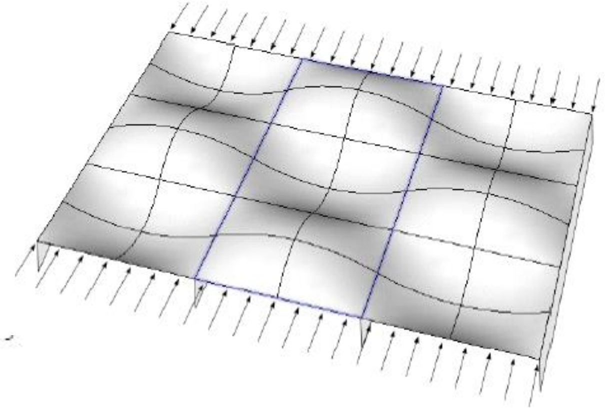


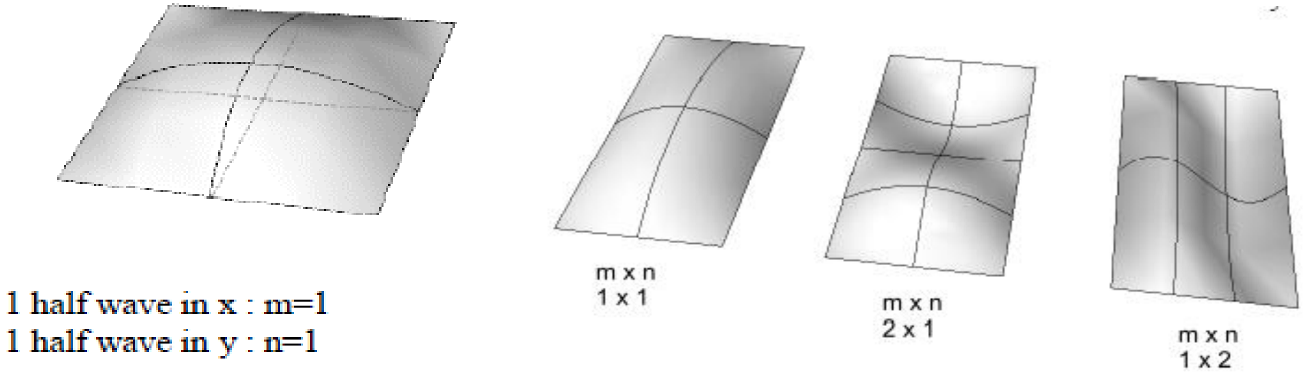


Buckling of Flat Plates (5/21)

- The assumption of pinned edges is reasonable in light of the anti-symmetry that is likely to develop in the buckling pattern. This means that the neighboring plates will not transmit a moment (though the frame may).

- Plates buckle into two dimensions. We will use the variables m and n to give the number of half sine waves in the x and y directions.





- The values of m and n (buckle shape) will depend in which shape has the lowest elastic potential energy. The shape will depend on the aspect ratio (a/b) of the plate.

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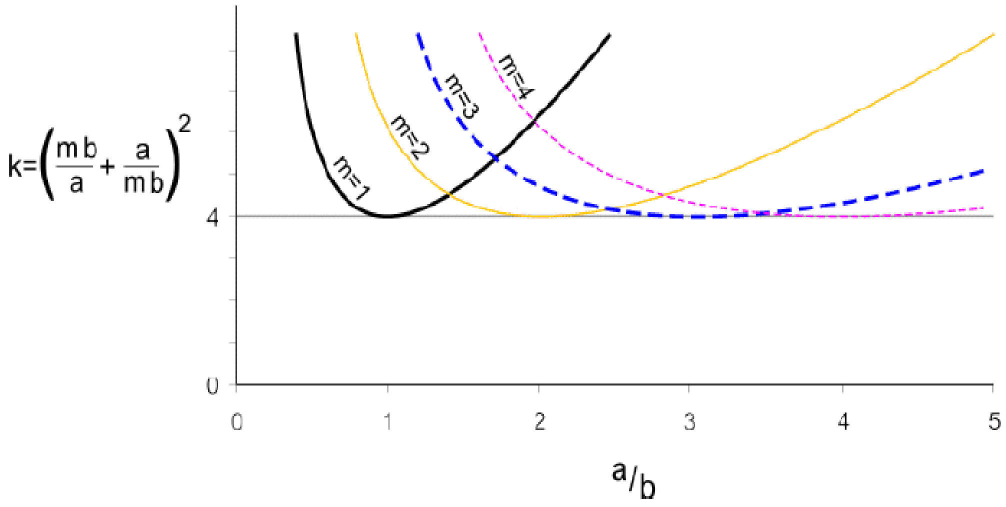
Buckling of Flat Plates (6/21)

- As shown in the lecturer notes, using an energy formulation we can derive the solution for the plate buckling as follows:

$$(N_x)_{min} = \frac{\pi^2 D}{b^2}\left(\frac{mb}{a} + \frac{a}{mb}\right)^2 \quad \text{where} \quad D = \frac{Et^3}{12(1-\nu^2)} \quad \Rightarrow \quad \sigma_E = \frac{(N_x)_{min}}{t}$$

$$\sigma_E = k\frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2 \quad \text{where} \quad k = \left(\frac{mb}{a} + \frac{a}{mb}\right)^2$$

When we plot k as a function of a/b, for various values of m, we see that the minimum value of k is always 4, and always occurring where a/b=m.

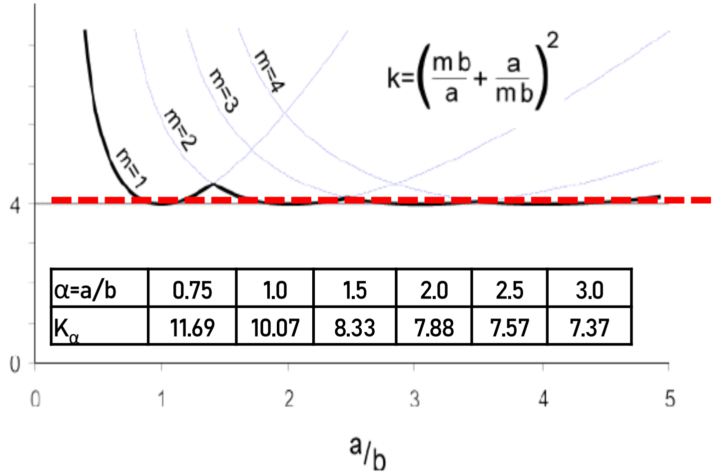


This means that for a plate with an aspect ratio of , say, 3 we will get the lowest buckling stress when we have m=3 (3 half-waves).

Buckling of Flat Plates (7/21)

The minimum buckling stress depends on the minimum k. It is reasonable to use k=4 for any aspect ratio.

For boundary conditions other than simple support the value of k in the critical buckling stress formulate will change



α=a/b	0.75	1.0	1.5	2.0	2.5	3.0
K_α	11.69	10.07	8.33	7.88	7.57	7.37

$$\sigma_{E(plate)} = k\frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2$$

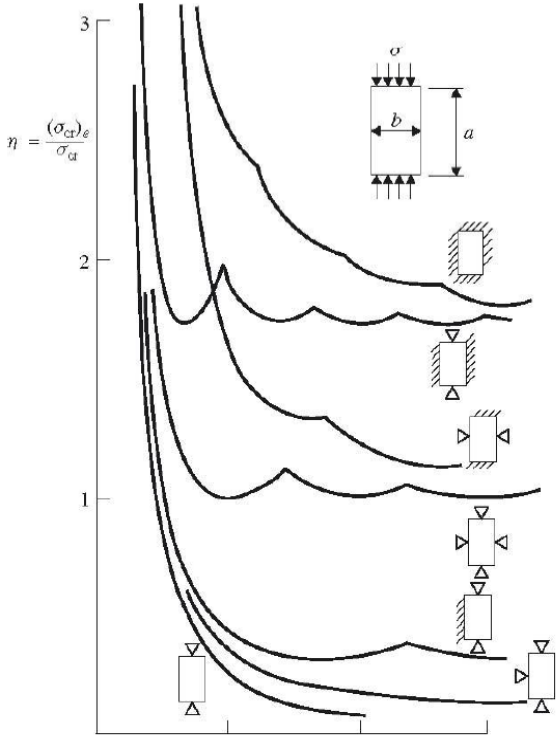


Plate buckling under compression

For a plate with fixed edges the coefficient k is shown in the above table:

The following design curves present the ratio between the critical elastic buckling stress for a plate with a particular edge fixings to the buckling stress for a simply supported plate with a similar aspect ratio (=a/b).

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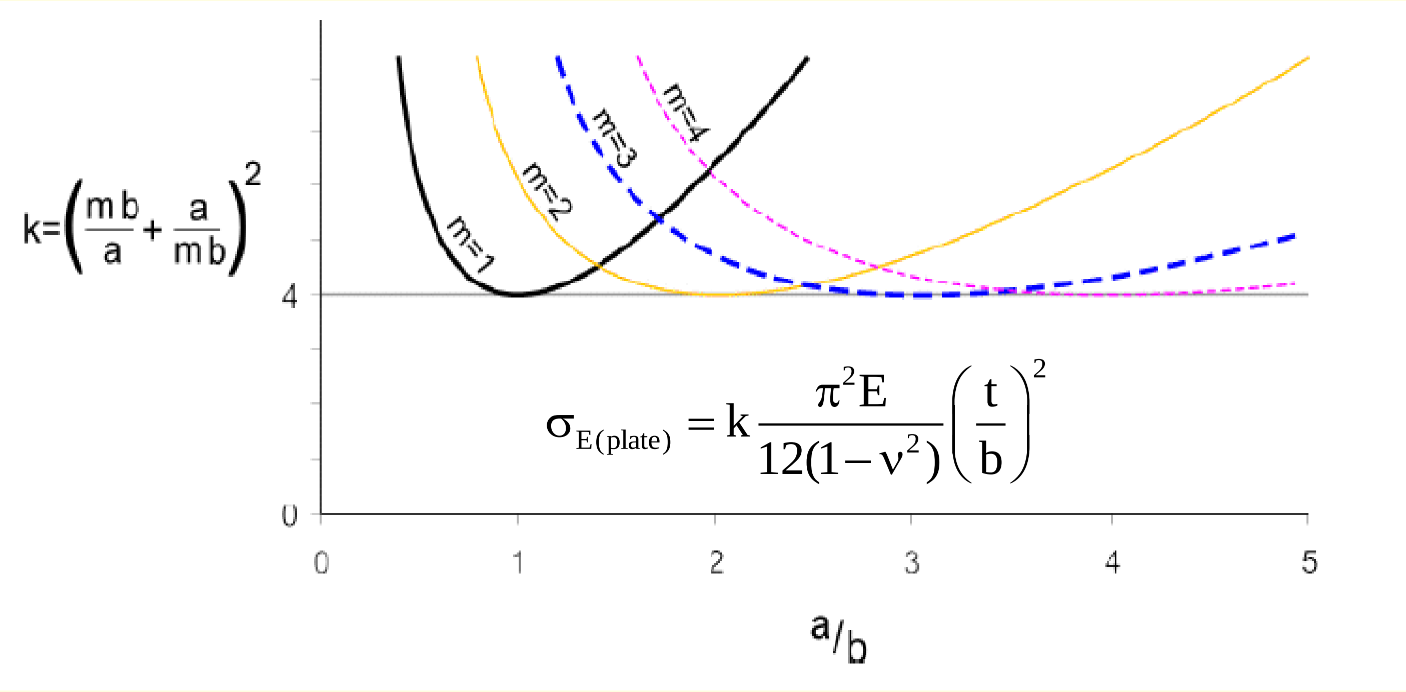
Simple problems (Boundary Condition = Simply supported):

1. For a longitudinal plate with an aspect ratio of 1.5, what is the minimum value of k? For this k, what is the value of m?

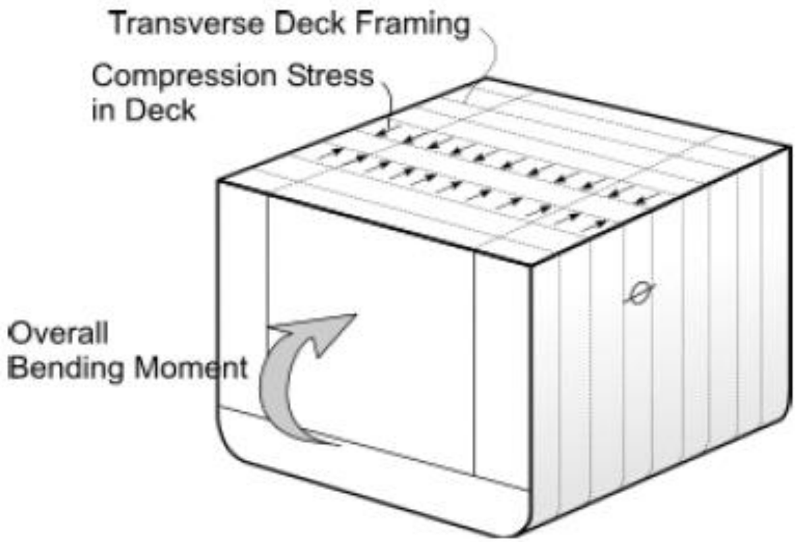
2. What is the buckling stress for a longitudinal steel deck plate of 6000mm x 1200mm x 12mm?

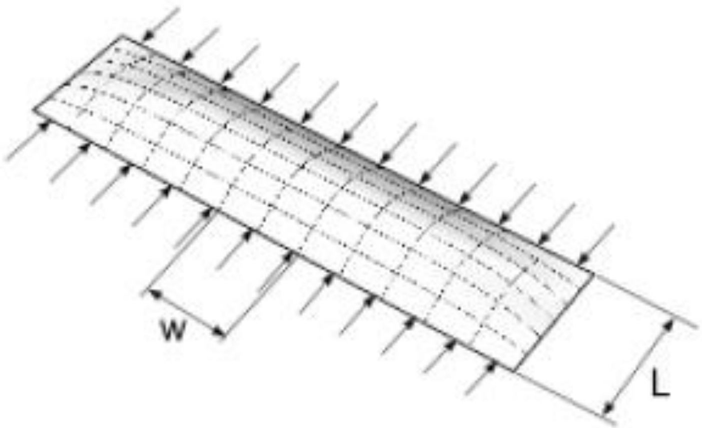


Buckling of Flat Plates (9/21)

Long plates/Transverse Buckling

When we consider a long plate loaded in the in-plane transverse direction. This will typically occur in the deck of a ship with transverse framing.





Buckling of Flat Plates (10/21)

- When we load a plate in this way, the buckling is simple. Each strip of the plate (of the some width 'w') buckles like each neighboring strip. We can write the Euler buckling load as:

$$P_E = \frac{\pi^2 EI}{L^2}$$

- We can rewrite this in terms of the stress in the plate:

$$\sigma_E = \frac{P_E}{A} = \frac{\pi^2 EI}{AL^2} = \frac{\pi^2 E r^2 A}{AL^2} \qquad \left(r = \sqrt{\frac{I}{A}} \right)$$

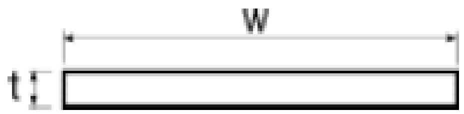
Which gives us:

$$\sigma_E = \pi^2 \left(\frac{r}{L} \right)^2 E$$

Buckling of Flat Plates (11/21)

Note: for a plate of width w,

$$I = \frac{wt^3}{12} = r^2 A \qquad r^2 = \frac{t^2}{12}$$



Normally we try to prevent buckling before the stresses reach yield by limiting the slenderness of the structural member. We will ensure that yield will occur first when we set;

$$\sigma_E > \sigma_Y \quad \text{or} \quad \sigma_E > k\sigma_Y \quad \text{for} \quad k > 1$$

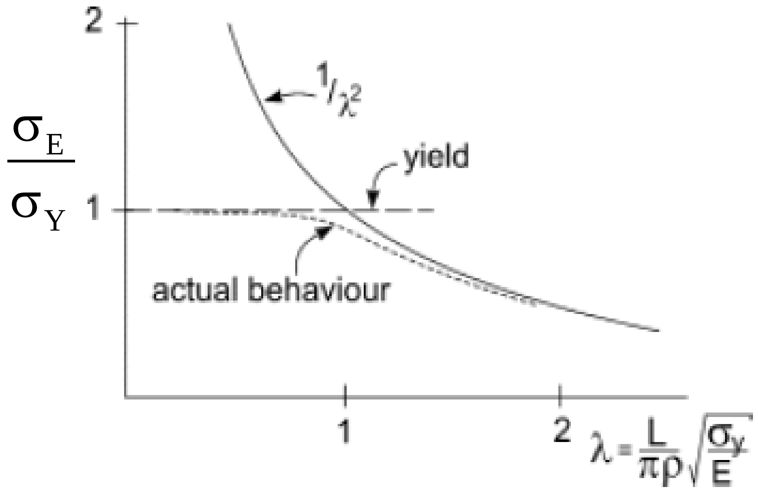
The ratio of buckling to yield stress is;

$$\frac{\sigma_E}{\sigma_Y} = \pi^2 \left(\frac{r}{L}\right)^2 \frac{E}{\sigma_Y} = \frac{1}{\lambda^2}$$

where λ is the slenderness ratio $\left(\lambda = \frac{L}{\pi r}\sqrt{\frac{\sigma_Y}{E}}\right)$

Buckling of Flat Plates (12/21)

The ratio is the Euler non-dimensional buckling stress. We can plot the Euler curves vs. slenderness. The curve is invalid for stresses above yield, due to imperfections (stress and geometry) the actual behavior tends to smoothly join the Euler curve and yield stress limit.



Johnson – Ostenfeld formulation

$$\sigma_{cr(column)} = \begin{cases} \sigma_E & \text{for } \sigma_E \le 0.5\sigma_Y \\ \sigma_Y\left(1 - \dfrac{\sigma_Y}{4\sigma_E}\right) & \text{for } \sigma_E > 0.5\sigma_Y \end{cases}$$

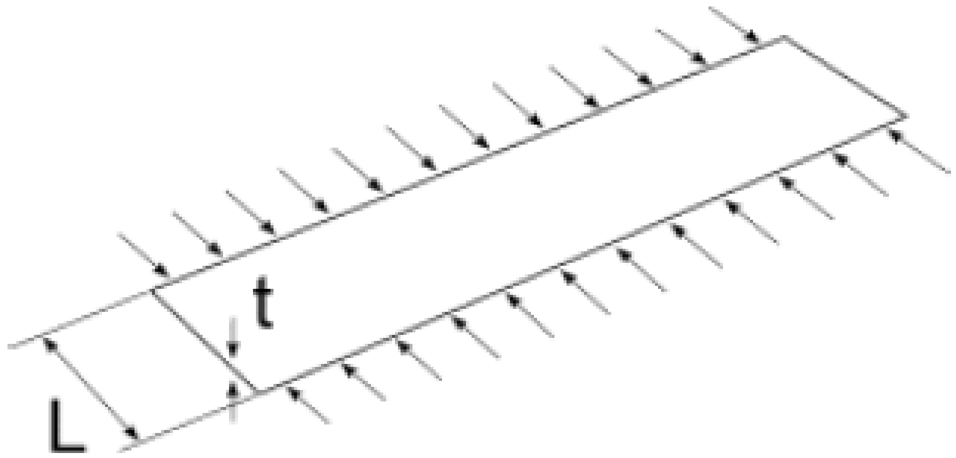
Buckling of Flat Plates (13/21)

- We can effectively prevent buckling by setting $\lambda > 1$;

$$\frac{L}{\pi r}\sqrt{\frac{\sigma_y}{E}} \leq 1 \quad \text{or} \quad \frac{L}{\pi r} \leq \sqrt{\frac{E}{\sigma_Y}} \quad \text{or} \quad \frac{L\sqrt{12}}{\pi t} \leq \sqrt{\frac{E}{\sigma_Y}}$$

$$\frac{L}{t} \leq 0.907\sqrt{\frac{E}{\sigma_Y}}$$

- We could use this formula to limit the buckling of a wide plate in compression (for pinned edges).



https://sdcverifier.com/wp-content/uploads/2018/03/FPI_Rules_e-July14.pdf

Buckling of Flat Plates (14/21)

Classification society rules have very similar formulae to prevent buckling in various plate elements.

For example see ABS rules part 5 App.5/2 AB for buckling restrictions for longitudinals;

7.9.5 Proportions of Webs of Longitudinals and Stiffeners

The depth-thickness ratio of webs of longitudinals and stiffeners is to satisfy the limits given below.

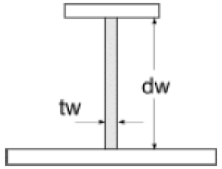
$d_w/t_w \leq 1.5(E/f_y)^{1/2}$ for angles and tee bars

$d_w/t_w \leq 0.85(E/f_y)^{1/2}$ for bulb plates

$d_w/t_w \leq 0.5(E/f_y)^{1/2}$ for flat bars

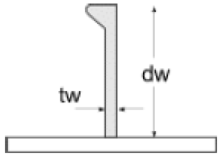
where d_w and t_w, are as defined in 5A-3-4/7.5.2 and E and f_y are as defined in 5A-3-4/7.3.

When these limits are complied with, the assumption on buckling control stated in 5A-3-4/5.1.2(e) is considered satisfied. If not, the buckling strength of the web is to be further investigated, as per 5A-3-4/7.3.



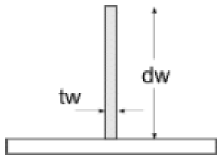
Tee Stiffeners

$$\frac{d_w}{t_w} \leq 1.5\sqrt{\frac{E}{\sigma_Y}}$$



Bulb stiffeners

$$\frac{d_w}{t_w} \leq 0.85\sqrt{\frac{E}{\sigma_Y}}$$



Flat bar

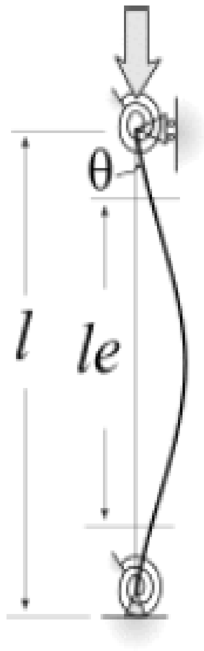
$$\frac{d_w}{t_w} \leq 0.5\sqrt{\frac{E}{\sigma_Y}}$$

Buckling of Flat Plates (15/21)

Q? why do the constants range from 0.5 to 1.5?

Ans: Because of the boundary conditions and effective length.

We've seen that the effective length for fixed ends is half that of the pinned ends. When the ends are elastically restrained from rotating, we get effective length somewhere between 0.5 L and 1.0 L (L= length).

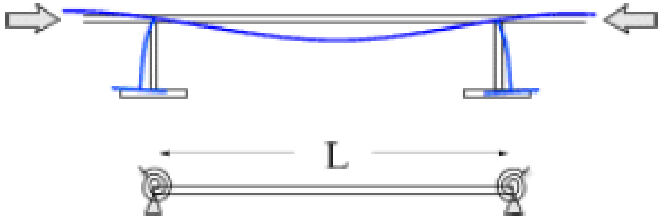


$$c = \frac{M}{\theta}$$

$n = \frac{cl}{EI}$	$\frac{l_e}{l}$	$\frac{P_{crit}}{P_E}$	Remarks
0	1	1	Pinned
1	0.86	1.35	
2.5	0.75	1.78	
5	0.66	2.3	Typical of ship structures
10	0.59	2.85	
∞	0.5	4	Fixed

Buckling of Flat Plates (16/21)

Plates supported by frames or bulkheads are closed to fixed than to pinned.



For example: a deck plate

Springs replace the effect of frames and neighboring plate.

Assume that $n = \dfrac{cL}{EI} = 10$ $\dfrac{P_{crit}}{P_E} = 2.85$ $\dfrac{l_e}{l} = 0.59$

This gives us a rule: $\dfrac{0.59L}{t} \le 0.907\sqrt{\dfrac{E}{\sigma_Y}}$

which can be written as: $\dfrac{L}{t} \le 1.53\sqrt{\dfrac{E}{\sigma_Y}}$ similar to ABS rule.

We can also write this as:

$$\frac{L}{t} \le \frac{698}{\sqrt{\sigma_Y}}$$

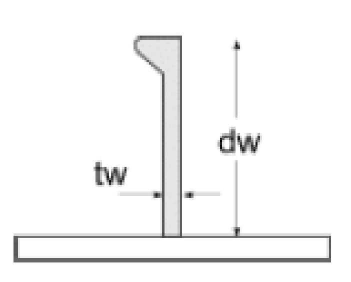
$$\frac{L}{t} \le 1.53\sqrt{\frac{E}{\sigma_Y}} = 1.53\sqrt{\frac{208000}{\sigma_Y}}$$

Which is similar to the format used in DNVGL rules.

Example

Problems:

1. For a bulb stiffener with a height (d_w) of 200mm, what is the minimum web thickness to ensure yield occurs prior to buckling (E=207GPa, Yield = 235MPa).



Bulb stiffeners

$$\frac{d_w}{t_w} \leq 0.85\sqrt{\frac{E}{\sigma_Y}}$$

2. Find λ for a plate 15mm on 650mm span. (E=207, Yield stress = 235 MPa).

$$\lambda = \frac{L}{\pi r}\sqrt{\frac{\sigma_Y}{E}}$$

Buckling of Flat Plates (18/21)

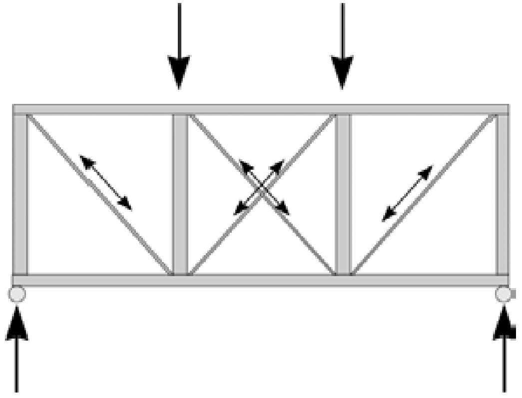
Local buckling of plate

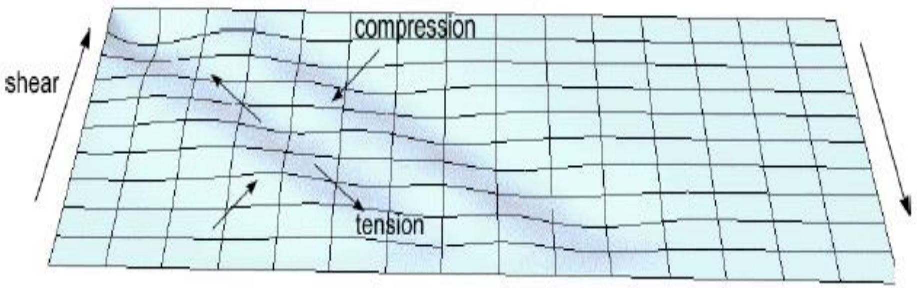
- Let's briefly discuss a variety of local buckling phenomena

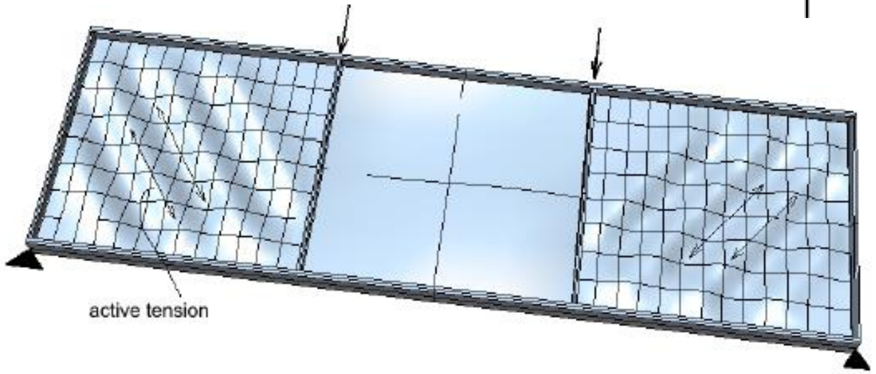
Local buckling refers to a variety of buckling mechanisms involving any part (web, flange, and bracket) of a frame. In all cases there is a local compression stress that buckles a section of plate. The compression may be result of bending shear or direct pressure.

Shear Buckling

When a plate (typically a web) experiences a shear stress, there is a compression filed on 45º diagonal.







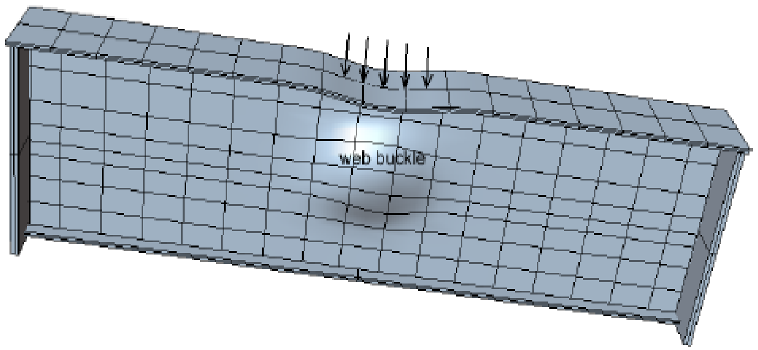
This type of shear buckling is actually common and expected in deep web girders (as might be found in Railway Bridge). As long as there are stiffeners on the surface to take compression. The buckled web will hold the tension in a kind of truss-like structure.

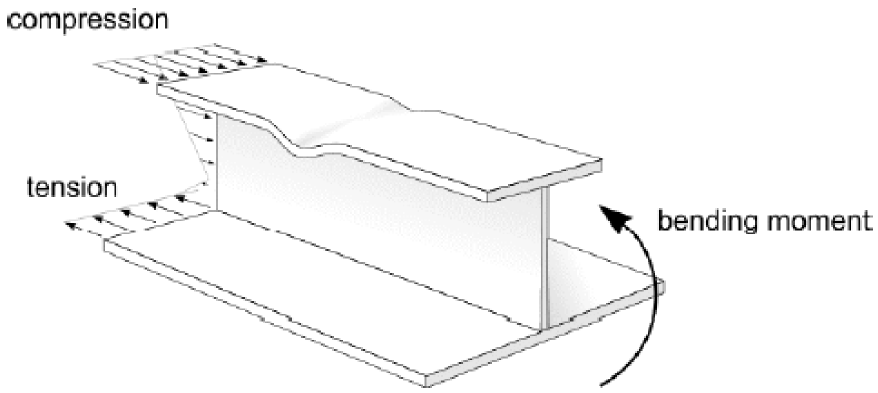
Buckling of Flat Plates (20/21)

Web and flange buckling

Another common type of local buckling is web buckling, which may be caused by direct compression due to an applied load.

Compression in the flange due to bending can cause the flange to buckle locally.





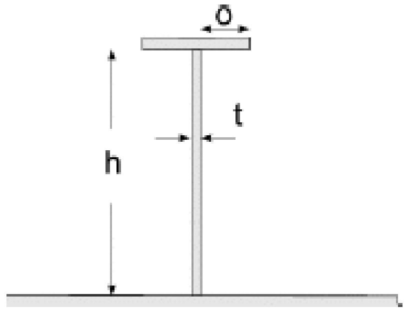
Buckling of Flat Plates (21/21)

Local buckling is prevented by limiting the local aspect ratios of parts of the construction. A typical rule for plate with flanges on both boundaries would be:

$$\frac{h}{t} \leq \frac{1000}{\sqrt{\sigma_Y}}$$

Or

$$\frac{h}{t} \leq 2.2\sqrt{\frac{E}{\sigma_Y}}$$



[σ and E in the same units]

When the plate is connected on one side only (as for a flange) it is called an 'outstand'. The typical local buckling rule is for an 'outstand' is:

$$\frac{o}{t} \leq \frac{250}{\sqrt{\sigma_Y}} \qquad \frac{o}{t} \leq 0.55\sqrt{\frac{E}{\sigma_Y}}$$

[o and E in the same units]

Learning Outcomes (Review)

- We have investigated the Failure Modes.

- Now we are able to:
  - Be aware of various modes of failure caused by in-plane compression of grillages.
  - Be familiar with energy method for derivation of critical buckling stress of a flat plate & column.
  - Evaluate the critical buckling stress of a flat plate & column.
  - Design an effective stiffening system against in-plane compression

- Details can be referred to topics 8 in the lecture notes.

[Theory of Plates and Grillages]

[Part I] Plastic Design of Structures

- Plastic theory of bending (Topic 1)
- Ultimate loads on beams (Topic 2)
- Collapse of frames and grillage structures (Topic 3)

[Part II] Elastic Plate Theory

- Basic (Topic 4)
- Simply supported plates under Sinusoidal Loading (Topic 5)
- Long clamped plates (Topic 6)
- Short Clamped plates (Topic 7)
- Additional (Low aspect ratio plates, strength & permanent set)

[Part III] Buckling of Stiffened Panels

- Failure modes (Topic 8)
- Tripping (Topic 9)
- Post-buckling behaviour (Topic 10)

Kam Sa Hab Ni Da

감사합니다

Thank you!

Questions?

Aerial View of Korean Presidential Archives in Sejong city
(Construction Completed in 2014)